

# Feasibility Test: Bond Return Forecasting in the German Financial Market

A Master Thesis Presented  
by

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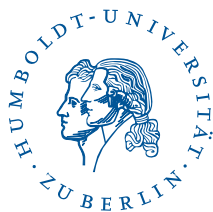
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to

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# Abstract

Understanding the composition of the bond return is always a popular topic in the financial markets. There are various factors that influence the bond returns. Therefore, a precise prediction of the bond returns is still under discussion. This paper is enlightened by the papers of Ilmanen (1995, 1997) and Ilmanen and Sayood(2002). They proposed six predictors in forecasting the US government bond excess returns. I analyze the rationale of using those predictors and attempt to calibrate the predictability of the German government bond returns. Firstly, a regression model is used for estimation. Then I use an additive model on the same financial market data set to further improve the model predictability.

*Key words: Excess bond return, term spread, inverse wealth, additive model*

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# Notation

EBR	excess bond return
EMH	efficient market hypothesis
IW	inverse wealth
RBV	real bond yield
TS	term spread
RRA	relative risk averse
INVRELW	inverse relative wealth
$K$	kernel function
$S_\alpha$	smoother matrix
$W_t$	stock market wealth level
$X$	explanary variable
$Y$	dependent variable
$c$	constant
$g_\alpha(\bullet)$	additive component function
$m(\bullet)$	regression function
$m$	year to maturity
$p_t$	bond price at period $t$
$x$	expected bond risk premia
$y_t$	bond yield at period $t$
$\Delta y_1$	change in one-perioed nominal yield
$\pi$	expected inflaiton rate
$\mu$	expected real rate of a one-period nominal bond

# 1 Introduction

Bonds have always been considered as *the* low risk investment. The bond market is traditionally the first choice for investors to hedge the effects of economic fluctuation. Until the 1960s, bond risk was low. This was always expected by bond market investors. However, for the last forty years, the bond market attracts more and more attention, as bonds play a role more important than just hedging the risk of macroeconomic fluctuation. The riskiness of the bond markets increased substantially in 1980s, with an increasing correlation of the movement together with the stock market. The volatility of interest rates has also increased during this period. These developments greatly impacted the bond risk, and have led to an increase of bond risk premiums.

Why should investors care about all the facts stated above? The fluctuation that took place during the last forty years indicates that there are much more investment opportunities in the traditional bond market nowadays. Furthermore, investors may be able to use the correlation between the stock and the bond market in predicting bond market returns.

Comparing with the stock markets, the movement of the bond markets is more difficult to be captured due to the larger universe of bond products and the complex individual bond character, defined by properties, such as maturity, coupon rate and bond embedded options. In this paper I will focus on the simplest bond type, the long-term German government bonds, and try to explain the movement of the bond returns.

Great amount of research has been carried out with the goal of identifying the factors that indicate the bond return. Bonds are one of the financial products that have the longest history. However, the prediction of movement of the bond return is still one of the most popular subjects in modern financial studies. My paper focuses on two questions: (1) Can the excess return of long-term German government bonds be predicted? and (2) Does active investment management make sense when investing in long-term German government bonds?

This paper starts out with an introduction of the basic market hypothesis behind the bond return. Next, I try to understand the correlation of the bond returns across borders and introduce the risk factors that impact the bond return. Afterwards I focus on calibrating the best bond return indicators and use a regression and an additive model to generate a fit model. Finally, I compare the forecasted return value with the realized value and carry out the model using different investment strategies in order to find the best application of the model.

To carry out my analysis, I use a subindex of the German bond market index REX with an average maturity of 10 years as the proxy of the total return of long term German government bonds. REX is the performance index of the German bond market. It consists mainly of German government bonds. "It is calculated by Deutsche Börse on the basis of 30 domestic bonds once daily. Each of the 30 bonds is weighted according to its market share, which is determined by the number of issues in each of the 30 maturity-/interest-rate categories over the past 25 years. Deutsche Börse reviews the weighting annually." Due to the lack of historical German government bond data, after discussing with Mr. Peter Schmidt from Landesbank Berlin, I use the REX as the proxy of long-term German government bond returns.

Since the German bond market lacks historical time series for short-term risk-free assets, the FIBOR rate (rating: AAA) is used as a substitute.

In order to forecast the excess bond returns for the German government bonds, I am also using a commodity price index. Unfortunately, a suitable index to represent the German commodity prices does not exist. However, German commodity prices are greatly driven by commodity prices in the US. Therefore, as a proxy for German commodity prices, I use the US year-on-year CRB index, for which reliable, historical time series exist.

## 2 Some Issues Before Modeling

### 2.1 Efficient Market Theory

Efficient Market Hypothesis is the one of most well known capital market hypothesis. An efficient capital market is defined as the current security prices in financial market fully reflect all the current information about that security in a timely manner.

The market efficiency is based on the following assumptions: 1) there are a large number of profit maximizing participants analyzing and valuing securities independently from each other; 2) new information comes into market in a random fashion and the releases of new information are independent from each other; 3) investors will adjust the valuing of the security rapidly to reflect the new information; 4) expected return implicitly include risk in the security price. If any of the assumptions cannot hold, there will exist the chances for gaining abnormal return.

There are three forms of the Efficient Market Hypothesis: weak, semistrong, and strong. Weak form EMH is defined as prices quickly adjust to all the available market information; semistrong form EMH assumes prices are inline with all public information, including both market and nonmarket information; strong form EMH assumes that prices reflect all information from public and private sources.

There are grand amount of literature on the tests used to examine each form of EMH. In a realistic world with imperfect information in the financial market, studies were made suggesting that for the time-series analysis, the semistrong form of EMH cannot hold. If the semistrong form of EMH holds, then investors should not be able to outperform the market in both short and long run. Therefore, time-series analysis could be applied in bond portfolio management with the intension of beating the market by estimating the future returns based on the long-run historical rate of return. The technical analysts believe that price reflect the market information not in a timely

manner. Technicians use historical data from the market to predict the prospective trends. Here in this paper, I hold the same assumption as the technicians and test the predictability of German government bond excess return.

## 2.2 Term Structure of Interest Rate

Term structure of interest rate is an important topic in economics. This traditional subject is still of interest in the bond market for analyzing financial products even with complicated characteristics.

**Pure Expectation Hypothesis** The pure expectation hypothesis assumes that all government bonds, regardless of maturities, have the same expected return. This simply states that there is no bond risk premium exists, which suggests a flat yield curve. An upward-sloping yield curve suggests that market expect an increasing change future rate. This hypothesis has been proved by historical financial market studies to be not true. Most of the time we could observe an positive bond risk premium.

**Risk Premium Hypothesis** In the risk premium hypothesis (also called liquidity premium hypothesis) is the earliest attack against the pure expectation hypothesis. It was made by Hicks (1939) based on Keynesian notion of "normal backwardation" and known as the Hicksian Liquidity-Preference Model. The liquidity preference model has three parts: first, borrowers would prefer to borrow long in order to hedge the their future supplies of loan capital (Hicks, 1939). Second, people that lend money have strong incentive to lent short in order to have free hands against economic fluctuation. Finally, the "speculators" can offset the gap of supply and demand but ask for compensation for the risk they endure. This theory assumes that investors are risk averse and only prefer to invest in the shortest holding period. In other words, In liquidity premium hypothesis the upward-sloping yield curve reflects only the required risk premium but no rate expectation. The liquidity habitat theory can be viewed as the special case of the preferred habitat hypothesis.

**Preferred Habitat Hypothesis** It is also called the "Market Segmentation

Theory". Modigliani and Sutch's (1966) preferred habitat hypothesis based their theory on the market segmentation hypothesis. The preferred habitat hypothesis implies that the expected return could either increase or decrease with duration. This hypothesis assumes that all investors have their preferred security duration, natural habitat. For instance, pension funds usually prefer longer duration than shorter duration due to the fact that long duration are less risky and could be tailored into portfolios that have similar duration as the pension funds. Sometimes the investors will even sacrifice some yield to invest in their preferred securities duration. According to the preferred habitat hypothesis, investors will only be tempted out of their natural habitat by the lure of higher expected returns or unless their own habitats change into other horizons. Perhold and Sharpe (1989) argued that the investors with long-term horizon are minority in the market. This theory is called Casual Empiricism. Casual Empiricism leads to a positive trend of risk premium associating with duration.

**Partial Equilibrium** On the contrary to the modern asset pricing models, partial equilibrium models view the risk premium of the securities independently from the risk they bear. Volatility is only a measure to the risk. Bonds returns are not correlated with other assets and other economic factors.

**Capital Asset Pricing Model (CAPM)** In the CAPM model, security return has a linear relation to the stock market sensitivity - the Beta. A security's risk premium is the product of that security's beta and the market risk premium of the security. Bond has a positive beta when bond return is positively correlated to the stock market return. But as stated above, CAPM implies a linear relation of the risk level and the risk premium. This linearization empirically does not exist. The security risk can not be observed one-to-one with duration. In fact, a concave curve of volatility with duration results in a concave return functions of with duration.

**General Equilibrium** The general equilibrium models allow the both risk and securities return varies over time and take the fundamental economic factors into account when analyzing the risk premium. All the general equilibrium models try to imply that there should be positive risk premium during the economic recession. During economic recession,

the marginal utility of money is higher than in economic expansion, therefore investment should earn positive risk premium.

In the financial markets, the shape of a yield curve probably reflect both rate expectations and required risk exposure. Cox, Ingersoll and Ross (1981) proved that the basic assumption of no risk premium does not hold and the universal risk neutrality would result in nonzero bond premiums. Cox, Ingersoll and Ross also point out that this offered the basis of the preferred habitat theory, and it is actually the investors' risk aversion act as the preferred habitat instead of the preferred rate or preferred investment duration. Apart from what have been stated above, other bond specific characteristics are also reasons for the risk premium, e.g. bond liquidity, bond maturity, bond currency etc. There are little statistical prove showing that there is a positive expected bond risk premium. On the other hand the positive slope of the yield curve indicates that there exists an positive average premium.

According to many theories, bond risk premium could relate to the following factors: stock market performance, bond market volatility, market risk aversion level etc.

According to theory of efficient market frontier investors require more return for bearing more risks. Investing in the long-term bonds will increase the risks associated with the underlying, such as the reinvestment risk and interest rate risk. Therefore, investors are usually compensated with higher return when investing in longer-term bonds. The term spread between the long and short-term bonds represents this theoretical background of the term structure of bonds. When analyzing the long-term government bond, term spread offers the insight into the movement of bond excess returns.

## **2.3 Multi-Country Model for the Bond Markets**

There are mass researches on the correlation of the bond return on the international market. These researches imply a positive correlation among bond returns across countries. Here the spread between the long-term government bonds and 3-month risk free rate of the US, Germany, UK, Japan and Canada are tested (Table 2.1).

At the same time, very highly correlation among the long-term bond total re-

US	Germany	UK	Japan	Canada
1	0.0540	0.6080	0.2810	0.4726
0.0540	1	0.0762	-0.2061	-0.0618
0.6080	0.0762	1	0.1760	0.4924
0.2810	-0.2061	0.1760	1	0.3013
0.4726	-0.0618	0.4924	0.3013	1

Table 2.1. Correlation Matrix of Excess Bond Return Across Countries

turns, indicating the co-movement of bond returns among all countries(Table 2.2).

US	Germany	UK	Japan	Canada
1	0.9921	0.9912	0.9659	0.9938
0.9921	1	0.9944	0.9635	0.9974
0.9912	0.9944	1	0.9770	0.9940
0.9659	0.9635	0.9770	1	0.9619
0.9938	0.9974	0.9940	0.9619	1

Table 2.2. Correlation Matrix of Long-Term Bond Return Across Countries

Table 2.1 and Table 2.2 indicates large correlation of the international bond markets. There could exist similar factors driven the bond returns. Later on in the my model, I also use the commodity price index from the US market to represent the movement of the German commodity price movement. On the other hand, the local predictors are proved to be also highly reliable for forecasting bond returns.

