Field Theory Dynamics
from Branes in String Theory

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Motivation

It is the main goal of physics to explain all ratios between measurable quantities. The hope is that in the end a very simple, beautiful and unique mathematical structure emerges, the theory of everything.

Modern physics is based on two major building blocks: general relativity and quantum physics. The former expresses all phenomena of the macroscopic world and especially gravity in terms of simple geometric concepts. The latter is powerful in explaining the microscopic world by replacing the notion of particles which occupy a certain position in space and carry a certain amount of momentum by the more abstract formalism of states in a Hilbert space and observables as operators acting on them. Since the distinction between microscopic and macroscopic world seems to be rather arbitrary, these two building blocks should be unified in an underlying theory. More than that, as they stand, the two concepts are even inconsistent. Straight forward quantization of general relativity leads to infinities in physical processes that can not be tolerated. A larger theory unifying gravity and quantum theory is hence not only desired from an aestethic point of view but indeed required for consistency.

After many years of extensive search for this unifying theory, a single candidate has emerged: string theory. String theory replaces the fundamental point like objects of particle physics by 1d strings thereby removing the infinities encountered in quantizing general relativity. General relativity reemerges as a low energy limit at large distances, where it was tested experimentally. However at small distances stringy physics takes over and even our concepts of space and time break down. Similarly ordinary particle physics as described by the standard model can reemerge in the limit where gravitational interactions between the particles can be neglected. The energy scale at which both gravity and quantum effects become important is set by the Planck scale and is roughly $10^{19} GeV$. Since this scale is so huge, it is impossible to just create the fundamental degrees of quantum gravity in an accelerator and then look what they are.

Even though string theory has all the ingredients required by modern physics it is difficult to make contact with physics as we know it. The major obstacle is that string theory allows for a variety of different vacua, each of which leads to different physics.

No precise predictions about the low-energy physics (like answers to the why questions left by the standard model) can be made without finding a process that determines the right vacuum. However the most important concepts appearing in string theory, namely gauge theory, gravity and supersymmetry, are indeed known or believed to dominate the real world. While the former two are the bases for all physics described by the standard model and general relativity, the latter is believed to be of similar importance for next generation collider physics. Experimental verification of supersymmetry in the real world would be even more support that string theory is not just the only known consistent quantum theory of gravity, but indeed the fundamental theory realized in our world.

Of special interest are also objects which probe the regime of quantum gravity, that is they are small enough for quantum effects to be important and heavy enough to require gravity. Examples of such objects are black holes close to their singularity or our universe in very early times, close to big bang. Treatment of these important issues as well as the vacuum selection problem requires non-perturbative information about string theory. So far the perturbative expansion of string theory in terms of worldsurfaces was the only definition we had of string theory. Only very recently tools have emerged that allow us control certain aspects of the non-perturbative physics behind string theory, raising the hope that these fundamental issues can finally be addressed.

Introduction

During the so called "second string revolution" it has become possible to gain control over aspects of string theory [1, 2, 3] that were not contained in its perturbation expansion in terms of worldsurfaces. The major achievements were the discovery of D-branes [4, 5] as one of the non-perturbative objects in string theory and the realization of the role of duality symmetries in string theory [6], relating two seemingly different theoretical descriptions to one and the same physical situation.

Dualities have been of similar importance in gauge theories. Using dualities it has been possible to solve the IR behaviour of certain quantum field theories exactly [7] and get a lot of non-perturbative information even in situations with less restrictive symmetries. However in all those setups supersymmetry is a vital ingredient and it is not clear yet how these methods can be generalized to non-supersymmetric situations.

Non-perturbative string theory and Super Yang Mills (SYM) gauge theories are indeed deeply related. The dynamics of the D-branes which are so important for our understanding of non-perturbative string-theory is basically governed by SYM. This connection can be used in a twofold way: dualities and other obscure aspects of field theory only discovered recently, like non-trivial fixed points of the renormalization group with mutual non-local objects becoming massless, find their natural place in string theory, where they can be easily visualized.

On the other hand some problems which seem intractable in string theory can be mapped to questions in gauge theory, which are under much better control. Indeed it has been proposed that all non-perturbative aspects of string theory can be encoded in large N SYM theory [8].

It is the purpose of this work to highlight some of the insights gained with the help of this deep connection between gauge theory and string theory.

In Chapter 2 I will review the construction of D-branes and explain how gauge theory determines their dynamics. I will also comment on the important role D-branes played in understanding string theory dualities, since these dualities lead to the discovery of an 11-dimensional theory called M-theory that basically summarizes all the non-perturbative insights we gained about string theory and is the natural arena for visualizing aspects of SYM which are hard to understand from the field theory point of view.

In Chapter 3 I will introduce a setup first used by Hanany and Witten to study 3d gauge theories embedded in string theory. In Chapter 4 I will use this setup to study certain aspects of 6-dimensional and 4-dimensional physics. The interplay will allow us to understand certain non-trivial fixed points and transitions which are obscure from the field theory point of view, to say the least. But we can also learn about string theory from the correspondence. Among many other things it will allow us to show that in string theory chiral vacua can be smoothly deformed into non-chiral vacua, perhaps taking a small step towards a more detailed understanding of the vacuum selection problem.

In Chapter 5 I will show how the other aspects of the gauge-theory / string-theory connection are related to the Hanany-Witten setups by a series of string-theory dualities. We will see that the different techniques used to explore the correspondence might be more or less powerful in various situations, but that in the end we are guaranteed to obtain the same results, no matter how we chose to embed our gauge theory under consideration into string theory.

In Chapter 6 I will discuss what open problems remain and where we can go from here.

Chapter 1

D-branes and non-perturbative effects in string theory

1.1 The breakdown of perturbation theory

String theory as we used to know it was only defined via its perturbation series. That is a given scattering process receives contributions from worldsheets of various topology. Higher genus surfaces are weighted with higher powers of g_s, the string coupling (which is the expectation value of a dynamical field, the dilaton: $g_s \sim e^{<\Phi>}$)

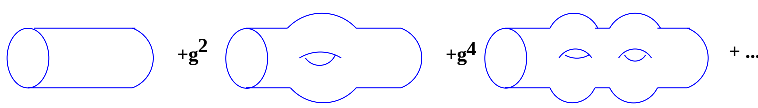

Figure 1: Propagation of a string from its perturbative definition

To calculate the contribution from a single diagram in the perturbation series depicted in Fig.1 , we have to solve a conformal field theory on the worldvolume of the given topology and than integrate over all possible deformations (moduli). This is often possible. 2d conformal field theories are very constraint due to the high amount of symmetry and many calculational tools are available. At weak coupling only diagrams of low genus contribute and we can actually calculate the amplitudes.

However this perturbative definition clearly fails when we are at strong coupling. Here we really have to calculate an infinite number of diagrams, since higher topologies are no longer suppressed. Worse than that, a generic scattering process may also receive contributions that are not even visible at all in the perturbation series, even if we would be able to sum up the infinite diagrams. Such contributions are suppressed as e^{-1/g_s} (or as e^{-1/g_s^2}). These contributions have a series expansion around strong coupling ($g_s \to \infty$)

$$e^{-1/g_s} = 1 - \frac{1}{g_s} + \frac{1}{2g_s^2} - \frac{1}{6g_s^3} + \dots$$

None of these terms has the right power to match any of those appearing in the ordinary perturbation series which only contains positive powers of g_s. These are purely non-perturbative effects.

From field theory it is well known that there are indeed phenomena giving rise to such non-perturbative contributions. The most famous example are instantons. Instantons are stable solutions to the Yang-Mills equations of motion that are centered in space and time. Their existence is due to the fact that Yang-Mills can have topologically distinct vacua. Instanton solutions interpolate between different vacua and their stabelness is therefore guaranteed by topology. Arguing on the bases of the cluster decomposition principle one can show [9] that in order to define a consistent quantum field theory we indeed have to sum over all possible instanton backgrounds when performing the path integral, so these configurations do contribute to scattering processes. Calculating the classical action corresponding to an instanton configuration one finds that it is indeed suppressed as e^{-1/g_{YM}^2}. This example also gives us an intuitive feeling why such things will never appear in the perturbative expansion: while perturbation theory expands around a given vacuum, non-perturbative contributions arise from tunnelling processes and interpolation between different vacua. But this is also why it is so crucial to understand non-perturbative states in string theory: solving the vacuum problem, that is what is the right string theory ground state and how did nature pick it, requires detailed understanding of precisely these processes.

Similar effects are due to solitonic objects like monopoles or domain walls. They are again stable solutions to the equations of motion centered in space. This enables us to interpret them as particles (or higher dimensional objects) in our theory. They have masses which go like $1/g_{YM}^2$. At weak coupling they are very heavy and can be neglected. However at strong coupling they should be included. Virtual monopoles running in loops will be suppressed by $\sim e^{-1/g_{YM}^2}$ due to the $e^{-\text{action}}$ factor in the path integral, signalling a non-perturbative contribution.

In the same spirit we can try to identify solitonic objects with mass $1/g^2$ in string theory in order to identify non-perturbative string states. By studying supergravity (SUGRA), the low-energy field theory limit of string theory, one indeed finds a whole zoo of such objects, generically called p-branes. Among more exotic objects there exists the magnetic dual of the string, the NS5 brane with tension $1/g_s^2$ and the so called Dirichlet (D) branes, whose tension at weak coupling only grows as $1/g_s$. Understanding those objects should enable us to learn about non-perturbative effects in string theory. They will be the topic of the rest of this work.

1.2 A string theoretic description of D-branes

To understand the contribution to the amplitude by a given brane configuration, one can study perturbative string theory in the background of the branes at weak string coupling. For the D-branes this is straight forward, once one realizes that D-branes are space-time defects at which open strings can end.

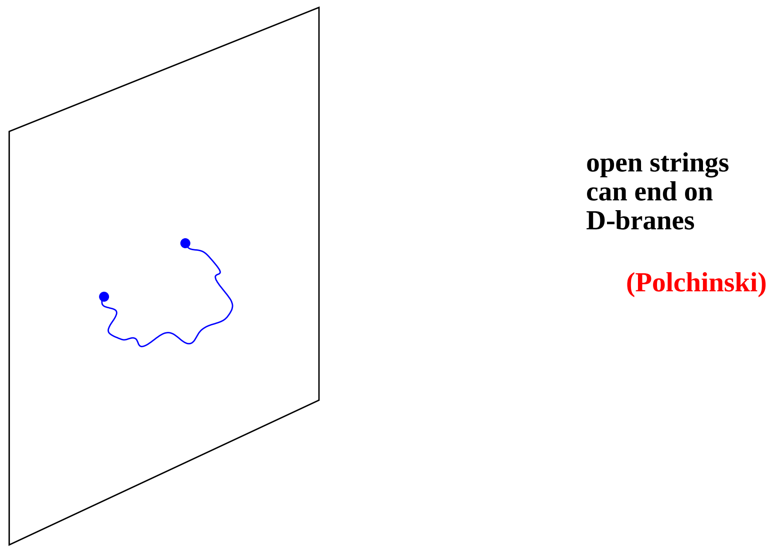

Figure 2: String in the background of a D-brane

Figure 2 shows this basic concept of D-brane physics. It was known since the early days of string theory that in open string theory one can as well impose Dirichlet boundary conditions (the end of the string is at a fixed position) as the usual Neumann boundary conditions (the end is free to move, no momentum is allowed to flow of the end). One usually neglected this possibility, since it introduced hyperplanes (the planes on which the endpoints are forced to stick) which break Lorentz invariance. It was the achievement of Polchinski to show [4, 5] via an explicit 1-loop open string calculation that these space-time defects carry charge under the RR gauge fields of string theory and calculating their tension to be

$$T_{Dp} = \frac{1}{l_s^{p+1} g_s} \tag{1.1}$$

where l_s and g_s denote the string coupling and length respectively. These properties allow us to identify them with the stringy version of the solitonic solutions of SUGRA which I already called D-branes before.

Now it is straight forward to do everything we are used to from perturbative string theory in the background of the D-branes. Quantizing the oscillator modes of the string theory in the presence of the modified boundary conditions one finds that the massless spectrum of the open strings ending on the D-brane are given by a SYM multiplet living on the brane worldvolume. That is, for the D9 brane we find the usual $\mathcal{N} = 1$ SYM multiplet consisting of the vector gauge field and the gauginos in 10d. All other branes yield dimensional reductions of those to the appropriate worldvolume dimension.

Demanding conformal invariance on the string worldsheet yields equations of motion for the space-time fields by setting the β function of the 2d conformal theory on the worldsheet to zero, order by order in the string tension (which plays the role of the coupling constant in the 2d theory), just like in the well known case of Neumann boundary conditions. Writing down an action that yields these equations one obtains as an effective action for the D-brane theory a supersymmetric Dirac-Born-Infeld action with Wess-Zumino couplings to the bulk fields [10]

$$S = S_{DBI} + S_{WZ} = -\int d^{p+1}\xi \; e^{-\Phi}\sqrt{-\det(g_{ij} - \mathcal{F}_{ij})} + \int d^{p+1}\xi \; Ce^{\mathcal{F}} \tag{1.2}$$

where $\mathcal{F} = F - B$, $C = \sum_{r=0}^{10} C^{(r)}$ is a formal sum over all the form fields present in the IIA/B supergravity and the integral always picks out the right form to go with the right power of $\mathcal{F}$ from the exponential. The fields should be understood as pullbacks from superspace to the worldvolume.

The low-energy approximation of this action, that is the expansion to lowest order in l_s^2 (the string length), yields SYM on the worldvolume. This is in accordance with the analysis of the massless spectrum. The gauge coupling can be read off from (1.2) to be

$$g_{YM}^2 = l_s^{p-3} g_s. \tag{1.3}$$

As in the case of fundamental string theory the scalars on the worldvolume define the position of the brane in the transverse space. Via the DBI action these are coupled to the worldvolume gauge fields. This is an important property of the D-brane action which we will explain in more detail in the following. Basically a brane can absorb the flux of a charged particle by bending in transverse space, balancing the force from the gauge fields with its tension, that is with the worldvolume scalars. A flat D-brane breaks half of the supersymmetries (since the open string spectrum only has half of the supersymmetries of the closed spectrum). The supersymmetries preserved have to satisfy [1]

$$\epsilon_L = \Gamma_0 \cdot \ldots \cdot \Gamma_p \epsilon_R \tag{1.4}$$

where the preserved supercharge is $\epsilon_L Q^L + \epsilon_R Q^R$ and Q^L, Q^R are the supercharges generated by left- and right- moving degrees of freedom in the surrounding type II string theory. Choosing a non-trivial embedding generically breaks all the supersymmetries. If

[1] which can e.g. be seen by analyzing the Killing spinor equations in the background of the D-brane soliton solution

the embedding geometry allows for some Killing spinors, lower fractions of supersymmetry may be preserved.

If we try to repeat this analysis for NS5 branes we run into trouble. The NS5 brane metric looks like a tube, the dilaton blows up if we move towards the core and any conformal field theory description breaks down. Only asymptotically, the NS5 brane can be described by a well known conformal field theory, a WZW model [11]. The NS5 brane worldvolume theory is not accessible by purely perturbative string techniques. However we will see later that we can deduce its properties by string dualities.

1.3 D-branes and gauge theory

By now we have gained some insights in the dynamics governing D-branes. We have learned that there is a very deep connection between D-branes and gauge theories. We will analyse how some of the most interesting aspects of D-branes are captured by simple field theoretic phenomena. This discussion will pave the way for the discussion in the following chapters, where I will exploit the D-brane / gauge theory correspondence to learn about string theory as well as about gauge theory. The general philosophy is that we consider certain limits of string theory, in which the gravity and heavy string modes (the bulk modes) decouple, leaving us just with the open string sector described by SYM. The basic quantities that control this limit are the Planck scale M_{pl} and the string scale M_s (the inverse of the string length). They satisfy

$$M_{pl}^4 g_s = M_s^4 \tag{1.5}$$

which just shows the relation between string frame and Einstein frame. Sending M_{pl} to infinity is the same as sending Newton's constant to zero, so gravity is decoupled. Taking M_s to infinity sends all excited string states to infinite mass effectively decoupling them, too. This can be done at finite string coupling, keeping an interacting SYM theory.

1.3.1 Gauge theory on the worldvolume

As we have seen, the effective theory on the worldvolume is given by a DBI action. We want to analyse this world volume theory in the limit, where the bulk physics decouples, that is we get rid of gravity and other closed string modes. We only keep the degrees of freedom on the brane. Expanding the DBI action in l_s^2 (which explicitely shows up together with every F) it is easy to see, that in the $l_s \to 0$ limit the theory on the worldvolume of the Dp-brane reduces to $U(1)$ SYM in p+1 dimensions. The amount of supersymmetry preserved by a given brane is determined by its embedding in space-time, as discussed above. A flat brane always preserves half of the 32 supercharges of type II theory, leading to maximally supersymmetric Yang-Mills on the worldvolume. Now let us consider what happens if N D-branes coincide. This situation was analysed by Witten [12].

A single D-brane supports on its worldvolume a single $U(1)$ multiplet. These massless states arise from a string starting and ending on the same brane. The mass of a state is given by the length of the string times the string tension. The massless vector hence arises from a zero length string starting and ending at the same point. Each of the ends of the strings carries a Chan-Paton label of the gauge group, that is an index in the fundamental representation, so the vector multiplet is correctly left with a fundamental and an antifundamental index, an adjoint field. Clearly nothing happens to these states if many D-branes coincide. However, whenever two branes are close, there are new states that become important. Strings stretching from one brane to the other yield states whose mass is determined by the distance between the branes. They carry a fundamental Chan Paton index of the U(1) of the brane they start and end on respectively. It is natural to identify those as W-bosons of a broken U(2) gauge group. The distance between the branes determines the Higgs expectation value. When N branes coincide all the W-bosons become massless and the full $U(N)$ gauge symmetry becomes visible.

In order to obtain different gauge groups one can consider D-branes coinciding on top of space-time singularities. An example of such a singularity which is under control from perturbative string theory is an orientifold plane, the fixed plane of a Z_2 orbifold action, that combines worldsheet parity with a space time reflection in r coordinates. The resulting $p+1=10-r$ space-time fixed plane is called an Op orientifold plane. A similar calculation like that of Polchinski's determination of the D-brane charge shows that the orientifold is also charged under the same RR field as the Dp, where the relative value of the charge is given by [2]

$$q_O = \pm 2^{p-4} q_D. \tag{1.6}$$

The sign is determined by a discrete choice of the precise way one performs the projection. When N D-branes coincide on-top of the orientifold (and hence also coincide with their N mirrors), only oriented strings stretching between the branes will yield new massless gauge bosons, leading to an $SO(2N)$ ($USp(2N)$) gauge theory on their worldvolume for an orientifold of negative (positive) charge. The best known example is the type I string. If we mod out IIB just by world-sheet parity we basically produce an O9. Since this is a space-filling brane we have to cancel the RR charge, forcing us to use the negatively charged orientifold with 32 D-branes on top of it, yielding an $SO(32)$ gauge theory, as expected.

[2] Here and in what follows I will always consider the D-brane and its Z_2 mirror as different objects, each carrying charge q_D. If one wants to consider only physical D-branes one should assign them charge $2q_D$ in these conventions.

1.3.2 Compactifications and D-branes

There is a seemingly different way that D-branes can be described by gauge theories. If we consider compactifications of string theory, we will have non-perturbative states in the resulting lower-dimensional theory from D-branes wrapping cycles of the compactification manifold. The mass of these states is just given by the tension of the brane times the volume of the cycle (and therefore has the $1/g_s$ dependence signalling a non-perturbative state). At certain points in the moduli space of compactifications some of these cycles may shrink to zero size, leading to new massless states in the low-energy theory. Some of these states are usually massless vectors, giving rise to non-perturbative gauge groups.

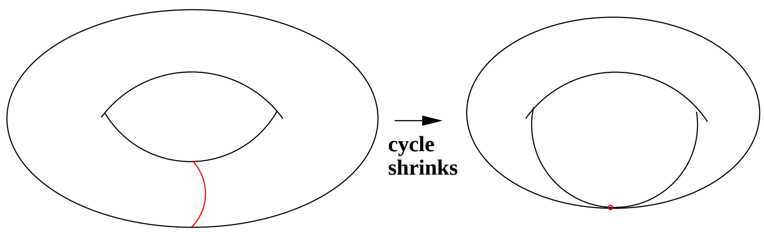

Figure 3: Non-perturbative states from D-branes on shrinking cycles

1.4 Engineering Gauge theories

With the two mechanisms at hand we can try to engineer gauge theories, that is we make up a string theory geometry with branes that realize a certain gauge theory we want to study. Combining the two basic mechanisms discussed above in various ways there are several possibilities to do so. Basically all these different approaches described in the literature can be separated in three classes. As I will discuss in the last chapter they are actually equivalent. There I will also give a more technical discussion for the specific case of $\mathcal{N} = 2$ theories in 4d.

1.4.1 Geometric Engineering

A geometric engineer tries to cook up a string background that captures all aspects of the gauge theory she wants to study in the geometry of the compactification manifold. In order to focus on the gauge theory modes, one has to decouple all stringy modes and all bulk modes. That is one has to send M_s and M_{pl} to infinity. Let me for simplicity of notation discuss the case of a K3 compactification (see [13] and references therein), engineering an $\mathcal{N} = (1,1)$ or $\mathcal{N} = (1,0)$ supersymmetric gauge theory in 6d for type IIA or the heterotic string respectively. It should be clear that these principles work the same way in other examples. The relevant scale is the 6d Planck scale given according to a KK ansatz by

$$M^4_{Pl,6} = V_{K3} M^8_{Pl,10}. \tag{1.7}$$

Decoupling gravity therefore effectively amounts to decompactifying the K3. Since the 6d gauge coupling of the perturbative gauge groups already present in 10d are also given via the KK ansatz as

$$g^{-2}_{YM,6} = V_{K3} g^{-2}_{YM,10} \tag{1.8}$$

they decouple in the same decompactification limit. The only gauge groups that survive are the non-perturbative gauge groups that arise via wrapping branes around vanishing cycles in the manifold. All information about the gauge theory is therefore encoded in the local singularity structure of the K3.

The basic example is IIA on an ALE space, that is a non-compact version of K3. An ALE is a blow-up of an R^4/Γ orbifold, where Γ is a discrete subgroup of $SU(2)$. Since spinors transform as a $(2,1)+(1,2)$ under the $SO(4) = SU(2) \times SU(2)$ spacetime rotations, embedding the orbifold in just one of the $SU(2)$ factors leaves half of the spinors invariant and hence also half of the supersymmetries unbroken. Since K3 can be written as an orbifold of T^4, these orbifold singularities can arise locally in the geometry of K3. The statement that Γ should be a subgroup of $SU(2)$ is equivalent to demanding that the holonomies of K3 only fill up $SU(2)$ and not the full $SO(4)$ of a generic 4d manifold. In order to obtain gauge dynamics, the local description in terms of the ALE is all we need. We expect new gauge dynamics when we move to the singular point, the orbifold itself.

The ALE space has topological non-trivial cycles.

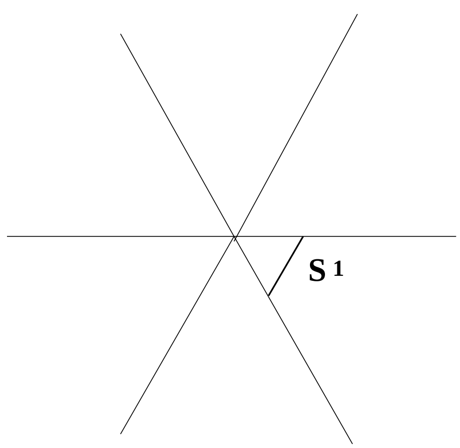

Figure 4: Non-trivial 1-cycle on R^2/Z_6.

Figure 4 illustrates non-trivial S^1s arising from an R^2/Z_6 orbifold. Similarly we get 2-spheres on the ALE. These 2-spheres shrink to zero size at the orbifold point. New massless states arise from D2 branes wrapping these cycles. The intersection pattern of the 2-cycles will determine the gauge group. Luckily all discrete subgroups of $SU(2)$ can be classified by an ADE pattern, where the corresponding Dynkin diagram gives us precisely the information about the intersection numbers of the vanishing spheres. The resulting gauge theory has a non-abelian ADE gauge group.

1.4.2 Branes as Probes

"Branes as Probes" is the most natural way if we want to learn something about string theory from Yang-Mills theory. The idea is that in order to study what happens to a given string background once one takes into account all the quantum effects, one probes the background with a D-brane [3]. On the worldvolume of the D-brane we will as usual find a gauge theory. The background geometry will be encoded in this gauge theory via the matter content, the amount of unbroken supersymmetry and the interaction potentials. Solving the quantum gauge theory will teach us about the quantum behaviour of the background. This technique has been very successfully used for probing D$p+4$ branes and O$p+4$ planes with Dp branes [14, 15, 16, 17, 18], as well as probing orbifold singularities with Dp branes [19, 20].

In both cases the matter content and classical superpotential of the gauge theory can be analyzed by perturbative string theory. For the higher p branes, we will find new states on the Dp worldvolume corresponding to the zero modes of strings stretching between Dp and D$p+4$ branes in addition to the gauge multiplet already present from the Dp-Dp strings.

In the case of the orbifold we first include all the twisted sectors required in string theory for consistency by including all the mirror D-branes and strings stretching in between them. Then we project onto states invariant under the orbifold group and this way obtain the corresponding spectrum.

1.4.3 Hanany-Witten setups

Hanany and Witten (HW) introduced a setup of intersecting branes realizing $d=3$ $\mathcal{N}=4$ gauge theories. The gauge theory again lives on the worldvolume of D-branes. The other branes make the gauge theory interesting by breaking SUSY and introducing new matter. Since we are now only dealing with flat branes in flat space, many things become very intuitive. Moduli and parameters just correspond to moving the branes around and are very easy to visualize. As advertised above I will show in the end, that

[3] In this language one could view string theory as we used to know it as probing space-time with a fundamental string.

all the 3 approaches are actually equivalent, so by studying the intuitive HW setups we can get non-trivial results about quantum string backgrounds by considering the "dual" branes as probes setup. The next chapter is devoted to an extensive review of the HW idea, so I won't go into any details at this point.

1.5 D-branes and dualities

1.5.1 String Dualities and M-theory

Probably the most important application of D-branes so far is the idea of string-dualities, the statement that one and the same physical system has two dual descriptions. The concept of duality was already discussed long ago in the context of field theories, as I will explain in more detail in the Chapter 3. In string theory duality was first detected in the form of T-duality [21]. Studying the spectrum of bosonic string theory on a circle of radius R

$$\begin{aligned} H &= \frac{1}{2}p_R^2 + p_L^2 + \text{oscillators} \\ p_R &= \frac{1}{\sqrt{2}}(\frac{l_s}{R}n - \frac{R}{l_s}m) \\ p_L &= \frac{1}{\sqrt{2}}(\frac{l_s}{R}n + \frac{R}{l_s}m) \end{aligned} \tag{1.9}$$

One sees that due to the presence of winding modes characterized by the integer m as well as momentum modes n around the circle, the states are invariant under an exchange of the two if one simultaneously takes R into l_s^2/R. This invariance under $R \to 1/R$ exchange can be shown to be a symmetry of amplitudes to all orders in perturbation theory and is believed to be valid even non-perturbatively. The two compactifications are T-dual to each other. For the superstring this T-duality works almost the same. For example type IIA on R is dual to IIB on l_s^2/R. In this case the $p+1$ form fields from the RR sector T-dualize into $p+2$ and p form fields, depending on whether we take the components along or transverse to the compact direction. Since the Dp branes couple to these fields, T-duality transverse to the worldvolume produces a $Dp+1$ brane while T-duality along a worldvolume direction leaves us with a $Dp-1$ brane.

