Humboldt-Universität zu Berlin

Chair of Statistics

Pricing Temperature Derivatives for Munich

Diploma Thesis

For Attainment of Diploma Degree in Economics

(Diplom-Volkswirt)

Maria Osipenko

Matriculation Number 507095

September 17th, 2009

Supervisor: Prof. Dr. Wolfgang Härdle

Abstract

Different weather events play an important role for industries with profits depending on temperature or other weather conditions. A market for trading on temperature events has recently emerged. The traded financial contracts allowing to transfer weather risks are called weather derivatives. The value of those contracts depend on a certain weather event, which is in the most cases average temperature.
The organized market for weather derivatives in Germany is still small. There are only two cities on which weather the derivatives are traded: Berlin and Essen. For many important locations of weather dependent industries no weather derivatives are traded. The problem of calculating arbitrage free prices for those regions arises.
In this diploma thesis temperature based weather derivatives on CAT index are considered, currently traded only for Berlin and Essen. They are used to construct CAT future prices for Munich where there is no organized trading on temperature events. Thereby the pricing model based on continuous time autoregressive model is adopted and used to calibrate the market price of risk as well as to estimate theoretical prices for the derivatives on Munich temperature.

Keywords: weather derivatives, derivative pricing, continuous time autoregressive process, market price of risk.

JEL Classification: C22, C51, G13.

Abstract

Verschiedene Wetterereignisse spielen eine wichtige Rolle für Industrien mit wetterabhängigen Gewinnen. In jüngster Zeit hat sich ein Markt fürs Handeln mit solchen Wetterereignissen entwickelt. Die Verträge, die gehandelt werden, sind die sogenannten Wetterderivate, die einen Transfer des Wetterrisikos ermöglichen. Ihr Wert hängt von einem Wetterereignis ab, was in der Regel die durchschnittliche Temperatur ist.
Der organisierte Markt für Wetterderivate in Deutschland ist relativ klein. Die Wertpapiere werden nur für zwei Städte gehandelt: Berlin und Essen. Für viele wichtige Standorte von wetterabhängigen Industrien werden keine Wetterderivate gehandelt. Darum entsteht das Problem arbitragefreie Preise für diese Regionen zu kalkulieren.
In dieser Diplomarbeit werden Wetterderivate auf CAT Index betrachtet, derzeit für Berlin und Essen gehandelt. Die werden dann dafür genutzt CAT Future Preise für München zu konstruieren. Dabei wird ein Preismodell basierend auf autoregressivem Prozess in stetiger Zeit angewandt um die Marktrisikoprämie zu kalibrieren und theoretische Preise für Wetterderivate in München zu gewinnen.

Schlagwörter: Wetterderivate, Derivatenbewertung, autoregressiver Prozess in stetiger Zeit, Marktrisikoprämie.

JEL Klassifikation: C22, C51, G13.

Contents

List of Figures

List of Tables

1 Introduction

Different weather events play an important role for industries with profits depending on temperature or other weather conditions. Examples of such industries are tourism, energy and agricultural sectors. Therefore a market for trading on temperature events has recently emerged. The traded financial contracts allowing to transfer weather risks are called weather derivatives. The value of those contracts depend on a certain weather event, which is in the most cases average temperature. Weather derivatives, available for the European cities and traded on the organized exchange – Chicago Mercantile Exchange (CME), are based on three main temperature indices:

- Cumulated Average Temperature index (CAT),
- Heating Degree Day index (HDD),
- Cooling Degree Day index (CDD).

However, the organized market for weather derivatives in Germany is still small. There are only two cities on which weather the derivatives are traded: Berlin and Essen. For many important locations of weather dependent industries no weather derivatives are traded. The problem of calculating arbitrage free prices for those regions arises.

Many authors investigated the problem of finding an appropriate pricing model for weather derivatives. See, e.g. Dornier and Querel (2000) or Alaton et al. (2002) who fitted an Ornstein-Uhlenbeck process to the temperature observations and demonstrated effect of mean, variance and market price of risk on CDD and HDD option prices. Campbell and Diebold (2005) applied an autoregressive process of higher order to model deseasonalized temperature and observed seasonality in temperature variation. Benth et al. (2007) and also Härdle and López Cabrera (2009) propose to model temperature dynamics with a deterministic seasonal component and a higher order continuous autoregressive process with seasonal variation. They also derive arbitrage free prices for future contracts on CDD and CAT indices.

In this diploma thesis temperature based weather derivatives on CAT index are considered, currently traded only for Berlin and Essen. They are used to construct CAT future prices for Munich, one of the most important economic and financial centers of Germany where there is no organized trading on temperature events. Thereby the pricing model as proposed by Benth et al. (2007) and as well by Härdle and López Cabrera (2009) is adopted and used to calibrate the market price of risk as well as to estimate theoretical prices for the derivatives on Munich temperature.

In the next section theoretical pricing model for the temperature based derivatives is considered. Section 3 continuous with applying the theoretical model to Berlin, Essen and Munich weather dynamics and deriving continuous time model coefficients for pricing. Calibration of market price of risk for Berlin and Essen as well as ways of constructing CAT derivative prices for Munich temperature are investigated in section 4. All the calculations for sections 3 and 4 were done with MATLAB version 2.7. After the empirical parts conclusions and further research suggestions are made.

2 Theoretical Model for Pricing Weather Derivatives

A reliable valuation procedure for weather derivatives plays a key role in the effectiveness of transferring risk with help of such type of derivatives. The pricing model for weather derivatives becomes more complicated by the fact that the underlying weather can not be traded and therefore weather derivatives can not be replicated by holding positions in the underlying and risk-free asset. Additionally, markets for weather derivatives are not very liquid and thus weather derivatives can't be efficiently replicated by other weather derivatives.

In this work we follow the pricing methodology introduced by Benth et al. (2007) and further developed and applied by Härdle and López Cabrera (2009). This pricing model is introduced in the following.

2.1 Temperature Dynamics in Discrete Time

In order to establish a pricing model for weather derivatives, one has to find an appropriate model for the evolution of the underlying, in this case – of the temperature.
From the observed temperature data, which is usually daily average temperature for a number of years, we construct following discrete time model for temperature dynamics as in Härdle and López Cabrera (2009).

- Let T_t be the average temperature in day t, $t = 1, \dots, M$. Assume T_t has dynamics described by

$$T_t = \Lambda_t + Y_t, \tag{1}$$

- Λ_t is set to be a deterministic seasonal function of the form

$$\Lambda_t = a + bt + \sum_{i=1}^{s} c_i \cdot \cos\left\{2\pi(t - d_i)/(i \cdot 365)\right\}, \tag{2}$$

- Y_t is a p-order autoregressive process

$$Y_t = \phi_0 + \sum_{i=1}^{p} \phi_i Y_{t-i} + \sigma_t \varepsilon_t, \tag{3}$$

- ε_t is white noise and σ_t is the variation of the average temperature on each day over all years smoothed by local linear regression (LLR). The LLR method is formalized in the equation below:

$$\hat{\sigma}_j^2 = \arg\min_{a,b} \sum_{j=1}^{365} \left\{ \tilde{\sigma}_j^2 - a(j) - b(j)(j - j_0) \right\}^2 K\left(\frac{j - j_0}{h}\right) \tag{4}$$

  with $\tilde{\sigma}_j = \frac{1}{N-1} \sum_{i=1}^{N} \hat{\varepsilon}_{i,j}^2$ temperature variation on each day over all years, $\hat{\varepsilon}_{i,j}$ residual of day j, $j = 1, \ldots, 365$, in year i, $i = 1, \ldots, N$ of the AR(p) process and $K(\cdot)$ is Gaussian kernel.

The seasonality function Λ_t is assumed to have a periodical form with time trend (2). Parameter a represents average temperature over the years, b stands for time trend and accounts for the effects of global warming. The set of parameters $c = \{c_i\}_{i=1}^{s}$ captures seasonal/periodical dynamic of temperature and the value of s is typically region/climate specific.

The residuals Y_t remaining after removing the seasonality Λ_t are to be modelled by an autoregressive process. However, before one can proceed, the resulting series $\{Y_t\}_{t=1}^{T}$ has to be tested for stationarity. For this purpose we apply two test: Augmented Dickey-Fuller (ADF) test for a unit root and KPSS test for stationarity.
ADF test is constructed as follows:

$$\Delta Y_t = \alpha Y_{t-1} + \xi_1 \Delta Y_{t-1} + \ldots + \xi_{p-1} \Delta Y_{t-p+1} + \varepsilon_t$$

where $\alpha = \phi_1 + \phi_2 + \ldots + \phi_p - 1$, $\xi_j = -(\phi_{j+1} + \phi_{j+2} + \ldots + \phi_p)$ and $\Delta Y_i = Y_i - Y_{i-1}$. The hypothesis to test is H_0: α=0 (unit root) vs. H_1: $\alpha < 0$ (stationarity) by means of regular t-test.

For the KPSS test we rewrite

$$Y_t = c + \mu t + k \sum_{i=1}^{t} \xi_i + \varepsilon_t,$$

and test H_0: μ=0, k=0 (stationarity) versus H_1: trend or level nonstationarity.

If for the residuals $\{Y_t\}_{t=1}^T$ the null hypothesis of ADF test (unit root) is rejected and the null hypothesis of KPSS test can't be rejected, we can proceed with modelling the autoregressive process AR(p), defined in equation (3). The order of the appropriate autoregressive process is chosen with help of Box-Jenkins Analysis see Box and Jenkins (1970) and Bayesian Information Criterion see Hurvich and Tsai (1989).

Since there is seasonal variation in the residuals, the process in (3) is heteroscedastic and we have to account for it in the model. Therefore we group the residuals in 365 groups, so that each group represents residuals of the same day over all years. Then we compute the variation of each day and smooth it as defined in (4).

Residuals of AR(p) process standardized by the seasonal standard deviation should be close to normal in order to apply the pricing methodology. This is usually tested by Jarque-Bera test with test statistic $JB = \frac{n}{6}\left(s^2 + \frac{(k-3)^2}{4}\right) \overset{as.}{\sim} \chi^2_2$, where s denotes skewness and k kurtosis of the tested random variable. Besides Kolmogorov-Smirnov test can be applied, it relies on the test statistic $d_n = \sup |F_n(x) - F_0(x)|$ with $F_n(x)$ and $F_0(x)$ empirical and theoretical cumulative distribution functions respectively and checks if the empirical distribution function is close enough to the theoretical one, in our case to normal cumulative distribution function. The null hypothesis of the tests assume normality. For precise explanation of both tests see e.g. Judge et al. (1988).

