

SFB 649 Discussion Paper 2006-002

# Calibration Design of Implied Volatility Surfaces

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This research was supported by the Deutsche  
Forschungsgemeinschaft through the SFB 649 "Economic Risk".

<http://sfb649.wiwi.hu-berlin.de>  
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin  
Spandauer Straße 1, D-10178 Berlin



SFB 649 ECONOMIC RISK BERLIN

# Calibration Design of Implied Volatility Surfaces

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## Abstract

The calibration of option pricing models leads to the minimization of an error functional. We show that its usual specification as a root mean squared error implies fluctuating exotics prices and possibly wrong prices. We propose a simple and natural method to overcome these problems, illustrate drawbacks of the usual approach and show advantages of our method. To this end, we calibrate the Heston model to a time series of DAX implied volatility surfaces and then price cliquet options.

**Key words:** calibration, data design, implied volatility surface, Heston model, cliquet option

**JEL codes:** C80, G13

**Acknowledgement:** This research was supported by Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk" and by Bankhaus Sal. Oppenheim. The authors would like to thank Bernd Engelmann and Peter Schwendner for helpful discussions.

# 1 Introduction

A stable calibration of option pricing models is of paramount importance for banks because of pricing of exotic options. Models are calibrated to the liquid market of plain vanillas, e.g. to price or implied volatility surfaces. But the data observed on this market are not uniformly distributed in the moneyness-maturity region instead they have a special design. This special data design should be taken care of in the calibration routine in order to avoid fluctuating or even wrong prices of exotic options.

We consider in Section 2 the special design of implied volatility surfaces. In Section 3 we calibrate DAX implied volatility surfaces by the usual approach and identify some problems. In Section 4 we describe our method and show how it deals with the problems of the usual approach. In Section 5 we compare the methods and see pricing differences for long times to maturity. Finally we draw our conclusions in Section 6.

## 2 Data design

Figure 1 shows the grid on which the EUREX reported the settlement volatilities of European DAX options on March 1st, 2004.

The figure shows that the data have a special design. They come in strings because only options with certain times to maturity are traded. These strings are not uniformly distributed in the time to maturity dimension because for short times there are more strings. The distribution within strings also differs: On the one hand the moneyness range changes on the other hand the frequency of the observations within a string differs. Moreover there are missing observations for out of the money calls. In addition the maximal time to maturity varies for different underlyings and changes even for one underlying if new products are issued by the exchange.

Another special feature becomes apparent in time series analysis. The grid of observations moves in time because on a new day all options have one day less to maturity than the day before. In this way the grid moves to zero days to maturity and whole strings disappear when their time to maturity is zero (or under a prescribed level). Moreover new strings appear in the grid when a new option with a long time to maturity is issued.

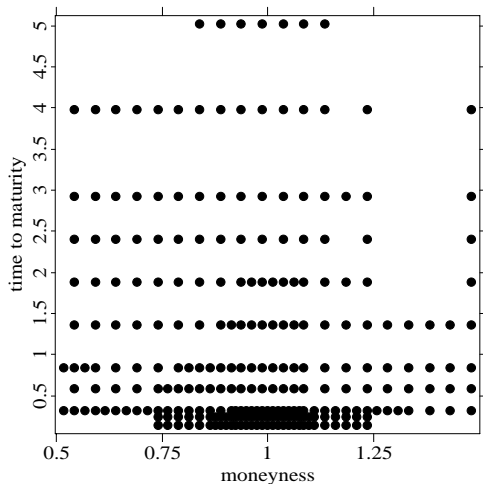


Figure 1: Grid of the DAX implied volatility surface of the EUREX on March 1st, 2004. (only for moneyness between 0.5 and 1.5)

Thus the grid of observations has a special design that changes in time. Both of these points should be taken care of in calibration methods in order to avoid wrong or fluctuating prices.