More interesting are dualities relating one string theory at weak coupling to another string theory at strong coupling. Many dualities of this type have been discovered over the recent years. However non of them can be proven by a direct calculation. Since by definition we compare a strongly coupled with a weakly coupled theory, only one side is accessible to calculations. Duality then amounts to a prediction for the strong coupling behaviour of the other theory. The reason why most string theorists nevertheless believe in the validity of these dualities is that they can be checked in several ways. The most important check is the matching of objects which are BPS. They preserve some fraction of the supersymmetry and are therefore protected by the superalgebra from any renormalization. We have already encountered some of these objects: D-branes. This way certain properties of these non-perturbative states which dominate the strong coupling theory can be calculated and they can be matched onto the perturbative states at weak coupling.

One of the examples I am going to consider several times in this work is the selfduality of type IIB string theory. Type IIB with coupling g_s is dual to type IIB with $1/g_s$. Including the axion a we can build a complex coupling $\tau = \frac{a}{2\pi} + \frac{i}{g_s}$. Combining the g_s to $1/g_s$ duality with the invariance of the axion under shifts of 2π, a whole $SL(2,Z)$ of dual theories can be constructed. The NS 2-form field combines with the RR 2-form into an $SL(2,Z)$ doublet. The objects coupling to them, the fundamental F1 and the D1 string are exchanged under the strong-weak coupling duality. More general $SL(2,Z)$ transformations take the F1 into a (p,q) bound state of p fundamental and q D-strings. Similarly their magnetic duals, the NS5 and the D5 brane form an $SL(2,Z)$ doublet. Since there is only one 4-form field, it has to be a singlet under $SL(2,Z)$ and hence the D3 brane stays invariant under all duality transformations. Since the low-energy effective actions of the dual theories are supposed to agree, the Planck scale has to remain invariant, therefore using (1.5) the dual string scale has to be $M_s g_s^{1/2}$.

Basically all string dualities can be summarized as the existence of an conjectural 11d theory, called M-theory, which contains all the string theories as well as 11d SUGRA as perturbative expansions in certain limits.

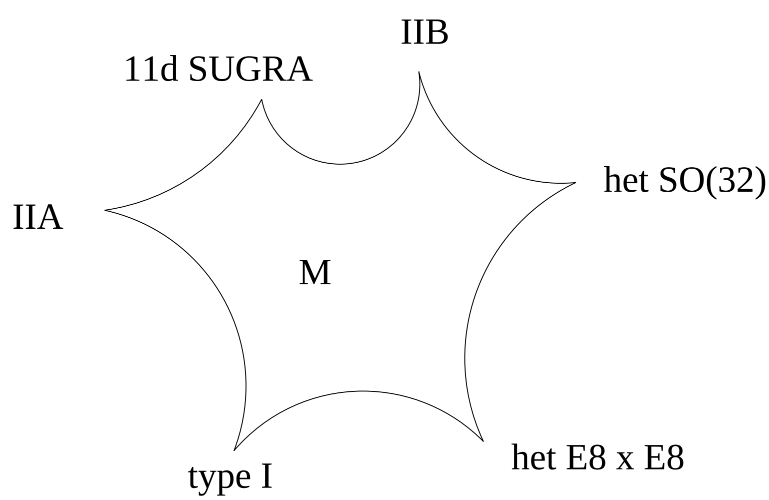

Figure 5: All known string theories as well as 11d SUGRA are just different perturbative expansions of an overarching 11d M-theory.

Since I am going to use this M-theory picture in what follows, let me briefly present as a defining duality of M-theory the duality between 11d SUGRA and type IIA, which originally led to the discovery of M-theory [6, 22]. According to this proposal M-theory on a circle is type IIA string theory with the IIA coupling and string length given in terms of the 11d Planck length and the radius of the 11th dimension R as

$$g_s^2 = \frac{R^3}{l_{pl}^3} \quad l_s^2 = \frac{l_{pl}^3}{R} \tag{1.10}$$

These relations can be obtained by comparing the low-energy effective actions. The relation really constitutes a strong-weak coupling duality: at very large R IIA becomes strongly coupled and we lose all control. However in the 11d picture as R becomes bigger the curvature becomes smaller and SUGRA becomes a good approximation. Similar at very small R the curvatures are Planckian in 11d, so SUGRA fails to capture the physics, however perturbative string theory is a good description. To describe couplings of order 1, we need the yet unknown full fledged M-theory.

The appearance of the 11th dimension can be seen from studying D-branes. D0 branes are non-perturbative states, whose mass goes to zero in the strong coupling limit. N D0 branes are believed to form a unique threshold bound state (that is with zero binding energy) [12, 23]. They therefore led to a tower of states with mass $\frac{N}{g_s l_s}$. It is natural to identify these as momentum modes around the 11th dimension of radius $R = \frac{1}{g_s l_s}$. M-theory also provides us with a nice organization principle for all the other branes. From 11d SUGRA we learn that M-theory has two extended objects, the M2 and the M5 brane. Together with three more complicated solutions that only arise upon compactification of at least one more direction, the wave (momentum mode around the circle) with mass $1/R$, its magnetic dual, the KK monopole 6 brane with tension R^2/l_{pl}^9 and an M9 brane with tension R^3/l_{pl}^{12}, they give rise to all brane solutions in the perturbative limits of M-theory.

1.5.2 Matrix Theory

Having said the above, it would clearly be desirable to find a microscopic definition of M-theory. The only candidate that has emerged so far is matrix theory [8]. The idea behind this approach is to quantize the theory in a special frame, called the infinite momentum frame, where only a very limited amount of the original degrees of freedom are visible. What we do is boost ourselves as observers infinitely along a compact direction, so that of all modes with momentum N/R only those with positive N survive. This has to be considered as N and R both go to infinity with N/R also going to infinity.

Since we want to work at finite N to do any realistic computation, one would like to study a reference frame that is described by finite N matrix theory and reduces to the IMF in the $N \to \infty$ limit. Such a frame exists, the discrete light cone frame. Therefore we want to study discrete lightcone quantization (DLCQ) of M-theory. This was conjectured to be described by the finite N matrix model in [24]. DLCQ formally can be thought of as quantizing the theory on a compact lightlike circle. This notion seems to be rather counterintuitive. Indeed it was shown in [25] that the best way to think about the DLCQ is to consider it as a limit of a compactification on an almost lightlike circle, that is

$$\begin{pmatrix} x \\ t \end{pmatrix} \sim \begin{pmatrix} x \\ t \end{pmatrix} + \begin{pmatrix} \sqrt{\frac{R^2}{2} + R_s^2} \\ -\frac{R}{\sqrt{2}} \end{pmatrix} \approx \begin{pmatrix} x \\ t \end{pmatrix} + \begin{pmatrix} \frac{R}{\sqrt{2}} + \frac{R_s^2}{\sqrt{2}R} \\ -\frac{R}{\sqrt{2}} \end{pmatrix} \tag{1.11}$$

where $R_s << R$. For $R_s \to 0$ this reduces to a lightlike compactification with radius R. Now this compactification is just a Lorentz boost transform of an ordinary spacelike compactification on R_s with boost parameter

$$\beta = \frac{R}{\sqrt{R^2 + 2R_s^2}} \approx 1 - \frac{R_s^2}{R^2}. \tag{1.12}$$

Therefore Seiberg's final result can be stated as follows: DLCQ of any system is Lorentz equivalent to the $R_s \to 0$ limit of a spacelike compactification on R_s. But we know what M-theory on a vanishing circle is: it is just weakly coupled IIA! Since this is a rather familiar theory, it is very easy to identify the relevant degrees of freedom.

So matrix theory is just the weak coupling limit of type IIA string theory. The relevant degrees of freedom surviving are the carriers of positive momentum around the (vanishing) circle in the 11th dimension. But we already identified them as D0 branes. Studying matrix theory of compactified M-theory, we find that the limit on the IIA side shrinks all the radii to zero, forcing us to perform a T-duality. This way the D0 branes turn into different branes. But after all one finds that matrix theory is defined via the worldvolume theory of a certain brane. Some of these are rather exotic. E.g. matrix theory on the T^4 is described by a D4 brane at very strong coupling, so that it turns into an M5 brane [26, 27]. Similar the T^5 compactification is described by D5 branes at strong coupling and hence IIB NS5 branes at weak coupling [28]. The T^6 is described by D6 branes at strong coupling [29, 30]. However in this case one automatically keeps some of the bulk modes, so that the interacting world volume theory is not decoupled from gravity [4]. Higher dimensional compactifications are plagued by similar problems. Even though we can still define matrix theory as the worldvolume theory of some brane, this description is not any more useful than saying M-theory is a consistent 11d theory of gravity, since the corresponding worldvolume theories do not decouple from the bulk gravitons in the matrix limit [5].

[4] Existence of limits decoupling the bulk while keeping an interacting theory on the brane are required for the "brane proof" of the existence of certain fixed points in higher dimensions. I will discuss the existence and non-existence of these limits in the following chapters.

[5] Some doubts have been voiced, whether it is legitimate to neglect all the effects of the modes with zero momentum around the compact circle, which became infinitely heavy and were integrated out by keeping

The matrix conjecture this way elevates the SYM / non-perturbative string theory correspondence to a principle: not only does the worldvolume SYM capture important aspects of non-perturbative string theory, it is used as a definition of "all of M-theory". So learning something about the worldvolume theory of branes will always automatically bring us a step further towards the goal of understanding the fundamental theory of everything.

By analyzing the size of D0 bound states some evidence can be found that matrix theory is holographic, that is information in a given space-time volume grows like the area surrounding the volume, not like the volume itself [31, 32]. This is supposed to be a genuine property of quantum gravity. The idea is that the best you can do is to fill up your volume with a big black hole and the entropy of the black hole grows with its horizon area. Until recently it has been totally unclear how such a principle could be implemented in string theory. Maldacena's proposal [33] led to a beautiful realization of holography in spaces with negative cosmological constant. So far matrix theory is our only candidate for a holographic description of Minkowski space.

1.5.3 Worldvolume theory of the NS5 brane

As we have seen it is easy to understand the worldvolume theory of the D-branes from analyzing the modes of strings with Dirichlet boundary conditions. The NS5 and M5 branes are a little bit more elusive, but using dualities we can say something about them, too. First consider the IIB NS5 brane. Type IIB is selfdual, that is type IIB with coupling g_s and string scale M_s is dual to type IIB with coupling $1/g_s$ and string scale $M_s g_s^{1/2}$, holding $M_{pl}^4 = M_s^4 g_s$ fixed. The dual theory is the theory of IIB D1 strings, which play the role of fundamental strings at strong coupling. This duality takes NS5 into D5 brane. From this we learn that as the D5 the NS5 will be governed by 6d SYM with gauge coupling

$$g_{NS5}^2 = \frac{1}{M_s^2}. \tag{1.13}$$

The supersymmetries preserved by an flat NS5 brane living in 012345 space will satisfy

$$\epsilon_L = \Gamma_0\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5\epsilon_L \quad \epsilon_R = \pm\Gamma_0\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5\epsilon_R \tag{1.14}$$

where the $\pm$ depends on whether we consider IIA or IIB.

The M5 brane can be best understood in the limit where M-theory is well described by 11d SUGRA. Here N coinciding M5 branes can just be thought of as a soliton given by the following SUGRA solution, with F being the field strength of the 3-form vector

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only the positive momentum modes. Doing field theory, these zero modes carry all the information about the non-trivial vacuum structure. In DLCQ the vacuum is trivial. The description we presented so far might have to be modified due to the effects of the zero modes.

potential [34]:

$$
\begin{aligned}
ds^2 &= f^{-1/3} dx_{||}^2 + f^{2/3}\left(dr^2 + r^2 d\Omega_4^2\right) \\
F_{\alpha_1\ldots\alpha_4} &= \frac{1}{2}\epsilon_{\alpha_1\ldots\alpha_4}\partial_{\alpha_5} H \\
f &= 1 + \frac{\pi N l_p^3}{r^3}.
\end{aligned}
\tag{1.15}
$$

Analyzing the zero modes of this soliton one can calculate that the worldvolume supports a 6d $\mathcal{N} = (2,0)$ supersymmetric tensor multiplet. From 11d SUGRA - IIA duality we immediately learn that the IIA NS5 brane hence also supports a (2,0) tensor multiplet. However this time one of the 5 scalars in the tensor multiplet lives on a circle (the one parametrizing the position of the 5brane in the 11th dimension). Using a normalization in which the scalar has mass dimension 2 [6] the radius of this circle is M_s^2.

[6] This is the natural normalization since the scalars sit in the same multiplet as the 2-form $B_{\mu\nu}$ which must have mass dimension 2, so that the B Wilson line is dimensionless

Chapter 2

Exploiting the SYM D-brane correspondence

2.1 Classical Hanany-Witten setups

After we have learned in principle how to use the SYM D-brane correspondence in order to engineer certain gauge theories in a stringy setup, we now want to exploit this correspondence and study possible applications. One of the most prominent and intuitive setups used to learn about gauge theories from string theory is the Hanany-Witten (HW) setup [35]. Let me briefly review the basic ideas. A very exhaustive review of these setups and their applications can be found in [36].

2.1.1 Branegineering

The idea behind Hanany-Witten setups is to study branes in flat space and get interesting gauge theories by having many flat branes intersecting each other. The matter content can be determined by simple, intuitive rules. Similarly deformations and moduli become easily visible. In the last chapter of this work I will establish a dictionary mapping HW setups to the other approaches, where the interesting dynamics is hidden in the background geometry. This way HW setups can be used to encode complicated looking information about deformations and phase transition in harmless looking brane moves. In several cases aspects of string theory in the background of the intersecting branes can be solved, leading to interesting results about the quantum gauge theory on the brane.

We start with the maximally supersymmetric (16 supercharges) SYM on the world-volume of a Dp brane. For definiteness let me discuss the case $p = 4$. [1] The idea will be the same in the other dimensions.

In order to go to more interesting physics with lower supersymmetry, we have to project

[1] As compared to $p = 3$ in the original work of [35].

out some of the degrees of freedom. This can be achieved by letting our D4 branes, which I henceforth will call color branes (N_c of those will give rise to SUSY QCD with N_c colors), end on another brane. The boundary conditions will do the job of projecting out certain states.

The concept of a brane ending on branes is a straight forward generalization of the defining property of D-branes as an object on which strings can end. Indeed all of the "brane ends on brane" configurations used in this work can be obtained from this defining setup via duality:

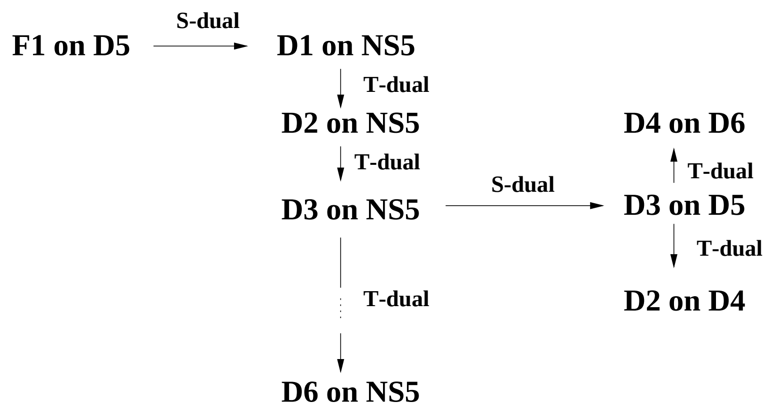

Figure 6: Configurations dual to a fundamental string ending on a D-brane.

Figure 6 illustrates this chain of dualities. Another way to understand the who-ends-on-whom rules is to study the worldvolume theories. The end of a brane is a charged object. In order for a given brane to be allowed to end on another brane, there should better be a field on the worldvolume that can carry away that charge. In the case of the fundamental string ending on the D-brane, the end of the string is charged electrically under the worldvolume gauge field. Similarly we can explain the other setups of figure 6 . For example the D2 brane ending on a IIA NS5 brane is the string like object charged under the 2-index tensor gauge field on the worldvolume, the D3 brane ending on the NS5 or D5 is the magnetic monopole on the worldvolume (which is a 2-brane in 6 dimensions).

In addition the non-vanishing field strength induced on the brane couples via the DBI (1.2) action to the scalars which describe the embedding of the brane and the brane has to bend in order to compensate the force exerted by the gauge field. Indeed it was shown in [37] that the DBI action allows for stable soliton solutions which can be interpreted as a string ending on a D-brane. Much can be understood about bending just on the base of symmetry arguments. As we will see in the following bending is really a quantum effect in the field theory. So for now I will neglect bending and only discuss the "classical" setup.

We choose to let our D4 branes end on two NS5 branes. This way the D4 brane stretches only over a finite interval bounded by the NS5 branes. Usually one chooses this finite interval to be in the 6 direction. This will do several things for us: for one certain degrees of freedom will be projected out by the boundary conditions, as desired. In addition, since our gauge theory now lives on a compact interval, the low energy theory will be governed by the KK-reduction on the interval. Thus effectively we will be dealing with a p dimensional theory. Last but not least the addition of the NS5 brane will break some more supersymmetry. We will remain with 8 supercharges, that is in the $p = 4$ case with $\mathcal{N} = 2$, as can be checked explicitly using (1.4) and (1.14). The following table displays the worldvolume directions of these branes. The additional D6 brane will be explained soon.

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
NS 5	x	x	x	x	x	x	o	o	o	o
D 4	x	x	x	x	o	o	x	o	o	o
D 6	x	x	x	x	o	o	o	x	x	x

The presence of these branes breaks the Lorentz symmetry group $SO(9,1)$ down to $SO(3,1) \times SO(2) \times SO(3)$. While the first part is our obvious Lorentz group in 4d, the rest should better have an interpretation as the R symmetry of the corresponding superalgebra. It has to be an R symmetry due to the fact that we interpreted the worldvolume scalars as positions of the branes in this "internal" part of spacetime, therefore the scalars and fermions on the worldvolume naturally transform like vectors and spinors under this internal Lorentz group. A symmetry group acting differently on the fermions and scalars in a supermultiplet is an R symmetry. Indeed the R symmetry of the $\mathcal{N} = 2$ SUSY algebra is $U(2) = SU(2) \times U(1) = SO(3) \times SO(2)$.

To determine from perturbative string theory which degrees of freedom are projected out would require an analysis in the background of NS5 branes. Luckily there is an easier way to figure out what is going on, by remembering once again that the scalars describe the position of the brane. Without the NS5 branes, we had 5 scalars in the 5d $\mathcal{N} = 2$ vectormultiplet, describing the position of the D4 brane in 45789 space. In addition we will get a 6th scalar after KK reduction from the 6-component of the vector itself. Under the 4d $\mathcal{N} = 2$ SUSY 4 of these scalars constitute the bosonic part of a hypermultiplet (HM), while the vector together with the 2 other scalars forms the bosonic part of the vectormultiplet (VM). Now requiring the D4 brane to end on the NS5 brane fixes its position in 789 space. The only scalars surviving are the 45 motion. From supersymmetry it than follows, that the surviving multiplet is the VM and the HM is projected out. The same will be true for any D brane ending on NS5 branes. So indeed in order to branegineer gauge theories a D brane ending on NS5 branes will be the starting building block. The gauge coupling of the gauge theory is also encoded in this simple setup. Due to the KK reduction the inverse gauge coupling is proportional to the distance L between the 2 NS5 branes. Using (1.3) the precise value for the gauge coupling of the p dimensional gauge theory from a Dp brane suspended between two NS5 branes is

$$g_{YM}^2 = \frac{l_s^{p-3} g_s}{L} \tag{2.1}$$

Moving the brane along the 45 direction turns on vevs for these scalars. This corresponds to moving on the Coulomb branch of the gauge theory. At generic points of the Coulomb branch all the D4 branes will be at a different 45 position and we are left with an unbroken $U(1)^{\text{rank}}$ gauge group, justifying the name.

As already indicated in the brane table above, using (1.4) and (1.14) there is one more brane one can add without spoiling any further SUSY, a D6 brane. Analyzing the possible brane motions one finds, that a D4 between two D6 branes supports a hypermultiplet while the vector is this time projected out. A D4 suspended from NS5 to D6 is stuck and hence supports no scalars and since we have $\mathcal{N} = 2$ this means no degrees of freedom at all. So in order to have a gauge theory we will keep having the D4 end on the two NS5 branes. Is there any new multiplet that arise from the presence of N_f "flavor" D6 branes? The D4 D6 system only contains D-branes, so it is easy to deal with. In addition to the strings ending only on the D4 brane, we will have strings ending on the D4 and on the D6. Analyzing their massless sector one finds N_f hypermultiplets in the fundamental representation of $SU(N_c)$. Naively one can understand this from the fact that such strings will have an N_c Chan Paton factor on their one end and an N_f Chan Paton factor on their other end.

The dynamics of this system in the IR will only be determined by the lightest branes. In the case of HW setups these will be the lowest dimensional objects around, that is the D4 branes. The $SU(N_f)$ symmetry is really a global symmetry, the gauge bosons from D6 D6 strings decouple. This general philosophy that only the smallest brane contributes to the dynamics carries over to HW setups in other dimensions, where we suspend Dp color branes between NS branes with $Dp + 2$ branes taking over the role of flavor branes. Motions of light branes will correspond to moduli, while motions of heavy branes are parameters of the theory.

Let us consider the limits involved in more detail. Let me discuss the case of D3 branes between NS5 branes and flavor D5 branes as in the original work of [35]. Since this is in type IIB the worldvolume of the NS5 brane supports just SYM and is easier to discuss.

In order to decouple gravity and higher string modes we send M_{pl} and M_s to infinity. We hold $g_{YM}^2 = g_s/L$ fixed. This sets the scale for all the gauge theory modes. In order to decouple the Kaluza Klein modes from the interval we want this to be much less than $1/L$, that is we have to consider weak string coupling. Indeed in this limit the gauge coupling on the NS5 and D5 branes, which is $1/M_s^2$ and g_s/M_s^2 goes to zero, justifying in a quantitative manner our assertion that at low energies the only surviving modes are those of the lowest dimensional brane, leading to a 3d gauge theory.

To summarize, we have shown that in the decoupling limit

$$l_s \to 0, \;\; l_{pl} \to 0, \;\; L \to 0, \;\; g_{YM}^2 \text{ fixed} \tag{2.2}$$

the HW setup with N_c Dp color and N_f D$p+2$ flavor branes describes an interacting p dimensional theory with $SU(N_c)$ gauge group and N_f flavors.

2.1.2 There's so much more one can do

So far we allowed ourselves to freely jump between dimensions in order to relate the classical setups. This is indeed possible by applying a simple T-duality to the original 3d setup. As long as we act inside the worldvolume of the NS5 brane it will stay an NS5 brane, while the color and flavor branes loose or gain a dimension whether we T-dualize a worldvolume or a transverse direction. The maximal dimension we can achieve this way is 6 from D6 branes between NS5 branes. These will be the main focus of this work. Gauge theories in more than 6 dimensions have a minimal of 16 supercharges. This is a classical constraint just following from the size of the spinor representation of $SO(6,1)$. Our branes know all this. In any dimension the 8 supercharge HW setup looks as follows:

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
NS 5	x	x	x	x	x	x	o	o	o	o
Dp	x	...	up	to	x^{p-1}	...	x	o	o	o
D$p+2$	x	...	up	to	x^{p-1}	...	o	x	x	x

In order to go down to 4 supercharges we need yet another kind of brane. One thing we can do is to rotate one of the players already present [38, 39]. Checking the unbroken supersymmetries according to (1.4) and (1.14) we see that for example an NS5' brane living in 012389 space will do the job. It will break 1/2 of the supersymmetries presented in the setup so far. In addition the $SO(3) = SU(2)$ part of the R-symmetry corresponding to rotations in the 789 plane is broken to $SO(2) = U(1)$ as required. Analyzing the massless modes on a D4 brane suspended between NS and NS' we now find that all the scalars are locked. Only the $\mathcal{N} = 1$ vector multiplet survives the projection. The same amount of SUSY will be preserved if we choose to rotate the second NS5 brane by any nonzero angle θ in the 4589 plane. The adjoint chiral multiplet from the decomposition of the $\mathcal{N} = 2$ vector multiplet under $\mathcal{N} = 1$ will receive a mass $m = \tan\theta$. For $\theta = 0$ one recovers the $\mathcal{N} = 2$ theory, for $\theta = \pi/2$ the adjoint decouples all together. In [38] they also found that when we use k coinciding NS5' branes a superpotential $W = X^{k+1}$ is created rather than the mass term X^2 which we obtained for $k = 1$. One can argue for a term like this by studying which flat directions such a term lifts in the classical field theory and than comparing with the possible brane motions. The second possibility is rotating the flavor branes [40]. This will leave the matter content untouched. Instead the superpotential $W = XQ\tilde{Q}$ required for an $\mathcal{N} = 2$ theory will be turned off continously with θ.

All these rotations are only possible for HW setups in 4 and lower dimensions, again reflecting the fact that 4 is the maximal dimension for SYM with 4 supercharges. Introducing even more branes with other rotations we can as well engineer gauge theories with 2 and 1 supercharges in 3 and 2 dimensions. Of course it is no problem to also write down configurations that preserve no supersymmetry at all. A generic setup will do so.

Since rotating branes as we have seen just corresponds to perturbing $\mathcal{N} = 2$ theories, only a very restricted class of $\mathcal{N} = 1$ theories may be constructed this way. For example (except for one exotic exception I will introduce later) no chiral gauge theories can be constructed this way. In order to do so it is necessary to generalize the idea of suspending a Dp brane on an interval in order to have a p dimensional gauge theory to having $Dp+1$ branes on a rectangle (a brane box) [41] [2]. This way generic models can be constructed, at least on the classic level. Recently it has been shown [43] that they are also good as a quantum theory as long as we choose an anomaly free matter content. This proof is done in the equivalent brane as probe picture. Since the relation between the various approaches will be one of my main subjects I will postpone a discussion of brane boxes to later chapters.

Let me at this point introduce another possible construction used to introduce flavors. It will also be the basic building block for product gauge groups. The basic question to analyze is the following: let us consider three NS5 branes at 3 different positions in the 6 direction. We will put N_c D4 branes on the first and N_c' D4 branes on the second interval. What is the low energy field theory corresponding to this? We will definitely get an $SU(N_c) \times SU(N_c')$ gauge theory. Additional matter will be produced from the D4 D4 strings stretching from the N_c to the N_c' branes. They have one Chan Paton factor in each gauge group. So we would expect to obtain bifundamental matter that is a hypermultiplet fundamental under both gauge groups. This can again be verified by checking that the allowed brane motions correspond to the classic moduli space of the gauge theory. Taking the third NS brane to infinity the second gauge group decouples and becomes a global symmetry. This way one can introduce flavors via semi-infinite D4 branes to the left or right.

