Spatiotemporal analysis of inflation in euro zone countries

Master Thesis Submitted to

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Mengshan Xu

Abstract

This paper applies the spatiotemporal technology to modeling inflation rates of eight euro zone countries in a time interval from 1998 to 2008. While applying Gaussian copula to increase the flexibility of the multivariate model, I use the weighted pairwise composite likelihood method suggested by Lindsay (1988), Cox and Reid (2004) to tackle the computational problem. Due to the existence of exogenous shock, the fitted series show noises and vibrate around the real series. These noises can be eliminated by a three-term moving average smoother, indicating that the model can explain the majority of the spatiotemporal dependence of inflation rates in these countries.

Keywords: Spatiotemporal Analysis, Gaussian Copula, Weighted Pair Composite Likelihood, Inflation Rates

JEL Classification: C21, C22, E31

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1 Introduction

Spatiotemporal analysis is generally related to analysis of data that are collected across time and space. An example is 20-year PM10 (particulate with diameter less than 10 micrometers) across the north-eastern USA from 1982 to 2002 (Diez Roux et al. (2008)). In this dataset, monthly values of PM10 are collected at different air pollution stations over a time span of 20 years. These values are spatially correlated in the sense that the correlation of values at two different locations is likely to be increasing with the decrease of distance between two stations, but decreasing with the increase of distance. On the other hand, values are correlated w.r.t time in the sense that at a station,the PM10 value today is likely highly correlated with the value of last week but less likely impacted by the value of 10 years ago. In addition, there are spatiotemporal interaction effects: if location A experiences a surge of PM10, it might take times for air circulation to carry these particulates to its neighboring location B and affect its PM10 value — a correlation across *both* time and space.

In this paper, I apply spatiotemporal model to analyzing a kind of economic data, inflation rate. Inflation is a macroeconomics phenomenon that is common in every country in the world and is tightly influenced by many factors, including quantity of money, unemployment, exchange rate and rational expectation. Without incorporating all these factors into a model, in this paper I based my analysis on a simple fact that inflation is a direct representation of weighted prices of a bundle of commodities. In this sense, inflation rate shares many things in common with PM10. Especially in a unified market where commodities are free to exchange such as Europe Union, trade of goods is similar to circulation of particulate. Commodities will be transported from those markets with lower prices to those markets where they are comparatively more expensive, just as particulates in the air will circulate from area with higher PM10 to those with lower PM10 by wind and Brown Motion. On the other hand, trade of goods are limited by spatial distance and time due to transportation cost and storage life, just as circulation of particulates are restricted by the mutability of wind and deposition process. Intuitively, the correlation between inflation rates at two spatiotemporal points is likely to fade with the increase of spatial distance and time lag. The empirical variogram of inflation rates of Europe Union countries from 1998 and 2015 has corroborated this intuition.

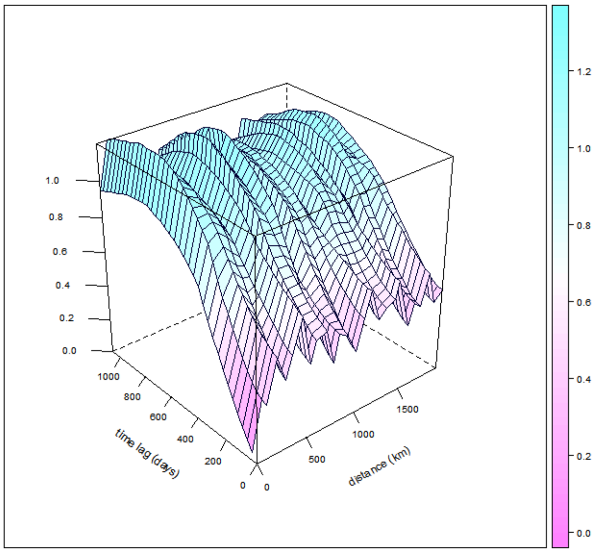


Figure 1.1: The (transformed[1]) empirical variogram of inflation rates in 19 Europe Countries from 1998 to 2015.

However, factors other than price will also influence inflation rate, and sometimes they are very significant. Mechanisms that determine the existence of these exogenous shocks, such as monetary policy or financial crisis, are not very likely to be fully explained by a model built on spatiotemporal interaction of prices (inflation). Two ways would probably tackle this challenge. The first one is to find a strategy to identify and separate these exogenous shocks. The second one is to constrain the analysis to those countries and time intervals, where these shocks do not play a significant role in forming inflation rate. In this paper I will take the second one, constraining my study to eight neighboring Europe counties in the middle Europe and excluding the periods of the financial crisis in 2008 as well as the following euro debt crisis. However, the fitted series still shows a moderate degree of vibration around the real series. These noises are probably due to the existence of persistent exogenous shocks. Finally, a moving average smoother of three terms helps to counteract almost completely these noises and produce nicely fitted series, indicating that the spatiotemporal model used in this paper can account for a large part of the variation and the interaction of inflation rates.

[1] The variogram here is calculated with *transformed* data. The detailed technique is discussed in Chapter 3. The γ (z-axis) is increasing and converging to 1 with the increase of time lag and distance, indicating the fade of covariance. The definition of variogram is given in the Chapter 3.

A combination of many spatiotemporal and multivariate technologies is applied in this paper. In recent decades, with the increasing importance of spatiotemporal analysis to many different areas in both natural and social sciences, various technologies are invented and developed to tackle the challenges that arise in spatiotemporal analysis. Fundamentally, there are two main challenges: *model setting* and *computational burden*. The first one is about how to find out an appropriate multivariate model to describe a spatiotemporal dataset and its correlation structure; the second one is about how to handle the tremendous computational burden in the estimation of high dimensional dataset, which is commonplace in spatiotemporal data analysis.

As pointed out by Song (2007), there are two basic strategies to deal with the model setting problem. In the first strategy, one can try to specify a full probability model to describe the spatiotemporal construction. The advantage of this strategy is that a correctly specified model combined with maximum likelihood method can offer the most efficient statistical inference. The disadvantage of it is that in many real cases where data types are not normal, it is difficult to find out a suitable multivariate distribution. In the second strategy, one only specifies the first two moments of the data and use the quasi-likelihood method to obtain the interested parameters. While maintaining model's robustness and flexibility, this strategy suffers a loss of estimate efficiency. In this paper I combined the ideas proposed by Bai, Kang, and Song (2014) and Bai, Song, Raghunathan (2012) together. While applying Gaussian copula (Song (2000)) to model the marginal distribution of inflation rates in different countries, I use the non-separable spatiotemporal covariance structure of Cressie and Huang (1999) to define the dependence structure of Gaussian copula. This combined strategy enjoys advantages of both sides: the flexibility and robustness of model, which is provided by copula construction; the capability of modeling dependence across time and location, which is provided by the spatiotemporal covariance structure.

As to computational burden, I plan to use pairwise composite marginal likelihood to reduce the computational burden. Bai et al. (2012) pointed out that pairwise composite marginal likelihood has three advantages: analytically simplicity, low requirement of distribution specification and no requirement of a metric combining both space and time. Specifically, in this paper I will use weighted pairwise composite marginal likelihood (Cox and Reid (2004) and Varin (2008), WCL hereafter) to analyze inflation rates of Europe countries.

In sum, the aim of this paper is to estimate the spatiotemporal dependence structure of Europe countries' inflation rate. In this process, I combined WCL, non-separable spatiotemporal covariance structure (Cressie and Huang (1999)) and Geo-copula (Bai et al. (2014)) (a copula-based spatial model for clustered data) into a copula-based spatiotemporal model. Furthermore, this paper is an attempt to apply spatiotemporal technology, which is already popular in natural scientific research, to the area of macroeconomics.

The paper proceeds as follows. In Chapter 2, I review literature and R packages which are related to the estimation in this paper, including spatiotemporal analysis, composite likelihood and inflation. In Chapter 3, after techniques applied in this paper are briefly summarized, I illustrate how these techniques can be combined to estimate parameters of spatiotemporal dependence. In Chapter 4 dataset used in this paper is described. In Chapter 5 estimation results are given and they are evaluated in Chapter 6. In Chapter 7 I give some further discussions and conclude the thesis.

2 Related Literature

The related literature in this paper is classified in four categories: Methodology and empirical studies on spatial and spatiotemporal dataset, theory about inflation, and relevant R packages that are applied in the study of this paper.

2.1 Methodology Studies

Methodologies applied in this paper are tightly related with the work of Bai et al. (2012) and Bai et al. (2014). The first work proposes a spatiotemporal estimating strategy based on multivariate normal model, and the second work proposes a spatial-cluster estimating strategy based on Gaussian copula. While combining the spatiotemporal setting of the first work and the Gaussian copula of the second work, this paper uses WCL to estimate parameters in spatiotemporal covariance function. Comprehensive reviews about composite likelihood and WCL can be found in Varin et al. (2011) and Bevilacqua et al. (2012). Detailed discussion about Gaussian copula can be found in Song (2007).

2.2 Empirical Studies

There are numerous empirical works about spatial or spatiotemporal data analysis in field of nature and social science. A brief summary of them can be seen in the introduction of Bai et al. (2014). Roughly, the methodologies used by all this literature can fall into two categories: those modelling spatial (temporal) variation with spatial (temporal) autocorrelation function and those modelling spatial (temporal) variation with spatial (temporal) covariance matrix. The methodology applied in this paper is the latter one.

In social science, literature on spatial (temporal) analysis is clustered on fields such as health and medicine. Works related to economic dataset account for only a small fraction. Recently, Schenker and Straub (2011) utilize a spatial time series model to explain the regional interdependence of unemployment in Switzerland. Holly et al. develop a spatiotemporal model to investigate house prices in USA (2010) and UK (2011) .

2.3 Inflation

Since this paper uses a stationary covariance function to explain the spatiotemporal dependence of inflation rates in Europe, the focus of inflation-related literature is on stationarity of inflation rates. List of literature as well as further discussion are given in Chapter 7.

