Trading on Deviations of Implied and Historical Distributions

A **Diploma Thesis** *Presented*

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Declaration of Authorship

I hereby confirm that I have authored this diploma thesis independently and without use of others than the indicated resources.

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Abstract

This diploma thesis tries to motivate the notion of state price densities (SPD's), surveys briefly some methods for estimating option implied state price densities and following investigates trading strategies designed to exploit deviations in skewness and kurtosis of an option implied SPD from a time series SPD of the german DAX index. While the option implied SPD is estimated by means of the Barle and Cakici implied binomial tree version using a cross section of DAX option prices, the historical density is inferred from the time series of the DAX index by a method applied to S&P500 by Ait–Sahalia et al.

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Contents

1 Introduction

In recent years a number of methods have been developed to infer implied state price densities (SPD) from cross sectional option prices. Previous studies in the literature have always compared this density to a historical density extracted from the observed time series of the underlying asset prices, i.e. a risk neutral density to an actual density. Ait–Sahalia et al. (2000) proposed not to compare a risk neutral density and an actual density, but to compare two risk neutral densities, one obtained from cross sectional S&P 500 option data and the other from the S&P 500 index time series. Furthermore, they proposed trading strategies designed to exploit differences in skewness and kurtosis of both densities. The goal of this diploma thesis is to apply the procedure to the german DAX index. While the option implied SPD is estimated by means of the Barle and Cakici implied binomial tree version, the time series density is inferred from the time series of the DAX index by applying a method used by Ait–Sahalia et al. Knowing both SPD the performance of skewness and kurtosis trades is investigated.

The options data utilized is obtained from the MD*BASE (http://www.mdtech.de) database which is included in XploRe. The time period is limited to data of the period between 01/01/97 and 12/31/99 for which the MD*BASE database contains daily closing prices of the DAX index, EUREX DAX option settlement prices and annual interest rates which are adjusted to the time to maturity of the above mentioned EUREX DAX options.

This diploma thesis begins in section 2 with a short review of option pricing theory as far as European call and put options are concerned. Section 3 tries to motivate briefly the theoretical background connecting option prices, state prices and risk neutral densities. The 4th section surveys some methods for estimating option implied state price densities. While the 5th section explains and applies the Barle and Cakici implied binomial tree algorithm version which estimates the option implied SPD using a two week cross section of DAX index options, the 6th section explains and applies the method to estimate DAX time series SPD from 3 months of historical index prices. Following, the 7th section compares the conditional skewness and kurtosis of both SPD's. The 8th section completes the thesis with the investigation of 4 trading strategies and section 9 concludes.

2 Option Pricing: A Short Review

Derivatives are financial instruments whose value depends on the values of other, more basic underlying variables or just the occurrence of a certain state in the future. One of the most famous derivatives is the option. A call option is the right to buy a particular asset (a stock S_T for example) for an agreed amount (strike K) at a specified time (expiry time T) in the future.

Option pricing theory focuses on the problem of pricing and hedging derivative securities. It is the first point which I am going to detail a further. The pricing problem can be restated in the following question. 'How much would an investor in time t be willing to pay for an option? What is it worth now, before expiry?'

At a first glance, one could state that the price C of an option with payoff function $h(S_T)$ is equal to the present value of the future cash flow which in turn is simply equal to the discounted expected value of the cash flow:

$$C = E(h) = e^{-r(T-t)} \int_0^\infty h(S_T)p(S_T)dS_T,$$

where p is the objective (historical) probability density function of the random variable S_T.

Even though such a pricing rule is appealing to economic intuition it does not need to be consistent with the concept of arbitrage. For example, Black and Scholes (1973) remarked that when the price of the underlying asset S_t follows a geometric Brownian motion diffusion, the expectation pricing rule defined above applied to European call options can create arbitrage opportunities. Furthermore, they showed that the absence of arbitrage opportunities is sufficient to define a unique price for a European call option, independently of the preferences of market agents. The methodology introduced by Black and Scholes was subsequently generalized to diffusion processes defined as solutions of stochastic differential equations.

Harrison and Kreps (1979) showed that, at the expense of calculating expectations with respect to another density q different from p a priori, it is still possible to price derivatives using expected payoffs. Harrison and Pliska (1981) proved that in a market where asset prices are described by stochastic processes, the absence of arbitrage opportunities implies the existence of an equivalent martingale measure Q, such that all (discounted) asset prices are Q–martingales. This martingale property then implies that the price of any European option can be calculated as the expectation of its payoff under the probability measure Q. In particular, the price C of a European call option is given by:

$$\begin{aligned} C(S_t, K, r, T-t) &= e^{-r(T-t)} \int_0^\infty \max(S_T - K, 0)dQ(S_T) && (1) \\ &= e^{-r(T-t)} \int_0^\infty \max(S_T - K, 0)q(S_T)dS_T && (2) \end{aligned}$$

where q is the density of the measure Q. This density has been given several names in the literature: state price density (SPD), risk neutral density (RND) and equivalent martingale density. One has to be aware not to confuse these notions. They coincide in the case of the Black–Scholes Model. However, in general, these notions correspond to different objects. A 'martingale measure' refers to the property that asset prices are expected to be Q–martingales. If markets are incomplete there exist in general infinitely many martingale measures. Such an equivalent martingale measure is uniquely defined if and only if markets are complete. A risk neutral density refers to the case where all contingent payoffs can be replicated by a self financing portfolio strategy. While the two former probability measures are abstract mathematical properties of the underlying asset stochastic process, a state price density should be viewed as a way of characterizing the prices of derivatives on these assets. It is rather the density used to price options using equation(1) and has therefore a 'forward looking character'. I will use the terms SPD, RND and equivalent martingale measure interchangeably.

To summarize, in the framework of arbitrage free markets each asset is characterized by two different probability densities, the historical density and the SPD. These two densities are different a priori and except in special cases such as the Black–Scholes model arbitrage arguments do not enable us to calculate one of them given the other. Let me mention, that this section profited a lot from Cont (1998).

3 The Relationship Between Option Prices, State Prices and RND's

An option is a special type of a derivative. Since there is no rule how a derivative has to be, other types of derivatives are imaginable. For example an Arrow–Debreu security, also called binary option, is an instrument which pays off 1 EURO if a certain state occurs and 0 EURO if not.

Let us consider a simple example to see how the prices of these instruments are related. Suppose we live in a 'binomial world' where we have one period starting in $t = 0$ and ending in $t = 1$, one risk free interest rate, r, for this period, two states ω_1 and ω_2 in $t = 1$, a bank account whose value in $t = 0$ is equal to B, a stock with value S in $t = 0$, an option C_K, with strike K, and two Arrow–Debreu securities, AD_1 and AD_2 with prices λ_1 and λ_2 in $t = 0$, related to states ω_1 and ω_2, respectively. That is, AD_1 pays off 1 EURO in ω_1 and nothing in ω_2. Conversely, AD_2 pays off 1 EURO in ω_2 and nothing in ω_1. Say, in $t = 1$ the stock either moves *up* and is worth

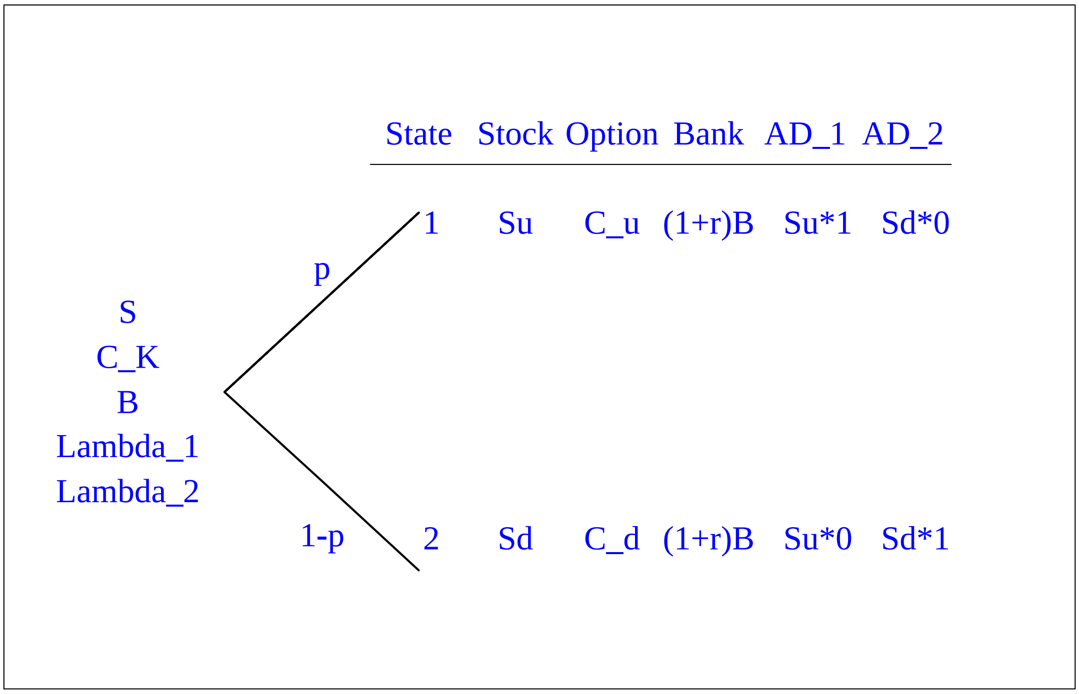

Figure 1: *Illustration of the one step binomial tree example.*

$S \times u$ in state ω_1 or moves *down* and has a price of $S \times d$ in ω_2. Furthermore, let us suppose $u > d$. Creating a portfolio A consisting of a long position of Su AD_1–securities with price λ_1 and Sd AD_2–securities with price λ_2 we generate the same payoff pattern as for the stock. The payoff in state ω_1 is $Su \times 1 + Sd \times 0 = Su$ and in ω_2 it is $Su \times 0 + Sd \times 1 = Sd$. From no–arbitrage arguments we conclude that in $t = 0$ portfolio A must have the same value as the stock:

$$S = Su \times \lambda_1 + Sd \times \lambda_2. \tag{3}$$

The bank account has in both states the value $(1+r)B$. The same reasoning as above leads us to:

$$B = (1+r)B \times (\lambda_1 + \lambda_2). \tag{4}$$

If we solve both equations for the prices of the two Arrow–Debreu securities, we find:

$$\lambda_1 = \frac{1+r-d}{(1+r)(u-d)} \tag{5}$$

$$\lambda_2 = \frac{u-(1+r)}{(1+r)(u-d)}. \tag{6}$$

In the binomial tree model the risk neutral probabilities p of an upmovement and $1-p$ of an downmovement of the stock index are given by (see Hull (1989)):

$$p = \frac{1+r-d}{u-d} \tag{7}$$

$$1-p = \frac{u-(1+r)}{u-d}. \tag{8}$$

Using (5) – (8) we derive the relationship between the state prices and the risk neutral probabilities:

$$\lambda_1 = \frac{1}{1+r} \times p \tag{9}$$

$$\lambda_2 = \frac{1}{1+r} \times (1-p). \tag{10}$$

We obtain the price of a call option C_K using the standard binomial pricing formula:

$$C_K = \frac{pC_u + (1-p)C_d}{1+r}, \tag{11}$$

where $C_u = \max(0, Su - K)$ and $C_d = \max(0, Sd - K)$. Finally, inserting (9) and (10) into (11) gives us a formula similar to the one for the stock derived above

$$C_K = C_u \times \lambda_1 + C_d \times \lambda_2. \tag{12}$$

Looking at (3) and (4), we see that the state prices are proportional to the risk neutral probabilities. They differ only by a proportionality factor $1/(1+r)$. In fact, using the risk neutral probability function state prices can be interpreted as the expected future payoff of 1 EURO. If we knew these risk neutral probabilities, we could, given r, deduce the state prices and calculate the price of the call option.

Contrary to this small example with two states, in the real world exists an infinite number of possible states making it very difficult to identify the specific states and their prices. Breeden and Litzenberger (1978), derived a similar result in a world with infinitely many states. Let $C(S_t, K, r, \tau)$ be the price of a call option on the underlying S_t with strike K, time to maturity τ and facing the risk free rate r. Then the state price density can be obtained by taking the second derivative of C with respect to the exercise price K:

$$q(S_T) = e^{r\tau} \frac{\partial^2 C}{\partial K^2}. \tag{13}$$

To see this, I sketch the 'demonstration' given in Breeden, Litzenberger (1978) (see also Bahra (1997)). Note that the one unit payoff of an elementary claim at a future state $S_T = K$ can be replicated (in the limit) by a butterfly spread centered on state $S_T = K$. That is, sell two call options with exercise price $K = S_T$ and buy one call option with strike $K = S_T - \Delta S_T$ and buy one call option with $K = S_T + \Delta S_T$. Note, ΔS_T is a small change in S_T and not any Greek delta. Thus, we build up a portfolio consisting of 4 call options expiring at the same future date T. The payoff is given by:

$$\frac{[C(S_t, K + \Delta S_T, r, \tau) - C(S_t, K, r, \tau)] - [C(S_t, K, r, \tau) - C(S_t, K - \Delta S_T, r, \tau)]}{\Delta S_T} = 1 \tag{14}$$

where the left hand expression is evaluated at $K = S_T$.

As ΔS_T goes to zero, the payoff function of the butterfly spread tends to a Dirac delta function with its mass at $K = S_T$. With other words, in the limit the butterfly becomes an Arrow–Debreu security paying 1 EURO if $S_T = K$ and zero otherwise.

Let $P(S_t, K, r, \tau)$ be the current (time t) price of an butterfly spread centered on K, then we can make appear the second order difference quotient of the call option pricing function:

$$\begin{aligned} \frac{P(S_t, K, r, \tau)}{\Delta S_t} &= \frac{[C(S_t, K + \Delta S_T, r, \tau) - C(S_t, K, r, \tau)]}{(\Delta S_T)^2} - \\ &\quad \frac{[C(S_t, K, r, \tau) - C(S_t, K - \Delta S_T, r, \tau)]}{(\Delta S_T)^2} \end{aligned} \tag{15}$$

evaluated at $K = S_T$. In the limit the right hand expression tends to the second derivative of the call pricing function with respect to the strike price K, evaluated at $K = S_T$:

$$\lim_{\Delta S_T \to 0} \frac{P(S_t, K, r, \tau)}{\Delta S_t} = \frac{\partial^2 C(S_t, K, r, \tau)}{\partial K^2}. \tag{16}$$

If we could observe prices of butterfly spreads across all states, each with infinitely small step sizes between strike prices, then we could deduce the state pricing function. Expressing the price of an Arrow–Debreu security as an expected future payoff, $e^{-r\tau} \times q(S_T)$, i.e. calculating the present value of 1 EURO multiplied by the risk neutral probability $q(S_T)$ of the state, $S_T = K$, giving rise to the payoff of 1 EURO. Equated with the right hand side of equation(16) gives us the result.

4 Methods for Estimating State Price Densities

Since Black and Scholes (1973) published their famous article, the options market emerged rapidly. Today, many options are liquid assets and their price is determined by the interaction of supply and demand. Thus market prices can be considered observations from the market and not quantities to be fixed by a mathematical approach. Since there is now a large body of evidence against the Black–Scholes assumption of a lognormal distribution for asset prices, a new direction emerged in research. Starting from a set of option prices search for a density q such that equation(1) holds. Such a distribution q is called an implied distribution. In the absence of arbitrage, this notion coincides with the concept of SPD. I will use these terms interchangeably in the following. I will now describe a few methods for extracting information about the SPD.

