Nonparametric Estimate for Conditional Quantiles of Time Series: An application for VaR

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This paper investigates a nonparametric approach for estimating conditional quantiles of time series for dependent data. The considered estimate is obtained by inverting a kernel estimate of the conditional distribution function. We implement the technique on four simulated samples with light and heavy-tailed distributions and on real financial data, by calculating VaR using the nonparametric procedure. The good performance of the estimator is illustrated with backtesting.

Keywords: Value at Risk, Conditional Quantiles, Nonparametric, Kernel Estimation, Backtesting

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1 Introduction

Risk management is an integral part of the entire management and controlling system of a company and financial institutions increasingly become aware that the major mission for risk management is adequate risk steering (in contrast to risk avoidance or risk minimization). Thus, an accurate method for measuring risk is necessary in order to enable competent and informed decision making in risk management.

Market risk represents the risk that the value of a financial asset will decrease due to the change in value of associated market risks, such as equity risk, interest rate risk, currency risk and commodity risk. Therefore, with appropriate tools in place, market risk management will encourage risk taking where it is most optimal given the rewards and capital consumption.

Together with an increased sophistication of financial instruments and an increasingly riskier financial market, the amount of potential loss for a financial institution is ascending. In addition, recent financial crisis have spawned the interest of researchers and economists in studying the risk in financial markets. The regulatory institutions have increased the pressure for companies to adopt reliable quantitative risk management tools, in their quest for a 'safe and sound' financial system. For example, the Basel Committee on Banking Supervision (1996) of the Bank for International Settlements has chosen value at risk (VaR) as the benchmark of risk measurement for capital requirements for financial institutions such as banks and investment firms, which implies that a correct evaluation of VaR enables efficient allocation of capital. As a result, VaR has become a standard tool in the financial industry to evaluate and manage financial risk.

The paper is structured as follows. Section 2 gives a short overview of the literature and different approaches in calculating VaR. In section 3 we present the nonparametric estimate for conditional quantiles of time series, together with its asymptotic behaviour. Theoretical aspects of the backtesting procedure is briefly presented. In Section 4, we illustrate the performance of the quantile function estimate with a small simulation study and a real data application and we evaluate its forecast power with baktesting. Section 5 concludes the paper.

2 Value at Risk Models

2.1 Background

During the recent financial crisis, an important source of losses and build up of leverage occurred in the trading book. Jorion (2001) notes that the volatility in the underlying financial variable and the exposure to this source of risk are the two main drivers of the losses for a financial institution and value at risk (VaR) is the appropriate method to infer the combined effect of the two factors. Jorion (2001) offers a comprehensive description of VaR and its applications in the field of risk management.

VaR is a well established risk management practice to measure the potential loss amount due to market risk employed in the financial industry for both the internal control and regulatory reporting. It is a measure which quantifies and controlls the risk of a portfolio. Moreover, in many companies the practice is to manage the market risk with a short-term focus, which means that long-term losses are prevented by avoiding losses from one day to the next. On a strategic level, organizations manage market risk by defining and monitoring risk limits in order to reduce the excessive exposure to losses. In this context, value at risk has become a standard benchmark in setting these limits, also because it enables comparisons across asset classes, as VaR can refer both to the total value of a position or to the risk per euro invested. Blanco and Blomstrom (1999) offer an overview of VaR applications in setting VaR based limits.

Berkowitz (2009) points out that VaR has become very popular because of its simplicity and that it usually does not make any assumptions of asset return normality and in addition, is a forward looking measure, as it can reflect for example the maximum loss over the next day with a certain probability. This idea is supported by empirical evidence of Taylor (2005), who uses VaR estimates to model volatility forecasts.

Within the framework of risk management, VaR is a key value for controlling and complying with external regulations. It provides the basis for the internal risk controlling models proposed by the Basel Committee on Banking Supervision. In particular, financial insitutions with activity in trading risky financial assets are required to maintain internally a minimum level of safe capital to counteract unforseen risk. The level of this capital can be calculated as a function of VaR. Basel II and III require a ten day holding period and a 99% confidence interval. For example, Deutsche Bank uses a 99% confidence level and a one-day holding period and one year historical market data for calculating the regulatory market risk capital for their general and specific market risks.

In statistical terms, VaR is a statistical risk measure that indicates how much a financial institution can lose on a financial asset (in terms of market value) with a given probability and over a given time horizon. In other words, is the quantile of the conditional asset return distribution. The VaR measure has the advantage of being a single estimate, which makes it accesible and easy to understand also by the less numerically literate management.

It is now obvious that to a risk manager, a good measure of market risk is more than necessary. There are many ways of calculating VaR for a financial asset, as a statistical estimator. In practice, the most traditional approaches to VaR computation are the variance-covariance method, historical simulation, Monte Carlo simulation and stress-testing.

Generically, VaR can be defined as the p-quantile of the return distribution at time $t + d$ conditioned on the information set $\mathcal{F}_t$

$$\mathrm{VaR}_{t+d}^{p} \stackrel{\text{def}}{=} \inf\{y \in R : \mathrm{P}(Y_{t+d} \leq y|\mathcal{F}_t) \geq p\} \tag{2.1}$$

were Y_t denotes the return and p is taking values such as 0.95, 0.99, or 0.05, 0.01 to reflect extreme risks.

The length of the time interval is specified by d and in practice it is typically chosen to be two weeks for the regulatory reporting (which usually means 10 business days for local applications or 12 business days for around-the-globe trading applications).

If we consider the conditional distribution function $F(y|x) = \mathrm{P}(Y_t \leq y|X_t = x) = \mathrm{E}[\mathbf{I}_{t,y}\,|X_t = x]$ of Y_t given given $X_t = x$, then the VaR can be expressed as the following:

$$\mathrm{VaR}_{t}^{p} \stackrel{\text{def}}{=} \inf\{y \in R : F(y|x) \geq p\} \tag{2.2}$$

where Y_t denotes the asset return, X_t can include both economic and market (exogeneous) variables and the lagged variables of Y_t. Thus, the VaR equation denotes practically the conditional quantile function, being concerned with the tail behaviour of the conditional distribution function $F(y|x)$.

Kuester et al. (2006) classify the approaches for constructing quantile estimates into the following: historical simulation, which calculates empirical quantiles from past data; fully parametric models, which describe the entire distribution of returns; extreme value theory uses parametric models only for the tails of the return distribution and quantile regression directly models a specific quantile, and not the whole distribution.

Quantile regression is one of the most established techniques in estimating the conditional quantile function and the seminal work of Koenker and Basset (1978) was a major step forward in estimating conditional quantiles. We offer more details about quantile regression model in Section 2.2.

Most of the existing risk management literature for VaR estimation has focused on parametric models and unconditional distributions. In practice, one of the most commonly used parametric method is the RiskMetrics model, pioneered by J.P. Morgan(1995) which assumes that returns of a financial asset follow a multivariate normal distribution (the mean change in the value of each variable is assumed to be zero and the variance is expressed as an exponentially weighted moving average of historical squared returns). The main criticism to this approach is that it does not capture the fat tails property of financial time series. A semiparametric approach is the conditional autoregressive value at risk (CAViaR) model of Engle and Manganelli (2004), which estimates VAR directly by quantile regression, but with no assumptions on distribution. We present more details about this model in Section 2.2.