Even though the co-integration of the bond returns are well-established, it can not explain the underlying factors of the change of the bond returns. When changes of the bond yield is analyzed, the change of exchange rate should also be taken into consideration, since it will influence the bond yield the change in an international capital market. By decomposing the excess long-term bond return using the Vector-autoregressive model (Campbell & Ammer 1993, Engsted & Tanggaard 2001, Engsted & Tanggaard 2005). This method introduced the channel through which the news of macroeconomic indicators impacts the asset price in the capital market. They found the excess stock and bond returns are largely driven by the news about future excess stock return and inflation respectively. As is known to all, bonds are



traded for a very long time as the product to hedge the inflation change, yet this feature could still be observed in the above studies. The inflation news accounts for 85% of the variation in the unexpected excess bond returns in the US and 69% in Germany (Engsted & Tanggaard 2005). However, there are also other factors contribute to the variation. Although the affect of news about real interest rate change is rather small, real interest rate does have impact on the short-term interest rate and slope of the term structure. The precise reason for the common movement is explained by Engsted and Tanggaard (2005).

At the same time, low correlation between the excess stock and bond returns can be observed, as suggested by Campbell and Ammer (1993). Therefore, factors that could influence the stock return will not be appropriate for analyzing the bond excess return. On the other hand, since the level of the stock markets directly associates to investors relative risk aversion, the stock market wealth will be considered as one of the factors, which influence the excess bond return.

## 2.4 Bond Return and Its Related Risk

Theoretically speaking, the return of the bond is subject to only three major factors, real interest rate, inflation rate premium, and risk premium. Among them the real interest rate depends on the risk averseness of the investor and inflation rate premium is in line with country's own economic condition. Risk premium consists of different kinds of risks, including interest rate risk, credit risk, reinvestment risk, option-related risks, liquidity risk, and exchange rate risk. Investors invest in bond by sacrificing the consumption in the current period in expecting of the future received return. This kind of sacrifice is rewarded by the bond return. There are great amount of research analyzing the bond return by decompose it into riskless return, which rewards the investors for the delayed consumption and is common for all bond, and the excess return, that compensate the investors for each bond-specific characters. Our target in the following model, excess bond return, is simply the total bond return exclude the riskless bond return. We will come back to this point in more detail in Section 3.1.

**Interest Rate Risk** Interest rate risk is the main driven factor of bond excess return. It refers to the effect of changes in the prevailing market

rate of interest on bond value. Investors should be rewarded for bearing interest rate risk. The commonly used method for calculating interest rate risk is Duration. Duration gives us a approximation of a bond's change in price for a given change in yield. Duration is influenced by three factors: maturity, coupon and embedded options. When comparing two bonds, the one with longer maturity has higher duration, therefore has a higher interest rate risk. A higher coupon directly leads to a higher bond yield. In the next section, we will introduce the pricing of a bond. A negative correlation between the bond price and the bond yield will be introduced. Embedded options will decrease the duration and lead to a low interest rate risk level.

Although the average reward for bearing interest rate risk is small, at times the reward is large. Investors' risk aversion is represented by the relative stock market performance. Government bonds, which are the study focus of my research here, are usually assumed to be free from default risk, because the government makes the payment guarantees.

**Yield Curve Risk** The yield curve risk is a risk of a security that is not captured by the duration measure. From the section above we already know that there exists a change in the shape of the yield curve. The yield curve illustrate the relation between the bond yield and maturity. The yield curve change is the possible change of the bond yield curve, which means that the yields change by different amounts for bonds with different maturities. There are two ways of changes of yield curve shape: parallel shift of the curve and non-parallel shift. The non-parallel shift of the curve cannot be captured by duration, therefore leads to the yield curve risk.

**Call Risk** This risk is for the bond embedded with a call option. For government bond there usually do not exist any call options. So I do not consider this type of risk in my study.

**Reinvestment Risk** A reinvestment risk refers to that when a bond has a call option and the option is executed, the investors will face the risk of loss generated from reinvesting in other securities. Since we do not consider call risk, there will be no reinvestment risk either.

**Credit Risk** The credit risk is also called the default risk. For government bonds, the default risk can be neglected, since the risks of failing in

paying back principle or coupons are almost zeros. It consists of the credit spread risk and the downgrade risk.

**Liquidity Risk** It is the risk that the sale of a fixed-income security must be made at a price less than its fair market value because of a lack of liquidity for a particular issue. Generally speaking, government bonds have perfect liquidity. Some investors would argue that they intend to hold the securities until the maturity dates, then they would not face any liquidity risk. It is true that investors do not need to suffer a loss of selling the security, but for some institutional investors, they sometimes need to marking their holdings to the market. With the less liquid asset held by investors, it would be difficult to find a comparable price. Sometimes investors have to accept lower pricing which could lead to higher cost and lower portfolio return.

**Exchange Rate Risk** In the model we will introduce later in this section, we will consider the exchange rate risk as one of the influential factors for excess bond return.

**Inflation Risk** It is the unexpected inflation risk, or in other words, the purchasing power risk. Inflation risk will be measured by the Consumer Price Index.

**Volatility Risk** For bonds embedded with options, the change in interest rate volatility will impact the value of the options, therefore influence the value of the bonds. Here we do not consider this risk.

**Event Risk** The risk encompasses outside the financial markets, such as natural disasters, regulatory issues or company restructuring.

## 2.5 Bond Pricing and Its Relating Factors

The value of a bond is the sum of all the present values of all the expected cash flow. A government bond's cash flows is consisted of coupon payments and principle repayments. When calculate the present value, we use the bond yield as the discount factor. It is the risk free rate of the corresponding bond. Therefore, we could observe a inversely related relation between the bond yield and the bond value. An increase in bond yield leads to a decreased the present value of the bond. The change of bond yield would

impact the bond value and the fair market value of bond would change over time until the maturity date. Here we have to pay attention that the bond yield is not equal to the bond return.

When the yield to maturity is lower than the coupon rate, we call the bond a "premium bond". While bond with higher yield to maturity is called the "discount bond". No matter the bond is issued as premium bond or discount bond. They all converge to their face value as maturity date approaches.

Bond yield is directly correlated to the bond price. For a bond with  $m$  years to maturity, relation between the bond price  $p_t$  and the bond yield  $y_t$  can be simply stated as:

$$p_t = \frac{1}{(1 + y_t)^m}$$

Bond yield is closely associated to the government's monetary policy. When the economy is developing, a potential increase in the interest rate will lead to higher interest rate risk of the bonds, which would increase the bond yield. For the bond investors this movement will make them suffer a loss with a decrease in the bond price. Therefore, a high bond yield and booming economy usually cause a less active bond market.

## 3 Modeling

### 3.1 Excess Bond Return

When analyzing bond return, researchers tend to use the bond risk premium instead of the bond return. Using risk premium avoids to include time value of bond and focus on the unpredictable return feature of the securities. The excess bond return is the main topic when analyzing the bond risk premium. It is defined as the realized bond risk premium, which is the long-term bond return over a risk free rate of a short term asset. Contradict to the theoretical view of the bond risk premium. The interest rate risk only takes up a small part of the excess bond return. This part is the expected part of the excess return. In Figure 3.2 we could observe that the long-term government bonds only earn meager margin for bearing the interest rate risk. Here we calculate the annualized average bond return by using the geomean of the bond return. Most of the excess return is unexpected - this is the risk investors have to bear when investing in the long-term securities. In the bond market we observe most of the time an positive bond risk premium.

It is difficult to sperate the unexpected part of excess return from the expected part. The one-period CPAM model and Liquidity Premium Hypothesis both assume constant expected risk premiums. However, recent analysis indicates that the expected risk premium varies over time. A historical average of bond return is a good measure for expected value only if the sample is long enough and unbiased. In the case of changing expected risk premium, even though the historical average of the bond return may be a good measure for the long-term excess bond return. It might not be counted as a good measure of the near-term excess return with its time-varying feature. If we could estimate excess bond return using available market information, the predicted excess return could be used as a proxy to the real excess return. As said before, the expected risk premium varies over time, analyzing the change of the excess bond return will offer us the overview of the expected risk premium in case it is abnormally high or low.

In my paper the sample period is 20 years. It helps to balance out the fluctuation of the inflation in a long-run. Excess bond return is calculated as the total bond return excludes the riskless return (Figure 3.1). It is our objective in this paper is to test the feasibility of predicting the excess bond return for German bond market. In our model we try to predict the prospect excess bond return by capturing the features of the historical excess return. Ilmanen suggested using a series of variables to predict the excess return (Ilmanen 1996c, p 53).

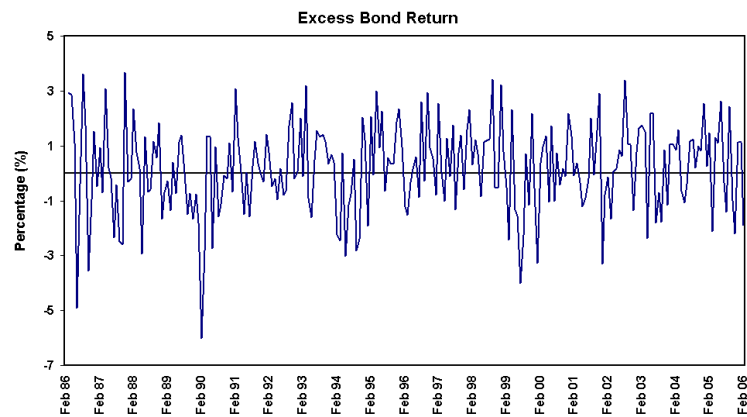


Figure 3.1. The business cycle pattern of the excess bond return, sample period Jan 1986-Dec 2006

The main goal in the model is test the predictability of the near term excess bond return by predicting the near-term direction of the excess return changes. If more than half of the change directions can be correctly predicted, it will offers portfolio managers the opportunities to beat the market by carrying out the active bond portfolio management.

## 3.2 Predictors

The descriptions below presents the focus of our predictors for forecasting the bond excess return. In the section above, we knew that if feasible predictors could be found to forecast the excess return, bond risk premium could also be estimated. Term spread is the first estimator considered as the excess return predictor. Fama and French (1989) and Jones (1992) proved the positive correlation of the term spread and the bond return. Ilmanen (1995b) and Mankiw (1986) analyzed the correlation among many countries. Term spread is used here to bring the curve effect into the excess bond return. Apart from the term spread, real bond yield and inverse wealth are also used in order to capture the risk aversion level of the market participants. The lagged bond return is the momentum variable I use to capture the market change. In addition, Change in trade weighted exchange rate and change in the CRB Index are also used as proxy for excess bond return. Falling commodity price and appreciating exchange rate indicates an increasing excess bond return due to a disinflationary pressure.

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**Term Spread** Difference between the estimated 10-year and three-month spot rate.

**Real Yield** Difference between the 10-year spot rate and the most recently published yearly CPI rate.

**Inverse Wealth** Ratio of the exponentially weighted past stock market level to the current stock market level ( $W_t$ ). Formally =  $(W_{t-1} + 0.9W_{t-2} + 0.9^2W_{t-3} + \dots) \times 0.1/W_t$ .

**Lagged Bond Return** Bond return from previous period.

**Change in Trade Weighted Exchange Rate** Change of the current exchange rate over previous period.

**Change in CRB Trend** Change of the current CRB Index over previous period.

**Excess Bond Return** Monthly return of a long-term Treasury bond in excess of the nominally riskless return of a one-month Treasury bill. Also called realized bond risk premium.

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### 3.2.1 Term Spread

In the traditional theory of fixed income securities, the bond value can be simply calculated by discount the future bond payments. Therefore, theoretically, bond valuation is clean and straightforward. However, practically a bond has exposure to a range of different kind of risks, e.g. the risk of default, including fail of a regular payment and default of the bond issuer, risk of changing interest rate etc. Many bonds deviate from their theoretical values if all these risks and factors are taken into account.

In my paper, our goal is to capture the variation of the bond return by forecasting the excess bond return. The excess return of bonds varies from time to time. The reward for bearing interest rate risk is relatively small on average, but it can vary significantly over time (Figure 3.2). When the economy is at the end of the recession excess return tends to be high, while at the end of expansion excess return is usually low. This reflects the risk aversion of the bond market investors. In economy recession investors try to avoid risky assets while in economy expansion investors will make more bold bets. In this case the risk aversion of investors becomes a very important proxy of the excess return. This term structure of the investment has received great attention both in theoretical analysis and in the investors' strategies. Some fund management companies are also using the portfolio managers' ability of using the term structure as a proxy of the fund performance. Term structure is defined as the yield differentials between long-term bonds and short-term bonds. Here we calibrate the term spread by using the difference between the 10-year government bonds rate and 3-month riskless rate (FIBOR rate).