2.2 Continuous Time Model for Temperature Dynamics

In order to apply a pricing model for temperature dynamics we need to switch to continuous time and transfer our obtained discrete time temperature dynamics to a continuous scale. For that purpose we use the continuous time autoregressive model (CAR(p)) with seasonal variation described by Benth et al. (2007).
CAR(p) Model:

$$T_t = \Lambda_t + X_{1t} \tag{5}$$

Where X_{qt} is the q-th coordinate of the stochastic process X_t, defined by the vectorial Ornstein-Uhlenbeck differential equation:

$$dX_t = AX_t dt + \mathbf{e}_p \sigma_t dB_t \tag{6}$$

With $\mathbf{e}_k$ denoting k-th unit vector in $\mathbb{R}^p$, $\sigma_t > 0$ is a real valued and integrable function and A is $(p \times p)$-matrix, defined as:

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -\alpha_p & -\alpha_{p-1} & -\alpha_{p-2} & \dots & -\alpha_1 \end{pmatrix}$$

The explicit solution to this stochastic differential equation is:

$$X_s = \exp(A(s-t)x) + \int_t^s \exp(A(s-u))\mathbf{e}_p \sigma_u dB_u. \tag{7}$$

For details see Gillespie (1996).
Stationarity condition of such a process is negativeness of the real parts of all eigenvalues of A and has to be checked before one proceeds with pricing. To identify the CAR(p) model associated with the fitted AR(p) model we make use of finite difference approximations of CAR model and obtain ap-

proximation for the dynamics of $X_{1,t}$ as in Härdle and López Cabrera (2009).

For $p = 1$,

$$dX_{1t} = -\alpha_1 X_{1t} dt + \sigma_t dB_t.$$

For $p = 2$,

$$\begin{aligned} X_{1(t+2)} &\approx (2 - \alpha_1) X_{1(t+1)}. \\ &+ (\alpha_1 - \alpha_2 - 1) X_{1(t)} + \sigma_t (B_{t-1} - B_t) \end{aligned}$$

For the purposes of this work we only need approximations defined up to $p = 3$. For CAR(3) following approximation holds as shown by Härdle and López Cabrera (2009):

$$\begin{aligned} X_{1(t+3)} &\approx (3 - \alpha_1) X_{1(t+2)} + (2\alpha_1 - \alpha_2 - 3) X_{1(t+1)} \\ &+ (-\alpha_1 + \alpha_2 - \alpha_3 + 1) X_{1(t)} + \sigma_t (B_{t-1} - B_t), \end{aligned} \tag{8}$$

where α_1, α_2 and α_3 denote the estimated autoregressive coefficients of the AR(3) model. In further we define $\beta_1 := 3 - \alpha_1$, $\beta_2 := 2\alpha_1 - \alpha_2 - 3$ and $\beta_3 := -\alpha_1 + \alpha_2 - \alpha_3 + 1$ as parameters of the CAR(3) model.

2.3 Pricing of Temperature Futures

An important feature of the market for temperature derivatives is its incompleteness, since the underlying, in our case temperature, is not tradeable and the derivatives can not be replicated.
However, although the markets for temperature derivatives are incomplete, their prices must be arbitrage-free, since they are tradeable assets. Therefore we assume that a pricing measure $Q = Q_{\theta_t}$ exists and can be parametrized by θ_t. Under Q_{θ_t} an arbitrage-free price of temperature future can be computed:

$$F_{(t,\tau_1,\tau_2)} = \mathrm{E}^{Q_{\theta_t}} \left[Y_T (T_t | \mathcal{F}_t) \right] \tag{9}$$

with $0 \leq t \leq T$, Y_T the payoff of the temperature index at $T > t$ and θ_t time varying market price of risk (MPR).
Risk neutral probability measure Q_{θ_t} can be parametrized via Girsanov's theorem, see Härdle et al. (2004):

$$B_t^\theta = B_t - \int_0^t \theta_u du,$$

where B_t^θ is a Brownian motion and a martingale under Q_θ. The temperature dynamics (6) under Q_θ becomes:

$$d\mathbf{X}_t = (A\mathbf{X}_t + \mathbf{e}_p \sigma_t \theta_t)dt + \mathbf{e}_p \sigma_t dB_t^\theta, \tag{10}$$

with explicit solution

$$\begin{aligned} X_s = \exp(A(s-t)x) \quad &+ \int_t^s \exp(A(s-u))\mathbf{e}_p \sigma_u \theta_u du \\ &+ \int_t^s \exp(A(s-u))\mathbf{e}_p \sigma_u dB_u^\theta. \end{aligned} \tag{11}$$

With this results we get a feasible model for pricing temperature futures.

2.4 The Pricing Model for CAT Futures and Options

As mentioned before in this diploma thesis we concentrate on the CAT index, however, similar methodology with minor modifications applies also to other temperature indices as HDD and CDD.
CAT index is defined below:

$$\mathrm{CAT}_{\tau_1,\tau_2} = \int_{\tau_1}^{\tau_2} T_u du, \tag{12}$$

where T_t is the average temperature of day t and is calculated by

$$T_t = \frac{T_t^{max} - T_t^{min}}{2},$$

with T_t^{max} and T_t^{min} being maximum and minimum of registered temperature on day t respectively.
As a result from (9) we obtain:

$$F_{CAT(t,\tau_1,\tau_2)} = \mathrm{E}^{Q_\theta}\left[\int_{\tau_1}^{\tau_2} T_s ds|\mathcal{F}_t\right]. \tag{13}$$

By inserting the temperature dynamics (5) in (13) and using the result of equation (11) we get the following formula:

$$\begin{aligned} F_{CAT(t,\tau_1,\tau_2)} &= \int_{\tau_1}^{\tau_2} \Lambda_u du + \mathbf{a}_{t,\tau_1,\tau_2}\mathbf{X}_t + \int_t^{\tau_1} \theta_u\sigma_u\mathbf{a}_{t,\tau_1,\tau_2}\mathbf{e}_p du \\ &+ \int_{\tau_1}^{\tau_2} \theta_u\sigma_u\mathbf{e}_1^\top A^{-1}\left[\exp\left\{A(\tau_2-u)\right\} - I_p\right]\mathbf{e}_p du, \end{aligned} \tag{14}$$

with $\mathbf{a}_{t,\tau_1,\tau_2} = \mathbf{e}_1^\top A^{-1}\left[\exp\left\{A(\tau_2-t)\right\} - \exp\left\{A(\tau_1-t)\right\}\right]$ and $I_p : p \times p$ identity matrix; see Benth et al. (2007) for proof.
It has to be mentioned that formula (14) is true for the contracts with trading date earlier than the beginning of the measurement period, i.e. $0 \leq t \leq \tau_1 < \tau_2$.
For the contracts with trading date between τ_1 and τ_2 a part of temperature dynamics in the time interval $[\tau_1, t]$ becomes deterministic and one has to account for that in the expression (14). I.e. for $\tau_1 < t < \tau_2$ holds:

$$\begin{aligned} F_{CAT(t,\tau_1,\tau_2)} &= \int_{\tau_1}^{t} T_u du + \mathrm{E}^{Q_\theta}\left(\int_t^{\tau_2} T_u du|\mathcal{F}_t\right) \\ &= \int_{\tau_1}^{t} T_u du + \int_t^{\tau_2} \Lambda_u du + \mathbf{a}_{t,t,\tau_2}\mathbf{X}_t \\ &+ \int_t^{\tau_2} \theta_u\sigma_u\mathbf{e}_1^\top A^{-1}\left[\exp\left\{A(\tau_2-u)\right\} - I_p\right]\mathbf{e}_p du \end{aligned} \tag{15}$$

with $\mathbf{a}_{t,t,\tau_2} = \mathbf{e}_1^\top A^{-1}\left[\exp\left\{A(\tau_2-t)\right\} - I_p\right]$.
Following Benth et al. (2007) for call options written on a CAT future with

strike K at exercise time $\tau < \tau_1$ during period $[\tau_1, \tau_2]$ it holds:

$$\begin{aligned} C_{CAT(t,\tau,\tau_1,\tau_2)} &= \exp\{-r(\tau - t)\} \\ &\times \left[\left(F_{CAT(t,\tau_1,\tau_2)} - K\right) \Phi\{d(t,\tau,\tau_1,\tau_2)\} \right. \\ &+ \left. \int_t^\tau \Sigma^2_{CAT(s,\tau_1,\tau_2)} ds \; \Phi'\{d(t,\tau,\tau_1,\tau_2)\} \right], \end{aligned} \tag{16}$$

with

$$d(t,\tau,\tau_1,\tau_2) = \frac{F_{CAT(t,\tau_1,\tau_2)} - K}{\sqrt{\int_t^\tau \Sigma^2_{CAT(s,\tau_1,\tau_2)} ds}}$$

where $\Sigma^2_{CAT(t,\tau_1,\tau_2)} = \sigma_t \mathbf{a}_{t,\tau_1,\tau_2} \mathbf{e}_p$ is CAT future volatility and Φ, Φ' denote standard normal cumulative and probability distribution functions respectively.

With the expressions above we obtained formulas for pricing temperature futures. The theoretical constructs for CAT futures depend however on the unknown parameter θ_t which has to be calibrated to the market prices.

2.5 Inferring the Market Price of Risk

As follows form equations (14) and (15) the value of θ_t is essential for pricing CAT futures but unknown. In the following we assume that θ_t is constant per contract and trading day.

Having observed market prices unknown parameters $\theta_t^i \in \Theta$ can be calibrated as

$$\begin{aligned} \hat{\theta}_t^i = \underset{\theta_t^i \in \Theta}{\arg\min} & \left[F_{CAT(t,\tau_1^i,\tau_2^i)} - \int_{\tau_1^i}^{\tau_2^i} \hat{\Lambda}_u du - \hat{\mathbf{a}}_{t,\tau_1^i,\tau_2^i} \widehat{\mathbf{X}}_t \right. \\ & - \theta_t^i \left\{ \int_t^{\tau_1^i} \hat{\sigma}_u \hat{\mathbf{a}}_{t,\tau_1^i,\tau_2^i} \mathbf{e}_p du \right. \\ & + \left. \left. \int_{\tau_1^i}^{\tau_2^i} \hat{\sigma}_u \mathbf{e}_1^\top A^{-1} \left[\exp\left\{ A(\tau_2^i - u) \right\} - I_p \right] \mathbf{e}_p du \right\} \right]^2, \end{aligned} \tag{17}$$

where $i = 1, \ldots, I$, I denotes the number of contract types, $t = 1, \ldots, M$ and Θ is bounded parameter space. After the values of MPR are calibrated for each contract and each trading day, following information can be extracted:

- prices for temperature futures for regions without weather market can be approximated by pricing under θ_t of existing markets,
- theoretical results about the relationship between MPR and variation of underlying can be applied by attempting to parametrize this relationship and thereby obtain estimates of θ_t for the corresponding regions without organized market knowing the variation of temperature.

This issues will be exploited in the empirical part of the thesis.

