Variance swaps

A Master Thesis Presented

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Contents

1 Introduction **5**

2 Variance swaps **7**
2.1 Volatility trading . . . 7
2.2 Definition of a variance swap . . . 8
2.3 Marking-to-market . . . 9
2.4 Pricing . . . 10
2.5 Discrete approximation of continuous strikes . . . 15
2.6 Setting up a replicating portfolio . . . 17

3 Dispersion trading with variance swaps **19**
3.1 Idea of dispersion trading . . . 19
3.2 Data description . . . 24
3.3 Implementation of the basic dispersion strategy on DAX Index . . . 25
3.4 Implementation of the improved dispersion strategy on DAX Index . . . 26

4 Conclusion **29**

5 Bibliography **30**

List of Figures

2.1 DAX and DAX 3-month realized volatility from 1990 to 2008, daily . . . 8
2.2 Variance swap: cash flow at expiry 9
2.3 Variance Vega of call options with strikes from 50 to 200 and equally weighted portfolio 11
2.4 Variance Vega of call options with strikes from 50 to 200 and portfolio inversely weighted by strikes 11
2.5 Variance Vega of call options with strikes from 50 to 200 and portfolio inversely weighted by squared strikes 11
2.6 Discrete approximation of a log payoff 15
2.7 3-month DAX variance swap strike, realized volatilty, payoff of a long swap position (March 2000 - December 2005) 18

3.1 Implied (black) vs. realized volatility of DAX index 21
3.2 Implied (black) vs. realized volatility of Deutsche Bank AG 21
3.3 Implied (black) vs. realized volatility of Commerzbank AG 21
3.4 Average implied correlation (dotted), average realized correlation (gray), payoff of the direct dispersion strategy (solid black) 25

List of Tables

2.1 Replication of a DAX variance swap with inception 07.09.2001 and expiry in 3 month 17

3.1 Comparison of basic and improved dispersion strategy payoffs for the period from January 2004 to December 2008 26

1 Introduction

Variance swap is a derivative instrument that offers an efficient way for trading volatility of the underlying asset. It introduced a new view on volatility not only as on a risk measure, but also an independent asset class.

First variance swap contracts were traded in late 1998. But only after the development of a replication argument using a portfolio of vanilla options variance swaps became really popular. For the relatively short period of time these OTC derivatives developed from simple contracts on future variance to much more sophisticated products. And today we already observe the emergence of the 3-rd generation of volatility derivatives: gamma swaps, corridor variance swaps and conditional variance swaps.

Like any other asset, volatility can be used in a variety of trading strategies. Here are some common applications:

Taking directional bets on volatility (variance). Variance swaps can be used by investors having beliefs about future volatility and willing to obtain simple direct exposure without taking positions in delta hedged vanilla options.

Trading spreads on indices. Variance swaps are used to trade a volatility spread between two indices.

Hedging volatility exposure. Variance swaps can be used to hedge investor's volatility exposure.

Dispersion trading. A popular trade consisting in going short in index variance and long in variances of some of index constituents.

Constant development of volatility instruments and improvement of their liquidity already allow to trade volatility almost as easy as traditional stocks and bonds. Initially traded OTC, now the number of securities having volatility as underlying are avaliable on exchanges. Thus the variance swaps idea is reflected in volatility indices, also called "fear" indices. These indices are often used as a benchmark of equity market risk and contain expectation of option market about future volatility. Among these indices are VIX - the Chicago Board Options Exchange (CBOE) index on the volatility of S&P 500, VSTOXX on Dow Jones EURO STOXX 50 volatility, VDAX - on the volatility of DAX. These volatility indices represent the theoretical prices of 1-month variance swaps on the corresponding index. they are calculated daily and on intraday basis by the exchange from the listed option prices. Also recently exchanges stared offering the derivative products, based on these volatility indices - options and futures.

This master thesis discusses the variance swaps market, pricing of variance swaps and their application.

The paper is organized as follows: the second chapter describes the idea of the volatility trading using variance swaps. Then some properties of variance swaps as well as valuation and hedging methodology with portfolio of vanilla options are described. The third chapter introduces and investigates the performance of one of the most popular volatility strategies - dispersion trading. The strategy was tested using variance swaps on DAX and its constituents during the 5-years period from 2004 to 2008. the second part of the chapter suggests some improvements to the basic strategy setup and measures their influence on strategy's profitability. The strategy showed on average positive payoffs during the examined period.

2 Variance swaps

2.1 Volatility trading

Traditionally volatility is viewed as a measure of variability, or risk, of an underlying asset. However recently investors began to look at volatility from a different angle. It happened due to emergence of a market for new derivative instruments - variance swaps.

At the end of 1990s these derivatives allowed to change investors' view on volatility from a simple risk measure to the separate equity class that can be directly traded on the market. Hence now is it reasonable for investors to form believes or expectations about the development of this new "asset".

One can distinguish some commonly observed properties of an equity volatility:

- reverts to it's long-term mean
- jumps when markets crash
- experiences different regimes
- negatively correlated with the underlying

Figure 2.1 illustrates some of these properties and support the thesis that volatility shows behavior which is possible to describe, predict and therefore utilize.

It is necessary to mention that volatility trading also existed before the introduction of variance swaps. It is also possible to obtain the exposure to volatility by taking and delta-hedging the positions in vanilla options. However this alternative approach has obvious weakness - the necessity of continuous delta-hedging. Delta-hedging (constant buying/ selling of underlying) generates transaction costs and can be connected with liquidity problems: some stocks and indices can be expensive to trade or they may lack liquidity. On the contrary, variance swaps do not have this drawback, they offer straightforward and direct exposure to the volatility of the underlying asset.

