

# Delta-Hedging a Hydropower Plant Using Stochastic Programming

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\* **Abstract** An important challenge for hydropower producers is to optimize reservoir discharges, which is subject to uncertainty in inflow and electricity prices. Furthermore, the producers want to hedge the risk in the operating profit. This article demonstrates how stochastic programming can be used to solve a multi-reservoir hydro scheduling case for a price-taking producer, and how such a model can be employed in subsequent delta-hedging of the electricity portfolio.

**Keywords:** Hydroelectric scheduling, stochastic programming, reservoir management, risk management, electricity markets, stochastic hydrology, cascaded reservoirs, case study, joint hedging and production, electricity prices, Nord Pool.

## 1 Introduction

The main challenge for a hydro producer with reservoir capacity is deciding on how much electricity to produce today versus future periods. To obtain the best possible balance between immediate and future costs of using the water, uncertain factors (inflow and electricity prices) must be considered. Stochastic optimization models for generation planning are in regular use in hydro-dominated systems [10].

An additional challenge for a hydropower generator is to reduce the risk of low profit from its entire operation. Risk management adds value by reducing the expected cost associated with financial distress.

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In this article, electricity prices and inflow are modelled as stochastic processes. The price sub-model is calibrated to future- and swap prices that are observed in the market. Price and inflow are assumed to be negatively correlated, and the price process is exogenous to the production optimization model, consistent with a price-taker assumption. The negative correlation is due to the positive relationship between local inflow and inflow for the whole system, and the negative relationship between system inflow and price. The price-taker assumption makes the analysis valid for a small hydropower producer operating in a well-functioning electricity market.

For the purpose of illustrating our modeling approach, relatively simple statistical models are fitted based on historical spot prices and inflow. Monte Carlo simulation is employed to generate an initial set of scenarios for price and inflow. Based on this high number of scenarios, a scenario tree is generated using the approach of Heitsch and Römisich [12]. In a real situation, more care should be taken modeling the scenarios in terms of analyzing historical data, incorporating expert judgments and existing forecasting models, and in the construction of event trees, making sure that the underlying data generating processes are well represented (including the information/stage structure) so that in turn the hydropower plants are operated efficiently. The corresponding stochastic program [13] is set up and solved as a large deterministic equivalent LP [24]. This creates acceptable solution times for a problem with 14 reservoirs and 10 power stations. The optimal objective function value converges as the discretization of the stochastic processes is refined.

Profit risk is sought reduced using so-called delta-hedging. Delta-hedging of a portfolio means buying and selling contracts so that the total hedged cash flows are insensitive to short-term movements in the contract prices. This is a standard method for risk hedging as explained in textbooks, e.g. McDonald [18]. In contrast to the approach of Fleten, Wallace, and Ziemba [9], or of [20, 2, 15, 5], with delta-hedging there is no need to expand the power optimization model with contract trading and a risk averse objective function. This means that less effort needs to be spent in model development and maintenance, and that the computing time will be shorter. We discuss how delta-hedging can be implemented in the context of an electricity portfolio and provide initial calculations.

The remainder of the article is structured as follows: The relevant markets are outlined in Sect. 2, while hedging is described in Sect. 3. Sect. 4 is devoted to modeling the stochastic processes as well as the decision problem itself. Results are given in Sect. 5, while we sum up in Sects. 6 and 7.

## 2 The Nordic Power Market

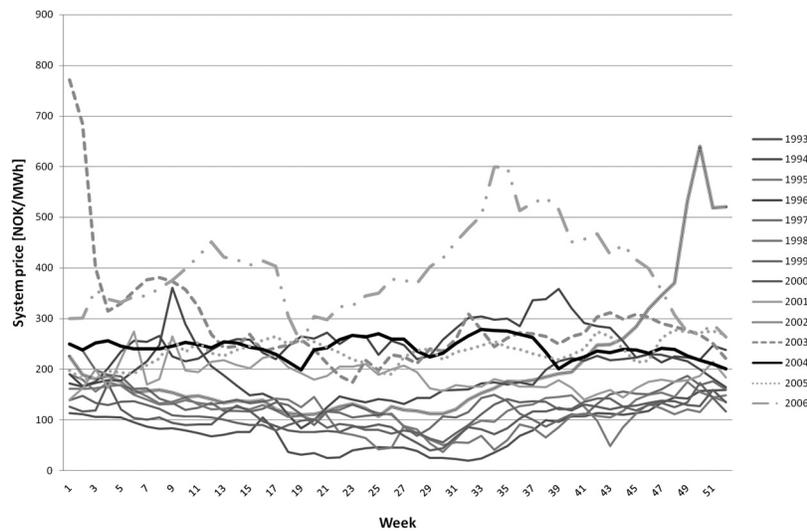
In the aftermath of the 1991 deregulation of the Norwegian power system, a Nordic power exchange was formed, Nord Pool. As the other Nordic countries joined Norway in the deregulation process, the scope of the exchange's activities steadily widened. Today the exchange is responsible for a number of markets, of which the

day-ahead market for physical delivery is central. There are also financial markets (Eltermin and Eloption) which enable future power trading.

