Essays on Incentive Contracts under Moral Hazard and Non-Verifiable Performance

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Abstract

This thesis consists of four self-contained essays that compare real-world incentive schemes used to mitigate moral hazard problems under non-verifiable performance.

The first essay contrasts the impact of the precision of performance measurement on wage costs in U- and J-type tournaments. In U-type tournaments prizes are fixed. In J-type tournaments only an overall wage sum is specified. The principal prefers a U-type tournament if workers receive a rent under limited liability and the costs of increasing precision are low. However, if workers are inequity-averse and have unlimited liability, the J-type tournament leads to lower wage costs.

The second essay analyzes optimal job design when there is only one contractible and imperfect performance measure for all tasks whose contribution to firm value is non-verifiable. Task splitting is optimal when relational contracts based on firm value are not feasible. By contrast, if an agent who performs a given set of tasks receives an implicit bonus, the principal always benefits from assigning an additional task to this agent.

The third essay compares an auction and a tournament in a procurement setting with non-contractible quality signals. Signals are affected by firms' non-observable investments in R&D and the procurer's precision of quality measurement. Although investments are always higher with the auction, the procurer may prefer the tournament if marginal costs of quality measurement are high or the production technology for quality is highly random.

In the last essay, a principal wants to induce two agents to produce an output. Agents can undertake non-contractible investments to reduce production cost of the output. Part of this innovation spills over and also reduces production cost of the other agent. Agents always underinvest with a general output price subsidy, while they may or may not do so with an innovation tournament. Strong spillovers tend to favor a general output price subsidy.

Keywords:
Tournaments, Relational Contracts, Multi-Tasking, Innovation Contests

Zusammenfassung

Diese Dissertation enthält vier Aufsätze zur Theorie der Anreizsetzung bei nicht-verifizierbaren Leistungsmaßen. Es werden positive Dominanzanalysen für Anreizmechanismen durchgeführt, die in realen wirtschaftlichen Situationen Anwendung finden.

Der erste Aufsatz analysiert zwei Bonus-Wettbewerbe in Unternehmen. Der Prinzipal kann entweder einen Bonuspool festlegen, dessen Aufteilung von der Leistung der Agenten abhängt, oder bereits ex ante die Höhe der Boni fixieren. Eine höhere Präzision der Leistungsmessung führt nur im zweiten Fall zu stärkeren Anreizen. Die optimale Wahl des Wettbewerbs hängt von den Präferenzen der Agenten, ihren Liquiditätsbeschränkungen und den Kosten der Leistungsmessung ab.

Der zweite Aufsatz untersucht die optimale Zuordnung von Aufgaben auf Stellen wenn relationale Verträge basierend auf subjektiven Leistungsmaßen explizite Anreizverträge ergänzen können. Die Spaltung von Aufgaben ist optimal, wenn die glaubhafte Bindung an relationale Verträge nicht möglich ist. Dagegen sollten Aufgaben immer dann gebündelt werden, wenn relationale Verträge bereits bestehen.

Im dritten Aufsatz möchte ein Käufer eine Innovation erwerben. Um qualitätssteigernde Investitionen bei potentiellen Anbietern zu induzieren, kann der Käufer entweder einen fixen Preis ausschreiben oder einen Auktionsmechanismus nutzen. Obwohl Investitionen unter der Auktion immer höher sind, bevorzugt der Käufer einen fixen Preis wenn die Grenzkosten der Qualitätsmessung hoch sind oder die Produktionstechnologie starken Zufallseinflüssen unterliegt.

Im letzten Aufsatz möchte ein Prinzipal das Produktionsergebnis zweier Agenten maximieren, die vor der Produktion in eine kostenreduzierende Innovation investieren können. Dabei kommt es zu Spillover-Effekten. Bei einer allgemeinen Preissubvention sind Investitionen stets zu gering, während ein Innovationswettbewerb zu Unter- und Überinvestitionen führen kann. Der Prinzipal bevorzugt eine Preissubvention bei starken Spillover-Effekten.

Schlagwörter:
Relative Leistungsturniere, Relationale Verträge, Mehraktionen-Modell, Innovationswettbewerbe

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Introduction

Most economic relationships are subject to moral hazard. A moral hazard problem arises whenever an agent chooses non-observable actions that affect his utility and that of the principal, and the objectives of principal and agent differ. The principal only observes the agent's performance, i.e., some imperfect signals of the actions taken.[1] If these signals are verifiable by a court, the principal can mitigate the moral hazard problem by designing a contract that rewards the agent for favorable performance.[2]

However, courts or other third parties are often not able to verify each piece of information that is available to the principal. For example, an employee's job usually has several dimensions such as production, cooperation, training, or innovation. Therefore, even the employer may find it difficult to assess the employee's overall contribution to the firm. Frequently, it will be too costly or even impossible to credibly communicate this contribution to an outside party. In such a case, at least some of the variables that are important to assess the agent's performance cannot be part of an enforceable contract. Then, the principal should try to find another way to incorporate those non-verifiable variables in an incentive scheme.

One well-known possibility to do this is the use of tournament schemes.[3] Tournaments can be applied if several agents perform comparable tasks for the principal. The principal fixes overall payments to agents ex ante. This prevents ex post opportunism of the principal because she cannot lower her costs by understating performance.[4] Agents are rewarded according to the ranks of their performance. Thus, they have incentives to exert effort.

Another possibility is the use of relational contracts. Relational contracts are informal agreements between principal and agent that are self-enforcing. Such agreements may exist in repeated principal-agent relationships. Then, both parties may prefer to stick to informal agreements if there is a credible future punishment threat in case they renege on the agreement.[5] For example, if an employer breaks her promise to pay a bonus for good performance, the employee might shirk in future

[1] Compare Salanié [1997], p. 107.

[2] Seminal works on optimal contracting under moral hazard and verifiable performance include, e.g., Arrow [1970], Mirrlees [1975], Homström [1979], Grossman and Hart [1983], and Sappington [1983]. For a survey see Salanié [1997], Laffont and Martimort [2002], or Bolton and Dewatripont [2005].

[3] See the seminal papers by Lazear and Rosen [1981], Green and Stokey [1983], Malcomson [1984], or Malcomson [1986].

[4] For clarity, I use the feminine pronoun for the principal and refer to an agent with the masculine pronoun.

[5] See, e.g., Holmström [1981] or Bull [1987].

periods.

Sometimes the principal can also use an auction mechanism to implement the desired actions. Consider, for example, the following situation. A procurer wants to buy a good that can be produced by different firms. The quality of the good, and thus its value for the procurer, increases in a firm's investment in R&D. After firms have sunk their investments, the procurer learns her non-verifiable valuation of the good that each firm is able to supply. Then firms bid prices at which they are willing to produce the good. The procurer will buy the good from the firm that offers the most favorable combination of quality and price. Since firms anticipate that they can bid higher prices if they offer a high-quality good, they have incentives to invest in quality.[6]

This thesis is composed of four essays that deal with principal-agent relationships under moral hazard and non-verifiable performance. In each essay I compare different incentive contracts that are stylized forms of real-world incentive schemes. The first essay analyzes two different tournament schemes in an employment relationship. In the second essay, I also consider an employment relationship. Here, the principal cannot distinguish agents' performances and thus cannot employ a tournament scheme. Instead, relational contracts and job design are used to mitigate the non-verifiability problem. The third and fourth essay deal with a situation in which the principal wants agents to invest in the development of a good or an innovation. I compare a tournament with an auction scheme and an output price subsidy, respectively. In the remainder of this introduction, I summarize the main results of each essay.

In the first essay, I contrast the impact of precision, i.e., the level of accuracy with which agents' performance is assessed, on wage costs in U- and J-type tournaments. In U-type tournaments, the principal fixes prizes ex ante. The prize that an agent receives only depends on his position in the employee ranking. This is, for example, the case in promotion tournaments where the winner is promoted to a different job with a higher salary. In J-type tournaments, the principal only specifies an overall bonus pool. Each agent's share of the bonus pool is determined by the extent to which his performance differs from those of the other agents. Such a tournament scheme is frequently used in Japanese firms.

In my model, the principal can increase precision by including more signals in performance measurement. These signals are non-verifiable. I find that increasing precision leads to stronger incentives in U-type tournaments, but not in J-type tournaments. The reason is that the *expected* share of the bonus pool that an agent receives does not depend on the number of signals. However, the marginal probability of winning the U-type tournament increases in precision.

To determine which tournament scheme the principal prefers, I analyze four different scenarios: Agents may be selfish or inequity-averse and may have unlimited

[6] Other solutions to the problem of moral hazard and non-verifiable performance include career concern models (see, e.g., Homström [1999], Gibbons and Murphy [1992], and Meyer and Vickers [1997]) and Nash implementation (see Moore [1992] for a survey). If the principal-agent problem contains some contractible variables (e.g., the amount and price of a good traded between buyer and seller), contract renegotiation plays an important role. See Bolton and Dewatripont [2005] for a survey of this strand of the literature. For the case that the principal can observe the agent's action and performance is verifiable, see Hermalin and Katz [1991].

or limited liability. If agents are selfish and have unlimited liability, the principal implements the first-best solution under both schemes. However, under selfishness and limited liability, the principal prefers a U-type tournament. This is because the principal can lower agents' rents by increasing precision only in U-type tournaments.

If agents are inequity-averse, the principal can reduce inequity costs by increasing precision in both tournaments. However, for all levels of precision, inequity costs are lower in a J-type tournament because in this tournament payments to agents do not necessarily differ strongly. Therefore, under inequity aversion and unlimited liability, the principal should always choose a J-type tournament. By contrast, if inequity-averse agents have limited liability, a U-type tournament may be superior. This is again due to the fact that only in U-type tournaments agents' rents decrease in the level of precision. As a result, the principal prefers a U-type tournament if the costs of increasing precision are small.

In the second essay, I analyze a multi-tasking problem. There are three different tasks, which the principal can assign to either one or two agents. The tasks' true contribution to firm value is non-verifiable but observable by the principal and the agents. Furthermore, there is a contractible but distorted performance measure. Such a situation occurs, for instance, when the principal cares about the quantity and the quality of a good that an agent produces and, additionally, the machine that is used for production has to be maintained. While the first two tasks – quantity and quality – are non-separable, the third task could be assigned to another agent. The only contractible variable may be output quantity, while firm value is also affected by quality and the life-span of the machine.[7]

If the principal pays bonuses based on the contractible performance measure, agents' objectives are, in general, not perfectly aligned with those of the principal. However, combining these bonuses with relational contracts can mitigate the incongruence problem.[8] A relational contract consists of an implicit bonus conditioned on the tasks' true contribution to firm value. The principal, who cares about her reputation in future periods, can commit to paying implicit bonuses if the expected firm value strongly responds to effort changes or the performance measure is sufficiently distorted. In both cases, the principal's loss from reneging on the relational contract is large and, therefore, commitment is credible.

Job design affects the characteristics of the incongruence problem and, thereby, the principal's ability to commit to implicit bonuses. Splitting tasks between agents has two effects. On the one hand, the principal can implement first-best effort in the one-task job by a pure explicit contract. On the other hand, an agent who performs two tasks receives a lower implicit bonus than an agent who performs three tasks. The reason is that the performance of an agent who is responsible for only two tasks is less important for the firm value. Therefore, the principal's loss from reneging on the relational contract with this agent is lower, so that the maximum implicit bonus she can credibly promise to pay decreases.

I find that task splitting is optimal if the principal cannot commit to paying an implicit bonus to an agent who performs three tasks. Under task splitting, im-

[7]In their seminal paper on multi-tasking, Holmström and Milgrom [1991] consider the same example.

[8]See also Baker et al. [1994], Pearce and Stacchetti [1998], and Demougin and Fabel [2004].

plicit bonuses remain infeasible, but explicit bonuses provide better incentives. By contrast, if the principal can commit to paying an implicit bonus to an agent who performs two tasks, the principal should also assign the third task to this agent. This leads to a strengthened relational contract, which always outweighs the negative effect of not having first-best effort in one task. Overall, the results in the second essay suggest that task splitting is less favorable in environments where firm value strongly responds to changes in effort but well aligned performance measures are not available.

In the third essay, I investigate a procurement setting where a procurer wishes to buy a good, which two firms can produce. A non-verifiable signal about the quality that each firm is able to supply is observed by the procurer and both firms. Signals are affected by firms' non-contractible investments in R&D and the procurer's precision of quality measurement. A typical example for such a situation is the procurement of a high-tech fighter plane by the Ministry of Defence. In this case, there are only a few potential suppliers, which are required to make investments in R&D to build a prototype. To determine the quality of the plane that each firm is able to supply, the procurer performs tests on the prototypes. The number and nature of these tests determines the precision of quality measurement.

I investigate whether the procurer should choose a tournament or an auction to maximize her expected profit. Both mechanisms are frequently used in procurement settings. In the tournament, the firm with the higher quality signal receives a prespecified prize and produces the good. In the auction, firms bid prices after investments have been made and quality signals have been observed. The firm that offers the most favorable combination of quality signal and price receives its bid and produces the good. Consequently, in the auction, firms' investments affect not only the probability of winning but also the price that the winning firm receives. Therefore, firms' investment incentives are higher in the auction than in the tournament.

Nevertheless, there are circumstances under which the procurer prefers the tournament. The price that the procurer has to pay in the auction is the difference between firms' quality signals weighted by the procurer's marginal valuation of quality. Therefore, the expected price in the auction increases in the variance of the quality signals, which is the higher the more random the production technology for quality and the more imprecise the quality measurement. Therefore, if increasing the precision of quality measurement is very costly or the production technology for quality is highly random, it may be better to fix a winner's prize ex ante. This result is in contrast to the previous literature which finds that auctions are superior to tournaments.[9]

In the fourth essay, which is joint work with Carsten Helm, we analyze a situation in which a principal wants to induce two agents to produce an output. Before production takes place, agents can invest in the development of an innovation that would reduce production costs. Part of this innovation spills over and also reduces the production costs of the other agent.

A topical problem that conforms to this general structure is the promotion of new technologies, where spillovers occur because firms learn from each others innovations. For example, many governments want to increase electricity production from

[9]See Che and Gale [2003] and Fullerton et al. [2002].

renewable energy sources. Renewable energy is not yet competitive, but the hope is that innovations will bring down production costs. Therefore, several European countries have passed legislation by which all producers of renewable energy receive a fixed price for power sold to the grid which lies above the market price. Some decision-makers have suggested that these subsidies should be focused on the most promising projects only.[10]

To investigate the advantages and disadvantages of these two subsidy schemes, we compare a general output price subsidy with an innovation tournament in which only the agent with the better innovation receives a positive output price. In our model, the only contractible variable is an agent's output. Investments are non-observable. The value of an innovation can be observed by the principal and the agents, but is non-verifiable to third parties.

The analysis provides some insights into the optimal choice between the two mechanisms if the principal's goal is to maximize overall output. Three effects can be distinguished. First, high spillovers favor a general output price subsidy because it induces both agents to produce in equilibrium. Second, if the stochastic innovation process is highly random, agents' expected realized innovations differ substantially. This increases the appeal of the tournament, in which resources for output production are concentrated on the most successful innovator. Finally, also the motivational effect of winning the tournament favors this scheme, although it may be detrimental if it induces excessive innovation.

[10] See "Clement sucht Konfrontation mit Trittin", in Frankfurter Allgemeine Zeitung, 02.09.2003, No. 203, p. 11. Such an approach had already been adopted in the UK with the NFFO (Non-Fossil Fuel Obligation), see Cleirigh [2001].

Essay 1

Precision in U-Type and J-Type Tournaments

1.1 Introduction

Real world labor tournaments are of two types (see Kanemoto and MacLeod [1989, 1992]). In the first type, the principal fixes prizes that correspond to each position in the agent ranking. A prominent example is a promotion tournament in which the winner is promoted to a different job with a higher salary, while the wages of the non-promoted agents remain the same. Thus, each agent's reward depends only on his relative position in the ranking. In the second type, the principal specifies an aggregate bonus pool. Each agent receives a share of the bonus pool that is not only determined by his position in the ranking, but also by the extent to which his performance differs from the one of his colleagues. For example, if there are two agents participating in the tournament and one of them performs twice as well as his colleague, he will get twice as much of the bonus pool. Kräkel [2002, 2003] calls the first tournament scheme *U-type tournament* and the second one *J-type tournament* because promotion tournaments are common in the U.S., but bonus pools are more frequently used in Japanese firms.[1] Most of the tournament literature focuses on U-type tournaments.[2] Kräkel [2002, 2003] was the first to compare both tournament types and shows that they differ substantially.

In this essay[3] I investigate another important characteristic in which the tournament schemes may differ: the impact of precision on wage costs, where precision denotes the accuracy with which the principal measures agents' performance. I assume that the principal decides on the number of signals that she collects about agents' performance, where collecting signals is costly. For example, the principal can decide to devote more time to the evaluation of agents' achievements by taking

[1] Of course, this is not to say that there are no promotion tournaments in Japanese firms or bonus pools in U.S. firms, respectively. For more on the importance of bonus payments and the assessment process of agents in Japanese firms, see Kräkel [2002, 2003] and further references therein, as well as Itoh [1991b], Endo [1994].

[2] See, e.g., Lazear and Rosen [1981], Green and Stokey [1983], Nalebuff and Stiglitz [1983], O'Keefe et al. [1984], Rosen [1986], Bhattacharya and Guasch [1988], and additional references in Prendergast [1999].

[3] This essay was first published in *Schmalenbach Business Review*, 57:167-192, 2005.

into account an increasing number of various sources.[4]

In Kräkel [2002], the principal can perfectly observe an agent's effort. Kräkel finds that J-type may dominate U-type tournaments if there is the possibility of collusion between agents, or if agents should invest in human capital prior to the tournament, or if many agents compete. In Kräkel [2003], effort is not perfectly observable. Then, U-type tournaments may lead to lower wage costs if agents are heterogenous. J-type tournaments may be preferable if agents are risk-averse, or if there is intermediate information.

Since measured performance is affected by chance, this essay is most closely related to Kräkel [2003]. Kräkel assumes that there are only two possible output levels that an agent can produce, and that the probability of high output is determined by agents' efforts. I make a similar assumption. By focusing on a two-agent tournament, I assume that signals on performance can take only two different values. Every signal provides information only about relative ranking. This assumption also captures the idea that agents' performance is difficult to measure, and therefore, only ordinal information can be acquired. The probability that a signal favors agent i is increasing in the effort of agent i and decreasing in the effort of agent j. Overall precision of performance measurement is composed of two elements. The first element is signal quality, which is exogenous and determines how strongly changes in effort affect the realization of each signal. The second element is the number of signals that the principal decides to collect.

In both tournament types, a higher signal quality increases incentives to work hard. By contrast, a higher number of signals increases incentives only in U-type tournaments. This is because the expected share of the bonus pool that an agent receives in a J-type tournament does not depend on the number of observations, but the marginal probability of winning the higher prize in a U-type tournament does. Nevertheless, if agents are selfish and have unlimited liability, wage costs are the same for both tournament types.[5] However, if agents have limited liability, the different possibilities for providing incentives matter. In this case, the principal prefers U-type to J-type tournaments for selfish agents, because in U-type tournaments agents' rents can be lowered by increasing the number of observations.

Presumably, J-type tournaments have a comparative advantage over U-type tournaments if agents are risk- or inequity-averse, since the wage payments to the winner and the loser of a J-type tournament need not differ strongly. In his framework, Kräkel [2003] shows that the conjecture on risk aversion is true. Applying the utility function developed by Fehr and Schmidt [1999], I investigate how tournament costs are affected if agents are inequity-averse.[6] When there is unlimited liability of agents,

[4]The underlying information system is crucial for analyzing incentive schemes. Which information structures make (U-type) tournaments superior to independent contracts is investigated, for instance, in Green and Stokey [1983], Nalebuff and Stiglitz [1983], Lazear and Rosen [1981]. Monitoring precision as an incentive device for the principal is examined, for instance, in O'Keefe et al. [1984], Demougin and Fluet [2003], and, in a non-tournament framework, Demougin and Fluet [2001]. In the latter two papers, monitoring is also costly.

[5]This finding corresponds to a result in Kräkel [2003], which says that first-best efforts are implemented in both tournament types when it is allowed to charge agents an entrance fee in the J-type tournament.

[6]Recently, there is an increasing amount of literature on inequity aversion. Compare, e.g., Fehr

I find that J-type tournaments always predominate because the inequity costs that the principal must bear are always lower. If agents have limited liability, J-type are likely to outperform U-type tournaments if agents' inequity aversion, their costs of effort, or their reservation utilities are large. However, if the costs of increasing precision are sufficiently low, a U-type tournament may lead to lower overall costs even if inequity aversion is relatively high.

The next section describes the model. Section 1.3 analyzes the possibilities of setting incentives in both tournaments. Section 1.4 derives the optimal wage costs for each tournament when agents have unlimited liability and there is a given precision level. Section 1.5 deals with limited liability and endogenous precision. Section 1.6 considers the case of inequity-averse agents. Section 1.7 concludes.

1.2 The model

I analyze a framework in which a risk-neutral principal employs two ex-ante identical agents indexed by i and j. Agents are risk-neutral and have a reservation utility $\bar{u} \geq 0$ from an outside option. An agent's cost of undertaking effort $e \geq 0$ is $c(e)$, where $c(e)$ is a strictly increasing and strictly convex function with $c(0) = 0$. Furthermore, $c(e)$ is differentiable for all $e > 0$ and $\lim_{e \to +0} c'(e) = 0$.

An agent's performance is difficult to measure in the sense that there is no verifiable signal on his contribution to the value of the organization.[7] However, the principal can observe noisy signals about which of the agents performs better, i.e., to some extent she is able to rank employees' performance relatively. This is modelled as follows: For the period in which agents compete, they choose effort levels e_i and e_j. Before e_i and e_j are chosen, the principal announces the precision with which she is going to measure performance, i.e., she specifies a positive integer n which is the number of signals she will include in the evaluation of agents' achievements. To avoid a commitment problem for the principal, I assume that n is contractible.[8] Furthermore, I assume for simplicity that the probability distribution is the same for all signals. A signal is in favor of agent i with probability $p_i(e_i, e_j)$, where p_i is strictly increasing in e_i, strictly decreasing in e_j, and partially differentiable in both arguments. A signal that is favorable for agent $j \neq i$ is observed with probability $p_j(e_i, e_j) = 1 - p_i(e_i, e_j)$. Furthermore, $p_j(x, y) = p_i(y, x)$ for all $x, y \geq 0$. This assumption ensures that both tournament types are *fair* in the sense of O'Keefe et al. [1984]. In particular, it implies that $p_i(x, x) = p_j(x, x) = 0.5$.

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and Schmidt [1999, 2003], Englmaier and Wambach [2002], Grund and Sliwka [2005], Demougin and Fluet [2003].

[7] This assumption excludes the implementation of contracts based on individual performance such as piece rates. Nevertheless, I do not claim that the tournament schemes considered here are optimal incentive mechanisms.

[8] This assumption is not unrealistic. In general, it is easier to verify whether some piece of information has been collected, than to verify the content of this information. For example, when evaluating the research of two assistant professors, it is easy to verify how many papers the commission in charge has considered carefully, but the assessment of each paper's individual quality is much more difficult to verify. That is, the precision of information collection is contractible, but the content of the collected information is not.

It is useful to have some examples for p_i in mind. For instance, p_i might take the form

$$p_i^T(e_i, e_j) \equiv \begin{cases} \frac{e_i}{e_i+e_j} & \text{if } e_i + e_j \neq 0 \\ 0.5 & \text{otherwise} \end{cases} . \tag{1.1}$$

Another possibility is that p_i depends only on the difference of effort levels, i.e., $p_i^D(e_i, e_j) = f(e_i - e_j)$. For example, this is the case if agents' output per day, q_i and q_j, serve as signals, where $q_i = e_i + \epsilon_i$, $q_j = e_j + \epsilon_j$, and ϵ_i and ϵ_j are individual (and exogenous) noise terms as in Lazear and Rosen [1981]. Then, p_i is the probability that agent i produces more on a particular day, and $p_i = G(e_i - e_j)$ where $G(.)$ denotes the cdf of $\epsilon_j - \epsilon_i$. In this case, n would be the number of days at which output is measured.

I assume that the realizations of the n different signals are independent of each other. It follows that the number of signals that favor agent i, denoted by K_i, is a binomially distributed random variable with parameters n and p_i, $K_i \sim \text{Bi}(n, p_i)$.

The principal wishes to minimize the overall costs of tournament design, which are composed of wage payments and expenditures for measuring performance.[9] If she uses a U-type tournament to create incentives for exerting effort, she fixes in advance a winner's prize of $w \geq 0$ and a loser's prize of l, $w > l$. Agent i receives w if the principal observes more signals that favor agent i instead of j, i.e., if for the realization of K_i, denoted by k_i, it holds that $k_i \geq (n+1)/2$. Otherwise agent i obtains l.[10] To avoid tedious case distinctions, I assume that n is odd, so that ties cannot occur.

Under a J-type tournament, the principal specifies a bonus pool $b \geq 0$ and a base wage $\bar{w}$ before the tournament starts. When the tournament is finished, each agent gets $\bar{w}$ plus a certain share of b depending on his measured performance.[11] Agent i receives

$$\bar{w} + \frac{k_i}{n} b, \tag{1.2}$$

while agent j receives

$$\bar{w} + \frac{k_j}{n} b = \bar{w} + \frac{n - k_i}{n} b. \tag{1.3}$$

According to this rule, if, for instance, two-thirds of all signals favor agent i, he also gets two-thirds of the bonus pool. Since the expected value of K_i is np_i, the expected payment to agent i is

$$\bar{w} + p_i b. \tag{1.4}$$

[9]Throughout this thesis, I assume that the principal has all the bargaining power. She moves first and proposes a contract, that the agent can either accept or reject. This simplifying assumption is prevalent in the literature. For alternative approaches see, e.g., Pitchford [1998], Mookherjee and Ray [2002], Inderst [2002], and Demougin and Helm [2005].

[10]During the course of the tournament, if one agent has a lead such that he will receive the winner's prize no matter what the realizations of the following signals, it will not be necessary to make further observations. However, since n is contracted and a court cannot observe the realizations of the signals, all n observations must be made.

[11]Such an assumption corresponds to reality, since according to the literature, bonus payments make up 18-30% of a Japanese worker's yearly income. Compare Kanemoto and MacLeod [1992], p.145, Itoh [1991b], pp. 348-350, and Ito [1992], p. 233.

Empirical tests show that individuals' preferences frequently exhibit a distaste for inequitable payoff distributions (see, e.g., Loewenstein et al. [1989], Fehr and Schmidt [2003]). Consequently, agents with such preferences feel dissatisfied when they receive different wages despite the fact that they have worked equally hard. This is exactly the case in my tournament framework, where agents will choose the same effort in equilibrium, but receive different wages. Therefore, it is relevant to investigate the effect of inequity aversion on both tournament schemes.

To do so, I use the concept introduced by Fehr and Schmidt [1999]. These authors develop a utility function such that an individual cares not only about its absolute payoff, but also about its relative payoff compared to a reference group. This utility function has already been applied by Grund and Sliwka [2005] and Demougin and Fluet [2003], who investigate U-type tournaments among inequity-averse agents. Agent i's utility is given by

$$u_i = w_i - [\alpha \max\{w_j - w_i, 0\} + \beta \max\{w_i - w_j, 0\}] - c(e_i) \tag{1.5}$$

if he earns w_i and agent j earns w_j. The parameters $\alpha \geq 0$ and $\beta \geq 0$ characterize the degree of inequity aversion. The first term in square brackets measures the utility loss from envy, which means that an agent dislikes earning less than his colleague. However, an agent's utility is also reduced if he earns more than his colleague, i.e., an agent feels compassion.[12] The utility loss from compassion is given by the second term in brackets.

As in Fehr and Schmidt [1999], I assume that $\beta \leq \alpha$ and $\beta < 1$. The first inequality says that the disutility from envy is stronger than that from compassion. The second inequality implies that an agent always benefits from an increase in his own wage when his colleague's wage is held constant.

1.3 Providing incentives

I first analyze the model for the (conventional) case of purely self-interested agents, i.e., I assume $\alpha = \beta = 0$. In section 1.6, I turn to the case of $\alpha, \beta \geq 0$. In this section, I derive agents' optimal effort choices given the number of observations n and the prizes w and l in the U-type tournament or the bonus pool b and the base wage $\bar{w}$ in the J-type tournament, respectively. My purpose is to identify similarities and differences in the way of setting incentives under both tournament schemes. Throughout my analysis, I restrict attention to symmetric Nash equilibria in pure strategies in the stage where agents choose their effort levels.

I start with considering the agents' optimal effort choices in the U-type tournament. Let Π denote the probability that agent i wins the higher prize w given his own effort e_i, the effort of his colleague e_j, and the number of observations n. Since this happens if and only if $k_i \geq (n+1)/2$, we have

$$\Pi \equiv \sum_{k=\frac{n+1}{2}}^{n} \pi_{ki}, \tag{1.6}$$

[12] For the possibility of a negative β, i.e., the possibility of feeling satisfied when being better off than others, see Grund and Sliwka [2005] and Demougin and Fluet [2003].

where π_{ki} denotes the probability that k out of n independent observations favor agent i, i.e.,

$$\pi_{ki} = \binom{n}{k} p_i^k (1-p_i)^{n-k}. \tag{1.7}$$

Both agents maximize their expected net incomes, i.e., their expected wages minus effort costs, so that the optimization problem of agent i and agent j, respectively, is

$$\max_{e_i} \; (1-\Pi)l + \Pi w - c(e_i), \tag{1.8}$$
$$\max_{e_j} \; \Pi l + (1-\Pi)w - c(e_j). \tag{1.9}$$

Defining Δw as the prize spread, i.e., $\Delta w = w - l$, the first-order conditions are

$$\frac{\partial \Pi}{\partial e_i}\Delta w - c'(e_i) = 0, \tag{1.10}$$
$$\text{and} \quad -\frac{\partial \Pi}{\partial e_j}\Delta w - c'(e_j) = 0, \tag{1.11}$$

where the partial derivatives of Π are

$$\frac{\partial \Pi}{\partial e_i} = \frac{n+1}{2}\binom{n}{\frac{n+1}{2}}[p_i(1-p_i)]^{\frac{n-1}{2}}\frac{\partial p_i}{\partial e_i}, \tag{1.12}$$
$$-\frac{\partial \Pi}{\partial e_j} = \frac{n+1}{2}\binom{n}{\frac{n+1}{2}}[p_i(1-p_i)]^{\frac{n-1}{2}}\frac{\partial p_j}{\partial e_j}. \tag{1.13}$$

The derivation is given in the appendix.