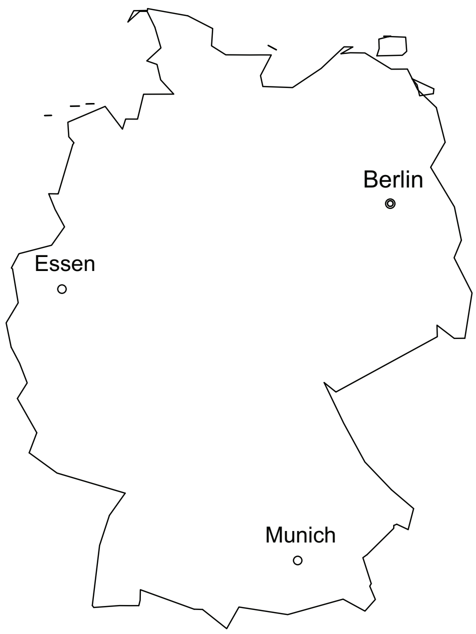

Figure 1: Location of Berlin, Essen and Munich on the map of Germany. map.R

3 Analysis of Weather Dynamics in Berlin, Essen and Munich

Parameters of weather dynamics have to be estimated from the real weather data for Berlin, Essen and Munich in order to apply the presented pricing model. Then, continuous time weather dynamics has to be constructed from the available data.

3.1 The Weather Data

The weather data to analyze in the following was downloaded from the website of Deutscher Wetterdienst[1]. Table 1 summarizes the time length and number of observations of the available datasets. The 29th February was removed from the three data sets in order for all the years to have the same length which makes the estimation procedure more consistent.

[1]Temperature data for Essen unavailable for free was replaced by temperature data of the closest location Düsseldorf.

City	Start Date	End Date	Observations
Berlin	19480101	20090731	22496
Essen	19690701	20090731	14641
Munich	19920517	20090731	6285

Table 1: Information about Weather Data. Source: Deutscher Wetterdienst

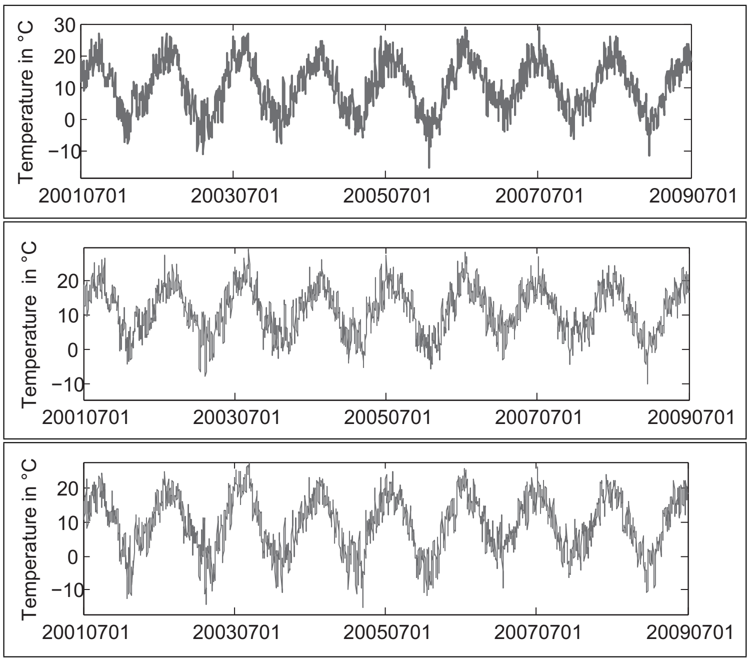

Figure 2: Daily temperature observations of the last eight years for Berlin (top), Essen (middle) and Munich (bottom). dynamics_temperature.m

The observations are daily average temperatures T_t which are computed as $T_t = \frac{T_t^{max} + T_t^{min}}{2}$. There were no missing values in the datasets. As seen from figure 2 temperature data indeed possesses strong seasonality and possibly seasonal variation which varies from month to month.

3.2 The Seasonality Effect

To remove seasonality we fit a deterministic periodic function Λ_t according to (2).

- For Berlin Λ_t has the form:

$$\Lambda_t = a + bt + \sum_{i=1}^{8} c_i \cdot \cos\{2\pi(t - d_i)/(i \cdot 365)\}$$

with parameters estimated with Nonlinear Least Squares and reported in table 2. The order of cosine series $s = 8$ was ascertained since longer cosine series appeared to be insignificant.

- For Essen Λ_t was found to be the same as for Berlin but with different parameters (see table 2).

- For Munich one additional periodic term had to be added to capture significant higher dynamics of temperature. The resulting parameter values are reported in table 2, too.

$$\Lambda_t = a + bt + \sum_{i=1}^{9} c_i \cdot \cos\{2\pi(t - d_i)/(i \cdot 365)\}$$

	Berlin	Essen	Munich		Berlin	Essen	Munich
$\hat{a}$	9.22	10.15	8.80	$\hat{c}_9$	-	-	-0.95
$\hat{b}$	$4 \cdot 10^{-5}$	$7 \cdot 10^{-5}$	$9 \cdot 10^{-5}$	$\hat{d}_1$	-165.19	-163.36	-168.82
$\hat{c}_1$	9.74	8.04	9.86	$\hat{d}_2$	-196.08	190.05	-18.22
$\hat{c}_2$	0.14	-0.07	0.21	$\hat{d}_3$	188.00	-128.83	143.27
$\hat{c}_3$	-0.29	-0.42	-0.56	$\hat{d}_4$	-155.93	-313.56	-31.13
$\hat{c}_4$	0.07	-0.16	-0.21	$\hat{d}_5$	153.49	338.28	255.54
$\hat{c}_5$	0.18	-0.12	0.54	$\hat{d}_6$	-219.60	309.38	-15.06
$\hat{c}_6$	0.29	0.33	-0.70	$\hat{d}_7$	-255.56	-735.27	-24.20
$\hat{c}_7$	0.10	0.13	1.02	$\hat{d}_8$	-371.38	353.91	37.35
$\hat{c}_8$	-0.56	-0.37	0.94	$\hat{d}_9$	-	-	-74.71

Table 2: Parameters of seasonal functions for Berlin, Essen and Munich estimated via nonlinear least squares. `dynamics_temperature.m`

The estimated parameter a represents the average of the temperature, which is the highest in Essen followed by Berlin and Munich. Parameter b accounts for the time trend, significant for all cities and represents the global warming effect. Vectors of parameters c_i and d_i are respectively amplitude and shift of cosine functions. Figure 3 shows temperature observations and fitted values for the three treated cities. The remaining residuals computed as difference of the observed and fitted values as well as their squared values are plotted in figure 4 and support the presence of some autoregressive process in the series.

3.3 The AR(p) Process

Before fitting any stationary process to the residual series we have to check for stationarity. Therefore we apply ADF test for unit root and KPSS test for stationarity as in Schwarz (1978).
Both test procedures report that residual series for all treated cities are stationary. For Munich ADF test regression provides test statistic -8.34 with critical value for $\alpha = 1\%$ equal to -2.58, for Berlin and Essen the values of test statistics are -13.42 and -12.38 respectively, so that the H_0 of unit root can be rejected on 1% significance level. The results of KPSS test are printed in table 3, the critical values for 1% significance level are given by 0.7390 for trend and 0.2160 for drift, due to the results the H_0 of stationarity can not be rejected for $\lambda = 1\%$.

City	trend	drift
Berlin	0.2020	0.2040
Essen	0.0480	0.0481
Munich	0.0111	0.0111

Table 3: Test statistics of KPSS test for stationarity. `dynamics_temperature.m`

Since residual series have proved to be stationary an autoregressive process can be fitted to this series. To find out the appropriate order of AR process we make use of Box-Jenkins analysis of partial autocorrelation functions (PACF) and Bayesian Information criterion (BIC) for model selection. Figure 5 plots partial autocorrelation functions for residuals of Berlin, Essen and Munich with 95% confidence intervals. They reveal that for all three cities PACF

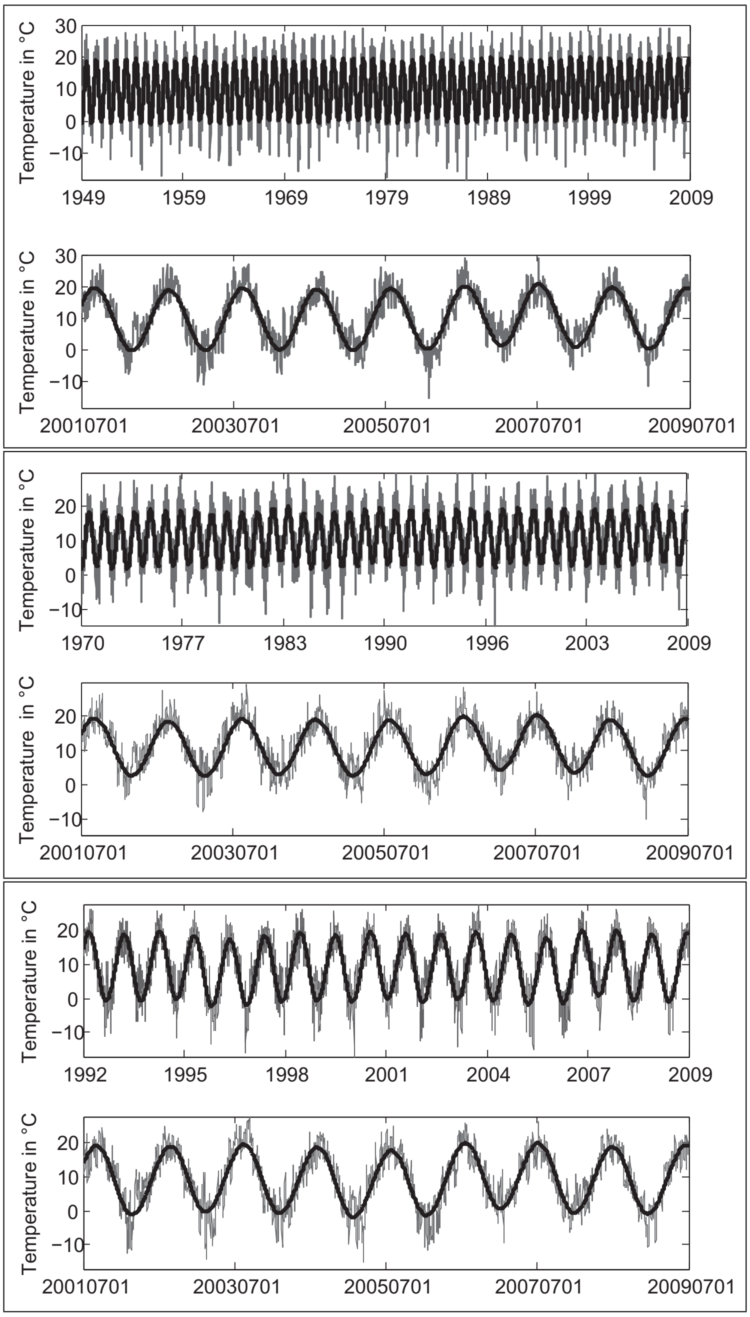

Figure 3: Observed and fitted values for temperature in Berlin (top), Essen (middle) and Munich (bottom). dynamics_temperature.m