4.1 Parametric Methods

4.1.1 Mixtures of Distributions

An approach which can be followed is to assume a particular parameterized stochastic process for the price of the underlying asset and to use observed option prices to recover the parameters which, in turn, can be used to infer the implied RND function. Under sufficiently strong assumptions about the underlyings' stochastic process, the RND function can be obtained in a closed form, as it is the case in the famous Black–Scholes model, where the underlying asset follows a geometric brownian motion with constant drift and volatility implying a lognormal RND function.

Rather than specifying the underlying asset price dynamics what implies a unique RND function once the parameters are known, it is less restrictive to make only assumptions about the functional form of the RND function itself. The parameters can be calculated so as to reproduce as closely as possible a set of option prices, for example using a least squares method. Using this method we cover several possible asset price dynamics.

Let the terminal payoff function of a European style option maturing at time T be $h(x)$. Assuming a constant risk free interest rate r, the price C of the option is the discounted expected payoff under the risk neutral probability measure Q:

$$C(S_T, K, r, \tau) = e^{-r\tau} \int_{-\infty}^{\infty} h(S_T) q(S_T) dS_T,$$

where $q(x)$ is the risk neutral probability density function. In theory any functional form for $q(x)$ can be used but the habit of working with the lognormal distribution by reference to the Black–Scholes model has led to models representing the state price density as a mixture of lognormals of different parameters.

$$\begin{aligned} q(S_T, K, r, \tau) &= \sum_{k=1}^{K} \theta_k LN(\alpha_k, \beta_k, S_T) \\ \sum_{k=1}^{K} \theta_k &= 1, \end{aligned}$$

where $LN()$ denotes a lognormal density function with parameters $\alpha_k = lnS_t + (\mu_k - \frac{1}{2}\sigma_k^2)\tau$ and $\beta_k = \sigma_k^2\sqrt{\tau}$.

Melick and Thomas (1997) use mixtures of three lognormal distributions. They apply the above framework to options on crude oil futures. Bahra (1997) uses a mixture of two lognormals on interest rates futures. Ritchey (1990) points out that a wide variety of shapes may be approximated with a mixture of lognormal distributions and gives examples of option prices where the RND of the log–return is a mixture of normal distributions.

A problem with mixtures is that the number of parameters increases fast. That is, mixing three lognormal distributions yields eight parameters. Given that there are often only ten or twenty observed option prices across different strike prices, mixture methods can easily overfit the data. Furthermore, Clews, Panigirtzoglou and Proudman (2000) point out that this parametric approach proved to generate SPD's characterised by sharp spikes. This occurs when one of two lognormal densities has a small variance while the other has a large one. Another point is that this method can generate implausible large changes in the shape of the SPD between consecutive days concerning in particular the skewness and kurtosis measures.

4.1.2 Generalized Distributions

Generalized methods use distributions with more inherent flexibility, which tend to be more parameterized than the normal or lognormal distributions. Generalized distributions often include common distributions as limiting cases for particular parameter constellations. Aparicio and Hodges (1998) use generalized beta functions of the second kind. This is a family of distributions which is described by four parameters and which includes the lognormal, gamma and exponential distributions, several Burr type distributions and others.

4.2 Expansions

Expansion methods are conceptually related to a series expansion of the SPD which is then truncated to give a parametric approximation. They typically start with a simple (often normal or lognormal) distribution, and then add correction terms in order to obtain a more flexible shape of the SPD. Therefore, considering the SPD as a general probability distribution we write:

$$P(S_T - S_t \leq x) = P_0(x) + \sum_{k=1}^{\infty} u_k P_k(x),$$

where $P_0(x)$ corresponds either to the normal or lognormal distribution. The parameters are then fitted to the observed option prices using statistical procedures or calibration methods. A general concern in this class of methods is that for certain parameter constellations the SPD can take on negative values, what can occur especially in the tails of the distribution. This problem is likely to arise since the SPD tails off to zero for large and small stock prices. If the correct density is close to zero an error with negative sign introduced by the approximation can lead to negative values. This drawback should not be viewed as prohibitive however. It only means that these methods should not be used to price options too far from the money (Cont (1998)). In the following, I will give a brief survey of a few expansion methods which I extracted mainly from Cont (1998) and Jackwerth (1999).

4.2.1 Cumulant Expansions

Potters, Cont, Bouchaud (1998) use cumulant expansions to add a single correction term to a normal distribution, which adjusts for the kurtosis of the SPD. The cumulants are defined as the coefficients of a Taylor expansion of the logarithm of the Fourier transform of a probability distribution q:

$$\ln \Phi_T(z) = \sum_{j=1}^{n} c_j(T) \frac{(iz)^j}{j!} + o(z^n),$$

where

$$\Phi_T(z) = \int q_{t,T}(x) e^{izx} dx$$

is the Fourier transform of q.
The cumulants are related to the central moments μ_j by the relations:

$$\begin{aligned} c_1 &= \mu_1 \\ c_2 &= \sigma^2 \\ c_3 &= \mu_3 \\ c_4 &= \mu_4 - 3\mu_2^2. \end{aligned}$$

Skewness s and kurtosis κ are defined as the third and fourth normalized cumulants:

$$s = \frac{c_3}{\sigma^2} \qquad \kappa = \frac{c_4}{\sigma^2}.$$

4.2.2 Edgeworth Expansions

An Edgeworth expansion is an expansion of the difference between two probability densities in terms of their cumulants:

$$\begin{aligned} q_1(x) - q_2(x) &= \frac{c_2(q_1) - c_2(q_2)}{2} \frac{d^2 q_2}{dx^2} - \frac{c_3(q_1) - c_3(q_2)}{3!} \frac{d^3 q_2}{dx^3} \\ &+ \frac{c_4(q_1) - c_4(q_2) + 3(c_2(q_1) - c_2(q_2))^2}{4!} \frac{d^4 q_2}{dx^4}. \end{aligned}$$

Jarrow and Rudd (1982), Corrado and Su (1996), Longstaff (1995) and Rubinstein (1998) use Edgeworth expansions, where the base distribution which can be normal, lognormal or binomial is augmented with correction terms that successively match each of the first four cumulants. The authors then search for the set of cumulants that gives the best fit between observed option prices and option prices based on the expanded SPD.

4.2.3 Hermite Polynomials Expansions

Abken, Madan and Ramamurtie (1996) use a Hermite polynomial expansion. These polynomials are related to successive derivatives of the normal densitiy function and provide four correction terms to the normal distribution. The kth Hermite polynomial is defined as:

$$\phi_k(x) = e^{x^2/2} \frac{d^k \phi_0}{dx^k} \qquad \phi_0(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}.$$

The approach is based on the properties of Hermite polynomials which form an orthonormal basis for the scalar product:

$$< f, g >= \frac{1}{\sqrt{2\pi}} \int g(x) f(x) e^{x^2/2} dx.$$

Expanding the SPD on this basis gives:

$$q_{t,T}(x) = \frac{1}{\sqrt{2\pi}} \sum_{k=0}^{\infty} b_k \phi_k(x) e^{x^2/2} \qquad b_k = \int q_{t,T}(x) \phi_k(x) dx.$$

Using an expansion in the Hermite polynomial basis for the payoff function h:

$$h(x) = \sum_{k=0}^{\infty} a_k \phi_k(x) \qquad a_k =< h, \phi_k >$$

we can express the price of an option C with payoff h as a linear combination of the coefficients q_k:

$$C = e^{-r(T-t)} \int q_{t,T}(x) h(x) dx = e^{-r(T-t)} \sum_{k=0}^{\infty} a_k b_k.$$

Since the coefficients a_k can be calculated analytically for a given payoff function h, one can truncate the expansion for the price C at order n and calculate b_k, $1 \leq k \leq n$, by minimizing the distance between a set of observed option prices and those that are generated by the assumed truncated expansion.

4.3 Non–Parametric Methods

Non–parametric methods enable to achieve greater flexibility in estimating the SPD by using model free statistical procedures based on very few assumptions about the data generating process. Thus, they allow more general functions. Several types of non–parametric methods have been proposed in the context of the study of option prices. In this section I divide the methods into four groups. First, implied risk neutral histograms are a simple way of approximating the SPD. Second, kernel methods used to fit the call pricing function directly. Third, fitting the implied volatility smile curve. Fourth, maximum entropy methods find a non–parametric probability density function minimizing an information criterion.

4.3.1 Histogram

As shown above, the discrete valuation equation(15) compounded at the risk free interest rate r can be used. It gives an approximation to the risk neutral probability of the underlying asset price lying at state $S_T = K$ at time T (see Bahra (1997)). That is, applying

$$\begin{aligned} q(S_T) &= e^{r\tau} \frac{[C(S_t, S_T + \Delta S_T, r, \tau) - C(S_t, S_T, r, \tau)]}{\Delta S_T} \\ &\quad - e^{r\tau} \frac{[C(S_t, S_T, r, \tau) - C(S_t, S_T - \Delta S_T, r, \tau)]}{\Delta S_T} \end{aligned}$$

evaluated at $S_T = K$ to call prices observed across a range of exercise prices results in the implied terminal risk neutral histogram of the underlying asset price.

One of the drawbacks of this method is that it relies on options being traded on equally spaced strikes. Also, there is no systematic way of modelling the tails of the histogram, which may not be observable due to the limited range of exercise prices traded in the market. Furthermore, nothing in this procedure can adjust for the existence of arbitrage opportunities indicated by negative probabilities which may be due to measurement errors or which may arise in cases where bid ask spreads are observed rather than actual traded prices.

4.3.2 Kernel Regressions

Kernel methods try to fit a function to a set of observed data points without specifying the parametric form of the function. These methods view each data point as the center of a region where the true function could be lying. The points in the neighbourhood of a certain data point are assumed to contain some amount of information that can contribute in the estimation of the value of the true function. The farther away a point is the less information it is assumed to contain and the less weight it will be attributed in the estimation procedure. A kernel function, which has the properties of a centered probability density function tailing off to zero, is used as a weighting operator. It measures the drop in likelihood that a data point far away from the data point to be estimated contributes to the value of the true function. Kernel methods are especially useful if the data set does not contain observations of the variable of interest for each possible data point.

Ait–Sahalia and Lo (1998) take a non–parametric approach and apply the 'Nadaraya–Watson' kernel estimator to estimate the entire option pricing formula $\hat{C}(\cdot)$. Then they differentiate this estimator twice with respect to the strike K to obtain $\partial^2 \hat{C}(\cdot)/\partial K^2$. Under suitable regularity conditions, the convergence (in probability) of $\hat{C}(\cdot)$ to the true option pricing formula $C(\cdot)$ implies that $\partial^2 \hat{C}(\cdot)/\partial K^2$ will converge to $\partial^2 C(\cdot)/\partial K^2$. Defining the parameter vector:

$$Z_i = [S_{t_i}, K_i, \tau_i, r_{t_i,\tau_i}, \delta_{t_i,\tau_i}],$$

where δ_{t_i,τ_i} denotes the dividend yield, the 'Nadaraya–Watson' kernel estimator is given by:

$$\hat{C}(Z) = \frac{\sum_{i=1}^{n} K((Z - Z_i)/h)C_i}{\sum_{i=1}^{n} K((Z - Z_i)/h)},$$

where C_i are the observed option prices associated to the parameters Z_i, h is the bandwidth which governs the smoothness and $K(\cdot)$ is a five dimensional kernel function that integrates to one.

The fact that the non–parametric regression approach involves a large number of regressors, coupled with the necessity to compute the second order derivative of $C(\cdot)$ makes this approach very data intensive and thus limiting its use.

4.3.3 Fitting the Implied Volatility Curve

The call price function has a large curvature for options near–the–money and very little curvature for options far out–of–the–money. A problem of a direct interpolation of the latter function is that it is difficult to fit its shape accurately. And since we are interested in the convexity of that function any small errors will tend to be magnified into large errors in the final estimated SPD. Shimko (1993) argued that Black–Scholes implied volatilities are more smooth. He interpolated the smile curve by fitting a quadratic polynomial of the strike price. He then used the Black–Scholes formula to translate the implied volatilities into option prices and diffentiated it twice with respect to the strike. Thus, he determined the SPD between the lowest and the highest strike options. Finally, Shimko extrapolated beyond the traded strike range by grafting lognormal tails onto the SPD.

Shimko's assumption that the smile can be represented by a quadratic function seems to be restrictive. Actual implied volatilities tend not to follow a parabolic form at deep out–of–the–money strikes (Bahra (1997)). Furthermore, grafting lognormal tails onto the SPD amounts to assigning a constant volatility structure to the smile outside of the traded strikes. Another problem arises from the fact that the final distribution is pieced together from three seperate parts. It is not always possible to ensure a smooth transition to the tails (see Bahra (1997)).

4.3.4 Maximum Entropy

Buchen & Kelly (1996) and Stutzer (1996) have proposed a method for estimating the SPD based on a statistical information theoretic approach, the maximum entropy method. These methods find a non–parametric probability density function which is as close as possible in terms of information content to a prior distribution. A measure of the information content is the entropy of a probability density q defined as:

$$S(q) = -\int_0^\infty q(x) \ln q(x) dx$$

which is maximized under the constraint:

$$\int_0^\infty q(x) dx = 1$$

and subject to correct pricing of options and the underlying.

Minimizing the Kullback–Leibler distance between q and the historical density p defined as:

$$S(p,q) = \int p(x) \ln \frac{p(x)}{q(x)} dx$$

is an extension of the method described above, where the historical density function is implicitly assumed to be the uniform density function.

However, one of the drawbacks of these methods results from the absence of smoothness constraints. One typically gets multimodal estimates of the SPD which may surprise since the historical PDF's of stock returns are always unimodal as Cont (1998) points out.

5 Estimation of the Option Implied SPD

While the previous section reviewed briefly SPD estimation techniques, this section details the Implied Binomial Tree (IBT) method which I will apply to DAX index data to obtain a proxy for the option implied SPD, denoted f^*. I will consider this SPD as the SPD used by investors to price options which will later be compared to a SPD, denoted g^*, estimated from historical time series DAX data (see section 6). Knowing these two SPD's I will implement and investigate trading strategies designed to exploit differences in skewness and kurtosis of both SPD's.

An Implied Binomial Tree is a numerical method whose algorithm is a data adaptive modification of the Cox–Ross–Rubinstein (CRR) algorithm. It can be used to price and hedge options and to compute implied probability distributions, among others. While the CRR binomial tree is caracterized by a constant volatility function and constant transition probabilities, the IBT tries to adapt the transition probabilities to the observed 'volatility smile'. Actually, the IBT techniques proposed by Rubinstein (1994), Derman and Kani (1994), Dupire (1994), and Barle and Cakici (1998) are discrete versions of a diffusion model in which the generalized volatility parameter is allowed to be a function of time and the underlying:

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma(S_t, t) dZ_t \tag{17}$$

where $\sigma(S_t, t)$ is the instantaneous local volatility function. Others models of this type often involve a special parametric form for the volatility function. But the IBT is constructed on the basis of observed option prices yielding $\sigma(S_t, t)$, i.e. take the observed smile as an input. Thus, the empirical fact that market implied volatilities decrease with the strike level and increase with time to maturity is better reflected.

