Three Essays on Auctions and Innovation

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Abstract

Innovation is central to development and economic growth. Innovation happens within some institutional framework. Auctions and auction-like mechanisms are institutions that organize transactions between economic agents. In the face of private information, they provide a means of revealing part of that information by inducing competition between agents. If well designed, they make use of the revealed information in order to achieve a certain objective, e.g., to maximize profit or to allocate efficiently.

In three essays, this dissertation studies the use of auctions in the context of innovation. Chapter 1 looks at the widely used practice of allocating government R&D subsidies to private companies. We point out flaws of that practice and propose improvements that can be adopted separately or in combination. Our proposals are tested in controlled lab experiments and by simulation. The results suggest that adopting the proposals might substantially improve the allocation of subsidies.

Chapter 2 revisits the literature on the sale, and, in particular, auctioning, of patent licenses by an innovator to a downstream oligopoly. It analyzes a modified auction that turns out to be more profitable than many other mechanisms that have been analyzed in the literature. There, a restricted number of royalty contracts is auctioned while all losers of the auction are granted the right to sign a royalty contract.

Chapter 3 looks at R&D tournaments. It starts from two well-known auction institutions, the fixed-prize tournament and the scoring auction. It combines both with an entry auction, a feature that has been proposed in the literature. We characterize Bayesian Nash equilibria such that both mechanisms are equivalent in a number of ways.

Zusammenfassung

Innovation ist von zentraler Bedeutung für Entwicklung und Wirtschaftswachstum. Innovation findet in einem institutionellen Rahmen statt. Auktionen und auktionsähnliche Mechanismen sind Institutionen. Sie organisieren Transaktionen zwischen ökonomischen Agenten. Auktionen enthüllen private Information indem sie Wettbewerb zwischen Agenten erzeugen. Wenn sie günstig gestaltet werden, dann nutzen sie die enthüllte private Information zur Erfüllung bestimmter Ziele, wie z.B. Gewinnmaximierung oder Effizienz. In drei Aufsätzen untersucht diese Dissertation die Verwendung von Auktionen im Kontext von Innovation. Kapitel 1 betrachtet die übliche Praxis der öffentlichen Subventionsvergabe an private Unternehmen. Wir weisen auf Nachteile dieser Praxis hin und machen Verbesserungsvorschläge, die man einzeln oder in Kombination anwenden kann. Unsere Vorschläge wurden mit Hilfe von Laborexperimenten und Simulation getestet. Die Ergebnisse lassen vermuten, dass unsere Vorschläge eine erhebliche Verbesserung der Subventionsvergabe ermöglichen.

Kapitel 2 betrachtet den Verkauf, bzw. die Versteigerung, von Lizenzen zur Nutzung einer patentierten Innovation an Unternehmen in einem oligopolistischen Markt. Das Kapitel analysiert eine modifizierte Lizenzauktion, die profitabler ist als die in der Literatur untersuchten Mechanismen. Bei dieser Auktion wird eine begrenzte Anzahl Royalty-Verträge (Royalty = mengenabhängige Lizenzgebühren) versteigert und die Verlierer der Auktion erhalten ebenfalls die Option auf Royalty-Verträge.

Kapitel 3 untersucht Innovations-Turniere. Zwei bekannte Mechanismen, das Festpreis-Turnier und die Scoring-Auktion, werden mit einer Auktion kombiniert, in der um die Teilnahme am Turnier geboten wird (entry auction). Diese Variante wurde in der Literatur vorgeschlagen. Wir charakterisieren Bayesianische Nash-Gleichgewichte bei denen die beiden Mechanismen zu gleichartigen Ergebnissen führen.

for my family

Contents

Introduction

In this dissertation we look at applications of auction mechanisms in the context of innovation. In three essays, we will see how auctions can be used in order to award R&D subsidies, how an innovator can profitably sell patent licenses, and how a procurer can employ auctions in order to select innovators for a tournament and reward the winner. Thus, in different ways, the use of auctions can provide incentives that *encourage* innovation.

Chapter 1 is joint work with Tim Grebe and Elmar Wolfstetter.[1] It points out flaws in public R&D subsidy programs as they are currently used in different countries. These flaws are mainly the selection based on a ranking of individual projects, rather than complete allocations, and the failure to induce competition among applicants in order to extract and use information about the necessary funding. We give two recommendations as to how that practice can be improved and support our proposals with Monte Carlo simulations and experimental results. In particular we propose the use of auction-like mechanisms where applicants compete for funding such that transparency and efficiency are improved and more innovative projects can be realized. The results suggest that adopting our proposals may improve the allocation considerably.[2]

Chapter 2 is joint work with Elmar Wolfstetter.[3] It revisits the licensing of a non-drastic process innovation by an outside innovator to a downstream Cournot oligopoly. We propose a mechanism that combines a restrictive license auction with royalty licensing. This mechanism is more profitable than standard license auctions, auctioning royalty contracts, fixed-fee licensing, pure royalty licensing, and two-part tariffs. In that sense, the proposed mechanism increases the incentives for innovation. The key features are that royalty contracts are auctioned and that losers of the auction are granted the option to sign a royalty contract. Combining royalties for winners and losers of the auction makes the integer constraint concerning the number of licenses irrelevant. Put differently, using royalty contracts for winners, rather than pure licenses, is a fine-tuning device in the face of an integer constraint on the number of licenses.

Chapter 3 deals with a procurer who needs an innovative good that can be provided by a number of sellers with different innovative abilities. Innovations are random but depend on unobservable effort and privately known

[1] Published as Giebe et al. (2006).

[2] This work is being continued in Ensthaler and Giebe (2009). There, we propose a simpler auction mechanism where bidders have a dominant strategy to bid their true required amount of funding.

[3] Published as Giebe and Wolfstetter (2008).

ability. We compare two procurement mechanisms: innovation tournaments with entry auction where an innovation is procured either employing a first-price auction or a fixed prize. In the entry auction of both mechanisms, innovators bid for the right to enter the tournament. We demonstrate existence of Bayesian Nash equilibria such that the outcomes of both mechanisms differ only in the exact monetary transfers to the sellers. Apart from that, both mechanisms exhibit the same efforts and innovations for all sellers (as functions of type), the same profit of the procurer, and the same *expected* profit of the sellers (as functions of type). Thus, in expectation, the mechanisms implement the same social choice. In the entry auction, innovators can adjust their bids in accordance with the expected profits of the subsequent tournament game. Thus, they bid more if they expect a larger profit. As a result, tournaments with different reward schemes turn out to be equally profitable for all players, including the procurer.

Chapter 1

Allocating R&D Subsidies

This chapter is based on Giebe et al. (2006).

1.1 Introduction

R&D subsidies to industry are an important part of research policy. For example, the federal government of Germany roughly spends €2 billion per year for supporting industry R&D. In 2002, this accounted for approximately 20% of its total R&D spending.[1] Typically, R&D subsidy programs are geared to a particular purpose such as job creation in particular regions or research intensity in particular industries. Some of these programs offer grants, others provide loans at subsidized interest rates or funding in return for a profit share. Most programs support small and medium sized businesses (SMEs).

In the present paper we analyze programs that offer non-refundable grants. This is a common form of subsidization. In most of these programs the allocation of funds is organized in competitions, as follows.[2] Applicants submit written project proposals to a program manager at some due date. These proposals are pre-screened and short-listed, and then evaluated by a team of experts on the basis of their scientific and economic merit. Based on the expert advice, a committee grades projects, using a small set of grades such as A, B, and C. And the committee selects projects in the order of the assigned grades, down from A to C, until the available budget is exhausted. Thereby, each funded project receives a subsidy equal to a predetermined percentage

[1] See, e.g., Czarnitzki and Fier (2001) or Federal Ministry of Education and Research (2004) for more details.

[2] A detailed description and analysis of some of the programs applied in Germany can be found in Federal Ministry of Education and Research (2004), Blum et al. (2001), Becker et al. (2004), and Eickelpasch and Fritsch (2005).

of the scheduled refundable project cost.[3]
We mention that similar R&D subsidy programs are employed in many other countries. For example, an account of programs employed in the U.K. and the U.S. can be found in Binks et al. (2003).
In the present paper we will not debate the merit of directly subsidizing firms' R&D activities.[4] However, we object to the way in which firms' projects are selected and subsidies are determined.
Specifically, we see two main deficiencies and propose to modify the currently used attribution procedure in two ways.

1. Funding the best projects until the budget is exhausted is inefficient. Instead, the selection should be based on a ranking of complete allocations of funds.

2. Funding the selected projects at a predetermined percentage of project cost is inefficient. Instead, one should induce applicants to compete by lowering their requests for funding.

In order to achieve these objectives, we propose to base the selection of projects on a ranking of allocations, and to embed that selection rule in a simple auction mechanism. The development of an auction-like mechanism for awarding subsidies has been suggested by Blum et al. (2001) and Blum and Kalus (2003). They propose that firms should compete with their requests for funding in order to economize on the amount of subsidies. Specifically, they propose to allocate a given budget of subsidies to those who request the lowest subsidy rates, as a share of their total project cost. However, this allocation rule can only give meaningful results if all projects that compete in one auction yield the same benefit per currency unit, which is rather restrictive.
In this paper, we develop the idea of auctioning subsidies with the goal to make it applicable to a larger class of applications, allowing for arbitrary quality and cost differences across projects. For this purpose we develop a slightly complicated selection rule that is based on a ranking of all possible allocations of subsidies and awards in such a way that the highest ranking feasible allocation is reached. Our proposed mechanisms match this allocation

[3] Typically, only part of the project cost, such as personnel cost, are eligible for subsidies. A frequently employed rule is the "matching grant" where 50% of the refundable project cost are reimbursed.

[4] Some researchers, such as Martin and Scott (2000), have suggested that one should subsidize venture capitalists rather than firms. Other researchers investigate into the overall effectiveness of R&D subsidies in promoting research in private enterprises. See, e.g., García-Quevedo (2005) for an international survey and Czarnitzki and Licht (2006) for a survey on the effects of R&D subsidies in Germany.

rule with either an open, descending-bid or a sealed-bid bidding procedure. We use Monte Carlo Simulations and controlled lab experiments to test the proposed mechanisms.

The plan of the paper is as follows. In section 2 we discuss selection rules and show why one should select on the basis of a ranking of complete allocations. In section 3, we explain two specifications of an auction mechanism and explain how our auction problem relates to the existing auction theory literature. Section 4 evaluates the proposed selection rule by a Monte Carlo Simulation. Section 5 describes the design of a lab experiment to test the two auction mechanisms and section 6 reviews its results. The paper closes in section 7 with a summary and discussion.

1.2 Ranking projects vs. ranking allocations

Before we design mechanisms that induce competition for funding, we explain how one should select who shall be subsidized, taking the subsidies to be paid to those who are selected as given.

Suppose applicants have submitted project proposals and the selection committee has evaluated them and has short-listed a set $P := \{1, \ldots, n\}$ of projects which are judged as eligible for funding. Project i shall receive a subsidy of s_i if selected. The selection committee has to choose a subset of projects that shall be funded within the limits of the given budget $\mathcal{B}$.

The standard selection rule is based on a ranking of individual projects, from the set of short-listed projects, as follows: 1) each project is assigned a grade from a given set of grades (for the moment one may assume that each project has a distinct grade); 2) projects are selected, moving from highest to lower grades, until the given budget is exhausted. As a result, no lower-grade project ever crowds out a higher grade. This may seem to be a desirable property; however, it is generally not optimal.

As an illustration consider the example of four projects, $P = \{P_1, \ldots, P_4\}$, which require the following subsidies if selected: $s = \{100, 50, 50, 50\}$ and a budget of 150. Suppose the selection committee has the preference order $P_1 \succeq P_2 \succeq P_3 \succeq P_4$. Then the selection based on the ranking of individual projects leads to the selection of projects $\{P_1, P_2\}$. However, if $\{P_2, P_3, P_4\}$ is preferred to $\{P_1, P_2\}$, it would be better to select $\{P_2, P_3, P_4\}$ since that allocation is also feasible at the given budget. This indicates that the selection based on the ranking of individual projects leads astray, because it does not take into account that a high-grade project may crowd out several lower-grade projects which are inferior in pairwise comparisons, but lead to a superior allocation. Indeed, that selection is equivalent to preferring every single higher-grade to any number of lower-grade projects.

Therefore, as a first step towards achieving a better selection process, the selection committee should think in terms of complete allocations, and apply the following selection rule.[5]

Proposal 1 *Select projects based on a ranking of allocations, rather than based on a ranking of projects, as follows: 1) determine all allocations that are feasible (can be funded with the given budget); 2) rank all feasible allocations and select the projects that are part of the highest ranking feasible allocation.*

In practical application this procedure may be fairly complex, since the number of allocations increases exponentially with the number of projects. Therefore, we recommend sticking to a fixed grading system, as it is typically used in the current system, consisting of at most three grades, such as $\{A, B, C\}$. Such a grading system treats projects of the same grade as perfect substitutes. We were also told by program managers that employing more than two or three grades was not sensible because this would only suggest a degree of precision that cannot be reached. In addition one may use constant equivalence rules that state how many higher-grade projects are equivalent to one lower-grade project. Using such constant equivalence rules corresponds to assuming linear indifference curves in the commodity resp. grade space. For example, for the grade set $\{A, B, C\}$ the equivalence rules $(e(b), e(c))$ state the number of grade-A projects that are equivalent to one grade-B, resp. grade-C, project. We also employ this practical device in our lab experiments which are described in section 1.5.
The proposal requires neither the use of grades nor of constant equivalence rules. Using these makes the selection simpler. However, it is only advisable if the underlying assumptions are justified as an approximation.
As a practical advice, one may consider to pre-specify the grade set and the fixed equivalence rules, and ask those who assess the quality of projects only to assign grades, taking into account the given equivalence rules.

1.2.1 Formal statement of the allocation ranking problem

We conclude this section with a precise statement of the allocation ranking problem. The notation introduced here will also be used to describe our auction mechanisms.
For this purpose, let $P := \{1, \ldots, n\}$ be the finite set of short-listed projects and $\mathcal{A}$ the set of subsets (i.e., the power set) of P. Therefore, $\mathcal{A}$ is the set of

[5]To an economist, this proposal may seem fairly obvious. However, in our experience, program managers are not aware of the flaws of the current selection procedure.

all conceivable allocations from which the committee has to select one, under some feasibility constraint.
Ideally, the selection committee has a complete preference ranking, "$\succeq$", of all allocations, such that for all $a, a' \in \mathcal{A}$ one has $a \succeq a'$ or $a' \succeq a$ that is reflexive and transitive. Such a preference ranking defined on a set of finite alternatives can be represented by an (ordinal) utility function, $U : \mathcal{A} \to \mathbb{R}$, such that $\forall a, a' \in \mathcal{A}$: $U(a) \geq U(a') \Leftrightarrow a \succeq a'$.
The promised subsidy for project i, if it is part of the allocation, is denoted by s_i.
The choice of allocation based on a ranking of individual projects (as in the *status quo* procedure) is denoted by a^s and the choice of allocation according to Proposal 1 is denoted by a^p. Thus, a^p is the maximizer of $U(a)$ over all feasible allocations that can be funded with the given budget $\mathcal{B}$:

$$a^p \in \arg\max_{a \in \mathcal{A}} \left\{ U(a) \mid \sum_{i \in a} s_i \leq \mathcal{B} \right\}. \tag{1.1}$$

As mentioned before, committees often employ a grading scheme as a simplifying device. Together with an equivalence rule of grades this may lead to a pragmatic construction of a utility function, as follows.
Let $G := \{g_1, \ldots, g_m\}$ be a set of grades, such as $G = \{A, B, C\}$ where $g_1 \succ g_2 \succ \ldots \succ g_m$. Then, the first step is to grade all projects, which is summarized by $\Gamma : P \to G$. Using Γ, one then computes, for each allocation, its frequency distribution of grades, denoted by $\gamma : \mathcal{A} \to \mathbb{N}^m$.
Next, the committee chooses an equivalence rule $e : G \to \mathbb{R}^m$, where $e(g_j)$ states the number of grade-g_1 projects that are equivalent to one grade-g_j project. Of course, $1 = e(g_1) > e(g_2) > \cdots > e(g_m)$.
Combining the grading scheme and the equivalence rule, one finds the utility function

$$U(a) := \sum_{j=1}^{m} \gamma_j(a) e(g_j). \tag{1.2}$$

Also notice that the selection based on ranking individual projects can be viewed as a special case of a ranking based on allocations if and only if the project manager has lexicographic preferences, which give first priority to grade g_1 projects, second priority to grade g_2 projects, etc.

1.3 Two auction mechanisms

We now turn to the second deficiency of the current subsidization policy: the funding of projects at a predetermined percentage of the refundable project

cost. Generally this leads to excessive funding of those who are selected, and thus tends to exclude other valuable projects.
Typically, the selection committee cannot know the amount of funding needed to induce the applicant to carry out its project. They only know that this unknown amount is not greater than s_i, the amount of subsidy that would be granted according to the current rules.[6] This suggests that one can reduce funding without losing valuable projects. It requires the design of a mechanism that induces applicants to compete by lowering their request for funding.
We propose two such mechanisms: one sealed-bid and one open, descending-bid mechanism. Both mechanisms are auction-like in the sense that applicants compete with their requests for funding which can be considered as their bids and the mechanism selects the best allocation that can be funded with the given budget.
To carry out their project as stated in the application, the applicant requires a certain amount of subsidization, which is denoted by z_i. The fact that z_i is private information motivates the use of the auction mechanisms. An auctioneer knowing z_i could directly implement the optimal allocation, namely

$$a^o \in \arg\max_{a \in \mathcal{A}} \left\{ U(a) \mid \sum_{i \in a} z_i \leq \mathcal{B} \right\}, \tag{1.3}$$

by funding each applicant in the allocation exactly at the required level to implement its project.
Each applicant now submits a bid b_i according to one of the following two mechanisms.

1.3.1 Sealed-bid mechanism

The sealed-bid mechanism is characterized by the following allocation and pricing rules:

1. Each applicant $i \in P$ makes a sealed bid $b_i \in [0, s_i]$,[7] without knowing the bids made by others. Bids are requests for funding.

2. On the basis of the given bids $b = (b_1, \ldots, b_n)$, the mechanism

[6] An applicant who requires more than s_i would not apply because he would not be able to carry out the project as stated in the application. In most programs there is close monitoring by program managers such that applicants are not able to change the nature of their project significantly once the application is accepted.

[7] Notice that the maximum bid is set to s_i. This maximum bid restriction is discussed in section 1.3.3

(a) selects the allocation, a^*, that solves the maximization problem[8]

$$a^* \in \arg\max_{a \in \mathcal{A}} \left\{ U(a) \mid \sum_{i \in a} b_i \leq \mathcal{B} \right\}. \tag{1.4}$$

(b) pays a subsidy equal to b_i if $i \in a^*$ and equal to zero otherwise.

1.3.2 Open, descending-bid mechanism

The second mechanism is an open, descending-bid auction which consists of several "rounds."

1. Each applicant i faces his own price clock that starts at s_i. Subsequently, the reading of the price clock declines at rate Δ in each round.

2. The final bid b_i of applicant i is the price where he stops his price clock. After stopping the price clock, applicants are not allowed to lower their bid any further. Applicants can see others' price clocks at any time and can always observe if other applicants have stopped in an earlier round.

3. On the basis of the given bids $b = (b_1, \ldots, b_n)$, the mechanism selects the allocation as in the sealed-bid mechanism.

Proposal 2 *Use either the sealed-bid or the open, descending-bid mechanism. This induces competition for funding.*

1.3.3 Maximum bid restriction

It is advisable to structure the auction in such a way that its outcome can never be inferior to the outcome that would be reached if one would apply Proposal 1 only, without an auction.
This can be achieved by setting individual maximum bids equal to the subsidy rates s_i that would be granted according to the current subsidy rules. Therefore, we propose:

Proposal 3 *If one uses one of the auction mechanisms, set each applicant's maximum bid equal to the subsidy rate that would be granted according to the current subsidy rules (which was denoted by s_i).*

[8] If a^* is not unique, it selects the allocation that minimizes $\sum_{i \in a^*} b_i$; if the result is still not unique, it selects at random.

In policy advice one should always try to make proposals that cannot yield an outcome inferior to that achieved by the *status quo* practice. To achieve this is the only purpose of Proposal 3.
Notice that we already incorporated this proposal in the two auction mechanisms described before.

1.3.4 An example of an open, descending-bid auction

The following example illustrates the working of the open, descending-bid mechanism (see Table 1.1). It assumes a budget of 70, a bidding decrement of 5, and five applicants (1 to 5). Projects are substitutes and have the utilities stated in column 2. The associated minimum subsidies (z_i) are stated in column 3, and the subsidies s_i that would be granted if no auction were used in column 4. Bold numbers indicate which applicants would be part of the allocation if the auction would stop at the current round. If no auction were used, the allocation would be $\{1, 2\}$, with total utility 100.

Applicant	Utility	z_i	s_i	Round 1	Round 2	Round 3
1	53	20	**40**	35	30	**25**
2	47	20	**30**	**25**	**20**	**20**
3	38	20	30	**25**	**20**	20
4	37	10	25	**20**	**15**	**10**
5	35	15	25	20	**15**	**15**
a			{1,2}	{2,3,4}	{2,3,4,5}	{1,2,4,5}
$U(a)$			100	122	157	172

Table 1.1: Example of an Open, Descending-Bid Auction

This example assumes that all applicants stop their price clocks at round 3. The auction ends with allocation $a^* = \{1, 2, 4, 5\}$. The example illustrates how an applicant, in the course of an auction, can be crowded out at some round and return to the allocation in a later round. The last row of the table states the total utility of the respective allocations. The optimal allocation is $\{1, 2, 3, 4\}$ and the maximum feasible utility is 175.

1.3.5 The nature of the auction problem

The present auction problem can be viewed as that of a multi-object procurer (auctioneer) facing several single-unit suppliers (bidders), where both the auctioneer and bidders are budget-constrained. To the best of our knowledge such a problem has not been studied before, neither in the theoretical nor in the experimental auctions literature.

There is a literature on standard single- and multi-unit auctions with budget constrained bidders (see Che and Gale, 1998, Laffont and Robert, 1996 for the single-unit case, and Benoît and Krishna, 2001 for the multi-unit case). However, a budget constrained auctioneer, which is of crucial importance in the present framework, has never been considered. Its game theoretic analysis raises a number of technical complications. The problem shares some features with package auctions (see Cramton et al., 2006). The common denominator is that the auctioneer faces a complex selection problem because he must have a ranking of all feasible allocations in order to select the best.

1.4 Monte Carlo simulation

The simulation is designed to assess the benefit of switching from a selection based on a ranking of individual projects to a global decision procedure that is based on a ranking of allocations, as recommended in Proposal 1. Notice that adding an auction mechanism, as proposed in Proposal 2, gives rise to further improvements.

