Term Structure of Interest Rate

- A EURIBOR Analysis

A Master Thesis Presented

by

Ying Chen
(169607)

to

Prof. Dr. Wolfgang Härdle

Institute for Statistics and Econometrics

in partial fulfillment of the requirements

for the degree of

Master of Arts

Humboldt-Universität zu Berlin

School of Business and Economics

Spandauer Str. 1

D-10178 Berlin

Berlin, September 26, 2002

Declaration of Authorship

I hereby confirm that I have authored this master thesis independently and without use of others than the indicated resources. All passages, which are literally or in general matter taken out of publications or other resources, are marked as such.

Ying Chen

Berlin, September 26, 2002

Abstract

Modelling term structure of interest rate has become important for investors. The classical term structure model (Vasicek model, Cox-Ingersoll-Ross model and Heath-Jarrow-Morton model) however have some drawbacks in application. Based on EURIBOR (Euro interbank offered rate) data in period from 23/11/1998 to 18/06/2002, we find EURIBOR curve abides by mean reversion and follows an upward sloping tendency. The standard deviation of EURIBOR, standard deviation and kurtosis of EURIBOR daily shift have a humped-similar distribution with a deviation toward the short-term rate.

This paper aims to follow a phenomenological model to describe the EURIBOR term structure from statistical and financial viewpoints. Earlier work indicates that the average spread of the FRC (forward rate curve) grows like the square-root of maturity, with a prefactor which is comparable to the spot rate volatility.

$$f(t,\theta) = r(t) + \sigma_r \sqrt{\frac{\theta}{\theta_{max}}} + \xi(t,\theta)$$

We denote this behavior as the $MAT1/2$ rule. Instead of a square-root law argued by the $MAT1/2$ rule, we obtain a $MAT2/3$ rule holds for the average spread curve, i.e. the average spread is a function of maturity power to $\frac{2}{3}$. We argue the $MAT2/3$ rule could illustrate data very well. For the quantitative analysis, we use the interactive statistical computing programm XploRe and apply MD*Book, which provides the creation of net based interactive electronic books from LaTeX files.

Thanks to

Professor Dr. Wolfgang Härdle (CASE, Humboldt-Universität zu Berlin), Dr. Zdeněk Hlávka (CASE, Humboldt-Universität zu Berlin), Dr. Peter Schwendner (Bankhaus Sal. Oppenheim) for all the gentle-hearted helps and advices.

Contents

1 Motivation **1**

2 Term Structure of Interest Rate Models **3**

2.1 Classical models 3

2.1.1 Vasicek model 4

2.1.2 Cox-Ingersoll-Ross (CIR) model 6

2.1.3 Heath-Jarrow-Morton (HJM) model 6

2.2 A Phenomenological model 7

3 Data description **10**

3.1 EURIBOR data 10

3.2 Notation 19

3.3 Statistical characteristics of EURIBOR 20

3.3.1 Spread and mean reversion 20

3.3.2 Standard deviation (SD) 26

3.3.3 Correlation 30

3.3.4 Summary 30

3.4 Statistical features of daily shift 35

3.4.1 Daily shift standard deviation 35

3.4.2 Daily shift kurtosis 38

3.4.3 Daily shift correlation 41

4 Spread Model **45**

4.1 Spread model 45

4.1.1 Data10y 46

5 Conclusion **55**

Bibliography **57**

Index **59**

List of Figures

3.1 Boxplot for Data30y 14

3.2 Boxplot for Data10y 16

3.3 Boxplot for Data10y2 18

3.4 Spot rate and long spread for Data30y 22

3.5 Spot rate and long spread for Data10y 23

3.6 Spot rate and long spread for Data10y2 24

3.7 Standard deviation for Data30y 28

3.8 Standard deviation for Data10y & Data10y2 29

3.9 Correlation for Data30y 32

3.10 Correlation for Data10y & Data10y2 33

3.11 Daily shift standard deviation for Data30y 36

3.12 Daily shift standard deviation for Data10y & Data10y2 . 37

3.13 Daily shift kurtosis for Data30y 39

3.14 Daily shift kurtosis for Data10y & Data10y2 40

3.15 Daily shift correlation for Data30y 42

3.16 Daily shift correlation for Data10y & Data10y2 43

4.1 Average spread for Data30y, Data10y and Data10y2 . . . 48

4.2 $MAT2/3$ rule 49

4.3 $MAT1/2$ rule 50

4.4 PCA 54

List of Tables

3.1 EURIBOR Datasets 12

3.2 Summary for Data30y 13

3.3 Summary for Data10y 15

3.4 Summary for Data10y2 17

3.5 Spot rate and spread (%) 21

3.6 SD and correlation with spot rate for EURIBOR 34

3.7 SD(%), Kurtosis and correlation(%) with spot rate for daily shift EURIBOR 44

4.1 XploRe grqq 47

4.2 XploRe gls 47

4.3 EURIBOR PCA 52

1 Motivation

Term structure of interest rate or yield curve illustrates the relationship between interest rates and their maturities. An obvious feature in nowadays money and financial markets is the boom of the derivative securities. The volume of trading in interest rate derivatives increased in both the OTC and exchange-traded markets very quickly in the 1980s and 1990s, Hull (2000). More investors, especially large institutional investors deal with interest rate derivatives such as futures, options and swaps to seek above-market-performance profits or to hedge their individual exposed risks. How to price these interest rate derivatives properly is interesting for issuers and investors, who both ask a fair-price due to the arbitrage free theory. Though some practitioners prefer pricing according to their many years' experience, more investors are persuaded by quantitative approaches applying mathematical, statistical and econometric tools. Modelling term structure of interest rate becomes a building block for pricing interest rate derivatives, since the payoff is dependent on the level of interest rate.

However we really meet quite difficulties in modelling term structure of interest rate. First of all, interest rate derivatives have different maturi-

ties. We should choose the interest rate with same maturity as derivatives' to compute the price. Subsequently we would concern the whole yield curve instead of one single rate. Secondly in the market we can't get interest rates of all maturities. More challenging is the daily shift of term structure, which means the yield curve of interest rate changes everyday.

This master thesis aims to follow a phenomenological model to describe the EURIBOR term structure from statistical and financial viewpoints, based on EURIBOR (Euro interbank offered rate) data in period from 23/11/1998 to 18/06/2002. The remainder of this thesis is organized as follows: in section 2 we introduce backgrounds of modelling term structure of interest rate. In section 3 the structure of EURIBOR, based on the statistical features of EURIBOR and its daily shift is interpreted. We then build up an average spread model with the help of least squares estimation. We argue the average spread grows as some power of maturity, which is called $MAT2/3$ rule in this paper. From principal component analysis we maintain that one independent component is enough to interpret the deviation of EURIBOR from the average spread curve.

2 Term Structure of Interest Rate Models

2.1 Classical models

The most well-known models are the Vasicek model, the Cox-Ingersoll-Ross model and the Heath-Jarrow-Morton model. Although classical models are motivated by their mathematical tractability, they have weak ability to describe the real data unless a so-called market-risk premium is included, this is even true for the popular Heath-Jarrow-Morton model. The main reason is that classical models ask for an arbitrage-free scenario. Once a risk-adjusted dynamics has been calibrated, such models can obviously not tell us anything useful about the real-world dynamics of interest rates. Cont (2000) argues that transaction costs will avoid arbitrage opportunity although arbitrage-free doesn't hold in a model.

2.1.1 Vasicek model

The Vasicek model (1977) assumes a risk-neutral scenario and focuses on the dynamics of the instantaneous-spot-rate dynamics, Hull (2000), Franke, Härdle and Hafner (2001), Brigo and Mercurio (2001). The spot rate is the zero-coupon interest rate with the minimum maturity. The Vasicek Model assumes the movement of spot rates follow a Brownian motion, which indicates that the spot interest rate is a continuous stochastic process and the difference of two interest rates with different maturities has mean 0 and variance exactly equal to the maturity gap. The yield curve is replicated from the dynamics of spot rate. The replicated curve only depends on the current time and instantaneous spot rate but not on the maturities of the claims constituting the portfolio.

$$dr(t) = k\{\alpha - r(t)\}dt + \sigma dW(t) \tag{2.1}$$

where $r(t)$ is the spot interest rate at time t, $W(t)$ is a standard Wiener process with volatility 1, the parameter k controls the mean-reversion speed with positive value, α is the long-term average level of the spot interest rate, around which $r(t)$ moves.

The reason of the Vasicek model's popularity is its analytical and mathematical tractability. Integrating (2.1), we get the expression for the interest rate as

$$r(s) = r(t)e^{-k(s-t)} + \theta\{1 - e^{-k(s-t)}\} + \sigma \int_t^s e^{-k(s-t)}\, dW(u) \tag{2.2}$$

where $r(s)$ is interest rate with a longer maturity $s > t$. From (2.2), we could replicate a yield curve. Once we get the yield curve, we could calculate the price of interest rate derivatives. For instance, for a European option on discount bonds, we first calculate the payoffs occurring in different time period.

$$P(t,T) = E_t[exp\{-\int_t^T r(s)\,ds\ \}] \tag{2.3}$$

where $P(t,T)$ means at time point t the present value of coupons or principle occurred in T, T is the delivery date, $E_t[\bullet]$ is the expected value of the occurred payoff at time point t. Then we could apply Black-Scholes model to calculate the call price. Replicating yield curve's mean-reversion is another achievement of Vasicek model. As t goes to infinity in equation (2.2), $r(t)$ will converge to α, which means α is a long-term average level. Mean-reversion is a vital feature of yield curve, otherwise a non-stop increase will happen, which obviously contradicts the practice.

The Vasicek model has an unavoidable drawback based on its assumption. The spot rate can be negative, since its movement obeys Brownian motion. However nominal interest rates are always positive in the market. Another drawback of the Vasicek model is the perfect correlation for yield curves from (2.2). Perfect correlation means that the correlation of two random interest rates with different maturity is always 1, which is rejected e.g. by Florent and Priaulet (2000).

2.1.2 Cox-Ingersoll-Ross (CIR) model

Cox, Ingersoll and Ross (1985) have proposed an alternative model based on Vasicek model. They introduce a $\sqrt{r(t)}$ to avoid the occurrence of negative short-term interest rate r.

$$dr(t) = k\{\alpha - r(t)\}dt + \sigma\sqrt{r(t)}dW(t) \tag{2.4}$$

The CIR model has the same assumptions and parameters as the Vasicek model except that the standard deviation is proportional to $\sqrt{r(t)}$, which guarantees the non negativeness of the spot rate $r(t)$.

2.1.3 Heath-Jarrow-Morton (HJM) model

Heath, Jarrow and Morton (1992) have derived an arbitrage-free framework for a stochastic evolution of the entire yield curve from an instantaneous forward rate. Using the spot curve observed in the market, the idea is to equate the underlying securities with the entire yield curve, the dynamics of which can be described by the dynamics of zero-coupon bonds or instantaneous forward rates. The assumption is based on the absence of arbitrage opportunities, which implies that the discounted prices of such products are martingales under a risk-neutral probability, Florent and Priaulet (2000).

$$df(t,T) = \alpha(t,T)dt + \sigma(t,T)dW(t) \tag{2.5}$$

where $f(t,T)$ is instantaneous forward rate of maturity $T-t$, T is delivery date, $\alpha(t,T) = \sigma(t,T)\int_t^T \sigma(t,\tau)\,d\tau$ is instantaneous drift, $\sigma(t,T)$ is standard deviation of $f(t,T)$ and $W(t)$ is a standard wiener process.

The HJM approach could be extended to a model with multi-factors:

$$df(t,T) = \alpha(t,T)dt + \sum_k \sigma_k(t,T)dW(t) \tag{2.6}$$

where $\alpha(t,T) = \sum_k \sigma_k(t,T) \int_t^T \sigma_k(t,\tau)\, d\tau$ is instantaneous drift for the multi-factors HJM model. It has gained a high degree of popularity, Brigo and Mercurio (2001). One reason is the correlation-dependent instruments such as European swaption traded alongside caps and floors have emerged in the last few years. Such instruments ask a model to replicate the imperfect correlation among rates. Interest rate is a variable influenced mostly by macroeconomics factors. The HJM approach can be extended to a multi-factor model to include these factors, which is tractable in application, Rebonato (1996). However Bouchaud, Sagna, Cont, El-Karoui and Potters (1997) found the HJM model is unsuitable to illustrate empirical data.

2.2 A Phenomenological model

Among the new contributions for modelling term structure of interest rate, a recently proposed phenomenological model is brilliant due to its mathematical tractability and good replication of the real structure of interest rate. Bouchaud et al. (1997) built up a heuristic model based on daily Eurodollar futures contracts from 1990 to 1996. They argued in favor of a $MAT1/2$ rule holding for average spread curve. The average spread, which is the average deviation of FRC (forward rate curve) from spot rate (3-month forward rate), grows as the square root of the maturity, with a prefactor which is comparable to the spot rate's volatility.

The model is:

$$f(t,\theta) = r(t) + s(t)\frac{Y[\theta]}{Y[\theta_{max}]} + \xi(t,\theta) \tag{2.7}$$

where $f(t,\theta)$ is the forward rate of maturity θ at time point t, the minimum maturity θ_{min} is 3 months and the maximum maturity θ_{max} is 8 years. $r(t) = f(t,\theta_{min})$ is the spot rate with minimum maturity 3 months and $s(t) = f(t,\theta_{max}) - r(t)$ is defined as long spread. $Y[.]$ is a certain time independent function. They defined $Y[\theta_{max}] = 1$, $Y[\theta_{min}] = 0$, and $\xi(t,\theta)$ represents the deviation from the empirical average curve. Apparently $\xi(t,\theta_{min}) = \xi(t,\theta_{max}) = 0$ and the time average of $\xi(t,\theta)$ is assumed $E_t[\xi(\bullet,\theta)] = 0$.

Based on these Eurodollar futures the authors found the average partial spread $E_t[s(\bullet,\theta)] = E_t[f(\bullet,\theta) - r(\bullet)]$ fits a square-root law respect to maturity and with a prefactor corresponding to the spot rate standard deviation, which strongly suggests that forward rate market prices include a risk premium.

$$E_t[s(\bullet,\theta)] = \sigma_r\sqrt{\frac{\theta}{\theta_{max}}} + \varepsilon(\theta) \tag{2.8}$$

Simultaneously they proved the deviation $\Delta(\theta) = \sqrt{E_t[\xi(t,\theta)^2]}$ from the average spread curve follows a humped distribution. For daily shift $\delta f(t,\theta) = f(t+1,\theta) - f(t,\theta)$, the authors defined volatility $\sigma(\theta) = \sqrt{E_t[\delta f(t,\theta)^2]}$, excess kurtosis of daily shift $\kappa(\theta) = \frac{E_t[\delta f(t,\theta)^4]}{\sigma^4(\theta)} - 3$ and spread correlation $C(\theta) = \frac{E_t[\delta f(t,\theta_min)(\delta f(t,\theta) - \delta f(t,\theta_min))]}{\sigma^2(\theta_min)}$, which measures the influence of the spot rate fluctuations on the other modes of motion. They found the earlier notation all follow a humped distribution and with a peak around 1 year, which indicates the instantaneous FRC departs from $MAT1/2$ rule with a maximum distortion around one year.