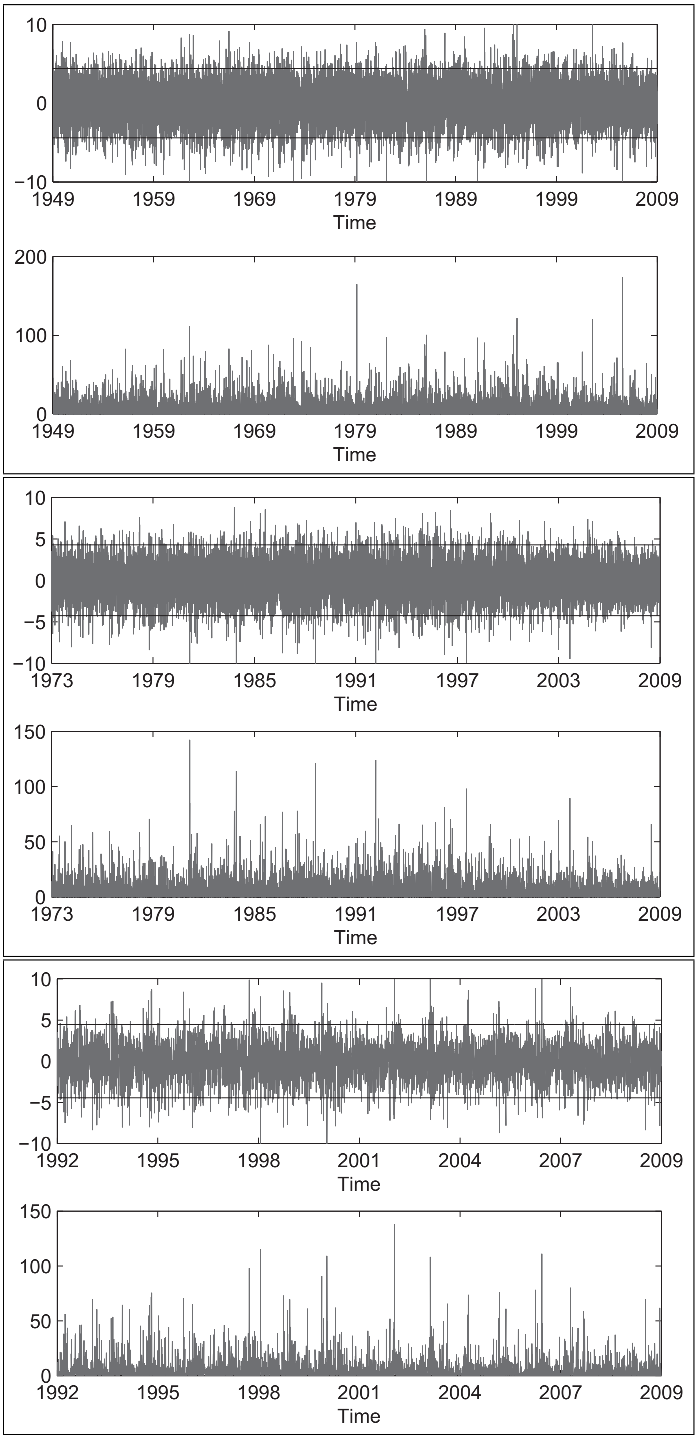

Figure 4: Residuals and squared residuals after removing seasonality in Berlin (top), Essen (middle) and Munich (bottom). `dynamics_temperature.m`

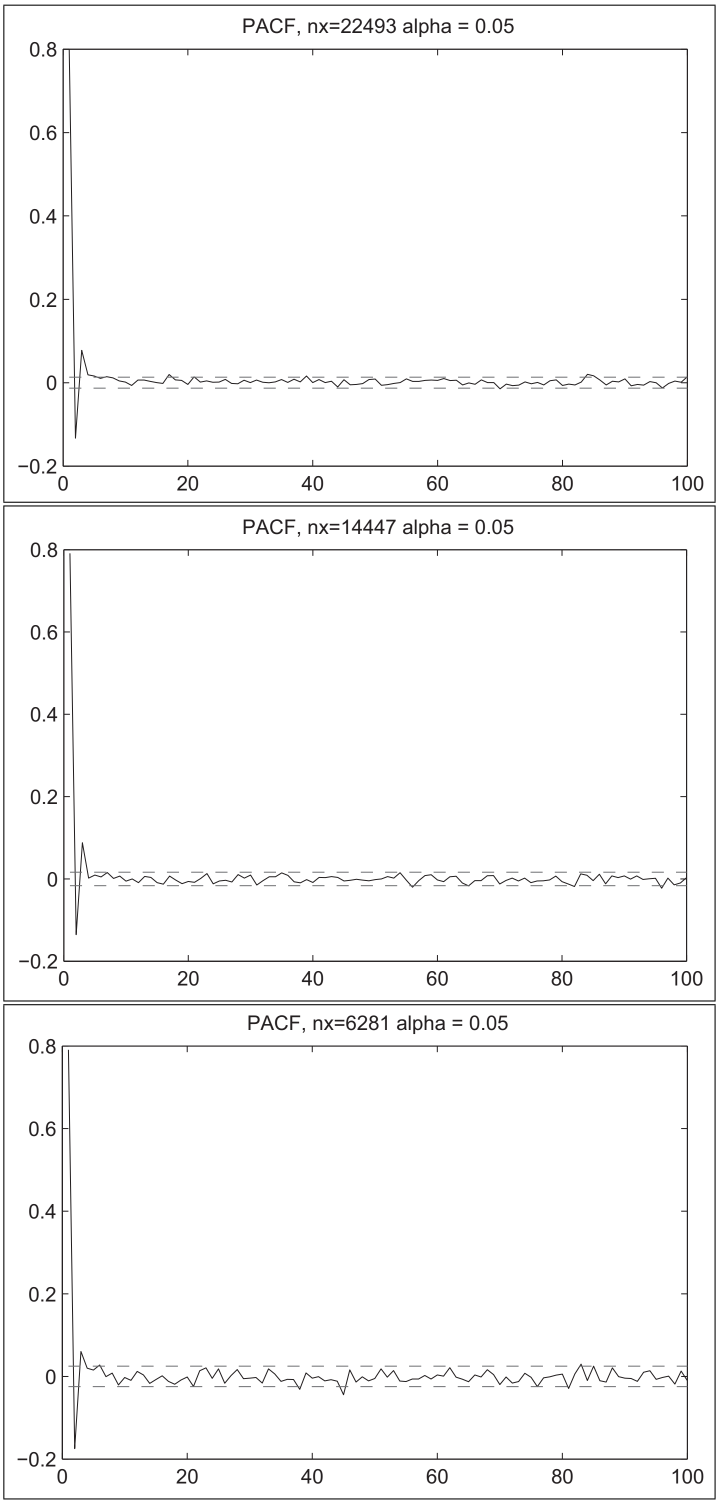

Figure 5: PACF of residuals after removing seasonality in Berlin (top), Essen (middle) and Munich (bottom). dynamics_temperature.m

are significant till the third lag and suggest therefore AR(3) as appropriate model. BIC supports this result and also suggest $p = 3$ as appropriate autoregressive order because this order maximizes BIC registered in table 4.

BIC	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)
Berlin	-1.6298	-1.6122	-1.6068	-1.6070	-1.6072
Essen	-1.5902	-1.5720	-1.5649	-1.5655	-1.5661
Munich	-1.6730	-1.6395	-1.6372	-1.6382	-1.6393

Table 4: BIC for different autoregressive orders p. `dynamics_temperature.m`

Estimated parameters of processes for all treated cities are reported in table 5. The autoregressive coefficients are quite close to each other and so they signal similarity in the autoregressive weather dynamics for the treated cities.

The plot in figure 6 shows PACF of the residuals and squared residuals of

City	α_1	α_2	α_3
Berlin	0.92	-0.20	0.08
Essen	0.91	-0.22	0.09
Munich	0.95	-0.24	0.06

Table 5: Estimated coefficients of AR(3) for Berlin, Essen and Munich. `dynamics_temperature.m`

AR(3) processes for the tree cities. As PACF of squared residuals shows some persistent seasonality, it makes sense to estimate the seasonal variation of the AR(3) process as formalized in equation (3). Therefore, we sort residuals $\hat{\varepsilon}_t$ of the estimated AR process in 365 groups corresponding to each day over all years of observations and estimate variance of the residuals for each day of a year as $\frac{1}{N-1}\sum_{i=1}^{N}\hat{\varepsilon}_{i,j}^2$ with $\hat{\varepsilon}_{i,j}$ residual in day j, $j = 1, \ldots, 365$, of year i, $i = 1, \ldots, N$.

As the second step we fit a local linear regression (LLR) to the resulting values with an optimal bandwidth suggested by Bowman and Azzalini (1997). The estimated residual variance and the fitted values of LLR are presented in figure 7.

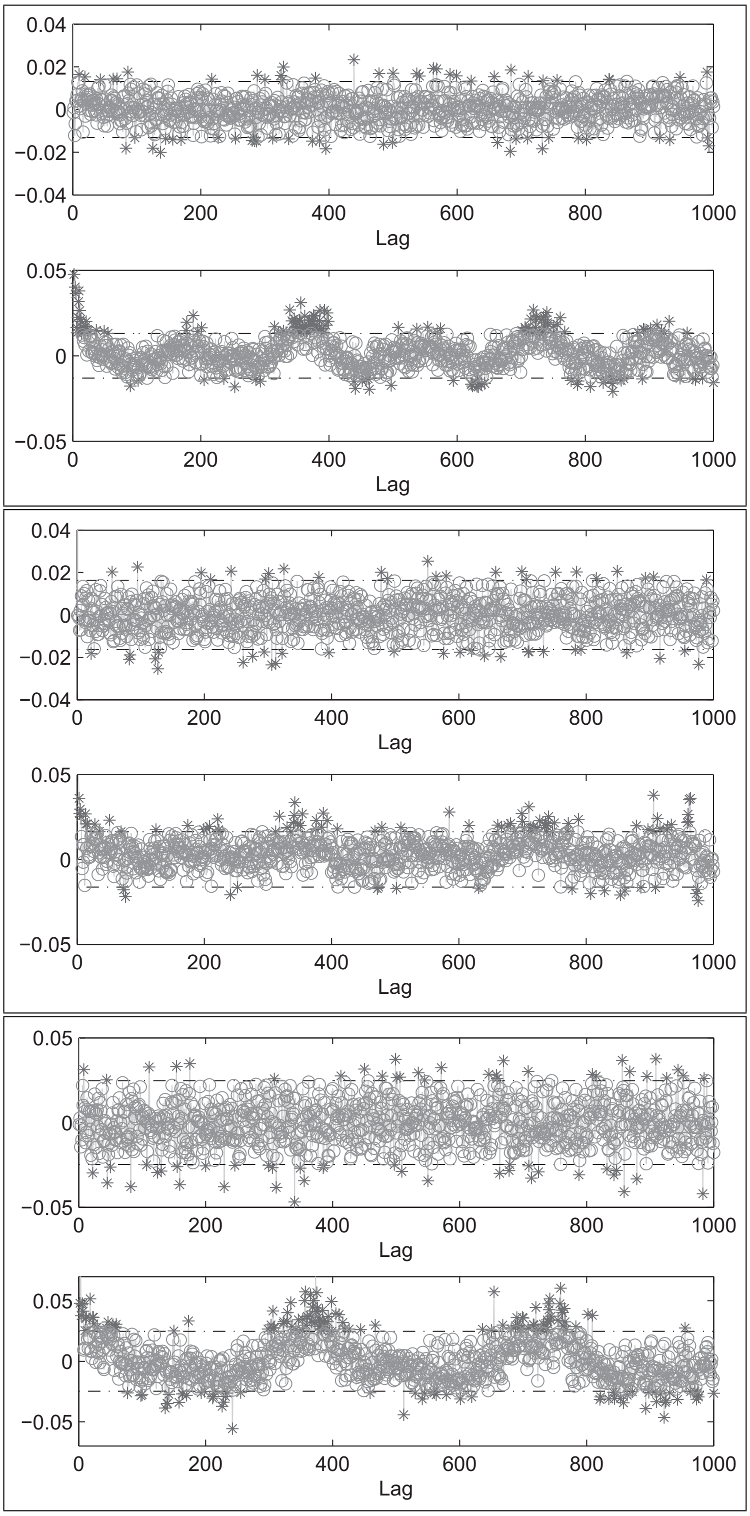

Figure 6: PACF of residuals and squared residuals of Berlin (top), Essen (middle) and Munich (bottom). `dynamics_temperature.m`