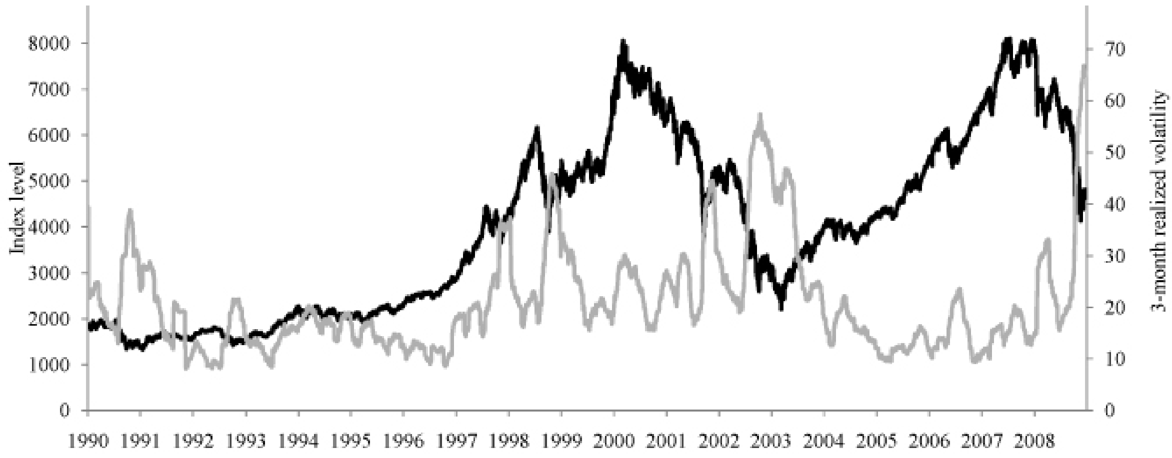

Figure 2.1: DAX and DAX 3-month realized volatility from 1990 to 2008, daily

2.2 Definition of a variance swap

Variance swap is a forward contract that at maturity pays the difference between realized variance σ_R^2 and predefined strike K_{var}^2 multiplied by notional N_{var}.

$$(\sigma_R^2 - K_{var}^2) \cdot N_{var} \tag{2.1}$$

When the contract expires the realized variance σ_R^2 can be measured in different ways, since there is no formally defined market convention. Usually variance swap contracts define a formula of a final realized volatility σ_R. It is a square root of annualized variance of daily log-returns of an underlying over a swap's maturity calculated in percentage terms:

$$\sigma_R = \sqrt{\frac{252}{T} \sum_{t=1}^{T} \left(\log \frac{S_t}{S_{t-1}} \right)^2} \cdot 100 \tag{2.2}$$

There are two ways to express the variance swap notional: variance notional and vega notional. Variance notional N_{var} shows the dollar amount of profit (loss) from difference in one point between the realized variance σ_R^2 and the variance swap strike K_{var}^2. But since market participants usually think in terms of volatility, vega notional N_{vega} turns to be a more intuitive measure. It shows the profit or loss from 1% change in volatility. Two measures are dependent and can substitute each other:

$$N_{vega} = N_{var} \cdot 2K_{var} \tag{2.3}$$

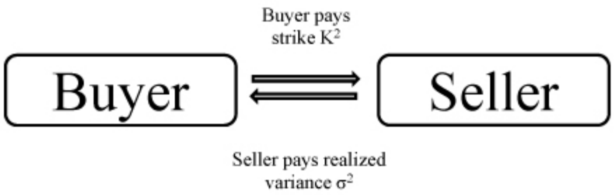

Figure 2.2: Variance swap: cash flow at expiry

Example: 3-month variance swap long

An investor wishes to gain exposure to the volatility of an underlying asset (e.g. DAX) over the next 3 month.

Trade size is 2,500 variance notional (represents a payoff of 2,500 per point difference between realized and implied variance).

If the variance swap strike is 20% ($K^2_{var} = 400$) and the subsequent variance realized over the course of the year is $(15\%)^2$ (quoted as $\sigma^2_R = 225$), the investor will make a loss because realized variance is below the level bought.

Overall loss to the long position: 437,500 = 2,500 x (400 - 225).

2.3 Marking-to-market

Marking-to-market of the variance swap is straightforward. If an investor wishes to close a variance swap position at some point t before maturity, he needs to define a value of the swap between inception 0 and maturity T. Here the additively property of variance is used. The variance at maturity $\sigma^2_{R,(0,T)}$ is just a time-weighted sum of variance realized before the valuation point $\sigma^2_{R,(0,t)}$ and variance still to be realized up to maturity $\sigma^2_{R,(t,T)}$. Since the later is unknown yet, we use it's estimate - the strike of a variance swap with inception in valuation point t and maturity at T. The value of the variance swap (per unit of variance notional) at time t is therefore:

$$\frac{1}{T}\left[t\sigma^2_{R,(0,t)} - (T-t)K^2_{var,(t,T)}\right] - K^2_{var,(0,T)} \tag{2.4}$$

2.4 Pricing

The pricing problem of a variance swap consists in finding the fair value of the strike. Following the logic of the futures contract pricing, the value of the variance swap at inception should be equal to zero. In case of a variance futures contract it can be achieved if the swap strike K^2_{var}, defined at inception, equals to the value of the realized variance σ^2_R, calculated at expiry.

In order to price a swap one needs to construct the replicating strategy that captures the future realized variance over swap's maturity. Then the cost of implementing this strategy will be the fair value of the future realized variance.

Assume investor has a long position in call option whose value is given by Black-Scholes equation: $C_{BS}(S, K, \sigma\sqrt{\tau})$.

Define the sensitivity to the underlying's variance (sometimes referred as "variance Vega") as:

$$V = \frac{\partial C_{BC}}{\partial \sigma^2} = \frac{S\sqrt{\tau}}{2\sigma} \frac{\exp(-d_1^2/2)}{\sqrt{2}\pi} \tag{2.5}$$

where

$$d_1 = \frac{\log(S/K) + (\sigma^2\tau)/2}{\sigma\sqrt{\tau}} \tag{2.6}$$

From 2.5 one can see that if we need a position in future realized variance then $C_{BS}(S, K, \sigma\sqrt{\tau})$ is imperfect vehicle since V is sensitive to stock price moves. The intuitive solution would be to construct an options portfolio with $V = const$.

What kind of portfolio is needed? Figures 2.3, 2.4, 2.5 show variance exposure of options with different strikes in differently weighted portfolios. First one can immediately observe that variance exposure of an option is the highest ATM then decreases with the price going deeper OTM or ITM. The second observation is that the variance exposure is higher for options with higher strikes. In portfolio on Figure 2.3 constituents are equally weighted, on Figure 2.4 they are weighted inversely, proportional to strike, on Figure 2.5 - to strike squared. The overall portfolio variance exposure is defined by a dotted line.