Matacz and Bouchaud (1999) proved that the phenomenology approach fitted for 3 forward rate curves (the Eurodollar CME-IMM contract, the Short Sterling LIFFE contract and the Bank Accepted Bills SFE contract) but didn't for DEM and JPY (the Euromark LIFFE contract and Euroyen TIFFE contract).

We will follow the ideas of the phenomenological model to explore EURIBOR (Euro Interbank Offered Rate) in the following sections.

3 Data description

3.1 EURIBOR data

Our study is based on daily EURIBOR (Euro Interbank Offered Rate) par rate. Par rate is a yield on a bond that makes its price equal to the principal. Par rate of EURIBOR is the internal rate of return from which financial instruments are actually traded. EURIBOR has been the benchmark rate of the large euro money market since it emerged in 1999. EURIBOR is sponsored by the European Banking Federation (FBE), which represents the interests of 3,000 banks in the 15 Member States of the European Union and in Iceland, Norway and Switzerland and by the Financial Markets Association (ACI). EURIBOR is the rate at which euro interbank term deposits are offered by one prime bank to another prime bank and is published at 11.00 a.m. CET (Central European Time) for spot value 2 days later. The choice of banks quoting for EURIBOR is based on market criteria. These banks are of first class credit standing. They have been selected to ensure that the diversity of the euro money market is adequately reflected, thereby making EURIBOR an efficient and representative benchmark. Since its launch, EURIBOR

has become a reality on the derivatives markets and is the underlying rate of many derivatives transactions, both OTC and exchange-traded. (see EURIBOR)

Our EURIBOR data comes from Dr. Schwendner (Bankhaus Sal. Oppenheim). There are 19 maturities for EURIBOR from 1 week to 30 years, which are mainly from 23/11/1998 to 18/06/2002 (900 trading days) except for the rates with maturities of 6 months, 9 months and 1 year. The minimum maturity is 1 week, which equals 1/52 year. We divide the original par rates into three data sets, for the following reasons:

1. **Data30y:** Data has 16 maturities from 1 week to 30 years without 6-month, 9-month and 1-year rates, since the rates of these 3 maturities have only 207 trading days.
2. **Data10y:** Data has 13 maturities from 1 week to 10 years without 6-month, 9-month and 1-year data. Since we want to compare our result with the empirical researches from Bouchaud et al. (1997) and Matacz and Bouchaud (1999), whose maximum maturities of interest rates are less than 10, i.e. 8 years, Bouchaud et al. (1997) and 9.5 years, Matacz and Bouchaud (1999) respectively.
3. **Data10y2:** Data has 16 maturities from 1 week to 10 years in period from 22/08/2001 to 18/06/2002, which are 207 trading days. In view of the fact that Bouchaud et al. (1997) find a humped distributions of forward rate daily shift's standard deviation, excess kurtosis and spread correlation with a peak around 1 year, we argue there maybe exist some special features around 1 year.

We give a short description of the 3 data sets in the following.

Maturity	Data30y	Data10y	Data10y2
1 week (D7)	x	x	x
1 month (M1)	x	x	x
2 months (M2)	x	x	x
3 months (M3)	x	x	x
6 months (M6)			x
9 months (M9)			x
1 year (Y01)			x
2 years (Y02)	x	x	x
3 years (Y03)	x	x	x
4 years (Y04)	x	x	x
5 years (Y05)	x	x	x
6 years (Y06)	x	x	x
7 years (Y07)	x	x	x
8 years (Y08)	x	x	x
9 years (Y09)	x	x	x
10 years (Y10)	x	x	x
15 years (Y15)	x		
20 years (Y20)	x		
30 years (Y30)	x		

Table 3.1: EURIBOR Datasets

Maturity	Minimum	Maximum	Mean	Median	Std. Error
1 week (D7)	2.5	5.17	3.7024	3.38	0.79479
1 month (M1)	2.51	5.03	3.7414	3.5	0.77909
2 months (M2)	2.51	5.12	3.7732	3.5	0.77082
3 months (M3)	2.51	5.14	3.8021	3.52	0.76497
2 years (Y02)	2.83	5.56	4.2611	4.37	0.72486
3 years (Y03)	3	5.65	4.4644	4.6	0.69355
4 years (Y04)	3.21	5.71	4.6359	4.76	0.66232
5 years (Y05)	3.4	5.76	4.7833	4.9	0.63135
6 years (Y06)	3.56	5.84	4.9196	5.03	0.60252
7 years (Y07)	3.69	5.91	5.0437	5.16	0.57604
8 years (Y08)	3.82	5.96	5.1497	5.25	0.55131
9 years (Y09)	3.93	6.01	5.2332	5.33	0.52788
10 years (Y10)	4.02	6.04	5.2986	5.39	0.51158
15 years (Y15)	4.36	6.27	5.5527	5.63	0.45665
20 years (Y20)	4.56	6.37	5.6748	5.77	0.41628
30 years (Y30)	4.75	6.39	5.7244	5.81	0.36963

Table 3.2: Summary for Data30y

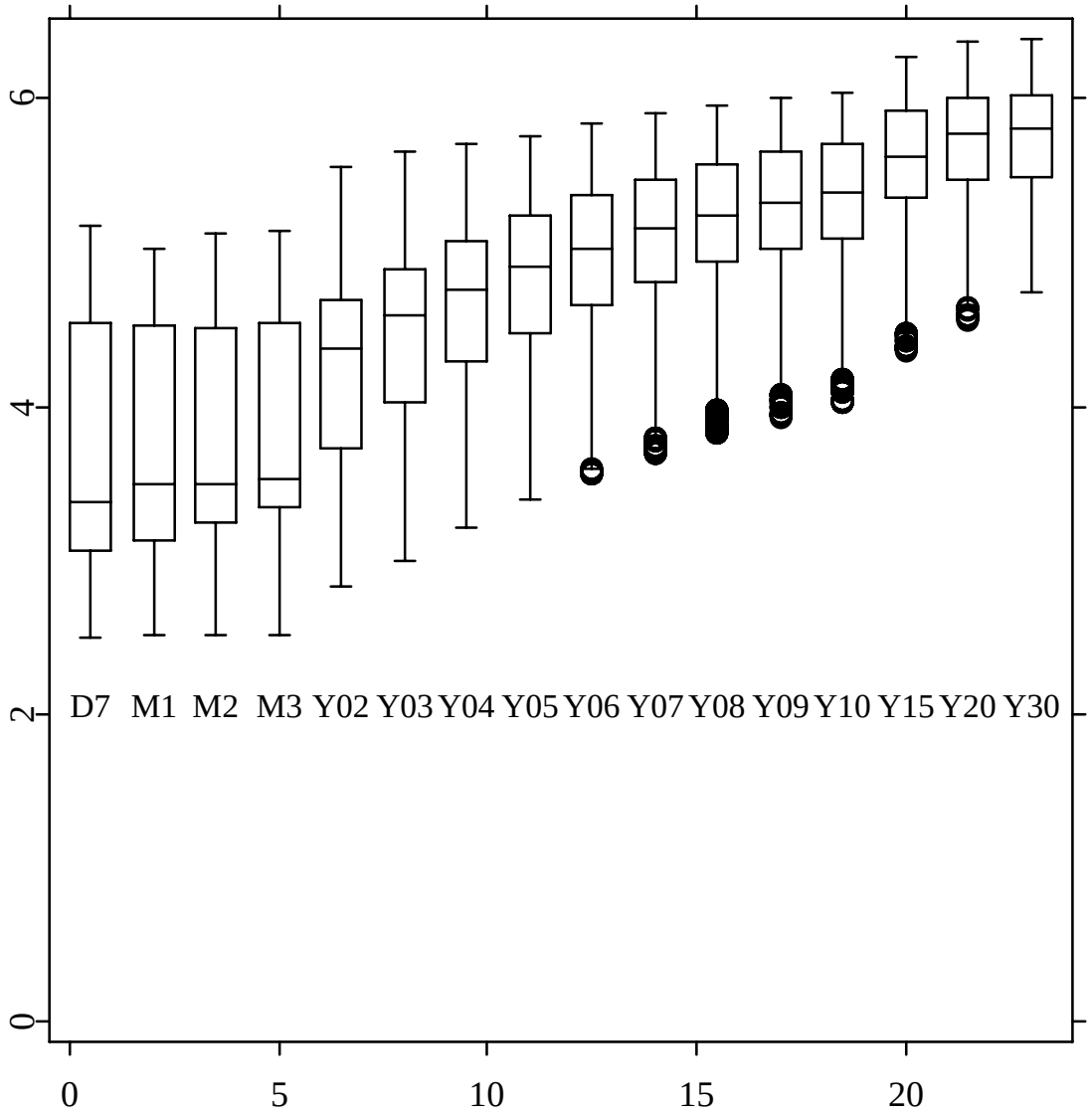

Figure 3.1: Boxplot for Data30y

Boxplot for Data30y EURIBORboxplot.xpl

Maturity	Minimum	Maximum	Mean	Median	Std. Error
1 week (D7)	2.5	5.17	3.7024	3.38	0.79479
1 month (M1)	2.51	5.03	3.7414	3.5	0.77909
2 months (M2)	2.51	5.12	3.7732	3.5	0.77082
3 months (M3)	2.51	5.14	3.8021	3.52	0.76497
2 years (Y02)	2.83	5.56	4.2611	4.37	0.72486
3 years (Y03)	3	5.65	4.4644	4.6	0.69355
4 years (Y04)	3.21	5.71	4.6359	4.76	0.66232
5 years (Y05)	3.4	5.76	4.7833	4.9	0.63135
6 years (Y06)	3.56	5.84	4.9196	5.03	0.60252
7 years (Y07)	3.69	5.91	5.0437	5.16	0.57604
8 years (Y08)	3.82	5.96	5.1497	5.25	0.55131
9 years (Y09)	3.93	6.01	5.2332	5.33	0.52788
10 years (Y10)	4.02	6.04	5.2986	5.39	0.51158
15 years (Y15)	4.36	6.27	5.5527	5.63	0.45665
20 years (Y20)	4.56	6.37	5.6748	5.77	0.41628
30 years (Y30)	4.75	6.39	5.7244	5.81	0.36963

Table 3.3: Summary for Data10y

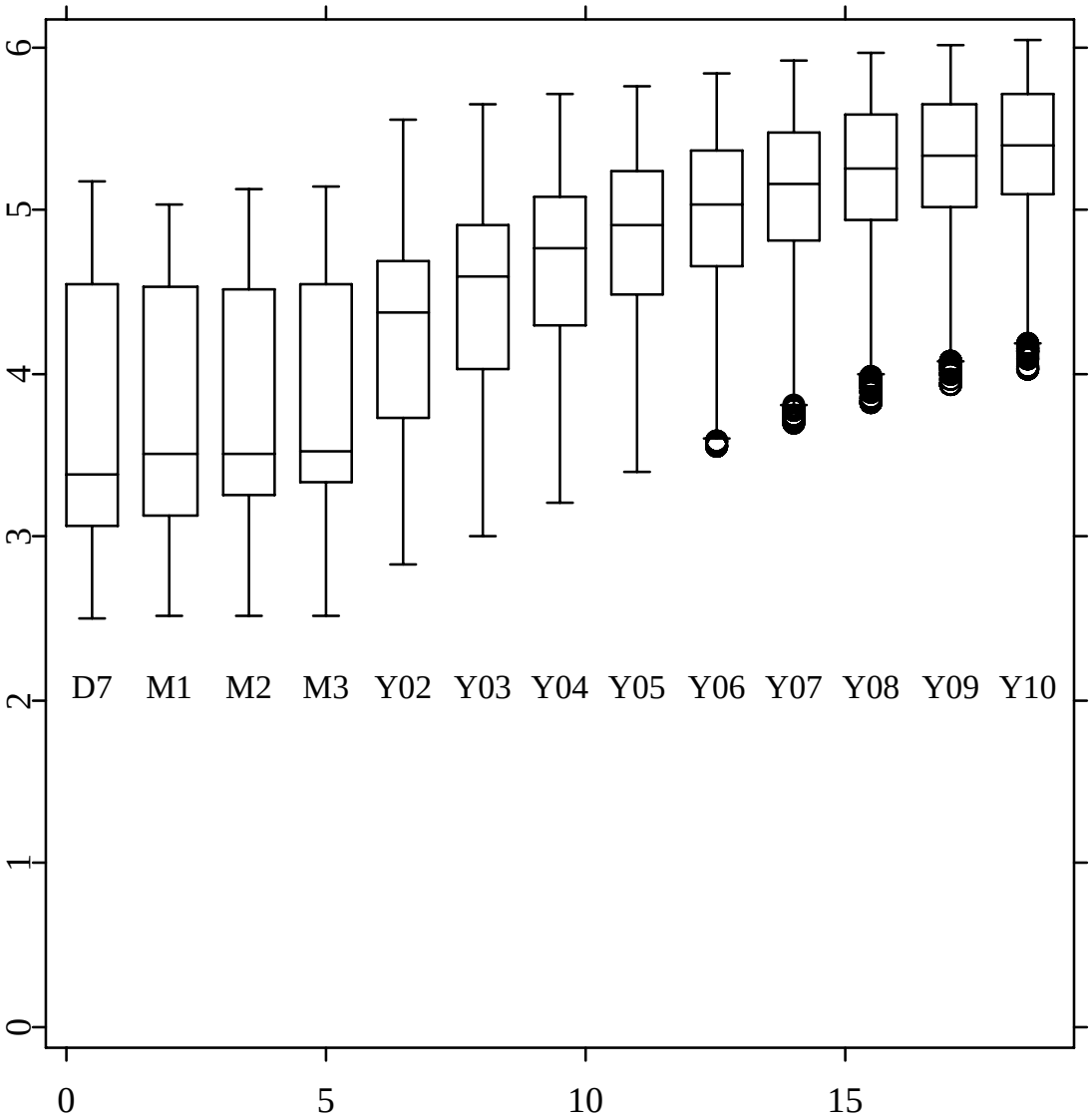

Figure 3.2: Boxplot for Data10y

Boxplot for Data10y EURIBORboxplot.xpl

Maturity	Minimum	Maximum	Mean	Median	Std. Error
1 week (D7)	3.23	4.55	3.5176	3.34	0.33287
1 month (M1)	3.31	4.43	3.5117	3.37	0.29689
2 months (M2)	3.29	4.36	3.5083	3.39	0.27112
3 months (M3)	3.28	4.31	3.5061	3.4	0.25687
6 months (M6)	3.11	4.19	3.5082	3.48	0.23753
9 months (M9)	2.99	4.12	3.5535	3.51	0.25681
1 year (Y01)	3.01	4.15	3.6762	3.67	0.29328
2 years (Y02)	3.15	4.57	4.0214	4.07	0.35666
3 years (Y03)	3.41	4.81	4.2859	4.34	0.35302
4 years (Y04)	3.66	4.99	4.4906	4.53	0.33309
5 years (Y05)	3.89	5.12	4.6618	4.68	0.30904
6 years (Y06)	4.09	5.23	4.8099	4.83	0.28433
7 years (Y07)	4.25	5.32	4.9336	4.96	0.26291
8 years (Y08)	4.38	5.39	5.0344	5.07	0.24553
9 years (Y09)	4.5	5.45	5.1164	5.14	0.23085
10 years (Y10)	4.59	5.5	5.1811	5.21	0.22