### 3 The mean squared error

Option pricing models have been calibrated for a long time. Usually the difference between the market and the model quantities is measured by a root mean squared error:

$$\sqrt{\frac{1}{n} \sum_{i=1}^n (Q_i^{mod} - Q_i^{mar})^2}$$

where  $Q$  denotes the price or the implied volatility and *mod* (*mar*) refers to a model (market) quantity. The index  $i$  runs over all observations of the surface that are considered in the calibration. Bakshi et al. (1997) minimize such a functional in their calibration and also Schoutens et al. (2004) use this error measure for their analysis. Variations such as relative errors instead of the absolute differences are preferred sometimes.

		24/02	25/02	25/02 with string
mean squared error	T=1	0.0621	0.0618	0.0620
	T=3	0.0730	0.0749	0.0728
volume error	T=1	0.0612	0.0615	0.0613
	T=3	0.0754	0.0750	0.0750

Table 1: Prices of cliquet options on 24th and 25th February 2004.

For independent and identically distributed observations this error measure converges to the  $L_2$ -norm. But the data from the EUREX are not independent and identically distributed because of the special design. Hence this error measure lacks this theoretical underpinning. Moreover its economic interpretation is not clear because it is some kind of average error at some points. This leads to two problems: The grid and therefore also the error measure are given by the exchange. The grid and hence the meaning of the error change over time. Preferable would be an error measure that has the same meaning for all surfaces and this meaning should not depend on the grid given by the exchange.

Some authors deal with this problem by disregarding whole strings. Despite its simplicity this method has some drawbacks: It is possible to choose on one day a grid that works well for the analysis. But this subjective approach can hardly be generalized sufficiently for a time series analysis. Moreover disregarding relevant observations cannot lead to a better statistic and often the resulting calibrations are significantly worse.

The above error measure is generally implemented by choosing a region of interest in the moneyness-maturity plane. As we consider here options with  $T = 1$  or  $T = 3$  years to maturity we use only observations with expiry between 4 months and 4 years. In the moneyness dimension we consider only option with moneyness between  $0.8 - 0.1T$  and  $1.2 + 0.1T$  where  $T$  denotes time to maturity. We analyze cliquet options whose strikes lie within this region of interest. In Section 5 we consider the question of wrong prices. Here we show only the fluctuating prices resulting from the movement of the grid.

In table 1 we present the prices of cliquet options in the Heston model. These cliquet options and all others considered in this work have local floors (caps) of  $-8\%$  ( $8\%$ ) and a global zero floor for the

periods  $t_i = iT/3$ ,  $i = 1, 2, 3$ . The model has been calibrated w.r.t. the quadratic errors of implied volatilities by a global optimization routine and the prices have been computed by Monte Carlo methods using 250 time steps a year and 1000000 simulations (max. relative std. error: 0.1%).

The table shows the prices on two subsequent days. On the second day a string drops below the maturity limit of 4 months and is not considered anymore by the above error measure. The table also gives the prices that would have been calculated if the string were still considered in the calibration. It becomes clear from the table that the prices on the first day and on the second day with the string still considered are quite close for both times to maturity. But the prices on second day without the string differ clearly from the other prices for the long time to maturity. The price of these cliquet options is about 2.5% higher. Thus these prices differences are significant compared to the usual profit marge. The prices of the cliquet options with 1 year to maturity do not fluctuate when the string disappears.

Similar problems occur when a new string appears in the grid. If no region of interest is specified the same problems arise because new strings still appear and the strings of expired options disappear.

## 4 The volume error

Although the input data for the calibration are observed only for a few maturities the calibrated model is often used for all times to maturity to price exotic options. Hence we want to measure the difference between the market and the model surface for all times to maturity and not only for a few maturities. This leads us to consider the volume between the two surfaces. This approach resolves the possibly wrong prices because the weight is distributed (more) uniformly over all times to maturity. We consider again a region of interest in the moneyness-maturity plane and compute the volume always over this area. To this end, the error surface which is the squared difference the market and the model surface has to be approximated. As the error measures should have the dimension price or implied volatility we divide the volume by the area of the region of interest.

