Multi-Asset Equity Options

A Master Thesis Presented

by

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to

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Declaration of Authorship

I hereby confirm that I have authored this master thesis independently and without use of others than the indicated resources. All passages, which are literally or in general matter taken out of publications or other resources, are marked as such.

Xia Su
Berlin, 26th September 2003
Abstract

The feature of several underlying assets requires traders to incorporate the correlation matrix of underlying assets in multi-asset equity options pricing. In this thesis, Monte Carlo simulation methods are used in order to quantify the precision of multi-asset equity options pricing. The developed quantlets in XploRe are specific to three standard types of multi-asset equity options. Due to the lack of a liquid market for implied correlations, this thesis then aims to understand the correlation risk and risk hedging. I demonstrate the correlation risk by an application to three-asset equity options of the three standard types. Correlation vegas, defined as the first derivative of the option price to its underlying asset correlation matrix, are calculated numerically using the finite diffusion approximation technique and presented in temperature plots.

Keywords: Multi-Asset Equity Options, Basket Options, Max Options, Min Options, Monte Carlo Simulation, Correlation Vega
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1 Introduction

An option is a contract between two parties that grants the holder the right to trade at a specified time at a previously agreed price. Since it is derived from other financial instruments, an option’s value depends on other underlying variables. Options have been created for many years, but only on 26th April 1973 they were first traded on an exchange (Wilmott, 1992). Nowadays first-generation options are already standardized and traded actively over 50 exchanges worldwide. Meanwhile, a number of nonstandard and complex products have been created and are traded on the over-the-counter derivatives market. One of such products is a multi-asset equity option. A multi-asset equity option, as its name implies, is an option on a portfolio of several underlying assets, e.g., stocks.

In order to gain diversification in investors’ portfolios, such new-emerged options are increasingly demanded. They are either sold to investors as an insurance product or packaged as a structured note for retail market, providing attractive multi-asset linked products. The $G – 7$ index-linked guaranteed investment certificates offered by Canada Trust Co. can serve as a good example, (Milevsky and Posner, 1998b). This certificate is basically a call option on a basket of international stock indexes. The basket is composed of a weighted average of seven stock indexes as shown
<table>
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<th>Country</th>
<th>Index</th>
<th>Weight</th>
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<td>Canada</td>
<td>TSE 100</td>
<td>10%</td>
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<tr>
<td>Germany</td>
<td>DAX</td>
<td>15%</td>
</tr>
<tr>
<td>Japan</td>
<td>Nikkei 225</td>
<td>20%</td>
</tr>
<tr>
<td>US</td>
<td>S&amp;P 500</td>
<td>25%</td>
</tr>
<tr>
<td>France</td>
<td>CAC 40</td>
<td>15%</td>
</tr>
<tr>
<td>Italy</td>
<td>MIB 30</td>
<td>5%</td>
</tr>
<tr>
<td>UK</td>
<td>FTSE 100</td>
<td>10%</td>
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Table 1.1: Composition of the $G - 7$ index-linked guaranteed investment certificates

With the emergence of options, an urgent and essential task is to precisely value them as well as to monitor the involved risks and then hedge them. This becomes settled since the well-known Black-Sholes (BS) formula was derived. In the formula, all the parameters can be observed directly from the market, except the actual volatility of the underlying price process which is assumed to be constant. However, as Fengler, Härdle and Schmidt (2002) show, the volatilities implied by observed market prices exhibit a "smile" pattern that is far different from the flat constant one used in the BS formula. Therefore, managing the volatility risk and then hedging it becomes the focus of interest both in volatility trading and in risk management.

However, one more problem arises when dealing with multi-asset equity options. Apart from the sophisticated payout structures, the new challenge of pricing and hedging multi-asset equity options is the involvement of implied correlations of underlying assets. This means that, in addition to the volatility risk as in the single underlying asset case, we now also bear the correlation risk.

As volatility, correlations first cannot be directly observed, but must be
estimated; moreover, correlations measured from financial time series data are notoriously unstable and vary over time as demonstrated by Neftci and Genberg (2002). However, the lack of standardized multi-asset contracts makes practitioners unable to invert a market quote to find out the “implied” correlations as the possibility with the volatility. Moreover, as opposed to volatility, correlation cannot be traded due to the absence of standard organized markets of multi-asset options. As a result, correlation risks cannot be hedged as precisely as volatility risks (Fengler and Schwendner, 2003). Multi Exchange Rates Options are an exception to this. In foreign exchange markets, hedging correlation is possible since the correlation can be determined completely by volatility via the exchange rate mechanism. This is however not possible for multi-asset equity options, as stocks are traded in cash and not linked in pairs as currencies are.

Thus, traders attempt to keep track of the statistics of the correlation risk and try to avoid risk peaks in certain correlations via dynamic price margins, (Fengler et al., 2002). In a competitive environment market forces are however driving the margins down, pushing practitioners to find efficient and robust ways of estimating equity correlations. Rapuch and Roncalli (2001) propose to investigate the dependence between two-asset options prices and the correlation parameter in the Black-Sholes model and generalize it in the framework of the copula construction of risk-neutral distributions. The monotone relationship and bounds of an option’s price with respect to the correlation are derived for most types of two-asset options. However, a monotone dependence of an option’s price on the correlation for the generalized case with more than two assets is restricted due to the stronger assumptions in multi-dimensions.
Consequently, one of the main tasks in this thesis is to find out how the \( n \)-asset equity options price is dependent on the correlations between the constitute stocks both quantitatively and qualitatively. In this thesis, the correlation risk is defined as Correlation Vega exposure. It is evaluated through the first order derivative of the option price with respect to correlation, \( \left( \frac{\partial C}{\partial \rho_{ij}} \right) \). Thus a triangular matrix \( \left( \frac{\partial C}{\partial \rho_{ij}} \right) \) with \( i < j \) will have to be calculated in the study. In order to make practitioners aware of the existence and impact of the correlation risk, a numerical application to three types of three-asset equity options is presented in detail. The sensitivity of the option to the three correlations is fully examined through a comparison of the correlation vega over various changes in the option’s underlying assets performance such as asset spot price and volatility.

In addition to a close study of the correlation risk, a good pricing model is also essential to be able to correctly value multi-asset equity options. This approach not only has to be well suited to high dimensions calculation, but can also incorporate both correlation and volatility into the valuation. Monte Carlo simulation methods not only fulfill the requirements, but are also easy to manipulate due to its easily understandable mathematical background. Therefore, European multi-asset options are in this thesis proposed to be priced through Monte Carlo simulation. Incorporating constant estimated correlations into a randomly generated stocks price distribution, we can obtain a relatively “precise” result. Then three quantlets in XploRe are created by the Monte Carlo simulation methods to value three prominent types of multi-asset options.

This thesis is organized as follows: Section 2 first discusses several popular multi-asset options pricing models. Based on the study, the Monte Carlo simulation methods are used for pricing European multi-asset eq-
uity options. Then three XploRe quantlets are developed and a simple example is presented. In Section 3, the correlation vega is calculated and demonstrated in temperature plots in the case of three-asset equity options. Then, the sensitivity of the Multi-asset equity option to three correlations is observed and analyzed. Finally, Section 4 provides a summary of the results of the thesis.
2 Multi-asset Equity Options

Pricing Model

2.1 Black-Scholes Framework

2.1.1 Black-Scholes Model for options with one underlying asset

The Black-Scholes (BS) model is usually used to calculate a theoretical call price (ignoring dividends paid during the life of the option). It is expressed as:

\[ C_t = S_t \Phi(d_1) - Ke^{-r\tau} \Phi(d_2), \]  
\[ d_1 = \frac{\ln(S_t/K) + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}, \]  
\[ d_2 = d_1 - \sigma\sqrt{\tau}, \]

where \( \Phi(x) \) is the cumulative probability distribution function for a standardized normal distribution, and the five key determinants of an option’s price are stock price \( S \), strike price \( K \), volatility \( \sigma \), time to expiration \( \tau \), and short-term (risk free) interest rate \( r \).

To derive the BS model, the first essential assumption is the lognormal
random walk

\[ dS_t = (r - d)S_t dt + \sigma S_t dW_t \]  

(2.4)

where the underlying asset follows a Wiener process with the drift of the difference between the dividend rate \( d \) and the risk-free rate \( r \) and volatility of \( \sigma \). Then a self-financing portfolio is constructed to replicate the payoff of the call option, which can be written as a differential equation:

\[ \frac{\partial C}{\partial t} + rS \frac{\partial C}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC \]

where \( C \) is the price of the call option. The final formula 2.1 is obtained by solving this differential equation with the boundary conditions \( C_T = \max(S - K, 0) \).

### 2.1.2 Black-Scholes Framework for options with several underlying assets

The Black-Scholes framework is utilized in most studies on complex or exotic options. Here, we also treat the discussed European multi-asset equity options in the standard Black-Scholes framework.

In a probability space \((\Omega, \mathcal{F}, P)\) and \((\mathcal{F}_t)_{t \geq 0}\), \( n \) correlated Brownian motions \( W_i \) are modeled for the \( n \) underlying assets with spot prices \( S_{it} \), constant correlations \( \rho_{ij} \) and volatilities \( \sigma_i \), dividend rate \( d_i \) and risk-free rate \( r \), \( i = 1, 2, \cdots, n \):

\[ dS_{it} = (r - d_i)S_{it} dt + \sigma_i S_{it} dW_{it} \]  

(2.5)

\[ \rho_{ij} dt = dW_{it} dW_{jt} \]  

(2.6)

This framework is set with the following assumptions:
• Markets are complete, in the sense that there are no transaction costs, no taxes, no restrictions on short-sales, no difference between the borrowing rate and the lending rate and that trading takes place continuously.

• Assets follow the standard Wiener processes and satisfy the stochastic differential equations.