The simulation is designed as follows: we consider a pool of 250 projects and a given budget of $\mathcal{B} = 1000$. Each project has two characteristics: its grade (either A or B), which is drawn independently with equal probability, and the *status quo* subsidy, s_i, which is drawn independently from a uniform distribution with support $(0, \mathcal{B})$. The program manager's preferences are characterized by a fixed equivalence rule e_B which indicates that e_B grade-A projects are equivalent to one grade-B project.

We wrote a VBA program for Microsoft Excel[9] that draws the grade of each project and its parameter s_i at random and then computes the two allocations, the *status quo* allocation and the allocation based on Proposal 1. This procedure is repeated 1000 times to obtain an empirical distribution of selected allocations for different equivalence rules $e_B \in \{0.3, 0.5, 0.7, 0.9, 0.99\}$.

The results of these simulations are presented in Figure 1.1 in the form of cumulative distribution functions, $F(x)$. There, $F(x)$ denotes the probability that Proposal 1 gives rise to a relative gain in the value of the selected allocation of x or less. The value of an allocation is computed as $n_A + e_B n_B$, where (n_A, n_B) denotes the number of A and B projects in the respective allocation. And the relative gain in value is computed by dividing the increment in value due to switching from *status quo* allocation, a^s, to the allocation based on Proposal 1, a^p, by the value of the *status quo* allocation, a^s.

[9]The program code is available upon request from the authors.

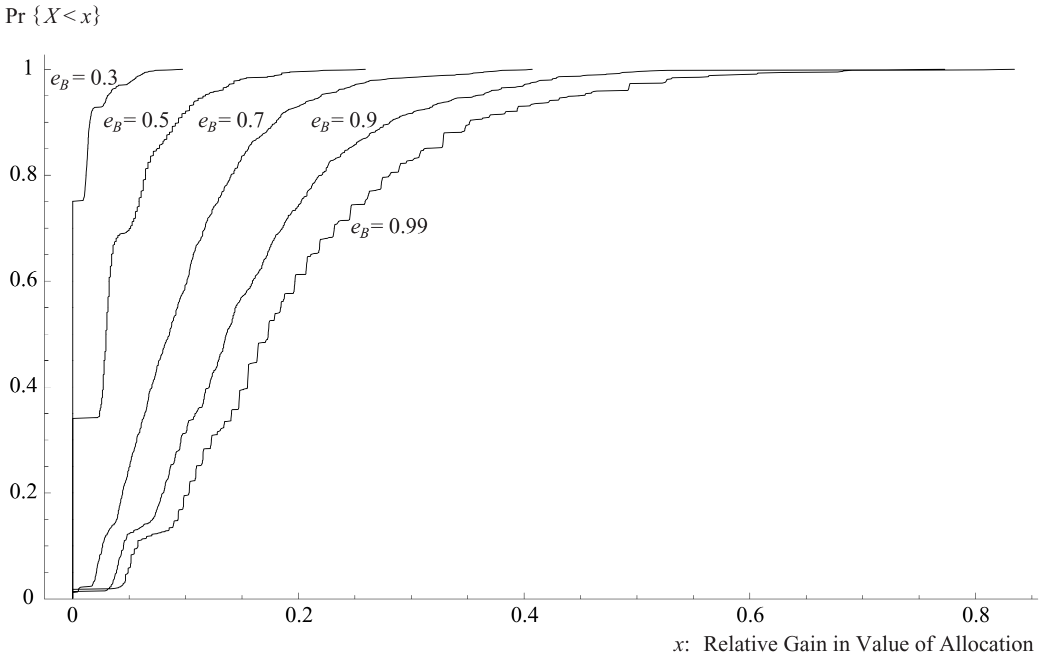

Figure 1.1: CDFs of relative value gains due to Proposal 1

Evidently, the relative value gains increase, in the sense of first-order stochastic dominance, if grade-B projects become more valuable relative to A projects. This is plausible, because giving absolute priority to grade-A projects, as the *status quo* preference rule does, becomes more costly as projects become closer substitutes.

1.5 Experiments

In order to test the two auction mechanisms we set up a series of computerized lab experiments.[10] There, subjects were assigned to play the role of a firm that applies for an R&D subsidy. They either participated in the sealed-bid or in the open, descending-bid mechanism. In the experiment, we used a simple grading scheme for projects as proposed above, with only two grades.

1.5.1 Experimental design

In the experiment, we formed groups of six subjects participating in one of the two mechanisms. Prior to the auction, each subject i was given the following private information $(z_i, \pi_i, s_i, g(i))$:

[10]For instructions and screenshots see Giebe et al. (2005).

1. the minimum subsidy needed to execute one's project, z_i;

2. the private profit earned in addition to the subsidy if one's project is executed, π_i;

3. the maximum (resp. starting) bid, s_i;

4. the grade of one's project, $g(i)$, either A or B.

The smallest monetary unit was 1 ECU (experimental currency unit).
Each subject was informed that (z_i, π_i, s_i) were independently drawn from uniform distributions with supports $z_i \in \{0, 1, \ldots, 5\}$, $\pi_i \in \{0, 1, \ldots, 10\}$, $s_i \in \{5, 6, \ldots, 10\}$, and that there would be three grade-A and three grade-B projects, assigned to subjects with equal probability.
The following information was given to all subjects:

1. the budget $\mathcal{B} = 20$,

2. the preference ranking over possible allocations:

$$\begin{aligned} &\{A, A, A, B, B, B\} \succ \{A, A, A, B, B\} \succ \{A, A, B, B, B\} \succ \{A, A, A, B\} \\ &\succ \{A, A, B, B\} \succ \{A, B, B, B\} \succ \{A, A, A\} \succ \{A, A, B\} \succ \{A, B, B\} \\ &\succ \{B, B, B\} \succ \{A, A\} \succ \{A, B\} \succ \{B, B\} \succ \{A\} \succ \{B\}. \end{aligned} \tag{1.5}$$

In the sealed-bid mechanism subjects were asked to enter their requested subsidy, b_i, referred to as "bid" in a computer screen window. After all bids were submitted, the software computed the best feasible allocation, based on the above preference ranking, according to the rules described in section 1.4. Those subjects who were part of the allocation received a credit equal to $b_i + \pi_i$ ECU; all others received no credit.
The open, descending-bid mechanism was set up as a clock auction. There, each subject had its own price clock, starting at the maximum bid s_i and decreasing at the fixed rate of one ECU per round. In each round, we first asked the grade-A subjects to make simultaneous bids; then, all grade-B subjects observed the bids of all A subjects, and made their own simultaneous bids. There, a bid means that one either freezes the current reading of one's price clock or accepts a reduction by one ECU. This procedure continued until all subjects had stopped their price clock.
A subject who stopped its price clock in one round was not able to "unfreeze" it later. In each round, the active grade-A subjects could see the current reading of the price clocks of all subjects and who had already stopped its

price clock in which previous round and at which price. Similarly, the active grade-B subjects could see the current reading of the price clocks of all subjects, which subjects had stopped in previous rounds, and, in addition, which grade-A subjects stopped in the current round.
When all subjects had stopped their price clock, the final bids b were the levels at which the individual price clocks had been stopped; the auction ended, and the software computed the best feasible allocation by the same rule as in the sealed-bid mechanism. Those subjects who were part of the allocation earned a credit of $b_i + \pi_i$ ECU; all others received no credit.

1.5.2 Experimental procedure

The experiments were conducted in November 2003 at the Department of Economics, Humboldt University at Berlin. The subjects were 96 student volunteers. They were recruited by advertisements in lectures and by mail shots. Most of them were undergraduate economics or business students.
The treatments were computerized using the experimental software "z-Tree" developed by Fischbacher (1999).
We conducted eight sessions. Four sessions were dedicated to the sealed-bid mechanism, and another four sessions to the open, descending-bid mechanism. In each session there were twelve distinct subjects.

Instructions and trial auction After being seated at a computer terminal, subjects were given written instructions including a detailed example.
In the instructions we referred to an allocation as a "combination," to a subsidy as a "grant," and to an applicant as a "bidder" in order to keep the terminology as neutral as possible without making it unduly difficult to understand the mechanism. We made clear that all decisions would be taken anonymously and that identities would not be revealed.
Two control questions checked whether the instructions were understood by all subjects. These control questions were computerized, with feedback for incorrect answers. Then, a "trial auction" was played which did not count for earnings.

Assignment of subjects to payoff-relevant auctions A session consisted of two parallel sequences of five auctions, each played by six subjects. After each auction subjects were randomly and anonymously reassigned to one of the two groups playing the next auction.
After each auction subjects were privately informed about their earnings. In order to reduce path dependencies, subjects were not told which allocation was selected.

At the end of the session subjects got a summary account of their earnings, and earnings were paid, including a show-up fee.

Payoffs A typical sealed-bid session took 40 and an open, descending-bid session 90 minutes. Each subject's earnings in ECU were converted into Euro at the rate 9 ECU = €1; in addition, subjects earned a show-up fee of €4 in a sealed-bid and €10 in an open, descending-bid session.
In sealed-bid sessions earnings were between €5.90 and €11, with an average of €8.40, and in the open, descending-bid sessions between €11.70 and €17.40, with an average of €14.40.

1.6 Results

Altogether, 96 subjects participated in eight sessions with a total of 78 payoff-relevant auctions.[11] The trial auctions are not considered in our analysis. As groups were rematched in every auction, subjects were able to learn from each other's behavior. Because of this, the results within a session are not independent. Hence, each treatment consists of four independent observations, one per session.
Since the set of independent observations is relatively small, we perform a mainly descriptive data analysis.
Of course, each auction resulted in one of the allocations stated in (1.5). These allocations are ranked by assigning a number $r \in \{1, \ldots, 15\}$, where $r = 1$ stands for $\{A, A, A, B, B, B\}$, $r = 2$ for $\{A, A, A, B, B, B\}$, etc. For convenience of notation we refer to the rank of the implemented allocation as r^*, that of the optimal allocation as r^o, and that of the allocation that would be implemented if all bids were equal to the maximum bids as r^p.
As it happened, the optimal allocation was $\{A, A, A, B, B, B\}$ in 70 of the 78 auctions and $\{A, A, A, B, B\}$ in the remaining eight auctions.
Table 1.2 indicates which allocations were implemented in the experiments. The further presentation and interpretation of the experimental results is ordered by the following hypotheses:

1. *The auction improves the allocation:* we explore to what extent the allocation improves relative to the allocation that would be reached if one adopted our Proposal 1 but not also Proposals 2 and 3.

[11] Actually, 80 auctions took place. However, due to a network problem, the data of two of the open, descending-bid auctions were lost. Subjects were only informed after the experiment. They received a lump-sum payment of €2 for the third auction where the problem occurred. We therefore think that the data from the remaining auctions can be analyzed.

Allocations (ordered by rank r)	Sealed-Bid Frequency	%	Open, Descending-Bid Frequency	%
$1: \{A, A, A, B, B, B\}$	3	7.5	6	15.8
$2: \{A, A, A, B, B\}$	22	55	20	52.7
$3: \{A, A, B, B, B\}$	6	15	3	7.9
$4: \{A, A, A, B\}$	8	20	9	23.7
$5: \{A, A, B, B\}$	1	2.5	0	0

Table 1.2: Frequency Distribution of Implemented Allocations

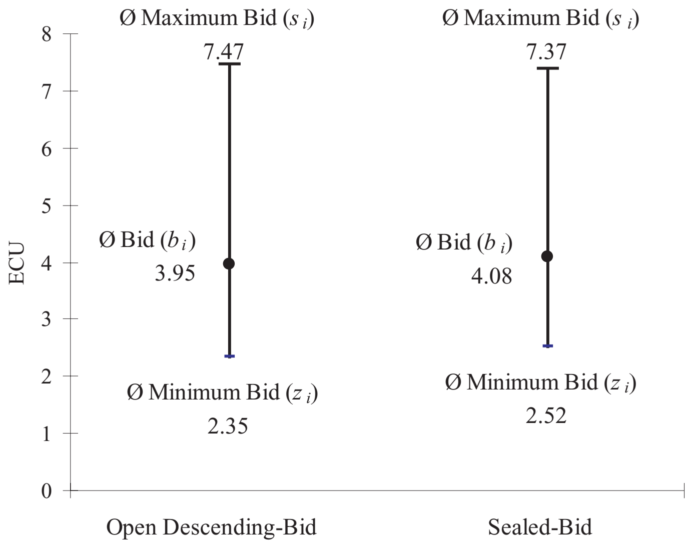

Figure 1.2: Average Bids

2. *The auction is almost efficient:* we explore how close the observed allocations are to the corresponding optimal allocations.

3. *"Handicapped" bidders play more aggressively:* we explore whether and if so to what extent grade-B bidders bid lower.

4. *Higher private profits give rise to more aggressive bidding:* we explore whether and if so to what extent bidders with a higher private profit submit lower bids.

5. *More experience gives rise to more aggressive bidding:* we explore whether bidders bid lower in later auctions in the sequence after gaining some experience.

Improvement due to the auction Figure 1.2 indicates that competition is effective. Bids are, on average, substantially below the maximum bids. Approximately 33% of all bids are even equal to the respective minimum bids. Average bids are slightly lower in the open, descending-bid mechanism.[12] Therefore, both mechanisms induce a remarkable intensity of competition.
We measure the improvement due to the auction by computing the average difference between the rank r^p and that of the implemented allocation, r^*, i.e., $|r^* - r^p|$. In the sealed-bid mechanism that measure is equal to 5.78 and in the open, descending-bid mechanism it is 5.89.[13] On average the auction increases the number of subsidized projects, relative to the allocation a^p, by 2.04. This indicates that adding the auction brings about a remarkable improvement.

Efficiency We call the outcome first-best if an auction implements the allocation a^o, i.e., if $r^* = r^o$. Similarly, we call it second-best or higher if $r^* = r^o + 1$ resp. $r^* > r^o + 1$.
We measure the deviation from the first-best by computing the average difference between the ranks of the optimal and the implemented allocations, $r^* - r^o$. In the sealed-bid mechanism that measure is equal to 1.45 and in the open, descending-bid mechanism it is 1.31. This indicates that the auctions implement allocations that are close to the efficient ones.

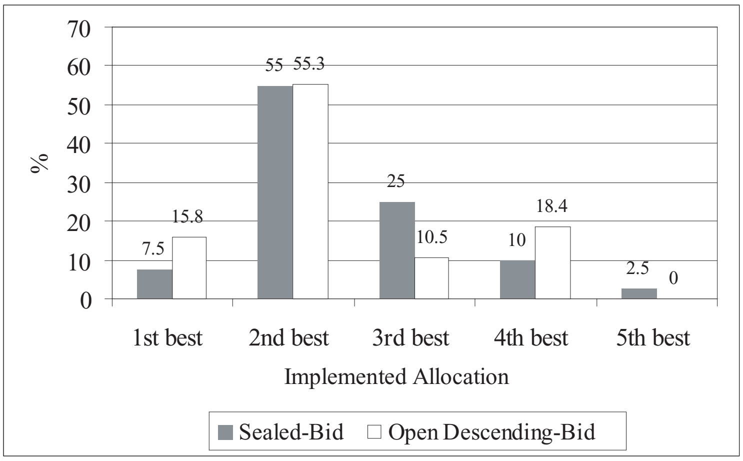

Figure 1.3: Efficiency Results

[12] Wilcoxon Rank-Sum tests using the difference in average bids in the two mechanisms ($n = 8$) confirm our result on the 10%-significance level.

[13] The average difference between r^* and the rank of the *status quo* allocation, i.e., the allocation that would be reached without using any of our proposals, is 6.1 on average.

Figure 1.3 summarizes the efficiency properties of both mechanisms. Without the auction, the implemented allocation would have been, on average, eighth-best. Thus, the deviation from efficiency is considerably smaller than the deviation from the allocations that would be reached without the auction.

"Handicapped bidders" Figs. 1.4 and 1.5 show that grade-A bidders bid higher on average. This applies to all eight sessions. Specifically, in the sealed-bid mechanism, grade-A bidders bid 27% higher on average and in the open, descending-bid mechanism 12% higher.[14]

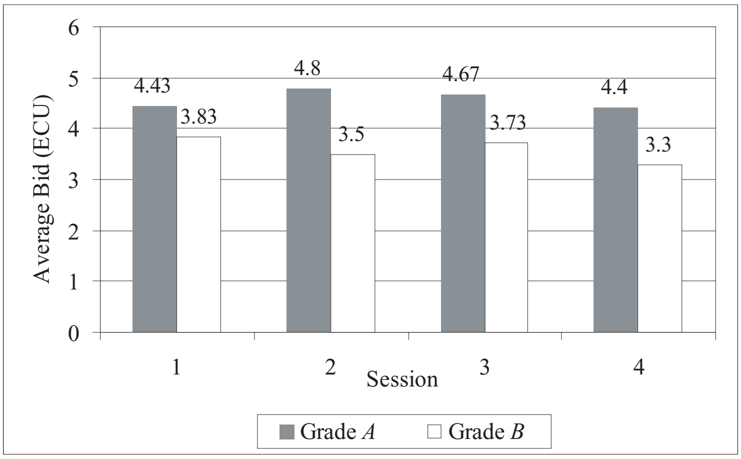

Figure 1.4: Average Bids in the Sealed-Bid Mechanism

Intuitively, higher private profits should induce lower bids because those bidders should care more about getting the minimum funding needed to get their project off the ground, rather than about collecting unnecessarily high subsidies.

Private profits The Pearson coefficients for the correlation between private profit and the bid are $\rho_{\pi,b} = -0.1$ for the sealed-bid mechanism and $\rho_{\pi,b} = -0.15$ for the open, descending-bid mechanism. The negative sign does indeed confirm this conjecture. However, the observed correlation is rather weak.

Experienced bidders In the sealed-bid mechanism, average bids remain fairly stable during a session. However, in the open, descending-bid mech-

[14] Wilcoxon Signed-Rank tests ($n = 4$) confirm these results for both mechanisms on a 5%-significance level.

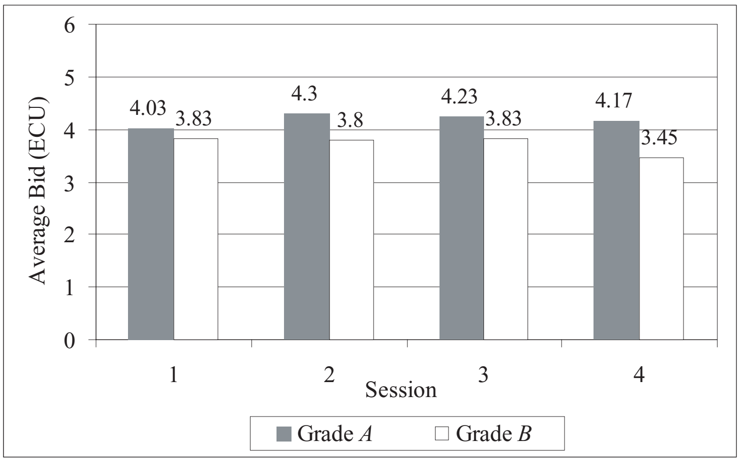

Figure 1.5: Average Bids in the Open, Descending-Bid Mechanism

anism the average bid in the first auction of each sequence is 15.7% higher than in the final one.

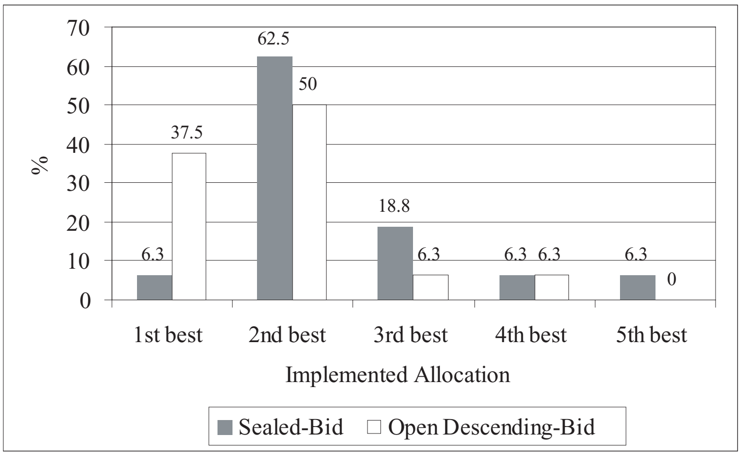

Figure 1.6: Efficiency in Late Auctions

Figure 1.6 states the outcomes of the fourth and fifth auction of each sequence, i.e., after bidders have acquired some experience. It indicates that experience induces more competitive bidding, resulting in a higher degree of efficiency. However, this improvement due to experience is more pronounced in the open, descending-bid mechanism.[15]

[15] A Wilcoxon Signed-Rank test on the 5%-significance level ($n = 4$) confirms our result: For the open, descending-bid mechanism, bids during the first two auctions of each

Not surprisingly, players lower their bids after they lose an auction. This learning effect is particulary strong in the sealed-bid auction. In fact, after losing an auction bids are on average reduced by 33.7% in the sealed-bid auction and by 16.7% in the open auction. And 73.7% of all losers respond in this way in the sealed-bid and 62.5% in the open auction.

We mention that the open mechanism reveals all individual maximum bids which remain hidden in the sealed-bid mechanism. Therefore, when we attribute differences between bidding behavior to the different auction mechanisms, one may object that we ignore the potential impact of the different information structure. Of course, we could have revealed all maximum bids also in the sealed-bid mechanism. Instead we checked whether there was any significant correlation between bids and observed maximum bids of competing bidders in the open, descending-bid mechanism. We found no significant correlation and therefore feel confident that this difference in information structure does not bias our findings.

1.7 Policy recommendations and discussion

The present paper analyzes the allocation of subsidies to fund socially valuable projects that are not feasible without subsidy. We focus on R&D subsidies, but possible applications of our mechanisms range from the funding of charitable projects to academic fellowships. Currently, these allocation decisions are based on a ranking of individual projects, and subsidies are awarded successively to the best projects until the budget is exhausted, which is not a good policy.

We identify two sources of inefficiency and propose better mechanisms. Specifically, we make two recommendations, the first of which can be implemented without the second:

- Select projects on the basis of a ranking of complete allocations rather than on a ranking of individual projects.

- Induce applicants to reveal information about their true need for funding and use that information. We advise to employ either an open, descending-bid or a sealed-bid auction-like mechanism in which applicants bid for subsidies.

sequence are significantly higher than in the final two. For the sealed-bid mechanism, the hypothesis of significantly higher bids in the first two auctions of each sequence is rejected. Our observations are based on only 32 "late" auctions, 16 per treatment. A more extensive series of experiments would be required to check the robustness of these results.