The challenge in finding an adequate estimate for VaR is to find a model which incorporates the special characteristics of financial time series. In modeling VaR, a classical assumption that the models rely on is that the returns are independent and identical distributed which means it is assumed the returns are uncorrelated over successive time intervals. Jorion (2001) relates this assumption with the *efficient markets* concept, which states that the current price includes all relevant information from the financial market. He states that in this context the prices should be uncorrelated and follow a random walk, as prices would only change as a result of news, which cannot be anticipated. However, in practice a series of statistical properties can be observed for financial returns, such as fat-tails (excess kurtosis), time-varying volatility and volatility clustering, indicated by high autocorrelation of the returns (large changes tend to be followed by large changes and small changes tend to be followed by small changes). Moreover, empirical applications consistently show that nonlinearity and changing volatility are very characteristic to financial time series. For example, DeBondt and Thaler (1985) showed that stock returns are serially correlated over long time horizons and Schwert and Stambaugh (1987) consider the changing volatility a stylized fact of stock market, when showing the positive relation between expected market risk premiums and the predictable volatility of stock returns. It follows that there is a necessity to find alternative models for VaR prediction, which are not restricted to the independent and identically distributed case and do not rely on the assumption that financial returns are normally distributed.

Nonparametric modeling takes a step further and addresses part of this challenges by constructing estimates without making assumptions on the form of the financial return distribution and allow for more flexibility and nonlinearity. For example, Yu and Jones (1998) introduced a nonparametric regression quantile estimation by kernel weighted local linear fitting, which inverts a local linear conditional distribution estimator. Cai and Wang (2008) proposed a nonparametric estimation of conditional VaR by inverting the weighted double kernel local linear estimate of the conditional distribution function.

Franke and Mwita (2003) proposed a nonparametric estimate for conditional quantiles of time series, which allows for investigation of quantiles, without being restricted to the independent and identically distributed case and without making any homoskedasticity assumption. As the motivation for this paper is to find a reliable and adequate VaR estimator, in the remaining of the paper we focus on the model proposed by Franke and Mwita (2003) and evaluate the performance by an empirical application.

In the following two subsections, we briefly present the parametric quantile regression model of Koenker and Basset (1978) and the CAViaR model of Engle and Manganelli (2004). In the empirical application, we use the two models to better illustrate the performance of different estimators for VaR.

2.2 Quantile Regression Model

Quantile regression models are designed to specify changes in the conditional quantile associated with a change in the explanatory variables. Therefore, they are very useful in predicting a given quantile of the conditional distribution and serve for estimating and conducting inference on conditional quantile functions (e.g VaR for a financial asset). The quantile regression was first introduced by Koenker and Basset(1978). For any random variable Y depending on X, they express the quantile regression model as following:

$$Q_{Y_t}^{(i)}(p|X) = \beta_0^{(p)} + \beta_1^{(p)} x_{1t} + \ldots + \beta_i^{(p)} x_{it} + \epsilon_t^{(p)} \tag{2.3}$$

with $0 < p < 1$ indicating the proportion of the asset returns values being below the quantile at p, $t \in 1, \ldots, n$, and ϵ_t are independent and identically distributed.

Quantile regression technique is a convenient tool to address the nonnormality characteristic of financial returns, as it estimates the conditional quantile function without the parametric assumptions of normality but considering only the linear functional form.

In particular, for the empirical application in section 4, we will calculate the VaR as follows. Let Y_t be the daily log return of the asset return and X_t be the first lag of Y_t. Then, the VaR for Y_t is predicted by:

$$\widehat{\text{VaR}}_t^p = \widehat{\alpha} + \widehat{\beta} X_t \tag{2.4}$$

As an alternative to linear quantile regression, the nonlinearity characteristic is addressed in the conditional autoregressive Value at Risk (CaViaR) model, briefly presented below.

2.3 Conditional Autoregressive Value at Risk by Regression Quantiles

Engle and Manganelli(2004) introduced the conditional autoregressive value at risk (CAViaR) model to estimate the VAR directly by quantile regression. They propose to estimate VaR by modeling the quantile directly instead of inverting a distribution function as in other parametric and nonparametric quantile regression models. Hence, they do not model the whole distribution of portfolio returns, but only the evolution of the quantile over time. The advantage of this model is that it makes no explicit distributional assumptions, which avoids possible model misspecification.

They note that it is important to find a model which accommodates the time-varying conditional quantiles, as VaR is a particular quantile of future portfolio values, conditional on current information and with a distribution of returns which changes over time.

Assume the observed data $X_t, Y_t, \quad t = 1, \ldots, T$, where Y_t is a vector of portfolio returns, X_t is a vector of time t observable variables and β_p is a vector of parameters to be estimated, with p denoting the probability associated with VaR. Let $f_t(\beta) \equiv f_t(x_{t-1}, \beta_p)$ denote the p-quantile of the distribution of portfolio returns at time t. The following generic CAViaR specification is proposed:

$$f_t(\beta) = \beta_0 + \sum_{i=1}^{q} \beta_i f_{t-i}(\beta) + \sum_{j=1}^{r} \beta_j l(x_{t-j}) \tag{2.5}$$

where β is of dimension $m = q + r + 1$. The function $l(x_{t-j})$, which contains a finite number of lagged values of observables, has the same role as the news impact curve for GARCH models introduced by Engle and NG (1993). For example, if we choose x_{t-1} to be the lagged returns, we would expect the VaR to increase as y_{t-1} becomes very negative, because one bad day makes the probability of the next greater.

Other specifications of the CAViar model are as follows:

Adaptive:

$$f_t(\beta_1) = f_{t-1}(\beta_1) + \beta_1\left\{ \left[1 + \exp\left(G[y_{t-1} - f_{t-1}(\beta_1)]\right)\right]^{-1} - p\right\}. \tag{2.6}$$

Symmetric absolute value:

$$f_t(\beta) = \beta_1 + \beta_2 f_{t-1}(\beta) + \beta_3 |y_{t-1}|. \tag{2.7}$$

Asymmetric slope:

$$f_t(\beta) = \beta_1 + \beta_2 f_{t-1}(\beta) + \beta_3 (y_{t-1})^+ + \beta_4 (y_{t-1})^-. \tag{2.8}$$

Indirect GARCH(1,1)

$$f_t(\beta) = \left\{\beta_1 + \beta_2 f_{t-1}^2(\beta) + \beta_3 (y_{t-1}^2)\right\}^{1/2}.$$

where G is some positive finite number and $(x)^+ = \max(x, 0), (x)^- = -\min(x, 0)$.

3 Nonparametric Estimate for Conditional Quantiles

3.1 Model setup

In this section we present a nonparametric approach for estimating conditional quantiles of time series for dependent data. As we already defined VaR as the p-quantile of the conditional return distribution of a financial asset, a main application of this technique is for calculating VaR.

The theoretical aspects of the nonparametric estimate have been proposed by Franke and Mwita (2003). In this part we present the model set up and some properties of the estimator and in the following section we expose the application of the estimation procedure on a simulated study and on real financial data.

Following Franke and Mwita (2003), we assume a stationary and α-mixing multivariate time series $\{V_t, t \in Z\}$ adapted to the sequence $\mathcal{F}_t$, -$\infty < t < \infty$, of σ-algebras. We partition the time series as $V_t = (Y_t, X_t)$, where the real valued response variable $Y_t \in R$ is $\mathcal{F}_t$-measurable and the covariate $X_t \in R^d$ is $\mathcal{F}_{t-1}$-measurable. For some $0 < p < 1$, we are interested in estimating the conditional p-quantile of Y_t, assuming that it is completely determined by X_t:

$$Y_t^p = \vartheta_\tau(X_t) + Z_t \tag{3.1}$$

where the conditional p-quantile of innovations Z_t given $\mathcal{F}_{t-1}$ is 0. The quantile innovations Z_t are *not* assumed to be independent of X_t. In this setup, we want to find a nonparametric estimate for the conditional quantile function $\vartheta_p(x)$, which apart from some regularity assumptions is rather arbitrary.