Three typical multi-asset equity options will be discussed in the following studies. They have different payout structures listed as follows:

• Basket Options: A European call option on an equally weighted basket of \( n \) assets
  \[
  \text{Payout} = \max\left(\frac{1}{n} \sum_{i=1}^{n} S_{iT} - K, \ 0\right)
  \]

• Max Options: An option on the maximum performance of \( n \) assets
  \[
  \text{Payout} = \max\left\{ \left(\max_{i=1}^{n} S_{iT}\right) - K, \ 0 \right\}
  \]

• Min Options: An option on the minimum performance of \( n \) assets
  \[
  \text{Payout} = \max\left\{ \left(\min_{i=1}^{n} S_{iT}\right) - K, \ 0 \right\}
  \]

where payouts at the terminal exercising date \( T \) are defined as \( S_{iT} \) with strike price \( K \). The notations employed here are further used throughout this thesis.

2.2 Available Pricing Methods

Various valuation techniques are proposed to price European multi-asset options. In order to obtain the qualitatively appropriate pricing method, an overview of these approaches is going to be given first.
2.2.1 Analytical Formulae Solution

It is extremely difficult to arrive at an analytical formula, since multi-asset equity options involve complex payout structures. However, it is still the most direct and accurate approach to find a “closed-form” solution for options with specific payout structures. Stulz (1982) studies European options on the minimum or maximum of two risky assets.

Stulz (1982) derives the formula for a call option on the minimum of two risky assets based on the idea of finding a self-financing portfolio with the same payoff at maturity $T$. The value of such a portfolio at date $t$ must be equal to the value of the option at $t$, $\forall \ t < T$, in order to prevent the possibility of arbitrage profits. By solving a partial differential equation and satisfying two boundary conditions the formula can be finally expressed as:

$$C_{\text{min}}(S_1, S_2, K, \tau) = S_1\Phi_2\{\gamma_1 + \sigma_1\sqrt{\tau}, d_1, (\rho_{12}\sigma_2 - \sigma_1)/\sigma\} + S_2\Phi_2\{\gamma_2 + \sigma_2\sqrt{\tau}, d_1, (\rho_{12}\sigma_1 - \sigma_2)/\sigma\} - Ke^{-rt}N_2(\gamma_1, \gamma_2, \rho_{12})$$ (2.7)

where $C_{\text{min}}(S_1, S_2, K, \tau)$ is a European call on two risky assets at spot price $S_1$ and $S_2$ with strike price $K$ and maturity $\tau$, $\Phi_2(\alpha, \beta, \rho)$ is the bivariate cumulative standard normal distribution with upper limits of integration $\alpha, \beta$, and correlation $\rho$, and $\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2$, $d_1 = \{\ln(S_2/S_1) - \frac{1}{2}\sigma^2\sqrt{\tau}\}/\sigma\sqrt{\tau}$, $\gamma_1 = \{\ln(S_1/K) + (r - \frac{1}{2}\sigma_1^2)\tau\}/\sigma_1\sqrt{\tau}$, $\gamma_2 = \{\ln(S_2/K) + (r - \frac{1}{2}\sigma_2^2)\tau\}/\sigma_2\sqrt{\tau}$ respectively.

With the value resulting from the above formula, we can simply calculate the price of an European call on the maximum of two risky assets by replicating a portfolio which has the same payout at the maturity date. The portfolio consists of holding a call option on $S_1$ and a call option
A long position in $C(S_1, K, \tau)$  
A long position in $C(S_2, K, \tau)$  
A short position in $M(S_1, S_2, K, \tau)$  
Total payoff

<table>
<thead>
<tr>
<th>Position</th>
<th>$S_1 &gt; S_2 &gt; K$</th>
<th>$S_1 &gt; K &gt; S_2$</th>
<th>$K &gt; S_1 &gt; S_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1 - K$</td>
<td>$S_1 - K$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$S_2 - K$</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$-(S_2 - K)$</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$S_1 - K$</td>
<td>$S_1 - K$</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1: Payoff of the replicated portfolio at maturity

on $S_2$, and short selling a call option on the minimum of $S_1$ and $S_2$. These three options all have the same exercise price $K$ and maturity $\tau$ as the call on the maximum of two assets. To verify the result, we can first assume that if $S_1$ is the maximum of the two risky assets, then the option on the maximum of two assets should be $S_1 - K$ when $S_1 > K$ and 0 when $S_1 - K$. The payout of the replicated portfolio are listed in Table 2.1 where $C(S, K, \tau)$ is an European call on asset $S$ with strike price $K$ and maturity $\tau$.

The above argument holds if $S_2$ is the maximum of the two assets. It follows that in all states of the world, the portfolio pays the same as the call on the maximum of two risky assets at maturity and therefore must have the same value as the call we studied. Therefore, an European call on the maximum of two risky assets can be priced as:

$$C_{\text{max}}(S_1, S_2, K, \tau) = C(S_1, K, \tau) + C(S_2, K, \tau) - C_{\text{min}}(S_1, S_2, K, \tau)$$ (2.8)

However, this analytical solution cannot be easily applied in practical use when dealing with options on the maximum or minimum of more than two assets. With the assumption of a joint lognormal random walk, Johnson (1987) generalizes it to the case of several assets. Although the distribution of a sum of correlated lognormal random variables is not
lognormal (Milevsky and Posner, 1998b), it turns out to be practical to find an approximated solution in the valuation of multi-asset options.

2.2.2 Approximated Basket Options Price Formulae

A variety of techniques are adopted when tackling the approximation problem of basket options valuation. Milevsky and Posner (1998b) present two different results based on reciprocal gamma and lognormal approximation respectively.

It was stated in the Pliska Harrison fundamental theorem of derivative assets pricing that, in a “frictionless” market, the no-arbitrage value of an option is equal to its expected payoff discounted at the risk-free rate, where the expectation is defined with respect to the risk-neutral probability density function (Cox and Ross, 1976). The risk-neutral probability density function of basket options is, unfortunately, not known in general. The commonly-used approximation method is to match the moment of the risk-neutral probability density function by a lognormal function:

$$C_{basket} = e^{-rT} \left( F \Phi\left\{ \frac{\ln\left(\frac{F}{K}\right) + \frac{\sigma^2}{2}}{\sqrt{\sigma}} \right\} - K \Phi\left\{ \frac{\ln\left(\frac{F}{K}\right) + \frac{\sigma^2}{2}}{\sqrt{\sigma}} \right\} \right)$$  \hspace{1cm} (2.9)$$

where $C_{basket}$ is a basket option with strike price $K$ and maturity at $T$, $F = \sum_{i=1}^{n} S_{0i}e^{(r-d_i)T}$ is the “pseudo-forward” price of the basket option, $\sigma^2$ is the variance of the basket, and $\Phi(x)$ is the cumulative standard normal distribution.

Assuming that the finite sum of lognormal variates follows a lognormal distribution is proven to be a convenient approximation but has little
theoretical justification. Valuations using the reciprocal gamma distribution as the risk-neutral probability density function are proven to perform better (Milevsky and Posner, 1998b). The justification originates in Asian option pricing theory, which shows that the sum of contemporaneously correlated lognormals converges to the reciprocal gamma distribution in the limit (Milevsky and Posner, 1998a). Therefore, reciprocal gamma is introduced as the risk-neutral probability density function and a closed-form formula can be obtained for the basket options pricing, employing moment matching techniques:

$$C_{basket} = e^{-rT} \left( FG\left(\frac{F}{K}, \alpha - 1, \beta\right) - KG\left(\frac{F}{K}, \alpha, \beta\right) \right) \quad (2.10)$$

which has a similar structure and parameters as (2.9) except that $G(x, \alpha, \beta)$ is the cumulative density function of the gamma distribution evaluated at $x$ with $\alpha = \frac{1}{\beta} + 1$, and $\beta = 1 - \frac{1}{M_2}$ ($M_2$ is the second moment of the basket.

### 2.2.3 Fast Fourier Transformation

Another research direction employs efficient numerical algorithms to evaluate options price, instead of computing analytical or approximate closed-form formulae. These methods can be classified into three groups: finite difference methods or other approaches dealing directly with partial differential equations (PDE), lattice binomial methods, and Monte Carlo simulation methods.

The Fast Fourier transformation is one method of the first of these groups. Andreas, Engelmann, Schwendner and Wystup (2002) demonstrate how to apply and implement the Fast Fourier transformation.
method in multi-asset options valuation to solve the multi-dimensional Black-Scholes PDE and the Greeks. As an illustration, they price exchange options, spread options and options on the maximum and minimum of 3 currencies.

Obviously, this approach is powerful since it can deal with multi-asset options of arbitrary payoffs. Moreover, it suits path-dependent options valuation due to its feature of the generalization of binomial tree methods. However, as Andreas et al. (2002) and other studies show, it outperforms Monte Carlo simulation methods only in the lower dimensions. Thus the Fast Fourier transformation method has a serious limitation of low efficiency in pricing multi-asset options.

2.2.4 Lattice Binomial Methods

The lattice binomial method was first proposed by Cox, Ross and Rubinstein (1979) and is shown to be a powerful and flexible tool for American options pricing. In a generalized lattice framework, Boyle, Evnine and Gibbs (1989) establish a model for multi-asset contingent claims. They solve the PDE that the value of a contingent claim satisfies numerically in a discrete-time setting: the multivariate lognormal distribution is approximated by a discrete probability distribution and then the value of the contingent claim is obtained by discounting its expected terminal value backwards in the discrete setting. As the original lattice binomial method, this extended model is applicable to handle the early exercise feature of American options in multivariate dimensions.

Stapleton and Subrahmanyam (1984) also examine multivariate contingent claims in discrete time models but through the derivation of a risk
neutral valuation relationship (RNVR), i.e., a formula yielding a fair price for multivariate contingent claims with the risk neutrality preference. When all investors are risk neutral, the expected return on all securities is the risk-free interest rate, $r$. This is because investors with risk neutrality preference do not require a premium to induce them to take risks. Then it is also true that the present value of derivatives in a risk neutrality world can be obtained by discounting its expected value at risk-free rate.