Thanks to research conducted by Härdle and Zheng (2001), XploRe provides two quantlets computing Derman–Kani's and Barle–Cakici's IBT's. Since Barle–Cakici's modification of Derman–Kani's IBT algorithm proved to be slightly more robust than Derman–Kani's original version, I decided to use the former to compute the option implied SPD. Jackwerth (1999) indicates already that Barle–Cakici's IBT exhibits arbitrage violations less frequently than Derman–Kani's IBT.

5.1 Barle–Cakici's IBT Algorithm

In this part, I will briefly explain Barle–Cakici's IBT algorithm. For deeper explanations and examples refer to Derman and Kani (1994), Barle and Cakici (1998), Jackwerth (1999) or Härdle and Zheng (2001).

Using induction an implied tree with N uniformly spaced time steps of length Δt is build up to calculate the underlyings' prices ($s_{n,i}$), transition probabilities ($p_{n,i}$) and Arrow–Debreu prices ($\lambda_{n,i}$) at each node i for each level n. Starting from the root at present time ($t_1 = 0$) and present underlying price ($S_{t_1=0} = S$) a recombining tree is constructed recursively forward, one time step Δt at a time, by adding the nodes for a new level at time t_{n+1} based on the information in a set of option prices expiring at t_{n+1}. Number the tree such that there are n nodes at the nth level, i.e. the node index i after the $n-1$th time step runs from 1 at the bottom to n at the top of the tree. The root of the tree is defined as $s_{1,1} = S$. Assume that the tree has already been implied from the root up to and including the nth level, i.e. time t_n. Let $r_{t_n,\Delta t}$

and $q_{t_n,\Delta t}$ denote the continuous risk free interest rate and dividend yield, respectively. The forward price $F_{n,i}$ of the underlyings' price $s_{n,i}$ is defined as $s_{n,i}e^{(r_{t_n,\Delta t}-q_{t_n,\Delta t})\Delta t}$. Barle and Cakici suggest to align the center node of the tree with the forward price $F_{n,i}$ and whenever a new option is needed to provide information about the next node of the tree they advocate to set the option's strike price K equal to the forward price of the previous node's underlyings' price.

At the nth level, the IBT requires the calculations as follows. If $(n+1)$ is odd, the price of the single central node $s_{n+1,n/2+1}$ is set to $F_{n,n/2+1}$. If $(n+1)$ is even, we have two central nodes. The lower central node price is:

$$s_{n+1,(n+1)/2} = F_{n,i}\frac{\lambda_{n,i}F_{n,i} - \Delta^C_{n,i}}{\lambda_{n,i}F_{n,i} + \Delta^C_{n,i}} \tag{18}$$

where

$$\Delta^C_{n,i} = e^{(r_{t_n,\Delta t}-q_{t_n,\Delta t})\Delta t}C(F_{n,i},t_{n+1}) - \sum_{j=i+1}^{n}\lambda_{n,i}(F_{n,j}-F_{n,i}) \tag{19}$$

and $C(F_{n,i},t_{n+1})$ is the call expiring in t_{n+1} which is computed using the Black–Scholes analytical pricing formula for European call options. Note that $\lambda_{1,1}=1$. The upper central node price is set to:

$$s_{n+1,(n+3)/2} = F^2_{n,(n+1)/2}/s_{n+1,(n+1)/2}. \tag{20}$$

Starting from these central nodes, repeated use of

$$s_{n+1,i+1} = \frac{\Delta^C_{n,i}s_{n+1,i} - \lambda_{n,i}F_{n,i}(F_{n,i}-s_{n+1,i})}{\Delta^C_{n,i} - \lambda_{n,i}(F_{n,i}-s_{n+1,i})} \tag{21}$$

builds the underlyings' prices corresponding to the upper part of that level. To get the lower part of the tree, an analogous recursion relation can be utilized:

$$s_{n+1,i} = \frac{\lambda_{n,i}F_{n,i}(s_{n+1,i+1}-F_{n,i}) - \Delta^P_{n,i}s_{n+1,i+1}}{\lambda_{n,i}(s_{n+1,i+1}-F_{n,i}) - \Delta^P_{n,i}} \tag{22}$$

with

$$\Delta^P_{n,i} = e^{(r_{t_n,\Delta t}-q_{t_n,\Delta t})\Delta t}P(F_{n,i},t_{n+1}) - \sum_{j=1}^{i-1}\lambda_{n,i}(F_{n,j}-F_{n,i}) \tag{23}$$

where $P(F_{n,i},t_{n+1})$ denotes the put option struck at $F_{n,i}$ and maturing at t_{n+1}. It is computed using the Black–Scholes analytical pricing formula for European put options. The transition probabilities can be computed by:

$$p_{n,i} = \frac{F_{n,i}-s_{n+1,i}}{s_{n+1,i+1}-s_{n+1,i}}. \tag{24}$$

Finally, the Arrow–Debreu prices are obtained solving the iterative formula:

$$\begin{aligned}
\lambda_{n+1,1} &= e^{-(r_{t_n,\Delta t}-q_{t_n,\Delta t})\Delta t}\big[(1-p_{n,1})\lambda_{n,1}\big] \\
\lambda_{n+1,i+1} &= e^{-(r_{t_n,\Delta t}-q_{t_n,\Delta t})\Delta t}\big[p_{n,i}\lambda_{n,i} + (1-p_{n,i+1})\lambda_{n,i+1}\big] \\
\lambda_{n+1,n+1} &= e^{-(r_{t_n,\Delta t}-q_{t_n,\Delta t})\Delta t}\big[p_{n,n}\lambda_{n,n}\big].
\end{aligned} \tag{25}$$

The discrete SPD approximation f^* is obtained by assigning $e^{(r_{t_0,\tau}-q_{t_0,\tau})\tau}\lambda_{N+1,i}$ to $s_{N+1,i}$, $i = 1, \dots, N+1$, at the final level $N+1$ with $\tau = T - t$. At some nodes the recursive application of equation(21) and (22) may sometimes give rise to a price $s_{n+1,i+1}$ outside the interval determined by:

$$F_{n,i} \quad \leq \quad s_{n+1,i+1} \quad \leq \quad F_{n,i+1},$$

leading to a negative transition probability $p_{n,i}$. To save the tree from such negative probabilities, Barle and Cakici override $s_{n+1,i+1}$ by setting it equal to the average of $F_{n,i}$ and $F_{n,i+1}$. Note that the mean of the IBT SPD is equal to the futures price by construction of the IBT.

5.2 Application to DAX Data

Now, it is time to apply the IBT method to real data. Using the DAX index data from the MD*BASE database (http://www.mdtech.de), I estimated the 3 month option implied IBT SPD f^* by means of the XploRe quantlets `IBTbc(.)` and `volsurf(.)` and a two week cross section of DAX index option prices for 30 periods beginning in April 1997 and ending in September 1999. For each period, I assumed a flat yield curve. I used the interest rate given in table(1) which precises some technical details. I measured time to maturity (TTM) in days and annualized it using the factor 360.

Let me describe the procedure in more detail for the first period. First of all, I estimated the implied volatility surface given the two week cross section of DAX option data and utilizing the XploRe quantlet `volsurf(.)` which computes the 3 dimensional implied volatility surface (implied volatility over time to maturity and moneyness) using a Kernel smoothing procedure. The volatility surface estimated on Monday, April 21, 1997, the day on which the IBT computation is initialized and which is immediately following the 3rd Friday of April 1997, used two weeks of option data from Monday, April 7, 1997, to Friday, April 18, 1997, the 3rd Friday of April 1997. The volatility surface is estimated for the moneyness interval $[0.8, 1.2]$ and the time to maturity interval $[0.0, 1.0]$. Following, the XploRe quantlet `IBTbc(.)` takes the volatility surface as input and computes the IBT, i.e. it calculates the tree of stock prices, the tree of transition probabilities and the tree of Arrow–Debreu prices using Barle and Cakici's method. Note that the observed smile enters the IBT via the analytical pricing formulae $C(F_{n,i}, t_{n+1})$ and $P(F_{n,i}, t_{n+1})$ which are functions of $S_{t_1} = s_{1,1}$, $K = F_{n,i}$, $r_{t_1,t_{n+1}}$, t_{n+1} and $\sigma_{implied}(F_{n,i}, t_{n+1})$. Let me mention, it may happen that at the edge of the tree option prices, with associated strike prices $F_{n,i}$ and node prices $s_{n+1,i+1}$, have to be computed for which the moneyness ratio $s_{n+1,i+1}/F_{n,i}$ lies outside the intverall $[0.0, 1.0]$ on which the volatility surface has been estimated. In these cases, I use the volatility at the edge of the surface.

Finally, I transform the SPD over $s_{N+1,i}$ into a SPD over log–returns $u_{N+1,i} = \ln(s_{N+1,i}/s_{1,1})$ what is quite easy, since:

$$\mathbb{P}(s_{N+1,i} = x) \quad = \quad \mathbb{P}\Big(\ln\big(\frac{s_{N+1,i}}{s_{1,1}}\big) = \ln\big(\frac{x}{s_{1,1}}\big)\Big) \tag{26}$$

$$= \quad \mathbb{P}\Big(u_{N+1,i} = u\Big) \tag{27}$$

where $u = \ln(x/s_{1,1})$. That is, $s_{N+1,i}$ has the same probability as $u_{N+1,i}$. Look at figure(2) for the SPD computed using $N = 10$ time steps and interest rate $r = 3.23$.

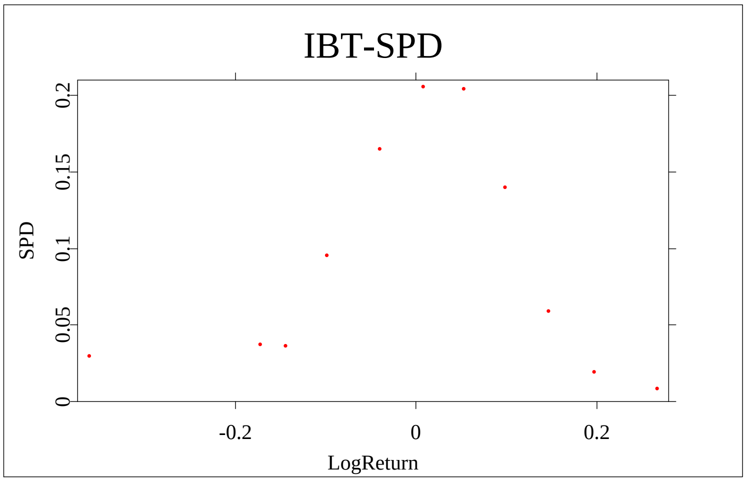

Figure 2: *Option implied SPD estimated on April 21, 1997, by an IBT with N=10 time steps, S=3328.41, r=3.23 and τ=88/360.*

A crucial aspect using binomial trees is the choice of N, the number of time steps N in which the time interval $[t, T]$ is divided. In general one can state, the more time steps are used the better is the discrete approximation of the continuous diffusion process and of the SPD. Unfortunately, the bigger N, the more node prices $s_{n,i}$ possibly have to be overridden in the IBT framework. Thereby we are effectively losing the information about the smile at the corresponding nodes. Furthermore, in contrast to the Monte Carlo method where a time step can be interpreted as a trading day or as a tick, for example, I have no knowledge of an 'economic' interpretation of a time step in a binomial tree. Therefore, I computed IBT's for different numbers of time steps. I found no hint for convergence of the variables of interest, skewness and kurtosis. Since both variables seemed to fluctuate around a mean, I computed IBT's with time steps $10, 20, \ldots, 100$ and took the average of these ten values for skewness and kurtosis as the option implied SPD skewness and kurtosis.

Applying this procedure for all 30 periods, beginning in April 1997 and ending in September 1999, I calculated the time series of skewness and kurtosis of the 3 month implied SPD f^* given in table(9) and shown in figures(3) and (4).

From table(9) given in the appendix we can extract that the implied SPD is clearly negatively skewed for all periods but one. In September 1999 it is slightly positively skewed. The pattern is similar for the kurtosis of f^* which is leptokurtic in all but one period. In October 1998 the SPD is platykurtic.