Ideally, the first proposal calls for a ranking of all allocations. But this is generally too complex. Therefore, we recommend to use simple grading schemes, combined with fixed equivalence rules. This will improve the allocation relative to the currently used selection procedure (which implicitly assumes lexicographic preferences), without raising the level of complexity.
We test the first proposal by means of a Monte Carlo simulation. We show that the efficiency gain from using a ranking of allocations is larger if projects are closer substitutes.
We test the second proposal in controlled lab experiments. We find that both proposed mechanisms induce a high level of competition among applicants and improve efficiency. The highest efficiency gains are realized by the open, descending-bid mechanism.
Of course, the proposals will only be implemented if they are favorably assessed by the representatives of the firms who apply for R&D subsidies and by the policymaker and his administrators.
As part of our study for the Germany Ministry of Economics and Labor (Becker et al., 2004) our coauthors from GIB and Fraunhofer ISI conducted an extensive opinion survey among German firms who had before applied for R&D subsidies in order to find out to what extent they would accept the implementation of the proposals. The results of this survey indicate that the majority of firms approves of the competitive selection procedure. Remarkably, acceptance among small and low equity firms, and those who complained about the current low success rate, is strongest. Altogether, the sealed-bid format has a higher acceptance rate than the open, descending-bid format.
With some exceptions, the program managers of the various R&D subsidy programs were less favorably disposed towards the proposed selection mechanisms. They objected that the proposals are not practicable, ranking allocations would be too complex, project quality would suffer if price counts in the selection, and that price competition would increase post-contractual opportunism.
Recently we learned about a project funded by the German Ministry of Education and Science in which a somewhat similar auction-like procedure has been tested for awarding public funds to farmers providing "ecological goods" such as grasslands with high ecological diversity. Although the allocation problem and the auction mechanism differ, some features are similar, e.g., the presence of budget constraints, quality classes for the provided goods, and a price discriminating award procedure. The results of the first field experiments reported in Groth (2005) suggest that these auction mechanisms are indeed practicable and can be implemented at reasonable cost.
The complexity of ranking allocations is certainly an issue. In this regard

there is a need to simplify (for example, as proposed above, by using a simple grading scheme combined with an equivalence rule) and for supporting software tools that facilitate the comparison of complex alternatives. A similar complexity issue comes up in package auctions when a number of heterogeneous goods are auctioned. However, this has not deterred users such as the Federal Communications Commission in the US to choose this highly complex auction format.

In our analysis we have evaluated the impact of our proposed mechanisms assuming a given set of projects. This ignores that the proposed change in selection and allocation rules may affect the proposed projects. If applicants anticipate that they compete not only in terms of project quality but also in terms of the requested amount of funding, they may propose different projects. However, this may very well be a change for the better.

The currently used selection rules favor “large” projects with high quality and high cost, which has been criticized by some applicants and programme managers.[16] The proposed new rules allow “small” projects, with low cost and lower quality, to compete with with the large ones because a combination of small projects may be preferred. This will surely induce participation of more small projects, probably to such an extent that the average quality of applications goes down. Nevertheless, as long as large projects are not discouraged from participation, the new selection rule will improve the quality of allocation.

Of course, if more “small” projects participate, and the success rate of “large” projects goes down, it is to be expected that less “large” projects participate. Therefore, it may be the case that the adoption of the proposed rules will adversely affect the quality mix of applications. If the selection rule accurately reflects the preferences, this change in the composition of submitted projects only indicates that proposals are better matched with these preferences. However, this puts a heavy burden on identifying preferences and translating them into the grading scheme.

We also mention that when more (on average smaller) projects are subsidized, the administrative costs will probably increase. Also, the adoption of auction mechanisms may raise transaction costs both to the administration and to proposers, especially if an open, descending-bid mechanism is employed. Of course, these issues have to be properly weighted in selecting one of the proposed bidding mechanisms.

Another concern was that switching to the proposed rules may exacerbate the problem of post-contractual opportunism, on the ground that the money saved on subsidies is compensated by a reduction of firms’ R&D activities.

[16] Eickelpasch and Fritsch (2005)

However, if firms can benefit from delivering less than the promised R&D activities, and get away without being sanctioned, one should expect that they do so regardless of the amount of the subsidy. We do not deny that there is a problem of post-contractual opportunism; but it is not clear why it should increase if one adopts the proposed selection and payment rules.
Finally, we mention that administrators are probably reluctant to change the current selection procedure also because the current practice gives them considerably more leeway. No one can be expected to give up such power on his own initiative. Therefore, the policymaker should exercise his power to make rules, and not delegate it to those who execute them. Unfortunately, this obvious principle is frequently violated in the public sector.

Chapter 2

License Auctions with Royalty Contracts

This chapter is based on Giebe and Wolfstetter (2008).

2.1 Introduction

This paper revisits the analysis of licensing an outside innovator's cost reducing innovation to a Cournot oligopoly. We propose a simple new mechanism that combines a license auction with royalty licensing in a particular way. This new mechanism is more profitable than the standard solutions evaluated in the literature such as standard license auctions, auctioning royalty contracts, fixed-fee licensing, pure royalty licensing, and two-part tariffs (see Kamien, 1992, Kamien and Tauman, 1984, 1986, Katz and Shapiro, 1985, 1986, Sen and Tauman, 2007).

The two key features of the proposed mechanism are that it grants the losers of the license auction the option to sign a royalty contract, and that it employs a royalty component in the auctioned contract. Like in the standard auction, the innovator auctions a restricted number of licenses; but, the auctioned licenses are royalty contracts, and, after the auction, those who lose the auction are granted the option to sign a pure royalty contract.

In equilibrium, the innovator sets the royalty rate for losers equal to the reduction of unit cost induced by the innovation. As a result, the royalty licensing granted in the second stage, after the auction, has no effect on equilibrium bids since losers of the auction have the same payoff functions as if no royalty option had been granted. Furthermore, in equilibrium the number of auctioned licenses is such that no loser is crowded out of the market. Thus, royalty income is collected which explains in part the superiority of the proposed mechanism.

Our analysis also takes into account that the number of licenses must be an integer which is particularly important if a small number of firms is involved. Here, the royalty component in the auctioned contracts comes into the picture as a tool for fine tuning. Essentially, that royalty component is used to implement the optimal mechanism without violating the integer constraint. Recently, Sen (2005) showed that the integer constraint can make pure royalty contracts superior to the standard license auction, contrary to a standard result of the literature. However, in our more general framework the integer constraint can be satisfied at no loss in profit.

The literature on patent licensing in oligopoly has branched out in various directions. Sen and Tauman (2007) analyzed an auction of royalty contracts. Wang (1998) and Kamien and Tauman (2002) analyzed the licensing problem from the perspective of an innovator who is also an incumbent player in the downstream product market. While an outside innovator is only interested in licensing income, an "inside" innovator must also take into account how giving access to his innovation affects his own downstream profit.

Muto (1993), Hernández-Murillo and Llobet (2006) dealt with other market games such as Bertrand or monopolistic competition with product differentiation in lieu of the Cournot competition assumed here. And Beggs (1992), Gallini and Wright (1990), Macho-Stadler and Pérez-Castrillo (1991) examined the benefits of royalty licensing either as a screening device in the face of incomplete information concerning the users' willingness to pay for the innovation or as a signaling device if the innovator has superior information concerning the cost reduction induced by his innovation.

The licensing mechanism proposed in the present paper stipulates royalty payments from all firms, and price discrimination between firms by offering different combinations of royalty rates and fixed fees. This raises the question: are royalty rates and discrimination employed in industry? Unfortunately, the empirical literature on licensing practices does not provide sufficient evidence to fully answer this question. A widely cited study of U.S. firms observes that "A down payment with running royalties method was used 46% of the time, while straight royalties and paid-up licenses accounted for 39% and 13%, respectively" (Rostoker, 1984, p.64). This finding is often interpreted as proving the predominance of royalty licensing (see also the study on foreign technology licensing to Indian firms by Vishwasrao, 2007).[1] That study also reports that the same innovator often employs different licensing schemes, possibly for licensing the same innovation to different customers.

Moreover, casual evidence suggests that discrimination is widely used in software licensing and in the sale of innovative products. A case in point is the

[1]Royalty contracts require inspection rights to monitor output. In a recent empirical study, Brousseau et al. (2007, p. 217) report that license agreements typically "grant inspection rights aimed at controlling the licensee's use of the licensed technology, to the licensor or to a third party."

"Original Equipment Manufacturer (OEM) Licensing" where PC manufacturers are charged different prices and sometimes are given a choice between a "one-time paid-up" license, which entitles the manufacturer to unlimited distribution of the software within a specified time period, and a per copy royalty license.

Similarly, new products are often sold to some users for unrestricted use while others are offered a leasing contract which is effectively a royalty licensing scheme. The only difference between these arrangements and the one proposed here is that customers are typically free to choose between these two arrangements, whereas the proposed mechanism assumes that the innovator limits that choice by offering a restricted number of licenses.

The plan of the paper is as follows. In Section 2.2 we state the licensing problem as a sequential game and introduce basic assumptions. Section 2.3 summarizes some general properties of the equilibrium, and Section 2.4 derives the optimal mechanism and explains the role of the integer constraint on the number of contracts. Section 2.5 offers a discussion of the main results and explores various extensions.

2.2 The model

There are $n \geq 2$ firms with the linear cost function $C_i(q_i) := cq_i$, $c > 0$, and the inverse demand function $P(Q)$ with $Q := \sum_{i=1}^{n} q_i$. They play a Cournot game.

An outside innovator owns a patented innovation that reduces the marginal cost from c to $c - \epsilon$ with $c > \epsilon > 0$. The innovator can permit the use of that innovation by issuing licenses. In general, a license is a two-part tariff contract, with a royalty rate per output unit and a fixed fee. This covers fixed-fee contracts, pure royalty contracts, and the auctioning of royalty contracts.

Throughout this text we employ the usual notion of a drastic vs. non-drastic innovation. An innovation is drastic if its exclusive use by one firm propels monopolization. Each innovation induces a natural oligopoly of a certain size, denoted by K, in the sense that if K or more firms operate with the new technology (at marginal cost $c - \epsilon$), all firms with marginal cost c exit, i.e. their equilibrium output is equal to zero because the equilibrium price is below c. In this text we assume that the innovation is non-drastic in the sense that $K > 1$.[2]

The following stage game (under complete information) is played: the inno-

[2]The notation is borrowed from Kamien (1992). The case of drastic innovation, $K \leq 1$, is trivial. There, the innovation induces a natural monopoly where issuing one fixed-fee license is optimal.

vator chooses a licensing mechanism; then firms play that mechanism as a non-cooperative game; finally, firms play a Cournot market game, after having observed the outcome of the previous play, knowing who gained access to the innovation and how.
We introduce the modified license auction (k, r_w, r_l). The innovator auctions a limited number of k royalty contracts (possibly with a minimum bid), with the royalty rate $r_w < \epsilon$, and gives those firms who lose the auction the option to sign a royalty contract with a royalty rate $r_l > 0$. Of course, $r_w < r_l$, and royalty contracts are not accepted if they exceed the cost reduction.
The mechanism m includes all other standard mechanisms considered in the literature as special cases, ranging from fixed-fee licensing, pure royalty licensing, to the auctioning of fixed-fee licenses or royalty contracts. For instance, fixed-fee licensing is equivalent to an auction of $k = n$ contracts with $r_w = 0$ and a minimum bid, and $r_l > \epsilon$. Pure royalty licensing is equivalent to $k = 0, 0 < r_l \leq \epsilon$. The case $k \in [1, n], r_w = 0, r_l > \epsilon$ is the standard license auction analyzed by Kamien (1992) and others, and with $r_w > 0$ the auction analyzed by Sen and Tauman (2007).
In the following we refer to those firms who win the auction as "winners" (w) and those who lose as "losers" (l).
The number of contracts k is an integer (which can make a difference but is typically ignored in the literature).
Throughout our analysis, the inverse market demand function P satisfies the following assumptions:

Assumption 1 *Inverse demand P is strictly decreasing in Q and continuously differentiable for all Q with $P(Q) > 0$, and $P(Q)Q$ is strictly concave in Q and $P(0) > c$, and $P(Q) = 0$ for all $Q \geq \bar{Q} > 0$ (satiation point).*

2.3 Equilibria of the Cournot market and licensing subgames

The equilibrium concept is subgame-perfect Nash equilibrium (SPNE) which is found by backward induction. Throughout we assume $r_l \leq \epsilon$ on the ground that $r_l > \epsilon$ is strictly dominated whenever that royalty rate is payoff relevant.

Cournot subgames The Cournot subgame is an asymmetric oligopoly game played between winners (w) and losers (l) who have exercised the royalty option or have no license at all. We focus on the particular subgames with k winners and $n - k$ losers, where $r_l \leq \epsilon$ and all losers have exercised the royalty option. Consequently, the unit costs are changed from c to $c_w := c - \epsilon + r_w$, resp. $c_l := c - \epsilon + r_l$. At the end of this section we briefly elaborate on the other subgames in which some firms have neither acquired a license nor exercised the royalty option.

Depending on the mechanism, in equilibrium either all losers are crowded out or coexist and produce positive outputs. The critical level of k above which all losers are crowded out depends upon effective unit costs, c_l, c_w. We denote this critical level by $\mathcal{K}(r_w, r_l)$, and mention that $\mathcal{K}(0, \epsilon) = K$, i.e. auctioning $\mathcal{K}(0, \epsilon) = K$ licenses establishes a natural oligopoly of size K. (Note that in a duopoly crowding out is impossible by assumption of a non-drastic innovation.)

Using the measure $\mathcal{K}(r_w, r_l)$ it follows that all firms, winners and losers alike, coexist in the Cournot market if there are less than n winners and $k < \mathcal{K}(r_w, r_l)$, whereas only winners are active in the Cournot subgame if $k = n$ or $k > \mathcal{K}(r_w, r_l)$.

Note that for $r_l \geq \epsilon$ all losers have an effective unit cost equal to c (since a contract with $r > \epsilon$ is never accepted), as in the standard license auction game without royalty contract option studied by Kamien (1992), Kamien et al. (1992) and others.

We denote the equilibrium Cournot outputs by q_w, q_l, Q, and the associated equilibrium profits of firms by π_w, π_l:

$$\pi_i := \Big(P(Q) - c_i\Big) q_i, \quad i \in \{l, w\} \tag{2.1}$$

$$Q := k q_w + (n-k) q_l \tag{2.2}$$

These equilibrium outputs are defined as solutions of the Kuhn-Tucker conditions $\partial\pi_i/\partial q_i \leq 0$, $q_i(\partial\pi_i/\partial q_i) = 0, i \in \{w, l\}$.

If both winners and losers coexist (i.e. if $q_w, q_l > 0$), one can eliminate the variable q_w and solve for (Q, q_l) as a function of average marginal cost, $\bar{c}$, by writing these conditions in the form[3]

$$0 = nP(Q) + QP'(Q) - n\bar{c} \tag{2.3}$$

$$0 = P(Q) + q_l P'(Q) - c_l \tag{2.4}$$

$$\bar{c} := \frac{k}{n} c_w + \frac{n-k}{n} c_l \tag{2.5}$$

Interestingly, Q can be solved uniquely as a function of $\bar{c}$, from (2.3), and then q_l can be computed by plugging Q into (2.4). Therefore, the equilibrium aggregate output Q is exclusively a function of $\bar{c}$, and losers' equilibrium output q_l is only a function of $\bar{c}$ and c_l.[4]

[3]With slight abuse of notation we use the same symbol for outputs and equilibrium strategies, whenever there is no risk of confusion.

[4]Since $(P(Q)Q)'$ is decreasing and $P'(Q) < 0$, it follows that $nP(Q) + P'(Q)Q$ is also

Whereas if losers are crowded out (i.e. if $P(Q)-c_l \leq 0$ and hence $q_l = 0$), the equilibrium aggregate output solves the obvious condition $kP(Q)+QP'(Q)-kc_w = 0$ and $q_w = Q/k$.

Lemma 1 *Equilibrium aggregate output Q is strictly decreasing in $\bar{c}$. If winners and losers coexist, both q_l and Q are (directly or indirectly) decreasing in c_l. If losers are crowded out, Q is decreasing in k and in c_w.*

Proof 1 *Suppose winners and losers coexist. Then, Q uniquely solves the condition $(nP(Q) + P'(Q)Q) = n\bar{c}$. By Assumption 1 $(P(Q) + P'(Q)Q)$ and $P(Q)$ are decreasing in Q; therefore, $(nP(Q) + P'(Q)Q)$ is also decreasing in Q. Hence, the equilibrium Q is decreasing in $\bar{c}$. The assertion concerning the effects of c_l are obvious. Now suppose losers are crowded out. Then, the market game is a symmetric k-firm oligopoly, and it follows immediately that Q is decreasing in c_w and in k.*

Another key property of the asymmetric oligopoly induced by licensing concerns gross profits $\bar{\pi}_i$, i.e. firms' profits before deducting royalty payments:

$$\begin{aligned} \bar{\pi}_i &:= \pi_i + R_i, \quad R_i := r_i q_i \\ &= \Big(P(Q) - c + \epsilon\Big) q_i, \quad i \in \{w, l\}. \end{aligned} \tag{2.6}$$

Lemma 2 *Aggregate gross profits, $\sum_{j=1}^{n} \bar{\pi}_j$, are strictly increasing in $\bar{c}$.*

Proof 2 *Aggregate gross profits are equal to $(P(Q)-c+\epsilon)Q$. By assumption 1, aggregate gross profits are concave in Q, and since the innovation is not drastic, the equilibrium Q is greater than the monopoly output. Therefore, the sum of gross profits is declining in Q, and since Q is decreasing in $\bar{c}$, by Lemma 1, aggregate gross profits are increasing in $\bar{c}$.*

Lemma 3 *If losers are crowded out, i.e. $P \leq c$, aggregate gross profits, $\sum_{j=1}^{n} \bar{\pi}_j$, are decreasing in k.*

Proof 3 *If losers are crowded out, then the market is a symmetric k-firm oligopoly with aggregate gross profits equal to $k\bar{\pi}_w$. Obviously, reducing the number of firms in a symmetric oligopoly increases the sum of gross profits for $k \geq 1$, and that sum of profits is maximized at $k = 1$ (monopoly).*

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decreasing. Together with the fact that $\lim_{Q\to 0}(P(Q) + P'(Q)Q) = P(0) > c$ we conclude that a positive solution $Q > 0$ exists and is unique. Given Q, existence and uniqueness of q_l follow immediately.

Finally, we mention that there are also subgames in which some firms have no license at all. In those subgames one must distinguish between three kinds of players: winners, losers who have exercised the royalty option, and losers who have not. In that case, the Cournot equilibrium solves an additional Kuhn-Tucker condition concerning those who have no license, where one has to change the definition of average marginal cost accordingly. A special case is the subgame where no loser has exercised the royalty option. This is particularly relevant if exercising the royalty option is unattractive because $r_l > \epsilon$.

Royalty licensing subgames After the auction has selected k winners and $n - k$ losers, each loser can either accept the royalty contract with the royalty rate r_l or refuse and operate under the initial marginal cost c. The SPNE of the royalty licensing subgame is to accept if $r_l < \epsilon$, to reject if $r_l > \epsilon$, whereas losers are indifferent if $r_l = \epsilon$. However, as will become clear, rejection cannot be part of the SPNE of the entire game. Therefore, in the SPNE of the entire game losers exercise the royalty option if and only if $r_l \leq \epsilon$.

Auction subgames In the auction subgames each firm is asked to bid on at most one of k license contracts in a standard auction (possibly with a minimum bid requirement) where the k highest bidders win, in case of a tie winners are selected at random, and each winner pays his bid. Each firm knows that if it does not obtain a license in the auction it can subsequently exercise the royalty contract option.

In equilibrium, all licenses are sold since $r_w < \epsilon$, provided the minimum bid is not set too high. Since firms have complete information, the equilibrium bid is equal to the value of the auctioned license contract. That value is the difference between the profit of a winner and that of a loser. In computing that value it is crucial to distinguish between the case when licenses are rationed because the innovator restricts the number of licenses ($k < n$) and when they are not rationed ($k = n$). If $k = n$, bidders' participation in the auction affects the structure of the subsequent oligopoly game, whereas if $k < n$ that market structure cannot be affected by an individual bidder.

As we show below, a minimum bid serves no purpose if $k < n$. However, if $k = n$, the innovator can only earn revenue if he charges a minimum bid. In that case, we assume that the innovator applies the most profitable minimum bid, which is equal to firms' maximum willingness to pay, $b_0 := \pi_w(n) - \pi_l(n-1)$ (as in Kamien, 1992).

Lemma 4 *Suppose k licenses with $r_w < \epsilon$ are auctioned, losers have the option to sign a royalty contract with the royalty rate $\epsilon \geq r_l > r_w$, and the auctioneer applies the minimum bid, b_0, if he sets $k = n$. The SPNE strategy of each firm is to participate in the auction and bid an amount equal to*[5]

$$b(k) = \begin{cases} \pi_w(k) - \pi_l(k) & \text{if} \quad k \leq n-1 \\ \pi_w(n) - \pi_l(n-1) & \text{if} \quad k = n. \end{cases} \tag{2.7}$$

Proof 4 *The value of an auctioned license is the difference between π_w and π_l. If $k < n$ no firm can unilaterally influence the subsequent market structure composed of k winners and $n - k$ losers. Because if a firm refrains from bidding, another bidder wins the license. Therefore, if $k < n$, each firm knows that it faces a given market structure in the subsequent oligopoly game, and therefore should participate in the auction and bid the amount $\pi_w(k) - \pi_l(k)$. This is different if $k = n$. Then, a firm that refrains from bidding thus changes the subsequent market structure from n winners and no loser to $n-1$ winners and one loser. In that case the auction can only generate revenue if the innovator sets an appropriate minimum bid because otherwise firms can buy a license with a zero bid. (Whereas, if $k < n$, a minimum bid serves no purpose.) Therefore, the innovator sets a minimum bid equal to firms' maximum willingness to pay, $\pi_w(n) - \pi_l(n-1)$, and firms participate in the auction and bid that amount.*

2.4 The optimal mechanism

The innovator chooses the licensing mechanism (k, r_w, r_l) that maximizes his income, Π, which is composed of auction revenue and royalty income from winners and losers. This format includes the standard mechanisms analyzed in the literature as special cases. In the SPNE all firms participate in the auction and bid according to the bid function $b(k)$ and all losers exercise the royalty option. Therefore,

$$\begin{aligned} \Pi &= kb(k) + kr_w q_w + (n-k) r_l q_l \\ &= k(\pi_w - \pi_l) + kR_w + (n-k)R_l \\ &= k(\bar{\pi}_w - \bar{\pi}_l) + nR_l. \end{aligned} \tag{2.8}$$

Proposition 1 (Optimal Licensing) *The optimal mechanism (k, r_w, r_l) exhibits:*

[5] There, $\pi_w(k), \pi_l(k)$ denote the equilibrium profits in the Cournot subgame with k winners and $n - k$ losers who exercise the royalty option.