Stapleton and Subrahmanyam (1984) prove that RNVR is obtained in two cases of multivariate normality with constant absolute risk aversion and multivariate lognormality with constant proportional risk aversion. The derivation of the RNVR with restrictions on preference in a discrete time setting is aimed at achieving a riskless hedge, thus the contingent claims price can be obtained by simply discounting the terminal expected payout at the risk-free interest rate under the assumption of universal risk neutrality. This serves as the theoretical background to establish the valuation formula for multivariate contingent claims.

2.2.5 Monte Carlo Simulation Methods

The third group, Monte Carlo simulation methods are considered as a powerful and flexible numerical tool for pricing multi-asset options on the following reasons: Firstly, it fits for higher dimensions ($n \geq 3$), and thus does not suffer the “curse of dimensionality” affecting other numerical methods; secondly it corresponds to several underlying stochastic factors and it also copes with complex path-dependency of options.

The two most important drawbacks of Monte Carlo simulation meth-
ods mentioned in the literature are: they converge to the true value at a lower speed of $O(1/\sqrt{n})$ compared to the Quasi Monte Carlo and the low-discrepancy sequence methods; also they are not very efficient in handling American-style early exercise, which nevertheless does not affect the pricing of European multi-asset options (Lüssem and Schumacher, 2002).

### 2.2.6 Other Pricing Approaches

Besides the above mentioned methods, other statistics tools are adopted to price multi-asset options. Copulas are introduced for derivatives pricing in 1999 by Rosenberg J. V.. He proposes first to use Plackett distributions, a special case of the copula construction of multidimensional probability distributions. Then his work is extended by using the general copula functions. Following the same idea, Coutant, Durrleman, Rapuch and Roncalli (2001) use copulas to define multivariate risk-neutral distributions and then derive the general pricing formulae for some multi-asset options. Here, the options price is calculated in the framework of copulas construction by assuming that the copula of $(S_{1t}, \cdots, S_{nt})$ is a normal copula with the correlation matrix.

### 2.3 Illustration of Monte Carlo Simulation Methods

Based upon the above discussion, Monte Carlo simulation methods are used in this thesis to value European multi-asset options due to its easily understandable mathematical background, in addition to its features as
mentioned above. Moreover, a sufficiently large number of simulations will be run in the model to gain a better accuracy.

Basically, the Monte Carlo simulation solves the integration problem by randomly sampling changes in the market variables. In the case of multi-asset equity options, this technique not only deals with higher dimensional integrals, but also has to model the stochastic behaviors of the underlying stocks and their correlation structure together. In our framework here a constant correlation matrix of stocks on an annual basis is taken into account. In this thesis only the technique is presented. One may turn to a long list of literature for the theoretical background, which is not covered here any more.

Basically, the Monte Carlo simulation methods work as follows (Schwendner, Martin and Papies, 2001):

1. Input the financial data, i.e. underlying asset prices $S_{t_0}$, $\sigma_i$ and assets correlation matrix $\rho_{ij}$.

2. The correlation matrix is decomposed into a lower triangular matrix through Cholesky factorization.

3. A large number of independent random underlying values $S_{1T}^1 \cdots S_{kT}^k$ ($k$ denotes the simulation number) are generated by computing the multivariate probability density function of a set of uniformly distributed pseudo numbers combined with the input inherent options values of $S_{t_0}$, $\sigma_i$, $r$ and $d_i$.

4. Obtain correlated multivariate random variables through the product of the lower triangular matrix decomposed from the correlation matrix in step(2) and the matrix of $S_{iT}^i$ ($i = 1, 2, \cdots, k$) obtained in step(3).
5. The option payoff $C_T(S_{iT})$ is calculated for each generated value scenario $S_{iT}$.

6. Take the arithmetic mean of the option payoffs as $\hat{C} = \frac{1}{n} \sum_{i=1}^{n} C_T(S_{iT})$.

Alternatively, the procedure can be described in short as follows:

Financial data
(underlying asset prices $S_{i0}$, $\sigma_i$ and assets correlation matrix $\rho_{ij}$)

↓

A lower triangle matrix of
the correlation matrix through Cholesky factorization

× Generated random variables

↓

Correlated variables

↓

Estimated financial data $S_{iT}$ distribution

↓

Compute and average the discounted option payout

### 2.4 Quantlets in XploRe

With the Monte Carlo methods, XploRe offers a fast and convenient numerical way to calculate the price of the three discussed multi-asset equity options.

The basic structure of quantlets [Basketpricer], [Maxpricer] and [Minpricer], which are wrapped with a dynamically linked library (DLL) written in C program language within XploRe, is given by
Price = \texttt{Basketpricer}(S, K, \tau, iv, r, d, Corr[, NumSim])

Price = \texttt{Maxpricer}(S, K, \tau, iv, r, d, Corr[, NumSim])

Price = \texttt{Minpricer}(S, K, \tau, iv, r, d, Corr[, NumSim])

The input data has to contain at least 7 essential underlying values of an \(n\)-asset option. The first is a \((n \times 1)\) underlying asset prices column vector \(S\), the second the strike \(K\), the third maturities \(\tau\) (expressed in years), the fourth a \((n \times 1)\) implied volatilities column vector \(iv\), the fifth the interest rates \(r\) (on a yearly basis), the sixth a \((n \times 1)\) dividend rate column vector \(d\), the seventh a \((n-1) \times 1\) column vector \(Corr\), showing the off-diagonal upper triangle correlation matrix row-wise. The last, but optional input parameter, \(NumSim\), stands for the number of numerical simulations. The default number is set to be 50,000.

2.5 An Example in XploRe

The following simple example serves as an illustration: consider three-asset European basket, max and min options at strike price \(K = 100\) with maturity \(\tau\) of half a year, where the interest rate is assumed to be \(r = 4\%\) and the three underlying assets are equally priced at \(S = 100\) with no dividends and volatility 0.3, 0.4, 0.25 respectively. The correlation
matrix is as follows:

\[
\begin{pmatrix}
1 & 0.3 & 0.3 \\
0.4 & 1 & 0.5 \\
0.8 & 0.5 & 1
\end{pmatrix}
\]

Then the command

\[
S = \#(100, 100, 100) \\
K = 100 \\
tau = 0.5 \\
niv = \#(0.30, 0.40, 0.25) \\
r = 0.04 \\
d = 0.0*\text{matrix}(3) \\
Corr = \#(0.30, 0.30, 0.50)
\]

Basketpricer(S, K, tau, iv, r, d, Corr) \\
Maxpricer(S, K, tau, iv, r, d, Corr) \\
Minpricer(S, K, tau, iv, r, d, Corr)

yields 7.7897, 19.876 and 2.2094 as prices for the basket, max and min options.
3 correlation vegas

Generally, the price of a multi-asset equity option is determined internally by the issuing financial institute after a close study of the model and the associated risks. Finding an appropriate pricing model is of course important, however, it is only a part of the whole story of global risk analysis. Quessette (2002) points out that some of the associated risks should be examined to be able to quantify prices correctly. Therefore, the measurement of the inherent correlation risk becomes another cutting-edge issue in option risk management. Analytically, the defined “Correlation Vega” exposure is the first derivative of the option price with respect to the correlation matrix $\rho_{ij} (i < j)$, and can be interpreted as the sensitivity of the option with respect to correlations.

3.1 Hints From Simple Calculations

First, a comparison of three scenarios leads to some hints for the existence of the correlation risk. Reconsider the example in section 2.5 the original scenario is set as a reference point, then two new scenarios are created with an overall upward and downward shift in the correlation matrix as below while keeping the other parameters stable as before.
<table>
<thead>
<tr>
<th>option type</th>
<th>upward shift scenario</th>
<th>reference scenario</th>
<th>downward shift scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basket Option</td>
<td>9.2475</td>
<td>7.7897</td>
<td>7.4141</td>
</tr>
<tr>
<td>Max Option</td>
<td>15.388</td>
<td>19.876</td>
<td>20.538</td>
</tr>
<tr>
<td>Min Option</td>
<td>4.9424</td>
<td>2.2094</td>
<td>1.7746</td>
</tr>
</tbody>
</table>

Table 3.1: Comparison of the prices over the change of correlations

\[
\begin{pmatrix}
1 & 0.9 & 0.8 \\
0.9 & 1 & 0.7 \\
0.8 & 0.7 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0.4 & 0.3 \\
0.4 & 1 & 0.1 \\
0.3 & 0.1 & 1
\end{pmatrix}
\]

Executing the correspondent commands leads to the results summarized in Table 3.1. Obviously, a shift in correlation can result in a significant change in the prices of options.

### 3.2 Correlation Risk of Multi Exchange Rates Options

Opposed to multi-asset equity options, the correlation risk involved in the valuation of Multi Exchange Rates Options can be estimated and consequently be hedged against. By using the interdependence of exchange rates, Wystup (2002) computes correlations explicitly via the known volatilities.

This can be illustrated by the use of a simple example of a triangular FX market: \( S_{1t} \) (BPD/USD), \( S_{2t} \) (USD/EUR) and \( S_{3t} \) (BPD/EUR). Based upon the assumption that the FX rates follow a geometric Brownian motion, the variance and covariance of currencies can be written as...
Var(\ln S_i) = \sigma_i^2 \text{ and Cov}(\ln S_i, \ln S_j) = \sigma_i \sigma_j \rho_{ij} \ (i, j = 1, 2, 3). \) Then taking the logarithm of the interdependence of the currencies \( S_{1t} S_{2t} = S_{3t} \) leads to

\[
\ln S_{1t} + \ln S_{2t} = \ln S_{3t}.
\]

Thus on the basis of the formula for the variance of the sum of two random variables \( \text{Var}(x + y) = \text{Var}(x) + 2 \text{Cov}(x, y) + \text{Var}(y) \), the correlation can be computed as:

\[
\text{Var}(\ln S_{3t}) = \text{Var}(\ln S_{1t}) + 2 \text{Cov}(\ln S_{1t}, \ln S_{2t}) + \text{Var}(\ln S_{2t}) \quad (3.1)
\]

\[
\sigma_3^2 = \sigma_1^2 + 2 \sigma_1 \sigma_2 \rho_{12} + \sigma_2^2 \quad (3.2)
\]

\[
\rho_{12} = \frac{\sigma_3^2 - \sigma_1^2 - \sigma_2^2}{2 \sigma_1 \sigma_2} \quad (3.3)
\]

In this way, the necessary correlation coefficients can be easily obtained through repeated calculations. This can be also interpreted in a geometrical way, (Wystup, 2002): if the three currencies are set as three corners of a triangle and the FX rates as three edge vectors, then the correlations are just the cosine of the three angles respectively. Thus the correlation structure turns out to be fully determined by the volatilities. This result has a striking implication: the correlation risk of multi exchange rate options can be easily hedged simply by trading FX volatilities. Unfortunately, this does not hold for stocks since they are traded in cash and not in pairs as currencies are.