If we choose $X_t = (Y_{t-1}, \ldots, Y_{t-q})$ as part of the past of the univariate time series Y_t, then model (3.1) would become a quantile autoregressive model of order q:

$$Y_t = \vartheta_p(Y_{t-1}, \ldots, Y_{t-q}) + Z_t, \tag{3.2}$$

It also includes the case of a nonparametric quantile regression where $(X_t, Z_t), -\infty < t < \infty$, are independent and identically distributed.

Franke and Mwita (2003) note that if we consider other financial time series models, equation (3.1) can be seen as a generalization of AR-ARCH-models, introduced in Weiss (1984), and their nonparametric generalizations reviewed by Härdle et al.(1997). For instance, they consider a financial time series model of AR(q)-ARCH(q)-type:

$$Y_t = \mu(X_t) + \sigma(X_t)e_t, \qquad t = 1, 2, \ldots \tag{3.3}$$

where $X_t = (Y_{t-1}, \ldots, Y_{t-q})$, μ and σ are arbitrary functions and e_t is a sequence of independent and identically distributed random variables with mean 0 and variance 1.

It follows that (3.3) can be written in the form of (3.1) with:

$$\vartheta_p(X_t) = \mu(X_t) + \sigma(X_t)q_p^e \tag{3.4}$$

and $Z_t = \sigma(X_t)(e_t - q_p^e)$, where q_p^e is the p-quantile of the distribution of e_t. The quantile innovations Z_t are independent of X_t only if Y_t has a homoskedastic error, which means the volatility function $\sigma(x)$ is constant.

In a parametric approach, the estimation of $\vartheta_p(X_t)$ based on the model (3.3) usually involves the estimation of $\mu(X_t)$ and $\sigma(X_t)$ and the calculation of q_p^e, for the latter assuming the distribution of e_t to be known, using historical simulation procedures or a combinaton of both.

Based on the more general model (3.1), Franke and Mwita (2003) derive a more straightforward estimate, without making any assumptions on the finiteness of the variance of Y_t which usually for financial time series, does not hold. A nonparametric estimate of $\vartheta_p(X_t)$ is obtained directly by first estimating the conditional distribution function of Y_t given X_t and then inverting it. This type of estimate is related to local medians as considered by Truong and Stone (1992) and Boente and Fraiman (1995).

For estimating the conditional distribution, the Nadaraya Watson kernel estimate is used. Despite its rather large bias and boundary effects, the Nadaraya Watson method has the advantage of being a constrained estimator between 0 and 1 and a monotonically increasing function, therefore the estimator is always a distribution function, which from obvious reasons is an important property when deriving quantile function estimators by the inversion of a distribution estimator.

3.2 Steps and methodology for nonparametric estimation for conditional quantiles

Following the methodology of Franke and Mwita (2003), based on a sample $(Y_t, X_t), t = 1, 2, \ldots, n$ from a stationary time series model, a nonparametric estimate for conditional quantiles is constructed with the following method:

1. Estimate the conditional distribution function of Y_t given $X_t = x$

$$F(y|x) \stackrel{\text{def}}{=} \mathrm{P}(Y_t \leq y|X_t = x) = \mathrm{E}[\mathbf{I}_{t,y}\,|X_t = x] \tag{3.5}$$

For estimating the conditional distribution, we use the standard Nadaraya-Watson kernel estimate:

$$\widehat{F}(y|x) = \frac{\sum_{t=1}^{n} K_h(x - X_t)\,\mathbf{I}_{t,y}}{\sum_{t=1}^{n} K_h(x - X_t)} \tag{3.6}$$

where $\mathbf{I}_{t,y} = \mathbf{I}_{\{Y_t \leq y\}}$ and $\mathbf{I}$ is the indicator function. Here, $K(u)$ is a d-dimensional kernel and $K_h(u) = h^{-d}K(u/h)$ is the rescaled kernel. For sake of simplicity, it is assumed that the bandwidth h is the same in all directions.

2. Obtain the nonparametric estimator for the conditional quantile function by inverting the estimated distribution function from above

$$\widehat{\vartheta}_t^p = \inf\{y \in R|\widehat{F}(y|x) \geq p\} \stackrel{\text{def}}{=} \widehat{F}^{-1}(p|x) \tag{3.7}$$

where $\widehat{F}^{-1}(p|x)$ denotes the usual generalized inverse of the distribution function $\widehat{F}(y|x)$ which is a pure jump function of y.

In the following section, the estimator's main asymptotic properties like asymptotic normality as the basis for inference and uniform strong consistency as a basis for a residual analysis are briefly presented.

3.3 Statistical properties: asymptotic behaviour and consistency

The good properties of the nonparametric estimate for conditional quantile, like asymptotic normality and consistency have been proved by Franke and Mwita (2003).

For dependent data, as it is the case for financial time series, it is necessary to impose certain conditions which control the dependence between X_i and X_j as the time distance

$i - j$ increases. In particular, if we consider observations falling in a small neighborhood of the fixed value x and assume that they are fairly separated in time, Franke et al.(2009) find it reasonable to expect that the asymptotics of the Nadaraya Watson estimator for time series data satisfying such a dependence assumption do not significantly differ from the asymptotics in the much simpler independent and identically distributed setting.

Therefore, for proving asymptotic properties of the Nadaraya Watson estimator for dependent data, we have to assume that the time series (Y_t, X_t) satisfies appropriate mixing conditions. Robinson (1983) and Masry and Tjøstheim (1995) prove strong consistency and asymptotic normality for α-mixing observations. In particular, a sequence $X_i, \ldots, X_j$ is said to be α-mixing, or strong mixing (Robinson 1983) if

$$\sup_{A \in \mathcal{F}_i^n, B \in \mathcal{F}_{n+k}^\infty} |\,\mathrm{P}(A \cap B) - \mathrm{P}(A)\,\mathrm{P}(B)| \leq \alpha_k \tag{3.8}$$

where $\alpha_k \to 0$ and $\mathcal{F}_i^j$ is the σ-field generated by $X_i, \ldots, X_j$.

Masry (1995) has demonstrated that under some mild conditions, both ARCH processes and nonlinear additive autoregressive models with exogeneous variables are stationary and α-mixing. Thus, choosing $X_t = (Y_{t-1}, \ldots, Y_{t-q})^\top$ in (3.3) and assuming the time series Y_t to be α-mixing would be an example of a quantile autoregressive process (3.1) for which (Y_t, X_t) and $\mathbf{I}_{t,y}$ in (3.6) are α-mixing as well.

Let (Y_t, X_t) be a stationary and α-mixing multivariate time series, as defined in Section 3.1, with mixing coefficients satisfying $\alpha(s) = \mathcal{O}(s^{(2+\delta)})$, for some $\delta > 0$. Choose $X_t = (Y_{t-1}, \ldots, Y_{t-q})^\top$, denote by $g(x)$ the stationary probability density of X_t and by $F(y|x)$ the conditional distribution function, with conditional density $f(y|x)$. As mentioned before, $K(u)$ is a d-dimensional kernel.

