

Rating Migrations

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1 Introduction

Ratings serve as an indicator of the financial strength of worthiness of a particular credit or corporate. That is why they constitute a very decisive criterion regarding the investment decision of the players in financial markets. This thesis deals with the assessment of the risk that is associated to company ratings assigned by agencies like Standard and Poor's (S&P) or Moody's. These ratings are assumed to reflect the true risk linked to the rated entity, containing both an idiosyncratic and a systematic or aggregate component. Hence, this thesis does not evaluate the work of the rating agencies. Rather, the aim is to provide information relevant for risk management by pinpointing the connection between this risk and the probability of a change in the rating of a particular credit.

In Section 2, I will present an introduction to rating agencies and the basic concepts of their work. In addition, I will introduce and discuss the different rating categories. These rating categories, ranging from AAA to D, can be assumed to be of ordinal order. This implies that the risk reflected by a particular rating category is higher for lower rating states and equivalently lower for higher rating states. Therefore, I focus my analysis on the probability of a migration from a given rating state to another in order to measure the risk. I will show the importance of risk associated to ratings for investors and obligors and give an overview on the field of research in credit risk and its development. Chapter 3 features a presentation of the most widely spread method used in practice for the analysis of credit risk: the cohort method. I will discuss the shortcomings associated with this method and will provide a framework which allows to account for these shortcomings. More precisely, using relative frequencies of rating changes as in the cohort method does not explain the existence of some correlation or dependency structure between rating transitions as is observed in empirical data. This empirical result implies that the assumption of the first-order Markov property for transition matrices is violated. In Section 4, I will point out a formal method to test the first-order Markov property and highlight a framework which allows to account for non-Markovian behaviour: the Aalen-Johansen estimator. Unfortunately, there is no data set available to me in order to apply the Aalen-Johansen estimator. However, because it can be shown that the estimator allows for time non-homogeneity and rating migrations within a period (in contrast to the cohort method), I will nevertheless present a short introduction to the underlying methodology. In addition, I debate why it is crucial to match the time horizon for transition matrices and the concerned risk horizon.

The discussion on the correlation structure between rating migrations is continued in Section 5. I prove that the cohort method is not an efficient estimator for pairwise correlated Bernoulli variables. In addition, more evidences for correlation given in the literature is exhibited. One hypothesis to explain this feature in the data is the dependence of rating migrations on economic variables like the state of the business cycle. As a first approach to account for this correlation, I present the Merton model. The Merton model suggests that changes in the asset value drive the underlying rating process. For calculation, one can partition the matrix of transition probabilities into a matrix of disjoint bins. Thus, one obtains threshold values for a rating migration corresponding to a certain asset value. For the construction of credit cycles, I follow the presentation of Belkin et al. (1998). I will display their method in Section 6 and apply it to annual rating data from S&P. The results are quite close to the ones obtained in the literature, but feature a longer time horizon. The main idea for the construction of credit cycles is to partition a change in a rating into an idiosyncratic and a systematic component. The latter component is then assumed to reflect the credit cycle and accordingly the cyclical behaviour of risk attributed to the rated entities.

One assumes these cycles to be driven by economic variables. Regarding my estimation results from the S&P data set, one can observe that the systematic component in fact mirrors the peaks and troughs of the business cycle. Hence, I try to shed more light on what drives the credit cycles by investigating the impact of different economic variables via autoregressive estimation. I will show that changes in GDP and the spread between Corporate Bond yields and the treasury yield do have a significant impact on the credit cycles.

Even though these results are quite convincing and back the afore mentioned theory, I have to point out that rating migrations are mainly driven by idiosyncratic, i.e. firm-specific components and thus it will be hard to obtain reliable forecasts on the transition matrices based on the systematic component only. Nevertheless, it is an interesting exercise to determine the economic influence on rating migrations as displaying aggregate risk. All computations within this thesis are done with the help of Xplore, a powerful tool for statistical analysis, including a lot of helpful procedures. All Xplore codes are provided in the Appendix. Besides Xplore, I used EViews for the autoregressive analysis of the constructed credit cycles.

2 Rating Migrations

The importance and influence of rating agencies has grown significantly over the last years, and thus has research. To see why, let's take a first look on the agencies and their work.

Although there exist several rating agencies, the market is dominated by the agencies Moody's and Standard & Poors. On July 1, 1914, Moody's Investors Service was incorporated. That same year, Moody's began expanding rating coverage to bonds issued by US cities and other municipalities. In 1916, Standard Statistics began to assign debt ratings to corporate bonds, with sovereign debt ratings following shortly thereafter. The number of applied rating categories range from "AAA" as the best grade to a single "C" for speculative grade and is almost equal for both agencies and hence highly comparable. Defaulted bonds, inducing, the amount outstanding can not be served by the issuer, are labelled "D". Although only a small number of bonds are "Not Rated" (N.R.), there exists a category for these bonds. Most credits that move to N.R. simply matured or performed at a value below which no rating is announced. N.R. may also occur as a result of mergers and acquisitions. Other credits are withdrawn due to a lack of cooperation. Ratings are often analyzed in the context of a portfolio perspective. Regarding the estimation of transition probabilities or relative frequencies it is thus important whether the information contained in the N.R. category is taken into consideration.

A rating agency mainly assigns a rating to a bond. But a company or country may also be rated. Hence Moody's and Standard and Poor's (S&P) evaluate the riskiness of corporate, municipal, and government issued securities and give each security a Bond Rating. The rating, or more accurately the risk, is based on two elements: the probability the organization will file for bankruptcy before the final bond payment is due and what percentage of the bondholder's claims creditors will receive if a bankruptcy takes place.

This thesis deals with the matrix of transition probabilities for corporate entity ratings. This matrix is defined to describe the probability for a change in an underlying rating. Thus "rating migrations" refers to a change from an initial rating to a new rating category. This section presents some of the most frequent methods to deal with rating migrations. Since not only the case of default is of interest in modern portfolio theory and risk management, the behavior of the entire transition matrix will be analyzed throughout this work.

In principle, the rating process focuses on the fundamental long-term

credit strength of a company. It is assumed that a good rating reflects a high probability for a firm to pay its debt. Thus the rating both reflects the risk and determines the cost of refinancing for a firm. Hence, the importance of rating for investors and companies becomes obvious.

In fact, the change in a rating of a bond or its probability of default has essential implications in terms of risk and portfolio management. To see this, consider the valuation of a bond by a portfolio or risk manager. Assigning a probability of a rating down or upgrade or even default is crucial because it reflects possible gains/losses. Thus, because the rating is assumed to reflect credit risk, a change in credit rating has direct performance implications. Hence, it is important to estimate not only the probability of default but also the likelihood of migration (from a given rating to any other).

Hence, it should be clear why interest and research on ratings increased over the past. Besides an increasing interest for risk management as a whole, there are some more reasons why ratings lie in the focus of research. Definitely, one reason is the connection between counter party risk and capital requirements as proposed by the Basel Committee on Bank Supervision (2003a). The new proposals of the Basel Capital Accord allow banks to link their capital requirements directly to the creditworthiness of counter parties and for those capital requirement on internal ratings. These alignment of economic and counter party risk clearly pushes up the interest and necessity for accurate estimation of default and transition probabilities.

Another reason why more research in credit risk is undertaken stems from the fast development in the market for credit risk which has become much more liquid in recent days (see for example Patel (2003)). Asset backed securities like Collateralized Bond and Loan Obligations (CBOs and CLOs) as well as credit derivatives and Credit Default Swaps (CDS) allow financial institutions to mitigate their credit risk exposure. Appropriate pricing, hedging and portfolio diversification of these new generation credit instruments, however, require an adequate description of the dynamic behavior of the underlying risk.

The adoption of a portfolio perspective to credit risk, as in Gupton et al. (1997) or Credit Suisse (1997), turns interest to systematic risk that is most important at a portfolio level. Suppose systematic risk can be assumed to be correlated with or caused by macroeconomic conditions. If, therefore, a link can be established between the macroeconomic environment and systematic credit risk factors, portfolio risk could be analyzed by the assumed behavior of macro-variables over time. This thesis will assess systematic risk.

Historically, the evolution of modern risk management and portfolio the-

ory goes back to Markowitz (1952) and his portfolio theory for investments. twenty two years later, Merton (1974), in his famous approach, threatened the company value as a deptholders call option; a milestone in the evolution of risk management. In his approach, a companies credit rating is assumed to be driven by its underlying asset value. Thus, Merton introduced a connection between credit risk and the underlying firm value. It is assumed that if the value of assets decline so much that the value is less than the amount of liabilities outstanding, the firm can not serve its dept and will thus default. In the generalization of his model, each rating category can be matched to a certain value of the firm. Because in the original model the company value is considered as a call option, one could use the Black Scholes option pricing model. The option based approach can be found in credit portfolio models such as Gupton, Finger and Bhatia's (1997) CreditMetrics, Moody's KMV and in CSFB's CreditRisk+ (Credit Suisse First Boston (1997)) and is clearly among the most famous and most widely used in literature.

Among the first constructing and publishing transition matrices for migration analysis were Altman (1989) and Altman and Kao (1992). They assess the changes from an initial bond rating, usually at the time of issuance, and show that for every rating and time horizon (one to five years) newly-rated issues from the earlier period exhibit greater stability. This is known as the aging effect. In more detail, the aging effect refers to the phenomenon that default probability rises as time to maturity decreases. At the time of issuance borrower's capital endorsement makes default unlikely while over time the probability of changing business conditions as well as unforeseen events increase and thus the case of default. Keenan et al. (1999) show that empirical default probabilities rise sharply after year two of issuance and peak in year four whereafter they drop again sharply and die out after year ten of issuance (see also Jonsonn and Fridson (1996) and Carty (1997)).

Another famous model was presented in a seminal study by Jarrow, Lando and Turnbull (1997) in which rating transitions were modeled as a time-homogenous Markov chain. This means that a firm's rating in the next period is not affected by its rating history (by the Markov property), and the probability of changing from one rating to another remains the same over time (time homogenous). In this model, default probabilities are infered from credit spreads; thus, the method can be considered as implicit. A general distinction between all models dealing with rating migrations consists in implicit and explicit (or historical) estimation of transition matrices. Here, implicit estimation refers to extracting transition and default information from market prices of risky zero-coupon bonds.

A comparison of the two most widely used models CreditRisk+ and CreditMetrics can be found in Gordy (1998). While in the framework of CreditMetrics transition matrices are considered, the CreditRisk+ model can be used for a more detailed discussion on modelling default probabilities. Here, the obligor is assumed to have only two possible end-of-period states, default and non-default. In the event of default, the lender suffers a loss of fixed size; this is the lender's exposure to the obligor. In Keenan et al. (1999) the model used by Moody's to predict future default rates is presented. The authors show that, using a poisson regression model (see Greene (1997)) for 12 months trailing default rates, it is possible to incorporate explanatory variables like macroeconomic conditions reflected by industrial production as well as the aging effect.

It should be clear that not all concepts relevant in the context of credit risk can be discussed within this work. For a comparison of different credit risk models including the concept of Value at Risk (VaR), confer to Crouhy et al. (2000). For the construction of loss distributions of credit portfolios (for financial institutions such as banks) conditioned on macroeconomic factors, see Bangia et al. (2000) and Pesaran et al. (2003). Besides a presentation of models threatening the probability of default, a discussion on Loss Given Default (LGD) can be found in Allen and Saunders (2002).

The goal of this thesis is to calculate the transition matrices of ratings and condition them on macroeconomic variables. For this, I first discuss the cohort method presented in chapter 3. It will be shown that there exists some systematic dependency structure between rating migrations. This dependency structure is assumed to be driven by economic variables. Therefore, I also present some methods in chapters 4 and 5 which are able to account for these correlation. In chapter 6 I will construct credit cycles myself and analyze the determinants of these cycles. Chapter 7 concludes.

3 The Cohort Method

In order to calculate transition probabilities from historical data, one can use the cohort method. The cohort method is most widely used and easy but, as will be shown, suffers an efficiency loss because of simplification.

The cohort method assigns transition probabilities to every initial rating. It does so by using the relative frequencies of migration from historical data. It

simply sums up the number of ratings in a certain state at the end of a period and divide it by the number of ratings at the beginning of the period. Table 1 shows the general construction for the corresponding transition matrix. Here p_{jk} is the probability of migrating from state j to k . Let the total number of firms be N . Then in rating state j there are n_j firms at the beginning of the period and n_{jk} migrated to state k at the end of the period. The estimated transition probability for stochastically independent migrations from initial rating j to k is: $p_{jk} = (n_{jk}/n_j)$. The last row consists of zeros for all elements but the last one. This stems from the assumption that once an entity defaulted it will not leave this state again. Hence, the last row is identical for every possible transition matrix. Displaying the last row can therefore be assumed to be optional. A transition matrix describes the probability of being in any of the various states in time $T + 1$ given state T . It is thus a full description of the probability distribution.

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{12} & p_{13} & \dots & p_{1k} \\ p_{21} & p_{22} & p_{23} & \dots & p_{2k} \\ \vdots & & & & \vdots \\ p_{j-1,1} & p_{j-1,2} & p_{j-1,3} & \dots & p_{j-1,k} \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$$

Table 1: Probability for rating migration from rating j to rating k .

Table 2 and 3 illustrate the cohort method for data provided by S&P in the year 2004. The rating categories are assumed to be *AAA*, *AA*, ... and *D*, where *D* stands for default¹. The resulting matrix is (7×8) or, in the general case, $(D - 1) \times D$ reflecting the absorbing property in the case of Default. That is the probability for leaving default once you defaulted is assumed to be zero. The entries in the first row of Table 2 show the behavior of initial *AAA* rated obligor's. That is from the 98 *AAA* rated obligor's at beginning of 2004, six moved to rating *AA* and 92 stayed in their initial rating class until the end of the year.