Indeed the two ways of including flavors are related if we take into account the HW effect: whenever a flavor D5 crosses and NS5 a D3 is created in between them. This

[2] An equivalent way of viewing this is to view NS 5 branes at an orbifold as it was suggested in [42]. For us it seems more natural to either have the advantages of branes as probes or the HW setup. However the hybrid construction yields the same answers.

process holds in all the T-dual configurations as well. That brane creation must occur can be seen by looking at the so called linking number of the NS5 and D5 brane, which is a topological invariant. In order to have it unchanged in brane crossing prcesses, a D3 brane has to be created when D5 and NS5 pass. So if we move all the flavor branes to the far left or right, we will create semi-infinite branes. The number of flavors didn't change, just their realization. There are many equivalent pictures yielding the same gauge theory.

There is one more classical parameter we can introduce: the relative position of the NS5 branes in the 789 space. This corresponds to turning on an FI term κ, that is adding

$$\int d^4x d^4\theta \kappa D \tag{2.3}$$

to the Lagrangian, where D is the auxiliary field in the vector multiplet. Once more one can prove this by analyzing the possible brane motions in the presence of this term and compare with classical gauge theory.

We have now all the tools in order to branegineer quite arbitrary classical gauge groups. The space of all possible brane motions corresponds to the classical moduli space of the gauge theory. We already identified the 45 motions as the Coulomb branch. The Higgs branch is seen if we split the color branes along the flavor branes. As shown earlier such a brane segment between two color branes supports the 4 scalars in a hypermultiplet, corresponding to moving the segment off, leaving a broken gauge group. A generic point on the Higgs branch then would look as follows:

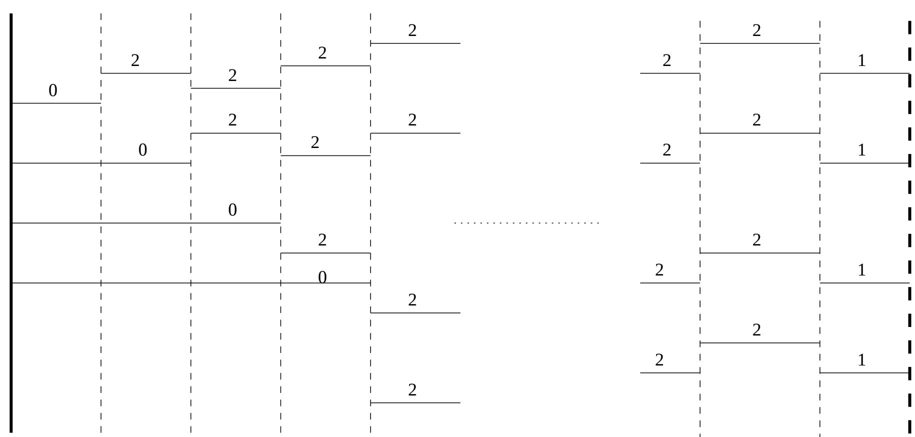

Figure 7: A maximally broken situation on the Higgs branch in a theory with 4 supercharges. The numbers denote the complex scalar degrees of freedom associated with motions of the given brane piece. The s-rule has been taken into account.

In the picture, the fat line is the NS brane, the fat broken line is the NS' brane, the horizontal lines denote color and the broken vertical lines denote flavor branes. For a theory with 8 supercharges, the NS' has to be replaced by an NS branes and both sides of the picture look the same. The picture does not depend on the dimension (it has to be 4 or less since we are only dealing with 4 supercharges). The numbers denote the complex degrees of freedom associated to moving around the brane pieces. In order to reproduce the results from classic gauge theory we have to take into account the so called s-rule [35]: given a single pair of NS and flavor brane, only a single color brane is allowed to end on them. Summing up all the degrees of freedom we find that the complex dimension of the Higgs branch is (for $N_f \geq N_c$)

$$d_{Higgs} = (2N_f - N_c)N_c = 2N_f N_c - N_c^2 \tag{2.4}$$

in perfect agreement with the field theory counting, where we just count the scalar fields that are not eaten by the Higgs mechanism,

$$d_{Higgs} = \text{number of scalars} \;-\; \text{number of vectors} = 2N_f N_c - N_c^2. \tag{2.5}$$

The Higgs branch is not visible when we use semi-infinite branes to realize the flavors. Turning on FI terms (that is separating the NS branes in 789 space for 8 supercharges or just in 7 space for 4 supercharges) kills the Coulomb branch of the $\mathcal{N}=2$ theories, as is expected. Since the color branes are only allowed to extend in the 6 direction in order to preserve SUSY they have to split on the flavor branes, that is we have to be on the Higgs branch.

2.2 Solving the quantum theory

2.2.1 Bending and quantum effects

Above I argued in a qualitative manner that a brane has to bend if another brane ends on it in order to balance the force exerted by the flux it has to support, since the end of the other brane represents a charge on its worldvolume. Let me discuss this point in a bit more of a quantitative manner following [44, 37]. Before doing so I'd like to show that bending indeed is a quantum effect from the point of view of the SYM.

Let me discuss the $\mathcal{N}=2$ setup in 4d since this was the main example I discussed on the classical level so far. The classical value of the Yang-Mills coupling is given by $g_{YM}^2 = \frac{g_s l_s}{L}$ where L again denotes the length of the interval. In order to focus on the SYM modes in this setup we want to take the decoupling limit (2.2). The tension of the D4 brane and the NS5 brane are $\propto \frac{1}{g_s}$ and $\propto \frac{1}{g_s^2}$ respectively. Since g_s is proportional to g_{YM}^2 we see that for zero coupling the NS5 brane is infinitely heavy and hence is not pulled or bent by the D4 brane. Turning on the loop corrections in the gauge theory is reflected in the bending of the branes.

As pointed out earlier, the DBI action couples the scalars and the vector multiplet, so a non-trivial flux on the brane (as it is induced by the end of another brane) can be supported by simultaneously turning on vevs for the scalars, that is bending the brane since the scalars describe the position of the brane. We are only interested in the $l_s \to 0$ limit, so the SYM approximation of the DBI is sufficient. We are looking for a stable setup with non-vanishing flux. One way to construct this is to look at BPS setups. One can construct them by demanding some preserved supersymmetry. The no force condition is then guaranteed. This discussion was carried out in [37]. The same results for the bending as found by Callan and Maldacena can be obtained in a more qualitative fashion following the discussion of Witten and noting that

- the bending should be proportional to the net charge, that is:
  branes ending on the left - brane ending on the right
- the scalar terms in the SYM action demands a minimal area embedding of the brane, therefore the bending should be a solution of a Laplace equation
- the bending can only depend on the worldvolume directions transverse to the end of the other brane and for symmetry reasons should only depend on the radial distance

From this discussion it follows that in order to solve for the bending of the NS branes in a p dimensional HW setup we should solve the Laplace equation in the $6-p$ NS5 brane worldvolume directions transverse to the end of the Dp brane, leading to r, log, $1/r$, ... bending in 5,4,3,... dimensions, where r denotes the radial coordinate along the NS5 brane worldvolume away from the end of the Dp brane. The coefficient in front of the r dependence will be proportional to the net number of branes. In 6d the transversal space is zero dimensional. No bending can occur, no field strength can be supported. The net number of branes has to be zero.

In theories with 8 supercharges the only perturbative corrections to the β function come from 1-loop. Since the gauge coupling is encoded in the length of the interval, the r^2, r, log, $1/r$, ... bending in 6,5,4,3,... dimensions reflects precisely the known 1-loop running of the gauge coupling in these dimensions.

There is one peculiar effect due to the bending. As I stated above, only motions of the light branes are moduli of the setup. Taking into account the bending we should make this slightly more precise: only such motions leaving the asymptotic form of the heavy branes untouched will correspond to moduli. Consider the simplest setup of N_c color branes on the interval. We argued above that the classic theory describes $U(N_c)$ gauge group. A generic point on the Coulomb branch will have an unbroken $U(1)^{N_c}$ corresponding to moving the N_c branes independently along the NS5 branes. If we have a $1/r$ or faster fall of in the bending, quantum mechanically the color branes only created a little dimple on the NS5 and the asymptotic of the NS branes will stay unchanged. However if we have logarithmic or linear bending, changing the center of mass would correspond to changing the asymptotic behaviour. Therefore on the quantum level the center of mass $U(1)$ part is frozen out and we are only left with an $SU(N_c)$ gauge theory. This reflects the field theory statement that only in 3 and lower dimensions a $U(1)$ gauge theory can lead to interacting IR physics.

Once we have frozen out the $U(1)$ we have to find a new interpretation for the 789 position of the branes. An FI term is only possible for abelian gauge groups (the auxiliary component of the vector multiplet transforms as an adjoint and is not gauge invariant unless we are dealing with $U(1)$). On the other hand SU theories have a new branch: the baryonic branch [3]. Interpreting the 789 position as a deformation that forces us on this baryonic branch yields the right moduli spaces when compared with gauge theory calculations.

In addition to the effects from loops there are also corrections due to instantons. These instantons can be seen directly in the brane picture. It is known that a D$p-4$ brane within a Dp brane satisfies the 4 dimensional YM instanton equations [45]. So D0 branes are instantons within D4 branes. To interpret these D0 branes as instantons in the 0123 spacetime we should consider Euclidean D0 branes whose world-line stretches along the 6 direction so that they are contained within the D4 branes between the NS5 branes [46, 47, 48]. This way the field theory objects and quantum effects have been mapped to D-branes and their properties. The problem is now to solve the theory after including all these effects.

2.2.2 Lifting to M-theory

So far we have seen how the quantum corrections manifest themselves in the stringy embedding. Now we are going to actually solve them. This analysis was performed in the remarkable work of Witten [44]. To implement this solution we use the IIA M-theory duality and view our setup as an 11d setup. For large values of the radius R of the eleventh dimension 11d SUGRA will be a good approximation. Since both the NS5 and the D4 lift to an M-theory M5 brane, our whole setup will be described by just a single M5 brane in 11d. For this brane to be a solution of the 11d equations of motion, it will be determined by the requirement that it lives on a minimal area cycle (this way Laplace equation sneaks in again). Therefore the shape of our branes will be solely determined by solving the problem of soap bubbles: we fix the asymptotics of the branes and they will arrange themselves to live on a minimal area cycle.

Before I move on and show that indeed all quantum effects, perturbative as well as non-perturbative, are indeed incorporated in the shape of the M5 brane in the 11d SUGRA limit, let me first discuss the validity of this limit. Let me discuss once more the precise

[3]Going from $SU(N_c)$ to $U(N_c)$ basically corresponds to gauging baryon number

decoupling limit from the IIA point of view. Here we want to decouple gravity, heavy string modes and KK modes from the finite interval, that is send M_s and M_{pl} to infinity and L to zero, while keeping the gauge coupling $g_{YM}^2 = \frac{g_s}{M_s L}$ fixed. Translating into 11d units $g_{YM}^2 = \frac{R}{L}$ is supposed to be fixed as L goes to zero. But this requires us to also take R to zero. This is the opposite limit of the one where we are able to solve! The only reason why I nevertheless will go on and do some calculations in what follows is that holomorphic quantities will be protected and we can calculate them at any value of R, even though they will only correspond to gauge theory quantities in the small R regime. Also qualitative aspects, like e.g. whether the theory confines, are supposed to agree. But it has been shown in [49] that in general unprotected terms do not agree. This is a pity, since the holomorphic information is encoded in the SW curve and was known already from pure field theory considerations [7]. The branes only give us a nice organizing principle for analyzing the holomorphic quantities in complicated situations, where field theory "guess and check" methods like they are usually employed in order to obtain SW curves do not work anymore, like in situations with many product groups [44] or matter content that leads to non hyperelliptic curves, e.g. two-index symmetric tensors [50].

In order to get new information we have to solve the full string theory in the background of NS and D4 branes, a task that seems too hard with today's tools. This is a problem that is common to most approaches of trying to get new gauge information out of string theory, including the most recent one, the Maldacena large N conjecture, about which I will make some more comments in what follows. There is always one regime where we can easily compute and another regime where we want the answer. However for Maldacena's case to get the full answer we have to do string theory on $AdS_5 \times S_5$ with some RR flux turned on. This hasn't been solved so far, but due to the large symmetries of the background at least there is hope.

Above I have argued that in M-theory we can solve for the exact shape of the M5 brane by solving the minimal area condition given a set of boundary conditions, which incorporate the classical input. Requiring SUSY of the low energy effective action amounts to restricting to supersymmetric cycles, a special subclass of minimal area cycles [51, 52]. A cycle is supersymmetric if a brane wrapping it preserves some amount of supersymmetry. For 2-cycles this condition directly translates into holomorphicity. A 2-cycle obtained as the zero-locus of a holomorphic function of 2 complex coordinates will preserve 1/2, as the zero locus of 2 equations in 3 variables 1/4 of the original supersymmetries.

Now let me go ahead and show that indeed all the quantum effects are incorporated in the shape of the brane. After this I will write down the solution as obtained in [44]. We already found that the bending of the brane incorporates the perturbative corrections. In the type IIA setup the non-perturbative corrections have to be put in by hand. They are represented by Euclidean D0 branes stretching along the 6 direction. In order to incorporate all instanton effects we would have to sum over all setups bound with a given number of D0s. What becomes of the D0 branes once we lift to 11d? Remember that the 11d origin of a D0 brane is momentum around the compact x^{10} circle, that is a bound state with D0 branes corresponds to adding 10 momentum. The 10 position of the D4 brane would be a function of time:

$$\dot{x}^{10} \neq 0 \quad \Rightarrow \quad x^{10} = x^{10}(t). \tag{2.6}$$

Our field theory instantons correspond to Euclidean D0 branes, that is their worldvolume stretches along the finite interval in the 6 direction instead of stretching in time. According to the same philosophy they should correspond to "twist" in the 11d setup, that is

$$x'^{10} \neq 0 \quad \Rightarrow \quad x^{10} = x^{10}(x^6). \tag{2.7}$$

All quantum effects lead to "bending" and "twisting" in 11d and are hence incorporated in the shape of the brane.

Now let me move on and present the solution. Let me discuss the general setup of $n+1$ NS5 branes leading to a product of n $SU(N_\alpha)$ gauge groups. We choose as complex coordinates $t = e^{-s} = e^{-(x^6+ix^{10})/R}$ and $v = x^4 + ix^5$, so that a D4 brane is at $v = const.$ and an NS5 brane at $t = const$. This is the only complex structure in which our two ingredients can be written as holomorphic functions. Since our 2-cycle asymptotically will look like NS5 or D4, we have to choose this structure. Taking the exponential in the definition of t beautifully takes into account the compactness of x^{10}. Using x^{10} as θ-parameter one can introduce a complex coupling constant

$$\tau_\alpha = \frac{\theta_\alpha}{2\pi} + i\frac{4\pi}{g_\alpha^2} = i(s_\alpha - s_{\alpha-1}). \tag{2.8}$$

where the s_α denote the positions of the various NS5 branes. At the one loop level the 1-loop running of the gauge coupling from the logarithmic bending is simply generalized to

$$s = \sum_i \log(v - a_{i,\alpha}) - \sum_j \log(v - b_{j,\alpha}), \tag{2.9}$$

where a and b denote the position of the D4 branes to the left and right of the NS5 brane under consideration.

In order to incorporate all the non perturbative effects we have to solve for a surface Σ in the four-manifold $\mathbf{R}^3 \times S^1$ which is parametrized by the coordinates s and v. As stated above, $\mathcal{N} = 2$ space-time supersymmetry requires that s varies holomorphically with v, such that Σ is a Riemann surface in $\mathbf{R}^3 \times S^1$. Using $t = \exp(-s)$, Σ is defined by the complex equation

$$F(t, v) = 0. \tag{2.10}$$

At a given value of v, the roots of F in t are the positions of the 5-branes, i.e. F is a polynomial of degree $n+1$ in t. On the other hand, for fixed t, the roots of F in v are the positions of the IIA 4-branes. Recall briefly the situation of a model with two 5-branes, i.e. $n = 1$. This model is described by the curve

$$F(t, v) = A(v)t^2 + B(v)t + C(v) = 0, \tag{2.11}$$

where A, B and C are polynomials in v of degree k. More specifically, the zeroes of $A(v)$ ($C(v)$) correspond to the positions of the semi-infinite 4-branes ending from the left (right) on the 5-branes. On the other hand, the polynomial $B(v)$ belongs to the k 4-branes suspended between the two 5-branes. After suitable rescaling and shifting of v and t, one obtains for pure $SU(k)$ gauge theory simply $A = C = 1$. $B(v)$ is then a polynomial of the following form:

$$B(v) = v^k + u_2 v^{k-2} + \ldots + u_k, \tag{2.12}$$

where the u_i are the order parameters of the theory. In order to include, for example, N_f flavors of hypermultiplets from the right, $C(v)$ takes the form

$$C(v) = f \prod_{j=1}^{N_f} (v - m_j). \tag{2.13}$$

It is important to note that these curves precisely coincide with the Seiberg-Witten Riemann surfaces [7]. The entire holomorphic $\mathcal{N} = 2$ prepotential is encoded in the curve $F(t, v) = 0$.

Chapter 3

Applications of the brane construction

As we have seen in the previous chapter there is a deep connection between gauge theories and string dynamics. In the following I will show how this connection can be used to understand certain aspects of gauge theory and the phase structure of string theory, especially the transitions between topologically distinct vacua.

3.1 Dualities in the brane picture

3.1.1 Exact S-duality

General Idea

In the previous chapter I discussed that there are certain symmetries in string theory, relating one theory at strong coupling to another theory at weak coupling. More generally speaking, in theories with a free parameter (e.g. the coupling) we identify theories with different values of this parameter. The easiest example is the self-duality of type IIB string theory, where this free coupling is the string coupling and the duality symmetry identifies the theory at g_s with the theory at $1/g_s$. If we also include the axion, g_s is enhanced to a complex coupling parameter and we have a whole $SL(2,Z)$ symmetry acting on it. This strong weak coupling duality exchanges fundamental string and D-string. Taking into account the full $SL(2,Z)$ one finds a whole zoo of (p,q) strings in type IIB. This kind of duality is usually called S-duality.

The idea that such a duality may also exist in field theory is very old. It is easy to convince oneself that classical Maxwell theory is invariant under the exchange of

$$E \leftrightarrow -B, \quad j_{el.} \leftrightarrow j_{magn.}.$$

Of course this "duality" can only be valid if we introduce magnetic charges as well as electric charges. Since in quantum mechanics consistency requires that electric charge e

and magnetic charge g satisfy the Dirac quantisation condition

$$eg = 2\pi n$$

we see that any implementation of this electric magnetic duality symmetry in a quantum theory would automatically provide an S-duality, since the charges also play the role of coupling parameters, and hence strong electric coupling corresponds to weak magnetic coupling. This idea of realizing a quantum version of electric magnetic duality was first voiced in [53] and is hence referred to as Montonen-Olive duality. The problem is that in a standard field theory, like QED, the coupling constant is not a good operator, but runs according to the renormalization group. Therefor it is expected that S-duality is only realized in finite theories. One way to guarantee finiteness is to consider maximally supersymmetric Yang-Mills where supersymmetry guarantees cancellation of all divergencies.

Indeed by now it is believed that $\mathcal{N} = 4$ SYM realizes Montonen-Olive duality. Like in IIB string theory the $g \leftrightarrow 1/g$ duality is enhanced to a full $SL(2, Z)$ duality once we include the theta angle and its invariance under 2π shifts. Since at least one of the two theories we want to identify with each other is at strong coupling, it is again impossible to directly prove the duality. But several consistency checks have been performed, the most convincing being Sen's calculation [54] establishing the existence of all the (p, q) dyon states $SL(2, Z)$ dual to the electron multiplet.

Another way to establish this Montonen-Olive duality is to exploit the gauge theory-string theory correspondence. The idea is that by embedding the gauge theories into string theory, string dualities directly translate down into dualities of the field theory. Of course this is not a "proof" of the duality, but one reduces the number of independent assumptions to just string dualities.

Indeed in the case of S-duality in $d = 4$, $\mathcal{N} = 4$ SYM it is straight forward to realize this idea. We just study flat D3 branes in uncurved space. According to (1.3) the gauge coupling on the D3 brane is just $g_{YM}^2 = g_s$. (p, q) dyons in the field theory are (p, q) strings ending on the brane. S-duality of IIB leaves the D3 brane invariant. Field theory duality is just what is left of the string theory duality after we decoupled the bulk modes.

S-duality in $\mathcal{N} = 1, 2$

There are some examples of finite theories with less than 16 supersymmetries. In $\mathcal{N} = 2$ the β function is solely determined at 1-loop, $\beta \propto 2N_c - N_f$. So just by choosing the right matter content the theory is finite. It is believed that all these theories do posses an S-duality acting on their coupling constant. This was first established for the $SU(2)$ case with 4 flavors in [55].

In order to find the S-duality in the brane picture one first performs the lift to M-theory. Finiteness translates into a no bending requirement. The duality group is then the homotopy group of the resulting Riemann surface [44]. Since this analysis only depends on having the curve and not whether we obtain it from brane physics or just from field theory this statement does not shed much new light on $\mathcal{N} = 2$ S-duality. The story becomes clearer when going to the branes as probes picture. Finiteness corresponds to cancellation of tadpoles in the orbifold background, as I will show when discussing the duality between these two approaches. The gauge theory can be realized as D3 branes on top of an ADE singularity. As in the $\mathcal{N} = 4$ case S-duality of the embedding IIB string theory directly translates down to S-duality of the gauge theory. In general the duality group will be bigger than just $SL(2, Z)$ since it will also include transformations that just relabel the gauge group factors. These are obvious symmetries. Since they in general do not commute with S-duality a large discrete duality group is generated.

The orbifold construction carries over straight forwardly to $\mathcal{N} = 1$. Now we have to consider D3 branes on top of an C^3/Γ singularity with $\Gamma \in SU(3)$. Here we have to distinguish two kinds of tadpoles [43]: tadpoles from twist elements that leave a 2d plane fixed (so that they look like $\mathcal{N} = 2$) cancel only in a finite theory, all other tadpoles have to cancel in order for the theory to be free of anomalies. This can be understood as follows: a non-vanishing tadpole corresponds to a net charge. In a compact space this has to vanish. If our orbifold has an r dimensional fixed plane, we are dealing with a charge in r dimensions. If r is bigger than 2 there are no problems with this. For r=2, as we have to deal with in the $\mathcal{N} = 2$ case and for the special tadpoles in $\mathcal{N} = 1$ the charge will lead to a logarithmic divergence. I will prove that this divergence is nothing but the running of the gauge coupling in Chapter 4. So cancellation of tadpoles of the first kind ($r = 0$) is a necessary requirement, tadpoles of the second kind ($r = 2$) vanish only in theories with no running. Again one can see the finiteness requirement in the dual brane box picture as a no-bending requirement [56]. S-duality is again established trivially by the embedding via D3 brane in type IIB.

There are certainly other $\mathcal{N} = 1$ S-dual pairs. The list of finite theories, which can be constructed using the methods of [57] or [58] exhibits many examples which can not be realized in any brany way so far, and probably most of them exhibit some kind of S-duality. One way one might hope to generate such S-dual pairs is to scan through all finite theories whose Seiberg dual is known. Before embarking on this discussion let me explain Seiberg duality and its brane realization.

3.1.2 Seiberg duality and Mirror symmetry

Universality

Another interesting aspect of field theory that can be addressed quite systematically in a brany language is universality. The phenomenon of universality is due to the effect that the renormalization group (RG) flow is irreversible. Evolving a theory towards the IR, certain information about the theory is lost. Several possible deformations of the theory are irrelevant and do not lead to new IR physics. Many different theories hence flow to the same fixed point. The detailed information encoded in the UV physics does not matter and all physical IR properties are just encoded in the structure of the fixed point theory. Since many different theories flow to the same fixed point their IR physics is described by the *universal* properties of the fixed point. All theories flowing to the same fixed point are usually referred to as a universality class. This is the closest we can get to duality in the context of theories with a running coupling. The dual theories (which are usually still referred to as electric and magnetic) no longer are identically, but nevertheless describe the same physics in the IR.

In field theory it is often very difficult to establish whether two different theories belong to one and the same universality class. Usually one identifies certain possible deformations of the theory as irrelevant. This is done analysing their quantum dimensions. Once we know the precise dimension (that is after quantum corrections have been taken into account) of a given operator we can read off from the dimension if the corresponding dimensionful coupling increases or decreases once we multiply all length scales involved with a certain scaling factor. The corresponding operators are called relevant or irrelevant. The former lead to a different IR description, latter leave the IR description unchanged. Of special interest are dimensionless operators. They are usually referred to as marginal. Existence of an exactly marginal operator leads to a series of FPs which can be smoothly deformed into each other by tuning the marginal coupling, that is a fixed line.

This way one can establish that theories which have the same Lagrangian up to some irrelevant operators belong to the same universality class. A much more spectacular example of such a matching was found in [59], where it was established that even two theories with a completely different gauge group can belong to the same universality class. Usually this phenomenon is called Seiberg duality. This nomenclature is due to the fact that one can view the two different UV descriptions as the dynamics of electric and magnetic variables respectively. The name duality might be a little bit misleading, since the two systems do describe different physics away from the fixed point. It is however quite impressive that they do describe the same physics in the IR.

Most statements about dualities of this kind are still conjectural, even though a huge amount of evidence has been accumulated. It would be desirable to gain a better understanding of this phenomenon of duality from the branes / SYM correspondence. The basic strategy is the following: we should identify certain brane moves as irrelevant. This can for example be done by matching the resulting gauge theories which are known to belong to the same universality class. Of course an intrinsic stringy explanation of why such an brane move should leave the IR physics unchanged would be desirable. It would basically complete the proof of Seiberg duality. But once we have identified a certain brane move as irrelevant, we can produce a vast variety of field theories belonging to common universality classes, by applying the irrelevant brane move to various configurations.

One example of such an irrelevant brane move is changing the 6 position of the various branes involved. Since the 6 direction is the one along which the color branes stretch, it seems reasonable to assume that the low energy physics shouldn't depend on the 6 positions involved. Effectively we performed a Kaluza Klein reduction along the 6 direction to obtain the p dimensional physics from the p+1 dimensional worldvolume of the color branes. In a Kaluza Klein reduction we throw away everything but the zero mode (that is the constant mode) along the compact direction. We do not expect that our low energy physics is sensitive to any structure (that is to the brane positions) in the KK reduced direction. Let me show in the following how one can use this assumption to obtain Seiberg duality and its 3d cousin, mirror symmetry, from this brane move. After that I will briefly comment on the validity of the assumption, that this brane move is indeed irrelevant.

Seiberg duality

Probably the most famous example of an IR duality of this type is the equivalence of SUSY QCD with N_f flavors and N_c colors with another SUSY QCD, also with N_f flavors, but with $N_f - N_c$ colors and additional singlet fields called mesons, coupling via a superpotential $Mq\bar{q}$ with the quark fields. Let me explain the phase diagram of SUSY QCD as it was discussed in [59]. For other gauge groups the phase structure will look very similar, with $r = N_f/N_c$ replaced by the ratio μ_{matter}/μ_{adj} where μ denotes the quadratic index.