1) restrictive licensing: $1 \leq k \leq n-1$*;*
2) maximum royalty rate for losers: $r_l = \epsilon$*;*
3) no crowding out: $q_l, q_w > 0$.

The proof is in a sequence of Lemmas below.

2.4.1 Pure royalty contracts are not optimal

Using a linear model, Kamien (1992) already showed that license auctions dominate pure royalty licensing. However, as Sen (2005) pointed out recently, if one takes into account that the number of licenses must be an integer (which has been ignored in the literature), pure royalty contracts may be more profitable than license auctions. However, as we now show, pure royalty contracts are not optimal, even if one accounts for the integer constraint concerning k. As our proof indicates, the proposed royalty option to losers plays a key role in establishing this result.

Lemma 5 (Exclusion of Pure Royalty Contracts) *Pure royalty licensing, i.e.* $k = 0$*, is not optimal (even if one accounts for the integer constraint concerning* k*).*

Proof 5 *Consider royalty licensing, i.e.* $k = 0$ *at the rate* $r_l \in (0, \epsilon]$ *(royalty rates greater than* ϵ *are never accepted). We prove the assertion by showing that the mechanism* $(k = 1, r_w = 0, r_l)$*, that issues one license and offers the same royalty rate* r_l *to all losers is more profitable for the innovator.*
Denote firms' equilibrium outputs under royalty licensing and the stated mechanism by q_R *resp.* (q_w, q_l)*, the associated aggregate outputs by* $Q_R := nq_R$*,* $Q_M := q_w + (n-1)q_l$*, and the equilibrium prices by* p_R*,* p_M*. Then, the innovator's profit is*

$$\begin{aligned}\Pi(1,0,r_l) &= (p_M - c + \epsilon)\, q_w - (p_M - c + \epsilon - r_l)\, q_l + r_l(n-1)q_l \\ &= (p_M - c + \epsilon - r_l)\,(q_w - q_l) + r_l Q_M \\ &> r_l Q_M > r_l Q_R = \Pi(0,0,r_l).\end{aligned}$$

The first inequality follows from three facts: 1) the innovation is non-drastic and therefore the one licensee cannot crowd out other firms which assures that the Cournot equilibrium price p_M *remains above the marginal cost c,* $p_M > c$*; hence, royalty income from losers is generated; 2)* $\epsilon \geq r_l$*; 3)* $q_w > q_l$ *because the licensee has lower marginal cost. To understand the second inequality, note that both regimes induce an n-firms oligopoly, where one firm has lower marginal cost in the modified license auction, which gives rise to a higher aggregate output, as we show in Lemma 1.*

2.4.2 Why it is not optimal to auction $k = n$ licenses

If the innovator auctions $k = n$ licenses, he sets a minimum bid equal to the amount $\pi_w(n) - \pi_l(n-1)$ (where, of course, $\pi_l(n-1)$ can be equal to zero), and every firm bids that amount, as already explained in Lemma 4. We now show that such a mechanism is not optimal. Instead, the optimal mechanism involves restrictive licensing, $k < n$.

Lemma 6 *The optimal mechanism sets $k < n$ (even if one accounts for the integer constraint concerning k).*

Proof 6 *Consider the mechanism $m := (n, r_w, r_l)$, supplemented by a minimum bid equal to $\pi_w(n) - \pi_l(n-1)$. We show that the modified mechanism $m' := (n-1, r_w, r_l)$, that differs from m only by replacing $k = n$ by $k = n-1$, is more profitable for the innovator:*

$$\Pi(m) := n\left(\bar{\pi}_w(n) - \bar{\pi}_l(n-1) + R_l(n-1)\right), \tag{2.9}$$
$$\Pi(m') := (n-1)(\bar{\pi}_w(n-1) - \bar{\pi}_l(n-1)) + nR_l(n-1), \tag{2.10}$$
$$\Pi(m') - \Pi(m) = (n-1)\bar{\pi}_w(n-1) + \bar{\pi}_l(n-1) - n\bar{\pi}_w(n) > 0. \tag{2.11}$$

To prove inequality (2.11)*, note that $\Pi(m') - \Pi(m)$ is the difference in aggregate gross profits in two market structures: one with $n-1$ winners and 1 loser and the other with n winners. If $\bar{\pi}_l(n-1) = 0$, the asserted inequality* (2.11) *follows from Lemma 3; and if $\bar{\pi}_l(n-1) > 0$, it follows from Lemma 2.*

2.4.3 Which royalty rate for losers?

Lemma 7 (Royalty rate for losers) *The optimal mechanism sets the royalty rate for losers equal to the cost reduction, $r_l = \epsilon$.*

Proof 7 *Suppose the royalty rate for losers is raised from r_l to r'_l with $r_l < r'_l \leq \epsilon$. Assume losers are not crowded out (we show in Lemma 10 that crowding out is not optimal). Therefore, the subsequent market game is an asymmetric oligopoly, characterized in* (2.3)-(2.5)*, with a higher average marginal cost, $\bar{c}' > \bar{c}$. Denote all equilibrium values induced by that change by a prime, the equilibrium royalty incomes by $R_i := r_i q_i, i \in \{w, l\}$, and gross profits (before deducting royalties) by $\bar{\pi}_i := \pi_i + R_i$.*
Recall that, by (2.8)*, $\Pi := k(\bar{\pi}_w - \bar{\pi}_l) + nR_l$, and use the fact that the sum of gross profits is increasing in $\bar{c}$, by Lemma 2, and that the increased royalty*

rate r_l' reduces losers' equilibrium profits, and one concludes,

$$\begin{aligned}
\Pi' - \Pi &= k\left((\bar{\pi}_w' - \bar{\pi}_w) - (\bar{\pi}_l' - \bar{\pi}_l)\right) - n(R_l - R_l') \\
&> (n-k)(\bar{\pi}_l - \bar{\pi}_l') - k(\bar{\pi}_l' - \bar{\pi}_l) - n(R_l - R_l'), \quad \textit{by Lemma } 2 \\
&= n(\bar{\pi}_l - \bar{\pi}_l') - n(R_l - R_l') \\
&= n(\pi_l - \pi_l') \\
&> 0.
\end{aligned} \tag{2.12}$$

$r_l > \epsilon$ is payoff dominated by $r_l = \epsilon$. Therefore, the optimal r_l is equal to the highest rate that is not rejected: $r_l = \epsilon$.

By charging maximum royalties to losers, the industry output is moved towards the monopoly output. This allows the innovator to extract maximum rent from the winners. Of course, raising royalties for losers may also imply lower royalty income from them. But this loss is more than compensated by the increased rent extracted from winners.

2.4.4 Why the optimal mechanism reduces to the choice of $\bar{c}$

We now show that the optimal mechanism, (k, r_w, r_l), can be reduced to the optimal choice of average marginal cost, $\bar{c}$. To prepare the proof, we first exclude the optimality of excessive crowding out. Excessive crowding out means that the mechanism gives rise to an equilibrium price lower than losers' marginal cost, $P < c$.

Lemma 8 (No "excessive crowding out") *The optimal mechanism exhibits $P \geq c$.*

Proof 8 *Suppose,* per absurdum*, that the optimal mechanism involves a price below c, which implies $q_l = 0$. In that case the innovator's profit is $\Pi = k\bar{\pi}$, i.e. the innovator collects the sum of winners' gross profits. Then, by Lemmas 2 and 3, the innovator's profit can be increased by raising $\bar{c}$ or by lowering k. In the present context the only way to raise $\bar{c}$ is to raise r_w up to the limit where $P = c$. It is optimal to raise that limit as much as possible, which implies $r_l = \epsilon$. Thus, a price below c is not optimal. In turn, lowering k is a move towards the monopoly outcome, i.e. it raises the price. This increases the innovator's profit up to the point where $P = c$. Again, $P < c$ is not optimal.*

Lemma 9 *The optimal choice of mechanism reduces to choosing the average marginal cost, $\bar{c}$ that maximizes*

$$\Pi(\bar{c}) := \left(P(Q(\bar{c})) - c\right)\left(Q(\bar{c}) - nq_L(\bar{c})\right) + Q(\bar{c})\epsilon. \tag{2.13}$$

Proof 9 *The innovator's profit* (2.8) *can be written in the form* (2.13) *if one replaces* q_w *by* Q *and* q_l *and uses the result* $r_l = \epsilon$*. By Lemma 8, excessive crowding out is not optimal. Therefore, the equilibrium solution of the Cournot market game,* (Q, q_l)*, solves equations* (2.3) *and* (2.4)*. These equilibrium outputs are exclusively a function of* $\bar{c}$*. It follows that* Π *is exclusively a function of* $\bar{c}$*. That function has a maximum by the Weierstrass extreme value theorem since the function is continuous and its domain can, without loss of generality, be taken to be the closed interval* $[\bar{c}_{min}, c]$*.*[6]

Using the definition of $\bar{c}$ it follows immediately:

Corollary 1 (Degree of freedom) *The optimal mechanism has a degree of freedom. Given the optimal* $\bar{c}$*, all combinations of* (k, r_w) *that satisfy the condition*

$$\bar{c} = c - \frac{k}{n}(\epsilon - r_w) \tag{2.14}$$

with $k \in [1, n-1]$ *and* $r_w < \epsilon$ *are optimal.*

This trade-off is illustrated in Figure 2.1 for the linear model $P = a - Q$, $a > c$.

2.4.5 Why is it not optimal to "crowd out" losers?

Lemma 10 (No "crowding out") *In the optimal mechanism winners and losers coexist, i.e.* $P > c$*, and* $q_w, q_l > 0$*.*

Proof 10 *We have already excluded* $P < c$ *(excessive crowding out). It only remains to be shown that a mechanism that induces* $P = c$ *cannot be optimal either.*
Suppose, per absurdum*, that the optimal mechanism involves a price equal to* c*, which implies that* $q_l = 0$ *is an interior solution of the losers' best-reply problem. Denote the* $\bar{c}$ *that gives rise to an equilibrium price equal to* c *by* $\bar{c}^*$*. Then, one must have:*

$$\begin{aligned} 0 = \left.\frac{\partial \Pi}{\partial \bar{c}}\right|_{\bar{c}=\bar{c}^*} &= (\epsilon + QP')Q' \quad \textit{(by (2.13))} \\ &= (\epsilon + n(\bar{c}^* - P))Q' \quad \textit{(by (2.3))} \\ &= (\epsilon + n(\bar{c}^* - c))Q' \quad \textit{(since } P = c\textit{).} \end{aligned} \tag{2.15}$$

[6]Thereby, $\bar{c}_{\min} := c - (\epsilon - r_w^{\min})$ and $r_w^{\min}$ is defined as the highest (negative) royalty rate r_w that together with $k = 1$ and $r_l = \epsilon$ propels a monopoly. Obviously, all royalty rates r_w below $r_w^{\min}$ are payoff dominated.

Therefore, $\bar{c}^* = c - \frac{\epsilon}{n}$*. Using the definition of* $\bar{c}$*, all combinations of* (k, r_w) *that solve the condition* $k(\epsilon - r_w) = \epsilon$ *are optimal. In particular,* $(k = 1, r_w = 0)$ *is optimal. However, together with* $P = c$*, this contradicts the assumption of a non-drastic innovation. Recall, a non-drastic innovation implies that the monopoly price is above the pre-innovation cost* c*.*

2.4.6 Why the proposed mechanism is superior

Compare the proposed optimal mechanism with the mechanisms considered in the literature. As we already pointed out, the class of mechanisms considered here includes all the other mechanisms as special cases. Therefore, our proposed optimal mechanism cannot be inferior. In Lemmas 6, 10 and 7 we have shown that it is optimal to choose $k < n$, to not crowd out losers, $q_l > 0$, and to set $r_l = \epsilon$. Therefore, unlike all these mechanisms, the proposed optimal mechanism generates royalty income from losers. This proves strict superiority, as long as one ignores that the number of licenses must be an integer.

2.4.7 Irrelevance of the integer constraint

We have shown that the optimal mechanism exhibits $k \in [1, n-1]$, $r_w < \epsilon$, $r_l = \epsilon$ and no crowding out. Now we show that the integer constraint concerning the number of contracts, k, is irrelevant in the sense that it can always be accommodated without affecting the innovator's equilibrium income.
Corollary 1 suggests that the degree of freedom allows us to implement the optimal $\bar{c}$ by choosing an arbitrary integer $1 \leq k \leq n-1$. But is this always feasible?
An integer k is feasible for a given optimal $\bar{c}$ if the corresponding royalty rate $r_w = \epsilon - \frac{n}{k}(c - \bar{c})$ is smaller than ϵ. By Lemma 5, which excludes $k = 0$, one has $\bar{c} < c$ and thus $r_w < \epsilon$. Therefore, the optimal $\bar{c}$ can always be implemented with an integer k, without loss in profit, as illustrated in Figure 2.1 for the inverse demand function $P(Q) = a - Q$.
This also indicates that royalties for winners are essentially useful to "fine tune" the optimal mechanism in the face of the integer constraint concerning k. If one ignores that integer constraint, royalties for winners serve no purpose.

Corollary 2 (Strict Superiority) *The optimal licensing mechanism is strictly more profitable than all standard licensing mechanisms such as pure royalty contracts, standard license auctions, auctions of royalty contracts, and take-it-or-leave-it two-part tariffs.*

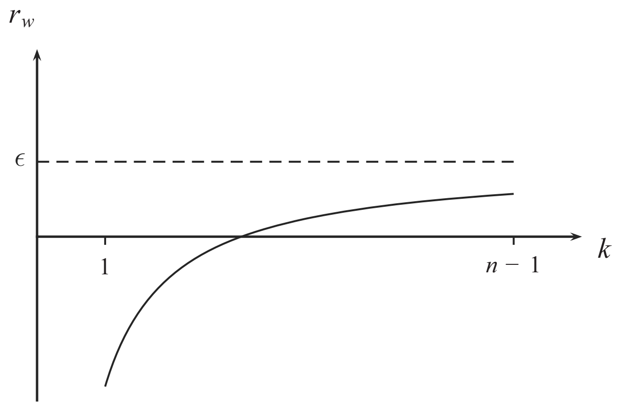

Figure 2.1: Optimal combinations of (k, r_w) for $(a, n, c, \epsilon) = (1, 20, .4, .05)$

We mention that if one does not include the proposed royalties for losers, as in the mechanism analyzed by Sen and Tauman (2007), the innovator's profit is not exclusively a function of $\bar{c}$, and one does not obtain the above degree of freedom. As a result one faces an integer problem that generically entails a loss in profit. The same holds true if one does not include royalties for winners. Therefore, adding royalties for winners as well as for losers is crucial for the above stated irrelevance of the integer constraint.

2.4.8 Illustration: linear demand

The literature on patent licensing usually assumes a linear model with $P := a - Q, a > c$. Due to the prominence of this model, we briefly illustrate our findings for that case.
In the linear model the optimal $\bar{c}$ is equal to $\bar{c} = c - \frac{a-c+\epsilon}{2n}$, and the optimal mechanism (k, r_w, r_l) is:

$$k \in \{1, \ldots, n-1\}, \quad \left(r_w, r_l\right) = \left(\epsilon - \frac{a-c+\epsilon}{2k}, \epsilon\right). \tag{2.16}$$

Moreover, $K = \frac{a-c}{\epsilon}$. Specifically, setting $k = n-1$ gives $r_w = \frac{2n-3-K}{2(n-1)}\epsilon$. For $K > 2n-3$ ("weak" innovations), r_w is negative, i.e. winners' outputs are subsidized.
If one ignores the integer constraint the royalty rate for winners serves no purpose and

$$(k, r_w, r_l) = \left(\min\left\{\frac{K+1}{2}, n-1\right\}, 0, \epsilon\right) \tag{2.17}$$

is an optimal mechanism. And if one would not use royalties for winners, then the integer constraint is generically binding, unless the innovation is sufficiently weak (i.e. $K \geq 2n - 3$). In that case the optimal k is the closest integer neighbor of $\min\left\{\frac{K+1}{2}, n-1\right\}$, at a loss in profit.

2.5 Discussion

2.5.1 Welfare comparisons

Welfare is completely determined by the adopted technologies and aggregate output. In the equilibrium of the Cournot subgame aggregate output is exclusively a function of the average marginal cost $\bar{c}$ if no crowding out occurs; whereas if crowding out occurs, aggregate output is exclusively a function of c_w.

Under the proposed mechanism the new technology is used by all firms; hence, social marginal cost is equal to $c - \epsilon$. No firm is crowded out, and thus the equilibrium aggregate output is determined by $\bar{c} < c$ (see (2.3)). Therefore, equilibrium aggregate output Q and thus welfare is greater than before the innovation, regardless of which optimal combination of k and r_w is applied.

Under optimal pure royalty licensing (with royalty rate r) the new technology is also used by all firms and hence social marginal cost is also equal to $c - \epsilon$. Aggregate output is completely determined by $c - \epsilon + r$. Therefore, if $r = \epsilon$ (which is optimal in the standard linear model), welfare is smaller than in the proposed mechanism, although greater than before the innovation.

The standard optimal license auction without royalties has several equilibrium outcomes, depending on the magnitude of the innovation.[7] For sufficiently strong innovations (reflected in a small number K), some firms are crowded out. All remaining firms use the new technology and social marginal cost is equal to $c - \epsilon$. Crowding out implies $P \leq c$. Whereas under our proposed mechanism, no firm is crowded out and $P > c$. Hence, in this case, the standard license auction entails greater welfare than our proposed mechanism. For weak innovations, the optimal standard auction does not crowd out. If all firms get a license, $k = n$, social marginal cost is equal to $c - \epsilon$ and the auction achieves the maximum welfare obtainable under Cournot competition.[8]

[7] See Kamien et al. (1992).

[8] If $k < n$, social marginal cost is between c and $c - \epsilon$, since those who do not get a license do not get the new technology. Nevertheless, the welfare comparison is ambiguous.

2.5.2 Why an auction instead of two-part tariffs?

One may ask: why is it not optimal to make a take-it-or-leave-it offer instead of an auction? Answering this question helps to understand the fundamental role of the auction in the present context.

Suppose the innovator makes a take-it-or-leave-it offer to each firm that gives firms the same allocation and transfers as the equilibrium of the auction. Then, a designated winner has an incentive to deviate and not accept the contract, for the following reason:

If the winner accepts, his payoff is the same as that of a firm that accepts a loser contract, by design of the auction. However, if he rejects the offer, each loser is made better off because one firm with low marginal cost has been replaced by one firm with high marginal cost. The deviator is now one of those losers. Therefore, his payoff is increased.

Why does this logic not apply if the innovator employs an auction to sell winner contracts? The reason is simple. If a firm does not bid or a designated winner deviates and rejects the offer, another bidder will become a winner instead. Therefore, the industry structure remains unchanged. Hence deviation does not pay.

Of course, deviation never pays for a loser because if a loser does not exercise the royalty option, the industry structure is not changed at all.

This explains why using auctions to sell a restricted number of licenses is a clever way to reduce the bargaining power of licensees, more than what the innovator could achieve with an ultimatum offer.[9]

2.5.3 Can a combinatorial auction do even better?

One may think that a combinatorial auction may be even more profitable for the innovator. However, this is not true, for the following reason.

Suppose the auctioneer runs the following combinatorial auction (ignoring royalties to winners): Each firm is invited to make a bid contingent on getting one out of $k \in \{1, 2, \ldots, n\}$ licenses. After all bids have been submitted, the auctioneer selects the most profitable k, awards k licenses, and all winners pay their bids. In equilibrium, bids are such that bidders are indifferent between winning and losing, as in (2.7). Of course, bids differ for different values of k, reaching a maximum profit of the innovator at some k. The auctioneer selects that k, which is precisely the number of licenses that he auctions in

[9] Of course, two-part tariffs are equally good if the innovator can credibly threaten to withdraw the innovation if one or more firms do not accept the offered two-part tariffs. In this light, the advantage of the auction is that it works without such "collective punishment" threats.

our simple auction. We conclude that using combinatorial auctions cannot further boost the innovator's profit.[10]

2.5.4 The "chutzpah mechanism" revisited

Having characterized the optimal mechanism in the class (k, r_w, r_l), one may ask: is it the best the innovator can do or is there a superior mechanism? Following Kamien (1992), Kamien et al. (1992) we address this question by looking at an extreme reference point that can be implemented by an appropriately generalized "chutzpah mechanism".[11] That reference point can be useful as an upper bound of the innovator's profit.

Taking (k, r_w, r_l) as given, the innovator cannot possibly extract more than the total industry gross profit, $\sum \bar{\pi} := k\bar{\pi}_w(k) + (n-k)\bar{\pi}_l(k)$. However, each firm can assure itself at least a (net) profit equal to $\pi_l(n-1)$, since $\pi_l(k) \geq \pi_l(n-1)$, for all $k \leq n-1$. Therefore, the innovator cannot extract more than:

$$\tilde{\Pi}(k) := \sum \bar{\pi} - n\pi_l(n-1).$$

Comparing this with (2.8) one finds, after a bit of rearranging,

$$\tilde{\Pi}(k) - \Pi(k) = n(\pi_l(k) - \pi_l(n-1) \geq 0,$$

with equality if and only if $k = n-1$. Therefore, if one chooses the optimal proposed mechanism, (k, r_w, r_l), the equilibrium innovator profit, Π, is bounded from above:

$$\Pi \leq \tilde{\Pi}(k).$$

This upper bound can be reached by our proposed mechanisms for $k = n-1$, which is an optimal number of licenses in the proposed mechanism.

Of course, the innovator can reach $\tilde{\Pi}(k)$ for all other k and even higher profits if he can extort additional transfers by threatening to trigger a collective penalty in the event when at least one firm fails to pay. This can be achieved by a generalized "chutzpah mechanism", which may be useful as a benchmark but offers no practically relevant guidance.[12]

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[10]Note, however, that combinatorial auctions are attractive if firms are heterogeneous, and firms are allowed to make bids contingent on who gets the innovation.

[11]The "chutzpah mechanism" was introduced by Kamien et al. (1990).

[12]There, 1) the innovator offers the mechanism $(k, r_w, r_l = \epsilon)$ supplemented by a participation fee equal to $\bar{\pi}_l(k) - \bar{\pi}_l(n-1)$, and the proviso that this offer is valid only if all n firms pay the requested participation fee. 2) If a bidder refuses to pay the participation fee, the innovator calls off the auction 1), refunds the collected participation fees (if any), and then 3) runs the unconditional license auction: $(k, r_w, r_l) = (n-1, r_w, r_l = \epsilon)$.