Table 3.4: Summary for Data10y2

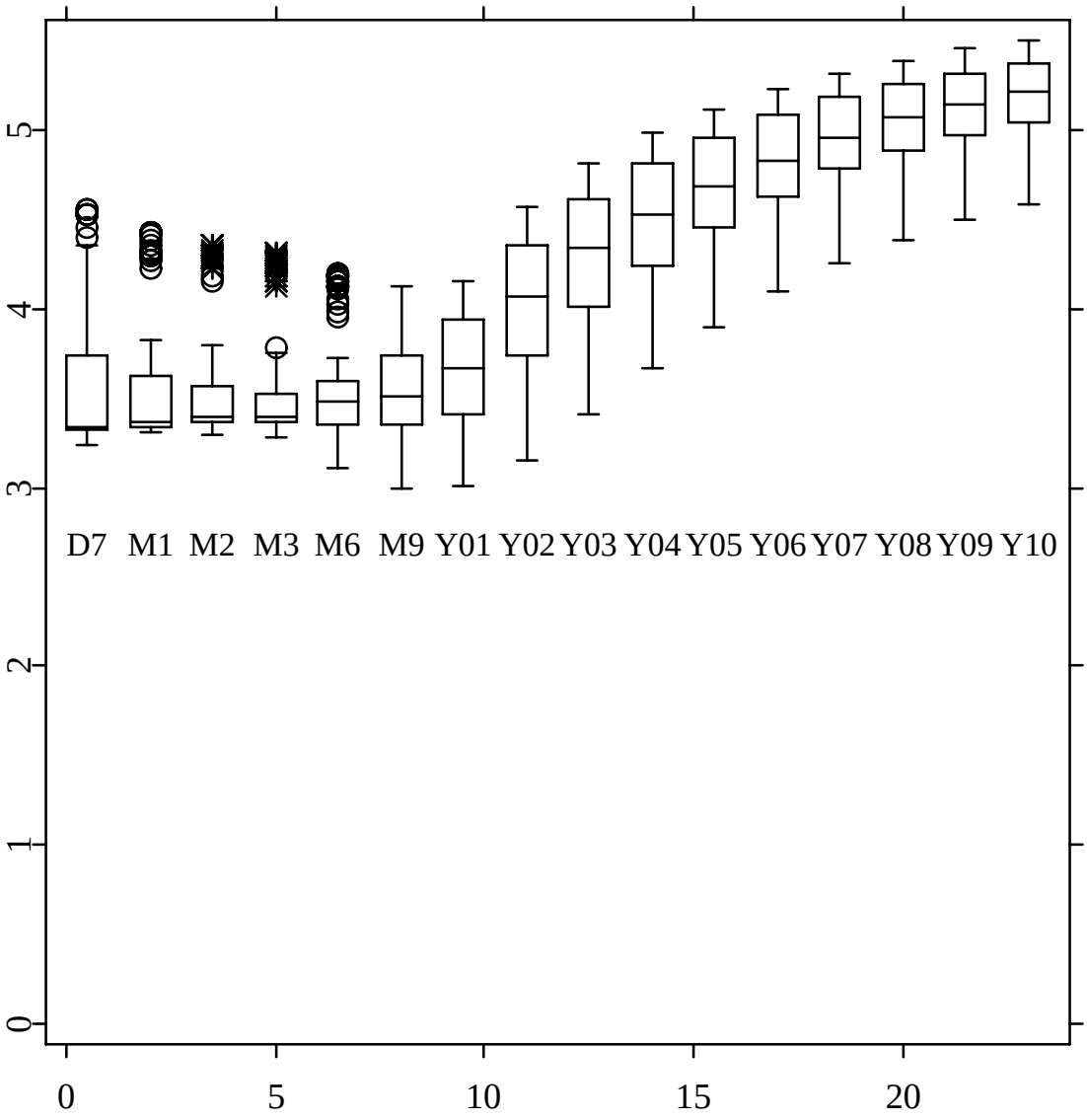

Figure 3.3: Boxplot for Data10y2

Boxplot for Data10y2 `EURIBORboxplot.xpl`

3.2 Notation

In our study, we choose 1-week rate as spot rate, i.e. interest rate of the minimum maturity. We define spot rate, long spread, partial spread, daily shift and some statistical notations as follows:

Spot rate:

$$r(t) = r(t, \theta_{min}) = r(t, \frac{1}{52}) \tag{3.1}$$

Long spread:

$$s(t) = r(t, \theta_{max}) - r(t) \tag{3.2}$$

Partial spread:

$$s(t, \theta) = r(t, \theta) - r(t) \tag{3.3}$$

Daily shift:

$$\delta(t, \theta) = r(t + 1, \theta) - r(t, \theta) \tag{3.4}$$

The standard deviation (Volatility) of EURIBOR:

$$\sigma(\theta) = \sqrt{E_t[r(\bullet, \theta) - E_t\{r(\bullet, \theta)\}]^2} \tag{3.5}$$

where, $E[r(t, \theta)]$ is the expected value over time point t.

The kurtosis of EURIBOR:

$$\kappa(\theta) = \frac{E_t[r(\bullet, \theta) - E_t\{r(\bullet, \theta)\}]^4}{\sigma^4(\theta)} \tag{3.6}$$

We use the correlation to measure the influence of the spot rate fluctuations on the other modes of motion

$$C(\theta) = \frac{E_t[r(\bullet) - E_t\{r(\bullet)\}] \cdot E_t[r(\bullet, \theta) - E_t\{r(\bullet, \theta)\}]}{\sigma^2(\theta_{min})} \tag{3.7}$$

3.3 Statistical characteristics of EURIBOR

3.3.1 Spread and mean reversion

The phenomenological model is based on the approximation of long spread (3.2). For data10y, the long spread varies between 0.19% and 3.33% and keeps always positive, see table(3.5). Analogously a positive long spread is found in data30y and data10y2. Related to risk and liquidity we observe such a situation is natural, since as a general rule, the longer the maturity of a security or an investment, the higher its risk and the worse its liquidity are. The long-term loan creditors will then ask a risk premium above the short-term rate at a particular time point, which indicates a positive spread in our research.

Sometimes we could also meet another situation, i.e. negative spread, for instance, Matacz and Bouchaud (1999) got a negative long spread for GBP data of daily prices of futures contracts on 3 month forward interest rate between 18/03/1994 and 18/02/1999, which could be produced by some macroeconomics variable's diffusion. This unusual condition of short-term rates exceeding long-term rates indicates a situation where the market expects a pronounced high in interest rates, Kern and Gutmann (1992). We think a positive spread however is more natural, since an upward sloping yield curves tends to be more often than the downward sloping.

Dataset	Mean of 1 week EURIBOR spread	Min. long	Max. long spread	Correlation of spot rate and long spread
Data30y	3.7024	0.65	3.75	-0.78023
Data10y	3.7024	0.19	3.33	-0.78023
Data10y2	3.5176	0.71	2.2	-0.87082

Table 3.5: Spot rate and spread (%)

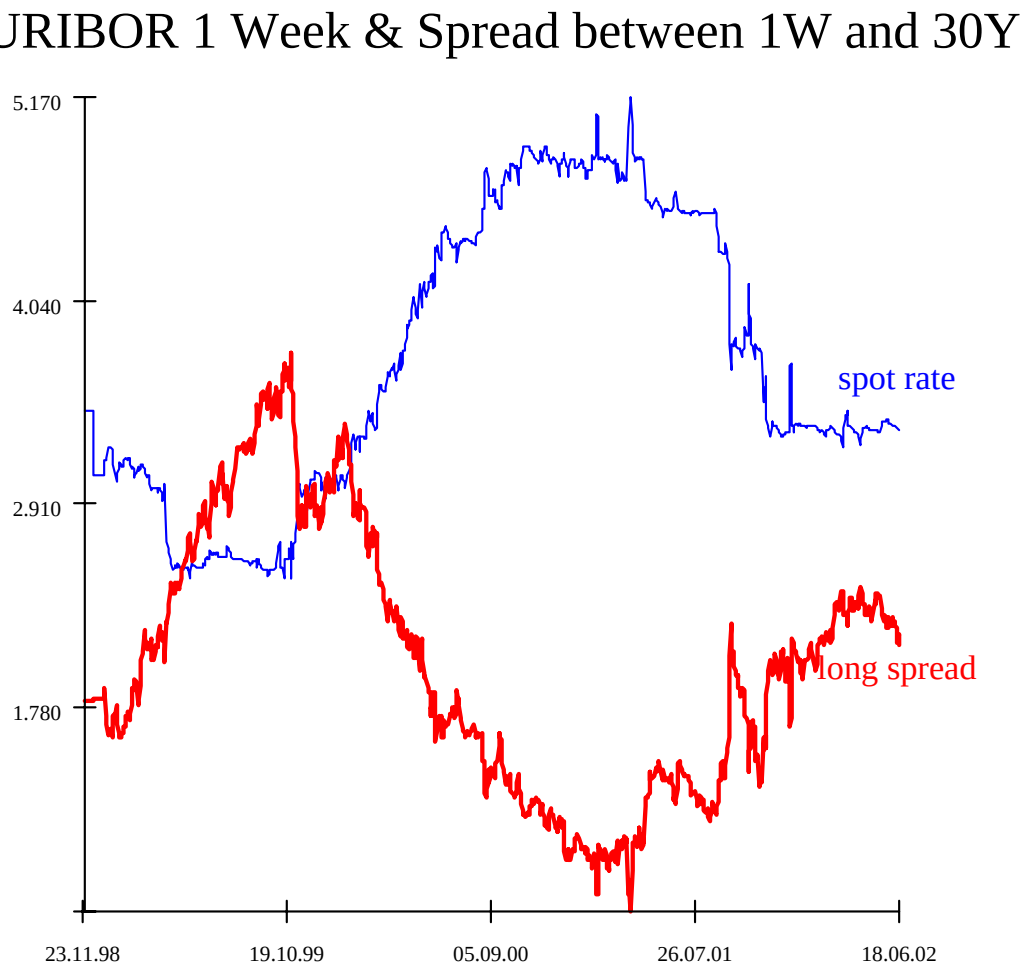

Figure 3.4: Spot rate and long spread for Data30y

Spot rate and long spread $s(t) = r(t, 30) - r(t)$ for Data30y with blue line is spot rate and red line is the long spread `EURIBORspread1.xpl`

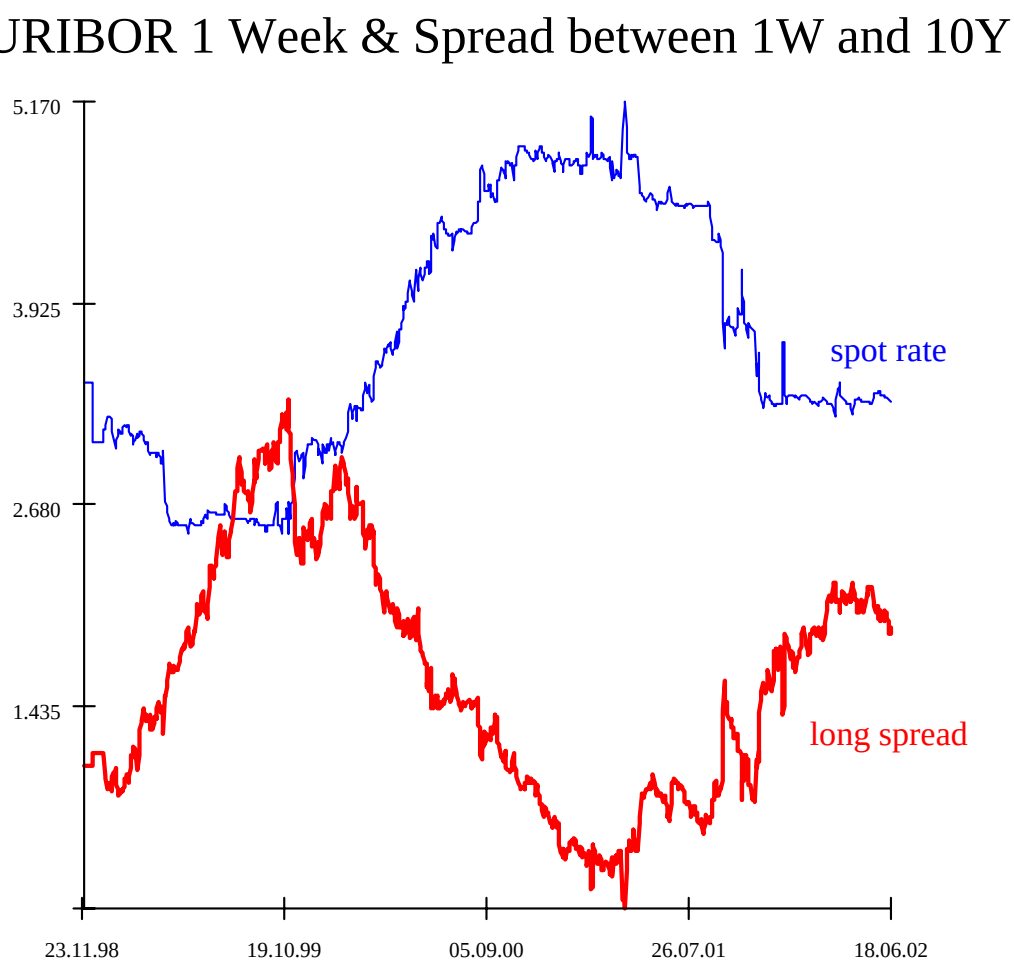

Figure 3.5: Spot rate and long spread for Data10y

Spot rate and long spread $s(t) = r(t, 10) - r(t)$ for Data10y with blue line is spot rate and red line is the long spread `EURIBORspread2.xpl`