Studies have found that term spread to be a significant predictor of the excess bond return (Jones and Roley, 1983; Shiller, Campbell and Shoenholtz, 1983; Campbell, 1986). Term structure can be viewed as a good estimator if the expected rate change is approximately equals zero. Therefore, term spread is an effective proxy if we use long-term sample period. The central bank has realized that an initially modest increase in the short-term rate usually could provoke a sharp response with long-term bond yield. The construction of a bond's yield curve is the curve of the bond yield on different maturities. Yields can be summarized into a graph of yields to maturity against the bond maturities, for example in Figure 3.2. Yield curves can be any shape, flat, concave or convex. In most cases, we can observe yield curve to be concave and this is also called the "normal" yield curve. In general, for



short maturity bond yield is low, given the risks and possible fluctuations are less uncertain, while for long maturity bond yield would usually be higher. In theory, a normal yield curve will have a positive correlation with the excess return, while a inverted yield curve leads to a negative correlation.

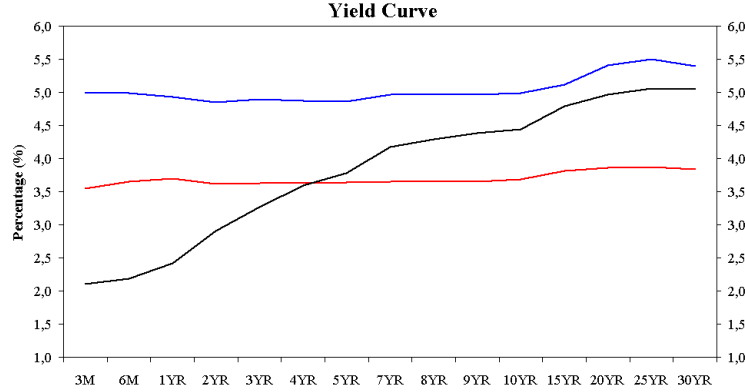


Figure 3.2. Yield Curve: as of 30 Nov 2006 (red line), as of 30 Nov 2003 (black line) and as of 30 Nov 2000 (blue line)

The term spread is not constant. It changes over time. The spread narrows when the yield curve flattens. We could observe a flattened curve over bond's maturities. When the yield curve gets steeper, investors are more risk averse and ask for more risk premium for risk bearing. Ilmanen (Ilmanen, 1995) suggested using the term spread as a proxy to capture the time structure of the bond risk premium in order to create a less noisy proxy for the bond excess return. Sometimes term spread is also being described as yield spread. Ilmanen defined the term spread as followed (Ilmanen, 1995):

One-month term spread can be calculated as:

$$TERMSP_{n,t} = y_{n,t} - y_{1,t}$$

$$= \left(\frac{1}{n}\right)E_t \sum_{i=0}^{n-1} [(n-1-i)(\Delta y_{1,t+2+i}) + x_{n-i,t+1+i}] \quad (3.1)$$

Maturity	Arithmetic Mean	Geometric Mean	Volatility	Sharpe Ratio
1-Mo.	3.13	3.13	2.95	1.06
6-Mo.	3.17	3.17	2.92	1.09
1-Yr.	3.33	3.33	2.99	1.11
2-Yr.	3.57	3.56	3.11	1.15
3-Yr.	3.81	3.81	3.16	1.21
4-Yr.	4.06	4.05	3.22	1.26
5-Yr.	4.20	4.19	3.17	1.32
6-Yr.	4.43	4.42	3.44	1.29
7-Yr.	4.59	4.58	3.53	1.30
8-Yr.	4.71	4.71	3.56	1.32
9-Yr.	4.79	4.78	3.57	1.34
10-Yr.	4.82	4.82	3.50	1.38
20-Yr.	5.27	5.27	3.97	1.33
30-Yr.	5.42	5.42	3.69	1.47

Table 3.1. German Bond Market Subsector Annual Returns and Other Statistics, Jan 1995 - Nov 2006

In the above context, I briefly introduced the Efficient Market Hypothesis. Bekdache based his study on the market segmentation or preferred habitat theory (Bekdache 2001). According to this theory, investors require a higher return to deviate from their own investment habit into investing in a different maturity. The basic market hypotheses are tested. The Pure Expectation Hypothesis is that the interest rates are supposed to move in the way they are expected, the returns on short and long term investment strategies are the same. The difference between the short and long term returns are zero. There is a weaker version of the Pure Expectation Hypothesis: Expectation Hypothesis. It states that the difference between the short and long term investment strategies is constant but not necessarily to be equal zero. By using a method of mixed model structure, he came to the conclusion that the pure expectation theory cannot hold for the term structure. Bekdache argued that the bond term spread vary over time, but this variation can not

be captured by the conditional heteroscedasticity time series models, e.g. ARCH-M or GARCH. These models are not able to include the economic factors but only the covariance between assets. In the long term, this investment habitat theory cannot be held, but it holds for the period that Federal Reserve uses money supply as the main tool to adjust economic fluctuation (Campbell, Kazemi and Nanisetty, 1999).

Previous research using the Expectations Hypothesis (Campbell, 1995, p 137), which tries to propose a constant return difference between the short and long term expected bond returns, can not be verified with the empirical data (Hardouvelis, 1994). But this classic paper by Campbell (Campbell, 1995) presented the time varying feature of the term structure. In Campbell's paper, the zero coupon yield for 6-month for zero coupon bonds was 3.5 percent, 6-year zero coupon yield was just over 5 percent, 10-year yield was 6 percent and 30-year yield was almost 6.5 percent, while the Fed fund rate is 3 percent. From this trend of the yield curve, we could conclude that most of the term structure was captured by the spread between short-term rate and 10-year rate. Some researchers chose 3-month to 7-year term spread for the reason that the average maturity of the government bonds is about 7 years. In our paper, we will use the term spread of 3-month government bond and 10-year government bond in the German market in order to capture more fluctuation of the yield curve.

When we use the steepness of the bond yield curve as a proxy of the bond risk premium, we are assuming that the market expectation of the rising rate is zero. A steep yield curve reflects that the investors require high risk premium for a rising rate, and when the expectation is zero the current yield curve is the best proxy for the future yield curve (Ilmanen, 1995). Obviously when we use term spread to forecast the excess bond return, this zero expectation assumption cannot always hold and expectation will be influenced by different kinds of market factors. In this case we need to introduce more proxy in order to make more precise forecast of the bond excess return.

Mankiw (1986), Bisignano (1987), and Solnik (1993) used countries' local instruments to analyze the term spread and found that the local instruments have very good predictability to the local excess bond return for many countries. Ilmanen's paper (1995, pp 482) suggests that the German bond market term spread would be an appropriate measure for estimating the German government bond's excess return. I chose term spread here as an

overall proxy for the excess bond return. This allows us to capture the change of the excess bond return even if we do not know the causes of the difference of excess returns over bonds.

TS > 0	TS < 0		
191	49		
EBR > 0	EBR < 0		
132	108		
TS > 0 and EBR > 0	TS < 0 and EBR < 0	TS > 0 and EBR < 0	TS < 0 and EBR > 0
106	23	85	26

Table 3.2. Comparison of Tern Spread and Excess Bond Return, total 240 months (Feb 1986 - Jan 2006)

We could see from the Table 3.2 that most of the time the term spread is positive (191 months out of total 240 ) and there is no certain relation between the sign of the term spread and the sign of the excess bond return.

In Year 2006 European Central Bank hikes its main interest rate to 3.5%, the sixth time hike, suggested that the yield curve of the German bond market flattens. Despite the increasing interest rate, interest rate is still much lower comparing with the level in Year 2000. A narrow spread together with a low level interest rate results in great challenge for the investors. If we have a look at the historical the yield curve in German bond market, we could find out that the yield curve sometimes is inverted. A inverted yield curve would lead to a negative relation with the bond excess return. Therefore, even though the term spread is a crucial proxy indicating the movement of the excess bond return, we observe in a long-run an uncertain relation between the yield curve and bond return in the German market.

### 3.2.2 Real Bond Yield

The second predictor we are bring in into our model is the real bond yield. Sometimes the real bond yield is also used as the overall proxy for the bond risk premium level. It is defined as the difference between the estimated ten-year spot rate of the government bond and the recently published yearly consumer price inflation rate. Real bond yield introduces the inflation rate

change into our model.

One-month real bond return can be written as:

$$\begin{aligned}
REALYLD_{n,t} &= y_{n,t} - \left(\frac{1}{n}\right)E_t \sum_{i=0}^{n-1} \pi_{t+1+i} \\
&= \left(\frac{1}{n}\right)E_t \sum_{i=0}^{n-1} (\mu_{t+1+i} + x_{n-i,t+1+i})
\end{aligned} \tag{3.2}$$

The formula above simply assumes that the inflation rate follows a random walk. If this assumption holds, the variation in the real bond yield reflects the variation in expected bond risk premium.

We use the Consumer Price Index (CPI) to calibrate the inflation in the German market. The CPI represents a basket of goods and services consumed by the urban consumers. The change in price level indicates the inflation level.

### 3.2.3 Inverse Wealth

The risk premium is guided and forecasted by using the wealth-dependent risk aversion. The level of risk aversion is closely correlated with the risk premium. Here a model is motivated with a positive subsistence level of investors.

$$U(W) = \frac{(W - \omega)^{1-\gamma}}{1 - \gamma} \tag{3.3}$$

This is the utility function proposed by Macus (1989). It states that the utility of identical agents using a two period, two-asset model. Here  $W$  is wealth level,  $\omega$  is subsistence wealth, and  $\gamma$  is a positive constant. The wealth dependent relative risk aversion (RRA) level can be measured by:

$$RRA = \frac{-WU_{WW}}{U_W} = \frac{\gamma}{1 - \frac{\omega}{W}} \tag{3.4}$$

When investors' wealth is declining, the risk aversion level will usually increase accordingly and investors have higher motivation of deviate to from the current product to the substitute, to a less risky asset. When the investors are more risk averse, they would require higher return for increasing risks.

The overall measure of investors' risk aversion level is "inverse wealth". The inverse wealth is defined as the ratio of past wealth (stock market level) to current level, where recent levels of wealth have greater weight in the "past wealth" than do distant levels of wealth. A high inverse wealth level (a depressed stock market) should reflect a high current risk aversion level and indicate a high risk premium in the near term.

On contrast to some previous researchers, Ilmanen stated that investor will be more risk averse if his wealth in previous period declined and would ask for a higher return given this situation. He suggested using the inverse relative wealth as an overall measure for the investors risk aversion (Ilmanen 1995, pp 482).

$$\begin{aligned}
 INVRELW_t &= \frac{ewaW_{t-1}}{W_t} \\
 &= \frac{(W_{t-1} + 0.9 * W_{t-2} + 0.9^2 * W_{t-2} + ...) * 0.1}{W_t}
 \end{aligned}
 \tag{3.5}$$

where Inverse relative wealth is calculated as the exponential weighted average of past wealth level. Ilmanen suggested that in order to capture the business cycle effect.

Marcus (1989) and Sharpe (1990) argued that the wealth-dependent risk aversion of the investors might cause the time variation of the expected risk premiums.

Ilmanen (1997) suggested that there is a inverse relation between stock market level and the subsequent bond return, which is caused by market participants' risk aversion or the lagged portfolio flows. The earlier declined wealth market level makes the investors more risk averse and increase their expected risk premium for holding risky assets. The recent poor performance of the stock market would make the investors to exit the market and switch

to the less risky bond market and expect the stock market continue to perform poorly.

#### **3.2.4 Lagged Bond Return**

The lagged bond return is the bond's lagged monthly return. It is used as an proxy for the market momentum. The basic assumption behind this choice is that the last month bond yield contains all the information of the past bond yields. Ilmanen (Ilmanen, 1997) suggested as well an alternative way to calibrate the market momentum. A dummy variable is set by the magnitude of the bond yield in excess of its six-month average.

#### **3.2.5 Change in Trade Weighted Exchange Rate**

Ilmanen (Ilmanen & Sayood, 2002) argued that appreciating exchange rate will also boost both the contemporary bond return and near-term rate. Taylor (Taylor, 1995) has suspected a correlation between the exchange rate change and risk aversion level of the market participants and argued that the uncovered risk premium for holding a foreign currency may be distorted by an exchange rate risk premium. The arbitrage theory will lead the return of holding a foreign currency bond equal to the sum of foreign currency risk premium and the bond risk premium. Fama & Farber (1979) relate the exchange market to the purchasing power among different countries, which has a close relation to the inflation rate of the countries.