2.5.5 Bertrand competition and differentiated goods

One may wonder to what extent our results rely on the assumptions of Cournot competition and homogeneous goods. If one maintains the homogeneous goods assumption but replaces Cournot by Bertrand competition, the optimal licensing mechanism obviously gives rise to monopoly, as already pointed out by Kamien (1992). However, as it is generally the case, the combination of homogeneous goods with Bertrand competition is only a borderline case. Therefore, Bertrand competition can only yield plausible results in the context of heterogenous goods.

If goods are sufficiently heterogeneous substitutes, each firm has its own market niche even if there is some dispersion of costs. Therefore, crowding out should be of less concern. Exclusive licensing is profitable to the innovator, as in the above model, since granting a license to a firm inflicts a negative externality on all other firms. Similarly, adding the royalty option to losers increases the innovator's profit (unless crowding out is optimal), since granting that option does not affect the payoffs of winners, and thus does not affect the equilibrium bids for the license. This suggests that the same logic that drives the superiority of the proposed mechanism in the above model applies equally well to Bertrand competition with sufficiently heterogeneous substitutes.

However, if goods are complements, the picture should change more drastically. In that case, firms mutually benefit from each others' cost reduction, since one firm's price reduction raises the other firms' demand. Therefore, giving one firm a license does not inflict a negative externality upon others which in turn implies that one cannot induce higher bids by restricting the number of licenses. This reasoning applies regardless of whether there is either Cournot or Bertrand competition. Therefore, one should expect that the attraction of auctioning a limited number of licenses vanishes, and it should be optimal to give all firms a license ($k = n$).

2.5.6 Incomplete information

The present analysis is confined to complete information in a common value setting, where the cost reduction induced by the innovation is independent of which firm uses that innovation. However, under certain conditions the proposed mechanism can also be viewed as an optimal mechanism under incomplete information.

If firms are subject to incomplete information, they know that the cost reduction induced by the innovation is the same for all users, but they do not know the size of that cost reduction. This gives rise to a common value auction problem.[13]

However, if that cost reduction becomes common knowledge among both firms and the innovator before the downstream market game is played, the innovator can announce a collection of mechanisms, one for each possible cost reduction, ask firms to make their bids contingent on these cost reductions, and then adopt the mechanism (k, r_w, r_l) that applies to the observed cost reduction. The *ex ante* optimal collection of mechanisms must be optimal pointwise, i.e. the mechanism prescribed for a certain cost reduction must be the one the innovator would choose *ex post*, after having observed that cost reduction. In this sense, the proposed mechanism and the associated equilibrium play of firms, looked at as a function of ϵ, can be viewed as the optimal collection of mechanisms under incomplete information.

[13]There is a small literature on the auctioning of one license under incomplete information that, however, assumes the private values framework (see Jehiel and Moldovanu, 2000, Das Varma, 2003, Goeree, 2003).

Chapter 3

Innovation Tournaments with Entry Auction

3.1 Introduction

Consider a procurement problem where a buyer needs an innovative good that can potentially be provided by many innovators (sellers). An innovation of any quality serves the procurer's needs, but the procurer's profit is increasing in innovation quality.

Innovation quality is random, but expected quality is increasing (in the sense of first-order stochastic dominance) in the seller's ability and R&D effort. Ability (or *type*) is private information of the seller. Ability and effort are not observable. Innovation quality is observable between the seller and the procurer, but not verifiable and thus not contractible.

The literature has repeatedly analyzed the profitability of two prominent procurement mechanisms that are also employed in real-world procurement settings:[1] innovation tournaments where an innovation is bought either employing a first-price (first-score) auction or a fixed prize.[2]

In a fixed-prize tournament, a prize is paid in return for the best of all innovations that are delivered at some due date. In the first-score auction, each innovator submits an innovation and a financial bid from which the procurer computes a score. The highest score wins and the winner is paid his financial bid.

We combine each of those two mechanisms with a discriminatory entry auction where the $n \geq 2$ highest-bidding participants pay their bids as an entry fee and then enter the tournament stage, where, in one mechanism, they compete for a fixed prize, and in the other, they compete in a scoring auc-

[1] Scotchmer (2004) provides many current and historical examples.

[2] The first-/second-score auction is a two-dimensional equivalent of a first-/second-price auction. We use these terms synonymously. A bid has a price and a quality dimension that are combined to a score. The highest score wins the auction. See Che (1993) for an analysis of these formats in a procurement setting.

tion. The losers of the entry auction do not pay anything and are not eligible for participation in the tournament. The aim of the entry auction is to (efficiently) select the most able innovators for the tournament and restrict entry, which is generally optimal.[3,4]

We will argue that, if the auction revenue does not accrue to the buyer, the entry auction can be interpreted in a way that makes those mechanisms similar to what we observe in real procurement applications: the procurer announces a shortlisting procedure and selects a few of the supposedly most able sellers to compete in a tournament where the winner will be rewarded with a prize (or a contract). Before entering the tournament, the shortlisted sellers face a cost of writing a detailed proposal or building a prototype.

For this setting, we characterize Bayesian Nash equilibria of both mechanisms with the following properties: a) although abilities are private information, signaling in the entry auction does not occur;[5] b) all sellers participate and the most able sellers enter the tournament stage, i.e., the entry auction is efficient; c) all sellers expect the same profit (as a function of type) in both mechanisms and this result does not depend on the optimal choice of the fixed prize.

Under a uniform distribution assumption, we demonstrate existence of the above equilibria, where, in addition to the above results, contestants choose the same efforts and produce the same expected innovations (as a function of their type) in both mechanisms. Moreover, if the procurer collects the entry auction revenue then her profit is the same in both mechanisms; and if the procurer is a welfare–maximizer, then the two mechanisms (with those equilibria) are optimal.

Given the huge literature on tournaments and on innovation (see, e.g., Konrad (2009) on tournaments and Scotchmer (2004) on the economics of innovation), we only mention work that is closely related to the present paper. Fullerton and McAfee (1999) also analyze the use of entry auctions for selecting the most able participants for a fixed-prize tournament. There, innovators have different marginal effort cost but the c.d.f. of their innovations is the

[3]The auction also generates revenue but that has to be compensated with a larger prize.

[4]Generally, it is optimal to restrict entry to a tournament in order to provide effort incentives and avoid duplication of cost. The procurer faces a tradeoff: The expected quality of the best innovation as well as total cost increase in the number of contestants. It is intuitive that, potentially, any number of innovators can be optimal: if the average innovation is very profitable for the procurer she might want to let many innovators engage in R&D while if this profit is low she might prefer only one or two of them.

[5]In our procurement setting, innovators in principle have an incentive to signal their types at the entry stage in order to influence their rivals' effort choice. In the particular equilibria we are going to discuss, a signaling issue does not arise. Nevertheless, signaling is a typical issue in these procurement problems.

same function of effort for all innovators. In contrast, in our model marginal cost is equal but innovators' types (as well as effort) affect the distribution of their innovations. Another difference is that Fullerton and McAfee (1999) assume that the private type information becomes common knowledge before the tournament begins, while in our model it remains private, but our focus is on equilibria where signaling does not occur. Fullerton and McAfee (1999) focus on the (in)efficiency of standard auctions but, as we will see, that issue does not arise in the present model.

Fullerton et al. (2002) is an experimental study that builds on the model of Taylor (1995). The tournament winner is awarded through a first-price auction. Taylor (1995) looks at a tournament as an optimal stopping problem, where identical innovators pay a fixed entry fee and then make a number of independent innovation draws where after each draw they decide whether to draw again.

Che and Gale (2003) look at the optimal design of R&D contests assuming a deterministic innovation technology. They find that a first-price auction outperforms a fixed prize. Schöttner (2008) asks why we observe both tournaments with a fixed prize and first-price auctions and presents a model where the fixed prize can be more profitable than the auction. Both assume that entry fees are not feasible and that the sellers' types are common knowledge. Che (1993) studies the use of first- and second-score auctions in procurement problems. Ding and Wolfstetter (2009) study the adverse selection problem that arises if the procurer cannot commit herself to never negotiating with inventors who circumvent the procurement mechanism. They also analyze the performance of tournaments with fixed prize and first-price auction.

Due to the prominence of the two procurement mechanisms (first-price auction and fixed prize), we take them as given and only add an entry auction as proposed by Fullerton and McAfee (1999). We do not attempt to design an optimal procurement mechanism. Nevertheless, we show that the (combined) mechanisms can be optimal for a welfare-maximizing buyer.

3.2 The model

There is a risk-neutral procurer (or buyer) who needs to buy an innovative good and there are $N \geq 3$ risk-neutral innovators (or sellers). The procurer commits to a mechanism as specified below.[6] For all mechanisms in the paper we assume random tie-breaking, e.g., when several innovators have the same

[6]This includes a commitment not to procure from players who circumvent the mechanism. See Ding and Wolfstetter (2009), where this assumption is relaxed which gives rise to an adverse selection problem.

innovation quality in the fixed-prize tournament or when two innovators have the same bid (or score) in an auction.

Seller i's innovation is denoted by the random variable Y_i with realizations $y_i \in (\underline{y}, \overline{y})$, $\underline{y} \geq 0$. There, y_i is the monetary value-added induced by the innovation for the procurer; also called the quality of the innovation. That value is not verifiable but observable by the innovator and the procurer. The innovation can only be used by the procurer and is worthless for the innovator. Also, the procurer can only employ one innovation.

In particular, innovator i independently draws an innovation from the c.d.f. $G^{a_i+e_i}$. There, G is a c.d.f. with support $(\underline{y}, \overline{y})$ and positive continuous density, $a_i > 0$ is i's *ability*, and $e_i \geq 0$ is i's research *effort*. For simplicity, we denote $k_i := a_i + e_i$. Thus, ability and effort are perfect substitutes with respect to innovation quality and larger values of k_i imply a draw from a "better" distribution in the sense of first-order stochastic dominance. All innovators draw from the same *class* of c.d.f.s, G^{k_i}, while k_i is (potentially) different for each innovator. Equivalently, k_i can be seen as the (non-integer) number of independent innovation draws from c.d.f. G where the best out of k_i draws is submitted to the procurer.[7]

Abilities (types) are private information of the sellers. Ability models expertise or a comparative advantage to solve the problem at hand. It is denoted by random variables A_i with realizations a_i. They are independently distributed with c.d.f. H, positive continuous density, and support $(\underline{a}, \overline{a})$, $\underline{a} \geq 0$. A seller who does not engage in R&D has zero profit, while R&D activity, i.e., $e_i > 0$, produces an innovation at cost $C(e_i) := ce_i + \gamma$ with $c, \gamma > 0$.[8] The fixed cost γ is the cost of employing one's ability. It can be interpreted as the cost of the minimum R&D scale. Effort can be "bought" at a constant marginal cost. Thus, additional effort can be interpreted as adding office space, hiring additional personnel, or paying overtime, which might have the same cost for all innovators within an industry. All of the above is common knowledge.

Throughout the paper, random variables are denoted by upper-case letters and the corresponding realizations by the respective lower-case letters. The first and second partial derivatives of a function $f(x, y)$ w.r.t. x are denoted by $\partial_x f(x, y)$ and $\partial_{x,x} f(x, y)$, respectively. The superscripts F, S, and B (e.g., in k^S) indicate mechanisms, *not powers.* W.l.o.g., innovators are labeled in decreasing order of abilities: $a_1 \geq \cdots \geq a_N$.

Order statistics are denoted as follows: The kth highest of K independent draws from c.d.f. H (ability) is $A_{(k:K)}$ and its c.d.f. is $H_{(k:K)}$. Seller i's

[7]If one makes k_i independent draws from c.d.f. G, then the highest order statistics (the best innovation draw) is distributed with c.d.f. G^{k_i}.

[8]Thus, by assumption, we exclude an innovation draw with zero effort.

innovation is drawn from c.d.f. $G^{a_i+e_i}$, where the kth highest of all the draws of innovators $i = 1, \ldots, K$ is denoted by $Y_{(k:K)}$, and, with a slight abuse of notation (suppressing the exponents) we write $G_{(k:K)}$ for the c.d.f. of $Y_{(k:K)}$. For example, $H_{(1:N-1)}$ is the c.d.f. of the highest ability, $A_{(1:N-1)}$, among seller i's $N-1$ rivals; $G_{(2:n)}$ is the c.d.f. of the second-best innovation, $Y_{(2:n)}$, generated among n sellers; and $a_{(1:N)}$ is the highest type (realization).

The timing of the basic game induced by the mechanisms is: At stage 0, the mechanism is announced by the buyer, at stage 1, sellers bid for entry to the tournament, at stage 2, innovations are drawn, and, finally, at stage 3, an innovation is procured using either a fixed-prize or a first-price auction.

The paper proceeds as follows. Section 3.3 analyzes the fixed-prize tournament with entry auction. Section 3.4 looks at the scoring-auction tournament with entry auction and derives a part of the central result. Section 3.5 gives more results assuming uniform distributions. Section 3.6 provides a welfare analysis. In section 3.7 we discuss results and related issues. Section 3.8 concludes the chapter. The Appendix contains a proof and results on order statistics.

3.3 Entry auction and fixed-prize tournament

Consider the following procurement mechanism, F (for "fixed" prize). The procurer announces a fixed prize P and a number $n \in [2, N-1]$ for an entry auction where the n highest-bidding participants win and pay an entry fee that is determined in the auction (details below).[9] Bids are published. The n auction winners ("contestants") compete for the fixed prize P that is awarded in return for the best innovation generated among them. All other sellers are excluded from the contest.[10]

Recall that contestant i has ability a_i, chooses unobservable effort e_i, and draws an innovation from c.d.f. $G^{a_i+e_i}$ at cost $C(e_i) := ce_i + \gamma$ if $e_i > 0$, while if $e_i = 0$, cost is zero and no innovation is drawn. Each contestant faces $n-1$ rivals. In the following, we suppose that efforts are positive and in section 3.5 we provide feasible parameters.

Consider stage 3, the procurement stage, where cost and effort are sunk. The best (out of n) innovations is awarded the prize P. Contestant i has produced innovation y_i and i's expected profit is (where the superscript $F3$ refers to the mechanism and the stage of the game)

$$\pi_i^{F3}(y_i) = PG^{\sum_{j \neq i} k_j}(y_i), \qquad k_j := a_j + e_j, \tag{3.1}$$

[9]Naturally, competition at the tournament stage requires $n \geq 2$ while the bidding equilibrium of the entry auction requires $n < N$.

[10]Whether or not the losers of the auction pay anything depends on the auction rules.

where $G^{\sum_{j\neq i} k_j}(y_i)$ is the probability that y_i is the best innovation.
At stage 2, contestant i chooses effort e_i and expects profit

$$\pi_i^{F2}(a_i, e_i) = E[\pi_i^{F3}(Y_i)] - ce_i - \gamma, \tag{3.2}$$

where[11]

$$E[\pi_i^{F3}(Y_i)] = \int_{\underline{y}}^{\overline{y}} PG^{\sum_{j\neq i} k_j}(y_i)dG^{k_i}(y_i) = \frac{k_i}{k_i + \sum_{j\neq i} k_j}P. \tag{3.3}$$

Thus,

$$\pi_i^{F2}(a_i, e_i) = \frac{k_i}{k_i + \sum_{j\neq i} k_j}P - ce_i - \gamma. \tag{3.4}$$

The profit $\pi_i^{F2}(a_i, e_i)$ is strictly concave and, if P is sufficiently large, positive. The interior solution is characterized by

$$\partial_{e_i}\pi_i^{F2}(a_i, e_i) = 0 \iff \frac{\sum_{j\neq i} k_j}{\left(k_i + \sum_{j\neq i} k_j\right)^2} = \frac{c}{P}. \tag{3.5}$$

Since the RHS of (3.5) is constant, we can equate the LHS of (3.5) for all $i = 1, \dots, n$ and the first-order condition simplifies to $k_1 = k_2 = \cdots = k_n$. Substituting back into (3.5), we obtain the (candidate) equilibrium effort

$$e_i^F = \frac{(n-1)P}{n^2c} - a_i. \tag{3.6}$$

Note that (3.6) characterizes the *unique* equilibrium that satisfies all contestants' first–order conditions, i.e., where all contestants choose positive effort. By (3.6), $e_i^F + a_i$ is the same constant for each i. For later use, define

$$k^F := e_i^F + a_i = \frac{(n-1)P}{n^2c}. \tag{3.7}$$

Insert (3.6) into (3.4), then i's expected tournament profit is[12]

$$\pi^{F2}(a_i) := \pi_i^{F2}(a_i, e_i^F) = \frac{P}{n^2} + ca_i - \gamma. \tag{3.8}$$

[11] The probability of winning the prize, $k_i/(k_i + \sum_{j\neq i} k_j)$, resembles the Tullock contest success function where each contestant's "contribution" is ability and effort, $k_i = a_i + e_i$. We neither assumed this function nor does it require a specific distribution assumption. See Jia (2008) on the stochastic foundations of the Tullock contest success function.

[12] Here, and in later sections, we drop the subscript i in the profit function since the equilibrium profit is the same function for all sellers.

Effort e_i^F is always positive if $P \geq (\bar{a}cn^2)/(n-1) =: P_{\min}(n)$. In the following we only consider prizes $P \geq P_{\min}(n)$, ensuring existence of the symmetric equilibrium, (3.6), for all n contestants. We discuss that issue in section 3.7.4. Also, we assume that the model parameters are such that (3.8) is positive. In section 3.5 we give an example of such parameters.

It follows that all sellers draw innovations from the same c.d.f., (3.9), and have the same probability of winning, $1/n$, regardless of ability.

$$G^{a_i+e_i^F} = G^{k^F} = G^{\frac{(n-1)P}{n^2c}}. \tag{3.9}$$

At stage 1, innovators bid for entry. A participant's maximum willingness to pay is equal to his equilibrium expected tournament profit conditional on entry. This profit, (3.8), is strictly increasing in ability. It only depends on a seller's own ability and is thus a pure private value, while, accordingly, rivals' profits are i.i.d. random variables. Thus, we have symmetric independent private values, which implies that there is no signaling issue in the entry game: sellers do not care about their rivals' abilities.

Thus, in particular, the discriminatory and the uniform-price auction formats are efficient and revenue-equivalent.[13] We analyze the uniform-price format because it is simpler, not because we recommend it or think it is the most appropriate format. There, bidders have the weakly dominant strategy to bid their expected tournament profits conditional on entry,

$$\beta^F(a_i) = \pi^{F2}(a_i). \tag{3.10}$$

This strategy guarantees a non-negative expected profit since if i wins, the price (entry fee) he pays is not above his expected tournament profit. Consider bidding more than $\beta^F(a_i)$. If i was previously a winner, he is still a winner with the same profit. If he was previously a loser, he is either still a loser or becomes a winner, in which case the previous price was at or above i's profit, and it is not lower now. Thus, i cannot be better off. A similar argument applies for bids below $\beta^F(a_i)$. The bid $\beta^F(a_i)$ is positive and strictly increasing in ability. Thus the auction is efficient if everybody participates.

[13] These formats are standard sealed-bid multi-unit auctions with single-unit demand. See, e.g., Krishna (2002, chs. 13, 14) for an analysis of these mechanisms in the symmetric independent private values framework.

Suppose they do, then seller i's equilibrium profit is

$$\begin{aligned}
\pi_i^{F1}(a_i) &= \Pr\left\{a_i > A_{(n:N-1)}\right\}\left(\pi^{F2}(a_i) - E\left[\beta^F(A_{(n:N-1)})\middle| a_i > A_{(n:N-1)}\right]\right) \\
&= H_{(n:N-1)}(a_i)\pi^{F2}(a_i) - \int_{\underline{a}}^{a_i} \pi^{F2}(a)dH_{(n:N-1)}(a) \\
&= \int_{\underline{a}}^{a_i} \partial_a \pi^{F2}(a)H_{(n:N-1)}(a)da \\
&= c\int_{\underline{a}}^{a_i} H_{(n:N-1)}(a)da =: \pi^{F1}(a_i) > 0, \qquad (3.11)
\end{aligned}$$

which confirms that all N sellers participate.
At stage 0, the procurer chooses P and n; the optimal choice is denoted by P^F and n^F. By (3.9), the expected value of the best innovation is

$$E\left[Y_{(1:n)}^F\right] = \int_{\underline{y}}^{\overline{y}} ydG(y)^{\sum_{j=1}^n \frac{(n-1)P}{n^2c}} = \int_{\underline{y}}^{\overline{y}} ydG(y)^{nk^F}. \qquad (3.12)$$

In order to obtain that innovation, the procurer pays P but also collects auction revenue, equal to n times the expected tournament profit of the seller with the n +1st highest ability. The procurer's objective is

$$\max_{n,P} \Pi^F(n,P) \quad \text{s.t. } n \in [2, N-1],\ P \geq P_{\min}(n), \qquad (3.13)$$

where

$$\Pi^F(n,P) = E\left[Y_{(1:n)}^F\right] - P + nE\left[\beta^F\left(A_{(n+1:N)}\right)\right]. \qquad (3.14)$$

Using (3.12) and (3.10), we write $\Pi^F(n,P)$ as

$$\Pi^F(n,P) = \int_{\underline{y}}^{\overline{y}} ydG^{\frac{(n-1)P}{nc}}(y) - P + n\left(\frac{P}{n^2} - \gamma + c\int_{\underline{a}}^{\overline{a}} adH_{(n+1:N)}(a)\right). \qquad (3.15)$$

Remark 1 *The choice of P is irrelevant in a certain sense: By* (3.12)*, the expected best innovation is characterized by $K := P(n-1)/nc$. In order to induce "innovation K", one sets the prize $P = Knc/(n-1)$. Then the procurer's only choice variables are the number of contestants, n, and expected innovation quality, K.*

$$\Pi^F(n,K) = \int_{\underline{y}}^{\overline{y}} ydG^K(y) - Kc - n\gamma + nc\int_{\underline{a}}^{\overline{a}} adH_{(n+1:N)}(a) \qquad (3.16)$$

We continue the discussion in section 3.5 and now turn to the second mechanism.

3.4 Entry auction, tournament, and scoring auction

Here, the procurer at stage 0 announces mechanism S ("scoring"): First, all interested innovators bid in a uniform-price entry auction (as in the previous section). Bids are published. The $n \in [2, N-1]$ highest bids win the auction, the winners pay the $n+1$st highest bid as a non-refundable entry fee and enter the tournament.[14] Then they simultaneously choose efforts and draw innovations. Finally, the procurer conducts a scoring auction and procures the innovation from the bidder with the highest score (details below).

Whereas the fixed prize is paid to the *best* innovator, the scoring auction allows all innovators to compete on price. Moreover, the auction provides an endogenous reward, while a fixed prize is a strategic variable and thus a source of errors. Our main result is driven by the fact that sellers anticipate the increased competition in a scoring auction and adjust their entry bids accordingly.