## 2.1 The Physical Market

Nord Pool offers exchange of electricity through the Elspot market, which is a day-ahead market with an hourly resolution. Participation in the market is voluntary. Participants submit their bids electronically on bidding forms, and a market clearing calculation is performed, determining the price for each of the 24 hours of the following day. A so-called system price is also calculated, as an average over the 24 hours, under an assumption of unlimited transmission capacity within the whole system. Price areas and counter-trading is used to handle congestion problems.

The system price typically shows seasonal and diurnal patterns [17]. More than 95% of Norwegian production comes from hydro plants. Consequently the system price will be very much dependent on reservoir levels and inflow. As electricity-based residential heating is the norm in Norway, load usually increases in periods with cold spells (and correspondingly low inflow). This may induce spikes in the spot price.



**Fig. 1** Weekly system price in NOK/MWh from 1993 to 2006, source: Nord Pool.  $1 \text{ €} \approx 8 \text{ NOK}$ .

## ***2.2 The Financial Market***

In the financial markets Nord Pool offers futures, swaps, options, electricity certificates, as well as emission allowance and certified emission reduction contracts. In the Eltermin market futures and swaps are traded. Future contracts are traded on a daily and weekly basis, swaps for months, quarters and years. All contracts are standardized and have a size of 1 MW during the delivery period. All these contracts have financial settlement, and the system price is used as a reference. The swap contracts are termed “forwards” on the exchange, and in this article the two terms will be used interchangeably. The Eloption market offers European options, or “swaptions”, with quarters and year forward contracts as the underlying.

For a hydro producer the information available in the financial markets of Nord Pool is useful when planning the optimal use of the water resources, as is investigated empirically by Fleten and Keppo [6].

## ***2.3 Electricity Price Characteristics***

Several studies on the characteristics of electricity prices have been made, e.g. Lucia and Schwartz [17]. Typical observed characteristics for the system price are price spikes, mean reversion, seasonality and excess kurtosis and skewness in price changes and log returns.

## **3 Hedging of Power Production**

The idea of hedging electricity portfolios via stochastic programming was introduced by Fleten, Wallace, and Ziemba [8], Fleten et al. [9]. The model was a “traditional” multistage stochastic program with a focus on the integration of production and financial trading. In this article our goal is to use a hydropower case to discuss a different approach to hedging, closer to what is learned by students in business schools and universities. A different but related approach is demonstrated in [22].

The key insight in modern option pricing theory is that it is possible to construct financial portfolios with exactly the same payoff structure as the underlying derivative. Hedging hydropower production means a search for a set of products that do exactly this - replicate the cash flows generated by hydro production. There are several possible reasons why a producer would prefer to hedge production, typically based on capital market imperfections (in a wide sense) that imply that risk averse behavior increases the value of the firm.

Wallace and Fleten [23] argue that hedging should not affect the actual production plan, given an efficient derivative market for hedging price risk. According to standard financial theory the market value of a financial contract is zero when first

entered into and will therefore not change the market value of production. A change in the production plan, will, however, change the market value of the production.

Risk is in this context typically related to price and inflow. It is not possible to hedge all risk related to the production, as no market for inflow risk exists. In addition, financial products needed to hedge high resolution price risk, e.g. spikes, are not available. Financial contracts that are liquid and available for price hedging have an increasing swap term (length of the delivery period) as the time to maturity increases. If the only available instrument is a one year contract, weekly price risk cannot be hedged, as the contract only reflects average risk over the entire period of one year. Furthermore it would be necessary to make some sort of assumption about the reservoir levels at the end of the period. Despite these shortcomings, we hereafter assume that the market for hedging price risk is complete.

The fact that it is nearly impossible to hedge all risk related to hydropower production complicates the methods used for risk-neutral pricing and thus also the hedging process itself. Merton [19] also encountered this problem when trying to find a price for options when underlying prices can jump. We are going to use the same idea as he introduced, namely that the additional risk over price risk, is assumed not systemic and can be diversified away.