#### **3.2.6 CRB Trend**

In addition to the four predictors Ilmanen suggested in his studies (Ilmanen, 1995; Ilmanen, 1997), we add two more predictors (Ilmanen & Sayood, 2002). According to modern economic studies the falling Commodity Research Index (issued by Commodity Research Bureau) suggests a disinflationary pressures, which lead to the increase of both current bond returns and near-term bond returns. The CRB index is the benchmark commodity index, and is considered as the standard for U.S. commodities prices.

The commodity price has the opposite movement from the bond price. Inflation is usually associated with increasing commodity price and at the

same time with devaluating the bond price. Inflation level has a direct impact on the nominal interest rate. A low inflation implies directly to a low nominal interest rate. With a falling CBR index level, which suggests a low inflation level, associates with a low interest rate level. This implies that the bonds durations will be longer than the fixed maturity (Campell 2000, pp. 1089). Bond duration has a positive impact on the pricing of the bond. Bond price increases with a higher duration and decrease with a lower duration. Therefore, the excess return increases together with a falling CRB index level. At low inflation level securities are more sensitive to the change of in change of inflation. In other word, investing in bonds is more risky. Investors would ask for higher risk premium for investing. Another explanation is that when the bond duration increases, the associated interest rate risk increases. An increased interest rate risk requires higher risk premium.

Another common proxy for inflation rate level is the Consumer Price Index (CPI). The CPI Index is constructed by a basket of goods prices. Fama and Schwert (Fama & Schwert, 1979) aruged that the expected inflation rate of different CPI components are different. These differential seasonals of different CPI components reflect the real costs of providing different goods. The seasonals of components of goods portfolio attribute to the seasonal feature of the CPI Index. Therefore, here we do not use the commonly used CPI as a proxy for inflation rate.

Another interpretation using the CPI Index is that in practise, when the central bank observes the CPI increase, it will adjust the interest rate to a higher level. This method is very commonly used by the central bank to control the inflation level. This would lead to a higher interest rate risk level for the government bonds. Therefore, we could in this case observe an increase in the bond yield, which leads directly to the decrease in the bond fair price.

### **3.2.7 Correlation of Predictors**

I examine the predictability of the German bond excess return from February 1986 to Jan 2006. I have in the section above introduced the predictors we use to forecast the bond excess return. The correlation between the bond excess return and the explanatory variables are illustrated in Figure 3.3. Three of the predictors have negative correlations with the dependent variable excess bond return. This result is different from the results Ilmanen



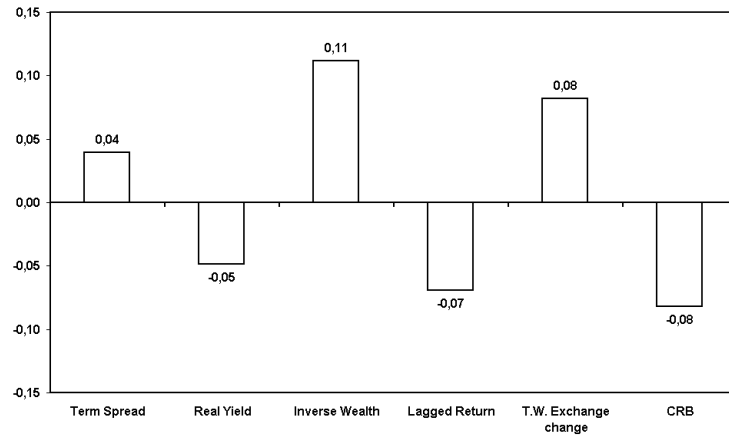


Figure 3.3. Correlation of Various Predictors with Subsequent Monthly Excess Return, Feb 1986 - Jan 2006

(1997) have for the US market. He showed both positive correlations on the real bond yield and the lagged bond return.

### 3.3 Model Selection

A great amount of researches were made by using regression models to analyze the market data of bond excess return. The advantage of the regression models is the simplicity in explaining the economic meaning. The predicability of the regression models on the bond returns are highly dependent on the macroeconomic conditions. In this paper we use the linear regression model first, in order to see the forecasting ability of the chosen predictors on dependent variable. Then we will try to bring in a semi-parametric model "Additive Model" to test the improvement can be made for model predictability.

#### 3.3.1 Linear Regression Model

We first bring all the predictors into a linear regression model to forecast the bond excess return. Linear regression model simply assumes that the dependent variable values are expected to follow the Gaussian distribution. That is, in linear regression function the dependent variable Y is linearly associated with the values of the independent variable X.

The least square algorithm is used in calculation to generate the regression equation. Before running the linear regression, I first test the stationarity of all the variables by using the Augmented Dickey-Fuller (ADF) Test.

From the test result in the Appendix we could see that among all the seven variables term spread and the real bond yield are not stationary. Therefore, I take the first derivatives for both variables and use the new variables in the regression model.

Expected Excess Bond Return =  $-0.83 + 0.03TS - 1.62RBY + 1.10IW - 0.07LBR + 0.09TWE - 0.07CRB$

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.83	0.644281	-1.288423	0.1989
C(2)	0.03	0.128100	0.209176	0.8345
C(3)	-1.62	0.836795	-1.936090	0.0541
C(4)	1.10	0.668599	1.648489	0.1006
C(5)	-0.07	0.063866	-1.155541	0.2491
C(6)	0.09	0.068986	1.299167	0.1952
C(7)	-0.07	0.048994	-1.370000	0.1720

### 3.3.2 Additive Model

The goal of my paper is to find the economic significant explanation of the movement of excess bond return and test the predictability of the excess bond return in German market for the use of the investors. The advantage of using the additive model is that it generates a regression model and allows for an interpretation of marginal changes (Härdle et al, 2004).

The additive models are widely used in economics due to the fact that they can be easily interpreted. From statistic point of view, additive models allow the componentwise analysis and combine flexible nonparametric modeling of multidimensional variables with a statistical precision that is typical of a one-dimensional explanatory variables (Härdle et al, 2004). For the dependent variable  $Y$  and  $X$  the as the  $d$ -dimensional vector of explanatory variable, an additive structure for the regression model  $m(\bullet)$ :

$$m(X) = c + \sum_{\alpha=1}^d g_{\alpha}(X_{\alpha}) \quad (3.6)$$

Stone (1985) showed that the optimal convergence rate of nonparametric regression function  $m(\bullet)$  is  $n^{-K/(2K+d)}$ , with  $K$  as the index smoothness of  $m(\bullet)$ . Different smoothers can be used. I will use both Nadaraya-Watson and local linear smoother. For additive models Stone showed that the optimal rate of convergence is  $n^{-K/(2K+1)}$ .  $g_{\alpha}(\bullet)$  are one-dimensional nonparametric functions operating on each element of the predictors.

The regression models of additive models were introduced and promoted greatly by Buja, Hastie and Tibshirani (1989) and Hastie and Tibshirani

(1990). The method they proposed was called the iterative backfitting, which is also the method I am using here in my paper.

Backfitting is largely used to estimate approximate  $g_\alpha(\bullet)$ . With the observed data set, using this model could lead to a system of normal equations with  $nd \times nd$  dimensions.

We start our process by assuming that

$$E_{X_\alpha} \{g_\alpha(X_\alpha)\} = 0 \quad (3.7)$$

for all  $\alpha$ . This directly leads to  $c = E(Y)$ . With formula

$$Y = m(X) + \epsilon$$

we allow the model to be heteroscedastic, i.e.  $E(\epsilon|X) = 0$  and  $(\epsilon|X) = \sigma^2(X)$ . The constant  $c$  can be then estimated with a faster rate than nonparametric ( $\sqrt{n}$ -rate). Then we can get

$$\hat{c} = \bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i$$

Hence, we get the constant  $c = 0$ .

Now we can start estimating additive component  $g_\alpha(\bullet)$ . As the general way in statistics we use the least square method to optimize problem

$$\min_m E \{Y - m(X)\}^2$$

such that

$$m(X) = \sum_{\alpha=1}^d g_\alpha(X_\alpha) \quad (3.8)$$

According to the projection theory, the equation above can be written as

$$g_\alpha(X_\alpha) = E \left[ \left\{ Y - \sum_{k \neq \alpha} g_k(X_k) \right\} | (X_\alpha) \right] \quad (3.9)$$

For the matrix dimension  $\alpha = 1, \dots, d$ . We could get the following matrix:

$$\begin{pmatrix} I & P_1 & \cdots & P_1 \\ P_2 & I & \cdots & P_2 \\ \vdots & & \ddots & \vdots \\ P_d & \cdots & P_d & I \end{pmatrix} \begin{pmatrix} g_1(X_1) \\ g_2(X_2) \\ \vdots \\ g_d(X_d) \end{pmatrix} = \begin{pmatrix} P_1 Y \\ P_2 Y \\ \vdots \\ P_d Y \end{pmatrix} \quad (3.10)$$

$$\underbrace{\begin{pmatrix} I & S_1 & \cdots & S_1 \\ S_2 & I & \cdots & S_2 \\ \vdots & & \ddots & \vdots \\ S_d & \cdots & S_d & I \end{pmatrix}}_{nd \times nd} \begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_d \end{pmatrix} = \begin{pmatrix} S_1 Y \\ S_2 Y \\ \vdots \\ S_d Y \end{pmatrix} \quad (3.11)$$

Here the operator  $P_\alpha(\bullet) = E(\bullet | X_\alpha)$ . Let  $S_\alpha$  be a  $(n \times n)$  smoother matrix. The smoother matrix  $S_\alpha$  has to meet the condition that  $S_\alpha Y$  estimates  $\{E(Y_1 | X_{1\alpha}), \dots, E(Y_n | X_{n\alpha})\}^T$ .

or abbreviated as,

$$\hat{P}g = \hat{Q}Y \quad (3.12)$$

In theory, we need  $d$  smoothers to estimate the all directions of  $X_1, \dots, X_d$ . However, Buja, Hastie & Tibshirani (1989) suggested a one-dimensional smoother to be used as a sufficient estimate to  $P_\alpha$ .

$$\hat{g}_\alpha^{(l)} = S_\alpha \left\{ Y - \sum_{k \neq \alpha} \hat{g}_k^{(l)} \right\} \quad (3.13)$$

where  $l = 1, 2, \dots$ . Vector  $g_\alpha$  is continuously refreshed until meeting the convergence criterion. In this case we are making a successive estimate using the partial residuals  $\{Y - \sum_{k \neq \alpha} \hat{g}_k^{(l-1)}\}$ .

For two-dimensional models, from 3.11 we could get that

$$\begin{aligned} g_1 &= S_1(Y - g_2) \\ g_2 &= S_2(Y - g_1) \end{aligned}$$

Using the same method, we could get

$$\begin{aligned} \hat{g}_1^{(l)} &= Y - \sum_{\alpha=0}^{l-1} (S_1 S_2)^\alpha (I - S_1) Y - (S_1 S_2)^{l-1} S_1 \hat{g}_2^{(0)} \\ \hat{g}_2^{(l)} &= S_2 \sum_{\alpha=0}^{l-1} (S_1 S_2)^\alpha (I - S_1) Y + S_2 (S_1 S_2)^{l-1} S_1 \hat{g}_2^{(0)} \end{aligned}$$

Since we have stated above that  $E_{X_\alpha}\{g_\alpha(X_\alpha)\} = 0$  and  $c = 0$ , the initialization  $\hat{g}_2^{(0)} = 0$  is reasonable, then we have

$$\begin{aligned} \hat{g}_1^{(l)} &= \{I - \sum_{\alpha=0}^{l-1} (S_1 S_2)^\alpha (I - S_1)\} Y \\ \hat{g}_2^{(l)} &= S_2 \sum_{\alpha=0}^{l-1} (S_1 S_2)^\alpha (I - S_1) Y \end{aligned}$$

The above process converges when the value of  $S_1 S_2$  is shrinking.

In our regression problem, we have six independent variables. Therefore, similar to the method above we will use six backfitting estimators. In figure below we plotted the realized values and the estimated curves.

Linton and Härdle (1996) used the marginal integration method and stated that the additive estimator converges at the rate of  $n^{-2/5}$  and asymptotically normal. Here we have to note that it requires the additive components to have an increasing number of derivatives as the dimension of  $X$  increases. Otherwise, model suffers from the curse of dimensionality. Horowitz and Mammen (2002) and Horowitz and Mammen (2005) argued that it is true regardless of the dimension of the explanatory variable and estimation is not penalized for not knowing the link function or other components of the additive model. Each additive component is asymptotically normally distributed with the same mean and variance as they would have if other components were known.

Assume that the observations are independent identical distributed. In our paper here I use the identity function. Therefore, as long as the link functions are twice differentiable and the second derivatives are sufficiently smooth. The convergence rate of the functions is  $n^{-2/5}$ .

