Essays in Contract Theory and Industrial Organization

DISSERTATION

zur Erlangung des akademischen Grades
doctor rerum politicarum
(Doktor der Wirtschaftswissenschaft)

eingereicht an der
Wirtschaftswissenschaftlichen Fakultät
der Humboldt-Universität zu Berlin

von
Andreas Frederik Asseyer, M.Sc.

Präsidentin der Humboldt-Universität zu Berlin:
Prof. Dr.-Ing. Dr. Sabine Kunst

Dekan der Wirtschaftswissenschaftlichen Fakultät:
Prof. Dr. Christian D. Schade

Gutachter:
1. Prof. Dr. Roland Strausz
2. Prof. Dr. Helmut Bester

Eingereicht am: 20.10.2016
Tag des Kolloquiums: 12.01.2017

Meinen Eltern Flora und Hans

Acknowledgements

I am very grateful to my supervisor Roland Strausz for the discussions and many helpful suggestions that guided my research. I also would like to express my sincere gratitude to Helmut Bester for many important comments that I often only fully appreciated some time after they were made. Moreover, I am very thankful to Ulrich Kamecke, Anja Schöttner and Sebastian Schweighofer-Kodritsch for agreeing to serve on my dissertation committee.

I am grateful for valuable feedback from Yves Breitmoser, Jana Friedrichsen, Matthias Lang, Vincent Meisner, Martin Pollrich, Shaofang Qi and many other participants of the Mikroökonomisches Kolloqium at Humboldt– and Freie Universität. Moreover, I would like to thank Philipp Dimakopoulos, Thomas Schacherer, and Jonas von Wangenheim for the very enjoyable time in our joint office. For all their administrative support, I am very thankful to Regine Hallmann, Viviana Lalli, Kristin Schwier, Myrna Selling, and Sandra Uzman .

I would like to thank Philipp Heller, Johannes Johnen, Sebastian Kohls, and Sebastian Schmitz for many inspiring conversations.

I am deeply grateful to Susanna for always supporting me and for being patient and impatient with me at the right points in time.

I thank my mother Flora Asseyer and my late father Hans Asseyer for all they have done for me. This dissertation is dedicated to them.

Contents

List of Figures

Introduction

This dissertation consists of three independent chapters in the fields of Contract Theory and Industrial Organization. The starting point of all three chapters is the observation that information is often asymmetrically distributed in contractual relationships. In many instances, one party to a contract knows more than the contractual partner about relevant parameters of the common environment. This asymmetry facilitates opportunistic behavior of the more knowledgeable party that threatens the efficiency of the contractual interaction and can reduce the benefit of the less knowledgeable party.

Chapters 1 and 2 are concerned with topics in Contract Theory. In these chapters, I study the form of information asymmetry that arises in contractual relationships where the less knowledgeable party can reduce its informational disadvantage vis-à-vis the contractual partner. In Chapter 1, I analyze the opportunity of the less knowledgeable party to consult a third party – an expert or supervisor – who can provide advice. In Chapter 2, the less knowledgeable party can itself engage in monitoring activities to gather additional information.

In Chapter 3, I explore the effect of information asymmetry within firms on discriminatory pricing in intermediate good markets. In particular, I study price discrimination and the associated welfare effects in an intermediate good market where a monopolistic upstream firms sells an input to downstream firms that vary in their exposure to the problem of asymmetric information due to different degrees of vertical integration. This chapter therefore contributes to a classical question in the literature on Industrial Organization.

Chapter 1 is concerned with contractual relationships where a party seeks advice from a supervisor but fears collusion between its contractual partner and the supervisor. In this context, I analyze what kind of information or data the party that seeks advice wants to give to the supervisor.

As an example of such a situation, one can imagine a city council that considers the construction of a new airport. Before offering a contract to a construction company, the city council can consult an expert who can provide additional information about the company's costs of building the airport. The city council fears that the expert could collude with the construction company and misrepresent her information in order to induce the city council to make a generous offer to the construction company. The city council decides what data to give to the expert. For instance, the city council can control whether the expert can inspect the blueprints for the planned airport, or whether she receives access to data about the past performances of the construction company. How should the city council optimally exert its informational control under the threat of collusion between the expert and the construction company?

An uninformed party with informational control – such as the city council in the example – faces a trade-off between information elicitation and collusion prevention. The more data it makes accessible to the supervisor, the better the advice that the supervisor could provide. However, a better informed supervisor can also organize collusion with the informed party more effectively.

As a result of this trade-off, I show that the uninformed party optimally withholds data from the supervisor. A partially informed supervisor can provide advice to the uninformed party. At the same time, there remains an information asymmetry between the supervisor and the informed party that complicates their efforts to find a collusive agreement against the uninformed party.

Furthermore, I show that if the uninformed party exerts informational control optimally, it can authorize the supervisor to contract with the informed party on its behalf without being harmed by the loss of control. Thus, delegation of contracting is optimal if informational control is exerted in the best way. Informational control is therefore a substitute to direct control.

Chapter 2 provides an analysis of dynamic contractual relationships where an uninformed party can engage in monitoring to gather additional information known to the contractual partner. I compare monitoring strategies that focus on actions of the informed party with monitoring strategies that concentrate on information that the informed party learns during the contractual relationship.

Revisiting the previous example, the analysis of Chapter 2 speaks to a situation where a city council considers to entrust a private company with the construction of an airport. The construction company can invest in the quality of the blueprint to lower its costs. The construction costs are also influenced by the ground conditions that the construction company only observes after it made the blueprint. The city council can monitor the investment in the quality of the blueprint or the ground conditions at a monitoring cost.

First, I show that the uninformed party never wants to monitor both the actions of the informed party and the new information that the informed party learns during the contractual relationship. If the city council observes the quality of the blueprint, it can shift all risk related to ground conditions to the construction company. Thus, the city council cannot gain from monitoring the ground conditions in addition to the quality of the blueprints.

Starting from this observation, I show that monitoring of actions always gives the uninformed party more control over the informed party than monitoring of new information. However, the two monitoring strategies turn out to be equivalent, if the actions taken by the informed party are close to efficient. In this case, the uninformed party simply chooses the monitoring strategy that leads to lower monitoring costs.

In Chapter 3, I analyze the welfare effects of price discrimination in intermediate good markets where a monopolistic upstream firm sells an input to downstream firms with different degrees of vertical integration. Vertically integrated downstream firms produce the final good themselves whereas vertically separated firms delegate the production of the final good to a subcontractor. The subcontractor has superior information about its costs to transform the intermediate into a final good. This information asymmetry results in an *agency cost* for the vertically separated downstream firm that is not present for vertically integrated downstream firms.

I show that – due to this asymmetry – the upstream firm wants to offer downstream firms different prices depending on their organizational form. I analyze the welfare effects of this form of price discrimination. It turns out that price discrimination is beneficial for total welfare under mild conditions on demand and cost functions.

I compare this result to the welfare effects of price discrimination in a model where all downstream firms are vertically integrated but have different production technologies. I show that the welfare gain of price discrimination is always larger in the model with asymmetric agency costs. Moreover, it turns out that price discrimination reduces welfare in the model with asymmetric production technologies under the same condition that ensures price discrimination to be beneficial for welfare in the model with asymmetric agency costs. Thus, the source of cost differences determines whether price discrimination in intermediate good markets is desirable from a welfare perspective.

Chapter 1

Informational Control and Collusive Supervision

This chapter is based on Asseyer (2016a).

1.1 Introduction

Collusion is a central concern in contractual relationships where a supervisor provides advice to a party that faces a trading partner with superior information. The manager of a division may overstate the difficulty of a project to the head of the organization in order to increase the wages of his subordinates. The board of directors of a public company may tolerate opportunistic behavior of the CEO instead of defending the interests of shareholders. Corrupted public procurement officers may pay private suppliers overly high prices to the detriment of the taxpayer.[1]

In many instances, the party who seeks advice can influence what the supervisor knows about its trading partner. The head of an organization decides whether to inform a manager about the education of her subordinates. Shareholders choose the number and expertise of outside directors on their board. A government agency determines how much data a public procurement officer receives about past performances of private suppliers.

In this chapter, I explore how the uninformed party should exert informational control under the threat of collusion between the supervisor and the trading partner. Furthermore, I analyze how optimal informational control influences the organizational form of the contractual relationship.

[1] Bertrand and Mullainathan (2003) provide empirical evidence for collusion between managers and workers. Hallock (1997) and Fracassi and Tate (2012) document friendly boards and show that they harm shareholders. Di Tella and Schargrodsky (2003) provide evidence for public procurement fraud.

I study a principal-supervisor-agent model where the agent can realize a project for the principal at a privately known cost, the supervisor observes a signal about the agent's costs, and the supervisor is protected by limited liability. I add collusion between the supervisor and the agent and informational control for the principal to the model. Following the literature on mechanism design with collusion, the supervisor and the agent can write an enforceable side-contract that specifies side-payments and coordinates their behavior under the contract proposed by the principal. In the spirit of the literature on Bayesian persuasion, the principal exerts informational control by freely choosing an information structure that generates the supervisor's signal.

I analyze the principal's optimal combination of an information structure and a contract. I show that the principal wants to inform the supervisor only partially about the agent.

The optimal information structure is shaped by a trade-off between information elicitation and collusion prevention. If the supervisor receives additional information about the agent, the informational advantage of the agent over the supervisor decreases. This is beneficial for the principal as long as the supervisor shares her information truthfully. However, it also reduces information asymmetry in the colluding coalition and therefore enables the supervisor and the agent to collude more efficiently. Due to this trade-off, the principal finds it optimal to withhold information from the supervisor.

Furthermore, I study the implications of informational control for the organizational form of the contractual relationship. I show that hierarchical delegation is an optimal organizational form if the principal exerts informational control optimally. Under the optimal information structure, the principal can authorize the supervisor to contract with the agent and achieve the same payoff as under the optimal centralized contract. This implies that informational control is a substitute for direct control.

Under delegation, the principal cannot directly influence the payoff of the agent. This is possible under centralized contracting and offers the principal an additional instrument to fight collusion. If the principal increases the agent's rent in the non-cooperative equilibrium of the contract, it becomes harder for the supervisor to find a profitable side-contract as she needs to compensate the agent. However, the principal needs to finance the rent payments to the agent which may make this strategy expensive.

I show that there exists an optimal combination of an information structure and a contract where the agent receives no rent. This implies that the principal does not make use of the additional instrument under the optimal information structure.

Thus, the principal does not benefit from direct control over the agent if it exerts informational control over the supervisor.

These results have two important implications for contractual relationships. First, even if data are abundant, information asymmetries can persist as long as the incentives of supervisors cannot be perfectly aligned with the interests of the party that seeks advice. Second, if technological developments facilitate the use of informational control, organizations are more likely to become decentralized and markets are more likely to be intermediated by experts.

In the next section, I discuss the related literature. Section 1.3 provides a simple example that illustrates the main results. In section 1.4, I introduce the model. In section 1.5, I characterize and analyze the optimal combination of an information structure and a contract. Section 1.6 presents a delegation game that implements the optimal centralized contract under the optimal information structure. Section 1.7 concludes the chapter.

1.2 Related Literature

This chapter builds on and contributes to two strands of literature. The first analyzes the implications of collusion under asymmetric information for mechanism design. The second studies optimal information design in games.

I model collusion as an enforceable side-contract between parties with soft asymmetric information. This approach is due to Laffont and Martimort (1997).[2] In Laffont and Martimort (2000), the authors study the provision of a public good under collusion. In a model with two agents, they show that collusion imposes no loss on the principal if the private information of agents is uncorrelated but reduces the principal's payoff if types are correlated. Che and Kim (2006) study a setup that encompasses the two player-two type models studied by Laffont and Martimort. They confirm their first result but demonstrate that collusion does also not harm the principal with correlated types if there are at least three agents with unlimited liability.[3] In the present chapter, the principal is harmed by collusion due to the limited liability of the supervisor.

More specifically, the present chapter belongs to the literature on collusive supervision with adverse selection.[4] Closely related are the works by Faure-Grimaud

[2]Collusion with verifiable information is studied in the seminal papers by Green and Laffont (1979) and Tirole (1986). The papers by Crémer (1996), McAfee and McMillan (1992), and Caillaud and Jehiel (1998) study collusion under asymmetric information in specific mechanisms.

[3]Che and Kim's result also requires risk-neutrality.

[4]Baliga and Sjöström (1998) and Laffont and Martimort (1998) study collusion and the optimality of delegation in a model with two productive agents and moral hazard (Baliga and Sjöström,

et al. (2003) and Celik (2009). Both papers study collusion in a model with a principal, an agent, and a supervisor who has imperfect information about the agent's type. Faure-Grimaud et al. (2003) analyze the optimal mechanism and show that it can be implemented through delegation. They derive this result in a model where the agent has two types and the supervisor observes a binary signal. In Celik (2009), the agent has more than two types and the supervisor's information is modelled as a partition of the agent's type space. In contrast to the result of Faure-Grimaud et al. (2003), Celik demonstrates that delegation is suboptimal. In the present chapter, the information structure is endogenously chosen by the principal. I show that the principal optimally selects an information structure under which delegation is optimal.

Mookherjee and Tsumagari (2004) study the optimal design of supplier networks and consider the possibility of supervision. Mookherjee et al. (2015) analyze optimal mechanisms in an environment where a supervisor and a producing agent can collude against a principal. In contrast to this chapter, they study the situation where the colluding coalition can enter a side-contract before accepting the mechanism offered by the principal. The supervisor and the agent can therefore include the participation decision in the side-contract.[5]

This chapter is also related to the literature on the optimal design of information and Bayesian persuasion. As in Kamenica and Gentzkow (2011), the principal can choose an information structure freely from the set of all information structures that satisfy a Bayesian consistency requirement, and the supervisor can evaluate any information structure at no cost. Bergemann et al. (2015) analyze the implications of a seller's information on a buyer's valuation for the possible distributions of profit for the seller and buyer surplus. In the delegated contracting game studied in section 1.6 of this chapter, the supervisor and the agent are in a similar situation as the seller and the buyer in Bergemann et al. (2015). However, there is a third party – the principal – who seeks to extract surplus from the supervisor and the agent without observing their information.

Bergemann and Pesendorfer (2007) study the joint optimal design of information structure and auction format when the seller can disclose information to bidders. As in Bergemann and Pesendorfer (2007), the current chapter analyzes static disclosure of information which occurs before the agents make their participation decision.[6]

1998) or adverse selection (Laffont and Martimort, 1998).

[5]Further papers that study this form of collusion are Dequiedt (2007), Pavlov (2008), Che and Kim (2009), and Che et al. (2014).

[6] In this respect, the current chapter differs from Eső and Szentes (2007a,b) and Li and Shi (2015), who consider sequential information disclosure, where agents first decide whether to par-

Finally, this chapter is connected to Ortner and Chassang (2015) and Ivanov (2010). Ortner and Chassang (2015) analyze a principal-monitor-agent model and show that corruption can be fought by introducing asymmetric information in the colluding coalition through the use of random transfers. In contrast to the present chapter, it is therefore the terms of the contract and not the type of the agent over which there is asymmetric information. This implies that the principal does not face a trade-off between information elicitation and collusion prevention in their setting.

Ivanov (2010) studies informational control and delegation in the model of Crawford and Sobel (1982). The uninformed party can design the signal of the informed party and decides whether to delegate decision making to the informed party. Ivanov shows that informational control and direct control – the uninformed party's decision to keep decision rights – can be substitutes or complements. This contrasts with the analysis under commitment in the present chapter where informational control and direct control are shown to be substitutes.

1.3 An illustrative example

In this section, I illustrate the main results of this chapter in a simple example.

The government agency P considers the construction of a new airport. The construction company A can build the airport for P. P does not know A's costs to realize the project. What it knows is that these costs depend on potential problems that A encounters during construction and on A's specialization. Suppose for simplicity that A encounters exactly one of two problems – problem a or b – and is either specialized in solving one type of the problem or is an all-rounder. If the encountered problem is A's speciality, his costs are low. If A encounters one type of the problem but is specialized in the other, costs are high. If A is an all-rounder, costs are intermediate independently of the type of the problem. A knows both the type of the problem and his specialization. From the perspective of P, all types of the problem and all specializations are equally likely and the type of the problem is independent from A's specialization. Figure 1.1 summarizes this description and specifies numbers for low, intermediate, and high costs.

P's gross benefit of the airport is 4. If P bargains directly with A, P's optimal price offer to A is the solution to the following monopsony problem:

$$\max_p \Pr(costs \leq p)(4 - p).$$

ticipate and then receive information, or from Bergemann and Wambach (2015) where agents receive new information sequentially.

Figure 1.1: The determinants of A's costs

	problem a: $^1/_2$	problem b: $^1/_2$
specialist a: $^1/_3$	1	3
specialist b: $^1/_3$	3	1
all-rounder: $^1/_3$	2	2

The table shows A's costs of realizing the project depending on the type of the problem and A's specialization. A encounters either problem a or problem b. Both problems are equally likely to occur. Each specialization is equally likely. The type of the problem is independent from A's specialization.

As low, intermediate, and high costs are all equally likely from P's perspective, the price $p = 2$ is optimal and gives P an expected payoff of $\frac{4}{3}$.

Instead of immediately offering a price to A, P can hire a consultant S. S can evaluate the type of the problem and the specialization of A at no cost. P can control whether S evaluates the problem or the specialization by granting her access to the blueprints of the planned airport or to data about past construction projects of A. However, P relies on truthful reports of S about the results of her evaluations. S owns assets of value 1 and can never be held liable for more than this value. S and A value the outside option of not participating in the construction project by zero.

If P can exclude the possibility of collusion between S and A, it optimally offers S a fixed fee equal to the value of her outside option and grants her access to all data about the type of the problem and A's specialization. Through the fixed fee, S becomes a disinterested party and shares all her knowledge with P. P then optimally offers A a price equal to his costs and makes an expected payoff of $4 - \mathbb{E}[costs] = 2$.

If S and A can collude, this contract is prone to manipulation. In exchange for a side-payment, S could promise A to report to P that costs are high independently of her information. A would be willing to make any side-payment up to the difference between his costs and the price that the firm pays. As S knows A's costs, she knows the maximal side-payment that A is willing to pay. Thus, S and A can find an efficient collusive agreement and behave as a single player. This is the case for any possible contract that P could propose. P is therefore back in the monopsony problem. It optimally offers a price $p = 2$ to A and receives an expected payoff of $\frac{4}{3}$.

If S is completely uninformed about A's costs, she cannot provide any helpful advice to P. If S is perfectly informed, she can efficiently organize collusion with A. This again renders her advice useless to P. These extreme cases illustrate that P faces a trade-off between information elicitation and collusion prevention. If S receives additional information about A's costs, this has two effects. On one hand, it can improve S's advice and reduce the informational advantage of A over P. On the other hand, it facilitates collusion as the information asymmetry within the colluding coalition is reduced. P needs to balance these two effects when it decides what data to provide to S.

Can P increase its payoff by informing S only partially about the costs of A? Suppose P allows S to learn only A's specialization. If A is an all-rounder, S knows that the costs are intermediate. If A is specialized, S knows that A has low or high costs with equal probability.

Furthermore, suppose that P commits to the following contract vis-à-vis S and A. Initially, S is required to post a *bond* of 1—the value of her assets. If S reports to P that A is specialized, P offers a price $p = 1$ to A. If S reports that A is an all-rounder, P offers a price $p = 2$. If A accepts the offered price, S can dissolve the bond. If A rejects the price, P liquidates the bond. If A accepts the low price offer of 1, S receives a *bonus* of 1.

If S shares her knowledge with P, S and A always make an expected profit of zero under this contract. The price offer to A never exceeds his costs. If A is an all-rounder, S knows that her bond is never liquidated and that she never receives the bonus. She therefore receives a payoff of zero. If A is specialized, S has an expected payoff of

$$\Pr(costs = 1|specialized) \times 1 + \Pr(costs = 3|specialized) \times (-1) = 0.$$

If A accepts the low price offer, S receives the bonus of 1. If A rejects the offer, S looses her assets of value 1. As both events are equally likely if A is specialized, S has an expected profit of zero.

If A is specialized, S has nothing to gain from a collusive agreement under which she promises A to tell P that he is an all-rounder. If A has low costs, he would pay a maximal side-payment of 1 for this promise. However, S would loose her bonus of 1 in this case. If A is an all-rounder, S has also no interest to induce P to offer a lower price by reporting that A is specialized. A would reject the lower price and S would always loose her assets.

S does also not benefit from a collusive agreement where A promises to reject a price offer above his costs, as S would loose her assets in this case. If A promises to accept a price below his costs, S needs to use her assets or the bonus to compensate A for the difference between price and costs.

The contract is therefore not prone to collusion and gives S the right incentives to report A's specialization truthfully to P. Under the contract, S and A receive a total payment of 2 if the airport is completed, and make a payment of 1 to P if the airport is not build. These payments are independent of S's report about A's specialization. This makes the contract robust against collusion.

Through the contract, P receives an expected payoff of

$$\Pr(costs = 3) \times 1 + \Pr(costs < 3) \times (4 - 2) = \frac{5}{3}.$$

This payoff is strictly greater than the expected payoff of $\frac{4}{3}$ that P receives if S is either uninformed or perfectly informed about the costs of A.[7] Thus, it is optimal for P to withhold information from S.

In the contract described above, P communicates with S and makes a price offer to A. P can implement this contract through *delegation*. P could offer S a delegation contract that requires S to post a bond of 1 and specifies a price of 2 to be paid to S if the airport is completed. P can liquidate the bond if the airport is not build.[8] If S accepts this delegation contract, she can offer a price to A for the construction of the airport.

It is optimal for S to offer a price $p = 1$ if she has observed that A is specialized and to set a price $p = 2$ if A is an all-rounder. It is easy to check that S's expected profit is zero in both cases. P receives the maximal expected payoff of $\frac{5}{3}$. Note that the delegation contract does not require any communication between P and S. Given that P controls S's information, it can delegate direct control over A to S without loosing money.

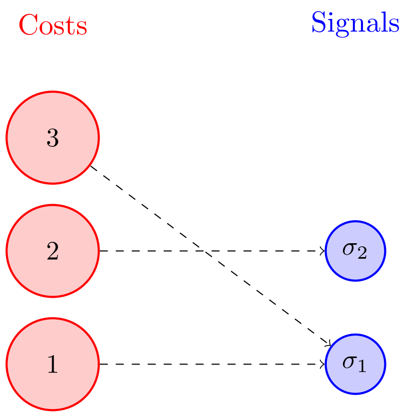
If S evaluates A's specialization, her learning process can be represented as an *information structure* as depicted in Figure 1.2.

The information structure generates a signal σ_1 if the costs of A are either high or low. It generates a signal σ_2 if the costs are intermediate. The signal σ_1 leads to the same posterior belief over the costs as if S learns that A is specialized. The signal σ_2 is equivalent to learning that A is an all-rounder. In the analysis of the general model, informational control is modelled as the possibility to choose an arbitrary information structure. This reduced-form approach allows to flexibly

[7]If S evaluates only the type of the problem, P's maximal expected payoff is also $\frac{4}{3}$. If the type of the problem is known, all cost levels of A are equally likely—as in the case where the type of the problem is unknown. Thus, a price of $p = 2$ remains optimal independently of the type of the problem.

[8]This arrangement corresponds to performance bonds widely used in procurement practice.

Figure 1.2: Information structure equivalent to learning A's specialization



This information structure is equivalent to learning A's specialization. There are two signals. The signal σ_1 is generated if A's costs are either 1 or 3. Observing this signal is equivalent to observing that S is specialized in one type of the problem. The signal σ_2 is generated if costs are 2 and is equivalent to observing that A is an all-rounder.

model any possible learning process of S.

As it will be shown in section 1.5, the information structure depicted above is the optimal way to inform S among all possible information structures. Under this information structure, P can avoid to pay rents to A. Importantly, it can also avoid rent payments to S. This follows from the fact that the information structure features two signals that have the same value for S. Under the first signal σ_1, S can receive a bonus but may also loose her assets. Under the second signal σ_2, S is certain neither to receive a bonus nor to loose her assets. I show in section 1.5 that this is a general feature of the optimal information structure. The better the good state after a certain signal, the higher the risk that the project is not realized in which case S looses money. The optimal information balances the positive and the negative content of signals to create signals that are equivalent to S. This allows P to reduce rent payments to S who can never claim to have received *bad news*.

1.4 The model

There are three players: the principal P ("it"), the supervisor S ("she"), and the agent A ("he"). P seeks the realization of a project. It values the completion of the project by $v \in \mathbb{R}$. Only A can realize the project for P. If A realizes the project, he incurs a cost of θ. A is privately informed about θ which belongs to the interval $\Theta = [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$. The cost θ is the realization of a random variable $\tilde{\theta}$ with cumulative distribution function (cdf) $F(\theta) = \Pr(\tilde{\theta} \leq \theta)$. The realization of the project is socially beneficial with a positive probability, i.e., $\Pr(\tilde{\theta} < v) > 0$. S can observe and interpret a signal σ at no cost. The signal is the realization of a random variable $\tilde{\sigma}$ that may be informative about A's cost. S is protected by limited liability, i.e., she can never incur losses that exceed the maximal loss $\ell \in [0, \infty)$. The signal σ is observed by S and A but not by P. P decides the informativeness of the signal by choosing an information structure.

Information structures

P has informational control and decides how S is informed about A's cost of project realization. Following the literature on Bayesian persuasion, I model informational control as the possibility to freely choose an information structure. An information structure is given by $I = (\Sigma, \mu)$ where Σ is a set of signals with generic element $\sigma \in \Sigma$ and $\mu \in \Delta(\Sigma \times \Theta)$ is a probability measure on the set of possible realizations of A's cost and S's signal. μ induces conditional cdfs of θ on σ and a marginal cdf of σ given by $G(\theta|\sigma) = \Pr(\tilde{\theta} \leq \theta|\sigma)$ and $H(\sigma) = \Pr(\tilde{\sigma} \leq \sigma)$. These have to be consistent with the unconditional distribution $F(\theta)$ and satisfy

$$\int_{\Sigma} G(\theta|\sigma) dH(\sigma) = F(\theta).$$

Let $\mathcal{I}$ be the set of all information structures. Given some information structure I, $Supp(\mu) \subset \Sigma \times \Theta$ denotes the support of the random variable $(\tilde{\sigma}, \tilde{\theta})$.

Allocations

An allocation describes whether the project is realized and what transfers are paid from P to S and A. An allocation is given by $(x, t_S, t_A) \in \{0, 1\} \times \mathbb{R}^2$. Project realization is denoted by $x = 1$. The transfer from P to i is given by t_i with $i \in \{A, S\}$. The allocation (x, t_S, t_A) leads to payoffs of $t_A - \theta x$ for A, t_S for S, and $vx - t_A - t_S$ for P. S and A value their outside options at zero.

Collusion

Following the literature on collusion in mechanism design, collusion is modelled as an enforceable side-contract between S and A that coordinates their communication with P and specifies side-transfers. Thus, P cannot observe any exchange of transfers or communication between the agents. In contrast, the realization of the project is observable and contractible. Starting from an allocation (x, t_A, t_S), S and A can modify the allocation to $(x, t_A + \tau, t_S - \tau)$. Furthermore, S and A can influence x, t_A, and t_S through their communication with P. If the signal σ is not perfectly informative about the cost θ, the side-contract needs to incentivize A to report his costs truthfully. I assume that S proposes the side-contract to A in a take-it-or-leave-it offer.

Contracts and side-contracts

I now describe the contract which P offers to S and A, and the side-contract which S can offer to A. Without loss of generality, P offers a contract that takes the form of a direct mechanism

$$\beta = \Big(x(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}), t_S(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}), t_A(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta})\Big),$$

which assigns an allocation to any profile of reports $(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta})$ where $\hat{\sigma}_i$ is the report of $i \in \{S, A\}$ about $\sigma \in \Sigma$, and $\hat{\theta}$ is the report from A about $\theta \in \Theta$.

Similarly, there is no loss of generality in focussing on direct side-contracts which S offers to A. Such side-contracts are given by

$$\gamma = \Big(\rho(\breve{\theta}; \sigma), \tau(\breve{\theta}; \sigma)\Big),$$

where $\rho : \Theta \times \Sigma \to \Sigma^2 \times \Theta$ is a reporting strategy to the mechanism β, and $\breve{\theta} \in \Theta$ is a report from A about the cost $\theta \in \Theta$. For a signal $\sigma \in \Sigma$ and a report $\breve{\theta} \in \Theta$ from A, the side-contract induces the allocation

$$\Big(x(\rho(\breve{\theta}; \sigma)), t_S(\rho(\breve{\theta}; \sigma)) - \tau(\breve{\theta}; \sigma), t_A(\rho(\breve{\theta}; \sigma)) + \tau(\breve{\theta}; \sigma)\Big).$$

The *null side-contract* is the side-contract that does not change the allocation and reports the information about σ and θ truthfully. Thus, the null side-contract satisfies $\rho(\breve{\theta}; \sigma) = (\sigma, \sigma, \breve{\theta})$ and $\tau(\breve{\theta}; \sigma) = 0$ for all $\sigma \in \Sigma$ and $\breve{\theta} \in \Theta$.

Timing and equilibrium concept

The timing of the game is as follows.

t=0: P chooses an information structure $I \in \mathcal{I}$ and offers a mechanism β to S and A.

t=1: S and A observe I, β, and the realization of the signal σ. A furthermore observes θ.

t=2: S and A each accept or reject P's offer. If either of them rejects, both agents receive their outside option. Otherwise the game continues.

t=3: S offers a side-contract γ to A.

t=4: A accepts or rejects S's offer. If A accepts, the side-contract and the mechanism are executed. If A rejects, both agents play the mechanism non-cooperatively.

I focus on *perfect Bayesian equilibria* (PBE) with *passive beliefs*. In these equilibria, S does not update her belief about θ if A rejects the side-contract off the equilibrium path. This approach follows Laffont and Martimort (1997) and the concept of *weak collusion-proofness* in Laffont and Martimort (2000).

1.5 Optimal contract with informational control

In this section, I analyze how P optimally designs an information structure and a contract under the threat of collusion. First, I state P's problem formally and present the benchmarks without supervision and without collusion as natural lower and upper bounds on P's expected profit. I then provide a solution to P's problem and analyze its properties.

P's problem

P optimally chooses an information structure I and a direct mechanism β in order to maximize her expected payoff under the constraints that S and A want to participate in the mechanism, that S never incurs a loss greater than ℓ, that S and A report their private information truthfully to the mechanism, and that there does not exist a feasible side-contract which gives S a strictly higher payoff than to participate non-cooperatively in the mechanism. Thus, I invoke a collusion-proofness principle: Any payoff that P can achieve in an equilibrium where S and A collude through a non-trivial side-contract, can also be attained in a collusion-proof mechanism. This approach follows Laffont and Martimort (1997). A proof of this statement in the current model can be provided along the lines of Proposition 4 in Laffont and Martimort (1997). Formally P's problem is the following:

$$\max_{I,\beta} \mathbb{E}\left[vx(\tilde{\sigma},\tilde{\sigma},\tilde{\theta}) - t_S(\tilde{\sigma},\tilde{\sigma},\tilde{\theta}) - t_A(\tilde{\sigma},\tilde{\sigma},\tilde{\theta})\right] \quad \text{subject to}$$

$$\mathbb{E}\left[t_S(\sigma,\sigma,\tilde{\theta})|\sigma\right] \geq 0, \tag{PC_S}$$

$$\mathbb{E}\left[t_S(\sigma,\sigma,\tilde{\theta})|\sigma\right] \geq \mathbb{E}\left[t_S(\hat{\sigma}_S,\sigma,\tilde{\theta})|\sigma\right], \tag{IC_S}$$

$$t_A(\sigma,\sigma,\theta) - \theta x(\sigma,\sigma,\theta) \geq 0, \tag{PC_A}$$

$$t_A(\sigma,\sigma,\theta) - \theta x(\sigma,\sigma,\theta) \geq t_A(\sigma,\hat{\sigma}_A,\hat{\theta}) - \theta x(\sigma,\hat{\sigma}_A,\hat{\theta}), \tag{IC_A}$$

$$t_S(\hat{\sigma}_S,\hat{\sigma}_A,\hat{\theta}) \geq -\ell; \tag{LL}$$

for all $(\sigma, \hat{\sigma}_S, \hat{\sigma}_A, \theta, \hat{\theta}) \in \Sigma^3 \times \Theta^2$ and subject to the Collusion-Proofness constraint

$$\mathbb{E}\big[t_S(\sigma,\sigma,\tilde{\theta})|\sigma\big] \geq \max_{\gamma} \mathbb{E}\left[t_S(\rho(\tilde{\theta};\sigma)) - \tau(\tilde{\theta};\sigma)|\sigma\right] \quad \text{subject to} \tag{CP}$$

$$t_A(\rho(\theta;\sigma)) + \tau(\theta;\sigma) - \theta x(\rho(\theta;\sigma)) \geq t_A(\sigma,\sigma,\theta) - \theta x(\sigma,\sigma,\theta), \tag{PC_A^γ}$$

$$t_A(\rho(\theta;\sigma)) + \tau(\theta;\sigma) - \theta x(\rho(\theta;\sigma)) \geq t_A(\rho(\breve{\theta};s)) - \tau(\breve{\theta};s) - \theta x(\breve{\theta};s), \tag{IC_A^γ}$$

$$t_S(\rho(\theta;\sigma)) - \tau(\theta;\sigma) \geq -\ell; \tag{LL^γ}$$

for all $(\sigma, \theta, \breve{\theta}) \in \Sigma^2 \times \Theta$. A mechanism β is called *feasible* if it satisfies the constraints (PC_S), (IC_S), (PC_A), (IC_A), and (CP). A side-contract γ is called feasible if it satisfies the constraints (PC_A^γ) and (IC_A^γ). If an information structure I is exogenously fixed, P's reduced problem of choosing an optimal contract is denoted by $\mathcal{P}_I$.

Benchmarks

Before I turn to study P's problem, it is insightful to examine the following three benchmark cases.

Complete information If θ is publicly observable, P implements the allocation $(x^*(\theta), t_S^*(\theta), t_A^*(\theta))$ where the project is realized whenever P's benefit exceeds A's cost, S receives no positive payment, and A is exactly compensated for his costs:[9]

$$x^*(\theta) = \mathbf{1}_{(\theta \leq v)}(\theta), \ t_S^*(\theta) = 0, \ \text{and} \ t_A^*(\theta) = \theta \cdot \mathbf{1}_{(\theta \leq v)}(\theta).$$

P achieves a payoff of $\max\{v - \theta, 0\}$. Its ex-ante expected payoff is the total expected social surplus

$$\overline{W} \equiv \int_{\underline{\theta}}^{v} (v - \theta) dF(\theta).$$

No supervision If P and A are the only players and no supervisor is available, P's problem is equivalent to that of a monopsonistic buyer. Thus, P optimally offers A a price $p = \theta^{ns}$ that is a solution to the maximization problem

$$\max_{p \in [\underline{\theta}, \overline{\theta}]} (v - p) F(p).$$

P can also achieve the monopsony payoff if S is present. For instance, P could choose an uninformative signal σ and make the mechanism independent of any report of S. Thus, the monopsony payoff is a lower bound on P's expected payoff.