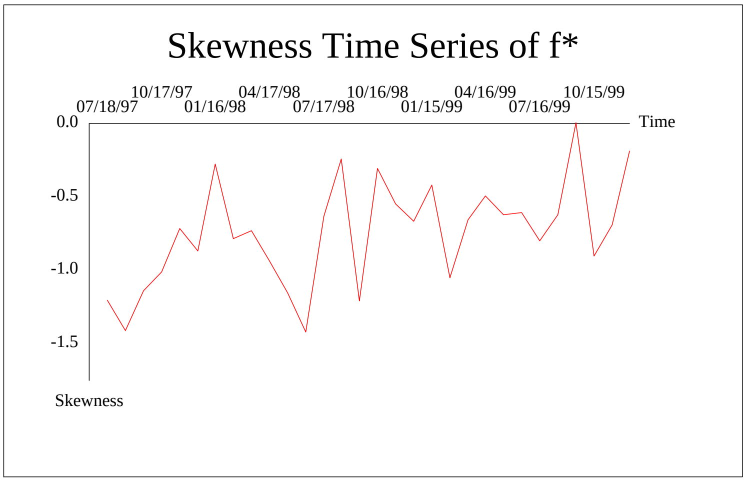

Figure 3: *Skewness time series of f^* for 30 periods.*

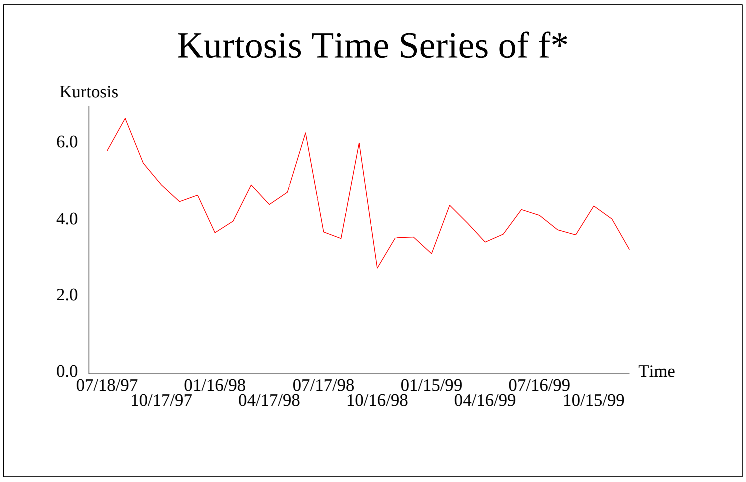

Figure 4: *Kurtosis time series of f^* for 30 periods.*

Month	3rd Friday	DAX on 3rd Fr	Mo after 3rd Fr	DAX on Mo	3rd Fr in 3 months	TTM from Mo to 3rd Fr	$r_{t,\tau}$	OptionData	IndexData
Apr 97	04/18/97		04/21/97	3328.41	88	07/18/97	3.23	04/07/97–04/18/97	01/20/97–04/18/97
May 97	05/16/97		05/20/97	3516.20	87	08/15/97	3.16	05/05/97–05/16/97	02/24/97–05/16/97
Jun 97	06/20/97		06/23/97	3748.79	88	09/19/97	3.12	06/09/97–06/20/97	03/24/97–06/20/97
Jul 97	07/18/97	4131.94	07/21/97	4139.96	88	10/17/97	3.18	07/07/97–07/18/97	04/21/97–07/18/97
Aug 97	08/15/97	4077.59	08/18/97	4080.55	95	11/21/97	3.27	08/04/97–08/15/97	05/20/97–08/15/97
Sep 97	09/19/97	3983.06	09/22/97	4096.85	88	12/19/97	3.30	09/08/97–09/19/97	06/23/97–09/19/97
Oct 97	10/17/97	4049.16	10/20/97	4124.86	88	01/16/98	3.66	10/06/97–10/17/97	07/21/97–10/17/97
Nov 97	11/21/97	3941.91	11/24/97	3832.10	88	02/20/98	3.75	11/10/97–11/21/97	08/18/97–11/21/97
Dec 97	12/19/97	4055.35	12/22/97	4125.54	88	03/20/98	3.71	12/08/97–12/19/97	09/22/97–12/19/97
Jan 98	01/16/98	4216.24	01/19/98	4290.05	88	04/17/98	3.55	01/05/98–01/16/98	10/20/97–01/16/98
Feb 98	02/20/98	4583.03	02/23/98	4610.66	81	05/15/98	3.49	02/09/98–02/20/98	11/24/97–02/20/98
Mar 98	03/20/98	5045.16	03/23/98	5014.13	88	06/19/98	3.52	03/09/98–03/20/98	12/22/97–03/20/98
Apr 98	04/17/98	5326.63	04/20/98	5407.93	88	07/17/98	3.64	04/06/98–04/17/98	01/19/98–04/17/98
May 98	05/15/98	5414.31	05/18/98	5343.66	95	08/21/98	3.63	05/04/98–05/15/98	02/23/98–05/15/98
Jun 98	06/19/98	5644.22	06/22/98	5648.11	88	09/18/98	3.55	06/08/98–06/19/98	03/23/98–06/19/98
Jul 98	07/17/98	6162.86	07/20/98	6186.09	88	10/16/98	3.54	07/06/98–07/17/98	04/20/98–07/17/98
Aug 98	08/21/98	5190.60	08/24/98	5253.38	88	11/20/98	3.48	08/10/98–08/21/98	05/18/98–08/21/98
Sep 98	09/18/98	4623.37	09/21/98	4439.13	88	12/18/98	3.48	09/07/98–09/18/98	06/22/98–09/18/98
Oct 98	10/16/98	4469.12	10/19/98	4466.18	88	01/15/99	3.56	10/05/98–10/16/98	07/20/98–10/16/98
Nov 98	11/20/98	4911.43	11/23/98	5024.51	88	02/19/99	3.65	11/09/98–11/20/98	08/24/98–11/20/98
Dec 98	12/18/98	4666.74	12/21/98	4826.70	88	03/19/99	3.32	12/07/98–12/18/98	09/21/98–12/18/98
Jan 99	01/15/99	4973.78	01/18/99	5076.85	91	04/16/99	3.14	01/04/99–01/15/99	10/19/98–01/15/99
Feb 99	02/19/99	4823.26	02/22/99	4887.70	92	05/21/99	3.11	02/08/99–02/19/99	11/23/98–02/19/99
Mar 99	03/19/99	5108.75	03/22/99	5034.68	91	06/18/99	3.06	03/08/99–03/19/99	12/21/98–03/19/99
Apr 99	04/16/99	5143.02	04/19/99	5252.40	91	07/16/99	2.65	04/05/99–04/16/99	01/18/99–04/16/99
May 99	05/21/99	5253.77	05/25/99	5165.72	90	08/20/99	2.61	05/10/99–05/21/99	02/22/99–05/21/99
Jun 99	06/18/99	5337.19	06/21/99	5468.47	91	09/17/99	2.67	06/07/99–06/18/99	03/22/99–06/18/99
Jul 99	07/16/99	5619.94	07/19/99	5624.74	91	10/15/99	2.73	07/05/99–07/16/99	04/19/99–07/16/99
Aug 99	08/20/99	5254.14	08/23/99	5301.98	91	11/19/99	2.72	08/09/99–08/20/99	05/25/99–08/20/99
Sep 99	09/17/99	5303.94	09/20/99	5351.98	91	12/17/99	2.72	09/06/99–09/17/99	06/21/99–09/17/99
Oct 99	10/15/99	5184.23	10/18/99						
Nov 99	11/19/99	5955.97	11/22/99						
Dec 99	12/17/99	6353.90	12/20/99						

Table 1: *Technical details for the application to DAX data*

6 Estimation of the Historical SPD

While the previous section was dedicated to finding a proxy for f^* used by investors to price options, this section tries to approximate the historical underlyings' density g^* at date $t = T$. Of course, if the process governing the underlying asset dynamics were common knowledge and if agents had perfect foresight, then by no arbitrage both SPD should be equal. However, in reality the econometrician does know g^* only in $t = T$. First, we will explain the estimation method for the underlyings' SPD applied to the S&P 500 by Ait–Sahalia et al. (2000). Second, we apply it to DAX data.

6.1 The Estimation Method

A number of previous studies in the literature have compared the density implied by observed option data to the density extracted from the observed underlying returns data. This approach amounts to comparing a risk neutral density to an actual density. Following Ait–Sahalia et al. (2000) these two densities are not comparable without assumptions on investors' preferences. As Härdle and Tsybakov (1995), they apply an estimation method which uses the observed asset prices to infer indirectly the time series SPD which should be equal to the option implied cross sectional SPD. Assuming the underlying S to follow an Îto diffusion process driven by a Brownian motion W:

$$dS_t = \mu(S_t)dt + \sigma(S_t)dW_t. \tag{28}$$

they rely on Girsanov's characterization of the change of measure from the actual density to the SPD. It says the diffusion function of the asset's dynamics is identical under both the risk neutral and the actual measure and only the drift function needs to be adjusted, leading to the risk neutral asset dynamics:

$$dS_t^* = (r_{t,\tau} - \delta_{t,\tau})S_t^* dt + \sigma(S_t^*)dW_t^*. \tag{29}$$

Let $g_t(S_t, S_T, \tau, r_{t,\tau}, \delta_{t,\tau})$ denote the conditional density of S_T given S_t that is generated by the dynamics defined in equation(28) and $g_t^*(S_t, S_T, \tau, r_{t,\tau}, \delta_{t,\tau})$ the conditional density generated by equation(29) then f^* can only be compared to g^* and not to g. In the Black–Scholes case with a constant drift rate $(r_{t,\tau} - \delta_{t,\tau})$ and diffusion function $\sigma(S_t^*) = \sigma S_t^*$ where σ is constant the option implied and time series risk neutral densities are identical and lognormal with expectation parameter equal to $(r_{t,\tau} - \delta_{t,\tau} - \sigma^2/2)\tau$ and variance parameter equal to $\sigma^2\tau$.

A crucial feature of this method is that the diffusion functions are identical under both the actual and the risk neutral dynamics (which follows from Girsanov's theorem). That is why it is not necessary to observe the risk neutral path of the DAX index $\{S_t^*\}$. The function $\sigma(\cdot)$ is estimated using N observed index values $\{S_t\}$ and applying Florens–Zmirou's (1993) (FZ) nonparametric version of the minimum contrast estimators:

$$\hat{\sigma}_{FZ}(S) = \frac{\sum_{i=1}^{N-1} K_{FZ}(\frac{S_i - S}{h_{FZ}})N\{S_{(i+1)/N} - S_{i/N}\}^2}{\sum_{i=1}^{N} K(\frac{S_i - S}{h_{FZ}})}, \tag{30}$$

where $K_{FZ}(\cdot)$ is a kernel function and h_{FZ} is a bandwidth parameter such that

$$
\begin{aligned}
(Nh_{FZ})^{-1} ln(N) &\rightarrow 0 \\
&and \\
Nh_{FZ}^4 &\rightarrow 0
\end{aligned}
$$

as $N \rightarrow \infty$. Without imposing restrictions on the drift function $\hat{\sigma}_{FZ}(.)$ is an unbiased estimator of $\sigma(.)$ in the model specified in equation(29). Since the DAX index is a performance index ($\delta_{t,\tau} = 0$), the risk neutral drift rate of equation(29) is equal to $r_{t,\tau}$.

Once the volatility function $\sigma(\cdot)$ is estimated, the time series SPD g^* can be computed by Monte Carlo integration. Use the Milstein scheme, simulate $M = 10,000$ paths of the diffusion function:

$$
dS_t^* = r_{t,\tau} S_t^* dt + \hat{\sigma}_{FZ}(S_t^*) dW_t^* \tag{31}
$$

with a time to maturity of 3 months, starting value $S_{t=0}$ equal to the DAX index value at the beginning of the period, collect the endpoints at T of these simulated paths $\{S_{T,m} : m = 1, \ldots, M\}$ and annualize the index log–returns. Then the SPD g^* is obtained by means of a nonparametric kernel density estimation of the index continuously compounded log–returns u:

$$
\hat{p}_t^*(u) = \frac{1}{Mh_{MC}} \sum_{m=1}^{M} K_{MC}\Big(\frac{u_m - u}{h_{MC}}\Big) \tag{32}
$$

where u_m is the log–return at the end of the mth path. The equation:

$$
\begin{aligned}
\text{Prob}(S_T \leq S) &= \text{Prob}(S_t e^u \leq S) \\
&= \text{Prob}(u \leq \log(S/S_t)) \\
&= \int_{-\infty}^{\log(S/S_t)} p_t^*(u) du
\end{aligned}
$$

relates this density estimator to the SPD g^*:

$$
\begin{aligned}
g_t^*(S) &= \frac{\partial}{\partial S} \text{Prob}(S_T \leq S) \\
&= \frac{p_t^*(\log(S/S_t))}{S}.
\end{aligned} \tag{33}
$$

This method results in a nonparametric estimator $\hat{g}^*$ which is $\sqrt{N}$–consistent even though $\hat{\sigma}_{FZ}$ converges at a slower rate (Ait–Sahalia et al. (2000)).

In the absence of arbitrage, the futures price is the expected future value of the spot price under the risk neutral measure. Therefore the time series SPD is translated such that its mean matches the implied future price. Then the bandwidth h_{MC} is chosen to best match the variance of the IBT implied SPD. In order to avoid over– or undersmoothing of g^*, h_{MC} is constrained to be within 0.5 to 5 times the optimal bandwidth implied by Silverman's rule of thumb. This procedure allows us to focus the SPD comparison on skewness and kurtosis of both SPD's.

6.2 Application to DAX Data

Having described the estimation method in section 6.1 I will now apply it to real data. Using the DAX index data from the MD*BASE database (http://www.mdtech.de) I estimated the diffusion function $\sigma^2(\cdot)$ by means of past index prices and following I simulated (forward) $M = 10,000$ paths to obtain the time series density.

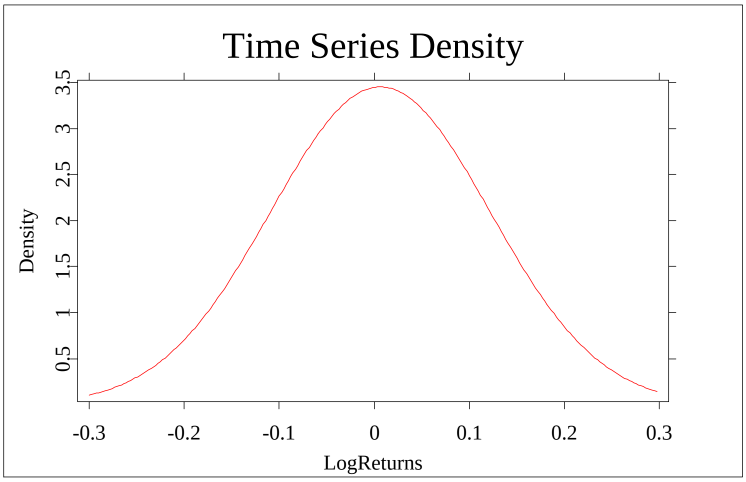

Figure 5: *Mean and variance adjusted estimated time series density on Friday, April 18, 1997. Simulated with M=10.000 paths,* $S_{t=0}$*=3328.41,* $r_{t,\tau}$*=3.23 and* τ*=88/360.*

To be more precise I will explain the methodology I applied for the first period in more detail. On Monday, April 21, 1997 (look at table(1)), I used 3 months of DAX index prices starting Monday, January 20, 1997, and ending Friday, April 18, 1997, the 3rd Friday immediately preceding Monday, April 21, 1997, the day on which the 3 months 'forward' Monte Carlo simulation starts. The bandwidth h_{FZ} is determined by the XploRe quantlet `regxbwcrit(.)` which determines the optimal bandwidth from a range of bandwidths by using the resubstitution estimator with the penalty function 'Generalized Cross Validation'. Figure(6) shows $\hat{\sigma}^2_{FZ}(\cdot)$ for the period from January 20, 1997, to April 18, 1997.

Knowing the diffusion function it is now possible to Monte Carlo simulate the index evolution. The Milstein scheme applied to equation(29) is given by:

$$S_i = S_{i-1} + rS_{i-1}\Delta t + \sigma(S_{i-1})\Delta W_i + \frac{1}{2}\sigma(S_{i-1})\frac{\partial \sigma}{\partial S}(S_{i-1})\Big((\Delta W_{i-1})^2 - \Delta t\Big) \quad (34)$$

where I set the drift equal to r, the interest rate given in table(1). The first derivative of $\sigma(\cdot)$ is approximated by:

$$\frac{\partial \sigma}{\partial S}(S_{i-1}) = \frac{\sigma(S_{i-1}) - \sigma(S_{i-1} - \Delta S)}{\Delta S}, \quad (35)$$

where ΔS is 1/2 of the width of the bingrid on which the diffusion function is estimated. Finally the estimated diffusion function is linearly extrapolated at both ends of the bingrid to accommodate potential outliers.

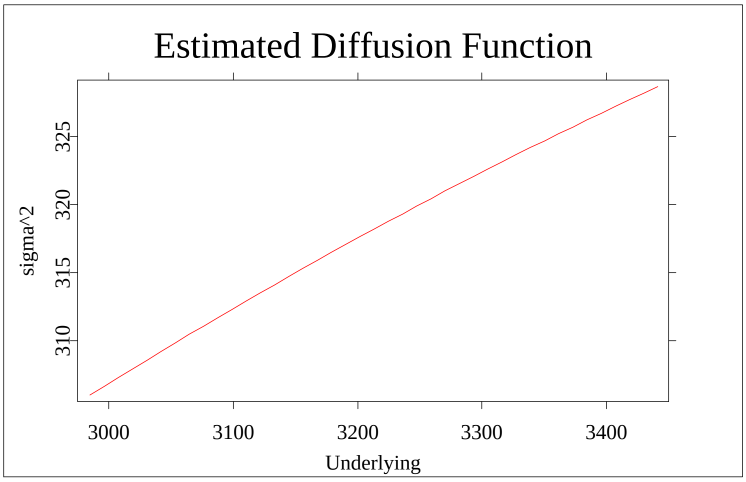

Figure 6: *Estimated diffusion function for the period 01/20/97 to 04/18/97.*

With these ingredients I started the simulation with index value $S_0 = 3328.41$ (Monday, April 21, 1997) and time to maturity $\tau = 88/360$ and $r = 3.23$. The expiration date $t = T$ is Friday, July 18, 1997. From these simulated index values I calculated annualized log–returns which were taken as input of the nonparametric density estimation mentioned in equation(32). The XploRe quantlet `denxest()` accomplished the estimation of the time series density by means of the Gaussian kernel function:

$$K(u) \quad = \quad \frac{1}{\sqrt{2\pi}} \exp\Big(-\frac{1}{2}u^2\Big).$$

The bandwidth h_{MC} is computed by the XploRe quantlet `denrot(.)` which applies Silverman's rule of thumb.