The corresponding probabilities are shown in Table 3. As expected, Table 3 shows that transition matrices exhibit higher default risk for lower quality grades. Specifically one can see that default likelihood increases exponentially with decreasing grade. A characteristic feature of rating transition matrices is the high probability mass on the diagonal. It shows that obligors are most likely to stay in their current rating. For the rating transition probability

¹Find some more detailed comments on the data in section 6.2.

	<i>AAA</i>	<i>AA</i>	<i>A</i>	<i>BBB</i>	<i>BB</i>	<i>B</i>	<i>CCC/C</i>	<i>D</i>
<i>AAA</i>	92	6	0	0	0	0	0	0
<i>AA</i>	1	393	15	1	0	0	0	0
<i>A</i>	0	17	1114	35	1	0	0	0
<i>BBB</i>	0	1	33	1331	27	2	0	0
<i>BB</i>	1	0	1	41	797	53	2	4
<i>B</i>	0	0	0	1	57	652	19	13
<i>CCC/C</i>	0	0	1	0	1	21	75	19

Table 2: Migration Counts; Source: S&P 2004 annual transition data.

	<i>AAA</i>	<i>AA</i>	<i>A</i>	<i>BBB</i>	<i>BB</i>	<i>B</i>	<i>CCC/C</i>	<i>D</i>
<i>AAA</i>	93.9	6.1	0.0	0.0	0.0	0.0	0.0	0.0
<i>AA</i>	0.2	95.9	3.7	0.2	0.0	0.0	0.0	0.0
<i>A</i>	0.0	1.5	95.5	3.0	0.1	0.0	0.0	0.0
<i>BBB</i>	0.0	0.1	2.4	95.5	1.9	0.1	0.0	0.0
<i>BB</i>	0.1	0.0	0.1	4.6	88.7	5.9	0.2	0.4
<i>B</i>	0.0	0.0	0.0	0.1	7.7	87.9	2.6	1.8
<i>CCC/C</i>	0.0	0.0	0.9	0.0	0.9	17.9	64.1	16.2
<i>D</i>	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0

Table 3: Migration Probabilities, calculated using cohort method.

distribution of an obligor given its initial rating, the second largest probability is usually in the direct neighborhood to the diagonal. One can conclude that the further away a cell is from the diagonal, the smaller is the likelihood for such an event to happen. In literature this rule is known as “monotonicity” (see Bangia et al. (2000) p.18).

In addition, it is interesting to consider the stability of each rating category over time. Stability is measured by the probability to stay in the initial state for every rating over the period of one year. It is shown in Figure 1 for annual transition data from S&P for the years 1981 to 2004. It can be observed that the higher a rating category the more stable the rating is, that is the lower the rating category the more unstable and volatile is the rating behavior. Especially the lowest rating category (*CCC/C*) shows a rather unstable behavior and is much more volatile than the other categories. In some periods the transition probability for all ratings belonging to this state sums to more than 50%.

The cohort method illustrated before is clearly an issuer-weighted es-

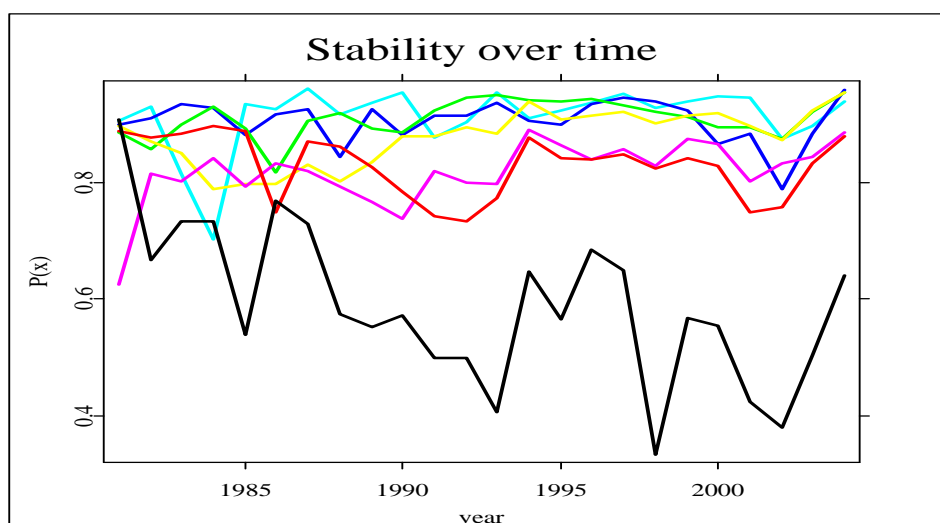


Figure 1: Transition stability over time. AAA cyan, AA blue, A green, BBB yellow, BB magenta, B red and CCC/C black.

Quantlet : migrate1.xpl

timator. That is, every observation represents one rated issuer and every rating has the same weight. In contrast, a dollar-weighted estimator would weight every rated issuer with the amount of outstanding debt. I will use an example from Standard & Poors (2005) “Annual Global Corporate Default Study 2004”² to illustrate why the issuer-weighted estimator is preferred over a dollar-weighted estimator.

Assume that it is known that two issuers each have a 50% probability of default. Further assume that one issuer has \$10 of principal outstanding while the other has \$90 of principal outstanding. If we estimate default probability using the issuer-weighted methodology, we have a 50% chance of estimating a 50% default probability (one of the two issuers defaults), a 25% chance of estimating a 0% default probability (neither defaults), and a 25% chance of estimating a 100% default probability (they both default). The standard deviation of the issuer-weighted estimator is 12.5%. If we estimate default probability using the dollar-weighted methodology, we have a 25% chance, each, of estimating 0% (neither defaults), 10% (the \$10 issuer defaults), 90% (the \$90 issuer defaults), and 100% (both default). The standard deviation of the dollar-weighted estimator is 20.5%. This issuer-weighted estimator (12.5% standard deviation) is clearly to be preferred over the dollar-weighted estimator (20.5% standard deviation).

²See Standard & Poors (2005), p.28.

Given the transition probabilities estimated with the cohort method one can assign a probability of migration to every issuer. Clearly for the method to be valid, some assumptions must be fulfilled. First, consider that the probability of migration only measures the movement for a single issuer or obligor. But in a portfolio context it would be important to know the probability of simultaneous rating transitions for more than one credit. Let's consider for example a portfolio consisting of two *BB* rated bonds. Given the transition probabilities above, single probabilities are consistent with these probabilities. In contrast, simultaneous probabilities for a migration from *BB* to *BBB* for one bond and migration from *BB* to *B* for another is just the product of the single probabilities ($0.046 * 0.059 = 0.0027 = 0.27\%$). The calculation of all 64 possible states as well as extending the calculation to the case of more than two bonds is straightforward. Note that this is only valid under the assumption of independent rating migrations! Section 5.1 will show consequences of the violation of independent rating migrations in detail.

Let's turn to some other shortcomings of the cohort method. One is censoring: This means that we do not know what happens to the firms/bonds after the sample period ends. Information carried by these firms/bonds is not available to the statistician, but may be important for the estimation. Another shortcoming is that rating changes within a period are ignored. To illustrate the effect of neglected migration within a period, consider an *AAA*-rated bond presented in Table 3. The probability of a direct move from *AAA* to *A* is zero but a movement from *AAA* to *A* via *AA* is positive. Thus, the true migration probability is underestimated. As stated by Carty (1997), large changes in credit quality have been very infrequent over the past 77 years. According to the author, only 11% of all rating changes since 1920 involved changes of more than one rating category. I will return to the problem of migrations within a period in Section 4.2.

4 Markov Chains

One of the most common assumptions for transition matrices is that they follow a first-order Markov process/chain. This is equal to say that transition matrices being time invariant or time homogenous. The prediction of transition probabilities for N periods would then simply be:

$$P(N) = P(0) * P(1) * \dots * P(N) = P(0)^N, \quad (1)$$

where $P(N)$ represents the transition matrix at time N and $P(0)$ is the initial transition matrix at the beginning of the period.

For the transition matrices of type $d \times d$, it is then possible to use an eigenvalue decomposition to calculate future transition matrices. The matrix $P(N)$ can be expressed according to

$$P(N) = \Gamma \Lambda^N \Gamma^{-1}. \quad (2)$$

Here, Λ is a diagonal matrix containing the eigenvalues and Γ contains the corresponding eigenvectors. Thus, under the assumption of time invariant transition matrices, computation is rather simple. In Xplore, computation can be done using the Quantlet “migrate2.xpl” (see Appendix A.2). The eigenvalues of a transition matrix are thus directly related to the evolution of the state vector over time. Hamilton (1994) shows that since every row of a transition matrix sums up to one, unity is an eigenvalue of the transition matrix P for any Markov chain. The Markov chain is said to be ergodic (i.e. a unique steady state distribution exists independent of the initial state) if all other eigenvalues are inside the unit circle. Since it was shown that an eigenvalue of one is obtained for every Markov chain, it is the second largest eigenvalue that determines the rate at which the ergodic system decays towards zero.

Bangia et al. (2000) state that one should observe a linear relationship between the logarithm of all eigenvalues and the transition horizon N . Surprisingly, they found that it is hard to reject the first order Markov assumption regarding a pure analysis of the eigenvalues.

4.1 Testing for Time Invariance

A formal test on time invariance is presented by Höse et al. (2002). The authors use a Chi-Square Test for homogeneity. The test performs as follows. Given migration data for m periods, the aggregated transition matrix is calculated. Again, analogous to the one period transition matrix one gets:

$$P(M) = \hat{p}_{jk}^+ \equiv \frac{c_{jk}^+}{n_j^+}, \quad (3)$$

with c_{jk}^+ for the number of migrations from initial rating j to rating k aggregated over all periods m . Here, n_j^+ is the aggregated number of observations from initial rating j .

The Null Hypothesis of constant transitions from any initial rating j is

$$H_0 = p_{jk}(t) = p_{jk}(m), \quad (4)$$

for $t = 1, \dots, m - 1$; $j = 1, \dots, d - 1$; $k = 1, \dots, d$, and may be tested via the test statistic

$$X_j^2 = \sum_{k=1}^d \sum_{t=1}^m \frac{[\tilde{c}_{jk}(t) - n_j(t)\hat{p}_{jk}^+]^2}{n_j(t)\hat{p}_{jk}^+}. \quad (5)$$

Under H_0 , X_j^2 is asymptotically χ^2 -distributed with $(d - 1)(m - 1)$ degrees of freedom.

Testing the combined Null Hypothesis of constant transition probabilities

$$H_0 : P(1) = P(2) = \dots = P(m), \quad (6)$$

the test statistic is:

$$X^2 = \sum_{j=1}^{d-1} X_j^2. \quad (7)$$

Under H_0 , X_2 is asymptotically χ^2 -distributed with $(d - 1)^2(m - 1)$ degrees of freedom.

As mentioned by the authors, two problems arise. First, the two tests (5) and (7) are based on the assumption of independent migration events. Second, the test statistics are only asymptotically Chi-Square distributed. This means that a sufficiently large sample sizes is required. As the authors point out, using the rule of thumb given in the literature, the sample size has to fulfill: $n_j(t)\hat{p}_{jk}^+ \geq 5$ for all j and k . Obviously, in practice this will rarely be the case. Bangia et al. (2002) unsurprisingly find that the diagonal elements are estimated with high precision while the off diagonal elements are estimated with large uncertainty. This is due to the low number of observations for off-diagonal elements.

Performing a test for the S&P 1981-2004 data set, one must reject the null hypothesis of time homogeneity for all rating states. Results are presented in Table 4. The test performs row wise. That is, for the first rating state *AAA* the hypothesis that all twenty-four years came from the same theoretical distribution is rejected at any usual confidence level with $7(24 - 1) = 161$ degrees of freedom. Results are presented in the first column of Table 4. Results for rating state *AA* can be found in the second column of Table 4. The last column shows the test for simultaneous transition of all matrices with $7^2(24 - 1) = 1127$ degrees of freedom. The test must clearly be rejected. Calculation is done with the Quantlet: XFGRatMig3.xpl which can be found in Xplore.

χ^2	186.53	419.75	497.78	639.97	587.12	566.13	237.71	3135
df	161	161	161	161	161	161	161	1127
$p - values$	0.9178	1	1	1	1	1	0.9999	1

Table 4: Transition stability for S&P 1981-2004 data with χ^2 test statistic, degrees of freedom (df) and corresponding probability values.

Quantlet : XFGRatMig3.xpl

4.2 The Aalen-Johansen Estimator

One possibility to account for non-Markovian behavior is the Aalen-Johansen estimator.