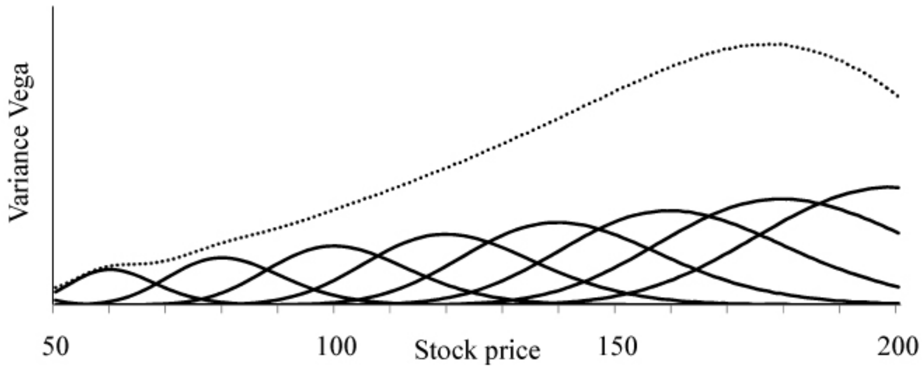

Figure 2.3: Variance Vega of call options with strikes from 50 to 200 and equally weighted portfolio

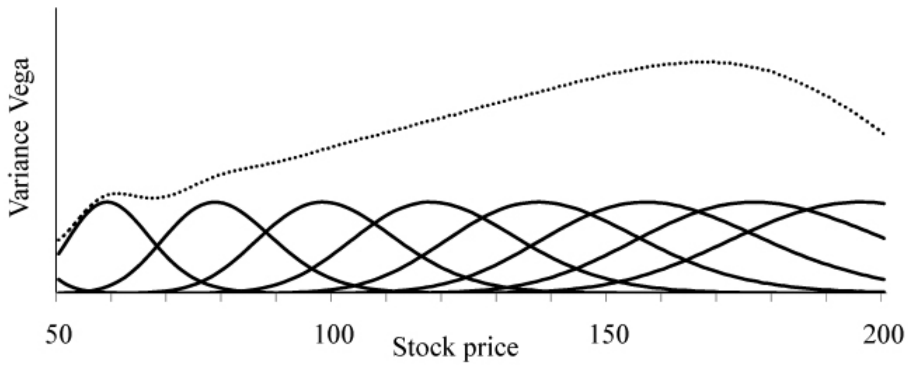

Figure 2.4: Variance Vega of call options with strikes from 50 to 200 and portfolio inversely weighted by strikes

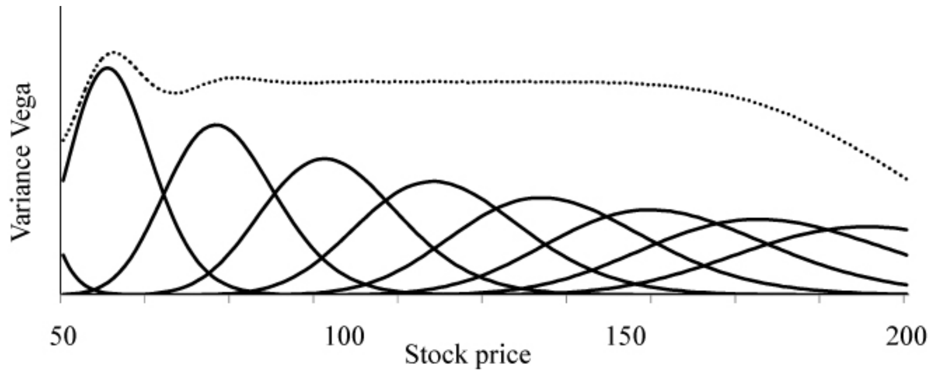

Figure 2.5: Variance Vega of call options with strikes from 50 to 200 and portfolio inversely weighted by squared strikes

It is visually clear, that in order to obtain V independent of underlying moves an investor should construct a portfolio similar to one shown on Figure 2.5.

In order to explain how the portfolio of vanilla options can capture the future realized variance more rigorous mathematical approach is necessary. This approach was first proposed by Carr and Madan (2002). The uniqueness of their idea first of all lies in fact that it requires very few assumptions about price development of the underlying.

The necessary assumptions are:

- existence of futures market with delivery dates $T' \geq T$
- futures contract F_t(underlying) follows a diffusion process with no jumps
- existence of European futures options market, for these options all strikes are available (market is complete)
- continuous trading is possible

Let's consider the following function:

$$f(F_t) = \frac{2}{T}\left[\log\frac{F_0}{F_t} + \frac{F_t}{F_0} - 1\right] \tag{2.7}$$

This function is twice differentiable with derivatives:

$$f'(F_t) = \frac{2}{T}\left[\frac{1}{F_0} - \frac{1}{F_t}\right] \tag{2.8}$$

and

$$f''(F_t) = \frac{2}{TF_t^2} \tag{2.9}$$

at time $t = 0$ the function $f(F_t)$ has a value of zero.

To find the dynamic of $f(F_t)$ use Itô's lemma. In general for every smooth twice differentiable function $f(F_t)$ Itô's lemma gives:

$$f(F_t) = f(F_0) + \int_0^T f'(F_t)dF_t + \frac{1}{2}\int_0^T F_t^2 f''(F_t)\sigma_t^2 dt \tag{2.10}$$

Substituting the above introduced function gives obtain expression for the realized variance:

$$\frac{1}{T}\int_0^T \sigma_t^2 dt = \frac{2}{T}\left[\log\frac{F_0}{F_T} + \frac{F_T}{F_0} - 1\right] - \frac{2}{T}\int_0^T \left[\frac{1}{F_0} - \frac{1}{F_t}\right] dF_t \tag{2.11}$$

Equation 2.11 shows that the value of a realized variance for the time interval from 0 to T equals to:

$$\frac{2}{T}\int_0^T \left[\frac{1}{F_0} - \frac{1}{F_t}\right] dF_t \tag{2.12}$$

- continuously rebalanced futures position. This position costs nothing to initiate and easy to replicate;

$$\frac{2}{T}\left[\log\frac{F_0}{F_T} + \frac{F_T}{F_0} - 1\right] \tag{2.13}$$

- **log contract**, static position of a contract that pays $f(F_T)$ at expiry and has to be replicated.