### 3.3 Correlation Risk Drivers

As observed in the above calculation, the correlation risk has different influences on options with different payoff structures. In order to gain an intuitive understanding of correlations’ influence on the price of the three
option types, Fengler and Schwendner (2003) present an interpretation of two different drivers for the correlation risk:

1. the influence of correlation on the volatility of the whole basket
2. the influence of correlation on the dispersion of individual assets in the basket

These two drivers affect the value of the three types of options in a different way as discussed in the following.

**Basket Option**
Basket options are affected only by the basket volatility. The dispersion of individual assets has no impact on the options price, as the payout is only dependent on the average of all the assets in the basket. If the correlation rises, the basket volatility increases and the options price will become more expensive. Thus, basket options are long in correlation.

**Max Option**
In addition to the basket volatility which increase the options price, the dispersion of individual assets also exerts an influence on the max options price. A higher dispersion of the individual assets implies more volatile changes in the prices of the underlying assets. That in turn implies a higher probability of any asset reaching particularly high levels at maturity. This effect decreases with rising correlations. Thus the two drivers have opposite effects on the price of max options. As can be observed from the numerical application in Section 3.4, max options can be both long and short in correlation, depending on the performance of the underlying assets, i.e., asset price, volatility and correlation levels. But
for most cases, especially, in the moderate correlation spectrum, options are short correlation. This results directly from the stronger effect of individual dispersion over that of basket volatility.

Min Option
Min options are also affected by the two drivers. But in this case both drivers work in the same direction. For min options, little dispersion of individual assets is preferred since it minimizes the probability of any individual asset shifting downward to particularly low levels and consequently increases the value of the options. Thus, the dispersion effect increases the options price with higher correlations. As the two drivers reinforce each other, min options are not only long in correlation but also highly sensitive to correlation.

3.4 Numerical Examples

In order to fully examine the impact of the correlation risk on the options price, a numerical application is then proposed in the case of three-asset equity options. Correlation vegas are calculated numerically in several scenarios and then compared and analyzed.

3.4.1 The General Set-up

The aim of the following numerical example is to observe and compare the correlation vega in terms of correlation change and in terms of the change of underlying assets performance. Therefore, I set three scenarios: options on identical assets, options on assets that differ only in prices and
options on assets that differ only in volatilities. Then for each scenario the correlation vega is estimated on varying correlations. The detailed values for each scenario are displayed in Table 3.2.

To keep things simple, I assume that interest rates are zero, assets pay no dividends, and the options are all exercised at strike price $K = 100$ in $T = 1$ year. Then in this way, the basket options are always at the ATM (at-the-moneyness) position as their strike always equals the average of the asset prices, no matter how they differ across the three scenarios.

The above settings are separately applied to all of the three option types. The price of the options is then computed via the discounted mean of the payoff after 50,000 simulations except for the min and max options on assets of different prices. In these cases, the correlation vega is somewhat changed in the three digits after the decimal point area. Therefore, 100,000 simulations were made to obtain more accurate estimations.

### 3.4.2 Numerical Approximation of correlation vegas

As defined, the correlation vega for three-asset equity options is an off-diagonal upper triangular matrix of first derivatives

$$
\begin{pmatrix}
1 & \frac{\partial C}{\partial p_{12}} & \frac{\partial C}{\partial p_{13}} \\
1 & \frac{\partial C}{\partial p_{23}} \\
1 & & 1
\end{pmatrix}
$$

Table 3.2: Scenarios Setup for the Numerical Example

<table>
<thead>
<tr>
<th></th>
<th>Identical Assets</th>
<th>Assets Differing in Prices</th>
<th>Assets Differing in Volatilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>(100, 100, 100)</td>
<td>(150, 100, 50)</td>
<td>(100, 100, 100)</td>
</tr>
<tr>
<td>$iv$</td>
<td>(0.30, 0.30, 0.30)</td>
<td>(0.30, 0.30, 0.30)</td>
<td>(0.60, 0.30, 0.15)</td>
</tr>
</tbody>
</table>
Thus in the following analysis, there are always three derivatives concerned. For convenience, they are noted in plots as CorVega(r12), CorVega(r13) and CorVega(r23).

The proposed method to calculate correlation vegas is known as finite diffusion approximation, a popular approach for estimating derivatives numerically. Then the effective three-point approximation formula is used:

\[ f'(x) \approx \frac{f(x + h) - f(x - h)}{2h} \]  \hspace{1cm} (3.4)

Since the data used are symmetrically placed relative to where the derivative is computed, this formula is also called a centered difference approximation. In practice, this formula is widely used because the three-point difference formula converges to the true value in a speed of \( O(1/h^2) \) and is more exact than the two-point formula with a convergence speed of \( O(1/h) \). Thus it lessens the necessity of choosing a very small \( h \) value. Here, in the numerical calculation, a distance \( h = 0.00005 \) is used in most cases in order to confirm a reliable estimate. Considering that the accuracy of Monte Carlo simulations declines with every extra digits and that the correlation vega is too small, I chose a larger distance \( h = 0.001 \) for the max and min options on assets that differ only in prices.

Thus, the first step to estimate correlation vegas is to calculate the prices of two options and then divide the difference by \( 2h \). Meanwhile, in order to achieve a smooth plot, the distance between the grid points of two coordinates \( \rho_{12} \) and \( \rho_{13} \) is set to be 0.01. Proceeding this way, a large data set of \( \left[ \frac{1-(1-0.01)}{0.01} + 1 \right]^2 = 40,401 \) correlation vegas is simulated for analysis.
The correlation sensitivity of a multi-asset option depends on all the correlations between constitute assets, as the underlying assets are correlated to each other. That is, the correlation vega of one correlation \( \left( \frac{\partial C}{\partial \rho_{ij}} \right) \) is a function of all correlations \( \rho_{ij} \) \( \forall \ i, j \) and \( i \neq j \). Geometrically, the correlation vega is one-to-one projected to a three-dimensional space with coordinates \( \rho_{12}, \rho_{13} \) and \( \rho_{23} \). As it is the common practice in presenting high dimensional data on a surface, I take a slice where the two axes \( \rho_{12} \) and \( \rho_{13} \) vary from \(-1\) to \(1\), but \( \rho_{23} \) is always fixed at a specific level. To better understand correlation vegas change in \( \rho_{23} \), three particular surfaces are chosen at \( \rho_{23} = 0 \), \( \rho_{23} = 0.5 \) and \( \rho_{23} = -0.5 \).

Furthermore in order to fully demonstrate the correlation risk, temperature plots are created in this work. The basic idea of temperature plots is similar to contour plots. But here the level of the correlation vega is demonstrated by different colors instead of contour lines. Basically, a cool color spectrum from purple to dark-blue is mapped to negative values from zero to the lowest; and a warm color spectrum from green, yellow to red indicates positive values to the highest. In this way, one can display the change of the correlation vega using the contrast of colors.

For the convenience of comparing different scenarios, I tried to set a common color scale for one option type. This is basically realized, but sometimes fails since the range of correlation vegas differs greatly. Thus the same color may indicate different levels even for the same type of option. Therefore, to avoid mistakes, a correspondent temperature scale is provided in each plot.
3.4.4 Analysis of the Result

Under different settings, altogether 69 plots are created and grouped in Appendix A.5, A.6 and A.7. Tables A.1, A.2 and A.3 summarize general statistics of the correlation vega for the three types of options in each scenario.

For all options, the correlation vega takes the form of a circle or an ellipse at different positions, depending on the actual value of $\rho_{23}$. This shows that the correlation vega is dependent on correlations. The correlation matrix should always be positive definite. This consequently defines the location of the correlation vega.

**Basket Option**

*Observation 1*: Since the correlation risk of basket options is driven only by the basket volatility and the effect is rather straightforward, only 15 plots are presented here. As is observed in all plots, the correlation vega of basket options is always positive. This is exactly the effect of basket volatilities and confirms the argument that basket options are long correlation. Although the correlation vega ranges widely from 0.5 up to 9.8, the median is only between 1 to 3. At the same time, the standard deviation of the correlation vega is less than 1. Thus generally, basket options display a moderate change in correlation exposure.

*Observation 2*: Reading from the plots, basket options become more sensitive to correlations in the diagonal direction and reach highest levels at the lower-right corner when $\rho_{12}$ and $\rho_{13}$ come to the lowest possible values. This leads to the conclusion that simultaneous movement of correlations is “dangerous”. This result is unsurprising since the shift in a larger number of correlations can cause a greater change in the basket
volatility calculated as

\[ \sigma^2(\text{basket}) = \sum_{i=1}^{n} \sigma_i^2 + 2 \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i \sigma_j \rho_{ij}, \quad n = 3, \quad i, j = 1, 2, 3. \]

Thus basket options are more sensitive under the collective shift of correlations, compared to the shift of a single correlation. That is why the dangerous area is expanded from \( \rho_{12} = -0.5 \) in Figure A.1 where \( \rho_{23} \) is equal to 0 to \( \rho_{12} = 0 \) in Figure A.7 where \( \rho_{23} \) is \(-0.5\).

Then the question arises, why options face great correlation exposures when correlations are small? Intuitively, a low correlation implies that any asset is highly probable to decline to a particularly low level. Therefore the option could be rather cheap. Then a subtle upward shift in correlation can reduce the probability and result in a relatively sharp change in options prices. In this sense, it can be regarded as the “slope” effect.