If inflow risk is assumed to be non-systemic, investors can hedge against it by holding a well-diversified portfolio. In a sufficiently liquid market a risk premium on inflow risk will represent an opportunity for excess profit. This window of opportunity would not last. In such a case the risk premium of inflow risk should be zero.

Even though it is nearly impossible to perfectly hedge hydropower production, it is possible to reduce the risk significantly. For a producer going short on futures and swaps is an effective way to lower the risk to an acceptable level. To achieve a consistent result the producer needs to plan and price production in such a way that it is possible to estimate how sensitive the production value is to changes in value of the available future and forward contracts.

Delta-hedging is usually explained in terms of hedging an option that has been sold. The hedger should try to maintain a position of  $delta = \Delta$  number of shares so that the risk in the total position is close to zero. Delta is simply the derivative of the option price with respect to the stock price. As the stock price changes, so does the delta, and the hedger must buy or sell to maintain a total position of zero risk. In theory, the position must be rebalanced continuously, but in practice a delta-hedger will wait for the position to become somewhat unhedged before trading. With  $F$  as the price of the underlying, and  $V$  as the value of the option (or portfolio) to be hedged, the delta is

$$\frac{\partial V}{\partial F} = \Delta . \quad (1)$$

If  $V$  depends nonlinearly on  $F$ , any change in the value of the underlying leads to a change in delta. Glassermann [11] describes a central-difference-estimator to find the approximate change in option value when the value of the underlying increases or decreases. If we assume that optimal expected cash flows from hydropower pro-

duction can be seen as an option with the forward curve as the underlying, the delta can be expressed in the following way:

$$\frac{\partial V}{\partial F} = \Delta \approx \frac{V^{\Delta+} - V^{\Delta-}}{2}, \quad (2)$$

where  $V^{\Delta*}$  is the electricity portfolio value resulting from a unit shift in the forward curve.

The portfolio value  $V$  depends in principle on all futures and swaps that together constitute the forward curve. One possibility is to define a vector of deltas, one for each traded product. However, the producer wants to be hedged against price risk, and not all moves in the forward curve are equally likely. It is natural to start with looking at the risk of a general shift in prices.

It is possible to go beyond the delta to consider other greeks such as the gamma, for changes in the delta, and the vega, for changes in the volatility. This is left for future work.

## 4 Production Models – Theory and Implementation

In this section we present the most important assumptions and explain the price and inflow models used for representing the stochastic processes. We then give a mathematical description of the medium to long term planning problem. Finally we give a short presentation of the actual power plant system from which input data has been extracted.

We assume that the hydro producer is a price-taker. Decisions made by the producer will not influence electricity prices. For most hydropower producers in the Nordic region this is a reasonably valid assumption. For large-scale producers such as Statkraft, it is more dubious. Such actors could employ their market power to manipulate prices. This is not a subject in this analysis, however.

### 4.1 Stochastic Models

Stochastic variables in the model are electricity spot prices and inflow. This is the norm for long term production planning models [10]. Furthermore a modest negative correlation between price and inflow is assumed. Again this makes sense, as load levels in Norway typically increase in periods with low inflow (during the winter), and on longer term, draughts leads to increased prices.

#### Price Model

Many price models have been suggested for the dynamics of electricity prices. Electricity companies tend to replace their models from time to time. The method of scenario generation we have chosen is not affected by the choice of price model,

so we opt for a simple one-factor mean-reverting process. The use of such a model enables us to capture some of the more important properties of electricity prices, in particular the tendency to revert to a long-run level. The price process is a variant of the Ornstein-Uhlenbeck process with time-dependent expectation and is expressed as:

$$\frac{d\Pi_t}{\Pi_t} = \kappa(\theta_t - \ln \Pi_t)dt + \sigma dZ_t . \quad (3)$$