2.4 R packages

R packages dealing with spatial (temporal) analysis have been developing for more than 15 years, but a set of widely accepted R classes for spatial (temporal) data had not been established until 2003 (Bivand et al. (2013)). From 2003 onwards, R packges handling spatial analysis start being equipped with unified classes that specify how spatial (temporal) data are organized and stored. In this paper, I also process data in accordance with the standard provided by these classes. Four spatial (temporal) R-packages are involved.

Package "spacetime" and "sp" by Pebesma et al. provides fundamental classes and methods for spatiotemporal data.

Package "gstat" by Pebesma et al. provides functions for calculating and fitting variograms. It is worth noting that in its latest version (1.0-22 (2015)) the set of spatiotemporal tools and functions is still limited and underdeveloped, compared to its comprehensive set of tools and functions in spatial dataset analysis.

Package "GeoCopula" by Kang et al. is not among those "standard" spatial (temporal) packages but an attached package of work of Bai et al. (2014). It provides a set of tools and functions for spatial analysis of clustered data with Joint Composite Estimation Function. Many of its functions can be applied to the estimation in this paper. However, there are two main differences between "GeoCopula" and the R code used for estimation in this paper. Firstly, "GeoCopula" is for spatial-cluster analysis, while this paper focuses on a spatiotemporal dataset. Secondly, "GeoCopula" is created for Joint Composite Estimation, while I use weighted pairwise composite likelihood method.

3 Methodology

In the first half of this Chapter I briefly introduce theoretical framework of the techniques applied in this paper. In the second half I will show how to combine these technologies together in order to estimate interested parameters from a real dataset. More details are given in Appendix.

3.1 Theoretical Framework

3.1.1 Spatiotemporal Process

To model the dataset across time and space, we need spatiotemporal process $Y(s,t)$. s is location index and t is time index. In a basic model, $Y(s,t)$ is usually decomposed into a deterministic part $\mu(s,t)$ and a spatiotemporal process $X(s,t)$.[1]

$$Y(s,t) = \mu(s,t) + X(s,t) \tag{3.1}$$

And the covariance of two observations at spatiotemporal coordinates (s_1, t_1) and (s_2, t_2) is given by:

$$C(s_1, s_2, t_1, t_2|\theta) = \mathrm{cov}\{X(s_1, t_1), X(s_2, t_2)|\theta\} \tag{3.2}$$

In the setting of this paper, s is the two-dimensional coordinate that represents the location of a country.

3.1.2 Gaussian Copula

The model in this paper is built on Gaussian copula:

$$C(F_1, F_2 ... F_n) = \Phi_n(\Phi^{-1}(F_1(x_1)), \Phi^{-1}(F_2(x_2)) ... \Phi^{-1}(F_n(x_n))|\Sigma) \tag{3.3}$$

[1] In literature a standard spatiotemporal process usually includes also a measurement error $\epsilon(s,t)$, which is independent of any other variables and the added variance is called nugget effect. I do not include this item in the analysis of this paper. As a result, exogenous shocks will be included into this missing error term. The impact of exogenous shocks will be discussed in Chapter 7.

F_i is the cumulative distribution function (CDF hereafter) of the i-th variable. $\Phi(.)$ is the CDF of the standard normal distribution. $\Phi_n(...|\Sigma)$ is CDF of a multivariate Gaussian distribution with covariance matrix Σ.

Compared to existing multivariate models whose forms and marginal distributions are strictly fixed, Gaussian copula has an enhanced flexibility in the sense of encompassing many existing models as specific cases. Bai et al. (2014) has summarized that how Gaussian copula can encompass multivariate Gaussian distribution (Cressie (1993)) and multivariate probit model (Heagerty and Lele (1998)) as specific cases. The second advantage of Gaussian copula is that the covariance matrix Σ that defines marginal dependence structure of Gaussian copula can be used to model the spatiotemporal dependence of dataset across time and space.

3.1.3 Spatiotemporal Covariance

In many spatiotemporal models, a flexible spatiotemporal covariance function plays a key role. A stationary spatiotemporal covariance function is a function $C(h, u)$, where h is space lag, and u is time lag. In this paper I use the non-separable spatiotemporal covariance suggested by Cressie and Huang (1999).

$$C(h,u;\theta) = \begin{cases} \frac{2\sigma^2\beta}{(a^2u^2+1)^{\nu}(a^2u^2+\beta)\gamma(\nu)}\left(\frac{b}{2}\left(\frac{a^2u^2+1}{a^2u^2+\beta}\right)^{\frac{1}{2}}h\right)^{\nu}K_{\nu}\left(b\left(\frac{a^2u^2+1}{a^2u^2+\beta}\right)^{\frac{1}{2}}h\right) & if\ h>0 \\ \frac{\sigma^2\beta}{(a^2u^2+1)^{\nu}(a^2u^2+\beta)} & if\ h=0 \end{cases} \tag{3.4}$$

Apparently it is a symmetric structure. After having proved that the covariance matrix following this structure is positive definite, Cressie and Huang (1999) did not give details of the properties of this structure in their work. Appendix A will briefly discuss the properties of this covariance structure.

3.1.4 Variogram and Empirical Variogram

Instead of using covariance function, people usually use *variogram* to describe autocorrelation of a spatial (temporal) analysis for historical reasons (Bivand et al. (2013)). Many conceptions and tools in spatial (temporal) analysis are tightly associated with variogram. A spatial variogram is:

$$\gamma(h) = \frac{1}{2}\,\mathrm{E}[(Y_{s_1} - Y_{s_2})^2] \qquad \text{with} \qquad \|s_1 - s_2\| = h$$

And its spatiotemporal version is:

$$\gamma(h,u) = \frac{1}{2}\,\mathrm{E}[(Y_{s_1,t_1} - Y_{s_2,t_2})^2] \qquad \text{with} \qquad \|s_1 - s_2\| = h, \|t_1 - t_2\| = u \tag{3.5}$$

Variogram function is tightly related with covariance function. From definition we can see, for a stationary process with zero mean and variance σ^2, the variogram is simply σ^2 minus the covariance of two spatial (temporal) locations.

$$\gamma(h,u) = \sigma^2 - \mathrm{cov}(Y_{s_1,t_1}, Y_{s_2,t_2}) \qquad \text{with} \qquad \|s_1 - s_2\| = h, \|t_1 - t_2\| = u \tag{3.6}$$

Figure 3.1 are two examples of spatial variogram and spatiotemporal variogram. Because in the spatiotemporal variogram both time and space lags are involved, we have a 3-D type graph and z-axis represents variogram value γ.

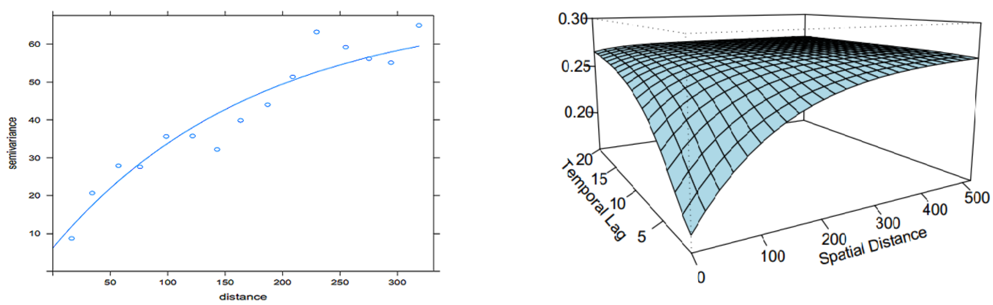


Figure 3.1: Examples of spatial variogram (left, source: Pebesma (2015)) and spatiotemporal variogram (right, source: Bai et al.(2012))

From these two variograms we can see that the value γ at the origin (same location and the same time) is 0, which can be derived from formula (3.5). When space lag or time lag goes large enough so that the covariance between two points is almost vanished, the γ converges to σ^2.

Empirical variogram is calculated directly from data and therefore can give us an overview of spatiotemporal dependence of data. An Empirical spatiotemporal variogram is defined as:

$$\widehat{\gamma}(h,u) = \frac{1}{2\|N_k\|} \sum_{\|(s_{i_1} - s_{i_2}\|, \|t_{j_1} - t_{j_2}\|) \in N_k} (Y_{s_{i_1},t_{j_1}} - Y_{s_{i_2},t_{j_2}})^2 \tag{3.7}$$

Where N_k is the set of pairs of points, whose time lags are u and space lags are h. In practice, the locations of observations are often irregularly distributed so that many pairs of observations might have their unique distance values. Thus in spatial (temporal) analysis those space lags that are close to each other are normally classified into an interval, for example, 0 to 10 km, 10 to 20 km, etc. And the value h for pairs within a certain interval is determined by the middle point of the interval or average space lags of all the pairs within this interval.

3.1.5 Weighted Least Square Estimate

Weighted Least Square Estimate (WLS hereafter) is a commonly used spatiotemporal estimation technology that obtains its estimates by minimizing the sum of the weighted square difference between the empirical variogram and the parametric variogram. WLS has comparatively low estimation efficiency but high computational efficiency. Follow the idea of Bai et al. (2012), I use WLS estimate as initial parameter for the further WCL estimation. The WLS is given by the following formula (Cressie (1993)):

$$\widehat{\theta}_{wls} = \arg\min_{\theta} \sum_{m=1}^{k} \sum_{n=1}^{j} \frac{\|N(h_m, u_n)\|}{2\gamma(h_m, u_n|\theta)^2} \{\widehat{\gamma}(h_m, u_n) - \gamma(h_m, u_n|\theta)\}^2 \tag{3.8}$$

$\widehat{\gamma}(h_m, u_n)$ is empirical variogram. $\gamma(h_m, u_n|\theta)$ is fitted parametric variogram. $\frac{\|N(h_m,u_n)\|}{2\gamma(h_m,u_n|\theta)^2}$ is weight. More details about WLS are given in the Appendix B.