We consider the first-score auction format, where bidders submit an innovation and a financial bid from which a score is computed. The highest score wins and the winner receives his financial bid as payment for his innovation. The procurer applies the ideal scoring rule $s = y - b$, where y is the innovation and b is the seller's financial bid.[15]

In order to simplify analysis, we analyze a revenue-equivalent auction format, the second-score auction.[16] There, the highest score wins and the winner is obliged to deliver the second-highest score to the procurer. Since the winner's innovation is fixed and different from all other innovations, an amount of money is paid to or by the winner in order to adjust his score to the second-highest score. At the auction stage, all innovations are drawn and cost and effort are sunk. Thus, a bidder's decision problem amounts to choosing a bid given one's innovation and supposed distributions of his $n-1$ rivals' innovations. For given abilities and efforts, the distribution of innovations is completely determined, which makes the standard auctions revenue-equivalent. In turn, for different standard auction formats at the procurement stage, the same equilibrium effort choice obtains.

In the auction, bidders have the weakly dominant strategy to bid $b_i = 0$, i.e., a score of $s_i = y_i$, equal to the value of their innovation. At the start of the

[14] Again, losers do not pay anything, do not enter the tournament, and are not allowed to submit an innovation.

[15] This scoring rule is ideal in the sense that it is the most credible: it reflects the true profit of the procurer. Thus the procurer has an incentive to select the most profitable innovation (taking into account the financial bid), which, in equilibrium, is equal to the best innovation.

[16] See, e.g., Che (1993) for an analysis of this auction format.

auction, all cost is sunk and, by assumption, the innovation is worthless for the innovator. Given the scoring rule, a bidder wants to win the auction iff the value of his *innovation* exceeds the second-highest *score*: only then would he be paid any money in the event of winning (otherwise he would have to pay in order to meet the second-highest score). The bid $b_i = 0$ ensures that he wins iff he wants to win.

Thus, the winner's equilibrium profit is the difference between his and the best rival's innovation, $y_i - y_{1:n-1}$.

At stage 3, the auction stage, contestant i's expected profit is[17]

$$\begin{aligned}\pi_i^{S3}(y_i) &= \Pr\{y_i > Y_{(1:n-1)}\}E[y_i - Y_{(1:n-1)}|y_i > Y_{(1:n-1)}] \\ &= \int_{\underline{y}}^{y_i} (y_i - y)dG^{\sum_{j\neq i} k_j}(y) \\ &= \int_{\underline{y}}^{y_i} G^{\sum_{j\neq i} k_j}(y)dy > 0.\end{aligned} \tag{3.17}$$

At stage 2, the tournament stage, contestant i chooses effort e_i at cost $ce_i + \gamma$ and has an expected profit of

$$\pi_i^{S2}(a_i, e_i) = E[\pi_i^{S3}(Y_i)] - ce_i - \gamma \tag{3.18}$$

$$= \int_{\underline{y}}^{\overline{y}} \pi_i^{S3}(y_i)dG^{k_i}(y_i) - ce_i - \gamma$$

$$= \int_{\underline{y}}^{\overline{y}} G^{\sum_{j\neq i} k_j}(y) \int_{y}^{\overline{y}} dG^{k_i}(y_i)dy - ce_i - \gamma$$

$$= \int_{\underline{y}}^{\overline{y}} G^{\sum_{j\neq i} k_j}(y) \left(1 - G^{k_i}(y)\right) dy - ce_i - \gamma. \tag{3.19}$$

We have

$$\partial_{e_i}\pi_i^{S2}(a_i, e_i) = -\int_{\underline{y}}^{\overline{y}} G^{k_i + \sum_{j\neq i} k_j}(y) \ln(G(y))dy - c, \tag{3.20}$$

$$\partial_{e_i,e_i}\pi_i^{S2}(a_i, e_i) = -\int_{\underline{y}}^{\overline{y}} G^{k_i + \sum_{j\neq i} k_j}(y) \left(\ln(G(y))\right)^2 dy < 0. \tag{3.21}$$

The interior maximizer is therefore given by the first-order condition[18]

$$-\int_{\underline{y}}^{\overline{y}} G^{k_i + \sum_{j\neq i} k_j}(y) \ln(G(y))dy = c. \tag{3.22}$$

Denote $K := k_i + \sum_{j\neq i} k_j$ as the solution of (3.22). If the model parameters are such that $K > a_i + \sum_{j\neq i} a_j$ then it characterizes the equilibrium effort.

[17]Recall the notation $k_j = a_j + e_j$.

[18]The LHS is positive since $\ln(G(y))$ is negative.

Then *total* effort is positive and the allocation of that effort among sellers is arbitrary. Thus, there are potentially many symmetric and asymmetric equilibria.[19]
As in the discussion of mechanism F, consider the symmetric candidate where $k_i = k_j =: k^S$ and thus $K = nk^S$ satisfies (3.22) and seller i chooses $e_i^S = k^S - a_i$. This requires that k^S is sufficiently large to ensure positive efforts for all abilities, i.e., $k^S > \bar{a}$ (for feasible parameter values, see section 3.5).
This equilibrium candidate is more appealing than others because, a), it is similar to the *unique* equilibrium of mechanism F where all contestants choose positive effort, b), the strategies are independent of rivals' private information and thus signaling does not occur (we discuss signaling in section 3.7.4), and, c), it has remarkable welfare properties (see section 3.6).
Recalling (3.19), seller i's equilibrium profit at stage 2, $\pi^{S2}(a_i)$, is[20]

$$\pi^{S2}(a_i) = \int_{\underline{y}}^{\overline{y}} G^{(n-1)k^S}(y) \left(1 - G^{k^S}(y)\right) dy - c\left(k^S - a_i\right) - \gamma. \tag{3.23}$$

Again, a seller's expected tournament profit is a pure private value, i.e., a function of own ability only, and, again, these symmetric effort strategies, $e_i^S = k^S - a_i$ imply that all contestants draw from the same c.d.f., G^{k^S}, and thus have the same probability of winning, $1/n$.
Consider stage 1, the entry auction stage. Similar to mechanism F, a bidder's the weakly dominant strategy is to bid

$$\beta^S(a_i) = \pi^{S2}(a_i). \tag{3.24}$$

If all sellers participate, seller i's expected profit is[21]

$$\begin{aligned}\pi_i^{S1}(a_i) &= \Pr\{a_i > A_{(n:N-1)}\} \left(\pi^{S2}(a_i) - E[\beta^S(A_{(n:N-1)})|a_i > A_{(n:N-1)}]\right) \\ &= c\int_{\underline{a}}^{a_i} H_{(n:N-1)}(a)da =: \pi^{S1}(a_i) > 0. \end{aligned}\tag{3.25}$$

Thus, in our symmetric equilibrium, all N innovators participate.
Comparing (3.25) and (3.11) and collecting the results so far, we state

Proposition 1 *Suppose the tournament equilibria of mechanisms F and S, characterized by $k^F = e_i^F + a_i$ and $k^S = e_i^S + a_i$, respectively, exist for each*

[19] For instance, all sellers might exert the same *absolute* effort, or effort is chosen in some relation to abilities.

[20] Again, the subscript i of the profit is dropped since in equilibrium the function is the same for all sellers. Also, for the moment, suppose that (3.23) is positive. We will give feasible parameter values in section 3.5.

[21] The computation is similar to that of (3.11).

contestant i. Then the games induced by F and S have Bayesian Nash equilibria, where all sellers participate and have the same expected profit in both mechanisms,

$$\pi^{F1}(a_i) = \pi^{S1}(a_i) = c \int_{\underline{a}}^{a_i} H_{(n:N-1)}(a)da > 0. \tag{3.26}$$

The n most able sellers innovate, and each is equally likely to win.

Remark 2 *Proposition 1 does not depend on the (optimal) choice of the fixed prize P, but we assumed $P \geq P_{min}(n)$. Increasing the prize does not make mechanism F more desirable for the sellers. Also, Proposition 1 does not imply that the procurer's expected profit is the same in both mechanisms.*

At stage 1, the procurer's expected equilibrium profit is

$$\begin{aligned} \Pi^S(n) &= E\left[Y^S_{(1:n)}\right] - \left(E\left[Y^S_{(1:n)}\right] - E\left[Y^S_{(2:n)}\right]\right) + nE\left[\beta^S\left(A_{(n+1:N)}\right)\right] \\ &= E\left[Y^S_{(2:n)}\right] + nE\left[\beta^S\left(A_{(n+1:N)}\right)\right] \\ &= \int_{\underline{y}}^{\overline{y}} ydG_{(2:n)}(y) + n\int_{\underline{a}}^{\overline{a}} \pi^{S2}(a)dH_{(n+1:N)}(a), \end{aligned} \tag{3.27}$$

where, in the first line, the first term is the expected best innovation, the second is the amount paid to the winner of the scoring auction, and the third term is the procurer's revenue from the entry auction.

3.5 Example: uniform distribution

Here, we provide model parameters for which our equilibria exist (see Proposition 1). With these parameters, the optimal choice of P is sufficient to make the procurer indifferent between both mechanisms, F and S. This result holds regardless of n, as long as the same n is used in both mechanisms. Assume uniformly distributed abilities and innovations with support $(0, 1)$. Thus, $(\underline{y}, \overline{y}) = (\underline{a}, \overline{a}) = (0, 1)$ and $H(x) = G(x) = x$ for $x \in (0, 1)$. We make a further assumption on the cost parameters c and γ:

$$0 < \gamma < \frac{c}{N-2} \leq c < \frac{1}{(N+2)^2} < 1. \tag{3.28}$$

This assumption makes the problem economically meaningful and ensures existence of the tournament equilibria (see Proposition 1) as well as the overall equilibria we are going to derive. These assumptions are chosen for convenience, i.e., they are sufficient but not necessary for our purposes.

3.5.1 Fixed-prize mechanism

By assumption (3.28), $\gamma < c/(N-2)$, and, therefore, (3.8) is positive for all $P \geq P_{\min}(n)$. The first term of (3.15) straightforwardly becomes

$$E\left[Y^F_{(1:n)}\right] = \int_0^1 y dy^{\frac{(n-1)P}{nc}} = \frac{(n-1)P}{(n-1)P+nc}. \tag{3.29}$$

Also (see the Appendix),

$$E[A_{(n+1:N)}] = \frac{N-n}{N+1}. \tag{3.30}$$

The procurer's decision problem becomes

$$\max_{n,P} \ \Pi^F(n,P) \quad \text{s.t. } n \in [2, N-1], \ P \geq P_{\min}(n) = \frac{cn^2}{n-1}, \tag{3.31}$$
$$\text{where } \ \Pi^F(n,P) = \frac{(n-1)P}{(n-1)P+nc} - \frac{n-1}{n}P + n\left(c\frac{N-n}{N+1} - \gamma\right).$$

Since $\Pi^F(n,P)$ is strictly concave in P, for any n it is optimal to set P equal to the interior maximizer, $P_{\text{int}}(n)$, unless $P_{\text{int}}(n) < P_{\min}(n)$ in which case $P_{\min}(n)$ is optimal.[22] By (3.31),

$$\partial_P \Pi^F(n,P) = \frac{1}{n} - 1 + \frac{nc(n-1)}{((n-1)P+nc)^2} = 0 \tag{3.32}$$

$$\iff \quad P = \frac{n\left(\sqrt{c}-c\right)}{n-1} =: P_{\text{int}}(n) > 0. \tag{3.33}$$

We have $P_{\text{int}}(n) > P_{\min}(n)$ iff $c < 1/(n+1)^2$, and this is satisfied since $c < 1/(N+2)^2$, see (3.28). Thus, the optimal prize (as a function of n) is

$$P^F = P_{\text{int}}(n) = \frac{n\left(\sqrt{c}-c\right)}{n-1}. \tag{3.34}$$

Applying (3.34) to (3.6) and (3.29), we get the equilibrium effort and the expected best innovation at the optimal prize, P^F,

$$e_i^F = \frac{1}{n}\left(\frac{1}{\sqrt{c}} - 1\right) - a_i, \tag{3.35}$$

$$k^F = \frac{1}{n}\left(\frac{1}{\sqrt{c}} - 1\right), \tag{3.36}$$

$$E\left[Y^F_{(1:n)}\right] = \int_0^1 y dy^{\frac{1}{\sqrt{c}}-1} = 1 - \sqrt{c}. \tag{3.37}$$

[22] $\partial_{P,P}\Pi^F(n,P) = -\frac{2nc(n-1)^2}{((n-1)P+nc)^3} < 0.$

Note that (3.37) is entirely determined by the optimal choice of P and does not depend on n. The profit at the optimal prize, $\Pi^F(n, P^F)$, is

$$\Pi^F(n, P^F) = \left(1 - \sqrt{c}\right)^2 + n\left(c\frac{N-n}{N+1} - \gamma\right). \quad (3.38)$$

Since $(N-n)/(N+1) > 0$, the above is positive if $(1-\sqrt{c})^2 > n\gamma$. This is satisfied by our assumptions (see the argument why (3.55) is positive).
The profit function is strictly concave in n and the interior maximizer is

$$n^F_{\text{int}} = \frac{1}{2}\left(N - (N+1)\frac{\gamma}{c}\right). \quad (3.39)$$

Recall the constraint $n \in [2, N-1]$. We have $n^F_{\text{int}} < N-1$, by (3.28), and

$$n^F_{\text{int}} \geq 2 \iff \frac{\gamma}{c} \leq \frac{N-4}{N+1}. \quad (3.40)$$

By the integer constraint on n, using n^F_{int} generates an upper bound on the maximum profit of mechanism F for $\gamma/c \leq (N-4)/(N+1)$, see (3.40),

$$\Pi^F(n^F_{\text{int}}, P^F) = \left(1-\sqrt{c}\right)^2 + \frac{1}{4}\left(N - (N+1)\frac{\gamma}{c}\right)\left(\frac{Nc}{N+1} - \gamma\right). \quad (3.41)$$

If $\gamma/c > (N-4)/(N+1)$, the corner solution $n = 2$ applies and the profit is

$$\Pi^F(2, P^F) = \left(1-\sqrt{c}\right)^2 - 2(\gamma - c) - \frac{6c}{N+1}. \quad (3.42)$$

3.5.2 Scoring-auction mechanism

Under the uniform distribution assumption, (3.19) becomes

$$\pi_i^{S2}(a_i, e_i) = \frac{1}{1 + \sum_{j\neq i} k_j} - \frac{1}{1 + k_i + \sum_{j\neq i} k_j} - ce_i - \gamma. \quad (3.43)$$

It is strictly concave in e_i and the first-order condition is

$$\partial_{e_i}\pi_i^{S2}(a_i, e_i) = 0 \iff k_i + \sum_{j\neq i} k_j = \frac{1}{\sqrt{c}} - 1. \quad (3.44)$$

Thus, in any equilibrium that satisfies all first-order conditions, all efforts and abilities sum up to $1/\sqrt{c} - 1$. In our symmetric equilibrium, each individual contestant's effort and ability sums up to the *same* constant, $k^S = e_i^S + a_i$, and i's equilibrium effort is

$$e_i^S = k^S - a_i = \frac{1}{n}\left(\frac{1}{\sqrt{c}} - 1\right) - a_i. \quad (3.45)$$

Remark 3 *Compare (3.45) with (3.35). If $P = P^F$, then, for any $n \in [2, N-1]$, the n most able innovators choose the same efforts and produce the same innovations at the same cost (as a function of type, respectively) in both mechanisms (under the uniform distribution assumption).*

Seller i's equilibrium tournament profit as a function of its ability, a_i, is

$$\pi^{S2}(a_i) := \pi_i^{S2}(a_i, e_i^S) = \frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})} + ca_i - \gamma. \tag{3.46}$$

It is strictly increasing in a_i and strictly decreasing in n. Effort e_i^S is positive by assumption $c < 1/(N+1)^2$, see (3.28), and (3.46) is positive.[23]
Recall (3.27). The expected second-best innovation is[24]

$$\begin{aligned} E\left[Y_{(2:n)}^S\right] &= \int_0^1 yG'_{(2:n)}(y)dy \\ &= \int_0^1 (n-1)\left(\frac{1}{\sqrt{c}}-1\right)\left(y^{\frac{n-1}{n}\left(\frac{1}{\sqrt{c}}-1\right)} - y^{\left(\frac{1}{\sqrt{c}}-1\right)}\right)dy \\ &= \frac{(n-1)\left(1-\sqrt{c}\right)^2}{n-1+\sqrt{c}}. \end{aligned} \tag{3.47}$$

The expected entry fee is, using (3.46),[25]

$$\begin{aligned} E\left[\pi^{S2}\left(A_{(n+1:N)}\right)\right] &= \frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})} + cE\left[A_{(n+1:N)}\right] - \gamma \\ &= \frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})} + c\frac{N-n}{N+1} - \gamma. \end{aligned} \tag{3.48}$$

Thus, (3.27) becomes

$$\begin{aligned} \Pi^S(n) &= \frac{(n-1)\left(1-\sqrt{c}\right)^2}{n-1+\sqrt{c}} + n\left(\frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})} + c\frac{N-n}{N+1} - \gamma\right) \\ &= \left(1-\sqrt{c}\right)^2 + n\left(c\frac{N-n}{N+1} - \gamma\right). \end{aligned} \tag{3.49}$$

[23]See the Appendix for a proof that (3.46) is positive.
[24]See the Appendix for a derivation of $G'_{(2:n)}(y)$.
[25]See the Appendix for a derivation of $E[A_{(n+1:N)}]$.

Since (3.49) is equal to (3.38), and recalling Remark 3, we state

Proposition 2 *Under the uniform distribution assumption and for a given a number $n \in [2, N-1]$ of contestants, model parameters exist such that there are Bayesian Nash equilibria where, in both mechanisms F and S, the n most able innovators choose the same positive efforts and produce the same innovations (as functions of type), and the procurer's profit is the same. Moreover, each seller's* expected *profit is the same in both mechanisms.*

Remark 4 *Note that, in these equilibria, the procurers profit is* always *the same in both mechanisms. Firt, the best innovation is the same, and, second, the sum of payments to and from the procurer is the same. The profit is only composed differently: In F, the entry fees are larger, and in S, the (endogenous) payment to the best innovator is lower (see section 3.7.1). In contrast, the innovators only* expect *the same profit, since equilibrium entry bids, as well as the reward for the best innovator, differ between both mechanisms.*

3.6 Welfare under uniform distribution

We look at the welfare properties of F and S under the uniform distribution assumption. We derive an efficiency benchmark and analyze both mechanisms from the point of view of a welfare-maximizing buyer.

3.6.1 First-best benchmark

Recall that the procurer announces the mechanism at stage 0, simultaneously with nature's draw of abilities. In particular, the number n of "active" innovators is fixed at that stage. We discuss this feature in section 3.7.5. Taking it as given, it is appropriate to define a welfare benchmark that also fixes n before abilities are realized. We compute the benchmark as follows. First, welfare is maximized over efforts, for given abilities and n. Second, the resulting expected maximum welfare is maximized over n. Note that fixing n has two consequences: the social fixed R&D cost of $n\gamma$ is incurred regardless of subsequent effort choices.[26]

Consider arbitrary realizations of abilities that w.l.o.g. are ordered $a_1 > \cdots > a_N$. Denote $\hat{a}_n := \sum_{i=1}^{n} a_i$ and $\hat{e}_n := \sum_{i=1}^{n} e_i$. For given n, expected welfare,

[26]Observe that it does not make sense to "fix" some n and later, after abilities become known, decide to make use of a lower number $n' < n$ of innovators in order to save the fixed cost γ if that is more profitable. Then we could as well say that we "fix" $n = N$ (all innovators) and later decide how many to employ. But then "fixing n" is meaningless.

$W(n, e_1, \dots, e_n)$, is the difference between the expected best innovation and the social effort cost,[27]

$$W(n, e_1, \dots, e_n) = E\left[Y_{(1:n)}\right] - c\sum_{i=1}^{n} e_i - n\gamma,$$
$$\text{where } \; E\left[Y_{(1:n)}\right] = \int_{\underline{y}}^{\overline{y}} y dG(y)^{\sum_{i=1}^{n} a_i + e_i}. \tag{3.50}$$

From (3.50), it is obvious that for given n only total effort, $\hat{e}_n$, and total ability, $\hat{a}_n$, matter, while individual efforts are inconsequential.[28] Thus, we can replace the choice variables $e_1, \dots, e_n$ by $\hat{e}_n$ in (3.50).
In the uniform distribution example, (3.50) simplifies to

$$W(n, \hat{e}_n) = \frac{\hat{a}_n + \hat{e}_n}{1 + \hat{a}_n + \hat{e}_n} - c\hat{e}_n - n\gamma. \tag{3.51}$$

Since $W(n, \hat{e}_n)$ is strictly concave in $\hat{e}_n$, the unique maximizer is

$$\hat{e}_n^* = \frac{1}{\sqrt{c}} - 1 - \hat{a}_n. \tag{3.52}$$

It is also positive: By $a_i < 1 \Rightarrow \hat{a}_n < n$,

$$\hat{e}_n^* = \frac{1}{\sqrt{c}} - 1 - \hat{a}_n > \frac{1}{\sqrt{c}} - 1 - n > 0 \iff c < \frac{1}{(n+1)^2}. \tag{3.53}$$

The latter is satisfied by assumption $c < 1/(N+2)^2$, see (3.28). The fact that $\hat{e}_n$ is positive implies that it is optimal to spend additional effort regardless of abilities $a_1, \dots, a_n$.
By (3.52), the expected value of the innovation is completely determined, i.e., the innovation is drawn from $G^{\hat{e}_n^* + \hat{a}_n} = G^{\frac{1}{\sqrt{c}} - 1}$, regardless of n:

$$E\left[Y_{(1:n)}\right] = \int_{\underline{y}}^{\overline{y}} y dG^{\frac{1}{\sqrt{c}} - 1}(y) = 1 - \sqrt{c}. \tag{3.54}$$

Inserting (3.52) into (3.51), we get[29]

$$W(n, \hat{e}_n^*) = \left(1 - \sqrt{c}\right)^2 + c\hat{a}_n - n\gamma. \tag{3.55}$$

[27]Note that by the assumed ordering of abilities, welfare $W(n, e_1, \dots, e_n)$ is produced by the n most able innovators, which is efficient.

[28]This is due to the constant and equal marginal cost of effort across sellers.

[29]By assumption (3.28), (3.55) is positive: Ignore the positive term $c\hat{a}_n$. By (3.28), $c < 1/(N+2)^2$. This can be written as $c < (1-\sqrt{c})^2/(N+1)^2$. Since $\gamma < c$ (by (3.28)), $\gamma < (1-\sqrt{c})^2/(N+1)^2$. And since $n \leq N-1$ and $N \geq 3$, this implies $\gamma < (1-\sqrt{c})^2/n$ which proves the assertion.