First of all, I calculate the optimum bandwidth h_{MC} given the vector of $10,000$ simulated index values. Then I search the bandwidth $h^{'}_{MC}$ which implies a variance of g^* to be closest to the variance of f^* (but to be still within 0.5 to 5 times h_{MC}). I stop the search if var(g^*) is within a range of 5% of var(f^*). Following, I translate g^* such that its mean matches the futures price F. To achieve this, I do the variable transformation: $S^{'}_i = S_i + F - E\big[S\big]$, then $E\big[S^{'}\big] = F$. Finally, I transform this density over DAX index values S_T into a density $g^{*'}$ over log–returns u_T. Since

$$\begin{aligned}
\mathbb{P}(S_T < x) &= \mathbb{P}(\ln S_T < \ln x) \\
&= \mathbb{P}\Big(\ln\big(\frac{S_T}{S_t}\big) < \ln\big(\frac{x}{S_t}\big)\Big) \\
&= \mathbb{P}(u_T < u)
\end{aligned}$$

where $x = S_t e^u$, we have

$$\mathbb{P}(S_T \in [x, x + \Delta x]) \quad = \quad \mathbb{P}(u_T \in [u, u + \Delta u])$$

and

$$\begin{aligned}
\mathbb{P}(S_T \in [x, x + \Delta x]) &\approx g^*(x)\Delta x \\
\mathbb{P}(u_T \in [u, u + \Delta u]) &\approx g^{*}\text{'}(u)\Delta u,
\end{aligned}$$

I set

$$\begin{aligned}
g^{*}\text{'}(u) &\approx \frac{g^*(S_t e^u)\Delta(S_t e^u)}{\Delta u} \\
&\approx \frac{g^*(S_t e^u)S_t e^u \Delta u}{\Delta u} \\
&\approx g^*(S_t e^u)S_t e^u
\end{aligned}$$

to transform the density. To simplify notation, I will denote both densities g^*. Figure(5) displays the resulting time series density over log–returns on Friday, April 18, 1997. At this day g^* has a skewness of $s = -0.0252$ and a kurtosis of $k = 2.6875$.

Proceeding in the same way for all 30 periods beginning in April 1997 and ending in September 1999, we obtain the time series of the 3 month 'forward' skewness and kurtosis values of g^* shown in figures(7) and (8) and given in table(10) in the appendix.

The figures reveal that the time series distribution is systematically slightly negatively skewed. Skewness is very close to zero. As far as kurtosis is concerned we can extract from table(10) that it is systematically smaller than but nevertheless very close to 3. Additionally, all time series density plots looked like the one shown in figure(5). Considering only the first four moments there are grounds for the assumption that the above estimation method produces quite normally distributed time series SPDs.

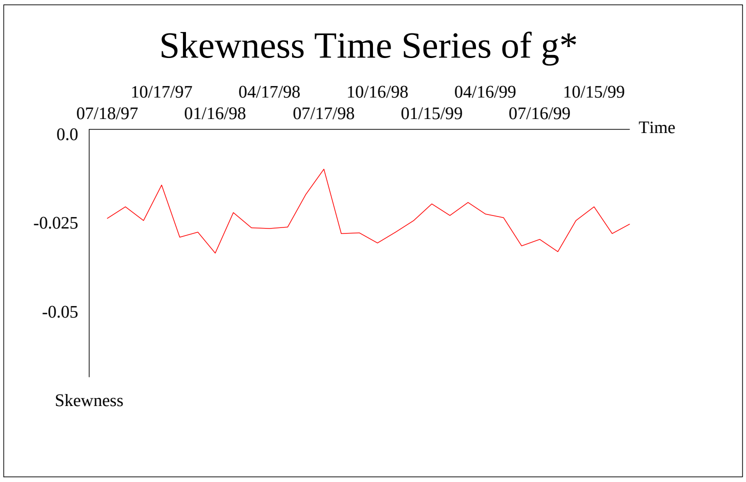

Figure 7: *Skewness time series of g^* for 30 periods.*

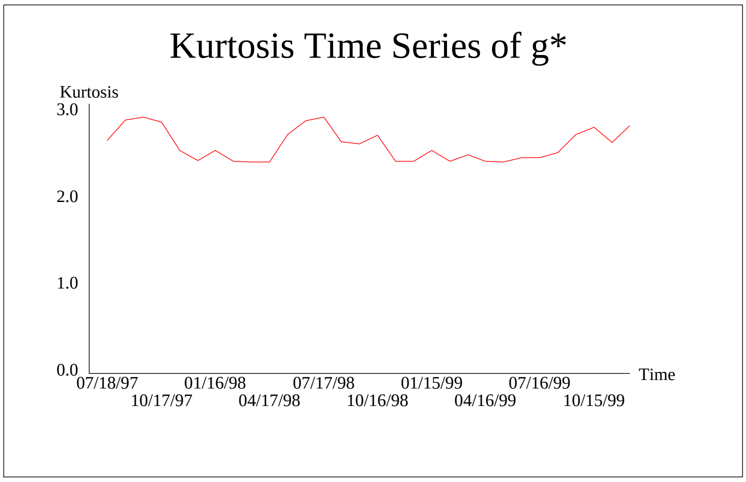

Figure 8: *Kurtosis time series of g^* for 30 periods.*

7 Comparison of Implied and Historical SPD

At this point it is time to compare implied and historical SPD. Since by construction, expectation and variance are adjusted, I focus the comparison on skewness and kurtosis. Starting with skewness, we can extract from figure(9) that except for one period the IBT implied SPD is systematically more negatively skewed than the time series SPD, a fact that is quite similar to what Ait–Sahalia et al. (2000) already found for the S&P 500. The 3 month IBT implied SPD for Friday, September 17, 1999 is slightly positively skewed. It may be due to the fact that in the months preceeding June 1999, the month in which the 3 month implied SPD was estimated, the DAX index stayed within a quite narrow horizontal range of index values after a substantial downturn in the 3rd quarter of 1998 (see figure(22)) and agents therefore possibly believed index prices lower than the average would be more realistic to appear. However, this is the only case where skew(f^*)>skew(g^*).

The kurtosis time series reveals a similar pattern as the skewness time series. The IBT implied SPD has except for one period systematically more kurtosis than the time series SPD. Again this feature is in line with what Ait–Sahalia et al. (2000) found for the S&P 500. The 3 month IBT implied SPD for Friday, October 16, 1998 has a slightly smaller kurtosis than the time series SPD. That is, investors assigned less probability mass to outliers. Note that the implied SPD was estimated in July 1998 after a period of 8 months of booming asset prices (see figure(22)). While it is comprehensible in such an environment that high index prices seem less realistic to appear, it is a bit confusing that the appearance of low index prices seems to be unrealistic as well.

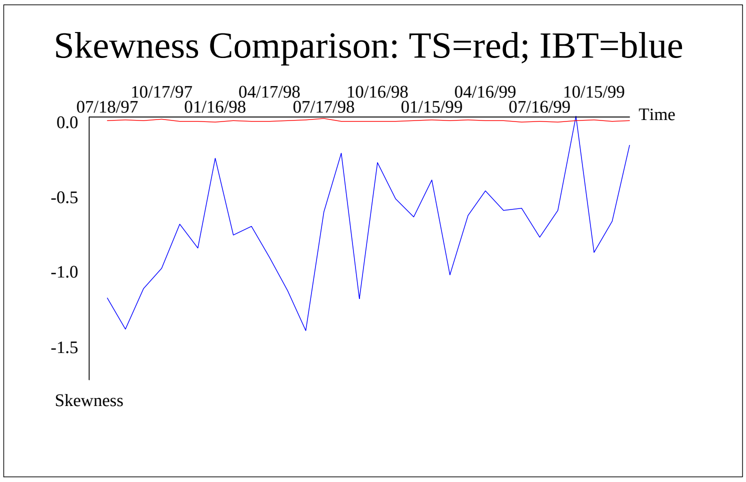

Figure 9: *Comparison of skewness time series for 30 periods.*

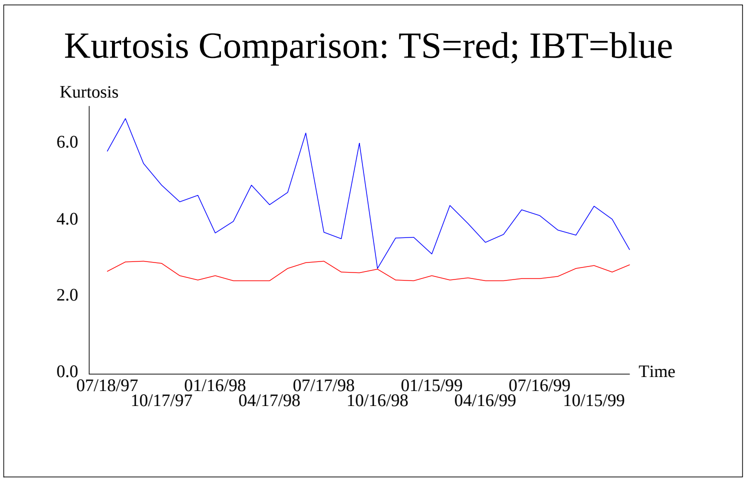

Figure 10: *Comparison of kurtosis time series for 30 periods.*

8 Trading Strategies

As already mentioned above, f^* and g^* should be equal if agents had perfect foresight and knowledge about the index process. However, in the previous section we learned that the implied and the time series SPD reveal differences in skewness and kurtosis. It seems that not all deviations have been traded away. Therefore, knowing both SPD's it is possible to construct trading rules to exploit differences in skewness and kurtosis (see Ait–Sahalia (2000)). In general, one is interested in what option to buy or to sell at the day at which both SPD's were estimated. I consider exclusively European call or put options. According to Ait–Sahalia (2000), all strategies are designed such that I do not change the resulting portfolio until maturity, i.e. I keep all options until they expire. Below I will explain in more detail what is meant to be a skewness and a kurtosis trade. In the sections to come I will use the following terms for moneyness which I define as $K/S_te^{(T-t)r}$:

		Moneyness(FOTM Put)	$<$	0.90
0.90	$\leq$	Moneyness(NOTM Put)	$<$	0.95
0.95	$\leq$	Moneyness(ATM Put)	$<$	1.00
1.00	$\leq$	Moneyness(ATM Call)	$<$	1.05
1.05	$\leq$	Moneyness(NOTM Call)	$<$	1.10
1.10	$\leq$	Moneyness(FOTM Call)		

Table 2: *Definitions of moneyness regions.*

where FOTM, NOTM, ATM stand for far out–of–the–money, near out–of–the–money and at–the–money respectively.

8.1 Skewness Trades

A skewness trading strategy is supposed to exploit differences in skewness of two distributions by buying options in the range of strike prices where they are underpriced and selling options in the range of strike prices where they are overpriced. More specifically, if the implied SPD f^* is less skewed (for example more negatively skewed) than the time series SPD g^*, i.e. skew(f^*) $<$ skew(g^*), I sell the whole range of strikes of OTM puts and buy the whole range of strikes of OTM calls. That is what Ait–Sahalia et al. (2000) call a S1 trade. Conversely, if the implied SPD is more skewed, i.e. skew(f^*) $>$ skew(g^*), I initiate the S2 trade by buying the whole range of strikes of OTM puts and selling the whole range of strikes of OTM calls. In both cases I keep the options until expiration.

8.1.1 Motivation

Skewness s is a measure of asymmetry of a probability distribution. While for a distribution symmetric around its mean $s = 0$, for an asymmetric distribution $s > 0$ indicates more weight to the left of the mean. Recalling from option pricing theory the pricing equation(1) for a European call option:

$$C(S_t, K, r, T-t) \quad = \quad e^{-r(T-t)} \int_0^{\infty} \max(S_T - K, 0) f^*(S_T) dS_T,$$

where f^* is the implied SPD, we see that when the two SPD are such that skew(f^*) $<$ skew(g^*), agents apparently assign a lower probability to high outcomes of the underlying than would be justified by the time series density (see figure(11)). Since for call options only the right 'tail' of the support determines the theoretical price, the latter is smaller than the price implied by equation(1) using the time series density. That is, I buy underpriced calls.

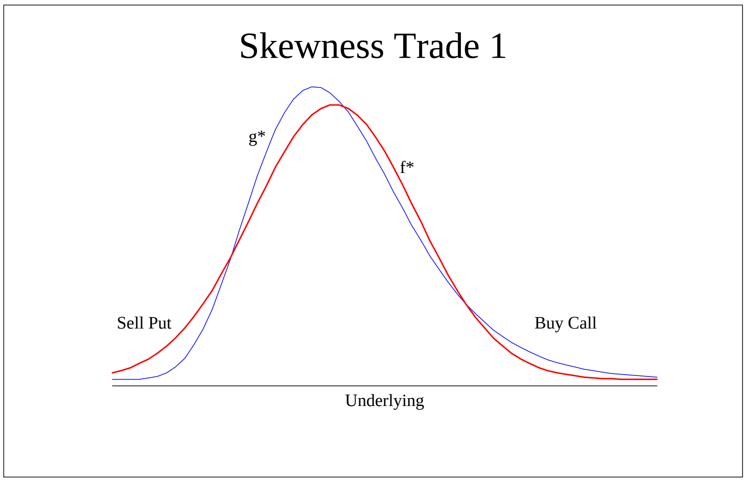

Figure 11: *Skewness trade 1.*

The same reasoning applies to European put options. Looking at the pricing equation for such an option:

$$P(S_t, K, r, T-t) \quad = \quad e^{-r(T-t)} \int_0^{\infty} \max(K - S_T, 0) f^*(S_T) dS_T, \tag{36}$$

I conclude that prices implied by this pricing equation using f^* are higher than the prices using the time series density. That is, I sell puts.

8.1.2 Payoff profile at Maturity

Since I hold all options until expiration and due to the fact that options for all strikes are not always available in markets we are going to investigate the payoff profile of this strategy for various compositions of the portfolio. To get an idea about the exposure at maturity let us begin with a simplified portfolio consisting of one short position in a put option with moneyness of 0.95 and one long position in a call option with moneyness of 1.05. To further simplify I assume that the future price F is 100. We see that this strategy implies an asymmetric payoff (refer to figure(12)) which is increasing in S_T the price of the underlying at maturity.