At a symmetric equilibrium[13] $e_i = e_j = e_U^*$, the first-order conditions reduce to

$$\gamma(n)\rho(e_U^*)\Delta w = c'(e_U^*), \tag{1.14}$$

where I define $\gamma(n)$ and $\rho(e)$ as

$$\gamma(n) \equiv \frac{n+1}{2^n}\binom{n}{\frac{n+1}{2}}, \quad \rho(e) \equiv \left.\frac{\partial p_i}{\partial e_i}\right|_{e_i=e_j=e} = \left.\frac{\partial p_j}{\partial e_j}\right|_{e_i=e_j=e}. \tag{1.15}$$

Starting from identical effort levels e, the term $\gamma(n)\rho(e)$ describes how strongly the probability of winning the tournament reacts to changes in effort. Therefore, one can interpret $\gamma(n)\rho(e)$ as overall precision of performance measurement. $\gamma(n)$ is endogenously determined by the number of observations that the principal chooses. Since $\gamma(n)$ is strictly increasing in n (see the appendix for a proof), overall precision is increasing in the number of observations to be made. $\rho(e)$ explains how sensitive each signal is to work effort. Therefore, I call $\rho(e)$ *signal quality.* For example, if the signals are the agents' output, $\rho(e)$ is given by the production technology. In this case, $\rho(e)$ is the higher the less that output is influenced by random factors beyond an agent's (and principal's) control. I say that signal quality increases if it changes

[13] Asymmetric equilibria might exist. However, for $p_i^T(e_i, e_j)$ and $p_i^D(e_i, e_j)$, there is at most one symmetric equilibrium.

from $\rho(e)$ to $\hat{\rho}(e)$, where $\hat{\rho}(e) \geq \rho(e)$ for all e. Note that $\rho(e)$ need not depend on e, e.g., if $p_i = G(e_i - e_j)$, $\rho(e) = G'(0)$. If $p_i = p^T(e_i, e_j)$, $\rho(e) = 1/(4e)$. I assume that $\rho'(e) \leq 0$.[14]

From equation (1.14), equilibrium effort e_U^* is strictly increasing in the prize spread Δw, the number of observations n, and the signal quality $\rho(e)$. However, e_U^* is not guaranteed to be a global maximum of an agent's objective function, since this function may not be concave. As it is clear for more common (U-type) tournament models, an equilibrium exists if either precision is not too great or effort costs are sufficiently convex (compare, e.g., Lazear and Rosen [1981], p. 845, fn. 2, and Bhattacharya and Guasch [1988], p. 871). This result also holds for my model. Therefore, I assume that the equilibrium exists for $n = 1$ (and given Δw). For example, this is the case for p_i^T, since p_i^T is concave in e_i for any given e_j. However, if n increases, the equilibrium may not be sustainable, so that in general there will be a maximum number of observations for which the equilibrium exists. I return to this problem in section 1.5.2, when n becomes endogenous.

In the next step, I consider an agent's optimization problem in the J-type tournament. Here, agent i's expected income net of effort costs is

$$\bar{w} + p_i(e_i, e_j)b - c(e_i), \tag{1.16}$$

and the first-order conditions for both agents are

$$\frac{\partial p_i}{\partial e_i} b - c'(e_i) = 0, \tag{1.17}$$

$$\frac{\partial p_j}{\partial e_j} b - c'(e_j) = 0. \tag{1.18}$$

At a symmetric equilibrium $e_i = e_j = e_J^*$, these conditions simplify to

$$\rho(e_J^*) b = c'(e_J^*). \tag{1.19}$$

Thus, e_J^* is increasing in the bonus pool b and in the signal quality, but it is not influenced by the number of observations n.[15] The following proposition summarizes the preceding observations.

Proposition 1.1 *Under both tournament schemes, an agent's effort increases in the signal quality. However, while in the U-type tournament effort increases in n, the number of observations does not affect effort in the J-type tournament.*

The intuition for proposition 1.1 can be explained by looking at the effect of a higher n in both tournament types. As we know from the literature (see for instance Lazear [1995]), the level of accuracy in measuring agents' performance has

[14] $\rho'(e) \leq 0$ if and only if $\left.\frac{\partial^2 p_i}{\partial e_i^2}\right|_{e_i=e_j=e} \leq 0$ since $\left.\frac{\partial^2 p_i}{\partial e_i \partial e_j}\right|_{e_i=e_j=e} = 0$ by the assumptions made on $p_i(e_i, e_j)$ and $p_j(e_i, e_j)$.

[15] The existence of the equilibrium is also independent of n. Incentives are identical in both tournaments if $n = 1$ in the U-type tournament and $\Delta w = b$. Therefore, e is implementable as a Nash equilibrium in pure strategies in the J-type tournament if and only if it is implementable in the U-type tournament for $n = 1$.

a critical impact on incentives in U-type tournaments. Given that both employees work equally hard, a higher accuracy means that the marginal probability of winning w increases. In my U-type tournament model, the marginal probability of winning increases in the signal quality and the number of observations. Consequently, if n is raised, agents exert a higher effort in equilibrium. This implies that a higher wage spread and a higher number of observations are substitutes for setting incentives in the U-type tournament.

By contrast, in the J-type tournament, the marginal expected utility from working harder also increases in the signal quality, but it is independent of n. Increasing the number of observations means that the expected number of signals that favor agent i increases because $E[K_i] = np_i$. However, a higher number of observations also decreases the share of the bonus pool that a agent receives per signal that favors him, which is $1/n$. In expected terms, both effects cancel out, so that the expected wage payment is not influenced by n.

1.4 Wage costs under unlimited liability

I now determine the principal's expected wage costs per agent for a given number of observations in both tournament types when the principal faces no wealth or limited liability constraints for agents. In this case, the principal can set a negative loser's prize l in the U-type tournament or a negative base wage $\bar{w}$ in the J-type tournament. The principal minimizes the expected wage per agent for a given effort level e and a given number of observations n. In doing so, she must take into account the incentive compatibility constraints (1.14) or (1.19), respectively, and the participation constraints, i.e., she has to ensure that an agent's expected utility is at least as high as his reservation utility $\bar{u}$. The following proposition gives the main result.

Proposition 1.2 *If agents have unlimited liability, the principal's wage costs per agent are $c(e) + \bar{u}$ under both tournament schemes, i.e., agents do not receive a rent and the principal implements first-best effort in both tournaments.*

Proof See appendix.

The intuition for proposition 1.2 is that in both mechanisms, the principal has at his disposal two instruments. He chooses the prize spread Δw and the bonus pool b, respectively, to provide incentives for agents to exert the desired effort, and she sets the loser's prize l and the base wage $\bar{w}$ so that no rent is left to agents. Thus, since in both tournaments the principal's marginal costs of implementing effort coincide with an agent's marginal effort costs, the principal implements first-best effort for each arbitrary valuation of effort.

1.5 Limited liability

I now assume that it is not feasible to impose payments on agents, i.e., the principal cannot set a negative loser's prize l in the U-type tournament or a negative base wage $\bar{w}$ in the J-type tournament. In this case, the principal may not be able to avoid leaving a rent to agents. As we know from the literature, in the U-type tournament the principal can lower this rent by increasing precision. The reason is that with a higher precision, the principal can reduce the wage spread and thus the expected payment to each agent. If the reduction in wages is greater than the additional costs from increasing precision, the principal benefits from lowering the rent payment to agents.

By contrast, section 1.3 shows that the number of observations has no impact on the required bonus pool in the J-type tournament. Therefore, the question arises whether wage costs differ in both tournaments when agents have limited liability.

1.5.1 Wage costs

When minimizing wage costs, in addition to the incentive compatibility and the participation constraints, the principal now has to take into account the limited liability conditions $l \geq 0$ and $\bar{w} \geq 0$, respectively. Hence, in the U-type tournament she faces the constraints

$$l + \frac{\Delta w}{2} - c(e) \geq \bar{u}, \quad l \geq 0, \tag{1.20}$$

where $\Delta w = c'(e)/[\gamma(n)\rho(e)]$ by (1.14). In the J-type tournament his constraints are

$$\bar{w} + \frac{b}{2} - c(e) \geq \bar{u}, \quad \bar{w} \geq 0, \tag{1.21}$$

where $b = c'(e)/\rho(e)$ by (1.19).

Since the principal wants to set l and $\bar{w}$ as low as possible, at least one of the two inequalities must be binding in each tournament. Therefore, we obtain

$$l = \max\left\{\bar{u} + c(e) - \frac{c'(e)}{2\gamma(n)\rho(e)}, 0\right\}, \tag{1.22}$$

$$\bar{w} = \max\left\{\bar{u} + c(e) - \frac{c'(e)}{2\rho(e)}, 0\right\}. \tag{1.23}$$

The first term in brackets refers to the case in which agents receive no rent, i.e., the participation constraint is binding. The second term is for the case with rent, i.e., the limited liability constraint is binding.

By adding $\Delta w/2$ and $b/2$ to l and $\bar{w}$, respectively, in each case, by denoting by $W_U(e, n)$ the principal's wage cost function in the U-type tournament, and by $W_J(e)$ the wage cost function in the J-type tournament, we obtain the following result:

Proposition 1.3 *When the principal faces limited liability constraints for agents, his wage cost function per agent is*

$$W_U(e, n) = \max\left\{c(e) + \bar{u}, \frac{c'(e)}{2\gamma(n)\rho(e)}\right\} \tag{1.24}$$

in the U-type tournament and

$$W_J(e) = \max\left\{c(e) + \bar{u}, \frac{c'(e)}{2\rho(e)}\right\} \tag{1.25}$$

in the J-type tournament.

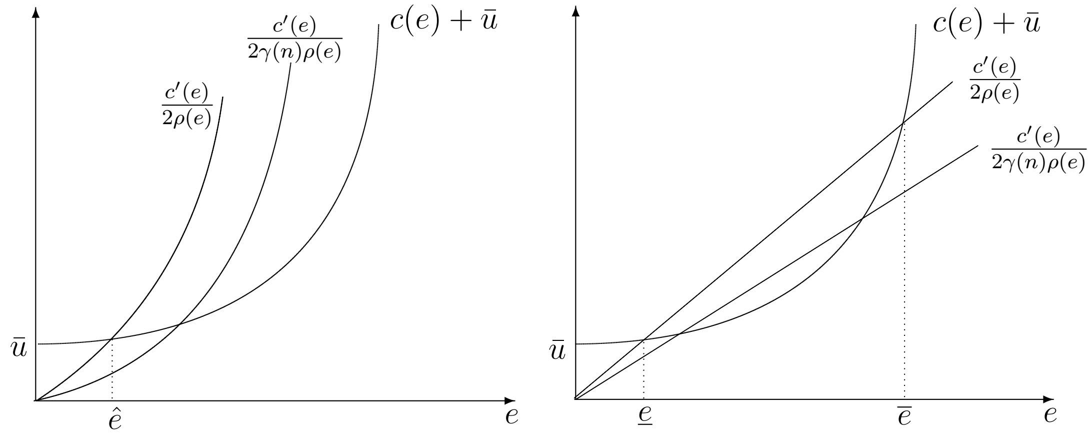

Figure 1.1: Possible wage cost functions.

The cost functions are identical as long as both participation constraints are binding. However, if the limited liability constraints are binding, the cost functions differ, because the prize spread in the U-type tournament depends on the number of observations n. Since $\gamma(n)$ is strictly increasing in n and $\gamma(1) = 1$, the wage payments in the rent case are identical if and only if $n = 1$, i.e., when the principal makes only one observation. If there is more than one observation, the wage costs of the U-type tournament are strictly lower.

For which effort levels a rent must be paid to agents depends on the particular form of p_i, $c(e)$ and $\bar{u}$. Assuming $n > 1$, I sketch two possible cases in figure 1.1. In both sketches, effort costs are $c(e) = Ce^2$, $C > 0$. In the left-hand picture, I consider the case $p_i = p_i^T(e_i, e_j)$, i.e., $\rho(e) = 1/(4e)$. The principal pays a rent for relatively high levels of e. The threshold $\hat{e}$ is the effort level where both the participation constraint and the limited liability constraint are binding in the J-type tournament. For all $e > \hat{e}$, the U-type tournament leads to strictly lower wage costs.

In the right-hand picture, $p_i = p_i^D(e_i, e_j)$, i.e., $\rho(e)$ is independent of e. Here, the principal pays a rent for intermediate values of e. For all e with $\underline{e} < e < \overline{e}$, the U-type tournament leads to strictly lower wage costs.

1.5.2 Overall tournament costs

So far, I have taken the number of observations n as given. I now assume that, before the tournament starts, the principal commits to a particular n, thus incurring costs $M(n)$. $M(n)$ is a strictly increasing function with $M(n+2) - M(n) > M(n) - M(n-2)$,[16] i.e., the higher n, the more strongly measurement costs increase if the principal switches to the next higher precision level $n + 2$.

Because increasing the number of observations has no effect on wage costs in the J-type tournament, the principal chooses the lowest possible n in this tournament, i.e., $n = 1$. By contrast, a higher number of observations may lower the wage costs in the U-type tournament for implementing a given e. Then, as long as the benefit

[16] This assumption replaces the convexity assumption $M' > 0, M'' > 0$ which one would make if the function $M(.)$ was differentiable. Remember that I assume that n is odd.

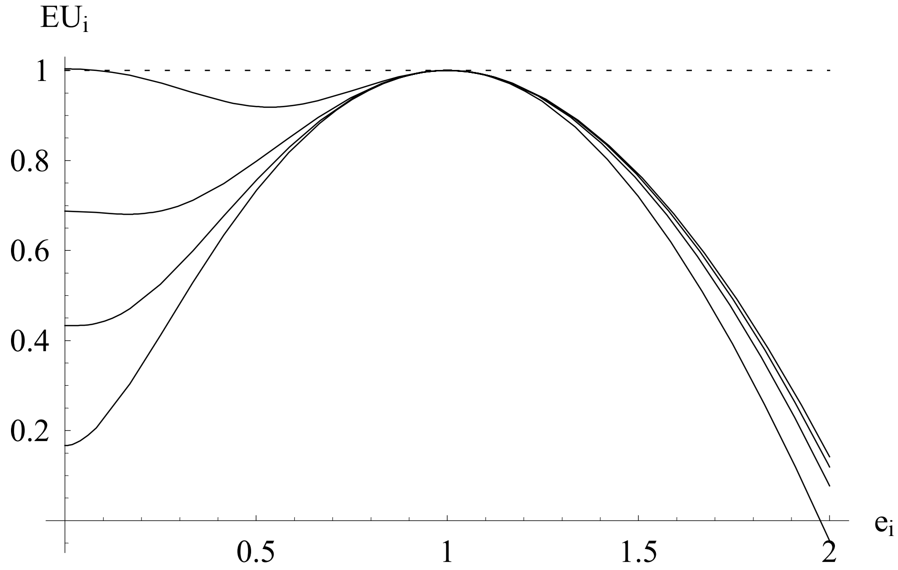

Figure 1.2: An agent's expected utility for different values of n.

from lowering wages is larger than the additional costs for increasing n, the principal should choose a higher n.

However, as I mentioned in section 1.3, there will in general be an upper bound on n, denoted by n_{max}, such that there is no equilibrium in pure strategies if and only if $n > n_{max}$. This upper bound n_{max} depends on the particular form of p_i and $c(e)$. For example, if $c(e) = e^2/2$, $p_i = p_i^T(e_i, e_j)$, and the principal wants to implement $e = 1$, one can show numerically that $n_{max} = 23$. For $n = 3, 5, 9, 25$, agent i's expected utility $EU_i(e_i)$ – given that $e_j = 1$, $\bar{u} = 1$, and w and l are optimally chosen for each n – is plotted in figure 1.2, where a higher n corresponds to a higher $EU_i(0)$. One can see that $e_i = 1$ is always a local maximum of agent i's objective function. However, if $n \geq 25$, agent i maximizes his expected utility by choosing $e_i = 0$. This is because for high n, the required prize spread Δw becomes very small, and therefore, l must increase to meet an agents's participation constraint. But then it is optimal for agent i to choose $e_i = 0$, i.e., collect the relatively high loser's prize and forgo the chance to win the relatively low Δw.

Let $n^*(e)$ denote the optimal number of observations in the U-type tournament. Since increasing the number of observations can be beneficial only if the limited liability constraint is binding for the given e when $n = 1$, one has[17]

$$n^*(e) = \begin{cases} 1 & \text{if} \quad c(e) + \bar{u} \geq \frac{c'(e)}{2\rho(e)} \\ \min\left\{\text{argmin}_n 2W_U(e,n) + M(n), n_{max}\right\} & \text{if} \quad c(e) + \bar{u} < \frac{c'(e)}{2\rho(e)} \end{cases}. \quad (1.26)$$

From the previous considerations, we immediately get the following proposition.

[17] The problem $\min_n 2W_U(e,n) + M(n)$ has a unique solution since $\frac{1}{\gamma(n-2)} - \frac{1}{\gamma(n)} > \frac{1}{\gamma(n)} - \frac{1}{\gamma(n+2)}$, i.e., the additional benefit of a higher n is strictly decreasing in n.

Proposition 1.4 *For all effort levels e, the U-type tournament leads to lower overall costs than does the J-type tournament. The U-type tournament leads to strictly lower overall costs for implementing a given e if and only if $n^*(e) > 1$.*

1.6 Inequity aversion

Under the assumptions of the previous sections, the J-type tournament never outperforms the U-type tournament. However, this result might no longer hold if agents care not only about their own absolute payoff. Presumably, if agents are inequity-averse, the J-type tournament might have a comparative advantage, since in this tournament agents' wages do not necessarily differ as strongly as in the U-type tournament.

Grund and Sliwka [2005] and Demougin and Fluet [2003] have already investigated U-type tournaments among inequity-averse agents. The main distinction between these two papers is that Grund and Sliwka assume that agents have unlimited liability, but Demougin and Fluet consider limited liability of agents and endogenize precision.[18] In both papers, the authors find that, for a given prize spread, inequity-averse agents exert higher effort than do purely self-interested agents. Nevertheless, Grund and Sliwka show that the principal implements less than first-best effort because his wage costs increase by some costs of inequity aversion. By contrast, Demougin and Fluet show in their model that when precision costs are high, the principal prefers inequity-averse agents. In this case, an agent's limited liability constraint is binding so that wage costs consist merely of the winner's prize, which is lower for inequity-averse agents. These results also hold for the U-type tournament in my framework. However, my purpose is to investigate the different effects of inequity aversion on the costs of U-type and J-type tournament.

Most of the mathematical derivations in this section are given in the appendix.

I start by deriving the incentive compatibility constraints for both tournament types for $\alpha, \beta \geq 0$. In the U-type tournament, the optimization problem of agent i is

$$\max_{e_i} \; \Pi(w - \beta\Delta w) + (1 - \Pi)(l - \alpha\Delta w) - c(e_i). \tag{1.27}$$

In the J-type tournament, agent i receives a lower wage than agent j if and only if fewer signals favor agent i, i.e., $k_i < (n + 1)/2$. Thus, i's expected utility is

$$\begin{aligned} &\bar{w} + p_i b - \alpha \textstyle\sum_{k=0}^{\frac{n+1}{2}-1} \pi_{ki} \left((n-k)\frac{b}{n} - k\frac{b}{n}\right) \\ &\quad -\beta \textstyle\sum_{k=\frac{n+1}{2}}^{n} \pi_{ki} \left(k\frac{b}{n} - (n-k)\frac{b}{n}\right) - c(e_i). \end{aligned} \tag{1.28}$$

At a symmetric equilibrium $e_i = e_j = e$, the first-order condition for the U-type tournament is

$$(1 + \alpha - \beta)\gamma(n)\rho(e)\Delta w = c'(e), \tag{1.29}$$

[18] Another difference is that Demougin and Fluet [2003] adopt exactly the same approach as in Fehr and Schmidt [1999] by defining inequity aversion on net income, i.e., wages minus costs of effort, but Grund and Sliwka [2005] define inequity aversion only on wages. To keep my analysis simple, I follow the approach by Grund and Sliwka [2005].

and for the J-type tournament it is

$$(1 + \alpha - \beta)\rho(e)b = c'(e). \tag{1.30}$$

Regarding the existence of the equilibria, I make the same assumption as in section 1.3.

Equation (1.29) gives the same results as Grund and Sliwka [2005] and Demougin and Fluet [2003] derive: The equilibrium effort in the U-type tournament is increasing in α and decreasing in β. Furthermore, by comparing (1.14) and (1.29) one sees that, for a given prize spread, inequity-averse agents exert higher effort than do purely self-interested agents. In the J-type tournament, by equation (1.30), inequity aversion has exactly the same effect on agents' equilibrium efforts as in the U-type tournament. In both cases, the marginal utility of an inequity-averse agent who exerts the same effort e as his colleague equals the marginal utility of an agent who is not inequity-averse multiplied by $1 + \alpha - \beta$.

Hence, due to the modified incentive structure under inequity aversion, the principal can implement the same effort level with a lower prize spread and a lower bonus pool, respectively. However, it must still be investigated how inequity aversion affects agents' participation decisions. An agent's participation constraint in the U-type tournament is

$$l + \frac{\Delta w}{2} - \frac{1}{2}(\alpha + \beta)\Delta w - c(e) \geq \bar{u}, \tag{1.31}$$

and in the J-type tournament the participation constraint is

$$\bar{w} + \frac{b}{2} - \frac{\gamma(n)}{2n}(\alpha + \beta)b - c(e) \geq \bar{u}. \tag{1.32}$$

Thus, the utility loss from inequity aversion amounts to $(\alpha + \beta)\Delta w/2$ in the U-type tournament and $\frac{\gamma(n)}{2n}(\alpha + \beta)b$ in the J-type tournament.

Here, there is an important difference between both tournament schemes: The number of observations n has no direct impact on an agent's expected utility loss in the symmetric equilibrium of the U-type tournament, but an agent's expected utility loss is decreasing in n in the J-type tournament.[19] The intuition is as follows: Since prizes are fixed in the U-type tournament, the difference in agents' incomes, and therefore the expected utility loss from inequity aversion, is independent of n. By contrast, as n increases in the J-type tournament, probability mass shifts from large to small income differences, i.e., large income differences become less likely. Thus, the expected utility loss from inequity aversion decreases in n, though the expected share of the bonus pool that an agent receives is constant for all precision levels.

The following proposition gives the principal's wage costs under unlimited and limited liability of agents.

Proposition 1.5 *(i) If agents are inequity-averse and have unlimited liability, the principal's wage costs are strictly lower in the J-type tournament for all $e \geq 0$ and $n > 1$. The wage costs per agent in the U-type tournament are*

$$W_U^{ui}(e, n) = c(e) + \bar{u} + \frac{1}{\gamma(n)} \frac{(\alpha + \beta)c'(e)}{2(1 + \alpha - \beta)\rho(e)}, \tag{1.33}$$

[19]This is because $\gamma(n)/n$ is decreasing in n. For the proof please see the appendix.

and in the J-type tournament the wage costs per agent are

$$W_J^{ui}(e,n) = c(e) + \bar{u} + \frac{\gamma(n)}{n}\frac{(\alpha+\beta)c'(e)}{2(1+\alpha-\beta)\rho(e)}. \quad (1.34)$$

(ii) If agents are inequity-averse and have limited liability, and the participation constraint (limited liability constraint) is binding in both tournaments, the principal's wage costs are strictly lower in the J-type tournament (U-type tournament) for all $e \geq 0$ *and* $n > 1$*. The wage costs per agent in the U-type tournament are*

$$\begin{aligned} W_U^{li}(e,n) \;=\; \max\Bigg\{ & c(e) + \bar{u} + \frac{1}{\gamma(n)}\frac{(\alpha+\beta)c'(e)}{2(1+\alpha-\beta)\rho(e)}, \\ & \frac{c'(e)}{2(1+\alpha-\beta)\gamma(n)\rho(e)}\Bigg\}, \end{aligned} \quad (1.35)$$

and in the J-type tournament the wage costs per agent are

$$\begin{aligned} W_J^{li}(e,n) \;=\; \max\Bigg\{ & c(e) + \bar{u} + \frac{\gamma(n)}{n}\frac{(\alpha+\beta)c'(e)}{2(1+\alpha-\beta)\rho(e)}, \\ & \frac{c'(e)}{2(1+\alpha-\beta)\rho(e)}\Bigg\}. \end{aligned} \quad (1.36)$$

Proof See appendix.

Compared to the case without inequity aversion, under unlimited liability the wage costs increase by a term that displays the inequity costs for each tournament. In both tournaments, inequity costs are decreasing in the number of observations, but for different reasons. In the J-type tournament, as n increases, large income differences become less likely, but the bonus pool that the principal needs to implement a given effort is unaffected. In the U-type tournament, inequity costs decrease in n because the wage spread for implementing a given effort is decreasing in n. However, the (direct) effect of increasing n on inequity costs is stronger in the J-type tournament. Therefore, the J-type tournament leads to strictly lower wage costs for all $n > 1$.

Naturally, wage costs do not change under limited liability when the participation constraints are binding. Hence, the cost advantage of the J-type tournament persists in this case. However, if instead the limited liability constraint is binding, we have the same situation as in section 1.5.1: The wage costs of the U-type tournament are strictly lower for all $n > 1$. This is due to the fact that inequity aversion affects incentives in both tournaments in the same way. Thus, compared to the case without inequity aversion, the only difference is that the wage spread in the U-type tournament and the bonus pool in the J-type tournament are both lowered by the factor $(1+\alpha-\beta)^{-1} < 1$.

Under limited liability, it is interesting to ask when a particular constraint is binding in each tournament. First consider the case $\alpha + \beta \geq 1$, i.e., the agents' inequity aversion is very high. Then, in the U-type tournament, the participation constraint is always binding. In the J-type tournament, the participation constraint is binding for small n. However, as n increases, depending on the values of $\alpha+\beta$, $c(e)$

and $\bar{u}$, the rent case may become the relevant one. Then, further increasing the number of observations cannot be beneficial for the principal in the J-type tournament. However, a higher n can still be profitable in the U-type tournament, since in this tournament wage costs are always strictly decreasing in n.

Such a case is illustrated in figure 1.3, where I assume that $p_i = p_i^T(e_i, e_j)$, $c(e) = e^2/2, e = 1, \alpha = 0.9, \beta = 0.2$, and $\bar{u} = 0$. The wage cost function of the U-type tournament is always decreasing in n, because the higher n, the lower are the inequity costs. In the J-type tournament, the limited liability constraint becomes binding for $n = 3$. Then, wage costs are constant because the required bonus pool is not affected by increasing n. Thus, if the costs of increasing the number of observations are low enough so that the principal benefits from increasing n above $n = 5$, she prefers the U-type tournament even if inequity aversion is high.[20]

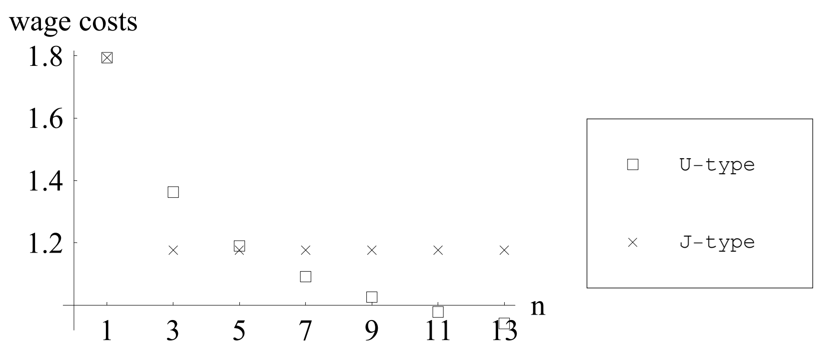

Figure 1.3: Possible wage costs functions under limited liability if $\alpha + \beta > 1$.

Now consider the case $\alpha + \beta < 1$. If the limited liability constraint is binding in the U-type tournament for $n = 1$, it is also binding in the J-type tournament, because both schemes are identical if the principal makes only one observation. Moreover, the rent case remains as the relevant one *for all* n in the J-type tournament. Therefore, the U-type tournament will always lead to lower wage costs if its liability constraint is binding for $n = 1$. For this to hold, it is necessary that an agent's effort costs and his reservation utility are not too great. On the other hand, if the participation constraint is binding in the U-type tournament for $n = 1$, it remains binding for all n, because the required wage spread decreases more strongly in n than do the inequity costs. Therefore, we are exactly in the same situation as described above for $\alpha + \beta \geq 1$.

The following corollary summarizes the preceding considerations.

Corollary 1.1 *Under limited liability and inequity aversion, if $\alpha + \beta$, $c(e)$ and $\bar{u}$ are sufficiently small and/or if the costs of increasing precision are sufficiently low, then the U-type tournament leads to lower overall costs than the J-type tournament for implementing e.*

[20] As in the case of $\alpha = \beta = 0$, the equilibrium exists in the U-type tournament for $n \leq 23$.

1.7 Conclusion

This essay gives two clear-cut results concerning relative payment schemes.

First, within the given framework, if agents care only about their own payoffs, then reward schemes based on fixed prizes minimize overall tournament costs for each arbitrary effort level. This may be the case if tournament participants do not work closely together, e.g., if they are managers of different divisions or work in the field. Furthermore, if agents earn rents, these rents can be lowered by increasing the accuracy of performance measurement only if prizes are fixed.

Second, if agents never earn rents and dislike earning wages different from those of their colleagues, then bonus pools lead to lower overall tournament costs. This may be more likely if employees are in close contact with each other.

However, if it is not always possible to pay inequity-averse agents just their reservation utility, neither of the two reward schemes dominates. Then, the costs of increasing the accuracy of performance measurement may be crucial. If these costs are low, fixed prizes may again be superior.

I used a stylized model, so I cannot capture all aspects that may be important for making the right choice between both tournament schemes. For instance, a drawback of U-type tournaments is that incentives may break down when during the course of the tournament it turns out that one agent has a large lead. This is not such a severe problem in a J-type tournament, since in this type of tournament each agent can still increase his share of the bonus pool by working hard. In my model, I avoid this problem by assuming that agents choose their effort levels once and for all at the beginning of the tournament. Such an assumption is justified if, for example, agents cannot evaluate a colleague's performance because they cannot acquire the same information as the principal.

Another important point is that if bribe payments are possible, then the principal cannot credibly commit himself to reward the agent who has performed better. In this case, incentives for hard work break down completely. In the U-type tournament, this problem is mitigated when the winner's prize is a promotion to a different job and the principal benefits from promoting the better agent to the higher position (see Fairburn and Malcomson [1994]).