**Observation 3:** When comparing options sensitivities to \( \rho_{12} \) in each scenario across the value \( \rho_{23} \), one observes that the correlation vega tends to be positively related to \( \rho_{23} \). Clearly, this is again the effect of basket volatility: the larger \( \rho_{23} \), the higher the volatility, the higher the options. However, such changes are relatively small so that a great contrast in color can not be observed.

**Observation 4:** In Figure A.1, A.10 and A.13 three derivatives, \( \frac{\partial C}{\partial \rho_{12}} \), \( \frac{\partial C}{\partial \rho_{13}} \) and \( \frac{\partial C}{\partial \rho_{23}} \), are displayed for basket options on identical assets. As it can be seen, the correlation exposures are moderate and the patterns of change are similar. That is, however, due to the fact that all the assets are identical. As explained in Observation 5 and 6, the correlation vega with respect to different correlations exhibits distinctly different movement tendencies when volatilities and prices differ across assets.
Observation 5: After the change in asset prices, asset 1 becomes the best performer with the highest price 150; while the price of asset 3 drops by one half. With its outstanding performance, asset 1 can contribute more to the option price. Consequently, $\frac{\partial C}{\partial \rho_{12}}$ exhibits an overall rise compared to the options on identical assets. While, $\frac{\partial C}{\partial \rho_{13}}$ decreases and $\frac{\partial C}{\partial \rho_{23}}$ drops by one half, proportional to the price change.

Observation 6: The correlation sensitivity is even more significantly affected when the assets differ in volatilities. Similar to the above observation, $\frac{\partial C}{\partial \rho_{12}}$ increases with doubled volatility of asset 1 and $\frac{\partial C}{\partial \rho_{13}}$ declines moderately. However, $\frac{\partial C}{\partial \rho_{23}}$ falls strikingly to about one third of the original level whereas the volatility of asset 3 is only halved. That is why we have almost the same color in Figure A.14. Thus, options become considerably less sensitive to the correlation between the assets with lower performance.

Based on the above two observations, options have a relatively larger sensitivity to the correlation of assets with higher performance. Therefore, the correlation of such assets deserve prior attention in pricing and hedging.

Max Option

Observation 1: Recall that max options are affected by two competing drivers so they could be both long and short in correlations. As is observed in our example, the correlation vega is in most cases negative. Seldom options are long correlation. Usually this is the case for the options which either differ in volatility or price and appears in the plots at the boundary with extreme correlation values close to ±1. This demonstrates that max options being both long and short in correlation highly depends on the specific levels of correlation, asset price and volatility.
Observation 2: Compared to basket options, max options exposes more greatly to the correlation risk. The median values are between $-0.006$ and $-5.5$, differing remarkably from case to case. They have the largest range in the three types of options. This results directly from the payoff structure where options payout is directed only at the performance of the best asset. Consequently, a higher dispersion between assets is preferred for max options.

Observation 3: Unlike basket options, max options are more sensitive to the movement of a single correlation. One may observe that the change tendency of $\frac{\partial C}{\partial \rho_{12}}$ parallels to the coordinate $\rho_{12}$, whereas $\frac{\partial C}{\partial \rho_{13}}$ changes vertically along the direction of coordinate $\rho_{13}$. Thus an option’s sensitivity to a certain correlation seems to be driven mainly by the correlation itself. Intuitively, this may be interpreted as the stronger dispersion effect over the basket volatility. However, the plots regarding $\frac{\partial C}{\partial \rho_{23}}$ look different and may contradict the argument. In fact these plots demonstrate the collective effect of $\rho_{12}$ and $\rho_{13}$. It can also be validated via the statistics of $\frac{\partial C}{\partial \rho_{23}}$ for options on identical assets, which is obviously different from $\frac{\partial C}{\partial \rho_{12}}$ and $\frac{\partial C}{\partial \rho_{13}}$ in the same scenario. Moreover, if one compares $\frac{\partial C}{\partial \rho_{23}}$ in each scenario across $\rho_{23}$, a strong dependence can still easily be observed.

Observation 4: Interestingly, the correlation vega of an option declines with correlation when underlying assets differ in price, otherwise it increases with correlation even when assets have different volatilities. For example, options become more sensitive from the left to right in Figure A.16 and A.17, but in the opposite way in Figure A.18.

The positive relationship between the correlation risk and correlations can be again interpreted as the “slope” effect. When correlations are large, options are fairly cheap due to the low dispersion between indi-
individual assets. However, a subtle change in correlation may significantly raise the possibility of any assets to arrive at a higher value, thus having a great impact on the options price.

In the case of the change in price, the price of asset 1 becomes extremely large. Then the asset should be the most likely candidate for entering into the payout function, if $\rho_{12}$ or $\rho_{13}$ is close to 1. If they are close to $-1$, the situation is more complex: it is likely that one of the other two assets outperform asset 1, thus increasing the options price. As a consequence, correlation diminishes options correlation exposure. And during this process, the basket volatility and dispersion effect compete with each other and one may take an absolute dominance at extreme correlation values. This helps to explain that a very small shift in correlations may change options with assets on different prices between being long and short correlation.

**Observation 5:** As basket options, correlation risk exposure of max options is moderate and of same structure for options on identical assets. But any change in asset price and volatility could produce a great effect on the options correlation exposure. Moreover such effect differs across assets with different performances. As expected, $\frac{\partial C}{\partial \rho_{12}}$ exhibits an overall rise compared to the options on identical assets, while $\frac{\partial C}{\partial \rho_{13}}$ and $\frac{\partial C}{\partial \rho_{23}}$ drops, whenever the price or volatility changes. The argument is, that it is unlikely for assets with extremely lower performance to outpace other assets. Consequently, the correlation of such assets does not matter so much.

**Observation 6:** One can notice that correlation sensitivity is significantly affected when the assets differ greatly in price: $\rho_{12}$ increases by over one half, at the same time $\frac{\partial C}{\partial \rho_{13}}$ and $\frac{\partial C}{\partial \rho_{23}}$ falls strikingly to almost 0. It can be
interpreted that the price of max options depends on the relative price of the assets, less strongly on volatility and even less on correlation. Based on this observation, one can conclude that correlation exposure of options on assets with extremely lower performance can be neglected in pricing and hedging when the underlying assets differ only in price.

Min Option

The payoff structure of min options is similar to that of max options, except that the worst, instead of best, asset is concerned. Therefore, these two types of options may possess some common features and at the same time exhibit significant distinctions.

Observation 1: As is examined in Section 3.3, min options are affected by the dispersion and basket volatility effect simultaneously. Since they reinforce each other, min options are always long in correlation. This is basically validated in the numerical application. Nevertheless the simulated data contain negative values. They are considered to be numerical errors since these fairly small negative values exist mostly in the case of a change in price, where the range of the correlation vega is extremely small around 0.

Observation 2: The median values of the correlation vega are small, ranging from 0.0007 to 1.5. However, the relative value of correlation vega to the options price turns out to be fairly high, compared to that of basket and min options. Meanwhile in the plots, an obvious contrast of color can be observed even in the area with moderate correlation values. These all demonstrate that min options are highly sensitive to correlations with the combined effect of two drivers.

Observation 3: Basically, the correlation vega is positively related to correlations. One may draw this conclusion from the observations that $\frac{\partial C}{\partial \rho_{12}}$. 

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\( \frac{\partial C}{\partial \rho_{13}} \) and \( \frac{\partial C}{\partial \rho_{23}} \) always increase as \( \rho_{23} \) rises. This is because increasing correlations result in beneficial effects of the both drivers on min options. Here, one can also see a large difference between the change patterns of the correlation sensitivity as \( \rho_{23} \) varies, which reinforces the conclusion obtained in Observation 2.

Observation 4: As seen in the plots, the change tendency of the correlation vega for min options is not so regular and the division between colors is not linear but curved. Consequently, some complex but interesting situations are created. For example in Figure A.43, two options with different correlation combinations as \( \rho_{12} = 0.5 \), \( \rho_{13} = 0.7 \) and \( \rho_{12} = \rho_{13} = 0 \) bear almost the same correlation risk. Thus, correlations of min options have to be closely monitored in pricing and hedging.

Observation 5: The correlation sensitivity of min options is again greatly affected when assets differ in volatilities or prices. As is shown in the plots, \( \frac{\partial C}{\partial \rho_{12}} \), \( \frac{\partial C}{\partial \rho_{13}} \) and \( \frac{\partial C}{\partial \rho_{23}} \) all decrease when assets change in volatilities and drop even more significantly as prices change. Obviously, min options also depend strongly on the relative price of the assets, less on volatility and least on correlation. Furthermore as in the numerical example, if the worst asset price decreases so greatly even below the strike price, correlations have almost no effect on options.

One issue worth to mention is that the decrease of the three derivatives are driven by different reasons. After the change in asset prices or volatilities, asset 3 turns out to be the worst asset. It is unlikely for asset 1 and 2 to decline significantly below the price of asset 3 and enter into the payoff formula. Therefore, \( \frac{\partial C}{\partial \rho_{12}} \) and \( \frac{\partial C}{\partial \rho_{13}} \) diminish greatly simply because the correlations of assets with higher performance do not matter so much any more. However this does not hold for asset 3. In fact, the options

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price is subject to any small change in the correlation and performance of asset 3. Thus, the decline of $\frac{\partial C}{\partial \rho_{p3}}$ is due to the decrease of its price or volatility, which deteriorate the options price and also the correlation sensitivity.

Observation 6: A big distinction between max and min options is the change pattern of the correlation vega. One may find that there are two somewhat symmetrical “dangerous” areas where min options are highly sensitive to correlations. One of the two areas is located at the right boundary of the plots for $\frac{\partial C}{\partial \rho_{p12}}$ (as in Figure A.43 and alike) or the top for $\frac{\partial C}{\partial \rho_{p13}}$ (as in Figure A.52 and alike). In such areas the examined correlation is extremely high close to 1, while the other is almost 0. Undoubtedly, the basket volatility effect of the examined correlation increases to rather high levels, so the price of options rises greatly. The other “dangerous” area is formed with the correlation combinations of one moderately high about 0.5 and one moderately low around −0.5. With such correlation combinations the three assets change greatly in opposite directions. Clearly, this is a dispersion effect: the more dispersed the assets, the more expensive options. This effect can be intensified if assets differ in volatilities. That is why this area becomes more obvious in the plots for the case of a change in volatility.