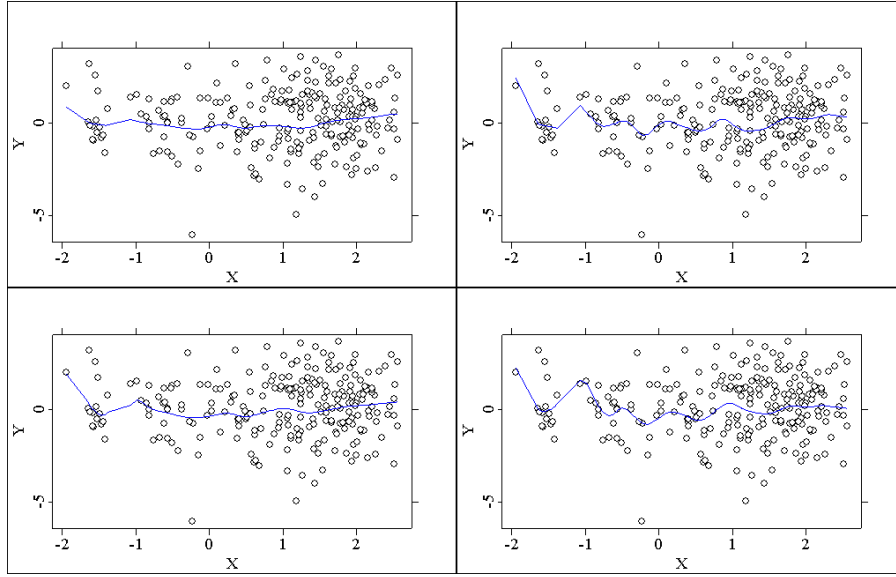


Figure 3.4. Additive component Term Spread, realized value (dots), estimated value (blue line). Upper left panel: Nadaraya-Watson, bandwidth: 0.5; upper right panel: Nadaraya-Watson, bandwidth: 0.3; lower left panel: local linear, bandwidth: 0.5; lower right panel: local linear, bandwidth: 0.3



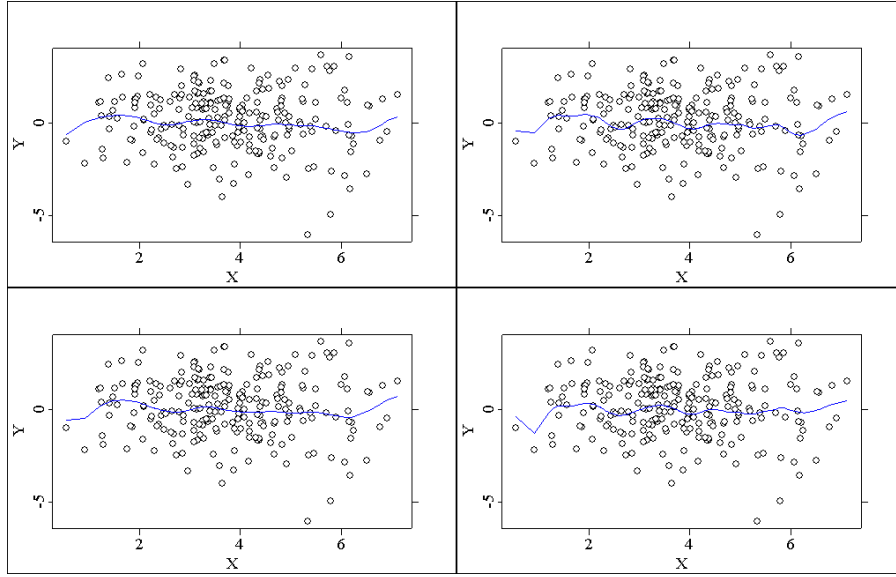


Figure 3.5. Additive component Real Bond Yield, realized value (dots), estimated value (blue line). Upper left panel: Nadaraya-Watson, bandwidth: 0.8; upper right panel: Nadaraya-Watson, bandwidth: 0.5; lower left panel: local linear, bandwidth: 0.8; lower right panel: local linear, bandwidth: 0.5

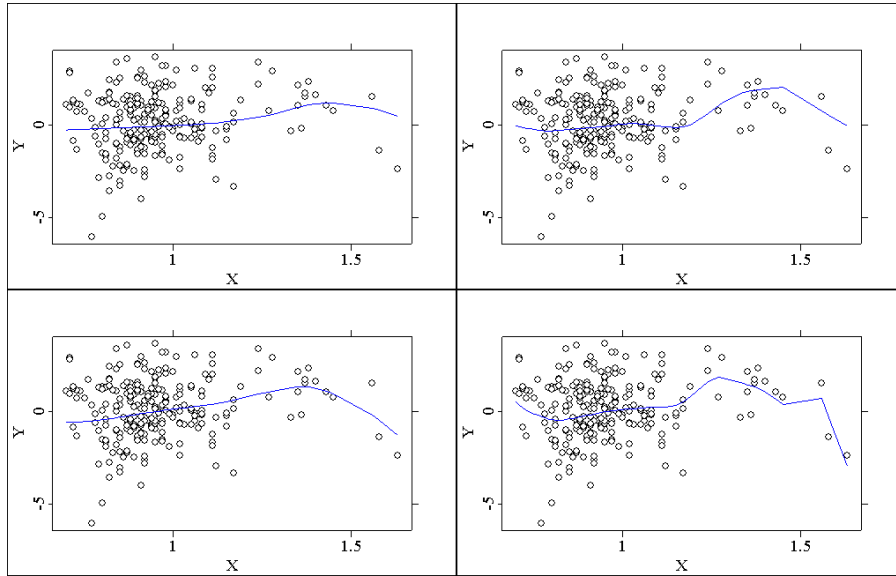


Figure 3.6. Additive component Inverse Wealth, realized value (dots), estimated value (blue line). Upper left panel: Nadaraya-Watson, bandwidth: 0.1; upper right panel: Nadaraya-Watson, bandwidth: 0.05; lower left panel: local linear, bandwidth: 0.1; lower right panel: local linear, bandwidth: 0.05

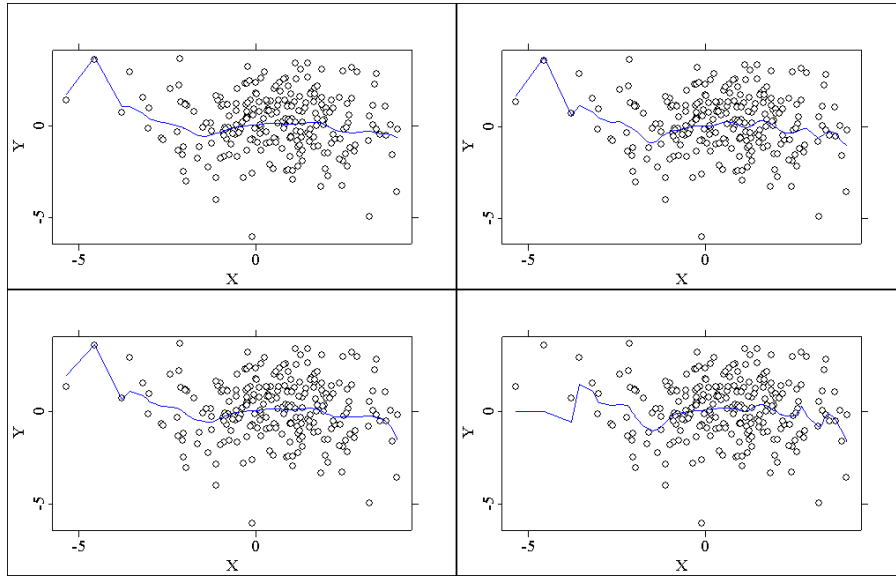


Figure 3.7. Additive component Lagged Bond Return, realized value (dots), estimated value (blue line). Upper left panel: Nadaraya-Watson, bandwidth: 0.8; upper right panel: Nadaraya-Watson, bandwidth: 0.5; lower left panel: local linear, bandwidth: 0.8; lower right panel: local linear, bandwidth: 0.5

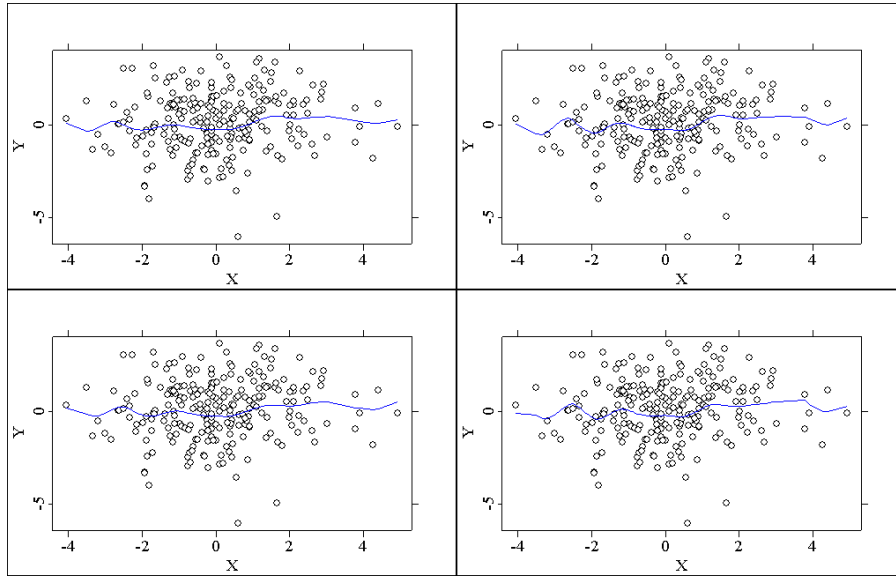


Figure 3.8. Additive component Change in Trade Weighted Exchange Rate, realized value (dots), estimated value (blue line). Upper left panel: Nadaraya-Watson, bandwidth: 0.8; upper right panel: Nadaraya-Watson, bandwidth: 0.5; lower left panel: local linear, bandwidth: 0.8; lower right panel: local linear, bandwidth: 0.5

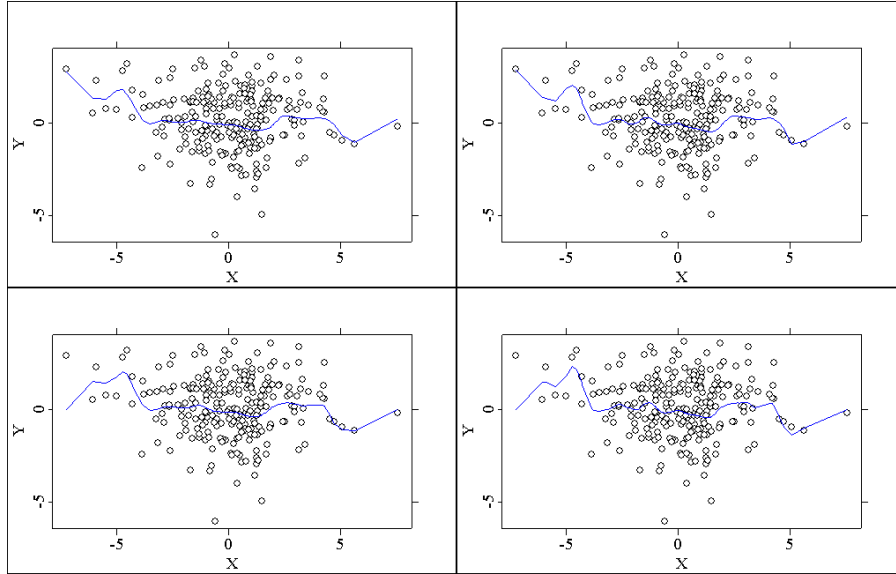


Figure 3.9. Additive component CRB Trend, realized value (dots), estimated value (blue line). Upper left panel: Nadaraya-Watson, bandwidth: 0.8; upper right panel: Nadaraya-Watson, bandwidth: 0.5; lower left panel: local linear, bandwidth: 0.8; lower right panel: local linear, bandwidth: 0.5

## 4 Data Description

The data source in this paper are from Datastream and Bloomberg. In our model I examine the predictability of the bond excess return by using term spread, real bond yield, inverse wealth, lagged bond return, change in trade-weighted exchange rate and the CRB trend. The data we are using is the monthly data from February 1986 to January 2006. The time series data for the German government bonds are less reliable comparing with the longer history in the US bond market. I check the bond data which are reasonably priced, risk-free or with little default risk. I tried to use the German government bond data as much as possible in the model. In Germany, we do not have the bond data for the liquid risk-free security market, therefore, we use the German interbank three months rate instead.