3.1.6 Weighted Pairwise Composite Likelihood

Composite likelihood is a type of pseudo likelihood method that construct its likelihood function by using marginal or conditional distribution of lower-dimensional component generated from a fully specified parametric distribution. With very few exceptions, composite likelihood cannot attain the efficiency of maximum likelihood, because composite likelihood usually takes a more or less misspecified distribution, leading to information loss. The trade-off brings considerable reduction of computational burden. In many cases, it makes many computationally prohibitive analyses of spatial or spatiotemporal datasets possible.

There are various forms of composite likelihood function. The one I use in this paper is Weighted Pairwise Composite Likelihood (WCL) (Cox and Reid (2004), Varin (2008)).

$$L_p(\theta; Y) = \prod_{i=1}^{m-1} \prod_{j=i+1}^{m} f(y_i, y_j|\theta)^{\omega_{ij}}$$

Its log-likelihood form is:

$$l_p(\theta; Y) = \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \omega_{ij} \log f(y_i, y_j|\theta) \tag{3.9}$$

The likelihood function is a product of weighted marginal densities. From its form we can see that it treats all these pairs as independent by multiplying them straightforward, where misspecification exists. According to Bai et al. (2014), this form includes at least the essential information of marginal and dependence parameters.

The weight ω_{ji} takes usually 1 or 0, indicating whether or not a certain pair is included in the composite likelihood function. A common feature shared by many spatiotemporal data is: the larger time lag or space lag is, the smaller correlation remains. Simulation results of Varin et al. (2005) and Bevilacque et al. (2012) show that better estimation efficiency is achieved by choosing pairs within short distance. That means we are going to find two proper cutoff values both for time lag and space lag and then focus our estimation only on those pairs, whose distance (both in time and space) is smaller than these pre-specified cutoffs. Other pairs are abandoned from the composite likelihood function.

3.1.7 Best Linear Prediction

The interest of our estimation is the parameters in spatiotemporal covariance function (3.4). Thus the evaluation of model fit in this paper is conducted with Best Linear Prediction Estimate (Toutenburg (1982)):

$$\widehat{\mathrm{E}}(Y|X) = \mathrm{E}(Y) + \mathrm{cov}(Y, X)\,\mathrm{cov}(X)^{-1}(X - \mathrm{E}(X)) \tag{3.10}$$

Y is the predicted (fitted) data. In this paper it is the inflation rate of one country or a vector of countries at the time point of prediction, say, $t + 1$. X is a vector representing the realized inflation rates of countries at time points before $t + 1$. The $\mathrm{cov}(Y, X)$ and $\mathrm{cov}(X)$ can be calculated with the estimated covariance function (3.4) , since the time lag and space lag between any element in Y and any element in X are known.

3.2 Estimation Strategy

In this Section I will illustrate how the above mentioned techniques are combined to work. In a word, I use Gaussian copula to build a multivariate model. Then I will use WLS to get a set of estimates for interest parameter. The WLS estimates will serve as the initial value [2] for further WCL estimation. Following are the detailed steps.

3.2.1 Model and Interested Parameter

The joint distribution of spatiotemporal process is captured by a Gaussian copula multivariate model.

$$\begin{aligned} &C(F_{c_1,t_1}(x_{11}), F_{c_1,t_2}(x_{12})...F_{c_i,t_j}(x_{ij})...) \\ &\qquad = \Phi_n(\Phi^{-1}(F_{c_1,t_1}(x_{11})), \Phi^{-1}(F_{c_1,t_2}(x_{12}))...\Phi^{-1}(F_{c_i,t_j}(x_{ij}))...|\Sigma) \end{aligned} \tag{3.11}$$

$F_{c_it_j}$ is the marginal CDF of the inflation rate in country i at time j. x_{ij} is the inflation rate of country i at time j. I use empirical CDF of country i, $\widehat{F}_{c_i}$, to calculate each $F_{c_it_j}(x_{ij})$. The advantage of this strategy is we do not need to specify the marginal model of inflation rate in each country, so that we can focus on the estimation of the parameters in Σ. (In Chapter 3.2.3 we will see that due to the dependence construction of Gaussian copula, the log-likelihood function is not affected by the specification in marginal CDF). However, since on each spatiotemporal point we have only one observation, I need to assume that the CDF of inflation rate in a certain country remains unchanged across time in order to calculate the empirical CDF value at each point. This additional assumption is formulated as:

$$F_{c_i,t_j} = F_{c_i,t_k} \quad \forall j,\, k \quad \text{or}$$
$$F_{c_i,t_j} \equiv F_{c_i} \qquad \forall j$$

Σ is the covariance matrix that defines the marginal dependence of Gaussian copula, which is the spatiotemporal covariance function (3.4) in the setting of this paper. To Gaussian copula it is also the covariance matrix of the multivariate normal density w.r.t transformed data (we can see the right side of formula (3.11)). Data are simply

[2] For non-linear optimization algorithms in R and other statistical program is a decent initial value critical for the quality of final estimates

transformed by formula (3.12) [3]

$$x_{ij}* = \Phi^{-1}(\widehat{F}_{c_i}(x_{ij})) \tag{3.12}$$

In spatiotemporal covariance function (3.4) the parameters of interest are a, b, β, σ and ν. There are three notable changes after the transformation. First, the spatiotemporal covariance is applied to the transformed inflation rates rather than to the original inflation rates. Second, σ^2 becomes exact one after the transformation, because data is transformed into standard normal density. Third, the $\mu(s, t)$ in formula (3.1) becomes 0 for the same reason. In brief, after the transformation, we get a new spatiotemporal process, and the next step is to use a multivariate normal model with covariance function Σ to fit it.

3.2.2 Estimate of ν

As pointed out by Stein (1999), the parameter ν in practice is difficult to estimate due to the requirement of dense spatial data and the emergence of identifiability problems. Following the idea from Bai et al. (2012), I conduct estimation of a, b and β with fixed ν and repeat the estimation at 10 different ν values that are equally spaced on [0,1] interval (from 0.1 to 1). Then I take the ν with the largest likelihood value as the estimate of ν.

3.2.3 Weighted Composite Likelihood Estimation

Next step is to construct the weighted composite likelihood (WCL) function. To determine the 1-0 weight in formula (3.9), appropriate cutoffs in time and space must be chosen. Here I choose cutoff in time by observing the WLS fitted variogram, i.e. finding out a point along the axis of time lag, from where correlation starts to vanish. All the observation pairs within this cutoff are included in the WCL function (The weight of them will be 1). As to space lag, since in this analysis we have only eight locations, in principle I can include pairs with all possible space lags, without adding too much computational burden.

WCL function is the sum of log likelihood functions of all the pairs that have a weight of 1. The likelihood function of a pair of observations is the marginal density of these two

[3] In practice, $\widehat{F}_{c_i}(x_{ij})$ is multiplied by $\frac{m}{m+1}$. m is the number of observations in country i used to calculate $\widehat{F}_{c_i}$, and $\frac{m}{m+1}$ can help to prevent the infinite value in the transformed data. Without multiplied by $\frac{m}{m+1}$, the largest observed value in country i will have an empirical CDF of 1, leading to the infinite value in transformed data. Note that this practice will bias the $\widehat{F}_{c_i}$.

observations. Due to the property of marginal closure of Gaussian copula, the marginal CDF derived from formula (3.11) is:

$$F(x_{ij}, x_{kl}|\theta) = \Phi_2(\Phi^{-1}(F_{c_i}(x_{ij})), \Phi^{-1}(F_{c_k}(x_{kl}))|\Sigma_{c_i t_j, c_k t_l|\theta}) \tag{3.13}$$

$\Sigma_{c_i t_j, c_k t_l|\theta}$ is a 2×2 matrix that is a sub-matrix of Σ. According to Song (2007), its density function can be formulated as:

$$\begin{aligned} & f(x_{ij}, x_{kl}|\theta) = c_\Phi\{F_{c_i}(x_{ij}), F_{c_k}(x_{kl})|\Sigma_{c_i t_j, c_k t_l|\theta}\} f_{c_i}(x_{ij}) f_{c_k}(x_{kl}) \\ & \text{with} \\ & c_\Phi\{F_{c_i}(x_{ij}), F_{c_k}(x_{kl})|\Sigma_{c_i t_j, c_k t_l|\theta}\} = |\Sigma_{c_i t_j, c_k t_l|\theta}|^{-1/2} \exp\{\frac{1}{2} q^\top (I_2 - \Sigma^{-1}_{c_i t_j, c_k t_l|\theta}) q\} \end{aligned} \tag{3.14}$$

Here $q = (q_i, q_k)$ with $q_i = \Phi^{-1}(\widehat{F}_{c_i}(x_{ij}))$, and I_2 is a two-dimensional identical matrix. In our setting we are only interested in parameters in Σ. Thus we can ignore the marginal density in marginal likelihood function (3.14).

$$f(x_{ij}, x_{kl}|\theta) \ \propto \ |\Sigma_{c_i t_j, c_k t_l|\theta}|^{-1/2} \exp(-\frac{1}{2} q^\top \Sigma^{-1}_{c_i t_j, c_k t_l|\theta} q) \tag{3.15}$$

Therefore, the WCL objective function can be simplified into the product of the right side of formula (3.15) for every pair that have a weight of 1. Its log-likelihood form is given by:

$$\widehat{\theta}_{wcl} = \arg \max_\theta \sum_{(i,j,k,l) \in \Omega_n} \{-\frac{1}{2} \log(|\Sigma_{c_i t_j, c_k t_l|\theta}|) - \frac{1}{2} q^\top \Sigma^{-1}_{c_i t_j, c_k t_l|\theta} q\} \tag{3.16}$$

Ω_n is the set of pairs with a weight $\omega_{ij} = 1$(in formula (3.9)). Objective function (3.16) is a simplified version of formula (3.9) in the setting of this paper.

Maximization process of objective function (3.16) requires an initial value. Following the idea of Bai et al. (2012), the weighted least square (WLS) that minimize the weighted difference between parametric variogram and empirical variogram is applied to obtain an initial value. In addition, WLS is also a non-linear optimization and therefore needs an initial value too. I set multiple initial values and compare the estimation results in order to determine an optimal WLS result.