However, this can only be observed in the plots of $\frac{\partial C}{\partial \rho_{p12}}$ and $\frac{\partial C}{\partial \rho_{p13}}$, as the plots of $\frac{\partial C}{\partial \rho_{p23}}$ demonstrate the collective effect of two correlations $\rho_{12}$ and $\rho_{13}$, thus displaying different images as the other plots.

Based upon the analysis above, the sensitivity of multi-asset equity options on correlations first depends on the payoff structure. Different options types display different change patterns of the correlation vega.
However no matter what the payoff functions look like, the correlation vega shifts strongly in the change of asset prices and volatilities.
4 Conclusion

As multi-asset equity options are derived from several underlying assets, the correlation matrix of the underlying assets is required to be fully incorporated in the options pricing. By randomly sampling the high dimensional stochastic distributions together with the correlation structure, Monte Carlo simulation methods “accurately” price multi-asset equity options. Therefore, Monte Carlo simulation methods are used in this thesis to develop three quantlets in XploRe, pricing three standard types of multi-asset options (basket options, min and max options).

Then, I discuss the correlation risk and present numerical results for three different scenarios: options on identical assets and options on assets differing in price or volatility only. The correlation vega is estimated by the finite diffusion approximation method and then demonstrated by temperature plots.

The shown differences in correlation vega can basically be attributed to two drivers of correlation risk acting differently for the three payout structures: basket volatility and dispersion. Therefore different types of options display significantly different change patterns of correlation vega. However, no matter how they change, options correlation exposure is greatly affected as asset prices or volatilities change.
A Appendix

A.1 XploRe Quantlets of Basketpricer

\textbf{proc} \text{(Price)}=\text{Basketpricer}(S, K, \tau, iv, r, d, Corr, \{, NumSim\})

\begin{table}[h]
\begin{tabular}{ll}
Library: & finance \\
See also: & Maxpricer, Minpricer \\
Macro: & \\
Description: & calculates the price of a multi-asset option which is based on the average performance of an equally weighted n underlying assets by Monto Carlo Simulation Methods. \\
Keywords: & multi-asset options pricing \\
Usage: & \text{P} = \text{Basketpricer}(S, K, \tau, iv, r, d, Corr\{, NumSim\})
\end{tabular}
\end{table}

Input
Parameter: S
Definition: 1 * n vector, starting price of the underlying assets

Parameter: K
Definition: scalar, strike price of the option

Parameter: tau
Definition: scalar, time to maturity (in years)

Parameter: iv
Definition: 1 * n vector, implied volatilities of the underlying options

Parameter: r
Definition: scalar, interest rate

Parameter: d
Definition: 1 * n vector, dividend rate of each underlying asset

Parameter: Corr
Definition: 1 * (n * (n - 1)/2) vector, off-diagonal upper triangle correlation matrix of the underlying assets excluding the diagonal elements, which are input row-wise. For example, #(0.2, 0.3, 0.6) means a correlation matrix as

\[
\begin{pmatrix}
1 & 0.4 & 0.5 \\
0.4 & 1 & 0.3 \\
0.5 & 0.3 & 1
\end{pmatrix}
\]

Parameter: NumSim
Definition: optional scalar, the number of simulations in the Monto Carlo Simulation method; if not given, 50,000 as the default value.

Output:
Parameter: Price  
Definition: scalar, the option price of basket options on an equally weighted basket of n assets

\[
\text{proc}(\text{Price}) = \text{Basketpricer}(S, K, \tau, \text{iv}, r, d, \text{Corr}, \text{NumSim})
\]

\[
h = \text{dlopen}(\text{“basketpricer.dll”})
\]

set the default number of simulations as 50,000 if no value is given

\[
\text{if} (\text{exist(“NumSim”) == 0})
\quad \text{NumSim} = 50000
\quad \text{endif}
\]

clarify the number of underlying assets

\[
n = \text{rows}(S)
\]

put weights on the asset price

\[
S = S./n
\]

guarantee rows(S) = rows(iv) = rows(d) = n, rows(Corr) = n*(n - 1)/2

\[
\text{if(rows(iv)! = n)}
\quad \text{error( n <> rows(iv), “Each asset should have its own volatility.”)}
\quad \text{endif}
\]

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if(rows(d)!= n)
    error( n <> rows(d), “Each asset should have its own dividend rate.” )
endif

clarify the elements number in the off-diagonal upper triangle correlation matrix

\[ N_{corr} = \frac{n*(n - 1)}{2} \]

if(rows(Corr) != Ncorr )
    error(rows(Corr) <> Ncorr, “The upper triangle correlation matrix must have n * (n-1)/2 elements.”)
endif

Corr = trans(Corr)

set the seed for simulations

istart = -5
Price = -99

rr = dlcall(h, “basketpricer”, n, S, K, tau, iv, r, d, NumSim, istart, Corr, Price)
dlclose(h)
A.2 XploRe Quantlets of Maxpricer

\texttt{proc}\texttt{(Price) = Maxpricer(S, K, tau, iv, r, d, Corr, {}, NumSim})

Library: finance

See also: Basketpricer, Minpricer

Macro:

Description: calculates the price of a multi-asset option which is based on the maximum performance of a basket of \( n \) underlying assets by Monte Carlo Simulation Methods.

Keywords: multi-asset options pricing

Usage: \( P = \texttt{Basketpricer}(S, K, \text{tau}, \text{iv}, r, d, \text{Corr}{}, \text{NumSim}) \)

Input

Parameter: \( S \)
Definition: \( 1 \times n \) vector, starting price of the underlying assets
Definition: scalar, strike price of the option
Parameter: tau
Definition: scalar, time to maturity (in years)
Parameter: iv
Definition: 1 * n vector, implied volatilities of the underlying options
Parameter: r
Definition: scalar, interest rate
Parameter: d
Definition: 1 * n vector, dividend rate of each underlying asset
Parameter: Corr
Definition: 1 * (n * (n - 1)/2) vector, off-diagonal upper triangle correlation matrix of the underlying assets excluding the diagonal elements, which are input row-wise. For example, #(0.2, 0.3, 0.6) means a correlation matrix as

\[
\begin{pmatrix}
1 & 0.4 & 0.5 \\
0.4 & 1 & 0.3 \\
0.5 & 0.3 & 1
\end{pmatrix}
\]

Parameter: NumSim
Definition: optional scalar, the number of simulations in the Monte Carlo Simulation method; if not given, 50,000 as the default value.

Output:

Parameter: Price
Definition: scalar, the price of max options on the maximum performance of n assets
proc(Price)=Maxpricer(S, K, tau, iv, r, d, Corr{, NumSim})

h=dlopen("basketpricer.dll")

set the default number of simulations as 50,000 if no value is given

if (exist("NumSim") == 0)
    NumSim = 50000
endif

clarify the number of underlying assets

n = rows(S)

guarantee rows(S) = rows(iv) = rows(d) = n, rows(Corr) = n*(n−1)/2

if(rows(iv)!= n)
    error( n <> rows(iv), “Each asset should have its own volatility.”)
endif

if(rows(d)!= n)
    error( n <> rows(d), “Each asset should have its own dividend rate.” )
endif

clarify the elements number in the off-diagonal upper triangle corre-
lation matrix

\[ N_{corr} = n^* (n - 1)/2 \]

if(rows(Corr) != Ncorr )
    error(rows(Corr) <> Ncorr, “The upper triangle correlation matrix must have n * (n-1)/2 elements.”)
endif

Corr = trans(Corr)

set the seed for simulations

istart = -5
Price = -99

\[ rr = dlcall(h, “maxpricer”, n, S, K, tau, iv, r, d, NumSim, istart, Corr, Price) \]

dlclose(h)

endp
A.3 XploRe Quantlets of Minpricer

\[ \text{proc}(\text{Price}) = \text{Minpricer}(S, K, \text{tau}, \text{iv}, r, d, \text{Corr}, \{, \text{NumSim}\}) \]

Library: finance

See also: Basketpricer, Maxpricer

Macro:

Description: calculates the price of a multi-asset option which is based on the minimum performance of a basket of \( n \) underlying assets by Monte Carlo Simulation Methods.

Keywords: multi-asset options pricing

Usage: \( P = \text{Minpricer}(S, K, \text{tau}, \text{iv}, r, d, \text{Corr}\{, \text{NumSim}\}) \)

Input

Parameter: \( S \)
Definition: \( 1 \times n \) vector, starting price of the underlying assets

Parameter: \( K \)
Definition: scalar, strike price of the option

Parameter: \( \text{tau} \)
Definition: scalar, time to maturity (in years)

Parameter: \( \text{iv} \)
Definition: \( 1 \times n \) vector, implied volatilities of the underlying options
Parameter: $r$
Definition: scalar, interest rate

Parameter: $d$
Definition: $1 \times n$ vector, dividend rate of each underlying asset

Parameter: Corr
Definition: $1 \times (n \times (n - 1)/2)$ vector, off-diagonal upper triangle correlation matrix of the underlying assets excluding the diagonal elements, which are input row-wise. For example, 
$\#(0.2, 0.3, 0.6)$ means a correlation matrix as

$$
\begin{pmatrix}
1 & 0.4 & 0.5 \\
0.4 & 1 & 0.3 \\
0.5 & 0.3 & 1
\end{pmatrix}
$$

Parameter: NumSim
Definition: optional scalar, the number of simulations in the Monte Carlo Simulation method; if not given, 50,000 as the default value.