Theorem 1 1 *Under a set of assumptions and as $n \to \infty$, let the sequence of bandwidths $h > 0$ converge to 0 such that $nh^d \to \infty$. Then, the conditional quantile estimate is consistent, $\widehat{\vartheta}_p(x) \xrightarrow{\mathcal{P}} \vartheta_p(x)$, and asymptotically unbiased:*

$$\mathrm{E}\left[\widehat{\vartheta}_p(x) - \vartheta_p(x)\right] = h^2 B_\vartheta\{\vartheta_p(x)\} + \mathcal{O}(h^2), \quad \textit{where} \quad B_\vartheta(y) = -\frac{B(y)}{f(y|x)}. \tag{3.9}$$

If, additionally, the bandwidths are chosen such that nh^{d+4} is either 1 or converges to 0, $\widehat{\vartheta}_p(x)$ is asymptotically normal

$$\sqrt{nh^d}\left[\widehat{\vartheta}_p(x) - \vartheta_p(x) - h^2 B_\vartheta\{\vartheta_p(x)\}\right] \xrightarrow{\mathcal{L}} \mathrm{N}\left[0, \frac{V^2\{\vartheta_p(x)\}}{f^2\{\vartheta_p(x)|x\}}\right] \tag{3.10}$$

Here, $B(y)$ and $V^2(y)$ are defined as the bias and variance expansion for the conditional distribution estimator and can be written as

$$B(y) = \frac{1}{g(x)} \nabla F(y|x)^\top \int u \nabla g(x)^\top u K(u) du + \frac{1}{2} \int u^\top \nabla^2 F(y|x) u K(u) du \quad (3.11)$$

$$V^2(y) = \frac{1}{g(x)} \left\{ F(y|x) - F(y|x)^2 \right\} \int K^2(u) du \quad (3.12)$$

The uniform convergence of the quantile estimator $\widehat{\vartheta}_p(x)$ is shown by the following theorem.

Theorem 2 1 *Under a set of assumptions and suppose that $h \to 0$ is a sequence of bandwidths such that $\widetilde{S}_n = nh^d (s_n \log n)^{-1} \to \infty$ for some $s_n \to \infty$. Let $S_n = h^2 + \widetilde{S}_n^{-\frac{1}{2}}$. Then we have*

$$\sup_{x \in \mathcal{G}} |\widehat{\vartheta}_p(x) - \vartheta_p(x)| = \mathcal{O}(S_n) + \mathcal{O}(\frac{1}{nh^d}) \quad a.s. \quad (3.13)$$

Here, $s_n, n \geq 1$ is an increasing sequence of positive integers such that for some finite A

$$\frac{n}{s_n} \alpha^{2s_n/(3n)}(s_n) \leq A, \quad 1 \leq s_n \leq \frac{n}{2} \quad for \quad all \quad n \geq 1. \quad (3.14)$$

and $\mathcal{G}$ is some compact set.

Franke and Mwita (2003) note that the uniform convergence of the nonparametric quantile function estimate allows for a detailed investigation of the quantile innovations Z_t of the model (3.1) based on the sample residuals $\widehat{Z}_t = Y_t - \widehat{\vartheta}_p(X_t)$ which is not restricted to the independent and identically distributed case.

Knowing the asymptotic distribution of the conditional quantile estimate, we are now able to construct pointwise asymptotic confidence intervals for the conditional quantile function. Based on Theorem 2.1, we can state the following result for the confidence interval for a fixed point x:

$$c(x) \stackrel{\text{def}}{=} \left[\widehat{\vartheta}_p(x) - \widehat{B}_\vartheta\{\vartheta_p(x)\} - \frac{z_\alpha \widehat{V}\{\vartheta_p(x)\}}{\sqrt{nh^d}}, \widehat{\vartheta}_p(x) - \widehat{B}_\vartheta\{\vartheta_p(x)\} + \frac{z_\alpha \widehat{V}\{\vartheta_p(x)\}}{\sqrt{nh^d}} \right] \quad (3.15)$$

$$\mathrm{P}\left(\widehat{c}(x) \supseteq \vartheta_p(x) \right) \to 1 - \alpha$$

where z_α is the α-quantile of the standard normal distribution.

3.4 Backtesting

Backtesting procedure is used for assessing the accuracy and forecast performance of the VaR models and to check how reliable the model is, so that risk managers of financial institutions can use it in the decision-making process. More precisely, the quality of the forecast estimator is evaluated by comparing the actual results to those obtained with the VaR model.

For this purpose, we follow the framework proposed by Berkowitz et al. (2009), which is designed for evaluating the accuracy of out-of-sample interval forecasts. The proposed procedure evaluates the VaR forecast by viewing them as one-sided interval forecasts. Each time the ex post return is lower than the VaR, a violation occurs. Formally, the violation time series can be defined as

$$\mathbf{I}_{t+1} \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if } R_{t+1} < \widehat{VaR_t^p}, \\ 0 & \text{otherwise.} \end{cases} \tag{3.16}$$

Berkowitz et al. (2009) note that if the VaR is estimated correctly, the probability that the VaR will be exceeded should be unpredictable, after using all past information.

The tests proposed by Berkowitz et al (2009) consider that the sequence of violations form a martingale difference, which means that the expectation of the violation at $t+1$, given the information set up to time t is zero. This property implies that the current violation is uncorrelated with any past variables. One of the ways they propose for testing the uncorrelatedness is by considering the CaViaR test of Engle and Manganelli (2004):

$$I_t = \alpha + \beta_1 I_{t-1} + \beta_2 VaR_t + u_t \tag{3.17}$$

Here, the error term u_t follows a Logistic distribution. Estimating the logit model, the coefficients $(\widehat{\beta}_1, \widehat{\beta}_2)^T$ are obtained. For testing the null hypothesis $\widehat{\beta}_1 = \widehat{\beta}_2 = 0$ the Wald's test is used.

Besides assesing the quality of the estimator, according to Lopez (1999), the backtesting technique can serve in establishing the required level of capital for market risk by including a multiplier based on the unconditional number of VaR violations.

4 Empirical Applications

In this section we evaluate the estimation performance of the nonparametric estimate for conditional quantiles. First, we implement the technique on simulated independent samples with four different distributions for the innovations and we show that the performance of the conditional quantile estimate does not depend strongly on the innovation distributions. Second, we make an application on real financial data, by using the nonparametric procedure for calculating VaR for three different stocks and we show that the proposed estimator produces better results when compared with CAViaR and linear quantile regression model, through backtesting.

All of the computations have been done in R, except tha CaViaR, which was done in MATLAB, using the original code from Engle and Manganelli (2004).

4.1 Monte Carlo simulation

Following Franke and Mwita (2003), we perform a Monte Carlo simulation to illustrate the performance of kernel estimates for the quantile autoregressive function. For that purpose, Franke and Mwita propose the following model for generating a sample $Y_t, t = 1, \ldots, n$, of size $n = 1000$ of the nonlinear AR(1)-ARCH(1) (1.2) process:

$$\mu(x) = a + bx + \frac{1}{\sqrt{2\pi}dx} \exp\Big(\frac{(x-c)^2}{d^2}\Big), \quad \sigma^2(x) = \omega + \alpha x^2 \tag{4.1}$$

with fixed parameters $a = 0.04, b = 0.03, c = 1.657, d = 0.1175, \omega = 0.007$, $\alpha = 0.2$. For the distribution of the innovations $e(t)$, we choose normal, exponential, student's t_2 and t_4-distribution, respectively.