A more sophisticated S1 strategy would be to buy (sell) several calls (puts) with increasing (decreasing) strikes as indicated in table(3). Figure(13) shows the payoff of a portfolio of 10 short puts with strikes ranging from 86 to 95 and of 10 long calls striking at 105 to 114, the future price is still assumed to be 100. The payoff is still increasing in S_T but it is increasing faster in the right tail and slower in the left tail as before. This is due to the fact that our portfolio contains, for example, at $S_T = 106$ two call options which are in the money instead of only one compared to the portfolio considered above. These options generate a payoff which is twice as much. At $S_T = 107$ the payoff is influenced by three ITM calls procuring a payoff which is three times higher as in the situation before etc. In a similar way we can explain the slower increase in the left tail. Just to sum up, we can state that this trading rule has a favorable payoff profile in a booming world where the underlying is increasing. But in bear markets it possibly generates negative cash flows. Buying (selling) two or more calls (puts) at the same strike would change the payoff profile in a similar way leading to a faster increase (slower decrease) with every call (put) bought (sold).

The S2 strategy payoff behaves in the opposite way. The same reasoning can be applied to explain its payoff profile which is shown in figures(14) and (15). In contradiction to the S1 trade the S2 trade is favorable in a falling market.

Table 1–S1

Option	Moneyness
short put	0.95
long call	1.05

Table 1–S2

Option	Moneyness
long put	0.95
short call	1.05

Table OTM–S1

Option	Moneyness
short put	0.86 – 0.95
long call	1.05 – 1.14

Table OTM–S2

Option	Moneyness
long put	0.86 – 0.95
short call	1.05 – 1.14

Table 3: *Portfolios of skewness trades.*

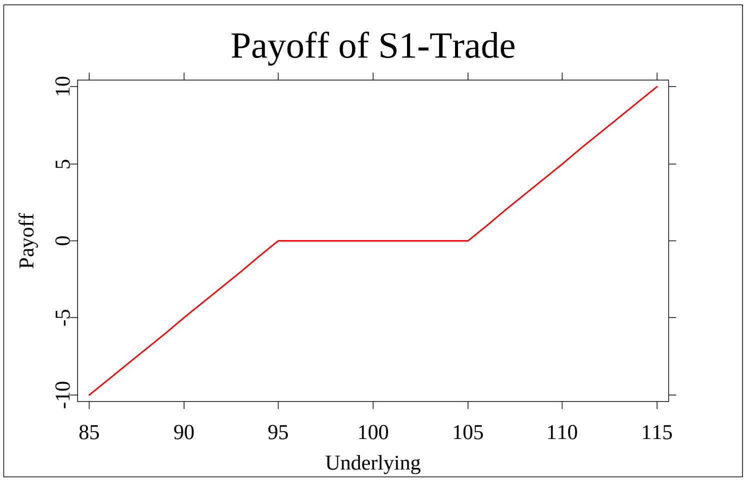

Figure 12: *Skewness trade 1 payoff at maturity of portfolio detailed in table(3).*

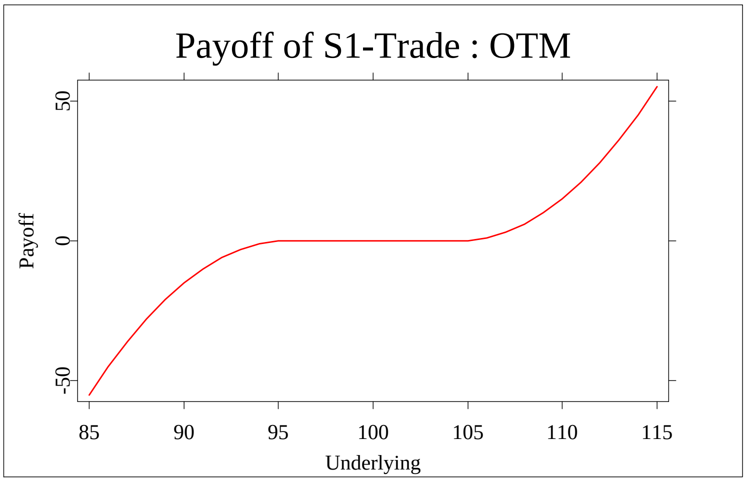

Figure 13: *Skewness trade 1 payoff at maturity of portfolio detailed in table(3).*

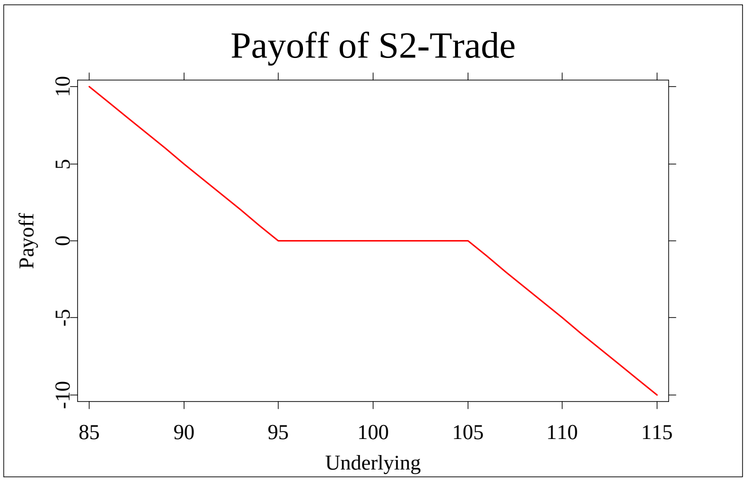

Figure 14: *Skewness trade 2 payoff at maturity of portfolio detailed in table(3).*

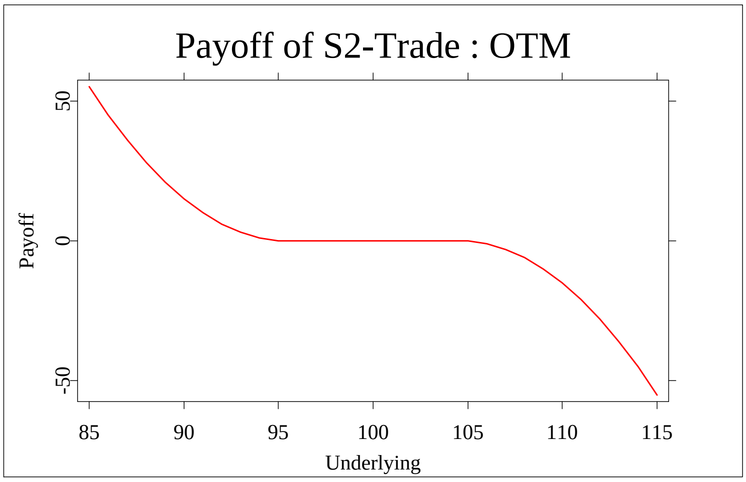

Figure 15: *Skewness trade 2 payoff at maturity of portfolio detailed in table(3).*

8.1.3 Performance

Given the skewness values for the implied and the time series SPD I now have a look on the performance of skewness trades type 1 and type 2. Performance is measured in net EURO cash flows which is the sum of the cash flows generated at initiation in $t = 0$ and at expiration in $t = T$. I ignore any interest rate between these two dates. Using EUREX settlement prices of 3 month DAX put and calls I initiated the S1 strategy at the Monday immediately following the 3rd Friday of each month, beginning in April 1997 and ending in September 1999. January, February, March 1997 drop out due to the time series density estimation for the 3rd Friday of April 1997. October, November and December 1999 drop out due since I look 3 months forward. The cash flow at initiation stems from the inflow generated by the written options and the outflow generated from the bought options and hypothetical 5% transaction costs on prices of bought and sold options. Since all options are kept in the portfolio until maturity (time to expiration is 3 months) the cash flow in $t = T$ is composed of the sum of the inner values of the options in the portfolio.

Table(4) shows the net EURO cash flows for each portfolio. We see that the sum of all cash flows is strongly positive (9855.50 EURO). Note that the net cash flow is always positive except for the portfolios initiated in June 1998 and in September 1998 where we incur heavy losses compared to the gains in the other periods. In other words, this strategy would have procured 28 times moderate gains and two times large negative cash flows. As figure(13) suggests this strategy is exposed to a quite substantial directional risk, a feature that appears in December 1997 and June 1998 where large payoffs at expiration (positive and negative) occur. Indeed, the period of November and December 1997 was a turning point of the DAX and the beginning of an 8 month bull market, explaining the large payoff in March 1998 of the portfolio initiated in December 1997. The same arguing explains the large negative payoff of the portfolio set up in June 1998 expiring in September 1998 (refer to figure(22)). Another point to note is that there is a zero cash flow at expiration in 24 periods. Periods with a zero cash flow at initiation and at expiration are due to the fact that there was not set up any portfolio, i.e. no option was bought or sold (there was no OTM option in the database). A positive cash flow both in $t = 0$ and $t = T$ is an indication for arbitrage opportunities.

Since there is only one period (June 1999), when the implied SPD is more (positively) skewed than the time series SPD an evaluation of the S1 trade with knowledge of the latter SPD's and without this knowledge is of no great use. For completeness look at table(5). We see that a comparison of the skewness measures would have filtered out a positive net cash flow. But to what extend this may be significant is uncertain.

For the same reason the S2 trade has no great informational content. Applied to real data it would have procured a negative net cash flow (see table(11))

While the S1 trade performance was independent of the knowledge of the implied and the time series SPD the S2 trade performance changes significantly if it had been applied in each period (without knowing both SPD). Table(12) seems to be the inverse of table(5) indicating that should there be an options mispricing it would probably be in the sense that the implied SPD is more negatively skewed than the time series SPD.

Month	CashFlow in $t = 0$	CashFlow in $t = T$	NetCashFlow
Apr 97	0.00	0.00	0.00
May 97	0.00	0.00	0.00
Jun 97	499.47	0.00	499.47
Jul 97	0.00	0.00	0.00
Aug 97	140.70	0.00	140.70
Sep 97	70.86	0.00	70.86
Oct 97	379.15	0.00	379.15
Nov 97	-243.08	916.06	672.99
Dec 97	420.39	4.196.76	$4,617.15$
Jan 98	407.93	0.00	407.93
Feb 98	110.01	0.00	110.01
Mar 98	957.43	726.88	1684.31
Apr 98	164.64	0.00	164.64
May 98	0.00	0.00	0.00
Jun 98	$1,570.64$	$-6,426.08$	$-4,855.44$
Jul 98	0.00	0.00	0.00
Aug 98	483.17	-127.14	356.03
Sep 98	$-1,813.08$	0.00	$-1,813.08$
Oct 98	0.00	0.00	0.00
Nov 98	519.37	0.00	519.37
Dec 98	465.57	0.00	465.57
Jan 99	562.88	0.00	562.88
Feb 99	185.16	0.00	185.16
Mar 99	789.97	0.00	789.97
Apr 99	293.84	0.00	293.84
May 99	0.00	0.00	0.00
Jun 99	0.00	0.00	0.00
Jul 99	0.00	0.00	0.00
Aug 99	140.22	0.00	140.22
Sep 99	932.62	$3,531.20$	$4,463.82$
Sum(NetCashFlows)	Mean(NetCashFlows)	Var(NetCashFlows)	Mean/$\sqrt{\text{Var}}$
9855.50	328.52	$2,430,221.51$	0.2107

Table 4: *Performance of S1 trade with 5% transaction costs. Cash flows are measured in EUROs.*

Month	CashFlow in $t = 0$	CashFlow in $t = T$	NetCashFlow
Apr 97	0.00	0.00	0.00
May 97	0.00	0.00	0.00
Jun 97	499.47	0.00	499.47
Jul 97	0.00	0.00	0.00
Aug 97	140.70	0.00	140.70
Sep 97	70.86	0.00	70.86
Oct 97	379.15	0.00	379.15
Nov 97	−243.08	916.06	672.99
Dec 97	420.39	4.196.76	4, 617.15
Jan 98	407.93	0.00	407.93
Feb 98	110.01	0.00	110.01
Mar 98	957.43	726.88	1684.31
Apr 98	164.64	0.00	164.64
May 98	0.00	0.00	0.00
Jun 98	1, 570.64	−6, 426.08	−4, 855.44
Jul 98	0.00	0.00	0.00
Aug 98	483.17	−127.14	356.03
Sep 98	−1, 813.08	0.00	−1, 813.08
Oct 98	0.00	0.00	0.00
Nov 98	519.37	0.00	519.37
Dec 98	465.57	0.00	465.57
Jan 99	562.88	0.00	562.88
Feb 99	185.16	0.00	185.16
Mar 99	789.97	0.00	789.97
Apr 99	293.84	0.00	293.84
May 99	0.00	0.00	0.00
Jun 99	899.68	0.00	899.68
Jul 99	0.00	0.00	0.00
Aug 99	140.22	0.00	140.22
Sep 99	932.62	3, 531.20	4, 463.82
Sum(NetCashFlows)	Mean(NetCashFlows)	Var(NetCashFlows)	Mean/$\sqrt{\text{Var}}$
10, 755.18	358.51	2, 436, 818.87	0.2297

Table 5: *Performance of S1 trade with 5% transaction costs and without knowledge of IBT and time series SPD's. Cash flows are measured in EUROs.*

8.2 Kurtosis Trades

A kurtosis trading strategy is supposed to exploit differences in kurtosis of two SPD by buying options in the range of strike prices where they are underpriced and selling options in the range of strike prices where they are overpriced. More specifically, if the implied SPD f^* has more kurtosis than the time series SPD g^*, i.e. $\text{kurt}(f^*) > \text{kurt}(g^*)$, we sell the whole range of strikes of FOTM puts, buy the whole range of strikes of NOTM puts, sell the whole range of strikes of ATM puts and calls, buy the whole range of strikes of NOTM calls and sell the whole range of strikes of FOTM calls. That is what Ait–Sahalia et al. (2000) call a K1 trade. Conversely, if the implied SPD has less kurtosis than the time series SPD g^*, i.e. $\text{kurt}(f^*) < \text{kurt}(g^*)$, we initiate the K2 trade by buying the whole range of strikes of FOTM puts, selling the whole range of strikes of NOTM puts, buying the whole range of strikes of ATM puts and calls, selling the whole range of strikes of NOTM calls and buying the whole range of strikes of FOTM calls. In both cases we keep the options until expiration.

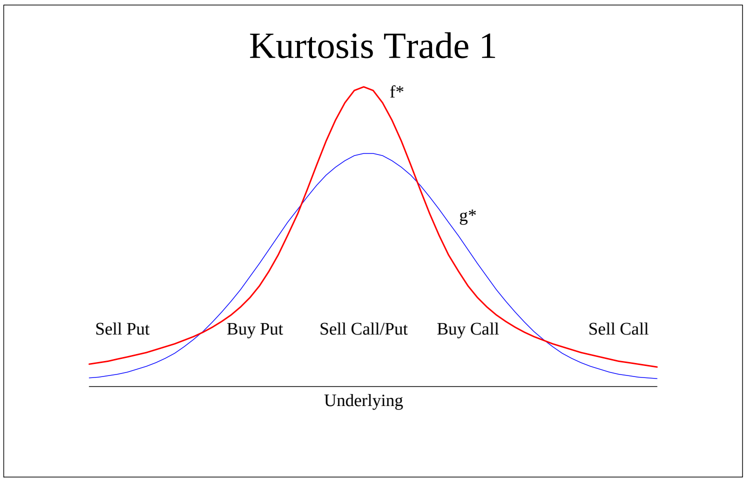

Figure 16: *Kurtosis trade 1.*

8.2.1 Motivation

The kurtosis κ measures the fatness of the tails of a distribution. For a normal distribution we have $\kappa = 3$. A distribution with $\kappa > 3$ is said to be leptokurtic and has fatter tails than the normal distribution. In general, the bigger κ is, the fatter the tails are. Again we consider the option pricing formulae (1) and (36) and reason as above using the probability mass to determine the moneyness regions where we buy or sell options. Look at figure(16) for a situation in which the implied SPD has more kurtosis than the time series SPD triggering to a K1 trade.