1.8 Appendix

Proofs for section 1.3

Derivation of $\frac{\partial \Pi}{\partial e_i}$ and $\frac{\partial \Pi}{\partial e_j}$.

$$\frac{\partial \Pi}{\partial e_i} = \sum_{k=\frac{n+1}{2}}^{n} \frac{\partial \pi_{ki}}{\partial e_i} \tag{1.37}$$

$$= \sum_{k=\frac{n+1}{2}}^{n} \frac{\partial p_i}{\partial e_i} \left(\binom{n}{k} k p_i^{k-1} (1-p_i)^{n-k} \right. \tag{1.38}$$

$$-\binom{n}{k}(n-k)p_i^k(1-p_i)^{n-k-1}\Bigg) \tag{1.39}$$

For $k = \frac{n+1}{2} + 1, \ldots, n$, the first term in brackets evaluated at k cancels with the second term in brackets evaluated at $k-1$, since

$$\binom{n}{k}kp_i^{k-1}(1-p_i)^{n-k} - \binom{n}{k-1}(n-k+1)p_i^{k-1}(1-p_i)^{n-k} = 0, \tag{1.40}$$

as some simple transformations show. Therefore, the only remaining term of the sum is the first term in brackets evaluated at $k = \frac{n+1}{2}$ and we have

$$\frac{\partial \Pi}{\partial e_i} = \frac{n+1}{2}\binom{n}{\frac{n+1}{2}}[p_i(1-p_i)]^{\frac{n-1}{2}}\frac{\partial p_i}{\partial e_i}. \tag{1.41}$$

Analogously, we obtain

$$\frac{\partial \Pi}{\partial e_j} = \frac{n+1}{2}\binom{n}{\frac{n+1}{2}}[p_i(1-p_i)]^{\frac{n-1}{2}}\frac{\partial p_i}{\partial e_j}. \tag{1.42}$$

And, since $p_i = 1 - p_j$,

$$-\frac{\partial \Pi}{\partial e_j} = \frac{n+1}{2}\binom{n}{\frac{n+1}{2}}[p_i(1-p_i)]^{\frac{n-1}{2}}\frac{\partial p_j}{\partial e_j}. \ \square \tag{1.43}$$

$\gamma(n)$ is strictly increasing in n if and only if

$$\frac{n+1}{2^n}\binom{n}{\frac{n+1}{2}} < \frac{n+3}{2^{n+2}}\binom{n+2}{\frac{n+3}{2}} \tag{1.44}$$

$$\Leftrightarrow \frac{4(n+1)n!}{\left(\frac{n+1}{2}\right)!\left(\frac{n-1}{2}\right)!} < \frac{(n+3)(n+2)!}{\left(\frac{n+3}{2}\right)!\left(\frac{n+1}{2}\right)!} \tag{1.45}$$

$$\Leftrightarrow 4\frac{n+3}{2}\left(\frac{n+3}{2}-1\right) < (n+3)(n+2) \tag{1.46}$$

$$\Leftrightarrow n+1 < n+2 \ \square \tag{1.47}$$

Proofs for section 1.4

Proof of proposition 1.2. In the U-type tournament, the principal's cost minimization problem per agent is

$$C_U^P(e) \equiv \min_{l,w} l + \frac{\Delta w}{2} \tag{1.48}$$

$$\text{s.t.} \quad \gamma(n)\rho(e)\Delta w = c'(e) \tag{1.49}$$

$$l + \frac{\Delta w}{2} - c(e) \geq \bar{u} \tag{1.50}$$

Since Δw can be eliminated using equation (1.49) and inequality (1.50) is binding at the optimal solution, we obtain $C_U^P(e) = c(e) + \bar{u}$. In the J-type tournament, the principal's cost minimization problem per agent is

$$C_J^P(e) \equiv \min_{\bar{w},b} \bar{w} + \frac{b}{2} \tag{1.51}$$

$$\text{s.t.} \qquad \rho(e)b \;=\; c'(e) \tag{1.52}$$

$$\bar{w} + \frac{b}{2} - c(e) \;\geq\; \bar{u} \tag{1.53}$$

Again, it is straightforward to derive that $C_J^P(e) = c(e) + \bar{u}$. Furthermore, for any value function of effort, $v(e)$, the principal will implement the effort level that maximizes $v(e) - c(e) - \bar{u}$ in both tournaments, which is also first-best effort.□

Proofs for section 1.6

Derivation of equation (1.30). I first show that

$$\text{(i)} \quad \sum_{k=0}^{n} \binom{n}{k}(n-2k)^2 = n2^n \quad \text{and} \quad \text{(ii)} \quad \frac{1}{2^{n+1}} \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k}(2k-n)^2 = \frac{n}{4}.$$

Proof of (i): We use:

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n, \tag{1.54}$$

$$\sum_{k=0}^{n} k\binom{n}{k} = \sum_{k=1}^{n} n\binom{n-1}{k-1} = \sum_{k=0}^{n-1} n\binom{n-1}{k} = n2^{n-1}, \tag{1.55}$$

$$\sum_{k=0}^{n} k^2\binom{n}{k} = \sum_{k=1}^{n} k^2\binom{n}{k} = n(n-1)\sum_{k=1}^{n}\binom{n-2}{k-2} + n\sum_{k=1}^{n}\binom{n-1}{k-1} \tag{1.56}$$

$$= n(n-1)\sum_{k=0}^{n-2}\binom{n-2}{k} + n\sum_{k=0}^{n-1}\binom{n-1}{k} \tag{1.57}$$

$$= n(n-1)2^{n-2} + n2^{n-1}, \tag{1.58}$$

where for the computation of the last sum it is used that

$$k^2\binom{n}{k} = (k(k-1)+k)\binom{n}{k} = n(n-1)\binom{n-2}{k-2} + n\binom{n-1}{k-1}. \tag{1.59}$$

Thus, we obtain

$$\sum_{k=0}^{n}\binom{n}{k}(n-2k)^2 = n^2 2^n - 4n^2 2^{n-1} + 4(n(n-1)2^{n-2} + n2^{n-1}) = n2^n. \;\square \tag{1.60}$$

Proof of (ii): One can easily verify that (ii) is true for $n = 1$. To prove the claim for $n \geq 3$ and n odd, first note that

$$k^2\binom{n}{k} = (k(k-1)+k)\binom{n}{k} = n(n-1)\binom{n-2}{k-2} + k\binom{n}{k}. \tag{1.61}$$

Furthermore,

$$\sum_{k=\frac{n+1}{2}}^{n} \binom{n-2}{k-2} = \sum_{k=\frac{n-3}{2}}^{n-2} \binom{n-2}{k} = 2^{n-3} + \binom{n-2}{\frac{n-1}{2}}, \tag{1.62}$$

and

$$\sum_{k=\frac{n+1}{2}}^{n} k\binom{n}{k} = n \sum_{k=\frac{n+1}{2}}^{n} \binom{n-1}{k-1} = n \sum_{k=\frac{n-1}{2}}^{n-1} \binom{n-1}{k} \tag{1.63}$$

$$= n\left(2^{n-2} + \binom{n-2}{\frac{n-1}{2}}\right). \tag{1.64}$$

It follows that

$$\sum_{k=\frac{n+1}{2}}^{n} k^2\binom{n}{k} = n(n-1)\left(2^{n-3} + \binom{n-2}{\frac{n-1}{2}}\right) + n\left(2^{n-2} + \binom{n-2}{\frac{n-1}{2}}\right) \tag{1.65}$$

$$= n(n+1)2^{n-3} + n^2\binom{n-2}{\frac{n-1}{2}}. \tag{1.66}$$

Thus,

$$\sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k}(2k-n)^2 = 4\left(n(n+1)2^{n-3} + n^2\binom{n-2}{\frac{n-1}{2}}\right) - 4n^2\left(2^{n-2} + \binom{n-2}{\frac{n-1}{2}}\right) + n^2 2^{n-1} \tag{1.67}$$

$$= 2^{n-1}(n(n+1) - 2n^2 + n^2) = 2^{n-1}n. \square \tag{1.68}$$

Now consider the expected utility of agent i. We have

$$\bar{w} + p_i b - \alpha \sum_{k=0}^{\frac{n+1}{2}-1} \pi_{ki}\left((n-k)\frac{b}{n} - k\frac{b}{n}\right) - \beta \sum_{k=\frac{n+1}{2}}^{n} \pi_{ki}\left(k\frac{b}{n} - (n-k)\frac{b}{n}\right) - c(e_i)$$

$$= \bar{w} + p_i b - \frac{b}{n}\left(\alpha \sum_{k=0}^{\frac{n+1}{2}-1} \pi_{ki}(n-2k) + \beta \sum_{k=\frac{n+1}{2}}^{n} \pi_{ki}(2k-n)\right) - c(e_i). \tag{1.69}$$

Thus, the first-order condition for a symmetric equilibrium is

$$\rho(e)b - \frac{b}{n}\rho(e)\left(\alpha \sum_{k=0}^{\frac{n+1}{2}-1} \binom{n}{k}\frac{2k-n}{2^{n-1}}(n-2k) + \beta \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k}\frac{2k-n}{2^{n-1}}(2k-n)\right) - c'(e) = 0. \tag{1.70}$$

From (i), it holds that

$$\sum_{k=0}^{\frac{n+1}{2}-1} \binom{n}{k} (n-2k)^2 = n2^n - \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} (n-2k)^2, \tag{1.71}$$

and, therefore, the first-order condition simplifies to

$$\rho(e)b \left[1 - \frac{1}{2^{n-1}n} \left(-\alpha n 2^n + (\alpha+\beta) \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} (2k-n)^2 \right) \right] = c'(e). \tag{1.72}$$

Then, by applying (ii), we obtain (1.30).□

Derivation of inequality (1.32). For any symmetric equilibrium, the participation constraint in the J-type tournament is

$$\bar{w} + \frac{b}{2} - \left(\alpha \sum_{k=0}^{\frac{n+1}{2}-1} \binom{n}{k} \frac{(n-2k)}{2^n} + \beta \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} \frac{(2k-n)}{2^n} \right) \frac{b}{n} - c(e) \geq \bar{u}. \tag{1.73}$$

By applying that

$$\gamma(n) = \frac{1}{2^{n-1}} \sum_{k=0}^{\frac{n+1}{2}-1} \binom{n}{k} (n-2k) = \frac{1}{2^{n-1}} \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} (2k-n), \tag{1.74}$$

one immediately gets (1.32). Hence, it remains to prove (1.74). To do so I first show that

$$\sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} (2k-n) = \binom{n}{\frac{n+1}{2}} \frac{n+1}{2}. \tag{1.75}$$

It is obvious that this equation holds for $n = 1$. Now I prove the claim for $n \geq 3$, n odd. It holds that

$$\sum_{k=\frac{n+1}{2}}^{n} k \binom{n}{k} = n \sum_{k=\frac{n+1}{2}}^{n} \binom{n-1}{k-1} = n \sum_{k=\frac{n-1}{2}}^{n-1} \binom{n-1}{k} \tag{1.76}$$

$$= n \left(2^{n-2} + \binom{n-2}{\frac{n-1}{2}} \right), \tag{1.77}$$

where the last equality follows from the fact that $\sum_{k=0}^{n} \binom{n}{k} = 2^n$ and n is odd. Hence, because $\sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} = 2^{n-1}$ for an odd n,

$$\sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} (2k-n) = 2n \left(2^{n-2} + \binom{n-2}{\frac{n-1}{2}} \right) - n2^{n-1} \tag{1.78}$$

$$= 2n \binom{n-2}{\frac{n-1}{2}} = \frac{2n(n-2)!}{\left(\frac{n-1}{2}\right)! \left(\frac{n-3}{2}\right)!} \tag{1.79}$$

$$= \frac{n(n-1)(n-2)!}{\left(\frac{n-1}{2}\right)!\left(\frac{n-3}{2}\right)!\frac{n-1}{2}} = \frac{n!}{\left(\frac{n-1}{2}\right)!\left(\frac{n-1}{2}\right)!} \tag{1.80}$$

$$= \frac{n!}{\left(\frac{n+1}{2}\right)!\left(\frac{n-1}{2}\right)!}\frac{n+1}{2} = \binom{n}{\frac{n+1}{2}}\frac{n+1}{2}. \tag{1.81}$$

Therefore, $\gamma(n) = \frac{1}{2^{n-1}}\sum_{k=\frac{n+1}{2}}^{n}\binom{n}{k}(2k-n)$. The other equation can be verified analogously.□

Proof of $\gamma(n)/n$ is decreasing in n.

$$\frac{1}{n}\left[\frac{n+1}{2^n}\binom{n}{\frac{n+1}{2}}\right] > \frac{1}{n+2}\left[\frac{n+3}{2^{n+2}}\binom{n+2}{\frac{n+3}{2}}\right] \tag{1.82}$$

$$\Leftrightarrow \frac{1}{n}\left[4\frac{(n+1)!}{\left(\frac{n+1}{2}\right)!\left(\frac{n-1}{2}\right)!}\right] > \frac{1}{n+2}\left[\frac{(n+3)!}{\left(\frac{n+3}{2}\right)!\left(\frac{n+1}{2}\right)!}\right] \tag{1.83}$$

$$\Leftrightarrow \frac{n+2}{n} > \frac{(n+3)(n+2)}{4\frac{n+3}{2}\frac{n+1}{2}} \tag{1.84}$$

$$\Leftrightarrow \frac{1}{n} > \frac{1}{n+1} \;\square \tag{1.85}$$

Proof of proposition 1.5. (i) For the U-type tournament, from (1.29) we get

$$\Delta w = \frac{c'(e)}{(1+\alpha-\beta)\gamma(n)\rho(e)}. \tag{1.86}$$

To minimize wage costs, the participation constraint (1.31) must be binding. Then, after substituting for Δw, we obtain

$$l = \bar{u} + c(e) - \frac{1}{2}\frac{c'(e)}{(1+\alpha-\beta)\gamma(n)\rho(e)} + \frac{1}{2}\frac{(\alpha+\beta)c'(e)}{(1+\alpha-\beta)\gamma(n)\rho(e)}. \tag{1.87}$$

By computing the expected wage per agent, $l+\Delta w/2$, using the above two equations, we obtain $W_U^{ui}(e,n)$. For the J-type tournament, equation (1.30) yields

$$b = \frac{c'(e)}{(1+\alpha-\beta)\rho(e)}. \tag{1.88}$$

$\bar{w}$ is derived by making the participation constraint (1.32) binding and substituting for b:

$$\bar{w} = \bar{u} + c(e) - \frac{1}{2}\frac{c'(e)}{(1+\alpha-\beta)\rho(e)} + \frac{1}{2}\frac{\gamma(n)(\alpha+\beta)c'(e)}{n(1+\alpha-\beta)\rho(e)}. \tag{1.89}$$

By computing $\bar{w} + b/2$, we obtain $W_J^{ui}(e,n)$. The costs of inequity aversion are higher under the U-type tournament if and only if

$$\frac{1}{\gamma(n)} \geq \frac{\gamma(n)}{n} \qquad \Leftrightarrow \qquad 1 \geq \frac{\gamma^2(n)}{n}. \tag{1.90}$$

Since $\gamma(1) = 1$, the costs of inequity aversion are the same under both mechanisms if $n = 1$. $\gamma^2(n)/n$ is strictly decreasing in n if and only if

$$\frac{1}{n}\left[\frac{n+1}{2^n}\binom{n}{\frac{n+1}{2}}\right]^2 > \frac{1}{n+2}\left[\frac{n+3}{2^{n+2}}\binom{n+2}{\frac{n+3}{2}}\right]^2 \tag{1.91}$$

$$\Leftrightarrow \frac{1}{n}\left[4\frac{(n+1)!}{\left(\frac{n+1}{2}\right)!\left(\frac{n-1}{2}\right)!}\right]^2 > \frac{1}{n+2}\left[\frac{(n+3)!}{\left(\frac{n+3}{2}\right)!\left(\frac{n+1}{2}\right)!}\right]^2 \tag{1.92}$$

$$\Leftrightarrow \frac{n+2}{n} > \left[\frac{(n+3)(n+2)}{4\frac{n+3}{2}\left(\frac{n+1}{2}\right)}\right]^2 \tag{1.93}$$

$$\Leftrightarrow \frac{1}{n} > \frac{n+2}{(n+1)^2} \tag{1.94}$$

$$\Leftrightarrow n^2 + 2n + 1 > n^2 + 2n, \tag{1.95}$$

where the last inequality is always true. Thus we have $1 > \gamma^2(n)/n$ for all $n > 1$.□
(ii) When agents have limited liability, wage costs depend on whether the participation constraint or the liability constraint is binding. In the former case, costs are the same as in (i). In the latter case, we have $l = 0$ and $\bar{w} = 0$, respectively. Therefore, we obtain average wage costs of $w/2$ and $b/2$. Thus, $W_U^{li}(e, n)$ and $W_J^{li}(e, n)$ follow. The comparison of $W_U^{li}(e, n)$ and $W_J^{li}(e, n)$ follows from (i), $\gamma(1) = 1$ and because $\gamma(n)$ is strictly increasing in n.□

Essay 2

Relational Contracts and Job Design

2.1 Introduction

Measuring employee performance is often difficult because objective performance measures only imperfectly reflect an employee's true contribution to the firm. Thus, if rewards depend on imperfect measures, employees' incentives are not perfectly aligned with the firm's objectives.[1] The use of subjective performance measures, i.e., measures that are observed only by the contracting parties, may mitigate this problem. Indeed, subjective performance evaluation plays an important role in incentive contracting (see, e.g., Gibbons [2005]). Lincoln Electric, for example, motivates its workforce by using piece rates in combination with bonuses based on supervisors' subjective assessments. Thereby, workers are not only rewarded for high output but also for more complex and subtle achievements such as cooperation, innovation, or dependability. Furthermore, Hayes and Schaefer [2000] find empirical evidence that there is subjective assessment in the determination of salary and bonus of chief executive officers.

Informal agreements based on subjective performance evaluation cannot be part of an enforceable (or explicit) employment contract but have to be self-enforcing. This may be the case if the principal cares about its reputation in future relationships (Holmström [1981], Bull [1987]). Baker et al. (1994) show that explicit and relational contracts[2] can be complements as well as substitutes. While in some circumstances only a combination of explicit and relational contracts generates nonnegative profits, relational contracts are infeasible if objective performance measures are sufficiently close to perfect.[3]

The aim of this essay is to investigate how the possibility to engage in relational

[1]See, e.g., Kerr [1975] for an extensive number of examples.

[2]The term relational contract denotes an informal agreement that is not enforceable by a court but is self-enforcing. Such contracts are also called "implicit"(Baker et al. (1994), MacLeod and Malcomson [1989]), "self-enforcing"(Klein [1996]) or "self-enforcing implicit"(Bull [1987]).

[3]For the interaction of explicit and relational contracts see also Pearce and Stacchetti [1998], Bernheim and Whinston [1998], Che and Yoo [2001], and Demougin and Fabel [2004]. More generally, contributions to the theory of relational contracts include e.g., MacLeod and Malcomson [1989], Klein [1996], Baker et al. [2002], MacLeod [2003], and Levin [2003].

contracts affects optimal job design, i.e., the optimal grouping of tasks into jobs. To do so, I first reformulate the model by Baker et al. (1994) for the case of multiple tasks. I assume that all tasks jointly affect non-verifiable firm value and a contractible but imperfect performance measure.

For a given set of tasks performed by a single agent, I find that relational contracts are feasible if the performance measure is sufficiently distorted[4] or firm value is sufficiently responsive to changes in effort. In both cases, the principal greatly benefits from better aligning incentives by paying an implicit bonus based on firm value. Employees anticipate that it is in the principal's interest not to renege on relational contracts since she wants to retain the possibility of using them in future periods.

In the next step, I examine when tasks should be split between agents. In my model, the only externality that can arise is due to the misallocation of effort across tasks. As a result, the first-best solution is implemented if the principal employs one agent for each task because this prevents misallocation of effort. Thus, in my framework, the solution to the job design problem is nontrivial only if employing one agent for each task is not possible. This is for example the case if some tasks are non-separable, e.g., quantity and quality in the production of a good.[5] For simplicity, I consider an environment in which three tasks are to be assigned to either one or two agents. Furthermore, agents cannot simultaneously perform the same task. Therefore, task splitting denotes the grouping of tasks in two different jobs where no task is part of both jobs.

Task splitting has two effects: On the one hand, effort in the one-task job is first-best. On the other hand, the agent in the two-task job receives a lower implicit bonus than an agent performing all three tasks would receive. By withdrawing a task from an agent, this agents' performance becomes less important for the firm value. Therefore, the principals' temptation to renege on a relational contract increases, leading to a lower feasible implicit bonus. This result always holds, even though an agent performing only two tasks may have more distorted explicit incentives than an agent performing three tasks.

The principal prefers to split tasks if she cannot commit herself to paying an implicit bonus to an agent performing all three tasks. Although implicit bonuses remain infeasible under task splitting, the effect of setting first-best incentives for the single-task job increases the principal's expected profit. By contrast, if the principal can commit to paying an implicit bonus to an agent who performs two tasks, expected profit increases if the third task is also assigned to this agent. This leads to a strengthened relational contract, which outweighs the loss from not having first-best incentives for the third task.

Thus, the results suggest that task assignments should be more complex when well aligned objective performance measures are not available or firm value is highly responsive to changes in effort, because then relational contracts are feasible. Frequently, this is the case on higher hierarchy levels within the firm. Then, jobs should

[4]This result is in line with Baker et al. (1994).

[5]Another possibility is that agents have positive opportunity costs. Since the principal must compensate agents for their opportunity costs, she might prefer to employ fewer agents than there are tasks.

be designed such that production workers specialize in a narrow range of tasks, while managers perform a broader range. The former group is more likely to be paid according to pure explicit contracts, while the latter one tends to be rewarded by a combination of explicit and relational contracts.

Furthermore, I also analyze how tasks should be grouped into jobs if all three tasks are separable and have to be split between two agents. For example, due to lack of time, it might not be possible that one agent performs all tasks. Since effort in the one-task job is first-best, one would expect that it is efficient to assign the task that affects firm value most strongly to this job. However, whether this is true or not depends on the characteristics of the corresponding two-task job. For example, if the agent who performs two tasks receives an implicit bonus, the principal may even want to assign the two most important tasks to him to be able to commit to a high-powered relational contract.

Agents are risk-neutral and have unlimited liability. However, it can be shown that a limited liability constraint for agents affects expected profits under task splitting and no task splitting in the same way. Therefore, the results regarding the principal's optimal decision on task splitting also hold if agents are protected by limited liability.

I further assume that agents' opportunity costs of working for the firm are zero so that there are, a priori, no additional costs of employing more than one agent. The extension to positive opportunity costs is discussed in section 2.6. Finally, most of the results can be generalized to the case of splitting n tasks between less than n agents, which I also explain in section 2.6.

With respect to job design, the problem considered in this essay is most closely related to Itoh (1994, 2001). He also shows that it is often optimal to group a broad range of tasks into an agent's job. In his framework, there is also one joint performance measure for all tasks. However, agents are risk-averse and the degree of cost substitutability between tasks varies. Assigning all tasks to one agent is optimal when the degree of substitutability is sufficiently low because then the effect of paying only one risk premium dominates.[6] By contrast, I focus on the impact of relational contracts in an environment with risk-neutral agents and independent tasks.

Among the first contributions to multi-tasking and job design are Holmström and Milgrom [1991] and Itoh (1991a, 1992).[7] They study static settings with risk-averse agents and one performance measure for each single task. Meyer et al. [1996] and Olsen and Torsvik [2000] extend the model by Holmström and Milgrom [1991] to a dynamic setting with limited intertemporal commitment of the principal. While focussing on the "ratchet effect", they show that rules for optimal job design in a static setting (such as sole responsibility for tasks, grouping hardest-to-monitor tasks in one job and easiest-to-monitor tasks in another, or a positive correlation between discretion and incentives) may no longer hold in a multi-period framework. This is also the case in my model in which task splitting is always optimal in a one-shot

[6] Moreover, Itoh (1994, 2001) also investigates under which circumstances the principal prefers to perform a task by herself.

[7] In particular, Itoh (1991a, 1992) examines when it is optimal to induce cooperation in multi-agent situations.

relationship but not necessarily in a long-term one. The reason is that in the former relational contracts are never feasible.

Valsecchi [1996] shows that appropriate job design can restrict the set of sequential equilibria to the Pareto optimal one if tasks are performed sequentially in a team production process. Through appropriate task grouping, the principal can exploit workers' private information about their own effort or the effort exerted by their colleagues. As in my framework, it is not possible to measure effort in each single task. However, the model is static and thus does not consider the use of non-verifiable information.

In the next section, the model is introduced. In section 2.3, I derive the optimal combination of implicit and explicit contracts for a given set of tasks. Section 2.4 analyzes the question of *when* tasks should be split between agents, while the section 2.5 examines *how* tasks should be grouped into jobs. Section 2.6 generalizes the results to an arbitrary number of tasks and agents and discusses the model assumptions. The last section of this essay concludes.

2.2 The model

I consider a relationship between a principal and one or two agents. All parties are risk-neutral. The principal is the owner of the firm in which the agents can be employed. In each period, the probability that the principal-agent(s) relationship will be repeated in the following period is exogenously given by $\rho \in (0, 1)$.

There are three tasks that jointly affect firm value Y. Y is either high or low, $Y \in \{0, 1\}$, and is realized at the end of each period. I define $N := \{1, 2, 3\}$ as the set of tasks and $e_t \geq 0$ as the non-observable effort exerted in task $t \in N$. Furthermore, e denotes the vector of all efforts, $e = (e_1, e_2, e_3)^T$.[8] The probability that firm value is high given e is assumed to be

$$\text{prob}[Y = 1|e] = \max\{f^T e, 1\} = \max\{f_1 e_1 + f_2 e_2 + f_3 e_3, 1\}, \tag{2.1}$$

where $f \in \mathbb{R}^3$ and $f_t > 0$ for all $t \in N$, i.e., all tasks are productive.

We could Y interpret alternatively as the value of a division or department in which the agents are employed. Then, the realization of Y would not only depend on the effort in the three tasks under consideration. For example, none of the following results would change if we had $\text{prob}[Y = 1|e] = \max\{y + f^T e, 1\}$, where y is determined by the contribution of other employees and is independent of e.

The realization of Y is observed by the principal and all employed agents but is non-verifiable. However, there is a verifiable performance measure $P \in \{0, 1\}$ that is also realized at the end of each period, where

$$\text{prob}[P = 1|e] = \max\{g^T e, 1\}, \tag{2.2}$$

$g \in \mathbb{R}^3$, $g_t > 0$ for all $t \in N$.[9] Similar to Y, the realization of P could also depend on the performance of other employees. Given f, g, and e, the realizations of Y and P are independent.

[8] All vectors are column vectors. Superscript T denotes transpose.

[9] Similar multi-tasking approaches are widely-used in the literature, see, e.g., Feltham and Xie [1994], Datar et al. [2001], and Baker [2002].

The principal cannot perform any task from N herself. She can hire either one or two homogeneous agents to perform the tasks. If she employs two agents, she must also decide how to group tasks into jobs. I assume that each task can be assigned to one agent only. Furthermore, a task assignment has to be maintained in all future periods. This also implies that the initially chosen number of agents is invariant over time.[10]

An agent's non-observable cost of exerting effort e is

$$c(e) = \frac{c}{2} e^T e, \quad c > 0, \tag{2.3}$$

i.e, $c(e)$ is separable and quadratic. Agents have unlimited liability. Their opportunity costs of working for the principal are zero in each period. Thus, there are no a priori costs of employing two rather than one agent. How results are affected by positive opportunity costs is discussed in section 2.6.

For simplicity, I assume that $f, g,$ and c are such that the probabilities $f^T e$ and $g^T e$ are always smaller than one at the optimal (first- and second-best) solution.[11] The vector of first-best efforts, denoted by e^{FB}, maximizes expected firm value minus costs of effort, i.e.,

$$e^{FB} = \text{argmax}_e f^T e - \frac{c}{2} e^T e. \tag{2.4}$$

Thus, $e^{FB} = c^{-1} f$ leading to an expected profit of $\frac{f^T f}{2c}$.

Timing is as follows in each period: At the beginning of the period, the principal individually offers each agent an explicit wage contract specifying some guaranteed fixed payment and an explicit bonus that will be paid at the end of the period if $P = 1$. Additionally, the principal may offer an implicit bonus that he promises to pay at the end of the period if $Y = 1$. However, since Y is non-verifiable, an agent will rely on such a promise only if he believes that it is in the principal's interest not to renege on it. Given the explicit and the relational contract, each agent chooses his effort level(s). Afterwards, Y and P are realized and each agent is rewarded according to his explicit contract. If $Y = 1$, the principal decides whether to pay the implicit bonuses to one or both agents.

In the reminder of this section and in the following one I analyze the case in which the principal employs only one agent who performs all three tasks. First assume that the agent does not trust the principal to pay any bonus based on the realization of Y.[12] Let α (fixum) and β (bonus) denote the components of the explicit wage contract that the principal offers to the agent. Then, the principal's optimization problem is

$$\max_{\alpha,\beta,e} f^T e - (\alpha + \beta g^T e), \tag{2.5}$$

$$\text{s.t.} \quad e = \text{argmax}_{\hat{e}} \alpha + \beta g^T \hat{e} - \frac{c}{2} \hat{e}^T \hat{e}, \tag{2.6}$$

[10]This can be justified if, for example, agents have to learn how to perform a task before production can take place. Then, changing the number of agents in future periods would lead to additional learning costs for at least one task. Such costs are often at least partly borne by the firm and might be prohibitively high. I will discuss the impact of this assumption in section 2.6.

[11]It can be shown that this is the case if $\max\{f^T f, f^T g\} < c$.

[12]The circumstances under which this happens are discussed in the next section.

$$0 \leq \alpha + \beta g^T e - \frac{c}{2} e^T e. \tag{2.7}$$

This problem has already been analyzed in similar forms, e.g., by Baker [2002] and Gibbons [2005].[13] From the incentive compatibility constraint (2.6), the agent exerts efforts $e(\beta) = \frac{\beta}{c} g$. For each β, the principal will choose α so that the agent's participation constraint (2.7) is binding. Thus, the optimal bonus $\tilde{\beta}$ maximizes $f^T e(\beta) - \frac{c}{2}(e(\beta))^T e(\beta)$, i.e.,

$$\tilde{\beta} = \frac{f^T g}{g^T g} = \frac{||f||}{||g||} \cos\theta, \tag{2.8}$$

where

$$||f|| := \sqrt{f_1 + f_2 + f_3} \quad \text{and} \quad \cos\theta := \frac{f^T g}{||f|| ||g||}. \tag{2.9}$$

By these definitions, $||f||$ and $||g||$ are the lengths of the vectors f and g, respectively, and θ is the angle between them. Because all components of f and g are assumed to be positive, $\cos\theta$ is also positive.[14] The resulting expected profit for the principal is

$$\tilde{\pi} = \frac{||f||^2}{2c} \cos^2\theta. \tag{2.10}$$

As pointed out by Baker [2002] and Gibbons [2005], there are two features that determine the optimal explicit bonus $\tilde{\beta}$: scaling, as given by $||f||/||g||$, and alignment, as given by $\cos\theta$. $\cos\theta$ can be interpreted as a measure of alignment (or congruity) between firm value and performance measure. The higher $\cos\theta$ the better aligned are f and g and, thus, the more useful is the performance measure for efficiently directing effort to the different tasks. Therefore, the optimal explicit bonus and expected profit increase in $\cos\theta$.

If $\cos\theta = 1$, i.e., f and g are perfectly aligned, the first-best solution is implemented by scaling the bonus appropriately. Thus, scaling only corrects for the difference in the lengths of f and g. For instance, if $\cos\theta = 1$ but firm value is more sensitive to changes in effort than the performance measure, i.e., $||f|| > ||g||$, we have $\tilde{\beta} > 1$.

I henceforth assume that the principal cannot implement first-best efforts by a pure explicit contract, i.e., $\cos\theta < 1$. I analyze the optimal combination of explicit and relational contracts in the next section.