For each of the four generated samples of the nonlinear AR(1)-ARCH(1) process, we estimate the conditional quantile $\vartheta_p(x)$ of Y_t given $Y_{t-1} = x$ by the kernel estimate of (2.2) for $p = 0.95$.

For the kernel estimate, we used the quartic kernel

$$K(u) = \frac{15}{16}(1-u^2)^2, \qquad |u| \le 1$$
$$= 0, \qquad |u| > 1 \tag{4.2}$$

For sake of simplicity, the bandwidths for the kernel estimates were chosen using cross-validation. However, we chose more than one bandwidth to better observe the performance of the proposed estimate (i.e we chose four different bandwidths for the simulated sample with normal and exponential distributed innovations and two different bandwidths for the simulated sample with student's t_2 and t_4-distribution).

Naturally, the true conditional 0.95-quantile is calculated as

$$\vartheta_p(x) = \mu(x) + \sigma(x) q_p^e. \tag{4.3}$$

where q_p^e denotes the p-quantile of the distribution of the innovations $e(t)$.

Both the estimator and the true conditional 0.95-quantile were calculated over grid points evenly distributed across the whole sample. To observe the accuracy of our estimator, we also constructed the 95% confidence interval, based on the asymptotic normality property shown in Section 3.3.

Figure 4.1 shows a typical sample with normally distributed innovations together with the true conditional quantile $\vartheta_p(x)$ and the nonparametric estimate for conditional quantile, for $p = 0.95$. For comparison reasons, we also fit a linear parametric quantile regression to the sample. As can be seen, the nonparametric estimate is closer to the true conditional quantile and lies completely inside the confidence interval. The same can be observed in Figure 4.2, 4.3 and 4.4 for the exponential, student's t_2 and t_4-distribution, respectively. As Franke and Mwita (2003) already noted, the estimate performs reasonably well for light- and heavy-tailed innovations, apart from the areas at the extreme right and left of the sample, where data is scarce.

To demonstrate the finite sample performance of the proposed nonparametric estimator, we evaluate it in terms of Mean Squared Error (MSE). The MSE for $(\widehat{\vartheta}_p(x_i)$ is defined as:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n} \widehat{\vartheta}_p(x_i) - \vartheta_p(x_i)\Big)^2 \tag{4.4}$$

where i are grid points evenly distributed across the whole sample.

For this purpose, we generated 100 random samples of size $n = 200$ and for each

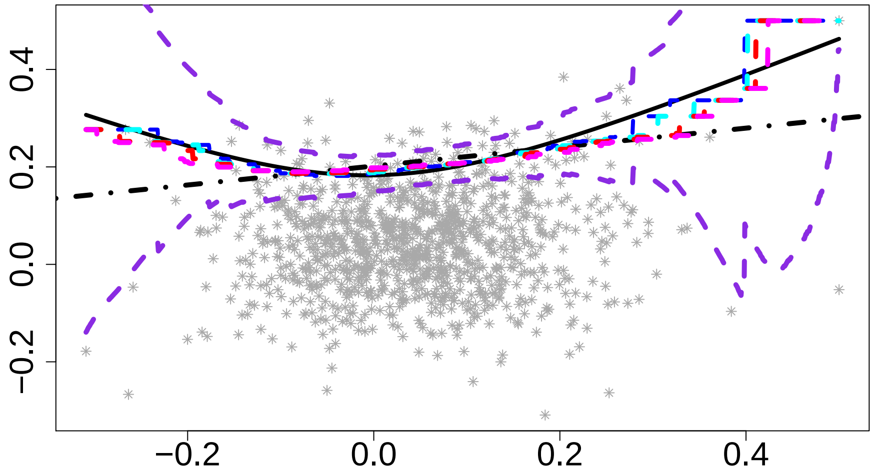

Figure 4.1: Simulated sample with *normally distributed innovations*, parametric quantile regression (p=0.95) (dotdash), true conditional 0.95-quantile (solid), nonparametric estimate for conditional quantile with h=0.15, h=0.18, h=0.21, h=0.24 . The violet dashed line is the 95% confidence interval. InvqSimN

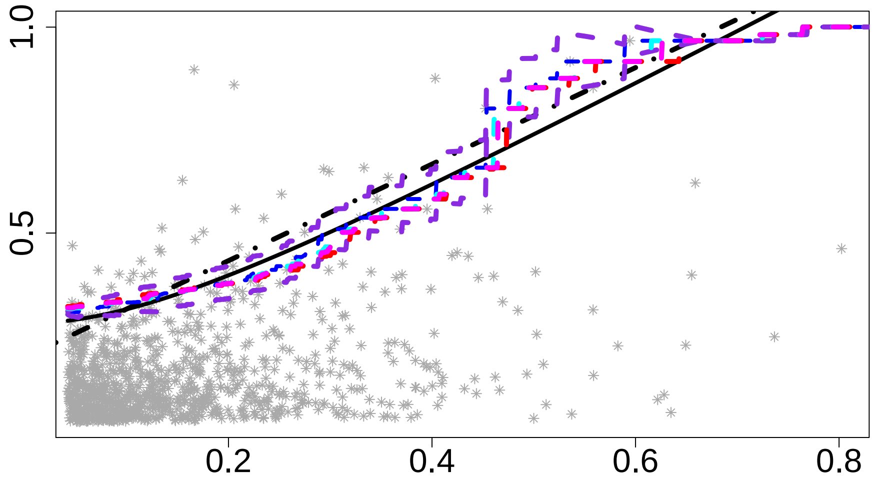

Figure 4.2: Simulated sample with *exponential distributed innovations*, parametric quantile regression (p=0.95) (dotdash), true conditional 0.95-quantile (solid), nonparametric estimate for conditional quantile with h=0.20, h=0.23, h=0.24, h=0.26. The violet dashed line is the 95% confidence interval. InvqSimE

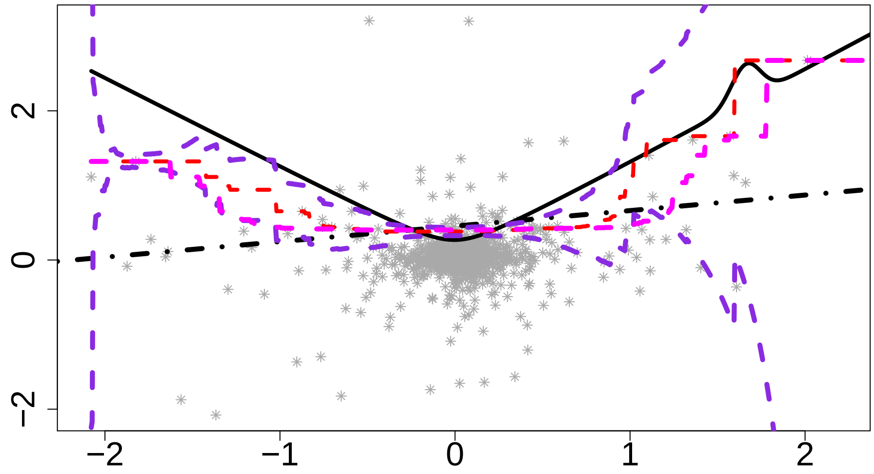

Figure 4.3: Simulated sample with *student's t_2 distributed innovations*, parametric quantile regression (p=0.95) (dotdash), true conditional 0.95-quantile (solid), nonparametric estimate for conditional quantile with h=1 and h=1.5. The violet dashed line is the 95% confidence interval. InvqSimT2