4 Data

4.1 Countries

The inflation data is downloaded from OECD website[1]. As mentioned in introduction, to analyze the inflation, we could not ignore the impact of exogenous shocks such as monetary policy or financial crisis. In so far as the method used in this paper is a multivariate Gaussian copula with a stationary spatiotemporal covariance matrix, I need to choose an appropriate range of countries and time span so that those non-stationary effect is minimum.

The countries in analysis in this paper are: Austria, Belgium, France, Germany, Italy, Luxembourg, Portugal, and Spain. These eight countries are among the first 11 eurozone countries and they are all on the Europe continent.

4.2 Time Span

Since 2000, all eight countries started to use euro and therefore the inflation rate of them began to calculate with the same currency. Rather than on Jan.2000, I set the starting point of my analysis on Jan.1998, when these eight countries met the Euro Convergence Criteria (1995), indicating the formation of euro zone in the sense that at that moment, these counties started to fulfill the common requirement of price stability and exchange rate stability .

The Figure 4.1 (left) represents the variation of inflation rate of these eight countries from Jan.1998 to Feb.2015. Table 4.1 represents the basic features of data by countries.

[1]https://data.oecd.org/

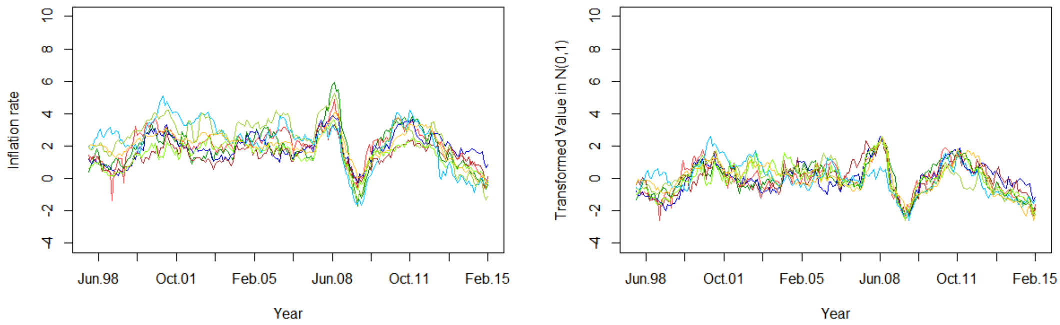


Figure 4.1: Inflation rates (left) and copula-transformed Inflation rates (right) of Austria, Belgium, France, Germany, Italy, Luxembourg, Portugal, and Spain

Figure 4.1 (left) and Table 4.1 together illustrate the distributions of inflation rates, at least in terms of first two moments, are different from country to country. Thus it is inappropriate to fit data with a multivariate normal model; the stationary assumption implicated by it is apparently violated. Through formula (3.12) we transform the data into a form that fits Gaussian copula. The Figure 4.1 (right) and Table 4.2 represent the transformed data. Apparently, the transformed data are marginally $N(0,1)$ distributed.

Countries	Austria	Belgium	France	Germany	Italy	Luxembourg	Portugal	Spain
mean	1.90%	1.93%	1.48%	1.46%	2.04%	2.10%	2.25%	2.46%
covariance	0.78%	1.59%	0.64%	0.51%	0.77%	1.08%	2.15%	1.93%
min	-0.27%	-1.69%	-0.73%	-0.50%	-0.56%	-1.35%	-1.66%	-1.37%
max	3.86%	5.91%	3.61%	3.32%	4.08%	4.87%	5.11%	5.27%

Table 4.1: Summary of inflation rates of eight Europe Countries

Although the transformed inflation rates are centered and have same variance, we can still note from Figure 4.1 (right) that there are many exogenous economic shocks especially in 2008 financial crisis, where a drastic steep fall follows a sudden rise. In the post-2008 period, there is an inverse u-shape common trend, indicating the existence of the persistent impact of financial crisis and other shocks such as euro debt crisis. These strong exogenous shocks will certainly have changed the distribution of inflation rates and they are very likely unable to be fully captured by the Gaussian copula with stationary covariance function. Therefore, in this paper I will choose the time interval from Jan.1998 to May 08, where no significant shocks dramatically influence our series. Table 4.3 gives the summary of transformed inflation rates from Jan.1998 to May.2008.

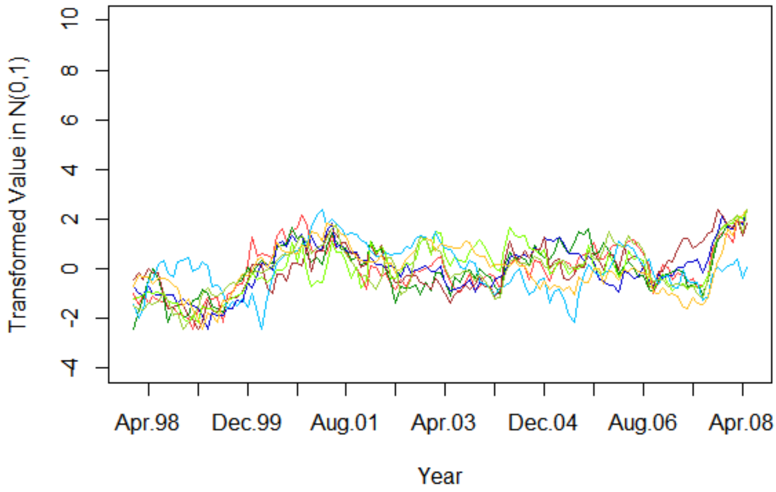


Figure 4.2: Copula-transformed inflation rates of Austria, Belgium, France, Germany, Italy, Luxembourg, Portugal, and Spain from 1995 to 2008

Countries	Austria	Belgium	France	Germany	Italy	Luxembourg	Portugal	Spain
mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00[2]
covariance	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
min	-2.58	-2.58	-2.58	-2.58	-2.58	-2.58	-2.58	-2.58
max	2.58	2.58	2.58	2.58	2.58	2.58	2.58	2.58

Table 4.2: Summary of $N(0,1)$ transformed inflation rates from Jan.1998 to Feb.2015

Countries	Austria	Belgium	France	Germany	Italy	Luxembourg	Portugal	Spain
mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
covariance	0.92	0.94	0.93	0.94	0.94	0.94	0.94	0.94
min	-2.41	-2.41	-2.41	-2.41	-2.41	-2.41	-2.41	-2.41
max	2.41	2.41	2.41	2.41	2.41	2.41	2.41	2.41

Table 4.3: Summary of $N(0,1)$ transformed inflation rates from Jan.1998 to May.2008

[2]All the value in this row is smaller than 0.005

4.3 Space Measure

In this paper I use the longitude and latitude coordinates of the *capital city* of each country to represent this country's location. For all these countries, capital city is one of their most significant national transportation centers and usually equipped with the largest train station or airport. An alternative is to set the location of a country as the center point of its longitude span and latitude span over the earth. Since only eight locations are involved in the analysis, the different choice of location measure will not alter the order of distance set. Thus the choice of locations measure of country will not significantly change the estimation results. The coordinates of each country are measured with Google Earth software. Table 4.4 and Table 4.5 give the coordinates and distances between these countries respectively.

Countries	Austria	Belgium	France	Germany	Italy	Luxembourg	Portugal	Spain
longitude	16.2	4.2	2.2	13.3	12.3	6.1	-9.1	-3.4
latitude	48.1	50.5	48.5	52.3	41.5	49.5	38.4	40.3

Table 4.4: The Coordinates of eight countries, in Degree.

Countries	Luxembourg	Belgium	Austria	Germany	France	Portugal	Italy	Spain
Luxembourg	0.00	175.81	757.50	594.10	306.20	1727.81	1011.13	1265.34
Belgium	175.81	0.00	911.43	663.63	265.30	1704.93	1178.58	1277.88
Austria	757.50	911.43	0.00	510.61	1038.18	2304.61	794.87	1782.79
Germany	594.10	663.63	510.61	0.00	893.80	2319.94	1202.08	1843.81
France	306.20	265.30	1038.18	893.80	0.00	1443.69	1111.01	1013.15
Portugal	1727.81	1704.93	2304.61	2319.94	1443.69	0.00	1855.78	534.48
Italy	1011.13	1178.58	794.87	1202.08	1111.01	1855.78	0.00	1327.75
Spain	1265.34	1277.88	1782.79	1843.81	1013.15	534.48	1327.75	0.00

Table 4.5: The distance matrix of eight countries, in Kilometer

5 Estimation and Results

5.1 Empirical Variogram

Figure 5.1 shows the empirical variogram of transformed inflation rates. The surface is calculated with formula (3.7); time lag is u and space lag is h. From Figure 5.1 we note that the γ (vertical axis) is increasing in with both time lag and distance. From formula (3.6) we know that an increased in γ means a decrease in covariance. Thus the variogram surface indicates that the empirical covariance is decreasing both in time and distance. This pattern justifies the usage of spatiotemporal covariance function (3.4).