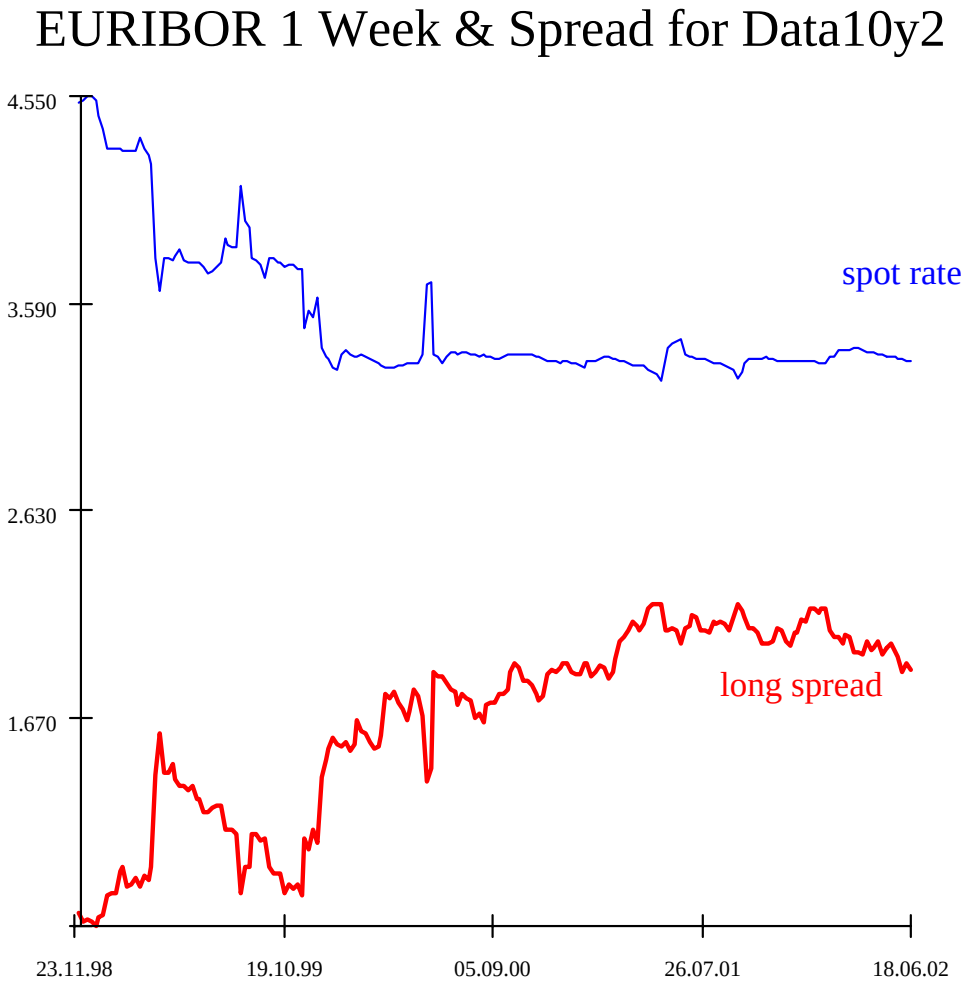

Figure 3.6: Spot rate and long spread for Data10y2

Spot rate and long spread $s(t) = r(t, 10) - r(t)$ for Data10y2 with blue line is spot rate and red line is the long spread `EURIBORspread3.xpl`

Another interesting point observed from the figures is the existence of an equilibrium level of the spot rate, which indicates a mean reversion feature of the interest rate. Mean reversion means that interest rates appear over time to be pulled back to some long-run equilibrium level. When the spot rate is too high, a negative drift tends to be caused by mean reversion and then the rates decrease, and when spot rate is low, a positive trend will appear to close to the long-run average level. We argue the mean reversion feature is economic reasonable, since as interest rate increases, borrowers will bear this high interest rate and be reluctant to enlarge their investment.

Simultaneously the supply of loans increase, as a result, the interest rate declines. However, an equilibrium of 3.7024% for 1-week rate is very smaller respect to other currency interest rate. For example, Matacz and Bouchaud (1999) found a general equilibrium rate of about 6.5% for USD, GBP, AUD and JPY except the DEM, which fluctuates around 3.93%. The possible reason could be the economic environment diffusion based on different regions. One possibility is that the government supervisory and intervention level for money markets are different.

Normally, USA and England more rely on the market adjustment while in Japan the government play a more important role. Another reason is based on the relationship between yield curve and inflation rate. Irving Fisher's theory, Brealey and Myers (2000)

$$NominalInterestRate \approx RealInterestRate + InflationRate$$

indicates the higher inflation rate is, the higher nominal interest rate and in turns the higher equilibrium level is. The small equilibrium value is maybe driven by a small inflation rate.

3.3.2 Standard deviation (SD)

The standard deviation of EURIBOR is a decreasing curve of the maturity. For instance the standard deviation reduces from 0.80% for 1-week rate to 0.51% for 10-year rate, which could be explained by mean reversion. The existence of a long-run average level means a convergence of the term structure curve, which from the statistical viewpoint means the interval of possible value of long-term rate drops off. Nevertheless the SD curve is not a single downward curve. For data10y2, which has more detailed maturities around 1 year, the SD curve declines first from 1-week to 6-month and then follows a humped distribution with a peak around 2 years. We argue the standard deviation of EURIBOR should be humped-similar distributed with some deviates towards smaller maturities. And the downward sloping curve for the data10y is caused by the shortage of the data with 6-month, 9-month and 1-year maturities. There is no general agreement on reasons for the existence of the hump. One possible explanation is as follows. Short-term rates are controlled by central banks, Hull (2000). By contrast, two- and three-year interest rates are determined to a large extent by the activities of traders. These traders may be over-reacted to the changes they observe in the short rate. It leads the volatility of these rates to be higher than the volatility of short rates. The deviation of humped distribution towards the short-term rate is caused by the frequent and sometime emergent short-term credit requirements. For maturities beyond two to three years the mean reversion of interest rates causes volatility to decline. A humped distribution of SD is also found by Cont (2000), Matacz and Bouchaud (1999)

and Rebonato (1996) but with a peak around 1 year.

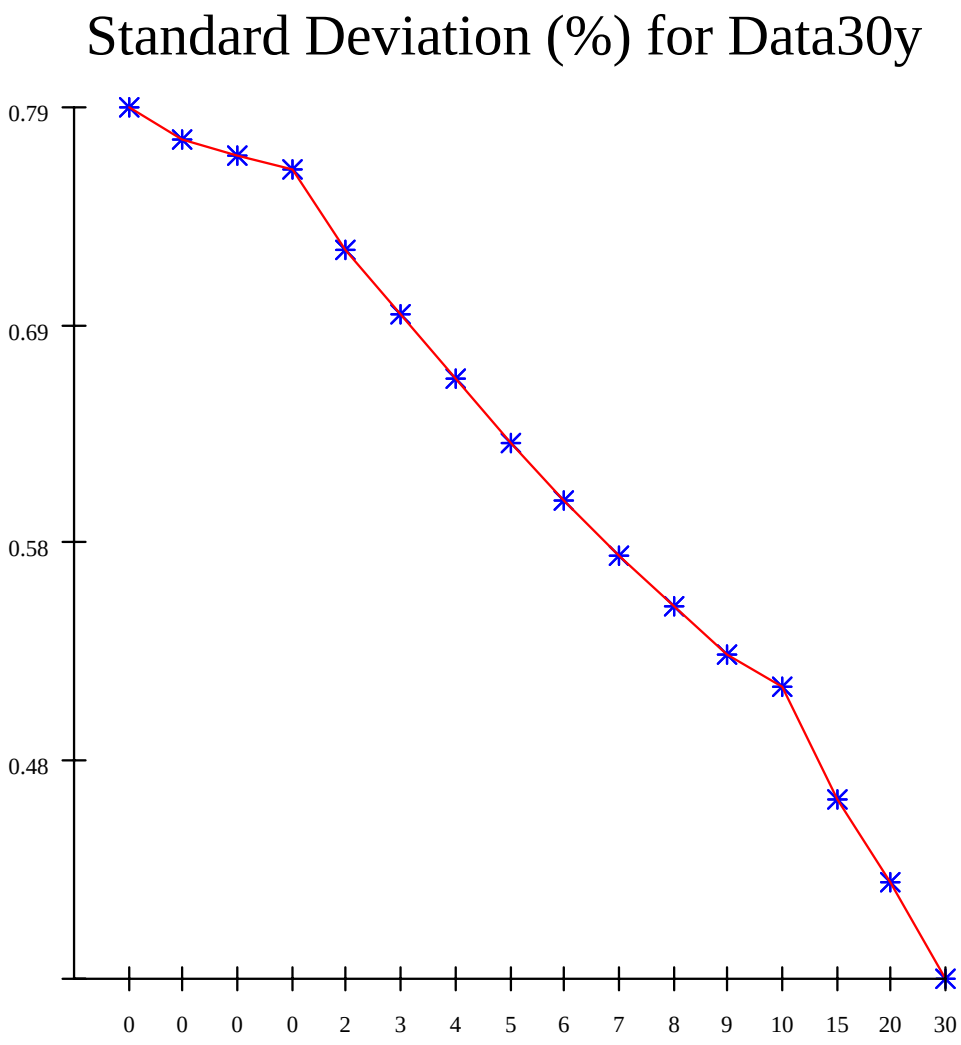

Figure 3.7: Standard deviation for Data30y

Standard deviation (%) for Data30y EURIBORsdcorr1.xpl

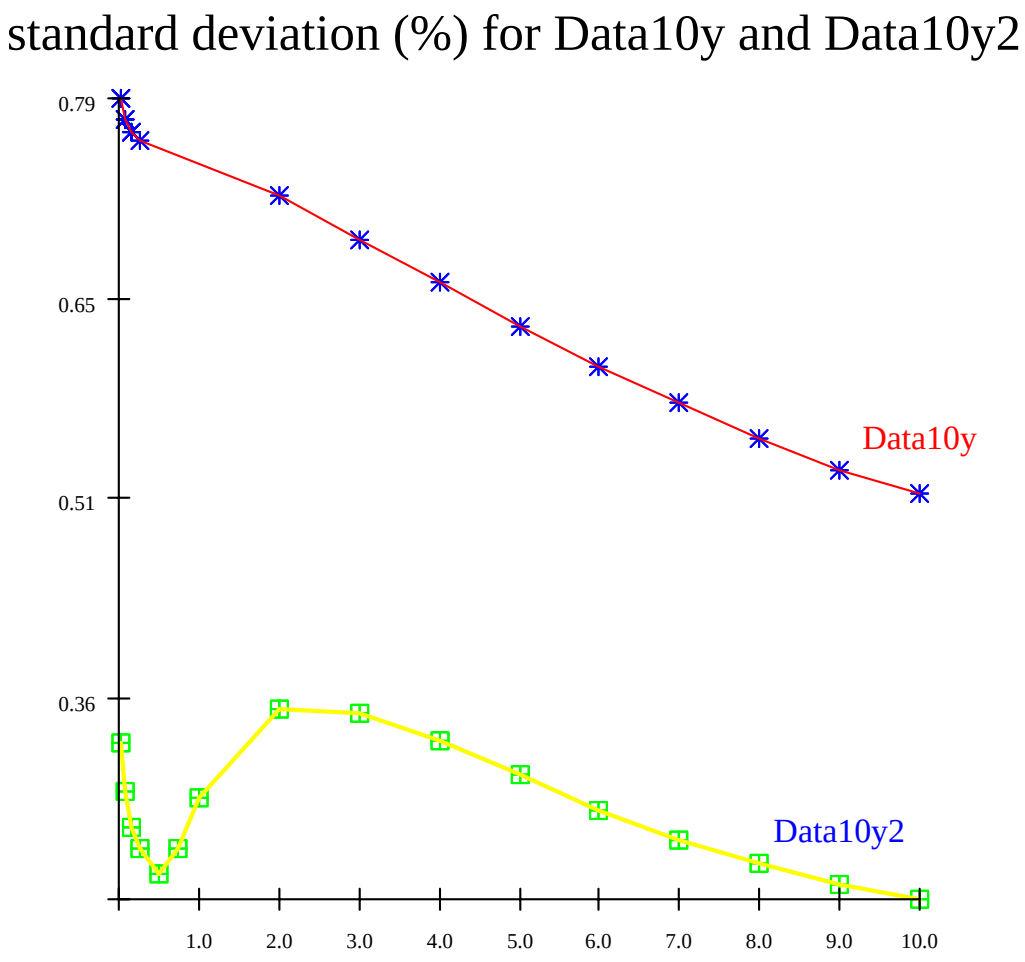

Figure 3.8: Standard deviation for Data10y & Data10y2

Standard deviation (%) for Data10y and Data10y2 EURIBORsd2.xpl

3.3.3 Correlation

We find a decreasing correlation curve of maturity gap generally holds. As many researchers have proven, the larger maturity gap between two rates, the lower the correlation between them. According to the segment theory, bond markets are segmented into long-term bond market and short-term bond market, where the interest rate value is decided by demand and supply. This indicates the long-term interest rate is totally uncorrelated with short-term rate. Although we think the segment theory is limited, it reveals some truth for the correlation curve. We also find for data10y2 there is an amplification of the correlation between 1-week and 2-year rate, i.e. the influence of spot rate is enlarged from 2 years with a negative effect. We argue such deviation would disappear with more samples.

3.3.4 Summary

We argue that the EURIBOR follows general term structure features. The main arguments are:

- EURIBOR's long spread is positive, i.e. the long-term rate is greater than the spot rate, which is normal based on the risk and liquidity theory. The EURIBOR has mean reversion features with an equilibrium level of 3.7024% for spot rate. Compared with interest rates of other currencies, this smaller equilibrium is maybe caused by some macroeconomics factor such as inflation rate.

- The distribution of standard deviation over θ is hump shaped with a peak around 2 years, which indicates that EURIBOR has a high

fluctuation around 2 years.

- We find the relationship between interest rates decreases as the maturity gap rises. In other words, the larger maturity gap between two rates is, the lower the correlation is.