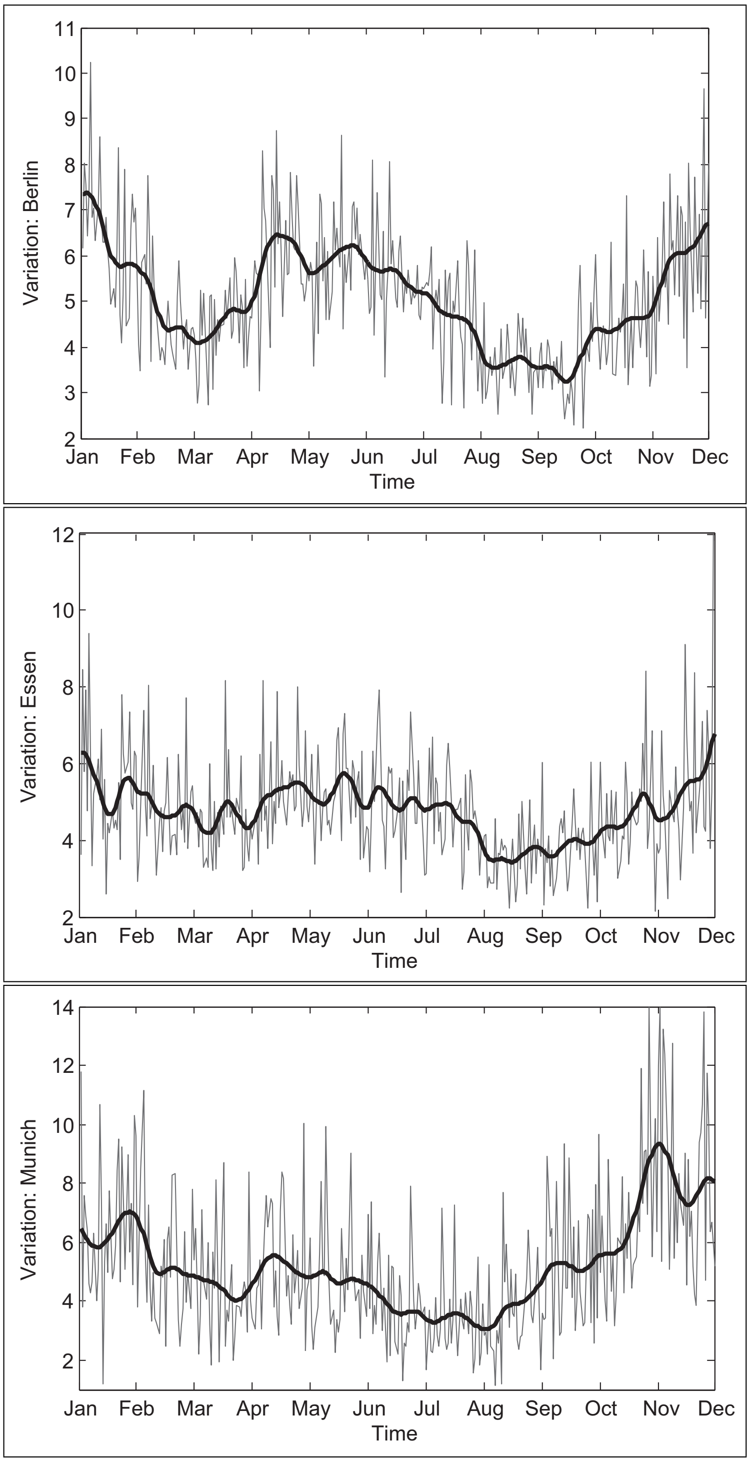

Figure 7: Estimated residual variation and fitted values of LLR in bold. `dynamics_temperature.m`

If our model is true then residuals standardized by the seasonal standard deviation should be at least close to normal. We check it by applying Jarque-Bera and Kolmogorov-Smirnov tests for normality. Results of these tests reject the null hypothesis of normality. Table 6 reports skewness, kurtosis and test statistics of the performed tests as well as critical values (in brackets). Figure 8 shows kernel density of the standardized residuals against normal

City	Skewness	Kurtosis	JB	KS
Berlin	-0.08	3.52	291.03 (14.11)	0.013 (0.011)
Essen	-0.07	3.49	160.63 (14.26)	0.015 (0.013)
Munich	-0.20	3.63	132.24 (14.88)	0.022 (0.021)

Table 6: Skewness, kurtosis and test statistics of Jarque-Bera and Kolmogorov-Smirnov normality tests. `dynamics_temperature.m`

density as well as the logarithm of the densities and indicate some deviations from normality. Nevertheless, the standardized residuals are reasonably close to normal so that we can continue with the normality assumption for the residual process which we will need for pricing weather derivatives.

We have obtained an explicit form of weather dynamics, it is now straight forward to forecast it for horizon of interest.

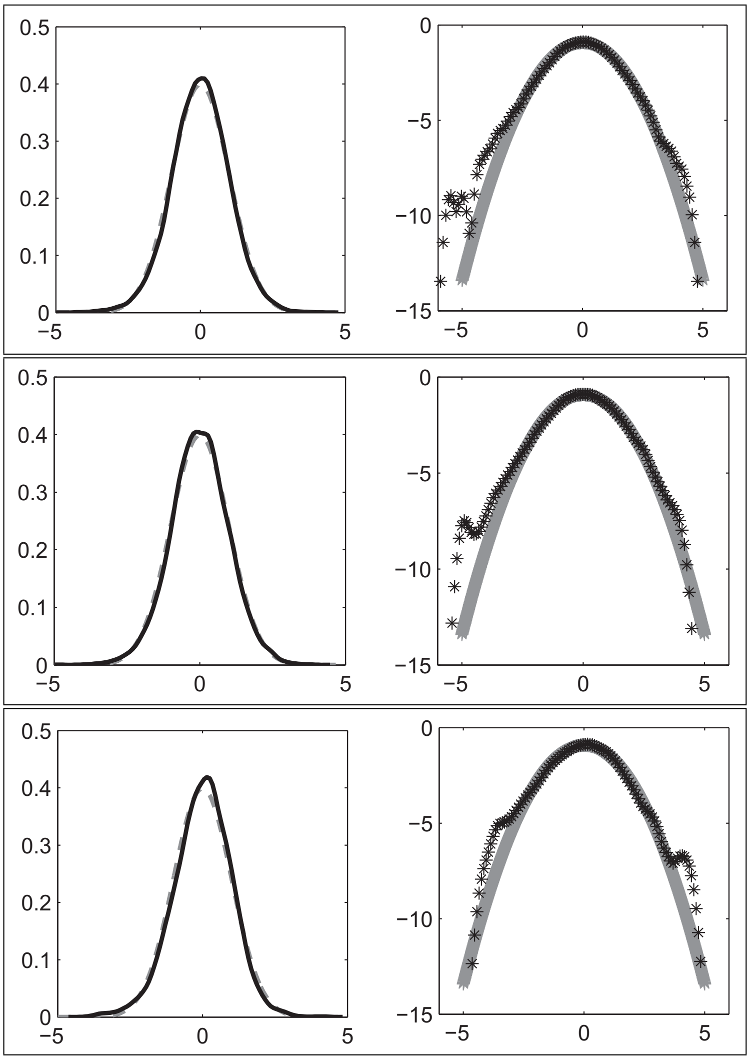

Figure 8: Empirical kernel estimated density of standardized residuals (black) against normal density (grey) on the left plots and the logarithm of the empirical density (black) and of the normal density (grey) on the right. Berlin (top), Essen (middle) and Munich (bottom). dynamics_temperature.m

4 Calibration of Market Price of Risk and Pricing CAT Derivatives for Munich

4.1 Data of CAT Future Prices for Berlin and Essen

CAT future prices for Berlin and Essen were downloaded from Bloomberg terminal in form of seven generics which summarize CAT contract future prices for contracts with measurement period in the current month (first generic), in the next month (second generic) and in two till six months (third to seventh generic) as traded on Chicago Mercantile Exchange (CME). For Berlin available CAT future prices start with 20031008 to 20090803 and for Essen the shorter period included prices from 20050617 to 20090803. The 29th February was removed from the dataset. The prices of CAT futures are traditionally measured in centigrade and their measurement periods run from April to October. An Example of data on CAT futures is shown in table 7.

	Trading Period		Measurement Period		CAT Index	
Code	First-trade	Last-trade	τ_1	τ_2	Berlin	Essen
J9	20080503	20090502	20090401	20090430	297	302
K9	20080603	20090602	20090501	20090531	464	437
M9	20080704	20090703	20090601	20090630	536	500
N9	20080803	20090802	20090701	20090731	630	575
Q9	20080903	20090902	20090801	20090831	607	572

Table 7: Berlin and Essen CAT future contracts listed on CME. Source: Bloomberg

The column "code" denotes the codes used by CME for temperature future contracts: J for April, K for May, M for June, N for July, Q for August, U for September and V for October, the last number of the code stands for the year in which the measurement takes place.
Figure 9 shows prices of CAT futures for Berlin and Essen increasing when temperature is high (June, July, August) and decreasing in colder months (April, May, September, October).