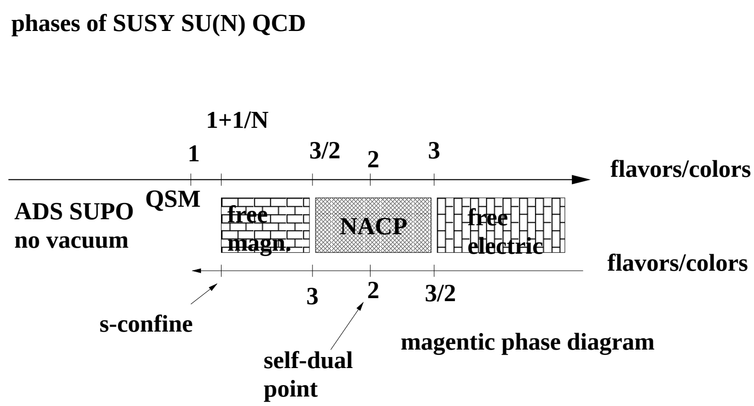

Figure 8: Phase diagram for SUSY QCD

For small r holomorphy and symmetries allow for a unique superpotential whose minimum is at infinity in all the moduli (the ADS superpotential). An instanton calculation at $N_f = N_c - 1$ shows that it is indeed generated for all $r < 1$. For $r = 1$ the quantum moduli space is described by mesons and baryons with a unique quantum constraint again fixed by symmetries and holomorphy. For $N_f = N_c + 1$ again baryons and mesons are the right degrees of freedom. The classical moduli space stays uncorrected. This behaviour is referred to as "s-confining" in the literature [60]. It is a special case of Seiberg duality with a trivial magnetic gauge group.

For r a little bit below 3 [1] one can establish the existence of a non-trivial IR fixed point following the arguments of Banks and Zaks [61]. The relation between R-charge and conformal dimension contained in the superconformal algebra tells us that this fixed point behaviour has to break down at $r \leq 3/2$. Assuming that the non-trivial fixed point theory, usually referred to as Non-abelian Coulomb phase, really holds in the whole regime $3/2 < r < 3$, a beautiful and consistent picture emerges. It is in this NACP that the dual description comes into play. Using the same borders in the dual group one can see that it is also in its NACP. The duality then states that this is indeed the same fixed point. There is plenty of evidence for this conjecture

- the 't Hooft anomaly matchings are satisfied
- the moduli spaces match
- under perturbations via superpotential terms we flow to new consistent dual pairs
- the ring of chiral operators matches

For $r \geq 3$ the electric theory loses asymptotic freedom and the theory is free in the IR. For $r \leq 3/2$ the electric theory is strongly coupled and intractable, however extending the duality conjecture to this regime one finds a free magnetic phase. For special values of r one encounters self-dual pairs. In the case of SUSY QCD this happens for $r = 2$. As shown in [62] in more general gauge theories self-duality might show up at $r = 1 + 1/n$ for some integer n (in this notation SUSY QCD realizes selfduality at $n = 1$).

It was shown by [38] that the two "dual" theories can indeed be connected via a motion of branes in the 6 direction. One important thing one has to remember from our discussion in Chapter 2 is the Hanany-Witten effect [35] of brane creation when an NS5 brane passes an D6 brane. Taking the NS5 brane all the way through the flavor branes and then "ϵ" around the NS5' brane one obtains a brane theory that is described by the dual SYM.

Mirror symmetry

Another example of universality is mirror symmetry in 3 dimensions. It was first discovered by Intriligator and Seiberg in their study of 3 dimensional $\mathcal{N} = 4$ theories (that is 8 supercharges). In 3 dimensions the vector is dual to a scalar, so that both the VM and

[1] To be more precise: N_f, $N_c \to \infty$, $g_{YM} \to 0$ with $g_{YM}^2 N_c$ fixed and $\frac{N_f}{N_c} = 3 - \epsilon$.

the HM just contain 4 scalars as bosonic degrees of freedom. Supersymmetry requires the Coulomb and the Higgs branch both to be hyper-Kahler manifolds. There might hence be a symmetry mapping the Higgs branch of one onto the Coulomb branch of another theory. Since the scalar one obtains from dualizing the vector lives on a circle of radius g_{YM}^2 while all the other scalars are in general non-compact, one should expect that such a symmetry can only exist at infinite coupling (that is in the far IR since the coupling has mass dimension 1).

Taking into account quantum corrections the Higgs branch remains untouched while the Coulomb branch is corrected at 1-loop. Intriligator and Seiberg constructed theories which are mirror to each other in the sense that their quantum moduli spaces agree in the far IR with the role of Coulomb and Higgs branch swapped.

The implementation of mirror symmetry in string theory uses $SL(2, Z)$ duality of type IIB and again the irrelevance of the motion in the 6 direction. This analyzes was performed in the original HW paper [35] and was one of the main motivations for introducing these brane setups. Acting with S-duality on a 3d HW setup changes NS5 branes and D5 branes. After rearranging branes in the 6 direction one winds up with a system that has again a SYM interpretation. If we start with a theory with a single gauge group and a lot of matter, we will end up with a product of many gauge groups and just two fundamental matter multiplets from the two original NS5 branes turned into D5 branes. I will present examples later on.

E.g. the mirror of $U(N)^k$ with bifundamentals and a single fundamental in one of the gauge groups is $U(k)$ with an adjoint and N flavors. In the HW realization we put the theory on a circle. The electric theory is obtained via k NS5 branes and one D5 brane in one of the gauge groups. The mirror has just one NS5 brane, which yields $U(k)$ with an adjoint [2]. The N D5 branes add the N matter multiplets.

Counting quaternionic dimensions of the branches is very simple. The dimension of the Coulomb branch is just the rank of the gauge group, so it is $N \cdot k$ in the original and k in the mirror. The Higgs dimension is obtained by counting the number of HMs not eaten by the Higgs mechanism, so it is the number of HMs minus the number of VMs, that is $Nk^2 + k - Nk^2 = k$ for the original and $Nk + k^2 - k^2 = Nk$. So we find the expected agreement. Doing a 1-loop calculation one can check that not only the dimension, but really the full hyper-Kahler metric agrees [63].

Generalization to USp groups needs some "song and dance" since we now have to deal with the S-dual of the O5. This analysis was performed in [64]. We can get mirror symmetry in this case also via combined S- and T-duality in the brane setup: consider the setup with a single NS5 and N D5s. T-dualizing this yields k D2 branes probing a

[2] Below I will give a more detailed discussion of HW setups with a compact x^6 direction, but it is straightforward to see that a single NS5 on the circle gives us a single gauge group with an adjoint from the "bifundamental" starting and ending in the same group.

background of N D6 branes. In the S-dual picture we have just a single D5 and N NS5s. T-dualizing now yields D2 branes probing an A_{N-1} singularity [65] with one extra matter multiplet coming from the D5. We will see later that these give indeed rise to the gauge groups presented above. I think this is a very beautiful realization of mirror symmetry, since it relates directly the two most prominent backgrounds which were studied using the brane probe technique. In this picture generalization to USp gauge groups is straight forward and yields a correspondence between D2 branes probing a D_N singularity and D2 branes probing an O6 $+N$ D6 system.

Combining both: gauge theory with $\mathcal{N} = 2$ in d=3

Mirror symmetry can be generalized to $\mathcal{N} = 2$ in d=3 [66, 67]. The mirror can be thought of as a theory of vortices. In addition one can still perform the EGK brane move that yielded Seiberg duality in 4 dimensions, so one might expect that these theories do exhibit two different kinds of dualities, mirror symmetry and Seiberg duality. Indeed it was argued in [68] and [69] from the field theory point of view, that the Seiberg duals suggested by the brane picture still hold in the 3d setup. This way one can produce not just two but really very many gauge theories that live in the same universality class.

Irrelevance of the 6 position?

I have presented several examples of theories in the same universality class by assuming that the 6 position in HW setups is irrelevant. This assumption was based on the fact, that in order to identify the low-energy field theory we effectively KK reduced on the interval thereby throwing away all modes that could probe any structure along this direction.

This argument could be spoiled if we encounter phase transitions when moving the branes around. Especially dangerous are points, where we move branes past each other. In the case of a NS5 brane crossing a D5 brane we are saved by the brane-creation mechanism and the s-rule. However there are situations when the IR physics indeed does change with the relative 6 position of branes. This happens when NS5 branes meet parallel flavor branes, like it is possible in HW setups with 4 supercharges (the NS5' is parallel to the D6 in the conventions used so far). As examples I will discuss the enhancement of the chiral global symmetry and the phenomenon of flavor doubling, both related to D6 branes crossing NS5' branes. Another example of a similar phase transition was discussed in [40] where it was shown that passing flavor branes which are rotated with respect to each other past each other changes the superpotential of the corresponding gauge theory.

As a first example let me discuss the appearance of enhanced chiral symmetry as suggested in [70, 71]. Let us consider a 4d gauge theory. The global symmetry of flavor rotations is visible as the decoupled $SU(N_f)$ gauge theory on the D6 worldvolume. With 8 supercharges that is all we expect since the rotations of fundamental and antifundamental

$\mathcal{N}=1$ chiral multiplets are linked via the $\mathcal{N}=2$ superpotential $QX\tilde{Q}$, where X is the adjoint scalar from the VM. For the same reason SO and USp gauge groups will have USp and SO global symmetry respectively (these are the subgroups of the full $SU(N_f)$ flavor rotations that leave the superpotential invariant). However in the situation with 4 supercharges no such superpotential is present. In the SU case we expect the full chiral $SU(N_F)_L \times SU(N_F)_R$ symmetry, while the orthogonal and symplectic gauge groups will have a full $SU(N_f)$ rotation group. Generically the branes only see the $\mathcal{N}=2$ remnant. If we move all D6 branes on top of the parallel NS5' brane they can split (this is just a 6d HW setup). The half D6 branes will realize the full global symmetry. For this to be possible we had to choose a very particular 6 position of the D6 branes.

This is just a special example of the more general phenomenon of "flavor doubling" [72] whenever a D6 brane meets an NS5'. Consider the following cross configuration:

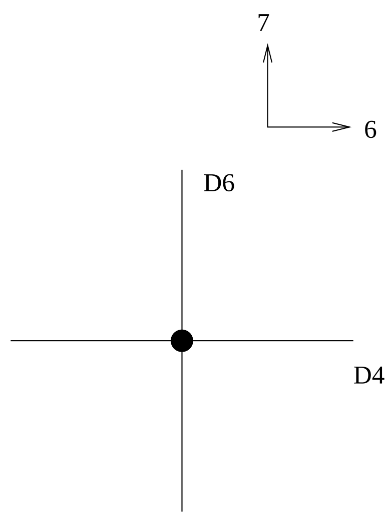

Figure 9: A "cross" configuration. An intersection of a D4 brane, a D6 brane and an NS5′ brane.

Let me identify the matter content corresponding to this configuration. First let us recall that when a D6 brane meets a D4 brane there is a massless hypermultiplet Q which transforms under the $U(1) \times U(1)$ symmetry groups which sits on the D4 and D6 branes. The number of supersymmetries for such a configuration is 8 and so there is a superpotential which is restricted by the supersymmetry to be $(m-x)\tilde{Q}Q$, where m is the 45 position of the D6 brane, x is the 45 position of the D4 brane.

We can slowly tune the position of an NS5′ brane to touch the intersection of the D4 and D6 branes. Locally the number of supersymmetries is now 4. At this point, both the D4 and the D6 branes can break and the gauge symmetry is enhanced to $U(1)_u \times U(1)_r \times$

$U(1)_d \times U(1)_l$. The u, d indices correspond to the two parts of the D6 branes and the l, r indices correspond to the two parts of the D4 branes.

As usual for the transitions which lead to breaking of the D branes, we should look for an interpretation as a Higgs mechanism. Since, the number of vector multiplets is increased by two, we need to look for two more massless chiral fields. By applying the same logic as in the case of the enhanced chiral symmetry, we see that we have four copies of chiral multiplets $\tilde{Q}, Q, \tilde{R}, R$. They carry charges (1,-1,0,0), (0,1,-1,0), (0,0,1,-1), (-1,0,0,1), respectively under the gauge groups.

In addition there are bi-fundamental fields for the intersection of the two new D4 branes, $\tilde{F}, F$ with charges (0,1,0,-1) and (0,-1,0,1), respectively. Two more bi-fundamental fields come from the two new D6 branes. For the moment, we will ignore the bi-fundamentals for the D6 branes, since they have six dimensional kinetic terms and so are not dynamical for the four dimensional system.

The system has 4 supercharges and we cannot exclude the possibility of a superpotential. Studying possible deformations and comparing with possible branches of field theory the superpotential can be uniquely fixed to be

$$\tilde{Q}\tilde{F}R - \tilde{R}FQ. \tag{3.1}$$

What happens if we rejoin the D6 brane and move it away from the NS5'? Depending on whether we move it to the left or right, either $\tilde{Q}Q$ or $\tilde{R}R$ will become massive, since the strings stretching between the one D4 piece and the D6 have to stretch a non-zero distance. If we embed this cross configuration in an HW setup realizing product gauge groups we find that any D6 brane basically contributes a fundamental hypermultiplet to all product factors! All but one of them will have a finite mass. At the special points when a D6 touches an NS5' and is allowed to split, the fundamental hypermultiplets in the two neighbouring groups will become massless simultaneously. We have created a situation in which the matter content does depend on the 6 positions of the branes involved.

3.1.3 S-dual $\mathcal{N} = 1$ pairs revisited

After this long discussion about Seiberg duality and all its cousins we can readress the question: what is the Seiberg dual of a finite theory? Many of the examples of Seiberg duality include theories that can be made finite upon adding an appropriate superpotential term. Is in these cases the Seiberg duality an S-duality?

Just checking through several examples one finds that in general the Seiberg dual of a finite theory is not finite [57, 73], so that a $g_{YM} \to 1/g_{YM}$ S-duality cannot be true. However in all examples the dual does have a very special property: it contains at least one marginal operator [74]. The existence of an exactly marginal operator corresponds to having an arbitrary coupling in the fixed point theory, parametrizing a whole fixed line. On the fixed line of marginal couplings the theory is superconformal, i.e. all β-functions are vanishing. Following the work of Leigh and Strassler [57], a simple criterion for the existence of exactly marginal operators is given by analyzing the exact Shifman-Vainshtein [75] formula for the β function. The gauge and Yukawa β functions get a 1-loop contribution and all higher loop and non-perturbative corrections enter as linear functions of the anomalous dimensions of the matter fields. In general setting the r β functions to zero yields r conditions on the r couplings, leaving at most a fixed point. If some of them are linearly dependent, we get a line of solutions and hence a marginal operator.

Now we can make the comparison: finite models are in general S-dual to a superconformal theory parametrized by a free coupling constant multiplying a marginal operator. In the special case that the dual is also finite (like in the well known $\mathcal{N}=4$ example) this dual free coupling is just the gauge coupling, whereas in general it is a combination of gauge and Yukawa couplings. Several examples along these lines have been presented in [57, 73, 74]. One of the examples found in [73] actually seems to give a finite dual of a finite theory. The electric model is based on the gauge group $SO(10)$ with matter fields V in $N_f = 8$ vector and $N_q = 8$ fields Q in the spinor representations. The model with this superfield content has vanishing one-loop gauge β function. Under addition of a superpotential

$$W = h \sum_{i=1}^{8} Q_i Q_i V_i, \tag{3.2}$$

the theory becomes finite. The conjectured S-dual is based on an $SU(9) \times USp(14)$ gauge group with a symmetric tensor in the $SU(9)$ factor and several bifundamentals and fundamentals. Again the 1-loop β functions vanish and the superpotential is such that according to [57] the whole theory is actually finite. For more technical details see [73].

3.2 Non-trivial RG fixed points

3.2.1 Appearance and applications

Interacting fixed point theories

An interesting phenomenon very familiar in 4 and lower dimensions is the appearance of non-trivial fixed points of the renormalization group. Since the β function by definition vanishes at the FP the theory at the FP is invariant under scale transformations. In the simplest cases the theory at the FP is free, that is the coupling vanishes. This is for example the case for the UV fixed point of asymptotically free field theories like QCD. Scale invariance of the free theory is somewhat trivial. There are just no interactions present that could set any scale. Sometimes however one encounters an interacting fixed point. In this case, at least in the supersymmetric version, the theory is believed to not only exhibit scale invariance, but the full conformal invariance (which includes scale invariance). Due to the lack of any scale and hence any mass in this (super)conformal theory, one has to deal with a continuum of states, making any particle interpretation impossible. All information about the theory is contained in its correlation functions. Such conformal theories are needed to describe critical systems and are of substantial interest in both particle and statistical physics.

One of the big surprises coming out of the SYM - string theory correspondence was that such fixed points can also exist in 5 and 6 dimensions. This was first noted in [76] for maximally supersymmetric ($\mathcal{N} = (2,0)$) 2-form "gauge" theory in 6 dimensions arising from compactifying IIB string theory on a K3. Similar fixed points where shown to also exist in 6d SYM theories. From the field theory point of view such fixed points were believed to be impossible. Since the dimension of a gauge field is 1 in any spacetime dimension, simple dimensional analysis tells us that the gauge coupling must be dimensionful in dimensions other than 4. While in lower dimensions the coupling becomes stronger at low energies, in 5 and higher dimensions it becomes weaker. Therefore all gauge theories in 5 and 6 (and higher dimensions) were believed to be infrared free, at the same time becoming ill defined in the UV, since for the same reasons the coupling blows up and the theory is non-renormalizable just from naive power counting.

The caveat in this argument is that we tacitly assumed that at least at some energy scale gauge theory is a valid description. Then it follows automatically that at all lower scales gauge theory is also a good description since the gauge coupling becomes even weaker and we hit the free fixed point in the IR. The only way to avoid this is to have a theory which is intrinsically strongly coupled, so that gauge theory is never a good description. A heuristic way to say this is that even though the F^2 gauge kinetic operator is an irrelevant operator (turning it on doesn't move us away from the free theory in the IR), we might be able to reach an interacting theory by taking the coupling parameter of this irrelevant operator (the gauge coupling) formally to infinity, leaving us with a strongly coupled gauge theory. From the field theory this only teaches us that gauge theory is not the right arena to discuss the appearance of non-trivial fixed points in 5 and 6 dimensions. All we can do is try to write down consistency conditions that have to be satisfied by the theory in order to have a chance to have a well defined strong coupling limit (where we expect the fixed point to be). These conditions where analyzed by Seiberg [17, 16]. In 6 dimensions one has to assure that the theory is anomaly free, while in 5 dimensions the relevant criterion is that once we make a small perturbation from the fixed point, that is we turn on small $1/g^2$, the resulting (infrared free) theory be free of UV divergencies. Surprisingly the branes then teach us that ALL theories satisfying these criteria in fact do give rise to non-trivial strong coupling fixed points, as I will partly show in the following for the 6 dimensional theories.

Another criterion is that there should exist a superconformal algebra in the corresponding dimension with the right amount of supersymmetry. These superconformal algebras where classified by Nahm [77]. Again one finds that the branes realize all possible superconformal algebras.

Applications

Just establishing the existence of these higher dimensional fixed point theories is certainly interesting on its own, since they were not expected to exist. But they also have some interesting applications. For one we can basically learn about 4d field theories from compactifying 6d FP theories on a torus. In a certain limit the physics of the compactified theory reduces to pure Yang-Mills theory. Let me briefly discuss the (2,0) FP compactified on a torus and how it reduces to pure (non-supersymmetric) QCD. Compactifying the A_{N-1} (2,0) theory (the theory on N coinciding M5 branes) on a circle of radius R_2 one obtains maximally supersymmetric $SU(N)$ Yang-Mills theory in 5d with coupling $g^2_{YM} = R_2$ (the theory on the resulting D4 worldvolume). In order to obtain 4 dimensional, non-supersymmetric QCD we compactify on yet another circle of radius R_1 and include a non-trivial twist by the R-symmetry, basically choosing anti-periodic boundary conditions for the fermions. The resulting $\mathcal{N} = 0$ $SU(N)$ gauge theory has gauge coupling

$$g^2_{QCD} = \frac{R_2}{R_1} \tag{3.3}$$

according to the standard KK ansatz. Besides the gauge bosons of QCD this theory certainly contains many other states. There are the KK modes with masses $1/R_2$ and $1/R_1$, the fermions with masses of order $1/R_1$ and the scalars, which get masses at one-loop from the fermions and hence have a mass of order

$$\frac{g_{QCD}}{R_1} = \sqrt{\frac{R_2}{R_1}}/R_1 = \sqrt{R_2/R_1^3}.$$

The question is whether we can find a limit in which all these states become very massive while keeping the QCD scale fixed.

This is indeed possible [78, 79]. Let us first read off the QCD scale from the information we have so far. This is we have to take into account the running of the gauge coupling with the energy scale μ,

$$g^{-2}_{QCD}(\mu) \sim \log \frac{\mu}{\Lambda_{QCD}}.$$

(3.3) gives us the gauge coupling at the compactification scale, hence

$$\frac{R_1}{R_2} = g^{-2}_{QCD}(1/R_1) \sim \log \frac{1}{R_1 \Lambda_{QCD}}.$$

From this we read off that

$$\Lambda_{QCD} \sim e^{-\frac{R_1}{R_2}}/R_1. \tag{3.4}$$

Now consider the limit $R_1 \to 0$, $R_2 \to 0$, with $R_1 >> R_2$. All KK states, the fermions and the scalars become very massive in this limit, the lightest ones being the scalars with mass $\sqrt{\frac{R_2}{R_1}} \cdot 1/R_1$. However due to the exponential suppression in (3.4) for sufficiently small R_2/R_1 the QCD scale will be much smaller than any of the other masses in the problem, leaving us with pure QCD as advertised. Note that this limit corresponds to very weak coupling in (3.3). This is not surprising, since as discussed above, (3.3) determines the coupling at the energy scale, where all the other fields become important. We want this to happen far above the QCD scale, that is at very weak coupling due to asymptotic freedom of QCD.

A second very important application is to study the deformations of the fixed points. As we will see in what follows this can be done very easily using branes. A given brane setup represents a certain phase of string theory. This will become more transparent once we have shown that brane setups are actually equivalent to the language of geometric compactifications. By tuning parameters of this compactification, that is by moving around the branes, we encounter critical points as certain branes collide, the non-trivial FP. Often at the FP we see new deformations that allow us to perform a phase transition into a topologically distinct vacuum of string theory.

Last but not least these FP theories play an important role in the recent matrix conjecture [8]. As explained in Chapter 2 this conjecture elevates the correspondence between gauge theory and non-perturbative string theory to a principle, defining all of M-theory in terms of the world-volume theory of certain branes. For Matrix compactifications on a T^4 the relevant brane theory is the strong coupling limit of the worldvolume of N coinciding D4 branes, that is the worldvolume theory of N M5 branes: the (2,0) FP theory [27].

3.2.2 Physics at non-trivial FP from branes

General idea

From what we have learned so far it should be clear that branes are the essential tool to prove the existence of strongly coupled fixed points in 5 and 6 dimensions. The principal idea is as follows: one considers string theory in the background of certain branes. Since this is a well defined theory, we can try to take the limit in which gravity and the other bulk modes decouple. In some cases this leaves us with an interacting theory on the brane. The strong coupling limit then corresponds to having some branes coincide, usually exhibiting an infinite tower of states becoming massless as we would expect from a conformal field theory. So proving the existence of an interacting fixed point amounts to analyzing whether string theory allows for a decoupling limit that leaves an interacting theory on the branes. To demonstrate this procedure let us briefly consider the brane realization of the (2,0) fixed point that was found by [76] using a geometric picture.

I will also discuss the maximally supersymmetric cases in 5,7 and higher dimensions as well as in 6d with (1,1) supersymmetry. In these cases the analysis of [77] tells us that there are no superconformal algebras. Indeed we will find in the brane picture that in these cases decoupling of the bulk modes leads automatically to a free theory on the brane.

The (2,0) theory from the M5 brane

Consider a system of N parallel M5 branes. In the same spirit as in (2.2) we want to decouple bulk gravity by taking the limit $M_{pl} \to$ infinity. We want this to do in such a fashion that the theory on the M5 branes stays interacting. Recall that the theory on the M5 is that of N $\mathcal{N} = (2,0)$ tensor multiplets. This theory does not have a coupling constant which we could keep fixed. However we do know that these tensor multiplets couple to the strings that describe the ends of M2 branes. The tension of these strings is $\frac{D}{l_{pl}^3}$ where D is the characteristic distance between two M5 branes. These tensions correspond to the vevs of the scalars in the tensor multiplet, since they are given by the M5 positions and have mass dimension 2 in the natural normalization. Therefor the decoupling limit will be $D \to 0$, $l_{pl} \to 0$, holding $u = \frac{D}{l_{pl}^3}$ fixed. At the origin of the moduli space, that is if all the u go to zero, we expect a superconformal fixed point. The strings become tensionless and provide the continuum of massless states of the conformal theory.

One might worry that this fixed point could be a free theory. One way to see this cannot be the case is to consider the theory on a large circle R. The resulting theory will be 5d SYM with gauge coupling $g_{YM}^2 = R$ [3]. We see that this is an interacting theory and the large R limit even corresponds to very strong coupling.

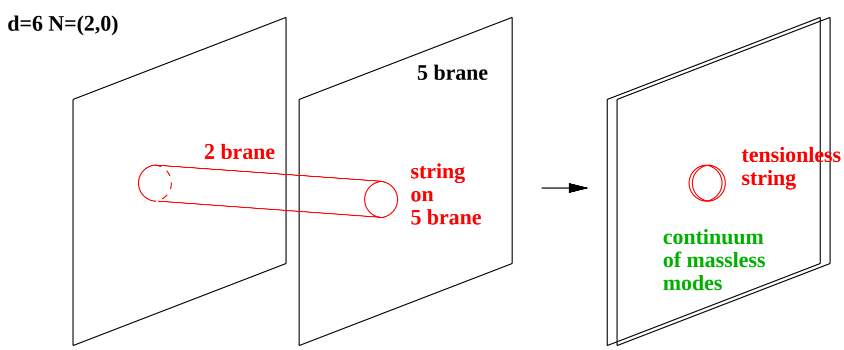

Figure 10: Brane realisation of the (2,0) theory.

[3]One way to see this relation is to consider the compactification in terms of branes. The M5 turns into a D4 with $g_{YM}^2 = l_s g_s = R$.

The other maximally supersymmetric theories

One can try to do a similar construction for SYM theories in 5,6,7 and higher dimensions, which are also maximally supersymmetric (where this time in 6d we have non-chiral (1,1) supersymmetry). These can be realized as the world volume theories of D4,5,6 and higher branes. We want to send M_{pl} and M_s to infinity in order to decouple all bulk modes [4]. If we would again predict non-trivial FPs we would be in trouble since the analysis of [77] shows that for these cases there are no superconformal algebras. We expect a non-trivial theory once we put several branes on top of each other. N colliding branes give rise to $U(N)$ gauge theory. In order to obtain an interacting theory on the branes we need to keep g^2_{YM} on the branes finite. According to (1.3)

$$g^{-2}_{YM} = M_s^{d-3}/g_s$$

for the d+1 dimensional gauge theory on the worldvolume of a Dd brane. Since

$$M_s^4 = g_s M_{pl}^4$$

We see that for the D7 brane $g^{-2}_{YM} = M^4_{pl}$ and hence g_{YM} goes to zero in the decoupling limit M_{pl}, $M_s \to \infty$. This still is true in higher dimensions. We are left with a free theory!

For the D6 $g^{-2}_{YM} = M^3_{pl}/g_s^{1/4}$ we see we can keep g_{YM} finite if we simultaneously with M_{pl} take g_s to infinity. According to [6] strongly coupled IIA string theory is better thought of as 11d SUGRA. The duality tells us that the 11d Planck scale $M_{pl,11}$ is given by

$$M^3_{pl,11} = M^3_{pl}/g_s^{1/4} = g^{-2}_{YM}.$$

In order to decouple the 11d bulk we again have to stick to a free theory on the worldvolume. For the D5 and D4 branes a similar story is true. In both cases g^2_{YM} can be kept finite only in the strong string coupling limit. S-dualizing, the D5 brane turns into an NS5 brane at weak coupling with gauge coupling $g^{-2}_{NS5} = M_s^2$ which is again free in the decoupling limit. One can slightly modify the decoupling limit by relaxing the condition that M_s is supposed to go to infinity. In this case still all the bulk modes decouple. However we are no longer left with just SYM on the brane. But whatever it is we are left with, string theory tells us that it exists. This way Seiberg proved [28] the existence of a 6d string theory. It has a string scale M_s and hence does not correspond to a conformal theory.