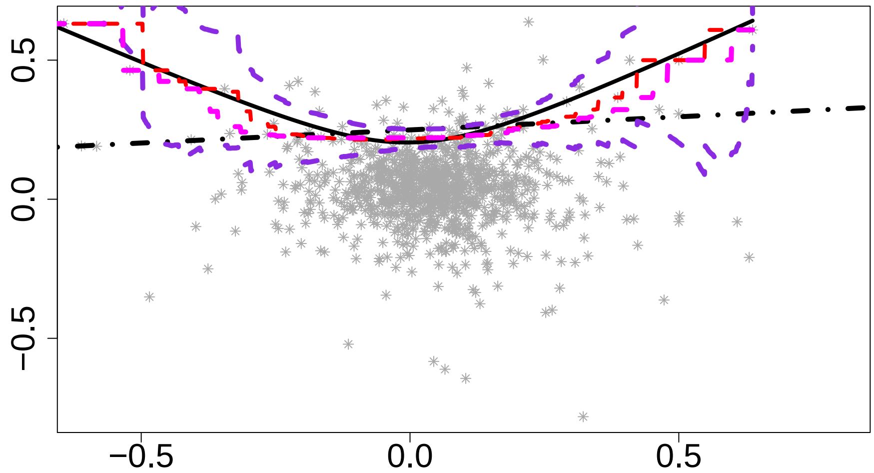

Figure 4.4: Simulated sample with *student's t_4 distributed innovations*, parametric quantile regression (p=0.95) (dotdash), true conditional 0.95-quantile (solid), nonparametric estimate for conditional quantile with h=0.3 and h=0.4. The grey dashed line is the 95% confidence interval. InvqSimT4

Distribution	Bandwidth	MSE
Normal	0.24	0.0042
Exponential	0.24	0.0450
Student's t2	1.50	0.2930
Student's t4	0.60	0.0706

Table 4.1: Mean squared error, calculated from 100 independent samples of the simulated process, with p=0.95.

independent sample we calculated the MSE, after estimating $\widehat{\vartheta}_p(x)$ and calculating the true conditional 0.95-quantile. In Table 4.1 we can see that for all four distributions the MSE value is small, which indicates that the proposed estimator has a small bias.

As already mentioned, it can be seen that the performance of the conditional quantile estimate does not depend strongly on the innovation distributions and it performs reasonably well for asymmetric (exponential), heavy-tailed (t_4) and infinite variance (t_2) innovations. As Franke and Mwita (2003) noted, the performance of the estimator could be improved by adapting the bandwidth to the local density of observations.

4.2 Application on financial data

To see how the proposed nonparametric estimate for conditional quantiles of time series performs on a real data set, we will estimate the VaR of three different stocks and compare it with the CAViaR model (Engle and Manganelli, 2004) and the parametric linear quantile regression (Koenker and Basset, 1978).

We examine the VaR forecasting performance for a portfolio that is short on IBM, HSBC and Ford. In this case, the holder of the portfolio suffers a loss when the value of the asset increases.

4.2.1 Data description

To implement the methodology, we have chosen the historical time series of returns for three stocks. The data set consists of 1512 daily adjusted closing prices from Yahoo Finance for the following stocks: IBM Corporation (component of S&P 500), HSBC Holding (component of FTSE 100 Index) and Ford Motor Company (component of S&P 500). The covered period is from March 1, 2005 to March 1, 2011. We computed the

Stock	Mean	Std. Dev.	Skewness	Kurtosis	Min	Max
IBM	-0.0004	0.0149	-0.0375	5.5040	-0.1090	0.0866
HSBC	0.0002	0.0209	1.6152	37.1312	-0.1823	0.2764
Ford	-0.0001	0.0357	0.0975	12.1477	-0.2553	0.2897

Table 4.2: Summary statistics for daily returns. The period is from March 11, 2005 to February 10, 2011. The number of observations is 1512

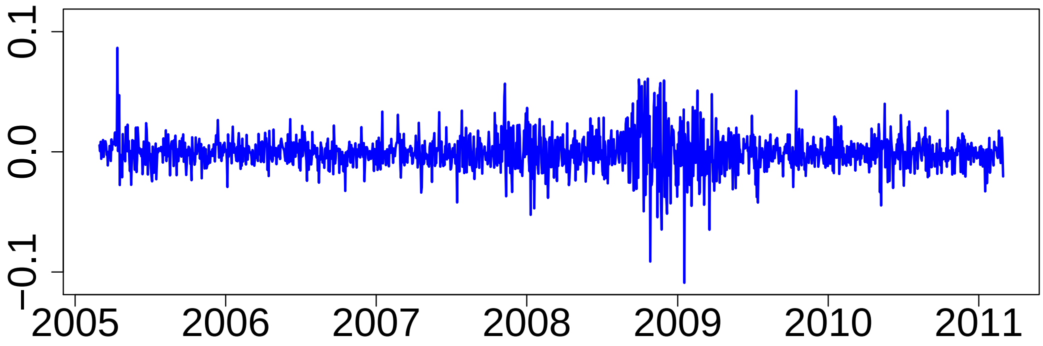

Figure 4.5: IBM log returns. The period is from March 1, 2005 to March 1, 2011. The number of observations is 1512

daily returns as the difference of the log of prices

$$R_t = \ln(P_t) - \ln(P_{t-1}) \tag{4.5}$$

Table 4.2 presents some relevant summary statistics for the calculated log returns of the chosen financial assets. It can be seen that IBM has negative skewness, while HSBC and Ford show positive skewness and across all three samples an excess kurtosis can be observed. Figure 4.5 shows the log returns of IBM and we can observe that volatility is not constant and that large changes tend to be followed by large changes and small changes tend to be followed by small changes. Therefore, the returns exhibit the typical behavior of financial time series: asymmetry in the data, violation of normality and volatility clustering, which motivates nonparametric estimation of VaR.

4.2.2 Application

In this section we show the estimating results for VaR prediction. For each stock, we compute three types of VaR, with $p = 0.95$.

First, we calculated VaR using the methodology of the nonparametric estimate for conditional quantiles proposed in Section 3. The method has been implemented in R programming software and for each of the stock, we obtained three series of nonparametric VaR forecasts: $\widehat{VaR}_{IBM}$, $\widehat{VaR}_{HSBC}$ and $\widehat{VaR}_{Ford}$. In calculating VaR, we use a moving window of N=252 (corresponding to approximately two years of trading data), which allows us to get an update for the estimator for each moving window with an increment of one trading day. This leaves us with 1259 VaR nonparametric estimates.

Second, we calculate the VaR series using the CaViaR model proposed by Engle and Manganelli (2004). From the different alternatives of the model, we present here only the results for the symmetric absolute value specification described in Section 3. We calculated 5% 1-day VaRs, using the first 1259 observations to estimate the model and the last 252 for out-of-sample testing.

Third, the linear quantile regression technique proposed by Koenker and Basset (1978) and described in Section 3, is used to calculate parametric estimates of VaR, with a moving window of N=252 and $p = 0.95$.

Figure 4.6, 4.7 and 4.8 show the forecasted 5% VaR sequence, estimated with the three techniques for IBM, HSBC and Ford. It can be seen that compared to CAViaR and linear quantile regression, the nonparametric VaR is much smoother, even for extreme values.

To check the accuracy of our estimator, we also constructed the 95% confidence interval, as described in Section 3.3. For all three stoks, the estimator lies inside the confidence interval.