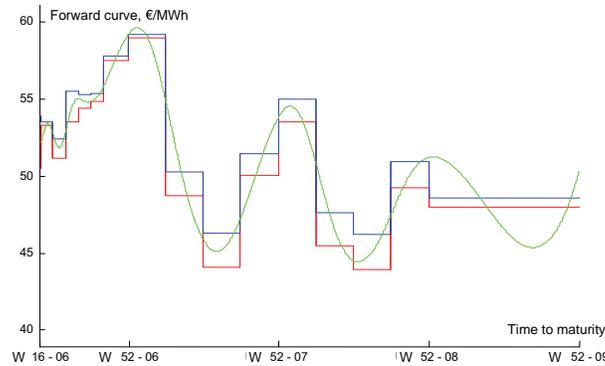
Here  $dZ_t$  is a Wiener process. Using Ito's Lemma and the log of the electricity price with the transformation  $g_t = \ln \Pi_t$  we get the discrete model:

$$\Delta g_t = \kappa(\theta_t - g_t)\Delta t + \sigma\sqrt{\Delta t}\varepsilon_t , \quad (4)$$

where  $\varepsilon_t$  is a standard normal random variable. To find the risk-adjusted process market prices of derivatives are used. According to Clewlow and Strickland [1], the relationship between market prices and the parameters of (3) is:

$$\theta_t = \frac{1}{\kappa} \frac{\partial \ln F_{0,T}}{\partial t} + \kappa \ln F_{0,T} + \frac{\sigma^2}{4\kappa} (1 - e^{-2\kappa T}) . \quad (5)$$

Here  $F_{0,T}$  is the current electricity forward curve, for forwards with delivery at time  $T$ . The electricity price model was calibrated using information from the term structure on 17 April 2006 for swap and future contracts from [www.nordpool.no](http://www.nordpool.no). The term structure was smoothed, see e.g. Fleten and Lemming [7]. If the term structure is displayed 2-dimensionally, with the bid-ask spread visible, as in Figure 2, the smoothed curve will pass through these bid-ask rectangles. Some adjustments were also made to the end of the term structure to achieve a more realistic seasonal effect. The parameters  $\sigma$  and  $\kappa$  in (5) were found using weekly spot prices from Trondheim in the period from 1996 to 2005.



**Fig. 2** Smoothed forward curve with bid-ask rectangles. Prices are in €/MWh.

### **Inflow Model**

The inflow process is in general multidimensional and has strong seasonal components. The main bulk of inflow to reservoirs comes during spring, whereas in winter the precipitation accumulates as snow. Forecasting the inflows and capturing the structure of the processes and their degree of predictability is of vital importance to hydro scheduling models. This issue is discussed by Tejada-Guibert, Johnson, and Stedinger [21].

The model for inflow has the same structure as for price (3). The model used also has seasonal expectation and variance. Autocorrelation is often present in inflow series. The one-factor mean-reverting process corresponds by discretization to an AR(1)-process, autocorrelated at lag 1. For the inflow the parameters have been estimated using weekly historical inflow data for the period 1951–2001 from the Nea-Nidelva river system in Norway. Weekly volatilities and mean-reversion coefficients were estimated using OLS.

## **4.2 Event Tree Modeling**

One might consider using stochastic dynamic programming to solve this problem. However, this is a multi-reservoir case, and price and inflow would need to be states as well. Due to the curse of dimensionality, this approach is not practical. Instead, we employ linear multistage stochastic programming [13].

To model how uncertainty (represented by stochastic inflow and electricity prices) unfolds over time, event trees are generated. The first node in the tree is called the root node. Each node in the tree represents a decision point, or equivalently a state, corresponding to a realization of the random variables up to the stage of state  $n$ , denoted by  $t(n)$ . Every state except the root node has a predecessor node, denoted  $a(n)$ .

Creating event trees which provide a satisfactory description of the stochastic processes is a considerable challenge. An overview up to about 2000 can be found in Dupačová, Consigli, and Wallace [3]. A reasonable update of later work is part of Kaut and Wallace [14].

Starting from fan scenarios, i.e. scenarios without a stage structure, has some advantages since it makes it easy for the problem owners to replace the price and/or inflow model with whatever they prefer, in particular simulators or “black boxes” they may have available. Heitsch and Römisch [12], Dupačová, Gröwe-Kuska, and Römisch [4] describe methods to construct event trees based on fans. Since our scenarios have been generated this way, we apply their approach. After all, the way we generate scenarios does not effect the main purpose of this article: To illustrate the use of delta-hedging. However, in a real setting, we would approach the scenario generation in a more careful way, as discussed by Kaut and Wallace [14], since the reduction technique may have weaknesses when starting from a fan (it should ideally be used to reduce a too large scenario tree with appropriate stage structure to a smaller one with the same structure.)

### 4.3 Deterministic Equivalent of the Stochastic Problem

In this section we present the mathematical program used for the production planning problem. Uncertainty for inflow and price is taken into account via joint discrete distributions, and are represented by an event tree with  $n$  nodes which represent different states in the stochastic process. This stochastic model is formulated as a deterministic equivalent linear program.