I use the difference of the 10 year total return index of German government bond, which has an approximately constant maturity of 10 years, and the 3 months German interbank lending rate (FIBOR - Frankfurter Interbank Offered Rate) as the proxy of bond excess return. Ilmanen (Ilmanen, 1995) suggested to use the liquid T-bill rate for the US bond market or one month risk-free rate as the short asset to forecast excess bond return in other countries. However, in our case due to recent data base date, one month FIBOR only exists for about ten years and therefore would not provide me with long-term stable feather. The bond excess return would be downwards biased because of the above mentioned reason. Another point we have to pay attention is that the FIBOR is not a real risk-free rate. A general rating of AAA suggests that it is almost risk-free, but there exists still default risk. The excess return is as well downwards biased due to this point. For term spread we use the spread between the 10 years German benchmark bond yield and 3 months FIBOR. We mentioned in section 3.2.1 that this spread well captures both the short end and the long end movement of the yield curve. Real bond yield is the difference between the 10 year German benchmark bond yield and the German CPI percentage year-over-year rate. There are large amount of stock market indices to calibrate the stock market

wealth level. I use the CDAX General Total Return Index. CDAX reflects the price development of stocks from both Prime Standard and General Standard. Hence, represent a broader universe comparing with for example DAX 30, which only include 30 stocks with large market capitalization. The index we chose for commodity price is the Commodity Research Bureau Index (CRB index), using the change in the CRB spot price index as another proxy for bond excess return. The change in Euro/Dollar trade-weighted exchange rate index is used as the last proxy. Data source is Bank of England trade weighted exchange rate index.

Data mining problem exists in all predictability studies. Even though completely avoid these bias is practically not possible, we tried to mitigate the bias by bringing in a logic economic explanation of the model.

**Length** 20 years and with 240 observations. It covers the longest possible period and avoids the data selection bias.

**Relevance** This quarter-century has been the change of technology, large government deficit, floating exchange rate. The sample data covers the period of the disinflation period in 1980's and 1990's and booming economy end of last century due to the technology advances.

**Neutrality** The selected period includes the disinflation periods in 1980's and 1990's. The yield trend in this case will not play important role when we calculate the geometric average return during this period.

## 5 Out-of-Sample Test

The result we have from the linear regression is based on the data from January 1986 to January 2006. It actually splits the excess bond return into two parts, the expected excess bond return and the residual. The equation is based on a in-sample forecast (Figure 5.1). But in reality an investor in Year 1986 or Year 1996 would not be able to use this result as he does not have the available data that I have now for until Year 2006. Therefore, an in-sample test will lead to an exaggerated predictability of the model and could, at worst, totally spurious the results.

Most of the investors are aware of these kind of data-snooping bias and could objectively treat the exciting results from empirical findings. In order to avoid the data-snooping bias, I run the in-sample test for another time for Year 1986 to Year 2000 then use the result to forecast excess return for January 2001. By doing this, I generate a expected value for investors for February 2001 and then compare the expected value with the real value. Then an out-of-sample test is generalized to test the model predictability (Figure 5.2).

I plot the 5 years (February 2001 to Jan 2006) both the expected bond excess return and the real value of excess return into 5.2. If all the forecasted values have the same sign as the real values, all observations will lie in the upper-right quadrant or the lower-left quadrant. The result might be not very impressive, implying the difficulties in predicting the short-term fluctuation of the excess bond returns. The longer term fluctuations are more predictable, as within a longer term the short term fluctuations tend to balance out themselves.

In an ideal model, all the points should lie in the upper right or lower left panels. It would mean that the model could correctly predict the direction of the excess bond return. However, we observe that there are great amount of data that lie in the other two panels. Overall, there are 57.08% of data lie the area where model has right prediction with in-sample test (Figure 5.1) while 51.67% with right prediction for the out-of-sample test (Figure 5.2).



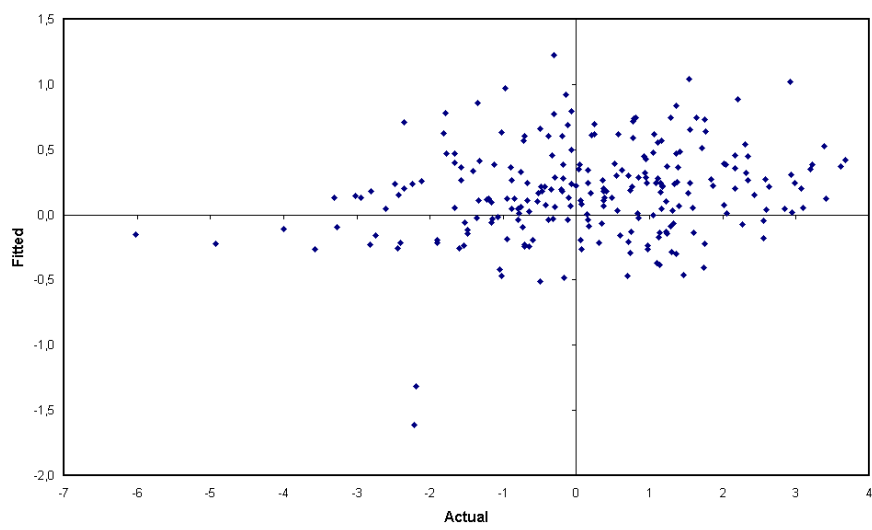


Figure 5.1. Model Prediction v.s. Realized EBR Value (In Sample), Feb 1986 - Jan 2006

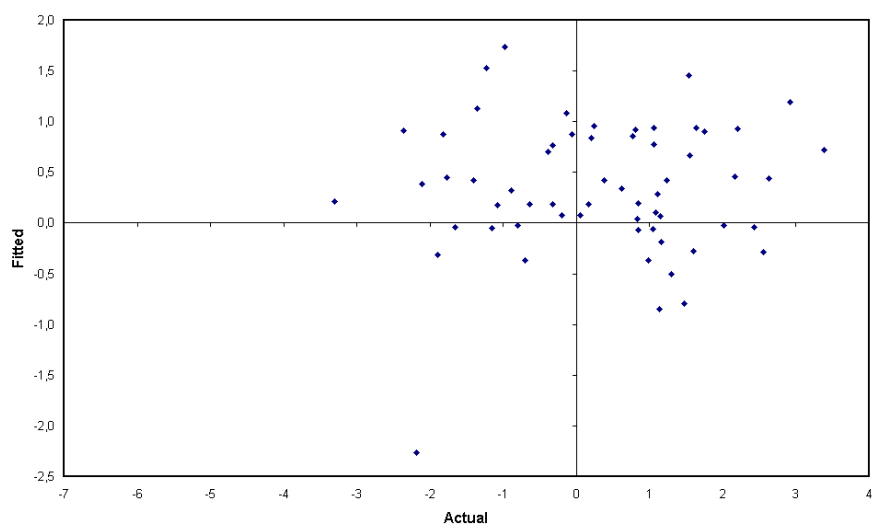


Figure 5.2. Out of Sample Test, Feb 2001 - Jan 2006

## 6 Strategies Implementations

The extension of the analytical studies in the real financial market is the main motivation of all theoretical analysis. Market participants and portfolio managers are more concerned about the financial significance than statistical significance. Based on the study above we will introduce the dynamic investment strategies to test their financial significance. In Section 3 we analyzed each predictors in relation to the dependent variable excess bond return and then pooled them into a multipredictor. Using the selected model we test the ability of this active approach. If the forecasting model generate a positive performance, the margin would be magnified by combining several strategies into a composite with a smoother performance over time.

In this section I try to apply the dynamic strategies using the model prediction and compare the performance of the dynamic strategies with the static strategy, which is the simple buy and hold strategy.

### 6.1 Buy and Hold Strategy

This strategy is the traditional way of investing in bonds. The strategy is self-explained from its name. Investor buys one unit of German government bond at the beginning of the sample period and hold until the end the period.

### 6.2 1/0 Strategy

This strategy is also called bond/cash strategy. It is the first dynamic strategy we use. It involves buying one unit of 10 year German government bond when the predicted value is positive and hold cash when the predicted value suggests negative excess bond return. Here we assume that investors' wealth will not increase by holding cash. This strategy ignores the magnitude

of the predicted value and try to capture Alpha from positively predicted market movement.

### **6.3 1/-1 Strategy**

This strategy is the second dynamic strategy that we carry out. Comparing with the 1/0 strategy, the 1/-1 strategy benefit from both positive and negative market movement. Therefore, it usually generates a more volatile performance. This strategy does not consider the magnitude of the prediction either, instead it tries to outperform by making the right move together with the market. When the model has a positive predictability power, 1/-1 strategy generates a positive Alpha by long a unit bond with a positive forecasted return or by short a unit bond with a negative forecasted return. Since this strategy tries to capture Alpha from both sides of the market movement, when the model's predictability power is poor, this will magnify a negative Alpha.

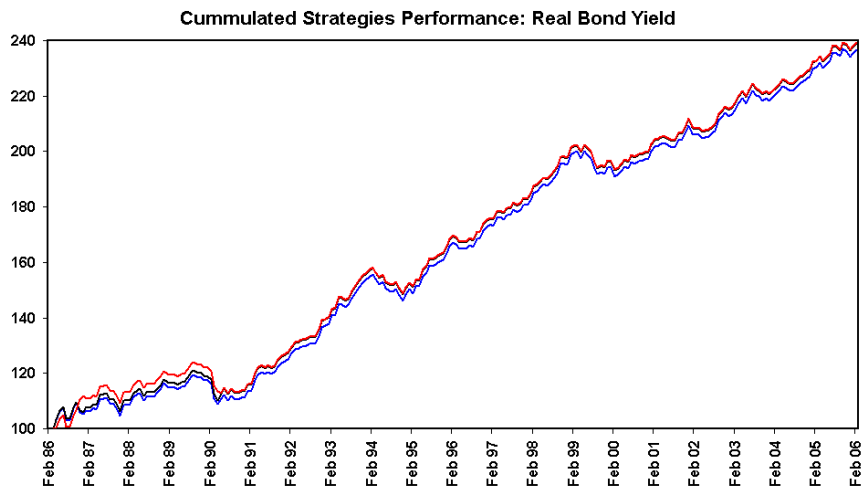


Figure 6.1. Rebased Cummulative Strategies Performance Using Real Bond Yield as Preidctor (Feb 1986 - Feb 2006), Buy and Hold Strategy (black line), 1/0 Strategy (red line), 1/-1 Strategy (blue line)

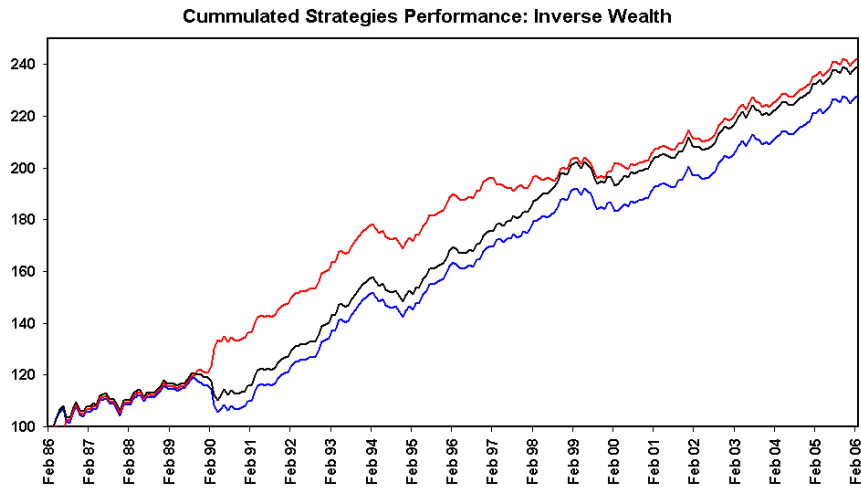


Figure 6.2. Rebased Cummulative Strategies Performance Using Inverse Wealth as Preidctor (Feb 1986 - Feb 2006), Buy and Hold Strategy (black line), 1/0 Strategy (red line), 1/-1 Strategy (blue line)