Hence, we compute first the observed points of the error surface

by

$$e_i \stackrel{\text{def}}{=} (Q_i^{\text{mod}} - Q_i^{\text{mar}})^2,$$

where the index  $i$  runs over all observations in the area of interest and  $Q$  denotes prices or implied volatilities. Then we approximate the volume  $V_e$  under the error surface based on the points  $e_i$  ( $i = 1, \dots, n$ ) and finally we compute the error measure  $\epsilon$  by

$$\epsilon \stackrel{\text{def}}{=} \sqrt{V_e/A}$$

where  $A$  denotes the area of the region of interest.

There are many ways to approximate the volume of the error surface. Non- or semiparametric methods (see e.g. Härdle et al. (2004)) give a good fit if there are enough data points. But in general implied volatility surface do not have enough data points for this approach (especially for long times to maturity). Moreover smoothing is relatively time consuming for computations and depends a lot on the smoothing parameter.

Another approach is to construct points of the implied volatility surface on a stable uniform grid and then to approximate the error in the usual way. This method depends critically on the constructed points and so far there is no generally accepted construction procedure. Arbitrage and especially calendar arbitrage make such a construction difficult (see e.g. Kahalé (2004)). Moreover a fine grid increases the calibration time significantly. Although this approach has advantages for some models (e.g. local volatility) our simpler method leads to similar results and requires less computation time. In addition it has the precise interpretation as mean height of the volume.

We consider as above the moneyness-maturity region

$$\{(m, T) : 1/3 \leq T \leq 4, 0.8 - 0.1T \leq m \leq 1.2 + 0.1T\}.$$

For this simple region of interest we use some linear interpolation in the maturity direction for calculating the volume. We compute the volume separately between the maturities. To this end, we first construct for each maturity boundary values of the error string by extrapolation. Then we approximate the area under this error string for each maturity. Afterwards, mean heights can be computed for these

areas so that the area under the error string can be represented by an equivalent rectangle. Now the volume between two adjacent maturities can be approximated by the volume between the corresponding rectangles and this volume can be calculated exactly from elementary solids. For the first and last maturity we consider only the part of the solid that lies over the region of interest (by linearly interpolating the heights).

The table 1 shows the prices of cliquet options when the model is calibrated w.r.t. the volume error. The prices of the cliquets with 1 year to maturity are quite similar to the corresponding prices calculated from the parameters of the other calibration. But the prices of the cliquet with 3 years to maturity differ significantly to the prices of the other calibration. Moreover the prices do not fluctuate when an implied volatility string disappears. Hence the volume error eliminates the fluctuating prices. The price differences to the usual approach are analyzed in the next section.

## 5 Pricing exotic options

We have seen that the prices of cliquet options jump when an implied volatility string disappears and the mean squared error is used for calibration.

The error that describes the goodness of fit jumps at the same time. This is shown in figure 2 that reports the errors for 20 trading days. After 10 trading days an implied volatility string falls out of the region of interest. The blue line jumps exactly at this point of time. As the blue line represents the root mean squared error the figure shows that these errors jump. Thus this error measure is hard to interpret and does not give reliable results. The volume error which is shown as red line shows no jumps and hence has the same interpretation for different surfaces or days.