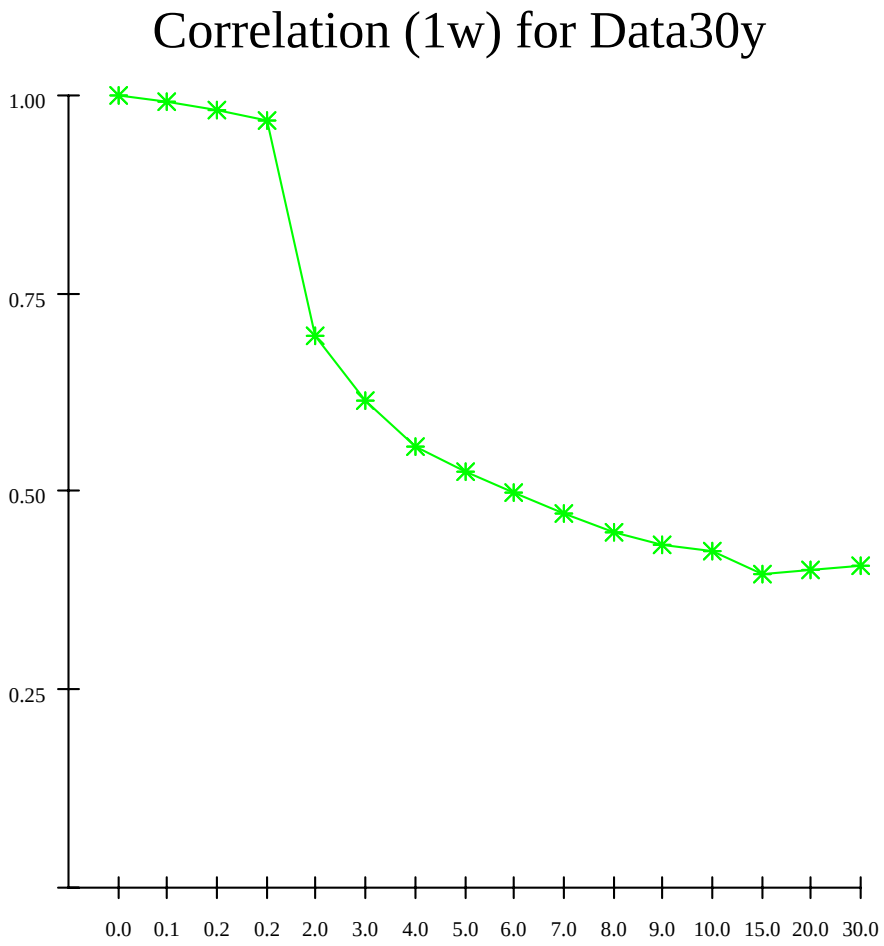

Figure 3.9: Correlation for Data30y

Correlation between spot rate and EURIBOR with other maturities for Data30y

`EURIBORsdcorr1.xpl`

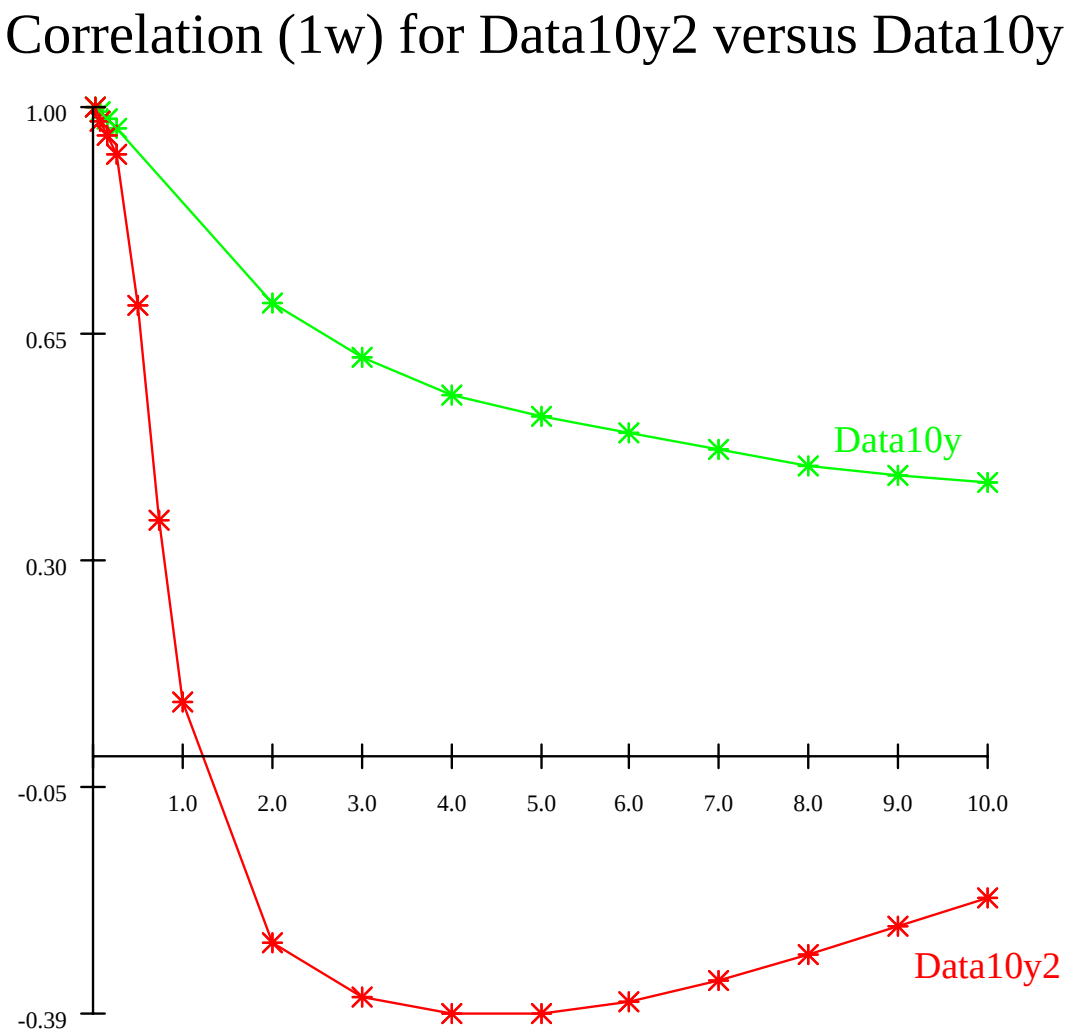

Figure 3.10: Correlation for Data10y & Data10y2

Correlation between spot rate and EURIBOR with other maturities for Data10y (green) and Data10y2 (red). `EURIBORcorr2.xpl`

Maturity	Data30y		Data10y		Data10y2	
	SD	Corr.	SD	Corr.	SD	Corr.
1w	0.80	1	0.80	1	0.33	1
1m	0.78	0.99	0.78	0.99	0.30	0.98
2m	0.77	0.98	0.77	0.98	0.27	0.96
3m	0.76	0.97	0.76	0.97	0.26	0.93
6m					0.24	0.69
9m					0.26	0.36
1y					0.29	0.08
2y	0.72	0.70	0.72	0.70	0.36	−0.29
3y	0.69	0.61	0.69	0.61	0.35	−0.37
4y	0.66	0.56	0.66	0.56	0.33	−0.39
5y	0.63	0.52	0.63	0.52	0.31	−0.39
6y	0.60	0.50	0.60	0.50	0.28	−0.38
7y	0.58	0.47	0.58	0.47	0.26	−0.34
8y	0.55	0.45	0.55	0.45	0.25	−0.30
9y	0.53	0.43	0.53	0.43	0.23	−0.26
10y	0.51	0.42	0.51	0.42	0.22	−0.22
15y	0.46	0.40				
20y	0.42	0.40				
30y	0.37	0.41				

Table 3.6: SD and correlation with spot rate for EURIBOR

3.4 Statistical features of daily shift

Daily shift of the term structure of interest rate is considered as one of the most risky factors for hedging. In this section, we will make some analysis on EURIBOR's daily shift.

3.4.1 Daily shift standard deviation

The daily shift standard deviation is defined as:

$$\sigma_\delta(\theta) = \sqrt{E_t[\delta(\bullet, \theta) - E_t\{\delta(\bullet, \theta)\}]^2} \tag{3.8}$$

From figure (3.11) and figure (3.12) we find the standard deviation of the daily shift EURIBOR has a similar structure as the SD curve of EURIBOR, i.e. a humped-similar distribution with a peak around 4 year for Data10y and Data30y versus a peak around 2 years for Data10y2. As same as the standard deviation of EURIBOR, see (3.8), there is a deviation from humped distribution, i.e. from 1 week to 3 or 6 months the daily shift standard deviation declines fast from a tremendously high value, which is even higher than the peak of the humped distribution.

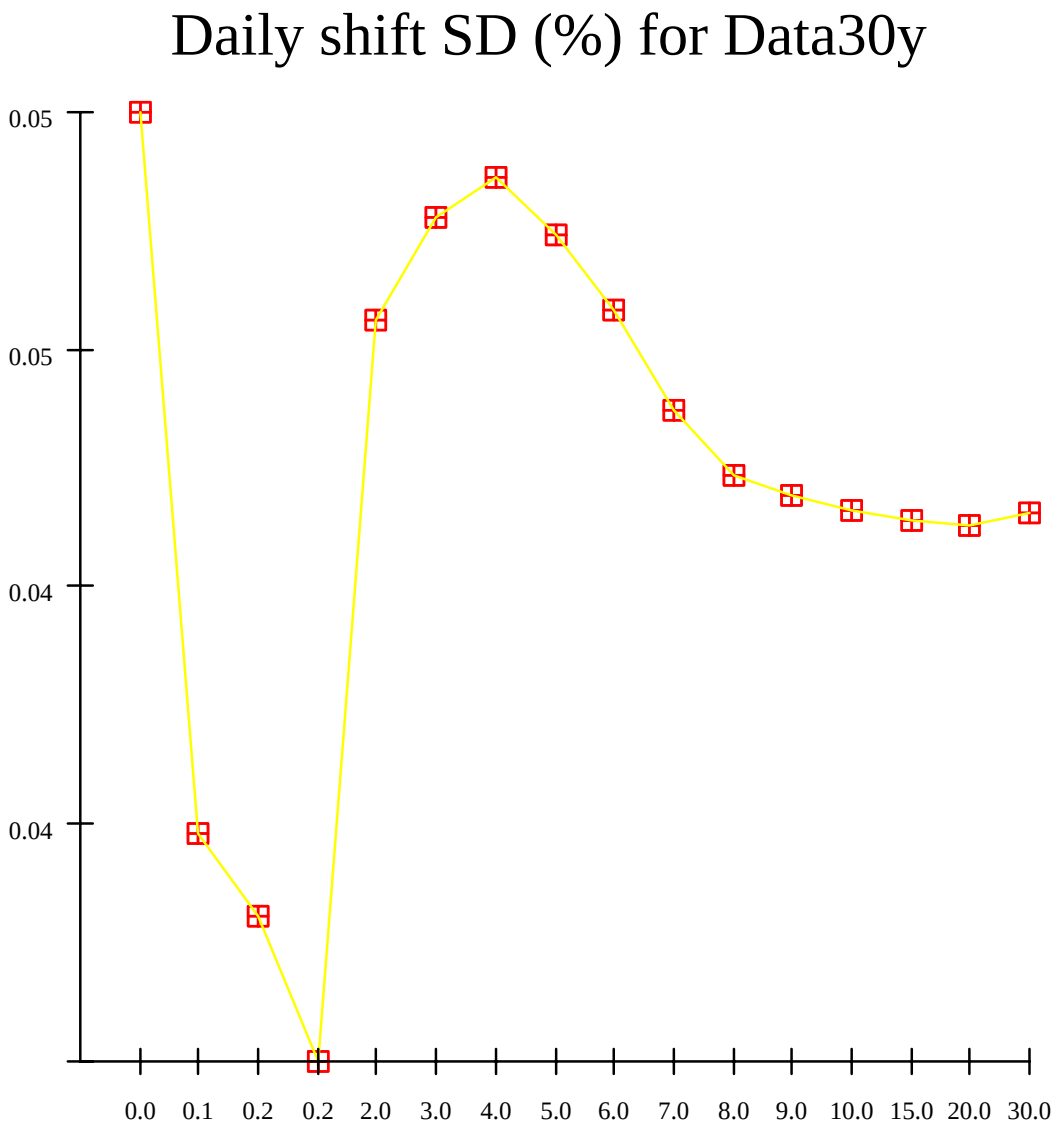

Figure 3.11: Daily shift standard deviation for Data30y

Standard deviation (%) for daily shift respect to Data30y. `EURIBORdaily1.xpl`

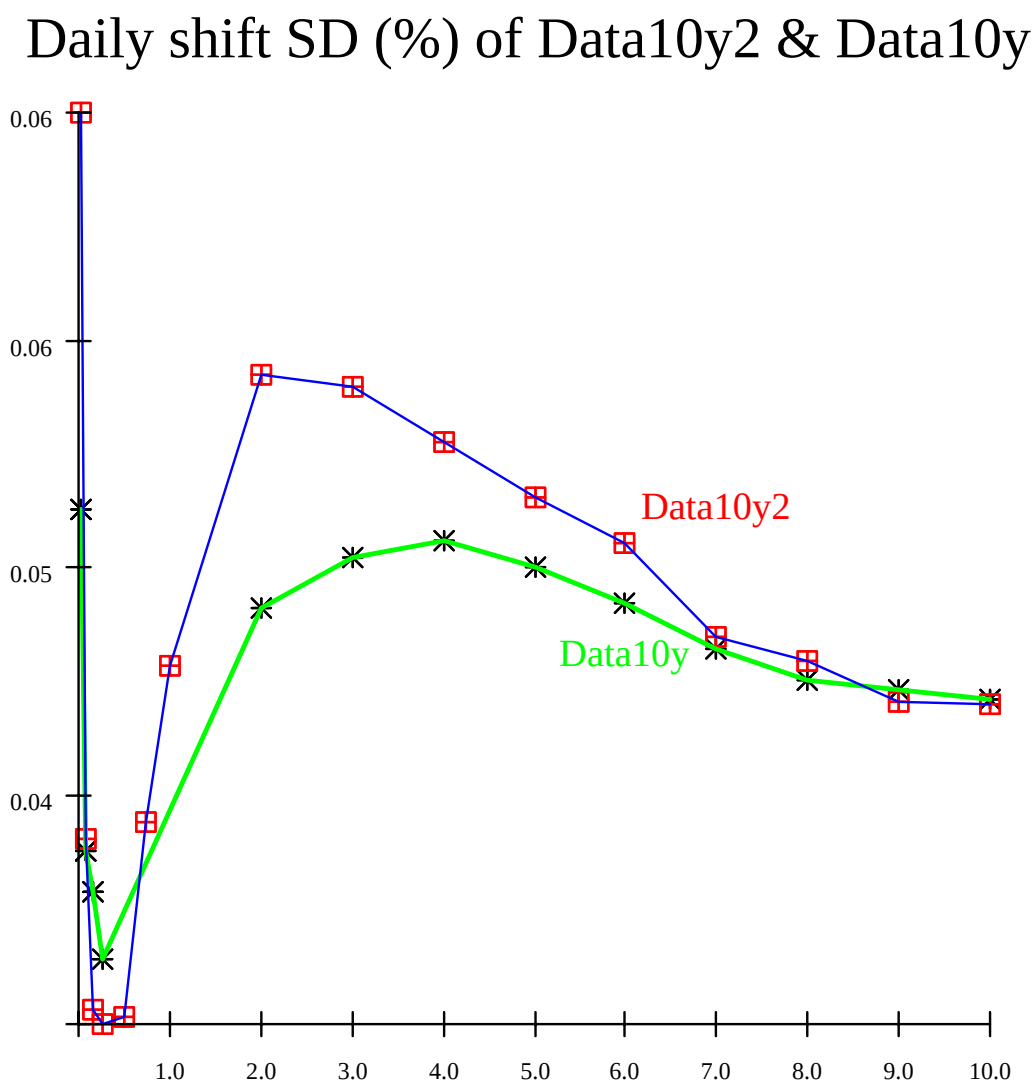

Figure 3.12: Daily shift standard deviation for Data10y & Data10y2

Standard deviation (%) for daily shift respect to Data10y2 (blue) and Data10y (green) . `EURIBORdaily2.xpl`

3.4.2 Daily shift kurtosis

The kurtosis of daily shift EURIBOR:

$$\kappa_\delta(\theta) = \frac{E_t[\delta(\bullet, \theta) - E_t\{\delta(\bullet, \theta)\}]^4}{\sigma_\delta^4(\theta)} \tag{3.9}$$

Lots of securities in financial market have fat tail distributions. We observe that the daily shift of EURIBOR follows a fat tail distribution too, since the daily shift kurtosis have a value larger than 3.