[9] The indicator function $\mathbf{1}_A(x)$ satisfies $\mathbf{1}_A(x) = 1$ if $x \in A$ and $\mathbf{1}_A(x) = 0$ if $x \notin A$.

No collusion If collusion is not possible, the principal's problem is equivalent to problem $\mathcal{P}$ without the collusion-proofness constraint (CP). In this case, P can achieve an expected payoff equal to the maximal social surplus $\overline{W}$. This can be done by choosing a signal which perfectly reveals θ. If P pays S a constant transfer of zero independently of her report, S is willing to share her information with P and the limited liability constraint is satisfied. P can then offer A a price exactly equal to his costs as long as $\theta \leq v$. A cannot do better than to accept this offer and P receives a payoff of $\max\{v - \theta, 0\}$. Its expected payoff is therefore $\overline{W}$.

In the remainder of this section, I characterize an optimal combination of an information structure and a contract. At first, I show that under any collusion-proof contract, the total sum of transfers to S and A depends only on the project realization decision. This simplification allows me to derive an upper bound on P's payoff which is independent of the information structure and the contract. I then present a combination of an information structure and a contract with which P can reach the upper bound.

Simplifying transfers

I start with an observation which considerably simplifies the structure of transfers which P pays to S and A in a collusion-proof mechanism.

Lemma 1.1. *Let I be an information structure and β be a feasible mechanism. It holds that*

$$\begin{aligned} & x(\sigma,\sigma,\theta) = x(\sigma',\sigma',\theta') \\ & \qquad \Rightarrow \quad t_S(\sigma,\sigma,\theta) + t_A(\sigma,\sigma,\theta) = t_S(\sigma',\sigma',\theta') + t_A(\sigma',\sigma',\theta') \end{aligned}$$

for all $(\sigma,\theta),(\sigma',\theta') \in supp(\mu)$.

The lemma implies that for any collusion-proof contract, the total transfer to S and A depends only on whether the project is realized or not. To see why this is the case, suppose there is a mechanism for which the two reporting profiles (σ,σ,θ) and $(\sigma',\sigma',\theta')$ both lead to the realization of the project but the sum of transfers is higher after the report $(\sigma',\sigma',\theta')$. S can then propose a side-contract which reports $(\sigma',\sigma',\theta')$ whenever the true types are (σ,σ,θ) and use the transfer τ to make A indifferent between participation in the side-contract and non-cooperative play of the mechanism. As the total sum of transfers is higher under the report $(\sigma',\sigma',\theta')$, the side-contract is strictly profitable for S. The original contract is therefore not collusion-proof.

It follows directly from Lemma 1.1 that for any feasible mechanism β, there exist two numbers $(T, r) \in \mathbb{R}^2$ such that the sum of transfers to S and A under β can be expressed as a function of the project realization decision:

$$t_S(\sigma, \sigma, \theta) + t_A(\sigma, \sigma, \theta) = T + x(\sigma, \sigma, \theta)r.$$

I call (T, r) the *collective transfers* associated with the mechanism β. As the sum of transfers to the two agents can only depend on whether the project is realized, it can at most take two values T_0 and T_1. T_0 is paid if the project is not realized, and T_1 is paid if the project is realized. Thus there exist T and r such that $T_0 = T$ and $T_1 = T + r$. T can be interpreted as a signing fee paid from P to the agents upon acceptance of the mechanism. r can be interpreted as a bonus that is additionally paid contingent on project realization. P's expected profit from the mechanism β can therefore be written as

$$(v - r)\mathbb{E}[x(\tilde{\sigma}, \tilde{\sigma}, \tilde{\theta})] - T.$$

Upper bound on *P*'s payoff

The results from the previous paragraphs can be used to derive an upper bound on P's payoff for all information structures and mechanisms. This upper bound is based on the social surplus and a lower bound on the joint payoff which A and A can secure through a side-contract.

Any mechanism β induces some ex-ante probability of project realization. This probability is given by $X = \mathbb{E}[x(\tilde{\sigma}, \tilde{\sigma}, \tilde{\theta})]$. The expected social surplus under this mechanism cannot be greater than the social surplus under a feasible mechanism β' where the same ex-ante probability of project realization X is reached with the lowest possible set of types. This means that A only realizes the project if his costs are weakly below the cutoff $\theta^c(X)$ that is defined as

$$\theta^c(X) \equiv \min \{\theta \in \Theta : F(\theta) \geq X\}.$$

The expected social surplus from the mechanism β' is given by

$$\begin{aligned} W^*(X) &\equiv \int_{\underline{\theta}}^{\theta^c(X)} (v - \theta) dF(\theta) - (F(\theta^c(X)) - X)(v - \theta^c(X)) \\ &= X(v - \theta^c(X)) + \int_{\underline{\theta}}^{\theta^c(X)} F(\theta) d\theta, \end{aligned}$$

where the second line follows from integration by parts. Due to the participation constraints of S and A, $W^*(X)$ is an upper bound on P's expected payoff under any mechanism β with an ex-ante probability of production X.

P's expected payoff from the mechanism β is given by

$$X(v - T - r) - (1 - X)T.$$

In the following lemma, I state a result that allows to derive a second upper bound on P's payoff based on this expression.

Lemma 1.2. *For any information structure I and any feasible mechanism β with $X = \mathbb{E}[x(\tilde{\sigma}, \tilde{\sigma}, \tilde{\theta})]$, the associated collective transfers $(T, r) \in \mathbb{R}^2$ satisfy i) $T \geq -\ell$, ii) $r \geq \theta^c(X)$, and iii) $T + r \geq \theta^c(X)$.*

The limited liability of S implies that she can never pay more than ℓ to P. When A decides whether to participate, he is informed about the signal σ and about his costs θ. This implies that A can always avoid negative payoffs by not participating in the mechanism. Point *i*) follows from these two observations.

A realizes the project for the cost level $\theta^c(X)$ after some signal σ^c. S and A need to receive a *bonus* of at least $\theta^c(X)$ to compensate A for the production costs. Otherwise, A either does not want to participate in the mechanism, or S and A can find a profitable side-contract where A does not produce if the costs are $\theta^c(X)$. This implies that point *ii*) needs to be satisfied.

Finally, point *iii*) is implied by the following argument. If the unconditional payment T is positive, point *ii*) implies *iii*). If it is negative, the total payment to S and A still needs to exceed the cutoff type $\theta^c(X)$. Otherwise, S would have a negative expected payoff after the signal σ^c and would not participate in the mechanism.

The lemma implies that P's expected profit from any mechanism with ex-ante probability of project realization of X is bounded from above by

$$W^\circ(X) = X(v - \theta^c(X)) + (1 - X)\ell.$$

This expression reflects that P needs to pay the coalition at least a transfer equal to the cutoff type, whenever the project is realized. If the project is not realized, P can extract at most the maximal liability of S.

P's expected profit is bounded from above by the social surplus $W^*(X)$ and the function $W^\circ(X)$. The following proposition states this result formally.

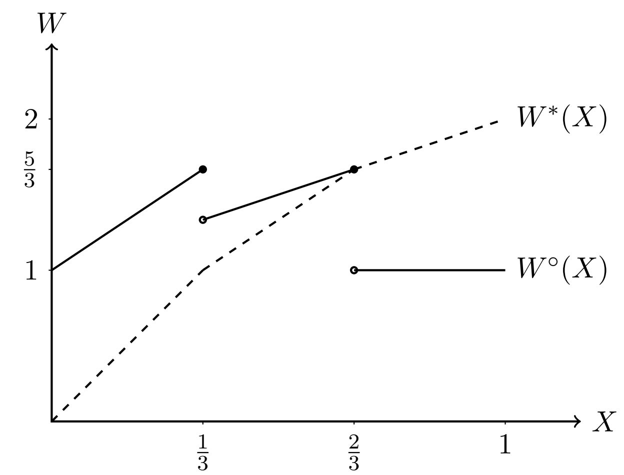
Proposition 1.1. *For any information structure I and any feasible mechanism β, P's expected profit $U_P(I, \beta)$ satisfies*

$$U_P(I, \beta) \leq W(X) \equiv \min\{W^*(X), W^\circ(X)\} \quad \textit{with} \quad X = \mathbb{E}[x(\tilde{\sigma}, \tilde{\sigma}, \tilde{\theta})].$$

The proposition states that there exists an upper bound $W(X)$ on P's expected profit which depends only on the ex-ante probability of project realization and is otherwise independent of the mechanism and the information structure. The minimal upper bound on the function $W(X)$ for any value of the ex-ante probability of project realization – given by $\sup_{X\in[0,1]} W(X)$ – constitutes a global upper bound on P's expected payoff for any information structure I and any feasible mechanism β.

Proposition 1.1 can be used to show that the partially revealing information structure and the contract proposed in the illustrative example are optimal. Figure 1.3 depicts the functions $W^*(X)$ and $W^\circ(X)$ for the illustrative example. The maximum of the minimum of the two upper bounds is attained for $X = \frac{2}{3}$ where $W(\frac{2}{3}) = \frac{5}{3}$. The information structure and the mechanism described in the illustrative example give P an expected payoff of $\frac{5}{3}$ and are therefore optimal.

Figure 1.3: Upper bound on P's expected payoff in illustrative example



The dashed line is the first upper bound $W^*(X)$. The solid line is the second upper bound $W^\circ(X)$. The minimum of the two functions $W(X)$ is maximized at $X = \frac{2}{3}$.

The upper bound $W(X)$ has a well defined maximum, i.e., the maximization problem $\max_{X\in[0,1]} W(X)$ has a solution. In order to characterize the maximizer, denote the intersection point of $W^*(X)$ and $W^\circ(X)$ by $X^\dagger$. The intersection point is defined by[10]

$$X^{\dagger} \equiv \Big\{ X \in [0,1] : \quad W^{\circ}(X') > W^{*}(X') \text{ for } X' < X, \qquad W^{\circ}(X') < W^{*}(X') \text{ for } X' > X \Big\}. \tag{1.1}$$

Furthermore, it is helpful to define the following constrained maximizer of the function $W^{\circ}(X)$

$$X^{\circ}(X) \equiv \max \left\{ \arg \max_{X' \in [X,1]} W^{\circ}(X') \right\}. \tag{1.2}$$

The following Lemma characterizes the ex-ante probability X^c at which the upper bound on P's profit attains its maximum.

Lemma 1.3. *The intersection point $X^{\dagger}$ is uniquely defined by equation (1.1) and $X^{\circ}(X)$ is well defined for all $X \in [0,1]$ by equation (1.2). Moreover, a solution to the optimization problem $\max_{X \in [0,1]} W(X)$ exists and is given by*

$$X^c = \begin{cases} X^{\dagger} & \text{if } \; X^{\dagger} < F(v) \text{ and } W^{*}(X^{\dagger}) > W^{\circ}(X^{\circ}(X^{\dagger})) \\ X^{\circ}(X^{\dagger}) & \text{if } \; X^{\dagger} < F(v) \text{ and } W^{*}(X^{\dagger}) \leq W^{\circ}(X^{\circ}(X^{\dagger})) \\ F(v) & \text{if } \; X^{\dagger} \geq F(v). \end{cases}$$

The intersection point $X^{\dagger}$ is well defined as the difference between the two upper bounds $W^{\circ}(X) - W^{*}(X)$ is decreasing in the ex-ante probability of project realization. The constrained maximizer $X^{\circ}(X)$ is well defined as $W^{\circ}(X)$ is left-continuous with downward jumps. This property is inherited from the cutoff type $\theta^c(X)$ that is increasing, left-continuous, and exhibits upward jumps at some value X if the cdf $F(\cdot)$ is constant and equal to X over a non-degenerate interval in $[0,1]$. As $W^{*}(X)$ is continuous and $W^{\circ}(X)$ is left-continuous, the upper bound $W(X)$ is left-continuous. At the intersection point $X^{\dagger}$, the upper bound $W(X)$ can exhibit a discontinuity if the function $W^{\circ}(X)$ makes a downward jump. In this case, it holds that

$$W^{\circ}(X^{\dagger}) \geq W^{*}(X^{\dagger}) > \lim_{X \searrow X^{\dagger}} W^{\circ}(X).$$

The discontinuity must be a downward jump. Thus, $W(X)$ is left-continuous with only downward jumps and therefore has a well defined maximizer—given by X^c. It follows that $W(X^c)$ is a global upper bound on P's expected payoff.

[10]As the functions $W^{*}(X)$ and $W^{\circ}(X)$ may not necessarily intersect, the intersection point may also describe the point where one function jumps above the other.

P can reach the upper bound

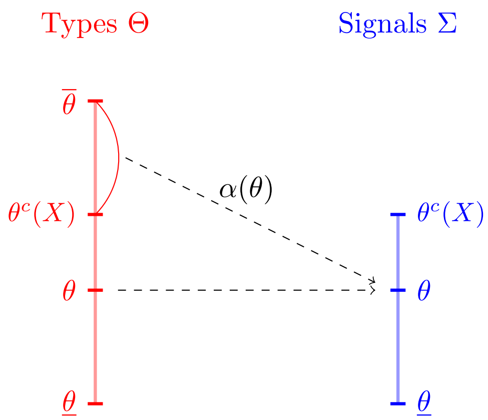
In the following, a solution to P's problem is presented. I construct a combination of an information structure and a mechanism with which P can implement any ex-ante probability of project realization X and achieve an expected payoff equal to the upper bound $W(X)$. Suppose for now that the cdf $F(\cdot)$ has a strictly positive density $f(\cdot)$. This assumption is made to clarify the exposition. All proofs in the appendix apply to the case where $F(\theta)$ may exhibit mass points. The construction presented in this section proves the following statement.

Proposition 1.2. *For any $X \in [0,1]$ there exists an information structure I and a mechanism β such that $\mathbb{E}[x(\tilde{\sigma}, \tilde{\sigma}, \tilde{\theta})] = X$ and $U_P(I, \beta) = W(X)$.*

The key idea behind the construction of the optimal combination of an information structure and a contract is to avoid rent payments to A without creating the possibility for S to earn high rents through collusion.

Consider an information structure as depicted in Figure 1.4. This information structure has a signal space equal to the lowest cost values of A such that the project is realized with ex-ante probability X, i.e. $\Sigma = [\underline{\theta}, \theta^c(X)]$. If A's costs θ lie below the cutoff $\theta^c(X)$, the signal $\sigma = \theta$ is drawn. The remaining mass of non-producing types is distributed over all signals in Σ. In particular, no type in $[\theta^c(X), \overline{\theta}]$ is more likely to generate a certain signal than any other type. The distribution of these types can therefore be described by a weighting function $\alpha : \Sigma \to \mathbb{R}_+$ which is positive and satisfies $\int_\Sigma \alpha(\sigma) d\sigma = 1$.

Figure 1.4: Weighted information structure



A weighted information structure has as many signals as types below the cutoff $\theta^c(X)$. If costs take the value θ below the cutoff, then the signal θ is generated. If costs take a value above the cutoff, a signal is generated according to the density $\alpha(\cdot)$ that depends only on the signal and not on the costs.

A weighting function $\alpha(\cdot)$ induces a cdf over costs conditional on a signal realization σ of

$$G(\theta|\sigma) = \begin{cases} 0 & \text{if} \quad \theta < \sigma \\ \frac{f(\sigma)}{f(\sigma)+(1-X)\alpha(\sigma)} & \text{if} \quad \theta \in [\sigma, \theta^c(X)] \\ \frac{f(\sigma)}{f(\sigma)+(1-X)\alpha(\sigma)} + \frac{(1-X)\alpha(\sigma)}{f(\sigma)+(1-X)\alpha(\sigma)} \frac{F(\theta)-F(\theta^c(X))}{1-F(\theta^c(X))} & \text{if} \quad \theta > \theta^c(X) \end{cases}$$

and a marginal cdf over signals given by

$$H(\sigma) = \int_{\underline{\theta}}^{\sigma} (f(\sigma') + (1-X)\alpha(\sigma'))d\sigma'.$$

I denote an information structure with a characterizing weighting function of $\alpha(\cdot)$ by I_α and refer to it as a *weighted information structure*

Such an information structure can be combined with a mechanism that induces project realization whenever S and A make the same report about the signal and this report coincides with A's report about his type:

$$x(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}) = \mathbf{1}_{(\hat{\sigma}_S = \hat{\sigma}_A = \hat{\theta} \leq \theta^c(X))}(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}). \tag{1.3}$$

The project is therefore realized whenever S and A both report the same signal and A reports to have the lowest possible cost level possible under the signal. Furthermore, the mechanism specifies a transfer to A that just compensates him for his costs in the case that he has the lowest possible cost level:

$$t_A(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}) = \hat{\theta} x(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}). \tag{1.4}$$

As the mechanism is required to be feasible, by Lemma 1.1, the transfer to S can be written as

$$t_S(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}) = T + x(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta})(r - \hat{\theta})$$

for the collective transfers (T, r).

Vis-à-vis A, the mechanism is equivalent to a price offer equal to the signal. Therefore, the mechanism satisfies A's participation constraint (PC_A) and his incentive compatibility constraint (IC_A). Under this mechanism, A never receives a rent.

On the equilibrium path after the signal σ, S receives an expected payoff of

$$U_S(\sigma) = T + \Pr(\theta = \sigma|\sigma)(r - \sigma) = T + \frac{f(\sigma)(r - \sigma)}{f(\sigma) + (1-X)\alpha(\sigma)}.$$

From Lemma 1.2 it is known that $r \geq \theta^c(X)$. Thus, S has nothing to gain from misreporting the signal, as this would reduce her expected payoff to T. Her incentive compatibility constraint (IC_S) is therefore satisfied.

S's participation constraint (PC_S) is satisfied if S has a positive expected payoff after all signal realizations. P wants to set the fix payment T as low as possible. Thus, S's participation constraint has to be binding after the signal for which the expected gain from project realization is smallest:

$$T = -\min_{\Sigma} \left\{ \frac{f(\sigma)(r - \sigma)}{f(\sigma) + (1 - X)\alpha(\sigma)} \right\}.$$

For a given variable payment r, P wants to choose the weighting function $\alpha(\cdot)$ which maximizes S's minimal expected gain from project realization over all signals. Thus, P would like to choose $\alpha(\cdot)$ such that S's expected gain from trade is constant across all signal realizations and extract the expected gain through the fix payment T. If this is possible, P can avoid rent payments to S.

A form of the weighting function which leaves S's expected payoff $U_S(\sigma)$ constant across all signals σ is given by

$$\alpha(\sigma) = \frac{f(\sigma)}{1 - X} \cdot \left(\frac{r - \sigma}{C} - 1 \right).$$

This weighting function gives S an expected gain of $U_S(\sigma) = T + C$ for a given positive constant $C \in \mathbb{R}_+$.

If there exists a constant $C \leq \ell$ such that the weighting function α is well defined, then P can extract the whole expected social surplus for a given ex-ante probability of project realization of X without violating S's limited liability constraint (LL). This turns out to be possible for any ex-ante probability X where the function $W^*(X)$ is the stricter upper bound on P's profit.

Lemma 1.4. *Consider the weighted information structure I_α and the mechanism β that are defined by the equations* (1.3), (1.4), *and*

$$T = -\frac{\int_{\underline{\theta}}^{\theta^c(X)} F(\theta) d\theta}{1 - X}, \quad r = \theta^c(X) - T, \quad \text{and} \quad \alpha(\sigma) = \frac{f(\sigma)}{1 - X} \cdot \frac{\theta^c(X) - \sigma}{r - \theta^c(X)}.$$

If $W^*(X) \leq W^\circ(X)$, I_α is well defined, β is feasible, and $U_P(I_\alpha, \beta) = W(X)$.

If the upper bound $W^\circ(X)$ is more restrictive, P can still find a constant C such that the weighting function $\alpha(\cdot)$ is a well defined. However, the constant has to be greater than the maximal loss ℓ. This is not feasible and implies that P has to leave an information rent to S. It turns out that it is optimal for P to set the weight of all signals between some value $\check{\theta}(X)$ and $\theta^c(X)$ to $\alpha(\cdot) = 0$ and to choose for the remaining signals in $[\underline{\theta}, \check{\theta}(X)]$ a weighting function which makes the expected payoff of S for these signals constant:

Lemma 1.5. *Consider the weighted information structure I_α and the mechanism β that are defined by the equations* (1.3), (1.4), *and*

$$T = -\ell, \quad r = \theta^c(X) + \ell, \quad \textit{and} \quad \alpha(\sigma) = \frac{f(\sigma)}{1-X} \cdot \frac{[\check{\theta}(X) - \sigma]_+}{r - \check{\theta}(X)},$$

where $\check{\theta}(X) \in [\underline{\theta}, \theta^c(X)]$ is uniquely defined by the equation

$$\int_{\underline{\theta}}^{\check{\theta}(X)} F(\sigma) d\sigma = (1 - X)(r - \check{\theta}(X)).$$

If $W^\circ(X) \leq W^*(X)$, I_α is well defined, β is feasible, and $U_P(I_\alpha, \beta) = W(X)$.

Under the information structures and the mechanisms defined in the two lemmata above, S cannot find a strictly profitable collusive agreement with A, i.e., the collusion-proofness constraint (CP) is satisfied. First, note that S has nothing to gain from a side-contract where the project is not realized even if A's costs lie below the cutoff $\theta^c(X)$. In this case, the colluding coalition looses a payoff of $r - \sigma$. Second, S does not gain from a side-contract where S and A misreport the signal and induce project realization if A's costs equal the true signal. In this case, the total expected payment to the coalition is the same with and without collusion and the probability of project realization does not change either. Finally, note that the collective transfers in both Lemmata satisfy that the total payment equals the cutoff type if the project is realized, i.e., $T + r = \theta^c(X)$. This implies that it is not profitable for the colluding coalition to extend project realization to cost levels above the cutoff $\theta^c(X)$ where S and A would jointly incur a loss.

Rents and distortions

It is optimal for P to implement the ex-ante probability of project realization X^c for which it achieves an expected payoff of $\max_{X \in [0,1]} W(X)$. Rent payments to S and A and potential distortions of the project realization decision depend on which of the two upper bounds $W^*(X)$ and $W^\circ(X)$ is more restrictive at the optimal ex-ante probability of project realization X^c.

Proposition 1.3. *If $W^*(X^c) \leq W^\circ(X^c)$, P's problem is solved by the information structure and the mechanism defined in Lemma 1.4 for $X = X^c$. Neither S nor A receives a rent. If $W^\circ(X^c) \leq W^*(X^c)$, P's problem is solved by the information* *structure and the mechanism defined in Lemma 1.5 for $X = X^c$. P leaves a rent to S and A.*

The project realization decision is inefficient if the maximal loss ℓ lies strictly below the value $\overline{\ell}$ that is given by

$$\overline{\ell} \equiv \begin{cases} \frac{\overline{W}}{1-F(v)} & \text{if} \quad v < \overline{\theta}, \\ \infty & \text{if} \quad v \geq \overline{\theta}. \end{cases}$$

If the upper bound $W(X)$ equals the social surplus at the maximizer X^c, P can extract the social surplus and neither S nor A receives a positive rent. If the upper bound $W(X)$ equals the second upper bound $W^{\circ}(X)$ at the maximizer X^c, P cannot extract the complete social surplus. In the solution to P's problem presented in Lemma 1.5, S receives the remaining surplus as a rent. However, this solution is not unique. There exist other optimal combinations of an information structure and a contract for which the remaining surplus is split between S and A.

The project realization decision is distorted if P implements an ex-ante probability of project realization that is smaller than the efficient probability $F(v)$. This is the case unless the supervisor's maximal loss ℓ exceeds the expected maximal social surplus from project realization by a factor of $\frac{1}{1-F(v)}$. If the project has a large value to P, the distortion is most likely to arise. Indeed, if P's value v exceeds the highest possible cost of A, then P can never extract the maximal social surplus.

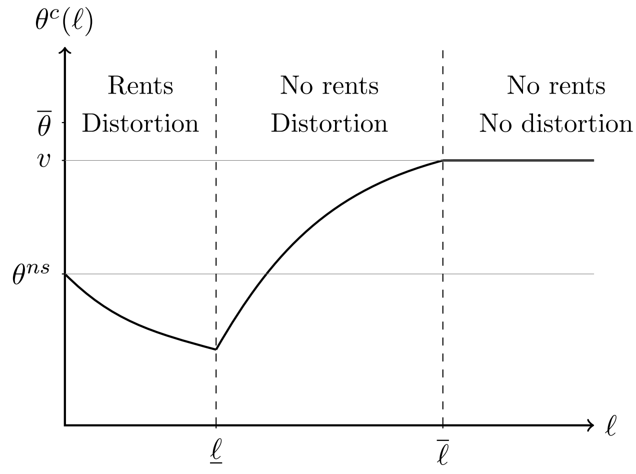
Comparative statics

In this section, I analyze the influence of the maximal loss ℓ on rents, distortions, and the informedness of S. I assume that the cdf $F(\theta)$ is strictly increasing and differentiable on Θ with a decreasing inverse hazard rate $F(\theta)/f(\theta)$. This relatively mild assumption allows to derive clear results on the effect of the maximal loss ℓ on rents, the probability of project realization, and the signal space under the optimal information structure. For a given maximal loss ℓ, let $\theta^c(\ell)$ denote the optimal cutoff value for project realization, the signal space under the optimal information structure I_α is denoted by $\Sigma(\ell)$. I say that $\Sigma(\ell)$ increases (decreases) in ℓ, if $\Sigma(\ell') \subset \Sigma(\ell'')$ for $\ell' < \ell''$ ($\ell' > \ell''$).

Proposition 1.4. *Suppose $F(\theta)$ has a strictly positive density $f(\theta)$ and $F(\theta)/f(\theta)$ is increasing. There exists a value of the maximal liability $\underline{\ell}$ with $\underline{\ell} < \overline{\ell}$ such that*

- *for $\ell \in [0, \underline{\ell}]$, the probability of project realization, total rent payments to S and A, and the signal space $\Sigma(\ell)$ are decreasing.*

Figure 1.5: Optimal cutoff for project realization as function of maximal liability



The cutoff for project realization depending on the maximal liability ℓ is $\theta^c(\ell) = F^{-1}(X^c(\ell))$. For $\ell = 0$, it equals the monopsony cutoff. For $\ell < \underline{\ell}$, the cutoff is decreasing, P has to pay rents. For $\ell > \underline{\ell}$, neither S nor A receives a rent. For $\ell \in (\underline{\ell}, \overline{\ell})$, the cutoff is increasing. For $\ell \geq \overline{\ell}$, the cutoff is efficient. If there are certain gains from trade, i.e. $\overline{\theta} \leq v$, efficiency is never attained as $\overline{\ell} = \infty$.

- *for $\ell \in (\underline{\ell}, \overline{\ell})$, the probability of project realization and the signal space $\Sigma(\ell)$ are increasing. Neither S nor A receives a rent.*
- *for $\ell \geq \overline{\ell}$, the project realization decision is efficient, the signal space $\Sigma(\ell)$ is constant, and neither S nor A receives a rent.*

The probability of project realization is not monotone in the maximal loss ℓ. Figure 1.5 depicts how the cutoff value of project realization changes with the maximal loss ℓ. For low levels of ℓ, an increase in the maximal loss reduces the probability with which the project is realized. For high levels of ℓ, the probability of project realization increases with ℓ.

For low levels of ℓ, P uses an increase in ℓ to reduce rent payments to S. This is optimally done by *deleting* the worst signals and extracting a higher fixed payment (equal to ℓ) from better signal realizations. If ℓ reaches $\underline{\ell}$, P can avoid rent payments to S. Any increase of ℓ above $\underline{\ell}$ is therefore used by P to increase the probability of project realization and to extract the additional social surplus.

The comparative statics with respect to the information structure shows that S does not necessarily become better informed if she can incur a higher loss. For low levels of ℓ, her informedness can decline as ℓ increases. Under the optimal weighted information structure I_α, the signal space is given by $\Sigma(\ell) = [\underline{\theta}, \theta^c(\ell)]$. The size of $\Sigma(\ell)$ therefore changes one-to-one with $\theta^c(\ell)$. The signal space is therefore smallest for $\ell = \underline{\ell}$.

The size of the signal space is a rough measure of the informativeness of an optimal information structure. In order to state whether S is better informed for a maximal loss ℓ compared with a maximal loss ℓ', the two optimal information structures should be ranked in the sense of a Blackwell ordering. An information structure I is more informative in the sense of Blackwell than another information structure I', if the posterior beliefs that are reached through the different signal realization of I' can be replicated by sending S a garbling of the signals of I.

It turns out that it is generally not possible to order the optimal information structures in the sense of Blackwell. Nevertheless, for the special case of a uniform distribution of $\tilde{\theta}$, Proposition 1.7 in the appendix shows that for low levels of ℓ, S's signal becomes less informative as ℓ increases.

1.6 Delegation

Under the optimal information structure, P can implement the optimal centralized mechanism through hierarchical delegation whereby P contracts with S who in turn contracts with A. Consider the following delegation game δ.

t=0: P chooses an information structure $I \in \mathcal{I}$ and offers a delegation contract (T, r) to S, where T is a transfer that is paid upon acceptance, and r is paid if the project is realized.

t=1: S and A observe I, (T, r), and σ, A observes θ.

t=2: S accepts or rejects the delegation contract. If S rejects, the game ends and the project is not realized.

t=3: If S accepts, she offers a price p to A.

t=4: A accepts the offer and realizes the project or rejects it and the project is not realized.

In the delegation game, P offers S a delegation contract which pays T conditional on acceptance and r if the project is realized. S offers A a price p for project realization. The reduced delegation game where the information structure I is exogenously fixed is denoted by δ_I.

If P chooses an optimal information structure, it can implement the optimal centralized mechanism in the delegation game.

Proposition 1.5. *There exists a PBE of the delegation game δ where P sets the optimal weighted information structure and a delegation contract equal to the optimal collective transfers, the project is realized with ex-ante probability X^c, and*

P attains the expected payoff $W(X^c)$. Following these choices of P at $t = 0$, the continuation equilibrium starting at $t = 1$ is unique.

Given an optimal information structure, P can delegate the interaction with A to S without a reduction of its payoff. Thus, informational control substitutes direct control. The proposition is an implication of the following Lemma.

Lemma 1.6. *For any information structure I, the solution to P's reduced problem $\mathcal{P}_I$ can be implemented in the reduced delegation game δ_I if A receives no rent on the equilibrium path, i.e. $t_A(\sigma, \sigma, \theta) - \theta x(\sigma, \sigma, \theta) = 0$.*

If P delegates contracting with A to S, then it foregoes one instrument which is present under the centralized mechanism. This instrument consists in providing rents to A. These rents make it harder for S to find a side-contract that is acceptable to A. In particular, P can relax the collusion-proofness constraint (CP) by tightening A's participation constraint in the side mechanism (PC_A^γ). However, this comes at the cost of providing rents to A. In other words, P may channel additional rents to A in order to make it expensive for S to *bribe* A into joining the colluding coalition.

If P can jointly choose the information structure and the mechanism, P does not make use of this instrument: Under the optimal combination of an information structure and a mechanism identified in Lemmata 1.4 and 1.5, A never receives a rent. P does not find it optimal to make it more expansive to bribe A. It follows that P is as well off under delegated contracting as under the centralized mechanism.

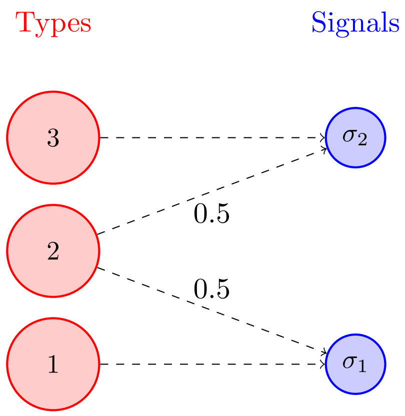
Delegation can be suboptimal for suboptimal information structures

Delegation is not optimal for any information structure. I show this by revisiting the illustrative example. Suppose S's signal is generated by the information structure depicted in Figure 1.6. This information structure consists of two signals σ_1 and σ_2, where σ_1 is always send for $\theta = 1$, σ_3 is always send for $\theta = 3$, and both signals are send with equal probability for $\theta = 2$.

Given this information structure, P strictly prefers the optimal centralized mechanism over delegation. P even prefers to ignore S and to contract directly with A over delegating to S the task to contract with A.

Proposition 1.6. *Suppose that θ is uniformly drawn from $\Theta = \{1, 2, 3\}$, $v = 4$, and $\ell \geq 0$. Given the information structure depicted in Figure 1.6, delegation is strictly suboptimal.*

Figure 1.6: Suboptimal information structure and suboptimal delegation



Information structure with suboptimal delegation consists of two signals. If costs are low, the signal σ_1 is generated. If costs are high, the signal σ_2 is generated. If costs intermediate, each signal is generated with equal probability.

The information structure in Figure 1.6 features one good and one bad signal. The presence of the bad signal harms P as it needs to ensure S's participation after the bad signal. For instance, consider the case where P sets a delegation contract under which the project is always realized. This implies that P receives an expected payment of $-T + 4 - r$. After the signal σ_2, S offers A a price of 3. The expected payoff of S after this signal is therefore $T + r - 3$. S only accepts the delegation contract after the signal σ_2 if $T + r \geq 3$ which implies that P's expected payoff cannot exceed 1. As I show in the proof of this proposition, P can never earn a higher profit than 1 for any delegation contract (T, r). In contrast, P can earn an expected payoff of $\frac{4}{3}$ if it directly offers a price to A. Delegation is therefore strictly suboptimal under the suboptimal information structure depicted in Figure 1.6.

1.7 Concluding remarks

In this chapter, I analyze the optimal use of informational control in a principal-supervisor-agent model under the threat of collusion between the supervisor and the agent.

The principal's optimal information structure is shaped by a trade-off between information elicitation and collusion prevention. If the supervisor learns additional information about the agent, the principal can potentially receive better informed advice from the supervisor. However, the supervisor can use the additional information to organize collusion with the agent more effectively. As a consequence, the principal optimally provides only partial information about the agent to the supplier.

Under the optimal information structure, the principal can delegate the interaction with the agent to the supervisor and still receive the same payoff as in the optimal centralized mechanism. Hierarchical delegation is therefore optimal and informational control substitutes direct control.