When the model is calibrated w.r.t. the mean squared error the prices and the errors jump when a volatility string expires. Figure 3 shows the prices of cliquet options on 10 trading days before and after the expiration of a string. The red line describes the prices for the calibration w.r.t. the volume error. These prices show a stable downward trend with small jumps that could be attributed to changes of the input data (spot, etc). On the other hand the prices from the calibration w.r.t. the mean squared error jump. After the jump the

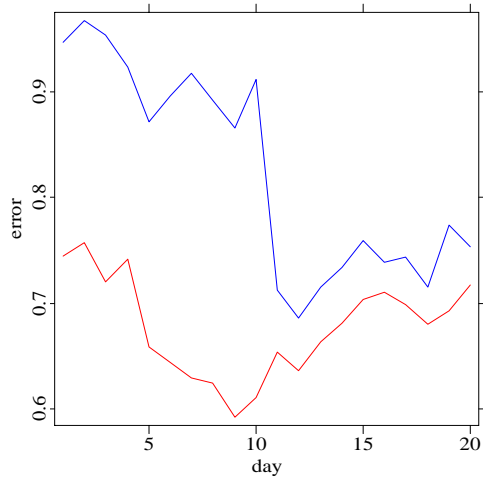


Figure 2: Calibration errors from 11/02/2004 to 09/03/2004. (blue: mean squared error, red: volume error)

two time series of prices are similar. But before the jump they differ about 2.5%. As the prices are similar for cliquets with small times to maturity and also for cliquets with long times to maturity after jumps the price difference before the jump can be seen as wrong prices.

Thus these plots show how prices jump and where possibly wrong prices appear. These two observations correspond to the movement of the strings and the special data design.

## 6 Conclusion

We have analyzed different calibrations of the Heston model and their implications for the pricing of exotic options. The model has been calibrated by minimization of different error functionals.

We have shown that the usual specification of this error measure as root mean squared error leads to fluctuating prices of exotic options. Moreover, the usual error has the shortcomings that it has no direct economic interpretation and this meaning is also changing over time. These problems stem from the special design of implied volatility surfaces.

In order to take care of this data design we have proposed an er-

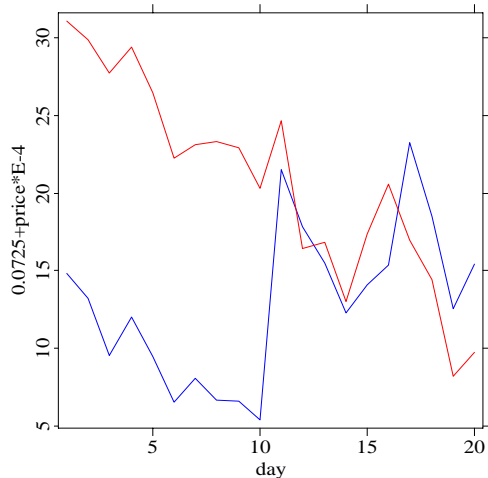


Figure 3: Prices of cliquet options from 11/02/2004 to 09/03/2004. (blue: mean squared error, red: volume error)

ror measure based on the volume between the market and the model surface. Calibration w.r.t. this error gives rise to continuous changes in the price and the error itself. Thus the fluctuation have been eliminated. Moreover this measure seems to correct wrong prices before expiration of implied volatility strings. In addition this error measure has the same interpretation as mean height of the volume between the model and the market surface for different days or surfaces.

Our approach can easily be generalized by weighting the errors observed between the market and the model. Such weighting by a 2 dimensional density function can be seen as a generalization of specifying a region of interest in the moneyness-maturity plane. The approach described in Section 4 puts more weight on long maturities because we consider a wider moneyness range for long times to maturity. Such a weighting makes sense for pricing options expiring after a long time.

Another approach for measuring the error is based on the interpolation of the observed implied volatility surface. This method leads to similar results but suffers from two drawbacks: There is no unique or generally accepted way for arbitrage free interpolation and the interpolated observations slow down calibration routines significantly. Moreover it leads to similar results as our approach.

Stable calibration routines are important for equity models because they imply prices of exotic options that are continuous in time. Such prices are the basis for trading exotic options. But stable parameters are even more important for calculating the greeks that are used by traders for hedging. Hence the stable and fast calibration method proposed helps reducing losses in derivatives trading resulting.

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This research was supported by the Deutsche  
Forschungsgemeinschaft through the SFB 649 "Economic Risk".