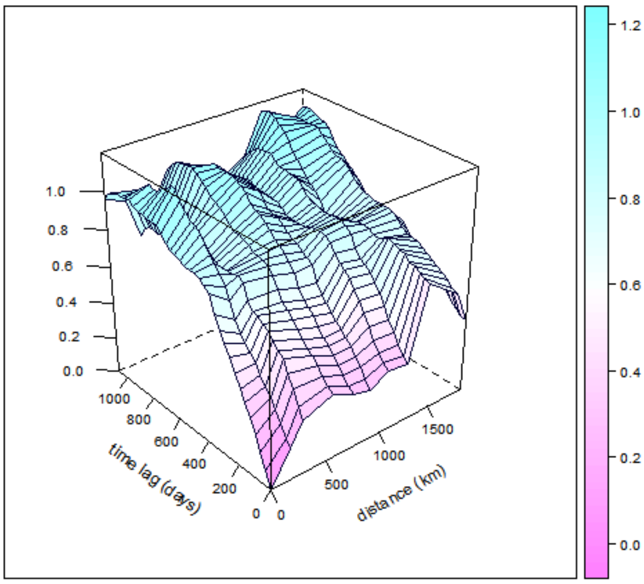


Figure 5.1: Empirical variogram of eight Europe counties

5.2 The Initial Value: WLS Estimate

Following the step mentioned in Chapter 3.2, I first use WLS to estimate an initial value. Formula (3.8) is applied. [1] The WLS result is a= 0.001673, b= 0.000106 and beta= 0.10054. Plugging them into the spatiotemporal covariance function (3.4), we have WLS fitted variogram:

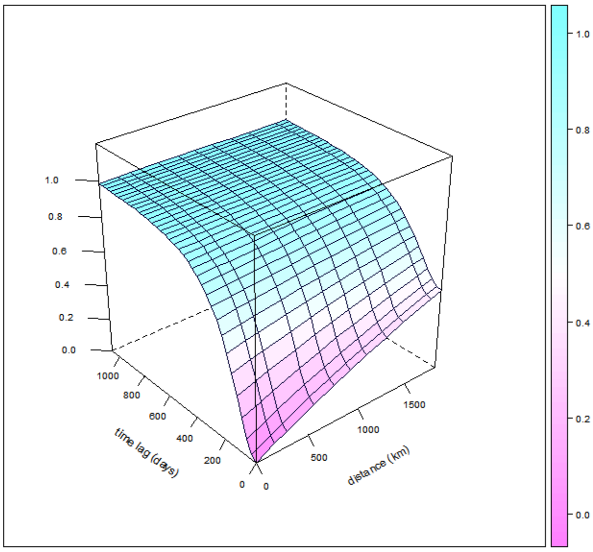


Figure 5.2: WLS fitted variogram

From the WLS fitted variogram we can observe that the surface is converge to 1 approximately from 600 days. Thus I choose 20 months as the maximum time lag for further WCL estimation. Any pair of observations whose time lag are within this maximum value are included in the weighted log-likelihood function (3.9) (that means the weight $w_{ij} = 1$). In practice I use its simplified form, objective function (3.16).

5.3 WCL Estimates and the Best ν

The WCL estimate is the θ that maximizes objective function (3.16). As mentioned in Chapter 3.2.2 I repeat the estimation at 10 ν values. The result are given in Table 5.1:

[1] The WLS is also a non-linear optimization problem and therefore its result is dependent on the quality of initial value too. In practice I set multiple initial values and pick the one with smallest value in Formula (3.8).

ν	a	b	β	WCL value
0.1	0.010018	0.000038	0.100000	-119257
0.2	0.002362	0.010075	0.100754	-119932
0.3	0.001896	0.006593	0.100399	-120068
0.4	0.002088	0.010072	0.100723	-119967
0.5	**0.001795**	**0.000298**	**0.102898**	**-114426**
0.6	0.001588	0.010090	0.100556	-120503
0.7	0.002209	0.010073	0.100726	-119933
0.8	0.002025	0.010075	0.100754	-119967
0.9	0.010419	0.000714	0.100895	-119938
1	0.005426	0.001150	0.098835	-119336

Table 5.1: To find the Best ν

The first column of Table 5.1 is different value of ν, and the following three columns are estimated parameters under fixed ν. It turns out that at $\nu = 0.5$ WCL has the maximum value and we have $\widehat{a} = 0.001795, \widehat{b} = 0.000298$ and $\widehat{\beta} = 0.1028$. Given $\widehat{\beta} = 0.1028$, $\widehat{a} = 0.001795$ means that the marginal temporal correlation decreases by around 8% with 1 month increase in time, and $\widehat{b} = 0.000298$ indicates that the marginal space correlation decays by around 9% with a 100-km increase in space. Figure 5.3 gives the fitted WCL variogram.

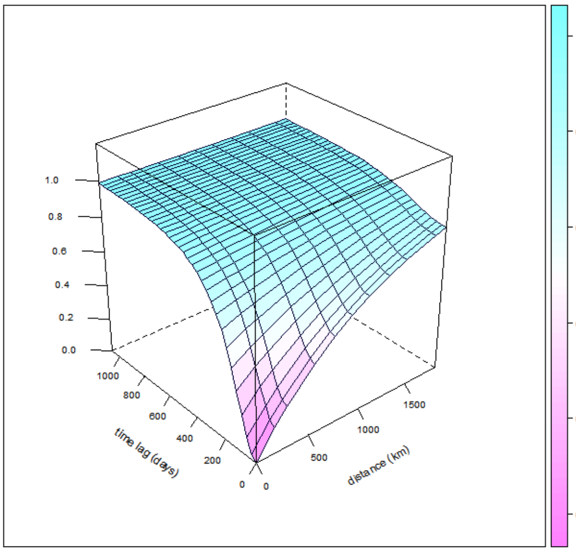


Figure 5.3: WCL fitted variogram

6 Model Fit

After getting WCL estimates, I use Best Linear Prediction (formula (3.10)) to generate predicted (fitted) data in order to evaluate the model fit. There are two kinds of predicted data points, i.e., $\widehat{\mathrm{E}}(Y|X)$ in formula (3.10). Firstly, we can use the historical data of transformed inflation rates to predict a full set of inflation rates of eight countries at $t+1$, which is mentioned as normal prediction in the following parts. Secondly, we can predict the inflation rates of country i at time t with inflation rates of other seven countries at time t and historical data (before time t) of eight countries, which is mentioned as missing value prediction in the following parts. In addition, I also evaluate both in-sample fit and out-of-sample performance.[1] The results are given in forms of figures of fitted series and Mean Squared Errors (MSE)[2].

6.1 Fitted Series

In this section I only present the fitted series of inflation rates of Germany. The series of other countries are given in Appendix D.

6.1.1 In-Sample Fit

Figure 6.1 illustrates fitted series and real series of transformed inflation rates in Germany (transformed by formula (3.12)). We can observe that while the fitted series show the same general trend as the real series does, they vibrate around the real series. This vibration could be caused by the exogenous shocks. After smoothing the series with a three-term moving average smother $x_t^* = \frac{x_{t-1}+x_t+x_{t+1}}{3}$, nicely fitted series are obtained (Figure 6.2), indicating that the model explains a large part of the spatiotemporal variation of the data. Chapter 7 will provide detailed discussion about the cause of this vibration.

[1] Instead of using 20 months lags that are used in estimation, I use 3 time lags to get a more parsimonious model. In case of three time lags, we will have, on the right side of formula (3.10), 24 items for normal prediction and 31 items for missing value prediction.

[2] The fitted series are from Aug.2002 to May.2008. MSE is calculated by $\frac{1}{n}\sum_{t=1}^{n}(\widehat{y_t} - y_t)^2$. For the fitted time interval, $n = 70$

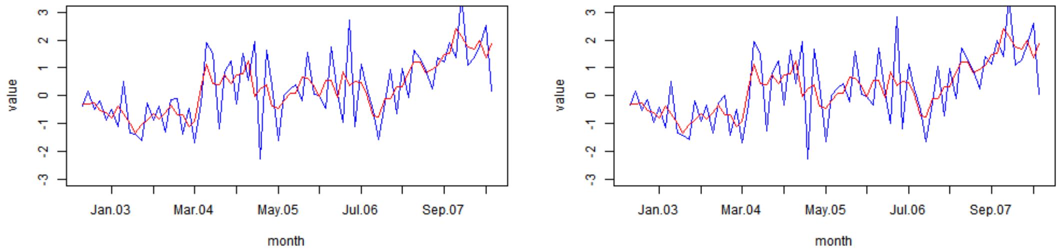


Figure 6.1: WCL Fitted transformed series and real transformed series of Germany (in-sample). Left is normal prediction and right is missing value prediction

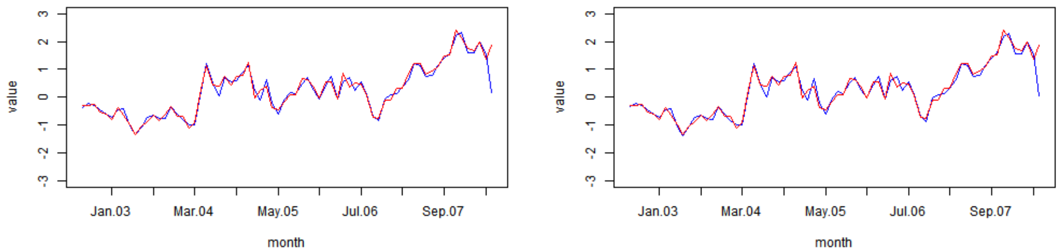


Figure 6.2: Smoothed WCL fitted transformed series and real transformed seriesof Germany (in-sample). Left is normal prediction and right is missing value prediction

Series in Figure 6.1 and Figure 6.2 are for *transformed* inflation rates. Inversing formula (3.12), we can get the smoothed *untransformed* fitted inflation rates, which are given in Figure 6.3.

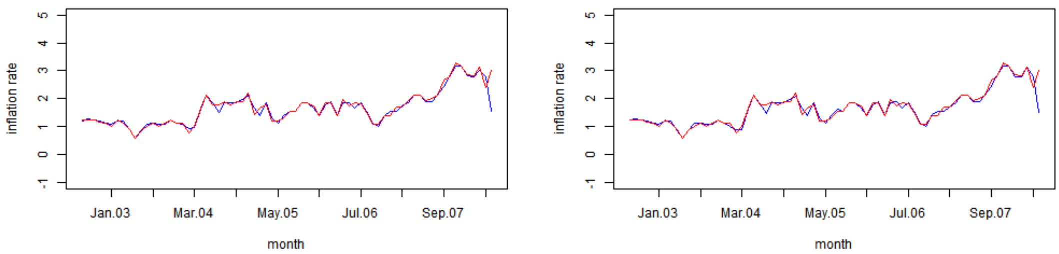


Figure 6.3: Smoothed WCL fitted inflation rate and real inflation rate of Germany (in-sample). Left is normal prediction and right is missing value prediction

6.1.2 Out-of-Sample Performance

The series of this section are fitted with parameters estimated from a subset of the data, in which the predicted spatiotemporal point is not included. [3] The fitted transformed inflation rates, fitted smoothed transformed inflation rates and fitted smoothed untransformed inflation rates of Germany are given in Figure 6.4, 6.5 and 6.6. The patterns illustrated in these Figures are similar to what we have in Figure 6.1, 6.2 and 6.3. The only difference, not a surprise, is that in-sample fitted series fit better than out-of-sample fitted series do.