These results have implications for the informational design in settings where a principal can interact with a group of agents only via intermediaries. For the situation where the principal faces a single intermediary and a single agent, the results of this chapter provide the optimal way to inform the intermediary. This is an implication of the fact that any form of delegation – a situation where the principal chooses an allocation only depending on communication with the supervisor who can in turn communicate with the agent – is a collusion-proof mechanism of the centralized game under collusion. The analysis of intermediated contractual relationships with several intermediaries and agents has interesting implications for the study of complex organizations and intermediated markets and is left for future research.

1.8 Appendix to Chapter 1

Proof of Lemma 1.1

Toward a contradiction, suppose there exists a mechanism β which satisfies all constraints. Moreover there exist $(\sigma', \theta'), (\sigma'', \theta'') \in supp(\mu)$ such that $x(\sigma', \sigma', \theta') = x(\sigma'', \sigma'', \theta'')$ and $t_S(\sigma', \sigma', \theta') + t_A(\sigma', \sigma', \theta') < t_S(\sigma'', \sigma'', \theta'') + t_A(\sigma'', \sigma'', \theta'')$.

Consider now the side-contract $\gamma = (\tau, \rho)$ which is defined as $\tau(\theta; s) = 0$ for $(\sigma, \theta) \neq (\sigma', \theta')$, $\tau(\theta'; \sigma') = t_A(\sigma'', \sigma'', \theta'') - t_A(\sigma', \sigma', \theta')$, $\rho(\theta; \sigma) = (\sigma, \sigma, \theta)$ for $(\sigma, \theta) \neq (\sigma', \theta')$, and $\rho(\theta'; \sigma') = (\sigma'', \sigma'', \theta'')$. This side-contract is feasible by construction, because (PC_A) implies (PC_A^γ) and (IC_A) implies (IC_A^γ). Furthermore

$$
\begin{aligned}
&\mathbb{E}\left[t_S(\rho(\theta; \sigma')) + \tau(\theta; \sigma')|\sigma'\right] \\
&\quad = \mu(\theta \neq \theta'|\sigma')\mathbb{E}\left[t_S(\sigma', \sigma', \theta)|\sigma', \theta \neq \theta'\right] \\
&\qquad\qquad + \mu(\theta'|\sigma')\left(t_S(\sigma'', \sigma'', \theta'') + t_A(\sigma'', \sigma'', \theta'') - t_A(\sigma', \sigma', \theta')\right) \\
&\quad > \mu(\theta \neq \theta'|\sigma')\mathbb{E}\left[t_S(\sigma', \sigma', \theta)|\sigma', \theta \neq \theta'\right] + \mu(\theta'|\sigma')t_S(\sigma', \sigma', \theta') \\
&\quad = \mathbb{E}\left[t_S(\sigma', \sigma', \theta)|\sigma'\right].
\end{aligned}
$$

Thus, (CP) is not satisfied for $\sigma' \in \Sigma$, which gives a contradiction. □

Proof of Lemma 1.2

$T \geq -\ell$ follows from the argument presented in the first paragraph after the lemma. It remains to show that $r \geq \theta^c(X)$ and $T + r \geq \theta^c(X)$.

Let σ^c be the signal after which the type $\theta^c(X)$ realizes the project. If $X = 1$, the expected payoff of S after the signal σ^c cannot be higher than $T + r - \theta^c(1)$. Thus, S's participation constraint is only satisfied after the signal σ^c if $T+r \geq \theta^c(1)$.

Suppose now that $X < 1$. Thus, there exist $(\sigma_0, \theta_0) \in \Sigma \times \Theta$ such that $x(\sigma_0, \sigma_0, \theta_0) = 0$. I first show that $r \geq \theta^c(X)$. Toward a contradiction suppose that $r < \theta^c(X)$. Consider now the side-contract γ which is defined as

$$
\rho(\hat{\theta}; \sigma) = \begin{cases} (\sigma_0, \sigma_0, \theta_0) & \text{if } (\sigma, \hat{\theta}) \in \{\sigma^c\} \times (r, \theta^c(X)], \\ (\sigma, \sigma, \hat{\theta}) & \text{otherwise,} \end{cases} \quad \text{and}
$$

$$
\tau(\hat{\theta}; \sigma) = \begin{cases} t_A(\sigma^c, \sigma^c, \hat{\theta}) - t_A(\sigma_0, \sigma_0, \theta_0) & \text{if } (\sigma, \hat{\theta}) \in \{\sigma^c\} \times (r, \theta^c(X)], \\ 0 & \text{otherwise.} \end{cases}
$$

This side-contract gives A always the same payoff than playing the mechanism non-cooperatively. It follows that the side-contract is feasible. S receives under the side-contract the same payoff as under the mechanism if $(\sigma, \theta) \notin \{\hat{\sigma}\} \times (r, \theta^c(X)]$.

If $(\sigma, \theta) \in \{\hat{\sigma}\} \times (r, \theta^c(X)]$, S's payoff under the side-contract exceeds her payoff from the mechanism by $\theta - r > 0$. Thus the mechanism β is not collusion-proof and it follows that $r \geq \theta^c(X)$.

S's expected payoff after the signal σ^c can be written as

$$\begin{aligned}
&\mathbb{E}[x(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c](T + r) + (1 - \mathbb{E}[x(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c])T - \mathbb{E}[t_A(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c] \\
&\leq \mathbb{E}[x(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c](T + r - \theta^c(X)) + (1 - \mathbb{E}[x(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c])T
\end{aligned}$$

where the second line follows from the fact that (PC_A) implies

$$\mathbb{E}[t_A(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c] \geq \mathbb{E}[x(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c])\theta^c(X).$$

It follows from (PC_S) that

$$\mathbb{E}[x(\sigma^c, \sigma^c, \tilde{\theta})|\hat{\sigma}](T + r - \theta^c(X)) + (1 - \mathbb{E}[x(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c])T \geq 0.$$

If $T \geq 0$, this is satisfied and $T + r \geq \theta^c(X)$. If $T < 0$, this can only be satisfied if $T + r \geq \theta^c(X)$. □

Proof of Proposition 1.1

The proof follows from the arguments in the main text. □

Proof of Lemma 1.3

Note that $W^\circ(X) - W^*(X) = (1-X)\ell - \int_{\underline{\theta}}^{\theta^c(X)} F(\theta)d\theta$ is strictly decreasing in X. Thus, $X^\dagger$ is uniquely defined.

Next, observe that $\theta^c(X)$ is an increasing and left-continuous function in X. This implies that $W^\circ(X)$ is also left-continuous and makes a downward jump at any point where $W^\circ(X)$ is discontinuous, i.e. $\lim_{X' \nearrow X} W^\circ(X') \geq \lim_{X' \searrow X} W^\circ(X')$ for any $X \in [0,1]$. Thus $\max_{X' \in [X,1]} W^\circ(X')$ has a solution for any $X \in [0,1]$ and $X^\circ(X)$ is well defined.

As, $W^*(X)$ is a continuous function, it follows that $W(X)$ is a continuous function for $X < X^\dagger$ and a left-continuous function which exhibits only downward jumps for $X > X^\dagger$. At $X = X^\dagger$, either $W^*(X^\dagger) \geq W^\circ(X^\dagger)$ by continuity of $W^*(X)$ and left-continuity of $W^\circ(X)$. Thus, $W(X)$ is also left-continuous with only downward jumps. From this it follows that the solution to $\max_{X\in[0,1]} W(X)$ is well defined.

If $X^\dagger \geq F(v)$, then for all $X \in [0,1]$, $W(X) \leq W^*(X) \leq W^*(F(v)) = W(F(v))$. Thus, $W(X)$ is maximized for $X = F(v)$ in this case. Suppose now that $X^\dagger < F(v)$.

Note that $W(X) = W^*(X)$ for $X \leq X^\dagger$ and therefore $X^\dagger = \arg\max_{X\in[0,X^\dagger]} W(X)$. $X^\circ(X^\dagger)$ maximizes $W^\circ(X)$ on $[X^\dagger, 1]$. If $W^*(X^\dagger) > W^\circ(X^\circ(X^\dagger))$, then $W(X)$ is maximized for $X = X^\dagger$. If $W^*(X^\dagger) \leq W^\circ(X^\circ(X^\dagger))$, then $W(X)$ is maximized for $X = X^\circ(X^\dagger)$. Thus, X^c as defined in the Lemma maximizes $W(X)$. □

Proof of Lemma 1.4

To accommodate the possibility of mass points, I define

$$\Delta_F(\theta) \equiv F(\theta) - \lim_{\theta' \nearrow \theta} F(\theta') \tag{1.5}$$

and

$$f(\theta) = \begin{cases} F'(\theta) & \text{if} \quad F'(\theta) \text{ exists;} \\ \Delta_F(\theta) & \text{if} \quad \Delta_F(\theta) > 0; \\ 0 & \text{otherwise.} \end{cases} \tag{1.6}$$

I denote the set of mass points by $\Theta_D \equiv \{\theta \in \Theta : \Delta_F(\theta) > 0\}$. As $F(\cdot)$ is a cdf, there can be at most countably many points θ where $F(\cdot)$ is not differentiable.

It needs to be checked whether I_α is a well defined information structure, whether β is a feasible mechanism, and whether P achieves an expected payoff of $W(X)$. This is the case if *i)* $\alpha(\cdot)$ is a well defined weighting function, *ii)* the constraints (LL), (PC_S), (PC_A), (IC_S) and, (IC_A) hold, *iii)* S and A cannot profit from a joint misrepresentation of the signal σ, and *iv)* $U_P(I, \beta) = W(X)$.

Note at first that $\alpha(\sigma) \geq 0$ for all $\sigma \in \Sigma$. Furthermore,

$$\int_\Sigma \alpha(\sigma) d\sigma = \int_\Sigma \frac{f(\sigma)(\theta^c(X) - \sigma)}{(1-X)(r - \theta^c(X))} d\sigma = \frac{\int_{\underline{\theta}}^{\theta^c(X)} F(\sigma) d\sigma}{-(1-X)T} = 1,$$

where the second equality follows from integration by parts. Condition *i)* is thus satisfied and I_α is a well defined information structure.

I now show that β is a feasible mechanism. β satisfies the limited liability constraint (LL):

$$\begin{aligned} T = -\frac{\int_{\underline{\theta}}^{\theta^c(X)} F(\theta) d\theta}{1 - X} &\geq -\ell \\ \Leftrightarrow \quad (1-X)\ell - \int_{\underline{\theta}}^{\theta^c(X)} F(\theta) d\theta &= W^\circ(X) - W^*(X) \geq 0. \end{aligned}$$

Note that under the proposed mechanism, A faces a posted price that equals σ in equilibrium. Thus, (PC_A) and (IC_A) are satisfied. (IC_S) is also satisfied, because a unilateral misreport from S results in a payoff of $T < 0$. It remains to check whether (PC_S) is satisfied and this is indeed the case as

$$\begin{aligned} U_S(\sigma) &= T + \Pr(\theta = \sigma|\sigma)(r - \sigma) \\ &= T + \frac{f(\sigma)(r-\sigma)}{f(\sigma) + (1-X)\alpha(\sigma)} \\ &= T + r - \theta^c(X) = 0, \end{aligned}$$

where the third equality follows from the definition of $\alpha(\cdot)$ and the last equality follows from the definition of r. Thus, condition *ii*) is satisfied.

Next, it is shown that S cannot find a feasible and profitable side-contract. Under any such side-contract, S essentially needs to make a take-it-or-leave-it offer to A. As the support of any signal $\sigma = \theta$ is $\{\theta\} \cup (\theta^c(X), \overline{\theta}]$, a deviation implies that S offers a price $\theta' \geq \theta^c(X)$. S's payoff under such a deviation takes the form $T + q(r - \theta)$ where $q \in [0, 1]$ is the probability of project realization under the deviation. Note that

$$T + q(r - \theta) \leq T + q(r - \theta^c(X)) = (1 - q)T \leq 0.$$

Thus, S cannot find a profitable and feasible side-contract and condition *iii*) is satisfied.

Finally, condition *iv*) follows from the following calculation:

$$\begin{aligned} U_P(I, \beta) &= X(v - r) - T \\ &= X(v - r - T) - (1 - X)T \\ &= X(v - \theta^c(X)) + \int_{\underline{\theta}}^{\theta^c(X)} F(\theta)d\theta \\ &= W^*(X) \\ &= W(X). \end{aligned}$$

□

Proof of Lemma 1.5

Consider $f(\cdot)$ as defined in the equations (1.5) and (1.6). It needs to be checked whether I_α is a well defined information structure, whether β is a feasible mechanism, and whether P achieves an expected payoff of $W(X)$. This is the case if *i*) $\alpha(\cdot)$ is a well defined weighting function, *ii*) the constraints (LL), (PC_S), (PC_A), (IC_S) and, (IC_A) hold, *iii*) S and A cannot profit from a joint misrepresentation of the signal σ, and *iv)* $U_P(I,\beta) = W(X)$.

Note at first that $\alpha(\sigma) \geq 0$ for all $\sigma \in \Sigma$. Furthermore,

$$\int_\Sigma \alpha(\sigma)d\sigma = \int_{\underline{\theta}}^{\check{\theta}(X)} \frac{f(\sigma)(\check{\theta}(X)-\sigma)}{(1-X)(r-\check{\theta}(X))}d\sigma = \frac{\int_{\underline{\theta}}^{\check{\theta}(X)} F(\sigma)d\sigma}{(1-X)(r-\check{\theta})} = 1,$$

where the second equality follows from integration by parts and the third equality follows from the definition of $\check{\theta}$, Condition *i)* is thus satisfied and I_α is a well defined information structure.

I now show that β is a feasible mechanism. β satisfies the limited liability constraint (LL) as $T = -\ell$.

Under the mechanism β, A faces a posted price that equals σ in equilibrium. Thus, (PC_A) and (IC_A) are satisfied. (IC_S) is also satisfied, because a unilateral misreport from S results in a payoff of $T = -\ell \leq 0$. It remains to check whether (PC_S) is satisfied. For $\sigma \in [\check{\theta}(X), \theta^c(X)]$, the expected payoff of S after the signal σ is given by

$$\begin{aligned} U_S(\sigma) &= T + \Pr(\theta = \sigma|\sigma)(r-\sigma) \\ &= T + r - \sigma \\ &= -\ell + r - \sigma \\ &= \theta^c(X) - \sigma \geq 0. \end{aligned}$$

For $\sigma \in [\underline{\theta}, \check{\theta}(X))$ S's expected payoff is given by

$$\begin{aligned} U_S(\sigma) &= T + \Pr(\theta = \sigma|\sigma)(r-\sigma) \\ &= T + \frac{f(\sigma)(r-\sigma)}{f(\sigma)+(1-X)\alpha(\sigma)} \\ &= T + r - \check{\theta}(X) \\ &= \theta^c(X) - \check{\theta}(X) \geq 0. \end{aligned}$$

where the third equality follows from the definition of $\alpha(\cdot)$ and the last equality follows from the definition of r. Thus, condition *ii)* is satisfied.

Next, it is shown that S cannot find a feasible and profitable side-contract. Under any such side-contract, S essentially needs to make a take-it-or-leave-it price offer to A. As the support of any signal $\sigma = \theta$ is $\{\theta\} \cup (\theta^c(X), \overline{\theta}]$, a deviation implies that S offers a price $\theta' \geq \theta^c(X)$. S's payoff under such a deviation takes the form $T + q(r-\theta)$ where $q \in [0,1]$ is the probability of project realization of the deviation.

Note that

$$T + q(r - \theta) \leq T + q(r - \theta^c(X)) = (1 - q)T \leq 0.$$

Thus, S cannot find a profitable and feasible side-contract and condition *iii)* is satisfied.

Finally, condition *iv)* follows from the following calculation:

$$\begin{aligned} U_P(I, \beta) &= X(v - r) - T \\ &= X(v - r - T) - (1 - X)T \\ &= X(v - \theta^c(X)) + (1 - X)\ell \\ &= W^\circ(X) \\ &= W(X). \end{aligned}$$

□

Proof of Proposition 1.3

The only point which remains to be proven is that production is inefficient as long as $\ell < \frac{W^*(F(v))}{1-F(V)}$. From the definition of X^c, it is known that $\theta^c(X) < F(v)$ unless $X^\dagger \geq F(v)$. This is the case if and only if

$$W^*(F(v)) \leq W^\circ(F(v)) \Leftrightarrow W^*(F(v)) \leq (1 - F(v))\ell \Leftrightarrow \ell \geq \frac{\overline{W}}{1 - F(v)}.$$

□

Proof of Proposition 1.4

In the proposition, the following assumption is stated.

Assumption 1.1. *$F(\theta)$ has a strictly positive density $f(\theta)$ and $F(\theta)/f(\theta)$ is increasing.*

I prove the result along the following sequence of Lemmata.

Lemma 1.7. *Under Assumption 1.1, $W^*(X)$ and $W^\circ(X)$ are both differentiable and strictly quasi-concave.*

Proof. Under the assumption, the inverse of the cdf $F^{-1}(X)$ exists and is differentiable. This implies that $\theta^c(X) = F^{-1}(X)$. $W^*(X)$ and $W^\circ(X)$ can therefore be written as

$$W^*(X) = X(v - F^{-1}(X)) + \int_{\underline{\theta}}^{F^{-1}(X)} F(\theta)d\theta \quad \text{and}$$
$$W^\circ(X) = X(v - F^{-1}(X)) + (1 - X)\ell.$$

As $F^{-1}(X)$ is differentiable, both functions are differentiable.

I show now that both functions are strictly quasi-concave if $F(\theta)/f(\theta)$ is increasing. The first derivatives of both functions are given by

$$\frac{dW^*(X)}{dX} = v - F^{-1}(X) \quad \text{and}$$
$$\frac{dW^\circ(X)}{dX} = v - F^{-1}(X) - X(F^{-1})'(X) - \ell.$$

These first derivatives are both strictly decreasing and change their sign at most once from + to −. This is obvious for $\frac{dW^*(X)}{dX}$ as $F^{-1}(X)$ is strictly increasing. For $\frac{dW^\circ(X)}{dX}$, this follows from the assumption that $F(\theta)/f(\theta)$ is increasing: Define the strictly increasing function $\phi(\theta) \equiv \theta + F(\theta/f(\theta)$. As $F^{-1}(X)$ is strictly increasing, it follows that $\phi(F^{-1}(X))$ is strictly increasing. As

$$\phi(F^{-1}(X)) = F^{-1}(X) + X/f(F^{-1}(X)) = F^{-1}(X) + X(F^{-1})'(X),$$

$F^{-1}(X)+X(F^{-1})'(X)$ is strictly increasing. Thus, $W^*(X)$ and $W^\circ(X)$ are strictly quasi-concave. □

Lemma 1.8. *Under Assumption 1.1, $W(X)$ is maximized by $X = X^c(\ell)$ which is given by*

$$X^c(\ell) = \begin{cases} X^\circ(\ell) & \text{if} \quad X^\dagger(\ell) \leq X^\circ(\ell); \\ X^\dagger(\ell) & \text{if} \quad X^\dagger(\ell) \in (X^\circ(\ell), F(v)); \\ F(v) & \text{if} \quad X^\dagger(\ell) \geq F(v). \end{cases}$$

Proof. The intersection point of $W^*(X)$ and $W^\circ(X)$ depends on ℓ and is denoted by $X^\dagger(\ell)$. The maximizer of $W^\circ(X)$ also depends on ℓ and is denoted by $X^\circ(\ell)$.

$X^c(\ell)$ is specified in its general form in Lemma 1.3. Here, it simplifies to the expression above due to the following three observations:

First, recall the definition of $X^\circ(X)$ and note that due to the strict quasiconcavity of $W^\circ(X)$, it holds that

$$X^\circ(X) = \begin{cases} X^\circ(\ell) & \text{if} \quad X \leq X^\circ(\ell), \\ X & \text{if} \quad X > X^\circ(\ell). \end{cases}$$

Second, the continuity of $W^*(X)$ and $W^\circ(X)$ implies that the intersection point $X^\dagger(\ell)$ satisfies $W^*(X^\dagger(\ell)) = W^\circ(X^\dagger(\ell))$. Finally, for all $\ell \geq 0$, it holds that $X^\circ(\ell) < F(v)$. This follows from the fact that

$$\frac{dW^\circ(F(v))}{dX} = v - F^{-1}(F(v)) - F(v)(F^{-1})'(F(v)) - \ell < 0.$$

□

Lemma 1.9. *Under Assumption 1.1, $X^\circ(\ell)$ is decreasing in ℓ, $X^\dagger(\ell)$ is increasing in ℓ, and $X^\circ(\underline{\ell}) = X^\dagger(\underline{\ell})$ for some $\underline{\ell} \in (0, \overline{\ell})$.*

Proof. $X^\circ(\ell)$ is implicitly defined by the first order condition

$$\frac{dW^\circ(X^\circ(\ell))}{dX} = v - F^{-1}(X^\circ(\ell)) - X^\circ(\ell)(F^{-1})'(X^\circ(\ell)) - \ell = 0.$$

As $F^{-1}(X)$ is differentiable, $dX^\circ(\ell)/d\ell$ exists and satisfies

$$\frac{dX^\circ(\ell)}{d\ell} = -\frac{1}{\phi'(F^{-1}(X^\circ(\ell)))} \leq -1.$$

$X^\dagger(\ell)$ is implicitly defined by

$$W^\circ(X^\dagger(\ell)) - W^*(X^\dagger(\ell)) = (1 - X^\dagger(\ell))\ell - \int_{\underline{\theta}}^{F^{-1}(X^\dagger(\ell))} F(\theta)d\theta = 0$$

Thus, $dX^\dagger(\ell)/d\ell$ exists and is given by

$$\frac{dX^\dagger(\ell)}{d\ell} = \frac{1 - X^\dagger(\ell)}{X^\dagger(\ell) + \ell} \geq 0.$$

Note that $X^\circ(0) = F(\theta^{ns}) > 0$ and $X^\dagger(0) = 0$. Thus, there exists some $\underline{\ell} \in (0, \infty)$ such that $X^\circ(\underline{\ell}) = X^\dagger(\underline{\ell})$. If $v < \overline{\theta}$, $X^\circ(\ell) < F(v)$ furthermore implies that $\underline{\ell} < \overline{\ell} \equiv \frac{W^*(F(v))}{1-F(v)}$, as $X^\dagger(\overline{\ell}) = F(v)$. □

Lemma 1.10. *Under Assumption 1.1, it holds that*

$$X^c(\ell) = \begin{cases} X^\circ(\ell) & \text{if} \quad \ell \in [0, \underline{\ell}); \\ X^\dagger(\ell) & \text{if} \quad \ell \in [\underline{\ell}, \overline{\ell}); \\ F(v) & \text{if} \quad \ell \geq \overline{\ell}. \end{cases}$$

$X^c(\ell)$ is decreasing for $\ell \in (0, \underline{\ell})$, increasing for $\ell \in (\underline{\ell}, \overline{\ell})$, and constant for $\ell \geq \overline{\ell}$.

Proof. Follows from the previous two Lemmata. □

Lemma 1.11. *Under Assumption 1.1 and for any $\ell < \ell'$, it holds that*

- $\Sigma(\ell) \subset \Sigma(\ell')$ *for* $\ell, \ell' \in [0, \underline{\ell}]$
- $\Sigma(\ell') \subset \Sigma(\ell)$ *for* $\ell, \ell' \in [\underline{\ell}, \overline{\ell}]$
- $\Sigma(\ell) = \Sigma(\ell')$ *for* $\ell, \ell' > \overline{\ell}$

Proof. From Lemmata 1.4 and 1.5, it follows that $\Sigma(\ell) = [\underline{\theta}, F^{-1}(X^c(\ell))]$. As $F^{-1}(X)$ is increasing under Assumption 1.1, the result follows from the previous Lemma. □

Lemma 1.12. *The total expected rents of S and A are positive and decreasing in ℓ for $\ell < \underline{\ell}$ and zero for $\ell \geq \underline{\ell}$.*

Proof. The total expected rents of S and A are given by

$$\begin{aligned} R(\ell) &\equiv [W^*(X^c(\ell)) - W^\circ(X^c(\ell))]_+ \\ &= \left[\int_{\underline{\theta}}^{F^{-1}(X^\dagger(\ell))} F(\theta) d\theta - (1 - X^\dagger(\ell))\ell \right]_+ . \end{aligned}$$

By Lemma 1.10 and the definition of the intersection point $X^\dagger(\ell)$, this expression is positive for $\ell < \underline{\ell}$ and zero for $\ell \geq \underline{\ell}$. For $\ell < \underline{\ell}$, it holds that

$$R'(\ell) = (X^c(\ell) + \ell) \cdot \frac{dX^c(\ell)}{d\ell} - (1 - X^c(\ell)).$$

By Lemma 1.10, this expression is negative. □

Proof of Proposition 1.5 and Lemma 1.6

Consider the continuation play of the game starting at $t = 1$ after P has set T and r and a weighted information structure I_α as defined in Lemma 1.4 and 1.5, respectively. In any PBE of the game, A accepts any price offer that lies weakly above his costs. After a signal σ, the collusion-proofness constraint (CP) in P's problem implies that a price $p = \sigma$ is optimal, as $t_A(\sigma, \sigma, \theta) - \theta x(\sigma, \sigma, \theta) = 0$.

Furthermore, this is the uniquely optimal price. Any price $p' < \sigma$ results in a payoff of $T < 0$. A price $p' \in (\sigma, \theta^c(X)]$ gives a payoff of $T + X(r - p') < T + X(r - \sigma)$ and any price $p' > \theta^c(X)$ gives a payoff $T + q(r - p') < T + q(r - \theta^c) < T + r - \theta^c = 0$ for some $q \in [0, 1]$. The continuation equilibrium at $t = 1$ is therefore unique and the ex-ante probability of project realization is X^c. As the delegation contract (T, r) equals the optimal collective transfers described in Lemmata 1.4 and 1.5, P attains an expected payoff of $W(X^c)$. □

Proof of Proposition 1.6

Under delegation, P offers to S a contract which prescribes an initial payment of T and an additional payment r contingent on project realization. After the signal σ_1, S offers A either a price $p = 1$ or a price $p = 2$. After the signal σ_2, S offers either $p = 2$ or $p = 3$. In any equilibrium, P's offer is accepted by S either after both signals or only after the *good* signal σ_1. This gives six types of equilibria[11] which need to be considered. I show that in all six cases, P's equilibrium payoff under delegation is lower than 1. If P ignores S and contracts directly with A, she can achieve an expected payoff of $\frac{4}{3}$. Thus, delegation is strictly suboptimal.

In the following, $p(\sigma)$ denotes the price that S offers in equilibrium after observing the signal σ. I assume that ℓ is large enough such that the limited liability constraint $T \geq -\ell$ is never violated. Under this assumption, I show that P's expected payoff can never exceed 1. This implies that P's payoff is also lower than 1 for smaller values of ℓ.

At first, I consider the equilibria where S accepts P's offer after both signals. In an equilibrium where S offers the prices $p(\sigma_1) = 1$ and $p(\sigma_2) = 2$, the expected payoffs of S are $U_S(\sigma_1) = T + \frac{2}{3}(r - 1)$ and $U_S(\sigma_2) = T + \frac{1}{3}(r - 2)$. As S optimally offers A a price of 2 for the signal σ_2, it needs to hold that $r \geq 2$. It follows that $U_S(\sigma_1) = U_S(\sigma_2) + \frac{1}{3}r > U_S(\sigma_2)$. P's expected payoff is

$$\begin{aligned}
&\Pr(\sigma_1)(-T + \Pr(\theta = 1|\sigma_1)(4 - r)) + \Pr(\sigma_2)(-T + \Pr(\theta = 2|\sigma_2)(4 - r)) \\
&\quad = \frac{1}{2}(-T + \frac{2}{3}(4 - r)) + \frac{1}{2}(-T + \frac{1}{3}(4 - r)) \\
&\quad = \frac{1}{2}(\frac{2}{3}(4 - 1) - U_S(\sigma_1)) + \frac{1}{2}(\frac{1}{3}(4 - 2) - U_S(\sigma_2)) \\
&\quad = \frac{4}{3} - \frac{1}{6}r - U_S(\sigma_2) \leq 1,
\end{aligned}$$

[11] Not all six types of equilibria must necessarily exist.

where the last inequality follows from the condition $r \geq 2$ and S's participation constraint $U_S(\sigma_2) \geq 0$.

Consider now an equilibrium in which S offers A a price of 2 after both signals, i.e. $p(\sigma_1) = p(\sigma_2) = 2$. This is optimal for S if

$$U_S(\sigma_1) = T + r - 2 \geq T + \frac{2}{3}(r-1) \Leftrightarrow r \geq 4 \text{ and}$$
$$U_S(\sigma_2) = T + \frac{1}{3}(r-2) \geq T + r - 3 \Leftrightarrow r \leq \frac{7}{2}.$$

Thus, we have a contradiction which implies that there does not exist an equilibrium with $p(\sigma_1) = p(\sigma_2) = 2$.

In the third type of equilibrium, A offers $p(\sigma_1) = 1$ and $p(\sigma_2) = 3$. This is optimal for S if

$$U_S(\sigma_1) = T + \frac{2}{3}(r-1) \geq T + r - 2 \Leftrightarrow r \leq 4 \text{ and}$$
$$U_S(\sigma_2) = T + r - 3 \geq T + \frac{1}{3}(r-2) \Leftrightarrow r \geq \frac{7}{2}.$$

As $U_1(\sigma_1) = U_1(\sigma_2) + \frac{1}{3}(7-r)$, P's payoff can be written as

$$\begin{aligned}
&\Pr(\sigma_1)(-T + \Pr(\theta = 1|\sigma_1)(4-r)) + \Pr(\sigma_2)(-T + 4 - r) \\
&= \frac{1}{2}(-T + \frac{2}{3}(4-r)) + \frac{1}{2}(-T + 4 - r) \\
&= \frac{1}{2}(\frac{2}{3}(4-1) - U_S(\sigma_1)) + \frac{1}{2}(4 - 3 - U_S(\sigma_2)) \\
&= \frac{3}{2} - \frac{1}{6}(7-r) - U_S(\sigma_2) \leq 1
\end{aligned}$$

where the last inequality follows from the participation constraint $U_1(\sigma_2) \geq 0$ and $r \leq 4$.

In an equilibrium where S offers $p(\sigma_1) = 2$ and $p(\sigma_2) = 3$, it holds that $U_S(\sigma_1) = T + r - 2$, $U_S(\sigma_2) = T + r - 3$, and $U_S(\sigma_1) = U_S(\sigma_2) + 1$. In this case, P's expected equilibrium payoff can be written as

$$\begin{aligned}
&\Pr(\sigma_1)(-T + 4 - r) + \Pr(\sigma_2)(-T + 4 - r) \\
&= \frac{1}{2}(-T + 4 - r) + \frac{1}{2}(-T + 4 - r) \\
&= \frac{1}{2}(4 - 2 - U_S(\sigma_1)) + \frac{1}{2}(4 - 3 - U_S(\sigma_2)) \\
&= 1 - U_S(\sigma_2) \leq 1
\end{aligned}$$

where the last inequality follows from the participation constraint $U_S(\sigma_2) \geq 0$.

I turn now to equilibria where S accepts P's offer only after the signal σ_1. In an equilibrium where S sets a price $p(\sigma_1) = 1$, P's expected payoff is

$$\Pr(\sigma_1)(-T + \frac{2}{3}(4 - r)) = \frac{1}{2}(\frac{2}{3}(4 - 1) - U_S(\sigma_1)) \leq 1$$

where the last inequality follows from the participation constraint $U_S(\sigma_1)) \geq 0$. In an equilibrium with $p(\sigma_1) = 2$, P's expected payoff is

$$\Pr(\sigma_1)(-T + 4 - r) = \frac{1}{2}(4 - 2 - U_S(\sigma_1)) \leq 1$$

by the participation constraint $U_S(\sigma_1)) \geq 0$.

Thus, delegation is strictly suboptimal under the information structure depicted in Figure 1.6. □

Statement and Proof of Proposition 1.7

Proposition 1.7. *Suppose $\tilde{\theta}$ is uniformly distributed on $[0, 1]$ and $v = 1$. If ℓ is low, S is becoming less informed as ℓ increases: For all ℓ, ℓ' with $0 \leq \ell < \ell' \leq \hat{\ell}$*

$$I_\alpha(\ell) \succ_{Blackwell} I_\alpha(\ell').$$

At first, I formally define a Blackwell ordering. Consider two information structures I_1 and I_2. Each information structure $I_i = (\Sigma_i, \mu_i)$ has an associated conditional cdf $G_i(\theta|\sigma)$ for $i \in \{1, 2\}$. A *garbling* $\Gamma \in \Delta(\Sigma_1 \times \Sigma_2)$ is a joint cdf over the two signal spaces. Γ induces a cdf over Σ_i conditional on $\sigma_j \in \Sigma_j$ which I denote by $\Gamma_{ij}(\cdot|\sigma_j)$.

Definition 1 (Blackwell ordering). $I_1 \succ_{Blackwell} I_2$ *if there exists a* garbling Γ *such that*

$$G_2(\theta|\sigma) = \int_{\Sigma_1} G_1(\theta|z) d\Gamma_{12}(z|\sigma).$$

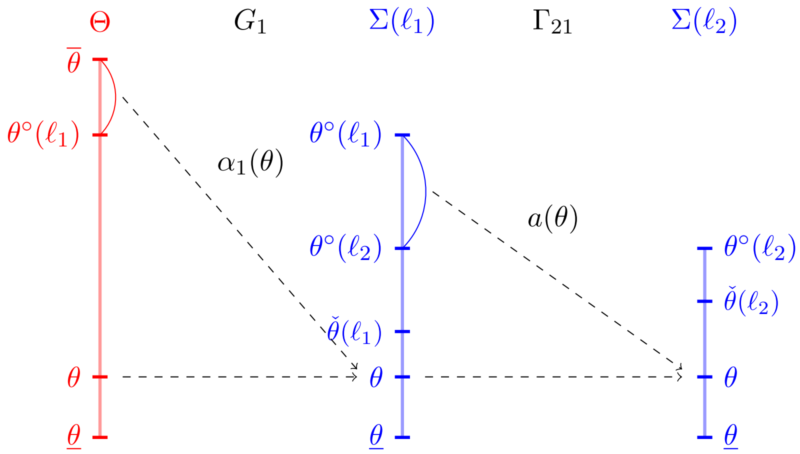
By Bayes' law

$$\Gamma_{12}(\sigma_1|\sigma_2) = \frac{\Gamma_1(\sigma_1)}{\Gamma_2(\sigma_2)} \Gamma_{21}(\sigma_2|\sigma_1)$$

where $\Gamma_i(\sigma_i) \equiv \int_{z_i \leq \sigma_i} \int_{\Sigma_j} d\Gamma(z_i, z_j)$.