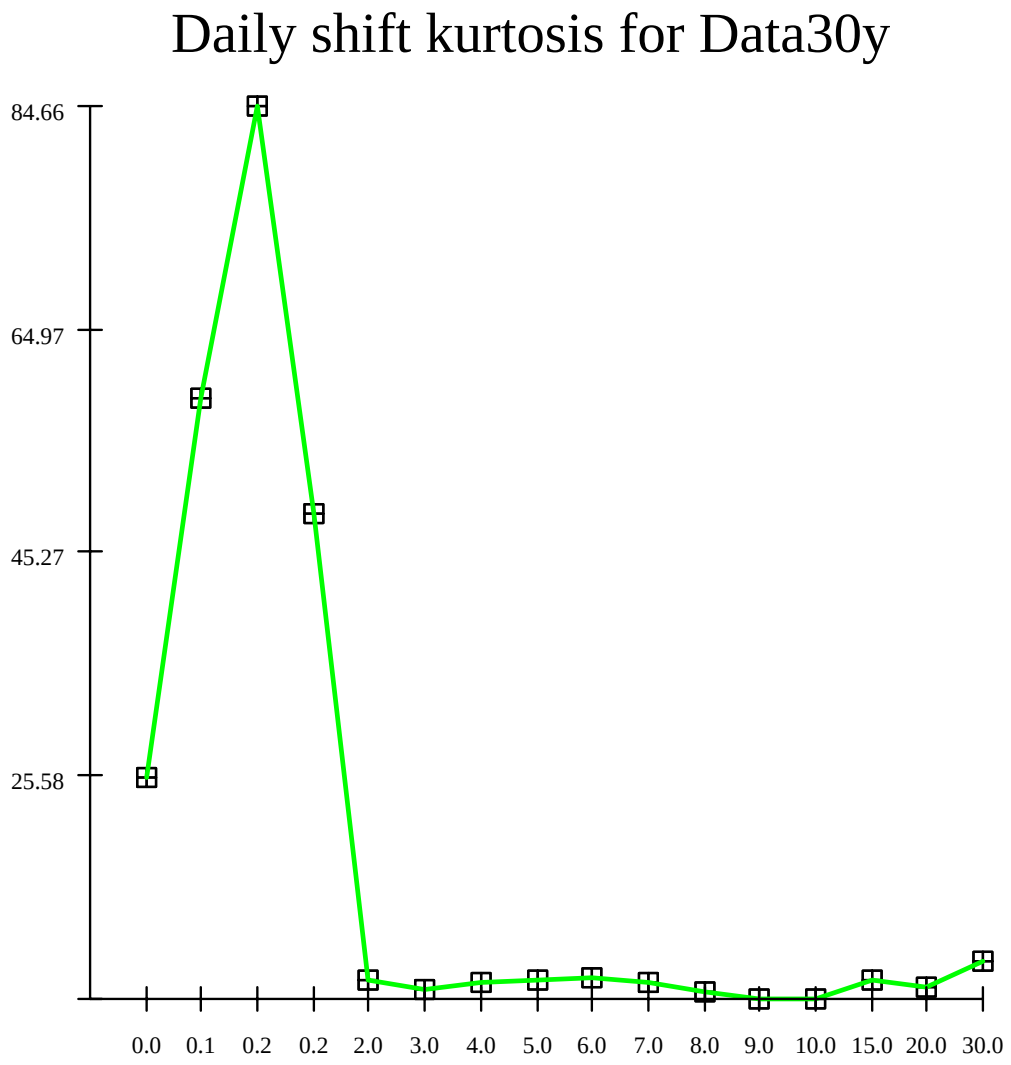

Figure 3.13: Daily shift kurtosis for Data30y

Kurtosis for daily shift respect to Data30y. `EURIBORdaily1.xpl`

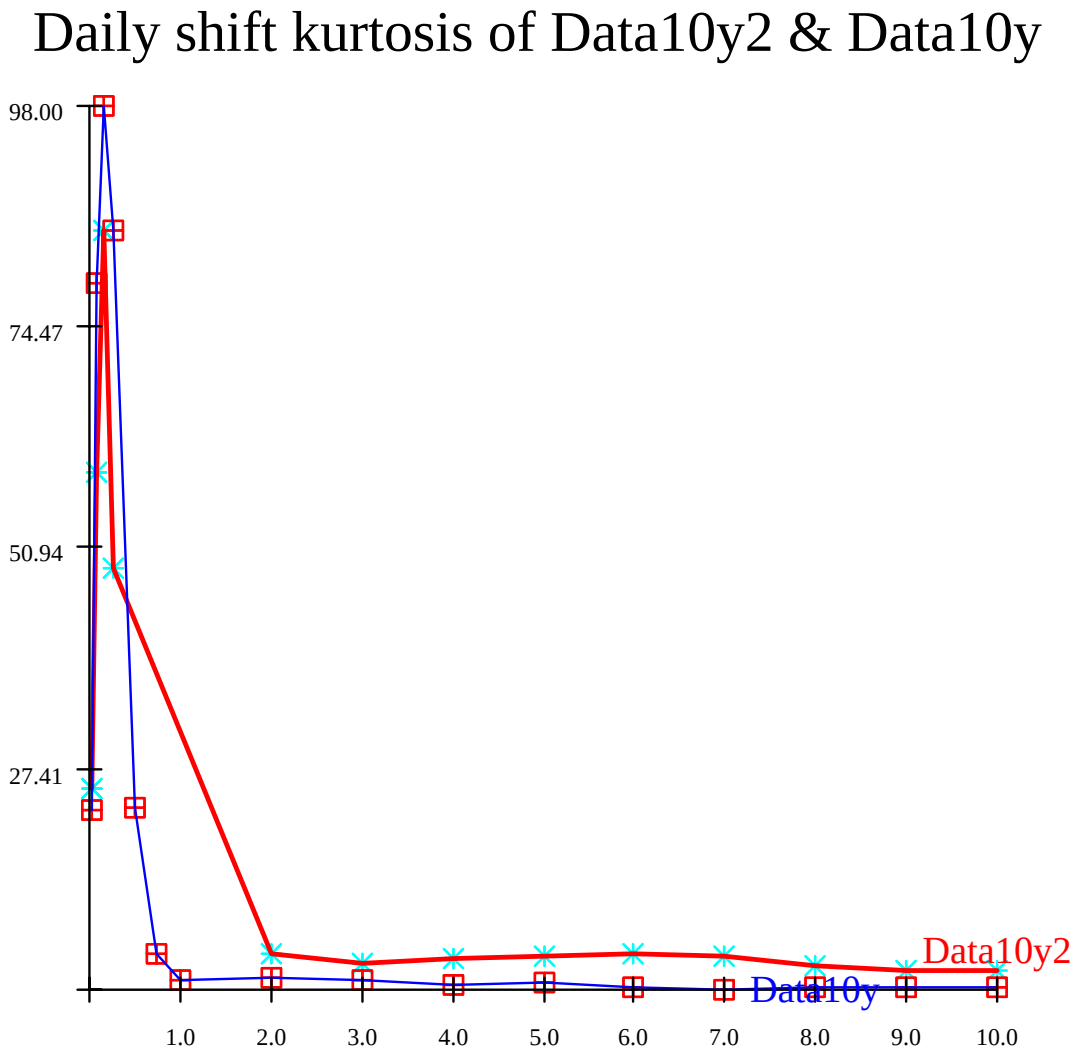

Figure 3.14: Daily shift kurtosis for Data10y & Data10y2

Kurtosis for daily shift respect to Data10y2 (blue) and Data10y (red).

`EURIBORdaily2.xpl`

3.4.3 Daily shift correlation

Analogously, we find an imperfect correlation structure for daily shift of EURIBOR, which indicates the spot rate fluctuations has a declined influence as the maturity gap increases.

$$C_\delta(\theta) = \frac{E[\delta(t) - E(\delta(t))] \cdot E[\delta(t,\theta) - E\{\delta(t,\theta)\}]}{\sigma_\delta^2(\theta_{min})} \tag{3.10}$$

Such daily correlation structures are also proved by other empirical research. A well-known example is the daily movements in zero-coupon rates derived from the interbank curve in France between 1995 and 1998 proven by Florent and Priaulet (2000) that the larger the maturity gap between two interest rates is, the less correlation of these two rates is.

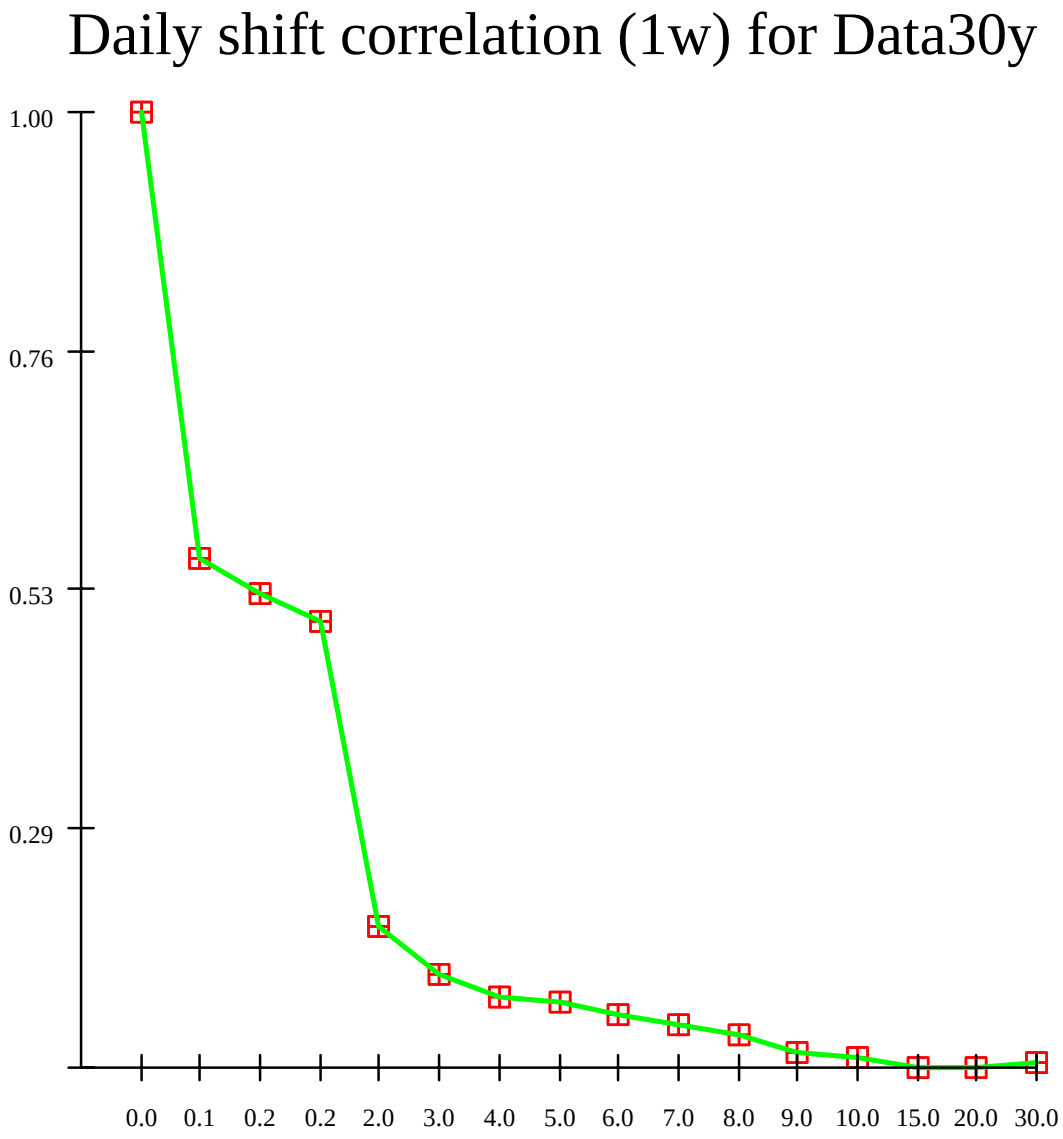

Figure 3.15: Daily shift correlation for Data30y

Correlation between 1-week daily shift and other maturities respect to Data30y.