Then Carr and Madan (2002) argue that the market structure assumed above allows to represent any twice differentiable payoff function $f(F_T)$ in a following way:

$$f(F_T) = f(k) + f'(k)\left([(F_T - k)^+ - (k - F_T)^+] + \int_0^k f''(K)(K - F_T)^+ dK + \right. \tag{2.14}$$

$$+\int_k^\infty f''(K)(F_T - K)^+ dK$$

For $\log\frac{F_0}{F_T} + \frac{F_T}{F_0} - 1$, expansion around F_0 we get:

$$\log\left(\frac{F_0}{F_T}\right) + \frac{F_T}{F_0} - 1 = \int_0^{F_0} \frac{1}{K^2}(K - F_T)^+ dK + \int_{F_0}^\infty \frac{1}{K^2}(F_T - K)^+ dK \tag{2.15}$$

Equation 2.15 represents the payoff of a log contract at maturity $f(F_T)$ as a sum of:

$$\int_0^{F_0} \frac{1}{K^2}(K - F_T)^+ dK \tag{2.16}$$

- portfolio of OTM puts (strikes are lower then forward underlying price F_0), inversely weighted by squared strikes;

$$\int_{F_0}^\infty \frac{1}{K^2}(F_T - K)^+ dK \tag{2.17}$$

- portfolio of OTM calls (strikes are higher then forward underlying price F_0), inversely weighted by squared strikes.

Now coming back to equation 2.11 we see that trader in order to obtain a constant exposure to future realized variance over the period 0 to T should an inception buy and hold the portfolio of puts 2.16 and calls 2.17. In addition he has to initiate and roll the futures position 2.12.

We are interested in costs of implementing the strategy. Since the initiation of futures contract 2.12 costs nothing, the cost of achieving the strategy will be defined solely by the portfolio of options. In order to obtain an expectation of a variance, or strike K_{var}^2 of a variance swap at inception, we take a risk-neutral expectation of a future strategy payoff:

$$K_{var}^2 = \frac{2}{T}e^{rT}\int_0^{F_0}\frac{1}{K^2}P_0(K)dK + \frac{2}{T}e^{rT}\int_{F_0}^{\infty}\frac{1}{K^2}C_0(K)dK \tag{2.18}$$

Although we obtained the theoretical expression for a future realized variance it is still not clear how to make a replication on practice. First, in reality we have a discrete price process. Second, the range of strikes is also discrete. Therefore one should make a discrete approximation. Moreover, even assuming discreteness of strikes, is impossible to find vanilla options with a complete strike range (from 0 to ∞ or close to ∞) traded on the market.

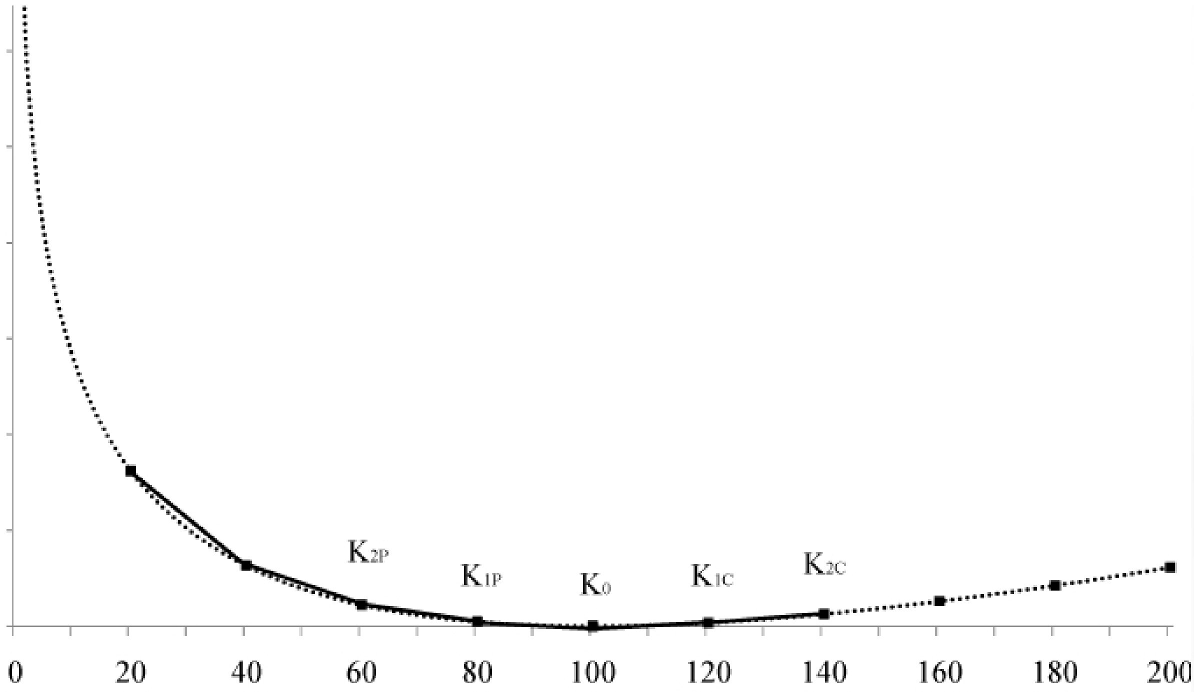

Figure 2.6: Discrete approximation of a log payoff

2.5 Discrete approximation of continuous strikes

In the previous section the assumption if infinite strikes was made. However this assumption is unrealistic. We can find only limited amount of strikes traded on the market. The availability depends on the underlying asset and some other market conditions. In order to solve a variance swap pricing problem one needs to make a discrete approximation. One approach was proposed by Dermann et al. (1998).

Taking a logarithmic payoff function 2.15 whose initial value should be equal to the weighted portfolio of puts and calls, we make a pairwise linear approximation.

This approach helps us to define how many options of each strike investor should purchase for the replication portfolio.

Figure 2.6 shows the logarithmic payoff (dotted line) and payoff if the replicating portfolio (solid black). Each linear segment on the graph represents the payoff of an option with strikes available for calculation. The slope of this linear segment will define the amount of options of this strike to be put in the portfolio.

For example, for the call option with strike $K_0 = 100$ the slope of the segment would be:

$$w_c(K_0) = \frac{f(K_{1c}) - f(K_0)}{K_{1c} - K_0} \tag{2.19}$$

The second segment - combination of call with strikes K_0 and K_{1c}:

$$w_c(K_{1c}) = \frac{f(K_{2c}) - f(K_{1c})}{K_{2c} - K_{1c}} - w_c(K_0) \tag{2.20}$$

The procedure for puts is similar. Following with all available strikes in a similar way one obtains the discrete approximation of a log-payoff, which is the true value of a future realized variance. The numerical example of replicating 3-month DAX variance swap using the introduced approximation is presented in the next section.