I show now that there exists such a garbling for the information structures $I_1 = I(\ell_1)$ and $I_2 = I(\ell_2)$ with $0 \leq \ell_1 < \ell_2 \leq \underline{\ell}$. Both information structures are characterized by a weight function denoted by $\alpha_i(\cdot)$ for the information structure $I(\ell_i)$. I describe the garbling in terms of Γ_{21} which is illustrated in Figure 1.7.

Figure 1.7: Garbling



The information structure I_1 is more informative in the sense of Blackwell if I_2 can be replicated by a garbling of the signals generated by I_1. If the signal θ of I_1 is smaller than the cutoff $\theta^\circ(\ell)$, I_2 generates the signal θ. If it is higher than the cutoff, a signal of I_2 is drawn according to the density $a(\cdot)$.

Suppose S receives the signal σ which is generated through the combination of G_1 and Γ_{21} as depicted in Figure 1.6. S's belief over the type of A can then be described by the cdf

$$G^\Gamma(\theta|\sigma) = \begin{cases} 0 & \text{if} \quad \theta < \sigma \\ \frac{f(\sigma)}{f(\sigma)+(1-X_1)\alpha_1(\sigma)+(X_1-X_2)a(\sigma)} & \text{if} \quad \theta \in [\sigma, \theta^\circ(\ell_2)] \\ \frac{f(\sigma)}{f(\sigma)+(1-X_1)\alpha_1(\sigma)+(X_1-X_2)a(\sigma)} \\ + \frac{(1-X_1)\alpha_1(\sigma)+(X_1-X_2)a(\sigma)}{f(\sigma)+(1-X_1)\alpha_1(\sigma)+(X_1-X_2)a(\sigma)} \frac{F(\theta)-F(\theta^\circ(\ell_2))}{1-F(\theta^\circ(\ell_2))} & \text{if} \quad \theta > \theta^\circ(\ell_2) \end{cases}$$

where $X_i = 1 - F(\theta^\circ(\ell_i))$.

This garbling Γ is characterized by the weighting function $a(\cdot)$. If there exists a weighting function $a(\cdot)$ such that $G^\Gamma(\theta|\sigma) = G_2(\theta|\sigma)$, then $I(\ell_1) \succ_{Blackwell} I(\ell_2)$. $G^\Gamma(\theta|\sigma) = G_2(\theta|\sigma)$ holds for

$$a(\theta) = \frac{(1 - F(\theta^\circ(\ell_2)))\alpha_2(\theta) - (1 - F(\theta^\circ(\ell_1)))\alpha_1(\theta)}{F(\theta^\circ(\ell_1)) - F(\theta^\circ(\ell_2))}.$$

The equation defines a weight function if $i)$ $\int_{\Sigma_2} a(\sigma)d\sigma = 1$ and $ii)$ $a(\sigma) \geq 0$ for all

$\sigma \in \Sigma_2$. $i)$ is satisfied as

$$\int_{\Sigma_2} a(\sigma)d\sigma = \frac{(1 - F(\theta^\circ(\ell_2))) \int_{\Sigma_2} \alpha_2(\sigma)d\sigma - (1 - F(\theta^\circ(\ell_1))) \int_{\Sigma_2} \alpha_1(\sigma)d\sigma}{F(\theta^\circ(\ell_1)) - F(\theta^\circ(\ell_2))}$$
$$= \frac{(1 - F(\theta^\circ(\ell_2))) - (1 - F(\theta^\circ(\ell_1)))}{F(\theta^\circ(\ell_1)) - F(\theta^\circ(\ell_2))} = 1$$

which follows from

$$\int_{\Sigma_2} \alpha_i(\sigma)d\sigma = \int_{\underline{\theta}}^{\check{\theta}(\ell_i)} \alpha_i(\sigma)d\sigma = 1.$$

$ii)$ is satisfied if $(1 - F(\theta^\circ(\ell_2)))\alpha_2(\theta) - (1 - F(\theta^\circ(\ell_1)))\alpha_1(\theta) \geq 0$. Using the definitions of T, r, $\alpha(\cdot)$, and $\check{\theta}$ from Lemma 1.5, this is equivalent to

$$f(\theta)\frac{\check{\theta}(\ell_2) - \theta}{\theta^\circ(\ell_2) + \ell_2 - \check{\theta}(\ell_2)} \geq f(\theta)\frac{\check{\theta}(\ell_1) - \theta}{\theta^\circ(\ell_1) + \ell_1 - \check{\theta}(\ell_1)}.$$

Thus, $I(\ell_1) \succ_{Blackwell} I(\ell_2)$ if $\frac{\check{\theta}(\ell) - \theta}{\theta^\circ(\ell) + \ell - \check{\theta}(\ell)}$ is non-decreasing in ℓ.

If $\tilde{\theta}$ is uniformly distributed on $[0, 1]$ and $v = 1$, it is easy to check from the definitions of $\theta^\circ(\ell)$ and $\check{\theta}(\ell)$ that $\theta^\circ(\ell) = 0.5(1 - \ell)$ and $\check{\theta}(\ell) = 0.5(\sqrt{3} - 1)(1 + \ell)$. It follows that

$$\frac{\check{\theta}(\ell) - \theta}{\theta^\circ(\ell) + \ell - \check{\theta}(\ell)} = \frac{\sqrt{3} - 1}{2 - \sqrt{3}} - \frac{2\theta}{(\sqrt{3} - 1)(1 + \ell)}$$

which is increasing in ℓ.

Thus, $a(\cdot)$ is a weight function and $I(\ell_1) \succ_{Blackwell} I(\ell_2)$. □

Chapter 2

Optimal Monitoring in Dynamic Procurement Contracts

This chapter is based on Asseyer (2016b).

2.1 Introduction

Public procurement constitutes a sizable part of economic activity in advanced economies. In 2013 the average OECD country spent 29% of total government expenditure on public procurement. This represents, on average, 12.1% of GDI (OECD, 2015a). The potential cost savings through more efficient public procurement procedures are large.[1] In the aftermath of the Great Recession – as many governments have been forced to reduce their debt – international organizations such as the OECD and national audit offices have urged public institutions to adopt better procurement procedures (NAO/OGC, 2008; GAO, 2013; OECD, 2015b). Among other measures for the effective management of public procurement, these organisations stress the importance of monitoring. In particular, they recommend monitoring the cost developments and other performance measures of private suppliers. However, monitoring is costly and not all sources of additional information help government institutions to manage public contracts effectively.[2]

This poses the question of what kind of information a government institution should monitor in order to optimally manage public contracts. I analyze this

[1] Cost savings of 1% of public procurement expenditure represents 43 billion EUR across OECD countries (OECD, 2015b).

[2] For example, a large number of performance indicators (NAO, 2014) did not prevent the overbilling of the companies G4S and Serco in the UK.

question in a principal-agent model of procurement where the agent can produce a good for the principal. Prior to production, the agent can make a cost-reducing investment at a privately known cost. Production costs are determined by the investment and a cost shock. This shock represents the agent's uncertainty of production costs at the time of making the investment. The principal can monitor the agent's investment decision, the cost shock or both at a cost.

I show that it is never optimal for the principal to monitor both. Furthermore, the principal achieves at least the same payoff – gross monitoring costs – if she monitors investment as though she was monitoring the shock. Both instruments perform equally well if the investment decision is close to being efficient.

The model captures two important aspects of public procurement projects. First, cost-reducing investments and the sequential, private learning of production costs are realistic features of many procurement contexts. Procurement contracts frequently run for several years. This is the case for construction projects of airports and high-speed internet networks as well as the management and maintenance of public facilities such as hospitals and highways.[3] Furthermore, suppliers can often make cost-reducing investments at an early stage of the project, e.g. at the planning phase. These investments influence the total costs of the project completion. Additionally, production costs are influenced by exogenous cost factors that suppliers observe after the investment decision. Examples of such factors include ground conditions and input prices for construction projects, and weather conditions for maintenance contracts. The model reflects both the dynamic aspect of learning during procurement contracts and the opportunity to invest in cost reductions.

Secondly, monitoring plays a central role in the management of public procurement projects. Suppliers' investment decisions can be monitored using key performance indicators such as quality milestones (Garvin et al., 2011). Exogenous cost factors can be monitored by third-party experts.[4] In practice, government agencies monitor a wide variety of outcomes. Typically, these outcomes can be classified as either endogenous or exogenous cost factors. In this chapter, I focus on the monitoring of investments and cost shocks as relevant examples for each type.

I solve the principal's problem by analyzing optimal contracts under the different monitoring regimes. If the agent's investment decision is monitored, the principal first has to elicit the investment cost and then the value of the shock. The principal's optimal contract induces underinvestment and efficient production

[3] In the UK, public-private partnerships under the scheme of the Private Finance Initiative usually run for 20 to 30 years (HMT, 2012).

[4] For instance, the Canadian government has employed KPMG as a third-party expert for a national shipbuilding procurement project (PWGSC, 2013).

of the good independently of the investment decision. The two sources of asymmetric information can be treated separately. The agent knows his investment cost before a contract is signed. Thus, the principal has to give an information rent to the agent and therefore distorts the investment decision. In contrast, the cost shock is realized after the parties have signed a contract. The principal does not have to give any rent to the agent for this information. She can induce efficient production and extract the surplus with a fixed fee. This implies that the principal has nothing to gain by monitoring both the cost shock and the investment decision.

If only the shock is monitored, the principal has to elicit the investment cost and induce the appropriate investment decision. Moral hazard concerning the investment decision is irrelevant if the distortion in the investment decision is not too large. The previously optimal contract is then still implementable. In contrast, if the principal wishes to implement a large distortion in investment, moral hazard becomes relevant. In this case, the optimal contract features distortions in the investment and the production decision.

Finally, I analyze the optimal contract without monitoring. The principal achieves a strictly smaller payoff than in the case where the shock is monitored. If the principal does not monitor, the agent can play *double deviations* by combining a deviation in the investment decision with a false report on the cost shock. Such deviations are not possible if the shock is monitored and they decrease the principal's payoff. This contrasts with the case where investment is monitored. Here, the agent cannot benefit from false reports about the cost shock. The unobserved investment decision 'connects' the two sources of private information. The principal cannot separate them as in the case of investment monitoring. The optimal contract without monitoring induces two types of inefficiencies: underinvestment in cost reduction and underproduction.

These results have the following implications for the optimal monitoring policy. First, the principal monitors either the shock or the investment decision. Second, monitoring investment is always at least as effective as monitoring the shock. Third, the principal can sometimes achieve the same payoff (gross monitoring costs) under the two monitoring regimes. This is the case if the distortion in the investment decision is not too high.

Following the seminal work by Laffont and Tirole (1986), an important part of the literature on procurement contracting assumes that the principal observes the production costs of the agent.[5] This chapter contributes to the literature on monitoring in principal-agent models, in which the principal's information is determined endogenously. Maskin and Riley (1985) and Khalil and Lawarrée (1995)

[5]See also Laffont and Tirole (1993).

analyze the question of input-vs-output monitoring, where a principal can monitor either an agent's effort or his realized return. Dewatripont and Maskin (1995) show that a principal may optimally restrict what he can monitor in order to avoid renegotiation. Khalil (1997) analyzes a principal-agent model where the principal cannot commit to the monitoring policy. Strausz (1997) looks at the delegation of monitoring to a third party under limited commitment. This chapter is the first to provide an analysis of the monitoring of post-contractual private information and moral hazard under full commitment.[6]

This chapter is also related to the literature on R&D and optimal procurement mechanisms which was initiated by Tan (1992), Piccione and Tan (1996), and Bag (1997). They analyze competitive procurement mechanisms when suppliers can invest in cost-reducing R&D. As the investment decision in this chapter can be interpreted as an investment in R&D, the contribution to this literature lies in analyzing the effect of unobserved and cost-reducing R&D on dynamic information rents. Cisternas and Figueroa (2015) analyze the optimal procurement mechanism when a buyer procures two projects sequentially and the winner of the first round can invest in cost reduction before the second round. Liu and Lu (2015) analyze optimal contracts in a model of procurement with an unobservable R&D effort in cost reduction under dynamic adverse selection.

A further related literature studies optimal contracts for *public-private partnerships* (PPP). Iossa and Martimort (2012, 2015) analyze when a public buyer should delegate the design and the implementation of a project to the same firm – which corresponds to a PPP – or different firms – such as under traditional forms of procurement. Hoppe and Schmitz (2013) analyze the implication of this decision for innovative procurement. Engel et al. (2013) study optimal contracts for PPPs from a public finance perspective. Closely related to the current chapter, Iossa and Martimort (2016) study optimal contracts for a PPP where a single supplier has private information and a moral hazard decision after contracting. In contrast to the current chapter, they consider a risk-averse supplier who has no private information at the outset of the contractual relationship. Furthermore, they study the benefit of complete contracts over incomplete contracts, where only complete contracts condition on communication with the supplier. Public-private partnerships are frequently used for the realization of mid to long-term procurement projects where learning of new information and cost-reducing investments by suppliers play an important role. This chapter contributes to the literature by providing an explicit analysis of the optimal monitoring policies for a government authority that

[6]In contrast to the literature on costly state verification initiated by Townsend (1979), the principal here decides ex-ante which information of the agent to observe.

is entering into a public-private partnership with a private supplier.

This chapter is furthermore related to the literature on information rents in dynamic principal-agent models. In models of dynamic adverse selection, Baron and Besanko (1984), Eső and Szentes (2007a,b), and Pavan et al. (2014) show that the principal does not have to give rents for the private information that agents learn after contracting. Krähmer and Strausz (2015) qualify this insight by showing that an agent receives post-contractual information rents if the set of signal realizations learned before contracting is discrete. Baron and Besanko (1984) and Eső and Szentes (2015) argue that privacy of post-contractual information may also be irrelevant in the presence of dynamic adverse selection and moral hazard. I show that their result does not extend to the setting in the current chapter. I elaborate on this point in the discussion section.

In the following section I introduce the model. Section 2.3 presents the optimal contract when the principal monitors the investment decision. Section 2.4 considers the case where the principal monitors the shock. Section 2.5 analyzes the optimal contracts without monitoring. Section 2.6 presents and analyzes the optimal monitoring policy. Section 2.7 discusses the role of post-contractual information rents and the robustness of the results. Section 2.8 concludes.

2.2 The model

A government institution (the principal) can procure a good from a supplier (the agent). The principal values the good by v. Prior to production, the agent can make a cost-reducing investment decision $x \in \{0, 1\}$. The investment decision leads to investment costs of $\kappa \cdot x$ to the agent. κ is the private information of the agent and is drawn from an interval $[\underline{\kappa}, \overline{\kappa}] \subset \mathbb{R}_+$ according to the distribution function F. F has a log-concave density function f, so that $F(\kappa)/f(\kappa)$ is weakly increasing and $(1 - F(\kappa))/f(\kappa)$ is weakly decreasing (Bagnoli and Bergstrom, 2005). The agent's production cost is determined by the investment decision x and a shock ε that is realized after the investment is made. The production cost is given by $c_x(\varepsilon)$. For both $x \in \{0, 1\}$, the function $c_x(\cdot)$ has the image $[\underline{c}, \overline{c}] \subset \mathbb{R}$, is strictly increasing and twice continuously differentiable. Without loss of generality I assume that the shock ε is uniformly distributed on the unit interval.[7] The investment is cost-reducing in the sense that $c_1(\varepsilon) < c_0(\varepsilon)$ for all $\varepsilon \in (0, 1)$. Furthermore, let $v \in (\underline{c}, \overline{c})$. There are two simple monitoring technologies. The principal can perfectly observe the

[7] If the shock ϵ leading to production costs $\hat{c}_x(\epsilon)$ is distributed according to a continuous and strictly increasing distribution function H on some interval, then the random variable $\varepsilon \equiv H(\epsilon)$ is uniformly distributed on $[0, 1]$. The cost functions can be redefined as $c_x(\varepsilon) = c_x(H(\epsilon)) \equiv \hat{c}_x(\epsilon)$. An assumption on $c_x(\cdot)$ would then translate into a joint assumption on $\hat{c}_x(\cdot)$ and H.

investment decision x at a monitoring cost $C^i > 0$, and the shock ε at a monitoring cost $C^s > 0$. I assume that the principal cannot monitor probabilistically.[8] Both parties are risk-neutral and have outside options associated with a payoff of zero. Let q be the probability of production and t be a transfer. The agent's payoff is $t - c_x(\varepsilon)q - \kappa x$ and the principal's payoff gross monitoring costs is $vq - t$. The timing of the game is as follows:

i) The agent learns κ.

ii) The principal decides what to monitor and offers a contract. The agent observes the principal's decision and accepts or rejects the contract. If the agent chooses reject, the game ends and both parties receive zero payoffs. Otherwise, the game continues.

iii) The agent makes the investment decision x.

iv) The shock ε is realized.

v) The agent can produce the good.

Complete information benchmark

Suppose the investment cost κ, the investment decision x, and the shock ε are publicly observed. In this case, the principal can extract the whole social surplus. For any κ, she chooses the investment decision and the probability of production, which maximize social surplus:

$$\max_{x, q_x(\cdot)} \int_0^1 (v - c_x(\varepsilon))q_x(\varepsilon)d\varepsilon - \kappa \cdot x$$

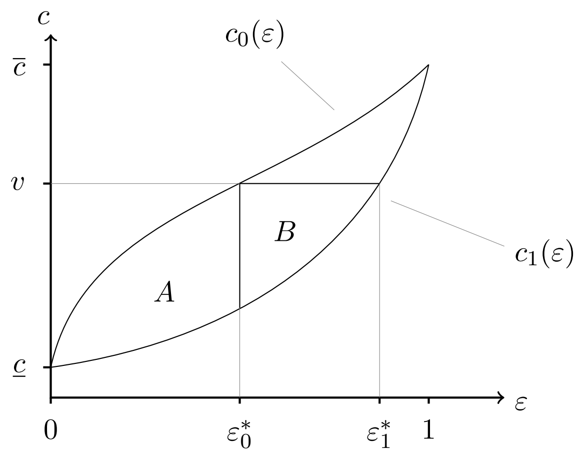
Independently of the investment decision, the principal procures the good from the agent if her valuation lies above the production costs: $v \geq c_x(\varepsilon)$ for $x \in \{0, 1\}$. I denote the thresholds of the shock by ε_x^* which satisfy $c_x(\varepsilon_x^*) = v$ with $x \in \{0, 1\}$. The principal induces the cost-reducing investment if the investment cost lies below the additional social value created by the investment, i.e., if $\kappa \leq \kappa^*$ where

$$\kappa^* \equiv \int_0^{\varepsilon_1^*} (v - c_1(\varepsilon))d\varepsilon - \int_0^{\varepsilon_0^*} (v - c_0(\varepsilon))d\varepsilon.$$

κ^* is the *efficient investment cutoff* and it equals the additional social surplus that is generated through the investment. I assume $\kappa^* \in (\underline{\kappa}, \overline{\kappa})$.

[8] An alternative assumption is that the principal has to spend C^i or C^s to install the monitoring technology independently of whether it is used later on and the agent observes whenever a monitoring technology is installed.

Figure 2.1: Efficient production and investment decisions



The costs of the agent are depicted as a function of the shock for both investment decisions. The areas A and B represents the social benefit of investment gross investment costs under efficient production decisions.

Graphically, κ^* can be represented as the areas A and B in Figure 2.1. In this figure, the production costs of the agent are depicted as functions of the cost shock for both investment decisions $x = 0$ and $x = 1$. Area A represents the expected cost savings of the investment for values of the cost shock under which the good is produced independently of the investment decision. Area B represents the additional social value of production for values of the cost shock where the good is only produced if the agent invests.

2.3 Monitoring investment

In this section, I analyze the principal's optimal contract when she monitors the agent's investment decision. For now, I assume that the principal monitors the investment and the cost shock. In that case, she only needs to elicit the investment cost from the agent. I solve for the optimal allocation. I then show that this allocation can be implemented with the same expected transfers even if the principal monitors only the investment and not the cost shock. This shows that the principal has nothing to gain from monitoring the cost shock when she is also monitoring the investment decision.

The principal offers the agent a menu of two contracts. The first contract requires the agent to invest and fixes a probability of production $q_1(\varepsilon)$ and an expected transfer $t_1(\varepsilon)$ as functions of the shock. The second contract requires the agent not to invest and fixes the probability of production $q_0(\varepsilon)$ and the expected transfer $t_0(\varepsilon)$. I denote the expected payoff of the contract $(q_x(\varepsilon), t_x(\varepsilon))$ – gross investment costs – by

$$U_x \equiv \int_0^1 (t_x(\varepsilon) - c_x(\varepsilon)q_x(\varepsilon))\, d\varepsilon, \tag{2.1}$$

for $x \in \{0, 1\}$. An agent with investment cost κ chooses the first contract if and only if $U_1 - \kappa \geq U_0$. I denote by $\hat{\kappa}$ the threshold at which the agent is indifferent between the two contracts. The threshold thus satisfies $\hat{\kappa} = U_1 - U_0$. The agent participates if at least one of the contracts gives him a higher payoff than his outside option. The principal's expected payoff from the menu of contracts is

$$\Pi \equiv F(\hat{\kappa}) \int_0^1 (vq_1(\varepsilon) - t_1(\varepsilon))\, d\varepsilon + (1 - F(\hat{\kappa})) \int_0^1 (vq_0(\varepsilon) - t_0(\varepsilon))\, d\varepsilon.$$

The first term represents the expected payoff if the agent invests, the second term is the expected payoff if the agent does not invest. Using equation (2.1), the principal's payoff can be expressed as a function of the probability of production, the investment threshold, and the expected payoffs of the agent from the two contracts. Using furthermore the relationship between expected payoffs and the investment threshold, the principal's expected payoff can be written as

$$\begin{aligned} \tilde{\Pi} \equiv F(\hat{\kappa}) &\left(\int_0^1 (v - c_1(\varepsilon))q_1(\varepsilon)d\varepsilon - \hat{\kappa} \right) \\ &+ (1 - F(\hat{\kappa})) \left(\int_0^1 (v - c_0(\varepsilon))q_0(\varepsilon)d\varepsilon \right) - U_0. \end{aligned} \tag{2.2}$$

The principal chooses the probabilities of production $q_1(\cdot)$ and $q_0(\cdot)$ and the investment threshold $\hat{\kappa}$ in order to maximize her expected payoff – subject to the participation constraint $U_0 \geq 0$. It is optimal for the principal to make the participation constraint of the non-investing agent binding. Furthermore, the principal procures the good from the agent if and only if it is efficient and sets $q_x^*(\varepsilon) \equiv \mathbf{1}(\varepsilon \leq \varepsilon_x^*)$ for $x \in \{0, 1\}$. The optimal investment threshold denoted by κ^i satisfies

$$\kappa^* = \int_0^{\varepsilon_1^*} (v - c_1(\varepsilon))d\varepsilon - \int_0^{\varepsilon_0^*} (v - c_0(\varepsilon))d\varepsilon = \kappa^i + \frac{F(\kappa^i)}{f(\kappa^i)}.$$

and $\kappa^i < \kappa^*$. The principal induces investment if the social surplus generated by investment exceeds the *virtual* investment costs. I denote by Π^i the payoff that the principal achieves. In contrast to the case with complete information, the principal needs to give an information rent to the agent for the private information on the investment cost. The principal faces an efficiency-rent extraction trade-off and distorts the investment decision. Whereas if the principal chooses efficient production conditional on the investment, the induced investment decision leads to underinvestment.

The optimal allocation can be implemented at the same revenue by a menu of contracts under which the production decision is delegated to the agent. The first contract prescribes investment, the second contract prohibits investment. In both contracts the agent decides whether to produce the good at price v. The first contract demands an initial payment of

$$T_1 = \int_0^{\varepsilon_1^*} (v - c_1(\varepsilon))d\varepsilon - \kappa^i.$$

The initial payment of the second contract is

$$T_0 = \int_0^{\varepsilon_0^*} (v - c_0(\varepsilon))d\varepsilon.$$

These contracts implement the principal's optimal investment decisions and allocations. As the principal can delegate the production decision to the agent, the contracts do not require the principal to observe the shock. I state this first result as a proposition.

Proposition 2.1. *If investment is monitored, the principal has nothing to gain from monitoring the shock.*

Because the shock is statistically independent of the investment cost, the principal can extract the private information on the shock from the agent without giving up any rent. Therefore, the principal cannot increase her payoff by monitoring the shock in addition to the investment. This reflects the result by Baron and Besanko (1984). In the following sections I show that the situation is different if investment is unobservable.

2.4 Monitoring the shock

In the following I suppose that the principal monitors only the shock. Under this monitoring policy, the optimal contract needs to give the agent incentives to reveal his costs of investment and to take the right investment decision. I show that moral hazard concerning the investment decision may be irrelevant. In this case, the principal can achieve the same payoff – gross monitoring costs – as in the case where she monitors investment. If moral hazard is relevant, the principal optimally chooses to introduce underproduction by non-investing agents in order to reduce rent payments to investing agents. In this case, the optimal contract features investment *and* production distortions.

The principal offers a menu of two contracts. One contract should be chosen by the agent if he invests, and the other contract is for the agent if he does not invest. Both contracts fix a probability of production $q_x(\varepsilon)$ and an expected transfer $t_x(\varepsilon)$ for $x \in \{0,1\}$.[9] Denote by K_x the set of all values of the investment cost for which the agent takes the investment decision x in equilibrium. A menu of contracts is incentive compatible if an agent with investment costs in K_x finds it optimal to choose the contract $(q_x(\cdot), t_x(\cdot))$ and the action x. A menu of contracts is individual rational if the agent always prefers one of the contracts over rejecting the principal's offer. Formally, if the principal monitors the shock, incentive compatibility and individual rationality require

$$\int_0^1 (t_x(\varepsilon) - c_x(\varepsilon)q_x(\varepsilon))\, d\varepsilon - \kappa x \geq$$
$$\max\left\{\int_0^1 (t_{x'}(\varepsilon) - c_{x''}(\varepsilon)q_{x'}(\varepsilon))\, d\varepsilon - \kappa x'', 0\right\}$$

for $\kappa \in K_x$ and $x, x', x'' \in \{0,1\}$.

Let U_x be the agent's expected payoff from the contract $(q_x(\cdot), t_x(\cdot))$ gross investment cost. The joint condition of incentive compatibility and individual rationality can then be expressed as

$$U_x - \kappa x \geq \max\left\{U_{x'} + \int_0^1 (c_{x'}(\varepsilon) - c_{x''}(\varepsilon))q_{x'}(\varepsilon)d\varepsilon - \kappa x'', 0\right\} \tag{2.3}$$

for $\kappa \in K_x$ and $x, x', x'' \in \{0,1\}$. Incentive compatibility and individual rationality can be characterized as follows.

Lemma 2.1. *If the principal monitors the shock, a menu of contracts is incentive compatible and individual rational if and only if for some* $\hat{\kappa} \in [\underline{\kappa}, \overline{\kappa}]$

1. $K_1 = [\underline{\kappa}, \hat{\kappa}]$, $K_0 = (\hat{\kappa}, \overline{\kappa}]$, *and* $U_1 - U_0 = \hat{\kappa}$;
2. $U_0 \geq 0$;
3. $\int_0^1 (c_0(\varepsilon) - c_1(\varepsilon))q_0(\varepsilon)d\varepsilon \leq \hat{\kappa}$;
4. $\int_0^1 (c_0(\varepsilon) - c_1(\varepsilon))q_1(\varepsilon)d\varepsilon \geq \hat{\kappa}$.

[9] The revelation principle due to Myerson (1986) allows us to focus on truthful direct mechanisms with random recommendations concerning the investment decision. The contracts studied here are mechanisms with deterministic recommendations, i.e., investment decisions. Under the assumptions on F and assumptions 2.1 and 2.2, this can be shown to be without loss of optimality.

If the agent optimally invests at some level of investment cost, then it is still optimal to invest if the investment costs are lower. This implies condition 1. Condition 2 guarantees individual rationality: Independently of the level of investment cost, the agent can always achieve a net benefit of U_0. If U_0 is better than the outside option, the agent always accepts one of the contract offers. Under condition 3, agents with low investment costs have no incentive to choose the contract aimed at non-investing agents and to invest nevertheless. With this deviation, the agent forgoes the rent provided under the contract $(q_1(\cdot), t_1(\cdot))$ but can keep any reduction in production costs. The rent therefore needs to exceed the reduced costs. Conversely, under condition 4, it is unprofitable for agents with high investment costs to pick the contract for investing agents and to abstain from investment. In this deviation, the agent picks the contract $(q_1(\cdot), t_1(\cdot))$ and receives the rent payment associated with it. However, he incurs a loss as he produces with the less-efficient production technology. The allocation is incentive compatible if this loss exceeds the rent.

By condition 1 in Lemma 2.1, the expected payoff of the principal can be expressed as in equation (2.2). The optimal menu of contracts for the principal is therefore the solution of the following problem:

$$\max_{U_0, \hat{\kappa}, (q_x(\cdot))_{x\in\{0,1\}}} \tilde{\Pi} \quad \text{s.t.} \quad \text{conditions 2 to 4 in Lemma 2.1} \tag{2.4}$$

This problem is equivalent to the principal's problem under the monitoring of investment and shock with the additional constraints 3 and 4. The principal cannot achieve a higher payoff (gross monitoring costs) than under investment monitoring. However, she achieves the same payoff if conditions 3 and 4 are not binding at the optimum. The optimal investment and production decisions from the optimal contract with investment monitoring always satisfy condition 4. Furthermore, condition 3 is satisfied if the distortion away from the first best investment decision is not too large:

Proposition 2.2. *If the shock is monitored, the principal can implement the investment decision characterized by the threshold κ^i and efficient production with the same expected transfers as under investment monitoring if and only if*

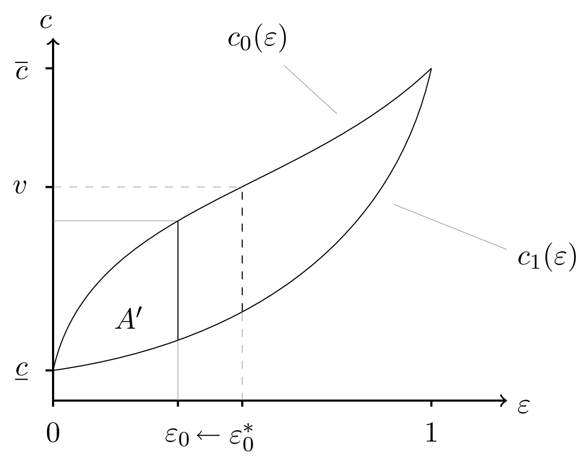
$$\kappa^i \geq \kappa^* - \int_{\varepsilon_0^*}^{\varepsilon_1^*} (v - c_1(\varepsilon))d\varepsilon. \tag{2.5}$$

If this condition is violated, *moral hazard is relevant.* In this case, the unobservability of the investment decision adds agency costs to the principal's problem. This is the case if the best deviation of an agent with low investment costs – i.e.,

$\kappa \in K_1$ – is to choose the contract $(q_0(\cdot), t_0(\cdot))$ and invest nevertheless. Under efficient production, the benefit of this deviation (gross investment cost) is given by the area A in Figure 2.1. A represents the expected cost savings that the agent can keep for himself when deviating. If this area is smaller than the rent κ^i that the agent receives in the optimal contract under investment monitoring, then moral hazard is irrelevant. Equation (2.5) expresses this comparison – using the fact that the sum of areas A and B in Figure 2.1 equal κ^*. The reformulation shows that moral hazard is irrelevant if the principal wants to implement a large investment threshold which is close to the efficient investment threshold. In contrast, if the principal aims to implement a small investment threshold, then moral hazard becomes relevant.

If moral hazard is relevant, the principal could still implement efficient production. In this case, the principal would have to give the investing agent a rent equal to the area A in Figure 2.1. This would imply an investment threshold equal to the area A. This threshold would be higher than the optimal threshold under monitoring of the investment decision.

Figure 2.2: Optimal contract under shock monitoring



If the principal monitors the shock and moral hazard is relevant, the principal reduces the cutoff from the efficient value ε_0^* to ε_0 to reduce the information rent of the agent from area A in Figure 2.1 to the area A'.

The principal can increase her payoff by reducing the probability of production for the agent who does not invest. As illustrated in Figure 2.2, this allows the principal to profitably reduce the rent of the investing agent to the area A'. The investment cost threshold is also affected because the agent's investment is optimal as long as the investment costs are smaller than the area A'. Note that it is never beneficial for the principal to push the threshold down to κ^i, the optimal threshold under monitoring of investment. At this threshold, the principal is indifferent between trading efficiently with an investing agent or with a non-investing agent. However, production with non-investing agents is inefficient under the threshold κ^i if moral hazard is relevant. The principal therefore prefers a strictly higher threshold. The optimal threshold is also strictly smaller than the efficient investment threshold κ^*. This follows from the observation that the area A' in Figure 2.2 is smaller than the area A in Figure 2.1, whereas the efficient investment threshold equals the sum of the areas A and B in Figure 2.1. It follows that the optimal contract under shock monitoring and the relevant moral hazard includes inefficient production by non-investing agents and a smaller distortion in the investment decision than in the optimal contract with investment monitoring.

In order to characterize the optimal contract, it is helpful to define – for any given incentive compatible and individual rational menu of contracts that implements an investment threshold $\hat{\kappa}$ – the principal's gain from investment by the agent with investment cost $\kappa \leq \hat{\kappa}$:

$$G(q_1(\cdot), q_0(\cdot), \hat{\kappa}) \equiv \int_0^1 (v - c_1(\varepsilon))q_1(\varepsilon)d\varepsilon - \hat{\kappa} - \int_0^1 (v - c_0(\varepsilon))q_0(\varepsilon)d\varepsilon. \qquad (2.6)$$

If an agent with investment cost κ invests, the principal can extract the expected social surplus of $\int_0^1 (v - c_1(\varepsilon))q_1(\varepsilon)d\varepsilon - \kappa$, reduced by the rent $\hat{\kappa} - \kappa$. If the same agent does not invest, the principal does not give an information rent and receives the social surplus of $\int_0^1 (v - c_0(\varepsilon))q_0(\varepsilon)d\varepsilon$. $G(\cdot, \cdot, \cdot)$ is therefore the part of the social value of the investment which the principal can appropriate.