Table 4.3 shows the summary statistics of the 5% VaR estimates. Across all three estimators, Ford has the highest mean and highest standard deviation, while IBM has the lowest mean and standard deviation. For all three stocks, the CaViaR estimates have the highest maximum value, while the parametric quantile regression have the lowest minimum value, as compared to the other two implemented models. The mean and standard deviation of the nonparametric estimate are very similar to the mean and standard deviation of the parametric quantile regression.

4.2.3 Forecast performance evaluated with backtesting

For evaluating the forecast performance of the proposed nonparametric estimator for conditional quantiles, we use the backesting procedure described in Section 3.4. We will compare the performance of the nonparametric estimate to the CaViaR and parametric quantile regression models, by applying the CAViaR test.

For the backtesting procedure, first we have to calculate the violation sequence (as defined in Section 3.4) for each stock. The estimated values of the VaR are compared

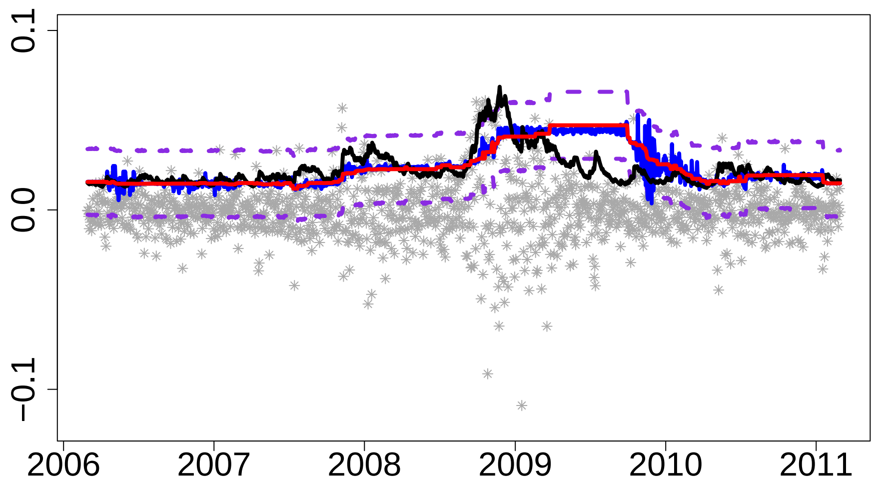

Figure 4.6: The $\widehat{VaR}_{IBM}$. The grey stars are daily returns of IBM Corporation, the blue line is the linear quantile regression, the black line is the $\widehat{CAViaR}_{IBM}$ and the red line shows the nonparametric estimate for conditional quantile $\widehat{VaR}_{IBM}$, with h=0.5. p=0.95. The violet dashed line is the 95% confidence interval. The moving window size is 252 days. VaRInvq

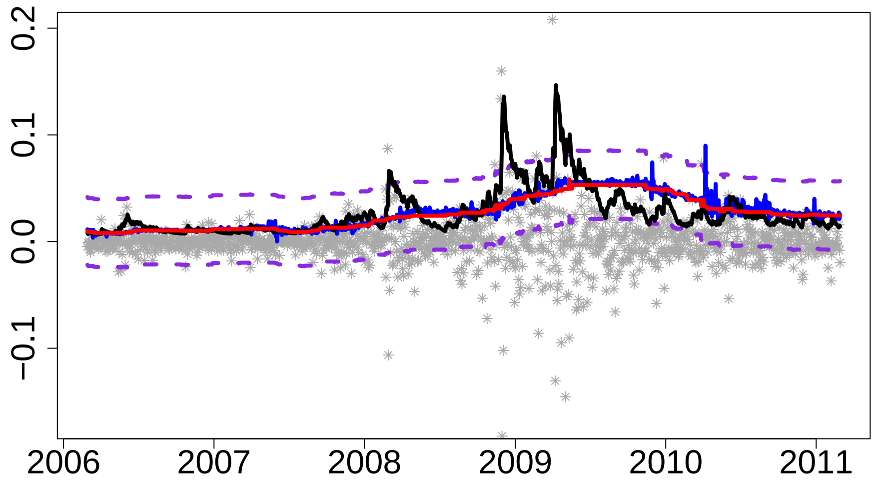

Figure 4.7: The $\widehat{VaR}_{HSBC}$. The blue stars are daily returns of HSBC Holdings, the blue line is the linear quantile regression, the black line is the $\widehat{CAViaR}_{HSBC}$ and the red line shows the nonparametric estimate for conditional quantile $\widehat{VaR}_{HSBC}$, with h=0.4. p=0.95. The violet dashed line is the 95% confidence interval. The moving window size is 252 days. VaRInvq

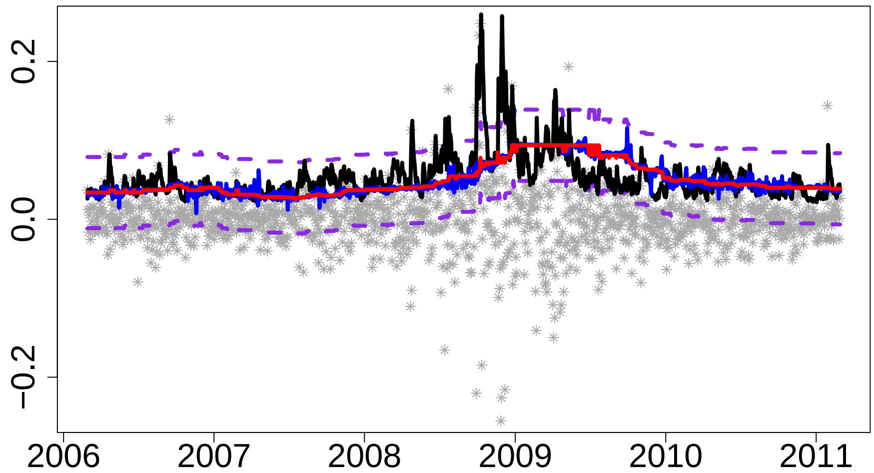

Figure 4.8: The $\widehat{VaR}_{Ford}$. The blue stars are daily returns of Ford Motor Company, the blue line is the linear quantile regression, the black line is the $\widehat{CAViaR}_{Ford}$ and the red line shows the nonparametric estimate for conditional quantile $\widehat{VaR}_{Ford}$, with h=0.5. p=0.95. The violet dashed line is the 95% confidence interval. The moving window size is 252 days. VaRInvq

Measure	Bandwidth	Mean	Std.Dev.	Min	Max
$\widehat{VaR}_{IBM}$	h=0.5	2.36	1.10	1.18	4.72
$\widehat{CAViaR}_{IBM}$		2.16	0.91	1.23	6.86
$\widehat{RQ\_VaR}_{IBM}$		2.35	1.10	0.36	5.30
$\widehat{VaR}_{HSBC}$	h=0.4	2.60	1.49	0.77	5.84
$\widehat{CAViaR}_{HSBC}$		2.38	2.10	0.74	14.70
$\widehat{RQ\_VaR}_{HSBC}$		2.70	1.58	0.01	8.99
$\widehat{VaR}_{Ford}$	h=0.3	4.83	1.83	2.53	14.17
$\widehat{CAViaR}_{Ford}$		5.03	2.77	2.11	25.96
$\widehat{RQ\_VaR}_{Ford}$		4.94	2.07	0.77	11.55