2.3 Combining explicit and relational contracts

The analysis in this section is similar to the one in Baker et al. (1994). The main difference is that these authors consider the case of a single agent performing only one task. The productivity of his effort with respect to the contractible performance

[13] In both papers, firm value is $f^T e$ plus a noise term, and the performance measure is $g^T e$ plus another noise term. In Baker [2002] the agent is also risk-averse.

[14] This implies that the performance measure is never completely useless. Allowing for zero components in f and g does not affect the results, but leads to tedious case distinctions under task splitting.

measure is observed only by the agent after the explicit contract has been signed. In general, this productivity is different from the effort's true contribution to firm value. This creates a congruity problem similar to the one considered here.[15]

Assume the principal offers the agent an explicit contract as described in the foregoing section. Additionally, suppose that she can credibly promise to pay an implicit bonus γ if $Y = 1$. Then the agent chooses $e(\beta, \gamma)$ to solve the problem

$$\max_e \alpha + \beta g^T e + \gamma f^T e - \frac{c}{2} e^T e, \tag{2.11}$$

i.e.,

$$e(\beta, \gamma) = \frac{1}{c}(\beta g + \gamma f). \tag{2.12}$$

For each combination of β and γ, the principal sets α so that the agent's participation constraint

$$\alpha + \beta g^T e(\beta, \gamma) + \gamma f^T e(\beta, \gamma) - \frac{c}{2}(e(\beta, \gamma))^T e(\beta, \gamma) \geq 0 \tag{2.13}$$

is binding.

When determining the optimal combination of explicit and implicit bonus, the principal must take into account that her promise to pay γ if firm value is high must be trustworthy to the agent. To model the role of trust, I assume that if the principal once reneges on the relational contract, the agent will never trust her again to pay an implicit bonus. Thus, if the principal breaks the relational contract, his fallback position is a pure explicit contract leading to profit $\tilde{\pi}$ (see (2.10)) in all future periods. Therefore, the principal chooses β and γ to solve the problem

$$\max_{\beta,\gamma} f^T e(\beta, \gamma) - \frac{c}{2} e(\beta, \gamma)^T e(\beta, \gamma) \tag{2.14}$$

$$\text{s.t.} \quad \gamma \leq \sum_{t=1}^{\infty} \rho^t \left[f^T e(\beta, \gamma) - \frac{c}{2} e(\beta, \gamma)^T e(\beta, \gamma) - \tilde{\pi} \right]. \tag{2.15}$$

Inequality (2.15) is the principal's commitment constraint. It says that the principal's short-term profit from reneging on the relational contract, γ, must not exceed the associated expected long-term loss, which is given by the term on the right-hand side of (2.15). If (2.15) was not satisfied, the agent would anticipate that the principal will not stick to the informal agreement if high firm value is realized.

When solving the principal's problem as given by (2.14) and (2.15), we get from the first-order condition for the optimal explicit bonus that

$$\beta(\gamma) = (1 - \gamma)\frac{||f||}{||g||} \cos\theta = (1 - \gamma)\tilde{\beta}. \tag{2.16}$$

Then the principal's expected profit can be written as[16]

$$\pi(\gamma) = \frac{||f||^2}{2c} \cos^2\theta + \frac{\gamma(2 - \gamma)}{2c} ||f||^2 (1 - \cos^2\theta). \tag{2.17}$$

[15]Furthermore, in Baker et al. (1994), the agent's opportunity costs are positive so that the principal's profit under a pure explicit contract can be negative.

[16]The derivations of (2.16) and (2.17) are given in the appendix.

If γ is credible, expected profit per period increases compared to a pure explicit contract by the second term on the right-hand side of (2.17). This term strictly increases in γ and is maximal at $\gamma = 1$. If $\gamma = 1$, the agent becomes the residual claimant and, thus, exerts first-best effort in each task. In return, he pays the expected profit to the principal, i.e., $\alpha = -\frac{||f||}{2c}$.

However, the principal will in general not be able to commit an implicit bonus of $\gamma = 1$. After substituting β, her commitment constraint (2.15) becomes

$$\phi\gamma \leq \frac{\gamma(2-\gamma)}{2c}||f||^2(1-\cos^2\theta), \tag{2.18}$$

where $\phi := (1-\rho)/\rho$.

The principal chooses the highest $\gamma \in [0,1]$ which satisfies (2.18) so that the optimal implicit bonus is

$$\gamma^* = \begin{cases} 1 & \text{if} \quad 2c\phi \leq ||f||^2(1-\cos^2\theta) \\ 2\left(1 - \frac{c\phi}{||f||^2(1-\cos^2\theta)}\right) & \text{if} \quad c\phi < ||f||^2(1-\cos^2\theta) < 2c\phi \\ 0 & \text{if} \quad ||f||^2(1-\cos^2\theta) \leq c\phi \end{cases}. \tag{2.19}$$

The principal's expected profit is

$$\pi(\gamma^*) = \begin{cases} \frac{||f||^2}{2c} & \text{if} \quad \gamma^* = 1 \\ \frac{||f||^2}{2c}\cos^2\theta + 2\phi\left(1 - \frac{c\phi}{||f||^2(1-\cos^2\theta)}\right) & \text{if} \quad 0 < \gamma^* < 1 \\ \frac{||f||^2}{2c}\cos^2\theta & \text{if} \quad \gamma^* = 0 \end{cases}. \tag{2.20}$$

The principal can commit to a high implicit bonus if her loss from breaking the relational contract is large. The per-period loss from reneging on the relational contract, which is given on the right-hand side of (2.18), increases in the term $||f||^2(1-\cos^2\theta)$. Thus, γ^* also increases in this term. Given $\cos^2\theta$, a high value of $||f||$ means that expected firm value strongly responds to changes in effort. Therefore, the benefit from better aligning incentives by paying an implicit bonus contingent on firm value Y is large. Given $||f||$, a low value of $\cos^2\theta$, i.e., a strongly distorted performance measure, also makes the use of implicit incentives more desirable. This leads to the first proposition.

Proposition 2.1 *Relational contracts exist in environments where well aligned performance measures are not available*[17] *or firm value is highly responsive to changes in effort.*

Furthermore, the optimal implicit bonus increases in ρ and decreases in c. The intuition is straightforward. A high probability that the principal-agent relationship will continue increases the expected loss from breaking the relational contract. Low effort costs increase the per-period benefit from using a relational contract (see (2.18)) and, therefore, also the loss from reneging on it.

In figure 2.1, the optimal implicit bonus and the resulting profit are depicted for fixed $||f||$ and varying $\cos^2\theta$. The higher $\cos^2\theta$ the less distorted the performance

[17] In their framework, Baker et al. (1994) derive the same result.

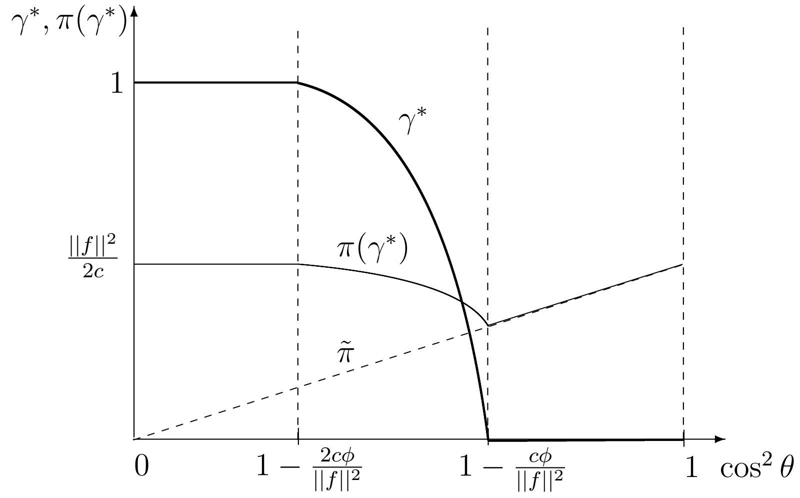

Figure 2.1: Optimal implicit bonus and expected profit.

measure, and, therefore, the smaller is the principal's loss from reneging on the relational contract. Thus, as already explained above, the maximal feasible implicit bonus decreases in $\cos^2\theta$. As a result, the principal benefits from a less distorted performance measure if and only if she is not able to pay an implicit bonus.

On the other hand, holding $\cos^2\theta$ constant while increasing $||f||$ fixes the distortion of the performance measure but increases each tasks' (expected) marginal productivity by the same factor. This means that firm value becomes more responsive to changes in effort. Therefore, both γ^* and $\pi(\gamma^*)$ increase in $||f||$.

I summarize these results in the following proposition.

Proposition 2.2 *(i) For fixed $||f||$, the optimal implicit bonus decreases in $\cos\theta$. The principal's expected profit strictly increases in $\cos\theta$ if and only if she cannot commit to an implicit bonus, i.e., if $\cos^2\theta \geq 1 - c\phi/||f||^2$. (ii) For fixed $\cos\theta$, the optimal implicit bonus and expected profit increase in $||f||$.*

2.4 When should tasks be split?

In the previous section, I have focussed on the case of one agent. In this section, I analyze under which circumstances the principal benefits from splitting tasks between two agents. Obviously, there is no use of task splitting if the principal can set first-best incentives when employing one agent, i.e., if $\gamma^* = 1$. Therefore, I henceforth consider the case $\gamma^* < 1$.

Assume that task i can be performed by another agent, while tasks j and k are non-separable, where $\{i, j, k\} = N$, $j < k$. If there is task splitting, agent 1 performs task i and agent 2 performs tasks j and k. I define

$$e^{-i} := (e_j, e_k)^T,\ f^{-i} := (f_j, f_k)^T,\ g^{-i} := (g_j, g_k)^T, \tag{2.21}$$

and

$$\cos\theta_{-i} := \frac{(f^{-i})^T g^{-i}}{||f^{-i}||||g^{-i}||}. \tag{2.22}$$

Furthermore, let β_i, γ_i and β_{-i}, γ_{-i} denote the explicit and implicit bonus for agent 1 and agent 2, respectively. Analogously, α_i and α_{-i} denote the fixed payments.

I assume that if the principal breaks a relational contract with one agent, both agents will not rely on relational contracts in all future periods. This implies that an agent can observe whether or not the principal kept an implicit agreement with his colleague.[18] Furthermore, I assume that agent 1 observes the explicit and relational contract offered to agent 2 and vice versa.

Suppose that implicit bonuses are credible. Then, given the effort levels of agent 2, e^{-i}, agent 1 chooses e_i to solve the problem

$$\max_{e_i} \alpha_i + \beta_i g^T e + \gamma_i f^T e - \frac{c}{2} e_i^2. \tag{2.23}$$

Analogously, given the effort of agent 1, e_i, agent 2 chooses e^{-i} to solve

$$\max_{e^{-i}} \alpha_{-i} + \beta_{-i} g^T e + \gamma_{-i} f^T e - \frac{c}{2} (e^{-i})^T e^{-i}. \tag{2.24}$$

It follows that

$$e_i(\beta_i, \gamma_i) = \frac{1}{c}(\beta_i g_i + \gamma_i f_i), \quad e^{-i}(\beta_{-i}, \gamma_{-i}) = \frac{1}{c}(\beta_{-i} g^{-i} + \gamma_{-i} f^{-i}), \tag{2.25}$$

i.e., an agent's effort choice does not depend on the effort choice of his colleague. However, the joint effort of both agents determines the probabilities of high firm value and a favorable performance measure and, therefore, also the expected payment to each agent. Thus, when deciding whether to accept the contract offered by the principal, each agent must anticipate the effort choice of his colleague. Moreover, each agent can trust the principal to pay his individual bonus only if the principal finds it beneficial to pay both implicit bonuses simultaneously. Thus, if an agent does not know the implicit bonus offered to his colleague, he cannot judge the credibility of the promise that the principal made to him. Therefore, the assumption that each agent observes the contract offered to his colleague simplifies the analysis.[19]

Let γ_i^* and γ_{-i}^* denote the optimal implicit bonuses that are paid to agent 1 and agent 2, respectively. As in the previous section, γ^* denotes the optimal implicit bonus with one agent performing all tasks alone. In order to compare the principal's profit under task splitting with her profit when all tasks are performed by one agent, I first derive the following proposition.

Proposition 2.3 *Assume that $\gamma^* < 1$ and task i is assigned to agent 1. Then agent 1 exerts effort e_i^{FB} and $\gamma_i^* = 0$. Furthermore, $\gamma_{-i}^* \leq \gamma^*$ and agent 2 exerts $(e^{FB})^{-i}$ if and only if $\cos\theta_{-i} = 1$. The principal's profit is*

$$\pi^S(\gamma_{-i}^*) = \begin{cases} \frac{f_i^2}{2c} + \frac{\|f^{-i}\|^2}{2c} \cos^2\theta_{-i} + 2\phi\left(1 - \frac{c\phi}{\|f^{-i}\|^2(1-\cos^2\theta_{-i})}\right) & \text{if} \quad 0 < \gamma_{-i}^* < 1 \\ \frac{f_i^2}{2c} + \frac{\|f^{-i}\|^2}{2c} \cos^2\theta_{-i} & \text{if} \quad \gamma_{-i}^* = 0 \end{cases}$$

[18] I make this assumption because it is prevalent in the literature (see, e.g., Bull [1987]). However, in the case of three tasks and two agents, it can be shown that the results do not change when only the agent who was cheated does no longer rely on relational contracts. I will discuss the impact of this assumption in more general settings in section 2.6.

[19] In the case of three tasks this assumption can be dropped. As we will see, agent 1 exerts first-best effort in his task under a pure explicit contract. Thus, by knowing f and g and the tasks assigned to himself, each agent can anticipate which contract will be offered to his colleague.

Proof See appendix.

Proposition 2.3 says that the principal provides first-best incentives for agent 1 by a pure explicit contract. Furthermore, an agent performing two tasks always receives a lower implicit bonus than an agent who performs three tasks.

When determining the optimal combination of an explicit and relational contract for agent 2, the principal can proceed as if he would solve the problem

$$\max_{\beta,\gamma}(f^{-i})^T e^{-i}(\beta,\gamma) - \frac{c}{2}(e^{-i}(\beta,\gamma))^T e^{-i}(\beta,\gamma) \tag{2.26}$$

$$\text{s.t.} \quad \phi\gamma \leq (f^{-i})^T e^{-i}(\beta,\gamma) - \frac{c}{2}(e^{-i}(\beta,\gamma))^T e^{-i}(\beta,\gamma) - \frac{||f^{-i}||}{2c}\cos^2\theta_{-i}, \tag{2.27}$$

which is equivalent to the problem analyzed in section 2.3. This is due to two reasons: Since agent 1 is responsible for only one task, he cannot misallocate effort between tasks. Consequently, there is no need to pay an implicit bonus to agent 1. Second, an agent's effort choice affects his colleague only in terms of his expected payment. This does not cause any problems, because the principal can always make agents' participation constraints binding by individually adjusting the fixed payments α_i and α_{-i}.

In the proof of proposition 2.3 it is shown that

$$||f^{-i}||^2(1-\cos^2\theta_{-i}) \leq ||f||^2(1-\cos^2\theta), \tag{2.28}$$

and therefore, by (2.19), $\gamma^*_{-i} \leq \gamma^*$. That is, if the principal withdraws an arbitrary task from an agent, the implicit bonus she can commit to paying to this agent decreases. The intuition for this result is as follows. Withdrawing a task from an agent affects his implicit bonus in two different ways. First, his performance becomes less important for the firm value because $||f^{-i}|| < ||f||$. Second, the congruency problem associated with this agent may become more or less severe, i.e., $\cos\theta_{-i}$ can be smaller or larger than $\cos\theta$.[20] The first effect decreases the agent's maximal feasible implicit bonus. The second effect works in the opposite direction if f^{-i} and g^{-i} are worse aligned than f and g. However, the first effect always dominates.

As a result, if $\gamma^* < 1$, first-best efforts can be implemented for agent 2 if and only if this is possible through a pure explicit contract, i.e., if f^{-i} and g^{-i} are perfectly aligned.

Although task splitting always leads to first-best incentives for one task, the next proposition shows that it is often optimal to assign all tasks to one agent.

Proposition 2.4 *(i) If $\gamma^* = 0$, the principal prefers task splitting. (ii) If $\gamma^*_{-i} > 0$, the principal prefers to assign all tasks to one agent.*

Proof See appendix.

By proposition 2.4, the principal prefers task splitting if relational contracts are infeasible no matter whether she employs one or two agents. However, if an agent

[20] At first sight one might think that $\cos\theta_{-i} \geq \cos\theta$, i.e., the congruency problem is always less severe with only two tasks. However, consider, e.g., $f = (1,0,x)^T$ and $g = (0,1,y)^T$. In this case, $\cos\theta_{-3} < \cos\theta$ for all $x, y > 0$.

who performs two tasks receives an implicit bonus, expected profit increases if the third task is also assigned to this agent.

To understand the intuition for these results, note that (2.28) is equivalent to

$$||f||^2 \cos^2\theta \leq f_i^2 + ||f^{-i}||^2 \cos^2\theta_{-i}. \tag{2.29}$$

From this inequality it follows immediately that the expected profit under pure explicit contracts is always larger under task splitting, i.e., $\pi(0) \leq \pi^S(0)$. The reason is that even if f^{-i} and g^{-i} are worse aligned than f and g, the misalignment between f^{-i} and g^{-i} cannot become so large that it dominates the positive effect of setting first-best incentives for task i. Thus, we obtain result (i). Furthermore, it is clear that $\pi(\gamma^*) = \pi^S(\gamma^*_{-i})$ if (2.28) binds.[21]

However, if (2.28) does not bind and $\gamma^*_{-i} > 0$, the increase in expected profits due to the use of an implicit bonus is larger when all tasks are performed by one agent, i.e.,

$$2\phi\left(1 - \frac{c\phi}{||f^{-i}||^2(1-\cos^2\theta_{-i})}\right) < 2\phi\left(1 - \frac{c\phi}{||f||^2(1-\cos^2\theta)}\right). \tag{2.30}$$

This is due to the fact that an agent who performs three tasks receives a higher implicit bonus. Therefore, as long as $\gamma^*_{-i} > 0$, the overall improvement of explicit contracts under task splitting never outweighs the loss due to a weakened relational contract for agent 2.

Only if f^{-i} and g^{-i} are so well aligned that the principal cannot commit to an implicit bonus for agent 2, pure explicit contracts under task splitting may dominate a combination of pure and relational contracts when tasks are not split. This is the only case that is not considered in proposition 2.4, namely, $\gamma^* > 0$ and $\gamma^*_{-i} = 0$. In this case, task splitting is optimal if f^{-i} and g^{-i} are sufficiently well aligned because then incentives with pure explicit contracts are close to first-best under task splitting. In the extreme case of $\cos\theta_{-i} = 1$, the principal implements first-best efforts if she employs two agents.

Formally, if $\gamma^* > 0$ and $\gamma^*_{-i} = 0$, it can be easily verified that assigning task i to another agent leads to a higher expected profit if

$$||f^{-i}||^2(1-\cos^2\theta_{-i}) \leq ||f||^2(1-\cos^2\theta) - 4c\phi\left(1 - \frac{c\phi}{||f||^2(1-\cos^2\theta)}\right). \tag{2.31}$$

The right-hand side of this inequality decreases in $||f||^2(1-\cos^2\theta)$ and increases in ϕ.[22] Thus, inequality (2.31) is likely to hold if $||f||^2(1-\cos^2\theta)$ is small and ϕ is large (i.e., γ^* is small), and/or if $||f^{-i}||^2(1-\cos^2\theta_{-i})$ is small (i.e., the congruency problem for agent 2 is not severe).

By combining propositions 2.1 and 2.4, we obtain that all tasks should be assigned to one agent if (I) the performance measure is not suitable to provide incentives for the two-task job (i.e., θ_{-i} is large) or (II) firm value strongly responds to effort

[21] This happens in the special case of $g_i/f_i = (g_j^2 + g_k^2)/(f_j g_j + f_k g_k)$ as can be seen from the proof of proposition 2.3.

[22] This is because $c\phi < ||f||^2(1-\cos^2\theta) < 2c\phi$ since $0 < \gamma^* < 1$.

changes in the two-task job. More loosely speaking, the principal should not split tasks if a pure explicit contract performs badly in the two-task job.

Under certain conditions, pure explicit contracts can induce production workers to allocate effort efficiently across tasks. For instance, Lazear [2000] shows that piece rates combined with some form of quality control can provide workers with incentives to produce high output without neglecting quality. If production workers can be closely monitored, even the number of hours worked may serve as a good proxy for performance. Usually, the output of supervisors and managers is less concrete, and, therefore, more difficult to measure. Thus, (I) suggests that jobs tend to consist of more tasks on higher hierarchy levels.

Furthermore, (II) implies that job assignments consisting of a broad range of tasks are more likely if these tasks strongly affect firm value. Suppose firm value Y depends on production and management tasks as explained in section 2.2. Then, (II) also leads to the conclusion that management tasks are more likely to be assigned to one agent than production tasks, because management tasks generally affect firm value more strongly.

Finally, since implicit bonuses also increase in the probability that the principal-agent relationship continues, employees that are more likely to stay with the firm should perform more tasks.

2.5 How should tasks be split?

In this section, I assume that all three tasks are separable and must be split between two agents. For example, due to lack of time, it might not be possible that one agent performs all tasks. Then, the question arises which task the principal should assign to agent 1, i.e., to the agent who is responsible for only one task.

There are two cases in which first-best effort is implemented in each task. First, if there is a task t such that $\cos\theta_{-t} = 1$, assigning this task to agent 1 is optimal. Then, by proposition 2.3, the principal induces first-best effort in each task by pure explicit contracts. Second, if it is possible to have $\gamma^*_{-t} = 1$ for some task t, assigning this task to agent 1 also leads to first-best incentives. Agent 2 then exerts first-best efforts under a pure relational contract.

Now assume the principal cannot implement first-best efforts under task splitting. Furthermore, I denote the task t for which f_t is maximal (minimal) as the most (least) important task. Let again task i be the task that is assigned to agent 1.

First consider the case that it is never possible to pay an implicit bonus to agent 2, i.e., $\gamma^*_{-t} = 0$ for all t. Then, by proposition 2.3, if agent 1 performs task i, the principal's expected profit is

$$\frac{||f||^2}{2c} - \frac{||f^{-i}||^2}{2c}(1 - \cos^2\theta_{-i}). \tag{2.32}$$

Thus, expected profit decreases in $||f^{-i}||(1 - \cos^2\theta_{-i})$.

If $0 < \gamma^*_{-t} < 1$ for all t, expected profit is

$$\frac{||f||^2}{2c} - \frac{||f^{-i}||^2}{2c}(1 - \cos^2\theta_{-i}) + 2\phi\left(1 - \frac{c\phi}{||f^{-i}||^2(1 - \cos^2\theta_{-i})}\right). \tag{2.33}$$

In contrast to (2.32), (2.33) strictly increases in $||f^{-i}||(1-\cos^2\theta_{-i})$.[23] We therefore get the following result.

Proposition 2.5 *(i) If* $\gamma^*_{-t} = 0$ *for all* t*, agent 1 should perform the task* i *which satisfies*

$$i = argmin_{t\in N}||f^{-t}||(1-\cos^2\theta_{-t}). \tag{2.34}$$

(ii) If $0 < \gamma^*_{-t} < 1$ *for all* t*, agent 1 should perform the task* i *which satisfies*

$$i = argmax_{t\in N}||f^{-t}||(1-\cos^2\theta_{-t}). \tag{2.35}$$

Although performance is first-best in the one-task job, assigning the most important task to agent 1 is in general not optimal. An exception is the case in which agent 2 never receives an implicit bonus and $\cos\theta_{-t}$ is independent of t. Usually, the latter condition does not hold. Then it will not be optimal to assign the most important task to agent 1 if the corresponding f^{-t} and g^{-t} are too badly aligned. In the special case in which all tasks are equally important, i.e., $f_1 = f_2 = f_3$, tasks should be assigned so that f^{-i} and g^{-i} are best aligned.

If agent 2 always receives an implicit bonus and $\cos\theta_{-t}$ is independent of t, it is even optimal to assign the least important task to agent 1. The reason is that, the more important the tasks assigned to agent 2, the larger is the principal's loss if she reneges on the relational contract with this agent. Therefore, the maximal feasible implicit bonus for agent 2 increases in the importance of the tasks performed by this agent. Moreover, a higher implicit bonus increases expected profit more strongly than having first-best effort in the most important task.

In the other extreme case, if agent 2 always receives an implicit bonus but $f_1 = f_2 = f_3$, tasks should be assigned so that f^{-i} and g^{-i} are worst aligned since the optimal implicit bonus also increases in misalignment.

If the optimal implicit bonus for agent 2 is not always either zero or positive under each possible task assignment, the improvement in explicit contracts (if $\gamma^*_{-i} = 0$) must be traded off against the benefit from having a relational contract with agent 2 (if $\gamma^*_{-i} > 0$). Optimal task splitting then depends on the particular form of f and g.

2.6 Discussion

In this section, I discuss the generalization of the analysis to more tasks and agents as well as some of the model assumptions. I also give some directions for further research.

The analysis of sections 2.3 and 2.4 can, under some restrictions, be generalized to the case of splitting n tasks between l agents, where $l < n$. Clearly, all results in section 2.3 apply for any arbitrary number of tasks performed by a single agent.

However, the analysis in section 2.4 becomes more complicated if the number of tasks increases. If there are, for instance, four tasks and two agents, it may be optimal to pay both agents an implicit bonus. Under the assumption that both

[23] This is due to the fact that $||f^{-i}||(1-\cos^2\theta_{-i}) < 2c\phi$ because $\gamma^*_{-i} < 1$.

agents lose trust if the principal reneges on a relational contract with one of them, the principal's only commitment constraint is

$$\phi(\gamma_i + \gamma_{-i}) \leq f^T e - \sum_{l=i,-i} (\alpha_l + \beta_l g^T e + \gamma_l f^T e) - \bar{\pi}, \tag{2.36}$$

where $\bar{\pi}$ denotes the profit under pure explicit contracts.[24] Naturally, the optimal implicit bonuses cannot be determined independently of each other. This means, in particular, that the derivation of the optimal contract for one agent cannot be boiled down to the problem analyzed in section 2.3 by just dropping the tasks performed by the other agent. This feature greatly simplified the analysis in the case of three tasks. However, it can be reestablished by changing the modelling of trust.

Assume that if the principal reneges on one relational contract only the agent who was cheated loses trust. This assumption leads to additional commitment constraints for the principal and, therefore, limits the set of implementable implicit bonuses relative to (2.36). Then it can be shown that agents' optimal contracts are independent of each other and all results of section 2.4 can be extended to the case of splitting n tasks between l agents.[25] In particular, assigning all tasks to one agent will be optimal if *at least* one agent receives an implicit bonus under each arbitrary task splitting. If, on the other hand, it is not credible to promise an implicit bonus to an agent who performs all tasks, task splitting always increases profits.

While I was not able to derive clear-cut results for the general optimal task assignment under the initial modelling of trust, it is clear how results will change relative to the case just described. If, under task splitting, *all* agents lose trust when the principal reneges on only one relational contract, the temptation to renege is smaller. As explained above, this results in a larger set of implementable relational contracts. Thus, task splitting will more frequently be preferred to assigning all tasks to one agent.

Given that tasks must be split, the optimal grouping of tasks into jobs depends on the particular form of f and g and the number of agents. Therefore, the results of section 2.5 more generally apply only for the splitting of n tasks between $n-1$ agents. Furthermore, if there is only the possibility of withdrawing one task from a particular agent, the results explain which one the principal should choose.

I made the assumption that the principal cannot change the task assignment in future periods. This affects her fallback position after breaking implicit agreements and, therefore, may be critical for the results derived. First consider the case in which the principal initially employs two agents and agrees on a positive implicit bonus with agent 2. If the principal reneged on the implicit agreement, she would not want to dismiss one agent because task splitting is superior under pure explicit contracts.

However, if there is initially a single agent performing all tasks, the principal would benefit from splitting tasks after breaking the relational contract. Thus, in this case the assumption of inflexible job design matters. It worsens the fallback position of the principal and, therefore, leads to a higher feasible implicit bonus for

[24] Compare constraint (2.58) in the appendix. Of course, all vectors are now four-dimensional.

[25] Proofs are available from the author upon request.

the single agent. Hence, ex ante it is in the principal's interest to commit to not splitting tasks in the future. I assumed that such a commitment is possible because the costs of hiring another agent (e.g., learning costs) are higher than the benefits.[26] However, if such a commitment is not possible, assigning all tasks to a single agent will be less often preferred.

Furthermore, I assumed that agents' reservation utility is zero. Now assume that an agent's alternative wage per period is $\bar{w} > 0$, where $\tilde{\pi} - 2\bar{w} > 0$, i.e., the expected profit under pure explicit contracts is still positive. Then it is easily verified that the principal's expected profit is $\pi(\gamma^*) - \bar{w}$ if all tasks are performed by one agent, and $\pi^S(\gamma^*_{-i}) - 2\bar{w}$ under task splitting.[27] Thus, task splitting becomes less attractive than with alternative wages of zero. At the end of section 2.4, I argued that jobs should be more complex on higher hierarchy levels. This conclusion is strengthened by positive but not too high opportunity costs since alternative wages will, in general, increase in the hierarchy level.

In this essay, I exclusively focussed on optimal job design under congruency problems. Of course, as the literature survey in the introduction shows, there are many other factors that influence the optimal assignment of tasks within an organization. In particular, there might be complementarities or substitutabilities between tasks. Consider, for example, the cost function

$$C(e_1, e_2, e_3) = c \sum_{i=1}^{3} e_i^2 + c\delta e_1 e_2 e_3, \quad \delta \in R. \tag{2.37}$$

Then, if $\delta < 0$ ($\delta > 0$), all tasks are complements (substitutes). If δ approaches zero, we come close to the case of independent tasks and the results derived in this essay apply. Presumably, if $\delta < 0$ ($\delta > 0$), task splitting becomes less (more) preferable. The cost function could also be extended to the case where some tasks are complements and others substitutes. The analysis of these problems may be subject to future research.

2.7 Conclusion

In this essay, I derived two main results concerning the optimal interplay between job design and relational contracts.

First, if the principal cannot commit to paying an implicit bonus to an agent who performs all tasks, the principal is better of by splitting tasks because this improves the performance of explicit contracts. This case occurs when the objective performance measure is not strongly distorted, or when firm value does not strongly respond to effort changes in the given set of tasks. Then, the principal's loss from reneging on a relational contract is small, so that she is not able to commit to paying an implicit bonus.

---

[26] Explicitly, I assume that $K \geq \frac{f_i^2}{2c} + \frac{||f^{-i}||^2}{2c} \cos^2 \theta_{-i} - \frac{||f||^2}{2c} \cos^2 \theta$, where K denotes the costs of learning how to perform a task which have to be borne by the firm, e.g., trainee programs or opportunity costs from having senior staff to teach the new colleague.

[27] The analysis becomes more complicated if $\tilde{\pi} - 2\bar{w} < 0$ because then optimal explicit and implicit bonuses depend on $\bar{w}$. Examining this problem may be subject to future research.