A potential deviation of an agent with a low investment cost is the following: Select contract $(q_0(\cdot), t_0(\cdot))$ instead of contract $(q_1(\cdot), t_1(\cdot))$ and choose the investment decision $x = 1$. The agent can then keep the reduction in production costs due to the investment for himself. If moral hazard is relevant then this deviation is attractive and the agent has to be given an information rent equal to the reduction in production costs. Formally, condition 3 of Lemma 2.1 is binding in the optimal contract. The investing agent therefore receives a rent (gross investment cost) of

$$U_1^s(q_0(\cdot)) \equiv \int_0^1 (c_0(\varepsilon) - c_1(\varepsilon))q_0(\varepsilon)d\varepsilon.$$

This rent is equal to the investment threshold by condition 1 in Lemma 2.1. With slight abuse of notation, I denote by $U_1^s(\varepsilon_0)$ the agent's rent for $q_0(\varepsilon) = \mathbf{1}(\varepsilon \leq \varepsilon_0)$, and by

$$u_1^s(\varepsilon_0) = c_0(\varepsilon_0) - c_1(\varepsilon_0)$$

the first derivative. $u_1^s(\varepsilon_0)$ is the marginal change in the agent's rent if the good is produced for a shock of size ε_0. The principal maximizes the virtual surplus–which is the difference between the social surplus and the rent of the agent. Under the following assumption, the virtual surplus is decreasing in the cost shock.

Assumption 2.1. $v - c_0(\varepsilon) - (F(\kappa^*)/(1 - F(\kappa^*))) \cdot u_1^s(\varepsilon)$ *is decreasing in* ε.[10]

The principal then chooses the following menu of contracts under shock monitoring.

Proposition 2.3. *If the shock is monitored and Assumption 2.1 is satisfied, the principal achieves an optimal payoff* Π^s *through the menu of contracts* $\{(q_1^s(\cdot), t_1^s(\cdot)), (q_0^s(\cdot), t_0^s(\cdot)\}$ *and the investment threshold* κ^s*:*

1. *If moral hazard is irrelevant, then production is efficient and the investment threshold is the same as with monitoring of the investment:* $q_x^s(\cdot) = q_x^*(\cdot)$ *for* $x \in \{0, 1\}$*, and* $\kappa^s = \kappa^i$*.*

2. *If moral hazard is relevant, then production is efficient if the agent invests. If the agent does not invest, there is underproduction. The investment threshold is lower than efficient and higher than with monitoring of the investment:*

$$q_1^s(\cdot) = q_1^*(\cdot) \quad \text{and} \quad q_0^s(\varepsilon) = \mathbf{1}(\varepsilon \leq \varepsilon_0^s) \quad \text{with} \quad \varepsilon_0^s < \varepsilon_0^*; \tag{2.7}$$

$$\kappa^s = \int_0^{\varepsilon_0^s} (c_0(\varepsilon) - c_1(\varepsilon)) d\varepsilon \in (\kappa^i, \kappa^*); \tag{2.8}$$

$$\begin{aligned}(1 - F(\kappa^s))(v - c_0(\varepsilon_0^s)) + f(\kappa^s) G(q_1^s(\cdot), q_0^s(\cdot), \kappa^s) u_1^s(\varepsilon_0^s) \\ = F(\kappa^s) u_1^s(\varepsilon_0^s).\end{aligned} \tag{2.9}$$

In both cases, optimal transfers satisfy $\int_0^1 t_x^s(\varepsilon) d\varepsilon = \int_0^1 (v - c_x(\varepsilon)) q_x^s(\varepsilon) d\varepsilon - x \cdot \kappa^s$ *for* $x \in \{0, 1\}$*.*

Moral hazard connects the production decision of non-investing agents and the investment cutoff. As illustrated in Figure 2.2, the principal can reduce the production probability of non-investing agent below the efficient cutoff. This reduces the information rent of the agent, but it also reduces the efficiency of production and the probability that the agent invests.

[10] This assumption is similar to standard assumptions made in the literature on sequential screening (Courty and Li, 2000). It is satisfied if κ^* is small enough.

Formally, the principal's trade-off can be seen from equation (2.9): The left-hand side captures the marginal beneficial effects on the principal's payoff when the good is procured for the shock ε and the right-hand side represents the marginal adverse effects. The first term on the left-hand side is the marginal increase in social surplus. The term on the right-hand side is the marginal increase in the agent's rent. The marginal effect on gross rents increases the fraction of agents who invest. This effect is beneficial for the principal and is captured by the second term on the left-hand side. In contrast to a standard adverse selection problem with an efficient and an inefficient type, the fraction of the efficient, i.e., investing, agents is endogenous to the mechanism.

2.5 No monitoring

In this section, I analyze the principal's optimal contract when there is no monitoring. In this case, the principal needs to elicit information on investment costs and the shock from the agent. At the same time, the optimal contract needs to provide incentives to the agent to take the right investment decision.

I show that without monitoring, the principal achieves a strictly lower payoff – gross monitoring costs – than by using either monitoring technology. Both sources of agency costs, moral hazard concerning the investment decision, and adverse selection concerning the cost shock, are therefore always relevant. The optimal contract induces underinvestment and underproduction of non-investing agents.

The principal offers a menu of two contracts. The first contract targets the agent who is making the investment, the second contract is designed for the agent who does not invest. Both contracts specify a probability of production $q_x(\varepsilon')$ and an expected transfer $t_x(\varepsilon')$ as functions of a report ε' on the shock for both investment decisions $x \in \{0, 1\}$.[11] K_x be the set of investment cost values for which the agent takes the decision x in equilibrium.

Incentive compatibility regarding the shock requires the following: A truthful report on the cost shock has to be optimal for an agent who has selected the correct contract and the appropriate investment decision. Formally, this implies

$$t_x(\varepsilon) - c_x(\varepsilon)q_x(\varepsilon) \geq t_x(\varepsilon') - c_x(\varepsilon)q_x(\varepsilon') \tag{2.10}$$

for all $\varepsilon, \varepsilon' \in [0, 1]$ and $x \in \{0, 1\}$.

Incentive compatibility regarding the whole menu of contracts requires that the agent with investment cost in K_x chooses the investment decision x, the contract

[11] See comment in footnote 9.

$(q_x(\cdot), t_x(\cdot))$, and reports ε truthfully. The menu of contracts is individually rational if the agent always prefers one of the contracts to his outside option. The joint condition of incentive compatibility and individual rationality under no monitoring can be expressed as

$$\int_0^1 (t_x(\varepsilon) - c_x(\varepsilon)q_x(\varepsilon))\, d\varepsilon - \kappa x \geq \tag{2.11}$$
$$\max\left\{\int_0^1 \max_{\varepsilon' \in [0,1]} \left(t_{x'}(\varepsilon') - c_{x''}(\varepsilon)q_{x'}(\varepsilon')\right) d\varepsilon - \kappa x'', 0\right\}$$

for all $\kappa \in K_x$, $x, x', x'' \in \{0, 1\}$. For the characterization of this constraint, it is helpful to make the following observation: An agent who has chosen the contract for the investment decision x but has made the decision x' falsely reports on the shock. The false report is optimally chosen such that the principal has a correct belief about production costs.

Lemma 2.2. *Incentive compatibility regarding the shock implies*

$$c_x^{-1}(c_{x'}(\varepsilon)) \in \arg \max_{\varepsilon' \in [0,1]} t_x(\varepsilon') - c_{x'}(\varepsilon)q_x(\varepsilon')$$

for all $x, x' \in \{0, 1\}$ *and all* $\varepsilon \in [0, 1]$.

After a deviation, the agent optimally corrects his previous lie. Suppose the agent picks the contract $(q_x(\cdot), t_x(\cdot))$ and deviates to the investment decision x'. For a report ε', the principal believes that the agent has the production costs $c_x(\varepsilon')$, whereas the agent has the true production costs $c_{x'}(\varepsilon)$. Lemma 2.2 implies that the agent optimally chooses ε' such that the principal holds the correct belief about the agent's production cost, i.e., $c_x(\varepsilon') = c_{x'}(\varepsilon)$. This result is most intuitive for the case where each contract stipulates a strike price r_x which the agent receives if the good is produced. On the equilibrium path, the agent makes a report that induces production if $c_x(\varepsilon) \leq r_x$. On a deviation path, the agent makes such a report if $c_{x'}(\varepsilon) \leq r_x$. It is then straightforward to see that the report ε' such that $c_x(\varepsilon') = c_{x'}(\varepsilon)$ is optimal for the agent on the deviation path.

Using the result from Lemma 2.2, incentive compatibility and individual rationality can be characterized.

Lemma 2.3. *If the principal does not monitor, a menu of contracts is incentive compatible and individual rational if and only if for some* $\hat{\kappa} \in [\underline{\kappa}, \overline{\kappa}]$

1. $K_1 = [\underline{\kappa}, \hat{\kappa}]$, $K_0 = (\hat{\kappa}, \overline{\kappa}]$, *and* $U_1 - U_0 = \hat{\kappa}$;

2. $U_0 \geq 0$;

3. $\int_0^1 c_0'(\varepsilon)q_0(\varepsilon)(c_1^{-1}(c_0(\varepsilon)) - \varepsilon)d\varepsilon \leq \hat{\kappa}$;

4. $\int_0^1 c_1'(\varepsilon)q_1(\varepsilon)(\varepsilon - c_0^{-1}(c_1(\varepsilon)))d\varepsilon \geq \hat{\kappa}$;

5. $q_x(\varepsilon)$ *is decreasing in* ε *and*

$$t_x(\varepsilon) = c_x(\varepsilon)q_x(\varepsilon) + t_x(1) - c_x(1)q_x(1) + \int_{\varepsilon}^{1} c_x'(z)q_x(z)dz.$$

Conditions 3, 4, and 5 differ from Lemma 2.1. Conditions 3 and 4 reflect that the agent optimally lies about the shock on a deviation path where the agent takes a different investment decision than in equilibrium. Condition 3 ensures that an agent with a low investment cost has no incentive to select contract $(q_0(\cdot), t_0(\cdot))$, to invest nevertheless, and to combine this deviation with a false report on ε. The left-hand side of condition 3 corresponds to the areas A' and B' in Figure 2.3, when the probability of production is $q_0(\varepsilon_0) = \mathbf{1}(\varepsilon \leq \varepsilon_0)$. The area A' represents the cost reduction which the agent realizes for values of the cost shock where the good is produced under both contracts. The area B' is an additional gain, since the probability of production is higher on the equilibrium path due to the opportunity to misreport the cost shock. Similarly, under condition 4 the agent with high investment costs does not gain from choosing the contract $(q_1(\cdot), t_1(\cdot))$, making the investment decision $x = 0$, and misreporting the cost shock. Condition 5 follows from standard monotonicity and revenue equivalence requirements that are necessary and sufficient for incentive compatibility regarding the shock.

Due to condition 1, the principal's payoff from an incentive compatible menu of contracts can be expressed as in equation (2.2). The principal's problem is then

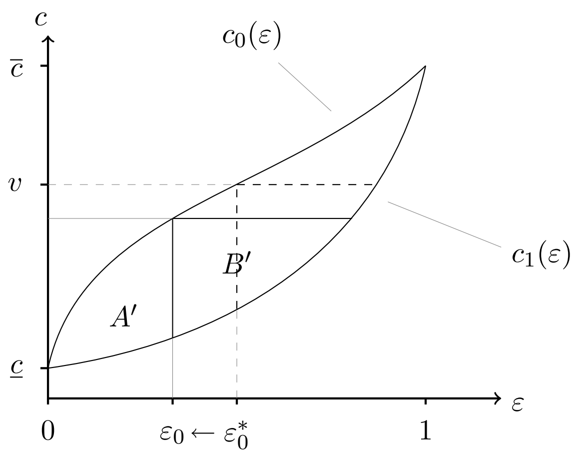
$$\max_{U_0, \hat{\kappa}, (q_x(\cdot))_{x\in\{0,1\}}} \tilde{\Pi} \quad \text{s.t.} \quad \text{conditions 2 to 5 in Lemma 2.3.} \tag{2.12}$$

Can the principal still implement the investment and production decisions from the optimal contracts with monitoring? This turns out to be impossible.

Proposition 2.4. *Without monitoring, efficient production* $q_x^*(\cdot)$ *for* $x \in \{0, 1\}$ *is incentive compatible only if the investment threshold is efficient:* $\hat{\kappa} = \kappa^*$. *Furthermore, the optimal investment and production decisions under monitoring the shock* $\{q_1^s(\cdot), q_0^s(\cdot), \kappa^s\}$ *do not satisfy the joint condition of incentive compatibility and individual rationality without monitoring.*

If the principal offers a contract that stipulates efficient production, incentive compatibility requires the agent to be willing to produce after either investment decision and to make a report that induces production as long as $c_x(\varepsilon) \leq v$. This implies that an agent who deviates by choosing the menu $(q_0(\cdot), t_0(\cdot))$ and the investment decision $x = 1$, finds it optimal to make a report $\varepsilon' \leq \varepsilon_0^*$ and to induce production as long as $c_1(\varepsilon) \leq v$, i.e., as long as production is efficient. On this deviation path, the agent receives the whole social surplus generated through the investment. This corresponds to the sum of the areas A and B in Figure 2.1. The principal could still implement efficient production but has to leave a rent equal to $A + B$ to an investing agent. Therefore, the agent will invest as long as his investment cost lies below the efficient investment cost threshold κ^*.

Figure 2.3: Optimal contract under no monitoring



Without monitoring the principal reduces the cutoff from the efficient value ε_0^* to ε_0 to reduce the information rent of the agent from area $A + B$ in Figure 2.1 to the area $A' + B'$.

Suppose now that the principal implements the production decisions from the optimal contract under monitoring the shock. Without monitoring, this results in a strictly higher investment threshold than under shock monitoring. Figure 2.3 illustrates this result: Given the production threshold ε_0, investing agents receive a rent (gross investment costs) equal to area A' if the principal monitors the shock. If the principal does not monitor, the rent increases to areas A' and B'. The optimal investment threshold under monitoring the shock is therefore no longer feasible.

In the optimal contract without monitoring, the principal introduces underproduction for non-investing agents. As illustrated in Figure 2.3, this reduces the information rent of an investing agent to the sum of areas A' and B'. The investment cost threshold equals the sum of these areas and therefore lies below the efficient threshold κ^*. However, it is never optimal for the principal to push the investment threshold below the optimal level with monitoring of investment κ^i, as production by non-investing agents is inefficient.

It follows that the agent's most attractive deviation strategy is to select the contract for non-investing agents, invest nevertheless, and misreport the cost shock such that the principal holds the correct belief about production costs. Formally, this implies that condition 3 in Lemma 2.3 is satisfied with equality in the optimal menu of contracts. The investing agent's rent, gross investment cost, is therefore determined by the non-investing agents' probability of production and given by

$$U_1^n(q_0(\cdot)) = \int_0^1 c_0'(\varepsilon)q_0(\varepsilon)(c_1^{-1}(c_0(\varepsilon)) - \varepsilon)d\varepsilon.$$

I denote by $U_1^n(\varepsilon_0)$ the agent's rent for $q_0(\varepsilon) = \mathbf{1}(\varepsilon \leq \varepsilon_0)$, and by

$$u_1^n(\varepsilon_0) = c_0'(\varepsilon)(c_1^{-1}(c_0(\varepsilon)) - \varepsilon)$$

the first derivative. I make an assumption that ensures that the principal benefits from a lower shock in the optimal contract. Technically speaking, this assumption ensures that the virtual surplus is decreasing in the shock ε.

Assumption 2.2. *$v - c_0(\varepsilon) - (F(\kappa^*)/(1 - F(\kappa^*))) \cdot u_1^n(\varepsilon)$ is decreasing in ε.*[12]

Without monitoring, the principal offers the following menu of contracts.

Proposition 2.5. *If there is no monitoring and Assumption 2.2 is satisfied, the principal achieves an optimal payoff Π^n through the menu of contracts $\{(q_1^n(\cdot), t_1^n(\cdot)), (q_0^n(\cdot), t_0^n(\cdot))\}$ and the investment threshold κ^n: If the agent invests, production is efficient. If the agent does not invest, there is underproduction. The investments threshold is lower than efficient and higher than with investment monitoring. For $x \in \{0, 1\}$*

$$q_1^n(\cdot) = q_1^*(\cdot) \quad \text{and} \quad q_0^n(\varepsilon) = \mathbf{1}(\varepsilon \leq \varepsilon_0^n) \quad \text{with} \quad \varepsilon_0^n < \varepsilon_0^*; \tag{2.13}$$

$$\kappa^n = \int_0^{\varepsilon_0^n} c_0'(\varepsilon)(c_1^{-1}(c_0(\varepsilon)) - \varepsilon)d\varepsilon \in (\kappa^i, \kappa^*); \tag{2.14}$$

$$\begin{aligned}(1 - F(\kappa^n))(v - c_0(\varepsilon_0^n)) + f(\kappa^n)G(q_1^n(\cdot), q_0^n(\cdot), \kappa^n)u_1^n(\varepsilon_0^n) \\ = F(\kappa^n)u_1^n(\varepsilon_0^n);\end{aligned} \tag{2.15}$$

$$t_x^n(\varepsilon) = c_x(\varepsilon)q_x^n(\varepsilon) + \int_\varepsilon^1 c_x'(z)q_x^n(z)dz - \int_0^1 zc_x'(z)q_x^n(z)dz + \kappa^n \cdot x. \tag{2.16}$$

The principal faces again an efficiency-rent extraction trade-off. If the principal reduces non-investing agents' probability of production below the efficient level then the rent payment to investing agents can be reduced. This is illustrated in Figure 2.3. However, this reduces the efficiency of the production decision

[12]Like Assumption 2.1, this is satisfied if κ^* is small enough.

for non-investing agents and decreases the fraction of investing agents. Formally, the principal's trade-off is reflected in equation (2.15). On the left-hand side are the marginal beneficial effects of production by non-investing types at shock ε: a marginal increase in efficiency and the marginal gain from the increase in the fraction of investing agent types. The right-hand side reflects the marginal increase in rent payments that have to be given to the agent. In the next section, I analyze the implications for the principal's optimal monitoring policy. Assumptions 2.1 and 2.2 are maintained throughout the section.

2.6 The optimal monitoring policy

It is straightforward to derive the implications for the optimal choice of a monitoring policy.

Proposition 2.6. *The principal's optimal monitoring policy is given by:*

- *no monitoring if* $C^i \geq \Pi^i - \Pi^n$ *and* $C^s > \Pi^s - \Pi^n$*;*
- *monitoring the shock if* $C^s \leq \min\left\{C^i + \Pi^s - \Pi^i, \Pi^s - \Pi^n\right\}$*;*
- *monitoring the investment if* $C^i < \min\left\{C^s + \Pi^i - \Pi^s, \Pi^i - \Pi^n\right\}$.

There exist monitoring costs $C^i > 0$ *and* $C^s > 0$ *such that each monitoring regime can be optimal.*

The principal finds her optimal monitoring policy by comparing the payoffs net of monitoring costs under the different monitoring regimes. This gives the conditions presented in the proposition. Apart from the policy where the principal monitors the investment and the shock, each monitoring regime is optimal for some values of the monitoring costs C^i and C^s.

The principal optimally monitors either the investment or the shock as long as monitoring costs are low enough. On one hand, this result builds on Proposition 2.1: if the principal can control the investment decision through monitoring, there is no additional gain from observing the shock. On the other hand, the result is implied by the fact that the principal achieves a strictly higher payoff under monitoring of the shock than without monitoring. If the principal does not monitor then she has to give a positive rent to the agent for private information of the cost shock. Without monitoring, the best deviation of the agent with low investment cost is to select the contract for non-investing agents, make the investment, and falsely report the cost shock. As Lemma 2.2 shows, the agent's false report is strict due to the fact that the investment on the deviation path is strictly different from zero. The agent can therefore secure a positive information rent for the private knowledge he acquires after signing a contract. In the discussion section, I relate this result to the insights on post-contractual information rents from the literature on dynamic mechanism design.

The next result shows that both monitoring technologies can be perfect substitutes. This is the case if moral hazard is irrelevant. In contrast, if moral hazard is relevant, then monitoring of the investment is the more effective instrument.

Proposition 2.7. *If moral hazard is irrelevant, both monitoring technologies give the same payoff gross monitoring costs. If moral hazard is relevant, the payoff gross monitoring costs is higher under investment monitoring; i.e., if equation* (2.5) *is satisfied, then* $\Pi^i = \Pi^s$*, otherwise* $\Pi^i > \Pi^s$*.*

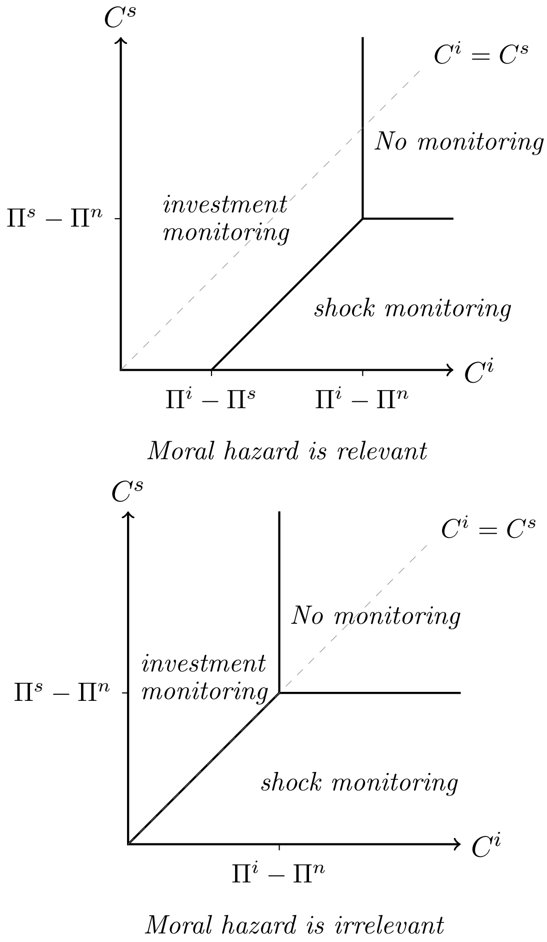
Without monitoring, the agent's optimal deviation exploits moral hazard regarding the investment and private information about the shock. The principal can directly eliminate moral hazard by monitoring the investment decision. As argued above, this also resolves the problem of private information. If moral hazard is irrelevant under shock monitoring then the principal can also eliminate both sources of agency cost by observing the shock. If moral hazard is irrelevant, then the agent's best deviation under shock monitoring does not make use of a deviation in the investment decision. In this case, both monitoring technologies are equally effective. The principal then simply chooses the monitoring technology with lower monitoring costs. If moral hazard is relevant, then monitoring the shock is not enough to eliminate the agency costs resulting from moral hazard regarding the investment. In this case, shock monitoring the shock is not as effective as investment monitoring. With shock monitoring, the agent's best deviation is to pick the contract for non-investing agents even if investment costs are low, and to make the investment nevertheless. Thus, the principal only monitors the shock if this is sufficiently cheaper than investment monitoring.

Figure 2.4 illustrates the optimal monitoring policy for the case where moral hazard is relevant and for the case where it is not.

Relevance of moral hazard

Moral hazard is relevant if the inequality (2.5) is satisfied. This condition is not formulated in terms of the fundamentals of the model. The condition can be satisfied under assumptions on the production cost functions $c_0(\cdot)$ and $c_1(\cdot)$ or the distribution function F. Here, I want to provide one such condition on the distribution function. Let $\{F_z(\kappa)\}_{z\in(\underline{z},\overline{z})}$ be a family of distribution functions of the investment cost which satisfies $F_z(\kappa) = F(\kappa - z)$ for all κ and z. Furthermore,

Figure 2.4: Optimal monitoring policy



The figure depicts the optimal monitoring policy depending on the costs of monitoring C^i and C^s for the cases where moral hazard is relevant and where it is irrelevant.

$F_{\underline{z}}(\kappa^*) = 1$ and $F_{\overline{z}}(\kappa^*) = 0$. These distribution functions are therefore generated by moving the support of distribution F. A higher z corresponds to a higher level of investment cost. I can then show the following result.

Proposition 2.8. *If the level of investment costs is high then moral hazard is irrelevant under monitoring of the shock, i.e.,* $\exists z' \in [\underline{z}, \overline{z})$*, such that inequality* (2.5) *is satisfied if* $z \geq z'$*.*

If the level of investment costs is high, then the probability that the agent will invest is small. The principal has to give an information rent to the agent only in this case. As the probability to pay this rent is small, the principal optimally induces a minor distortion in the investment decision. If this distortion is small, the information rent of investing agents is large. Thus, all deviations for which the agent selects the contract for non-investing agents become relatively less attractive. If the level of investment costs is large enough, moral hazard therefore becomes irrelevant.

Monitoring and efficiency

Monitoring often has ambiguous effects on efficiency. If the principal moves from no monitoring to investment monitoring, production efficiency increases whereas investment efficiency decreases. The optimal contracts under monitoring of investment and monitoring the shock lead either to identical investment and production decisions or imply lower investment efficiency and higher production efficiency under monitoring of investment.

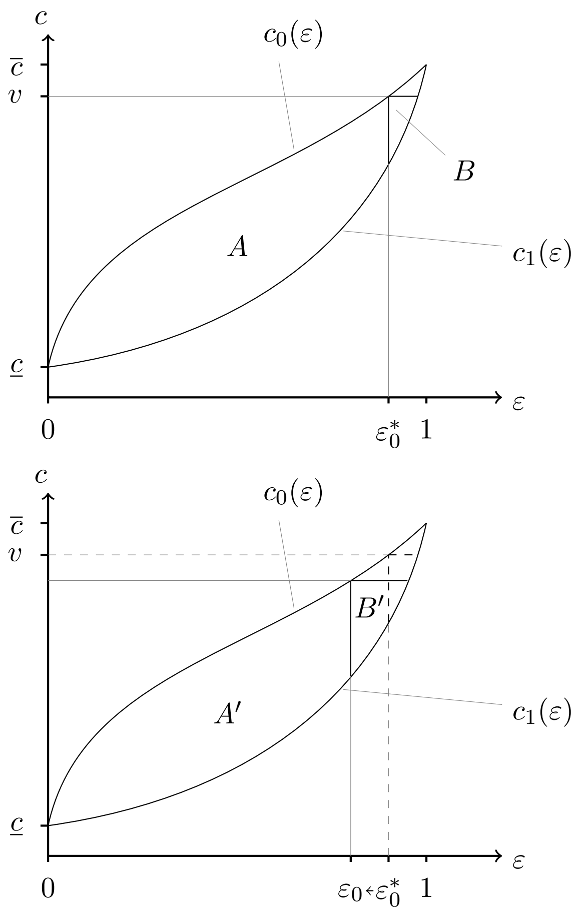
However, shock monitoring can have unambiguously negative effects on efficiency.

Proposition 2.9. *If the principal's value for the good is high, then investment and production are more efficient without monitoring than with monitoring the shock: There exists* $\hat{v} \in (\underline{c}, \overline{c})$ *such that* $\varepsilon_0^s < \varepsilon_0^n$ *and* $\kappa^s < \kappa^n$ *for* $v > \hat{v}$*.*

Perhaps surprisingly, private learning of the shock increases total efficiency if the value of the good is high. Privacy of post-contractual information may therefore increase the efficiency in optimal contracts.

In optimal contracts without monitoring and with shock monitoring, the principal reduces the information rent of the investing agent by trading less frequently with the non-investing agent. The effect of a small reduction in the probability of production on the information rent differs between the two cases. Proposition 2.9 follows from the fact that the marginal effect on rents is smaller without monitoring for high realizations of the shock. This is illustrated in Figure 2.5: When the production threshold for the non-investing agent is reduced from the efficient level ε_0^* to the smaller level ε_0 then the agent's rent decreases from A to A' under monitoring of the shock. If the principal does not monitor, the rent decreases from $A + B$ to $A' + B'$. Note that for high values of v, B' is larger than B. The reduction of the production threshold is therefore less effective in reducing information rents without monitoring than with shock monitoring. Thus, the principal finds it optimal to induce a smaller production distortion without monitoring. The agent therefore receives a higher total value of information rent without monitoring, which implies a higher investment threshold. It follows that the optimal contract is more efficient without monitoring.

Figure 2.5: The beneficial effect of privacy of information on efficiency



The upper part of the figure is equivalent to Figure 2.1 for the case where v is high. The lower part shows the effect of a reduction of the cutoff from ε_0^* to ε_0. With shock monitoring this reduces the agent's information rent from A to A'. Without monitoring it reduces the rent from $A + B$ to $A' + B'$. As B' is larger than B, the reduction in rent is larger with shock monitoring.

2.7 Discussion

Relevance of private post-contractual information

In the literature on dynamic mechanism design, a central question concerns the relevance of the privacy of post-contractual information. Baron and Besanko (1984), Eső and Szentes (2007a,b) and Pavan et al. (2014) show that in many setups of dynamic adverse selection, the principal can costlessly elicit private information which the agent learns after contracting. Baron and Besanko (1984) and Eső and Szentes (2015) argue that this insight also holds in models of dynamic adverse selection and moral hazard.

In contrast, Krähmer and Strausz (2015) show in a model of pure adverse selection, that the privacy of post-contractual information matters if the agent's private information learned before contracting (ex-ante type) is drawn from a discrete set. In this chapter, privacy of post-contractual information turns out to be relevant in a model of dynamic adverse selection and moral hazard. Whereas the agent's private information, i.e., the investment cost and the cost shock, are drawn from continuous distributions, the agent chooses an unobservable action from a discrete set. As in Krähmer and Strausz (2015), the agent's best deviation strategy under no monitoring includes a strict lie about post-contractual information. As such a deviation is not feasible if post-contractual information is publicly observed, privacy of post-contractual information is relevant. However, there are a two notable differences to the result in Krähmer and Strausz (2015). First, the envelope theorem[13] can be applied in the setup of the current chapter in order to determine the agent's expected utility as a function of his ex-ante type. This is not feasible in Krähmer and Strausz (2015) due to the discreteness of the agent's ex-ante type. Second, Krähmer and Strausz (2015) show that the principal's optimal allocation under observable post-contractual information can still be implemented if ex-post information is unobservable, though at a lower revenue. In this chapter, I show in Proposition 2.4, that the optimal allocation with shock monitoring – consisting of the investment and production decisions – cannot be implemented without monitoring. Privacy of post-contractual information restricts the set of implementable allocations for the principal, but does not change the principal's ability to extract surplus from a given allocation, as in Krähmer and Strausz (2015). This also explains the relation to Eső and Szentes (2015). They study a general model of dynamic adverse selection and moral hazard. They show that the principal can achieve the same revenue under public and private post-contractual information, as long as the optimal allocation and the optimal actions under public information remain implementable with private information. In the model of the current chapter, the principal cannot implement the optimal production and investment decisions with shock monitoring if she does not monitor (Proposition 2.4). The precondition of the result by Eső and Szentes (2015) is therefore not satisfied in this model.

Binary investment decision and fixed costs of investment

All results were presented using a simple model with a binary investment decision. A binary investment decision reflects non-convexities in the investment decision. In practice, many investment decisions are non-convex. If the investment represents

[13] Theorem 2 in Milgrom and Segal (2002)

the adoption of a new technology, the supplier is likely to face fixed costs of learning the new technology. Astebro (2004) empirically documents such fixed costs of learning. More generally, *lumpy* investment behavior is often attributed to fixed costs of investment and such non-convex investment behavior is widely observed among firms.[14]

2.8 Conclusion

What kind of information should government institutions monitor in order to manage public contracts efficiently? This chapter provides an analysis of this question in a principal-agent model where the principal can decide whether to monitor an agent's investment in cost reduction or a shock to production costs that the agent learns after contracting. I show that the principal optimally monitors either the investment decision or the cost shock. Under investment monitoring, the principal achieves at least the same payoff – gross monitoring costs – as under shock monitoring. However, moral hazard may be irrelevant under shock monitoring. In this case, both monitoring technologies are equally effective.

The results suggest that government institutions should monitor endogenous cost factors (such as investments) rather than exogenous cost factors (such as cost shocks) if the costs of monitoring are similar across outcomes. However, if exogenous cost factors are cheaper to monitor then doing so can give a government institution the same control over private suppliers at lower costs.

[14] Doms and Dunne (1998) provide empirical evidence and Caplin and Leahy (2010) survey the history of applications of the (S, s)-model, an investment model with fixed costs.

2.9 Appendix to Chapter 2

Proof of Proposition 2.1 The proof follows from the discussion in the main text. □

Proof of Lemma 2.1 Condition 1 is equivalent to (2.3) for $x = 1$, $x' = x'' = 0$, and for $x = 0$, $x' = x'' = 1$. Condition 2 is equivalent to (2.3) for $x = x' = x'' = 0$. Conditions 3 and 4 are implied by condition 1, (2.3) for $x = x'' = 1$, $x' = 0$, and (2.3) for $x = x'' = 0$, $x' = 1$. Conversely, conditions 1, 3, and 4 imply (2.3) for $x = x'' = 1$, $x' = 0$, and for $x = x'' = 0$, $x' = 1$. Conditions 1 and 2 imply (2.3) for $x = x' = x'' = 1$. It remains to prove that conditions 1 to 4 imply (2.3) for $x = x' = 1$, $x'' = 0$, and for $x = x' = 0$, $x'' = 1$. The first constraint can be rewritten as

$$\int_0^1 (c_0(\varepsilon) - c_1(\varepsilon))q_1(\varepsilon)d\varepsilon \geq \kappa$$

for $\kappa \in K_1$. By condition 1, $\kappa \leq \hat{\kappa}$ for $\kappa \in K_1$, and the constraint is implied by condition 4. The second constraint can be written as

$$\int_0^1 (c_0(\varepsilon) - c_1(\varepsilon))q_0(\varepsilon)d\varepsilon \leq \kappa$$

for $\kappa \in K_0$. By condition 1, $\kappa \geq \hat{\kappa}$ for $\kappa \in K_0$. The constraint is therefore implied by condition 3. □

Proof of Proposition 2.2 Plug $\hat{\kappa} = \kappa^i$ and efficient production rules in conditions 3 and 4 of Lemma 2.1. The left-hand side of condition 3 can be rewritten as

$$\begin{aligned}
&\int_0^{\varepsilon_0^*} (c_0(\varepsilon) - c_1(\varepsilon))d\varepsilon \\
&= \int_0^{\varepsilon_1^*} (v - c_1(\varepsilon))d\varepsilon - \int_0^{\varepsilon_0^*} (v - c_0(\varepsilon))d\varepsilon - \int_{\varepsilon_0^*}^{\varepsilon_1^*} (v - c_1(\varepsilon))d\varepsilon \\
&= \kappa^* - \int_{\varepsilon_0^*}^{\varepsilon_1^*} (v - c_1(\varepsilon))d\varepsilon.
\end{aligned}$$

Condition 3 is therefore satisfied if the condition in the lemma is satisfied. The left-hand side of condition 4 can by the same steps be written as $\kappa^* - \int_{\varepsilon_0^*}^{\varepsilon_1^*}(v - c_0(\varepsilon))d\varepsilon$ which is greater than κ^*. Condition 4 is therefore always satisfied. Note that conditions 3 and 4 do not restrict the choice of U_0 and U_1. One can therefore set U_0 and U_1 as in the optimal contract under monitoring of the investment, so that expected transfers are identical in both cases. □

Proof of Proposition 2.3 If moral hazard is irrelevant, the result is immediate. Suppose moral hazard is relevant and consider the principal's problem (2.4). Neglect condition 4 of Lemma 2.1. It is optimal to set $U_0 = 0$ and $q_1(\cdot) = q_1^*(\cdot)$.