Implied Volatility	Moneyness	Type of option	B-S Price	Weight	Contribution
25.57	0.9250	Put	124.83	0.3481	34.81
24.95	0.9375	Put	142.25	0.3389	33.89
24.38	0.9500	Put	162.41	0.3300	33.00
23.82	0.9625	Put	185.14	0.3215	32.15
23.28	0.9750	Put	210.68	0.3133	31.33
22.78	0.9875	Put	239.75	0.3054	30.54
22.32	1.0000	Put	272.54	0.2979	29.79
21.87	1.0125	Call	232.11	0.2905	29.05
21.38	1.0250	Call	194.84	0.2835	28.35
20.99	1.0375	Call	162.20	0.2767	27.67
20.66	1.0500	Call	133.80	0.2702	27.02
20.36	1.0625	Call	109.06	0.2638	26.38
20.09	1.0750	Call	88.00	0.2577	25.77

Table 2.1: Replication of a DAX variance swap with inception 07.09.2001 and expiry in 3 month

2.6 Setting up a replicating portfolio

In this section we present an example of a variance swap replication using real market data. Using introduced in section 2.5 methodology we are replicating a 3-month DAX variance swap starting from 07.09.2001. Current index value is 4730, 3-month risk free rate (EURIBOR) 4.2%.

The replicating portfolio consists of 13 OTM vanilla options: 7 puts and 6 calls with strikes 85 ip apart. Black-Scholes option prices are calculated using data from implied volatility surfaces of DAX vanilla options tradet on EUREX exchange.

The prices, and numbers of options required for each strike are given in Table 2.1.

The value of options portfolio constructed with weights from the Table 2.1 is the total cost of replicating future variance and variance swap strike: $K_{var}^2 = 534.67$ or $K_{var} = 23.12$.

Figure 2.7 represents the strikes of 3-month DAX variance swaps, calculated daily for the period from March 2000 to December 2005 (black line) and 3-month realized variance for the same period (grey line). The difference between strike and realized variance is a swap

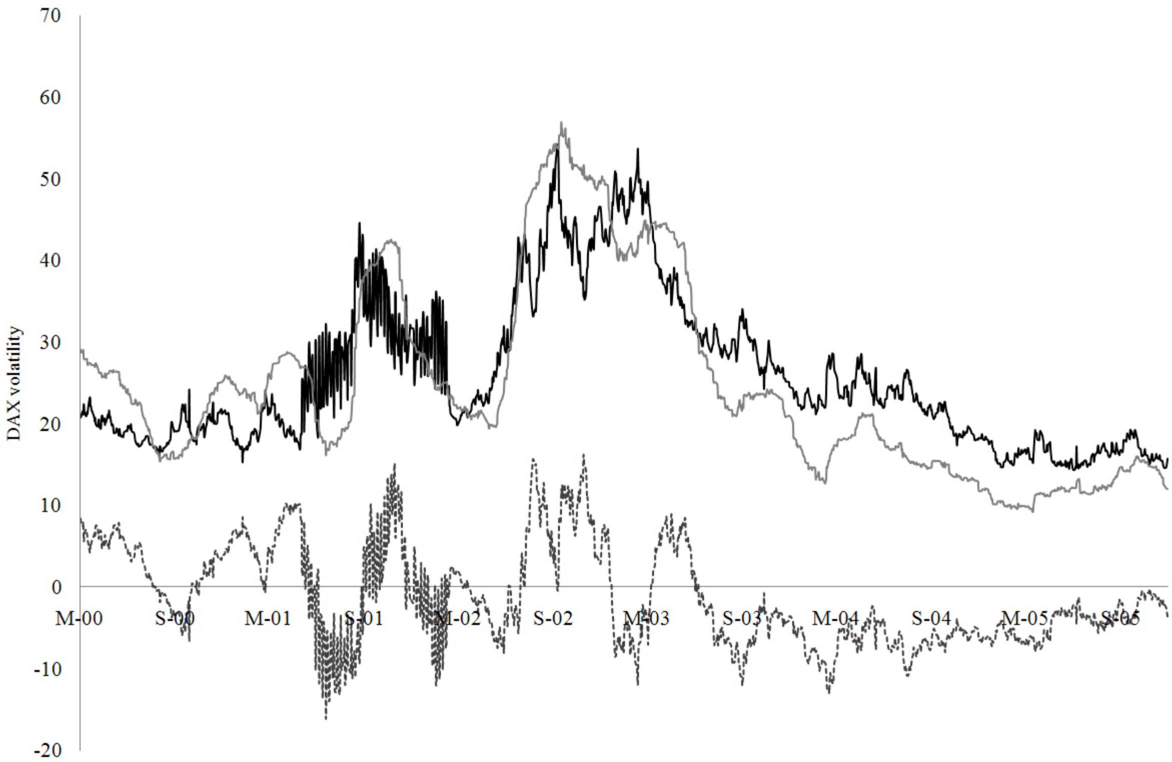

Figure 2.7: 3-month DAX variance swap strike, realized volatilty, payoff of a long swap position (March 2000 - December 2005)

payoff at expiry (dotted line). As one can see, on average the payoff of a long variance swap is negative, that means that the value of future variance implied (predicted) by the DAX options markets is often higher that its realized value. This is a known fact and concerns all index options traded on exchange. It was already investigated on the market and now is used for more sophisticated, then simple variance swaps, volatility strategies. One of these strategies that utilizes this observation is called **dispersion trading** and will be described in details in the next chapter.

3 Dispersion trading with variance swaps

3.1 Idea of dispersion trading

In modern portfolio theory the return on asset is a random variable. The return on assets portfolio is a sum of weighted returns of portfolio constituents. The risk of the portfolio (or basket of assets) can be measured by the variance (or alternatively standard deviation) of its return. Portfolio variance can be calculated using the following formula:

$$\sigma^2_{Basket} = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} w_i w_j \sigma_i \sigma_j \rho_{ij} \tag{3.1}$$

where σ_i - standard deviation of the return of an i-th constituent (also called volatility), w_i - weight of an i-th constituent in the basket, ρ_{ij} - correlation coefficient between the i-th and the j-th constituent.