For the D4 brane the strong string coupling limit once more decompactifies the 11th dimension. The D4 brane becomes an M5. Hence the 5d SYM at strong coupling grows an extra dimension. We do obtain a non-trivial strong coupling fixed point, but it is again the 6 dimensional (2,0) theory which we encountered before.

[4] The former is required to decouple gravity while the latter decouples the higher order terms from the DBI action.

3.2.3 6d Hanany-Witten setups

Motivation

In order to study less supersymmetric theories we need more involved brane setups. In what follows I will present a Hanany-Witten setup describing fixed points in $d = 6$ with $\mathcal{N} = 1$. This setup was presented in [80] and further analyzed in [81, 71, 82]. The same fixed points where analyzed from the branes as probes point of view by [83, 20] and were geometrically engineered from F-theory in [84]. The motivation for analyzing these theories instead of their more supersymmetric cousins is threefold.

- As described above we can learn about 4d gauge theory by putting these fixed points on a torus. Since the dynamics of maximally supersymmetric SYM in 4d is rather constrained it is definitely interesting to do this with less supersymmetric theories. If we again want to study non-supersymmetric QCD by imposing anti-periodic boundary conditions on the fermions, we have this time the choice to put different boundary conditions on fermions coming from vector and hypermultiplets, opening up the possibility to realize QCD with matter.

- The $(2, 0)$ fixed point has only one possible deformation, which corresponds to moving the M5 branes apart. To really study phase transitions we need to study the more elaborate fixed points with only 8 supercharges, which in general do allow several different perturbations, leading to a transition between distinct phases.

- When used as matrix models these fixed points should be used to describe DLCQ string theories with 16 supercharges, like the heterotic string on a T^4 or type II string on a K3. These theories have a much richer dynamics and are hence more interesting than their more supersymmetric cousins.

The consistency requirement: Anomaly cancellation

As discussed above, even though field theory is not the right arena to discuss the appearance of non-trivial FPs, it nevertheless imposes several constraints on the existence of consistent strong coupling limits. In 6 dimensions this criterion is anomaly cancellation. Let me briefly review the relevant mechanism from the field theory point of view. In what follows we will see that the branes actually know about this mechanism and only yield anomaly free theories.

The anomaly in six dimensions can be characterized by an anomaly eight form. For a theory with a product gauge group $G = \prod_\alpha G_\alpha$ with matter transforming in the representation R, the anomaly polynomial reads

$$I = \sum_\alpha \left(\mathrm{Tr} F_\alpha^4 - \sum_R n_R \mathrm{tr} F^4 \right) - 6 \sum_{\alpha\alpha' \; RR'} n_{R,R'} \mathrm{tr}_R F_\alpha^2 \mathrm{tr}_{R'} F_{\alpha'}^2 . \tag{3.5}$$

Here, Tr denotes the trace in the adjoint and tr_R is the trace in Representation R. The symbol tr is reserved for the trace in the fundamental representation. n_R denotes the number of matter multiplets transforming in a particular representation R and $n_{RR'}$ is the number of multiplets transforming in the representation $R \times R'$ of a product group $G_\alpha \times G_\beta$. For a consistent theory, the anomaly should be cancelled in some way: Either the anomaly polynomial vanishes or we cancel the anomaly by a Green–Schwarz mechanism [85]. So let us look at the anomaly in more detail. First, we want to rewrite the anomaly polynomial using only traces in the fundamental representation. Formally, the polynomial then looks like

$$I = \sum_\alpha a_\alpha \mathrm{tr} F_\alpha^4 + \sum_{\alpha\alpha'} c_{\alpha\,\alpha'} \left(\mathrm{tr} F_\alpha^2\right) \left(\mathrm{tr} F_{\alpha'}^2\right) \tag{3.6}$$

For concreteness let me discuss the case of a single gauge group factor. The anomaly reduces to

$$I = a \mathrm{tr} F^4 + c(F^2)^2 \tag{3.7}$$

For $a = c = 0$ the anomaly cancels completely. If $a \neq 0$ the anomaly can't be cancelled and the theory is sick. However if $a = 0$ we can cancel the anomaly by the Green-Schwarz mechanism by coupling it to a two-index tensor field. In 6 dimensions the tensor splits into an anti-selfdual and an selfdual part. While the former sits in the gravity multiplet, the latter is part of the tensor multiplet. Depending on the sign of c we have to use one or the other. If $c > 0$ we can cure the anomaly by coupling to a tensor multiplet, for $c < 0$ the theory is sick without coupling to gravity. If realized in terms of branes, in the latter case there can't be a consistent decoupling of the bulk modes. For a very detailed discussion of the anomaly polynomials for product gauge groups see e.g. [86].

In the case of $SU(N)$ gauge theory using the group theoretical identity [87]

$$\mathrm{Tr} F^4_{SU(N)} = 2N \mathrm{tr} F^4_{SU(N)} + 6 \left(\mathrm{tr} F^2_{SU(N)}\right)^2 \tag{3.8}$$

we find that

$$I = (2N - N_f) \mathrm{tr} F^4 + 6 \left(\mathrm{tr} F^2\right)^2 . \tag{3.9}$$

Therefore, the deadly F^4 term cancels precisely for $N_f = 2N$. The prefactor c of the $\left(F^2\right)^2$ term is always bigger than zero, so that we can cancel the anomaly by coupling to a single tensor multiplet.

Adding the Green-Schwarz counterterm, the action contains a coupling of the scalar ϕ in the tensor multiplet to the gauge fields, so that the gauge kinetic term becomes [17]

$$\frac{1}{g^2} \mathrm{tr} F_{\mu\nu}^2 + \sqrt{c}\phi \mathrm{tr} F_{\mu\nu}^2 .$$

We see that one can absorb the bare gauge coupling into the expectation value of ϕ by just a shift of the origin. We obtain an effective coupling

$$\frac{1}{g_{eff}^2} = \sqrt{c}\phi .$$

At $\phi = 0$ we expect the possibility of a strong coupling fixed point [17].

The brane configuration

In order to study Hanany-Witten setups in 6 dimensions, we need as fundamental ingredients D6 branes, on whose worldvolume we realize the gauge theory. In addition one has the obligatory NS5 branes, between which the 6 branes are suspended. As usual this compact interval gets rid of one of the 7 worldvolume dimensions of the D6 brane via KK reduction. In addition the boundary conditions for the D6 branes ending on the NS5 branes projects out some of the fields. In total we break 3/4 of the supercharges (1/2 by the D6 and another 1/2 by the NS5) leaving $\mathcal{N} = 1$ in d=6.

As usual there are two ways to introduce flavors in the setup. One is to include the additional flavor branes, which in our case will be D8 branes. I will explain how to deal with them later. For now I will include flavors via semi-infinite D6 branes to the right and left. With this the players in our game are:

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
NS 5	x	x	x	x	x	x	o	o	o	o
D 6	x	x	x	x	x	x	x	o	o	o
D 8	x	x	x	x	x	x	o	x	x	x

The presence of these branes breaks the $SO(9,1)$ Lorentz symmetry of type IIA down to $SO(5,1) \times SO(3)$ corresponding to rotations in the 789 space and Lorentz transformations in 012345 space. The $SO(3)$ can be identified with the $SU(2)$ R-symmetry of the d=6 $\mathcal{N} = 1$ SUSY algebra. To understand the basic issues that are different in 6d from the "usual" Hanany-Witten setups discussed above let me first focus on the simplest setup.

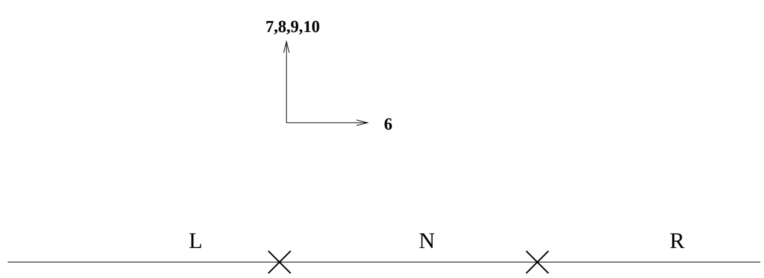

Figure 11: The brane configuration under consideration, giving rise to a 6 dimensional field theory. Horizontal lines represent D6 branes, the crosses represent NS5 branes.

Figure 11 shows a configuration of N D6 branes suspended between two NS5 branes. There are L (R) semi-infinite D6 branes to the left (right). Let us first discuss the matter content corresponding to this setup. As usual we get an $SU(N)$ vector multiplet from the color branes and $L+R$ fundamental hypermultiplets from the semi-infinite flavor branes. But these are not all the matter multiplets we get. According to our standard philosophy we keep all the light modes coming from the lowest dimensional branes. In all other dimensions this meant decoupling the worldvolume fields of the NS5 branes. But here the NS5 branes live inside the D6 brane, they also have a 6d worldvolume as the finite D6 brane pieces we are considering. Hence we should also include the matter from the NS5 branes.

This can also be discussed in a more quantitative manner, looking at the precise decoupling limit. As discussed in our introduction to Hanany-Witten setups we want to take M_{pl} and M_s to infinity in order to decouple the bulk, sent the length L of the interval to zero in order to decouple the KK modes and do this all in such a way to keep the gauge coupling of the 6d theory fixed. According to (1.3) we get $g_{YM}^2 = \frac{g_s}{M_s^3 L}$. The theory on the NS5 branes is a theory of tensor multiplets, so there is no gauge coupling. To see what is going on let us translate into 11d units via (1.10). We see that $g_{YM}^2 = \frac{l_{pl,11}^3}{L}$. But this is precisely the tension of the M2 branes stretching between the 5 branes. When discussing the decoupling of the (2,0) theory we found that this quantity basically governs the interaction strength of the theory on the 5 brane. Keeping it fixed means that we should indeed keep the interacting modes on the 5 branes as well as those on the worldvolume of the finite D6 brane.

So now what is this additional matter? The theory on a IIA NS5 brane is the theory of a (2,0)– tensor multiplet. This multiplet consists of a tensor and 5 scalars (and fermions). Because of the presence of the D6 branes, one half of the SUSY is broken and we are left with a (1,0) theory. The tensor multiplet decomposes into a (1,0) tensor, which only contains one scalar, and a hypermultiplet, which contains 4 scalars. The hypermultiplet is projected out from the massless spectrum because the position of the semi-infinite D6 branes fixes the position of the NS5 branes. Moving the NS5 brane out of the D6 brane (remember that we have the 5 brane embedded in the D6 brane) corresponds to turning on the 3 FI terms and a theta angle. The scalar in the tensor multiplet corresponds to motions of the 5 branes in the x_6 direction. This is going to be a modulus of our theory. We have two NS5 branes and therefore two tensor multiplets, but effectively we keep only one of them because one of the scalars can be taken to describe the center of mass motion of the system. The vev of the other scalar gives us the distance between the NS5 branes. On the other hand, we know that the distance between the NS5 branes is related to the inverse Yang-Mills coupling of the six-dimensional gauge theory according to the usual KK philosophy. The vev of the scalar hence plays the role of a gauge coupling, as expected from field theory.

Now let us turn to the quantum picture. As in the other theories with 8 supercharges, the 1-loop information will be encoded in the bending of the NS5 branes and there won't be any further corrections. We in particular expect the bending to provide us with the information about the anomalies, since those are a 1-loop effect. Indeed the analysis of the bending is really easy in 6d, as discussed before. Since the NS5 branes are embedded in the D6 branes there is no transverse worldvolume. The NS5 brane can't absorb any flux coming from the charged ends of a D6 brane. The only way we can have a consistent brane setup is if the net charge cancels. The net charge is given by the number of D6 branes ending from one side minus the number of D6 branes ending from the other side. Thus, we only get a consistent picture if:

$$N = L = R,$$

where L (R) denotes the number of D6 ending from the left (right). The total number of flavors is

$$N_f = L + R = 2N. \tag{3.10}$$

Together with the tensor multiplet from the NS5 branes this is indeed the (only) right [5] matter content to cancel the anomaly.

Realization of the fixed point

Our theory has a Coulomb branch parametrized by the vev of the scalar in the tensor multiplet. At the origin, the effective gauge coupling becomes infinite and we find the strong coupling fixed point. Since the tensor multiplet corresponds to the distance between the NS5 branes this happens precisely when they coincide. The strings from D2 branes stretching between the NS5 branes become tensionless. Building product groups is straight forward. Again we find FPs whenever the NS5 branes coincide.

An interesting generalization is to put the theory on a circle. In this case we will see an $SU(N)^k$ gauge group, where k denotes the number of NS5 branes. Tuning the moduli in such a way that we obtain the fixed point theory amounts to moving all the 5 branes on top of each other. Since the distance between the 5 branes is the effective coupling constant of the corresponding gauge theory we see that there is always one gauge group who's color branes stretch around the entire circle, no matter where we choose to bring our 5 branes together. This means that we will always have one gauge factor with a finite gauge coupling that becomes free in the IR and hence becomes a global symmetry. From the field theory point of view this statement can also be seen to be true. Defining

[5] For $SU(2)$ and $SU(3)$ there are also some other possibilities since they do not have an independent fourth order Casimir and hence a vanishes automatically. Global anomalies [88] restrict us to $N_f = 4, 10$ for $SU(2)$ and $N_f = 0, 6, 12$ for $SU(3)$. The 10 flavor case can be achieved by realizing $SU(2)$ as $USp(2)$ with an orientifold. The other $SU(3)$ theories do not have an HW interpretation.

the effective gauge couplings by absorbing the classic gauge couplings in the vev of the scalars in the tensor multiplets, one finds that there is always one subgroup for which g_{eff} becomes negative at large ϕ [20], so that we should start of with a finite value for this coupling.

The value of ϕ at which this sign change happens is given by $g^{-2}_{dec.fact.} = \frac{R_6}{l_s^3 g_s} = \frac{R_6}{l^3_{pl,11}}$ (remember that ϕ has mass dimension 2). That is in the decoupling limit, where we send $l_{pl,11} \to 0$ keeping $\frac{L}{l_s^3 g^s}$ fixed, this critical value goes off to infinity, so we do not see any effect of the finite radius. If however we choose to also send R_6 to zero as well, keeping $\frac{R_6}{l^3_{pl,11}}$ fixed, we can engineer a theory with the following properties:

- it decouples from all bulk modes, hence it is 6d
- gauge theory breaks down at energy scales $\frac{R_6}{l^3_{pl,11}}$, new degrees of freedom become important
- the theory contains string like excitations with tension $\frac{R_6}{l^3_{pl,11}}$

One usually refers to those theories as little string theories. The way we constructed them is the T-dual of the original construction of [89] as a generalization of Seiberg's work [28] on the theory with 16 supercharges.

Including 8 branes

So far we have seen how to realize fixed points in branes. Now I will move on to more complicated setups in order to classify all fixed points realizable by HW setups. In [90] all anomaly free matter contents in 6d have been classified. Strictly speaking this list should be seen as the list of all theories that could possibly have a consistent strong coupling fixed point. The nice stringy result is that indeed all of them do. HW setups will not be able to generate the exceptional gauge groups, but will be very efficient at classifying those FP allowed with classical gauge groups and products thereof. The few missing cases can be realized via geometry [84, 91].

In order to build more general fixed point theories, let us now look at the second possibility to include flavors, via D8 branes living in 012345789, as discussed in [71]. D8 branes introduce additional complication, since D8 branes are not a solution of standard IIA theory but require massive IIA [92], that is the inclusion of a cosmological constant m. The D8 branes then act as domain walls between regions of space with different values of m. m is quantized and hence in appropriate normalization can be chosen to be an integer.

The presence of m makes itself known to the brane configuration via a coupling

$$-m \int dx^{10} B \wedge *F^{(2)}, \tag{3.11}$$

where B is the 2 form NS gauge field under which the NS5 brane is charged and F is the 8 form field strength for the 7 form gauge field under which the D6 is charged. From the Bianchi identity for the 2 form field strength dual to $F^{(8)}$ in the presence of a D6 brane ending on a 5 brane, $dF^{(2)} = d * F^{(8)} = \theta(x^6)\delta^{(789)} - mH$ (where $H = dB$), one gets $mdH = \delta^{(6789)}$. The δ function comes from the charge of the D6 brane-end. From here we get back the old result, that for $m = 0$ D6 branes always have to end on a given NS5 brane in pairs of opposite charge (that is from opposite sides). However for arbitrary m this relation shows that RR charge conservation requires that we have a difference of m between the number of branes ending from the left and from the right [6].

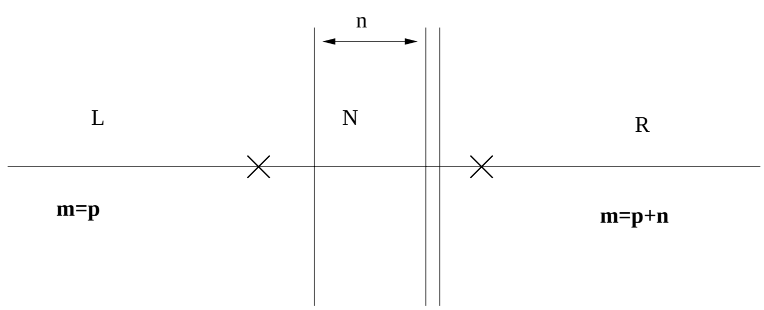

Figure 12: Basic Hanany Zaffaroni Setup

Figure 12 shows the basic brane configuration involving D8 branes. The n D8 branes in the middle give rise to n flavors for the $SU(N)$ gauge group. In addition they raise the cosmological constant m from p to $p+n$. In the background of these values of m the modified RR charge conservation tells us

$$\begin{aligned} N &= L - p \\ R &= N - (p+n) \end{aligned}$$

With the $R + L$ flavors from the semi-infinite D6 branes the total number of flavors is $L + R + n = 2N$ still in agreement with the gauge anomaly considerations on the 6 brane for every possible value of p. So far we have not succeeded in obtaining any new FPs, since SUSY QCD with $N_f = 2N_c$ is just what we were already able to realize with semi-infinite D6 branes. However as we move on we will need the D8 branes in more elaborate setups.

[6]Throughout this work I will use the following conventions to fix the signs: by passing through an D8 from left to right m increases by 1 unit. In a background of a given m the number of D6 branes ending on a given NS5 from the left is by m bigger than the number of D6 branes ending from the right.

Orientifolds

There is one more element we can incorporate in the HW brane setup, the orientifold. Since an orientifold breaks the same supersymmetries as the corresponding D-brane, we can introduce O8, O6 or both. Each of them comes with two possible signs. Let us first consider the O8. We have to distinguish two possibilities: O8 planes with negative or positive D8 brane charge (that is -16 or +16) [4]. The former are the T-dual of the O9 projecting IIB to type I. On the D8 worldvolume they project the symmetry group to a (global) SO group, while on the D6 we get a local USp group. The positively charged O8 projects onto global USp and local SO. If we want to have vanishing total D8 charge, we should restrict ourselves to either 2 negatively charged O8s with 32 D8 branes or one O8 of each type.

There is indeed a physics reason that we should restrict ourselves to satisfy this requirement. What becomes important is that not only the cosmological constant jumps when we cross a D8 brane, but also the dilaton. If the dilaton is constant to the right of a given D8 brane, $e^{-\Phi}$ (the inverse string coupling) will rise linearly on the left. For symmetry reasons this means that the dilaton will run with slope -8 into an O8 from the left and leave with slope +8 on the other side. In order to be back to slope -8 again before hitting the next O8, one has to put precisely 16 physical D8 branes in between. One interesting application of this behaviour is that this way the coupling may diverge at the orientifold planes, while it is finite everywhere else and even constant in between the 8th and the 9th physical D8. Usually one refers to this whole setup as type I' string theory. Of course it is no problem to include just one O8 and fewer D8 branes by just keeping far away from the others.

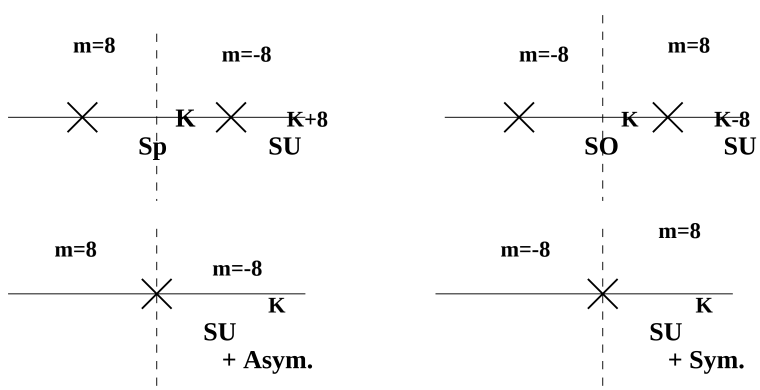

Figure 13: Various possibilities to introduce O8 planes

The gauge groups corresponding to brane setups in Figure 13 have been analyzed in the equivalent setup for 4d in [50]. If the O8 is in between two 5 branes the 'center' gauge group is projected to $USp(K)$ or $SO(K)$ respectively. All other gauge groups stay SU, however the SU groups to the right are identified with those to the left and one effectively projects out half of the SU groups. In addition to this, in 6d we get the special situation that the O8 also changes the cosmological constant. For symmetry reasons we have to choose $m = \pm 8$ on the two sides of the orientifold. Therefore we obtain in total

$$\begin{aligned} USp(k) \quad &\times \quad \prod_i SU(k+8i) \quad \text{or} \\ SO(k) \quad &\times \quad \prod_i SU(k-8i) \end{aligned}$$

respectively. From strings stretching in between neighbouring gauge groups we get bifundamentals $(\Box, \Box)$. We have taken all the D8 branes that are required to cancel the O8 plane charge to be far away. Of course including them we get some more fundamental matter fields and new contributions to the cosmological constant.

The other possible situation considered in [50] is that one of the 5 branes is stuck to the O8. In this case all the groups stay SU, again with the left and the right ones identified, leading to effectively half the number of gauge groups. In addition the middle gauge group has a matter multiplet in the antisymmetric or symmetric representation for positive/negative charge O8 planes. In 6d we again have the effect of the cosmological constant and as a result get

$$\prod_i SU(k \pm 8i)$$

with antisymmetric/symmetric tensor matter in the first gauge group factor in addition to the bifundamentals.

The other realization of SO and USp groups is to introduce an O6 along the D6 branes. In discussing O6 planes we have to be careful to take into account an effect first discussed in [93]: whenever an O6 passes through an NS5 brane, it changes its sign. Originally this was argued for O4 and NS5 based on consistency of the resulting 4d physics. Here we could basically do the same: without including this effect on would construct anomalous gauge theories. The sign flip miracously removes all anomalies, providing strong arguments in favor of this assumption. There also exist some worldsheet arguments, that further support the existence of the flip [94].

Due to this sign flip the O6 also contributes to the RR charge cancellation. Since its charge is +4 on one side and -4 on the other, the number of D6 branes ending to the left and right also has to jump by 8. We therefor are left with product gauge groups forming an

$$\prod_i SO(N+8) \times USp(N)$$

chain with bifundamentals. Bringing the 3 ingredients - D8, O8 and O6 - together, we can clearly cook up very complicated models. I will write down some hopefully illustrative examples later on.

All these gauge theories I constructed with the help of orientifolds are anomaly free [86, 90, 83] upon coupling to the tensor multiplets associated to the independent motions of the 5 branes. Note that in the case of the O8, already a single NS5 brane gives rise to a tensor multiplet, parametrising the distance to the fixed plane. There is no decoupled center of mass dynamics since presence of the orientifolds breaks translation invariance along the 6 direction.

Again it is straight forward to put the theory on a circle. I will restrict myself to the discussion of O8s. The O6 case is pretty obvious. The only thing one has to take care of is that only an even number of NS5 branes is allowed, so that the "global" groups from semi-infinite D6 branes are either both USp or both SO and we are able to close the circle. We should either take 2 negatively charged O8s with 32 D8 branes (that is the 16 physical D8 branes and their 16 mirrors) or a pair of oppositely charged O8s.

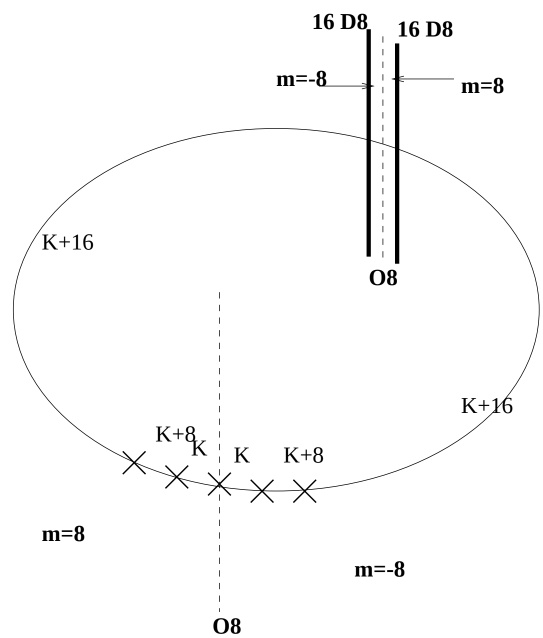

Figure 14: Brane configuration yielding a product of two USp and several SU gauge groups with bifundamentals and an antisymmetric tensor.

Encoding the position of the D8 branes in a vector w_μ, where the entries in w denote the number of D8 branes in the gauge group between NS5 brane number μ and number $\mu+1$, one can write down formulas for the resulting gauge groups. Clearly $\sum_\mu w_\mu = 32$. In addition we introduce a quantity D_μ that encodes in a similar way the information about the orientifold: the entry corresponding to the gauge group that includes an orientifold is 16, since an O8 carries 16 times the charge of a D8. All other entries are zero.

Let me only discuss the case of 2 negatively charged O8s with 32 D8 branes. This will have a dual description in terms of $SO(32)$ small instantons. The case of oppositely charged orientifolds describes a disconnected part of the moduli space. It is dual to string theory compactifications with reduced rank of the gauge theory, like the CHL string [95].