I now show that the optimal threshold κ^s satisfies $\kappa^s \in (\kappa^i, \kappa^*)$. Suppose $\kappa^s \leq \kappa^i$. As moral hazard is relevant, $q_0(\cdot) = q_0^*(\cdot)$ is not feasible. It follows that the marginal gain of an additionally investing agent type exceeds the marginal information rent: $G(q_1^*, q_0, \kappa^s) - F(\kappa^s)/f(\kappa^s) > G(q_1^*, q_0^*, \kappa^i) - F(\kappa^i)/f(\kappa^i) = 0$. It is therefore profitable to increase κ^s which also relaxes condition 3. It follows that $\kappa^s > \kappa^i$. Suppose next $\kappa^s \geq \kappa^*$. The proof of Proposition 2.2 implies that $q_0(\cdot) = q_0^*(\cdot)$ is feasible. The marginal gain of an additionally investing agent type is then lower as the marginal information rent: $G(q_1^*, q_0^*, \kappa^s) - F(\kappa^s)/f(\kappa^s) = \kappa^* - \kappa^s - F(\kappa^s)/f(\kappa^s) \leq -F(\kappa^s)/f(\kappa^s) < 0$. Decreasing κ^s is profitable. It follows $\kappa^s < \kappa^*$.

Furthermore, note that $G(q_1^*, q_0, \kappa^s) < F(\kappa^s)/f(\kappa^s)$ in any optimum. If this does not hold it is always profitable to increase κ^s and set q_0 closer to q_0^*.

Moreover, condition 3 of Lemma 2.1 is satisfied with equality at the optimum. If condition 3 is satisfied with strict inequality, it is possible to set $q_0(\cdot)$ closer to $q_0^*(\cdot)$ and increase the payoff.

One can therefore write the threshold $\hat{\kappa}$ as a function of $q_0^s(\cdot)$. Plugging this into (2.2) and taking the pointwise first-order derivative with respect to q_0 for any ε gives

$$f(\hat{\kappa}(q_0)) \left\{ \frac{1 - F(\hat{\kappa}(q_0))}{f(\hat{\kappa}(q_0))} (v - c_0(\varepsilon)) - \left(\frac{F(\hat{\kappa}(q_0))}{f(\hat{\kappa}(q_0))} - G(q_1^*, q_0, \hat{\kappa}(q_0)) \right) u_1^s(\varepsilon) \right\}. \tag{2.17}$$

For any fixed threshold $\hat{\kappa}$ that satisfies $\hat{\kappa} \in (\kappa^i, \kappa^*)$ and $G(q_1^*, q_0, \hat{\kappa}) < F(\hat{\kappa})/f(\hat{\kappa})$, this expression is decreasing in ε under Assumption 2.1. If $u_1^s(\varepsilon)$ is increasing, this follows from $G(q_1^*, q_0, \hat{\kappa}) < F(\hat{\kappa})/f(\hat{\kappa})$. If $u_1^s(\varepsilon)$ is decreasing, it follows from Assumption 2.1 as $F(\kappa^*)/(1-F(\kappa^*) > F(\hat{\kappa})/(1-F(\hat{\kappa}) - f(\hat{\kappa})/(1-F(\hat{\kappa})) \cdot G(q_1^*, q_0, \hat{\kappa})$. Thus, the threshold $\hat{\kappa}$ is optimally implemented by a step function $q_0(\varepsilon) = \mathbf{1}(\varepsilon \leq \varepsilon_0)$. Since κ^s satisfies $\kappa^s \in (\kappa^i, \kappa^*)$ and $G(q_1^*, q_0, \kappa^s) < F(\kappa^s)/f(\kappa^s)$, there exists a cutoff ε_0^s which optimally implements κ^s and this implies (2.8). There is a unique combination of κ^s and ε_0^s which equates the first-order derivative to zero and satisfies (2.9). This follows from κ^s being increasing in ε_0^s, $(1 - F(\hat{\kappa}))/f(\hat{\kappa})$ being increasing in κ^s, and $G(q_1^*, q_0^s, \kappa^s) - F(\kappa^s)/f(\kappa^s)$ being increasing in ε_0^s and κ^s, for $q_0^s(\varepsilon) = \mathbf{1}(\varepsilon \leq \varepsilon_0^s)$.

It remains to show that $\varepsilon_0^s < \varepsilon_0^*$. For $\varepsilon_0^s \geq \varepsilon_0^*$, there is no first-order loss from decreasing ε_0^s whereas there is a first-order gain from a lower fraction of investing agent types as $F(\kappa^s)/f(\kappa^s) - G(q_1^*, q_0^s, \kappa^s) > 0$. Finally, one can easily check that condition 4 of Lemma 2.1 is satisfied as $\kappa^s < \kappa^*$. Optimal transfers can be derived from the definitions of U_1 and U_0. □

Proof of Lemma 2.2 For $\tilde{\varepsilon}(\varepsilon) = c_x^{-1}(c_{x'}(\varepsilon))$, (2.10) implies

$$\begin{aligned} t_x(\varepsilon') - c_{x'}(\varepsilon)q_x(\varepsilon') &= t_x(\varepsilon') - c_x(c_x^{-1}(c_{x'}(\varepsilon)))q_x(\varepsilon') \\ &\leq t_x(c_x^{-1}(c_{x'}(\varepsilon))) - c_x(c_x^{-1}(c_{x'}(\varepsilon)))q_x(c_x^{-1}(c_{x'}(\varepsilon))) \\ &= t_x(\tilde{\varepsilon}(\varepsilon)) - c_{x'}(\varepsilon)q_x(\tilde{\varepsilon}(\varepsilon)) \end{aligned}$$

□

Proof of Lemma 2.3 By standard mechanism design arguments, one can show that condition 5 of the lemma is sufficient and necessary for (2.10). (2.11) can be rewritten as follows. The left-hand side equals $U_x - \kappa x$. Using Lemma 2.2 and condition 5 one can rewrite the right-hand side as follows (where I use the notation $\tilde{\varepsilon}(\varepsilon) = c_{x'}^{-1}(c_{x''}(\varepsilon))$):

$$\begin{aligned} &\int_0^1 \max_{\varepsilon'} \left(t_{x'}(\varepsilon') - c_{x''}(\varepsilon)q_{x'}(\varepsilon')\right) d\varepsilon - \kappa x'' \\ &\quad= \int_0^1 \left(t_{x'}(c_{x'}^{-1}(c_{x''}(\varepsilon))) - c_{x'}(c_{x'}^{-1}(c_{x''}(\varepsilon)))q_{x'}(c_{x'}^{-1}(c_{x''}(\varepsilon)))\right) d\varepsilon - \kappa x'' \\ &\quad= \int_0^1 \left(t_{x'}(1) - c_{x'}(1)q_{x'}(1) + \int_{\tilde{\varepsilon}(\varepsilon)}^1 c_{x'}(z)q_{x'}(z)dz\right) d\varepsilon - \kappa x'' \\ &\quad= U_{x'} + \int_0^1 \int_{\tilde{\varepsilon}(\varepsilon)}^{\varepsilon} c'_{x'}(z)q_{x'}(z)dzd\varepsilon - \kappa x'' \\ &\quad= U_x + \int_0^1 \varepsilon c'_{x'}(\tilde{\varepsilon}(\varepsilon))q_{x'}(\tilde{\varepsilon}(\varepsilon))\frac{d\tilde{\varepsilon}}{d\varepsilon}d\varepsilon - \int_0^1 \varepsilon c'_{x'}(\varepsilon)q_{x'}(\varepsilon)d\varepsilon - \kappa x'' \\ &\quad= U_x + \int_0^1 c'_{x'}(\varepsilon)q_{x'}(\varepsilon)(c_{x''}^{-1}(c_{x'}(\varepsilon)) - \varepsilon)d\varepsilon - \kappa x''. \end{aligned}$$

where the first inequality follows from Lemma 2.2, the second from condition 5, the fourth from integration by parts, and the fifth from a change of variable from ε to $\tilde{\varepsilon}$. Under condition 5, (2.11) is therefore equivalent to

$$U_x - \kappa x \geq \max\left\{U_x + \int_0^1 c'_{x'}(\varepsilon)q_{x'}(\varepsilon)(c_{x''}^{-1}(c_{x'}(\varepsilon)) - \varepsilon)d\varepsilon - \kappa x'', 0\right\}$$

for $\kappa \in K_x$ and $x, x', x'' \in \{0,1\}$. The equivalence of this condition to the conditions 1 to 4 of the lemma can be shown by taking exactly the same steps as in the proof of Lemma 2.1. □

Proof of Proposition 2.4 Plug $q_x^*(\varepsilon)$ into the left-hand sides of conditions 3 and 4 of Lemma 2.3.

$$\begin{aligned}\int_0^{\varepsilon_x^*} c_x'(\varepsilon)(\varepsilon - c_{x'}^{-1}(c_x(\varepsilon)))d\varepsilon &= \int_0^{\varepsilon_x^*} c_x'(\varepsilon)\varepsilon d\varepsilon - \int_0^{\varepsilon_x^*} c_x'(\varepsilon)c_{x'}^{-1}(c_x(\varepsilon))d\varepsilon \\ &= \int_0^{\varepsilon_x^*} c_x'(\varepsilon)\varepsilon d\varepsilon - \int_0^{\varepsilon_{x'}^*} c_{x'}'(\varepsilon)\varepsilon d\varepsilon \\ &= \int_0^{\varepsilon_x^*} (v - c_x(\varepsilon))d\varepsilon - \int_0^{\varepsilon_{x'}^*} (v - c_{x'}(\varepsilon))d\varepsilon \\ &= (-1)^{\mathbf{1}(x=1)}\kappa^*\end{aligned}$$

where the second equality follows from a change of variable and the third equality follows from integration by parts. κ^* is therefore the only threshold that is incentive compatible and individual rational under efficient production.

In order to prove the second part of the lemma I show that conditions 3 and 4 in Lemma 2.1 are less restrictive than conditions 3 and 4 in Lemma 2.3. In order to see this, note that for $\tilde{\varepsilon}(\varepsilon) = c_x^{-1}(c_{x'}(\varepsilon))$

$$\int_0^1 c_x'(\varepsilon)q_x(\varepsilon)(c_{x'}^{-1}(c_x(\varepsilon)) - \varepsilon)d\varepsilon = \int_0^1 \int_{\tilde{\varepsilon}(\varepsilon)}^{\varepsilon} c_x'(z)q_x(z)dzd\varepsilon \tag{2.18}$$

$$\begin{aligned}&= \int_0^1 (c_x(\varepsilon) - c_{x'}(\varepsilon))q_x(\varepsilon)d\varepsilon - \int_0^1 (c_x(\varepsilon) - c_x(\tilde{\varepsilon}(\varepsilon)))q_x(\varepsilon)d\varepsilon \\ &\quad + \int_0^1 \int_{\tilde{\varepsilon}(\varepsilon)}^{\varepsilon} c_x'(z)q_x(z)dzd\varepsilon \\ &= \int_0^1 (c_x(\varepsilon) - c_{x'}(\varepsilon))q_x(\varepsilon)d\varepsilon + \int_0^1 \int_{\tilde{\varepsilon}(\varepsilon)}^{\varepsilon} c_x'(z)(q_x(z) - q_x(\varepsilon))dzd\varepsilon \\ &> (<) \int_0^1 (c_x(\varepsilon) - c_{x'}(\varepsilon))q_x(\varepsilon)d\varepsilon \quad \text{for } x = 0, x' = 1 \quad (x = 1, x' = 0)\end{aligned}$$

where the last inequality holds if $q_x(\varepsilon)$ is not constant on $[0,1]$. This is the case for q_0^s and q_1^s. Since condition 3 of Lemma 2.1 is satisfied with equality for q_0^s and κ^s, the stricter condition 3 of Lemma 2.3 cannot be satisfied for q_0^s and κ^s. □

Proof of Proposition 2.5 The proof is only sketched as it essentially follows the same steps as the proof of Proposition 2.3. Consider the principal's problem defined in (2.12) and neglect conditions 4 and 5. It is optimal to set $U_0 = 0$

and $q_1(\cdot) = q_1^*(\cdot)$. Using the same arguments as in the proof of Proposition 2.3, it can be shown that the optimal investment threshold κ^a satisfies $\kappa^a \in (\kappa^i, \kappa^*)$, $G(q_1^*, q_0, \kappa^a) < F(\kappa^a)/f(\kappa^a)$, and that condition 3 is satisfied with equality. The last result allows to derive a first-order condition analogous to (2.17) with $u_1^a(\varepsilon)$ instead of $u_1^s(\varepsilon)$. By the same arguments as in the proof of Proposition 2.3, it can be derived that there is a unique optimal pair κ^a and ε_0^a with $q_0^a(\varepsilon) = \mathbf{1}(\varepsilon \leq \varepsilon_0^a)$ which satisfies (2.14) and (2.15), and $\varepsilon_0^a < \varepsilon_0^*$. Clearly $q_0^a(\varepsilon)$ is decreasing in ε, transfers can be chosen to satisfy the requirement of condition 5, and it can be shown that condition 4 is satisfied. □

Proof of Proposition 2.6 By Proposition 2.1, it is never optimal to monitor the shock and the investment as this would give the payoff Π^i which can also be achieved if only investment is monitored. The conditions for optimality are derived from the payoffs $\Pi^i - C^i$, $\Pi^s - C^s$, and Π^n that can be achieved under monitoring of investment, the shock, and no monitoring. From Proposition 2.3 it follows that $\Pi^i \geq \Pi^n$. If $\Pi^s > \Pi^n$, there exist $C^i > 0$ and $C^s > 0$ such that any of the three monitoring choices can be optimal. $\Pi^s > \Pi^n$ follows from the fact that condition 3 of Lemma 2.3 is more restrictive than condition 3 of Lemma 2.1, which is implied by (2.18). □

Proof of Proposition 2.7 The proof follows from the discussion in the main text. □

Proof of Proposition 2.8 By Proposition 2.3, $\Pi^i = \Pi^s$ if moral hazard is irrelevant and $\Pi^i > \Pi^s$ is moral hazard is relevant. It only remains to show that there exists a $z' \in [\underline{z}, \overline{z})$ such that moral hazard is relevant (i.e., (2.5) is satisfied) iff $z \geq z'$. The optimal investment threshold κ_z^i is a function of z implicitly defined by

$$\kappa_z^i + \frac{F_z(\kappa_z^i)}{f_z(\kappa_z^i)} = \kappa^*$$

From the definition of F_z it follows that $F_z(\kappa)/f_z(\kappa) = F(\kappa - z)/f(\kappa - z)$. By log-concavity of f, $F_z(\kappa)/f_z(\kappa)$ is increasing in κ and decreasing in z. This proves that κ_z^i is increasing in z. Moreover $\kappa_{\overline{z}}^i = \kappa^*$ as $F_{\overline{z}}(\kappa^*) = 0$. As $\kappa^* - \kappa_z^i$ is therefore decreasing in z and zero at $\overline{z}$, this establishes the existence of z'. □

Proof of Proposition 2.9 For $v = \overline{c}$, (2.9) and (2.15) are solved by $\varepsilon_0^s = \varepsilon_0^n = 1$. As $u_1^s(1) = u_1^n(1) = 0$, there is by (2.8) and (2.14) no first-order effect on κ^s and κ^n for v smaller but close to $\overline{c}$. However, $u_1^s(\varepsilon) > u_1^n(\varepsilon)$ for ε close to one.

This is implied by the following argument: Note that $u_1^s(1) = u_1^n(1) = 0$ and $\partial u_1^s(\varepsilon)/\partial\varepsilon = c_0'(\varepsilon) - c_1'(\varepsilon)$. For ε close to one, the following approximation holds

$$\begin{aligned}\partial u_1^n(\varepsilon)/\partial\varepsilon_0 &= c_0''(\varepsilon)(c_1^{-1}(c_0(\varepsilon)) - \varepsilon) + c_0'(\varepsilon)\left(\frac{c_0'(\varepsilon)}{c_1'(c_1^{-1}(c_0(\varepsilon)))} - 1\right)\\ &\simeq \frac{(c_0'(\varepsilon))^2}{c_1'(\varepsilon)} - c_0'(\varepsilon)\end{aligned}$$

as $c_1^{-1}(c_0(\varepsilon)) \simeq \varepsilon$ for ε close to one. It follows for ε close to one

$$\begin{aligned}\partial u_1^n(\varepsilon)/\partial\varepsilon - \partial u_1^s(\varepsilon)/\partial\varepsilon_0 &\simeq \left(\frac{(c_0'(\varepsilon))^2}{c_1'(\varepsilon)} - c_0'(\varepsilon)\right) - \left(c_0'(\varepsilon) - c_1'(\varepsilon)\right)\\ = \frac{1}{c_1'(\varepsilon)}\left(c_0'(\varepsilon)^2 - 2c_0'(\varepsilon)c_1'(\varepsilon) + c_1'(\varepsilon)^2\right) &= \frac{1}{c_1'(\varepsilon)}(c_0'(\varepsilon) - c_1'(\varepsilon))^2 > 0\end{aligned}$$

It follows by continuity that $u_1^s(\varepsilon) > u_1^n(\varepsilon)$ for ε close to one. This implies

$$\begin{aligned}&\frac{1-F(\kappa^n)}{f(\kappa^n)}(v - c_0(\varepsilon_0^s)) - u_1^n(\varepsilon_0^s)\left(\frac{F(\kappa^n)}{f(\kappa^n)} - G(q_1^*, q^s, \kappa^n)\right)\\ &\qquad\simeq \frac{1-F(\kappa^s)}{f(\kappa^s)}(v - c_0(\varepsilon_0^s)) - u_1^n(\varepsilon_0^s)\left(\frac{F(\kappa^s)}{f(\kappa^s)} - G(q_1^*, q^s, \kappa^s)\right)\\ &\qquad> \frac{1-F(\kappa^s)}{f(\kappa^s)}(v - c_0(\varepsilon_0^s)) - u_1^s(\varepsilon_0^s)\left(\frac{F(\kappa^s)}{f(\kappa^s)} - G(q_1^*, q^s, \kappa^s)\right) = 0.\end{aligned}$$

Thus, $\varepsilon_0^n > \varepsilon_0^s$ and $\kappa^n > \kappa^s$ for v sufficiently close to $\bar{c}$. □

Chapter 3

Price Discrimination in Intermediate Good Markets with Asymmetric Agency Costs

This chapter is based on Asseyer (2016c).

3.1 Introduction

Competition authorities in both the United States and the European Union consider discriminatory pricing policies a potential abuse of a dominant market position.[1] On this basis, a monopolistic upstream firm can be restrained from offering different tariffs to different downstream firms. The extant literature on price discrimination in intermediate good markets analyzes the welfare implications of banning price discrimination (Katz, 1987; DeGraba, 1990; Yoshida, 2000; Inderst and Shaffer, 2009; Inderst and Valletti, 2009; Herweg and Müller, 2014). This literature mainly focusses on the case where downstream firms differ with respect to their costs. Thereby, it is implicitly assumed that differences in costs reflect solely differences in the social costs of production, i.e., the opportunity costs for labor and other inputs into the production process. I refer to these costs as *production costs*. However, the costs of a firm are also determined by *agency costs* that arise if different stakeholders of a firm have conflicting interests.

In many industries, different firms have different forms of vertical organization. The coexistence of vertically integrated and vertically separated firms in the same industry is documented for the US petroleum refining industry (Bindemann, 1999),

[1] The Patman-Robinson Act regulates price discrimination in the United States. In the EU, Article 82c) of the CE treaty restricts the use of price discrimination.

the UK package holiday industry (Buehler and Schmutzler, 2005), the beer industry in the UK (Slade, 1998a), gasoline retailing in Canada (Slade, 1998b), the US cable television industry (Waterman and Weiss, 1996; Chipty, 2001), and the Mexican footwear industry (Woodruff, 2002).[2] With vertical separation, different vertically related firms need to bargain over production and pricing decisions. The examples above therefore suggest that agency costs are often asymmetric among different downstream firms in intermediate good markets. In these industries, upstream firms may therefore wish to charge downstream firms different prices depending on whether they are vertically integrated or vertically separated.

It therefore arises the question how price discrimination affects the total welfare on a market if cost differences are implied by agency problems, and whether the source of cost differences matters for the welfare implications of discriminatory pricing policies.

In order to study this question, I analyze a model of an intermediate good market where a monopolistic upstream firm sells an intermediate good to two downstream firms that serve independent and identical consumer markets. The upstream firm can offer the downstream firms arbitrary tariffs that determine how much the downstream firms have to pay for a given quantity.[3] In a first step, I study a model where downstream firms have different agency costs due to asymmetric degrees of vertical integration. One downstream firm is vertically integrated and produces the final good itself. The other downstream firm is vertically separated and delegates production to a subcontractor. The integrated downstream firm and the subcontractor have private information about their production costs. Due to the informational advantage of the subcontractor, the separated downstream firm has to give an information rent to its subcontractor. This rent is an agency cost which leads to a cost disadvantage for the vertically separated downstream firm.

I show that it is optimal for the upstream firm to discriminate between the downstream firms by favoring the vertically separated downstream firm. As the vertically separated downstream firm is *weaker* and reacts more sensitively to a price increase, the upstream firm offers the vertically separated firm a lower tariff.

Furthermore, I demonstrate that price discrimination increases total welfare under mild conditions. Due to asymmetric agency costs, the vertically integrated downstream firm produces more of the final good than the vertically separated firm even if it has the same cost function as the subcontractor. With price dis-

[2] These examples are taken from Buehler and Schmutzler (2005).

[3] Throughout the chapter, price discrimination refers to third degree price discrimination, i.e., a situation where the upstream firm offers the downstream firm different tariffs. This approach is in line with the regulatory policy of the EU that does not consider quantity discounts as an abuse of a dominant market position, c.f. (Herweg and Müller, 2012).

crimination, the separated firm faces a lower tariff than the integrated firm. This partially offsets the cost disadvantage of vertical separation. If price discrimination is banned, both downstream firms receive the same tariff that improves the terms of trade of the integrated firm and deteriorates the terms for the separated firm. Consequently, the separated downstream firm reduces its production whereas the vertically integrated firm extends its production. A ban on price discrimination therefore increases the difference in production between the firms even if they have the same production costs. Under mild conditions that are satisfied for the case of linear demand and cost functions, a ban on price discrimination reduces the total production of the downstream firms with identical cost functions. Together with the marginal effect described above, this implies that price discrimination increases total welfare.

In a next step, I compare this result to the welfare effects of price discrimination when downstream firms have different production costs. I consider a model as in Herweg and Müller (2014) where both downstream firms produce a final good and have private information about their production costs. One downstream firm uses a production technology which is less efficient than the production technology of the other downstream firm. In particular, I consider the case where the cost disadvantage due to the different technology is exactly equivalent to the cost disadvantage due to vertical separation in the first model.

From a positive perspective, cost differences between downstream firms lead to the same market outcomes independently of whether they are caused by agency or production costs. The incentives are independent of the source of the cost difference. However, the source of cost differences matters from a welfare perspective. Agency costs arise if stakeholders within a firm or a vertical relationship can secure rents for themselves. Rent payments constitute a cost from the perspective of the firm. From a welfare perspective, rent payments have only a redistributive effect. In contrast, production cost have a direct impact on total welfare.

I show that price discrimination has always a more positive effect on welfare in the model with asymmetric agency costs. In the model with asymmetric production costs, price discrimination implies that the downstream firms produce different quantities of the final good even if they have the same social costs of production. Under a ban on price discrimination, both downstream firms produce the same quantity if they have the same production costs. Thus, a ban on price discrimination reduces the difference in production between the downstream firms.

If both final good markets are covered, the same conditions as before imply that price discrimination reduces the total production of the downstream firms with identical cost functions. Together with the marginal effect described before, this implies that price discrimination reduces welfare in the model with asymmetric production costs.

Related literature There is a long-standing interest in the welfare effects of price discrimination. Robinson (1933) provides the first formal analysis of the welfare effects of price discrimination in final good markets. Aguirre et al. (2010) and Cowan (2012) extend and generalize her insights to more general demand functions.

The literature on price discrimination in intermediate good markets starts with Katz (1987) who shows that price discrimination can be detrimental for welfare if larger downstream firms have the possibility to engage in inefficient backward integration by producing an input instead of buying it. In a model with linear demand curves, DeGraba (1990) shows that price discrimination reduces welfare in the short and in the long run. Yoshida (2000) analyzes a model with linear production technologies and shows that price discrimination often reduces welfare. Inderst and Valletti (2009) analyze the short and long run welfare effects of price discrimination when downstream firms have the opportunity to change their supplier. They show that price discrimination reduces consumer surplus in the short run but can increase consumer surplus in the long run. Arya and Mittendorf (2010) demonstrate that price discrimination can increase social welfare in a model where downstream firms serve several final good markets and lower demand markets are less competitive.

In contrast to the previous papers, and as in the current chapter, O'Brien and Shaffer (1994), Inderst and Shaffer (2009), and Herweg and Müller (2014) analyze the case where the upstream firm sets non-linear tariffs. O'Brien and Shaffer (1994) and Inderst and Shaffer (2009) analyze settings under complete information where the upstream firm offers two-part tariffs to the downstream firms. Both papers show that price discrimination is usually welfare increasing. Herweg and Müller (2014) is the closest to the current chapter. They analyze a model where an upstream firm sells an input to two downstream firms which have private information about their production costs. The upstream firm offers the optimal screening contracts to the downstream firms. Herweg and Müller show that price discrimination is welfare-reducing as long as markets are covered under uniform pricing. The model with asymmetric production costs studied in the current chapter is a version of their model.

As most of the literature, the current chapter takes the industry structure as given. Herweg and Müller (2012) analyze the welfare effects of price discrimination if new buyers can enter the market. Again following most of the papers in the literature, the seller has all bargaining power in the current chapter. O'Brien (2014) allows for more general forms of bargaining between buyers and sellers.

In the next section, I present the model with asymmetric agency costs. For this model, section 3.3 provides the equilibrium analysis when price discrimination is permitted. In Section 3.4, the welfare effects of price discrimination are analyzed under asymmetric agency costs. Section 3.5 presents a comparison to the welfare result under asymmetric production technologies. Section 3.6 concludes.

3.2 A model with asymmetric agency costs

An upstream firm U produces an intermediate good at zero marginal cost for two downstream firms which serve two independent markets. The first downstream firm D_1 serves market 1 and the second downstream firm D_2 serves market 2. The inverse demand function is identical on both markets and given by $P(q_i)$ where q_i is the quantity which is sold on market i. The inverse demand function $P(q_i)$ is strictly decreasing in the quantity q_i.

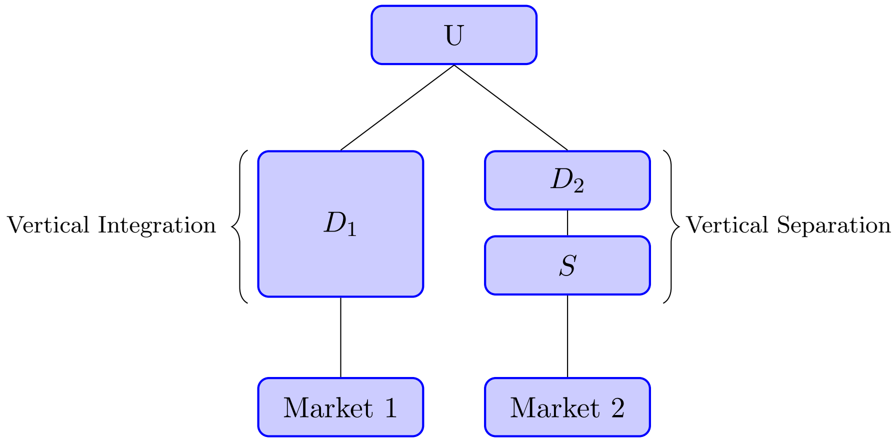
D_1 can produce the final good with a one-to-one technology which transforms q_1 units of the intermediate good into q_1 units of the final good at a cost $c_1 k(q_1)$. The variable c_1 is a parameter of the cost function which is private information to D_1. The cost parameter c_1 is drawn according to the cumulative distribution function $F(c_1)$ from the interval $[\underline{c}, \infty) \subset \mathbb{R}_+$. $F(\cdot)$ has a twice differentiable density $f(\cdot)$.

D_2 buys the input from U and delegates production to a subcontractor S. Using a one-to-one production technology, S produces the quantity q_2 of the final good at production cost $c_2 k(q_2)$ where the cost parameter c_2 is private information to S and drawn from the interval $[\underline{c}, \infty)$ according to the cumulative distribution function $F(c_2)$.

I refer to D_1 as the vertically integrated downstream firm and to D_2 and S as the vertically separated downstream firm. The structure of the industry is illustrated in Figure 3.1.

Profit Functions If the transfer t_i is paid from D_i to U with $i \in \{1, 2\}$, t_s is a payment from S to D_2, and q_i is the quantity sold on market i, U makes a profit of $t_1 + t_2$, D_1's profit is $q_1 P(q_1) - c_1 k(q_1) - t_1$, D_2 receives a profit of $t_s - t_2$, and S makes a profit of $q_2 P(q_2) - c_2 k(q_2) - t_s$. These definitions implicitly assume that S sells the final good on market 2. Since I do not restrict the set of possible subcontracts between D_2 and S, and since quantities are contractible, this setting is equivalent to the case where D_2 delegates production to S but serves the market 2 itself.

Figure 3.1: Industry structure with different degree of vertical integration



The upstream firm U sells an input to the vertically integrated downstream firm D_1 and the vertically separated downstream D_2 that delegates production to the subcontractor S. D_1 and D_2 serve independent markets.

The contracting game I study the following contracting game between U, D_1, D_2, and S. At the beginning of the game, the cost parameters c_1 and c_2 are drawn. The parameter c_1 is only observed by D_1 and c_2 is only observed by S. Next, U offers a tariff $T_1(q_1)$ to D_1 and a tariff $T_2(q_2)$ to D_2. These tariffs imply that D_i can order a quantity q_i of the intermediate good at a total payment of $T_i(q_i)$. Then, D_2 offers a tariff $T_s(q_2)$ to S. With this tariff, D_2 commits to deliver a quantity q_2 of the intermediate good at a payment $T_s(q_2)$. S can accept or reject this offer. If S accepts and orders a positive quantity q_2, D_2 is committed to accept the tariff $T_2(q_2)$. After S has made the participation decision, D_1 chooses whether to accept or reject $T_1(q_1)$. Finally, production takes place and payoffs realize. I impose no restrictions, such as linearity, on the tariffs $T_1(q_1)$, $T_2(q_2)$, and $T_s(q_2)$.

I analyze U's most preferred Perfect Bayesian Equilibrium of this game. I make the following assumptions about the demand and cost functions.

Assumption 3.1. *The inverse demand function $P(q)$ and the cost function $k(q)$ satisfy the following conditions:*

1. $R(q) \equiv qP(q)$ *is strictly concave with finite maximizer;*

2. $k(0) = 0$, $k'(q) > 0$ *if* $q > 0$, $k''(q) \geq 0$*;*

3. $R'(0) - \underline{c}k'(0) > 0$.

Parts 1 and 2 assume that revenue is concave and that the cost function is increasing and weakly convex. Condition 3 ensures that production is profitable if the cost parameter is sufficiently low.

An important role is played by the *virtual cost parameter* $h(c_i)$ which is defined as

$$h(c_i) \equiv c_i + \frac{F(c_i)}{f(c_i)}.$$

Note that $h(c_i)$ is distributed on $[\underline{c}, \infty)$ according to the cumulative distribution function $G(c) \equiv \Pr(h(c_i) \leq c) = F(h^{-1}(c))$ with the density $g(c) = G'(c)$. I make the following assumption:

Assumption 3.2. *The distribution function $F(\cdot)$ satisfies the following conditions:*

1. *$h(c)$ is strictly increasing and convex;*
2. *$g(c)/f(c)$ is weakly increasing;*
3. *$c + \frac{F(c)+G(c)}{f(c)+g(c)}$ is weakly increasing.*

Assumption 3.2 is satisfied for a large class of standard distribution functions such as the normal, logistic, beta, gamma, Gumbel, Fréchet, Weibull, chi squared, and chi distributions. The first part of the assumption states that the virtual cost parameter be increasing and convex. Note that this is satisfied if the inverse hazard rate $F(c)/f(c)$ is increasing and convex. This assumption corresponds to conditions on pass-through rates of demand functions when demand is interpreted as a distribution function in the tradition of Bulow and Pfleiderer (1983). An increasing hazard rate is equivalent to a pass-through rate greater than one, and a convex hazard rate is equivalent to an increasing pass-through rate. Weyl and Fabinger (2009) show that these two assumptions are satisfied for a large class of standard distribution functions. The second part of the assumption states that the distribution of the virtual cost parameter dominates the distribution of the real cost parameter in the likelihood ratio order. For the last part of the assumption, note that the inverse hazard rate of virtual cost parameter, $G(c)/g(c)$ is increasing, if the inverse hazard rate of the real cost parameter $F(c)/f(c)$ is increasing and convex. The assumption is therefore satisfied if also the linear combination $\alpha(c)F(c)/f(c) + (1 - \alpha(c))G(c)/g(c)$ with $\alpha(c) = f(c)/(f(c) + g(c))$ is increasing.[4]

Assumptions 3.1 and 3.2 are maintained throughout the chapter.

[4] The distribution function $F(c) = e^{-1/a(c-b)^2}$ on $[b, \infty)$ is an example for which it can be easily checked that Assumption 3.2 is satisfied. Note that this distribution is a special case of a Fréchet distribution. I present the calculations in the Appendix.

Industry optimum If there is no conflict of interest within the industry, the four players maximize the total industry profit. For the production cost parameters c_1 and c_2, the total profit is given by

$$\sum_{i=1}^{2} R(q_i) - c_i k(q_i).$$

Assumption 3.1 implies that there is a pair of unique and finite quantities $(q_1^*(c_1), q_2^*(c_2))$ that maximizes this expression and satisfies $q_1^*(c) = q_2^*(c) = q^*(c)$.