Second, the principal prefers not to split tasks whenever it is possible to pay an implicit bonus under task splitting. The reason is that, if an agent performing a given set of tasks receives an implicit bonus, assigning an additional task to this agent allows the principal to commit to an even higher implicit bonus. This is due to the fact that the principal's loss from breaking a relational contract increases in the number of tasks that an agent performs. The strengthened relational contract always outweighs the loss from not having first-best effort in the additional task.

Overall, broad task assignments are optimal if objective performance measurement is difficult, or firm value is highly responsive to effort changes in the given tasks. This implies that task assignments tend to be more complex on higher hierarchy levels within a firm.

I assumed that there is only one exogenously given contractible performance measure. In many situations, the principal can invest in generating additional performance measures, thereby improving the performance of explicit contracts. However, doing so increases the costs of performance measurement. The analysis in this essay shows that job design can be a substitute to generating performance measures. When there is a second performance measure for three tasks, first-best incentives can be implemented for each tasks under task splitting.[28] However, instead of incurring costs for better objective performance measurement, the principal might prefer to assign all tasks to one agent if this leads to a high-powered relational contract.

2.8 Appendix

Derivation of $\beta(\gamma)$ as given in (2.16) and $\pi(\gamma)$ as given in (2.17). Define λ as the Lagrange multiplier of (2.15). The first-order conditions are

$$(1+\lambda)[f^T g - (\beta g + \gamma f)^T g] = 0 \quad (2.38)$$

$$(1+\lambda)[f^T f - (\beta g + \gamma f)^T f] - \lambda\phi = 0. \quad (2.39)$$

From the first equation, the optimal explicit bonus for a given γ is

$$\beta(\gamma) = (1-\gamma)\frac{f^T g}{g^T g} = (1-\gamma)\frac{||f||}{||g||}\cos\theta = (1-\gamma)\hat{\beta}. \quad (2.40)$$

The principal's profit becomes

$$f^T e(\beta,\gamma) - \frac{c}{2} e(\beta,\gamma)^T e(\beta,\gamma) \quad (2.41)$$

$$= \frac{1}{c}\left[f^T(\beta g + \gamma f) - \frac{1}{2}(\beta g + \gamma f)^T(\beta g + \gamma f)\right] \quad (2.42)$$

$$= \frac{1}{c}\left[\beta f^T g + \gamma f^T f - \frac{1}{2}(\beta^2 g^T g + \gamma^2 f^T f + 2\beta\gamma f^T g)\right] \quad (2.43)$$

[28]The two performance measures must be linearly independent. That is, if the performance measures are characterized by the vectors g and h, we must have $g \neq \lambda h$ for all $\lambda \in \mathbb{R}$. Then, the principal can always implement first-best incentives in the two-task job if she appropriately weights the two performance measures in a pure explicit contract.

$$
\begin{aligned}
&= \frac{1}{c}\left[(1-\gamma)\frac{(f^Tg)^2}{g^Tg} + \gamma f^T f \right.\\
&\qquad \left. -\frac{1}{2}\left((1-\gamma)^2\frac{(f^Tg)^2}{g^Tg} + \gamma^2 f^T f + 2(1-\gamma)\gamma\frac{(f^Tg)^2}{g^Tg}\right)\right] \qquad (2.44)\\
&= \frac{1}{2c}\frac{(f^Tg)^2}{g^Tg} - \frac{\gamma}{c}\frac{(f^Tg)^2}{g^Tg} + \frac{\gamma}{c}f^Tf\\
&\qquad -\frac{\gamma}{2c}\left(\gamma f^Tf + (-2+\gamma+2(1-\gamma))\frac{(f^Tg)^2}{g^Tg}\right) \qquad (2.45)\\
&= \frac{1}{2c}\frac{(f^Tg)^2}{g^Tg} + \frac{\gamma}{c}f^Tf\left(1-\frac{\gamma}{2}\right) - \frac{\gamma}{2c}(2-\gamma)\frac{(f^Tg)^2}{g^Tg} \qquad (2.46)\\
&= \frac{||f||^2}{2c}\cos^2\theta + \frac{\gamma(2-\gamma)}{2c}||f||^2(1-\cos^2\theta). \qquad (2.47)
\end{aligned}
$$

Proof of proposition 2.3. First consider the principal's fallback position when she reneges on one or both relational contracts. In this case, she will offer in each following period the explicit contracts that solve the problem

$$\max_{\substack{\alpha_l,\beta_l,e_l\\ l=i,-i}} f^Te - \sum_{l=i,-i}(\alpha_l + \beta_l g^T e), \qquad (2.48)$$

$$
\begin{aligned}
\text{s.t.} \qquad e_i &= \frac{1}{c}\beta_i g_i, \quad e^{-i} = \frac{1}{c}\beta_{-i}g^{-i}, \qquad (2.49)\\
0 &\leq \alpha_i + \beta_i g^T e - \frac{c}{2}e_i^2, \qquad (2.50)\\
0 &\leq \alpha_{-i} + \beta_{-i} g^T e - \frac{c}{2}(e^{-i})^Te^{-i}. \qquad (2.51)
\end{aligned}
$$

The solution to this problem is straightforward and leads to optimal explicit bonuses of $\beta_i = f_i/g_i$, $\beta_{-i} = (f^{-i})^Tg^{-i}/((g^{-i})^Tg^{-i})$ and an expected profit of

$$\bar{\pi} := \frac{f_i^2}{2c} + \frac{||f_{-i}||^2}{2c}\cos^2\theta_{-i}. \qquad (2.52)$$

Thus, the optimal combination of explicit and relational contracts is determined by solving

$$\max_{\substack{\alpha_l,\beta_l,\gamma_l,e_l\\ l=i,-i}} f^Te - \sum_{l=i,-i}(\alpha_l + \beta_l g^T e + \gamma_l f^T e), \qquad (2.53)$$

$$
\begin{aligned}
\text{s.t.} \qquad e_i &= \frac{1}{c}(\gamma_i f_i + \beta_i g_i), \qquad (2.54)\\
e^{-i} &= \frac{1}{c}(\gamma_{-i} f^{-i} + \beta_{-i} g^{-i}), \qquad (2.55)\\
0 &\leq \alpha_i + \beta_i g^T e + \gamma_i f^T e - \frac{c}{2}e_i^2, \qquad (2.56)\\
0 &\leq \alpha_{-i} + \beta_{-i} g^T e + \gamma_{-i} f^T e - \frac{c}{2}(e^{-i})^Te^{-i}, \qquad (2.57)\\
\phi(\gamma_i + \gamma_{-i}) &\leq f^Te - \sum_{l=i,-i}(\alpha_l + \beta_l g^T e + \gamma_l f^T e) - \bar{\pi}. \qquad (2.58)
\end{aligned}
$$

Note that (2.58) also implies that the principal will not break the relational contract with only one of the two agents. It is easy to verify that the agents' participation constraints (2.56) and (2.57) bind at the optimal solution. Thus, by substituting e_i and e^{-i} and defining

$$\begin{aligned}\pi(\beta_i,\gamma_i,\beta_{-i},\gamma_{-i}) \;:=\; & \frac{1}{c}\left[f_i(\beta_i g_i+\gamma_i f_i)+(f^{-i})^T(\beta_{-i}g^{-i}+\gamma_{-i}f^{-i})\right]- \\ & \frac{1}{2c}\left[(\beta_i g_i+\gamma_i f_i)^2+(\beta_{-i}g^{-i}+\gamma_{-i}f^{-i})^T(\beta_{-i}g^{-i}+\gamma_{-i}f^{-i})\right],\end{aligned} \tag{2.59}$$

the problem can be simplified to

$$\max_{\substack{\beta_i,\gamma_i\\ \beta_{-i},\gamma_{-i}}} \pi(\beta_i,\gamma_i,\beta_{-i},\gamma_{-i}) \tag{2.60}$$

$$\text{s.t.}\quad \phi(\gamma_i+\gamma_{-i})\le \pi(\beta_i,\gamma_i,\beta_{-i},\gamma_{-i})-\frac{f_i^2}{2c}-\frac{||f^{-i}||^2}{2c}\cos^2\theta_{-i}. \tag{2.61}$$

From the first-order condition for β_i,

$$\beta_i(\gamma_i)=(1-\gamma_i)\frac{f_i}{g_i}\quad\text{for}\quad 0\le\gamma_i\le 1. \tag{2.62}$$

It follows from (2.54) that $e_i=e_i^{FB}$. After substituting β_i, the principal's problem becomes

$$\max_{\substack{\gamma_i\\ \beta_{-i},\gamma_{-i}}} \frac{f_i^2}{2c}+\pi(0,0,\beta_{-i},\gamma_{-i}) \tag{2.63}$$

$$\text{s.t.}\quad \phi(\gamma_i+\gamma_{-i})\le \pi(0,0,\beta_{-i},\gamma_{-i})-\frac{||f^{-i}||^2}{2c}\cos^2\theta_{-i}. \tag{2.64}$$

Thus, the principal cannot do better than setting $\gamma_i=0$. The remaining optimization problem corresponds to the one considered in section 2.3 so that $\pi^S(\gamma_{-i}^*)$ follows from (2.20).

It remains to show that $\gamma_{-i}^*\le\gamma^*$. By (2.19), this will be the case if

$$||f^{-i}||^2(1-\cos^2\theta_{-i})\le ||f||^2(1-\cos^2\theta). \tag{2.65}$$

For parsimony, define $x:=\cos^2\theta$ and $x_{-i}:=\cos^2\theta_{-i}$. Then, (2.65) is equivalent to

$$\begin{aligned} ||f||^2x \;&\le\; f_i^2+||f^{-i}||^2x_{-i} \\ \Leftrightarrow\; \frac{(f_ig_i+f_jg_j+f_kg_k)^2}{g_i^2+g_j^2+g_k^2} \;&\le\; f_i^2+\frac{(f_jg_j+f_kg_k)^2}{g_j^2+g_k^2}.\end{aligned} \tag{2.66}$$

By defining $A:=f_jg_j+f_kg_k$ and $B:=g_j^2+g_k^2$ we receive

$$\frac{(f_ig_i+A)^2}{g_i^2+B}\;\le\; f_i^2+\frac{A^2}{B}. \tag{2.67}$$

This inequality can be transformed to

$$0\;\le\; g_i^2-2f_ig_i\frac{B}{A}+f_i^2\frac{B^2}{A^2}=\left(g_i-f_i\frac{B}{A}\right)^2. \tag{2.68}$$

Thus, $\gamma^*_{-i} \leq \gamma^* < 1$, i.e., $(e^{FB})^{-i}$ is implemented if and only if $\cos\theta_{-i} = 1$ (by setting $\gamma^*_{-i} = 0$ and $\beta^*_{-i} = \frac{||f^{-i}||}{||g^{-i}||}$).[29] □

Proof of proposition 2.4. (i) By proposition 2.3, from $\gamma^* = 0$ it follows that $\gamma^*_{-i} = 0$. Thus, task splitting leads to a weakly higher expected profit iff

$$\frac{||f||^2}{2c}x \leq \frac{f_i^2}{2c} + \frac{||f^{-i}||^2}{2c}x_{-i} \tag{2.69}$$

which holds by (2.66).□

(ii) By proposition 2.3, $\gamma^* \geq \gamma^*_{-i}$. By assumption, $\gamma^* < 1$. Thus, task splitting leads to a lower expected profit than no task splitting iff

$$\frac{||f||^2 - ||f^{-i}||^2}{2c} + \frac{||f^{-i}||^2}{2c}x_{-i} + 2\phi\left(1 - \frac{c\phi}{||f^{-i}||^2(1-x_{-i})}\right) \leq \frac{||f||^2}{2c}x + 2\phi\left(1 - \frac{c\phi}{||f||^2(1-x)}\right). \tag{2.70}$$

Because $\gamma^*_{-i} < 1$, we have $||f^{-i}||^2(1 - x_{-i}) < 2c\phi$ and, therefore, the left-hand side of (2.70) strictly increases in $||f^{-i}||^2(1 - x_{-i})$. Furthermore, (2.70) binds iff $||f^{-i}||^2(1 - x_{-i}) = ||f||^2(1 - x)$. Thus, by (2.65), (2.70) holds.□

[29] From $\gamma^*_{-i} < 1$ it follows that $\gamma^*_i = 0$ is the uniquely optimal implicit bonus for agent 1.

Essay 3

Procurement Mechanisms under Non-Verifiable Quality

3.1 Introduction

The procurement of goods or services is frequently accomplished by holding contests among potential suppliers. In this essay, I analyze two such contest mechanisms, a tournament and an auction, in a setting in which the procurer and the competing firms observe a non-verifiable signal about the quality of the good that each firm is able to supply. In contrast to the literature, I find that the procurer prefers the tournament to the auction under certain circumstances.

In most procurement settings, the procurer cares about the quality of the good or service that she wishes to buy. This quality is usually affected by suppliers' investments in R&D. In many cases, these investments are non-observable and the resulting quality is non-verifiable by outsiders. Moreover, even the procurer may find it difficult to measure quality, which is often multi-dimensional and may be perfectly observable only after the good has been consumed.

To fix ideas, consider the procurement of a new high-tech fighter plane by the Ministry of Defence. In this case, there are only a few potential suppliers, which are required to make investments in R&D to build a prototype. To determine the quality of the plane that each firm is able to supply, the procurer performs tests on the prototypes. However, these tests do not result in a complete assessment of all properties of the plane because receiving and processing all relevant pieces of information would be too costly or even impossible. Thus, the procurer's decision on to whom to award the production contract is based on a *signal* about quality. Additionally, the procurer decides on the precision of this signal. For example, she can increase precision by performing more tests or hiring more specialists. Of course, investment decisions are affected by firms' expectations about the precision of quality measurement.

A similar situation occurs in architectural contests, in which contestants develop proposals for the design of new buildings or the reconstruction of historical sights. A jury assesses these proposals and awards the winner prize to the contestant whose proposal seems to best fit the purpose of the building.

An important advantage of contests is that they provide incentives for investments in R&D even if these investments and the resulting qualities are non-verifiable to third parties, so that incentive contracts based on these variables are not enforceable.[1] In a tournament, the procurer fixes a prize scheme ex ante. In an auction mechanism, each firm bids a price at which it is willing to supply the good, and the procurer awards the production contract to the firm that offers the most favorable combination of quality and price. Both mechanisms prevent opportunistic behavior ex post. The reason is that the procurer cannot lower the payments to firms by understating quality, and firms do not benefit from overstating their costs.

I consider a two-firm setting in which firms have the same non-deterministic production technology for quality, i.e., quality is determined by firms' non-obser-vable investments in R&D and some random factors. After investments have been made, the procurer assesses the quality that each firm is able to supply. This assessment process results in a quality signal for each firm. Quality signals are non-verifiable but observable by the procurer and both firms. This is, for example, the case if employees of both firms are present when prototypes are tested and are able to evaluate the information revealed in the assessment process because they have acquired the necessary knowledge at the R&D stage. This may be impossible for an outsider who does not have the required knowledge about technology or the procurer's preferences.

Once signals have been observed, parties play according to the pre-specified mechanism and the contract is awarded. An important difference between the two mechanisms is that, in the tournament, the procurer fixes the prize structure *before* investments are made, while in the auction mechanism the price is determined *after* investments have been made and signals have been observed. In the tournament, the firm with the higher quality signal receives the pre-specified winner prize. In the auction, each firm bids a price which the procurer has to pay if she awards the production contract to this firm. The procurer chooses the firm that offers the most favorable combination of price and quality instead of using a pure quality criterion. As a consequence, a firm's investment does not only affect its quality signal but also its bidding strategy in the auction.

My setting is similar to the one studied by Che and Gale [2003]. However, these authors assume that the production technology for quality is deterministic and that the procurer perfectly observes the quality of a firm's innovation, while all other firms observe nothing.[2] Furthermore, they do not restrict attention to tournaments and auctions, but study all contest mechanisms in which only the winner receives a prize. They find that a first-price auction with two firms is optimal if firms are homogeneous. With heterogeneous firms, an auction with the two most efficient firms is optimal, where the more efficient firm should be handicapped through a maximum allowable prize. The reason for the superiority of the auction is that a

[1] A prominent real-world example of a procurement contest is the 1829 contest where Liverpool and Manchester Railway announced a prize of £500 for the best performing engine for the first passenger line between two British cities (see Fullerton and McAfee [1999]). Other examples include the development of fuel-efficient refrigerators (see Langreth [1994]), standards for digital televisions (see Schwarz [1991]), or high-speed train systems. Contests are also common in internal labor markets where only those candidates are promoted who exert the highest effort on the job or in acquiring firm-specific human capital. For further examples, see, e.g., Che and Gale [2003].

[2] In contrast to my setting, they also allow for more general investment cost functions and heterogeneous firms.

firm which can offer only a low-quality good can still compete effectively against a firm with a high-quality good by bidding a lower price. This would not be the case if the set of allowable prizes is limited, like in a tournament with one ex ante fixed prize.

By contrast, in my setup the procurer may prefer the tournament to the auction mechanism. In the auction, the price that the procurer has to pay is the difference between firms' quality signals weighted by the procurer's marginal valuation of quality. Therefore, a higher investment increases not only the probability of winning the production contract but also the expected price that a firm receives. Consequently, investment incentives are always higher in the auction than in the tournament, where increasing investments does not affect the winner prize. As a result, the procurer cannot implement low investment levels when using the auction mechanism. This turns out to be a drawback if the expected price in the auction is so high that firms earn rents.

The expected price in the auction increases in the variance of the quality signals, which is the higher the more random the production technology for quality and the more imprecise the quality measurement. Thus, the procurer prefers a tournament if quality is highly random or increasing the precision of quality measurement is very costly. In both cases, the procurer must set a high winner prize in the tournament to implement high investment levels. However, she can also implement low investments by decreasing the winner prize. This may lead to a higher expected profit than having high investments but also a high price as in the auction.

Fullerton et al. (2002) also exclusively study the use of auctions and fixed prizes to procure an innovation. They consider the same information structure like Che and Gale [2003] but adopt the research tournament model from Taylor [1995]. In that model, the production technology for generating innovations is a sequential search process with recall, i.e., the technology is stochastic as in my framework. However, I assume that firms invest only once. As Che and Gale [2003], Fullerton et al. (2002) also find that the auction mechanism will generally reduce the procurer's expenditure.

Besides the aforementioned papers, the literature on the design of research contests is still limited. Two other important contributions are Fullerton et al. (1999) and Fullerton and McAfee [1999]. In the former work, the predictions of the search model of tournaments are tested in laboratory experiments. The latter paper is concerned about the optimal number of participants in research tournaments and how the contestants should be selected. It generalizes results from Taylor [1995].

Apart from contest mechanisms there is another well-known possibility for providing adequate incentives to induce research efforts when investments are non-verifiable. Specifically, Edlin and Reichelstein (1996a,b) show that under certain conditions a simple trade contract specifying a fixed price and quantity in combination with a renegotiation of the contract can induce efficient investments. However, Che and Hausch [1999] show that contracting has no value if investments are sufficiently cooperative[3] and parties cannot commit not to renegotiate. This is precisely

[3] An investment is said to be cooperative if not only the investor benefits from his investment (for example, in terms of reducing production cost), but also the non-investing party, i.e., the procurer.

the case in my procurement setting so that the results from Edlin and Reichelstein (1996a,b) are not applicable.

In section 3.2, the model is introduced. Section 3.3 and section 3.4 analyze the tournament and the auction, respectively. I compare both mechanisms in section 3.5. In section 3.6, I endogenize the precision of performance measurement. Section 3.7 concludes.

3.2 The model

A procurer wants to buy one unit of a good. The procurer's valuation of the good increases in its quality. There are two ex ante identical firms that are able to produce the good. All parties are risk-neutral. Due to legal restrictions or firms' limited capacity to pay, payments to firms must be non-negative. This also implies that the procurer is not able to extract surplus by charging an ex ante fee. In what follows, I will refer to this assumption as a firm's limited liability constraint.

First, the procurer specifies the mechanism that is going to be used, i.e., either a tournament or an auction. Afterwards, each firm builds a prototype (or develops a design proposal etc). The quality of a prototype randomly depends on the firm's specific investment in R&D. In the next step, the procurer assesses the quality of the prototypes. Given this quality assessment, the production contract is awarded to one firm[4] according to the pre-specified mechanism.

This sequence of events is modelled as follows. In the investment stage, firm i, $i = 1, 2$, chooses a non-observable investment strategy $x_i \geq 0$ incurring non-observable investment costs $c(x_i) + \bar{c}$. I assume that $c(0) = 0$, $\bar{c} \geq 0$, $c'(x_i) > 0$ for $x_i > 0$, $c'(0) = 0$, and $\inf_{x_i \geq 0} c''(x_i) = D$, $D > 0$. A positive D is required to guarantee the existence of pure strategy equilibria in the investment stage.[5]

If firm i chooses $x_i = 0$, it does not make a specific investment in R&D but builds a prototype using its already available knowledge.[6] In this case, the firm spends $\bar{c}$, which includes the firm's opportunity costs. That is, investments in R&D are not essential to produce the good. The quality of firm i's prototype is $q_i = x_i + \mu_i$, where μ_i is the value of a random variable which is realized after the investment strategy has been chosen. Thus, expected quality linearly increases in the investment strategy x_i. The assumption of strictly convex investment costs implies that the production technology for quality exhibits decreasing returns to scale. A firm's investments strategy, investment costs, and quality are non-observable.

Since the focus of this essay is the implementation of investments in R&D, I do not consider any problems related to post-contractual opportunism. In particular, I assume that q_i is also the quality of the good that firm i will supply if it is chosen to produce the good.[7] Furthermore, production costs are independent of investments,

[4] I assume that the good cannot be produced in a cooperation of both firms.

[5] This assumptions excludes some cost functions, e.g., $c(x) = x^\alpha$, where $\alpha > 2$. However, there is of course a wide range of functions that satisfy this assumption, e.g., $c(x) = \gamma(x) + x^\alpha$ or $c(x) = \gamma(x) + e^x$, where $\gamma(x)$ is an arbitrary convex function and $1 < \alpha \leq 2$.

[6] Alternatively, $x_i = 0$ might represent a minimum required investment in R&D leading to costs of $\bar{c}$.

[7] E.g., it might be verifiable that the good is identical to the prototype, or that the building is

and any incentive problem that might arise in the production stage is independent of the events in the previous stages, which are examined in this paper. For simplicity, I normalize production costs to zero.

The procurer's assessment of firm i's prototype leads to a quality signal $s_i = q_i + \nu_i$. The term ν_i is the realization of a random variable. The procurer and both firms observe the quality signals, i.e., each firm observes not only its own signal but also the one of the other firm. However, these signals are non-verifiable to third parties.

The random variables μ_1 and μ_2 as well as ν_1 and ν_2 are identically distributed with variance σ_μ^2 and σ_ν^2, respectively, where $0 < \sigma_\mu^2 < \infty$ and $0 \leq \sigma_\nu^2 \leq \bar{\sigma}_\nu^2 < \infty$. All random variables are independent. The expected value of μ_i is non-negative. For parsimony, I define a new random variable $\epsilon_i := \mu_i + \nu_i$ so that $s_i = x_i + \epsilon_i$, and denote the corresponding cdf by $F(\epsilon_i)$. $F(.)$ is known to the procurer and the firms and is assumed to be once differentiable with $f(.) := F'(.)$.

Thus, a firm's quality signal is the sum of true quality and some noise occurring when quality is assessed. The lower the variance of ϵ_i, $\text{var}(\epsilon_i) = \sigma_\mu^2 + \sigma_\nu^2$, the more strongly quality signals respond to changes in investments. While σ_μ^2 is exogenously given by the production technology, σ_ν^2 reflects the precision of quality measurement.

The procurer can always observe a signal of precision $\bar{\sigma}_\nu^2$. For simplicity, I assume that this signal is costless. However, by running more tests on the prototypes or hiring more experts, thereby incurring higher measurement costs, the procurer can increase signal precision. In the first stage, when the procurer specifies the mechanism to be used, she also commits to a particular precision, e.g., to certain tests to be run on the prototypes.

The procurer's valuation of a good of quality q is vq, where $v > 0$. The procurer observes his valuation only after the good has been consumed. If the good is not produced, the procurer's utility is zero.

In equilibrium, firms will choose the same investment strategy x. Then, given that the firm with the higher quality signal produces the good, the procurer's expected valuation *before* quality signals are observed is

$$v\text{E}[q_i|s_i \geq s_j, x] = v(x + \text{E}[\mu_i|\epsilon_i \geq \epsilon_j]). \tag{3.1}$$

It holds that

$$\text{E}[\mu_i] \;<\; \text{E}[\mu_i|\epsilon_i \geq \epsilon_j] \;\leq\; \text{E}[\mu_{(2)}], \tag{3.2}$$

where $\text{E}[\mu_{(2)}]$ denotes the expected value of the second-order statistic of the sample μ_1, μ_2.

If $\sigma_\nu^2 = 0$, quality is perfectly observable. Then, given investments x, expected quality attains its maximum value, i.e., the second inequality in (3.2) is binding. The reason is that the firm that can supply the higher quality always produces the good. By contrast, if $\sigma_\nu^2 > 0$, there is the risk of awarding the production contract to the low-quality firm. However, since $\sigma_\nu^2 < \infty$, the signals improve the procurer's information on the quality that each firm can supply, i.e., $E[\mu_i|\epsilon_i \geq \epsilon_j] > E[\mu_i]$.

The term $E[\mu_i|\epsilon_i \geq \epsilon_j]$ increases in σ_μ^2 and decreases in σ_ν^2. This is because a more random production technology for quality increases the expected quality that

identical to the design proposal.

the high-quality firm can offer, but less precise quality measurement increases the risk that the low-quality firm has the higher quality signal.

Given σ_μ^2 and $\bar{\sigma}_\nu^2$, I assume that the procurer's expected valuation of the good if both firms do not invest is at least as high as the firms' costs in this case, i.e.,

$$v\mathrm{E}[\mu_i|\epsilon_i \geq \epsilon_j] \geq 2\bar{c}. \tag{3.3}$$

As will become clear later, given the two procurement mechanisms, this assumption ensures that ex ante the procurer always wants to procure the good.

Furthermore, I denote by x^* the socially optimal investment level given that two firms invest and choose the same investment strategy, i.e.,

$$x^* := \mathrm{argmax}_x v(x + E[\mu_i|\epsilon_i \geq \epsilon_j]) - 2(c(x) + \bar{c}). \tag{3.4}$$

Since $c'(0) = 0$, x^* is positive and $c'(x^*) = v/2$.[8] Because of assumption (3.3), the corresponding social surplus is positive.

In real world, the procurer cannot spend an arbitrarily large amount of money on the procurement process. For example, if the procurer is the government or a firm, there will in general be a budget that cannot be exceeded. I assume that the procurer can spend at most $B > 0$. Technically, this budget constraint combined with sufficiently high exogenous noise σ_μ^2 will ensure the existence of pure strategy equilibria in both mechanisms in the investment stage. To do so it is not necessary that the budget constraint is restrictive in the sense that it is binding at the optimal solution. For example, consider the investment $\tilde{x}$ such that

$$v(\tilde{x} + E[\mu_{(2)}]) = 2(c(\tilde{x}) + \bar{c}). \tag{3.5}$$

Then, the procurer would never want to implement an $x > \tilde{x}$. The reason is that for any such x expected procurement costs, which are at least $2(c(x) + \bar{c})$, exceed the expected valuation of the good. Thus, if we define $B := v(\tilde{x} + E[\mu_{(2)}])$, the procurer can implement all x that may lead to a non-negative expected profit.

Until section 3.5, I do not explicitly model the procurer's decision on the precision of quality measurement. Instead, I take precision as given and abstract from measurement costs, i.e., the budget B can be spend entirely on payments to firms.

3.3 Tournament

In this section, I determine the minimum payments to firms which are necessary to implement a given investment x under a given precision of quality measurement in a tournament.

In a tournament, the procurer offers both firms a contract specifying the winner prize $w \geq 0$ and a guaranteed fixed payment $l_t \geq 0$ to ensure firms' participation. Furthermore, the contract says that, after observing quality signals s_i and s_j, firm i receives $l_t + w$ if $s_i > s_j$, and l_t if $s_i < s_j$. Ties are broken by flipping a fair coin. The firm that receives the winner prize produces the good.

[8]Note that x^* does not depend on the precision of quality measurement.

Although quality signals are non-verifiable, the contract is incentive compatible for the procurer. This is because w and l_t are contractible and independent of the signals. Moreover, the procurer strictly prefers to pay the winner prize to the firm with the higher quality signal because the good that this firm will produce is of higher expected quality.[9]

In the stage where firms choose their investment strategies, I look for a Nash equilibrium in pure strategies. As will become clear later, ex ante identical firms choose the same investment in equilibrium. Since the procurer wishes to implement a given investment x bearing minimal costs, his optimization problem can be defined as:

$$
\begin{aligned}
C_t(x) &:= \min_{w,l_t} \; w + 2l_t && (3.6)\\
\text{s.t.} \quad x &= \text{argmax}_{\hat{x}\geq 0}\; l_t + \text{prob}[s_i > s_j | x_i = \hat{x}, x_j = x]w - c(\hat{x}) && (3.7)\\
0 &\leq l_t + \frac{1}{2}w - c(x) - \bar{c} && (3.8)\\
B &\geq w + 2l_t && (3.9)\\
0 &\leq l_t,\; l_t + w && (3.10)
\end{aligned}
$$

The expression $\text{prob}[s_i > s_j | x_i = \hat{x}, x_j = x]$ denotes the probability that firm i wins the tournament conditional on $x_i = \hat{x}$ and $x_j = x$, $i, j = 1, 2$, $i \neq j$. Condition (3.7) is the incentive compatibility constraint which ensures that firm i finds it optimal to invest x given that firm j also chooses x. Condition (3.8) is the participation constraint. It says that the expected payment that a firm receives less its investment costs must be non-negative (recall that $\bar{c}$ already includes a firm's opportunity costs). The expected payment takes the form $l_t + \frac{1}{2}w$ because a firm obtains l_t for sure and the probability of winning w is $\frac{1}{2}$ in any symmetric equilibrium. Condition (3.9) is the procurer's budget constraint and (3.10) is due to firms' limited liability.

In order to solve the procurer's optimization problem, I first simplify the incentive compatibility constraint.[10] The winning probability of firm i can be rewritten as

$$
\begin{aligned}
\text{prob}[s_i > s_j | x_i, x_j] &= \text{prob}[x_i + \epsilon_i > x_j + \epsilon_j]\\
&= \text{prob}[x_i - x_j > \epsilon_j - \epsilon_i]\\
&=: G(x_i - x_j),
\end{aligned} \qquad (3.11)
$$

where $G(.)$ denotes the cdf of the random variable $\eta := \epsilon_j - \epsilon_i$. The corresponding density function is $g(.)$. Because ϵ_i and ϵ_j are iid, $g(.)$ is symmetric around zero. Thus, $\text{prob}[s_j > s_i | x_i, x_j] = 1 - G(x_i - x_j) = G(x_j - x_i)$. Therefore, firms' optimization problems are symmetric. Unless otherwise stated, I assume that $g(.)$ is differentiable and that $g(\eta) > 0$ for all $\eta \in \mathbb{R}$.[11]

Taking x_j as given, firm i chooses its investment x_i such that

$$x_i = \text{argmax}_{\hat{x}\geq 0}\; l_t + G(\hat{x} - x_j)w - c(\hat{x}) - \bar{c}. \qquad (3.12)$$

[9] I assume that firms cannot bribe the procurer.