The description of the Aalen-Johansen estimator (Aalen and Johansen (1978)) follows the one of Jafry and Schuermann (2003). Unfortunately, it was not possible for me to obtain an appropriate data set in order to perform an accurate estimation of the presented Aalen-Johansen estimator. However, because the estimator gives a possibility to deal with time invariance, accounts for migrations within a period and was shown to improve estimations significantly, a short introduction will be presented³. As Jafry and Schuermann show, the Aalen-Johansen or nonparametric product-limit estimator is consistent. The estimator for the transition matrix from time s to time t , $P(s, t)$ is given by

$$\hat{P}(s, t) = \prod_{i=1}^m (I + \Delta \hat{A}(T_i)). \quad (8)$$

Here, $\hat{P}(s, t)$ is the transition probability matrix from time s to time t and the jk^{th} element of the matrix denotes the probability that the Markov process starting in state j at date s will be in state k at date t . Furthermore, m is the number out of total days on which at least one transition occurs. Thus, for annual data matrices, m would be the number of days in that year on which at least one transition occurs. T_i is the jump in time interval from s to t . The estimator, which is clearly a duration approach, thus allows for time non-homogeneity while fully accounting for all movements within the sample period (estimation horizon). The Matrix $\Delta \hat{A}(T_i)$ is given by

For each fraction, the nominator denotes the number of transitions from

³Focusing on one-year horizon, Jafry and Schuermann compare this approach to the cohort method. Comparison is based on a singular value decomposition of transition matrices. The authors find great impact and differences between the Aalen-Johansen approach and the cohort method. These differences are clearly in favor of the duration approach.

$$\Delta\hat{A}(T_i) = \begin{pmatrix} -\frac{\Delta N_{1,1}(T_i)}{Y_1(T_i)} & \frac{\Delta N_{1,2}(T_i)}{Y_1(T_i)} & \frac{\Delta N_{1,3}(T_i)}{Y_1(T_i)} & \dots & \frac{\Delta N_{1,p}(T_i)}{Y_1(T_i)} \\ \frac{\Delta N_{2,1}(T_i)}{Y_2(T_i)} & -\frac{\Delta N_{2,2}(T_i)}{Y_2(T_i)} & \frac{\Delta N_{2,3}(T_i)}{Y_2(T_i)} & \dots & \frac{\Delta N_{2,p}(T_i)}{Y_2(T_i)} \\ \vdots & \vdots & \ddots & \dots & \vdots \\ \frac{\Delta N_{j-1,1}(T_i)}{Y_{j-1}(T_i)} & \frac{\Delta N_{j-1,2}(T_i)}{Y_{j-1}(T_i)} & \dots & -\frac{\Delta N_{j-1}(T_i)}{Y_{j-1}(T_i)} & \frac{\Delta N_{j-1,k}(T_i)}{Y_{j-1}(T_i)} \\ 0 & 0 & \dots & \dots & 0 \end{pmatrix}.$$

Table 5: Probability for migration from j to k .

Source: Jafry and Schuermann (2003) eq. (5).

state j to k over the time interval from s to t , T_i . The diagonal elements count the number of transitions away from initial state j . $Y_j(T_i)$ is, similar to the cohort method, the number of exposed (or at risk) firms, i.e. the number of firms in initial state j at the beginning of the time interval. Note that adding the identity matrix yields again the assumed absorbing property for the default state. This reflects the assumption of staying in default once you defaulted. Further, the diagonal elements can then be interpreted as the probability to stay in the initial rating. The Aalen-Johansen estimator is thus equal to the cohort method for short time intervals. For a short time horizon one could expect the differences between the cohort estimator and the Aalen-Johansen approach to be neglecting. As the time horizon extends however differences between the two estimators increase because of the higher migration potential for longer time horizons. As before, the last row consists of zeros since it reflects the absorbing state. It is easy to see that the rows of $(I + \Delta\hat{A}(T_i))$ sum up to one.

To illustrate the major consequences of the Aalen-Johansen estimator consider an AAA rated obligor. Suppose that for the time interval under consideration no AAA rated obligor was observed to default. Using the cohort method thus would not contribute any probability of default to initially AAA rated obligors. When using the Aalen-Johansen estimator however it suffices that an obligor migrated from AAA to AA to A and then defaulted to contribute some probability mass to the migration from AAA to D, $P_{AAA,D}$. This shows that the cohort method would underestimate the true probability of default for AAA rated obligor's and thus improvement can be achieved by using the Aalen-Johansen estimator.

4.3 Time Horizon

As Gupton et al. (1997) point out, the transition matrix should be estimated for the same time interval as the risk horizon we want to estimate. This should be very intuitive, since a manager facing a risk horizon of, let's say, two years would like to have information about the underlying risk structure over the same time period. Systematic changes in credit risk over time as the general time dependence of credit risk would clearly better be reflected. So far, I used one-year risk horizons for computations as it is a common time interval for most applications and therefore often considered in praxis. For a two-year risk horizon, however, one would use a two-year rather than one-year transition matrix. Table 6 shows the two-year transition matrix over the years 2003 and 2004 for the same S&P data. Computation of different annual time horizons can be done with the Quantlet "migrate3.xpl" (see Appendix A.3).

	<i>AAA</i>	<i>AA</i>	<i>A</i>	<i>BBB</i>	<i>BB</i>	<i>B</i>	<i>CCC/C</i>	<i>D</i>
<i>AAA</i>	91.7	7.3	0.1	0.0	0.0	0.0	0.0	0.0
<i>AA</i>	0.4	9.2	7.3	0.4	0.0	0.0	0.0	0.0
<i>A</i>	0.0	1.2	93.9	4.8	0.1	0.0	0.0	0.0
<i>BBB</i>	0	0	2.0	94.0	3.6	0.2	0.0	0.1
<i>BB</i>	0.1	0.0	0.1	4.0	86.6	8.3	0.5	0.5
<i>B</i>	0.0	0.0	0.0	0.1	7.6	85.8	3.4	3.1
<i>CCC/C</i>	0.0	0.0	0.4	0.0	0.8	15.2	56.4	27.3
<i>D</i>	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0

Table 6: Transition matrix over two year risk horizon, S&P years 2003/2004 data.

5 Dependent Migration

5.1 Consequences of Dependent Migration

This section will discuss consequences of correlated rating migrations. Thereby rating migrations are thought to be correlated if one issuer improves in credit quality over the course of one year, and this induces another issuer to be more likely to improve in credit quality as well. Equivalently, if one issuer

deteriorates in credit quality, the other issuer would also be more likely to deteriorate in credit quality. For example consider positive credit quality correlation between the two *BB*-rated issuers. The probability of uncorrelated joint migration from *BB* to *BBB* for the first credit and from *BB* to *B* for the second credit at the same time was calculated to be 0.27%.

Now assume that both rating migrations are correlated with a coefficient of correlation $\rho = 0.3$. Bivariate probabilities for known correlation and transition probabilities can easily be computed. The formula is:

$$p_{(BBB,B)} = p_{BBB} p_B + (\rho \sqrt{p_{BBB}(1 - p_{BBB})p_B(1 - p_B)}), \quad (9)$$

where p_{BBB} is the single probability for the first credit to migrate to *BBB* and p_B the probability that the second credit rating migrates to *B*; $p_{BBB,B}$ is then simply the joint probability.

Using the data from above, the joint migration likelihood would then be 1.75%. This is about six times higher than the uncorrelated joint migration! Neglecting correlation can therefore have substantial influence on the expected outcome.

One can show that if correlation in fact exists the cohort estimator is not the best estimator. Let's assume correlation exists. Consider the probability p_{jk} for a credit to migrate from initial rating j to rating k . Because migration either occurs or not, one can assume the migration to follow a Bernoulli distributed indicator variable $1\{e_{i2} = k\}$. The indicator becomes one if a migration occurs and is zero otherwise. The probability p_{jk} can then be written as:

$$p_{jk} = P(e_{i2} = k | e_{i1} = j) \quad \text{and} \quad \sum_{k=1}^d p_{jk} = 1. \quad (10)$$

For Bernoulli distributed variables the standard deviation of p_{jk} is:

$$\sigma_{jk} = \sqrt{\frac{p_{jk}(1 - p_{jk})}{n_j}}. \quad (11)$$

However, Huschens and Locarek-Junge (see Huschens and Locarek-Junge (2000) p.44) show that for pairwise correlated Bernoulli variables with correlation ρ_{jk} one obtains the variance:

$$\sigma_{jk}^2 = \frac{p_{jk}(1 - p_{jk})}{n_j} + \frac{n_j - 1}{n_j} \rho_{jk} p_{jk}(1 - p_{jk}). \quad (12)$$

Thus, for positive pairwise correlation ρ_{jk} the law of large numbers fails and one obtains:

$$\lim_{n_j \rightarrow \infty} \sigma_{jk}^2 = \rho_{jk} p_{jk}(1 - p_{jk}). \quad (13)$$

In consequence, the cohort method is not a consistent estimator, because it does not account for pairwise correlation.

Note, that accounting for correlation between credit classes has important implications for the standard deviation of portfolio value. The standard deviation is a symmetric measure of dispersion around the average portfolio value. The greater the dispersion around the average value, the larger the standard deviation, and the greater the risk. If the portfolio value is measured in Euros, the calculated standard deviation also results in Euros. Correlation thus increases the standard deviation of the portfolio value which has negative implications in terms of modern portfolio theory.

It is well known that the parameter of correlation is bounded between $-1 < \rho < 1$. However, for identically distributed and symmetrically correlated random variables and the numbers of observations n going to infinity, there is a lower bound of correlation which is zero. To see this, assume the variance-covariance matrix of n random variables to be:

$$\mathbf{V}_n = \sigma^2 \begin{pmatrix} 1 & \rho & \dots & \rho \\ \rho & \ddots & \ddots & \vdots \\ \vdots & \ddots & 1 & \rho \\ \rho & \dots & \rho & 1 \end{pmatrix}.$$

As can be seen, one assumes an equicorrelation structure for the whole matrix. The determinant is $\det(V_n) = (1 + (n - 1)\rho)(1 - \rho)^{n-1}$. For the matrix to be positive definite, all submatrices should be positive definite. The expression $\det(V_n)$ captures all the determinants of the n submatrices. Hence it follows

$$\rho^* = -\frac{1}{n-1} < \rho < 1, \quad (14)$$

which yields a lower bound of zero for $n \rightarrow \infty$.

5.2 Evidence for Correlation

It is often pointed out that macroeconomic variables seem to influence transition probabilities in a systematic way. Studies which explicitly recognize the impact of business cycles on rating transitions are Bangia, Diebold and Schuerman (2000), Belkin, Suchower, and Forest, Jr (1998) and Nickell, Perraudin and Varotto (2000) (see also Gupton et al. (1997) and Carty (1997), which also suggest the likelihood of correlation). Their main contribution is to show that rating transitions depend on business cycles. Thus, using the

cohort method neglects the correlation with macroeconomic variables. Assuming a zero correlation like this is too simplistic and unrealistic. Because operating conditions adjust dynamically and movements in any one macroeconomic variable may affect several issuers, the credit ratings of different obligors are likely to be linked and therefore to move together. The failure of stochastic independent rating migration is key in estimating transition matrices. It causes the binomial distribution to break down. Further the law of large numbers and the Central Limit Theorem are not valid.

Because the study of Nickell et al. (2000) is very appealing I will shortly introduce it. The authors divide transition matrices into three states of the business cycle; normal times, peaks and troughs depending on whether real GDP in the country in question was in the middle, upper or lower third of the growth rates recorded in the sample period. By the same methodology one can divide the data sample in different industry categories, for example Banks, Industrial, Insurance or use different domicile characteristics like US, Japan and Europe. Depending on the size of the data sample, fine enough sub samples could be constructed to point out the dependence regarding all of these characteristics.

The analysis is performed by using simply historical transition probabilities as well as by constructing an ordered Probit model. The later one allows for detection of marginal effects and confirms the former findings of using the cohort method. They found that, particularly for low rated borrowers, business cycle effects are important. Generally, for investment grade bonds (*BBB* or better) the volatility falls sharply in business cycle peak years and rises in business cycle troughs. Insofar as the reference asset for most credit derivatives is company/institution specific, the ability to condition a transition matrix on the industry (to which the company belongs) is definitely desirable. Assuming that changes in the business cycles are time independent Markov chains, the computation of multi-period transition matrices can be done as well. To do so however, requires some cumbersome calculation. The major shortcoming clearly is the need for a large data set which makes this method hard to apply with the common level of precision.

When dealing with the estimation of transition matrices, one should try to cover migration dependencies nevertheless. Another method to do so is to assign historical transition probabilities to all possible outcomes as is suggested by Gupton et al. (1997) (see Gupton et al. (1997) p. 84). One could start with all obligors rated *AAA* and *AA* at the beginning of the period and calculate the number of assigned ratings in any of the possible states at the end of the period. This ensures any pair of possible credit moves

to be accounted. Hence, for seven non-default categories 28 joint migration matrices have to be calculated. Thus it would be possible to account for dependencies without having to specify any particular correlation structure or to rely calculations on any specific kind of model. However one faces the same problems as Nickel et al. (2000), since too many observations are required for reliable transition probabilities. Furthermore, all credits are considered to be identical. But it is critical to assume the banking industry to behave in the same way as any other industry. Thus in contrast to Nickel et al. (2000), the estimation is not sensitive to firm specific characteristics. Therefore, it is not applicable and presented only for the purpose of illustration.

5.3 Calculating Correlations

The aforementioned shortcomings of direct estimation of transition matrices make it necessary to develop a model which is capable of considering joint movements in a consistent way. In the framework of CreditMetrics the use of a generalized Merton approach is suggested (see also Bangia et al. (2000)). Furthermore, it is also quite easy to implement. Hence, after presenting the generalized Merton model, it will be shown how correlation between rating migrations can be calculated within this framework.

The problem is that for credit quality no market or information for the correlation structure exists. The solution suggested by CreditMetrics is to calculate correlations for two assets in a liquid market and use it as a drawback for correlation in credit risk. For example, if observable, one could use bond prices and assume that rating migrations are linked to the bond price (credit spread) through some pricing model. CreditMetrics however points out that practical problems may arise again regarding the availability of data. Bond prices would be needed for all rated bonds over the full time horizon. For low rated bonds or bonds without observable market prices (e.g. when no market making exists) over the full time horizon, this would lead to some difficulties in estimation. However, in high liquid markets such as the equity market, it is possible to estimate the correlation directly from observed market prices. Hence, in the framework of CreditMetrics, a companies credit rating is assumed to be driven by its underlying asset value. This delivers already the basic assumption of the Merton model.

The Merton approach only assumes that company i 's rating is determined by its asset value y_i which is assumed to be a latent random variable.