#### Data

$t$	Index for periods. Let $t(n)$ be the period belonging to node $n$ .
$i, j \in \mathcal{I}$	Indices for reservoirs in set $\mathcal{I}$
$\mathcal{U}_i$	Set of reservoirs upstream of reservoir $i$ whose outflow will go to reservoir $i$
$\mathcal{R}_i$	Set of reservoirs upstream of reservoir $i$ whose spill will go to reservoir $i$
$n, \mathcal{N}$	Index, set for nodes in the event tree
$S_t$	Set of nodes in period $t$
$a(n)$	Index of predecessor to node $n$
$P_n$	Unconditional probability of the state in node $n$
$\pi_n$	Electricity price in node $n$
$D_t$	Discount factor for period $t$
$K_i$	Water-to-energy coefficient for reservoir $i$
$v_{i,n}$	Inflow in node $n$ for reservoir $i$
$L_{max,i,t}$	Upper bound in period $t$ for reservoir level in reservoir $i$
$L_{min,i,t}$	Lower bound in period $t$ for reservoir level in reservoir $i$
$L_{end,i,n}$	End level for reservoir $i$ in node $n$
$Q_{max,i,t}$	Upper bound in period $t$ for discharge through the station for reservoir $i$
$Q_{min,i,t}$	Lower bound in period $t$ for discharge through the station for reservoir $i$
$r$	Risk free interest rate

#### Decision Variables

$V$	Value of production for the whole planning period
$l_{i,n}$	Reservoir level in node $n$ at the start of period $t(n)$ for reservoir $i$
$r_{i,n}$	Spill in node $n$ during period $t(n)$ for reservoir $i$
$w_{i,n}$	Hydropower generation in node $n$ during period $t(n)$ , $w_{i,n} = K_i q_{i,n}$
$q_{i,n}$	Production discharge in $n$ from reservoir $i$

#### Objective Function

$$\max V = \sum_{i \in \mathcal{I}} \sum_{n \in \mathcal{N}} P_n \pi_n D_{t(n)} K_i q_{i,n} \quad (6)$$

#### Constraints

$$l_{i,n} - l_{i,a(n)} + q_{i,n} + r_{i,n} - \sum_{j \in \mathcal{U}_i} q_{j,n} - \sum_{j \in \mathcal{R}_i} r_{j,n} = v_{i,n}, \quad n \in \mathcal{N}, i \in \mathcal{I} \quad (7)$$

$$l_{i,n} = L_{end,i,n}, \quad i \in \mathcal{I}, n \in S_T \quad (8)$$

$$L_{min,i,t} \leq l_{i,n} \leq L_{max,i,t}, \quad n \in \mathcal{N}, i \in \mathcal{I} \quad (9)$$

$$Q_{min,i,t} \leq q_{i,n} \leq Q_{max,i,t}, \quad n \in \mathcal{I}, i \in \mathcal{I} \quad (10)$$

$$q_{i,n}, r_{i,n} \geq 0, \quad n \in \mathcal{N}, i \in \mathcal{I} \quad (11)$$

The objective function (6) is the sum of the discounted expected future revenues from each period. There are no direct variable costs of hydropower generation; all costs are fixed. We use the risk free interest rate for discounting, because risk is already adjusted for in the stochastic process for the cash flows, in that it is calibrated to the forward curve. In (7) the reservoir in each node  $n$  is dependent on the reservoir level in the predecessor node  $a(n)$ . The initial storage levels are  $l_{i,0} = L_{init,i}$  for all reservoirs  $i$ . End reservoir levels are fixed by (8). The constraints on the flow of water in (10) have time indices since the time intervals have different lengths. With *discharge* we mean water being used for electricity production. *Spill* is the amount of water that is not utilized. This could typically occur in situations where the reservoir is full. Time-varying bounds on reservoir levels (9) and discharges (10) reflect physical, technical and environmental concerns.

#### 4.4 Model Implementation

The deterministic equivalent described in the previous section has been used to solve the hydropower production problem for a number of plants and reservoirs in Mid-Norway, in the Nea-Nidelva waterway. The optimization itself was done on a 2.4 GHz Intel Celeron CPU with 3.71 GB RAM. Scenarios were generated as described in Sect. 4.2.

##### Period of Analysis

The typical horizon for hydro scheduling is a few months to a few years. A typical length of the first time step ranges from one week to a month. The hydro scheduling model gives signals to hydro unit commitment via marginal values of stored water in the reservoirs and/or via total generation during the first week.