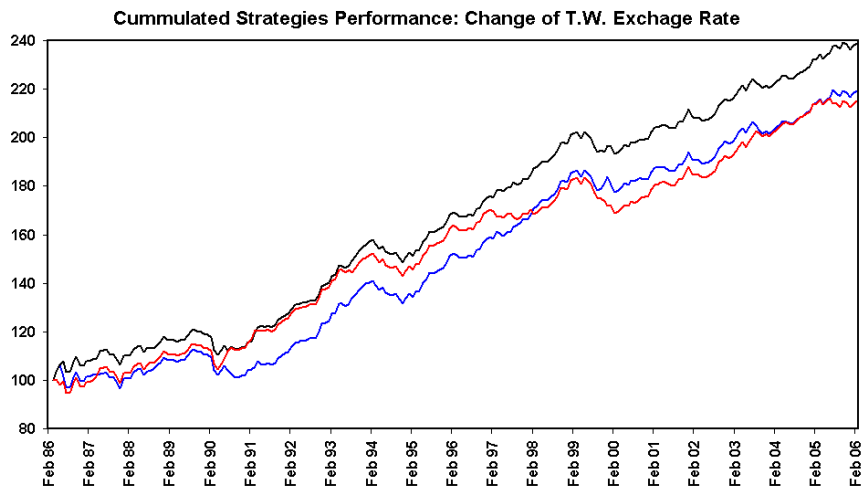


Figure 6.3. Rebased Cumulative Strategies Performance Using Change in Trade Weighted Exchange Rate as Predictor (Feb 1986 - Feb 2006), Buy and Hold Strategy (black line), 1/0 Strategy (red line), 1/-1 Strategy (blue line)

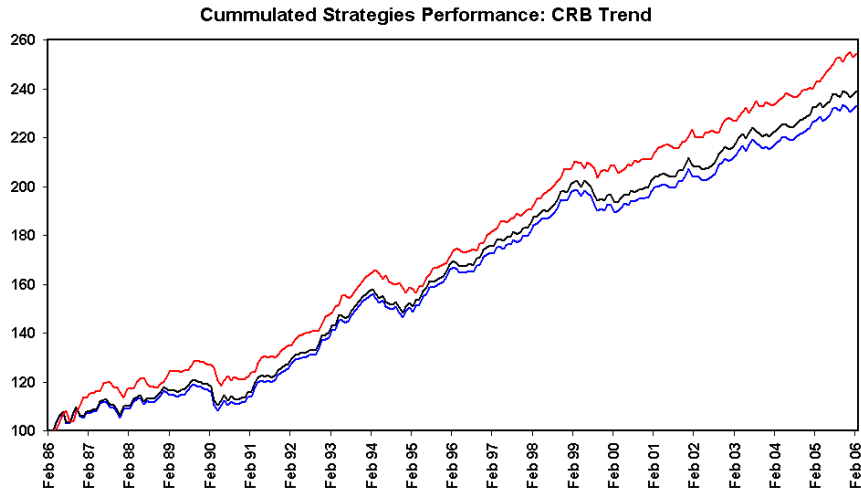


Figure 6.4. Rebased Cumulative Strategies Performance Using CRB Trend as Preidctor (Feb 1986 - Feb 2006), Buy and Hold Strategy (black line), 1/0 Strategy (red line), 1/-1 Strategy (blue line)

## 6.4 Critiques

Consequently, if investors predict a booming market, strategies with a bigger weight on the positive expected return could be carried out as well, for example 2/-1 combined with one unit loan. The performance of the above three strategies are illustrated below on a rolling base in comparison to the US treasury. We could observe that the performance of the US treasury outperform the German government Bunds. It is due to the higher return volatility of the US treasury. In the above sections we mentioned that the sample period we are using is over 20 years and the model predictability power is with a long-term focus. It would be normal that when the model is carried out using dynamic strategies, there exists some periods when negative Alpha is generated for a month-long period. Such a period length is considered to be short comparing to the whole sample period, however, the



performance of a portfolio manager is usually estimated rather frequently. Generating a negative Alpha for 3-4 months in a roll would be quite possible to make him lose his job. In this case, investors should cautiously treat the model predictability.

The impact of transaction cost has to be taken into consideration as well. It reduces the profitability of the investment strategies and sometimes even becomes the source of negative alpha. Normally the transaction cost of the government bonds are relatively small. The transaction cost for the always bond strategy can be neglected, as it only happens once. The influence of transaction cost on the 1/0 strategy is greater than holding the bond until the end, but still relatively small comparing with the 1/-1 strategy. 1/-1 strategy involves also shorting bond. Therefore, the performance for using the 1/-1 strategy is exaggerated.

As we could see from the figures above, the outperformance of the strategies using each single predictor is not always certain. For strategies using real bond yield to predict bond excess return generate similar performance. The dynamic strategies do not outperform the static strategy. When using the inverse wealth as the predictor for bond excess return, it pays off for actively manage the portfolio, as the 1/-1 strategy outperform the buy and hold static strategy almost all the time. However, the 1/0 strategy underperform the static strategy most of time. Active strategies do not pay for using the change in trade weighted exchange rate in any time point in history. The result for using CRB as predictor generate us similar results as the ones for the inverse wealth. 1/-1 strategy outperforms static strategy, while 1/0 strategy underperform the static strategy.

In general, we could observe a mixed picture by using single predictor to forecast the excess bond return. The dynamic strategies do not necessarily outperform the static strategy, however, the 1/0 strategy always underperform the 1/-1 strategy.

## 7 Conclusion

The results suggest that the term spread is a significant indicator for the excess bond return. A normal, even steep yield curve suggests a high positive future abnormal return. An inverted yield curve tends to be followed by a negative abnormal return. Inverse wealth has great power of predictability, and it pays to use inverse wealth as indicator for active bond management. The same conclusion can be made for using CRB trend.

Using a single predictor to carry out investment strategies provides a mixed result. Here, the outperformance of using active investment strategies is not ensured. The combination of predictors enhances the predictability of the model. In this case, the excess return is not completely unpredictable: 4.8% can be predicted (the expected excess return). More than 95% cannot be forecasted (the unexpected excess return).

The prediction result of the regression model is similar to the result of using the additive model. The rather weak predictability power is due to the lack of suitable German bond financial market data. With a more reliable data set, Ilmanen (1995, 1997) provided a slightly better return predictability (10%) for the US bond market. Furthermore, the lack of suitable time series has also impacted the results. Another reason for the unexciting result is that there are various factors that influence the bond return, and I introduced six of them into the model. There could be other factors that also impact the excess bond return, which have not been included.

Additionally, bond returns are influenced by different factors in different macroeconomic status. Monetary policy used by the government is crucial when analyzing bond return. This is not generated by the market itself, but is dependent on the decision of the central bank. It increases the prediction difficulties as well.

In future research, further improvements can be made. More predictors can be analyzed and introduced into the model. Other statistic models can be tested for their power of predictability. However, there exists a trade-off of

the model predictability and the explanatory power of the model.

# Appendix

## Analysis results using Eviews

### 7.1.1 Regression Results

#### Regression model 1: on original data

Dependent Variable: EBR

Method: Least Squares

Date: 12/23/06 Time: 22:31

Sample: 1 240

Included observations: 240

EBR=C(1)+C(2)\*TS+C(3)\*RBY+C(4)\*IW+C(5)\*LBR+C(6)\*TWE+C(7)  
\*CRB

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.679984	0.720958	-0.943168	0.3466
C(2)	0.091904	0.098321	0.934735	0.3509
C(3)	-0.088669	0.080600	-1.100107	0.2724
C(4)	1.202078	0.668146	1.799123	0.0733
C(5)	-0.079901	0.064823	-1.232590	0.2190
C(6)	0.084091	0.069255	1.214214	0.2259
C(7)	-0.078804	0.049728	-1.584699	0.1144
R-squared	0.039729	Mean dependent var	0.173485	
Adjusted R-squared	0.015001	S.D. dependent var	1.623504	
S.E. of regression	1.611281	Akaike info criterion	3.820670	
Sum squared resid	604.9211	Schwarz criterion	3.922188	
Log likelihood	-451.4803	Durbin-Watson stat	1.919232	

**Regression model 2:** on stationary data

Dependent Variable: EBR

Method: Least Squares

Date: 12/28/06 Time: 20:48

Sample: 1 240

Included observations: 240

$$\text{EBR} = \text{C}(1) + \text{C}(2) * \text{DTS} + \text{C}(3) * \text{DRBY} + \text{C}(4) * \text{IW} + \text{C}(5) * \text{LBR} + \text{C}(6) * \text{TWE} + \text{C}(7) * \text{CRB}$$

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.830107	0.644281	-1.288423	0.1989
C(2)	0.026795	0.128100	0.209176	0.8345
C(3)	-1.620111	0.836795	-1.936090	0.0541
C(4)	1.102178	0.668599	1.648489	0.1006
C(5)	-0.073800	0.063866	-1.155541	0.2491
C(6)	0.089624	0.068986	1.299167	0.1952
C(7)	-0.067122	0.048994	-1.370000	0.1720
R-squared	0.048262	Mean dependent var	0.173485	
Adjusted R-squared	0.023753	S.D. dependent var	1.623504	
S.E. of regression	1.604107	Akaike info criterion	3.811744	
Sum squared resid	599.5459	Schwarz criterion	3.913263	
Log likelihood	-450.4093	Durbin-Watson stat	1.974865	

**Small sample regression:** on stationary data (180 months, from Feb 1986 – Jan 2006)

Dependent Variable: EBRs

Method: Least Squares

Date: 01/02/07 Time: 20:00

Sample: 1 180

Included observations: 180

EBRS=C(1)+C(2)\*DTSS+C(3)\*DRBYS+C(4)\*IWS+C(5)\*LBRS  
+C(6)\*TWES+C(7)\*CRBS

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-1.257666	0.992497	-1.267175	0.2068
C(2)	-0.114753	0.186196	-0.616303	0.5385
C(3)	-2.956834	1.109008	-2.666198	0.0084
C(4)	1.543080	1.071156	1.440574	0.1515
C(5)	-0.064921	0.072203	-0.899145	0.3698
C(6)	0.065406	0.078811	0.829910	0.4077
C(7)	-0.073269	0.060152	-1.218072	0.2249
R-squared	0.070737	Mean dependent var		0.124962
Adjusted R-squared	0.038508	S.D. dependent var		1.668356
S.E. of regression	1.635918	Akaike info criterion		3.860397
Sum squared resid	462.9872	Schwarz criterion		3.984568
Log likelihood	-340.4358	Durbin-Watson stat		1.949958

**TS**

Dependent Variable: EBR

Method: Least Squares

Date: 12/23/06 Time: 23:09

Sample: 1 240

Included observations: 240

EBR=C(1)+C(2)\*TS

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.117790	0.138949	0.847719	0.3974
C(2)	0.057885	0.094661	0.611497	0.5415
R-squared	0.001569	Mean dependent var		0.173485
Adjusted R-squared	-0.002626	S.D. dependent var		1.623504
S.E. of regression	1.625635	Akaike info criterion		3.817973
Sum squared resid	628.9600	Schwarz criterion		3.846978
Log likelihood	-456.1567	Durbin-Watson stat		1.845156

**RBV**

Dependent Variable: EBR

Method: Least Squares

Date: 12/23/06 Time: 23:11

Sample: 1 240

Included observations: 240

EBR=C(1)+C(2)\*RBV

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.396310	0.313985	1.262195	0.2081
C(2)	-0.058971	0.078322	-0.752925	0.4522
R-squared	0.002376	Mean dependent var		0.173485
Adjusted R-squared	-0.001815	S.D. dependent var		1.623504
S.E. of regression	1.624977	Akaike info criterion		3.817163
Sum squared resid	628.4513	Schwarz criterion		3.846169
Log likelihood	-456.0596	Durbin-Watson stat		1.852528



## IW

Dependent Variable: EBR  
Method: Least Squares  
Date: 12/23/06 Time: 23:15  
Sample: 1 240  
Included observations: 240  
EBR=C(1)+C(2)\*IW

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.924343	0.641198	-1.441587	0.1507
C(2)	1.146623	0.660768	1.735288	0.0840
R-squared	0.012494	Mean dependent var	0.173485	
Adjusted R-squared	0.008345	S.D. dependent var	1.623504	
S.E. of regression	1.616716	Akaike info criterion	3.806970	
Sum squared resid	622.0776	Schwarz criterion	3.835975	
Log likelihood	-454.8363	Durbin-Watson stat	1.865404	

## LBR

Dependent Variable: EBR  
Method: Least Squares  
Date: 12/23/06 Time: 23:18  
Sample: 1 240  
Included observations: 240  
EBR=C(1)+C(2)\*LBR