We have to distinguish three cases

- the number of NS5 branes is odd, so one of them has to be stuck at one of the O8s and the others live in mirror pairs on the circle
- the number of NS5 branes is even, one is stuck at each of the orientifolds and the others appear in mirror pairs
- the number of NS5 branes is even and all of them appear in mirror pairs on the circle and are free to move

Let me discuss in some detail the case where $k+1$, the number of NS5 branes, is odd. The other two cases can be worked out the same way [81, 83]. As in Figure 14 we have two orientifolds on the circle, both with negative charge. An example also explaining the notation is illustrated in the following figure:

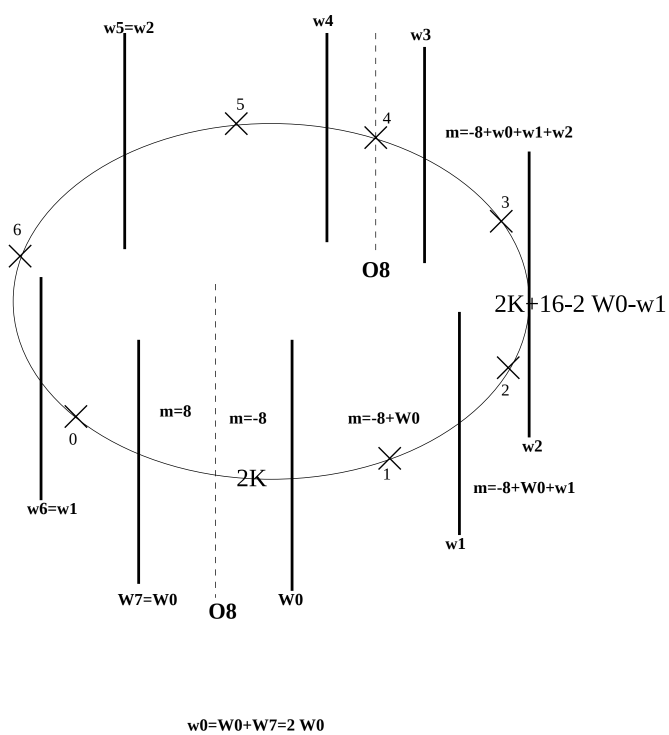

Figure 15: Notation in the example of 7 NS5 branes on the circle

One of the 5 branes is stuck on the one orientifold. We label the 5 branes with μ running from 0 to k as in Figure 15 [7]. In addition there are 32 D8 branes. Let again w_μ denote the number of 8 branes in between the μth and $(\mu+1)$th NS5 brane. Obviously $\sum_\mu w_\mu = 32$. Also symmetry with respect to the orientifold plane requires $w_\mu = w_{k+1-\mu}$. It is useful to slightly modify the definition of the vector counting the D8 branes by introducing W_μ in order to take care of the effect that for every gauge group whose color branes stretch over the orientifold half of the w_μ branes have to be located on either side. Therefore in these cases we define $W_\mu = \frac{1}{2} w_\mu$ while in all other cases $W_\mu = w_\mu$.

According to the rules of how to introduce 8 branes and orientifolds [8], we find that the gauge group is

$$USp(V_0) \times \prod_{\mu=1}^{k/2} SU(V_\mu) \tag{3.12}$$

with

$$V_\mu = 2K - \sum_{\nu=0}^{\mu-1} (\mu - \nu) W_\nu + 8\mu$$

We have $\frac{1}{2} w_0 \Box_0, \oplus_{\mu=1}^{k/2} w_\mu \Box_\mu, \oplus_{\mu=1}^{k/2} (\Box_{\mu-1}, \Box_\mu)$ and $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_{k/2}$ matter multiplets (subscripts label the gauge group) and $k/2$ tensor multiplets.

Using the symmetry property $W_\mu = w_\mu = w_{k+1-\mu} = W_{k+1-\mu}$, $W_0 = \frac{1}{2} w_0$ the requirement of having a total of 32 D8 branes $\sum_{\nu=0}^{k} w_\nu = 32$ can be rewritten as

$$\sum_{\nu=0}^{k/2} W_\nu = 16$$

which just says that 16 of the 32 D8 branes have to be on either side of the orientifold. Therefore

$$\sum_{\nu=0}^{\mu-1} \mu W_\nu = 16\mu - \mu \sum_{\nu=\mu}^{k/2} W_\nu$$

and (3.12) becomes

$$V_\mu = 2K - 8\mu + \sum_{\nu=0}^{\mu-1} \nu W_\nu + \sum_{\nu=\mu}^{k/2} \mu W_\nu$$

It should come as no surprise that all these theories are anomaly free. This was explicitely checked in [83].

[7] Note that in this numbering the stuck 5 brane is number $(k+2)/2$.

[8] The D8 branes have two effects: first they introduce a fundamental hypermultiplet in the gauge group they are sitting in, second they decrease the number of colors in every following gauge group to their right by one per 5 brane in between, since the value of the cosmological constant m is changed.

3.3 Extracting information about the FP theory

3.3.1 Global Symmetries

So far I have discussed how to realize FP theories in a given brane setup in the limit that all the bulk modes decouple. In the spirit of Seiberg [16, 17], identifying a decoupling limit in which interacting physics survives on the brane is basically an existence proof for these FP theories. As I discussed earlier, this is a result, that could not be obtained from pure field theory reasoning. Field theory gave us consistency requirements, branes show us the existence.

But now we can move on and ask ourselves whether we can actually learn something about the FP theories from their brane realization and our (partial) knowledge of the "embedding" string theory. One of the facts we can learn about are enhanced global symmetries. Some information about the global symmetries can be gotten already from field theory: once we perturb the FP by turning on the relevant perturbation $1/g^2F^2$, as discussed in length above, we will flow to the free IR fixed point. Dealing with a free theory, it is straight forward to identify the global symmetries. In our case there will be for example a global flavor symmetry $SU(N_f)$. It will be realized in the branes as the gauge group on the higher dimensional branes, e.g. those branes whose worldvolume fields decouple together with the bulk modes. In our case the $SU(N_f)$ global flavor symmetry is realized on the worldvolume of the N_f D8 branes.

Once we go to the fixed point (that is turn off the $1/g^2$ perturbation and go to infinite coupling) these symmetries are still present. But sometimes it happens that we gain some new or enhanced global symmetries at the fixed point. This can occur if some symmetry breaking operators become irrelevant at the fixed point. This phenomenon is well known from field theory where it goes under the name accidental symmetries: some symmetries which appear to be valid in the IR may be broken by higher dimensional operators. So while in the full theory they are no good symmetries, they become better and better approximate symmetries when we go to the IR and at the fixed point they are exact.

Enhanced global symmetries in the brane setup are again realized as worldvolume gauge symmetries on the decoupled heavy branes. Consider an easy example: Take an HW setup, realizing pure $U(1)$ gauge theory with $\mathcal{N} = 4$ in d=3. That is we just suspend a single D3 brane between two type IIB NS5 branes. The coupling is given by the separation between the NS5 branes. This theory is known to lead to a nontrivial IR fixed point, as can be seen following arguments like those in [96]. Going to the IR fixed point is equivalent to going to infinite coupling, since in 3d the coupling has dimension of mass and therefor sets the scale. Taking the coupling to infinity amounts to looking at energies way below the intrinsic scale, that is the far IR of our theory. The strong coupling fixed point corresponds to taking the two NS5 branes to coincide. Since the worldvolume theory of IIB NS5 branes is just SYM, this leads to an enhanced global $SU(2)$ symmetry at the fixed point. This same mechanism works in almost all other HW-like setups.

However in 6d the situation is special and we won't get away with this trick: as shown above in 6d the dynamics of the NS5 branes does not decouple from the D6 brane gauge theory. In fact we needed to include the tensors from the NS5 branes in the dynamics in order to cancel anomalies. So coinciding NS5 branes do not correspond to global symmetries. On the other hand the global $SU(N_f)$ symmetries on the D8 branes are already visible at finite coupling, that is in the free field theory. It therefore seems that in the 6d setup we do not see enhanced global symmetries. There is however a way to obtain such enhanced global symmetries. Roughly speaking one can go to the fixed point by taking the "surrounding" IIA string coupling to infinity instead of letting the NS5 branes coincide [82].

It is well known that in the strong coupling limit the $SO(2N)$ global symmetries on N D8 branes coinciding on top of an O8 plane (with $N = 1, \ldots, 7$) is enhanced to E_{N+1} [97]. The missing gauge bosons are realized by D0 branes stuck on the D8 brane, whose mass goes like $\frac{1}{g_s l_s}$ and hence they become massless in the strong coupling limit [98]. This leads to fixed points with exceptional enhanced global symmetries. The best known example is the one with no D6 branes and just one NS5 brane and 7 D8 branes all sitting on top of the O8. At strong coupling this becomes Horava-Witten M-theory with one M5 brane sitting on the "end of the world brane". This is the well known realization of the small E_8 instanton [99].

3.3.2 Deformations and Phase Transitions

It is very easy to see deformations of the fixed point theories in the brane picture. For example we can move one of the NS5 branes away from the fixed point. If we do this along the 6 direction, we go back on the Coulomb branch of our theory, if we move it off along 789 we turn on some FI terms. For the FI terms it doesn't matter at all whether we turn them on at the fixed point or at any point on the Coulomb branch.

More interesting are deformations that are not possible away from the FP, that is if the Coulomb branch and a Higgs branch meet and the FP sits at the common origin. In this case we can actually perform a phase transition. That is we start with branes realizing the theory on its Coulomb branch as one phase of string theory. Tuning the moduli we can go to the fixed point. At the fixed point we see a new deformation that was not possible before, taking us out to the Higgs branch. Tuning the Higgs moduli away from the fixed point, we have performed a transition to a topologically distinct phase of string theory.

Let me show one example of such a transition. In hindsight of the relation to geometry which I will exhibit in the next Chapter, I will call this a small instanton transition. It was first discussed in the context of 6d brane setups in [82].

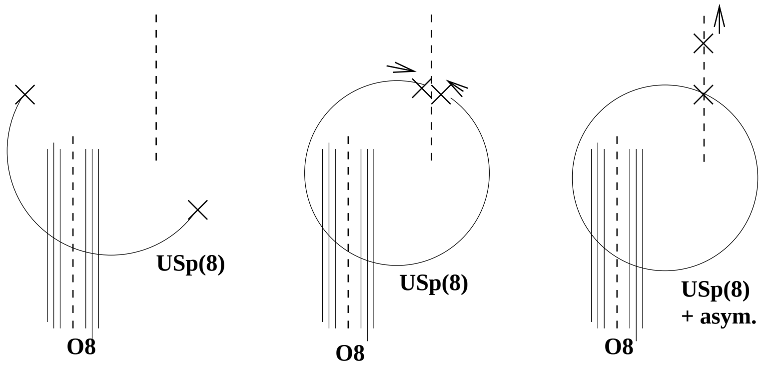

Figure 16: The small instanton transition trading 1 tensor for 29 hypers.

We start with a special case of a theory with 2 O8s and 32 D8 branes on a circle. All of the D8s are on top of a single O8, so we will see an $SO(32)$ global symmetry. In addition we will study a single pair of NS5 branes. The cosmological constant on each side will be ± 8 respectively, so that we have to put at least 8 D6 branes on this interval. They will all terminate on the NS5 branes. The corresponding gauge group can easily be read off to be $USp(8)$. There are going to be 16 fundamental hypermultiplets from the D8 branes and one tensor multiplet describing the position of the NS5 and its mirror on the circle. Tuning the scalar in the tensor multiplet we can reach a fixed point by letting the NS5 collide with its mirror at the far orientifold. At this point we see a new deformation: one of the NS5 branes can be moved of in 789 space. According to our rules this will still be a $USp(8)$ gauge theory. However one obtains an additional hypermultiplet in the antisymmetric tensor representation from the far orientifold. This gives us $27 + 1 = 28$ new hypermultiplets. In addition one of the NS5 branes moves freely in 789 space, corresponding to a 29th hypermultiplet. But now the second NS5 brane is stuck at the orientifold, so there is no tensor multiplet left [9]. All in all this describes a phase transition trading 1 tensor for 29 hypers. These numbers are required for any theory that can be coupled consistently to gravity: the contribution of a single tensor multiplet to the gravitational anomaly in 6d is the same as that of 29 hypers. So if the gauge theory can be coupled consistently to gravity in one phase (and since our setup is a limit of string theory we should definitely be able to do so), then the other phase which lost one tensor and has the same number of vectors has to have 29 more hypermultiplets in order to have a consistent coupling to gravity, too.

[9]The gauge theory has $a = c = 0$ and hence doesn't need a Green Schwarz mechanism to cancel the anomaly.

3.3.3 Correlators from branes

So far we have seen that quite a lot can be learned about 6d fixed points using the brane realization I have presented in this work. But the things we are really interested in are the correlation functions defining the superconformal field theory. So far they have escaped our control.

Recently a tool has emerged that might finally help us to tackle this goal. According to Maldacena there is a duality that relates the theory on the worldvolume of a given brane setup to gravity in the background of the branes. At large N the gravity calculation reduces to a classical calculation. It will be of great interest to apply these techniques to the theories I have been discussing. Some progress in this direction has been made in [100] where the dimensions of the chiral operators have been found. In this way the brane setups I have presented serve as the natural starting point for any more elaborate analysis of the physics of these 6 dimensional theories.

3.3.4 A chiral / non-chiral transition

By the very nature of Hanany-Witten setups there is a close connection between physics in $d+2$ and d dimensions. As I explained, if we realize our desired d dimensional gauge theory on the worldvolume of Dd branes on an interval, matter multiplets can easily be incorporated by orthogonally intersecting $D(d+2)$ branes. Now instead of just using $D(d+2)$ branes we can use a full Hanany-Witten setup, corresponding to a gauge theory in two more dimensions with twice the amount of supersymmetry. Applying this idea let me use our 6d brane setups to learn something about 4d physics. We will follow what happens to our setup under the same brane motions we have performed above, again giving rise to phase transitions, this time for $\mathcal{N}=1$ supersymmetric 4 dimensional vacua. As a highlight we will present a transition between a chiral and a non-chiral phase [72] [10].

Let me start with the standard ingredients realizing an $\mathcal{N}=1$ theory in 4d, that is we will have to deal with NS5 branes, D6 branes and D4 branes. We will allow NS5 and D6 to be rotated from 45 into 89 space by an arbitrary angle. As usual we will refer to NS5 branes if they live along 012345 and NS5' if the angle is 90 degrees, that is they live in 012378 space. When using more general branes the we will call them A,B or C NS5 branes and the following conventions will be used for the angles

[10] The brane realization of this chiral theory was also discussed in [94, 101].

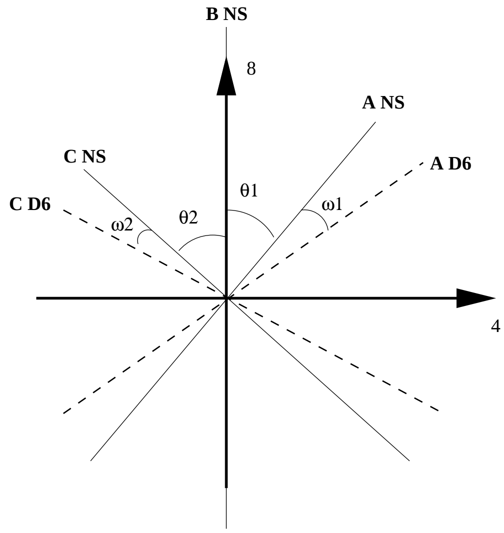

Figure 17: Notation for the angles in the product group setup.

In what follows I will at some point replace a D6 by a full 6d HW setup, that is by D6 branes with NS5' branes embedded in them. In our 6d analysis we already realized that in order to get interesting phase transitions we should really include an orientifold. This way a single NS5' brane will be stuck, while bringing it together with another one we will realize a non-trivial fixed point which has a new branch opening up.

Hence the easiest setups we are going to study have at least 2 group factors and 3 NS5 branes: the NS5' stuck at the orientifold and then another NS5 and its mirror. In order to study the resulting theory it is very helpful to first discuss the matter arising without the orientifold. That is we first study the following setup for a product gauge group analyzed in[70]:

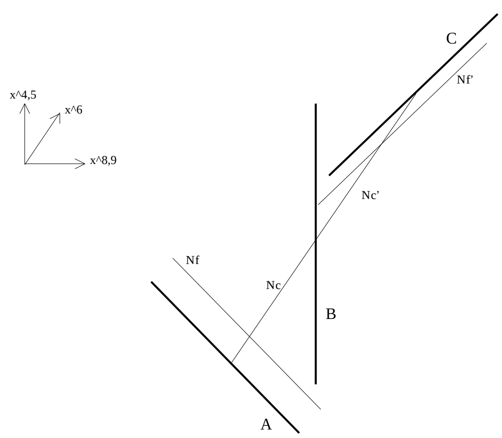

Figure 18: The product gauge group considered in [70]. The thick lines are NS branes at arbitrary angle in 45-89 direction. The D6 branes are parallel to the A and C NS5 branes.

Let me choose the B branes to be still NS5' branes (just fixing the overall orientation). Let (k, k', k'') denote the number of A, B and C type NS5 branes respectively. The resulting SYM will have $SU(N_c) \times SU(N_c')$ gauge group with bifundamental matter F and $\tilde{F}$ and fundamental chiral multiplets Q, $\tilde{Q}$ and Q', $\tilde{Q}'$ from the D6 branes in the first and second group factor respectively. In addition there will be adjoints X_1 and X_2. For $k = k' = k'' = 1$ they will be massive and can be integrated out.

According to [70] the (k, k, k) case then leads to a superpotential

$$W = m_1 X_1^{k+1} + m_2 X_2^{k+1} + X_1 \tilde{F} F + X_2 \tilde{F} F + \lambda_1 Q X_1 \tilde{Q} + \lambda_2 Q' X_2 \tilde{Q}' \tag{3.13}$$

while the $(k, 1, k)$ case leads to

$$W = -\frac{1}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)(F\tilde{F})^{k+1} + \lambda_1 Q X_1 \tilde{Q} + \lambda_2 Q' X_2 \tilde{Q}'. \tag{3.14}$$

In the following, I will identify these superpotentials by their triple number, $(\cdot, \cdot, \cdot)$. The coefficients in the superpotential are determined in terms of the angles as

$$\lambda_1 = \sin\omega_1 \tag{3.15}$$

$$\lambda_2 = \sin\omega_2 \tag{3.16}$$

$$m_1 = \tan\theta_1 \tag{3.17}$$

$$m_2 = \tan\theta_2. \tag{3.18}$$

As usual all these statements can be verified by studying allowed brane motions and comparing with the classical moduli spaces of the gauge groups in the presence of these superpotential terms.

Now we are in a position to introduce the orientifold. I will put an O6 on top of the middle NS5' brane, so that the NS5' is embedded inside the O6 and we are basically dealing with a 6d HW setup. In principle we can also include an O6'. However this won't be related to any 6d theory. In order to be symmetric with respect to the orientifold we have to restrict ourselves to

- $N_c = N_c'$
- $N_f = N_f'$
- $\theta_1 = -\theta_2$ and $\omega_1 = -\omega_2$ (the A branes have to be mirrors of the C branes)
- equal number of A branes and C branes, that is we consider only (k, k', k) configurations
- one can only include an O6 or an O6$'$ (the B NS5$'$ brane, carrying one unit of NS charge, has to be self-mirror)

Since one possible deformation will be to move the NS5' brane along the orientifold (corresponding to a baryonic branch of the gauge theory) I will also have to study setups with $k' = 0$. This will again allow me to uniquely fix the matter content and the interactions by comparing to the flat directions expected from classical field theory. So we want to consider the following possible setups:

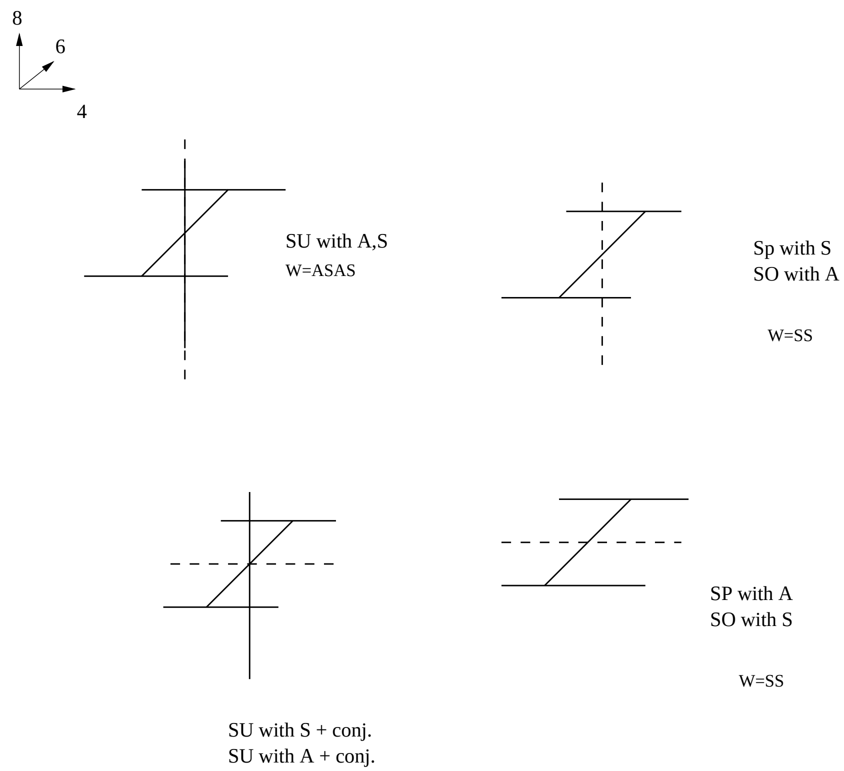

Figure 19: Theories with orientifolds and their deformations

On the left in figure 19 we find the two possible projections of the product gauge group setup, the right hand side displays two possible setups involving just 2 NS5 branes and an orientifold, which arise as possible deformations of the theories on the left, as we will see in the following. Let us briefly summarize the resulting gauge groups and matter contents. A more detailed discussion of this identification will be presented in the following sections.

The configuration in the upper right corner of figure 19 is just the $\mathcal{N} = 2$ setup analyzed in [50]. The corresponding gauge group is SO (USp) depending on the sign of the orientifold projection. Rotating the NS5 branes breaks $\mathcal{N} = 2$ to $\mathcal{N} = 1$ by giving a mass to the adjoint chiral multiplet in the $\mathcal{N} = 2$ vector multiplet. We are left with an SO (USp). Note that this mass is already infinite at $\theta = \frac{\pi}{4}$, since this time it is given by the angle between the outer NS5 branes which is twice the angle θ between outer NS5 and NS5$'$ that determined the mass before. Rotating further I claim that instead of the adjoint tensor that became infinitely heavy a new tensor with the opposite symmetry properties is coming down from infinite mass. As usual the way to prove this is to compare the possible brane moves with results from classical field theory. Note that if we were dealing with D branes instead of the NS5 branes the rotation we performed would precisely change their worldvolume gauge theory from SO to Sp. It's therefore reasonable to assume that something similar happens to the NS5 branes, too. This way we identify the gauge group corresponding to the brane configuration in the lower right corner of figure 19 to be an SO (USp) gauge theory with a symmetric (antisymmetric) tensor.

In the two pictures on the left the two SU factors are identified under the orientifold projection. One adjoint field is present, whose mass is given by the angle θ between NS5 and NS5$'$ brane. In addition there are degrees of freedom that gave rise to bifundamentals in the product gauge groups. According to the analysis of [72] one finds that in the orientifolded theories the O6' will give rise to a full flavor [11] of symmetric or antisymmetric tensors for the O6$'$, depending on the sign of the projection. The really interesting case is the one with the O6. This now is part of a 6d HW setup. The O6 changes sign when passing through the NS5'. This means that in order to conserve RR charge it has to have 8 half D6 branes embedded in it on one side. The corresponding matter content will be an antisymmetric tensor, a conjugate symmetric tensor and 8 fundamentals. Analyzing the field theory one indeed finds two baryonic branches corresponding to moving the NS5' to the SO or USp side of the O6 and leaving one of the two theories on the right side of figure 19 respectively. Note that this is a chiral theory with anomaly free matter content. The 8 fundamentals are precisely what we need to cancel the contribution from the tensors.

Now we have all the ingredients we need in order to perform the transition. We perform the "small instanton transition" very similar to how it was performed in the 6d case and follow through what happens to our 4d physics.

[11] that is the tensor and its conjugate

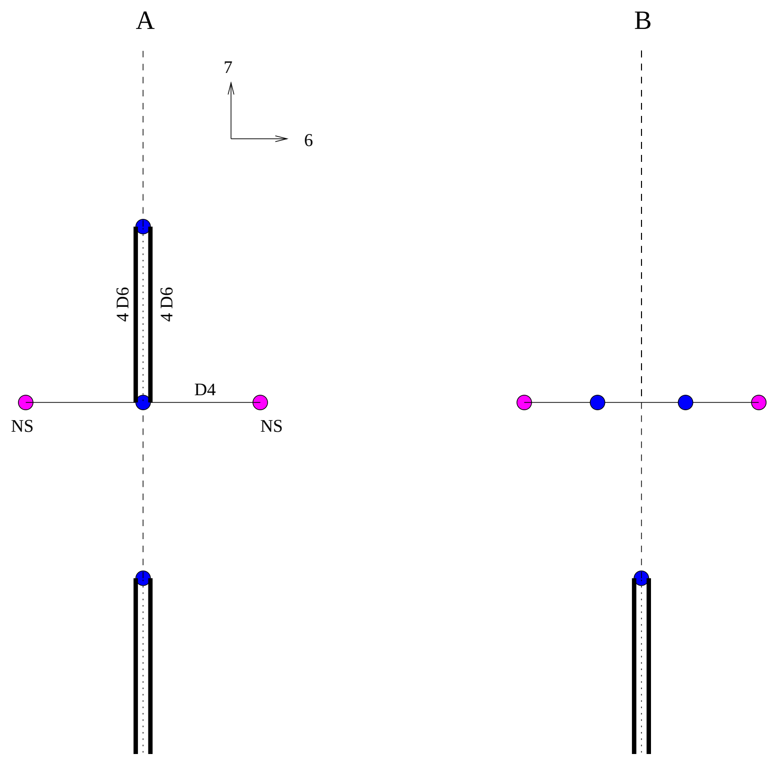

Figure 20: A small instanton transition which lead to chirality change in the spectrum. An NS5′ brane comes from infinity in the x^7 direction and combines with the stuck NS5′ at the origin. At this point they can both leave the origin in the x^6 direction. The resulting four dimensional theory is no longer chiral.

Consider the configuration depicted in figure 20 A. It realizes the chiral theory I just described. I included two other NS5' branes far away so that they have no effect on the dynamics. We would like to bring them in from infinity in order to perform the transition. We will call them upper and lower branes, respectively. The original NS5′ brane, to which the four dimensional system is attached, will be called central.

The NS5′ branes are not allowed to move in the 456 directions, since they can only move off the orientifold as a mirror pair. On the other hand, the relative motion of the NS5′ branes along the x^7 direction corresponds to a change in the two tensor multiplets in the 6d theory. There is, however, an option for a pair of NS5′ branes to move in the 456 directions. We can move two NS5′ branes to touch in the x^7 direction. At this point, the orientifold planes from above and below the pair of NS5′ branes are identical, as is clear from figure 20 . From a six dimensional point of view such a motion corresponds to taking one of the gauge couplings to infinity and thereby obtaining strings with vanishing tension - a non trivial fixed point. The pair of NS5′ branes can then move in the 456 directions, as in figure 20 B. The resulting six dimensional gauge group is now completely broken.

Let us go back to reinterpret this transition in our four dimensional system. When the NS5$'$ branes are far, we have our chiral theory. When the NS5$'$ branes move in the 456 directions, the theory is different. We can read it off from figure 20 B. We have $SO(N_c) \times SU(N_c)$ with a symmetric tensor for the SO group and a pair (chiral and its conjugate) of bi-fundamental fields. Note that this is a non-chiral theory as advertised.