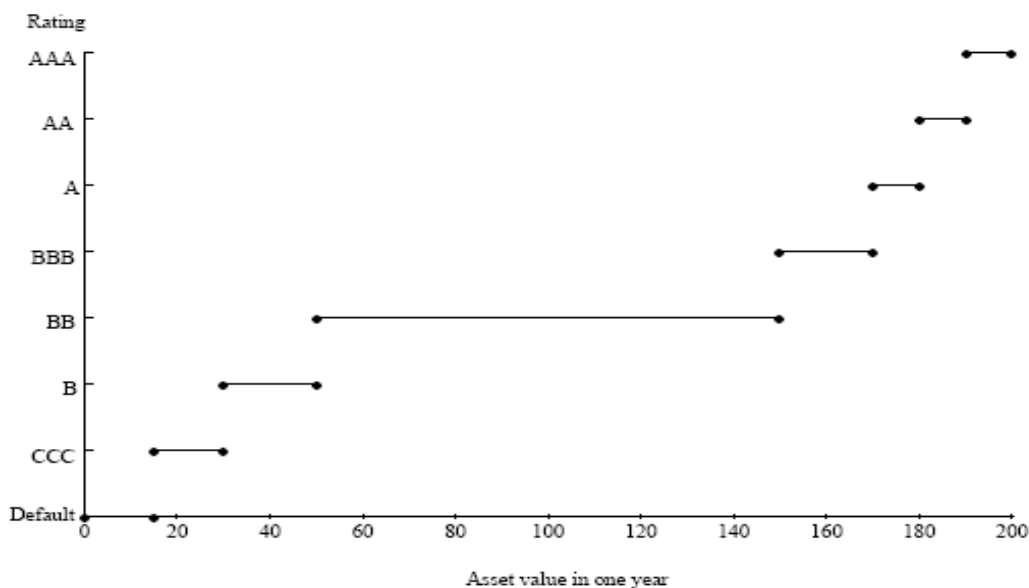


Figure 2: Rating Migration driven by underlying Asset Value for Rating *BB*.
Source: CreditMetrics (1997), Chart 8.1.

Furthermore, the state of obligor i at the risk-horizon depends on the location of y_i relative to a set of cut-off values. In the full version of the model, the cut-offs divide the real number line into bins for each end-of-period rating grade. Each change in company rating could then be associated to a certain change in the company's asset value. This means that a drop in asset value would lead to a downgrade in its rating. See Figure 2 for an illustration on a firm with underlying rating *BB*. The firm's asset value is labelled on the x -axis.

Assuming known asset thresholds and normally distributed percent changes in asset value, one only needs to model the company's change in asset value. Then it is possible to describe the evolution of the credit rating. That is, a rating migration takes place if the corresponding value of the standard normal distribution for the asset threshold is reached. Figure 3 illustrates this idea for a credit with rating *BBB*. A change in a firm's asset value would lead to a rating shift if a certain threshold value is reached. The probability to stay in the initial rating is represented by the area between the bars *BB* and *A*. Correspondingly, the probability for a rating migration to *A* is determined by the area between bars *A* and *AA*. This method allows to capture not only defaults, but migrations across non-default grades as well. If the value of the firm falls below a certain threshold, e.g. the face value of its debt B , the call option will not be executed and the shareholders will put the firm to the

debtholders, determining the default state.

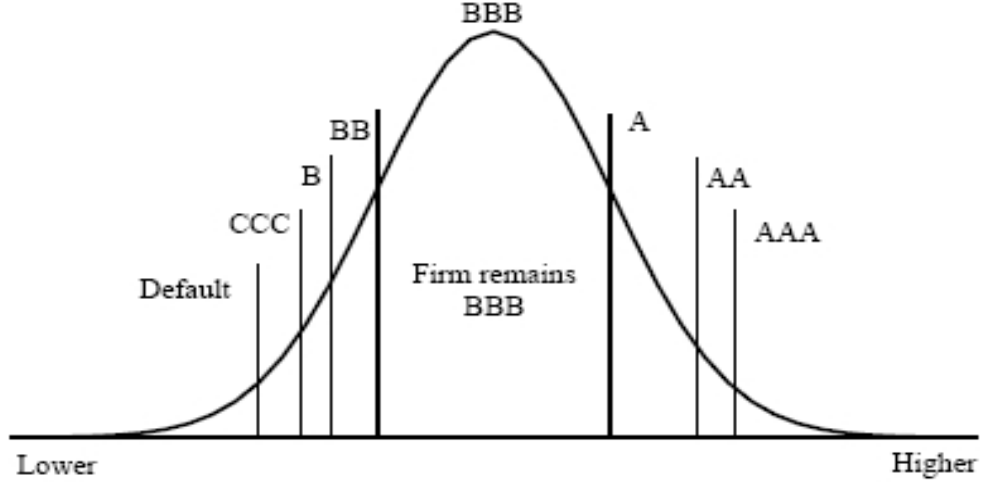


Figure 3: Normal distributed rating thresholds for firm values
Source: CreditMetrics (1997), Chart 6.1.

Consider the transition probability matrix for the S&P data in 2004 presented in Table 3. First, let's partition the probability matrix in a matrix of disjoint bins $(X_j^J, X_{j+1}^J]$, where X is assumed to be a normally distributed underlying credit change indicator and the initial rating category is denoted as J . Here j and $j + 1$ are the end of period ratings. That is, for the cumulated probability of a migration event the corresponding value of the normal distribution is computed. For the probability matrix from above, the matrix of disjoint bins is shown in Table 7.

Consider for example a threshold of 2.815 for a rating change from AA to AAA . The assigned transition probability would then be $1 - \Phi(X_{AAA})$, where Φ stands for the standard normal cumulative distribution function, or 0.2%. Note that the threshold of the first and last column are ∞ and $-\infty$. Every threshold value between ∞ and 2.815 would therefore correspond to a rating migration from AA to AAA . Given X_{AA} , one can determine the probability of staying in rating category AA by $\Phi(X_{AAA}) - \Phi(X_{AA})$. Equivalent, one may calculate the probabilities for the other categories.

Next, one may like to describe the behavior of two credits jointly, e.g. the probability for the first credit to stay in rating B and for the second credit to stay in A at the same time. For this, one assumes the asset returns r, r' of the two credits to be bivariate normally distributed. For any given parameter of correlation ρ , it is possible to calculate the bivariate normal distribution

	AAA	AA	A	BBB	BB	B	CCC/C	D
AAA	(∞ ; -1.55)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)
AA	(∞ ; 2.82)	(2.82; -1.76)	(-1.76; -2.82)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)
A	(∞ ; ∞)	(∞ ; 2.18)	(2.18; -1.87)	(-1.87; -3.14)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)
BBB	(∞ ; ∞)	(∞ ; 3.19)	(3.19; 1.97)	(1.97; -2.04)	(-2.04; -2.98)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)	($-\infty$; $-\infty$)
BB	(∞ ; 3.06)	(3.06; 3.06)	(3.06; 2.84)	(2.84; 1.67)	(1.67; -1.51)	(-1.51; -2.47)	(-2.47; -2.617)	(-2.62; $-\infty$)
B	(∞ ; ∞)	(∞ ; ∞)	(∞ ; ∞)	(∞ ; 3.00)	(3.00; 1.42)	(1.42; -1.72)	(-1.717; -2.11)	(-2.11; $-\infty$)
CCC/C	(∞ ; ∞)	(∞ ; ∞)	(∞ ; 2.39)	(2.39; 2.39)	(2.39; 2.12)	(2.12; 0.85)	(0.85; -0.99)	(-0.99; $-\infty$)

Table 7: Disjoint bins corresponding to transition probabilities calculated from S&P 2004 data.

according to

$$P\{X_B < R < X_{BB}, X'_A < R' < X'_{AA}\} = \int_{X_B}^{X_{BB}} \int_{X'_A}^{X'_{AA}} f(r, r'; \Sigma) dr' dr. \quad (15)$$

In the framework of CreditMetrics, $f(r, r'; \Sigma)$ represents the density function for the bivariate normal distribution with covariance matrix σ . The variables r and r' represent the values that the two asset returns may take within the specified intervals. The probability for the first credit to stay in the initial rating B is given by the probability for the asset return to fall between the interval X_B and X_{BB} and for the second credit to stay in A the interval between X_A and X_{AA} .

Note that assuming normally distributed asset returns merely facilitates calculation. It is not a necessary assumption. Any multivariate distribution capable to characterize joint movements of asset values by a single correlation would be applicable.

Thus, given the parameter of correlation ρ this establishes a nice framework for the calculation of bivariate migrations. However, some problems arise. For pairwise correlation of joint credit moves the calculation is cumbersome. In general, for N observations in the sample (i.e. N obligors in a portfolio), $N(N - 1)/2$ possible pairwise correlations must be considered. For a portfolio consisting of 100 obligors, 4950 pairwise credit correlations would have to be estimated. Furthermore, given eight rating categories including default, 64 possible joint migrations are to be calculated. Storing a correlation matrix of such a large size makes this approach difficult to handle. As suggest by Gupton et al. (1997), index correlations could be used to reduce required calculations. Correlation between a firm operating in the German finance sector and another operating in the industry sector could then be approximated by using the calculated correlation among the indices. More specific correlations clearly could be obtained. Thus, one would be left with having to determine how specific the calculation of correlation should be. One has to compromise between the exact correlation (which implies a high degree of calculation) and a rough approximation of correlation (which in turn implies less calculation).

Another method for the calculation of bivariate correlation is presented by Höse et al. (2002). The authors show that the parameter of bivariate correlation ρ_{jk} can be consistently estimated for a given $\rho \geq 0$ using a threshold normal model. Here the single parameter ρ may be interpreted as a mean asset return. The corresponding restricted Maximum-Likelihood estimator

used is:

$$\hat{\rho}_{jk} = \max \left\{ 0; \frac{\beta(\hat{x}_{j,k-1}, \hat{x}_{jk}; \rho) - \hat{p}_{jk}^2}{\hat{p}_{jk}(1 - \hat{p}_{jk})} \right\}, \quad (16)$$

where \hat{x}_{jk} is distributed, as before, according to the inverse of the normal distribution for the accumulated estimated transition probability \hat{p}_{ji} ,

$$\hat{x}_{jk} = \Phi^{-1} \left(\sum_{i=1}^k \hat{p}_{ji} \right) \quad (17)$$

and $\beta(\hat{x}_{j,k-1}, \hat{x}_{jk}; \rho)$ is the estimator for the probability of a simultaneous migration from initial rating $(j, k-1)$ to rating (j, k) .

Thus for the migration correlation

$$\rho_{jk} = \frac{p_{jj:kk} - p_{jk}^2}{p_{jk}(1 - p_{jk})}, \quad (18)$$

the parameter $p_{jj:kk}$ of simultaneous migration (e.g. $p_{jj:dd}$ would be the probability of simultaneous default from initial ratings jj) is simply replaced by its estimator

$$p_{jj:kk} = \beta(\hat{x}_{j,k-1}, \hat{x}_{jk}; \rho). \quad (19)$$

The probability β is calculated by stochastic integration.

The variance is estimated by

$$\hat{\sigma}_{jk} = \sqrt{\frac{\hat{p}_{jk}(1 - \hat{p}_{jk})}{n_j} + \frac{n_j - 1}{n_j} \hat{\rho}_{jk} \hat{p}_{jk}(1 - \hat{p}_{jk})}. \quad (20)$$

Calculation can be done with the Quantlet ‘‘VaRRatMigRate.xpl’’ which is implemented in Xplore.

6 Credit Cycles

It was already pointed out that rating migrations are subject to some correlation or dependency structure with economic variables. Macroeconomic conditions (economic cycles) were proved to have some influence on the behavior of transition probabilities (Nickell et al. (2000)). Motivated by these findings, one would like to specify the economic or systematic influence on migration behavior.

Therefore I will present a framework which allows to extract the systematic impact on rating migrations. This section comprises as follows. First, in

section 6.1 an introduction to the underlying model to create credit cycles is presented. The constructed credit cycles can be interpreted to reflect some systematic influence. Then, in section 6.2 some comments on the data used for estimation are given. Thereafter, in section 6.3 the estimation process is described in detail, estimation results are presented and some interpretations on the results are given. Finally, in section 6.4 I try to detect specific economic determinants which drive the constructed credit cycles.

6.1 Model Description

The model for the construction of credit cycles is based on the CreditMetrics approach (which itself is based on the option theoretic model of Merton). The Merton approach was already presented in Section 5.3. The main feature is that for credit migrations which are determined by standard normal distributed asset returns one can easily construct probability intervals for every credit rating j as in Table 7.

As before, X is the corresponding bin (the estimated threshold value) of the underlying standard normal distributed process and determines the migration events. Bangia et al. (2000), Belkin et al. (1998), Wei (2000) and Huschens et al. (2005) assume the underlying variable X to be decomposed according to

$$X = \sqrt{\varphi}Z + \sqrt{1 - \varphi}Y. \quad (21)$$

Here, Z represents the systematic component or systematic credit risk and Y reflects the idiosyncratic risk of a borrower. Further Z and Y are assumed to be independent unit normal random variables. The parameter φ works like a weighting factor of the parameters Y and Z . Because φ determines the influence of systematic risk, it can be interpreted as a measure of correlation. Hence, the migration process is subject to both firm-specific events and systematic influences equal to all firms. In this framework one may think of systematic influences as being economic variables such as GDP.

In order to determine the parameters in equation (21) and thus the quantitative influence they have on rating migrations, consider the transition probability $p(J, j)$ from Table 7 for a migration from initial rating J to rating j :

$$p(J, j) = \Phi(X_{j-1}^J) - \Phi(X_j^J). \quad (22)$$

Here again, Φ is the standard normal cumulative distribution function. Note that the default bin has a lower threshold of $-\infty$ and the AAA bin an upper

threshold of ∞ . For an initial AAA rated bond, a transition to A occurs according to $p(AAA, A) = \Phi(X_{AA}^{AAA}) - \Phi(X_A^{AAA})$.