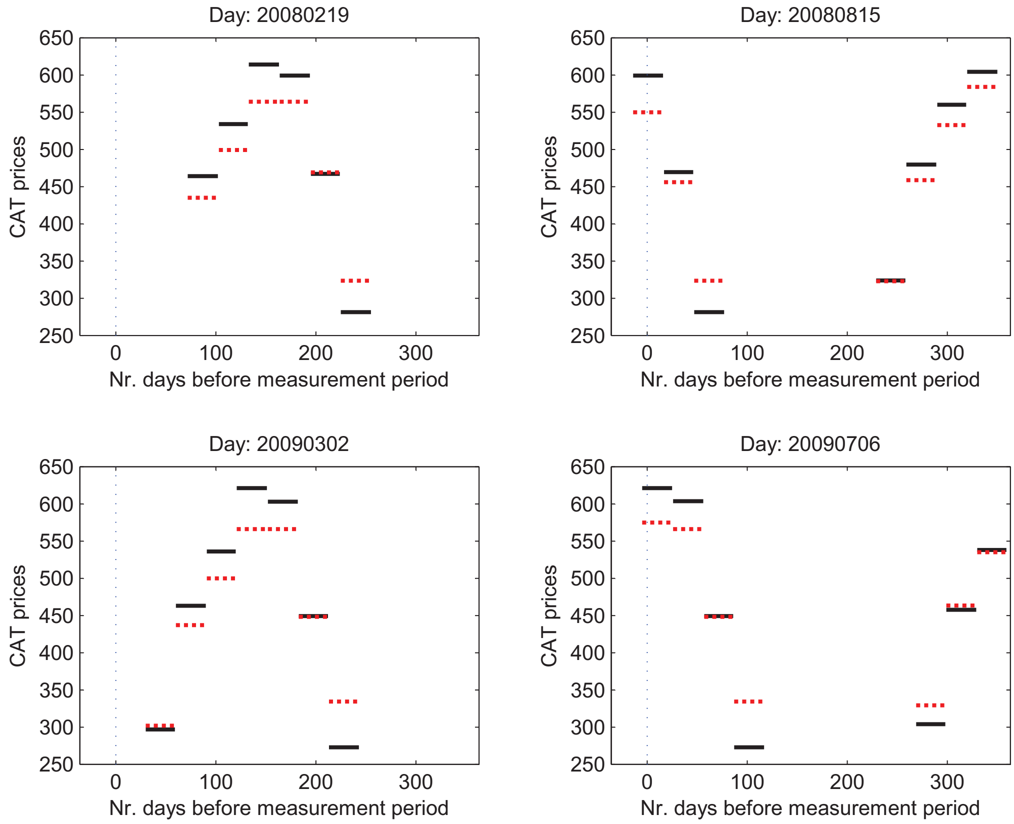

Figure 9: CAT prices for Berlin (solid line) and Essen (dotted line) on four randomly chosen dates in 2008 and 2009. cat_pricing.m

4.2 Calibration of Market Price of Risk for Berlin and Essen

Having observed market prices and having estimated weather dynamics we can actually calibrate the market price of risk (MPR) for the two cities under the assumption of existence of constant MPR per contract and day. This assumption implies that θ_t varies over time and also depends on the type of contract and on the time left to measurement period.
First of all we compute the coefficients of CAR(3) process according to (6). Table 8 reports the coefficients computed as in (8). Since the real parts of eigenvalues of A are negative, condition for stationarity is fulfilled.

We calibrate θ_t^i from the observed CAT future prices for Berlin and Essen

City	β_1	β_2	β_3	λ_1	$\lambda_{2,3}$
Berlin	2.08	1.37	0.21	-0.22	$-0.93 \pm 0.31i$
Essen	2.09	1.39	0.22	-0.22	$-0.94 \pm 0.33i$
Munich	2.05	1.34	0.23	-0.26	$-0.90 \pm 0.27i$

Table 8: CAR(3) parameters β_k and eigenvalues λ_k, $k = 1, 2, 3$ of matrix A for Berlin, Essen and Munich. cat_pricing.m

and the estimated temperature dynamics for these cities solving (17) with respect to θ_t^i. Assume θ_t^i equal zero in cases where CAT future prices are missing (no trade took place). The calibrated θ_t's show indeed time varying nature and dependence on the time to measurement period. Figures 10 and 11 show prices of CAT futures and calibrated values of θ_t for the seven contracts on two randomly chosen time points. MPR tend to take negative values for April, May and October and in the same time positive values for July and August, for June and September sign of MPR changes.

This phenomena is interpreted as the influence of temperature variation σ_t^2. At trading day 20090327 MPR of contracts for April, May and October is negative because in those periods σ_t decreases in faster rates than it does in summer months. The same effect can be seen for trading day 20090521. This findings became already standard in the literature, e.g. Härdle and López Cabrera (2009).

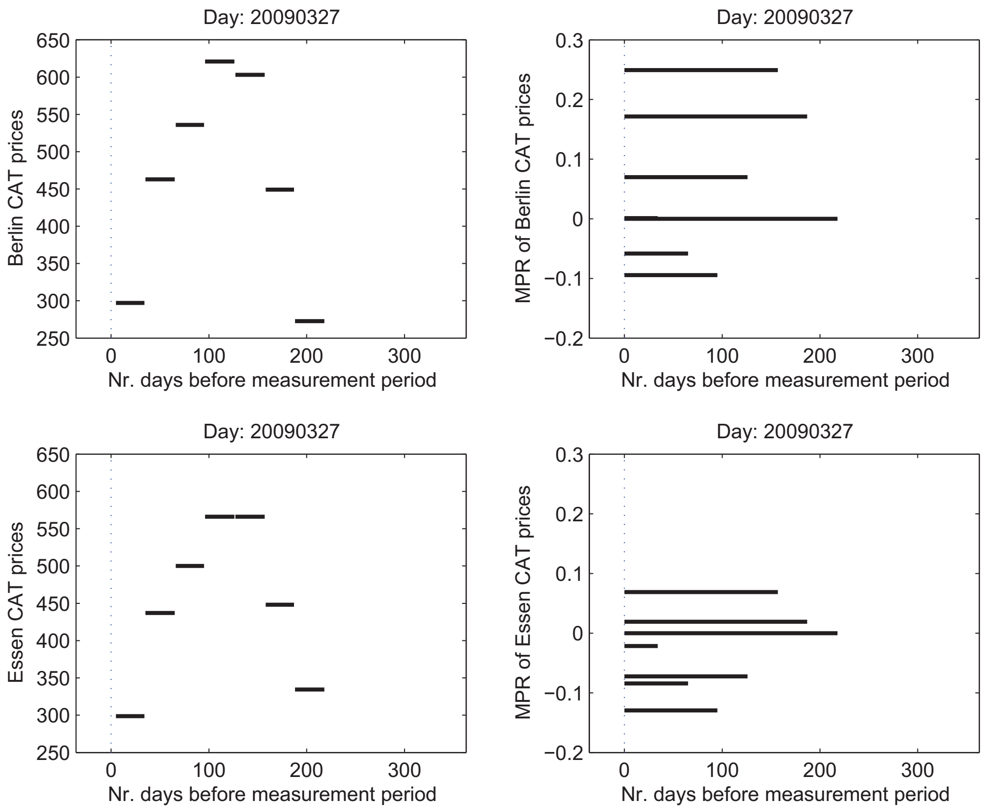

Figure 10: Observed prices of CAT futures (left panels) and calibrated θ_t (right panels) for Berlin and Essen on 20090327. `cat_pricing.m`

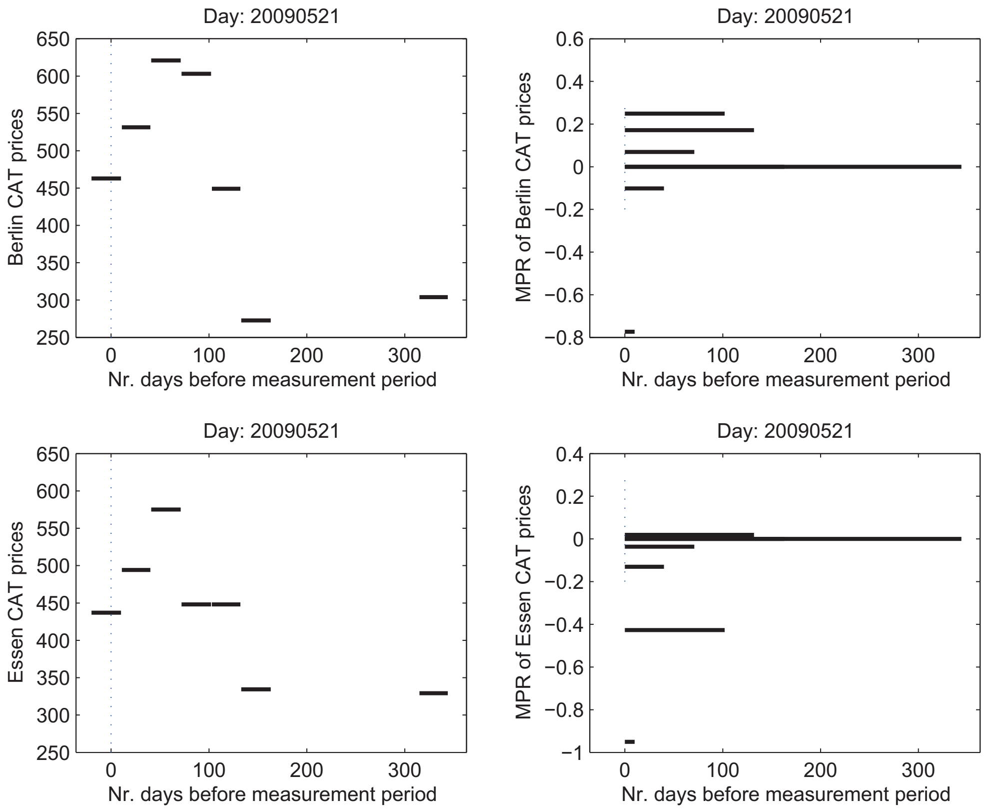

Figure 11: Observed prices of CAT futures (left panels) and calibrated θ_t (right panels) for Berlin and Essen on 20090521. cat_pricing.m

4.3 Pricing Weather Derivatives in Munich

Having calibrated the values for θ_t^i we obtained the pricing measure Q_{θ_t}. However, this measure is not known for Munich. There are several approaches possible, two of them are presented in the following section.

4.3.1 Pricing under Equivalent Pricing Measures Q_{θ_t} of Berlin and Essen

One solution could be to price CAT futures for Munich under $Q^B_{\theta_t}$ MPR of Berlin and $Q^E_{\theta_t}$ MPR of Essen separately and then take an average with weights equal to $\frac{1}{2}$. Then, price of CAT future for Munich would be described by (18) under $Q_{\theta_t^B}$ and by (19) under $Q_{\theta_t^E}$. The estimated price of CAT future for Munich would be a simple average of the two estimators (20).

$$\widehat{F}^M_{CAT(t,\tau_1,\tau_2)} = \mathrm{E}^{Q^B_\theta}\left(\int_{\tau_1}^{\tau_2} T_u du|\mathcal{F}_t\right) \tag{18}$$

$$\widetilde{F}^M_{CAT(t,\tau_1,\tau_2)} = \mathrm{E}^{Q^E_\theta}\left(\int_{\tau_1}^{\tau_2} T_u du|\mathcal{F}_t\right) \tag{19}$$

$$\overline{F}^M_{CAT(t,\tau_1,\tau_2)} = \frac{1}{2}\widehat{F}^M_{CAT(t,\tau_1,\tau_2)} + \frac{1}{2}\widetilde{F}^M_{CAT(t,\tau_1,\tau_2)} \tag{20}$$

The estimated prices of CAT futures for Munich in comparison to Berlin and Essen are presented in figure 12. The estimated prices in spring and autumn months are lower than those of Berlin and Essen, whereas the prices for summer months are of the same magnitude or higher than in Berlin and Essen. Possibly it reflects more continental character of climate in Munich and corresponds to on average lower temperatures in autumn and spring and higher temperatures in summer in comparison to Berlin and Essen possessing milder climate.