Chapter 4

The equivalence of the various approaches

As mentioned in the beginning there are several ways discussed in the literature to embed gauge theories in string theory. So far I have only discussed in detail the Hanany-Witten approach. The advantage of this approach is that the setups and especially their moduli and deformations have a very intuitive meaning. One just has to move around flat branes. In this section I'd like to show that indeed all the approaches are equivalent, in the sense that the resulting string theories are dual to each other. This way I will show that the phase transitions I studied in the previous chapter by moving branes together and apart again in a different fashion can indeed be interpreted on the dual side as a transition from one geometrical compactification to a topologically distinct vacuum. This analysis will hopefully provide one further step towards an understanding of the dynamical processes relating the various string vacua, finally leading to a determination of the true vacuum of string theory.

4.1 The duality between orbifold and NS5 brane

The basic relation for our discussion will be the observation of [65] that string theory in the background of an NS5 brane is T-dual to string theory on an A-type ALE space. This is already an example of an equivalence between branes as probes and geometrical engineering in the restricted case of theories with 16 supercharges. Embedding a given 6d field theory in string theory via probing flat IIA/B with an NS5 brane or by engineering it via IIB/A on an ALE space is guaranteed to give the same results, since these two setups are actually equivalent as string backgrounds.

Let me discuss this duality in some detail. I will mostly follow the reasoning of [102, 103]. In order to apply T-duality we should look for a $U(1)$ isometry of our solution. C^2/Z_{k+1} does have some $U(1)$ isometries, but the corresponding radius grows as we move away from the orbifold point, so that at infinity it becomes infinite. The would be dual should then be some geometry with a vanishing circle at infinity and hence would be hard to interpret.

In order to make some progress we consider a slight modification of the original geometry to something that looks asymptotically like an S^1 fiber over R^3 and still has the A_k singularity at the origin [1]. These are precisely the properties of a $k+1$ centered KK monopole in the limit that all the centers coincide. Now we just T-dualize the $U(1)$ isometry we introduced by hand. One finds that the dual metric is the NS5 brane metric as advertised. A short-cut argument is that T-duality exchanges the two $U(1)$ fields in 9d generated by the component of the metric and the B-field around the circle respectively. Since the KK-solution is the monopole of the former while the NS5 brane is the monopole of the latter, the two solutions get interchanged under T-duality, too.

4.2 Hanany Witten versus branes at orbifolds

4.2.1 The orbifold construction

Now we want to move on to the more interesting case of 8 supercharges. Let me first demonstrate that Hanany-Witten setups are equivalent to branes at orbifolds, a particular realization of the branes as probes idea. Here I will follow the discussion in [82, 81]. We consider D5 branes moving on top of a C^2/Γ orbifold space.

At this point it is necessary to actually work out the field theory of K Dp [2] branes moving on the orbifold. So let me briefly review the relevant analysis due to Douglas and Moore [19].

Even though the orbifolded space is singular, string theory physics on the orbifolded space is smooth. String theory automatically provides additional modes in the twisted sectors that resolve the singularity. In the case of an orbifold these twisted sectors are taken care of by including also the k Z_{k+1} mirror images of each original Dp brane. At the orbifold point the original Dp coincides with all its mirrors, so that we start off with maximally supersymmetric SYM in $p+1$ dimensions and $U((k+1)K)$ gauge group. In this theory we impose the projection conditions by throwing away everything that is not invariant under Z_{k+1}. Since we chose our Z_{k+1} action in such a way that it is a subgroup of $SU(2)$ (remember that this is what we mean by the orbifold limit of an ALE space), we are guaranteed that the remaining spectrum will be that of a theory with 8 supercharges,

[1] The decoupled physics in 6d won't depend on the radius of the circle, as one could see by recalling once more the decoupling limit. It is only encoded in the local singularity and not in the global structure.

[2] Of course we have to restrict ourselves to $p \leq 5$ so that the transverse space is big enough to carry an 4d ALE space. This restriction again just reflects the classical fact that theories in above 6 dimensions have at least 16 supercharges.

so it is enough to study only the bosons and the fermions just follow by SUSY. The Z_{k+1} acts

- on the $SO(4)$ subgroup of the R-symmetry, acting on the scalars and fermions given in the obvious way by embedding of the geometric action in internal spacetime (the scalars are the spacetime Xs!)
- on the $U((k+1)K)$ Chan Paton indices according to some representation of Z_{k+1}.

If we want to describe D3 branes which are free to move away from the orbifold fixed point we should restrict ourselves to embed the orbifold group into the Chan Paton factors via the regular representation R of Γ, that is the $|\Gamma|$ dimensional representation that accounts for every mirror once. Note that this representation is reducible and decomposes in terms of the irreducible representations r_i as $R = \oplus_i dim(r_i) r_i$. Doing so we take into account the D3 branes and all its mirrors, as is required if we want to have branes that are free to move away from the fixed point. Other representations can be considered as well (at least at a classical level) and lead to fractional branes [19, 104, 105, 106]. We will have more to say about these in what follows. For the moment let's restrict ourselves to the regular representation.

We get the following actions on the fields (i, j labelling the columns, of length K, of vector indices transforming under the same irreducible representation r_i of Γ and $a = 1, \ldots, 4$ a vector index of the $SO(4)$ R-symmetry.

Vectors:

$$A^i_j \to r_i \times \bar{r}_j A^i_j$$

leaving a $\prod_i U(dim(r_i)N)$ gauge group, since $r_i \times \bar{r}_j$ contains the identity only for $i = j$. For our case of $\Gamma = Z_{k+1}$ all the r_i are one dimensional, since Z_{k+1} is abelian. Together with the gauginos and eventually the scalars corresponding to motions transverse to the ALE space and transverse to the brane for $p < 5$ [3], they will combine into full VMs.

Scalars:

$$\Phi^{ia}_j \to a^4_{ik} r_k \times \bar{r}_j \Phi^{ia}_j$$

where a^4_{ik} denotes the Clebsch Gordan coefficient in the decomposition of $r_i \times \mathbf{4} = \oplus_k a^4_{ik} r_k$, where where $\mathbf{4}$ denotes the 4 dimensional representation of the scalars under the R symmetry. We hence obtain a^4_{ik} scalars transforming as bifundamentals under the ith and kth gauge group. For $i = k$ this is interpreted as an adjoint. For Z_{k+1}, $\mathbf{4} \to 2r_1 + 2r_{-1}$ and we obtain bifundamentals in neighbouring gauge groups. Together with the fermions these scalars will form HMs.

[3] e.g. a D3 brane in 0123 with the ALE space in 6789 will have a complex scalar corresponding to 45 motions that does not transform under the spacetime action of the orbifold and is hence projected as the vectors.

4.2.2 The classical branches

Having constructed the field theory corresponding to D5 branes on top of the singularity we can move on and study its classical branches. The Higgs branch corresponds to moving the branes away from the singularity, its metric will be the metric of the ALE space (the resolved orbifold) itself. Non-trivial fixed points will occur when the branes actually sit on top of the singularity. For $p < 5$ we will also have a Coulomb branch from the scalars in the VM. This will have to be interpreted in terms of "fractional branes". For $p = 5$, that is in 6d, there is no such Coulomb branch associated to the VM. However we already found before that in 6d we need to couple the gauge theory to tensor multiplets in order to cancel the anomalies. These bring in new scalars which parametrize a branch that is now referred to as the Coulomb branch (even though it has quite a different interpretation from in the other cases). In the HW setup these tensors were automatically provided by the NS5 brane motions. In the orbifold construction they correspond to degrees of freedom coming from reducing the 10d two-form on the vanishing two cycles. Of course these degrees of freedom are also there in fewer than 6 dimensions. But as in the HW case, taking the limit in which the bulk fields decouple we find in $d < 6$ that these extra matter multiplets decouple as well, whereas in $d = 6$ we have to keep them since their interaction strength is of the same magnitude as the 6d SYM coupling.

4.2.3 Including D9 branes and orientifolds

Let me for a moment just discuss the classical theory, so that we do not have to worry about anomalies and charge cancellation. We are hence free to study K D5 branes in the background of N D9 branes and an ADE singularity. Without the D9 branes the Higgs branch still describes the ALE space itself. Without the ADE singularity, the Higgs branch corresponds to the moduli space of K $U(N)$ instantons, since a Dp brane is just the instanton within the Dp+4 brane [45]. This can be put together naturally to the statement that the theory with ALE space and the D9 branes together yield a theory whose Higgs branch is the moduli space of K $U(N)$ instantons on the ALE [19, 83]. Kronheimer and Nakajima [107] have introduced these theories as a hyper-Kahler quotient construction previously, exactly with the goal of describing these moduli spaces. It is beautiful to see that they naturally appear in the classical analysis of brane theory. Of course the same Higgs branch arises in any other Dp D$p+4$ system with the D$p+4$ brane wrapping the ALE space.

Besides the obvious data N and K we need one more piece of information: the Wilson lines at infinity. Since π_1 at infinity is the discrete ADE group Γ such non-trivial Wilson lines can exist. They are characterized by a matrix ρ_∞ representing Γ in the gauge group. That is ρ_∞ is given by $\rho_\infty = \oplus_\mu w_\mu R_\mu$, where R_μ are the irreducible representations of Γ and w_μ some integers, so that ρ_∞ really is a representation of Γ. In addition for ρ_∞

to be an element of the $U(N)$ gauge group it better be an $N \times N$ matrix, so we have to demand that $\sum_\mu n_\mu w_\mu = N$ where the n_μ are the dimensions of the corresponding representation. To keep the notation readable I will from now on focus on A-type (that is abelian Z_{k+1}) orbifold groups only, so that all $n_\mu = 1$. The formulas for the general case can be found in [83]. In addition one could also include non-trivial first Chern classes. They would correspond to turning on B fluxes in the type IIB background. For simplicity I will also neglect those. From here on it is a tedious but straightforward analysis to find the resulting gauge group. It is $\prod_{\mu=0}^{k} U(V_\mu)$ where [4]

$$V_0 = 2K, \quad V_{i\neq 0} = 2K + \sum_{j=1}^{k} C^{-1}_{ij} w_j.$$

As matter we have bifundamentals under neighboring gauge groups and w_μ fundamentals in the μth gauge group. This indeed coincides with the hyper-Kahler construction. Now let us discuss the quantum theory. According to [20] anomaly freedom of the above gauge theory demands $N = 0$. This is no big surprise: we are not allowed to put any D9 branes in type IIB.

Similarly one can discuss K $SO(N)$ instantons. In the brane picture this is done by introducing an additional O9 plane. Here anomaly freedom will demand $N = 32$, since we are now dealing with type I. However the classical construction can still be carried out for any N, yielding again a description of instanton moduli spaces in terms of the classical Higgs branch of a given gauge theory. The resulting gauge group is modified slightly:

- For $k+1$ even there is an additional discrete choice for ρ_∞ due to the fact that the space-time gauge group is really $Spin(32)/Z_2$ and not $SO(32)$. The Z_2 is generated by the element w in the center of $Spin(32)$ which acts as -1 on the vector, -1 on the spinor of negative chirality, and $+1$ on the spinor of positive chirality. Because only representations with $w = 1$ are in the $Spin(32)/\mathbb{Z}_2$ string theory, the identity element $e \in \Gamma_G$ can be mapped to either the element 1 or w in $Spin(32)$.

- The orientifold projection will of course change the gauge group. This is again a straight forward exercise in applying projection operators and one finds for example for $k+1$ odd:

$$Sp(V_0) \times \prod_{\mu=1}^{k/2} SU(V_\mu). \tag{4.1}$$

  The matter content consists of hypermultiplets transforming according to $\frac{1}{2} w_0 \Box_0$, $\oplus_{\mu=1}^{k/2} w_\mu \Box_\mu$, $\oplus_{\mu=1}^{k/2} (\Box_{\mu-1}, \Box_\mu)$ and $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_{k/2}$

[4] C^{-1} denotes the inverse Cartan metric of Γ, that is for Z_{k+1} $C^{-1}_{i<j} = i(k+1-j)/(k+1)$.

- the V_μs pick up an additional contribution from a vector D_μ basically reflecting the orientifold charge so that we obtain

$$V_0 = 2K, \quad V_{i\neq 0} = 2K + \sum_{j=1}^{k} C_{ij}^{-1}(w_j + D_j)$$

Choosing $D_\mu = -\delta_{\mu,0}$ one reproduces the right classical theory whose Higgs branch is the moduli space of instantons, while $D_\mu = -16\delta_{\mu,0} - 8\delta_{\mu,k/2} - 8\delta_{\mu,(k+2)/2}$ yields an anomaly free gauge theory once we couple to the additional tensor multiplets from the 2-form fluxes. Choosing the anomaly free theory one finds that one misses some degrees of freedom that are necessary to describe the Higgs branch. All in all 29 hypermultiplet degrees of freedom a missing for every tensor present on the Coulomb branch. We again interpret this as a small instanton transition. The Higgs branch describes the D5 dissolved as an instanton inside the D9. If the instanton shrinks down to a point we reach a non-trivial fixed point. From there the Coulomb branch opens up, described by the anomaly free gauge theory living on the worldvolume of the D5. The new branch is parametrized by the vev of the scalars in the tensor multiplets, which as I mentioned arise in this setup from reducing the 10d two form on the vanishing cycles.

More intuitive is the same process for the E_8 small instanton. Here the Higgs branch still corresponds to an M5 dissolved inside an M-theory end of the world brane carrying an E_8 gauge theory [108, 109]. However the Coulomb branch this time has a real geometric interpretation, moving of the 5 brane into the bulk. As we have seen, both these cases are also beautifully realized in the HW setup. Here the $SO(32)$ as well as the $E_8 \times E_8$ small instanton's Coulomb branch are visible as motions of the 5 brane.

4.2.4 Applying T-duality

Of course it is no coincidence that we find the same small instanton transition in both realizations. In fact the two approaches are related by a simple T-duality. Considering an HW setup on a circle and T-dualizing this 6-direction, the NS5 branes turn into the orbifold singularity as above [65], the D6 becomes the the D5, the small $SO(32)$ instanton. The O8s and D8s turn into O9s and D9s. We reproduce precisely the same low-energy theories if we identify the w_μ and D_μ I defined in the HW setup as the number of D8 branes in the μth gauge group and the contribution of the orientifold charge to the μth gauge group, with the w_μ and D_μ defined above, basically carrying the information about the non-trivial Wilson lines at infinity. The Z_2 choice for even $k+1$ (number of NS5 branes in the HW setup) corresponds to the choice of having all NS5 branes free to move or one stuck at each of the orientifold planes. These identifications reflect the well known fact [4], that under T-duality the Wilson lines translate into brane positions.

Note that this way a small $SO(32)$ instanton transforms into a D6 brane in the HW setup with the NS5 branes playing the role of the singularity. On the other hand taking the strong coupling limit of the HW the O8 turns into an M-theory end of the world brane. So the strong coupling limit of HW perfectly captures the small $E_8 \times E_8$ small instanton as well, but this time the NS5 branes are the instantons (after all they become M5 branes in 11d) while the D6 branes in 11d turn into the singularity. This way one can reproduce the theories of [91] describing small E_8 instantons colliding on top of an A or D type singularity by HW setups.

4.2.5 Adiabatically expanding to d=4 $\mathcal{N} = 1$

As I explained in the last chapter, a HW setup in d dimensions is closely related to a HW setup in d+2 dimension with twice the amount of supersymmetry, by replacing the flavor system in d dimensions by a whole HW setup in d+2. This was how we realized the chiral / non-chiral transition in 4d by performing the 6d small instanton transition. This correspondence of course has an equivalent in the geometric language: having analyzed string theory on ALE spaces, one can next consider theories on an ALE fibration over a P^1. These will also lead to theories with half the amount of supersymmetry in 2 less dimensions. In the limit of large base size one can apply the 6d results fibrewise. This is usually referred to as the adiabatic argument.

4.3 Branes and geometry: obtaining the Seiberg-Witten curve

4.3.1 The 3 approaches

As we have seen, the branes as probes and the HW approach to field theories are actually equivalent. In both cases we cook up a certain string background whose low energy field theory realizes the gauge setup we want. But then it turns out that these two string backgrounds are dual to each other. This duality allowed us to analyze certain aspects in one or the other frame: e.g. deformations were obvious in the HW setup, the identification of the Higgs branch as the moduli space of $SO(32)$ instantons on an ALE space can easily be motivated from the orbifold point of view. Let me finally show in the example of Seiberg-Witten theory that this kind of relation also extends to the third approach, geometrical engineering. This will complete our dictionary from phase transitions enforced by brane motions to the purely geometrical language of string compactifications [5].

The three approaches to engineer gauge theories have already been mentioned in general in the introduction. Let me show in more detail how they work in the special case of $\mathcal{N} = 2$ theories in 4d. I will especially focus on the quantum aspects, that is I will identify the quantum contributions and show how they can sometimes be solved for.

[5] A similar connection has been established in 5d in [110].

Geometrical Engineering

In the introduction I already discussed how geometric engineering works in principle and how to engineer 6d theories with 16 supercharges. Now we want to go down to $\mathcal{N}=2$ in d=4. So we have to compactify two more dimensions and break half of the supersymmetry. This is done via the adiabatic expansion. That is we look at non-compact Calabi-Yau manifolds, which locally look like an ALE fibration over a 2d base. The simplest base is just a P^1, but we will also need bases built out of several P^1s with non-trivial intersection patterns.

Let me first discuss the case of a P^1 base. If we choose the ALE fiber to be of A_k type, we will start with an $SU(k+1)$ gauge theory in 6 dimensions. The massless states arise from D2 branes wrapping the vanishing cycles of the ALE in the orbifold limit. Taking into account the fibration structure only the monodromy invariant states survive. It turns out that if one deals with a genus g base with ADE singularity one expects to have ADE gauge symmetry with g adjoint hyper multiplets [111, 112, 113]. That is for our case of genus zero we only keep a pure $\mathcal{N}=2$ SYM as desired. For $g=1$ one obtains the finite $\mathcal{N}=4$ spectrum and for higher g we would loose asymptotic freedom.

Decoupling gravity is again achieved by effectively decompactifying the Calabi-Yau manifold and focusing on the local singularity structure. However this time there are two scales involved, the size of the base P^1 and the characteristic size of the fibers, so that some care is required.

As an illustrative example [114] consider the geometric engineering of a pure $SU(2)$ gauge symmetry which is related to an A_1 singularity in $K3$. Locally, one needs a vanishing 2-sphere P^1, around which the D-branes, being the $W^{\pm}$ bosons, are wrapped. This P^1_f has to be fibered over the base P^1_b in order to have $\mathcal{N}=2$ supersymmetry in four dimensions. The different ways to perform this fibration are encoded by an integer n, and the corresponding fiber bundles are the Hirzebruch surfaces F_n. The mass (in string units) of the $W^{\pm}$ bosons corresponds to the area of the fiber P^1_f, whereas the area of the base P^1_b is proportional to $1/g^2$ (g^2 is the four-dimensional gauge coupling at the string scale). Now let us perform the field theory limit which means that we send the string scale to infinity. Asymptotic freedom implies that g^2 should go to zero in this limit; thus the Kähler class of the base P^1_b must go to zero: $t_b \to \infty$. Second, in the field theory limit the gauge boson masses should go to zero, i.e. $t_f \to 0$. In fact these two limits are related by the running coupling constant,

$$\frac{1}{g^2} \sim \log \frac{M_W}{\Lambda}, \tag{4.2}$$

and the local geometry is derived from the following double scaling limit:

$$t_b \sim -\log t_f \to \infty. \tag{4.3}$$

Clearly, this picture can be easily generalized to engineer higher rank ADE gauge groups. In this case there is not only one shrinking P_f in the fiber, but several such that the fiber acquires a local ADE singularity. For the close comparison with the Hanany-Witten set up, it is also interesting how product gauge groups can be geometrically engineered [115]. For concreteness consider the group $SU(n) \times SU(m)$ with a hypermultiplet in the bifundamental representation. To realize this we have an A_{n-1} singularity over P_b^1 and an A_{m-1} singularity over another P_b^1. The two P_b^1's intersect at a point where the singularity jumps to A_{m+n-1}. This can be seen in 6 dimensions as symmetry breaking of $SU(n+m)$ to $SU(n) \times SU(m) \times U(1)$ by the vevs of some scalars in the Cartan subalgebra of $SU(n+m)$. It is straightforward to generalize this procedure to an arbitrary product of SU groups with matter in bi-fundamentals. One associates to each gauge group a base P_b^1 over which there is the corresponding SU singularity; to each pair of gauge groups connected by a bi-fundamental representation one associates an intersection of the base P_b^1's, where over the intersection point the singularity is enhanced to $SU(n+m)$.

Now it is interesting to see how the quantum effects are incorporated. Basically we took care of the 1-loop effects by the double scaling limit, the non-perturbative effects are due to worldsheet instantons, that is non-trivial embeddings of the string worldsheet in the target space. Actually the way this appears in string theory is slightly more involved. From the field theory point of view all the information we want is encoded in the so called prepotential

$$\mathcal{F}(A) = \frac{1}{2}\tau_0 A^2 + \frac{i}{\pi} A^2 \log\left(\frac{A}{\Lambda}\right)^2 + \frac{1}{2\pi i} A^2 \sum_{l=1}^{\infty} c_l \left(\frac{\Lambda}{A}\right)^{4l}. \tag{4.4}$$

Here $\tau_0 = \frac{1}{g^2}$ is the classical gauge coupling, the logarithmic term describes the one-loop correction due to the running of the gauge coupling and the last term collects all non-perturbative contributions from the instantons with instanton numbers c_l. This is the function that is solved for by the SW solution.

The way this prepotential appears in IIA string theory is by a classical computation (that is no g_s corrections). We only get corrections from worldsheet instantons. This is basically due to the fact that the dilaton, which controls the g_s corrections, sits in a hypermultiplet and therefore doesn't talk to the prepotential which controls the vector-multiplets. For a Calabi-Yau it has the following structure [116]:

$$\mathcal{F}^{\rm II} = -\frac{1}{6} C_{ABC} t_A t_B t_C - \frac{\chi\zeta(3)}{2(2\pi^3)} + \frac{1}{(2\pi)^3} \sum_{d_1,\dots,d_h} n_{d_1,\dots,d_h} Li_3(\mathbf{exp}[i \sum_A d_A t_A]), \tag{4.5}$$

where we work inside the Kähler cone $\mathrm{Re}(t_A) \geq 0$. The polynomial part of the type-IIA prepotential is given in terms of the classical intersection numbers C_{ABC} and the Euler number χ, whereas the coefficients $n_{d_1,\dots,d_h}$ of the exponential terms denote the rational instanton numbers of genus 0 and multi degree d_A. For our non-compact version of the

CY there are just two moduli, namely t_b and t_f and the corresponding instanton numbers n_{d_b,d_f} denote instantons wrapping the base d_b times and the fiber d_f times.

Now we want to see that in the double scaling limit (4.3) the string prepotential gives us back the field theory prepotential with the right logarithmic one-loop behaviour. First note that in this limit the n_{0,d_f} yield precisely the logarithmic terms from the 1-loop corrections (well, after all that's how we chose our limit.). The worldsheet instantons wrapping non-trivially around the base will yield the non-perturbative contributions.

But how can we sum them up? What comes to our rescue is mirror symmetry. Type IIA on a given Calabi Yau is dual to IIB on the mirror Calabi Yau. The two hodge numbers counting "shape" and "size", $h^{2,1}$ and $h^{1,1}$ get interchanged. It is interesting to see where the quantum corrections come from. As I already mentioned, the g_s corrections will only affect the HM moduli space. In addition worldsheet instantons will correct moduli associated with $h^{1,1}$, since they are associated with the non-trivial two cycles. But this means that the vector multiplet moduli space on the IIB side is exact!

IIA	α'	g_s
$VM_{h^{1,1}}$	x	0
$HM_{h^{2,1}}$	0	x

IIB	α'	g_s
$VM_{h^{2,1}}$	0	0
$HM_{h^{1,1}}$	x	x

All we have to do now is find the mirror of our non-compact Calabi Yau. There is a well defined description of how to perform this so called "local mirror map" [114]. This way we can solve for the SW curve by a classical computation in string theory. To give the SW curve a geometrical interpretation we once more perform a T-duality, taking the singularity of the IIB fiber into an NS5 brane [117]. This way the beautiful final answer is obtained: The SW curve appears as the physical object a IIA NS5 brane wraps on.

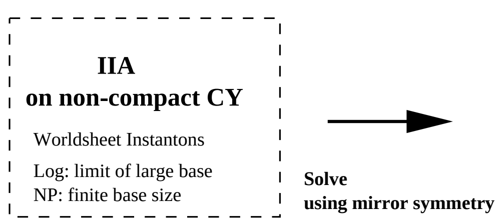

Figure 21: Solving in Geometric Engineering via Local Mirror Symmetry

Fractional Branes

Above I have shown how to construct the field theory of K Dp branes on top of an orbifold singularity. Their Higgs Branch was identified as moving the branes away from the singularity, breaking the $U(K)^{k+1}$ gauge group to its diagonal $U(N)$ subgroup. But, as I mentioned, for $p < 5$ we will also see a Coulomb branch, corresponding to the scalars in the VM, along which the gauge group is generically broken to $U(1)^{K(k+1)}$. How do we see this Coulomb branch in the brane at orbifolds description? I will explain this in the case of D3 branes relevant for the discussion of $\mathcal{N} = 2$ in 4d. Generalizations to other Dp branes should be obvious.

The answer was given in [104] (see also [19]): on the Coulomb branch, a single D3 brane splits into $|\Gamma|$ fractional branes of equal mass $\frac{m_{D3}}{|\Gamma|}$. These fractional branes can be interpreted as D5 branes wrapping the vanishing cycles of the orbifold. Therefore they are stuck to the singularity in the orbifolded part of spacetime, but are free to move in the transverse space, giving rise to the Coulomb branch. The example considered in [104] was D0 branes moving on an ALE space. In the orbifold limit these new degrees of freedom supported on the Coulomb branch are precisely those that are necessary for the Matrix description of enhanced gauge symmetry for M-theory on the ALE space.

Looking just at a single fractional brane is achieved by embedding the orbifold group into the Chan Paton factors via a representation other than the regular one. This is consistent with the fact, that only by choosing the regular representation can we describe objects that are free to move away from the orbifold. This identification can be proven by an explicit world-sheet calculation [19] showing that the fractional branes carry charge under the RR fields coupling to D5 branes. This charge vanishes if and only if one chooses the regular representation. So instead of just obtaining fractional branes by splitting D3 branes to move on to the Coulomb branch, one can just add these fractional branes by hand. This is the approach chosen in [118, 119] to explain the wrapped membranes in Matrix theory. We will give a more detailed analysis of the branches, parameters and the quantum behaviour of these theories as we proceed.