`EURIBORdaily1.xpl`

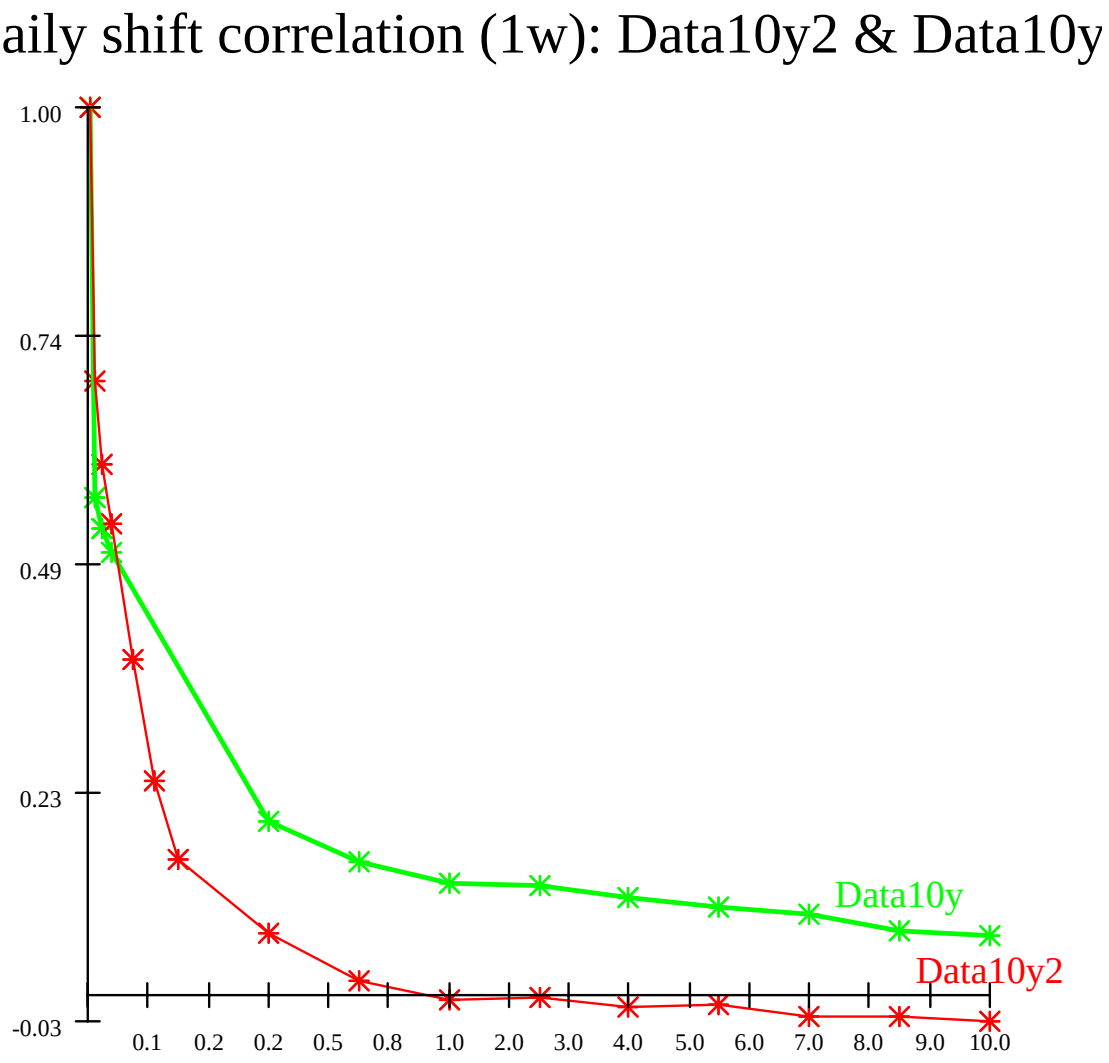

Figure 3.16: Daily shift correlation for Data10y & Data10y2

Correlation between 1-week daily shift and other maturities respect to Data10y2 (red) and Data10y (green). `EURIBORdaily2.xpl`

Maturity	Data30y			Data10y			Data10y2		
	SD	Kurtosis	Corr.	SD	Kurtosis	Corr.	SD	Kurtosis	Corr.
1w	0.05	25.36	1	0.05	25.36	1	0.06	22.96	1
1m	0.04	58.99	0.56	0.04	58.99	0.56	0.04	79.05	0.69
2m	0.04	84.66	0.52	0.04	84.66	0.52	0.03	98.00	0.60
3m	0.03	48.73	0.50	0.03	48.73	0.50	0.03	84.77	0.53
6m							0.03	23.35	0.38
9m							0.04	7.81	0.24
1y							0.04	4.88	0.15
2y	0.05	7.64	0.20	0.05	7.64	0.20	0.05	5.05	0.07
3y	0.05	6.62	0.15	0.05	6.62	0.15	0.05	4.82	0.01
4y	0.05	7.23	0.13	0.05	7.23	0.13	0.05	4.47	−0.01
5y	0.05	7.56	0.12	0.05	7.56	0.12	0.05	4.56	−0.00
6y	0.05	7.75	0.11	0.05	7.75	0.11	0.05	4.11	−0.01
7y	0.04	7.38	0.10	0.04	7.38	0.10	0.04	3.88	−0.01
8y	0.04	6.50	0.09	0.04	6.50	0.09	0.04	4.06	−0.02
9y	0.04	5.95	0.07	0.04	5.95	0.07	0.04	4.03	−0.03
10y	0.04	5.89	0.07	0.04	5.89	0.07	0.04	4.02	−0.03
15y	0.04	7.55	0.06						
20y	0.04	6.89	0.06						
30y	0.04	9.15	0.06						

Table 3.7: SD(%), Kurtosis and correlation(%) with spot rate for daily shift EURIBOR

4 Spread Model

4.1 Spread model

In this section, we will build up a heuristic model with the help of Least Squares estimation and explain the deviation from the spread model.
We find that the $MAT1/2$ model can't describe EURIBOR well, even though it fits US FRC 1994-1996 Bouchaud et al. (1997) and GBP (Short Sterling LIFFE future contract) 1994-1999, Matacz and Bouchaud (1999). According to our research, we find that there are three distinct aspects of EURIBOR average spread curve respect to US FRC (forward rate curve).

$$E_t[s(\bullet,\theta)] = A \cdot [\frac{\theta}{\theta_{max}}]^{\beta} + \epsilon(\theta) \tag{4.1}$$

with $E_t[s(\bullet,\theta)]$ the average partial spread over t

- We argue that the average spread of EURIBOR follows a $MAT2/3$ **rule** instead of a square-root law as indicated by the phenomenological model of Bouchaud et al. (1997). We call this approach the $MAT2/3$ rule.

- We find that the $MAT2/3$ rule fits very well for Data10y with $\beta = \frac{2}{3}$, but is faint for describing Data30y and Data10y2's average spread curve. A possible reason is as follows. According to the liquidity theory, long- term maturity securities have worse liquidity, subsequently long-term investors follow a different strategy of investment from short-term investor, which makes the market for over-10-year maturity deviates from the $MAT2/3$ rule. The small sample size of Data10y2 makes it hard to regress the structure of the average spread curve.

- In the $MAT1/2$ model, the prefactor is argued comparable to the spot rate volatility, which is not the case of $MAT2/3$ rule. Similarly, the prefactor for DEM is not comparable with the spot rate volatility by Matacz and Bouchaud (1999).

4.1.1 Data10y

Regression average spread as a function of maturity

The average spread could be described as a function of maturity. From figure (4.1), we argue that the function follows a $MAT2/3$ rule. Instead of $\beta = \frac{1}{2}$ argued by the $MAT1/2$ rule, we find $\beta = \frac{2}{3}$ fits the average spread curve perfectly, which could be also seen from the quantile-quantile plot (see Figure 4.3), which compares distributions of EURIBOR average spread and our approximation.

$$E_t[s(\bullet,\theta)] = A \cdot [\frac{\theta}{\theta_{max}}]^{\frac{2}{3}} + \epsilon(\theta) \tag{4.2}$$

Description:	Generates a quantile-quantile-plot to compare two variables.
Usage:	$bp = grqq(y, x, col)$

Table 4.1: XploRe grqq

Applying XploRe quantlet **gls**, we get prefactor $A = 1.6649$ ($A/\sqrt{250} = 0.1053$) for the $MAT2/3$ rule.

Description:	Computes the Generalized Least Squares estimate for the coefficients of a linear model when the errors have as covariance matrix $sigma^2 * om$.
Usage:	$b = gls(x, y, om)$
Note:	The estimate is given by $b = inv(x' * om * x) * x' * om * y$.

Table 4.2: XploRe gls

Härdle, Klinke and Müller (2000)

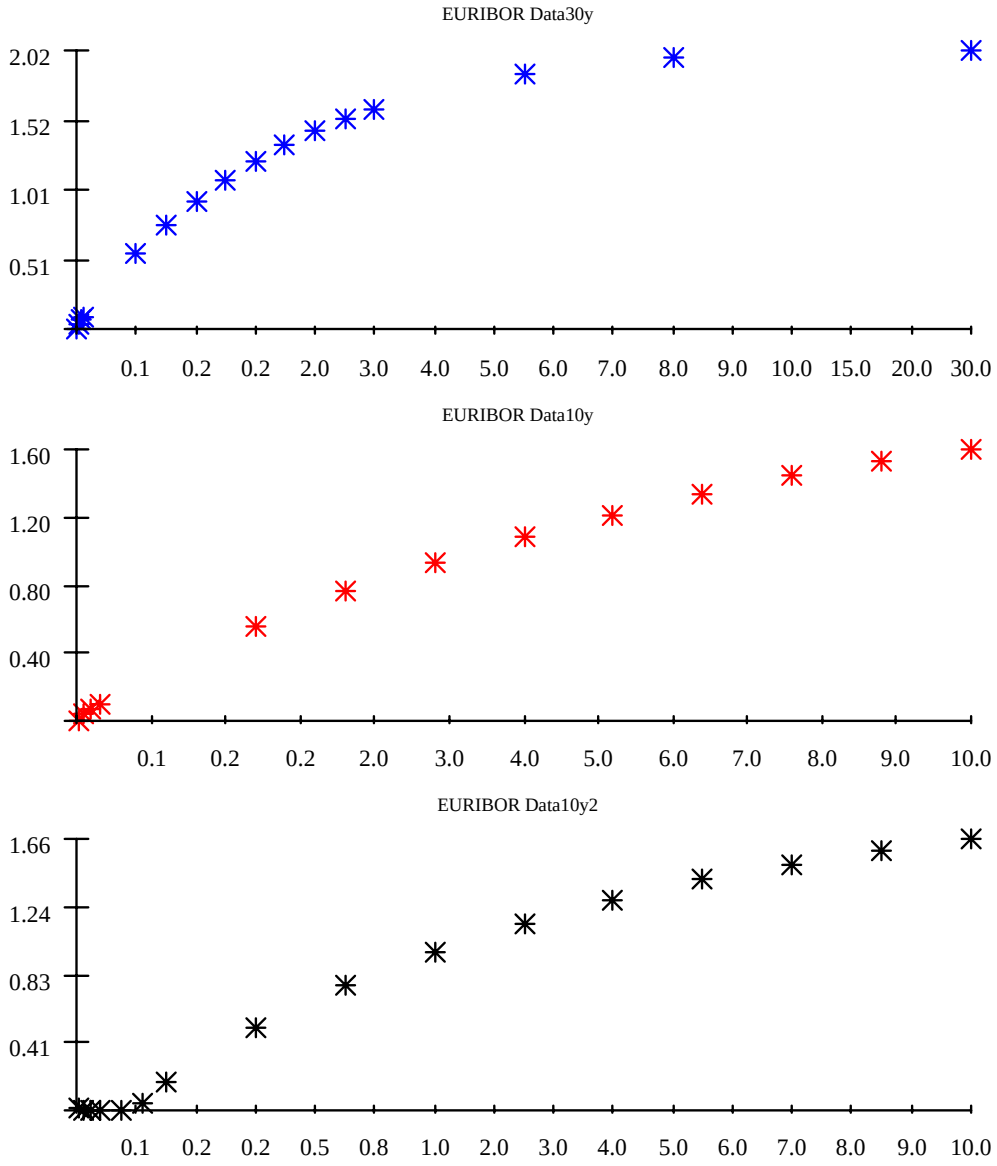

Figure 4.1: Average spread for Data30y, Data10y and Data10y2

Average spread as a function of the maturity for Data30y, Data10y and Data10y2 .

`EURIBORdata.xpl`

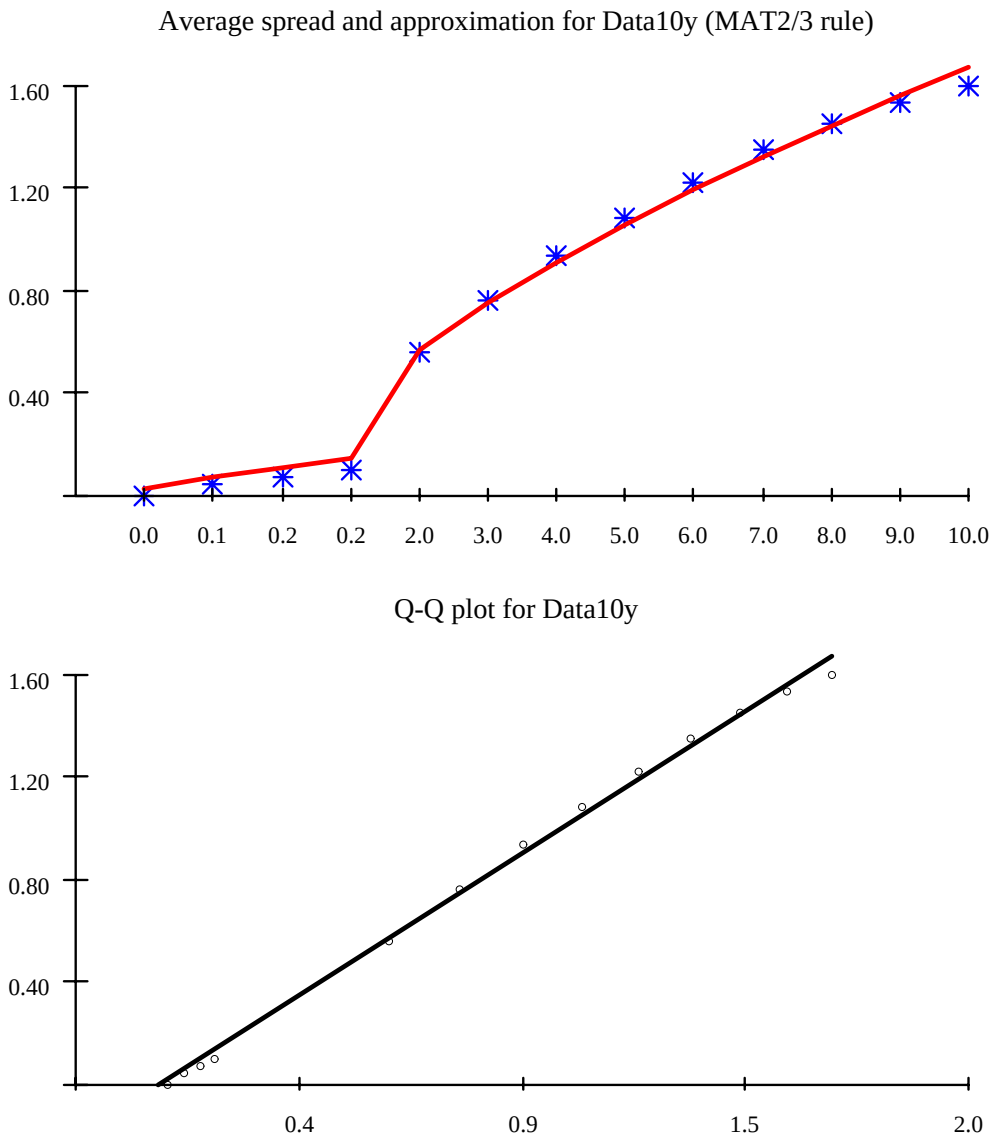

Figure 4.2: $MAT2/3$ rule

Average spread and approximation as a function of the maturity for Data10y with $\beta = \frac{2}{3}$ and QQ plot. `EURIBORapp.xpl`