Draw-backs of this approach is uncertainty about possibly spatial characteristics of MPR or dependence on seasonal variation which is unequal for considered locations.

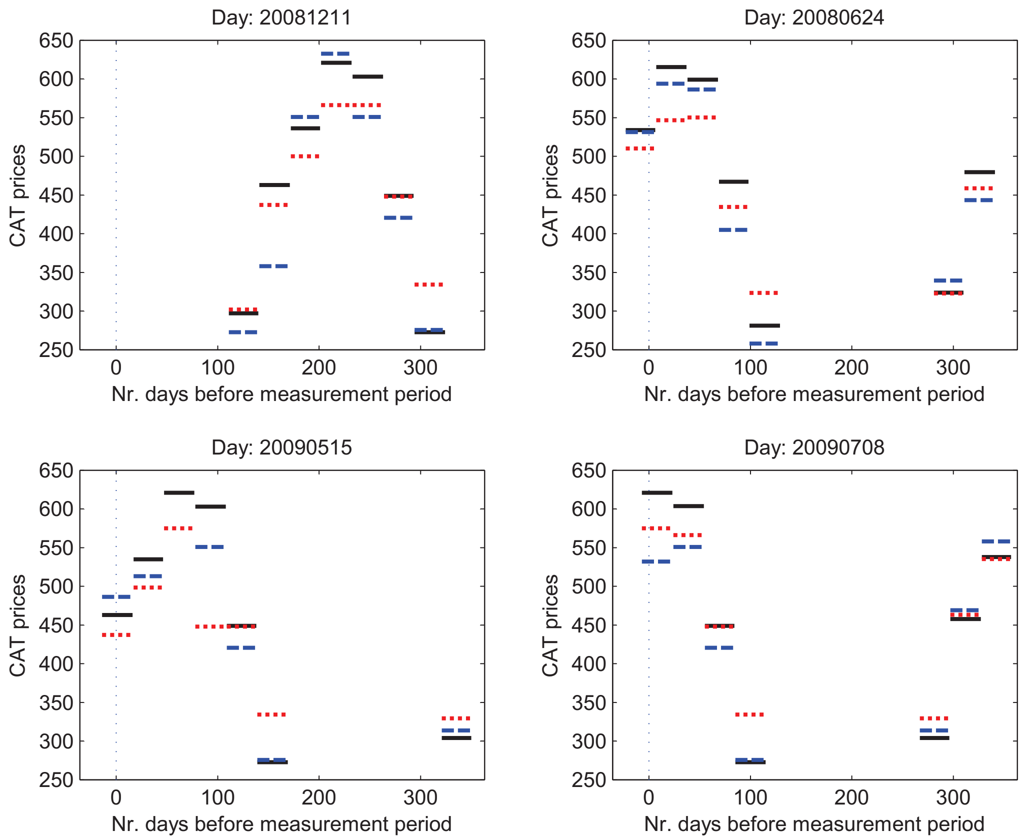

Figure 12: Observed prices of CAT futures for Berlin (solid line) and Essen (dotted line) and estimated CAT future prices for Munich (dashed line). cat_pricing.m

4.3.2 Parametrization of MPR for Munich

The other way of inferring MPR for Munich is parametrization of MPR by regressing it on seasonal variation. Since the contracts listed on CME always have a measurement period of one month, it makes sense to assume MPR is affected by the average or cumulated variation in the measurement month. This is the motivation for treating θ's averaged over trading period together with average monthly variation of temperature.
In further, we define

- average MPR $\hat{\theta}^i_{\tau_1,\tau_2}$ as an average of the calibrated $\hat{\theta}^i_t$ traded over time period $[t_{\tau_1,\tau_2}, T_{\tau_1,\tau_2}]$ with measurement interval $[\tau_1, \tau_2]$:

$$\hat{\theta}^i_{\tau_1,\tau_2} = \frac{1}{T_{\tau_1,\tau_2} - t_{\tau_1,\tau_2}} \sum_{t=t_{\tau_1,\tau_2}}^{T_{\tau_1,\tau_2}} \hat{\theta}^i_t,$$

  where t_{τ_1,τ_2} and T_{τ_1,τ_2} indicate the first and the last trade for the contracts with measurement month $[\tau_1, \tau_2]$.

- Variation of month $[\tau_1, \tau_2]$:

$$\hat{\sigma}^2_{\tau_1,\tau_2} = \frac{1}{\tau_2 - \tau_1} \sum_{t=\tau_1}^{\tau_2} \hat{\sigma}^2_t.$$

We consider only contracts which have measurement period in one and in two months from 20080801 to 20090731. In the following we treat the values of $\hat{\theta}^i_t$ for contracts on Berlin and Essen as generated out of the same population and regress $\hat{\theta}^i_{\tau_1,\tau_2}$ on $\hat{\sigma}^2_{\tau_1,\tau_2}$. Knowing from theory of derivative pricing that the relation between MPR and variation is reverse, we guess that the relationship is either linear of the form $y = a + bx$, where $b < 0$ or it possesses diminishing marginal effects and is specified as $y = a + bx + cx^2$, where $b < 0$ and $c > 0$.
Figures 13 and 14 show plotted points for generic 2 and generic 3 with coordinates $\hat{\theta}_{\tau_1,\tau_2}$ and $\hat{\sigma}^2_{\tau_1,\tau_2}$ and the fitted values $\hat{\hat{\theta}}_{\tau_1,\tau_2}$ for both specifications. Characteristics of the fitted model are reported in tables 9 and 10.

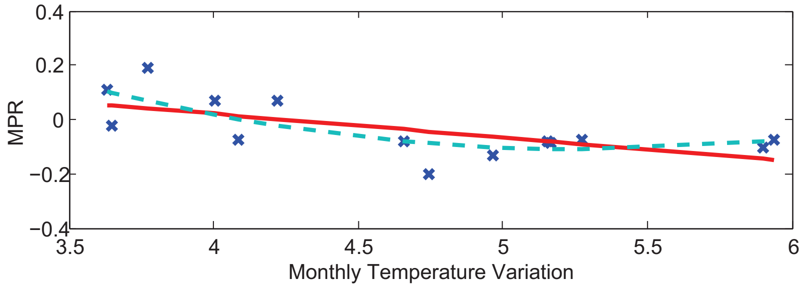

Figure 13: Calibrated MPR and temperature variation of CAT futures for Berlin and Essen from July 2008 to June 2009 with measurement period in one month and fitted values of regression specifications 1 and 2 (solid and dotted lines respectively). `mpr_parameter.m`

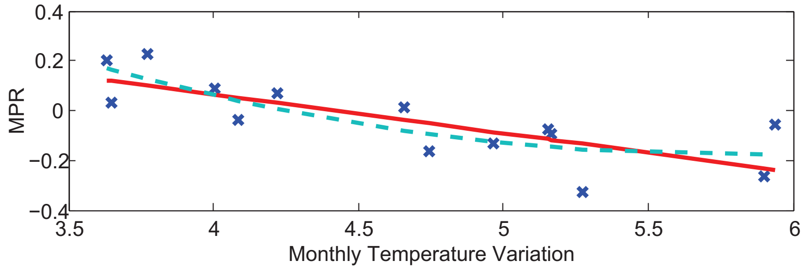

Figure 14: Calibrated MPR and temperature variation of CAT futures for Berlin and Essen from June 2008 to May 2009 with measurement period in two months and fitted values of regression specifications 1 and 2 (solid and dotted lines respectively). `mpr_parameter.m`

The second specification which accounts for diminishing marginal effects fits better according to R^2_{adj} and root mean squared error criterion. This justifies the use of second specification for inferring MPR of Munich. Therefore, for contracts with measurement period in one month

$$\hat{\hat{\theta}}_{\tau_1,\tau_2} = 2.064 - 0.822 \cdot \hat{\sigma}^2_{\tau_1,\tau_2} + 0.078 \cdot \hat{\sigma}^4_{\tau_1,\tau_2}, \tag{21}$$

for contracts with measurement in two months holds

$$\hat{\hat{\theta}}_{\tau_1,\tau_2} = 2.248 - 0.832 \cdot \hat{\sigma}^2_{\tau_1,\tau_2} + 0.071 \cdot \hat{\sigma}^4_{\tau_1,\tau_2}. \tag{22}$$

The estimated values of $\hat{\hat{\theta}}_{\tau_1,\tau_2}$ for Munich for contracts with measurement period in one ($\hat{\hat{\theta}}^{(1)}_{\tau_1,\tau_2}$) and in two months ($\hat{\hat{\theta}}^{(2)}_{\tau_1,\tau_2}$) are shown in table 11. With obtained values of MPR for Munich temperature derivatives can be priced. Advantage of this method is the independence of the measurement period.

Parameters	$\hat{\hat{\theta}}_{\tau_1,\tau_2} = a + b \cdot \hat{\sigma}^2_{\tau_1,\tau_2}$	$\hat{\hat{\theta}}_{\tau_1,\tau_2} = a + b \cdot \hat{\sigma}^2_{\tau_1,\tau_2} + c \cdot \hat{\sigma}^4_{\tau_1,\tau_2}$
a	0.3714	2.0640
b	-0.0874	-0.8215
c	-	0.0776
R^2_{adj}	0.4157	0.4902

Table 9: Estimated parameters of linear regression for MPR of CAT futures with measurement period in one month. `mpr_parameter.m`

Parameters	$\hat{\hat{\theta}}_{\tau_1,\tau_2} = a + b \cdot \hat{\sigma}^2_{\tau_1,\tau_2}$	$\hat{\hat{\theta}}_{\tau_1,\tau_2} = a + b \cdot \hat{\sigma}^2_{\tau_1,\tau_2} + c \cdot \hat{\sigma}^4_{\tau_1,\tau_2}$
a	0.6906	2.2480
b	-0.1565	-0.8319
c	-	0.0714
R^2_{adj}	0.5668	0.5963

Table 10: Estimated parameters of linear regression for MPR of CAT futures with measurement period in two months. `mpr_parameter.m`

Weather derivatives with shorter or longer measurement periods than one month can be priced. Draw-backs of the parametrization methodology are large confidence intervals since the number of values to regress is small and therefore risk of misspecification. Also assumption that MPR of Berlin and Essen are generated by the same process is restrictive and doesn't have to hold in reality.

measurement month	variation	$\hat{\hat{\theta}}^{(1)}_{\tau_1,\tau_2}$	$\hat{\hat{\theta}}^{(2)}_{\tau_1,\tau_2}$
April	5.06	-0.0983	-0.1441
May	5.07	-0.0985	-0.1450
June	4.31	-0.0300	-0.0193
July	3.15	0.2483	0.3312
August	3.23	0.2230	0.3017
September	4.33	-0.0334	-0.0242
October	5.16	-0.1006	-0.1542

Table 11: Estimated values of θ_{τ_1,τ_2} for Munich for contracts with measurement period in one ($\hat{\hat{\theta}}^{(1)}_{\tau_1,\tau_2}$) and in two months ($\hat{\hat{\theta}}^{(2)}_{\tau_1,\tau_2}$). `mpr_parameter.m`

Table 12 reports estimated prices of CAT futures for Munich under risk neutral pricing measures of Berlin and Essen (fourth column) and under parametrized via linear regression MPR for Munich (fifth column). The results show the estimation under the pricing measures of Berlin and Essen reports prices that are not significantly different from those estimated by parametrization method.