It remains to determine the integer n, i.e., the maximization problem is

$$\max_{n\in[1,N]} E[W(n,\hat{e}_n^*)], \tag{3.56}$$

where $W(n,\hat{e}_n^*)$ is given by (3.55). We have[30]

$$\begin{aligned} E[W(n,\hat{e}_n^*)] &= E\left[\left(1-\sqrt{c}\right)^2 + c\sum_{i=1}^{n} A_{(i:N)} - n\gamma\right] \\ &= \left(1-\sqrt{c}\right)^2 + c\frac{n(2N-n+1)}{2(N+1)} - n\gamma, \end{aligned} \tag{3.57}$$

which is quadratic and strictly concave in n. The global maximizer is

$$n_{\text{int}}^* = \frac{1}{2} + N - (N+1)\frac{\gamma}{c}. \tag{3.58}$$

Inserting n_{int}^* into (3.57) gives an upper bound on maximum welfare:

$$W(n_{\text{int}}^*,\hat{e}_n^*) = \left(1-\sqrt{c}\right)^2 + \frac{(c(2N+1)-2\gamma(N+1))^2}{8c(N+1)}. \tag{3.59}$$

Since (3.57) is strictly concave, quadratic in n, and positive for $n \in [1,N]$, the welfare maximum is found by rounding n_{int}^* to the nearest integer (of the range $1,\ldots,N$) and inserting into (3.57).
In the following sections, we consider a welfare-maximizing buyer who employs mechanisms F and S under the same informational and incentive constraints as the profit-maximizing buyer in sections 3.3 to 3.5.

3.6.2 Fixed-prize mechanism

Consider a welfare-maximizing buyer who employs mechanism F. We have shown that equilibrium effort is given by (3.6) as long as $P \geq cn^2/(n-1)$. Expected equilibrium welfare is the expected difference of the best innovation, $Y_{(1:n)}^F$, and total R&D cost given that the n most able sellers provide effort as in (3.6). Also recall (3.29).

$$\begin{aligned} W^F(n,P) &= E\left[Y_{(1:n)}^F - \sum_{i=1}^{n}\left(c\left(\frac{(n-1)P}{n^2c} - A_{(i:N)}\right) + \gamma\right)\right] \\ &= \frac{(n-1)P}{(n-1)P+nc} - \frac{n-1}{n}P + c\sum_{i=1}^{n} E\left[A_{(i:N)}\right] - n\gamma, \end{aligned} \tag{3.60}$$

[30] See the Appendix for a derivation of $E[\sum_{i=1}^{n} A_{(i:N)}]$. Also note that (3.57) is positive since (3.55) is positive.

where, again, $\sum_{i=1}^{n} E\left[A_{(i:N)}\right] = n(2N-n+1)/2(N+1)$, see (3.87). Comparing (3.31) and (3.60) as functions of P, both differ only in a constant and thus the same optimal P, (3.34), results. Inserting (3.34) into (3.60), we get

$$W^F(n) = \left(1-\sqrt{c}\right)^2 + c\frac{n(2N-n+1)}{2(N+1)} - n\gamma. \tag{3.61}$$

This is equal to (3.57). Thus, the procurer achieves the welfare benchmark, as long as the welfare-optimal n (obtained by rounding (3.58) to the nearest integer) is in the range $2, \ldots, N-1$. The benchmark permits $n = 1$ and $n = N$ which is not feasible in F. Thus, F can "almost always" implement first best.

Remark 5 *Compare* (3.39) *with* (3.58). *The welfare-maximizing buyer sets a larger n than the profit-maximizing buyer (ignoring the integer constraint). The buyer pays informational rents to the sellers: their expected profits are positive and increasing in ability. In addition, R&D cost is reimbursed in expectation. For the welfare-maximizing buyer, those informational rents are not counted as a "loss" and thus she employs more innovators than the profit-maximizing buyer.*

3.6.3 Scoring-auction mechanism

Recall two results of the previous section: If the buyer is a welfare maximizer and employs mechanism F, then equilibrium efforts and the optimal prize are the same as with a profit-maximizing buyer. This implies that, for both types of buyer and for the same n, sellers submit the same bids in the entry auction. Thus, F and S, again, induce the same expected procurer's profit, as we confirm below.
Consider the welfare-maximizing buyer and recall the equilibrium effort for mechanism S, (3.45). Expected welfare $W^S(n)$ in S is the expected difference of the best innovation, $Y^S_{(1:n)}$, and the corresponding social R&D cost given that in equilibrium the n most able sellers provide effort as in (3.45). Recall that $E[Y^S_{(1:n)}] = 1 - \sqrt{c}$ is independent of n.

$$W^S(n) = E\left[Y^S_{(1:n)} - \sum_{i=1}^{n}\left(c\left(\frac{1}{n}\left(\frac{1}{\sqrt{c}}-1\right) - A_{(i:N)}\right) + \gamma\right)\right] \tag{3.62}$$

$$= \left(1-\sqrt{c}\right)^2 + cE\left[\sum_{i=1}^{n} A_{(i:N)}\right] - n\gamma, \tag{3.63}$$

where, again, $E\left[\sum_{i=1}^{n} A_{(i:N)}\right] = n(2N-n+1)/2(N+1)$, see (3.87). Thus, $W^S(n)$ is equal to (3.57) and we conclude that mechanism S implements the welfare benchmark as long as the welfare-maximizing n (obtained by rounding (3.58) to the nearest integer in $[1, N]$) is in the range $[2, N-1]$.[31]

3.7 Discussion

3.7.1 Comparison of stage 3

By Proposition 2, given the optimal prize P^F and the same number of contestants, the expected innovations are the same in F and S. Is the procurer's profit at stage 3 the same, too, i.e., does she procure the innovations at the same price (ignoring the entry auction)? In fact, the buyer's expected profit at stage 3 of mechanism S exceeds that of F, $\Pi^{S3}(n) > \Pi^{F3}(n, P^F)$, for any feasible n (ignoring the integer constraint), where, collecting previous results,

$$\begin{aligned}\Pi^{F3}(n, P^F) &= E\left[Y^F_{(1:n)}\right] - P^F \\ &= 1 - \sqrt{c} - \frac{n(\sqrt{c} - c)}{n-1}, \qquad (3.64)\\ \Pi^{S3}(n) &= E\left[Y^S_{(1:n)}\right] - \left(E\left[Y^S_{(1:n)}\right] - E\left[Y^S_{(2:n)}\right]\right) \\ &= E\left[Y^S_{(2:n)}\right] \\ &= \frac{(n-1)(1-\sqrt{c})^2}{n-1+\sqrt{c}}. \qquad (3.65)\end{aligned}$$

Thus, sellers expect a larger reward in mechanism F. But that is anticipated and results in larger bids in the entry auction (as can be easily checked). Put differently, mechanism S is more competitive since sellers compete in price, while in the fixed-prize mechanism the best innovator always wins. But, as a result, in mechanism S sellers are less aggressive at the entry stage since there is less to gain. This endogenous adjustment of entry fees drives our main result.

3.7.2 Social choice

We briefly discuss our results in the spirit of *social choice*.[32] In our setting, a social choice consists of efforts for all sellers and transfers for all players, including the procurer. In addition it must be decided which (if any) of the resulting innovations is employed by the procurer.[33] Of course, it is

[31]Note, again, that the welfare benchmark permits $n = 1$ and $n = N$ which is not feasible in S.

[32]See, e.g., Mas-Colell et al. (1995, ch. 23).

[33]Recall the assumption that the innovation is worthless for a seller.

optimal to use the best available innovation. By Propositions 1 and 2, both mechanisms implement the same social choice in expectation.[34] In particular, under the uniform distribution assumption, the allocation of efforts among the N innovators (and the resulting innovations) and the procurer's profit are *exactly* the same for both mechanisms. Thus, the result goes beyond "revenue equivalence" (which is a statement about *expected* payoffs). The mechanisms differ only in transfers to the innovators which are, however, the same in expectation. Moreover, seller's expected transfers are exclusively functions of a seller's own type and that function is the same for all sellers. As we have shown, the difference is that in F the bids in the entry auction, paid by the n winners, are larger while the expected reward for the best innovator is smaller. Payments to the procurer are made by the same n (most able) innovators and the reward is paid to the same (best) innovator.

3.7.3 A direct procurement mechanism

Given the equivalence results derived so far (Propositions 1 and 2), we want to shed some light on the common structure of both mechanisms, F and S. For this purpose, we argue that both mechanisms implement the outcome of a particular equilibrium of a direct incentive-compatible procurement mechanism. We briefly sketch what that direct mechanism looks like.[35]

In the given setting, incentive compatibility has two dimensions: truthful reporting of types and choice of the prescribed effort. In order to induce effort, payments must depend on innovation quality (i.e., payments that only depend on type do not work).

Consider a direct mechanism that maps types into efforts and sets transfers for all sellers. The mechanism induces a sequential game. At stage 0, nature draws abilities and the mechanism is announced, including the number n of contestants. At stage 1, sellers report their types, $\hat{a}_i$. Reports are published and become common knowledge.[36] The procurer prescribes positive efforts for the n (reportedly) most able contestants and zero effort for the other sellers (they also get zero payments). At stage 2, contestants choose effort and draw innovations. At stage 3, they present their innovations, payments are made and the game ends. (The procurer employs the best innovation.)

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[34] We cannot state a social choice *function* for our setting since a part of the transfers induced by the mechanisms (the reward for the winner) depends on random events. Thus there is no mapping from types into transfers. However, we can say that the mechanisms assign the same *expected* transfer to each type.

[35] For the sake of this discussion assume that the equilibria exist.

[36] Recall that in our symmetric equilibria, it does not matter whether type reports are published since the equilibrium strategies do not make use of type information.

In particular, each seller pays a type-dependent fee and there is an additional reward $p(y)$ for the best innovation. The reward is a positive function of all innovations, $p(y) := p(y_1, \ldots, y_n)$.[37] Sellers who are prescribed zero effort receive no payments and do not pay anything.
At stage 3, contestant i's expected profit is (denote the vector of rivals' innovations by y_{-i} and denote $k_j := a_j + e_j$)[38]

$$\begin{aligned}\pi_i^3(y_i) &= \Pr\left\{y_i > Y_{(1:n-1)}\right\} E\left[p(Y)\,\middle|\, y_i > Y_{(1:n-1)}\right] \\ &= \int_{\underline{y}}^{y_i} \cdots \int_{\underline{y}}^{y_i} p(y_i, y_{-i}) dG^{k_{(1)}}(y_{(1)}) \cdots dG^{k_{(n-1)}}(y_{(n-1)}).\end{aligned} \tag{3.66}$$

At stage 2, contestant i's expected profit is

$$\begin{aligned}\pi_i^2(a_i, e_i) &= E\left[\pi_i^3(Y_i)\right] - ce_i - \gamma \\ &= \int_{\underline{y}}^{\overline{y}} \pi_i^3(y_i) dG^{k_i}(y_i) - ce_i - \gamma.\end{aligned} \tag{3.67}$$

Consider (3.67). The expected reward, $E[\pi^3(Y_i)]$, is entirely determined by each seller's k_i. Since $\frac{dk_i}{de_i} = 1$, the first derivative of $E[\pi^3(Y_i)]$ w.r.t. e_i is again entirely a function of all k_i. The derivative of the remaining terms of (3.67) w.r.t. e_i is equal to c, a constant. Thus, if there is a pure-strategy equilibrium that satisfies each player's first-order condition (derived from (3.67)), then i's optimal choice is some k_i as a function of the rivals' $k_{(1)}, \ldots, k_{(n-1)}$ only. Again, we restrict attention to the symmetric equilibrium candidate where $k_i = k_j =: k^*$ for all contestants i, j. Contestant i's probability of winning the reward is[39]

$$\begin{aligned}&\Pr\left\{Y_i > Y_{(1:n-1)}\right\} := \int_{\underline{y}}^{\overline{y}} G^{\sum_{j\neq i} k_j}(y) dG^{k_i}(y) = \frac{k_i}{k_i + \sum_{j\neq i} k_j}, \\ &k_i = k_j = k^* \;\Rightarrow\; \Pr\left\{Y_i > Y_{(1:n-1)}\right\} = \frac{1}{n}.\end{aligned} \tag{3.68}$$

Thus, in equilibrium, each contestant is equally likely to win. After inserting the candidate, (3.67) becomes a pure private value, characterized by the constant k^* and one's own ability a_i. Thus, the expected tournament profit function is the same for all contestants, denoted by $\pi^2(a_i)$.
Revelation of types is achieved by setting an appropriate payment for each of the n contestants that depends on the ability of the (reportedly) most able player who is *not* selected for the R&D stage.[40] Thus, a contestant pays the

[37] The reward may be constant in one, several or all arguments. Thus, a fixed prize, $p(y) := P$, is included.

[38] It can be straightforwardly verified that (3.1) and (3.17) can be computed from (3.66). Also note that the subscripts of $k_{(j)}$, $y_{(j)}$ in (3.66) refer to i's $n-1$ rivals.

[39] Again, this resembles the Tullock contest success function.

[40] In auction settings, that player is sometimes called the marginal bidder.

expected profit of the player that is crowded out by him.
Suppose the procurer selects the n (reportedly) most able players as contestants for the prize $p(y)$ (with random tie breaking) and each of them has to pay a fee of $f(n) := \pi^2\left(a_{(n+1:N)}\right)$, while the other $N - n$ players do not pay anything. Then truthful reporting of types is a weakly dominant strategy: it ensures a non-negative profit since conditional on being selected as a contestant, the fee $f(n)$ never exceeds the expected continuation profit. If truthful reporting makes player i a winner, then reporting a higher ability does not change anything. If a lower report changes anything, then player i becomes a loser but prefers being a winner. A similar argument applies if truthful reporting makes i a loser.
Thus, the game has a Bayesian Nash equilibrium where all N sellers participate, reveal their types, and the n most able sellers innovate as requested. At stage 1, player i's expected profit is

$$\begin{aligned}\pi^1(a_i) &= \Pr\left\{a_i > A_{(n:N-1)}\right\}\left(\pi^2(a_i) - E\left[f(n)|\, a_i > A_{(n:N-1)}\right]\right)\\ &= H_{(n:N-1)}(a_i)\pi^2(a_i) - \int_{\underline{a}}^{a_i}\pi^2(a)dH_{(n:N-1)}(a)\\ &= \int_{\underline{a}}^{a_i}\partial_a\pi^2(a)H_{(n:N-1)}(a)da\\ &= c\int_{\underline{a}}^{a_i}H_{(n:N-1)}(a)da > 0,\end{aligned} \tag{3.69}$$

equal to (3.11) and (3.25). In order to derive the last step, note that in equilibrium, $\pi^2(a_i)$ (obtained by inserting k^* into (3.67)) contains the argument a_i only in the middle term, ce_i, where $e_i = k^* - a_i$. We know that a_i is eliminated from the first term (see the discussion above), $E[\pi^3(Y_i)]$, since a_i enters only as part of the sum $a_i + e_i$ which in equilibrium is replaced by the constant k^*. Since (3.69) is positive, it is confirmed that everybody participates.

Remark 6 *The general reward, $p(y)$, includes the fixed prize and the scoring auction as special cases. The above analysis shows that, as long as the best innovator gets a reward, its exact form is inconsequential: The fees, $f(n)$, are functions of contestants' expected continuation profit. The reward, $p(y)$, is part of that expected profit. A more generous reward is compensated by larger fees.*

We conclude that our two mechanisms, in expectation, can implement the above equilibrium outcome of the direct incentive-compatible procurement mechanism.

3.7.4 Signaling

Our procurement problem, in principle, exhibits a signaling issue. Players may want to signal their ability at the entry stage in order to influence their potential tournament rivals' effort choices (or induce them to drop out of the tournament). In the present paper, we focused on symmetric equilibria where strategies do not require predicting rivals' types. Although these equilibria are appealing, we do not want to downplay the importance of signaling in real procurement settings.

However, the relevance of an equilibrium without signaling can be justified: When a game has multiple equilibria, one has to decide which, if any, equilibrium is the "solution" of the game. In complex decision problems, players may have to revert to simple heuristic strategies. This may be due to time, cognitive or cost constraints, etc. In this sense, simple strategies, like the ones we derived (based on one's own information), might form the appropriate solution.

The literature sometimes assumes that the private information becomes common knowledge before the tournament stage (e.g., Fullerton and McAfee, 1999). This assumption might be justified, e.g., in industries where players know each other such that they are sufficiently well informed as soon as the identity of their rivals is revealed.

In section 3.4 we found that mechanism S has many equilibria at the tournament stage, where signaling is indeed an issue. For that reason, mechanism F might be preferred. Also note that we employed convenient assumptions on the parameters c and γ as well as the fixed prize P in order to avoid that contestants drop out of the tournament after inferring their rivals' abilities from the entry auction bids.

3.7.5 Why not choose n after the entry auction?

In real procurement settings, we often observe that procurers announce in advance how many sellers will be shortlisted (i.e. allowed to compete) before eliciting their bids. For sellers, this is important since they need to decide whether it is worthwhile to participate and, usually, participation is costly. Also, fixing the number of contestants ex ante makes the game considerably simpler to play. Government procurers are often restricted to rules of procurement, like specifying the (minimum) number of offers to elicit. This is meant to increase transparency and prevent corruption by government agents to whom the task of procurement is delegated. It is also a typical feature of multi-unit auctions that the number of objects to be sold is announced before the auction. In our case, an object means entry to the tournament.

Taking the above into account, our procurer chooses the number of contestants, n, before the entry auction, i.e., before being able to infer the seller's abilities.[41] This is also reflected in our welfare benchmark.

However, from a theoretical perspective, one might ask if this design is optimal since it prevents the procurer from using information collected in the auction to optimally adjust the number of (costly) contestants. In order to justify this design, we consider two modifications of the mechanisms.

Suppose mechanisms F and S are modified such that the procurer announces n after the entry auction (and, in F, the prize P^F; alternatively, the buyer might announce a prize function $P^F(n)$ at stage 0; then the uncertainty is only about n). The weaknesses of this design are that sellers cannot express their willingness to pay for different n (which might lead to cautious bidding) and that predicting the choice of n is complicated. Nevertheless, there might be an efficient equilibrium.

Now consider the more appealing modification of F and S where sellers bid contingent on the subsequent choice of n (and in mechanism F, the buyer announces a prize function $P^F(n)$ at stage 0). There, seller i submits bids $\beta_i(n)$ for each $n \in [2, N-1]$. The buyer selects the most profitable n and collects the entry fees of an entry auction with the bids $\beta_1(n), \ldots, \beta_N(n)$. This is equivalent to saying that the sellers take part in $N-2$ different auctions and then the buyer selects one of them (and the corresponding n and $P^F(n)$) to be payoff-relevant.

A complication of that design is that the sellers need beliefs about how the buyer chooses n if signals are inconsistent (e.g. suppose the observed bid functions cross, and bidder i submits the highest bid for $n = 2$ but the second-highest bid for $n = 3$).

Although the dominant strategies we derived for the uniform-price entry auction might still be an intuitive equilibrium candidate (i.e., bidding one's expected tournament profit conditional on entry for each n), there is a potential incentive to deviate: Suppose everybody bids as in our basic games (for each n). Suppose the buyer then chooses some $n = \tilde{n}$. Then, say, seller j's bid, $\beta_j(\tilde{n})$ is the $\tilde{n}+1$st highest bid for $\tilde{n}$ which implies that j sets the entry fee while not being selected as a contestant. If j deviates by reducing his bid $\beta_j(\tilde{n})$ then the entry fee for $n = \tilde{n}$ decreases and $\tilde{n}$ becomes less attractive for the buyer.[42] If this induces the buyer to choose a larger n then j enters the tournament with a positive expected continuation profit.

[41] The same is done in Fullerton and McAfee (1999).

[42] The feasible range for that "deviation" depends on the next-lowest bid. If j undercuts the next-lowest rival then that rival sets the entry fee and j's bid is irrelevant.

3.7.6 Entry fees

In our procurement setting with heterogeneous types, it is vital to select the most able innovators. Similar to Fullerton and McAfee (1999), we adopted an entry auction. The alternative, a fixed entry fee, would be an additional strategic variable and thus a source of errors. The auction provides an endogenous entry fee that is "adjusted" to the given realizations of abilities.
In the following we interpret the entry auction in a way that makes our two mechanisms look more similar to real procurement settings. Recall that we analyzed the uniform-price auction format only because of its simplicity. In the equilibria we discussed, a discriminatory auction, where winners pay their bids, is revenue-equivalent.
Consider the discriminatory auction and suppose the auction revenue does not accrue to the procurer but, instead, the bids are sunk cost of writing a proposal, or building a prototype. Then one might expect that the most able sellers have the best proposals or the most promising prototypes and can thus be identified. They bear this entry cost and then compete for a prize (or a contract). The fact that losers do not pay anything in that auction format can be interpreted as a reimbursement of the losers' cost.[43]
For this setting, the results of Proposition 1 apply, i.e. sellers expect the same profit in both mechanisms.[44] Moreover, the sellers would still choose the efforts and draw the innovations as we showed (for given P and n).
The result that the procurer is indifferent between both mechanisms, however, requires that she collects the auction revenue.

3.7.7 Bilateral contracts

Competition at the tournament stage requires $n \geq 2$. Mathematically, the analysis of the tournament mechanisms F and S, however, extends to lower n, say, $n = 1$, as well, with similar results, as we show below. Economically, we interpret the case $n = 1$ as a bilateral contract.
In this section we assume that the innovation is verifiable and, thus, innovation quality is contractible. We briefly discuss the use of bilateral contracts (mechanism B) under our uniform distribution assumption in order to compare bilateral contracts with our two tournament mechanisms, F and S.
We also use the uniform-price entry auction, that, for $n = 1$, collapses to a Vickrey auction. The auction winner, say, i, pays the second-highest bid,

[43] In some procurement settings, bid preparation is very costly and bids can be attracted by reimbursing (part of) that cost. As an alternative, one might also look at the all-pay auction format where there is no reimbursement of losers.