Equity indices traded on the stock market also can be viewed as baskets of their constituents. The weights in this basket are defined in a different way, which depends on a particular index. Thus one can distinguish **price-weighted** indices such as the Dow Jones Industrial Average, where weights are defined according to the current market price of assets included in the index basket, and **market-value weighted** indices, such as DAX and S&P500, with weights according to the market capitalization of the company.

Let's take an arbitrary market index. We know the index value historical development as well as price development of each of index constituents. Using this information we can calculate the historical index and constituents' volatility using for instance formula 2.2. The constituents weights (market values or current stock prices, depending on the index) are also known for us. The only parameter to be defined are correlation coefficients of every pair of constituents ρ_{ij}. For simplicity assume $\rho_{ij} = const$ for any pair of i, j and call this parameter $\overline{\rho}$ - **average index correlation, or dispersion**. Then having index volatility σ_{index} and volatility of each constituent σ_i, we can express the average index correlation:

$$\overline{\rho} = \frac{\sigma_{index}^2 - \sum_{i=1}^{n} w_i^2 \sigma_i^2}{2\sum_{i=1}^{n}\sum_{j=i+1}^{n} w_i w_j \sigma_i \sigma_j} \tag{3.2}$$

Hence it appears the idea of **dispersion trading**, consisting of buying the volatility of index constituents and selling the volatility of the index. Corresponding positions in variances can be taken by buying (selling) variance swaps.

By going short index variance and long variance of index constituents we go short dispersion, or enter the direct dispersion strategy. Why can this strategy be attractive for investors? Dispersion strategy can show profitability because selling future realized variance we receive a price which usually turns out to be higher then the realized variance itself. This is due to the fact, that the price for variance is defined as a variance swap strike K_{var}^2. The strike (as explained in details in section 2.4) can be replicated by prices of options traded on the market. Therefore it reflects the volatility expectations that investors have at the time of entering the strategy.

Index options appear to be more expensive than their theoretical Black-Scholes prices, in other words they are overpriced, therefore the implied volatility, used for strike calculation, is too high and investors will pay too much for realized variance on the contract expiry. However in the case of single equity options one observes no volatility distortion. Both empirical findings are illustrated on Figures 3.1, 3.2 and 3.3.

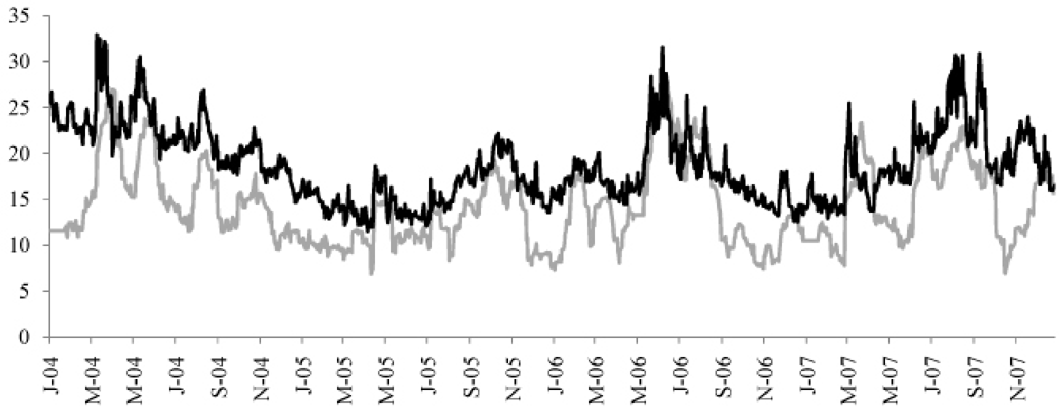

Figure 3.1: Implied (black) vs. realized volatility of DAX index

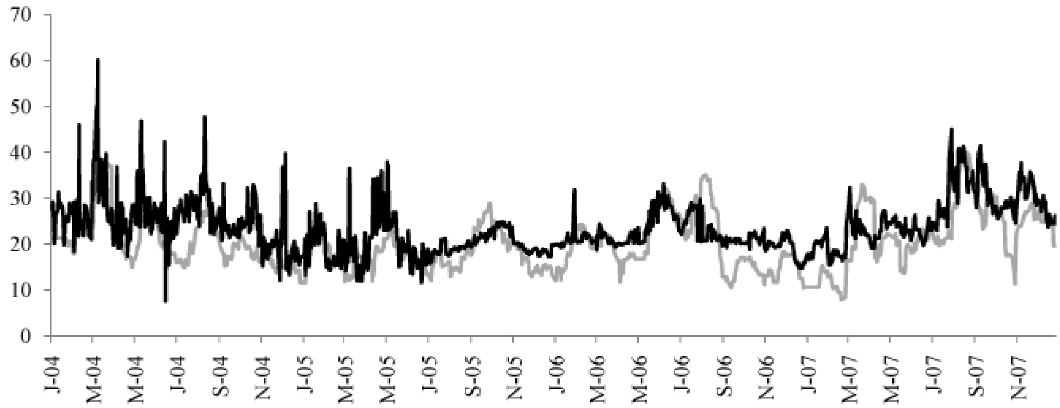

Figure 3.2: Implied (black) vs. realized volatility of Deutsche Bank AG

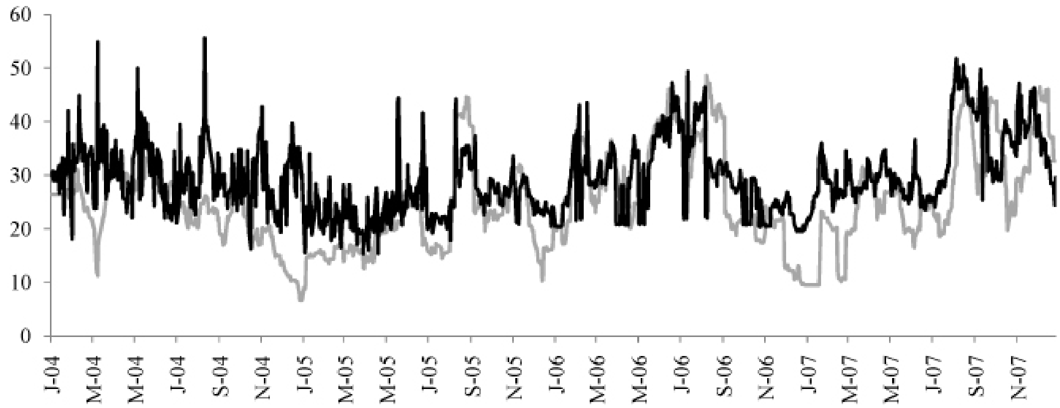

Figure 3.3: Implied (black) vs. realized volatility of Commerzbank AG

This empirical observation is used in dispersion trading. The mostly widespread dispersion strategy, direct strategy, is a long position in constituents' variances and short in variance of the index. This strategy should have on average positive payoffs. Hoverer under some market conditions it is profitable to enter the trade in opposite direction. This will be called - inverse dispersion strategy.