Furthermore, equation (21) may be rewritten as:

$$Y = \frac{X - \sqrt{\varphi}Z}{\sqrt{1 - \varphi}}. \quad (23)$$

Following Belkin et al. (1998), I replace the X in the transition matrix $p(J, j)$ of equation (22) with the parameter Y from equation (23) to get:

$$\Delta(Y) = \Phi(Y_{j-1}) - \Phi(Y_j). \quad (24)$$

Equivalently,

$$\Delta(X_{j-1}^J, X_j^J, Z_t) = \Phi\left(\frac{X_{j-1}^J - \sqrt{\varphi}Z_t}{\sqrt{1 - \varphi}}\right) - \Phi\left(\frac{X_j^J - \sqrt{\varphi}Z_t}{\sqrt{1 - \varphi}}\right). \quad (25)$$

The expression (25) can be interpreted as the transition probability from initial rating J to rating j for the idiosyncratic component. Note that here, the threshold values X are derived from the matrix of average transition probabilities for the years $1, \dots, T$.

Now, one is interested in a value of Z that best approximates the probabilities associated with the defined bins and the observed transition probabilities for every year t . For that, one needs a Z that minimizes the weighted, mean-squared discrepancies between the observed transition probabilities and those defined in equation (25). To quantify this influence the least squares problem is defined according to

$$\min Z_t \sum_J \sum_j \frac{n_{t,J}[p_t(J, j) - \Delta(x_{j-1}^J, X_j^J, Z_t)]^2}{\Delta(x_{j-1}^J, X_j^J, Z_t)(1 - \Delta(x_{j-1}^J, X_j^J, Z_t))}, \quad (26)$$

with $n_{t,J}$ as the number of observed migrations in rating class J for year t . The denominator contains the sample variance to weigh the observations. Thus the authors obtained an expression for the minimum deviation between the transition probability in year t , denoted $p_t(J, j)$, and the idiosyncratic component conditional on the parameters Z and φ .

Minimizing equation (26) with respect to Z and φ shows to what extend transition probabilities in year t can be explained by the idiosyncratic component. One thus considers the single influence of the idiosyncratic component on rating migrations. In any year t , the transition matrix will deviate from the average. The interest now lies in those values of Z and φ that minimize the “distance” between the year t transition probabilities and the idiosyncratic

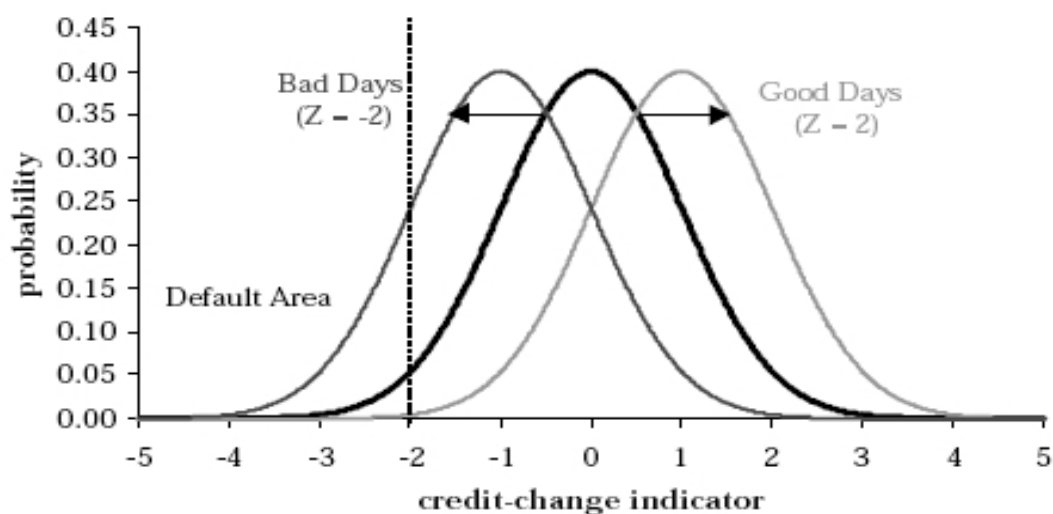


Figure 4: Impact of Z on rating transitions
Source: Kim (1999), Chart 3.

component. As a consequence, the parameter Z determines the systematic influence on migrations in year t and can be interpreted as a credit cycle. The parameter φ can be used to determine the impact of Z on X . Hence, it can be interpreted as a parameter of correlation between the transition probabilities X and the systematic components Z .

The parameters Z and φ will be estimated using the methodology of Belkin et al. (2003). That is, the minimization problem in equation (26) is applied to the entire transition matrix for every year t . One then obtains one parameter estimate Z for each year t . Loosely speaking, a positive Z can be interpreted as a “good year” in terms of credit risk conditions. More intuitively than analytically correct, one can think of a positive Z -score as reflecting a higher ratio of upgrades to downgrades, while for a negative Z -score the opposite is true. The distribution effect of rating transitions conditional on Z is illustrated in Figure 4. During bad days, a negative Z is obtained and the average transition probability is moved to the left. This means, that on average the transition probabilities more likely rate downgrades than upgrades. For good days, this works the other way round.

In a next step, following the methodology of Wei (2000), multiple estimates of Z are provided for every year. This method estimates the Z -score for every single rating category instead of averaging over the categories. As a result, the scores are more differentiated and trends within a state can be detected. Since seven rating categories are considered, the same number of Z -scores is obtained. From the obtained seven Z -scores, Wei takes simply the

average of Z to get one single Z for every year. Recalling Table 2, one can observe that the number of rated credits is distributed rather unequal among the rating categories. Therefore, in contrast to Wei, I use weights to calculate the average Z -score in every year. For this I multiply the Z -scores for every row/rating with the relative number of transitions (relative frequencies). In the year 2004 for example, the rating category *AAA* shows 98 initially rated credits. But there are only 6 movements away from the initial rating. That is just 6/98 or 0.0612 of all *AAA* rated credits were subject to rating migrations. Applying this process for every rating category in every year t , I end up with seven numbers of relative frequencies for every year t shown in Table 8.

6/98	= 0.061
17/410	= 0.042
53/1167	= 0.045
63/1394	= 0.045
102/899	= 0.113
90/742	= 0.121
42/117	= 0.359
<i>Sum</i>	= 0.787

Table 8: Relative frequencies; calculated from S&P 2004 data.

Finally, to ensure that the sum of all weights equals one for every year t , a normalization is necessary. The impact of rating *AAA* for the Z -score in 2004 is thus $0.061/0.787 = 0.078$ or 7.8%. Hence, a relatively high number of migrations contribute more weight to the average Z -score and overall risk contained in the portfolio sample is reflected more precisely.

6.2 Some Comments on the Data

For a statistical analysis of credit cycles, I use the data presented in the S& P's "Annual Global Corporate Default Study 2004" (2005). The report offers transition matrices for the years 1981 to 2004, based on the Static Pool Methodology.

In this method pools of credits are constructed for the first day of each year and are observed from that point forward. If not withdrawn, called, or defaulted, the membership of issues in a pool remains static. The ratings are compared on the first and last day of each year in order to construct the

transition matrix for that pool. Every year, a new static pool is formed taking together the new issuers and the active issuers from the previous pool.

The data used for estimation consists of seven non-defaulting categories ranging from *AAA* representing the best grade to *CCC/C*. The three subcategories *CCC*, *CC* and *C* are pooled to one single category *CCC/C*. In general, bonds falling in the rating categories between *BB* and *CCC/C* are referred to as “junk bonds”, whereas bonds rated *BBB* or better are usually referred to as “investment grade” bonds. Finer subcategories distinguishing between *AA+*, *AA* and *AA-* would possibly allow to reflect risk in more detail. In practice, those subcategories exist for every given rating category from *AA* to *CCC*. Thus, it would in principle be possible to use 21 non-defaulting categories for estimation. However, there exists the problem of low sample size in most categories. Low sample size lowers the value of statistical inference. Mapping ratings to eight categories as done by S&P thus seems to be reasonable in this framework.

Besides these eight categories, the S&P report presents one further category including all “Not Rated” (N.R.) bonds. Most credits that move to N.R. simply matured or performed at a value below which no rating is announced. They may also occur as a result of mergers and acquisitions. Others are withdrawn because of a lack of cooperation. This could be the case when a company is experiencing financial difficulties and refuses to provide all the information needed to continue surveillance on the ratings indicating a possible near term credit default. However, as Carty (1997) points out, only 1% of all transitions to “not rated” may have been due to deteriorating credit quality. I omit a further analysis of the “Not Rated” category. The process of estimation is therefore based on the observations of all categories but “Not Rated”.

Unlike Wei, I will not smooth the transition probabilities in order to reach a monotonically decreasing migration structure. Investigating rating categories *BBB* to *D*, one sees that defaults are more likely than downgrading. A smoothing of transition probabilities would therefore lead to an underestimation of default risk.

However, in order to get better numerical solutions, I assign a value of $1 * 10^{-4}$ to every entry in the transition matrix where no migration has occurred. By doing so, one gets bins distinct from ∞ and $-\infty$. Exhibit 2 in Belkin et al. (1998) suggests a similar manipulation. Note that this does not affect the qualitative outcome of transition probabilities. It merely generates numerical transition threshold values (bins) that improve the estimation results of the minimization algorithm. As S&P comments, their study ana-

lyzed the rating histories of 11,150 companies that were rated by Standard & Poor's on December 31st, 1980, or that were first rated between that date and December 31st, 2004. These companies include industry, utilities, financial institutions, and insurance companies around the world with long-term local currency ratings. The analysis excludes public information ratings and ratings based on the guaranty of another company. Structured finance vehicles, public-sector issuers, and sovereign issuers are also excluded from this study.

6.3 Estimating the Z -score

In a first step, I calculate the average transition matrix for the years 1981 – 2004. Analogous to the one-period transition matrix presented in the framework of the cohort approach one gets:

$$P(M) = \hat{p}_{jk}^+ \equiv \frac{c_{jk}^+}{n_j^+}, \quad (27)$$

where c_{jk}^+ is the number of migration counts from initial rating j to k aggregated over all periods m and n_j^+ is the total number of observations from initial rating j aggregated over all periods. Then, the bins/threshold values are calculated and multiplied by Z and φ according to equation (23). Furthermore, I compute the historical transition probabilities for every year t .

Evaluation of the minimum in equation (26) is done like in Belkin et al. (1998). I assume the parameter φ to be known and fixed. By minimizing equation (26), one then gets a time series of Z -scores for every year t conditional on the parameter φ . I compute the variance of this series and look for that particular φ that yields a standardized variance of one, like in Belkin et al. For computation, I use the Nelder Mead Simplex Algorithm implemented in Xplore. Global optima are reached in fact and results are stable (see Appendix A.6).

For the period 1984–2004, φ was calculated to be 0.3. The corresponding series of Z -scores is shown in Figure 5. Computation of the Z -scores uses the Quantlet ZminUni.xpl (see Appendix A.5). Due to the fact that on average Z should be zero and without trend, the initial estimate was chosen to be zero. Furthermore, trying different initial estimates for Z shows that the algorithm is not sensitive to different starting values and converges (on average) after about ten iterations.

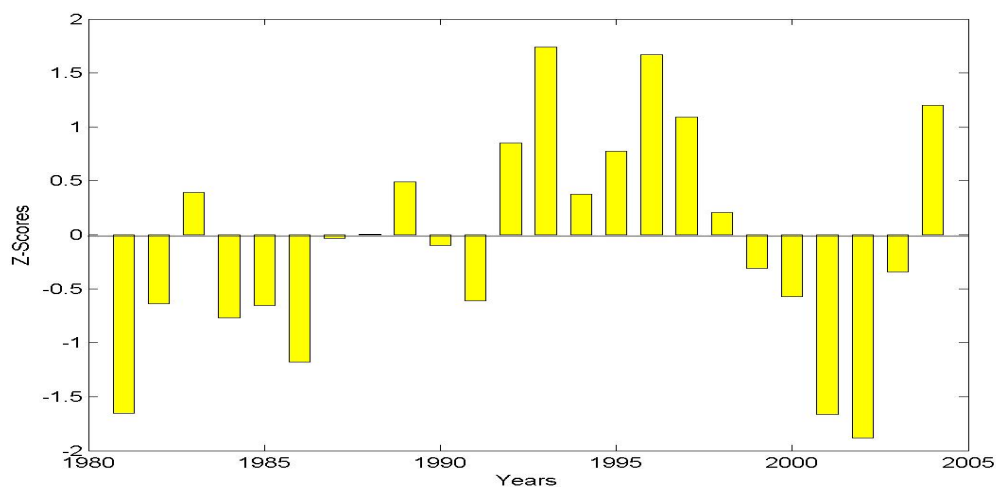


Figure 5: Estimated Z -scores from S&P 2004 data.

Note, for the calculation of the variance I start with a small value of φ and increase it successively until the computed variance becomes smaller or equal to one for the first time. This is justified, since the variance decreases monotonically with φ as is illustrated in Table 9.

Since the optimal Z -score shows almost no reaction to small changes in φ , the ad hoc computation of the variance has no significant impact. Exact calculations do not improve the estimation results or induce a significant change in the quality of estimations.

Figure 5 shows the complete series of Z -scores for the years 1981 to 2004. One can interpret a positive Z -score as a “good” year in terms of credit conditions while for a negative Z -score the opposite is assumed. A Z -score thus gives a signal regarding average credit conditions. For example, for the year 2004, the obtained Z -score is 1.2031 and $\varphi = 0.03$. Compared to the average credit conditions in years 1981-2004, the year 2004 can thus be regarded as a good year in terms of credit conditions. The average risk associated to rating downgrades was considerable low. The variance of Z is one by definition.