Public information Private information is an essential ingredient of the model. With public information, the difference in the vertical organization of the downstream firms is irrelevant because the seller can charge nonlinear tariffs. To see this, suppose that the cost parameters c_1 and c_2 are public information. In this case, U can extract the whole industry profit. U optimally offers D_1 and D_2 the two-part-tariffs (w, Φ) where the piece-rate w equals U's marginal cost of zero and the fixed payment Φ is set to extract the whole profit, i.e.

$$\Phi = R(q^*(c_i)) - c_i k(q^*(c_i)).$$

D_1 cannot do better than to accept the tariff and to choose the quantity $q_1(c_1) = q^*(c_1)$. For D_2 it is optimal to pass on the contractual terms to S. S is then in the same situation as D_1 and produces the quantity $q_2(c_2) = q^*(c_2)$. U discriminates between D_1 and D_2 only on the basis of their realized cost parameters, not on the basis of their organizational form.[5] The analysis in this chapter shows that asymmetric information about cost parameters renders discrimination on the basis of the organizational form profitable. This holds even if the upstream firm can offer arbitrary tariffs.

Price discrimination Due to the presence of private information, U may choose tariffs which reflect two different forms of price discrimination. First, U may wish to offer D_1 and D_2 different contracts based on the observable difference in their vertical organization. This would be an instance of third degree price discrimination. Second, U may find it optimal to offer nonlinear contracts to the downstream firms in order to screen their private information in the most profitable way, i.e. to engage in second degree price discrimination. Here, I follow the literature on price discrimination with nonlinear tariffs (Inderst and Shaffer, 2009; Herweg and Müller, 2014) and focus on third degree price discrimination and its welfare effects.

[5]The analysis of this model under public information is a special case in Inderst and Shaffer (2009) who analyze price discrimination under symmetric information with two part tariffs.

In the following, the term price discrimination always refers to third degree price discrimination.

Definition 2. *U price discriminates between D_1 and D_2 if and only if $T_1(q) \neq T_2(q)$ for some quantity q. U favors D_i if $T_i(q) \leq T_j(q)$ for all quantities q and $T_i(q) < T_j(q)$ for some quantity q, with $i, j \in \{1, 2\}$ and $i \neq j$.*

I analyze the normative question whether an upstream firm with market power should be allowed to discriminate among his customers when nonlinear tariffs can be used.

3.3 Optimal tariffs

In this section, I provide the equilibrium analysis of the game when price discrimination is allowed and when it is banned.

Optimal tariffs under price discrimination

I analyze the optimal wholesale tariffs that U offers to the downstream firms D_1 and D_2 when U can discriminate between the downstream firms. In the vertical chain serving market 2, a double marginalization problem arises: D_2 has to give an information rent to S for information about production costs, and U has to give an information rent to D_2 for the private knowledge which D_2 gains through communication with S. If the cost parameters c_1 and c_2 are the same, the costs of D_1 are lower than the costs of D_2 as D_1 does not have to pay an information rent.

I show that U discriminates between D_1 and D_2 by favoring the vertically separated firm D_2 if the virtual cost parameter $h(\cdot)$ is strictly convex. If $h(\cdot)$ is linear, U offers the downstream firms the same tariff. The curvature of $h(\cdot)$ measures how strongly D_2 increases the tariff for S if U increases the tariff for D_2. If $h(\cdot)$ is linear, increasing prices for D_1 and D_2 reduces the induced demand from markets 1 and 2 in the same proportion. Thus, the price elasticities of the induced demand functions are the same. However, if $h(\cdot)$ is strictly convex, then an increase in the tariff for D_2 induces D_2 to strongly increase the tariff for S. This strongly reduces the demand from market 2. This implies that it is optimal for U to give D_2 a more attractive tariff than D_1.

I now analyze the tariffs that U optimally offers to the two downstream firms. As the two markets are independent, the optimal design of the tariff for D_1 is independent of the tariff for D_2 and vice versa. I can thus analyze the two problems separately.

Optimal wholesale tariff for $\mathbf{D_1}$ U chooses the tariff $T_1(q_1)$ in order to maximize its expected profit. The optimal tariff needs to satisfy the incentive constraint that D_1 orders its preferred quantity for any value of the cost parameter, and the participation constraint that D_1 accepts the tariff $T_1(q_1)$. U's optimal tariff is therefore a solution to the following maximization problem.

$$\mathcal{P}_1^d: \quad \max_{T_1(\cdot)} \int T_1(q_1(c_1))dF(c_1)$$

$$\text{s.t.} \quad q_1(c_1) \in \arg\max_q R(q) - c_1 k(q) - T_1(q), \qquad (IC_1)$$

$$\max_q R(q) - c_1 k(q) - T_1(q) \geq 0 \quad \text{for all } c_1 \in [\underline{c}, \infty). \qquad (PC_1)$$

This problem can be solved using standard techniques as provided in a clear exposition by Martimort and Stole (2009). The solution to this problem is presented in the following Lemma.

Lemma 3.1. *If price discrimination is permitted, U offers the tariff $T_1^d(q_1)$ to D_1, D_1 accepts the offer, and orders a quantity $q_1^d(c_1)$. $T_1^d(q_1)$ and $q_1^d(c_1)$ are given by*

$$T_1^d(q) = \int_0^q (R'(x) - {(q_1^d)}^{-1}(x)k'(x))dx, \tag{3.1}$$

$$q_1^d(c_1) = \arg\max_q R(q) - h(c_1)k(q), \tag{3.2}$$

where ${(q_1^d)}^{-1}(\cdot)$ is the inverse of $q_1^d(c_1)$ with

$${(q_1^d)}^{-1}(0) = \overline{c}_1 \equiv \min\left\{c \in [\underline{c}, \infty) : q_1^d(c) = 0\right\}.$$

U optimally screens the private information of D_1 by offering a tariff for which D_1 orders a quantity that maximizes the *virtual* industry profit on market 1. This is reflected by the definition of the quantity $q_1^d(c_1)$ in equation (3.2). The optimal tariff $T_1(\cdot)$ is the tariff which induces D_1 to choose this quantity.

Optimal wholesale tariff for $\mathbf{D_2}$ I now turn to the analysis of the tariff which U optimally offers to D_2. For a given tariff $T_2(q_2)$, D_2 offers a tariff $T_s(q_2)$ to S. The optimal offer of U to D_2 depends on the tariff which D_2 offers to S in response.

The game can be solved backwards. I first analyze the optimal offer of D_2 to S for any given tariff $T_2(q_2)$. D_2 optimally offers a tariff $T_s(q_2)$ which maximizes its expected net payments, ensuring that the anticipated equilibrium quantities $q_2(c_2)$ are indeed optimal for S and guaranteeing S a non-negative profit. For the given tariff $T_2(q_2)$, D_2 thus offers a tariff $T_s(q_2)$ which is a solution to the following problem.

$$\mathcal{P}_s^d: \quad \max_{T_s(\cdot)} \int (T_s(q_2(c_2)) - T_2(q_2(c_2)))dF(c_2)$$

$$\text{s.t.} \quad q_2(c_2) \in \arg\max_q R(q) - c_2 k(q) - T_s(q), \qquad (IC_s)$$

$$\max_q R(q) - c_2 k(q) - T_s(q) \geq 0 \quad \text{for all } c_2 \in [\underline{c}, \infty). \qquad (PC_s)$$

Note that a quantity schedule $q_2(c_2)$ can be optimal for S under some tariff $T_s(q_2)$ if and only if $q_2(c_2)$ is non-increasing. D_2's optimal response to the offered tariff $T_2(q_2)$ satisfies the following conditions.

Lemma 3.2. *Given any tariff $T_2(q_2)$ offered by U, D_2 optimally offers a tariff $T_s(q_2)$ to S such that S orders a quantity $q_2(c_2)$ and D_2 makes a non-negative profit for any $c_2 \in [\underline{c}, \infty)$:*

$$q_2(c_2) = \arg\max_q R(q) - h(c_2)k(q) - T_2(q), \tag{3.3}$$

$$\max_q R(q) - h(c_2)k(q) - T_2(q) \geq 0. \tag{3.4}$$

D_2 offers a tariff which induces S to order a quantity that maximizes the virtual joint profit of D_2 and S. This is reflected in equation (3.3). Furthermore, D_2 can guarantee itself a non-negative profit. To see this, suppose D_2's profit is negative for some realization of the cost parameter c_2'. By the envelope theorem, D_2's profit for cost parameter c_2, given by $\max_q R(q) - h(c_2)k(q) - T_2(q)$ is decreasing in c_2. Thus D_2's profit is negative for all cost parameters higher than c_2'. D_2 can then offer a different tariff under which S orders zero quantity for all values of the cost parameter above c_2', D_2 rejects U's offer for these values of c_2, and ordered quantities remain the same as before for cost parameters below c_2'. The new quantity schedule remains non-increasing and therefore there exists a tariff which makes these quantities optimal for S. Furthermore, this tariff increases D_2's expected profit.

Thus, U optimally offers a tariff $T_2(q_2)$ to D_2 which anticipates the quantity that S orders under an optimal response tariff $T_s(q_2)$, and which guarantees D_2 a non-negative profit for any realization of the cost parameter c_2. Formally, the optimal offer $T_2(q_2)$ solves the following optimization problem.

$$\mathcal{P}_2^d: \quad \max_{T_2(\cdot)} \int T_2(q_2(c_2))dF(c_2)$$

$$\text{s.t.} \quad q_2(c_2) \in \arg\max_q R(q) - h(c_2)k(q) - T_2(q), \qquad (IC_2)$$

$$\max_q R(q) - h(c_2)k(q) - T_2(q) \geq 0 \quad \text{for all } c_2 \in [\underline{c}, \infty). \qquad (PC_2)$$

The solution to this optimization problem completes the characterization of equilibrium behavior if price discrimination is allowed.

Lemma 3.3. *If price discrimination is permitted, U offers the tariff $T_2^d(c_2)$ to D_2, D_2 offers the tariff $T_s^d(q_2)$ to S, S and D_2 accept the offers, and S orders the quantity $q_2^d(c_2)$. $T_2^d(q_2)$, $T_s^d(q_2)$, and $q_2^d(c_2)$ are given by*

$$T_2^d(q) = \int_0^q (R'(x) - h((q_2^d)^{-1}(x))k'(x))dx, \tag{3.5}$$

$$T_s^d(q) = \int_0^q (R'(x) - (q_2^d)^{-1}(x)k'(x))dx, \tag{3.6}$$

$$q_2^d(c_2) = \arg\max_q R(q) - (h(c_2) + h'(c_2)(h(c_2) - c_2))k(q), \tag{3.7}$$

where $(q_2^d)^{-1}(\cdot)$ is the inverse of $q_2(c_2)$ with

$$(q_2^d)^{-1}(0) = \bar{c}_2 \equiv \min\{c \in [\underline{c}, \infty) : q_2(c) = 0\}.$$

In equilibrium, the quantities $q_2(c_2)$ maximize a virtual joint profit of the industry on market 2, where the expression in equation (3.7) reflects an informational double marginalization problem. If U produces a quantity q_2 for market 2, a margin of $h(c_2) - c_2$ has to be given to S, and a margin of $h'(c_2)(h(c_2) - c_2)$ has to be granted to D_2, in order to extract the information about the cost parameter c_2. This double marginalization problem implies that if the cost parameters c_1 and c_2 take identical values, the quantity on market 1 is larger than the quantity on market 2.

Price discrimination between $\mathbf{D_1}$ and $\mathbf{D_2}$ I am now in a position to compare the tariffs $T_1^d(q_1)$ and $T_2^d(q_2)$. This allows me to answer the questions whether there is price discrimination between the downstream firms and which form of vertical organization is favored under price discrimination. Apart from the fact that D_1 has a cost function of $c_1k(q)$ whereas D_2 has a cost function of $h(c_2)k(q)$, U's optimization problems $\mathcal{P}_1^d$ and $\mathcal{P}_2^d$ are equivalent. If D_1 and D_2 face the same tariff $T(q)$, D_1 and D_2 choose the same quantity whenever the cost parameters satisfy the condition $c_1 = h(c_2)$. Whether U discriminates between the downstream firms can now be simply tested by comparing the quantities which D_1 and D_2 buy for $c_1 = h(c_2)$.

Lemma 3.4. *U price discriminates between D_1 and D_2 if and only if*

$$q_1(h(c)) \neq q_2(c) \text{ for some } c \in [\underline{c}, \infty).$$

This test can now be applied.

Proposition 3.1. *U discriminates between D_1 and D_2 by favoring D_2 if and only if $h(\cdot)$ is strictly convex for some values of the cost parameter which satisfy $q_1(c) > 0$. If $h(\cdot)$ is linear, U offers D_1 and D_2 the same tariff.*

Proof. From equations (3.2) and (3.7), it follows that $q_1(h(c)) = q_2(c)$ if and only if $h(h(c)) = h(c) + h'(c)(h(c) - c)$. This is satisfied for all $c \in [\underline{c}, \infty)$ if and only if $h(\cdot)$ is linear. In contrast, if $h(\cdot)$ is strictly convex for some c', $h(h(c)) > h(c) + h'(c)(h(c) - c)$ holds for c close to c', and $q_2(c) > q_1(h(c))$. As $h(\cdot)$ is assumed to be convex, $q_2(c) \geq q_1(h(c))$ for all $c \in [\underline{c}, \infty)$ and from the definitions of $T_1^d(q_1)$ and $T_2^d(q_2)$ in equations (3.1) and (3.5) it follows that D_2 is favored by price discrimination. □

Price discrimination arises if the virtual cost parameter function $h(\cdot)$ is curved. How does this result come about? The curvature of $h(\cdot)$ influences how the vertically separated firm D_2 changes the tariff for the subcontractor $T_s(\cdot)$ when U changes the tariff $T_2(\cdot)$. If $h(\cdot)$ is linear, an increase in the marginal price charged by U to D_2 changes the induced demand on market 2 in the same way it changes the demand on market 1. If $h(\cdot)$ is convex, then D_2 increases the tariffs to S more strongly such that the induced demand on market 2 reduces more than the induced demand on market 1 for the same price change.

To see the relationship with price elasticity, suppose that $R(q) - c \cdot k(q)$ is linear in q and $q \in [0, 1]$. Furthermore assume that U charges a linear tariff $T(q) = p \cdot q$. The induced (expected) demand on market 1 then satisfies $D_1(p) = F(\frac{R(1)-p}{k(1)})$ and the induced (expected) demand on market 2 satisfies $D_2(p) = F(h^{-1}(\frac{R(1)-p}{k(1)}))$. For $P = \frac{R(1)-p}{k(1)}$, the price elasticity on market 1 is then given by

$$\varepsilon_1(p) = -\frac{D_1'(p)p}{D_1(p)} = \frac{p \cdot f(P)}{k(1) \cdot F(P)} = \frac{p}{k(1)\,(h(P) - P)}$$

and the price elasticity on market 2 is

$$\varepsilon_2(p) = -\frac{D_2'(p)p}{D_2(p)} = \frac{p \cdot f\left(h^{-1}(P)\right)(h^{-1})'(P)}{k(1) \cdot F\left(h^{-1}(P)\right)} = \frac{p \cdot (h^{-1})'(P)}{k(1)\left(P - (h^{-1})'(P)\right)}.$$

It follows that

$$\begin{aligned} \varepsilon_2(p) \geq \varepsilon_1(p) \Leftrightarrow \quad & \frac{p \cdot (h^{-1})'(P)}{k(1)\left(P - (h^{-1})'(P)\right)} \geq \frac{p}{k(1)\left(h(P) - P\right)} \\ \Leftrightarrow \quad & \frac{h(P) - h(h^{-1}(P))}{P - h^{-1}(P)} \geq \frac{1}{(h^{-1})'(P)} = h'(h^{-1}(P)). \end{aligned}$$

The condition in the last line is satisfied with equality if $h(\cdot)$ is linear. It is satisfied with a strict inequality if $h(\cdot)$ is strictly convex. In this case, the induced demand on market 2 is more elastic than the demand from market 1.

The relationship between the curvature of the virtual cost parameter to price elasticity is less clear if $R(q) - c \cdot k(q)$ is not linear. Nevertheless, the curvature of the virtual cost parameter determines the favored downstream firm. This is similar to price discrimination on separated consumer markets where relative price elasticity determines the favored market.

Optimal tariff when price discrimination is banned

I now analyze the equilibrium behavior in the game when price discrimination is banned. If price discrimination is banned, U has to offer the same tariff to the two downstream firms. The optimal tariff needs to satisfy incentive and participation constraints for both downstream firms. These constraints are the same as in the case when price discrimination is allowed. In particular, Lemma 3.2 still describes the optimal tariff which D_2 offers to S in response to the offer from U. The only additional constraint for U is the ban on discrimination.

Formally U's optimization problem is given as follows:

$$\mathcal{P}^n: \quad \max_{T(\cdot)} \int T(q_1(c_1))dF(c_1) + \int T(q_2(c_2))dF(c_2)$$

$$\text{s.t.} \quad q_1(c_1) \in \arg\max_q R(q) - c_1 k(q) - T(q), \qquad (IC_1^n)$$

$$q_2(c_2) \in \arg\max_q R(q) - h(c_2)k(q) - T(q), \qquad (IC_2^n)$$

$$\max_q R(q) - c_1 k(q) - T(q) \geq 0 \quad \text{for all } c_1 \in [\underline{c}, \infty), \qquad (PC_1^n)$$

$$\max_q R(q) - h(c_2) k(q) - T(q) \geq 0 \quad \text{for all } c_2 \in [\underline{c}, \infty). \qquad (PC_2^n)$$

The problem $\mathcal{P}^n$ is technically equivalent to the problems $\mathcal{P}_1^d$ and $\mathcal{P}_2^d$ with the additional non-discrimination constraint that $T_1(q) = T_2(q)$. This constraint connects these two problems. The equilibrium behavior under a ban on price discrimination is then given in the following lemma.

Lemma 3.5. *If price discrimination is banned, U offers the tariff $T^n(q)$ to D_1 and D_2, D_2 offers the tariff $T_s^n(q)$ to S, D_1 and S accept the offers, D_1 orders the quantity $q_1^n(c_1) = q^n(h^{-1}(c_1))$, and S orders the quantity $q_2^n(c_2) = q^n(c_2)$. $T^n(q)$, $T_s^n(q)$, and $q^n(c)$ are given by*

$$T^n(q) = \int_0^q (R'(x) - h((q^n)^{-1}(x))k'(x))dx, \tag{3.8}$$

$$T_s^n(q) = \int_0^q (R'(x) - (q^n)^{-1}(x)k'(x))dx, \tag{3.9}$$

$$q^n(c) = \arg\max_q R(q) - \left(h(c) + \frac{F(h(c)) + h'(c)F(c)}{f(h(c)) + f(c)}\right) k(q), \tag{3.10}$$

where $(q^n)^{-1}(\cdot)$ is the inverse of $q^n(c)$ with

$$(q^n)^{-1}(0) = \bar{c}^n \equiv \min\{c \in [\underline{c}, \infty) : q^n(c) = 0\}.$$

3.4 Welfare effects under asymmetric agency costs

In this section, I analyze the welfare effects of price discrimination if downstream firms differ with respect to their agency costs.

For this purpose, I first determine the effects of a ban on price discrimination on consumers and downstream firms on the two markets. I show that the vertically separated firm D_2 and the consumers on market 2 are harmed by a ban on price discrimination whereas the vertically integrated firm D_1 and consumers on market 1 benefit.

I then compare the total welfare arising under the optimal uniform tariff with the total welfare in the case where U sets the optimal discriminating tariffs. I show that under mild conditions on cost and demand functions price discrimination increases welfare. Intuitively, price discrimination is beneficial to welfare as it partially offsets the inefficiency which arises through double marginalization on market 2. Price discrimination shifts production from market 1 to market 2. As total production is smaller on market 2, the marginal consumer on market 1 has a lower valuation for the good than the marginal consumer on market 2. The shift in production from market 1 to market 2 is therefore welfare increasing.

The social welfare on market i for a given cost parameter c_i and a quantity q_i

is

$$w_i(c_i, q_i) \equiv \int_0^{q_i} P(x)dx - c_i k(q_i).$$

This is the sum of the consumer surplus, given by $\int_0^{q_i} P(x)dx - P(q_i)q_i$, and the producer surplus $P(q_i)q_i - c_i k(q_i)$. In a given equilibrium with equilibrium quantities $q_1(c_1)$ and $q_2(c_2)$, the expected welfare on market i can be written as

$$W_i(q_i(\cdot)) \equiv \int_{\underline{c}}^{\infty} w_i(c_i, q_i(c_i))dF(c_i).$$

Equivalently, one can define the expected consumer surplus on the two markets. The total expected welfare is the sum of expected welfare on the two markets, i.e., $W(q_1(\cdot), q_2(\cdot)) = W_1(q_1(\cdot)) + W_2(q_2(\cdot))$.

The effects of price discrimination on the two markets If price discrimination is allowed, U favors the separated downstream firm D_1 over the integrated firm D_2. This is not allowed anymore once price discrimination is banned. U then has to treat D_1 and D_2 the same. This implies that the quantity on market 1 increases if price discrimination is banned whereas the quantity on market 2 decreases.

Proposition 3.2. *Price discrimination decreases the quantity produced by the vertically integrated downstream firm D_1, increases the quantity produced by the vertically separated downstream firm D_2, and the vertically integrated downstream firm produces a higher quantity with and without price discrimination:*

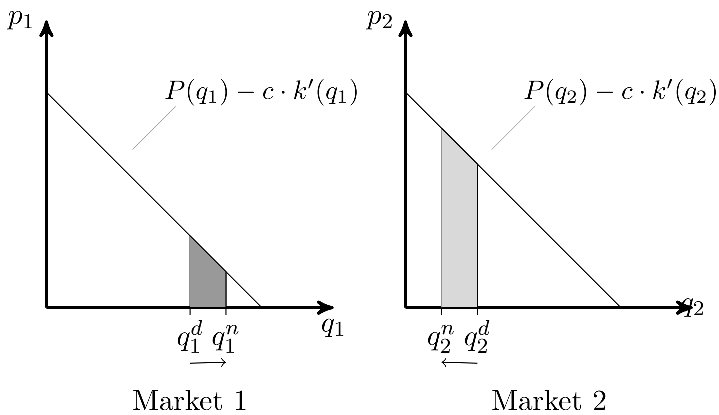
$$q_2^n(c) \leq q_2^d(c) < q_1^d(c) \leq q_1^n(c).$$

As a consequence of the effect on market quantities, a ban on price discrimination increases the expected profit of D_1 and the expected consumer surplus on market 1. It decreases the expected profits of D_2 and S and decreases the expected consumer surplus on market 2.

If price discrimination is banned, D_2 faces a tariff which is everywhere steeper than the tariff under price discrimination. Thus, D_2 increases the tariff in the subcontract and the quantity ordered by S decreases for any realization of the cost parameter. It follows that not only D_2 and S are worse off, but also the consumers on market 2 who face a higher price. The converse holds true on market 1. After a ban on price discrimination, D_1 faces a flatter tariff than before. D_1 orders a higher quantity of the input and the price on market 1 decreases to the benefit of consumers.

Effects on total welfare A ban on price discrimination increases welfare on market 1 but decreases welfare on market 2. Thus, price discrimination increases total welfare, if the welfare gain on market 2 outweighs the welfare loss on market 1. If price discrimination is banned, production for market 2 is shifted to market 1. Whether price discrimination is permitted or not, the quantity on market 1 is higher than the quantity on market 2 even if the cost parameters are the same for D_1 and S. This implies that the price on market 1 is lower than the price on market 2. Thus, the marginal consumers on market 1 have a lower marginal value for the good than the marginal consumers on market 2. This suggests that price discrimination should increase total welfare since it shifts the good from low to high valuation consumers. However, the total quantity of production may be different with and without price discrimination. If total quantity remains constant in both situations, or is higher under price discrimination, then price discrimination should be allowed.

Figure 3.2: Effect of price discrimination



The inverse demand functions net of marginal costs are depicted for both markets and identical cost parameters $c_1 = c_2 = c$. The quantity q_i^d is sold on market i if price discrimination is allowed, q_i^n is sold if price discrimination is banned. A ban on price discrimination increases the quantity on market 1 and reduces the quantity on market 2. The dark shaded area is the resulting welfare gain on market 1, the light shaded area represents the welfare loss on market 2.

Figure 3.2 illustrates this argument: In the figure, the inverse demand function net of marginal costs is depicted for both markets and identical cost parameters $c_1 = c_2 = c$. The quantities q_1^d and q_2^d are the quantities if price discrimination is allowed. Due to the double marginalization problem on market 2, $q_1^d > q_2^d$ even though the cost parameters are identical. This implies that the marginal consumer on market 1 has a lower marginal value for the good than the marginal consumer on market 2. If price discrimination is banned, the quantity on market 1 increases to q_1^n and the dark shaded area represents the welfare gain on market 1. However, the quantity on market 2 decreases to q_2^n and the light shaded area represents the reduction in welfare on market 2. If the total quantity on both markets is similar with and without price discrimination, i.e. $q_1^n - q_1^d \simeq q_2^d - q_2^n$, then the light shaded area is clearly larger than the dark shaded area and a ban on price discrimination reduces welfare.

If marginal revenue per marginal costs is concave, then total quantity on both markets for identical cost parameters c_1 and c_2 is higher under price discrimination. This implies that total welfare is higher under price discrimination. As the welfare $w(c_i, q_i)$ is concave in quantity, total welfare tends to increase if the quantities on both markets become more similar. This implies that price discrimination remains welfare increasing, even if it slightly reduces total quantity on the two markets.

Proposition 3.3. *If $R'(q)/k'(q)$ is concave, then price discrimination increases welfare when downstream firms have different agency costs.*

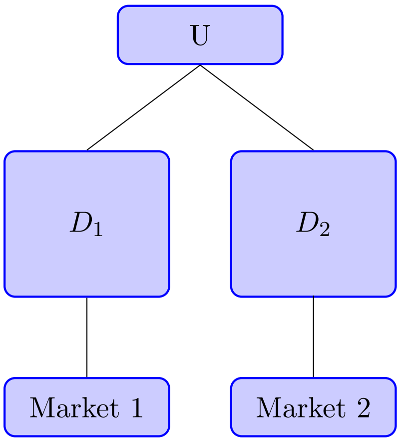
Proposition 3.3 implies that for a linear cost function $k(q) = K \cdot q$ with $K > 0$, price discrimination increases welfare if marginal revenue is concave. One example of a demand function for which marginal revenue is concave is the linear inverse demand function $P(q) = \max\{A - B \cdot q, 0\}$ for $A, B > 0$.

3.5 Agency versus production costs

In this section, I want to relate the welfare results of the previous section to the welfare implications of price discrimination in a setting where all downstream firms produce the final good themselves but differ with respect to production costs. In this case, the cost differences between downstream firms are not caused by different forms of vertical organization but are due to the fact that downstream firms use different production technologies.

Thus, I compare the welfare effects of price discrimination in a model with *asymmetric agency cost* to the welfare effects in a model with *asymmetric production costs*. In particular, I analyze the welfare effects of price discrimination under asymmetric production costs in a setting in which the upstream firm U optimally offers the same tariffs as in the model with asymmetric agency costs studied in the previous sections. This allows a clean comparison of the welfare effects of price discrimination in the two models. Furthermore this approach points to a practical problem in the regulation of price discrimination: The source of cost differences cannot be identified from the tariffs and quantities observed on the market. This poses the question whether a competition authority can make the right regulatory decision without knowledge of the source of cost differences.

Figure 3.3: Industry structure with asymmetric production costs



The upstream firm U sells to two downstream firms that are vertically integrated and serve independent markets.

First, I show that price discrimination has always a more positive effect on welfare in the model with asymmetric agency costs. I then show that under mild conditions on demand and cost functions, price discrimination is welfare increasing under asymmetric agency costs and welfare decreasing under asymmetric production costs. A competition authority can therefore not choose the right regulatory regime without knowledge of the source of cost differences between the downstream firms.

Herweg and Müller (2014) introduce the model of asymmetric production costs studied in this section. For specific assumptions on the distribution function, their model with asymmetric production costs is equivalent to the model with asymmetric agency costs studied in the previous section. In order to prove the result that price discrimination has different welfare effects in the two models, I extend Proposition 7 in Herweg and Müller (2014) to more general demand and cost functions.

A model with asymmetric production costs

The industry structure of the model with asymmetric production costs is given in Figure 3.3. In contrast to the model with asymmetric agency costs, D_2 produces the final good itself using a one-to-one technology. D_2's cost function is given by $c_2 \cdot k(q_2)$. Whereas the cost parameter of D_1 is distributed according to the cumulative distribution function $F(\cdot)$, the cost parameter c_2 is now distributed according to the distribution function $G(c_2) = F(h^{-1}(c_2))$.

For the upstream firm U, this situation is perfectly equivalent to the model where D_2 delegates production to the subcontractor S, has a virtual cost function of $h(c_2)$, and c_2 is distributed according to $F(\cdot)$. Furthermore, the downstream firms order the same quantities in both models. The model with asymmetric production costs and the model with asymmetric agency costs therefore give rise to the same tariffs and quantities.

Proposition 3.4. *In the equilibrium of the model with asymmetric production costs, U sets the tariffs $T_1^D(q_1)$ and $T_2^D(q_2)$ if price discrimination is permitted, and $T^N(q_i)$ with $i \in \{1,2\}$ if price discrimination is banned. These tariffs are the same as in the model with asymmetric agency costs: $T_1^D(q) = T_1^d(q)$, $T_2^D(q) = T_2^d(q)$, and $T^N(q) = T^n(q)$ for all $q \geq 0$.*

The downstream firms order the quantities $q_1^D(c_1)$ and $q_2^D(c_2)$ if price discrimination is permitted, and $q^N(c_i)$ with $i \in \{1,2\}$ if price discrimination is banned. For D_1 these quantities are the same as in the model with asymmetric agency costs: $q_1^D(c_1) = q_1^d(c_1)$ and $q^N(c_1) = q_1^n(c_1)$ for all $c_1 \in [\underline{c}, \infty)$. For D_2 the quantity ordered for cost parameter c_2 is the same as the quantity in the model with asymmetric agency cost for the cost parameter $h^{-1}(c_2)$: $q_2^D(c_2) = q_2^d(h^{-1}(c_2))$ and $q^N(c_2) = q_2^n(h^{-1}(c_2))$ for all $c_2 \in [\underline{c}, \infty)$.

For the same cost parameter $c = c_1 = c_2$, a ban on price discrimination increases the quantity produced by D_1 and decreases the quantity produced by D_2: $q^N(c) \in [q_1^D(c), q_2^D(c)]$ for all $c \in [\underline{c}, \infty)$.

This result follows from the observation that it does not matter for the incentives of D_2 whether the cost parameter c_2 is real – as in the model with asymmetric production costs – or virtual – as in the case of asymmetric agency costs. Furthermore the result arises as the real cost parameter in the model with asymmetric production costs and the virtual cost parameter in the model with asymmetric agency costs are identically distributed, due to the definition of $G(c_2)$ given by $\Pr(h(c_2) \leq c) = \Pr(c_2 \leq h^{-1}(c)) = G(c)$. For the same cost parameters $c_1 = c_2 = c$, the quantity produced under a ban on price discrimination is bracketed by the quantities produced under price discrimination, i.e. $q^N(c) \in [q_1^D(c), q_2^D(c)]$. A ban on price discrimination therefore reduces the difference in the quantities on the two markets for the same real cost parameter. I now turn to the comparison of the welfare effects of price discrimination in the two models.

A comparison of the welfare effects of price discrimination

In this section, I compare the welfare effects of price discrimination in the models with different agency costs and different production costs. I first show that price discrimination has always more favorable welfare effects in the model with asymmetric agency costs. The welfare effect of price discrimination under asymmetric production costs is given by

$$\Delta W_P = \int_{\underline{c}}^{\infty} \left[w(q_1^D(c), c) - w(q^N(c), c)\right] dF(c) + \int_{\underline{c}}^{\infty} \left[w(q_2^D(c), c) - w(q^N(c), c)\right] dG(c).$$

The welfare effect of price discrimination under asymmetric agency costs is given by

$$\Delta W_A = \int_{\underline{c}}^{\infty} \Big[w(q_1^d(c), c) - w(q_1^n(c), c)\Big] dF(c) + \int_{\underline{c}}^{\infty} \Big[w(q_2^d(c), c) - w(q_2^n(c), c) + (c - h^{-1}(c))(k(q_2^d(c)) - k(q_2^n(c)))\Big] dF(c).$$

This expression can be rewritten using the equilibrium quantities of the model with asymmetric production costs. This gives

$$\Delta W_A = \int_{\underline{c}}^{\infty} \left[w(q_1^D(c), c) - w(q^N(c), c)\right] dF(c) + \int_{\underline{c}}^{\infty} \Big[w(q_2^D(c), c) - w(q^N(c), c) + (c - h^{-1}(c))(k(q_2^D(c)) - k(q^N(c)))\Big] dG(c).$$

Thus, the difference between the welfare effects in the two models is given by

$$\Delta W_A - \Delta W_P = \int_{\underline{c}}^{\infty} \left[(c - h^{-1}(c))(k(q_2^D(c)) - k(q^N(c)))\right] dG(c). \qquad (3.11)$$

This expression reflects the fundamental difference of production costs and agency costs with respect to welfare. If the downstream firms differ with respect to production costs, their cost functions are directly represented in the expression of total welfare. However, if the downstream firms have asymmetric agency costs, then their cost differences include rent payments to subcontractors. These rent payments are a private cost to the delegating downstream firm but not a social cost since they are the profit of the subcontractor.

In particular, this implies that if the downstream firm D_2 has production costs of $c \cdot k(q)$, then the social costs of production are $c \cdot k(q)$ in the model with asymmetric production costs. In the model with asymmetric agency costs, the social costs are $h^{-1}(c) \cdot k(q)$. The difference of these terms, $(c - h^{-1}(c))k(q)$, are the rent payments to the subcontractor.

The difference in the welfare effects of a ban on price discrimination under different agency cost and different production cost is positive, since downstream firm D_2 is favored under price discrimination.

Proposition 3.5. *The welfare gain from price discrimination is higher in the model with asymmetric agency costs:* $\Delta W_A > \Delta W_P$*.*

Price discrimination has very different effects in the two models. In the model with asymmetric agency costs, a ban on price discrimination increases the difference in quantities produced by the downstream firms for the same real cost parameters. In contrast, in the model with asymmetric production costs, a ban on price discrimination decreases the difference in quantities for the same real cost parameter. If the quantities on the markets become more similar, then production is shifted from the market with high quantity and low price to the market with low quantity and high price. In other words, the good is redistributed from the marginal consumer on the first market to the marginal consumer on the second market. As the price is lower on the first market, the marginal consumer on this market has a lower valuation than the marginal consumer on the second market. Due to this redistribution effect, total welfare tends to be higher if the quantities on the two markets are more similar for the same real cost parameters.

If marginal revenue per marginal costs is concave and both markets are covered, the total quantity for the same cost parameter is higher if price discrimination is banned. Together with the marginal effect described above, this implies the following result.