Hanany-Witten

The 4d Hanany-Witten setup and its solution via the M-theory lift has already been discussed in great detail in Chapter 2. Let me summarize Witten's solution I reviewed there in the form of a diagram like in the other case, so that we are able to compare:

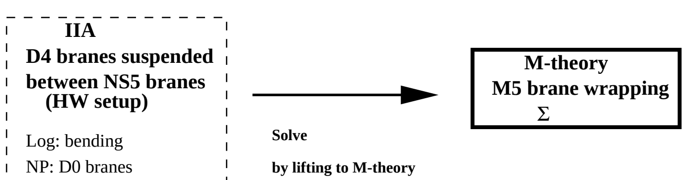

Figure 22: Solving HW setup via M-theory

4.3.2 A T-duality for bent branes and branes ending on branes

From our discussion of the 6d case we are by now used to the idea that branes as probes are dual to HW setups. The T-duality as in the 6d setup will work the same for D4 branes suspended between NS5 branes and D3 branes at ADE singularities, as long as in the HW setup the brane connects all the way around the circle. In 6d we were forced to only consider those setups from anomaly reasons. Here they only correspond to the very special case of finite theories at the origin of the Coulomb branch. To understand the more general cases we will need a generalization of the notion of T-duality to the case of branes ending on branes and bent branes. This will be the purpose of this section. To summarize what we will find: the dual of a brane living on a interval bounded by two other branes will turn out to be the fractional brane discussed in the context of branes at orbifolds. Or again phrased in our diagrammatic language:

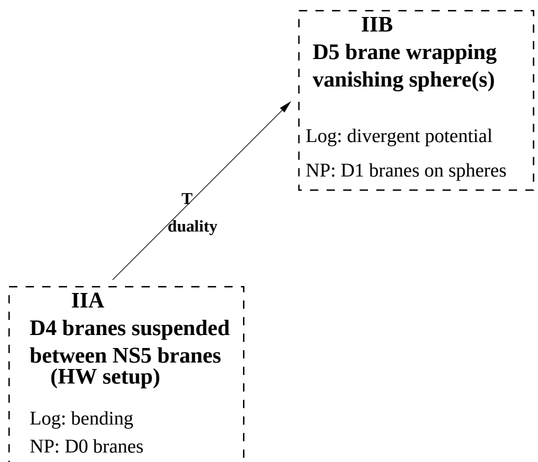

Figure 23: Duality relation between HW setup and fractional branes

Consider again K D3 branes at a transverse A_k singularity, that is we take the D3 branes to live in the 0123 space and put these on top of an A_k singularity. Since the D3 branes can move away from the singularity we should choose to embed the orbifold group in the Chan Paton factors via the regular representation, that is include the D3 brane and all its k images. The corresponding gauge group is $U(K)^{k+1}$. Upon T-duality in the 6 direction this can be mapped into an elliptic Hanany-Witten setup, that is $k+1$ NS5 branes living in the 012345 directions, with K D4 branes living in 01236 space suspended between every pair of consecutive NS5 branes on the circle. Let us label the NS5 branes with $i = 0, \ldots, k$. This is just the T-duality between branes at orbifolds and HW setups I discussed above in detail for the 6d case.

But now consider a setup where we have just K D4 branes stretching between the ith and the $(i+1)$th NS5 brane and no other D4 branes. By just comparing the resulting gauge theory (a single $U(K)$) it is natural to assume that the same T-duality in the 6 direction as we used above now translates this into N fractional branes in the sense of [104], that is N D5 branes wrapping the vanishing homology two-cycle σ_i [6] of the A_k orbifold in the dual picture. Or again in the language of [104], we consider the same orbifold group, but this time choose to embed the orbifold group into the Chan Paton factors via the 1d irreducible representation r_i associated with the ith node of the extended Dynkin diagram. Or more generally, if we have K_i D4 branes connecting the ith NS5 brane with the $(i+1)$th NS5 brane the dual setup will again be given by a Z_{k+1} orbifold, this time with the orbifold group embedded in the Chan Paton indices via a general representation R given by

$$R = \oplus_i N_i r_i. \tag{4.6}$$

As a first check note that even in this more general case the resulting gauge groups and matter contents agree: both constructions yield a $\prod_i U(K_i)$ gauge group with hypermultiplets transforming as bi-fundamentals under neighboring gauge groups. Similarly we can compare the classical parameters and moduli. First consider the case of all $K_i = K$, where the duality is well established. This theory has two branches, a Coulomb branch and a Higgs branch. As free parameters we can add FI terms ξ_i, for each gauge group factor which are triplets under the $SU(2)_R$ symmetry and totally lift the Coulomb branch. Since we have $k+1$ gauge group factors we have in addition k gauge coupling constants g_i^2, which at least in the classical theory are additional free parameters. Together with the θ-angles they form the complex coupling constants $\tau_i = \frac{\theta_i}{2\pi} + \frac{4\pi i}{g_i^2}$

On the orbifold side the Higgs branch just corresponds to moving the D3 branes away from the orbifold point in the ALE space. Therefore the Higgs branch metric coincides with the metric of the transverse ALE space. Since all the N_i are equal in the HW setup all the D4 brane pieces can connect to form N full D4 branes which can then move away from the NS5 branes in the 789 direction. As usual, the 4th real dimension of this branch is given by the A_6 component of the gauge field on the D4 branes.

When the D3 branes meet the NS5 branes in 789 space, that is at the origin of the Higgs branch, they can separate into the pieces which are then free to move around 45 space along the NS5 branes giving rise to the Coulomb branch. In the orbifold picture the same process is described by the K D3 branes splitting up into $(k+1)K$ fractional branes. These are now localized at the singularity in 6789 space, but are also free to move in the 45 space. Now consider turning on the FI terms ξ_i. For the orbifold this corresponds to resolving the singularity by blowing up the ith vanishing sphere σ_i. The mass of the states on the Coulomb branch (which are after all D5 branes wrapping these spheres) become of order ξ_i^2. The Coulomb branch is removed as a supersymmetric vacuum. In the dual

[6] where $\sigma_{k+1} = -\sum_i \sigma_i$ [104]

picture the FI terms correspond to motion of the NS5 branes in 789 space. This resolves the singularity since the 5 branes no longer coincide and again the mass of states on the Coulomb branch (which in this case are D4 branes stretching between the displaced NS5 branes) is of order ξ_i^2.

Last but not least we have to discuss the coupling constants. On the HW side they are simply given in terms of the distances Δ_i between the NS5 branes in the 6 direction, or to be more precise

$$g_i^{-2} = \Delta_i / g_s^A l_s \tag{4.7}$$

where g_s^A and l_s denote the string coupling constant and string length of the underlying type IIA string theory respectively. Since we are dealing with a theory with compact 6 direction we have

$$\sum_i \Delta_i = R_6 \tag{4.8}$$

where R_6 is the radius of the 6 direction. The easiest case to consider is if all $\Delta_i = \Delta$, that is the NS5 branes are equally spaced around the circle. In this case we get $g_i^{-2} = g^{-2} = R_6/g_s^A k l_s$. Applying T-duality to type IIB this means that $g_i^{-2} = 1/g_s^B k$ since $g_s^B = g_s^A l_s / R_6$, in accordance with the orbifold analysis [19, 120]. The same configuration can be also viewed as N_i fractional branes, that is N_i D5 branes wrapping the ith vanishing cycle. According to [120] in this case the coupling constant is given in terms of fluxes of two-form charge on the vanishing sphere

$$\tau_i = \int_{\sigma_i} B^{RR} + i \int_{\sigma_i} F - B^{NS} \tag{4.9}$$

At the orbifold the values of the B-fields are fixed to yield again $g_i^{-2} = 1/g_s^B k$ [104]. Next consider the case where all the NS5 branes coincide. In 6d this will correspond to an enhanced gauge symmetry on the NS5 brane. As shown in [121] this only happens if the B-flux is zero on the dual side, again in accordance with our formulas. [7] Under T-duality these $\int_{\sigma_i} F - B^{NS}$ Wilson lines translate into the positions of the NS5 branes on the circle, as expected.

It is easy to see that this discussion generalizes without any problems to arbitrary values of K_i. In addition we can introduce D6 branes in the HW setup living in 0123789 space to give extra matter. These simply T-dualize into D7 branes wrapping the ALE space in the dual picture.

One may wonder whether this is consistent as a quantum theory. Since, as we discussed above, the fractional branes are charged under the RR gauge fields we need 'enough' non-compact space in the transverse dimensions for this to be consistent. Since we are dealing

[7] The one gauge factor with the finite gauge coupling is asymptotically non-free. In the IR it is just a global symmetry. At higher energies new 6 dimensional degrees of freedom come in from the self-intersection of the NS5 branes - it is impossible to put bent branes on a circle without any selfintersection.

with $\mathcal{N} = 2$ SUSY there are only two kinds of corrections, one-loop and non-perturbative corrections. All higher loop contributions vanish exactly. The one loop beta function gives rise to the usual logarithmic running of the coupling constant. The instanton contributions have been summed up by the Seiberg-Witten solution.

On the HW side it is well known how to incorporate quantum corrections. In the full string theory the D4 brane ending on the NS5 brane actually has a back-reaction on the NS5 brane leading to a logarithmic bending of the NS5 brane. Since the coupling constant is given in terms of distance between the NS5 branes this fact just reflects the 1-loop correction, the running of the gauge coupling.

The non-perturbative effects in this case are given by Euclidean D0 branes stretching along the 6 direction [46, 47, 48] using the well known relation that SYM instantons inside a Dp brane are represented by D$p-4$ branes [45]. Like the D4 branes themselves these Euclidean D0 branes can of course split on the NS5 branes and become fractional D0 branes [48]. According to the same T duality in the 6 direction we applied to the D4 branes they should become fractional D(-1) branes, that is Euclidean D1 branes wrapping the vanishing cycles. This fits nicely with the picture that this way the instanton corrections on the orbifold side are again given by D$p-4$ branes living inside the Dp brane.

But how does the 1-loop effect arise in the orbifold picture? Note that the fractional branes carry charge under the appropriate RR fields, as can be born out by a world-sheet computation of the corresponding tadpoles [19]. They represent a charge sitting in the space transverse to the D3 brane and transverse to the orbifold. In our example this is the 2 dimensional 45 space. In two dimensions the Green's functions are given by logarithms. Therefore we would expect a theory of a charged object in 2 dimensions to be plagued by divergences. To be more precise we have the following trouble: we want to consider effectively a 3 brane sitting in 6d spacetime. Since the transverse space is only 2d, the corresponding 4-form vector potential $C^{(4)}$ grows logarithmically.

From the D5 brane point of view the 3 brane vector potential couples to the world-volume theory via

$$\int dx^6 \mathcal{F} \wedge C^{(4)}. \tag{4.10}$$

After compactification on σ_i this leads to a term in the 3 brane action of the form

$$\int_{\sigma_i} \mathcal{F} \cdot \int dx^4 C^{(4)}. \tag{4.11}$$

But recall that we identified $\int_{\sigma_i} \mathcal{F}$ as the gauge coupling $\frac{4\pi}{g_i^2}$. The growing of $C^{(4)}$ can be absorbed by introducing an effective running coupling constant. The divergences we encounter are just due to the 1-loop running of the gauge coupling! Charge neutrality, which is obtained if we choose the regular representation, corresponds to finiteness [19, 120]. In this way we can see that the correct dependence of the beta function on N_c and N_f is reproduced. This can most easily be seen by observing that the charge clearly depends linearly on the number of fractional branes and is zero when $N_f = 2N_c$, corresponding to the case where the fractional branes can all combine to form a full brane. The relative contributions originate from the (self-)intersection numbers of the spheres σ_i given by the (extended) Cartan matrix (here for A_{k-1}).

In this way the bending of the branes directly translates into the fall-off of the RR field in transverse space in the orbifold picture. The correspondence also holds in other dimensions (for D5 branes the transverse space is 0 dimensional and we need neutrality reflecting anomaly freedom of the underlying gauge theory, in 5,4,3,2,... dimensions we get linear, logarithmic, $1/r$, $1/r^2$, ... fall off for the field strength). This reflects the appropriate running of the gauge coupling just as in the HW picture.

4.3.3 Unifying the different approaches

Now we are in a position to put all the three approaches together into an unifying picture. To visualize what we are about to do, let me first show a diagram that summarizes the connection. I will then explain the dualities involved step by step.

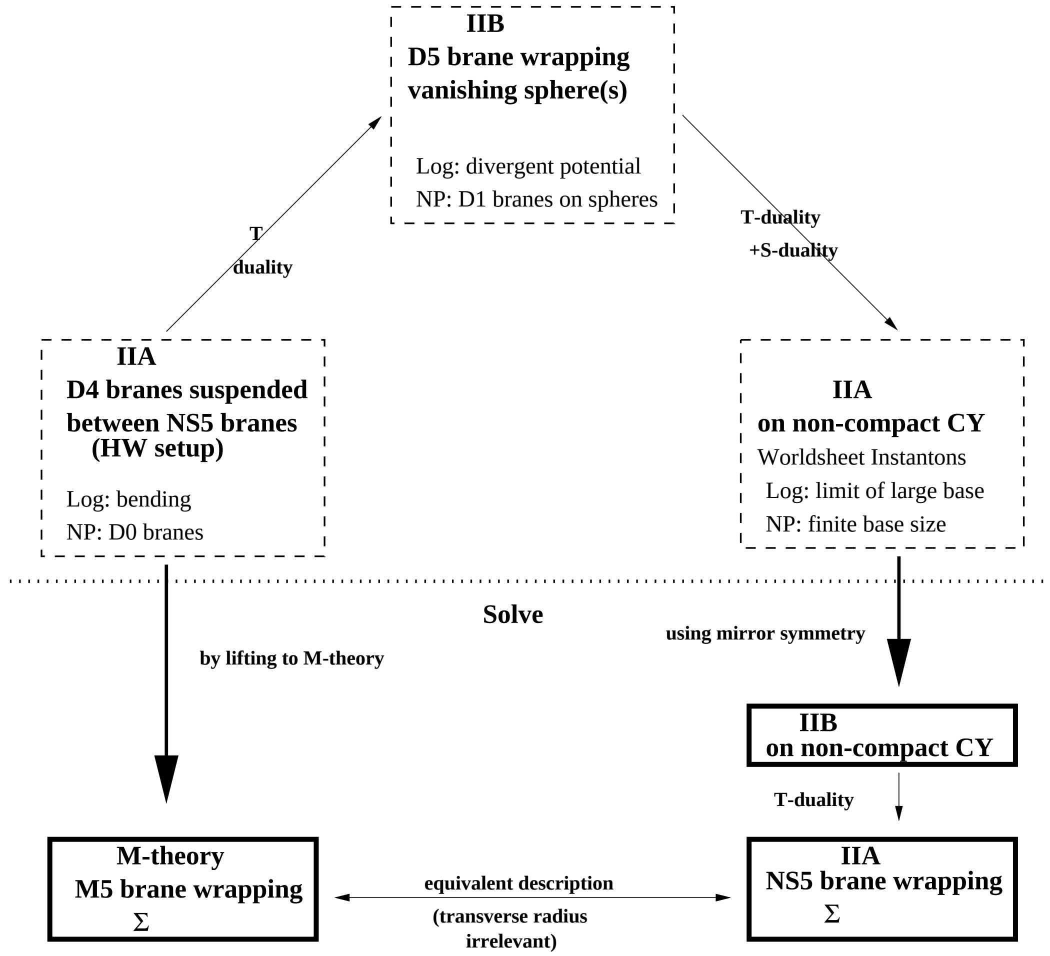

Figure 24: Connection between the various approaches. The non-compact Calabi Yau spaces are ALE spaces fibered over a sphere (or several intersecting spheres). These can be T-dualized via T-duality on the fiber. The last T-duality, connecting to the HW picture is the one discussed in this work.

In Figure (24) the connections are displayed diagrammatically, putting together the pieces of Figures (21), (22) and (23). For the geometric engineering approach of [114] the starting point is IIA string theory on a non-compact Calabi-Yau which is an ALE space fibered over some base (a non-compact version of a K3 fibration). Quantum corrections can be solved by using mirror symmetry to a IIB picture. A T-duality on the ALE fiber can be used to map this to a IIA NS5 brane wrapping the Seiberg-Witten curve [117, 114]. While in this approach the solution is obtained via mirror symmetry, this last T-duality leads to the very nice interpretation of the SW curve: it appears as the physical object the 5 brane is wrapping.

Applying the same T-duality, which we have just applied to the IIB solution, directly on the ALE fiber in the IIA setup (which is the starting point for the geometric engineering approach), followed by an S-duality in the resulting IIB string theory, it is straight-forward to connect this to a setup of D5 branes wrapping the base of the fibration.

Using the T-duality between fractional branes and branes ending on branes proposed above one can translate this into the very intuitive language of a HW setup [35], which finally can be solved for by lifting to M-theory [44]. This way we not only obtain the same solution (the SW curve), but also the same physical interpretation: the SW curve appears directly as the space an M5 brane wraps on [8].

In the following we will discuss the steps in the above chain of dualities in more detail. There are two important points one should account for. For one thing there are the quantum corrections. Since we are dealing with $\mathcal{N}=2$ there are only two types of these corrections: the 1-loop contribution which gives rise to the logarithmic running of the gauge coupling and the instanton corrections. It is interesting to see how these quantum effects are incorporated in the various pictures and how they are finally solved for.

The other thing one should treat with some care is how the singularity type affects the gauge group. In all the examples we are considering there are always two pieces of information, which we will refer to as the singularity types determining the "gauge group" and the "product structure". This is easiest to understand in a particular example. Consider N D3 branes at an A_{k-1} singularity. This gives rise to an $SU(N)^k$ gauge group. In this example N determines the "gauge group" (the size and type of the single gauge group factors) and k the "product structure" (we have k $SU(N)$ factors). Since in the various duality transformations we are using we repeatedly map branes into singularities and vice versa it is quite important to distinguish which ingredient in the picture determines gauge group and which determines product structure. In the cases where the gauge group is determined by a geometric singularity, generalization to D or E type singularities leads to SO or $E_{6,7,8}$ gauge groups, while in the cases where the product structure is determined by a geometric singularity the generalization to D or E type just leads to a more involved products of $SU(N)$ groups determined by the corresponding extended Dynkin diagram. For example in the D3 brane case N D3 branes at an E_6 singularity lead to an $SU(N)^3 \times SU(2N)^3 \times SU(3N)$ gauge group.

HW $\rightarrow$ fractional branes: This is the duality between branes ending on branes and fractional branes I have discussed above. A system of k NS5 branes with N_i D4 branes suspended between the ith and the $(i+1)$th NS5 brane is dual to an A_{k-1} orbifold singularity with N_i fractional branes associated to the ith shrinking cycle. While in the HW setup the gauge group is determined by the D4 branes and the NS5 branes determine the product structure, the corresponding roles are played by D3 branes and the orbifold type in the dual picture. That is, the vanishing 2-cycles are given by spheres intersecting according to the extended Dynkin diagram. The gauge group can be generalized to D type

[8] In the spirit of [122] the difference between IIA and M5 brane is irrelevant, since the radius of the transverse circle does not affect the low-energy SYM on the brane.

by including orientifold planes on top of the D-branes. It is not known how to achieve E type gauge groups this way. Using an E type orbifold in the fractional branes only affects the product structure.

The 1-loop quantum effects correspond to bending of the NS5 branes in HW language and get mapped to the logarithmic Green's functions in the orbifold picture, as discussed in the previous section. Non-perturbative effects are due to Euclidean D0 branes, which are mapped by the same T-duality into Euclidean D1 branes wrapping the vanishing 2-cycles (fractional D-instantons).

Fractional branes → IIA on non-compact CY: We can now apply a different T-duality on the setup described by the fractional branes. First we S-dualize, taking the N_i D5 branes into NS5 branes and the Euclidean D1 branes into fundamental strings (that is world-sheet instantons). Performing a T-duality in the overall transverse space (that is 45 space) the NS5 branes turn into an A_{N_i} singularity according to the duality of [65] which we have already used several times. The vanishing 2 cycle, which is still the space built out of the k spheres intersecting according to the extended Dynkin diagram, stays unchanged. The fact that the NS5 branes only wrap parts of this base (N_i NS5 branes on the ith sphere σ_i) translates into the fact that the type of the ALE fibers over the base changes from one sphere to the other. From the IIA point of view this looks like T-duality acting on the fibers as described in [117]. Since now everything is geometric, generalizations to E type are straight forward: the product structure is determined by the intersection pattern of the k spheres, the ADE type of the fiber determines the gauge group. This way we can even engineer products of exceptional gauge groups.

The non-perturbative effects are now due to world-sheet instantons, as expected. The log-corrections coming from one loop are incorporated in the particular limit one has to choose to decouple gravity [123]. The system can be solved via local mirror symmetry.

Chapter 5

Open problems and directions for further research

5.1 Brane Boxes

As I mentioned in the beginning there is a natural generalization of HW setups: the brane box [41]. In order to branegineer generic models with 4 supercharges, one should consider living on a rectangle bounded by two kinds of NS5 branes. Only recently it has become clear [43] that these models are indeed consistent also at the quantum level. This calculation is done using a generalization of the HW - branes as probes T-duality I have presented here. Brane boxes are T-dual to D3 branes at a C^3/Γ orbifold, where this time $\Gamma \in SU(3)$, so that we are left with $\mathcal{N}=1$ instead of $\mathcal{N}=2$. Again a brane on a single box is T-dual to fractional branes characterized by a given irreducible representation of the orbifold group [124], just as in the case I was discussing.

A tadpole associated to a generic orbifold element without any fixed plane corresponds to a charge in a 0d space. So it has to vanish. Indeed it was shown that vanishing of these generic tadpoles is equivalent to anomaly freedom. However there are also tadpoles corresponding to a twists that leave a 2d fixed plane. For the same reasons as I discussed in the previous chapter one should interpret the resulting logarithmic divergence as the running of the gauge coupling. Vanishing of these tadpoles therefore is not necessary for consistency but implies finiteness. This condition once more corresponds to a no-bending condition for the brane boxes.

It would definitely be desirable to perform the "lift to M-theory" for the brane boxes. For one this should explain the anomaly in terms of bending and hence explore some yet unknown aspects of brane physics. It is still the wrong limit to actually solve the theory, but again it should be possible to solve for all the holomorphic quantities. Some intrinsic $\mathcal{N}=1$ phenomena such as dynamical supersymmetry breaking can be studied this way and find their natural place in string theory. It is obvious that such a lift has to be performed via a SUSY 3-cycle instead of the SUSY 2-cycle. The conditions for SUSY 3-cycles have also been worked out in [51, 52]. These equations are technically much more involved, but hopefully the lift can be performed.

5.2 Maldacena Conjecture

Recently a really remarkable proposal has been made by Maldacena [33] generalizing the aspects of the brane / SYM correspondence I have been discussing so far. It states that the worldvolume theory of a brane is really dual to string theory in the background of the brane. For macroscopic systems (that is large number of coinciding branes) the curvature of the brane solution becomes small, so that supergravity is a good approximation of string theory. Taking the limit $M_s \to \infty$ reduces the worldvolume theory to SYM, while we focus in to the near horizon region of the soliton, leading to the statement that large N $\mathcal{N} = 4$ $SU(N)$ SYM is dual to supergravity in an $AdS_5 \times S^5$ background.

This conjecture already led to a variety of beautiful results. Again it basically helps to understand aspects string theory as well as of field theory. As I explained in Chapter 2 it is the most promising approach for actually solving some of aspects of field theory that so far have escaped our control. On the other hand we have learned several new aspects about quantum gravity. As I mentioned it is the first realization of the concept of holography, which is supposed to be a genuine property of quantum gravity. So it seems that there are still many aspects of brane physics that need to be explored. Hopefully in the end we will find a true understanding of the fundamental problems we set out to solve.

Summary

In this thesis I discussed several applications of the connection of non-perturbative string theory and SYM theory. In Chapter 1 I reviewed the physics of D-branes as one example of a non-perturbative effect in string theory. Their dynamics is dominated by gauge theory. This fact can be used to engineer certain string backgrounds which yield interacting SYM theories as their low-energy description.

In Chapter 2 I then introduced one of the approaches in detail, the HW setup. I gave a summary of the identification of the classical gauge theory, showed how quantum effects manifest themselves in the brane picture and how to solve them.

This way of embedding gauge theories into string theories has several interesting applications. These were the topic of Chapter 3. First I discussed dualities in field theory and showed how they arise as a natural consequence of string duality. As a second application I used branes to prove the existence of non-trivial fixed point theories in 6 dimensions and to study their properties. Some of these fixed points describe phase transitions between two different brane configurations. From a 4d point of view these 6d transitions can induce a chiral non-chiral transition.

In Chapter 4 I discussed the relation of the HW setup with the other approaches of embedding gauge theory into string theory, especially the branes as probes approach. The different ways of embedding gauge theories in string theory are shown to be actually T-dual as string backgrounds. For one this allowed us to explore several new aspects of T-duality, like T-duality for bended branes and branes ending on branes. In addition this relation can be used to show that the transitions found in the brane picture can as well be understood as transitions between topologically distinct compactifications of string theory.

Some open problems and directions for further research were mentioned in Chapter 5.

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Zusammenfassung

Nach jahrelanger Suche hat sich bis heute Stringtheorie als einziger Kandidat einer konsistenten Quantentheorie der Gravitation herauskristallisiert. Um aus der Stringtheorie präzise Vorhersagen für unsere Niederenergiewelt zu gewinnen, ist es notwendig, das Vakuumproblem zu lösen, das heißt einen Mechanismus zu finden, der aufzeigt, in welchem Stringvakuum wir leben und warum die Natur dieses ausgewählt hat. Die Beantwortung dieser Frage benötigt nicht-perturbative Informationen. Diese wurden erst in jüngster Zeit zugänglich.

Eine besondere Rolle in dieser Entwicklung spielten die sogenannten D-branes. Sie stellen mögliche nicht-perturbative Beiträge zu Stringamplituden dar. Die Identifizierung, daß D-branes einfach Objekte sind, auf denen Strings enden können, ermöglicht sie zu handhaben und zu zeigen, daß ihre Dynamik im wesentlichen durch Eichtheorien erfaßt wird. D-branes erlaubten, zahlreiche Dualitätssymmetrien zu etablieren, deren Hauptaussage zu sein scheint, daß alle 5 Stringtheorien sowie 11d Supergravitation nur verschiedene perturbative Limites einer fundamentalen 11d Theorie sind, M-Theorie.

In dieser Arbeit habe ich mich mit einigen Anwendungen dieser Ideen beschäftigt. Die Tatsache, daß D-branes durch Super Yang-Mills Theorien beschrieben werden, erlaubt uns einen Stringhintergrund derart zu präparieren, daß wir nahezu jede Eichtheorie als relevante Niederenergiebeschreibung erhalten können. Eine besonders verbreitete Variante dieser Idee sind die sogenannten “Hanany Witten setups”, in denen dieser Stringhintergrund nur aus flachen branes im flachen Raum besteht. Mit Hilfe dieser Technik habe ich verschiedene Dualitätssymmetrien in Feldtheorien auf Stringdualitäten zurückgeführt. Ferner ist es möglich, mit Hilfe der branes die Existenz nicht trivialer Fixpunkt Theorien in sechs Dimensionen zu beweisen und einige ihrer Eigenschaften zu analysieren. Einige dieser Fixpunkte beschreiben Phasenübergänge zwischen verschiedenen brane Hintergründen. Unter anderem läßt es sich auf diese Weise zeigen, daß es in 4 Dimensionen Übergänge zwischen chiralen und nicht chiralen Vacua gibt.

Ferner wurde gezeigt, daß alle anderen Zugänge zu dem Problem, Eichtheorien in Stringtheorie einzubetten, im wesentlichen äquivalent zum HW Ansatz sind, in dem Sinn, daß die entsprechenden Stringhintergründe dual zueinander sind. Dadurch können neue Aspekte der String T-Dualität verstanden werden, so wie z.B. T-Dualität für brane Segmente und gebogene branes. Außerdem erlaubt uns diese Verbindung, die Phasenübergänge, die wir im HW Bild entdeckt haben, tatsächlich als Übergänge zwischen topologisch verschiedenen Stringkompaktifizierungen zu verstehen.

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Selbständigkeitserklärung

Hiermit versichere ich, die vorliegende Arbeit selbständig angefertigt zu haben und keine weiteren als die angegebenen Hilfsmittel verwendet zu haben.

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