Output:

Parameter: Price
Definition: scalar, the price of min options on the minimum performance of n assets

```
proc(Price)=Minpricer(S, K, tau, iv, r, d, Corr, NumSim)
```

```
h=dlopen(“basketpricer.dll”)
```
set the default number of simulations as 50,000 if no value is given

    if (exist(“NumSim”) == 0)
        NumSim = 50000
    endif

clarify the number of underlying assets

    n = rows(S)

guarantee rows(S) = rows(iv) = rows(d) = n, rows(Corr) = n*(n – 1)/2

    if(rows(iv)!= n)
        error( n <> rows(iv), “Each asset should have its own volatility.”
    endif

    if(rows(d) != n)
        error( n <> rows(d), “Each asset should have its own dividend rate.”
    endif

clarify the elements number in the off-diagonal upper triangle correlation matrix

    Ncorr = n*(n - 1)/2

    if(rows(Corr) != Ncorr )
error(rows(Corr) <> Ncorr, “The upper triangle correlation matrix must have n * (n-1)/2 elements.”)
endif

Corr = trans(Corr)

set the seed for simulations

istart = -5
Price = -99

rr = dcall(h, “minpricer”, n, S, K, tau, iv, r, d, NumSim,
           istart, Corr, Price)
dlclose(h)
endp

A.4 Statistics Table of correlation vegas

Please note: up and dn indicate \( \rho_{23} = 0.5 \) and \( \rho_{23} = -0.5 \) respectively, otherwise \( \rho_{23} = 0 \). IV shows that in this scenario the underlying assets differ in implied volatility only, and \( S \) in prices only. Otherwise assets are identical.
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<thead>
<tr>
<th>scenario</th>
<th>Mean</th>
<th>Median</th>
<th>Standard Deviation</th>
<th>Max</th>
<th>Min</th>
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<td>$\rho_{12}$</td>
<td>2.44262</td>
<td>2.19590</td>
<td>0.79665</td>
<td>6.34940</td>
<td>1.42581</td>
</tr>
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<td>$\rho_{12}IV$</td>
<td>3.45140</td>
<td>3.21900</td>
<td>0.81568</td>
<td>6.13281</td>
<td>2.24663</td>
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<td>$\rho_{12}S$</td>
<td>3.40958</td>
<td>3.06818</td>
<td>1.11977</td>
<td>9.43762</td>
<td>1.95159</td>
</tr>
<tr>
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<td>2.99353</td>
<td>2.66616</td>
<td>0.98661</td>
<td>8.08212</td>
<td>1.99248</td>
</tr>
<tr>
<td>$\rho_{12}dnIV$</td>
<td>3.58136</td>
<td>3.42561</td>
<td>0.64192</td>
<td>5.31923</td>
<td>2.56896</td>
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<td>3.32203</td>
<td>0.94907</td>
<td>7.19098</td>
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<td>1.93474</td>
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<td>4.41417</td>
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<td>1.09994</td>
<td>9.87982</td>
<td>1.84576</td>
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<td>$\rho_{13}$</td>
<td>2.44574</td>
<td>2.19539</td>
<td>0.76119</td>
<td>6.27897</td>
<td>1.45825</td>
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<td>$\rho_{13}IV$</td>
<td>1.75019</td>
<td>1.63166</td>
<td>0.42059</td>
<td>3.11102</td>
<td>1.11676</td>
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<td>0.28490</td>
<td>1.90053</td>
<td>0.59294</td>
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<td>3.45219</td>
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Table A.1: Statistics for Basket Options in 15 scenarios
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<td>$\rho_{12}$</td>
<td>-3.66797</td>
<td>-3.10312</td>
<td>2.47107</td>
<td>0.23420</td>
<td>-22.75251</td>
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<tr>
<td>$\rho_{12IV}$</td>
<td>-4.70338</td>
<td>-4.33076</td>
<td>2.43380</td>
<td>0.39857</td>
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<td>$\rho_{12S}$</td>
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<td>-5.24413</td>
<td>1.07486</td>
<td>0.01379</td>
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<tr>
<td>$\rho_{12dn}$</td>
<td>-4.12190</td>
<td>-3.51118</td>
<td>2.51370</td>
<td>-0.00956</td>
<td>-21.21325</td>
</tr>
<tr>
<td>$\rho_{12dnIV}$</td>
<td>-4.97548</td>
<td>-4.58442</td>
<td>2.43717</td>
<td>0.34086</td>
<td>-14.21220</td>
</tr>
<tr>
<td>$\rho_{12dnS}$</td>
<td>-4.83866</td>
<td>-5.24810</td>
<td>1.05486</td>
<td>0.04225</td>
<td>-5.55715</td>
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<td>$\rho_{12up}$</td>
<td>-4.12190</td>
<td>-3.51118</td>
<td>2.51370</td>
<td>-0.00956</td>
<td>-21.21325</td>
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<td>-4.97548</td>
<td>-4.58442</td>
<td>2.43717</td>
<td>0.34086</td>
<td>-14.21220</td>
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<td>$\rho_{12upS}$</td>
<td>-4.83866</td>
<td>-5.24810</td>
<td>1.05486</td>
<td>0.04225</td>
<td>-5.55715</td>
</tr>
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| $\rho_{13}$  | -3.68931  | -3.17768  | 2.37213            | 0.16448  | -22.01344|
| $\rho_{13IV}$ | -1.98773  | -2.03320  | 0.75281            | 0.13676  | -4.08981 |
| $\rho_{13S}$  | -0.02917  | -0.02787  | 0.02397            | 0.03767  | -0.11776 |
| $\rho_{13dn}$ | -4.14366  | -3.58927  | 2.39564            | -0.13856 | -20.78074|
| $\rho_{13dnIV}$ | -2.33957 | -2.35056  | 0.80147            | -0.01764 | -4.51116 |
| $\rho_{13dnS}$ | -0.04292  | -0.04313  | 0.03297            | 0.0296  | -0.12324 |
| $\rho_{13up}$  | -4.14366  | -3.58927  | 2.39564            | -0.13856 | -20.78074|
| $\rho_{13upIV}$ | -1.57206 | -1.63384  | 0.63193            | 0.01240  | -3.47984 |
| $\rho_{13upS}$ | -0.00583  | -0.00528  | 0.00586            | 0.02170  | -0.03628 |

| $\rho_{23}$  | -3.03162  | -3.14645  | 0.68155            | -0.93135 | -4.44484 |
| $\rho_{23IV}$ | -1.81123  | -1.86049  | 0.50844            | -0.61143 | -2.90168 |
| $\rho_{23S}$  | -0.02420  | -0.01053  | 0.02798            | 0.00921  | -0.10459 |
| $\rho_{23dn}$ | -1.99438  | -2.04359  | 0.59024            | -0.56416 | -3.58730 |
| $\rho_{23dnIV}$ | -1.36873 | -1.39319  | 0.41399            | -0.42779 | -2.30937 |
| $\rho_{23dnS}$ | -0.00640  | -0.00154  | 0.01033            | 0.02215  | -0.05282 |
| $\rho_{23up}$  | -4.97176  | -5.11830  | 0.78866            | -2.24967 | -6.41353 |
| $\rho_{23upIV}$ | -2.54322 | -2.62788  | 0.65083            | -0.88272 | -3.74385 |
| $\rho_{23upS}$ | -0.01976  | -0.01024  | 0.02122            | 0.00298  | -0.07576 |

Table A.2: Statistics for Max Options in 27 scenarios
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<th>Standard Deviation</th>
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<th>Min</th>
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<td>$\rho_{12}$</td>
<td>1.12622</td>
<td>1.13349</td>
<td>0.61011</td>
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<td>0.74248</td>
<td>0.78908</td>
<td>0.37588</td>
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<td>0.00820</td>
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<td>$\rho_{12} dn$</td>
<td>0.64182</td>
<td>0.67235</td>
<td>0.31924</td>
<td>2.52470</td>
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<td>$\rho_{12} dn IV$</td>
<td>0.46080</td>
<td>0.48894</td>
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<td>$\rho_{12} dn S$</td>
<td>0.00058</td>
<td>0.00070</td>
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<td>$\rho_{12} up$</td>
<td>1.59058</td>
<td>1.50731</td>
<td>0.90004</td>
<td>6.77738</td>
<td>0.01775</td>
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<td>$\rho_{12} up IV$</td>
<td>0.93096</td>
<td>0.98716</td>
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<td>3.41195</td>
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<td>$\rho_{13}$</td>
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<td>1.29268</td>
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<td>0.79699</td>
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<td>$\rho_{23} up IV$</td>
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<td>0.98138</td>
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<td>2.76696</td>
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<td>$\rho_{23} up S$</td>
<td>0.03932</td>
<td>0.04618</td>
<td>0.02164</td>
<td>0.09741</td>
<td>-0.01687</td>
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Table A.3: Statistics for Min Options in 27 scenarios
A.5 Temperature Plots of Basket Options

Basket Opt.: CorVega(r12), r23=0

Figure A.1: CorrVega of $\rho_{12}$ at $\rho_{23} = 0$ for Basket Options
Figure A.2: CorrVega of $\rho_{12}$ at $\rho_{23} = 0$ for Basket Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
Figure A.3: CorrVega of $\rho_{12}$ at $\rho_{23} = 0$ for Basket Options with $S_0 = (150, 100, 50)$
Basket Opt.: CorVega(r12), r23=0.5

Figure A.4: CorrVega of $\rho_{12}$ at $\rho_{23} = 0.5$ for Basket Options
BasOpt. of various IV: CorVega(r12), r23=0.5

Figure A.5: CorrVega of $\rho_{12}$ at $\rho_{23} = 0.5$ for Basket Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
BasOpt. of various S: CorVega(r12), r23=0.5

Figure A.6: CorrVega of $\rho_{12}$ at $\rho_{23} = 0.5$ for Basket Options with $S_0 = (150, 100, 50)$
Basket Opt.: CorVega(r12), r23=-0.5

Figure A.7: CorrVega of $\rho_{12}$ at $\rho_{23} = -0.5$ for Basket Options
Figure A.8: CorrVega of $\rho_{12}$ at $\rho_{23} = -0.5$ for Basket Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
BasOpt. of various S: CorVega(r12), r23=-0.5