Proposition 3.6. *Suppose* $R'(q)/k'(q)$ *is concave. Price discrimination decreases welfare in the model with asymmetric production costs if the exclusion of market* 1 *is unlikely, i.e., if* $\Pr(q_1^D(c_1) = 0) \leq Q$ *for some* $Q \in (0, 1)$*.*

For very high cost parameters, market 2 is still served under price discrimination whereas it is not served if price discrimination is banned. If this case is sufficiently unlikely, price discrimination reduces welfare if marginal revenue per marginal costs is concave.

The conditions of Proposition 3.6 are satisfied for the linear inverse demand function $P(q) = \max\{A - B \cdot q, 0\}$ and the linear cost function $K \cdot q$ if K is sufficiently small.

For a wide range of demand and cost functions, price discrimination is beneficial for welfare in the model with asymmetric agency costs and detrimental for welfare in the model with asymmetric production costs. Thus, the source of cost differences crucially influences the welfare implications of price discrimination.

3.6 Conclusion

This chapter provides an analysis of the welfare effects of price discrimination in intermediate good markets when downstream firms have different costs. I first analyze a model where the cost differences are implied by different agency costs due to asymmetric vertical integration. I show that price discrimination increases social welfare for a wide range of demand and cost functions. I then compare this result with the welfare effects of price discrimination in a model where cost differences are the consequence of different production costs. I demonstrate that price discrimination has more positive effects on welfare in the model with asymmetric agency costs. Furthermore, I show that price discrimination reduces welfare in the model with asymmetric production costs for the same condition on demand and cost functions as long as both markets are unlikely to be excluded. The welfare effects of price discrimination therefore crucially depend on the source of cost differences between firms.

The analysis in this chapter was restricted to the case where downstream firms serve separate markets. The results can be expected to extend to the case where downstream firms compete in the same market but offer very differentiated products. It remains an open and very interesting question to which extent the results from the current chapter extend to the case where downstream competition is intense. This question is left for future research.

3.7 Appendix to Chapter 3

Example for a distribution function which satisfies Assumption 3.2

Lemma 3.6. *The distribution function $F(c) = e^{-1/a(c-b)}$ satisfies Assumption 3.2.*

Proof. As $f(c) = e^{-1/a(c-b)} \cdot \frac{1}{a(c-b)^2}$, the virtual cost parameter is $h(c) = c + a(c-b)^2$ which is increasing and convex. Point 2. of the Assumption is satisfied if $\frac{g(c)}{f(c)} = \frac{f(h^{-1}(c))/h'(h^{-1}(c))}{f(c)}$ is weakly increasing. As $h(c)$ is increasing, this is the case if $\frac{f(c)}{h'(c))f(h(c))}$ is increasing. It can be shown that

$$\frac{f(c)}{h'(c))f(h(c))} = \frac{e^{-\frac{1}{a(c-b)}} \cdot \frac{1}{a(c-b)^2}}{e^{-\frac{1}{a(c-b)(1+a(c-b))}} \cdot \frac{1+2a(c-b)}{a(c-b)^2(1+a(c-b))^2}}$$
$$= e^{-\frac{1}{1+a(c-b)}} \cdot \frac{(1+a(c-b))^2}{1+2a(c-b)}.$$

This term is increasing. It remains to check that 3. of Assumption 3.2 is satisfied. Note that $c + \frac{F(c)+G(c)}{f(c)+g(c)} = c + \frac{F(c)+F(h^{-1}(c))}{f(c)+f(h^{-1}(c))/h'(h^{-1}(c))}$ is increasing if $h(c) + \frac{F(h(c))+F(c)}{f(h(c))+f(c)h'(c)} = h(c) + h'(c)\frac{F(h(c))+F(c)}{f(h(c))h'(c)+f(c)}$ is increasing. After some tedious calculations, it can be shown that

$$h(c) + h'(c)\frac{F(h(c)) + F(c)}{f(h(c))h'(c) + f(c)}$$
$$= c + a(c-b)^2 + (1 + 2a(c-b))\frac{e^{-1/a(c-b+a(c-b)^2)} + e^{-1/a(c-b)}}{\frac{e^{-1/a(c-b+a(c-b)^2)}}{a(c-b+a(c-b)^2)^2} + \frac{e^{-1/a(c-b)}}{a(c-b)^2}}$$
$$= c + a(c-b)^2 + (1 + 2a(c-b))a(c-b)^2\frac{1 + e^{-1/a(c-b)}}{1 + e^{-1/a(c-b)}\frac{1+2a(c-b)}{(1+a(c-b))^2}}$$

This expression is increasing if $\frac{1+e^{-1/a(c-b)}}{1+e^{-1/a(c-b)}\frac{1+2a(c-b)}{(1+a(c-b))^2}}$ is increasing. This is the case as $\frac{1+2a(c-b)}{(1+a(c-b))^2}$ is increasing. □

Proof of Lemma 3.1

Define the variable $\Pi_1(c_1) = \max_q R(q) - c_1 k(q) - T_1(q)$. By standard arguments the incentive compatibility constraint IC_1 is equivalent to $\Pi_1'(c_1) = -k(q_1(c_1))$ and $q_1(c_1)$ being non-increasing. Using integration by parts, the problem $\mathcal{P}_1^d$ can be restated as

$$\max_{q_1(c_1),\bar{c}_1,\Pi_1(\bar{c}_1)} \int_{\underline{c}}^{\infty} [R(q_1(c_1)) - h(c_1)k(q_1(c_1))]\, dF(c_1) - \Pi_1(\bar{c}_1)$$

subject to $\Pi_1(\bar{c}_1) \geq 0$ and $q_1(c_1)$ non-decreasing in c_1. The solution to this problem is given by $\Pi_1(\bar{c}_1) = 0$, $q_1^d(c_1)$ which is the pointwise maximizer of the expression above and is non-increasing in c_1, and $\bar{c}_1 = \min\{c \in [\underline{c}, \infty) : \arg\max_q R(q) - h(c)k(q) = 0\}$. $T_1^d(\cdot)$ can be computed using the condition $R'(q_1^d(c)) - c_1 k'(q_1^d(c)) = (T^d)'(q_1^d(c))$ which implies $R'(q) - (q_1^d)^{-1}(q)k'(q) = (T^d)'(q)$ where $(q_1^d)^{-1}(x)$ is the inverse function of $q_1^d(c)$, precisely defined in the Lemma. Using this and $T^d(0) = 0$ gives the result. □

Proof of Lemma 3.2

Define $\Pi_s(c_2) \equiv \max_q R(q) - c_2 k(q) - T_s(q)$. By essentially the same arguments as in the Proof of Lemma 3.2, problem $\mathcal{P}_s^d$ can be written as

$$\max_{q_2(c_2),\bar{c}_2,\Pi_s(\bar{c}_2)} \int_{\underline{c}}^{\infty} [R(q_2(c_2)) - h(c_2)k(q_2(c_2)) - T_2(q_2(c_2))]\, dF(c_2) - \Pi_s(\bar{c}_2)$$

subject to $\Pi_s(\bar{c}_2) \geq 0$ and $q_2(c_2)$ non-decreasing in c_2. The quantity $q_2(c_2)$ which maximizes the expression above pointwisely in non-decreasing in c_2. Thus, it is part of any solution. I now show that D_2 can guarantee himself a positive payoff Note first that $\max_q R(q) - h(c_2)k(q) - T_2(q)$ is non-increasing in c_2. If in any BPE D_2 incurs a negative profit for some c_2, it also incurs a negative profit for all $c_2' > c_2$. Define $\hat{c}_2 = \inf\{c \in [\underline{c}, \infty) : \max_q R(q) - h(c_2)k(q) - T_2(q) < 0\}$. For any $c_2 \geq \hat{c}_2$, D_2 could commit vis-à-vis S not to accept U's offer. As the quantity is then zero for $c_2 \geq \hat{c}_2$, and positive and non-decreasing for $c_2 < \hat{c}_2$, the new quantity schedule is still implementable but can be implemented at a higher profit. D_2 therefore has a profitable deviation. □

Proof of Lemma 3.3

The proof follows the same steps the proof of Lemma 3.1 with the only differences that c_1 is replaced by $h(c_2)$. $T_s^d(q)$ can be computed as $T_1^d(q)$ above, $T_2^d(q)$ can be computed as $T_1^d(q)$ above where c_1 is replaced by $h(c_2)$. □

Proof of Lemma 3.5

Note at first that the results of Lemma 3.2 still hold if price discrimination is banned, thus $\mathcal{P}^n$ is the correct optimization problem. Define $\Pi_1(c_1) = \max_q R(q) -$

$c_1 k(q) - T(q)$ and $\Pi_2(c_2) = \max_q R(q) - h(c_2)k(q) - T(q)$. By standard arguments, (IC_1^n) is equivalent to $\Pi_1'(c_1) = -k(q_1(c_1))$ and $q_1(c_1)$ non-increasing and (IC_2^n) is equivalent to $\Pi_2'(c_2) = -h'(c_2)k(q_2(c_2))$ and $q_2(c_2)$ non-increasing. One can now make a change of variable from $T(\cdot)$ to $\Pi_1(c_1)$ and $\Pi_2(c_2)$ where $\Pi_1(c_1) = \Pi_2(c_2)$ if $c_1 = h(c_2)$. This is equivalent to $q_1(c_1) = q_2(c_2)$ if $c_1 = h(c_2)$. Using integration by parts, the problem can now be written as

$$\begin{aligned}\max_{q_1(c_1),q_2(c_2),\overline{c},\Pi_1(\overline{c})} & \int_{\underline{c}}^{\overline{c}} \left[R(q_1(c_1)) - h(c_1)k(q_1(c_1))\right] dF(c_1) \\ & + \int_{\underline{c}}^{h^{-1}(\overline{c})} \big[R(q_2(c_2)) - (h(c_2) + h'(c_2)(h(c_2) - c_2))k(q_2(c_2))\big] \, dF(c_2) \\ & - 2\Pi_1(\overline{c})\end{aligned}$$

subject to $q_i(c_i)$ non-increasing for $i \in \{1,2\}$, $\Pi_1(\overline{c}) \geq 0$, and $q_1(h(c)) = q_2(c)$ for all $c \in [\underline{c}, \infty)$. It is optimal to set $\Pi_1(\overline{c}) = 0$. Neglecting the monotonicity constraints, one can write the Lagrangian

$$\begin{aligned}\max_{q_1(c_1),q_2(c_2),\overline{c},\Pi_1(\overline{c})} & \int_{\underline{c}}^{\overline{c}} \left[R(q_1(c_1)) - h(c_1)k(q_1(c_1))\right] dF(c_1) \\ & + \int_{\underline{c}}^{h^{-1}(\overline{c})} \big[R(q_2(c_2)) - (h(c_2) + h'(c_2)(h(c_2) - c_2))k(q_2(c_2))\big] \, dF(c_2) \\ & + \int_{\underline{c}}^{h^{-1}(\overline{c})} \lambda(c)(q_1(h(c)) - q_2(c)) dc.\end{aligned}$$

Maximizing the Lagrangian in a pointwise way gives the quantities $q_1^n(c_1)$ and $q_2^n(c_2)$ stated in the Lemma. Under Assumption 3.2, these quantities are non-increasing and therefore satisfy the neglected monotonicity constraints. The tariffs $T^n(q)$ and $T_s^n(q)$ can be computed as in the proofs of Lemmata 3.2 and 3.3. □

Proof of Proposition 3.2

I show at first that $q_1^n(c) = q^n(h(c)) \geq q_1^d(c_1)$. This holds as

$$\begin{aligned} & c + \frac{F(c) + h'(h^{-1}(c))F(h^{-1}(c))}{f(c) + f(h^{-1}(c))} \leq c + \frac{F(c)}{f(c)} \\ \Leftrightarrow \quad & h'(h^{-1}(c))(c - h^{-1}(c)) \leq h(c) - c\end{aligned}$$

which is satisfied due to the convexity of $h(\cdot)$.

Next, I show that $q_2^n(c) = q^n(c) \leq q_2^d(c_2)$. This holds as

$$h(c) + \frac{F(h(c)) + h'(c)F(c)}{f(h(c)) + f(c)} \geq h(c) + h'(c)(h(c) - c)$$
$$\Leftrightarrow h(h(c)) - h(c) \geq (h(c) - c)h'(c)$$

which is also satisfied because $h(\cdot)$ is convex.

Finally, $q_1^d(c) > q_2^d(c)$ follows from

$$h(c) < h(c) + h'(c)(h(c) - c).$$

□

Proof of Proposition 3.3

The welfare effect of price discrimination is given by

$$\Delta W = \int_{\underline{c}}^{\infty} \left(w(c, q_1^d(c)) + w(c, q_2^d(c)) - w(c, q_1^n(c)) - w(c, q_2^n(c)) \right) dF(c).$$

Due to the concavity of the welfare function, total welfare is higher under price discrimination if (i) $|q_1^d(c) - q_2^d(c)| \leq |q_1^n(c) - q_2^n(c)|$ and (ii) $q_1^d(c) + q_2^d(c) \geq q_1^n(c) + q_2^n(c)$ for all cost parameters c. (i) is satisfied because $h(\cdot)$ is convex. This is shown in the proof of Proposition 3.2. To show that (ii) holds, define $\phi(x)$ as the inverse function of $R'(q)/k'(q)$, i.e. $R'(\phi(x))/k'(\phi(x)) = x$. For all c such that all quantities are positive, (ii) can be written as

$$\phi(h(c)) + \phi\left(h(c) + h'(c)(h(c) - c)\right) \geq$$
$$\phi\left(c + \frac{F(c) + h'(h^{-1}(c))F(h^{-1}(c))}{f(c) + f(h^{-1}(c))}\right) + \phi\left(h(c) + \frac{F(h(c)) + h'(c)F(c)}{f(h(c)) + f(c)}\right).$$

If $R'(q)/k'(q)$ is decreasing and concave, $\phi(x)$ is decreasing and concave and (ii) holds if

$$h(c) + h(c) + h'(c)(h(c) - c) \leq$$
$$c + \frac{F(c) + h'(h^{-1}(c))F(h^{-1}(c))}{f(c) + f(h^{-1}(c))} + h(c) + \frac{F(h(c)) + h'(c)F(c)}{f(h(c)) + f(c)}.$$

This is equivalent to

$$\frac{F(c)}{f(c)} - \frac{F(c) + h'(h^{-1}(c))F(h^{-1}(c))}{f(c) + f(h^{-1}(c))} \leq \frac{F(h(c)) + h'(c)F(c)}{f(h(c)) + f(c)} - \frac{F(c)}{f(c)}$$

which can equivalently be expressed as

$$\frac{F(c)f(h^{-1}(c)) - f(c)F(h^{-1}(c))h'(h^{-1}(c))}{f(c) + f(h^{-1}(c))} \leq \frac{F(h(c))f(c) - f(h(c))F(c)h'(c)}{f(c) + f(h(c))}.$$

The last inequality is satisfied if

$$\frac{F(c)f(h^{-1}(c)) - f(c)F(h^{-1}(c))h'(h^{-1}(c))}{f(h^{-1}(c)) + f(c))}$$

is non-decreasing. Note at first that

$$\begin{aligned}
&\frac{\partial}{\partial c}\left(\frac{F(c)f(h^{-1}(c)) - f(c)F(h^{-1}(c))h'(h^{-1}(c))}{f(h^{-1}(c)) + f(c)}\right) \\
&\geq \frac{\partial}{\partial c}\left(\frac{F(c)f(h^{-1}(c)) - f(c)F(h^{-1}(c))h'(h^{-1}(c))}{f(h^{-1}(c)) + f(c)h'(h^{-1}(c))}\right) \\
&= \frac{\partial}{\partial c}\left(\frac{F(c)g(c) - f(c)G(c)}{f(c) + g(c)}\right)
\end{aligned}$$

because $h'(\cdot) \geq 1$ and convex. Furthermore

$$\frac{\partial}{\partial c}\left(\frac{F(c)g(c) - f(c)G(c)}{f(c) + g(c)}\right) = \frac{(F(c) + G(c))(f(c)g'(c) - f'(c)g(c))}{(f(c) + g(c))^2}$$

which is positive under Assumption 3.2. Until here, I made the assumption that both markets are always covered. However, note that market 2 is more likely to be not served if price discrimination is banned. In that case, the calculations made above are a lower bound on the welfare gain from price discrimination, and this lower bound was shown to be positive. Because welfare is strictly concave in q, a continuity argument implies that the result holds as long as $R'(q)/k'(q)$ – and thus $\phi(x)$ – is not too convex. □

Proof of Proposition 3.4

The results of the first two paragraphs follow from the observation that the downstream firm D_2 in the model with asymmetric production costs behaves for the cost parameter c_2 as the vertically separated downstream firm in the model with asymmetric agency costs for the cost parameter $h^{-1}(c_2)$.

It remains to show that $q^N(c) \in [q_1^D(c), q_2^D(c)]$. $q^N(c) \geq q_1^D(c)$ follows from $q_N(c) = q_1^n(c)$, $q_1^D(c) = q_1^d(c)$, and Proposition 3.2. $q^N(c) \leq q_2^D(c)$ follows from $q_N(c) = q_1^n(c)$, $q_2^D(c) = q_2^d(h^{-1}(c))$, and the ranking of virtual cost parameters for $q_1^n(c)$ and $q_2^d(h^{-1}(c))$ given by

$$c + h'(h^{-1}(c))(c - h^{-1}(c)) \leq c + \frac{F(c) + h'(h^{-1}(c))F(h^{-1}(c))}{f(c) + f(h^{-1}(c))}$$

which follows from

$$\frac{F(h^{-1}(c))}{f(h^{-1}(c))} \leq \frac{F(c)}{f(c)}.$$

□

Proof of Proposition 3.5

The derivation of ΔW_P is straightforward. The expression for ΔW_A can be derived as follows

$$\begin{aligned}
\Delta W_A &= \int_{\underline{c}}^{\infty} \Big[w(c_1, q_1^d(c_1)) - w(c_1, q^n(c_1))\Big]\, dF(c_1) \\
&\quad + \int_{\underline{c}}^{\infty} \Big[w(c_2, q_2^d(c_2)) - w(c_2, q^n(c_2))\Big]\, dF(c_2) \\
&= \int_{\underline{c}}^{\infty} \big[w(c_1, q_1^D(c_1)) - w(c_1, q^N(c_1))\big]\, dF(c_1) \\
&\quad + \int_{\underline{c}}^{\infty} \big[w(c_2, q_2^D(h(c_2))) - w(c_2, q^N(h(c_2)))\big]\, dF(c_2) \\
&= \int_{\underline{c}}^{\infty} (w(c_1, q_1^D(c_1)) - w(c_1, q^N(c_1))) dF(c_1) \\
&\quad + \int_{\underline{c}}^{\infty} \int_{q^N(z_2)}^{q_2^D(z_2)} P(x)dx - h^{-1}(z_2)(q_2^D(z_2) - q^N(z_2)) dG(z_2) \\
&= \int_{\underline{c}}^{\infty} \big[w(c_1, q_1^D(c_1)) - w(c_1, q^N(c_1))\big]\, dF(c_1) \\
&\quad + \int_{\underline{c}}^{\infty} \Big[w(z_2, q_2^D(z_2)) - w(z_2, q^N(z_2) \\
&\qquad + (z_2 - h^{-1}(z_2))(k(q_2^D(z_2)) - k(q^N(z_2)))\Big] dG(z_2),
\end{aligned}$$

where the second equality follows from a change of variable from c_2 to $z_2 = h(c_2)$ and where $F(c_1) = F(c_1)$ and $G(c_2) = F(h^{-1}(c_2))$. For $z_2 = c_2$, this expression is equal to the expression in the main text. It is now immediate to observe that $\Delta W_A > \Delta W_P$ if D_2 is favored under price discrimination, i.e. if $q_2^d(c_2) > q^n(c_2)$ for all c_2. □

Proof of Proposition 3.6

I show that $\Delta W_P < 0$ if $R'(q)/k'(q)$ is concave and $Pr(q_2^d(c_2) = 0) \leq Q$ for some $Q \in [0,1]$. Suppose at first that $R'(q)/k'(q)$ is concave and define as above the concave function $\phi(x)$ by $R'(\phi(x))/k'(\phi(x)) = x$. The welfare effect of price discrimination is given by

$$\Delta W_P = \int_{\underline{c}}^{\infty} \Big(f(c)w(c, q_1^d(c)) + g(c)w(c, q_2^d(c)) - (f(c)+g(c))w(c, q^n(c)) \Big)\, dc.$$

For a given c and due to the strict concavity of the function $w(c,q)$ in q, the expression below the integral is strictly positive if $f(c)q_1^d(c) + g(c)q_2^d(c) \leq (f(c) + g(c))q^n(c)$. This is equivalent to

$$f(c)\phi\left(c + \frac{F(c)}{f(c)}\right) + g(c)\phi\left(c + \frac{G(c)}{g(c)}\right) \leq (f(c)+g(c))\phi\left(c + \frac{F(c)+G(c)}{f(c)+g(c)}\right) \tag{3.12}$$

for any $c \in [\underline{c}, \infty)$ such that $q_1^D(c) > 0$. Note that Equation (3.12) is satisfied as $\phi(x)$ is concave. If the probability $\Pr(q_1^D(c_1) = 0)$ that the market 2 is excluded is sufficiently small, the total welfare effect of price discrimination is negative, i.e., $\Delta W_P < 0$. □

Bibliography

Aguirre, I., Cowan, S., and Vickers, J. (2010). Monopoly price discrimination and demand curvature. *American Economic Review*, 100(4):1601–1615.

Arya, A. and Mittendorf, B. (2010). Input price discrimination when buyers operate in multiple markets. *Journal of Industrial Economics*, 58(4):846–867.

Asseyer, A. (2016a). Informational control and collusive supervision. *Working paper, Humboldt-Universitt zu Berlin.*

Asseyer, A. (2016b). Optimal monitoring in dynamic procurement contracts. *Working paper, Humboldt-Universitt zu Berlin.*

Asseyer, A. (2016c). Price discrimination in intermediate good markets with asymmetric agency agency costs. *Working paper, Humboldt-Universitt zu Berlin.*

Astebro, T. (2004). Sunk costs and the depth and probability of technology adoption. *Journal of Industrial Economics*, 52(3):381–399.

Bag, P. K. (1997). Optimal auction design and R&D. *European Economic Review*, 41(9):1655–1674.

Bagnoli, M. and Bergstrom, T. (2005). Log-concave probability and its applications. *Economic Theory*, 26(2):445–469.

Baliga, S. and Sjöström, T. (1998). Decentralization and collusion. *Journal of Economic Theory*, 83(2):196–232.

Baron, D. P. and Besanko, D. (1984). Regulation and information in a continuing relationship. *Information Economics and Policy*, 1(3):267–302.

Bergemann, D., Brooks, B., and Morris, S. (2015). The limits of price discrimination. *American Economic Review*, 105(3):921–957.

Bergemann, D. and Pesendorfer, M. (2007). Information structures in optimal auctions. *Journal of Economic Theory*, 137(1):580–609.

Bergemann, D. and Wambach, A. (2015). Sequential information disclosure in auctions. *Journal of Economic Theory*, 159:1074–1095.

Bertrand, M. and Mullainathan, S. (2003). Enjoying the quiet life? corporate governance and managerial preferences. *Journal of Political Economy*, 111(5):1043–1075.

Bindemann, K. (1999). Vertical integration in the oil industry: A review of the literature. *Journal of Energy Literature*, 5:3–26.

Buehler, S. and Schmutzler, A. (2005). Asymmetric vertical integration. *Advances in Theoretical Economics*, 5(1).

Bulow, J. I. and Pfleiderer, P. (1983). A note on the effect of cost changes on prices. *Journal of Political Economy*, 91(1):182–185.

Caillaud, B. and Jehiel, P. (1998). Collusion in auctions with externalities. *Rand Journal of Economics*, 29(4):680–702.

Caplin, A. and Leahy, J. (2010). Economic theory and the world of practice: A celebration of the (S, s) model. *Journal of Economic Perspectives*, 24(1):183–202.

Celik, G. (2009). Mechanism design with collusive supervision. *Journal of Economic Theory*, 144(1):69–95.

Che, Y.-K., Condorelli, D., and Kim, J. (2014). Weak cartels and collusion-proof auctions. *Working paper, Columbia University, University of Essex, and Seoul National University.*

Che, Y.-K. and Kim, J. (2006). Robustly collusion-proof implementation. *Econometrica*, 74(4):1063–1107.

Che, Y.-K. and Kim, J. (2009). Optimal collusion-proof auctions. *Journal of Economic Theory*, 144(2):565–603.

Chipty, T. (2001). Vertical integration, market foreclosure, and consumer welfare in the cable television industry. *American Economic Review*, 91(3):428–453.

Cisternas, G. and Figueroa, N. (2015). Sequential procurement auctions and their effect on investment decisions. *RAND Journal of Economics*, 46(4):824–843.

Courty, P. and Li, H. (2000). Sequential screening. *Review of Economic Studies*, 67(4):697–717.

Cowan, S. (2012). Third-degree price discrimination and consumer surplus. *Journal of Industrial Economics*, 60(2):333–345.

Crawford, V. P. and Sobel, J. (1982). Strategic information transmission. *Econometrica*, 50(6):1431–1451.

Crémer, J. (1996). Manipulations by coalitions under asymmetric information: The case of groves mechanisms. *Games and Economic Behavior*, 13(1):39–73.

DeGraba, P. (1990). Input market price discrimination and the choice of technology. *American Economic Review*, 80(5):1246–1253.

Dequiedt, V. (2007). Efficient collusion in optimal auctions. *Journal of Economic Theory*, 136(1):302–323.

Dewatripont, M. and Maskin, E. (1995). Contractual contingencies and renegotiation. *RAND Journal of Economics*, 26(4):704–719.

Di Tella, R. and Schargrodsky, E. (2003). The role of wages and auditing during a crackdown on corruption in the city of buenos aires. *Journal of Law and Economics*, 46(1):269–292.

Doms, M. and Dunne, T. (1998). Capital adjustment patterns in manufacturing plants. *Review of Economic Dynamics*, 1(2):409–429.

Engel, E., Fischer, R., and Galetovic, A. (2013). The basic public finance of public–private partnerships. *Journal of the European Economic Association*, 11(1):83–111.

Eső, P. and Szentes, B. (2007a). Optimal information disclosure in auctions and the handicap auction. *Review of Economic Studies*, 74(3):705–731.

Eső, P. and Szentes, B. (2007b). The price of advice. *Rand Journal of Economics*, 38(4):863–880.

Eső, P. and Szentes, B. (2015). Dynamic contracting: An irrelevance theorem. *Theoretical Economics*, forthcoming.

Faure-Grimaud, A., Laffont, J.-J., and Martimort, D. (2003). Collusion, delegation and supervision with soft information. *Review of Economic Studies*, 70(2):253–279.

Fracassi, C. and Tate, G. (2012). External networking and internal firm governance. *Journal of Finance*, 67(1):153–194.

GAO (2013). Agencies need to strengthen oversight of multibillion dollar investments in operations and maintenance. *Report "GAO-14-66" by the United States Government Accountability Office (GAO).*

Garvin, M., Molenaar, K., Navarro, D., and Proctor, G. (2011). Key performance indicators in public-private partnerships. *Report for the US Department of Transportation, report no. FHWA-PL-10-029.*

Green, J. and Laffont, J.-J. (1979). On coalition incentive compatibility. *Review of Economic Studies*, 46(2):243–254.

Hallock, K. F. (1997). Reciprocally interlocking boards of directors and executive compensation. *Journal of Financial and Quantitative Analysis*, 32(3):331–344.

Herweg, F. and Müller, D. (2012). Price discrimination in input markets: Downstream entry and efficiency. *Journal of Economics & Management Strategy*, 21(3):773–799.

Herweg, F. and Müller, D. (2014). Price discrimination in input markets: Quantity discounts and private information. *Economic Journal*, 124(577):776–804.

HMT (2012). A new approach to public private partnerships. *HM Treasury, UK.*

Hoppe, E. I. and Schmitz, P. W. (2013). Public-private partnerships versus traditional procurement: Innovation incentives and information gathering. *RAND Journal of Economics*, 44(1):56–74.

Inderst, R. and Shaffer, G. (2009). Market power, price discrimination, and allocative efficiency in intermediate-goods markets. *RAND Journal of Economics*, 40(4):658–672.

Inderst, R. and Valletti, T. (2009). Price discrimination in input markets. *RAND Journal of Economics*, 40(1):1–19.

Iossa, E. and Martimort, D. (2012). Risk allocation and the costs and benefits of public–private partnerships. *RAND Journal of Economics*, 43(3):442–474.

Iossa, E. and Martimort, D. (2015). The simple microeconomics of public-private partnerships. *Journal of Public Economic Theory*, 17(1):4–48.

Iossa, E. and Martimort, D. (2016). Corruption in PPPs, incentives and contract incompleteness. *International Journal of Industrial Organization*, 44:85–100.

Ivanov, M. (2010). Informational control and organizational design. *Journal of Economic Theory*, 145(2):721–751.

Kamenica, E. and Gentzkow, M. (2011). Bayesian persuasion. *American Economic Review*, 101(6):2590–2615.

Katz, M. L. (1987). The welfare effects of third-degree price discrimination in intermediate good markets. *American Economic Review*, 77(1):154–67.

Khalil, F. (1997). Auditing without commitment. *RAND Journal of Economics*, 28(4):629–640.

Khalil, F. and Lawarrée, J. (1995). Input versus output monitoring: who is the residual claimant? *Journal of Economic Theory*, 66(1):139–157.

Krähmer, D. and Strausz, R. (2015). Ex post information rents in sequential screening. *Games and Economic Behavior*, 90:257–273.

Laffont, J.-J. and Martimort, D. (1997). Collusion under asymmetric information. *Econometrica*, 65(4):875–911.

Laffont, J.-J. and Martimort, D. (1998). Collusion and delegation. *RAND Journal of Economics*, 29(2):280–305.

Laffont, J.-J. and Martimort, D. (2000). Mechanism design with collusion and correlation. *Econometrica*, 68(2):309–342.

Laffont, J.-J. and Tirole, J. (1986). Using cost observation to regulate firms. *Journal of Political Economy*, 94(3):614–641.

Laffont, J.-J. and Tirole, J. (1993). *A Theory of incentives in procurement and regulation*. Cambridge, MA: The MIT Press.

Li, H. and Shi, X. (2015). Discriminatory information disclosure. *Working paper, University of British Columbia and University of Toronto.*

Liu, B. and Lu, J. (2015). Optimal two-stage procurement with private r&d efficiency. *Working paper, National University of Singapore.*

Martimort, D. and Stole, L. (2009). Market participation in delegated and intrinsic common-agency games. *Rand Journal of Economics*, 40(1):78–102.

Maskin, E. and Riley, J. (1985). Input versus output incentive schemes. *Journal of Public Economics*, 28(1):1–23.

McAfee, R. P. and McMillan, J. (1992). Bidding rings. *American Economic Review*, 82(3):579–99.

Milgrom, P. and Segal, I. (2002). Envelope theorems for arbitrary choice sets. *Econometrica*, 70(2):583–601.

Mookherjee, D., Motta, A., and Tsumagari, M. (2015). Weak ex ante collusion and design of supervisory institutions. *Working paper, Boston University, University of New South Wales, and Keio University.*

Mookherjee, D. and Tsumagari, M. (2004). The organization of supplier networks: effects of delegation and intermediation. *Econometrica*, 72(4):1179–1219.

Myerson, R. B. (1986). Multistage games with communication. *Econometrica*, 54(2):323 – 358.

NAO (2014). Transforming government's contract management. *Report "HC269" by the National Audit Office (NAO), UK.*

NAO/OGC (2008). Good practice contract management framework. *National Audit Office (NAO) and Office of Government Commerce (OGC), UK.*

O'Brien, D. P. (2014). The welfare effects of third-degree price discrimination in intermediate good markets: the case of bargaining. *RAND Journal of Economics*, 45(1):92–115.

O'Brien, D. P. and Shaffer, G. (1994). The welfare effects of forbidding discriminatory discounts: A secondary line analysis of Robinson-Patman. *Journal of Law, Economics, and Organization*, 10(2):296–318.

OECD (2015a). Public procurement spending. *Government at a Glance.*

OECD (2015b). Recommendation of the council on public procurement. *Directorate for Public Governance and Territorial Development.*

Ortner, J. and Chassang, S. (2015). Making corruption harder: Asymmetric information, collusion, and crime. *Working paper, Boston University and Princeton University.*

Pavan, A., Segal, I., and Toikka, J. (2014). Dynamic mechanism design: A Myersonian approach. *Econometrica*, 82(2):601–653.

Pavlov, G. (2008). Auction design in the presence of collusion. *Theoretical Economics*, 3(3):383–429.

Piccione, M. and Tan, G. (1996). Cost-reducing investment, optimal procurement and implementation by auctions. *International Economic Review*, 37(3):663–685.

PWGSC (July 11, 2013). Government of canada selects third-party expert to support upcoming national shipbuilding procurement strategy projects. *News Release: Public Works and Government Services Canada, retrieved from www.tpsgc-pwgsc.gc.ca/app-acq/sam-mps/nouvelles-news-eng.html on May 11, 2015.*

Robinson, J. (1933). *Economics of imperfect competition.* London, UK: Macmillan and Co.

Slade, M. E. (1998a). Beer and the tie: Did divestiture of brewer-owned public houses lead to higher beer prices? *The Economic Journal*, 108(448):565–602.

Slade, M. E. (1998b). Strategic motives for vertical separation: Evidence from retail gasoline markets. *Journal of Law, Economics, and Organization*, 14(1):84–113.

Strausz, R. (1997). Delegation of monitoring in a principal-agent relationship. *Review of Economic Studies*, 64(3):337–357.

Tan, G. (1992). Entry and R&D in procurement contracting. *Journal of Economic Theory*, 58(1):41–60.

Tirole, J. (1986). Hierarchies and bureaucracies: On the role of collusion in organizations. *Journal of Law, Economics, and Organization*, 2(2):181–214.

Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2):265–293.

Waterman, D. and Weiss, A. A. (1996). The effects of vertical integration between cable television systems and pay cable networks. *Journal of Econometrics*, 72(1):357–395.

Weyl, E. G. and Fabinger, M. (2009). Pass-through as an economic tool. *Working paper, Microsoft Research New York City and Unversity of Tokyo.*

Woodruff, C. (2002). Non-contractible investments and vertical integration in the mexican footwear industry. *International Journal of Industrial Organization*, 20(8):1197–1224.

Yoshida, Y. (2000). Third-degree price discrimination in input markets: output and welfare. *American Economic Review*, 90(1):240–246.

Selbständigkeitserklärung

Für diese Dissertation habe ich keine anderen Hilfsmittel außer der angeführten Literatur benutzt.
Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung meiner Dissertation benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

Berlin, 20. Oktober 2016

Andreas Asseyer