[10] The analysis corresponds to the one in Lazear and Rosen [1981].

[11] These assumptions simplify the analysis, but are not critical for the results. In particular, in section (3.5), I also consider exponentially and uniformly distributed random variables ϵ_i, ϵ_j.

Assuming an interior solution, equation (3.12) implies

$$g(x_i - x_j)w - c'(x_i) = 0. \tag{3.13}$$

The corresponding first-order condition for firm j is symmetrical with (3.13). Since $g(x_i - x_j) = g(x_j - x_i)$, a Nash equilibrium must be symmetric, i.e., $x_i = x_j =: x_t$. Thus, condition (3.13) simplifies to

$$g(0)w - c'(x_t) = 0. \tag{3.14}$$

A higher investment increases the probability of winning the tournament, and therefore a firm's expected payment. Condition (3.14) says that the increase in expected payments must equal the increase in investment costs when raising quality starting from x_t. The investment x_t increases not only in the winner prize w but also in $g(0)$, which reflects how sensitive the winning probability is to changes in the investment level.

Intuitively, the winning probability should react more strongly to changes in investments if the randomness of the production technology or the noise in quality assessment decreases. Indeed, for several distributions there is a one-to-one relation between $g(0)$ and σ_μ^2, σ_ν^2. For example, if μ_i and ν_i are normally distributed, $\eta = (\mu_j + \nu_j) - (\mu_i + \nu_i)$ is also normally distributed with variance $2(\sigma_\mu^2 + \sigma_\nu^2)$, and

$$g(0) = \frac{1}{2\sqrt{\pi}\sqrt{\sigma_\mu^2 + \sigma_\nu^2}}. \tag{3.15}$$

Thus, x_t decreases in σ_μ^2 and σ_ν^2. This means that the procurer can increase incentives not only by increasing w but also by increasing the precision of quality assessment, i.e., by lowering σ_ν^2.

However, firm i's objective function (3.12) is not necessarily concave, so that x_t as given by (3.14) is not guaranteed to be a Nash equilibrium without any further assumptions. Lazear and Rosen [1981], Nalebuff and Stiglitz [1983], and McLaughlin [1988], among others, have already discussed this problem. Since $w \leq B$, a sufficient condition for the strict concavity of (3.12) is that

$$\sup_\eta g'(\eta)B < \inf_x c''(x) \equiv D. \tag{3.16}$$

This inequality holds if $g'(\eta)$ does not become too large. For many distributions, this is the case if $\text{var}(\eta) = 2(\sigma_\mu^2 + \sigma_\nu^2)$ is sufficiently high. For example, if μ_i and ν_i are normally distributed and $\sigma_\nu^2 = 0$, then there is a $\sigma_\mu^2(B, D)$ such that (3.16) is satisfied for all $\sigma_\mu^2 > \sigma_\mu^2(B, D)$. This claim is proved in the appendix. For the remainder of this section, I assume that (3.16) holds, i.e., I restrict attention to the class of problems for which the exogenous parameters σ_μ^2, B, and D are such that a firm's objective function is concave.

Concavity of (3.12) combined with the fact that (3.12) is strictly increasing at $\hat{x} = 0$ for $w > 0$, implies that both firms choose a positive investment x_t in a pure strategy Nash-equilibrium for each prize w, $0 < w \leq B$.[12] In particular, given that

[12] Note that condition (3.16) ensures that firm i's objective function (3.12) is concave *for all* x_j.

one firm chooses investment $x_t > 0$, it is never optimal for the other firm to choose $x = 0$, i.e.,

$$\frac{w(x_t)}{2} - c(x_t) > G(-x_t)w(x_t), \tag{3.17}$$

where $w(x_t)$ denotes the prize that satisfies equation (3.14) for $x_t > 0$. In other words, (3.16) ensures that σ_μ^2 is so high that $w(x_t)$ is large enough to lead to sufficiently high expected payments if both firms invest x_t.

Since I ensured that firms' equilibrium investment strategies are given by (3.14), I can replace the incentive compatibility constraint (3.7) in the procurer's problem with (3.14), so that I obtain

$$\begin{array}{lrcl} C_t(x) := \min_{w,l_t} & w + 2l_t & & \\ \text{s.t.} & g(0)w - c'(x) & = & 0 \\ & l_t + 1/2w - c(x) - \bar{c} & \geq & 0 \\ & w + 2l_t & \leq & B \\ & w + l_t, l_t & \geq & 0. \end{array} \tag{3.18}$$

By eliminating w using (3.14), the problem reduces to:

$$\begin{array}{lrcl} & \min_{l_t} \frac{c'(x)}{g(0)} + 2l_t & & \\ \text{s.t.} & l_t + \frac{c'(x)}{2g(0)} - c(x) - \bar{c} & \geq & 0 \\ & \frac{B}{2} - \frac{c'(x)}{2g(0)} \geq l_t & \geq & 0 \end{array} \tag{3.19}$$

The procurer cannot do better than choosing the lowest l_t that satisfies both the participation and the limited liability constraint, i.e.,

$$l_t = \begin{cases} \bar{c} + c(x) - \frac{c'(x)}{2g(0)} & \text{if } \bar{c} + c(x) - \frac{c'(x)}{2g(0)} \geq 0 \\ 0 & \text{otherwise} \end{cases}. \tag{3.20}$$

In the first case, a firm's participation constraint binds. In the second case, the limited liability constraint binds. The procurer's budget constraint only determines whether the available resources are sufficient to implement x, i.e., the particular value of B has no impact on the optimal choice of w and l_t. By eliminating l_t from the objective function using (3.20), we obtain the following proposition.

Proposition 3.1 *The procurer's costs of implementing an investment x under the tournament scheme are*

$$C_t(x) = \max\left\{2(c(x) + \bar{c}), \frac{c'(x)}{g(0)}\right\}, \tag{3.21}$$

provided that $C_t(x) \leq B$.

Naturally, an investment x is implementable only if $C_t(x) \leq B$. If $2(c(x) + \bar{c}) \geq c'(x)/g(0)$, firms' participation constraints are binding. Then, payments to firms equal the sum of firms' costs, and thus firms receive no rents. From (3.17) we obtain

$$\left[\frac{1}{2} - G(-x)\right]\frac{c'(x)}{g(0)} > c(x) \quad \text{for all} \quad x > 0. \tag{3.22}$$

Since $\frac{1}{2} > G(-x) \geq 0$, we have $c'(x)/g(0) > 2c(x)$ for positive investments. Nevertheless, if $\bar{c} > 0$, the participation constraint is binding in some interval $[0, \bar{x}]$, where $\bar{x}$ is sufficiently small but positive. This is due to the assumption that $c'(0) = 0$. The participation constraint might also be binding for larger investments because I do not assume that $c'(x)$ is convex.

If $2(c(x) + \bar{c}) < c'(x)/g(0)$, firms' liability constraints are binding. The required prize spread for implementing x is larger than the sum of firms' investment costs and firms earn rents.

3.4 Auction

I now repeat the above analysis for the auction mechanism. In the auction, the procurer offers to both firms the following contract. There is a minimum price $r \geq 0$ that will be paid to the firm which produces the good. Furthermore, each firm receives a fixed payment l_a to ensure participation. After observing quality signals s_i and s_j, firms submit bids p_i and p_j, where p_i, $p_j \geq r$. Firm i produces the good if it offers the higher expected surplus to the procurer, i.e., if $vE[q_i|s_i, s_j, x_a] - p_i > vE[q_j|s_j, s_j, x_a] - p_j$, where x_a denotes the symmetric equilibrium in the investment stage of the auction.[13] In this case, firm i receives the payment $l_a + p_i$, and firm j receives l_a. If both firms offer the same expected surplus, the firm with the higher quality signal wins the auction. If signals are also identical, the winner is chosen by flipping a fair coin.

The variables r, p_i, and p_j are verifiable. However, in contrast to the tournament, the payment to the winning firm is not fixed ex ante, but will depend on the quality signals. Nevertheless, the contract is incentive compatible for the procurer. Given firms' verifiable bids, she wants the firm to produce the good that offers the higher expected surplus.[14]

I focus on a variation of the first-price auction since a second-price auction would not perform well in this framework. To see this, first suppose that the procurer states that she is going to award the contract to the firm bidding the lowest price, and this firm is paid the price that the losing firm bid. Then it is a weakly dominant strategy for every firm to bid r. If ties are broken by flipping a fair coin, every firm wins with probability $\frac{1}{2}$ regardless of its investment. Consequently, there are no incentives to make a positive investment. If the mechanism says that in the case of a tie the firm with the higher signal wins, the second-price auction is identical to a tournament with $w = r$. The variant of the second-price auction in which the firm that bids the highest expected surplus wins and is required to match the surplus of the losing firm (see Che [1993]) is not feasible since surpluses are non-verifiable.

[13] Since firms are symmetric ex ante, it is convincing to restrict attention to symmetric Nash equilibria in the investment stage. Of course, x_a is not observable. However, the procurer knows by her choice of the incentive structure which investments have been made.

[14] Note that the procurer cannot commit herself to a "reservation surplus", i.e., she cannot state credibly that she is only willing to buy if at least one firm offers an expected surplus exceeding a predetermined amount. This is because quality signals are non-verifiable, so that ex post the procurer would always understate firms' expected qualities to lower the price she has to pay. Anticipating this behavior, firms' would not participate in the mechanism.

To determine firms' optimal behavior in the bidding stage, suppose certain quality signals s_i and s_j have been observed. Then, the expected difference between the qualities that firms can offer is

$$\begin{aligned} & E[q_i - q_j | s_i, s_j, x_a] \\ = \; & E[\mu_i - \mu_j | \mu_i + \nu_i = s_i - x_a, \mu_j + \nu_j = s_j - x_a] \\ = \; & \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [(s_i - x_a - \nu_i) - (s_j - x_a - \nu_j)] h(\nu_i) h(\nu_j) d\nu_i d\nu_j \\ = \; & s_i - s_j, \end{aligned} \tag{3.23}$$

where $h(.)$ denotes the density function of ν_i.

If $s_i = s_j$,[15] expected qualities are identical for both firms, so that the procurer awards the contract to the firm bidding the lowest price. Because in the last stage of the game investment costs are sunk, each firm prefers receiving an arbitrary small payment to receiving no payment at all. Thus, the unique Nash equilibrium in pure strategies is $p_i = p_j = r$.

Now suppose that $s_i > s_j$ has been observed, i.e., if firm i instead of firm j produces the good, the procurer's expected benefit from quality increases by $v(s_i - s_j)$. Since firm j cannot offer an expected surplus exceeding $vE[q_j|s_i, s_j, x_a] - r$, in equilibrium firm i wins the contract choosing p_i according to

$$vE[q_i|s_i, s_j, x_a] - p_i = vE[q_j|s_i, s_j, x_a] - r \quad \Leftrightarrow \quad p_i = v(s_i - s_j) + r. \tag{3.24}$$

Consequently, the payment that firm i receives in addition to l_a in the last stage of the game is:

$$\begin{cases} v[x_i + \epsilon_i - (x_j + \epsilon_j)] + r & \text{if } x_i + \epsilon_i > x_j + \epsilon_j \\ \frac{1}{2} r & \text{if } x_i + \epsilon_i = x_j + \epsilon_j \\ 0 & \text{if } x_i + \epsilon_i < x_j + \epsilon_j \end{cases} \tag{3.25}$$

Thus, in equilibrium the contract is awarded to the firm with the higher quality signal. This firm receives the minimum price r plus a *quality premium* $v|s_i - s_j|$.

The procurer's cost minimization problem for implementing the investment x can now be defined as

$$C_a(x) := \min_{r, l_a} r + v\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}] + 2l_a \tag{3.26}$$

$$\text{s.t.} \quad x = \mathrm{argmax}_{\hat{x}} G(\hat{x} - x) \mathrm{E}[v(\hat{x} + \epsilon_i - (x + \epsilon_j)) + r | \hat{x} + \epsilon_i > x + \epsilon_j] - c(\hat{x}) \tag{3.27}$$

$$0 \leq \frac{1}{2} \left(r + v\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}] \right) + l_a - c(x) - \bar{c} \tag{3.28}$$

$$B \geq r + v\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}] + 2l_a \tag{3.29}$$

$$0 \leq r, l_a, \tag{3.30}$$

where $\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}]$ denotes the expected value of the difference of the second-order statistic and the first-order statistic of the sample ϵ_i, ϵ_j. Given symmetric investments, $r + v\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}]$ is the expected price that the procurer has to pay to the

[15] Of course, $s_i = s_j$ is observed with probability zero. But considering this case is helpful for deriving the optimal bidding strategies if $s_i > s_j$.

auction winner. When minimizing the expected payment to firms, the procurer has to take into account that the guaranteed minimum price r has to be chosen according to the incentive compatibility constraint (3.27). Conditions (3.28) to (3.30) are the participation constraint, the procurer's budget constraint[16], and the limited liability constraint, respectively.

I first simplify the incentive compatibility constraint. Firm i chooses its investment level x_i given the investment decision of firm j, x_j, such that

$$\begin{aligned} x_i &= \text{argmax}_{\hat{x}\geq 0} \int_{-\infty}^{\hat{x}-x_j} (v(\hat{x} - x_j - \eta) + r) g(\eta) d\eta - c(\hat{x}) \\ &= \text{argmax}_{\hat{x}\geq 0}\, rG(\hat{x} - x_j) + v \int_{-\infty}^{\hat{x}-x_j} (\hat{x} - x_j - \eta) g(\eta) d\eta - c(\hat{x}). \end{aligned} \tag{3.31}$$

Given x_i, firm j chooses x_j such that:[17]

$$\begin{aligned} x_j &= \text{argmax}_{\hat{x}\geq 0} \int_{x_i-\hat{x}}^{\infty} (v(\hat{x} - x_i + \eta) + r) g(\eta) d\eta - c(\hat{x}) \\ &= \text{argmax}_{\hat{x}\geq 0}\, rG(\hat{x} - x_i) + v \int_{x_i-\hat{x}}^{\infty} (\hat{x} - x_i + \eta) g(\eta) d\eta - c(\hat{x}) \end{aligned} \tag{3.32}$$

The expected payment to each firm is composed of two parts. On the one hand, a firm receives r with the probability of having the higher quality signal given investments. Additionally, a firm obtains an expected quality premium. This is the expected difference between quality signals times v, given that the firm had the higher quality signal.

Assuming an interior solution, (3.31) implies

$$rg(x_i - x_j) + vG(x_i - x_j) - c'(x_i) = 0 \tag{3.33}$$

as a necessary condition for (x_i, x_j) to be a Nash equilibrium. The corresponding condition for firm j is symmetrical with (3.33). For a symmetric equilibrium $x_i = x_j =: x_a$, (3.33) simplifies to

$$rg(0) + \frac{v}{2} - c'(x_a) = 0. \tag{3.34}$$

Because $c(.)$ is strictly convex, there is at most one symmetric equilibrium.

Similarly to the tournament mechanism, we have to guarantee that x_a is indeed a Nash equilibrium. This is ensured if firms' objective functions (3.31) and (3.32) are strictly concave. Since $r \leq B - v\text{E}[\epsilon_{(2)} - \epsilon_{(1)}]$, this is the case if

$$\sup_{\eta}\{(B - v\text{E}[\epsilon_{(2)} - \epsilon_{(1)}])g'(\eta) + vg(\eta)\} < \inf_{x} c''(x). \tag{3.35}$$

As in the tournament, firms' objective functions are strictly concave for normally distributed random variables if σ_μ^2 is sufficiently high. I prove this claim in the appendix. For the remainder of this section, I assume that condition (3.35) is satisfied.

Since firm i's objective function (3.31) is strictly increasing at $\hat{x} = 0$ for all $r \geq 0$, both firms choose a positive investment in equilibrium even if the procurer does

[16]In the auction, the procurer's budget constraint is met only in expected terms. However, this is the case in many real world situations. Furthermore, in this model, the budget is necessary only to have an upper bound on r, which is fixed ex ante.

[17]Recall that $1 - G(\eta) = G(-\eta)$ since g(.) is symmetric around zero.

not set a positive minimum price. This is because, compared to the tournament, increasing investments has an additional effect on a firm's profit in the auction (compare (3.14) and (3.34)). In the auction, a firm not only increases its probability of winning but also its expected quality premium.

The latter effect causes another difference to the tournament: In the auction, there is a lower bound on the set of implementable quality levels. In any symmetric equilibrium, if $r = 0$, a firm's marginal expected payment equals $v/2$ (see (3.34)). This is because a firm's expected quality premium increases when it raises investments. Therefore, equilibrium investments are at least as high as the socially optimal investment level, i.e., $x_a \geq x^*$, and $x_a = x^*$ if and only if $r = 0$.

Assuming that (3.35) holds, the procurer's problem of implementing x can now be simplified as follows:

$$\begin{aligned} C_a(x) := \min_{r,l_a} r + v\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}] + 2l_a & & & \quad (3.36) \\ \text{s.t.} \quad rg(0) + v/2 - c'(x) &= 0 & & \quad (3.37) \\ \frac{1}{2}\left(r + v\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}]\right) + l_a - c(x) - \bar{c} &\geq 0 & & \quad (3.38) \\ r + v\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}] + 2l_a &\leq B & & \quad (3.39) \\ r, l_a &\geq 0 & & \quad (3.40) \end{aligned}$$

After eliminating r by using equation (3.37), we obtain for l_a:

$$l_a = \begin{cases} \bar{c} + c(x) - \frac{1}{2}\left(\frac{c'(x)}{g(0)} - \frac{v}{2g(0)} + v\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}]\right) & \\ \qquad \text{if} \quad \bar{c} + c(x) - \frac{1}{2}\left(\frac{c'(x)}{g(0)} - \frac{v}{2g(0)} + v\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}]\right) \geq 0 & \\ 0 \qquad \text{otherwise} & \end{cases} \quad (3.41)$$

In the first case, a firm's participation constraint is binding, i.e., l_a can be chosen so that firms are just compensated for their investment costs. In the second case, the expected price that a firm receives in the auction is larger than this firm's costs, and, therefore, fixed payments are not necessary. Thus, the liability constraint is binding and firms earn rents. The procurer's budget constraint only decides whether x can be implemented.

Proposition 3.2 *Suppose $c'(x) \geq v/2$. The procurer's costs of implementing the investment x in the auction mechanism are*

$$C_a(x) = \max\left\{2(c(x) + \bar{c}), \frac{c'(x)}{g(0)} - \frac{v}{2g(0)} + vE[\epsilon_{(2)} - \epsilon_{(1)}]\right\}, \quad (3.42)$$

provided that $C_a(x) \leq B$.

The procurer can implement an investment $x \geq x^*$ only if $C_a(x) \leq B$. Firms receive rents if the second term in braces, which is the minimum price r required for implementing x plus the expected quality premium, is larger than the first term in braces.

In the auction, firms invest at least x^*. The procurer can implement an investment $x > x^*$ if she chooses a positive minimum price r. However, if the participation constraint is binding at $x = x^*$, the procurer maximizes her expected profit

$$v(x + \mathrm{E}[\mu_i | \epsilon_i \geq \epsilon_j]) - C_a(x) \tag{3.43}$$

by implementing x^* setting $r = 0$.

Furthermore, since (3.35) must hold for $\eta = 0$ and $g'(\eta) = 0$, we have $v < c''(x)/g(0)$ for all x. That is, if the limited liability constraint is binding at $x = x^*$, the procurer will also implement x^* since her marginal valuation of quality is smaller than the marginal costs of quality at any investment $x \geq x^*$. This holds no matter which constraint is binding at x.

The next proposition follows immediately.

Proposition 3.3 *When using the auction mechanism, the procurer maximizes her expected profit by implementing x^*. The resulting expected profit is*

$$v(x^* + E[\mu_i | \epsilon_i \geq \epsilon_j]) - \max\{2(c(x^*) + \bar{c}), vE[\epsilon_{(2)} - \epsilon_{(1)}]\}. \tag{3.44}$$

The optimal investment in the auction is independent of the precision of the quality signals. Nevertheless, increasing the precision of quality measurement increases the procurer's expected profit. On the one hand, the procurer's expected valuation of the good increases because mistakenly choosing the low-quality firm for production becomes less likely. On the other hand, the expected payment to firms (weakly) decreases, because the expected difference in quality signals and, therefore, the expected quality premium $v\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}]$ decreases. Consequently, when choosing the precision of quality measurement in the first stage, the procurer should increase precision as long as the increase in expected profits is larger than the increase in measurement costs.

By contrast, the effect of a less random production technology on expected profits is ambiguous. The procurer's expected valuation of the good decreases because the expected quality that the high-quality firm can offer is not much higher than the average expected quality, $x + \mathrm{E}[\mu_i]$. However, the expected quality premium also decreases because it is more likely that firms can offer similar qualities.

Of course, the procurer will prefer not to procure the good using the auction mechanism if the resulting profit is smaller than zero, which is the procurer's reservation utility. Furthermore, x^* is not implementable, and therefore, the auction mechanism cannot be used, if the procurer's available budget is to small, i.e., $B < C_a(x^*)$.

Condition (3.35) implies that it is never optimal for one firm to choose $x = 0$ if the other firm chooses x^*, i.e., given that $r = 0$,

$$v \int_{-\infty}^{0} -\eta g(\eta) d\eta - c(x^*) > v \int_{-\infty}^{-x^*} (-x^* - \eta) g(\eta) d\eta. \tag{3.45}$$

Since $\int_{-\infty}^{0} -\eta g(\eta) d\eta = \int_{0}^{\infty} \eta g(\eta) d\eta = 1/2\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}]$, the last inequality implies that

$$2c(x^*) < v\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}]. \tag{3.46}$$

Therefore, the participation constraint is binding at $x = x^*$ if and only if $\bar{c}$ is sufficiently high.

3.5 Comparison of the mechanisms

The purpose of this section is to investigate under which circumstances the procurer prefers the tournament to the auction mechanism.

To compare the auction and the tournament, I restrict attention to the class of problems for which firms' objective functions are strictly concave in both mechanisms. That is, I assume that the values of the exogenous parameters v, B, D, and σ_μ^2 are such that the conditions (3.16) and (3.35) are satisfied. A sufficient condition for (3.16) and (3.35) to hold simultaneously is that

$$\sup_\eta \{Bg'(\eta) + vg(\eta)\} < \inf_x c''(x). \tag{3.47}$$

Again, this is in general the case if the production technology for quality is sufficiently random.[18]

Furthermore, I assume that the budget B is high enough to implement x^* in the auction if the resulting profit is non-negative, i.e.,

$$B \geq v(x^* + \mathrm{E}[\mu_{(2)}]). \tag{3.48}$$

I first derive the investments levels that the procurer implements under the tournament scheme, denoted by x_t^*. If firms' participation constraints are binding at $x = x^*$, the procurer maximizes her expected profit

$$v(x + \mathrm{E}[\mu_i | \epsilon_i \geq \epsilon_j]) - C_t(x) \tag{3.49}$$

by implementing $x_t^* = x^*$.

If firms' participation constraints are not binding at x^*, the optimal investment x_t^* is given by

$$x_t^* = \max\left\{x : x < x^* \text{ and } 2(c(x) + \bar{c}) = \frac{c'(x)}{g(0)}\right\}. \tag{3.50}$$

That is, expected profit is maximized by implementing the highest investment $x < x^*$ at which both the limited liability and the participation constraints are binding. The reason is that, by (3.47), $v < c''(x)/g(0)$ for all x,[19] so that the marginal costs of quality exceed the marginal benefit for all investment levels at which only the limited liability constraints are binding. Figure 3.1 illustrates x_t^* for the case of strictly convex marginal investment costs.

Proposition 3.4 *Suppose that (3.47) holds. In the tournament, the procurer implements the highest investment x such that $x \leq x^*$ and firms' participation constraints are binding at x. The procurer's expected profit is always positive.*

[18]For normally distributed random variables, it is sufficient that condition (3.35) holds. See inequality (3.72) and the subsequent explanation in the appendix.

[19]Note that although (3.47) is sufficient (and not necessary) for the existence of pure strategy equilibria in both mechanisms, $v < c''(x)/g(0)$ is necessary for the concavity of a firm's objective function in the auction. This is the case even if we already restrict r to zero (see (3.31)).

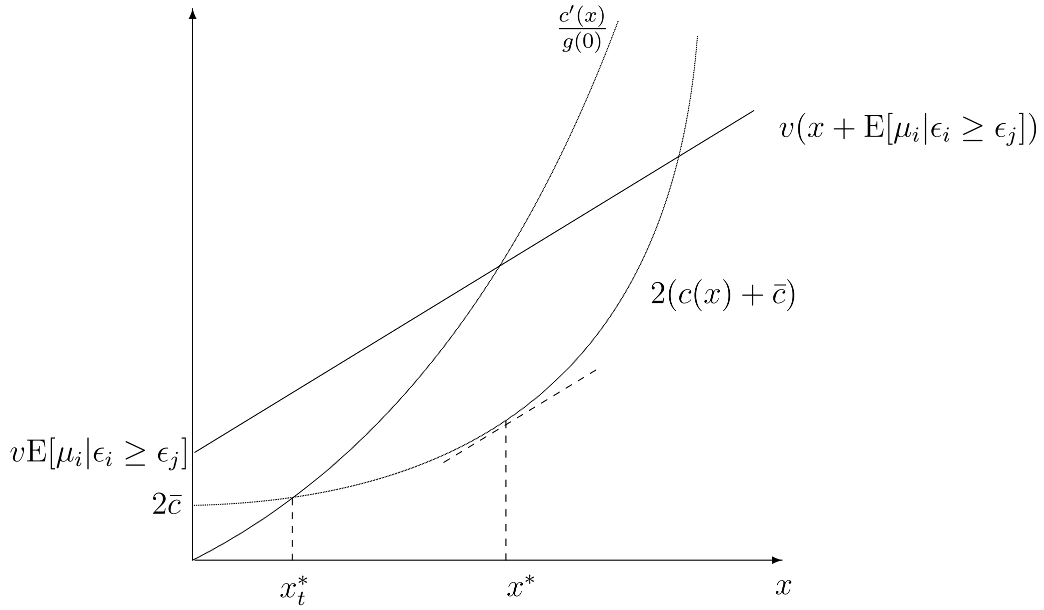

Figure 3.1: Optimal investment in the tournament.

The procurer's profit is positive because the participation constraint is binding at the optimal investment x_t^* and because of assumption (3.3). Due to the binding participation constraint, firms never earn rents. Optimal investments are positive if and only if $\bar{c} > 0$. The reason is that, by (3.22), if $\bar{c} = 0$, $x = 0$ is the only investment at which firms' participation constraints are binding.[20] Because of (3.48), the procurer's budget is high enough to implement x_t^*.

The procurer's expected profit in the tournament is

$$v(x_t^* + \mathrm{E}[\mu_i|\epsilon_i \geq \epsilon_j]) - \max\left\{2(c(x_t^*) + \bar{c}), \frac{c'(x_t^*)}{g(0)}\right\}. \tag{3.51}$$

In contrast to the auction, the optimal investment in the tournament depends on both the randomness of the production technology and the precision of quality measurement. The lower $\mathrm{var}(\epsilon_i) = \sigma_\mu^2 + \sigma_\nu^2$ the higher is $g(0)$, and therefore also x_t^*. As in the auction, a higher precision of quality measurement increases the procurer's expected profit, while the effect of a less random production technology is ambiguous.

An advantage of the tournament is that the procurer can implement investments that are smaller than the socially optimal investment level x^*. She will do so whenever she has to pay a rent to firms for investing x^*. She would also like to do so in the auction. However, this is not possible because the auction mechanism generates too strong investment incentives.

When comparing the procurer's expected profit under both mechanisms, one has to take into account that the optimal precisions of quality measurement usually differ for both mechanisms. For this reason, I now introduce measurement costs $M(\sigma_\nu^2)$, where $M(\sigma_\nu^2) \geq 0$ for all $\sigma_\nu^2 \leq \bar{\sigma}_\nu^2$, $M(\bar{\sigma}_\nu^2) = 0$ and $M(.)$ is strictly decreasing in σ_ν^2.

[20] Note that the procurer's expected profit is positive in the case $\bar{c} = 0$ because $\mathrm{E}[\mu_i|\epsilon_i \geq \epsilon_j] > 0$.

Let $\sigma^2_{\nu_a}$ denote an optimal precision level in the auction, i.e., $\sigma^2_{\nu_a}$ maximizes the function

$$\Pi_a(\sigma^2_\nu) := v(x^* + \mathrm{E}[\mu_i|\epsilon_i \geq \epsilon_j, \sigma^2_\nu]) \\ - \max\{2(c(x^*) + \bar{c}), v\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}|\sigma^2_\nu]\} - M(\sigma^2_\nu). \quad (3.52)$$

A sufficient condition for the superiority of the tournament is that the expected profit when implementing x^* is higher in the tournament than in the auction, when the precision in the tournament is also $\sigma^2_{\nu_a}$, i.e.,

$$v(x^* + \mathrm{E}[\mu_i|\epsilon_i \geq \epsilon_j, \sigma^2_{\nu_a}]) - \max\left\{2(c(x^*) + \bar{c}), \left.\frac{v}{2g(0)}\right|_{\sigma^2_{\nu_a}}\right\} - M(\sigma^2_{\nu_a}) \geq \Pi_a(\sigma^2_{\nu_a}). \quad (3.53)$$

This leads to the following proposition.

Proposition 3.5 *A sufficient condition for the principal to prefer the tournament to the auction is that*

$$\left.\frac{1}{2g(0)}\right|_{\sigma^2_\nu} \leq E[\epsilon_{(2)} - \epsilon_{(1)}|\sigma^2_\nu] \quad \textit{for all} \quad 0 \leq \sigma^2_\nu \leq \bar{\sigma}^2_\nu. \quad (3.54)$$

Inequality (3.54) says that the required prize spread for implementing x^* in the tournament is lower than the expected quality premium in the auction for each given precision of quality measurement. Thus, (3.54) ensures that condition (3.53) is satisfied.

The precision of quality measurement can in general not serve as an indicator for the fulfillment of inequality (3.54). The reason is that both the left-hand side and the right-hand side of (3.54) are decreasing in the precision of quality measurement. In the tournament, a higher precision means that the probability of winning the prize is more sensitive to changes in investments. Therefore, the required prize for implementing x^* decreases. In the auction, if measurement errors decrease, the expected quality premium also decreases because similar quality signals become more likely.

Whether (3.54) holds depends on the specific distribution from which $\epsilon_i = \mu_i + \nu_i$, $i = 1, 2$, is drawn. For example, if ϵ_i is exponentially distributed with mean λ and variance λ^2, one obtains that (the derivation is given in the appendix)

$$\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}|\sigma^2_\nu] = \left.\frac{1}{2g(0)}\right|_{\sigma^2_\nu} = \lambda. \quad (3.55)$$

Thus, in this case, inequality (3.54) is always binding.