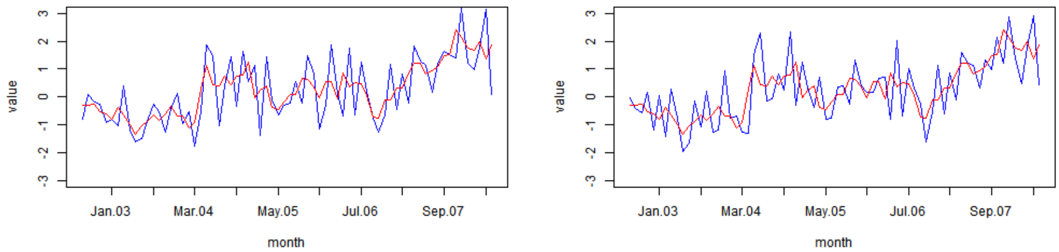


Figure 6.4: WCL Fitted transformed series and real transformed series of Germany (out-of-Sample). Left is normal prediction and right is missing value prediction

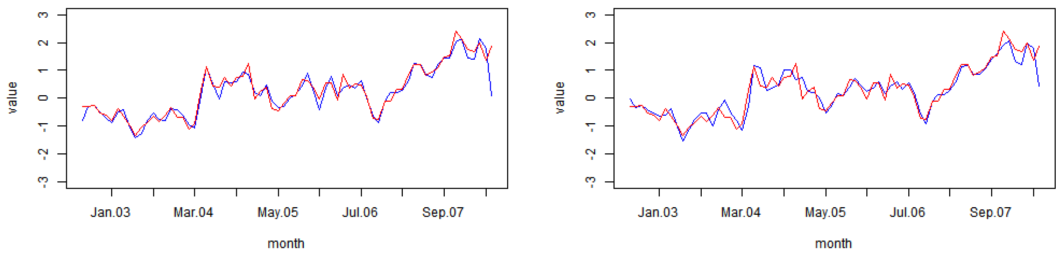


Figure 6.5: Smoothed WCL Fitted transformed series and real transformed series of Germany (out-of-Sample). Left is normal prediction and right is missing value prediction

[3]For example, the fitted transformed inflation rate at Sep.2002 is calculated with the parameter estimated with data from Jan.1998 to Aug.2002.

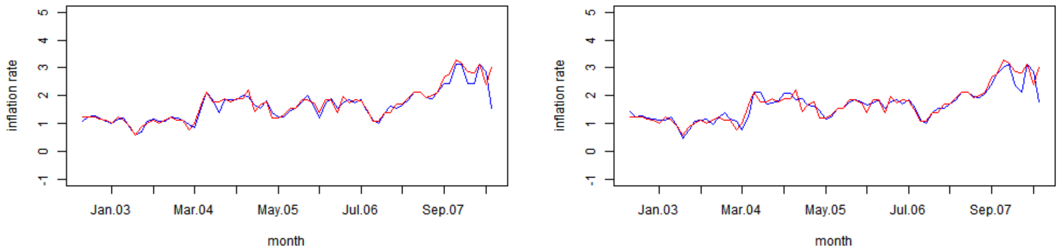


Figure 6.6: Smoothed WCL fitted inflation rate and real inflation rate of Germany (out-of-Sample). Left is normal prediction and right is missing value prediction

6.2 Mean Square Error

Table 6.1 gives MSEs of the above illustrated series.

	normal prediction		missing value prediction	
	in-sample	out-of-sample	in-sample	out-of-sample
unsmoothed transformed	0.78343	0.60415	0.83258	0.70258
smoothed transformed [4]	0.02140	0.04050	0.02310	0.07280
smoothed untransformed	0.00996	0.02151	0.01048	0.03705

Table 6.1: MSE of WCL fitted inflation rates in Germany

The first row of Table 6.1 corresponds to Figure 6.1 and Figure 6.4, illustrating that the vibration of fitted transformed series around real transformed series results in a large MSE value. The second row corresponds to Figure 6.2 and Figure 6.5, and its small MSE values indicates a better fitted transformed series after smoothing. The third row corresponds to Figure 6.3 and Figure 6.6, indicating that MSE for untransformed inflation rates is smaller than that for transformed data. This smaller MSE is due to the facts that MSE is scale-dependent (Hyndman et al. (2006)) and that real inflation rates have a small range than $N(0,1)$ distribution. To our surprise, missing value predictions have normally larger MSE than those of normal predictions. This could also be attributed to exogenous shocks, and Chapter 7 will give a detailed explanation. MSEs of other countries are given in Table 8.4, 8.5, 8.6 in Appendix D. From these tables we can see that the MSEs' patterns of other countries are similar to that of Germany.

[4] The last value is excluded from the MSE calculation for smoothed series, for it is not averaged. This applies to all MSEs for smoothed series from this place onwards.

6.3 Comparison with Other Models

6.3.1 Comparison with WLS

First I compare the model fit with WLS estimates. Although WLS estimates serve in this paper as initial values for further WCL estimation, WLS itself is also probably the most popular method in spatial data analysis (Bai et al. (2012)). We would like to see whether WCL really has improved the model performance of its initial value. Figure 6.2 gives MSEs of WLS estimates, and Table 6.3 gives a more clean pattern of MSEs *difference* by subtracting Figure 6.1 from Figure 6.2.

	normal prediction		missing value prediction	
	in-sample	out-of-sample	in-sample	out-of-sample
unsmoothed transformed	0.86935	0.86938	0.80145	0.85651
smoothed transformed	0.02378	0.06645	0.02134	0.03398
smoothed untransformed	0.01148	0.03115	0.01063	0.01322

Table 6.2: MSE of WLS fitted inflation rates in Germany

	normal prediction		missing value prediction	
	in-sample	out-of-sample	in-sample	out-of-sample
unsmoothed transformed	0.01801	0.25236	0.03677	0.16680
smoothed transformed	-0.00005	-0.00653	0.00067	-0.00636
smoothed untransformed	0.00067	-0.00829	0.00100	-0.00590

Table 6.3: MSE difference between WLS and WCL fitted inflation rates in Germany

We can see that WCL has considerably reduced the MSE in the unsmoothed series (the first row of Figure 6.3). After smoothing, MSE differences become very small, for smoothing counteracts noises, such as exogenous shocks or those price effects that are not explained by the model. A fair comparison should be established according to the first row in Figure 6.3, since it is directly associated with untreated model outcome. This improvement of WCL on WLS is in accordance with the finding of Lele and Taper (2002).

6.3.2 Comparison with Time Series Model on Inflation

Although many complicated time series model on Inflation are developed, in this section I take AR model to conduct the comparison. The reason for this choice is to establish a fair comparison, in which the alternative method should be able to work on the same dataset. Recent complicated time series model on inflation, for example the one proposed by Dossche and Everaert (2005), takes other factors such as central bank inflation target, expectation, intrinsic and extrinsic persistence into account and therefore needs many other datasets to be involved. AR model, however, only requires the same dataset that I use in this paper. Moreover, it has a linear prediction equation similar to formula (3.10). AR process is:

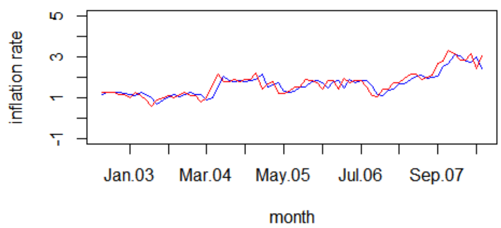
$$x_t = \mu + \sum_{j=1}^{p} \alpha_j x_{t-j} + \epsilon_t$$

I choose order $p = 3$, which is the same as the numbers of historical terms I used to fit WCL estimates in Chapter 6.1.[5] Table 6.4 shows MSEs of AR fitted inflation rates of Germany and its smoothed version:

	in-sample	out-of-sample
unsmoothed	0.07688	0.08217
smoothed	0.04384	0.04422

Table 6.4: MSE of AR fitted inflation rates in Germany

Comparing Table 6.4 to Table 6.1, we find that unsmoothed AR fitted series performed better than unsmoothed WCL series (we can see that even if the two series are not of the same scale). After smoothing, however, MSEs of AR model are not as dramatically reduced as those of WCL model. Figure 6.7 and 6.8 help to explain this difference.



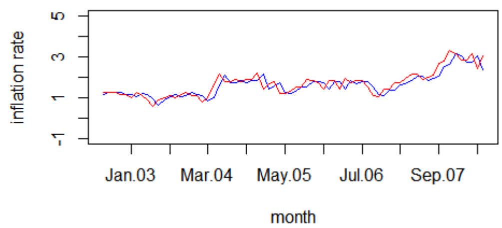


Figure 6.7: AR fitted inflation rate and real inflation rate of Germany . Left is in-sample and right is out-of-sample

[5]AIC test (Akaike (1973)) shows that the optimal orders for eight countries range from 1 to 5. In practice I test all these order and find that MSEs changes are very small in response to the changes of order within the range from 1 to 5. And these small changes in MSE will not change the comparison results that follow.