In our case the planning horizon is divided into 14 periods and spans April 2006 to October 2007. The first six periods are weeks, the next four periods are months, and the final four periods are quarters. This corresponds to the swap term of the products traded at Nord Pool at the beginning of the first period. The stage structure is illustrated in Fig. 3.



**Fig. 3** Stage structure

The production facilities in Nea-Nidelva currently consists of a catchment area of 3100 km<sup>2</sup>, 10 reservoirs and 14 plants with a total installed production capacity of 614 MW. The waterway has its origin in Sweden and ends in the city of Trondheim, a distance of 160 km. A general view of parts of the waterway is shown in Figure 4. The ratio of aggregate reservoir capacity to annual inflow is relatively high (64%), which makes this system of reservoirs well suited for a production planning/risk management analysis. To simplify the problem somewhat we have used fixed water flow to energy conversion coefficients for the power stations. This means that the energy efficiency is not affected by the actual reservoir level. The topology of this system is such that the real efficiency does not vary much with weekly flow and reservoir levels, so dealing with this issue in more detail is left for future work.

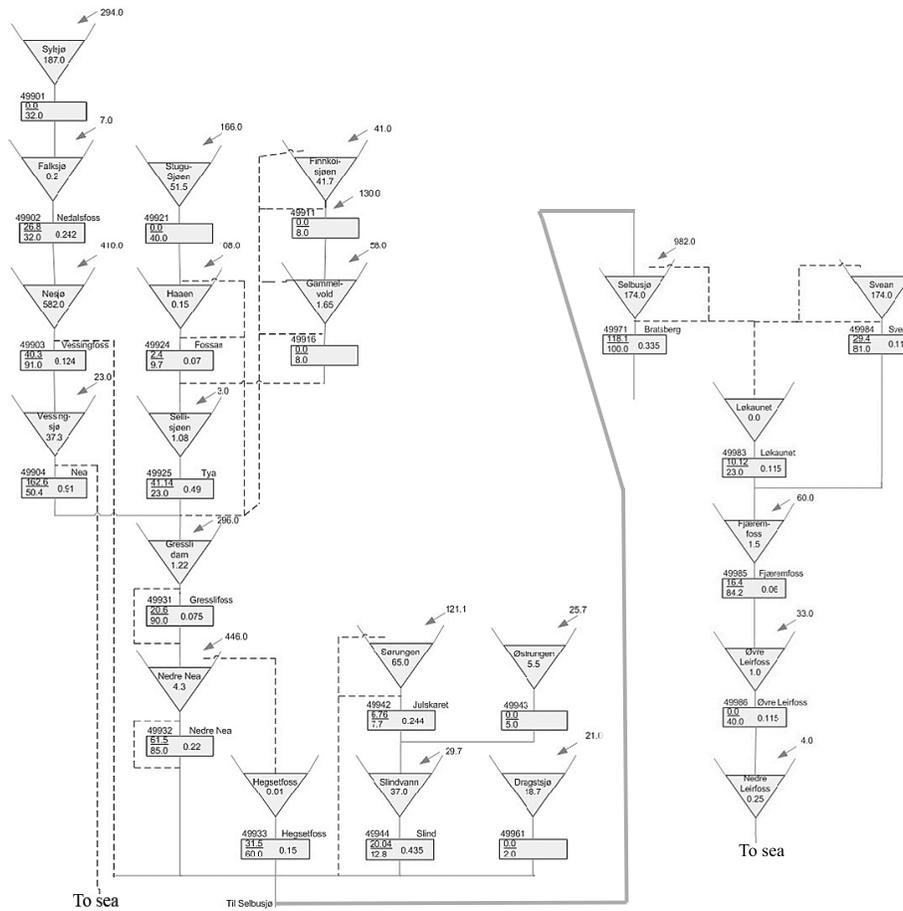


Fig. 4 View of the Nea-Nidelva water system

The value of water at the end of the planning horizon depends on the time of year, the reservoir levels and price levels. This value function is hard to estimate, however, and instead of using such a function we have chosen to set target levels for the reservoir at the end of the planning horizon. End reservoir levels have been set according to:

$$L_{end,i,n} = f_T L_{max,i}, i \in \mathcal{I}, n \in S_T, \quad (12)$$

where  $f_T \in [0, 1]$  is a parameter governing the relative end levels. Starting levels were set to the actual historical levels in week 16 2006, at approximately half full.