Figure A.9: CorrVega of $\rho_{12}$ at $\rho_{23} = -0.5$ for Basket Options with $S_0 = (150, 100, 50)$
Figure A.10: CorrVega of $\rho_{13}$ at $\rho_{23} = 0$ for Basket Options
Figure A.11: CorrVega of $\rho_{13}$ at $\rho_{23} = 0$ for Basket Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
Figure A.12: CorrVega of $\rho_{13}$ at $\rho_{23} = 0$ for Basket Options with $S_0 = (150, 100, 50)$
Basket Opt.: CorVega(r23), r23=0

Figure A.13: CorrVega of $\rho_{23}$ at $\rho_{23} = 0$ for Basket Options
Figure A.14: CorrVega of $\rho_{23}$ at $\rho_{23} = 0$ for Basket Options with $\sigma = (0.6, 0.3, 0.15)$
Figure A.15: CorrVega of $\rho_{23}$ at $\rho_{23} = 0$ for Basket Options with $S_0 = (150, 100, 50)$
A.6 Temperature Plots of Max Options

Max Opt.: CorVega(r12), r23=0

Figure A.16: CorrVega of $\rho_{12}$ at $\rho_{23} = 0$ for Max Options
Figure A.17: CorrVega of $\rho_{12}$ at $\rho_{23} = 0$ for Max Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
MaxOpt. of various S: CorVega(r12), r23=0

Figure A.18: CorrVega of $\rho_{12}$ at $\rho_{23} = 0$ for Max Options with $S_0 = (150, 100, 50)$
Figure A.19: CorrVega of $\rho_{12}$ at $\rho_{23} = 0.5$ for Max Options
MaxOpt. of various IV: CorVega(r12), r23=0.5

Figure A.20: CorrVega of $\rho_{12}$ at $\rho_{23} = 0.5$ for Max Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
MaxOpt. of various S: CorVega(r12), r23=0.5

Figure A.21: CorrVega of $\rho_{12}$ at $\rho_{23} = 0.5$ for Max Options with $S_0 = (150, 100, 50)$
Max Opt.: CorVega(r12), \( r_{23} = -0.5 \)

Figure A.22: CorrVega of \( \rho_{12} \) at \( \rho_{23} = -0.5 \) for Max Options
MaxOpt. of various IV: CorVega(r12), r23=-0.5

Figure A.23: CorrVega of $\rho_{12}$ at $\rho_{23} = -0.5$ for Max Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
MaxOpt. of various S: CorVega(r12), r23=-0.5

Figure A.24: CorrVega of $\rho_{12}$ at $\rho_{23} = -0.5$ for Max Options with $S_0 = (150, 100, 50)$
Figure A.25: CorrVega of $\rho_{13}$ at $\rho_{23} = 0$ for Max Options
MaxOpt. of various IV: CorVega(r13), r23=0

Figure A.26: CorrVega of $\rho_{13}$ at $\rho_{23} = 0$ for Max Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
MaxOpt. of various S: CorVega(r13), r23=0

Figure A.27: CorrVega of $\rho_{13}$ at $\rho_{23} = 0$ for Max Options with $S_0 = (150, 100, 50)$
Figure A.28: CorrVega of $\rho_{13}$ at $\rho_{23} = 0.5$ for Max Options
MaxOpt. of various IV: CorVega(r13), r23=0.5

Figure A.29: CorrVega of $\rho_{13}$ at $\rho_{23} = 0.5$ for Max Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
MaxOpt. of various $S$: CorVega($r_{13}$), $r_{23}=0.5$

Figure A.30: CorrVega of $\rho_{13}$ at $\rho_{23} = 0.5$ for Max Options with $S_0 = (150, 100, 50)$
Max Opt.: CorVega(r13), r23=-0.5

Figure A.31: CorrVega of $\rho_{13}$ at $\rho_{23} = -0.5$ for Max Options
MaxOpt. of various IV: CorVega(r13), r23=-0.5

Figure A.32: CorrVega of ρ_{13} at ρ_{23} = −0.5 for Max Options with \( \hat{\sigma} = (0.6, 0.3, 0.15) \)
MaxOpt. of various $S$: CorVega(r13), $r_{23} = -0.5$

Figure A.33: CorrVega of $\rho_{13}$ at $\rho_{23} = -0.5$ for Max Options with $S_0 = (150, 100, 50)$
Figure A.34: CorrVega of $\rho_{23}$ at $\rho_{23} = 0$ for Max Options
MaxOpt. of various IV: CorVega(r23), r23=0

Figure A.35: CorrVega of $\rho_{23}$ at $\rho_{23} = 0$ for Max Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
MaxOpt. of various S: CorVega(r23), r23=0

Figure A.36: CorrVega of $\rho_{23}$ at $\rho_{23} = 0$ for Max Options with $S_0 = (150, 100, 50)$
Figure A.37: CorrVega of $\rho_{23}$ at $\rho_{23} = 0.5$ for Max Options
MaxOpt. of various IV: CorVega(r23), r23=0.5

Figure A.38: CorrVega of $\rho_{23}$ at $\rho_{23} = 0.5$ for Max Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
MaxOpt. of various S: CorVega(r23), r23=0.5

Figure A.39: CorrVega of $\rho_{23}$ at $\rho_{23} = 0.5$ for Max Options with $S_0 = (150, 100, 50)$
Max Opt.: CorVega(r23), r23=-0.5

Figure A.40: CorrVega of $\rho_{23}$ at $\rho_{23} = -0.5$ for Max Options
MaxOpt. of various IV: CorVega(r23), r23=−0.5

Figure A.41: CorrVega of $\rho_{23}$ at $\rho_{23} = -0.5$ for Max Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
MaxOpt. of various S: CorVega(r23), r23=-0.5

Figure A.42: CorrVega of $\rho_{23}$ at $\rho_{23} = -0.5$ for Max Options with $S_0 = (150, 100, 50)$
A.7 Temperature Plots of Min Options

Min Opt.: CorVega(r12), r23=0

Figure A.43: CorrVega of $\rho_{12}$ at $\rho_{23} = 0$ for Min Options
Figure A.44: CorrVega of $\rho_{12}$ at $\rho_{23} = 0$ for Min Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
Figure A.45: CorrVega of $\rho_{12}$ at $\rho_{23} = 0$ for Min Options with $S_0 = (150, 100, 50)$
Figure A.46: CorrVega of $\rho_{12}$ at $\rho_{23} = 0.5$ for Min Options
Figure A.47: CorrVega of $\rho_{12}$ at $\rho_{23} = 0.5$ for Min Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
Figure A.48: CorrVega of $\rho_{12}$ at $\rho_{23} = 0.5$ for Min Options with $S_0 = (150, 100, 50)$
Figure A.49: CorrVega of $\rho_{12}$ at $\rho_{23} = -0.5$ for Min Options
MinOpt of various IV: CorVega(r12), r23=-0.5

Figure A.50: CorrVega of $\rho_{12}$ at $\rho_{23} = -0.5$ for Min Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
Figure A.51: CorrVega of $\rho_{12}$ at $\rho_{23} = -0.5$ for Min Options with $S_0 = (150, 100, 50)$
Min Opt.: CorVega(r13), r23=0

Figure A.52: CorrVega of $\rho_{13}$ at $\rho_{23} = 0$ for Min Options
Figure A.53: CorrVega of $\rho_{13}$ at $\rho_{23} = 0$ for Min Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
Figure A.54: CorrVega of $\rho_{13}$ at $\rho_{23} = 0$ for Min Options with $S_0 = (150, 100, 50)$
Min Opt.: CorVega(r13), r23=0.5

Figure A.55: CorrVega of $\rho_{13}$ at $\rho_{23} = 0.5$ for Min Options
MinOpt of various IV: CorVega(r13), r23=0.5

Figure A.56: CorrVega of $\rho_{13}$ at $\rho_{23} = 0.5$ for Min Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
Figure A.57: CorrVega of $\rho_{13}$ at $\rho_{23} = 0.5$ for Min Options with $S_0 = (150, 100, 50)$
Min Opt.: CorVega(r13), r23=-0.5

Figure A.58: CorrVega of $\rho_{13}$ at $\rho_{23} = -0.5$ for Min Options
MinOpt of various IV: CorVega(r13), r23 = -0.5

Figure A.59: CorrVega of $\rho_{13}$ at $\rho_{23} = -0.5$ for Min Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
Figure A.60: CorrVega of $\rho_{13}$ at $\rho_{23} = -0.5$ for Min Options with $S_0 = (150, 100, 50)$
Figure A.61: CorrVega of $\rho_{23}3$ at $\rho_{23} = 0$ for Min Options
MinOpt of various IV: CorVega(r23), r23=0

Figure A.62: CorrVega of $\rho_{23}$ at $\rho_{23} = 0$ for Min Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
Figure A.63: CorrVega of $\rho_{23}$ at $\rho_{23} = 0$ for Min Options with $S_0 = (150, 100, 50)$
Figure A.64: CorrVega of $\rho_{23}$ at $\rho_{23} = 0.5$ for Min Options
MinOpt of various IV: CorVega(r23), r23=0.5

Figure A.65: CorrVega of $\rho_{23}$ at $\rho_{23} = 0.5$ for Min Options with $\hat{\sigma} = (0.6, 0.3, 0.15)$
Figure A.66: CorrVega of $\rho_{23}$ at $\rho_{23} = 0.5$ for Min Options with $S_0 = (150, 100, 50)$
Figure A.67: CorrVega of $\rho_{23}$ at $\rho_{23} = -0.5$ for Min Options
MinOpt of various IV: CorVega(r23), r23=-0.5

Figure A.68: CorrVega of \( \rho_{23} \) at \( \rho_{23} = -0.5 \) for Min Options with \( \hat{\sigma} = (0.6, 0.3, 0.15) \)
MinOpt of various $S$: CorVega$(r_{23})$, $r_{23} = -0.5$

Figure A.69: CorrVega of $\rho_{23}$ at $\rho_{23} = -0.5$ for Min Options with $S_0 = (150, 100, 50)$
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