The payoff of the direct dispersion strategy is a sum of variance swap payoffs of each of i-th constituent

$$(\sigma^2_{R,i} - K^2_{var,i}) \cdot N_i \tag{3.3}$$

and of short position in index swap

$$(K^2_{var,index} - \sigma^2_{R,index}) \cdot N_{index} \tag{3.4}$$

where

$$N_i = N_{index} \cdot w_i \tag{3.5}$$

The payoff of the overall strategy is:

$$N_{index} \cdot \left(\sum_{i=1}^{n} w_i \sigma^2_{R,i} - \sigma^2_{R,Index} \right) - ResidualStrike \tag{3.6}$$

The residual strike

$$ResidualStrike = N_{index} \cdot \left(\sum_{i=1}^{n} w_i K^2_{var,i} - K^2_{var,Index} \right) \tag{3.7}$$

is defined by using methodology introduced before by means of replication portfolios of vanilla OTM options on index and all index constituents.

However on practice implementing this kind of strategy investors can face a number of problems. First, for indices with a large number of constituent stocks (such as S&P 500) it would be problematic to initiate large number of variance swap contracts. This is due to the fact that the market for some variance swaps didn't reach the required liquidity yet. Second, there is still the problem of hedging vega-exposure created by these swaps. It means a bank should not only virtually value (use for replication purposes), but also physically acquire and hold the positions in portfolio of replicating options. These options in turn require dynamic delta-hedging. So, such a big variance swap trade (as for example in case of S&P 500) requires from the bank additional human capital and can be connected with big transaction costs. The remedy would be to make a stocks selection and to form the offsetting variance portfolio only from a part of index constituents.

Also it was mentioned before that, sometimes the payoff of the strategy could be negative. In order words sometimes it is more profitable to buy index volatility and sell volatility of constituents. So the procedure which could help in decision about trade direction may also improve the overall profitability.

If we summarize, the success of the volatility dispersion strategy lies in correct determining:

- the direction of the strategy
- constituents for the offsetting variance basket

Next sections will present the results of implementing the dispersion trading strategy on DAX and DAX constituents' variances. First we implement its classical variant meaning short position in index variance against long positions in variances of all 30 constituents. Then the changes to the basic strategy discussed above are implemented and the profitability of these improvements is measured.

3.2 Data description

The dispersion trading strategy is implemented using DAX index options and options on DAX constituents (both puts and calls).

Options data

The data is obtained from the Eurex Tick-Data (Dataset No. 2002). Options data covers the 5 years period from January 2004 to December 2008.

Risk-free rate

EURIBOR 1-month rate is used as risk free rate for option prices and implied volatilities calculation. The data was obtained from Bloomberg database for the period from January 2004 to December 2008.

DAX and constituents end-of-the-day daily price series

Daily prices are obtained from Bloomberg database for the period January 1998 - December 2008 in order to make a covariance matrix estimation necessary for PCA and GARCH model estimation.

DAX composition over examined period and constituent weights

The DAX index is a market value weighted index. The list of constituents, as well as the weights, are time variant. For each trading date the current weights and constituents lists were taken. The data was obtained from Bloomberg database and covers period from January 2004 to December 2008.

Option prices, EURIBOR rates and underlying prices were used to calculate the implied volatility smiles for each trading date. Implied volatilities obtained were afterwards used for calculation of option prices for variance swap replication portfolio.

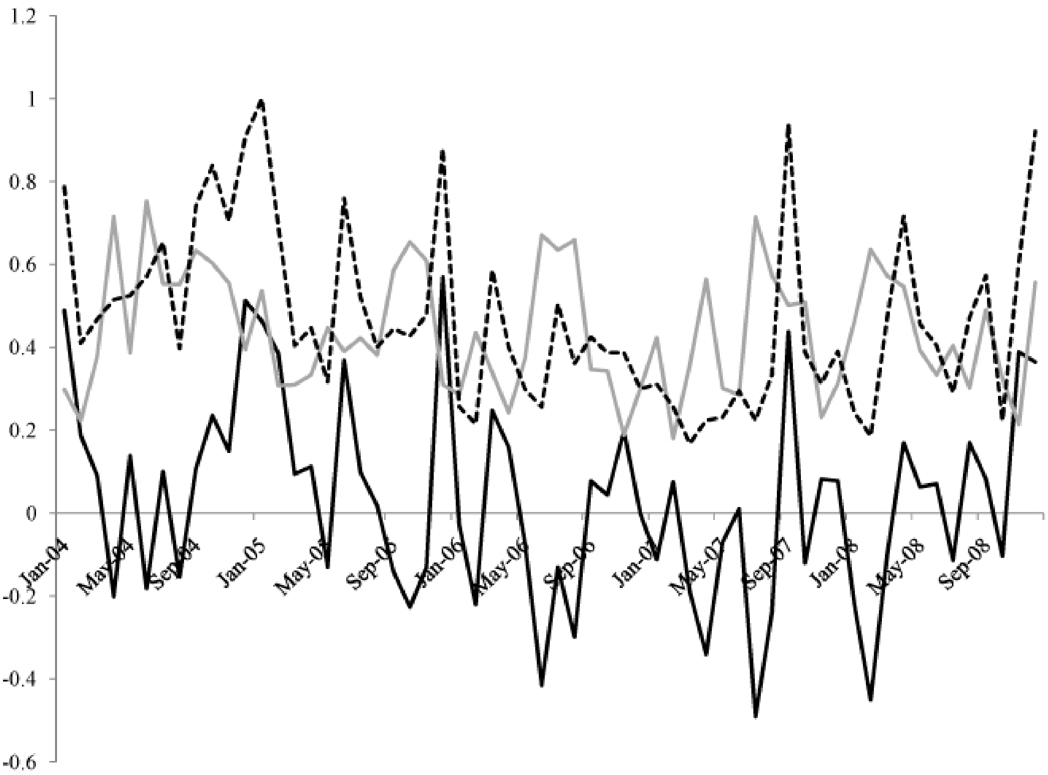

Figure 3.4: Average implied correlation (dotted), average realized correlation (gray), payoff of the direct dispersion strategy (solid black)