[44] For the sellers it does not matter if the procurer collects revenue or not.

draws an innovation y_i and, in return, receives the prize $p(y_i)$.
Consider the prize function $p(y) = y$ where the procurer hands over the entire profit of the innovation to the innovator. Thus, the innovator has the monopoly rights on the innovation profit (residual claimant); in contrast to the tournaments, the auction winner gets the prize for sure. These are the optimal effort incentives, since the innovator has all relevant information (about ability), bears all cost, and receives the entire profit of the innovation.[45] The procurer's profit is equal to the single entry fee that is paid by the auction winner.
Note that with prize function $p(y) = y$ the innovator's reward is equal to the reward he would get in the equilibrium of the second-score auction when the best rival has a zero innovation (or there are no rivals): it is equal to the difference between the best and second-best (i.e., zero) innovation submitted to the procurer. That in part explains why the results for mechanism B resemble those of the tournament mechanisms, F and S.
Since the analysis is similar to that of the other mechanisms, we abbreviate the presentation. At stage 3, the innovator receives profit $\pi^{B3}(y_i) = y_i$. At stage 2, i's expected profit is

$$\pi^{B2}(a_i, e_i) = E[Y] - ce_i - \gamma = \frac{k_i}{1 + k_i} - ce_i - \gamma. \tag{3.70}$$

It is strictly concave and the interior maximizer is $e_i^B = 1/\sqrt{c} - 1 - a_i$. Again, $e_i^B > 0$ since $c < 1/(N+2)^2$, see (3.28), and the resulting expected innovation is the same as in mechanisms F and S. Reinserting into (3.70) we get the equilibrium profit

$$\pi^{B2}(a_i) := \pi^{B2}(a_i, e_i^B) = \left(1 - \sqrt{c}\right)^2 + ca_i - \gamma. \tag{3.71}$$

The equilibrium bid is $\beta^B(a_i) = \pi^{B2}(a_i)$. At stage 1, bidder i expects profit

$$\begin{aligned}\pi^{B1}(a_i) &= H_{(1:N-1)}(a_i)\pi^{B2}(a_i) - \int_{\underline{a}}^{a_i} \pi^{B2}(a)dH_{(1:N-1)}(a) \\ &= c\int_{\underline{a}}^{a_i} H_{(1:N-1)}(a)da. \end{aligned} \tag{3.72}$$

Comparing (3.72) with (3.11) and (3.25), these expressions are equivalent with the exception that $n = 1$ in (3.72), while $n \geq 2$ in (3.11) and (3.25). Thus, (3.72) is smaller, i.e., a seller's expected profit is lower in the bilateral contract mechanism, B.

[45] Alternatively, one can show that $p(y) = y$ is optimal within a class of increasing prize functions $p(y)$ that contain it as an element, e.g., it is easily shown that for $p(y) = y^s/s$ with $s > 0$ it is optimal to set $s = 1$.

The procurer collects a single entry fee (equal to the equilibrium bid of the seller with second-highest ability), gets the innovation y_i, and pays out the prize $p(y_i) = y_i$ to the innovator. Expected profit is

$$\Pi^B = E[Y] - E[Y] + E\left[\beta\left(A_{(2:N)}\right)\right] = E\left[\beta\left(A_{(2:N)}\right)\right], \qquad (3.73)$$

where

$$E\left[\beta\left(A_{(2:N)}\right)\right] = E\left[\pi^{B2}\left(A_{(2:N)}\right)\right] = (1-\sqrt{c})^2 + cE\left[A_{(2:N)}\right] - \gamma \qquad (3.74)$$

and (see the Appendix) $E[A_{(2:N)}] = (N-1)/(N+1)$. Therefore,[46]

$$\Pi^B = (1-\sqrt{c})^2 + c\frac{N-1}{N+1} - \gamma. \qquad (3.75)$$

Lemma 11 *If innovations are verifiable then (under the unform distribution assumption) the bilateral contract, B, is superior to the tournament mechanisms, F and S, if the R&D cost exhibits a sufficiently large "economies of scale" effect. In particular,*

$$B \succeq \{F, S\} \iff \frac{\gamma}{c} \geq \frac{N-3}{N+1}. \qquad (3.76)$$

Proof 11 *Recall the equivalence results for F and S and denote both by T for "tournament". Consider* $n^T := n^F = n^S$*. For* $n^T_{int} < 2$ *(which is equivalent to* $\gamma/c > (N-4)/(N+1)$*), we get the corner solution* $n^T = 2$*. Then the maximum tournament profit is, see* (3.42)*,*

$$\Pi^T(2) = \left(1-\sqrt{c}\right)^2 - 2(\gamma - c) - \frac{6c}{N+1}. \qquad (3.77)$$

Then,

$$\Pi^B < \Pi^T(2) \iff \frac{\gamma}{c} < \frac{N-3}{N+1}. \qquad (3.78)$$

Thus, T is superior in the range $(N-4)/(N+1) < \gamma/c < (N-3)/(N+1)$ *while the bilateral contract is superior for* $\gamma/c > (N-3)/(N+1)$*. Now consider* $n^T_{int} > 2$ *(which is equivalent to* $\gamma/c < (N-4)/(N+1)$*), taking into account that n is an integer. We show that* $\Pi^T(n^T_{int}+1) > \Pi^B$*. Together with the facts that 1) the tournament profit* $\Pi^T(n)$ *is strictly concave in n, and 2)* $n^T_{int} < N-1$*, this implies that there always exists an integer number of*

[46] Π^B is positive: recall the argument why (3.55) is positive and note $(N-1)/(N+1) > 0$.

contestants in the range $[n_{int}^T, n_{int}^T + 1]$ *that does not exceed* $N-1$*, where* T *is superior.*

$$\Pi^T\left(n_{int}^T + 1\right) = \left(1-\sqrt{c}\right)^2 + \frac{1}{4}\left(N - (N+1)\frac{\gamma}{c} + 2\right)\left(c\frac{N-2}{N+1} - \gamma\right), \tag{3.79}$$

$$\Pi^T\left(n_{int}^T + 1\right) - \Pi^B = \frac{((N-4)c - (N+1)\gamma)\left(Nc - (N+1)\gamma\right)}{4c(N+1)}. \tag{3.80}$$

Consider the RHS of (3.80)*. The denominator is a product where both factors are positive by the assumption* $n_{int}^T > 2$*.*

One might think that whenever B is superior to T, this is due to the prize function $p(y) = y$. However, it can be shown that the bilateral contract ($n = 1$) strictly profit-dominates a tournament ($n \geq 2$) if the winner is rewarded using the same prize function $p(y) = y$ is used in both.[47]
Now consider the welfare-maximizing buyer. It is straightforward to verify that B implements the welfare benchmark if the welfare-maximizer is $n^* = 1$, i.e., if $n_{\text{int}}^* \leq 3/2$, see (3.57) and (3.58). We have $E[Y] = 1 - \sqrt{c}$.

$$W^B = E\left[Y - c\left(\frac{1}{\sqrt{c}} - 1 - A_{(1:N)}\right) - \gamma\right] \tag{3.81}$$

$$= \left(1-\sqrt{c}\right)^2 + c\frac{N}{N+1} - \gamma. \tag{3.82}$$

Thus, whenever $n^* = 1$ is the welfare-maximizer (in the sense of our benchmark), mechanism B is optimal.[48] Note that since the interior optimal n, n_{int}^*, is rounded to the nearest integer, B is optimal if $n_{\text{int}}^* \leq 3/2$ (since then $n^* = 1$), while F and S are optimal if $3/2 \leq n_{\text{int}}^* \leq N - 1/2$ (since then $2 \leq n^* \leq N-1$).

Remark 7 *Comparing the above results with those of section 3.6, we note that the welfare-maximizing buyer finds an optimal mechanism among* F*,* S*, and* B*, unless it is optimal to let all sellers innovate* ($n^* = N$)*.*[49] *In particular,* F *and* S *are optimal if* $n^* \in [2, N-1]$ *while* B *is optimal if* $n^* = 1$*.*

[47] The proof is omitted since the computation is tedious and does not provide much insight. A corresponding Mathematica file is available from the author.
[48] In order to see this, insert $n = 1$ into (3.57).
[49] The bidding equilibrium in the entry auction breaks down if all N inventors are admitted to the tournament.

3.8 Conclusion

We considered procurement of an innovation when innovations are random, non-verifiable, and depend on innovators' privately known ability and unobservable effort.
We looked at entry auctions that select contestants for R&D tournaments where an innovation is bought either in return for a fixed prize or a payment determined in a scoring auction. The purpose of the entry auction is to select the most able innovators and restrict entry.
We characterized particular Bayesian Nash equilibria where the mechanisms differ only in the *exact* transfers (entry fees and rewards) to the innovators, while they are the same in expectation. In these equilibria, the allocation of efforts and thus innovations (as functions of type) as well as the procurer's profit are the same under both mechanisms. These equilibria do not involve a signaling issue since equilibrium efforts only depend on the respective player's own type. At the tournament stage, that symmetric equilibrium is the unique equilibrium with positive efforts of all contestants in the fixed-prize mechanism while the scoring-auction mechanism has multiple symmetric equilibria. Thus, one would favor the fixed-prize design.
The results imply that, as long as the best innovation receives a reward, the particular form of the reward does not matter. We also saw that there was a degree of freedom in choosing an auction format for the entry auction.
As an interpretation, we argued that both mechanisms can be seen as implementations of an equilibrium (outcome) of a larger class of mechanisms. This, again, implies that some features of the mechanisms are arbitrary, in particular the method of selecting the contestants and rewarding the winner.
The result that sellers have the same expected profit in both mechanisms neither relies on the uniform distribution assumption nor does it depend on the optimal choice of the fixed prize. The latter might be somewhat surprising. In the entry auction sellers bid according to the expected profits that arise from the subsequent game. The bidding equilibrium is a result of the sellers' competition with each other. A more generous reward leads to more aggressive bidding. This might explain why sellers' expected profits do not depend on the choice of the fixed prize: different profit opportunities are "competed away" between the sellers.
It seems intuitive that a more competitive procurement stage, i.e. a scoring auction instead of a fixed prize, reduces the willingness to pay for participation. In that sense, a competitive stage where the size of entry fees is determined endogenously makes the choice of the procurement method more arbitrary. Using an alternative interpretation of the entry auction, we might say that lower expected profits at the tournament stage lead to less invest-

ment (e.g., into prototypes) at the entry stage. We discussed the practical relevance of the entry auction stage and argued that one does not need to take the bids literally, i.e., as payments to the procurer. Some of the results also hold if one interprets the bids as sunk cost of bid preparation or prototypes. In particular, sellers still expect the same profit in both mechanisms.
Under the uniform distribution assumption, these equilibria make the mechanisms optimal for a welfare-maximizing procurer. Effort and the expected innovation are the same for welfare- and profit-maximizing procurers. This is another reason making these particular equilibria interesting.

3.9 Appendix

3.9.1 Order statistics for uniform distribution on $(0, 1)$

1. Demonstrate $E[A_{(i:N)}] = (N+1-i)/(N+1)$. We have

$$E[A_{(i:N)}] = \int_{\underline{y}}^{\overline{y}} a dH_{(i:N)}(a) = 1 - \int_0^1 H_{(i:N)}(a)da. \tag{3.83}$$

By a standard result on order statistics,

$$H_{(i:N)}(a) = \sum_{j=0}^{i-1} \binom{N}{j} a^{N-j}(1-a)^j. \tag{3.84}$$

Thus (using repeated integration by parts for every term),

$$\begin{aligned}
&\int_0^1 H_{(i:N)}(a)da \\
=&\int_0^1 a^N da + \int_0^1 N a^{N-1}(1-a)da \\
&\quad + \cdots + \int_0^1 \frac{N!}{(i-1)(N-i+1)!} a^{N-i+1}(1-a)^{i-1}da \\
=&\frac{1}{N+1} + \frac{1}{N+1} + \cdots + \frac{1}{N+1} \\
=&\frac{i}{N+1}.
\end{aligned} \tag{3.85}$$

Thus,

$$E[A_{(i:N)}] = 1 - \frac{i}{N+1} = \frac{N+1-i}{N+1}. \tag{3.86}$$

2. Derive $E\left[\sum_{i=1}^{n} A_{(i:N)}\right]$.

$$
\begin{aligned}
E\left[\sum_{i=1}^{n} A_{(i:N)}\right] &= \sum_{i=1}^{n} E\left[A_{(i:N)}\right] \\
&= \frac{1}{N+1}\left(N + (N-1) + \cdots + (N-(n-1))\right) \qquad (3.87) \\
&= \frac{n(2N-n+1)}{2(N+1)}
\end{aligned}
$$

3. Derive $G'_{(2:n)}(y)$. For mechanism S, in equilibrium $k := k^S = a_i + e_i^S = (1/\sqrt{c}-1)/n$ is a constant for the $i = 1, \ldots, n$ tournament participants, see (3.45). Similar to (3.84), we get

$$
\begin{aligned}
G_{(2:n)}(y) &= \sum_{j=0}^{1} \binom{n}{j} \left(y^k\right)^{n-j} \left(1-y^k\right)^j \\
&= ny^{k(n-1)} - (n-1)y^{kn}, \qquad (3.88) \\
G'_{(2:n)}(y) &= kn(n-1)\left(y^{k(n-1)-1} - y^{kn-1}\right) \\
&= (n-1)\left(\frac{1}{\sqrt{c}}-1\right)\left(y^{\frac{n-1}{n}\left(\frac{1}{\sqrt{c}}-1\right)-1} - y^{\left(\frac{1}{\sqrt{c}}-2\right)}\right). \qquad (3.89)
\end{aligned}
$$

3.9.2 Proof that (3.46) is positive

Proof 12 *It is sufficient to show that* $\gamma < \sqrt{c}(1-\sqrt{c})^2/n(n-1+\sqrt{c})$. *Since* $\gamma < c/(N-2)$ *by assumption* (3.28), *we only need to show that*

$$\frac{c}{N-2} < \frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})}. \qquad (3.90)$$

Recall that $n \leq N-1$. *Replace* n *by* $N-1$ *in* (3.90). *We get*

$$\frac{c}{N-2} < \frac{\sqrt{c}(1-\sqrt{c})^2}{(N-1)(N-2+\sqrt{c})}, \qquad (3.91)$$

which implies (3.90). *Write* (3.91) *as*

$$\frac{\sqrt{c}(N-1)(N-2+\sqrt{c})}{N-2} < (1-\sqrt{c})^2. \qquad (3.92)$$

Next, we use assumption $c < 1/(N+2)^2$ *(see* (3.28)*): We replace* $\sqrt{c}$ *by* $1/(N+2)$ *on both sides. Note that this makes the LHS larger and the RHS smaller. We get an inequality in* N *that is satisfied for* $N \geq 3$ *(as we assume).*

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Bibliography

C. Becker, R. Bierhals, K. Blind, S. Ekert, T. Giebe, T. Grebe, and E. Wolfstetter. Effizienzsteigerung bei der FuE-Projektförderung durch wettbewerbliche Vergabeverfahren, 2004. Report for the German Ministry of Economics and Labor[50].

A. W. Beggs. The licensing of patents under asymmetric information. *International Journal of Industrial Organization*, 10:171–191, 1992.

J.-P. Benoît and V. Krishna. Multiple-object auctions with budget constrained bidders. *Review of Economic Studies*, 68:155–179, 2001.

M. Binks, A. Lockett, D. Siegel, and C. Wessner. Policies promoting innovation in small firms: Evidence from the UK and US. *Small Business Economics*, 20:121–127, 2003.

U. Blum and F. Kalus. Auctioning public financial support incentives. *International Journal of Technology Management*, 26 (2/3/4):270–276, 2003.

U. Blum, H. Bertheid, U. Draugelates, A. Kleinknecht, W. Leonhardt, W. Ruhrmann, H. Scheibner, and M. Weck. Endbericht der Kommission 'Systemevaluation der wirtschaftsintegrierenden Forschungsförderung', 2001. Report for the German Ministry of Economics and Labor[51].

E. Brousseau, R. Coeurderoy, and C. Chaserant. The governance of contracts: Empirical evidence on technology licensing agreements. *Journal of Institutional and Theoretical Economics*, 163:205–235, 2007.

Y.-K. Che. Design competition through multidimensional auctions. *RAND Journal of Economics*, 24:668–680, 1993.

Y.-K. Che and I. Gale. Standard auctions with financially constrained bidders. *Review of Economic Studies*, 65:1–22, 1998.

[50] www.wiwi.hu-berlin.de/wt1/research/index.html
[51] www.bmwi.de/bmwi/navigation/service/publikationen,did=21772.html

Y.-K. Che and I. Gale. Optimal design of research contests. *American Economic Review*, 93:646–671, 2003.

P. Cramton, Y. Shoham, and R. Steinberg, editors. *Combinatorial Auctions*. MIT Press, 2006.

D. Czarnitzki and A. Fier. Do R&D subsidies matter? – evidence for the german service sector. *ZEW Discussion Paper*, No. 01-19, 2001. Mannheim[52].

D. Czarnitzki and G. Licht. Additionality of public R&D grants in a transition economy: The case of eastern germany. *The Economics of Transition*, 14:101–131, 2006.

G. Das Varma. Bidding for a process innovation under alternative modes of competition. *International Journal of Industrial Organization*, 21:15–37, 2003.

W. Ding and E. Wolfstetter. Prizes and lemons: Procurement of innovation under imperfect commitment. *Humboldt University Working Paper*, 2009.

A. Eickelpasch and M. Fritsch. Contests for cooperation – a new approach in german innovation policy. *Research Policy*, 34:1269–1282, 2005.

L. Ensthaler and T. Giebe. Subsidies, knapsack auctions and dantzig's greedy heuristic. *SFB/TR15 Discussion Paper No. 254, Berlin*, 2009.

Federal Ministry of Education and Research. Report of the federal government on research 2004, 2004. Germany[53].

U. Fischbacher. z-Tree. Zurich toolbox for readymade economic experiments – Experimenter's Manual. *Working Paper 21, University of Zürich*, 1999.

R. L. Fullerton and R. Preston McAfee. Auctioning entry into tournaments. *The Journal of Political Economy*, 107:573–605, 1999.

R. L. Fullerton, B. G. Linster, M. McKee, and S. Slate. Using auctions to reward tournament winners: Theory and experimental investigations. *RAND Journal of Economics*, 33:62–84, 2002.

N. T. Gallini and B. D. Wright. Technology transfer under asymmetric information. *RAND Journal of Economics*, 21:147–161, 1990.

[52] ftp://ftp.zew.de/pub/zew-docs/dp/dp0119.pdf
[53] www.bmbf.de/pub/bufo_2004_eng_full_version.pdf

J. García-Quevedo. Do public subsidies complement business R&D? a meta-analysis of the econometric evidence. *Kyklos*, 57:87–102, 2005.

T. Giebe and E. Wolfstetter. License auctions with royalty contracts for (winners and) losers. *Games and Economic Behavior*, 63:91–106, 2008.

T. Giebe, T. Grebe, and E. Wolfstetter. How to allocate R&D (and other) subsidies: An experimentally tested policy recommendation – instructions and screenshots from the lab experiment. *Humboldt University at Berlin*[54], 2005.

T. Giebe, T. Grebe, and E. Wolfstetter. How to allocate R&D (and other) subsidies: An experimentally tested policy recommendation. *Research Policy*, 35:1261–1272, 2006.

J. Goeree. Bidding for the future: Signaling in auctions with an aftermarket. *Journal of Economic Theory*, 108:345–364, 2003.

M. Groth. Auctions in an outcome-based payment scheme to reward ecological services in agriculture – conception, impelementation and results. *Georg-August-University Göttingen*[55], 2005.

R. Hernández-Murillo and G. Llobet. Patent licensing revisited: Heterogeneous firms and product differentiation. *International Journal of Industrial Organization*, 24:149–175, 2006.

P. Jehiel and B. Moldovanu. Auctions with downstream interaction among buyers. *RAND Journal of Economics*, 31:768–791, 2000.

H. Jia. A stochastic derivation of the ratio form of contest success functions. *Public Choice*, 135:125–130, 2008.

M. I. Kamien. Patent licensing. In R. Aumann and S. Hart, editors, *Handbook of Game Theory*, volume I, pages 331–354. Elsevier Science Publicers (North–Holland), 1992.

M. I. Kamien and Y. Tauman. The private value of a patent: a game theoretic analysis. *Journal of Economics*, 4:93–118, 1984.

M. I. Kamien and Y. Tauman. Fee versus royalties and the private value of a patent. *Quarterly Journal of Economics*, 101:471–491, 1986.

[54]www.wiwi.hu-berlin.de/wt1/research/index.html
[55]www.feweb.vu.nl/ersa2005/final_papers/180.pdf

M. I. Kamien and Y. Tauman. Patent licensing: The inside story. *The Manchester School*, 70:7–15, 2002.

M. I. Kamien, Y. Tauman, and S. Zamir. On the value of information in a strategic conflict. *Games and Economic Behavior*, 2:129–153, 1990.

M. I. Kamien, S. Oren, and Y. Tauman. Optimal licensing of cost-reducing innovation. *Journal of Mathematical Economics*, 21:483–508, 1992.

M. L. Katz and C. Shapiro. On the licensing of innovations. *Rand Journal of Economics*, 16:504–520, 1985.

M. L. Katz and C. Shapiro. How to license intangible property. *Quarterly Journal of Economics*, 101:567–590, 1986.

K. Konrad. *Strategy and Dynamics in Contests*. Oxford University Press, 2009.

V. Krishna. *Auction Theory*. Academic Press, 2002.

J.-J. Laffont and J. Robert. Optimal auction with financially constrained buyers. *Economics Letters*, 52:181–186, 1996.

I. Macho-Stadler and D. Pérez-Castrillo. Contrats de licences et asymmétrie d'information. *Annales D'Economie et de Statistique*, 24:189–208, 1991.

S. Martin and J. T. Scott. The nature of innovation market failure and the design of public support for private innovation. *Research Policy*, 29: 437–447, 2000.

A. Mas-Colell, M. Whinston, and J. Green. *Microeconomic Theory*. Oxford University Press, 1995.

S. Muto. On licensing policies in Bertrand competition. *Games and Economic Behavior*, 5:257–267, 1993.

M. Rostoker. A survey of corporate licensing. *IDEA*, 24:59–92, 1984.

A. Schöttner. Fixed-prize tournaments versus first-price auctions in innovation contests. *Economic Theory*, 35:57–71, 2008.

S. Scotchmer. *Innovation and Incentives*. MIT Press, 2004.

D. Sen. Fee versus royalty reconsidered. *Games and Economic Behaviour*, 53:141–147, 2005.

D. Sen and Y. Tauman. General licensing schemes for a cost-reducing innovation. *Games and Economic Behavior*, 59:163–186, 2007.

C. R. Taylor. Digging for golden carrots: An analysis of research tournaments. *American Economic Review*, 85:872–890, 1995.

S. Vishwasrao. Royalties vs. fees: How do firms pay for foreign technology? *International Journal of Industrial Organization*, 25:741–759, 2007.

X. H. Wang. Fee versus royalty licensing in a cournot duopoly model. *Economics Letters*, 60:55–62, 1998.

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Selbständigkeitserklärung

Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung meiner Dissertation benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

Berlin, den 31. März 2009

Thomas Giebe