The overall results for the Z -scores are similar to those of Belkin et al. (1998). But, while the sample period in Belkin et al. ends in 1997, estimations presented here show results until the year 2004. The results can be summarized as follows. During the 80’s credit conditions are weak for almost all years. However, in contrast to the nineties the trend is not that clear. A good year, that is a positive Z , can be observed for the year 1983

φ	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04	0.045	0.05
$var(\varphi)$	5.95	2.98	1.99	1.5	1.2	1	0.86	0.75	0.67	0.61

Table 9: Variance for different φ .*Quantlet : migrate4.xpl*

although Z is negative for all other years around. The years 1990 and 1991 are known as years which exhibit high downgrade risk. This is also reflected in the corresponding Z -score. The following cycle shows best credit conditions until 1998 whereafter credit conditions turn rather bad. That is, the boom of the economy and the rise in stock markets associated to the new economy is very well reflected by the Z -scores. As well is the crash in the year 2000 and the years thereafter. Declining stock markets, bankruptcies etc. lead to deteriorating economic growth. The effects to the credit market are shown by the negative Z -scores from 1998 to 2003. It was in 2004 when credit conditions turned positive for the first time since 1998.

Some remarks on the limitation of a single parameter approach must be made. As Wei (2000) points out, a single parameter only shows the overall effects. Here, this is equivalent to the average Z -score for every year. It is easy to see that different rating categories may react in different ways within a given year. While the probability to stay in rating category *AAA* may keep unchanged or even increase, lower rated obligors could suffer remarkable downgrade probabilities. While the overall Z -score is negative, not all obligors may react in the same way. Wei (2000) shows that only for business cycle peaks and troughs a clear rating drift can be observed.

I estimate a Z -score for every rating/row similar to Wei (2000). This yields seven Z -scores for every year t as presented in Table 10. For computation see the Quantlet: *ZminMult.xpl* (see Appendix A.8). Unlike Wei (2000), I weigh every Z -score with the relative frequency of transitions for that row in order to calculate a single Z for every year. I expect the average Z for any year t to be a more specific risk measure for the portfolio view than it would be under equal weighting of every row. This method was presented in detail in Section 6.1. Estimation results in Table 10 show the distinct behavior of different initial ratings within a year t . Trend behavior can be observed only for the years 1996, 1997, 2002 and 2004. For the years 1989, 1992, 1993, 1995 and 2003 behavior is equal for almost all rating categories. For the estimations presented here, a rating drift for business cycle peaks, like in 1984 and

1997 and for the troughs, like in 1982, 1991 and 2001 (as they are suggested by Figure 7) can be observed as well. However, in contrast to Wei this drift is not always clear. It can be observed, that the first and the last rating category often behave in contrast to the trend.

<i>Year</i>	<i>AAA</i>	<i>AA</i>	<i>A</i>	<i>BBB</i>	<i>BB</i>	<i>B</i>	<i>CCC/C</i>	<i>Average</i>
1981	-0.5	0.44	0.5	0.31	-2.56	1.25	2.08	-0.55
1982	-0.58	-1.52	-0.75	-0.08	-0.59	0.56	0.61	-0.12
1983	-2.52	0.66	1.5	1.06	0.06	0.55	2.58	0.60
1984	-4.32	0.98	0.69	0.99	1.15	1.09	0.34	-0.49
1985	-1.22	-1.63	-0.81	-0.67	-0.39	0.77	3.09	0.73
1986	0.14	-0.19	-1.16	-0.91	0.20	-1.73	0.66	-0.54
1987	0.04	1.12	-1.13	-0.95	0.47	0.98	1.80	0.54
1988	-1.14	-1.34	-0.45	0.66	0.31	0.56	2.09	0.64
1989	0.52	0.625	-1.38	0.06	2.11	0.91	0.34	0.59
1990	1.38	-0.81	-0.98	-0.19	-1.40	0.32	1.69	0.20
1991	-1.25	-0.02	-0.25	-0.09	-0.34	-1.13	0.23	-0.31
1992	-0.56	0.25	0.44	0.63	1.11	1.13	0.66	0.69
1993	0.21	0.75	0.97	-0.21	1.26	2.69	3.61	2.38
1994	-0.31	-0.45	0.20	0.56	1.30	0.72	0.90	0.59
1995	0.08	-0.31	0.90	0.77	0.88	0.97	1.41	0.93
1996	0.58	0.81	1.61	1.44	1.33	1.46	3.5	2.04
1997	1.25	1.03	0.38	0.58	1.23	1.04	2.14	1.42
1998	-0.27	0.98	0.02	-0.58	0.14	0.24	0.81	0.45
1999	0.66	-0.30	0.09	0.28	-0.11	-0.66	-1.08	-0.52
2000	0.33	-1.08	-0.53	0.03	-0.09	-0.59	-0.60	-0.50
2001	0.94	-0.88	-0.74	-0.74	-1.47	-2.11	-2.00	-1.55
2002	-1.55	-2.77	-1.80	-1.64	-0.59	-1.43	-1.76	-1.70
2003	-0.97	-0.78	-0.38	-0.47	-0.69	0.58	-0.31	-0.35
2004	0.63	1.75	0.89	0.42	0.47	1.21	1.98	1.37

Table 10: Multiple Z -score estimates form S&P 2004 data.

Figure 6 shows the joint plot of the single parameter Z and the average annual Z obtained from the multiple estimates with relative weights. During the eighties, there exist some disagreement between the two measures. However, only for the year 1985 the series show clear contrasting signs. From 1991 onwards, estimations are almost equal. Small differences only exist in the magnitude of the estimated Z -scores.

Note that φ explains the average correlation between the systematic variable Z and the underlying process X . It thus shows the fraction of change in rating to economic or credit cycles. The overall findings are close to those

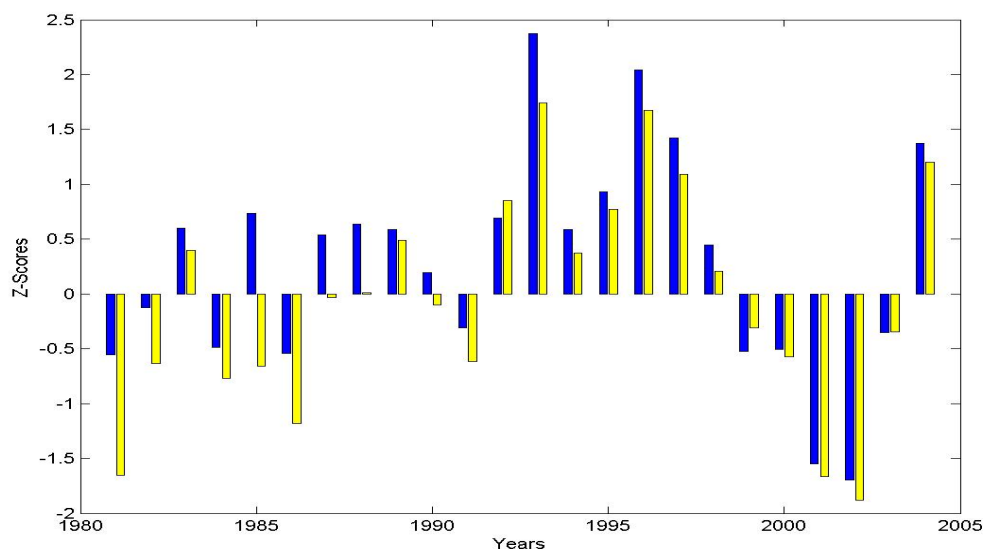


Figure 6: Estimated single and multiple Z -scores.

of Belkin et al. and Wei. They calculated a parameter of correlation of 0.016 and 0.013. That is, only 1.3% to 1.6% of all rating transitions are explained by economic cycles. Applying an equal weighting to all Z -scores, I get $\varphi = 0.021$ for the period 1981 to 2004, which is close to their findings. For the weighting method described above, φ was calculated to be 0.04 and is thus slightly higher. Although a credit correlation between 1.4% and 4% seems to be very low, Gupton et al. (1997) show that for default correlations to be in the range of 2% to 4%, asset correlation needs to be 40% to 60%. This is, as the authors point out, a measure consistent to observable asset correlation (see Gupton et al. (1997) p.91). The Basel Committee on Banking Supervision finds evidence for an average asset correlation of 20% (see Basel Committee on Banking Supervision p.36).

By separating industries or geographic regions, one could clearly construct more detailed credit cycles. Credit cycles conditioned on the industry would probably show a higher systematic correlation and a higher degree of trend behavior. Unfortunately, this is not applicable for the given data sample. In general, it will be hard to find data samples that are large enough to construct such specific cycles. This is a problem common to most approaches as was already addressed before. It is easy to see that one can use the estimated parameter φ and the Z -score for year t to fit a transition matrix. Table 11 exhibits the realized and the fitted transition matrices for the year 2004. The parameter $Z = 1.38$ stems from the specific weighting of the multiple Z ,

the calculated φ is 0.04.

	AAA	AA	A	BBB	BB	B	CCC/C	D
Observed Transition Matrix 2004								
AAA	93.9	6.1	0.0	0.0	0.0	0.0	0.0	0.0
AA	0.2	95.9	3.7	0.2	0.0	0.0	0.0	0.0
A	0.0	1.5	95.5	3.0	0.1	0.0	0.0	0.0
BBB	0.0	0.1	2.4	95.5	1.9	0.1	0.0	0.0
BB	0.1	0.0	0.1	4.6	88.7	5.9	0.2	0.4
B	0.0	0.0	0.0	0.1	7.7	87.9	2.6	1.8
CCC/C	0.0	0.0	0.9	0.0	0.9	17.9	64.1	16.2
D	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
Fitted Transition Matrix 2004								
AAA	95.5	4.3	0.2	0.0	0.0	0.0	0.0	0.0
AA	1.2	93.9	4.6	0.2	0.0	0.0	0.0	0.0
A	0.1	3.7	92.8	3.1	0.2	0.1	0.0	0.0
BBB	0.0	0.4	6.6	89.8	2.6	0.4	0.1	0.1
BB	0.1	0.2	0.7	9.1	84.1	4.8	0.5	0.5
B	0.0	0.2	0.4	0.6	9.1	83.3	3.1	3.4
CCC/C	0.2	0.0	0.7	0.8	2.5	15.8	56.8	23.2
D	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
$Z = 1.38, \varphi = 0.04$								

Table 11: Observed and fitted Migration Probabilities.

Quantlet : migrate5.xpl

6.4 Conditioning the Credit Cycle

It was shown that systematic influences on migrations are captured by the variable Z which can be interpreted as a credit cycle variable. Hence, it would be interesting to determine what influences Z . Therefore, I start with a presentation of some economic variables which may be able to determine the credit cycles. I will give some economic intuition for their influence on Z and use linear regression in order to detect significant explanatory power.

Because the S&P sample mainly consist of US-ratings, I will use US-data in the following. In a first glance, one may look for some obvious correlation

with economic variables. This is visualized in Figure 7. The series of Z is shown in red. The variables under consideration are the annual percentage changes (based on chained dollars, year 2000) of the US Gross Domestic Product (GDP) which is the black line. The spread between Moody's seasoned *Baa* Corporate Bond yield and the 3 year yield for the US treasury bill which is the blue line⁴. The same spread but for Moody's seasoned *Aaa* Corporate Bond yield, labelled green⁵. Similar to these series, the annual percentage changes for the DOW-Jones industrial index (annual changes from calculated 12 month average) seem to be correlated with Z as well. Here, the expected positive correlation clearly holds for the peak in stock markets during the nineties, as well as for the period thereafter when stock markets crashed. However, because changes were as large as 35% the series is not plotted in order to get a nice scaling.

Furthermore, I tested the Federal Reserve fund rate, US-consumer prices, US-producer prices and Moody's Seasoned *Aaa* and *Baa* Corporate Bond Yields which, however, does not show any meaningful correlation. For any of these series, the calculated correlation is less than 0.2. Furthermore, the graphical presentation shows no joint pattern neither. Only for the plotted series and the DOW-Jones index a parameter of correlation greater than 0.2 could be found. While the correlation between Z and GDP is 0.27 and between Z and the *Baa* spread -0.3 , a correlation of 0.46 can be found between Z and the DOW-Jones industrial index. Although the graphical presentation suggests the *Aaa* spread to be correlated with Z , computation yields a coefficient of correlation of only -0.09 . This delivers a first hint that explaining Z by real variables might in fact be difficult.

The idea to forecast Z conditional on macro economic variables was first introduced by Kim (1999). Using a one parameter approach as presented by Belkin et al. (1998), Kim shows that it is possible to determine the influence of different economic and financial variables on Z ⁶. Much more experience exists in forecasting these variables and thus one can use these forecasts in order to construct future transition matrices. The limitations of this approach are obvious, as the non-monotonous behavior within most rating categories was shown above. Nevertheless, one would still like to know how credit cycles are affected by economic variables. Analyzing the correlation structure between the different Z 's shown in Table 10 shows that a further analysis could

⁴Moody's *Baa* equals *BBB* in the notation of S&P.

⁵Moody's *Aaa* equals *AAA* in the notation of S&P.

⁶He showed the spread between *Aaa* and *Baa* rated bonds, the yield of the 10-year treasury bond, the quarterly CPI-inflation and the quarterly growth on GDP to be significant.

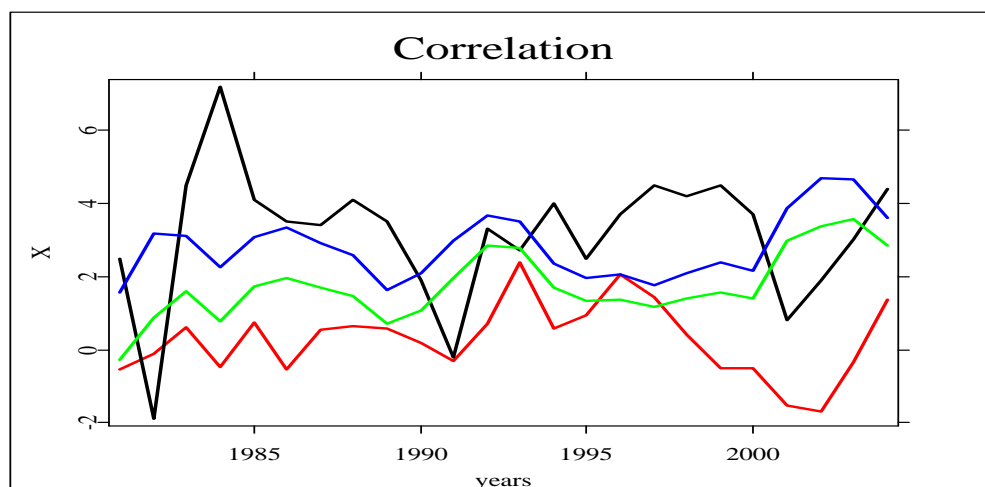


Figure 7: Correlation of Z with economic variables.