It follows that the auction and the tournament lead to the same expected profit if and only if the participation constraints are binding at x^* and the same level of precision is optimal in both mechanisms. However, if it is optimal to choose a precision level different from $\sigma^2_{\nu_a}$ in the tournament, the procurer strictly prefers the tournament. Furthermore, if the principal must pay a rent to firms for investing x^*, i.e.,

$$2(c(x^*) + \bar{c}) < v\lambda, \quad (3.56)$$

expected profit is also strictly higher under the tournament scheme, where the principal implements $x_t^* < x^*$ as given by (3.50). Since $\lambda^2 = \sigma_\mu^2 + \sigma_\nu^2$, this case occurs when the precision of quality measurement is sufficiently low and/or the production technology for quality is sufficiently random.

By contrast, if μ_i and ν_i are normally distributed, we have

$$\left.\frac{1}{2g(0)}\right|_{\sigma_\nu^2} = \sqrt{\pi}\sqrt{\sigma_\mu^2 + \sigma_\nu^2} \quad \text{and} \quad \mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}|\sigma_\nu^2] = \frac{2}{\sqrt{\pi}}\sqrt{\sigma_\mu^2 + \sigma_\nu^2}. \qquad (3.57)$$

Therefore, the prize in the tournament for implementing x^* is always strictly higher than the expected price in the auction for a given precision of quality measurement. The same is true for uniformly distributed ϵ_i and ϵ_j. A proof is given in the appendix.

3.6 Endogenous precision

Even if the expected payments to firms for investing x^* are higher in the tournament, this does not necessarily imply that the procurer prefers the auction. To illustrate this point, I now consider an example in which the principal endogenously determines the optimal precision of quality measurement for both mechanisms.

Assume that the random variables μ_i and ν_i, $i = 1, 2$, are normally distributed with $\mathrm{E}[\mu_i] = 0$. Firms' investment costs are $c(x) = 1/4x^2$. Therefore, the socially optimal investment is $x^* = v$. The budget is $B = v(x^* + \mathrm{E}[\mu_{(2)}])$, i.e., B is high enough to implement all x, such that $x \leq x^*$ and the procurer's expected profit is non-negative.

The procurer can choose between the precision levels $\sigma_\nu^2 = 0$ and $\sigma_\nu^2 = \bar{\sigma}_\nu^2$. Costs of precision are $M(0) = m > 0$ and $M(\bar{\sigma}_\nu^2) = 0$. That is, if the procurer spends m on quality measurement, quality becomes perfectly observable. I define

$$\overline{E} := \mathrm{E}[\mu_i|\epsilon_i \geq \epsilon_j, \sigma_\nu^2 = 0] = \mathrm{E}[\mu_{(2)}] \quad \text{and} \quad \underline{E} := \mathrm{E}[\mu_i|\epsilon_i \geq \epsilon_j, \sigma_\nu^2 = \bar{\sigma}_\nu^2]. \qquad (3.58)$$

Furthermore, I assume that $\sigma_\mu^2 \geq 1$ and $v \in (0, 1.38]$. For these parameter values, condition (3.72) in the appendix holds and, therefore, the existence of equilibria in pure strategies is ensured.

The assumption of $\bar{c} = 0$ leads to the extreme result that the procurer implements $x_t^* = 0$ in the tournament but greatly simplifies the analysis of this example. The qualitative results remain unchanged if $\bar{c}$ is positive but small (relative to the parameters v and σ_μ^2).

In the tournament, the procurer obtains the expected profit

$$\Pi_t(\sigma_\nu^2) := v\mathrm{E}[\mu_i|\epsilon_i > \epsilon_j, \sigma_\nu^2] - M(\sigma_\nu^2). \qquad (3.59)$$

He chooses the high precision of quality measurement, $\sigma_\nu^2 = 0$, if the expected valuation of quality increases more strongly than the measurement costs, i.e.,

$$v(\overline{E} - \underline{E}) \geq m. \qquad (3.60)$$

In the auction, since $\bar{c} = 0$, the limited liability constraint is binding by (3.46). Thus, by (3.57), the procurer's expected profit is

$$\Pi_a(\sigma_\nu^2) = v(v + \mathrm{E}[\mu_i|\epsilon_i > \epsilon_j, \sigma_\nu^2]) - \frac{2v}{\sqrt{\pi}}\sqrt{\sigma_\mu^2 + \sigma_\nu^2} - M(\sigma_\nu^2). \qquad (3.61)$$

He chooses high precision if

$$v(\overline{E} - \underline{E}) + \frac{2v}{\sqrt{\pi}}(\sqrt{\sigma_\mu^2 + \bar{\sigma}_\nu^2} - \sigma_\mu) \geq m. \tag{3.62}$$

In the auction, a higher precision not only increases expected quality but also decreases the price. Therefore, compared to the tournament, the procurer's expected profit increases more strongly when she chooses the high precision of quality measurement.

When determining optimal precision levels, three cases must be distinguished. First, if the costs of increasing precision are small, i.e., if (3.60) holds, the procurer chooses $\sigma_\nu^2 = 0$ in both mechanisms. Second, if the costs of increasing precision are high, i.e., if (3.62) does not hold, precision is low in both mechanisms, $\sigma_\nu^2 = \bar{\sigma}_\nu^2$. Finally, for intermediate values of m, i.e., if only (3.62) holds, precision is high in the auction but low in the tournament.

When precision is identical in both mechanisms, we obtain $\Pi_t(\sigma_\nu^2) \geq \Pi_a(\sigma_\nu^2)$ if and only if

$$\frac{2}{\sqrt{\pi}}\sqrt{\sigma_\mu^2 + \sigma_\nu^2} \geq v, \tag{3.63}$$

where the term on the left-hand side of the inequality is the expected price in the auction divided by v. In the first case, when m is small, (3.63) simplifies to $\frac{2}{\sqrt{\pi}}\sigma_\mu \geq v$. Therefore, the procurer prefers the tournament to the auction if her marginal valuation of quality is low or the production technology for quality is highly random. If v is low, firms' investments in the auction mechanism are also low, and the procurer does not strongly benefit from these investments. If the production technology is highly random, the expected qualities that firms can offer vary strongly, so that the price in the auction is high.

In the second case, when optimal precision is low in both mechanisms, the price in the auction increases. Therefore, if (3.63) holds for high precision, it is also satisfied when precision is low. Otherwise, the tournament becomes superior under low precision if the price of the auction increases strongly, i.e., $\bar{\sigma}_\nu^2$ is high.

In the third case, when precision is low in the tournament but high in the auction, the tournament leads to a higher expected profit than the auction if

$$m \geq v(v + \overline{E} - \underline{E}) - \frac{2v}{\sqrt{\pi}}\sigma_\mu, \tag{3.64}$$

i.e., if measurement costs are above some lower bound. From (3.62), which must hold in this case, we also have an upper bound for m. It is easy to check that the inequalities (3.62) and (3.64) can be simultaneously satisfied for some m only if (3.63) holds.

Thus, in the given example, a low precision of quality measurement, a highly random production technology for quality, and a low marginal valuation of quality favor the tournament.

The result that the procurer prefers a tournament if the precision of quality measurement is sufficiently is not restricted to the example but holds in general. The reason is that the expected profit in the auction is negative whenever the expected valuation of quality is lower than the expected quality premium, i.e., if the inequality

$$x^* + \mathrm{E}[\mu_i|\epsilon_i \geq \epsilon_j, \sigma_\nu^2] < \mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}|\sigma_\nu^2] \tag{3.65}$$

holds. Since $\mathrm{E}[\mu_i|\epsilon_i \geq \epsilon_j] \leq \mathrm{E}[\mu_{(2)}]$, there is an upper bound on the left-hand side of (3.65) that is independent of the level of precision σ_ν^2. By contrast, the right-hand side, and therefore, the quality premium, can become arbitrarily large when the precision decreases. The optimal precision is low if the costless signal has a high variance $\bar{\sigma}_\nu^2$ and increasing precision is expensive.

A low precision of quality measurement also increases the payments to firms in the tournament. Firms anticipate that the winner of the tournament is determined by luck rather than performance, and, therefore, the winner prize must increase for implementing a given investment. However, because the winner prize is fixed ex ante, investment incentives in the tournament are not as strong as in the auction. Therefore, the procurer can implement low investments, which she will prefer if increasing investment incentives is very costly because of imprecise quality measurement. As a result, the tournament always leads to a positive expected profit (see proposition 3.4).

Furthermore, since x^* increases in v, inequality (3.65) is more likely to hold if the procurer's marginal valuation of quality is small. The foregoing observations are summarized in the following proposition.

Proposition 3.6 *The procurer prefers the tournament to the auction if the optimal precision of quality measurement in the auction is sufficiently low. Furthermore, a low valuation of quality also favors the tournament.*

3.7 Conclusion

In this essay, I analyzed and compared the characteristics of two procurement mechanisms, an auction and a tournament. In contrast to the previous literature, I showed that there are circumstances under which the procurer prefers a tournament scheme to an auction mechanism.

In the auction, the firm with the higher quality signal wins. This firm receives a price that reflects the increase of the procurer's expected valuation of the good when she buys from the firm with the higher expected quality. If quality signals vary strongly because quality is difficult to measure or highly random, the expected qualities that firms supply also differ strongly. Therefore, the expected price that the procurer has to pay in the auction mechanism are high.

In this case, the tournament has an advantage over the auction. Because the prize that the firm with the higher quality signal receives in the tournament is fixed ex ante, investment incentives are smaller than in the auction. Therefore, by adjusting the winner prize, the procurer can implement lower investments in the tournament, thereby decreasing the payments to firms. As a result, the procurer's expected profit in the tournament is higher than in the auction if the precision of quality measurement is sufficiently low. This is the case if the costs of increasing precision are high.

Furthermore, the procurer always implements the socially optimal investment level x^* in the auction. Consequently, she prefers the tournament for such distributions of the noise terms for which the prize for implementing x^* in the tournament is always smaller than the corresponding expected price in the auction (e.g., an exponential distribution).

There are several interesting extensions of the model, which might be subject to future research. For example, in the auction, the procurer might benefit from concealing quality signals. Firms may have different production technologies for quality and/or investment costs. Furthermore, there may be more than two firms that are able to produce the good. In this case, the procurer may want to restrict entry.

3.8 Appendix

Existence of the equilibrium in the tournament for normally distributed random variables. If $\mu_i \sim N(\bar{\mu}, \sigma_\mu^2)$ and $\nu_i \sim N(\bar{\nu}, \sigma_\nu^2)$, $\eta = (\mu_j + \nu_j) - (\mu_i + \nu_i)$ is also normally distributed with mean 0, variance $\sigma_\eta^2 = 2(\sigma_\mu^2 + \sigma_\nu^2)$, and density

$$g(\eta) = \frac{1}{\sqrt{2\pi}\sigma_\eta} \exp\left(-\frac{\eta^2}{2\sigma_\eta^2}\right). \tag{3.66}$$

We obtain that

$$g'(\eta) = -\frac{\eta}{\sqrt{2\pi}\sigma_\eta^3} \exp\left(\frac{-\eta^2}{2\sigma_\eta^2}\right), \tag{3.67}$$

which attains its maximum value at $\eta = -\sigma_\eta$, where

$$g'(-\sigma_\eta) = \frac{1}{2\sqrt{2\pi}(\sigma_\mu^2 + \sigma_\nu^2)} \exp\left(-\frac{1}{2}\right) \le \frac{1}{2\sqrt{2\pi}\sigma_\mu^2} \exp\left(-\frac{1}{2}\right). \tag{3.68}$$

Thus, (3.16) is satisfied for all $\sigma_\nu^2 \ge 0$ if

$$\frac{B}{2\sqrt{2\pi}\sigma_\mu^2} \exp\left(-\frac{1}{2}\right) < D. \tag{3.69}$$

Given B and D, this condition holds if σ_μ^2 is sufficiently high.[21] □

Existence of the equilibrium in the auction for normally distributed random variables. We have that

$$\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}] = \mathrm{E}[\eta | \eta \ge 0] = \frac{2}{\sqrt{2\pi}\sigma_\eta} \int_0^\infty \eta \exp\left(-\frac{\eta^2}{2\sigma_\eta^2}\right) d\eta = \sqrt{\frac{2}{\pi}}\sigma_\eta. \tag{3.70}$$

Since $g(\eta) \le g(0)$, inequality (3.35) holds for all $\sigma_\nu^2 \ge 0$ if

$$\frac{B - \frac{2v}{\sqrt{\pi}}\sigma_\mu}{2\sqrt{2\pi}\sigma_\mu^2} \exp\left(-\frac{1}{2}\right) + \frac{v}{2\sqrt{\pi}\sigma_\mu} < \inf_x c''(x) \tag{3.71}$$

$$\Leftrightarrow \frac{B}{2\sqrt{2\pi}\sigma_\mu^2} \exp\left(-\frac{1}{2}\right) + \frac{v}{\sigma_\mu}\left(\frac{1}{2\sqrt{\pi}} - \frac{\exp(-0.5)}{\sqrt{2}\pi}\right) < \inf_x c''(x). \tag{3.72}$$

[21] Kräkel and Sliwka [2004] derive a similar result.

Given B, D, v, inequality (3.72) is satisfied if σ_μ^2 is sufficiently high. Since the second term on the left-hand side of (3.72) is positive, condition (3.72) implies that (3.69) is satisfied.□

Proof of the claim that (3.54) is binding for exponentially distributed ϵ_i, ϵ_j. If ϵ_i is exponentially distributed, it has the density

$$f(\epsilon_i) = \begin{cases} \frac{1}{\lambda}\exp\left(-\frac{\epsilon_i}{\lambda}\right) & \text{if } \epsilon_i \geq 0 \\ 0 & \text{otherwise} \end{cases}, \tag{3.73}$$

where $\lambda \geq 0$ is the mean. The random variable $\epsilon_{(2)} - \epsilon_{(1)}$ is also exponentially distributed with mean λ, see for example Arnold et al. [1992], p. 72. Furthermore, we have

$$g(0) = \int_0^\infty \frac{1}{\lambda^2}\exp\left(-\frac{2t}{\lambda}\right)dt = \frac{1}{2\lambda}. \ \square \tag{3.74}$$

Proof of the claim that (3.54) does not hold for uniformly distributed ϵ_i, ϵ_j. If ϵ_i and ϵ_j are uniformly distributed in an interval of length u, the composed random variable $\eta = \epsilon_j - \epsilon_i$ is subject to a triangular distribution in the interval $[-u, u]$ with density function

$$g(\eta) = \begin{cases} \frac{1}{u} + \frac{\eta}{u^2} & \text{if } \quad -u \leq \eta \leq 0 \\ \frac{1}{u} - \frac{\eta}{u^2} & \text{if } \quad 0 < \eta \leq u \end{cases}. \tag{3.75}$$

Thus, we have

$$\mathrm{E}[\epsilon_{(2)} - \epsilon_{(1)}] = \mathrm{E}[\eta|\eta \geq 0] = 2\int_0^u \left(\frac{1}{u} - \frac{\eta}{u^2}\right)\eta d\eta = \frac{u}{3} \tag{3.76}$$

and (3.54) does not holds since $u/2 > u/3$.□

Essay 4

Subsidizing Technological Innovations in the Presence of R&D Spillovers

4.1 Introduction

In this essay, which is joint work with Carsten Helm, we analyze a situation where a principal wants to induce two agents to produce an output. The agents can undertake a costly investment to reduce production cost of the output. Part of this 'innovation' spills over and also reduces production cost of the other agent. We compare different incentive structures and examine the principal's choice between directing financial incentives towards cost-reducing investments and output production.

A topical problem that conforms to this general structure is the promotion of new technologies, where spillovers occur because firms learn from each others innovations. To fix ideas, consider the following problem. Many governments want to increase electricity production from renewable energy sources in order to combat climate change and to reduce dependency on fossil energy. Renewable energy is not yet competitive, but the hope is that innovations will bring down production costs Manne and Richels [2004]. Therefore, several countries like Germany, France and Spain have passed legislation by which all producers of renewable energy receive a fixed price for power sold to the grid which lies above the market price. Some decision-makers have suggested that these subsidies should be focused on the most promising projects only.[1] Such an approach had been adopted in the UK with the NFFO (Non-Fossil Fuel Obligation), where the guaranteed market price had only been paid to the firms who presented the best bids (Cleirigh [2001]).

Much of the following analysis proceeds with this specific example in mind. Nevertheless, the model setup is substantially more general and the findings provide insights on other topical issues too. One example is the provision of incentives within firms. Often workers can increase productivity by investing in their human capital. Through the interaction at the workplace, part of this investment spills

[1] See the recent debate between the German ministers for the economy and the environment ("Clement sucht Konfrontation mit Trittin", in Frankfurter Allgemeine Zeitung, 02.09.2003, No. 203, p. 11.)

over to other workers. The firm has to decide about the optimal mix of providing incentives for the accumulation of human capital and for output production.

Another example is research funding, which often involves tournaments for which researchers submit project proposals. The subsequent selection process makes it possible to concentrate funding on the most promising proposals. However, if returns to scale are decreasing and if there are strong spillovers, it may be optimal to have several research teams working on the same project. There are also motivational effects which have to be taken into account. If incentive payments are conditioned mainly on the quality of research proposals, too much effort may be devoted to the writing of such proposals as compared to the research after the proposal has been accepted. Obviously, these arguments apply for public research funding as well as for research within firms.

We model agents' choices as a two stage game. In the first stage, agents invest into an innovation that reduces the cost of producing output. In the second stage, stochastic innovations are observed and production takes place. While we assume that output can be contracted upon, contracts based on the value of innovation are not feasible. The reason is that even if the principal and the agents can evaluate the innovation, such information is usually difficult to verify by a court. Moreover, we assume that agents' investments are not observable. Therefore, we have a moral hazard problem and the first-best innovation/output profile will not be implementable.

The principal has a given set of funds that she wants to use to maximize output, e.g., electricity from renewable energies. We focus on two policy instruments: (i) a general output price subsidy, and (ii) an innovation tournament such that only the winner receives an output price subsidy. We restrict our analysis to the two extremes of either subsidizing both agents to the same extent or subsidizing only one agent because these two schemes seem to be the most relevant. In particular, guaranteeing firms different prices for electricity that has been generated from the same renewable energies would probably constitute illegal price discrimination.

With a general output price subsidy, agents always underinvest in innovations since they disregard beneficial spillovers. By contrast, under the innovation tournament agents may even overinvest because they try to outperform the other agent (motivational effect). Given suboptimal investment decisions, expected overall output is always lower than in the first-best solution.

Whether output is higher under the tournament or under the general output price subsidy mainly depends on three effects. First, high spillovers favor a general output price subsidy because it induces both agents to produce in equilibrium. Second, a high variance of the stochastic innovation process means that the expected realized innovations of the two agents differ substantially. This increases the appeal of the tournament, in which resources for output production are concentrated on the most successful innovator. Finally, also the motivational effect described above favors a tournament, although it may be detrimental if it induces excessive innovation.

Our basic setup is similar to the seminal contribution by d'Aspremont and Jacquemin [1988], who also consider the interaction among firms that invest in cost-reducing innovation. The innovation is not completely appropriable due to spillovers, which may be substantial even in the presence of patent protection due to, e.g., personnel movements and informal networks (see Arrow [1962] and Mansfield [1985]).

This leads to underinvestment in R&D. d'Aspremont and Jacquemin [1988] analyze for an oligopolistic market structure whether cooperation among firms can alleviate this problem.[2]

Our work differs from d'Aspremont and Jacquemin [1988] and the related literature in several important respects. First, there is no problem of imperfect competition in our framework. Second, innovation is stochastic and the related investment is non-contractible. Third, there is an active regulator who can use his budget to provide incentives for innovation investments and/or output production.

Hinloopen [1997] also considers an active government, but he focuses on R&D subsidies, which are non-contractible in our framework. Related papers that analyze stochastic innovation are Martin [2002] and Gehrig [2004]. In Martin [2002] uncertainty is modelled as an uncertain discovery time. Essentially, he analyzes a patent racing model of cost-saving innovation in a quantity-setting duopoly. In Gehrig [2004] the development of an idea succeeds with a certain probability, and the firm invests resources to find out the likely success of the innovation. In our model, innovation depends on related investments and a random term.

There is also a substantial environmental economics literature on the stimulation of technological innovation. However, most of this literature focuses on firms' decisions to adopt a known technology under different instruments such as permits, taxes and standards (e.g., Requate and Unold [2003], as well as Jung et al. [1996]). A rare exception are Biglaiser and Horowitz [1995], who consider binary choices whether to undertake research into a technology that reduces the emission intensity of production.

Section 4.2 introduces the model. Section 4.3 determines socially optimal levels of innovation investment and output production. Sections 4.4 and 4.5 analyze the general output price subsidy and the tournament, respectively. Section 4.6 compares these two policy instruments, and section 4.7 concludes.

4.2 The model

The basic structure of our model is similar to the two-stage game of cost-saving innovations in d'Aspremont and Jacquemin [1988]. There are two ex-ante identical agents, indexed alternatively $i, j = 1, 2$. In the first stage, the principal commits to a mechanism. In the second stage, each agent undertakes a non-observable investment, $x_i \geq 0$, into the development of a process innovation that reduces production cost. The uncertain innovation output of this investment is $e_i x_i$, where $e_i \in (0, 2\bar{e}]$ is the realization of a random variable with a continuous and once differentiable cdf $F(e_i)$, symmetric density function $f(e_i)$, and expected value $\mathrm{E}[e_i] = \bar{e}$. The two random variables are identically and independently distributed. Innovations are non-verifiable but can be observed by the principal at no cost. In the third stage, both agents observe $e_i x_i$ and $e_j x_j$ and produce the verifiable output $q_i, q_j \geq 0$.[3]

[2] The literature on knowledge spillovers in an imperfectly competitive market environment is large. Examples are Suzumura [1992] who provides a generalization of d'Aspremont and Jacquemin [1988], as well as Kamien et al. [1992] who consider spillovers of research inputs (rather than research outputs). For surveys see DeBondt [1997] and Amir [2000].

[3] Under the tournament scheme, it will be required that the principal also observes innovations.

Agent i's total production cost after accounting for the process innovation are given by

$$\frac{q_i^2}{2[e_i x_i + s(q_j) e_j x_j]} + \frac{c}{4} x_i^2, \tag{4.1}$$

where

$$s(q_j) = \begin{cases} s & \text{if} \quad q_j > 0 \\ 0 & \text{otherwise} \end{cases}. \tag{4.2}$$

Accordingly, cost of output and cost of innovation are both quadratic, reflecting diminishing returns to innovation investments and output production. Production cost are reduced not only by own innovation, $e_i x_i$, but also by innovation of the other agent, $e_j x_j$, given that the other agent also produces. The spillover parameter $s \in [0, 1]$ indicates the extent of the latter effect. Note that the model focuses on innovation which is 'essential' in the sense that production cost rise to infinity in the absence of innovation.

All parties are risk-neutral and agents' reservation utility is zero. Furthermore, agents are wealth constrained so that the principal cannot charge entry fees. For parsimony, we assume that agents receive payments for their output and innovation only from the principal.[4]

It remains to specify the principal's objective function. We assume that she wants to maximize output, $q_1 + q_2$, for a given budget m. In our opinion, this better reflects actual decision processes than a maximization of social welfare. Especially since the monetarized benefits of producing electricity from renewable rather than 'conventional' energy are essentially not known. It also emphasizes our focus on problems where the principal is not interested in innovation per se, but in an output that can be produced more cheaply if innovation occurs. Furthermore, none of our main results depends on the level of the budget m. Finally, note that exactly the same results are obtained by solving the alternative problem of minimizing the principal's cost for a given expected overall output.

Before we turn to the two mechanisms that the principal considers – an output price subsidy that is paid either to all agents, or only to the winner of an innovation tournament – we first derive the first-best solution.

4.3 The first-best solution

Assume that it is possible to write binding contracts that specify investments x_i, x_j and, for given innovations $e_i x_i$, $e_j x_j$, output levels $q_i(e_i x_i, e_j x_j)$, $q_j(e_i x_i, e_j x_j)$. Then the principal's problem is to spend his budget m so as to maximize output $q_1 + q_2$ subject to the participation constraint that agents obtain a non-negative expected utility. Since agents are identical ex ante, optimal investment levels as chosen in the first stage of the game will be the same for both agents. However, optimal output levels as chosen in the second stage of the game will differ in general since agents realize different innovations.

[4]In the renewable energies example, electricity could be sold for a positive price even without government intervention. However, accounting for this possibility would substantially complicate the analysis without providing interesting insights.

By the principle of backwards induction, we first solve the principal's problem of maximizing output, q_1+q_2, for given innovations, $e_ix_i, i=1,2$. Since production cost are quadratic in output, it is optimal that both agents produce. Accordingly, the principal has to respect the budget constraint that payments m must be sufficient to induce participation of both agents,

$$m-\frac{q_1^2}{2(e_1x_1+se_2x_2)}-\frac{q_2^2}{2(e_2x_2+se_1x_1)}-\frac{c}{4}x_1^2-\frac{c}{4}x_2^2\geq 0. \tag{4.3}$$

Denoting the multiplier of the corresponding Lagrangian by λ, the first-order conditions with respect to q_1 and q_2 are

$$1-\lambda\frac{q_i}{e_ix_i+se_jx_j}=0, \quad i,j=1,2,\, i\neq j. \tag{4.4}$$

Together with the binding budget constraint this can be solved for optimal output levels[5]

$$q_i(e_ix_i,e_jx_j)=\left(\frac{2m-0.5c(x_i^2+x_j^2)}{(1+s)(e_ix_i+e_jx_j)}\right)^{1/2}(e_ix_i+se_jx_j), \quad i,j=1,2,\, i\neq j. \tag{4.5}$$

Accordingly, the agent who realizes the better innovation will produce more output unless $s=1$. Turning to the decision about innovation investments, x_i and x_j, the realization of the random variables is not known. Hence, the principal maximizes expected output, $\mathrm{E}[q_1+q_2]$, thereby anticipating the relation between the realized innovation and output as given by (4.5):[6]

$$\max_{x_1,x_2}\,[(1+s)(2m-0.5c(x_1^2+x_2^2)]^{1/2}E\left[(e_1x_1+e_2x_2)^{1/2}\right]. \tag{4.6}$$

In the appendix, we verify that for both agents the first-best investment level is

$$x^*=\left(\frac{2m}{3c}\right)^{1/2}. \tag{4.7}$$

Note that x^* depends neither on spillovers s nor on the expected value of the random term, $\bar{e}$. This is due to the fact that higher values of s and $\bar{e}$ have two effects: they reduce production cost and they increase returns to innovation investments. The first effect favors a reallocation of resources to output production, the second one a reallocation of resources to innovation investments. In our model, these two effects just cancel out in the first-best solution.

Upon substitution of x^* into (4.5), we get for $i,j=1,2,\, i\neq j$,

$$q_i^*(e_i,e_j)=\frac{2(e_i+se_j)}{((1+s)(e_i+e_j))^{1/2}}\left(\frac{2}{c}\right)^{1/4}\left(\frac{m}{3}\right)^{3/4}. \tag{4.8}$$

Overall expected output is

$$\mathrm{E}[q_i^*(e_i,e_j)+q_j^*(e_i,e_j)]=2(1+s)^{1/2}\left(\frac{2}{c}\right)^{1/4}\left(\frac{m}{3}\right)^{3/4}\mathrm{E}[(e_i+e_j)^{1/2}]. \tag{4.9}$$

We summarize the comparative statics in the following proposition.

[5] Output levels are well-defined since $2m\geq 0.5c(x_1^2+x_2^2)$ according to (4.3).

[6] Note that the budget constraint is satisfied by construction of $q_i(x_i,x_j,e_i,e_j)$.

Proposition 4.1 *A higher budget m and a lower investment cost parameter c increase x^*, q_1^* and q_2^*. A higher spillover parameter s increases q_i^* if and only if $e_i < (2+s)e_j$, $i, j = 1, 2$, $i \neq j$.*

The last statement implies that we can distinguish three cases. First, suppose that the realized innovation of firm i is substantially better than that of firm j, i.e., $e_i x^* > (2+s)e_j x^*$. Then firm j's production cost are reduced substantially more through spillovers than firm i's. Accordingly, as spillovers increase it is efficient to shift some production activity from firm i to firm j. This provides a rational why the most successful innovators may also be the most reluctant to share their innovations. Second, if firm j realizes a considerably better innovation, i.e., $e_j x^* > (2+s)e_i x^*$, we simply have the reverse case for which j's output falls in spillovers. Finally, if e_i and e_j are sufficiently close to each other, the output of both firms increases in s.

Implementation of the first-best solution depends on verifiable information about innovation investment x_i or the realized innovation $e_i x_i$. However, as discussed in the introduction such contractible information will often not be available. Therefore, we assume in the following that innovation $e_i x_i$ and investment x_i are non-verifiable and compare two alternative incentive contracts: (i) a general output price subsidy, and (ii) a tournament where subsidies are concentrated on the agent with the best innovation.

4.4 General output price subsidy

Given the principal's target of output maximization, an obvious instrument is to offer agents a linear incentive contract which consists of a fixed payment F_a, and a price p_a per output unit. Accordingly, the sequence of events is as follows. In the first stage, the principal commits to an output price p_a and a fixed payment F_a, taking into account his budget constraint. In the second stage, agents simultaneously choose an investment x_i. Afterwards, each agent observes his production costs, chooses his output level q_i, and payments occur. Since output is a random variable ex ante, we assume that the budget constraint is soft, i.e., *expected* payments to firms must not exceed m.

The game is solved by backward induction. In the last stage, given innovations $e_i x_i$ and $e_j x_j$, agent i chooses output q_i to maximize his payoff

$$\pi_i^a = F_a + p_a q_i - \frac{q_i^2}{2(e_i x_i + s e_j x_j)} - \frac{c}{4} x_i^2. \tag{4.10}$$

From the first-order condition, output is chosen according to

$$q_i(p_a, e_i x_i, e_j x_j) = p_a(e_i x_i + s e_j x_j). \tag{4.11}$$

In the second stage, anticipating $q_i(\cdot)$, agent i maximizes his expected payoff

$$\max_{x_i} F_a + E\left[\frac{p_a^2}{2}(e_i x_i + s e_j x_j)\right] - \frac{c}{4} x_i^2 \tag{4.12}$$

by choosing

$$x_i = \frac{p_a^2 \bar{e}}{c}. \tag{4.13}$$

We can now state the principal's problem in the first stage game as

$$\max_{p_a, F_a} \mathrm{E}[q_i + q_j] \tag{4.14}$$

$$\text{s. t. } q_i = \frac{p_a^3 \bar{e}}{c}(e_i + s e_j), \qquad i, j = 1, 2;\ i \neq j \tag{4.15}$$

$$m = \mathrm{E}[q_i + q_j] p_a + 2F_a \tag{4.16}$$

$$0 \leq \mathrm{E}[\pi_i^a], \qquad i = 1, 2 \tag{4.17}$$

$$0 \leq F_a, \tag{4.18}$$

where the incentive compatibility constraint (4.15) follows from substitution of (4.13) into (4.11), (4.16) is the principal's budget constraint, (4.17) is the agents' participation constraint, and (4.18) is the liability limit. Solving this problem (see the appendix) yields contract elements $F_a = 0$ and

$$p_a = \left(\frac{cm}{2(1+s)\bar{e}^2} \right)^{1/4}. \tag{4.19}$$

Upon substitution, investment and expected output levels are

$$x_a = \left(\frac{m}{2(1+s)c} \right)^{1/2}, \tag{4.20}$$

$$\mathrm{E}[q_i^a + q_j^a] = \left(\frac{2(1+s)}{c} \right)^{1/4} m^{3/4} \bar{e}^{1/2}. \tag{4.21}$$

Proposition 4.2 *Under a general output price subsidy, agents always underinvest and expected overall output is lower than in the first-best solution. Furthermore, agents always receive a rent.*

Proof See appendix.