Table 4.3: VaR 5% summary statistics. The period is from March 1, 2005 to March 1, 2011. The numbers in the table are scaled up by 10^2

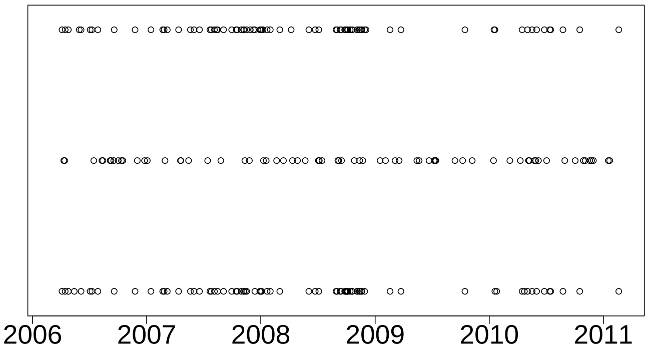

Figure 4.9: The timings of violations. The top circles are for $\widehat{VaR}_{IBM}$ (80 violations), the middle ones are for $\widehat{CAViaR}_{IBM}$ (81 violations) and the bottom ones are for $\widehat{RQ\_VaR}_{IBM}$ (77 violations). VaRInvq

with the actual returns, a violation occuring for each observation larger than the VaR estimate. Because we are interested in evaluating the forecast performance, each time we compare the ex post return to the VaR estimate. The violations are calculated using moving windows, with a window size of 252 days.

Figure 4.9 shows the timings of the violations $t : It = 1$ of $\widehat{VaR}_{IBM}$, $\widehat{CAViaR}_{IBM}$ and $\widehat{RQ\_VaR}_{IBM}$. The figure shows that the total number of violations for nonparametric VaR and CAViaR are similar, but nonetheless both have more violations than the parametric quantile regression. Figure 4.10 shows the violations of $\widehat{VaR}_{HSBC}$, $\widehat{CAViaR}_{HSBC}$ and $\widehat{RQ\_VaR}_{HSBC}$. For this stock, CAViaR has the least violations, while nonparametric VaR and parametric quantile regression are similar. Figure 4.11 depicts the violations of $\widehat{VaR}_{Ford}$, $\widehat{CAViaR}_{Ford}$ and $\widehat{RQ\_VaR}_{Ford}$. In this case, parametric quantile regression has the most violations, while the other two models are very similar.

The backtesting procedure is performed separately for each sequence of It. The null hypothesis is that each sequence It forms a series of martingale difference. The out of sample CAViaR test has been applied. The results of the test are shown in Table 4.4. The highest p-values have been obtained by $\widehat{VaR}_{IBM}$, $\widehat{VaR}_{HSBC}$ and $\widehat{VaR}_{Ford}$. The best result is obtained for $\widehat{VaR}_{IBM}$. The $\widehat{CAViaR}_{Ford}$ and $\widehat{RQ\_VaR}_{Ford}$ are rejected at 5% and 1% significance level, respectively, by the CAViaR test. This indicates that overall, the nonparametric VaR performs better than CAViaR and parametric quantile regression.

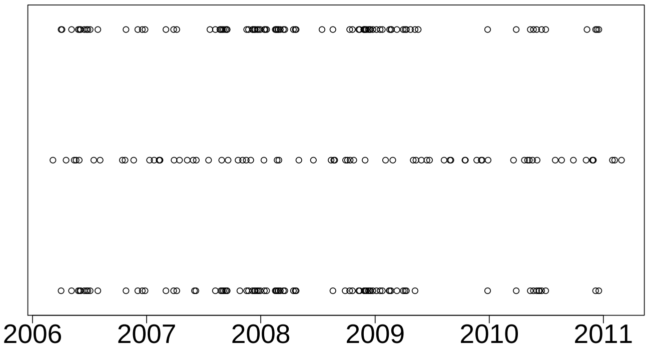

Figure 4.10: The timings of violations. The top circles are for $\widehat{VaR}_{HSBC}$ (89 violations), the middle ones are for $\widehat{CAViaR}_{HSBC}$ (78 violations) and the bottom ones are for $\widehat{RQ\_VaR}_{HSBC}$ (84 violations). VaRInvq

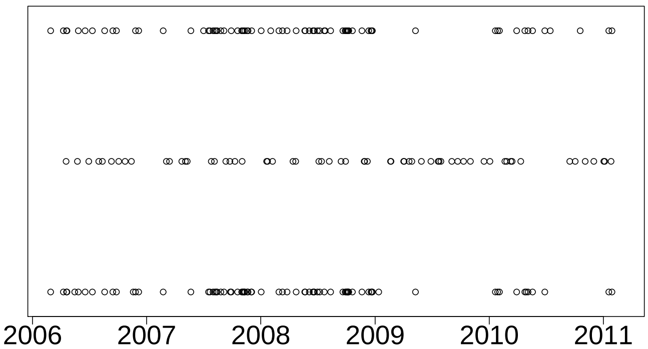

Figure 4.11: The timings of violations. The top circles are for $\widehat{VaR}_{Ford}$ (77 violations), the middle ones are for $\widehat{CAViaR}_{Ford}$ (81 violations) and the bottom ones are for $\widehat{RQ\_VaR}_{Ford}$ (78 violations). VaRInvq

Measure	Bandwidth	CAViaR test
$\widehat{VaR}_{IBM}$	h=0.5	0.2147
$\widehat{CAViaR}_{IBM}$		0.1139
$\widehat{RQ\_VaR}_{IBM}$		0.1529
$\widehat{VaR}_{HSBC}$	h=0.4	0.1572
$\widehat{CAViaR}_{HSBC}$		0.0865
$\widehat{RQ\_VaR}_{HSBC}$		0.0511
$\widehat{VaR}_{Ford}$	h=0.3	0.0770
$\widehat{CAViaR}_{Ford}$		0.0234*
$\widehat{RQ\_VaR}_{Ford}$		0.0010**

*, ** denotes significance at 5 and 1 percent level, respectively

Table 4.4: VaR, CAViaR and quantile regression estimates backtesting *p*-values, obtained with CAViaR test

5 Conclusion

In this paper we present a nonparametric estimate for conditional quantile functions of time series, which is not restricted to independent and identically distributed case. After presenting the theoretical setup, as an illustration we applied the estimator first to a simulation study, considering four different distributions for the innovations and we showed that the estimator performs well, independent of the chosen distribution. Secondly, we made an application for VaR on a real financial data set and we showed that the nonparametric estimate for VaR is smoother and delivers better results than the CAViaR model and the parametric quantile regression models for all three stocks that we considered in our application. Another useful application for the nonparametric estimate of conditional quantiles is the estimation of nonparametric predictive intervals as explained by Koenker (1994), Zhou and Portnoy (1996).

List of Figures

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Declaration of Authorship

I hereby confirm that I have authored this master thesis independently and without use of others than the indicated sources. Where I have consulted the published work of others, in any form (e.g. ideas, equations, figures, text, tables),this is always explicitly attributed.

Berlin, June 12th, 2012

Ioana Balcau

Erklärung zur Urheberschaft

Hiermit erklare ich, Ioana Balcau, dass ich die vorliegende Arbeit allein und nur unter Verwendung der aufgeführten Quellen und Hilfsmittel angefertigt habe. Die Prüfungsordnung ist mir bekannt. Ich habe in meinem Studienfach bisher keine Masterarbeit eingereicht bzw. diese nicht endgültig nicht bestanden.

Berlin, June 12th, 2012

Ioana Balcau