Z red, GDP black, Aaa spread green and Baa spread blue.

Quantlet : migrate6.xpl

still have explanatory power. Table 12 displays the calculated correlation for the Z -scores presented above. Correlation seems to be significant between almost all rating classes and of the same magnitude independent of the distance between rating categories. Only for the AAA rating, the assumption of independence could be fulfilled. Here, correlation is close to zero and thus far less than for the other states.

It should be intuitive to investigate the influence of economic variables on transition probabilities. Hence, it remains to determine which variables may have explanatory power. Obviously, besides the historical correlation a theoretic justification for the systematic influence on Z should exist. Let's consider the case of GDP. For high positive GDP growth rates, general operating conditions for firms should be good on average. Thus one would expect earnings to increase, making it easier to serve debt obligations. A positive connection to Z could clearly be assumed. Unfortunately, the graphical presentation in Figure 7 shows similar movements only since the beginning of the nineties. This is clearly not enough for a reliable autoregressive analysis of annual data.

In addition, the spread between Moody's seasoned Baa Corporate Bond yield and the 3 year yield of the treasury bill seems to be positively related to the Z -score until 1996. Thereafter, the relationship seems to be reversed. Theoretically, an inverse relationship would be much more intuitive. While the Bond price reflects the financing costs to the firm, the Bond spread (the

	AAA	AA	A	BBB	BB	B	CCC/C
AAA	1	0.09	-0.14	-0.14	-0.02	-0.05	0.04
AA	0.09	1	0.60	0.46	0.44	0.51	0.44
A	-0.14	0.60	1	0.80	0.29	0.57	0.53
BBB	-0.14	0.46	0.80	1	0.41	0.52	0.43
BB	-0.02	0.44	0.29	0.41	1	0.43	0.27
B	-0.05	0.51	0.57	0.52	0.43	1	0.76
CCC/C	0.038	0.44	0.53	0.43	0.27	0.76	1

Table 12: Correlation between Z .

difference in yield) is thought to exhibit more precise information about this rating class in general ⁷. Thus, a spread widening should therefore reflect increasing systematic downgrade or default risk. This leaves spreads to be of major influence in credit cycles. Note that looking on average *Baa* Bond yields does not show idiosyncratic information but average credit conditions for *Baa* bond yields.

Finally, consider the growth rates of the DOW-Jones industrial index. By a similar argumentation as for GDP, a positive relationship is expected. Normally one assumes the index to reflect expected future operating conditions for the firms contained. Thus, an increasing index shows that future earnings are expected to rise. Note that the correlation between GDP and the DOW-Jones industrial series here was calculated to be 0.33. However, as is often stated in literature, I assume both series to be uncorrelated.

Estimation Results

It was shown that the credit cycle variable Z may in fact be determined by economic variables. The correlation between the different Z -scores showed that, despite the limitations of a single parameter approach, further analysis may be able to provide more interesting information. Therefore, I will run a linear regression to analyze the assertions made above. The model is:

$$\begin{aligned}
 Z_t = & \beta_1 X_{1,t} + \beta_2 X_{1,t-1} + \dots + \beta_p X_{2,t} + \beta_{p+1} X_{2,t-1} + \dots \\
 & + \beta_n Z_{t-1} + \beta_{n+1} Z_{t-2} + \dots + \varepsilon_t,
 \end{aligned} \tag{28}$$

⁷The Yield is a inverse function of the bond price. Hence a lower Bond price is associated to a higher yield.

where ε_t is assumed to be independent and identical distributed. The parameters X contains explanatory variables and the different β must be estimated.

Here, $Z_{t-1}\beta_n$ is the lagged value of the depend variable. Hence, the optimal lag structure for every series must be determined.⁸ I do so by minimizing the appropriate information criteria (IC) such as the Akaike Information Criterium (AIC), the Schwarz Information Criterium (SIC) and the Hannan-Quinn (HQ) criterium.⁹ When running the linear regression I start for every series that is included with the optimal lag structure determined by the Information criteria. The dependent variable is included in order to account for possible autoregressive behavior. Consider the case of GDP for example. For the annual changes the optimal lag structure suggested by the IC is three. Thus, the GDP enters the regression on Z with three lags. Every lag that results insignificant, will be excluded from the regression. In order to determine the regression in equation (28), I apply this methodology to all series under consideration.

Dependent Variable: ZBAR
Method: Least Squares
Sample(adjusted): 1982 2004
Included observations: 23 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.359956	0.190310	1.891422	0.0724
DSPREADBAA	-0.595287	0.250804	-2.373514	0.0272
R-squared	0.211521	Mean dependent var		0.307437
Adjusted R-squared	0.173975	S.D. dependent var		0.997407
S.E. of regression	0.906503	Akaike info criterion		2.724496
Sum squared resid	17.25670	Schwarz criterion		2.823235
Log likelihood	-29.33171	F-statistic		5.633571
Durbin-Watson stat	1.482896	Prob(F-statistic)		0.027239

Figure 8: Estimated OLS regression for average Z ; Eviews output.

The dependent variable regarded first is the average weighted Z computed in section 6.3. Following the aforementioned methodology, I end up with the estimation shown in Table 8. The low R^2 ($= 0.17$) confirms what was already suggested before. The Z -scores are the averages over all rating

⁸Stationarity of all series is tested by Augmented Dicky Fuller Test. Differences are taken if a unit root was detected.

⁹The IC results are better with an intercept included in the regression, although the constant does not result to be significant.

states and thus not specific enough to yield good estimates. The only variable which is not excluded from the regression is the first difference of the spread between Moody's *Baa* Corporate Bond yield and the yield of the three year treasury bill.

I try to improve the estimation results by choosing the Z corresponding to rating category *BBB*, Z_{BBB} , as dependent variable. This is motivated by the fact that, as was shown in Figure 1, ratings of lower rated entities seem to be less stable over time and to react more direct to economic conditions. The Z_{BBB} is chosen because data about the corresponding average corporate bond yield is available. This yield (the yield spread) is assumed to carry information on the behavior of Z_{BBB} . Thus I expect estimation results to improve. Again, in order to account for the autoregressive pattern of Z_{BBB} , the optimal lag structure of is determined by the IC.

Dependent Variable: ZBBB
Method: Least Squares
Sample(adjusted): 1983 2004
Included observations: 22 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.336414	0.339872	0.989825	0.3354
DSPREADBAA(-1)	-0.877989	0.195325	-4.495022	0.0003
GDPCHG	0.301899	0.071371	4.229982	0.0005
GDPCHG(-1)	-0.382775	0.079900	-4.790663	0.0001
R-squared	0.673122	Mean dependent var		0.046591
Adjusted R-squared	0.618642	S.D. dependent var		0.772174
S.E. of regression	0.476850	Akaike info criterion		1.519734
Sum squared resid	4.092939	Schwarz criterion		1.718106
Log likelihood	-12.71708	F-statistic		12.35548
Durbin-Watson stat	2.137448	Prob(F-statistic)		0.000126

Figure 9: Estimated OLS regression for Z_{BBB} ; Eviews output.

The estimation results are shown in Figure 9. Estimations for GDP changes and lagged changes show significant, trend reverting coefficient estimates. For the spread between *Baa* and the 3 year treasury bill, the parameter for the lagged value of the first difference result significant. Thus, including the lagged difference for the spread, the changes in GDP and lagged changes in GDP seem to deliver a good model. With an estimated R^2 of 0.62 the model improves significantly compared to the one regarded before.

Both estimations show the negative impact of changes in the Corporate Bond spread on Z . The role of Corporate Bond spreads as a main influencer on credit cycles can therefore be confirmed. Unfortunately, the GDP enters the regression with trend reverting coefficients. Forecasts on changes in GDP, as they may be obtainable from macroeconomic research, will therefore not allow to construct reliable forecast on Z .

Again, one of the major shortcomings of this analysis is the use of annual data. Using quarterly Z -score estimates (and thus more observations) would undoubtedly improve the accuracy of estimation. Because the estimation of Z needs many observations for statistically reliable outcomes, the problem arises again in data. Hence, together with the results of autoregressive analysis it is difficult to reliably account for systematic behavior.

7 Conclusion

This thesis deals with the assessment of the risk that is associated to corporate ratings. These ratings are assumed to reflect the true risk linked to the rated entity. Hence, this thesis does not evaluate the work of the rating agencies.

I show the importance of risk associated to ratings for investors and obligors and give an overview on the field of research in credit risk and its development. The analysis focuses on the probability of a migration from a given rating state to another in order to measure the risk. I present the cohort method and show that using relative frequencies of rating changes does not explain the existence of some correlation or dependency structure between rating transitions. A formal method to test the first-order Markov property shows that this property is violated for the transition matrices that I constructed from the S&P data sample. To account for these shortcomings, I present the Aalen-Johansen estimator. Unfortunately, there is no data set available to me in order to apply the Aalen-Johansen estimator. However, because it can be shown that the estimator allows for time non-homogeneity and rating migrations within a period (in contrast to the cohort method), I nevertheless present a short introduction to the underlying methodology.

As another approach to account for this correlation, I present the Merton model. The Merton model suggests that changes in the asset value drive the underlying rating process. For calculation, one can partition the matrix of transition probabilities into a matrix of disjoint bins. Thus, one obtains threshold values for a rating migration corresponding to a certain asset value. For the construction of credit cycles, I follow the presentation of Belkin et al. The results are quite close to the ones obtained in the literature, but feature a longer time horizon. The main idea for the construction of credit cycles is to partition a change in a rating into an idiosyncratic and a systematic component. The latter component is then assumed to reflect the credit cycle and accordingly the cyclical behaviour of risk attributed to the rated entities. I show that although a clear rating drift seems to exist, certain rating categories behave distinct to this trend. Systematic risk seems to be allocated unequal among the different rating categories. Only for a few years the drift and thus the risk is unambiguous.

I try to shed more light on what drives the credit cycles by investigating the impact of different economic variables via autoregressive estimation. I show that changes in GDP and the spread between Corporate Bond yields and the treasury yield do have a significant impact on the credit cycles. Furthermore, I tested different economic variables such as US Consumer Prices,

US Producer Prices, the Federal Reserve Fund Rate and the DOW-Jones industrial index. But none of these series show significant impact on the constructed credit cycles.

A major shortcoming here is clearly the use of annual data. One would expect estimation results to improve for shorter frequencies as for quarterly data. Because the estimation of Z needs many observations for statistically reliable outcomes, the problem arises again in data.

I have to point out that rating migrations are mainly driven by idiosyncratic, i.e. firm-specific components and thus it will be hard to obtain reliable forecasts on the transition matrices based on the systematic component only. Nevertheless, it is an interesting exercise to determine the economic influence on rating migrations as displaying aggregate risk.

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A Appendix

A.1 Time stability

```

proc()=migrate1()
;-----
; Description   shows annual rating stability over time
;               for 7 non defaulting rating categories
;-----
; Input
;   Parameter   s
;   Description start of sample period
;   Parameter   e
;   Description end of sample period
; Output
;   Graphical object showing stability over time
;-----
;--reading data-----
s=1981           ;sample starts in 1981
e=2004           ;sample ends in 2004
d=7              ;number of rating categories
p=e-s+1
mig=matrix(d,d+1,p)
fmt=".\\data\\mig%.0f.dat"   ;data source
y=s:e
str = string(fmt, y)
i=0
do
  i=i+1
  mig[:,i]=read(str[i])
until (i==p)
;--start computation-----
etp=matrix(d,d+1,p)
diagments=matrix(d)
j=0
do
  j=j+1
  counts=mig[:,j]
  nstart=sum(counts,2)      ;portfolio weights before migration

```

```

    etp[, ,j]=counts./nstart      ;derives estimated transition prob.
    diagment=xdia(etp[,1:d])
    until (j==p)

prob=matrix(d,p)
n=0
z=0
do
  n=n+1
  do
    z=z+1
    prob[z,n]=diagment[z,1,n]
  until (z==d)
z=0
until (n==p)

a=prob[1]'      ; AAA
b=prob[2]'      ; AA
c=prob[3]'      ; A
d=prob[4]'      ; BBB
e=prob[5]'      ; BB
f=prob[6]'      ; B
g=prob[7]'      ; CCC/C

Display1=createdisplay(1,1)
  set1=setmask(y~a, "line", "cyan")
  set2=setmask(y~b, "line", "blue")
  set3=setmask(y~c, "line", "green")
  set4=setmask(y~d, "line", "yellow")
  set5=setmask(y~e, "line", "magenta")
  set6=setmask(y~f, "line", "red")
  set7=setmask(y~g, "line", "black")

show(Display1,1,1,set1,set2,set3,set4,set5,set6,set7)
setgopt(Display1,1,1,"title","Stability over time","xlabel","year",
        "ylabel","P(x)","dispsize",500|400,"disppos",500|200)

endp

```

```

library("xplora")
library("VaR")
library("graphic")
library("plot")

```

```

  migrate1()