Underinvestment occurs because spillovers constitute an externality for the individual agent. The rent arises from the limited liability constraint. In the absence of it, the principal could charge an entrance fee $F_a < 0$ and use the additional funds to stimulate output. However, even then the first-best would not be attainable since agents disregard spillovers in their investment decision.

Proposition 4.3 *Higher spillovers increase expected overall output but reduce investment. Furthermore, they increase output of agent i, q_i^a, if and only if*

$$e_i < \left(\frac{4}{3} + \frac{1}{3}s \right) e_j, \quad i, j = 1, 2,\ i \neq j. \tag{4.22}$$

At first sight, the negative tradeoff between investments and expected output seems surprising because a higher output makes cost reducing innovations more profitable. However, due to the budget constraint a higher expected output implies a lower price p_a (see 4.16), which leads to lower investments x_i (see 4.13). In particular, an agent bases his innovation decision on the *marginal* profitability of x_i, which increases in the output price. Accordingly, spillovers that increase incentives to produce output force the principal to reduce p_a, thereby reducing the agents' incentives for innovation investments. The intuition for the last statement in proposition 4.3 is similar to the one for the first-best solution (see proposition 4.1).

Given that agents underinvest, the principal may consider research tournaments in order to stimulate innovation investments. This instrument requires that realized innovations are observable by the principal, but they need not be verifiable to a third party. Tournaments are analyzed in the next section.

4.5 Tournament

In a tournament, only the winner, i.e., the agent with the better innovation, receives a fixed output price.[7] The losing agent receives no price subsidy and will not produce. Hence there are no spillovers.[8] The sequence of moves is the same as in the previous section: In stage 1, the principal commits to an output price $p_t > 0$ for the winner and a fixed payment F_t that will be paid to both agents. In stage 2, agents invest and the tournament winner is determined. In stage 3, the winner produces output.

We first consider the last stage and assume w.l.o.g. that agent i realizes the better innovation, i.e., $e_i x_i > e_j x_j$. Then, $q_j = 0$ and agent i chooses output q_i to maximize his payoff

$$\pi_i^t = p_t q_i - \frac{q_i^2}{2e_i x_i} - \frac{c}{4}x_i^2 + F_t, \tag{4.23}$$

where $F_t \geq 0$ denotes side payments to ensure participation. From the first-order condition,

$$q_i(p_t, e_i x_i) = p_t e_i x_i. \tag{4.24}$$

In the second stage, agents choose investments taking into account that agent i will receive p_t if and only if $e_i x_i > e_j x_j$, or $e_i > x_j/x_i e_j$. Anticipating the payoff maximizing output of the winning agent, agent i chooses x_i to maximize his expected payoff,

$$\max_{x_i} \frac{p_t^2}{2} x_i \int_0^{2\bar{e}} \left[\int_{\frac{x_j}{x_i}e_j}^{2\bar{e}} e_i f(e_i) de_i \right] f(e_j) de_j - \frac{c}{4}x_i^2 + F_t, \tag{4.25}$$

where the integral term represents the expected realization of the random variable e_i weighted by the chance of winning the tournament. Put differently, for symmetric investments $x_i = x_j$, it represents the expected value of the second-order statistic

[7] We assume that agents cannot bribe the principal.

[8] Obviously, an optimal mechanism would imply positive but different output prices for both agents (and, possibly, a fixed prize for the best innovator). However, for the reasons discussed in the introduction we focus on the two extremes of identical output prices and only one positive price.

of the sample e_i, e_j, which we denote by $\bar{e}_{(2)} \equiv \mathrm{E}[\max\{e_i, e_j\}]$, times the chance of winning the tournament.[9]

From the first-order condition it follows that in a symmetric equilibrium

$$x_t = \frac{p_t^2}{c}\Gamma, \tag{4.26}$$

where $\Gamma = \bar{e}_{(2)}/2 + \gamma$, and γ is defined as

$$\gamma \equiv \int_0^{2\bar{e}} (e_j f(e_j))^2 de_j. \tag{4.27}$$

Accordingly, $\bar{e}_{(2)}/2$ is the expected e_i of the tournament winner times the probability of winning. As $\bar{e}_{(2)}$ increases relative to $\bar{e}$, the tournament's *selection effect* of concentrating subsidies on the most successful innovator becomes more beneficial. Similarly, γ represents the increase of an agent's expected realization of the random variable due to a higher probability of winning the tournament as he raises x_i.[10] Hence, a high γ represents a strong *motivational effect* that arises from the incentive to outperform the other agent in a tournament. Intuitively, both effects increase the profitability of innovation investments.

Denoting output of the tournament winner by q_t, the principal's problem in the first stage game becomes

$$\max_{p_t, F_t} \mathrm{E}[q_t] \tag{4.28}$$

$$\text{s. t.} \quad q_t = \max\{e_i, e_j\}\frac{p_t^3}{c}\Gamma, \qquad i \neq j \tag{4.29}$$

$$m = \mathrm{E}[q_t]p_t + 2F_t \tag{4.30}$$

$$0 \leq \mathrm{E}[\pi_i^t], \qquad i = 1, 2 \tag{4.31}$$

$$0 \leq F_t, \tag{4.32}$$

where (4.29) – the incentive compatibility constraint of the tournament winner – follows from substitution of (4.26) into (4.24), while (4.30) to (4.32) represent the budget, participation, and wealth constraint, respectively.

Since expected output is increasing in p_t, the principal prefers to spend the entire budget m on the output price and to set lump-sum transfers $F_t = 0$. However, in contrast to a general output price subsidy doing so may violate agents' participation constraints so that we need to distinguish whether they bind or not.

Proposition 4.4 *Under a tournament scheme agents earn rent if and only if* $\bar{e}_{(2)} > 2\gamma$.

[9] See the appendix for a formal derivation of $\bar{e}_{(2)}$ and for the calculation of the first-order condition. Asymmetric equilibria may exist, but since agents are identical we restrict attention to symmetric equilibria. The second-order condition, $\frac{p_t^2}{2}\int_0^{2\bar{e}} \frac{e_j^2 x_j^2}{x_i^3}\left(-f\left(\frac{x_j}{x_i}e_j\right) - \frac{x_j}{x_i}e_j f'\left(\frac{x_j}{x_i}e_j\right)\right) f(e_j)de_j - \frac{c}{2} < 0$, is assumed to hold.

[10] This interpretation follows from the first-order condition as given in the Appendix (eq. 4.49).

Proof See appendix.

Due to the limited liability constraint, an agent receives rent if expected profits from output production exceed cost of innovation investment. Whether this is the case depends on the characteristics $\bar{e}_{(2)}$ and γ of the given innovation technology. The proposition reflects that a high $\bar{e}_{(2)}$ increases profits by reducing production costs, while a low γ reduces innovation investments (see 4.26).

Obviously, optimal contract elements as well as investment and output levels will differ for the cases with and without rent. In particular, if the participation constraint does not bind, the principal chooses the highest output price that his budget constraint (4.30) allows. Noting that $\mathrm{E}[q_t] = p_t^3 \bar{e}_{(2)} \Gamma / c$ (by 4.29), this yields contract elements with rent

$$F_t^r = 0, \qquad p_t^r = \left(\frac{cm}{\Gamma \bar{e}_{(2)}} \right)^{1/4}. \tag{4.33}$$

Upon substitution, investment and expected output levels are

$$x_t^r = \left(\frac{m\Gamma}{c\bar{e}_{(2)}} \right)^{1/2}, \tag{4.34}$$

$$\mathrm{E}[q_t^r] = \left(\frac{\bar{e}_{(2)} \Gamma m^3}{c} \right)^{1/4}. \tag{4.35}$$

By contrast, if agents do not earn rent, optimal contract elements follow from the binding budget and participation constraints (see eqs. 4.55 and 4.56 in the appendix) as

$$F_t = \frac{m(\Gamma - \bar{e}_{(2)})}{2(\Gamma + \bar{e}_{(2)})}, \qquad p_t = \left(\frac{2cm}{\Gamma(\Gamma + \bar{e}_{(2)})} \right)^{1/4}. \tag{4.36}$$

Upon substitution, investment and expected output levels are

$$x_t = \left(\frac{2m\Gamma}{c(\Gamma + \bar{e}_{(2)})} \right)^{1/2}, \tag{4.37}$$

$$\mathrm{E}[q_t] = \bar{e}_{(2)} \left(\frac{\Gamma}{c} \right)^{1/4} \left(\frac{2m}{\Gamma + \bar{e}_{(2)}} \right)^{3/4}. \tag{4.38}$$

4.6 Comparison of tournament and general price subsidy

In the preceding sections we have determined investment and output levels for three solutions: the output maximizing optimum, a general output price subsidy, and a research tournament where only the winner receives an output price subsidy. The next proposition compares investment levels in these three solutions.

Proposition 4.5 *In the tournament scheme, agents invest less than in the first-best solution if and only if* $\bar{e}_{(2)} > 6\gamma$. *They always invest more than with a general output price subsidy.*

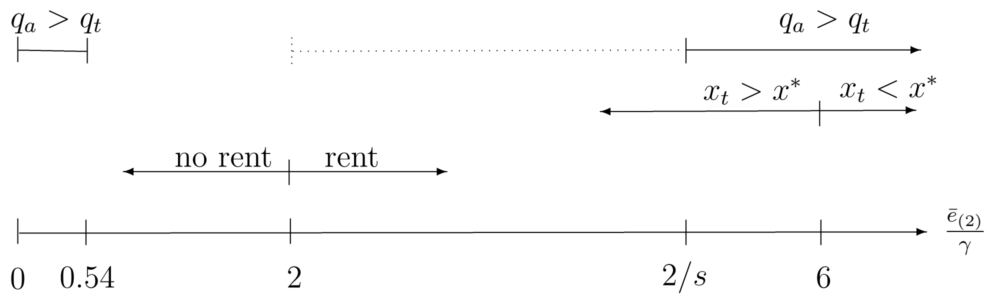

Figure 4.1: Investment and output levels.

Proof See appendix.

After proposition 4.4 it has already been explained that a high $\bar{e}_{(2)}$ reduces cost of output production, while a low γ reduces incentives to invest in innovations. Therefore, the quotient $\bar{e}_{(2)}/\gamma$ indicates the strength of agents' incentives to allocate resources to output production rather than investments. If this quotient is larger than 2, net benefits from output production exceed investment costs so that agents earn rent (see proposition 4.4). If it is larger than 6, agents also underinvest. Figure 4.1 illustrates these findings.

We now turn to a comparison of output levels, where we denote the overall expected output under the general price subsidy by $E[q_a]$, i.e., $E[q_a] \equiv E[q_i^a + q_j^a]$. The first result shows that the general output price subsidy always leads to more output for low values of $\bar{e}_{(2)}/\gamma$ and for high values of $\bar{e}_{(2)}/\gamma$ (see Figure 4.1).

Proposition 4.6 *$E[q_a] > E[q_t]$ for all $\bar{e}_{(2)}/\gamma \leq 0.54$ and all $\bar{e}_{(2)}/\gamma \geq 2/s$. For intermediate values of $\bar{e}_{(2)}/\gamma$, either the general price subsidy or the tournament may yield a higher output. In particular, $E[q_a] > E[q_t]$ is more likely if spillovers are large and if the variance of innovations is small.*

Proof See appendix.

Intuitively, under a tournament low values of $\bar{e}_{(2)}/\gamma$ lead to excessive investment incentives which implies that only few resources (i.e., a small fraction of m) are directed to output production. On the other hand, if $\bar{e}_{(2)}/\gamma$ becomes sufficiently high, investments in the tournament are no longer high enough to compensate for the loss of spillovers and shared output production. By contrast, investment and production incentives under a general price subsidy are independent of $\bar{e}_{(2)}/\gamma$.

For intermediate values of $\bar{e}_{(2)}/\gamma$, the comparison of output under the two mechanisms depends on spillover levels and on the variance of the random term. In particular, in the case with rent, comparing (4.21) and (4.35) yields

$$\mathrm{E}[q_t^r] > \mathrm{E}[q_a] \quad \Longleftrightarrow \quad \left(\frac{\bar{e}_{(2)}}{\bar{e}^2}\right)\left(\frac{\bar{e}_{(2)}}{2} + \gamma\right) > 2(1+s). \tag{4.39}$$

Similarly, for the no-rent case a comparison of (4.21) and (4.38) yields

$$\mathrm{E}[q_t] > \mathrm{E}[q_a] \quad \Longleftrightarrow \quad 4\left(\frac{\bar{e}^4_{(2)}}{\bar{e}^2}\right)\left(\frac{0.5\bar{e}_{(2)} + \gamma}{\left(1.5\bar{e}_{(2)} + \gamma\right)^3}\right) > (1+s), \tag{4.40}$$

where it follows from differentiation that the left-hand side increases in $\bar{e}_{(2)}$ and decreases in γ.

A high variance implies that the expected innovation of the more successful agent is substantially above average, i.e., that $\bar{e}_{(2)}/\bar{e}$ is large. This 'selection effect' favors the tournament because it increases the benefits of concentrating subsidies on the most successful innovator. Furthermore, high values of γ imply that the tournament exhibits a strong 'motivational effect' when agents choose innovation investments. In the rent case, this effect again favors the tournament (see 4.39). By contrast, innovation investments in the no-rent case are so high that the principal has to pay lump sum transfers $F_t > 0$ to assure the agents' participation. Accordingly, he has less resources to subsidize production. This reduces output under the tournament (see 4.40).

Finally, high spillovers favor the general price subsidy because they increase the benefits from having both agents producing output. In particular, the next result shows that the general output price subsidy always leads to more output if spillovers $s \in [0, 1]$ are sufficiently large.

Proposition 4.7 *There exists a level of spillovers $s_a < 1$ such that the general price subsidy leads to a higher expected output than the tournament for all $s > s_a$.*

Proof See appendix.

4.7 Conclusion

The main motivating example for our analysis was the problem of promoting new technologies such as renewable energies. While not being competitive yet, energy production from renewables is characterized by steep learning curves. We have captured this characteristic by assuming that production cost can be reduced by non-contractible investments into innovations, which partly spill over to other firms. These spillovers, together with our assumption of diminishing returns to scale, provide a strong rationale for inducing production from both agents in our model. This is achieved by guaranteeing a fixed output price for renewables. Such an instrument has indeed been applied rather successfully in several EU countries (WBGU 2004). For example, the share of renewables in the consumption of electricity increased in Germany from 4.6% in 1998 to about 8% in 2003 (Deutscher Bundestag [2004]).

However, with non-contractible innovation investments, agents disregard the beneficial effect of spillovers, resulting in underinvestment. In order to strengthen innovation incentives the principal may consider a tournament where only the winner receives an output price subsidy. This has the further advantage that price subsidies can be targeted at the most successful innovator. A similar system has been applied with the NFFO in the UK.[11] However, the NFFO had only limited suc-

[11] Under the NFFO, renewable energy production projects are awarded to the firm who asks the lowest price for producing a specified output. Obviously, the firm which has realized the better innovation will win the bidding competition. Furthermore, the problems of minimizing costs for a given output target and maximizing output for a given cost target are equivalent to each other (see section 4.2). Hence the NFFO bidding system is quite similar to our tournament model.

cess (Cleirigh [2001]), and it has been replaced recently by a quota system (see www.dti.gov.uk/energy/renewables).

In our model, the tournament system does always induce more innovation than the general price subsidy. Nevertheless, a comparison of output under the two mechanisms is ambiguous and mainly depends on three effects.

First, high spillovers favor the general price subsidy; and there always exists a spillover rate smaller than one such that this mechanism outperforms the tournament. However, problems may arise when agents have an influence on the spillover rate. Consider our second introductory example where workers can increase productivity by investing in their human capital. Obviously, the principal has an interest that workers share their improved knowledge. Unfortunately, doing so is most beneficial for the agent who has acquired less knowledge, as his production cost are reduced more substantially through spillovers. If this difference is large enough, output of the more successful investor may even fall as more knowledge is exchanged (see propositions 4.1 and 4.3). Accordingly, he has little incentives to do so.

Second, if the expected innovation of the more successful agent substantially exceeds that of the other agent, then this favors the tournament. Such a situation is more likely to occur if the variance of innovations is large so that its realizations are likely to differ by a large amount. In such a situation the tournament benefits from the ability to select the best innovator for production.

Finally, the tournament provides stronger innovation incentives because agents want to outperform each other. Given that the general price subsidy leads to underinvestment, this effect is beneficial; but only as long as it is not too strong. In particular, if marginal increases in innovation investments have a large effect on the probability of winning the tournament, then agents will invest too much. Due to the budget constraint, this implies that too small a share of the available budget is used for output production. Such an argument is sometimes brought forward in the context of research funding, our third introductory example. Funding agencies often invite the submission of research proposals, of which only the most promising are funded. This tournament system induces researchers to allocate substantial resources to proposal writing. Some of these resources might be better used carrying out the project itself. A system where the allocation of funds is based on research output – like our general price subsidy – may be better suited to achieve this.

In contrast to the general output price subsidy, the tournament requires the assessment of agents' realized innovations. For simplicity, we assumed that the principal can observe agents' innovations at no cost. However, the assessment of an innovation may require a substantial amount of time and some special knowledge. The costs involved must be born by the principal under a tournament scheme. Thus, high costs of assessing innovations also favor a general output price subsidy.

4.8 Appendix

The first-order conditions for problem (4.6) are

$$\frac{-E\left[(e_i x_i + e_j x_j)^{1/2}\right] c x_i}{2(2m - 0.5c(x_i^2 + x_j^2))^{1/2}} + \frac{\partial E}{\partial x_i}(2m - 0.5c(x_i^2 + x_j^2))^{1/2} = 0, \quad i, j = 1, 2, i \neq j \quad (4.41)$$

Since

$$\frac{\partial E}{\partial x_i} = \int_0^{2\bar{e}} \int_0^{2\bar{e}} \frac{e_i}{2(e_i x_i + e_j x_j)^{1/2}} f(e_i) f(e_j) de_i de_j = \frac{\partial E}{\partial x_j}, \tag{4.42}$$

optimal investment levels are identical for both agents, $x_i = x_j = x$. After substituting $x_i = x_j = x$ into problem (4.6), the new first-order condition becomes

$$\frac{2m - 3cx^2}{2(2mx - cx^3)^{1/2}} = 0, \tag{4.43}$$

which gives x^*. It is easy to verify that (4.6) is concave for $x \in [0, (2m/c)^{0.5}]$, which must hold according to (4.3).

Calculation of optimal contract with general output price subsidy. Eliminating q_i from the principal's problem, (4.14) to (4.18), and using $x_i = p_a^2 \bar{e}/c$, the problem becomes

$$\max_{p_a, F_a} \frac{2(1+s)\bar{e}^2 p_a^3}{c} \tag{4.44}$$

$$\text{s. t. } m = \frac{2(1+s)\bar{e}^2 p_a^4}{c} + 2F_a \tag{4.45}$$

$$0 \leq \frac{1}{2}(1+s)\frac{(p_a^2 \bar{e})^2}{c} - \frac{1}{4}c\left(\frac{p_a^2 \bar{e}}{c}\right)^2 + F_a \tag{4.46}$$

$$0 \leq F_a. \tag{4.47}$$

Agents' participation constraint (4.46) is satisfied for all $p_a \geq 0$ and $F_a = 0$. Since fixed payments provide no incentives, the principal sets $F_a = 0$, and p_a follows from (4.45).

Proof of proposition 4.2. Comparing (4.7) and (4.20), investment is lower under a general price subsidy iff

$$\frac{m}{2(1+s)c} < \frac{2m}{3c}, \tag{4.48}$$

which always holds since $s \in [0, 1]$. Since investments are suboptimal, expected output is smaller than in the first-best solution. Finally, as $p_a > 0$ the participation constraint (4.46) holds with strict inequality.□

Derivation of $\bar{e}_{(2)}$ and equation (4.26). The first-order condition of (4.25) is

$$\frac{p_t^2}{2}\left(\int_0^{2\bar{e}} \left[\int_{\frac{x_j}{x_i}e_j}^{2\bar{e}} e_i f(e_i) de_i\right] f(e_j) de_j + x_i \int_0^{2\bar{e}} \frac{x_j}{x_i} e_j f\left(\frac{x_j}{x_i} e_j\right) \frac{x_j}{x_i^2} e_j f(e_j) de_j\right) - \frac{c}{2} x_i = 0. \tag{4.49}$$

At a symmetric equilibrium $x_i = x_j \equiv x$, it simplifies to

$$\frac{p_t^2}{2} \int_0^{2\bar{e}} \left[\int_{e_j}^{2\bar{e}} e_i f(e_i) de_i + e_j^2 f(e_j)\right] f(e_j) de_j - \frac{c}{2} x = 0. \tag{4.50}$$

Furthermore, the expected value of the second-order statistic is

$$\bar{e}_{(2)} \equiv 2\int_0^{2\bar{e}} e_i F(e_i) f(e_i) de_i \tag{4.51}$$

$$= 2\int_0^{2\bar{e}} e_i \int_0^{e_i} f(e_j) de_j f(e_i) de_i \tag{4.52}$$

$$= 2\int_0^{2\bar{e}} \int_{e_j}^{2\bar{e}} e_i f(e_i) f(e_j) de_i de_j, \tag{4.53}$$

where the last equation follows because the ranges of integration are identical. Substitution into (4.50) and rearranging yields (4.26).

Proof of propositions 4.4. From (4.29), $\mathrm{E}[q_t] = p_t^3 \bar{e}_{(2)}\Gamma/c$. Using this and $x_i^t = \Gamma p_t^2/c$ (from 4.26), the principal's problem becomes

$$\max_{p_t, F_t} \frac{\bar{e}_{(2)}\Gamma p_t^3}{c} \tag{4.54}$$

$$0 = m - \bar{e}_{(2)}\frac{p_t^4}{c}\Gamma - 2F_t \tag{4.55}$$

$$0 \leq \bar{e}_{(2)}\frac{p_t^4}{c}\Gamma - \frac{p_t^4}{c}\Gamma^2 + 4F_t. \tag{4.56}$$

The participation constraint does not bind at $F_t = 0$ iff $\bar{e}_{(2)} > \Gamma \Leftrightarrow \bar{e}_{(2)} > 2\gamma$.□

Proof of proposition 4.5. In the case with rent, comparing (4.34) and (4.7) yields the result regarding the first-best solution. From the comparison of (4.34) and (4.20) it follows that the tournament leads to higher investments than the general price subsidy iff $\bar{e}_{(2)}/\Gamma < 2(1+s)$ which holds since $\bar{e}_{(2)}/\Gamma < 2$. Furthermore, comparing (4.34) and (4.37) shows that agents invest more in the no-rent case than in the rent case, i.e.,

$$x_t^r = \left(\frac{m\Gamma^r}{c\bar{e}_{(2)}^r}\right)^{1/2} < \left(\frac{2m\Gamma}{c(\Gamma + \bar{e}_{(2)})}\right)^{1/2} = x_t \tag{4.57}$$

$$\Longleftrightarrow \frac{\Gamma^r}{\bar{e}_{(2)}^r} < \frac{2\Gamma}{\Gamma + \bar{e}_{(2)}}, \tag{4.58}$$

where the last inequality holds since by proposition 4.4 the l.h.s. is smaller than 1 and the r.h.s. is larger than 1.□

Proof of proposition 4.6. The first condition concerns the no-rent case ($\bar{e}_{(2)} \leq 2\gamma$). Comparing (4.21) and (4.38) yields

$$\mathrm{E}[q_a] = \left(\frac{2(1+s)}{c}\right)^{1/4} m^{3/4}\bar{e}^{1/2} > \bar{e}_{(2)}\left(\frac{\Gamma}{c}\right)^{1/4}\left(\frac{2m}{\Gamma + \bar{e}_{(2)}}\right)^{3/4} = \mathrm{E}[q_t] \tag{4.59}$$

$$\Longleftrightarrow \frac{\bar{e}_{(2)}}{\Gamma} + \left(\frac{\Gamma}{\bar{e}_{(2)}}\right)^2 + \frac{3\Gamma}{\bar{e}_{(2)}} + 3 > \left(\frac{\bar{e}_{(2)}}{\bar{e}}\right)^2 \frac{4}{1+s}. \tag{4.60}$$

Define $y \equiv \bar{e}_{(2)}/\Gamma$ and note that the l.h.s. of (4.60) is decreasing in $\bar{e}_{(2)}/\Gamma$ as

$$\frac{d}{dy}\left(y + y^{-2} + 3y^{-1}\right) = 1 - 2y^{-3} - 3y^{-2} < 0 \qquad \text{for all } y \leq 2. \tag{4.61}$$

Furthermore, $\bar{e}_{(2)} < 2\bar{e}$. Since (i) the r.h.s. of (4.60) is strictly smaller than 16, and (ii) the l.h.s. equals 16 at $\bar{e}_{(2)}/\Gamma \approx 0.425$ and is decreasing in $\bar{e}_{(2)}/\Gamma$, $\bar{e}_{(2)}/\Gamma < 0.425$ is sufficient for (4.60) to hold. The proposition follows after substituting $\Gamma = \bar{e}_{(2)}/2+\gamma$. The second condition concerns the rent case ($\bar{e}_{(2)} > 2\gamma$). Comparing (4.35) and (4.21) we get

$$\mathrm{E}[q_a] = \left(\frac{2(1+s)}{c}\right)^{1/4} m^{3/4}\bar{e}^{1/2} \quad > \quad m^{3/4}\left(\frac{\bar{e}_{(2)}\Gamma}{c}\right)^{1/4} = \mathrm{E}[q_t^r] \tag{4.62}$$

$$\iff 2(1+s)\frac{\bar{e}_{(2)}}{\Gamma} \quad > \quad \left(\frac{\bar{e}_{(2)}}{\bar{e}}\right)^2. \tag{4.63}$$

The r.h.s. is strictly smaller than 4. Upon substituting for Γ, the l.h.s. is larger than 4 if $\bar{e}_{(2)}/\gamma > 2/s$.□

Proof of proposition 4.7. We first consider the case with rent, where the condition for $\mathrm{E}[q_a] > \mathrm{E}[q_t^r]$ can be rewritten as (see 4.63)

$$s > \frac{\bar{e}_{(2)}\Gamma}{2\bar{e}^2} - 1. \tag{4.64}$$

Define

$$s_a \equiv \frac{\bar{e}_{(2)}\Gamma}{2\bar{e}^2} - 1. \tag{4.65}$$

Then $s_a < 1$ because

$$\bar{e}_{(2)}\Gamma < \bar{e}_{(2)}^2 < (2\bar{e})^2 = 4\bar{e}^2, \tag{4.66}$$

where the first inequality holds because $\Gamma < \bar{e}_{(2)}$ in the rent case of the tournament. Turning to the no-rent case, note that the maximum value of the l.h.s of (4.60) is 8, which it obtains at $\bar{e}_{(2)}/\Gamma = 1$. It follows that a sufficient condition for $\mathrm{E}[q_a] > \mathrm{E}[q_t]$ is

$$\left(\frac{\bar{e}_{(2)}}{\bar{e}}\right)^2 \frac{1}{1+s} < 2. \tag{4.67}$$

Finally, from the fact that $\bar{e}_{(2)}/\bar{e} < 2$ it follows that there exists an $s_a < 1$ such that $\mathrm{E}[q_a] > \mathrm{E}[q_t]$ for all $s \geq s_a$.□

Summary

Each of the four essays of this thesis was concerned with incentive schemes that help to mitigate moral hazard problems if agents' performances are non-verifiable. The aim was not to characterize optimal incentive mechanisms but to identify advantages and drawbacks of real-world incentive schemes. Although the analyzed situations are, naturally, stylized representations of reality, the results derived provide some guidance for choosing between the incentive devices under consideration.

The first essay points out some important aspects for the design of relative incentive schemes in employment relationships. Bonus pools do, in general, not lead to highly unequal payments to agents. This is an advantage if agents are inequity-averse. By contrast, fixing prizes ex ante increases inequity costs. However, under fixed prizes, the principal can lower the prize spread by increasing the precision of performance measurement. Therefore, fixing prizes may be beneficial if agents earn rents, because then the principal can lower the overall wage payment by collecting more information about agents' performances. As a result, the principal should use bonus pools if agents are inequity-averse but do not earn rents. In the other extreme, when selfish agents earn rents, fixing prizes maximizes profit. For cases in between these two extremes, i.e., with inequity-averse agents that may earn rents, the principal's optimal choice depends on her costs of measuring agents' performances. If additional information on performance is available at low costs, fixing prizes is optimal. These rules apply under the assumptions that agents are risk-neutral and homogeneous, do not receive intermediate information about the performance of their colleagues, and cannot bribe the principal.

The second essay provides some insights into the optimal interplay of job design and relational contracts. The existence of relational contracts favors broader task assignments because thereby relational contracts are further strengthened. The principal benefits from a high-powered relational contract even if this contract prevents first-best effort in one task. Relational contracts exist if contractible performance measures are highly distorted or firm value strongly responds to effort changes in the given tasks. Thus the principal benefits from assigning additional tasks to agents who already perform tasks that are important for the firm but whose contribution to firm value is difficult to measure.

The last two essays deal with situations in which agents should invest in the development of a good or an innovation. In the third essay, the principal cares about the quality of a good that she wishes to procure. In the fourth essay, the principal wants to maximize the output of a good whose production costs can be lowered if agents invest in innovations.

In the procurement setting, to induce investments in quality, the procurer can

either fix a prize for the firm with the best prototype ex ante, or hold an auction ex post. Furthermore, the procurer decides on the precision with which she assesses the quality of the prototypes. It turns out that fixing a prize ex ante is optimal if increasing the precision of performance measurement is very costly, or if random influences on quality are high. In this case, the price in the auction, which is determined by the difference between firms' quality signals, is high. High randomness of quality and performance measurement also increase the prize in the tournament. However, in the tournament the procurer also benefits from the fact that he can lower firms' investment incentives by choosing a lower prize.

In the fourth essay, the principal always fixes output prices ex ante. Innovations are observable by the principal at no cost. The principal can either only reward the agent with the best innovation by a positive output price, or choose the same positive output price for both agents. In the latter case, in which both agents produce, there is a further decrease of production costs due to spillovers. Therefore, high spillovers favor a general price subsidy. A highly random innovation process makes it more worthwhile to concentrate subsidies on the more efficient agent, because his expected innovation is substantially better than the one of the less efficient agent.

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Selbständigkeitserklärung

Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung meiner Dissertation benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

Berlin, den 30. März 2005 Anja Schöttner