8.2.2 Payoff profile at Maturity

To form an idea of the K1 strategy's exposure at maturity we start once again with a simplified portfolio containing one short put with moneyness 0.90, one long put with moneyness 0.95, one short put and one short call with moneyness 1.00, one long call with moneyness 1.05 and one short call with moneyness 1.10. Figure(17) reveals that this portfolio inevitably leads to a negative payoff at maturity regardless the movement of the underlying.

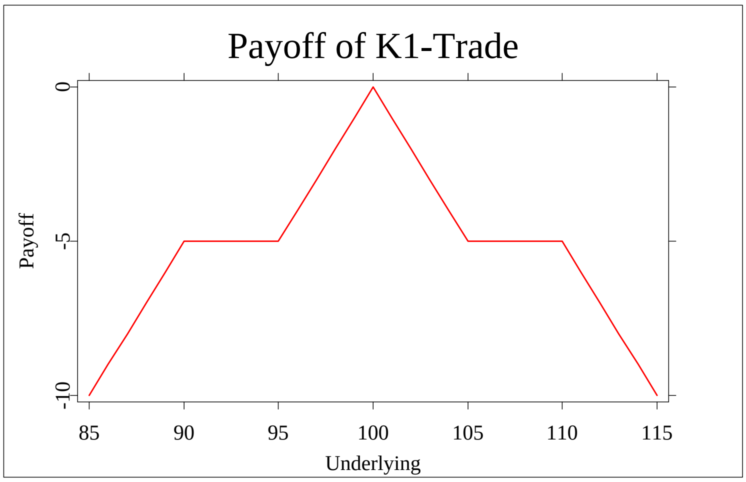

Figure 17: *Kurtosis trade 1 payoff at maturity of portfolio detailed in table(6).*

What happens if we increase the number of short FOTM puts and short FOTM calls? For example, sell puts at strikes K from 86 to 90 and sell calls at strikes K from 110 to 114 and keep the portfolio unchanged in the NOTM and ATM regions. The answer is quite the same as for the S1 trade (see figure(18)). The payoff in the left tail increases slower (the graph is concave in this region) and the payoff in the right tail decreases faster compared to the straight lines from figure(17). The payoff is still always negative.

Changing the number of long puts and calls in the NOTM regions can produce a positive payoff. Setting up the portfolio given in table(6) results in a payoff function shown in figure(19).

It is quite intuitive that the more long positions the portfolio contains the more positive the payoff will be. Conversely, if we add to that portfolio FOTM short puts and calls as detailed in table(6), FOTM–NOTM, the payoff decreases in the FOTM regions as figure(20) reveals.

Finally, should we be able to buy the whole range of strikes as the K1 trading rule suggests (the portfolio is given in table(6), FOTM–NOTM–ATM), we get a payoff profile which is quite similar to the one from figure(17). In fact, the payoff function looks like the ‘continuous’ version of figure(17).

As a conclusion we can state that the payoff function can have quite different shapes depending heavily on the specific options in the portfolio. If it is possible to implement the K1 trading rule as proposed the payoff is negative. But it may happen that the payoff function is positive in case that more NOTM options (long positions) are available than FOTM or ATM (short positions) options.

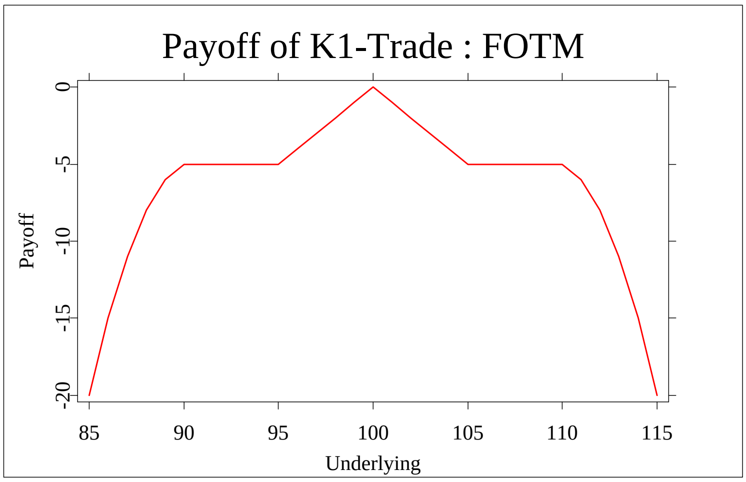

Figure 18: *Kurtosis trade 1 payoff at maturity of portfolio detailed in table(6).*

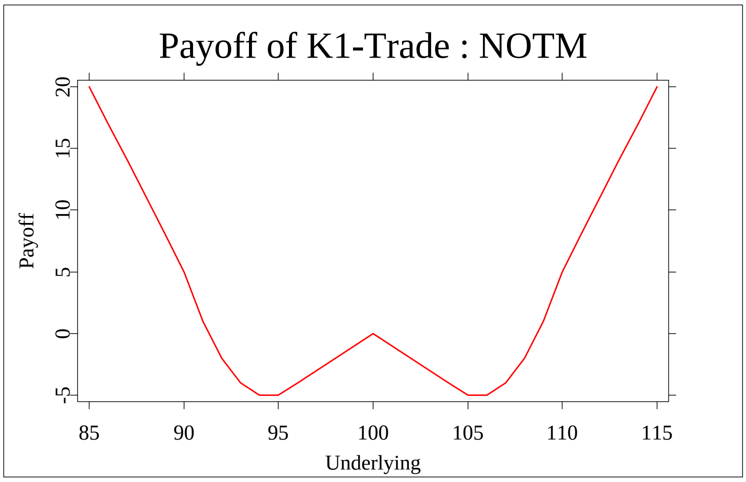

Figure 19: *Kurtosis trade 1 payoff at maturity of portfolio detailed in table(6).*

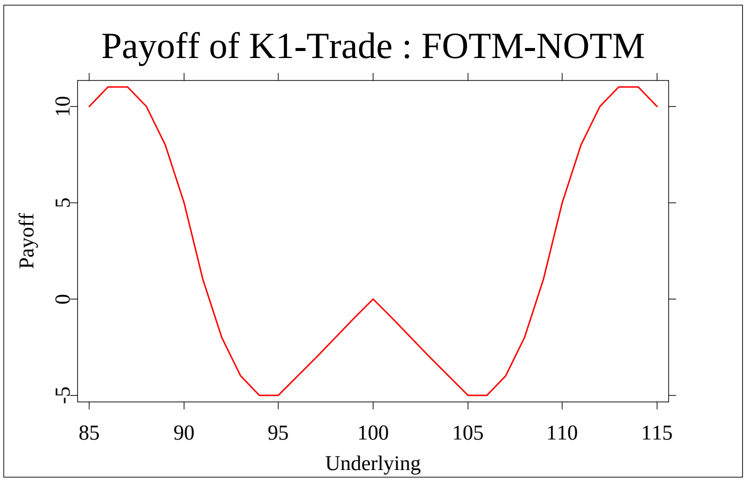

Figure 20: *Kurtosis trade 1 payoff at maturity of portfolio detailed in table(6).*

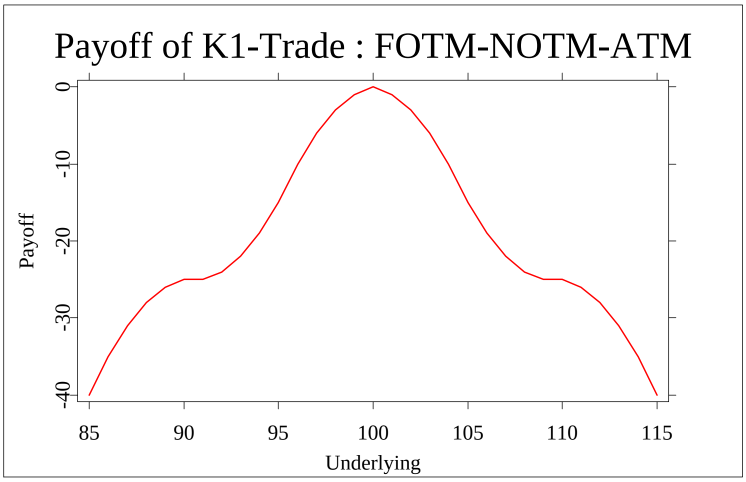

Figure 21: *Kurtosis trade 1 payoff at maturity of portfolio detailed in table(6).*

Table 1–K1	
Option	Moneyness
short put	0.90
long put	0.95
short put	1.00
short call	1.00
long call	1.05
short call	1.10

Table FOTM–K1	
Option	Moneyness
short put	$0.86 - 0.90$
long put	0.95
short put	1.00
short call	1.00
long call	1.05
short call	$1.10 - 1.14$

Table NOTM–K1	
Option	Moneyness
short put	0.90
long put	$0.91 - 0.95$
short put	1.00
short call	1.00
long call	$1.05 - 1.09$
short call	1.10

Table FOTM–NOTM–K1	
Option	Moneyness
short put	$0.86 - 0.90$
long put	$0.91 - 0.95$
short put	1.00
short call	1.00
long call	$1.05 - 1.09$
short call	$1.10 - 1.14$

Table FOTM–NOTM–ATM–K1	
Option	Moneyness
short put	$0.86 - 0.90$
long put	$0.91 - 0.95$
short put	$0.96 - 1.00$
short call	$1.00 - 1.04$
long call	$1.05 - 1.09$
short call	$1.10 - 1.14$

Table 6: *Portfolios of kurtosis trades.*

8.2.3 Performance

To investigate the performance of the kurtosis trades, K1 and K2, I proceed in the same way as for the skewness trades. From table(7) we can extract that the total net EURO cash flow of the K1 trade, applied when $\text{kurt}(f^*) > \text{kurt}(g^*)$, is strongly positive $(10,915.77$ EURO). As the payoff profile already suggested all portfolios generate a negative cash flow at expiration. In contrast to that, the cash flow at initiation in $t = 0$ is always positive. Given the positive total net cash flow, we can state that the K1 trade earns its profit in $t = 0$. Looking at the DAX evolution shown in figure(22), we understand why the payoff of the portfolios set up in the months of April 1997, May 1997 and in the months from November 1997 to June 1998 is relatively more negative than for the portfolios of June 1997 to October 1997 and November 1998 to June 1999. The reason is that the DAX is moving up or down for the former months and stays within an almost horizontal range of quotes for the latter months (see the payoff profile depicted in figure(21)). In July 1998 no portfolio was set up since $\text{kurt}(f^*) < \text{kurt}(g^*)$.

What would have happened if we had implemented the K1 trade without knowing both SPD's? Again, the answer to this question can only be indicated due to the rare occurrences of periods in which $\text{kurt}(f^*) < \text{kurt}(g^*)$. But look at table(7). Contrarily to the S1 trade, the SPD comparison has filtered out a strongly negative net cash flow that would have been generated by a portfolio set up in July 1998. But the significance of this feature is again uncertain.

About the K2 trade can only be said that without a SPD comparison it would have procured heavy losses. It can not be evaluated completely since there was only one period in which $\text{kurt}(f^*) < \text{kurt}(g^*)$. For completeness you find the performance in table(13) and table(14) in the appendix.

Month	CashFlow in $t=0$	CashFlow in $t=T$	NetCashFlow
Apr 97	1186.46	−4166.64	−2980.18
May 97	874.10	−2790.54	−1916.45
Jun 97	1070.54	−698.36	372.18
Jul 97	1488.75	−254.20	1234.54
Aug 97	1265.22	−332.36	932.86
Sep 97	2040.11	−133.95	1906.16
Oct 97	639.15	−16.24	622.91
Nov 97	1023.66	−1516.06	−492.40
Dec 97	1171.24	−2716.12	−1544.88
Jan 98	722.66	−1903.26	−1180.60
Feb 98	1049.10	−2757.24	−1708.15
Mar 98	1620.18	−1582.66	37.51
Apr 98	1606.69	−1275.72	330.98
May 98	1961.85	−337.60	1624.25
Jun 98	2448.11	−3239.67	−791.56
Jul 98	0.00	0.00	0.00
Aug 98	1690.20	−1065.71	624.50
Sep 98	4354.56	−366.96	3987.61
Oct 98	2787.59	−1395.12	1392.47
Nov 98	880.98	−663.70	247.29
Dec 98	3514.44	−543.75	2970.69
Jan 99	1839.99	0.00	1839.99
Feb 99	1727.85	−761.31	966.53
Mar 99	1987.80	−685.95	1301.85
Apr 99	1125.79	−598.88	535.91
May 99	1754.17	−58.28	1695.89
Jun 99	1448.94	−484.24	964.70
Jul 99	1631.62	−2044.62	−413.00
Aug 99	1351.15	−1667.91	−316.76
Sep 99	1636.54	−2965.60	−1329.05
Sum(NetCashFlows)	Mean(NetCashFlows)	Var(NetCashFlows)	Mean/$\sqrt{\text{Var}}$
10915.77	363.86	$2,231,551.71$	0.2436

Table 7: *Performance of K1 trade with 5% transaction costs. Cash flows are measured in EUROs.*

Month	CashFlow in $t = 0$	CashFlow in $t = T$	NetCashFlow
Apr 97	1186.46	−4166.64	−2980.18
May 97	874.10	−2790.54	−1916.45
Jun 97	1070.54	−698.36	372.18
Jul 97	1488.75	−254.20	1234.54
Aug 97	1265.22	−332.36	932.86
Sep 97	2040.11	−133.95	1906.16
Oct 97	639.15	−16.24	622.91
Nov 97	1023.66	−1516.06	−492.40
Dec 97	1171.24	−2716.12	−1544.88
Jan 98	722.66	−1903.26	−1180.60
Feb 98	1049.10	−2757.24	−1708.15
Mar 98	1620.18	−1582.66	37.51
Apr 98	1606.69	−1275.72	330.98
May 98	1961.85	−337.60	1624.25
Jun 98	2448.11	−3239.67	−791.56
Jul 98	1846.61	−9635.28	−7788.67
Aug 98	1690.20	−1065.71	624.50
Sep 98	4354.56	−366.96	3987.61
Oct 98	2787.59	−1395.12	1392.47
Nov 98	880.98	−663.70	247.29
Dec 98	3514.44	−543.75	2970.69
Jan 99	1839.99	0.00	1839.99
Feb 99	1727.85	−761.31	966.53
Mar 99	1987.80	−685.95	1301.85
Apr 99	1125.79	−598.88	535.91
May 99	1754.17	−58.28	1695.89
Jun 99	1448.94	−484.24	964.70
Jul 99	1631.62	−2044.62	−413.00
Aug 99	1351.15	−1667.91	−316.76
Sep 99	1636.54	−2965.60	−1329.05
Sum(NetCashFlows)	Mean(NetCashFlows)	Var(NetCashFlows)	Mean/$\sqrt{\text{Var}}$
3,127.10	104.24	4,449,111.12	0.0494