```

A.2 Future transition matrices

```

proc(etpt)= migrate2(t,counts)
;-----
; Description  uses spectral decomposition to calculate
;              future transition matrices for t periods
;-----
; Input
; Parameter   t
; Description number of periods to compute
; Parameter   counts
; Description matrix containing the data
; Output
; d x d matrix of est. transition prob. in period t
;-----
;--deriving prob. for initial period-----
  nstart=sum(counts,2) ;portfolio weights before migration
  etp=counts./nstart  ;derives estimated transition prob.
;-- arranging dimension -----
  d=cols(etp)
  null=matrix(1,d)-1
  null[,d]=1
  etp=etp|null
;-- start decomposition -----
  y= eiggn(etp)      ;computes eigenvalues/vectors of etp
  e=y.values.re
  e=diag(e)
  z=y.vectors.re

  etpt=z*(e^t)*inv(z) ;computes trans. prob. for period t

```

```

    etpt=round(etpt,4)

endp

library("xplore")
library("math")
library("VaR")

counts=read("./data/average.dat") ;choose dataset
t=5      ;choose # of periods

migrate2(t,counts)

```

A.3 Time Horizon

```

proc(etp,esd)= migrate3(t,y,d)
;-----
; Description  computes transition matrices over a
;              t period horizon
;-----
; Input
; Parameter    t
; Description  # number of years over which to calculate
; Parameter    y
; Description  year when calculation starts
; Parameter    d
; Description  # of rating categories incl. default
; Output
; d-1 x d matrix of transition prob. for choosen time horizon
; d-1 x d matrix of estimated standard deviations
;-----
counts=matrix(d-1,d,t)
fmt="./data/mig%.0f.dat"    ; data source

x=y+(t-1)
j=y:x
str = string(fmt, j)

```

```

i=0
do
  i=i+1
  counts[,i]=read(str[i])
until (i==t)

  sumc=sum(counts,3)      ;counts data over all periods
  nstart=sum(sumc,2)      ;portfolio weights before migration
  nend=sum(sumc,1)'       ;portfolio weights after migration
  etp=sumc./nstart        ;derives estimated transition prob.
  esd=sqrt((etp.*(1-etp))./nstart)
  etp=round(etp,3)
  esd=round(esd,3)

endp

library("VaR")

t=2      ; choose time horizon t
y=2003   ; choose first year
d=8      ; # of rating categories incl. default

out=migrate3(t,y,d)
out

```

A.4 Smoothing transition probabilities

```

proc(etp,etv,not,relnot)= migrateS(counts)
;-----
; Description  manipulates quantitatively the transition
;              matrix in order to get numeric threshold values
;-----
; Input:
;   counts    transition data for year t
; Output:
;   etp       estimated transition probabilities

```

```

;      etv   estimated threshold values
;      not   number of transitions
;      relnot relative number of transitions
;-----
;-- computes dimensions (begin) -----
di=dim(counts)
d=matrix(3,1)
d[1:2]=di[1:2]
if (dim(di)==2)
  d[3]=1
else
  d[3]=di[3]
endif
;-- start (manipulation) -----
k=0
i=0
j=0
do
  k=k+1
  do
    i=i+1
    do
      j=j+1
      if (counts[i,j,k]==0)
        counts[i,j,k]=1e-4
      endif
    until(j==d[2])
    j=0
  until (i==d[1])
  i=0
until (k==d[3])
;-- end (manipulation) -----
nstart=sum(counts,2)      ;weights before migration
nend=sum(counts,1)'      ;weights after migration
etp=counts./nstart       ;estimated transition prob.
diagelements=xdiag(counts[,1:d[2]-1])
not=nstart-diagelements  ;number of transitions
div=not/nstart
sumdiv=sum(div)

```

```

relnot=div/sumdiv      ;relative number of trans.
;-- derive threshold values -----
cdfwert=cumsum(etp,2)
null=matrix(d[1],1,d[3])-1
eins=matrix(d[1],1,d[3])
neu=null~cdfwert[,1:d[2]-1]~eins
etv=qfn(neu)          ;derives threshold values
etv=-etv
;-- end -----
not=round(not,0)
etv=round(etv,5)

endp

```

```
library("VaR")
```

```

counts=read(".\data\mig2004.dat")
out=migrateS(counts)
out

```

A.5 Z-scores matrix wise

```

proc(minz)=f(z)
;-----
; Description   minimizes Z matrix wise
;-----
etvave=getglobal("etvave")
phi= getglobal("phi")
etp=getglobal("etp")
not=getglobal("not")
;--(begin)-----
delta1=cdfn((etvave - sqrt(phi)*z[1,1].*
              matrix(rows(etvave),cols(etvave)))/sqrt(1-phi))
delta2=diff(-delta1')
delta=delta2'
;---(min)-----
not2=not.*matrix(7,8)

```

```

    m1=(not2.*(etp-delta)^2)./(delta.*(1-delta))
    m2=sum(m1,2)
    minz=sum(m2)

endp

proc()= opt(phi)
;--(read data)-----
counts=read(".\data\average.dat")
p=migrateS(counts)
etvave=p.etv
putglobal ("etvave")

mig=matrix(7,8,24)
fmt=". \data\mig%.0f.dat"
i=1981:2004
str = string(fmt, i)

i=0
do
    i=i+1
    mig[:,i]=read(str[i])
until (i==24)

out=migrateS(mig)
etp1=out.etp
not1=out.not
zmin=matrix(24,3)
;--(call algorithm)-----
t=0
do
    t=t+1
    etp=etp1[:,t]
    putglobal ("etp")
    not=not1[:,t]
    putglobal ("not")
    z=nelmin(0, "f", 100, 1.0e-9)
    zmin[t,1]=z.minimum
    zmin[t,2]=z.iter

```

```

        zmin[t,3]=z.converged
    until (t==24)
    varz=var(zmin[,1])
    varz=round(varz,2)
    meanz=mean(zmin[,1])
    putglobal ("varz")
    putglobal ("zmin")
    putglobal ("meanz")
endp

;--(compute phi)-----
proc(zmin,phi,varz,meanz)=minphi()

phi=0.026      ;initial phi
do
    phi=phi+0.001
    putglobal ("phi")
    opt(phi)
    varz= getglobal ("varz")
    until(varz<=1)
    zmin= getglobal ("zmin")
    zmin=round(zmin,5)
    meanz=getglobal("meanz")

endp

library("nummath")
library("VaR")
func(".\migrateS")

minphi()

```

A.6 Proof for Global Minimum

```

proc(minz)=f(z)
;-----
; Description  shows that Nelder Mead algorithm finds

```

```

;           global minimum
;-----
;  Input
;    phi   initial parameter of systematic influence, e.g. 0.03
;    year  from data set      e.g.  \mig1984.dat
;-----
;    first interval (-10;10)
;    second interval (-2;2)
;-----
counts=read( ".\data\average.dat" )
p=migrateS(counts)
etvavg=p.etv

phi=getglobal("phi")
etp=getglobal("etp")
not=getglobal("not")
z=getglobal("z")

;--(begin)-----
delta1=cdfn((etvavg - (sqrt(phi)*z)*
            matrix(rows(etvavg),cols(etvavg)))/sqrt(1-phi))
delta2=diff(-delta1')
delta=delta2'
;-----
not2=not.*matrix(7,8)

m1=(not2.*(etp-delta)^2)./(delta.*(1-delta))
m2=sum(m1,2)
minz=sum(m2)

endp

proc(g)= globalopt()

g=matrix(81,2)
z= -10.25
i=0
do

```

```
        i=i+1
        z = z + (0.25)
        putglobal ("z")
        g[i,1]=z
        g[i,2]=f()

    until(z==10 )
endp

proc(g)= localopt()
    g=matrix(41,2)
    z= -2.1
    i=0
    do
        i=i+1
        z = z + (0.1)
        putglobal ("z")
        g[i,1]=z
        g[i,2]=f()
    until(i==41 )
endp

library("nummath")
library("VaR")
\func(".\migrateS")

phi= 0.03 ;choose phi
putglobal("phi")
counts=read(".\data\mig1984.dat") ;choose year
x=migrateS(counts)
etp=x.etp
not=x.not
putglobal ("etp")
putglobal ("not")

globalopt()
localopt()
```

For Year 1984 and $\varphi = 0.03$ the result clearly shows a global minimum in the neighborhood of -0.8 . The Nelder Mead algorithm calculates -0.77 , after 11 iterations.

A.7 Monotonically decreasing variance

```

proc()=migrate4()
;-----
; Description   shows that variance of Z
;               decreases with phi
;-----
plot=matrix(10,2)
u=0
phi=0.0
do
  phi=phi+0.005
  u=u+1
  putglobal ("phi")
  opt(phi)           ;from ZminUni/ZminMult
  varz= getglobal ("varz")
  plot[u,1]=phi
  plot[u,2]=varz
plot
  until(u==10)

endp

library("nummath")
library("VaR")
func(".\ZminUni")

migrate4()

```

A.8 Z-scores row wise

```

proc(zmin)=f(z)
;-----

```

```

; Description   row wise minimization of z
;-----
etvavg= getglobal("etvavg")      ; avg. estimated transition prob.
phi= getglobal("phi")
etp= getglobal("etp")           ; observed transition prob.
;--(begin)-----
delta1=cdfn((etvavg - sqrt(phi)*z[1,1].*
              matrix(rows(etvavg),cols(etvavg)))/sqrt(1-phi))
delta2=diff(-delta1')
delta=delta2'
;---(min)-----
m1=((etp-delta)^2)./(delta.*(1-delta))
zmin=sum(m1,2)
;---(end)-----
endp

```

```

proc()= opt(phi)

counts=read(".\data\average.dat")
p=migrateS(counts)
etvavg1=p.etv

mig=matrix(7,8,24)
fmt=". \data\mig%.0f.dat"
y=1981:2004
str = string(fmt, y)

i=0
do
  i=i+1
  mig[:,i]=read(str[i])
until (i==24)

out=migrateS(mig)
etp1=out.etp
not1=out.not
relnot=out.relnot

```

```

zmin=matrix(7,3,24)
avg=matrix(7)
avgz=matrix(24)
;-----
  t=0
  r=0
  do
    t=t+1
    do
      r=r+1
      etvavg=etvavg1[r,]
      putglobal ("etvavg")
      etp=etp1[r,,t]
      putglobal ("etp")
      z=nelmin(0, "f", 100, 1.0e-12) ;calls algorithm
      zmin[r,1,t]=z.minimum
      zmin[r,2,t]=z.iter
      zmin[r,3,t]=z.converged
      notsum=sum(not1[,1,t],1)
      avg[r]=relnot[r,1,t].*zmin[r,1,t] ;weighting
      until(r==7)
      r=0
      avgz[t]=sum(avg)
    until (t==24)
  varz=var(avgz)
  meanz=mean(avgz)
  putglobal("varz")
  putglobal("zmin")
  putglobal("avgz")
  putglobal("meanz")
;-----
endp

proc(zmin,avgz,phi,varz,meanz)=minphi()

phi=0.038 ;innitial phi
do
  phi=phi+0.001

```

```

    putglobal ("phi")
    opt(phi)
    varz= getglobal ("varz")
    until(varz<=1)
zmin= getglobal ("zmin")
zmin=round(zmin,5)
getglobal("avgz")
getglobal("meanz")
endp

library("nummath")
library("VaR")
\func(".\migrateS")

minphi()

```

A.9 Fitted transition matrix

```

proc()=migrate5(phi)
;-----
; Description   calculates fitted transition matrix
;-----
; Input
;   r   realized trans.
;   c   average transition matrix
;   phi weight of systematic comp.
;   Z   state of the cycle
; Output
;   realized transition matrix
;   fitted transition matrix
;-----
z=getglobal("z")
c=read(".\data\average.dat")
p=migrateS(c)
etvavg=p.etv

delta1=cdfn((etvavg - sqrt(phi))*z[1,1].*

```

```

        matrix(rows(etvavg),cols(etvavg))/sqrt(1-phi))
    adelta=delta1[,2:9]
    bdelta=delta1[,1:8]
    fit=bdelta-adelta
    fit=round(fit,3)
    fit

endp

library("xplora")
\func(".\migrateS")

r=read(".\data\mig2004.dat")    ;realized matrix
tru=migrateS(r)
truetsp=tru.etp
realized=round(truetsp,3)
realized

zmin=read(".\data\zscore.dat")
z=zmin[24]                    ;choose est. Z-score form years
                                ;1981=1 to 2004=24 or type new Z

putglobal("z")
phi=0.04                      ;choose phi

migrate5(phi)

```

A.10 Correlation of Z and economic variables

```

proc()=migrate6()
;-----
; Calculates correlation between the estimated Z-scores
; and economic variables and shows a graphical presentation
;-----
z=read(".\data\zscore.dat")
gdp=read(".\data\GDP.dat")
cpi=read(".\data\USCPI.dat")
ppi=read(".\data\PPI.dat")

```

```
DOW=read(".\data\DOW.dat")
fedfund=read(".\data\FedFund.dat")
aaayield=read(".\data\yieldsAaa.dat")
baayield=read(".\data\yieldsBaa.dat")
spreadbaa=read(".\data\spreadbaa.dat")
spreadaaa=read(".\data\spreadaaa.dat")

a=z~gdp~cpi~aaayield~fedfund~baayield~ppi~DOW~spreadbaa~spreadaaa
correl=corr(a)
correl

t=1981:2004
setsize(700,500)
Display1=createdisplay(1,1)
set1=setmask(t~z, "line", "red")
set2=setmask(t~gdp, "line", "black")
; set3=setmask(t~DOW, "line", "yellow") ;!scaleing!
set4=setmask(t~spreadbaa, "line", "blue")
set5=setmask(t~spreadaaa, "line", "green")
show(Display1,1,1,set1,set2,set4,set5)
setgopt(Display1,1,1,"title","Correlation","xlabel","years","ylabel",
        "X","dispsize",550|400,"disppos",400|200)

endp

library("xplore")
library("plot")

migrate6()
```

Erklärung zur Urheberschaft

Hiermit erkläre ich, dass ich die vorliegende Arbeit allein und nur unter Verwendung der aufgeführten Quellen und Hilfsmittel angefertigt habe.

Malte Kleindiek

Berlin, den 20. Juni 2005