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.212739	0.110984	1.916855	0.0565
C(2)	-0.068489	0.063911	-1.071628	0.2850
R-squared	0.004802	Mean dependent var	0.173485	
Adjusted R-squared	0.000620	S.D. dependent var	1.623504	
S.E. of regression	1.623001	Akaike info criterion	3.814729	
Sum squared resid	626.9232	Schwarz criterion	3.843734	
Log likelihood	-455.7675	Durbin-Watson stat	1.832989	

## TWE

Dependent Variable: EBR

Method: Least Squares

Date: 12/23/06 Time: 23:19

Sample: 1 240

Included observations: 240

EBR=C(1)+C(2)\*TWE

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.169484	0.104709	1.618616	0.1069
C(2)	0.087466	0.068805	1.271202	0.2049
R-squared	0.006744	Mean dependent var		0.173485
Adjusted R-squared	0.002571	S.D. dependent var		1.623504
S.E. of regression	1.621416	Akaike info criterion		3.812776
Sum squared resid	625.6999	Schwarz criterion		3.841781
Log likelihood	-455.5331	Durbin-Watson stat		1.847613

## CRB

Dependent Variable: EBR

Method: Least Squares

Date: 12/23/06 Time: 23:20

Sample: 1 240

Included observations: 240

EBR=C(1)+C(2)\*CRB

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.180552	0.104810	1.722664	0.0862
C(2)	-0.062654	0.049291	-1.271111	0.2049
R-squared	0.006743	Mean dependent var		0.173485
Adjusted R-squared	0.002570	S.D. dependent var		1.623504
S.E. of regression	1.621417	Akaike info criterion		3.812777
Sum squared resid	625.7005	Schwarz criterion		3.841782
Log likelihood	-455.5332	Durbin-Watson stat		1.868313

## 7.1.2 ADF tests

### 1. EBR

Null Hypothesis: EBR has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic based on SIC, MAXLAG=14)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-14.39676	0.0000
Test critical values: 1% level	-3.457630	
5% level	-2.873440	
10% level	-2.573187	

\*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(EBR)

Method: Least Squares

Date: 12/23/06 Time: 18:14

Sample(adjusted): 2 240

Included observations: 239 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
EBR(-1)	-0.930391	0.064625	-14.39676	0.0000
C	0.149254	0.105220	1.418491	0.1574
R-squared	0.466537	Mean dependent var	-0.020217	
Adjusted R-squared	0.464286	S.D. dependent var	2.208496	
S.E. of regression	1.616454	Akaike info criterion	3.806680	
Sum squared resid	619.2627	Schwarz criterion	3.835771	
Log likelihood	-452.8982	F-statistic	207.2666	
Durbin-Watson stat	1.996137	Prob(F-statistic)	0.000000	

## 2. TS

Null Hypothesis: TS has a unit root

Exogenous: Constant

Lag Length: 1 (Automatic based on SIC, MAXLAG=14)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-1.883113	0.3399
Test critical values: 1% level	-3.457747	
5% level	-2.873492	
10% level	-2.573215	

\*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(TS)

Method: Least Squares

Date: 12/23/06 Time: 18:18

Sample(adjusted): 3 240

Included observations: 238 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
TS(-1)	-0.022963	0.012194	-1.883113	0.0609
D(TS(-1))	0.235870	0.063067	3.740027	0.0002
C	0.020650	0.017883	1.154680	0.2494
R-squared	0.065130	Mean dependent var	-0.002143	
Adjusted R-squared	0.057174	S.D. dependent var	0.214557	
S.E. of regression	0.208333	Akaike info criterion	-0.286833	
Sum squared resid	10.19962	Schwarz criterion	-0.243065	
Log likelihood	37.13311	F-statistic	8.185968	
Durbin-Watson stat	2.047204	Prob(F-statistic)	0.000366	

### 3. RBY

Null Hypothesis: RBY has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic based on SIC, MAXLAG=14)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-1.670063	0.4452
Test critical values: 1% level	-3.457630	
5% level	-2.873440	
10% level	-2.573187	

\*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(RBY)

Method: Least Squares

Date: 12/23/06 Time: 18:25

Sample(adjusted): 2 240

Included observations: 239 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RBY(-1)	-0.026866	0.016087	-1.670063	0.0962
C	0.084434	0.064612	1.306774	0.1926
R-squared	0.011632	Mean dependent var	-0.017364	
Adjusted R-squared	0.007461	S.D. dependent var	0.332567	
S.E. of regression	0.331324	Akaike info criterion	0.636895	
Sum squared resid	26.01686	Schwarz criterion	0.665987	
Log likelihood	-74.10893	F-statistic	2.789112	
Durbin-Watson stat	1.844455	Prob(F-statistic)	0.096227	

#### 4. IW

Null Hypothesis: IW has a unit root

Exogenous: Constant

Lag Length: 1 (Automatic based on SIC, MAXLAG=14)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-3.655500	0.0054
Test critical values: 1% level	-3.457747	
5% level	-2.873492	
10% level	-2.573215	

\*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(IW)

Method: Least Squares

Date: 12/23/06 Time: 18:27

Sample(adjusted): 3 240

Included observations: 238 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
IW(-1)	-0.090111	0.024651	-3.655500	0.0003
D(IW(-1))	0.175644	0.063982	2.745218	0.0065
C	0.086773	0.023943	3.624131	0.0004
R-squared	0.070488	Mean dependent var		0.000432
Adjusted R-squared	0.062577	S.D. dependent var		0.060818
S.E. of regression	0.058884	Akaike info criterion		-2.813967
Sum squared resid	0.814823	Schwarz criterion		-2.770199
Log likelihood	337.8621	F-statistic		8.910352
Durbin-Watson stat	2.029033	Prob(F-statistic)		0.000186

## 5. LBR

Null Hypothesis: LBR has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic based on SIC, MAXLAG=14)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-14.50179	0.0000
Test critical values: 1% level	-3.457630	
5% level	-2.873440	
10% level	-2.573187	

\*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LBR)

Method: Least Squares

Date: 12/23/06 Time: 18:42

Sample(adjusted): 2 240

Included observations: 239 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LBR(-1)	-0.938669	0.064728	-14.50179	0.0000
C	0.505738	0.110226	4.588199	0.0000
R-squared	0.470157	Mean dependent var		-0.0206
Adjusted R-squared	0.467921	S.D. dependent var		2.2058
S.E. of regression	1.608994	Akaike info criterion		3.7974
Sum squared resid	613.5604	Schwarz criterion		3.8265
Log likelihood	-451.7927	F-statistic		210.30
Durbin-Watson stat	1.989307	Prob(F-statistic)		0.0000

## 6. TWE

Null Hypothesis: TWE has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic based on SIC, MAXLAG=14)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-14.26830	0.0000
Test critical values: 1% level	-3.457630	
5% level	-2.873440	
10% level	-2.573187	

\*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(TWE)

Method: Least Squares

Date: 12/23/06 Time: 18:32

Sample(adjusted): 2 240

Included observations: 239 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
TWE(-1)	-0.919719	0.064459	-14.26830	0.0000
C	0.032847	0.098292	0.334178	0.7385
R-squared	0.462078	Mean dependent var	-0.008251	
Adjusted R-squared	0.459808	S.D. dependent var	2.066596	
S.E. of regression	1.518901	Akaike info criterion	3.682184	
Sum squared resid	546.7730	Schwarz criterion	3.711275	
Log likelihood	-438.0210	F-statistic	203.5845	
Durbin-Watson stat	2.000505	Prob(F-statistic)	0.000000	



## 7. CRB

Null Hypothesis: CRB has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic based on SIC, MAXLAG=14)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-13.20443	0.0000
Test critical values: 1% level	-3.457630	
5% level	-2.873440	
10% level	-2.573187	

\*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(CRB)

Method: Least Squares

Date: 12/23/06 Time: 18:33

Sample(adjusted): 2 240

Included observations: 239 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CRB(-1)	-0.845827	0.064056	-13.20443	0.0000
C	0.107259	0.136301	0.786929	0.4321
R-squared	0.423858	Mean dependent var		0.017593
Adjusted R-squared	0.421427	S.D. dependent var		2.766811
S.E. of regression	2.104546	Akaike info criterion		4.334410
Sum squared resid	1049.700	Schwarz criterion		4.363502
Log likelihood	-515.9620	F-statistic		174.3570
Durbin-Watson stat	1.994216	Prob(F-statistic)		0.000000

## 8. DTS

Null Hypothesis: DTS has a unit root

Exogenous: Constant

Lag Length: 2 (Automatic based on SIC, MAXLAG=14)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-7.048038	0.0000
Test critical values: 1% level	-3.457865	
5% level	-2.873543	
10% level	-2.573242	

\*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(DIFFTS)

Method: Least Squares

Date: 01/04/07 Time: 07:26

Sample(adjusted): 4 240

Included observations: 237 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DTS(-1)	-0.770457	0.109315	-7.048038	0.0000
D(DFTS(-1))	-0.178535	0.088095	-2.026627	0.0438
D(DTS(-2))	-0.244166	0.063524	-3.843690	0.0002
C	-0.060498	0.052320	-1.156295	0.2487
R-squared	0.514863	Mean dependent var		0.000837
Adjusted R-squared	0.508616	S.D. dependent var		1.132851
S.E. of regression	0.794115	Akaike info criterion		2.393555
Sum squared resid	146.9340	Schwarz criterion		2.452088
Log likelihood	-279.6363	F-statistic		82.42553
Durbin-Watson stat	1.945975	Prob(F-statistic)		0.000000

## 9. DRBY

Null Hypothesis: DIFFRBY has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic based on SIC, MAXLAG=14)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-15.83386	0.0000
Test critical values: 1% level	-3.457630	
5% level	-2.873440	
10% level	-2.573187	

\*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(DIFFRBY)

Method: Least Squares

Date: 01/04/07 Time: 07:29

Sample(adjusted): 2 240

Included observations: 239 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DIFFRBY(-1)	-1.024459	0.064701	-15.83386	0.0000
C	0.001663	0.008126	0.204639	0.8380
R-squared	0.514057	Mean dependent var	-0.000550	
Adjusted R-squared	0.512006	S.D. dependent var	0.179810	
S.E. of regression	0.125609	Akaike info criterion	-1.302951	
Sum squared resid	3.739302	Schwarz criterion	-1.273859	
Log likelihood	157.7027	F-statistic	250.7110	
Durbin-Watson stat	2.002464	Prob(F-statistic)	0.000000	

## XploRe code

```
1 library("xplore")
2 library("stats")
3 library("plot")
4 library("times")
5 library("finance")
6 library("gam")
7
8 data = read("GGB.txt")
9 EBR = data[,1]
10 TS = data[,2]
11 RBY = data[,3]
12 IW = data[,4]
13 LBR = data[,5]
14 TWE = data[,6]
15 CRB = data[,7]
16 X = TS ~ RBY ~ IW ~ LBR ~ TWE ~ CRB
17 ;{beta,se,betastan,p} = linreg(X,EBR)
18
19 h1 = 0.8
20 h2 = 0.5
21 opt = gamopt("x",X,"shf",1)
22 {m,b,const} = backfit(X,EBR,h1,0,"qua")
23 opt = gamopt("x",X,"shf",1)
24 {q,c,constt} = backfit(X,EBR,h2,0,"qua")
25 opt = gamopt("x",X,"shf",1)
26 {n,b,const} = backfit(X,EBR,h1,1,"qua")
27 opt = gamopt("x",X,"shf",1)
28 {p,b,const} = backfit(X,EBR,h2,1,"qua")
29
30 pic = createdisplay(2,2)
31 d1 = sort(X[,2]~m[,2],1)
32 d2 = sort(X[,2]~q[,2],1)
33 d3 = sort(X[,2]~n[,2],1)
34 d4 = sort(X[,2]~p[,2],1)
35 d1=setmask(d1,"line","thin","blue")
36 d2=setmask(d2,"line","thin","blue")
37 d3=setmask(d3,"line","thin","blue")
38 d4=setmask(d4,"line","thin","blue")
39 show(pic,1,1,d1,X[,2]~EBR)
40 show(pic,1,2,d2,X[,2]~EBR)
41 show(pic,2,1,d3,X[,2]~EBR)
42 show(pic,2,2,d4,X[,2]~EBR)
```

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# Declaration of Authorship

I hereby confirm that I have authored this master thesis independently and without use of other than the indicated resources. All passages, which are literally or in general taken out of publications or other resources, are marked as such.

Chen Xu  
Berlin, January 5th, 2007