Table 8: *Performance of K1 trade with 5% transaction costs and without knowledge of IBT and time series SPD's. Cash flows are measured in EUROs.*

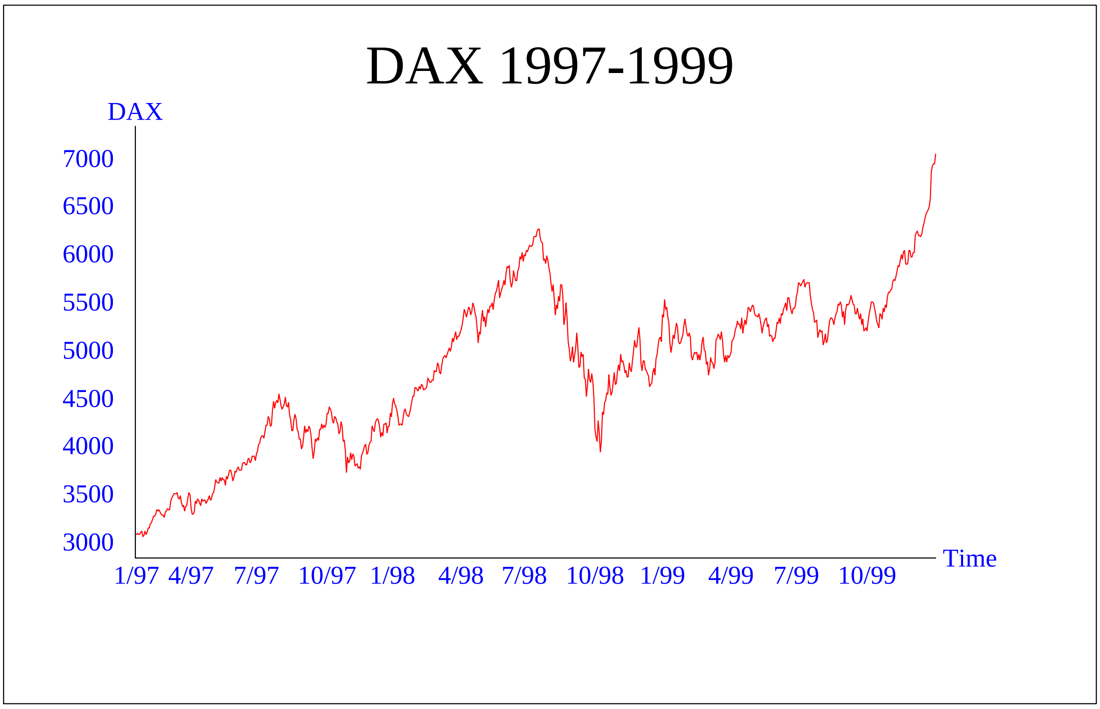

Figure 22: *Evolution of DAX from January 1997 to December 1999*

9 Conclusion

Interpreting the implied SPD as the SPD used by investors to price options, the historical density as the 'real' underlyings' SPD and assuming that no agent but one know the underlyings' SPD one should expect this agent to make higher profits than all others due to its superior knowledge. That is why, exploiting deviations of implied and historical density appears to be very promising at a first glance. Of course, if all market agents knew the underlyings' SPD, both SPD would be equal. In view of the high net cash flows generated by both skewness and kurtosis trades of type 1, it seems that not all agents are aware of discrepancies in the third and fourth moment of both SPD. However, the strategies seem to be exposed to a substantial directional risk. Even if the dataset contained bearish and bullish market phases, both trades have to be tested on more extensive data. Considering the current political and economic developments, it is not clear how these trades will perform being exposed to 'peso risks'. Given that profits stem from highly positive cash flows at portfolio initiation, i.e. profits result from possibly mispriced options, who knows how the pricing behavior of agents changes, how do agents assign probabilities to future values of the underlying?

We measured performance in net EURO cash flows. This approach does not take risk into account as, for example the Sharpe ratio which is a measure of the risk adjusted return of an investment. But to compute a return an initial investment has to be done. However, in the simulation above, some portfolios generated positive payoffs both at initiation and at maturity. It is a challenge for future research to find a way how to adjust for risk in such situations.

The SPD comparison yielded the same result for each period but one. The implied

SPD f^* was in all but one period more negatively skewed than the time series SPD g^*. While g^* was all in periods platykurtic, f^* was in all but one period leptokurtic. In this period the kurtosis of g^* was slightly greater than that of f^*. Therefore, there was no alternating use of type 1 and type 2 trades. But in more turbulent market environments such an approach might prove useful. The procedure could be extended and fine tuned by applying a density distance measure (as in Ait–Sahalia et al. (2000)) to give a signal when to set up a portfolio either of type 1 of type 2. Furthermore, it is tempting to modify the time series SPD estimation method such that the Monte Carlo paths be simulated drawing random numbers not from a normal distribution but from the distribution of the residuals resulting from the nonparametric estimation of $\sigma_{FZ}(.)$ (see Härdle and Yatchew (2001)).

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A Table: Skewness and Kurtosis of Implied SPD

Month	Skewness	Kurtosis
Jul 97	-1.2044	5.8235
Aug 97	-1.4131	6.6749
Sep 97	-1.1429	5.4931
Oct 97	-1.0101	4.9309
Nov 97	-0.7160	4.5001
Dec 97	-0.8709	4.6656
Jan 98	-0.2774	3.6744
Feb 98	-0.7868	3.9864
Mar 98	-0.7292	4.9341
Apr 98	-0.9373	4.4271
May 98	-1.1558	4.7437
Jun 98	-1.4207	6.2847
Jul 98	-0.6338	3.6951
Aug 98	-0.2437	3.5332
Sep 98	-1.2088	6.0198
Oct 98	-0.3043	2.7416
Nov 98	-0.5478	3.5555
Dec 98	-0.6648	3.5756
Jan 99	-0.4187	3.1339
Feb 99	-1.0525	4.3999
Mar 99	-0.6551	3.9228
Apr 99	-0.4920	3.4420
May 99	-0.6244	3.6367
Jun 99	-0.6087	4.2930
Jul 99	-0.8023	4.1396
Aug 99	-0.6233	3.7597
Sep 99	$+0.0023$	3.6290
Oct 99	-0.9020	4.3792
Nov 99	-0.6926	4.0425
Dec 99	-0.1878	3.2438

Table 9: *Skewness and kurtosis of f^* for all 30 periods.*

B Table: Skewness and Kurtosis of Historical SPD

Month	Skewness	Kurtosis
Jul 97	−0.0252	2.6875
Aug 97	−0.0219	2.9188
Sep 97	−0.0258	2.9518
Oct 97	−0.0158	2.8932
Nov 97	−0.0305	2.5674
Dec 97	−0.0290	2.4539
Jan 98	−0.0351	2.5716
Feb 98	−0.0236	2.4390
Mar 98	−0.0279	2.4354
Apr 98	−0.0280	2.4368
May 98	−0.0276	2.7539
Jun 98	−0.0184	2.9066
Jul 98	−0.0112	2.9504
Aug 98	−0.0295	2.6706
Sep 98	−0.0292	2.6388
Oct 98	−0.0321	2.7426
Nov 98	−0.0291	2.4450
Dec 98	−0.0258	2.4430
Jan 99	−0.0210	2.5707
Feb 99	−0.0244	2.4450
Mar 99	−0.0207	2.5143
Apr 99	−0.0239	2.4384
May 99	−0.0250	2.4340
Jun 99	−0.0330	2.4880
Jul 99	−0.0311	2.4875
Aug 99	−0.0345	2.5397
Sep 99	−0.0258	2.7493
Oct 99	−0.0219	2.8314
Nov 99	−0.0294	2.6611
Dec 99	−0.0269	2.8538

Table 10: *Skewness and kurtosis of g^* for all 30 periods.*

C Table: Performance of S2 trade

Month	CashFlow in $t = 0$	CashFlow in $t = T$	NetCashFlow
Apr 97	0.00	0.00	0.00
May 97	0.00	0.00	0.00
Jun 97	0.00	0.00	0.00
Jul 97	0.00	0.00	0.00
Aug 97	0.00	0.00	0.00
Sep 97	0.00	0.00	0.00
Oct 97	0.00	0.00	0.00
Nov 97	0.00	0.00	0.00
Dec 97	0.00	0.00	0.00
Jan 98	0.00	0.00	0.00
Feb 98	0.00	0.00	0.00
Mar 98	0.00	0.00	0.00
Apr 98	0.00	0.00	0.00
May 98	0.00	0.00	0.00
Jun 98	0.00	0.00	0.00
Jul 98	0.00	0.00	0.00
Aug 98	0.00	0.00	0.00
Sep 98	0.00	0.00	0.00
Oct 98	0.00	0.00	0.00
Nov 98	0.00	0.00	0.00
Dec 98	0.00	0.00	0.00
Jan 99	0.00	0.00	0.00
Feb 99	0.00	0.00	0.00
Mar 99	0.00	0.00	0.00
Apr 99	0.00	0.00	0.00
May 99	0.00	0.00	0.00
Jun 99	$-1,029.52$	0.00	$-1,029.52$
Jul 99	0.00	0.00	0.00
Aug 99	0.00	0.00	0.00
Sep 99	0.00	0.00	0.00

Table 11: *Performance of S2 trade with 5% transaction costs. Cash flows are measured in EUROs.*

D Table: Performance of S2 Trade without Knowing the SPD's

Month	CashFlow in $t=0$	CashFlow in $t=T$	NetCashFlow
Apr 97	0.00	0.00	0.00
May 97	0.00	0.00	0.00
Jun 97	−584.13	0.00	−584.13
Jul 97	0.00	0.00	0.00
Aug 97	−155.51	0.00	−155.51
Sep 97	−339.54	0.00	−339.54
Oct 97	−419.06	0.00	−419.06
Nov 97	219.93	−916.06	−696.14
Dec 97	−613.42	−4,196.76	−4,810.18
Jan 98	−450.87	0.00	−450.87
Feb 98	−121.59	0.00	−121.59
Mar 98	−1,126.57	−726.88	−1,853.45
Apr 98	−181.97	0.00	−181.97
May 98	0.00	0.00	0.00
Jun 98	−1,810.96	6,426.08	4,615.12
Jul 98	0.00	0.00	0.00
Aug 98	−534.03	127.14	−406.89
Sep 98	1,305.93	0.00	1,305.93
Oct 98	0.00	0.00	0.00
Nov 98	−574.04	0.00	−574.04
Dec 98	−963.04	0.00	−963.04
Jan 99	−622.13	0.00	−622.13
Feb 99	−204.65	0.00	−204.65
Mar 99	−1,057.84	0.00	−1,057.84
Apr 99	−324.77	0.00	−324.77
May 99	0.00	0.00	0.00
Jun 99	−1,029.52	0.00	−1,029.52
Jul 99	0.00	0.00	0.00
Aug 99	−154.98	0.00	−154.98
Sep 99	−1,106.18	−3,531.20	−4,637.38
Sum(NetCashFlows)	Mean(NetCashFlows)	Var(NetCashFlows)	Mean/$\sqrt{\text{Var}}$
−13,666.58	−455.55	2,427,903.29	−0.2924

Table 12: *Performance of S2 trade with 5% transaction costs and without knowledge of IBT and time series SPD's. Cash flows are measured in EUROs.*

E Table: Performance of K2 trade

Month	CashFlow in $t = 0$	CashFlow in $t = T$	NetCashFlow
Apr 97	0.00	0.00	0.00
May 97	0.00	0.00	0.00
Jun 97	0.00	0.00	0.00
Jul 97	0.00	0.00	0.00
Aug 97	0.00	0.00	0.00
Sep 97	0.00	0.00	0.00
Oct 97	0.00	0.00	0.00
Nov 97	0.00	0.00	0.00
Dec 97	0.00	0.00	0.00
Jan 98	0.00	0.00	0.00
Feb 98	0.00	0.00	0.00
Mar 98	0.00	0.00	0.00
Apr 98	0.00	0.00	0.00
May 98	0.00	0.00	0.00
Jun 98	0.00	0.00	0.00
Jul 98	−2040.99	9635.28	7594.29
Aug 98	0.00	0.00	0.00
Sep 98	0.00	0.00	0.00
Oct 98	0.00	0.00	0.00
Nov 98	0.00	0.00	0.00
Dec 98	0.00	0.00	0.00
Jan 99	0.00	0.00	0.00
Feb 99	0.00	0.00	0.00
Mar 99	0.00	0.00	0.00
Apr 99	0.00	0.00	0.00
May 99	0.00	0.00	0.00
Jun 99	0.00	0.00	0.00
Jul 99	0.00	0.00	0.00
Aug 99	0.00	0.00	0.00
Sep 99	0.00	0.00	0.00

Table 13: *Performance of K2 trade with* 5% *transaction costs. Cash flows are measured in EUROs.*

F Table: Performance of K2 Trade without Knowing the SPD's

Month	CashFlow in $t=0$	CashFlow in $t=T$	NetCashFlow
Apr 97	−1311.35	4166.64	2855.29
May 97	−966.11	2790.54	1824.44
Jun 97	−1300.47	698.36	602.11
Jul 97	−1645.46	254.20	−1391.25
Aug 97	−1429.58	332.36	−1097.22
Sep 97	−2612.69	133.95	−2478.74
Oct 97	−790.45	16.24	−774.21
Nov 97	−1180.14	1516.06	335.91
Dec 97	−1520.16	2716.12	1195.95
Jan 98	−889.13	1903.26	1014.13
Feb 98	−1183.91	2757.24	1573.34
Mar 98	−1967.62	1582.66	−384.96
Apr 98	−1812.31	1275.72	−536.59
May 98	−2168.36	337.60	−1830.76
Jun 98	−2953.09	3239.67	286.85
Jul 98	−2040.99	9632.28	7594.29
Aug 98	−1975.19	1065.71	−909.49
Sep 98	−5161.63	366.96	−4794.68
Oct 98	−3081.02	1395.12	−1685.90
Nov 98	−1088.81	663.70	−455.12
Dec 98	−4312.36	543.75	−3768.61
Jan 99	−2158.41	0.00	−2158.41
Feb 99	−1950.76	761.31	−1189.44
Mar 99	−2479.20	685.95	−1793.26
Apr 99	−1309.41	598.88	−719.53
May 99	−1938.83	58.28	−1880.54
Jun 99	−1738.66	484.24	−1254.42
Jul 99	−1803.38	2044.62	241.25
Aug 99	−1524.45	1667.91	143.46
Sep 99	−2000.85	2965.60	964.74
Sum(NetCashFlows)	Mean(NetCashFlows)	Var(NetCashFlows)	Mean/$\sqrt{\text{Var}}$
−11,675.84	−389.19	4,854,041.16	0.1767

Table 14: *Performance of K2 trade with 5% transaction costs and without knowledge of IBT and time series SPD's. Cash flows are measured in EUROs.*