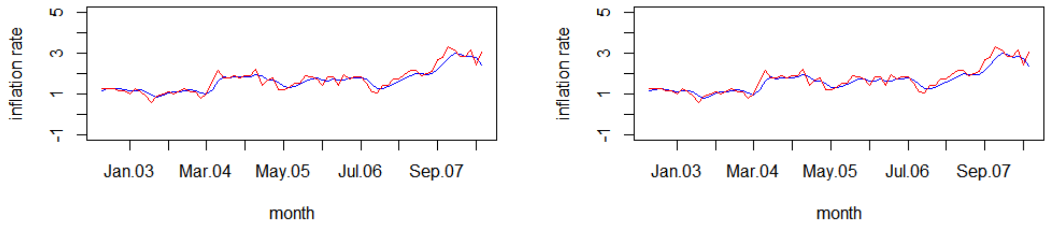


Figure 6.8: Smoothed AR fitted inflation rate and real inflation rate of Germany. Left is in-sample and right is out-of-sample

From Figure 6.7 we can see that AR fitted series is of a fashion that "the best prediction for tomorrow's price is simply today's price"(Campbell et al. (1997)). This pattern is not surprising, because high correlation (low γ) near the origin, which is illustrated in Figure 5.1, implicates that AR regression will give a large weight to the last period. Since AR prediction model works differently than Best Linear Prediction model (formula (3.10)), smoothing process pulls AR fitted series to its center and therefore increases the squared error at some places. Finally, a fair comparison between AR and WCL can be established by comparing Table 6.4 with the last row of Table 6.1, because this comparison is conducted on the same scale[6]. It is clear that neither unsmoothed nor smoothed AR fitted inflation rates will provide a MSE value smaller than those provided by smoothed WCL fitted inflation rates. Table 8.7 in Appendix D gives MSEs of AR fitted inflation rates of all the eight countries. Comparing Table 8.7 to Table 8.6, we can conclude that WCL fitted series outperform AR fitted series.

[6]Note that the first two rows in Table 6.1 are reported in value in $N(0,1)$, and the last row is reported in inflation rate

7 Discussion and Conclusion

From the results we can see that although the smoothed predicted (fitted) series indicates a nice fit, there is still a moderate degree of noise in the non-smoothed series. Since the spatiotemporal model applied in this paper is equipped with a stationary covariance structure, the existence of noise in fitted data probably indicates the non-stationary features in the original data. Two candidates for non-stationarity are unit-root and changing level (trend).

7.1 Unit-root or Changing Level and Trend

The existence of unit root will greatly influence the way people forecast future values of a process. Historically, there is a long dispute about the nature of inflation and the focus is whether inflation is I(1) process. Johansen (1992) finds that log prices have two unit roots and inflation therefore has one; and Barsky (1987) holds the opinion that inflation is non-stationary after 1960, but stationary before it. On the other hand, Rose (1988) find US inflation is stationary; Levin and Lin (1992) find evidence of stationarity by using panel data unit test.

More recently, literature is increasingly in favor of the opinion that inflation is stationary process with changing level and trend. Culver and Pepell (1997) find strong evidence that inflation rates in 13 OECD countries are stationary. Basher and Westerlund (2006) improved Culver and Pepell's result further by applying panel data unit root test that permit cross-sectional dependency and structural breaks. Im et al. (2010) apply a panel unit root test, which allows the existence heterogeneous breaks in both level and trend, to 22 OECD countries and find strong evidence that world-wide inflation are stationary. The countries used for analysis in this paper are included in both above mentioned works.

7.2 An Explanation for Noise

Assuming that the non-stationarity of inflation rates is caused by changing level and trend, I present an explanation for the cause of observed noises of results and conclude this paper. For transformed data, both $\mathrm{E}(Y)$ and $\mathrm{E}(X)$ are 0. So the formula (3.10) is simplified into:

$$\widehat{\mathrm{E}}(Y|X) = \mathrm{cov}(Y, X)\,\mathrm{cov}(X)^{-1}X \tag{7.1}$$

By observing the matrix $\mathrm{cov}(Y, X)\,\mathrm{cov}(X)^{-1}$ calculated with WCL estimates, I find two properties (the calculated matrix is given in Table 8.1, 8.2 and 8.3 in Appendix C). Firstly, the predicted (fitted) data of each country Y_i is mainly determined by its own historical value, and the impacts of other countries are small. Secondly, the signs of neighboring parameters of a country's own historical values in prediction formula (7.1) are opposite.[1]

The exogenous shocks will produce a vibrating pattern due to the opposite signs of neighbouring parameter. To see this point, say $\widehat{Y}_i \equiv \widehat{X}_{c_i,t+1}$ is the predicted (fitted) inflation rate of country i at time $t+1$, we have:

$$\widehat{X}_{c_i,t+1} = \beta_{c_i}^{\top} X = \beta_{c_{i,own}}^{\top} X_{own} + \beta_{c_{i,other}}^{\top} X_{other}$$

β_{c_i} is the i-th row of matrix $\mathrm{cov}(Y, X)\,\mathrm{cov}(X)^{-1}$, which are parameters deciding the value of $\widehat{X}_{c_i,t+1}$. $\beta_{c_{i,own}}$ is a vector of those parameters in front of country i's own historical values, X_{own}, and $\beta_{c_{i,other}}$ is a vector of parameters of other countries's values, X_{other} . According to the two observed properties, we have:

$$\widehat{X}_{c_i,t+1} \approx \beta_{c_{i,own}}^{\top} X_{own} = \beta_{c_i,t} x_{c_i,t} + \beta_{c_i,t-1} x_{c_i,t-1} + \beta_{c_i,t-2} x_{c_i,t-2} \tag{7.2}$$

Note that I use three terms to fit WCL estimates, so there are three items on the right side of formula (7.2). $\beta_{c_i,t}$ has the same sign as $\beta_{c_i,t-2}$, and their sign is opposite to that of $\beta_{c_i,t-1}$. Note that $\beta_{c_i,t}$ is an element of matrix $\mathrm{cov}(Y, X)\,\mathrm{cov}(X)^{-1}$ that is calculated from covariance function (3.4). Thus $\beta_{c_i,t}$ is not a function of time t but a function of time lag u. Thus we can rewrite $\beta_{c_i,t} = \beta_{c_i,u_1}$, $\beta_{c_i,t-1} = \beta_{c_i,u_2}$ and $\beta_{c_i,t-2} = \beta_{c_i,u_3}$.

In case of absence of exogenous shocks, formula (7.1) works well and therefore the fitted

[1] In this paper I do not investigate the origin of these two properties, but simply derive these properties from calculated results. These properties are probably associated with the property of covariance function (3.4).

values $\widehat{X}_{c_i,t+1}$ should be close to real series $X_{c_i,t+1}$. Now assume that we have a shock α in time $t-2$. Then we have:

$$\begin{aligned}
\tilde{X}_{c_i,t-1} &= \widehat{X}_{c_i,t-1} + \beta_{c_i,u_1}\alpha = \beta_{c_i,u_1}(x_{c_i,t-2}+\alpha) + \beta_{c_i,u_2}x_{c_i,t-3} + \beta_{c_i,u_3}x_{c_i,t-4} \\
\tilde{X}_{c_i,t} &= \widehat{X}_{c_i,t} + \beta_{c_i,u_2}\alpha = \beta_{c_i,u_1}x_{c_i,t-1} + \beta_{c_i,u_2}(x_{c_i,t-2}+\alpha) + \beta_{c_i,u_3}x_{c_i,t-3} \\
\tilde{X}_{c_i,t+1} &= \widehat{X}_{c_i,t+1} + \beta_{c_i,u_3}\alpha = \beta_{c_i,u_1}x_{c_i,t} + \beta_{c_i,u_2}x_{c_i,t-1} + \beta_{c_i,u_3}(x_{c_i,t-2}+\alpha)
\end{aligned}$$

Since β_{c_i,u_2} has a different sign to those of β_{c_i,u_1} and β_{c_i,u_3}, the series $\tilde{X}_{c_i}$ vibrates around the series $\widehat{X}_{c_i}$.

This could also help to explain the reason that the MSE of missing value prediction is larger than that of normal prediction. In case of using three months as X in formula (7.1), X in missing value prediction has $3 \times 8 + 7 = 31$ items, while X in normal prediction has only $3 \times 8 = 24$ items. Thus more shocks are accumulated in the missing value prediction, leading to larger MSE.

7.3 Conclusion

This paper applies spatiotemporal covariance matrix to model the inflation rates of eight Europe countries. Multivariate technologies such as Gaussian copula and WCL are utilized to enhance the flexibility of the model and to find a balance between estimation efficiency and computational efficiency.

The model is built on two assumptions. First, the distribution of inflation rates in a country remains unchanged over time. Second, the parameters of spatiotemporal covariance matrix are constant across time and space, i.e. stationary. From the estimation results we can see that while the model can explain a large part of spatiotemporal variance of inflation rates of 8 Europe countries, the fitted series demonstrate vibrating pattern around the real series. With the theory that inflation rates are stationary with changing level and trend, this vibrating pattern can be attributed to exogenous shocks.

In sum, the model provides a proper flexible structure to explain a large part of spatiotemporal interaction of inflation rates. While the model itself is unable to capture the exogenous shocks that change levels and trends of inflation rates, a moving average smoother helps to counteract these exogenous shocks and to obtain a smoothed WCL fitted series, which outperforms AR fitted series.

8 Appendix

Appendix A: 3.1.3 Spatiotemporal Covariance Function of Cressie and Huang (1999)

Spatiotemporal covariance function (3.4) is reproduced here:

$$C(h,u;\theta) = \begin{cases} \frac{2\sigma^2\beta}{(a^2u^2+1)^\nu(a^2u^2+\beta)\gamma(\nu)}(\frac{b}{2}(\frac{a^2u^2+1}{a^2u^2+\beta})^{\frac{1}{2}}h)^\nu K_\nu(b(\frac{a^2u^2+1}{a^2u^2+\beta})^{\frac{1}{2}}h) & if \quad h>0 \\ \frac{\sigma^2\beta}{(a^2u^2+1)^\nu(a^2u^2+\beta)} & if \quad h=0 \end{cases}$$

Firstly, this is a non-separable spatiotemporal covariance function. The term *separable* means:

$$\exists\, C_1,\, C_2 :\; C(u,h) = C_1(u)C_2(h)$$

In the case of $h=0$, $C(u,0;\theta)$ is a time series auto correlation function. And $C(0,0;\theta) = \sigma^2$. σ^2 represent the variance of every single spatiotemporal point. Apparently, $C(u,0;\theta)$ is decreasing with u, indicating that covariance is fading with the time lag.