3.3 Implementation of the basic dispersion strategy on DAX Index

In this section we investigate the performance of a dispersion trading strategy over the 5 years period from January 2004 to December 2008. The dispersion trade was initiated at the beginning of every moth over the examined period. Each time the 1-month variance swaps on DAX and DAX constituents were traded. At the end of the month the realized volatilities were exchanged on the variance swap strikes (defined at the beginning of the month) and the new dispersion trade was initiated. The procedure was repeated 60 times.

First we implement the basic dispersion strategy, which shows on average positive payoffs over the examined period (Figure 3.4). Then several improvements to the strategy are proposed. The first extension helps to define at initial point the direction of the strategy (if one should go long of short dispersion). The second improvement mostly concerns dispersion strategies on indices with many constituents and helps to chose stocks for the

Strategy	Mean	Median	Std. Dev.	Skewness	Kurtosis	J-B	Probability
Basic	0.032	0.067	0.242	0.157	2.694	0.480	0.786
Improved	0.077	0.096	0.232	-0.188	3.012	0.354	0.838

Table 3.1: Comparison of basic and improved dispersion strategy payoffs for the period from January 2004 to December 2008

offsetting variance portfolio.

The basic dispersion strategy consists of selling short volatility of the index and purchasing volatilities of index constituents through selling/buying corresponding variance swaps. Descriptive statistics shows that the average payoff of the strategy is higher, but close to zero. Therefore in the next section several changes are introduced that would help to improve the strategy profitability.

3.4 Implementation of the improved dispersion strategy on DAX Index

It was discussed already that index options are usually overestimated (which is not the case for single equity options), the future volatility implied by the index options will be higher then realized volatility meaning that the direct dispersion strategy is on average profitable. However the reverse scenario may also take place. Therefore it is necessary to define weather to enter a dispersion (short index variance, long constituents variance) or reverse dispersion (long index variance and short constituents variances) strategy.

This can be done by making a forecast of a future volatility with GARCH (1,1) model and multiplying the result by 1.1, which was implemented in the paper of Deng (2008) for dispersion strategy for S&P500 correlation. In this paper we do the estimate of a GARCH (1,1) every 60 times the trade was initiated. The 252 nearest days price history was always taken for the estimation. Then on basis of the estimated model a forecast for 1 month (21 days) in advance was made. The benchmark number obtained was then used for comparison with a strike of an index variance swap replicated for a strategy. If the average correlation, calculated using index and weighted constituents variance swap strikes is higher than the average correlation, calculated using variances predicted by the GARCH (1,1) model, then one should enter a direct dispersion strategy - sell index variance and buy constituents variance. If the variance predicted by GARCH is higher than variance implied by the option market one should enter the reverse dispersion trade (long index variance and short constituents variances).

Table 3.1 shows that the applied method helped to improve the profitability of the strategy. The average payoff increased on 41.7%. Therefore we conclude that one can capitalize the dispersion strategy in both directions. The second improvement serves to decrease transaction cost and cope with low market liquidity.

Since transaction costs should also be taken into consideration, it is reasonable to make a stocks selection for the offsetting variance portfolio. The selection can be made using Principal Components Analysis (PCA). The Principal Components Analysis is made in order to define the most "effective" constituent stocks, which help to capture the most of index variance variation.

Applying PCA procedure to the variance-covariance matrix of the historical index constituents returns (market-value weighted) one can define the vectors of principal components that capture the most of the variability of the analyzed portfolio.

The procedure can be implemented in the following steps:

- estimating the variance-covariance matrix of weighted log return of index constituents
- defining eigenvector which corresponds to the highest eigenvalue of the matrix
- defining the first n principal components the explain the 90% of the index returns variation
- and finally selecting stocks to the offsetting portfolio which have the highest correlation with these principal component

This procedure was applied to the DAX constituents each of the 60 trading days, each time 10 stocks were selected, which have the highest correlation with the first PC.

According to our results, the 1-st PC explains on average 50% of DAX variability, thereafter each next PC adds only 2-3% to the explained index variability, so it is difficult to distinguish the first several that explain together 90%. If we take stocks, highly correlated only with the 1-st PC, we can significantly increase the offsetting portfolio's variance, because by excluding 20 stocks from the portfolio we make it less diversified, and therefore more risky.

However it was shown that one still can obtain reasonable results after using the PCA procedure. Thus in the paper of Deng (2008) it was successfully applied to S&P500 index representing lager then DAX portfolios of stocks. Index is already considered to be a good diversified portfolio of stocks. But trading all 500 variance swaps can generate large transaction costs, which will negatively influence the return on trade. Therefore the subset selection is necessary. After calculating the principal components of S&P500 it was easier to define the first group that explains around 90% of index variation. Then selection of stock having the highest correlation with these components for the offsetting portfolio made more sense then in case of DAX.

4 Conclusion

In this paper we described variance swaps - recently emerged derivative instruments on future realized volatility.

Today the wide range of volatility-dependent instruments are available for investors. One can bet on volatility directly, on correlation, on smile etc.

One of the popular volatility derivative trades developed recently is dispersion trading. The idea of this trade was in detail discussed in this paper and then implemented on DAX and DAX constituents' volatilities for the period 2004-2008. On average the trade showed positive payoffs, however some further improvements were propose. Thus, prediction of future volatility with GARCH (1,1) helped to increase the profitability of the strategy. On the other hand, the PCA procedure for constituents' selection didn't show satisfactory results.

In conclusion, variance swaps discussed in the paper can be characterized as a good alternative to the traditional methods of trading volatility (delta hedged options, straddles, strangles). The relative simplicity of trade initiating helped to develop and implement more sophisticated strategies such as dispersion trading discussed. Moreover, new volatility instruments are bound to emerge in future which shows that the liquidity of the market will grow and every investor will soon find the product answering his needs.

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Berlin, January 26, 2009