## 5 Results

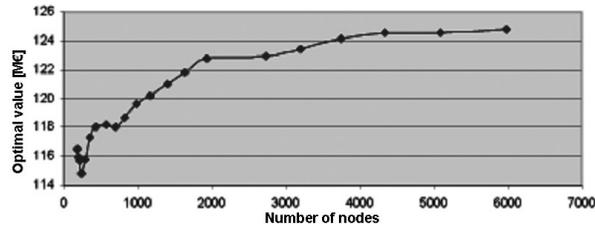
### 5.1 Optimization Analysis

The optimization is performed using the dual simplex algorithm in Mosel Xpress. The risk free rate of interest is set at 3.5% in all analyses. The analysis of convergence is done with the parameter set in Table 5.1:  $S_{fan}$  represents the number paths

**Table 1** Input data for convergence analysis

$f_T$	$S_{fan}$	Correlation
0.725	500	-0.2

constructed via Monte Carlo sampling of (4) and the corresponding inflow equation, before the construction of the event trees. The optimal value of expected production during the planning period seems to converge with an increasing number of nodes in the event tree, as shown in Figure 5.



**Fig. 5** Optimal value of production (vertical axis) for specified number of nodes in the event tree

## 5.2 Value of Production

The subsequent analysis is done using the parameters presented in Table 2.

**Table 2** Scenario generation parameters for analysis of production value

$S_{fan}$	Reduction level	$S_{tree}$	Nodes
1000	0.35	771	3701

The reduction level in Table 2 is a parameter used in the scenario generation process. The number of nodes in the resulting event tree decreases with an increasing reduction level. Table 3 shows the value of operating revenues for various reservoir levels at the end of the planning period. As expected the value is higher for lower end reservoir levels.

**Table 3** Value of operating revenues for different reservoir end levels

$f_T$	Correlation	Value[M€]
0.750	-0.2	123.7
0.725	-0.2	126.0
0.700	-0.2	128.3
0.675	-0.2	130.5

By assuming no correlation between inflow and prices (Table 4) the resulting values do not differ much from the case with assumed negative correlation. This could indicate that the low negative correlation does not necessarily have a significant effect on the value of production<sup>1</sup>. It can not be ruled out that it may have an effect on hedging.

**Table 4** Value of production for different reservoir end levels, no correlation between price and inflow

$f_T$	Correlation	Value[M€]
0.750	0.0	124.1
0.725	0.0	126.4
0.700	0.0	128.6
0.675	0.0	130.9

<sup>1</sup> An absolute value of correlation of 0.2 may be too low to draw conclusions from. Further analysis is left for future work.

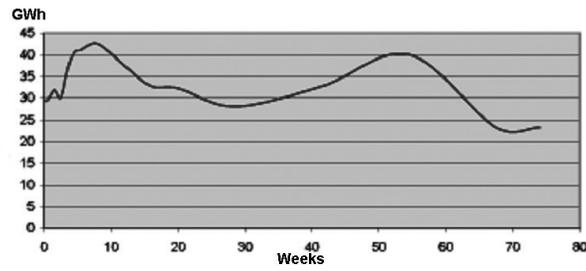
### 5.3 Expected Production Strategy

For the analysis below we used the following parameters:

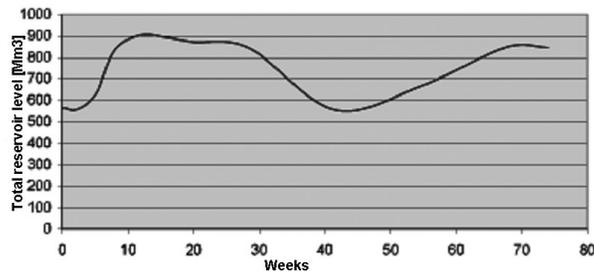
**Table 5** Parameters for analysis of the production strategy

$f_T$	Correlation	$S_{fan}$	Reduction level	Nodes	$S_{free}$
0.7	-0.2	1000	0.12	7395	937

Figures 6 and 7 present examples of expected production and reservoir level respectively. Expected production displays a clear seasonal variation in addition to a downward trend. The seasonality is partly a result of the seasonal variation in the term structure of futures prices.



**Fig. 6** Expected weekly production for case in Table 5



**Fig. 7** Expected reservoir levels for case in Table 5

### 5.4 Expected Cash Flow

The expected operating revenue in each period is presented in Figure 8. From this figure it is clear that there is a certain resemblance between the cash flow and forward curves. Intuitively, this makes sense, as the forward curve is the main source of information on future spot price levels.