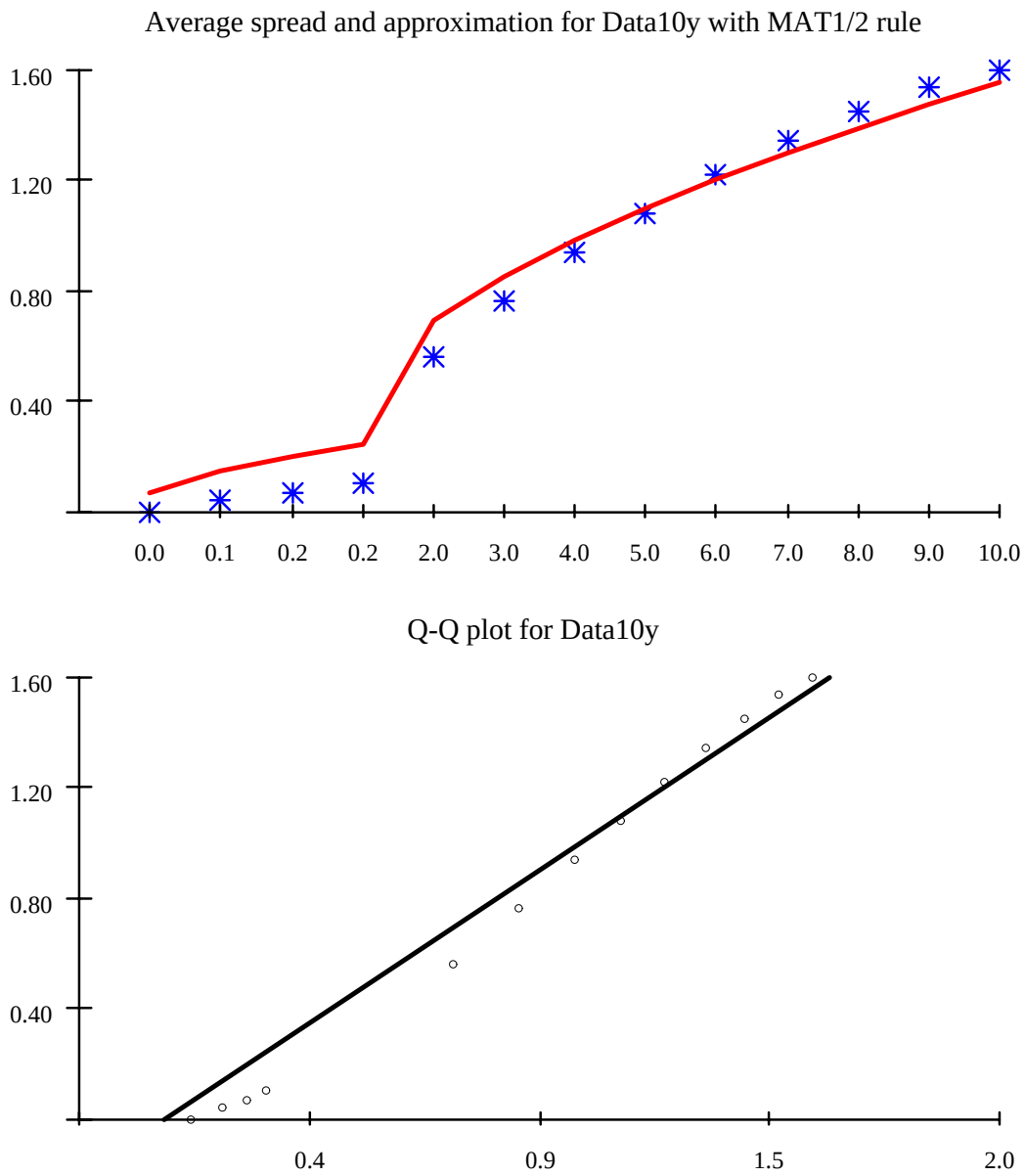

Figure 4.3: $MAT1/2$ rule

Average spread and approximation as a function of the maturity for Data10y with $\beta = \frac{1}{2}$ and QQ plot. `EURIBORapp.xpl`

PCA (Principal Component Analysis) for error

There is a deviation of the partial spread $s(t,\theta)$ from the approximated average spread curve. We apply the principal component analysis to reduce dimensions of the deviation.

$$s(t,\theta) - A \cdot [\frac{\theta}{\theta_{max}}]^{\frac{2}{3}} = \hat{\varepsilon}(t,\theta) \tag{4.3}$$

We define:

the estimated deviation

$$\hat{\varepsilon}(t) = \begin{pmatrix} \hat{\varepsilon}(t,\theta_1) \\ \hat{\varepsilon}(t,\theta_2) \\ \hat{\varepsilon}(t,\theta_3) \\ \hat{\varepsilon}(t,\theta_4) \\ \hat{\varepsilon}(t,\theta_5) \\ \hat{\varepsilon}(t,\theta_6) \\ \hat{\varepsilon}(t,\theta_7) \\ \hat{\varepsilon}(t,\theta_8) \\ \hat{\varepsilon}(t,\theta_9) \\ \hat{\varepsilon}(t,\theta_{10}) \\ \hat{\varepsilon}(t,\theta_{11}) \\ \hat{\varepsilon}(t,\theta_{12}) \\ \hat{\varepsilon}(t,\theta_{13}) \end{pmatrix}$$

where θ_i is maturities, $i = 1, 2, .., 13$, e.g. θ_1 is 1-week maturity,

the empirical correlation

$$\gamma = corr\{\hat{\varepsilon}(t)\} \tag{4.4}$$

where $\hat{\varepsilon}(t)$ is a vector of $\hat{\varepsilon}(t,\theta_i)$, $i = 1, 2, .., 13$.

We find the first PC can explain 96.965% of the empirical correlation.

Then the error corresponding to the maturity with the first eigenvalue is enough to explain the deviation of the partial spread from the $MAT2/3$ rule.

Error	Eigenvalue	Proportion of variances	Cumulated %
1-week	4.4002	0.96965	0.96965
1-month	0.10512	0.023166	0.99281
Others	0.032621	0.0071886	1

Table 4.3: EURIBOR PCA

The spread model could be transformed into

$$s(t,\theta) = s(t) \cdot [\frac{\theta}{\theta_{max}}]^{\frac{2}{3}} + \zeta_1(t)\epsilon(t,\theta_1) \tag{4.5}$$

where $\zeta_1(t)$ is the first eigenvector and $\epsilon(t,\theta_1)$ is the element with the largest eigenvalue.

$$\zeta_1(t) = \begin{pmatrix} -1.8433e-34 \\ 0.020959 \\ 0.041991 \\ 0.065446 \\ 0.27034 \\ 0.30897 \\ 0.32962 \\ 0.33786 \\ 0.34298 \\ 0.34662 \\ 0.34898 \\ 0.34888 \\ 0.34768 \end{pmatrix}$$

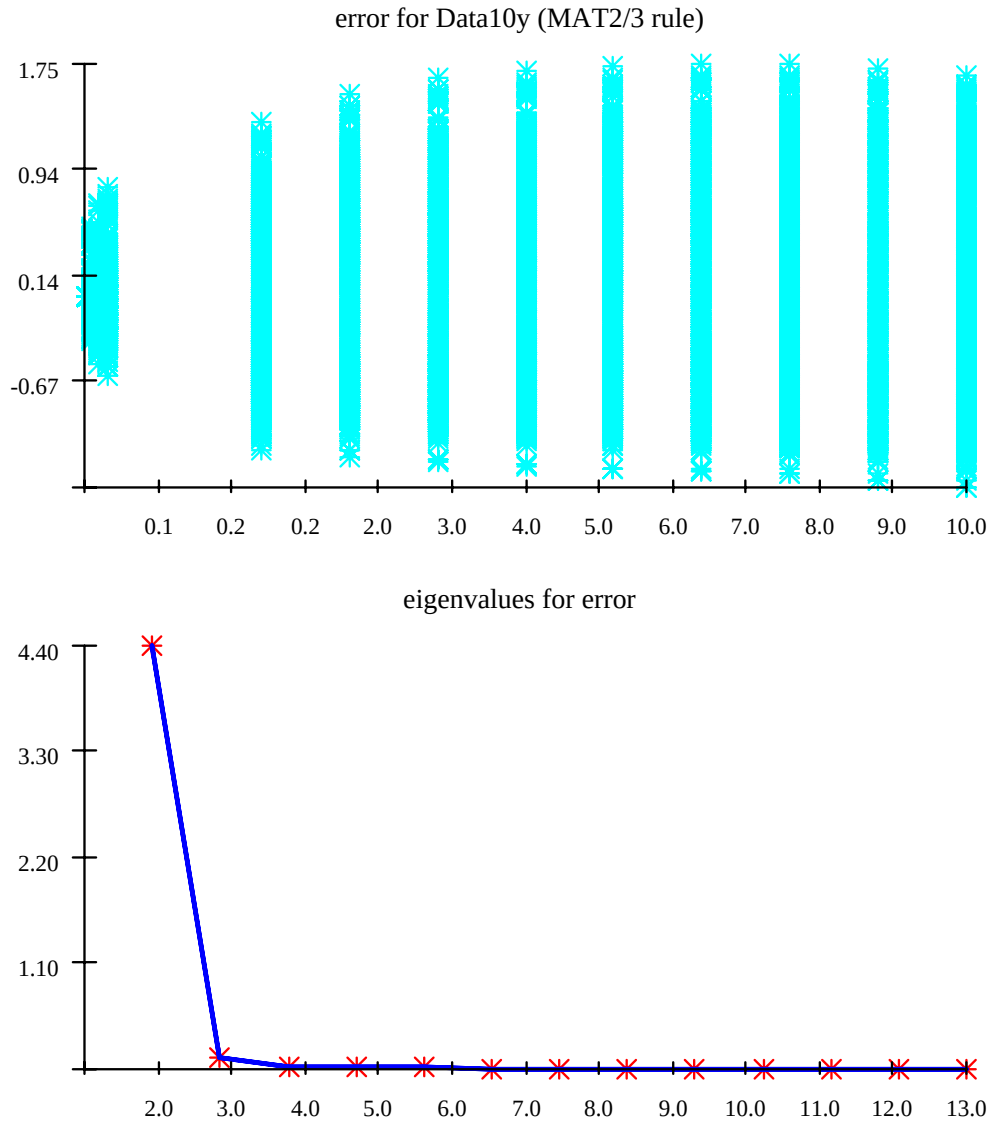

Figure 4.4: PCA

Error for average spread curve and the eigenvalues for the covariance of error.

`EURIBORapp.xpl`

5 Conclusion

We find that the EURIBOR curve follows general yield curve features. For instance, the spot rate has mean reversion with an equilibrium level of about 3.5% to 3.7%, the long spread keeps positive, which indicates an often appearing upward sloping trend of the term structure. The distribution of EURIBOR standard deviation is humped-similar with deviation towards short-term rate; the peak of the hump is around 2 years. The influence of the spot rate declines as the rate maturity increases, where the correlation curve declines as maturity gap rise up. Analogously we find a similar structure in the distribution of the daily shift standard deviation and kurtosis.

For the Data10y average spread curve we maintain a **MAT2/3** rule with $\beta = \frac{2}{3}$ holds instead of a square-root law argued by phenomenology model. We argue there should be a distortion of the instantaneous average spread of EURIBOR curve around 4 year. The volume of independent components needed to interpret the deviation of the average spread is very small. The first eigenvalue could already explain over 90% covariance of error. We argue a general **MAT2/3** rule for term structure of interest rate instead a constant β holds for all interest rate respect to

all markets.

Bibliography

Bouchaud, J., Sagna, N., Cont, R., El-Karoui, N. and Potters, M. (1997). Phenomenology of the interest rate curve.

Brealey, R. and Myers, S. (2000). *Principles Of Corporate Finance*, 6 edn, The McGraw-Hill Companies, Inc.

Brigo, D. and Mercurio, F. (2001). *Interest Rate Models Theory and Practice*, Springer, Heidelberg.

Cont, R. (2000). Modeling term structure dynamics: An infinite dimensional approach., *International Journal of Theoretical and Applied Finance* .

Florent, I. and Priaulet, P. (2000). Focus on interest rates: Comparing the approaches of portfolio managers and option writers., *Quants* **38**.

Franke, J., Härdle, W. and Hafner, C. (2001). *Einführung in die Statistik der Finanzmärkte*, Springer, Heidelberg.

Härdle, W., Klinke, S. and Müller, M. (2000). *Xplore-Learning Guide*, Springer, Heidelberg.

Hull, J. (2000). *Options, Futures, & Other Derivatives*, 4 edn, Prentice-hall, Inc.

Kern, D. and Gutmann, P. (1992). *Interest Rate Analysis And Forecasting*, woodhead-Faulkner.

Matacz, A. and Bouchaud, J. (1999). An empirical investigation of the forward interest rate term structure., *cond-mat/9907297* .

Rebonato, R. (1996). *Interest-Rate Option Models*, 2 edn, John Wiley & Sons.

Index

CIR model, 6
Classical models, 3
Conclusion, 55
Correlation, 30

Daily shift correlation, 41
Daily shift kurtosis, 38
Daily shift standard deviation, 35
Data description, 10
Data10y, 46

EURIBOR data, 10

HJM model, 6

Motivation, 1

Notation, 19

PCA, 51
Phenomenological model, 7

Regression average spread as a function of maturity, 46

Spread and mean reversion, 20
Spread Model, 45
Spread model, 45
Standard deviation, 26
Statistical characteristics of EURIBOR, 20
Statistical features of daily shift, 35
Summary, 30

Term Structure of Interest Rate Models, 3

Vasicek model, 4