Trading date	τ_1	τ_2	$\overline{F}^M_{CAT}$	$\widehat{F}^M_{CAT}$
20080801	20080901	20080930	411.06	404.12
20080901	20081001	20081031	274.35	264.38
20081001	20090401	20090430	258.19	264.11
20090401	20090501	20090531	414.05	422.13
20090501	20090601	20090630	535.73	513.54
20090601	20090701	20090731	571.15	568.19
20090701	20090801	20090831	520.54	522.04
20090801	20090901	20090930	390.92	392.57

Table 12: Estimated prices of CAT futures for Munich under risk neutral pricing measures of Berlin and Essen ($\overline{F}^M_{CAT}$) and under parametrized MPR for Munich ($\widehat{F}^M_{CAT}$). `cat_pricing.m`

With obtained CAT future prices for Munich and also other derivatives such as call options on CAT futures can be priced for Munich using formula (16). An example of call options on CAT future is given in table 13. In this example $\overline{F}^M_{CAT(t,\tau_1,\tau_2)}$ CAT future price on Munich temperature was calculated by formula (20). $\widehat{F}^M_{CAT(t,\tau_1,\tau_2)}$ was obtained by plugging in (14) the values of $\hat{\hat{\theta}}_{\tau_1,\tau_2}$ from (21) ($\hat{\sigma}^2_{\tau_1=20090701,\tau_2=20090731} = 3.15$ implies $\hat{\hat{\theta}}_{\tau_1=20090701,\tau_2=20090731} = 0.25$ and $\hat{\sigma}^2_{\tau_1=20090701,\tau_2=20090707} = 3.21$ implies $\hat{\hat{\theta}}_{\tau_1=20090701,\tau_2=20090707} = 0.23$) and temperature dynamics.

Figure 15 below shows calculated call prices for $\widehat{F}_{CAT(t,\tau_1=20090701,\tau_2=20090707)} = 132.89$ with $t = 20090601$, $r = 1\%$ and $\tau = 20090607$ for different level of strike K and call prices with $t = 20090601$, $r = 1\%$ and $K = 125$ for different time to maturity periods. The plotted values correspond to developed theory of call options behavior in this variables, see Hull (2006) and Härdle et al. (2004).

Derivative	Example 1	Example 2
Index	CAT	CAT
r	5%	1%
t	1. June 2009	1. June 2009
Measurement Period	1.-31. July 2009	1.-7. July 2009
Strike	550°C	125°C
Tick Value	1°C=£15	1°C=£15
$\overline{F}^M_{CAT(t,\tau_1,\tau_2)}$	571.15	128.97
$\overline{C}_{CAT(t,\tau=20090607,\tau_1,\tau_2)}$	15.67	3.74
$\overline{C}_{CAT(t,\tau=20090614,\tau_1,\tau_2)}$	11.04	3.49
$\widehat{F}^M_{CAT(t,\tau_1,\tau_2)}$	568.18	132.89
$\widehat{C}_{CAT(t,\tau=20090607,\tau_1,\tau_2)}$	13.47	7.43
$\widehat{C}_{CAT(t,\tau=20090614,\tau_1,\tau_2)}$	9.49	6.93

Table 13: Example of pricing CAT future call options. $\overline{F}^M_{CAT(t,\tau_1,\tau_2)}$ denotes CAT future price on Munich temperature computed by (20), $\widehat{F}^M_{CAT(t,\tau_1,\tau_2)}$ – by (14) with $\hat{\hat{\theta}}_t$ obtained using (21); $\overline{C}_{CAT(t,\tau,\tau_1,\tau_2)}$ and $\widehat{C}_{CAT(t,\tau,\tau_1,\tau_2)}$ denote corresponding call option with maturity τ computed according to (16). `pricing_example.m`

Thereby pricing of CAT futures for Munich with other than monthly measurement period is restricted since MPR of Berlin and Essen was assumed to be tight to the measurement month. The use of parametrization method also relies on the assumption that the underlying relationship between MPR and

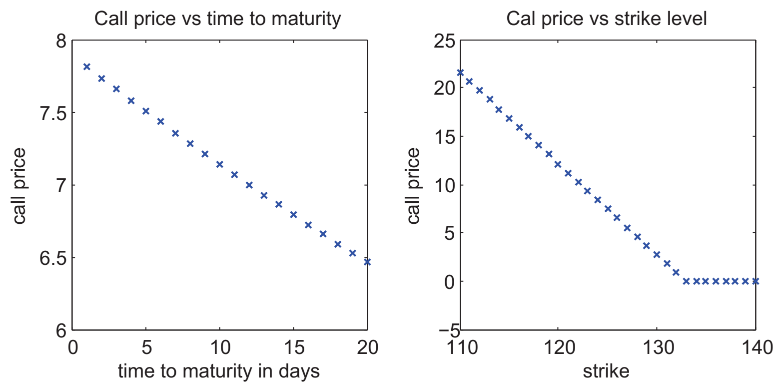

Figure 15: Example of pricing CAT future call option. Left panel shows call price for $\widehat{F}_{CAT(t=20090601,\tau_1=20090701,\tau_2=20090707)} = 132.89$ with $r = 1\%$ and $K = 125$ for different time to maturity in days $\tau - t$. Right panel plots call prices with $\tau = 20090607$ for different levels of strike K. `pricing_example.m`

seasonal variation is independent of the length of the measurement period, which is a restrictive assumption.
In general, it may be concluded that presented methods can be applied to pricing temperature derivatives in regions without a market for temperature and these prices can be used at least as pilot prices for forward contracts on temperature indices beyond the organized temperature markets.

5 Conclusion

In this diploma thesis a pricing model for temperature derivatives was applied to CAT futures of Berlin, Essen and Munich. In order to be able to construct a feasible pricing methodology, discrete and continuous time temperature dynamics was parametrized for the considered locations.
The temperature dynamics was proved to have a seasonal component with seasonally heteroscedastic autoregressive process in residuals. Seasonal component consisted of an intercept, time trend and combinations of cosine functions for every of three cities and was estimated with nonlinear least squares. The order of the autoregressive processes for the considered locations was equal to three lags of temperature residuals and was chosen according to Bayesian Information criterion. The coefficients of the autoregressive processes were estimated via ordinary least squares and were similar for the treated cities. Seasonal variation was smoothed with means of local linear regression and showed an increase in winter, early autumn and spring and a decrease in summer months. Munich temperature exhibited higher variation in winter, autumn and spring than Berlin or Essen.
Further, coefficients for continuous time model in form of the solution to Ornstein-Uhlenbeck differential equation were derived from the estimated discrete time autoregressive processes to switch over to the pricing model.
The described pricing model was applied to the CAT future prices of Berlin and Essen in order to calibrate parameter θ_t representing the market price of risk per contract and trading day. It was shown that θ_t exhibits reversal dependence on the seasonal variation of the temperature and tends to be positive in months with low temperature variation and negative in periods with high variability of temperature.
Finally, two approaches to constructing weather derivative prices for regions without organized markets were applied to the example of Munich.
First approach was based on pricing temperature derivatives for Munich under the pricing measures Q_{θ_t} of Berlin and Essen. The estimated prices of CAT futures for Munich were then the simple averages of the two resulting estimates.

Second method incorporated parametrization of θ_t in monthly seasonal variation. Two parametrization schemes were chosen on the basis of pricing theory for derivatives: linear specification and parabolic form reflecting diminishing marginal effects. Both specification gave plausible results in the sense that the coefficient of the partial effect of the seasonal variation was estimated to be negative. However, second specification gave a better fit in terms of R^2_{adj} and root mean squared error and was therefore applied to infer market price of risk for Munich.

Both approaches to pricing reported, nevertheless, similar prices for CAT futures of Munich and can be useful in practice to get at least pilot price estimates for CAT futures and options on CAT futures for regions without an organized market for temperature derivatives.

However, the applied methods for constructing weather derivatives for Munich have their draw-backs since the underlying assumption are quite restrictive. Therefore suggestions for further research in this area would be attempts to relax the restrictive assumptions on θ_t and consider e.g. its possibly spatial nature and its autoregressive dynamics in time.

References

ALATON, P., B. DJEHICHE, AND S. D. (2002): "On Modelling and Pricing Weather Derivatives," *Appl. Math. Finance*, 1, 1–20.

BENTH, F., J. SALTYTE BENTH, AND S. KOEKEBAKKER (2007): "Putting a Price on Temperature," *Scandinavian Journal of Statistics*, 747–767.

BOWMAN, A. AND W. AZZALINI (1997): *Applied Smoothing Techniques for Data Analysis*, Oxford: Claredon Press.

BOX, G. AND G. JENKINS (1970): *Time Series Analysis - Forecasting and Control*, San Francisco: Holden Day.

CAMPBELL, S. AND F. DIEBOLD (2005): "Weather Forecasting for Weather Derivatives," *American Stat. Assoc.*, 469, 6–16.

CIZEK, P., W. HÄRDLE, AND R. WERON (2005): *Statistical Tools for Finance and Insurance*, Springer.

DORNIER, F. AND M. QUEREL (2000): "Caution to the Wind," *Energy Power Risk Management, in: Weather Risk Special Report*, 30–32.

GILLESPIE, D. (1996): "Exact numerical simulation of the Ornstein-Uhlenbeck process and its integral," *Physical Review*, 2084–2091.

HÄRDLE, W., J. FRANKE, AND C. HAFNER (2004): *Statistics of Financial markets*, Sprigner.

HÄRDLE, W. AND B. LÓPEZ CABRERA (2009): "Implied Market Price of Weather Risk," SFB 649 Discussion Paper 2009-001.

HULL, J. (2006): *Option, Future and Other Derivatives*, New York: Prentice Hall.

HURVICH, C. AND C. TSAI (1989): "Regression and time series model selection in small samples," *Biometrica*, 297–307.

Jewson, S., A. Brix, and C. Ziehmann (2005): *Weather Derivative Valuation: The Meteorologimal, Statistical, Financial and Mathematical Foundations.*, Cambridge University Press.

Judge et al. (1988): *Introduction to the Theory and Practice of Econometrics, 3rd edn.*, Oxford.

Schwarz, G. (1978): "Estimating the dimension of a model," *Annals of Statistics*, 2, 461–464.

Statement of Authorship

I hereby certify that this diploma thesis has been composed by myself, and describes my own work, unless otherwise acknowledged in the text. All references and verbatim extracts have been quoted, and all sources of information have been specifically acknowledged. It has not been accepted in any previous application for a degree.

Berlin, September 17th, 2009

Maria Osipenko

Declaration of Consent

After positive appraisal of this thesis, I agree that one copy of my presented thesis may remain at the disposal of the library of Humboldt- Universität zu Berlin.

Berlin, September 17th, 2009

Maria Osipenko