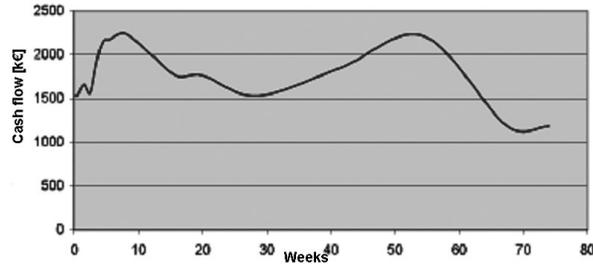


Fig. 8 Expected weekly cash flows for case in Table 5

### 5.5 Delta-Hedging

By adding and subtracting one unit for all contracts in the forward curve it is possible to obtain an expression for the sensitivity of the cash flows and the total production value in the subperiods of the planning problem. This can be achieved by using (2).

Here the value of the cash flows  $V$  is in € and delta is in MWh. The delta is calculated from model instances created using the parameters in Table 6.

Table 6 Parameters for delta-hedging

$f_T$	Correlation	$S_{fan}$	Reduction level	Nodes	$S_{tree}$
0.7	-0.2	500	0.12	4058	472

The results from the sensitivity analysis for each interval in the planning period are shown in Table 7. For each period the delta can be observed as a quantitative discrepancy in the expected cash flows. The conclusion from the table is that the producer is recommended to go short in the swap and future products that spans the next 3.5 months, and to go long in the products that span the rest of the planning horizon.

**Table 7** Cash flow sensitivities for parallell shifts in the forward curve

Period[t]	1	2	3	4	5	6	7
$V^{\Delta^+}$ [k€]	1382	1223	1241	1640	1990	1970	8197
$V^{\Delta^-}$ [k€]	1780	2009	1741	1896	2123	1995	8431
$\Delta$ [GWh]	-199	-393	-250	-128	-66	-12	-117

8	9	10	11	12	13	14	Whole period
7777	7668	7619	20299	26036	30412	15760	133214
9047	7362	7224	17904	24393	27202	14228	127336
-635	153	197	1198	821	1605	766	2939

## 6 Discussion

Much has been left for future work, since this is a first attempt at delta-hedging of a portfolio with hydropower. Further sensitivity analyses with respect to correlation, end reservoir level and number of fan scenarios before scenario reduction is a natural next step. An interesting future possibility is to compare integrated risk management such as in Fleten et al. [9] with delta-hedging. One could also measure the performance of the delta-hedging over time, and compare the performance with e.g. delta hedges that come from deterministic generation scheduling, or scheduling heuristics. Furthermore, it would be interesting to investigate the effects of the long-term negative relationship between local inflow and local prices, that leads to a natural hedge meaning the risk-minimizing position is less than expected electricity sales.

The electricity price model takes many of the typical empirical characteristics of electricity prices into account. However, since we only need weekly average prices, the need for short term characteristics, such as spikes, is reduced.

Any real use of this approach would have to be more careful about how scenarios are generated from the stochastic models of price and inflow.

The current status of the development of this model is that it remains a case study. The owner of the power plant, Trondheim Energi, has been taken over by Statkraft, who may be less interested in hedging cash flows.

A hedging strategy should be based on second-order market information, in the form of a term structure of volatility. The model in (3) has a simple volatility structure that does not fully reflect real market dynamics. It would also be preferable to have a model providing a better representation of the correlation between prices and inflow. It is possible to update and upgrade the models for prices and inflow used in this analysis, albeit possibly at the cost of keeping the disadvantages associated with creating reduced multistage event trees from two-stage scenario fans. Using more sophisticated models for the stochastic processes will also allow for more realistic and more effective hedging strategies. For example, there is just one random factor driving (3). In reality, a model with many factors is needed to capture a large part of the variance [16].

A hedging strategy involving frequent trading could lead to large transaction costs. On the other hand, if too much time pass between each time the portfolio is updated, it could lead to unnecessary losses. The optimal trading frequency must be found in future work.

## 7 Conclusion

The model gives reasonable results. For an increasing number of nodes in the event tree the optimal value of the production converged towards a stable level. This production value increased when the fixed reservoir level at the end of the planning horizon was lowered. An assumed negative correlation between price and inflow did not seem to have a significant effect on the expected production value.

Even with 6000 nodes in the event tree the model did not need more than a couple of minutes to solve. This suggests that there could be a considerable potential for using such models to plan the hydro production and risk management.

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