In the case of $h>0$, the covariance function can be divided into two parts $\frac{\sigma^2\beta}{(a^2u^2+1)^\nu(a^2u^2+\beta)}$ and $\frac{2}{\gamma(\nu)}(\frac{b}{2}(\frac{a^2u^2+1}{a^2u^2+\beta})^{\frac{1}{2}}h)^\nu K_\nu(b(\frac{a^2u^2+1}{a^2u^2+\beta})^{\frac{1}{2}}h)$. The second part is actually Matérn covariance function $\frac{1}{\gamma(\nu)2^{\nu-1}}(x)^\nu K_\nu(x)$ with $x = b(\frac{a^2u^2+1}{a^2u^2+\beta})^{\frac{1}{2}}h$. Matérn covariance function is popular for spatial analysis. It's value decreases with space lag h.

a is the scaling parameter of time and b is the scaling parameter of space. The $\beta>0$ is a space-time parameter. In case of $\beta=1$, the covariance function (3.4) degenerates into a separable covariance function. With $\beta\neq 1$, $\frac{a^2u^2+1}{a^2u^2+\beta}$ reflect a time-effect adjusted space dependence.

Appendix B: 3.1.5 Weighted Least Square Estimate

The motivation of WLS is to minimize the weighted square sum of distance of empirical variogram $\widehat{\gamma}(h,u)$ and parametric variogram $\gamma(h,u,\theta)$. The Generalized least squares (GLS) objective function is:

$$(\widehat{\gamma}(h,u)-\gamma(h,u,\theta))^\top R(\theta)^{-1}(\widehat{\gamma}(h,u)-\gamma(h,u,\theta)) \tag{8.1}$$

As pointed out by Schabenberger and Gotway (2005), it is often difficult to obtain $R(\theta)$. The minimizing process of formula (8.1) is also computational demanding, because $R(\theta)$ is a function of θ and therefore iterative process must include the updating of $R(\theta)$. Cressie (1993) suggests an inferior but computational efficient substitute of formula (8.1) by replacing $R(\theta)$ by diagonal matrix $W(\theta)$. The weighted least square objective function in spatiotemporal case is:

$$\begin{aligned}&(\widehat{\gamma}(h,u)-\gamma(h,u,\theta))^\top W(\theta)^{-1}(\widehat{\gamma}(h,u)-\gamma(h,u,\theta))\\ &=\sum_{m=1}^{k}\sum_{n=1}^{j}\frac{\|N(h_m,u_n)\|}{2\gamma(h_m,u_n,\theta)^2}\{\widehat{\gamma}(h_m,u_n)-\gamma(h_m,u_n,\theta)\}^2\end{aligned} \tag{8.2}$$

WLS estimate is obtained by minimize formula (8.2) w.r.t θ.

Appendix C: 7.2 An Explanation for Noise

The matrix $\operatorname{cov}(Y,X)\operatorname{cov}(X)^{-1}$ calculated with WCL estimates is given in Table 8.1, 8.2 and 8.3. Historical data of three months are included, so it is a 8×24 matrix. The matrix is long, so I split it into three tables. Each column name represents an element in X that multiplies this column according to formula (7.1). The fitted $\widehat{\mathrm{E}}(Y|X)$ is at time $t+1$. Chapter 7.2 discusses the properties revealed by this matrix.

	$X_{c_1,t}$	$X_{c_2,t}$	$X_{c_3,t}$	$X_{c_4,t}$	$X_{c_5,t}$	$X_{c_6,t}$	$X_{c_7,t}$	$X_{c_8,t}$
c_1	2.4004	0.0266	0.0235	0.0322	0.0312	0.0112	0.0266	0.0209
c_2	0.0147	2.4139	0.0180	0.0316	0.0334	0.0121	0.0206	0.0203
c_3	0.0105	0.0097	2.4345	0.0353	0.0116	0.0042	0.0361	0.0106
c_4	0.0103	0.0204	0.0341	2.4362	0.0130	0.0045	0.0192	0.0091
c_5	0.0132	0.0260	0.0155	0.0222	2.4239	0.0168	0.0242	0.0286
c_6	0.0009	0.0049	0.0015	0.0020	0.0150	2.4504	0.0116	0.0387
c_7	0.0100	0.0036	0.0298	0.0119	0.0145	0.0082	2.4514	0.0225
c_8	0.0043	0.0064	0.0052	0.0048	0.0243	0.0455	0.0241	2.4399

Table 8.1: The matrix $\mathrm{cov}(Y, X)\,\mathrm{cov}(X)^{-1}$ calculated with WCL estimates

	$X_{c_1,t-1}$	$X_{c_2,t-1}$	$X_{c_3,t-1}$	$X_{c_4,t-1}$	$X_{c_5,t-1}$	$X_{c_6,t-1}$	$X_{c_7,t-1}$	$X_{c_8,t-1}$
c_1	-2.1505	-0.0342	-0.0304	-0.0411	-0.0409	-0.0128	-0.0338	-0.0266
c_2	-0.0206	-2.1659	-0.0230	-0.0401	-0.0437	-0.0138	-0.0260	-0.0256
c_3	-0.0148	-0.0119	-2.1935	-0.0451	-0.0148	-0.0041	-0.0459	-0.0129
c_4	-0.0146	-0.0257	-0.0440	-2.1949	-0.0165	-0.0044	-0.0239	-0.0110
c_5	-0.0185	-0.0332	-0.0198	-0.0278	-2.1800	-0.0198	-0.0306	-0.0367
c_6	-0.0014	-0.0054	-0.0014	-0.0018	-0.0187	-2.2120	-0.0134	-0.0499
c_7	-0.0137	-0.0041	-0.0381	-0.0145	-0.0187	-0.0090	-2.2146	-0.0281
c_8	-0.0060	-0.0074	-0.0062	-0.0053	-0.0312	-0.0572	-0.0297	-2.2007

Table 8.2: The matrix $\mathrm{cov}(Y, X)\,\mathrm{cov}(X)^{-1}$ calculated with WCL estimate (continued)

	$X_{c_1,t-2}$	$X_{c_2,t-2}$	$X_{c_3,t-2}$	$X_{c_4,t-2}$	$X_{c_5,t-2}$	$X_{c_6,t-2}$	$X_{c_7,t-2}$	$X_{c_8,t-2}$
c_1	0.7298	0.0096	0.0085	0.0113	0.0118	0.0025	0.0091	0.0071
c_2	0.0068	0.7330	0.0063	0.0109	0.0126	0.0028	0.0067	0.0068
c_3	0.0048	0.0029	0.7414	0.0125	0.0039	0.0004	0.0124	0.0030
c_4	0.0049	0.0069	0.0124	0.7414	0.0043	0.0004	0.0060	0.0025
c_5	0.0061	0.0092	0.0053	0.0072	0.7376	0.0044	0.0081	0.0101
c_6	0.0005	0.0010	0.0000	0.0000	0.0048	0.7456	0.0028	0.0140
c_7	0.0043	0.0007	0.0105	0.0035	0.0051	0.0015	0.7470	0.0073
c_8	0.0019	0.0015	0.0013	0.0009	0.0087	0.0153	0.0074	0.7435

Table 8.3: The matrix $\mathrm{cov}(Y, X)\,\mathrm{cov}(X)^{-1}$ calculated with WCL estimate (continued)

Appendix D: 5.4 Model Fit of other countries

In Chapter 5 I only give the fitted series of Germany. Here I show the fitted series of other countries. For each countries, unsmoothed transformed inflation rates and smoothed untransformed inflation rates are given.

In-Sample Fit

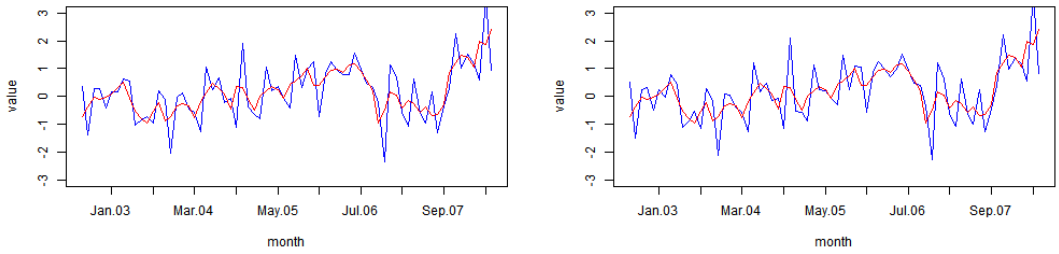


Figure 8.1: WCL Fitted transformed series and real transformed series of Luxembourg (in-sample). Left is normal prediction and right is missing value prediction

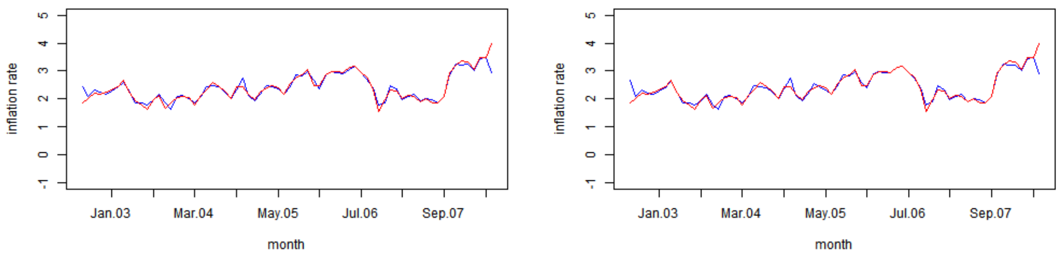


Figure 8.2: Smoothed WCL fitted inflation rate and real inflation rate of Luxembourg (in-sample). Left is normal prediction and right is missing value prediction

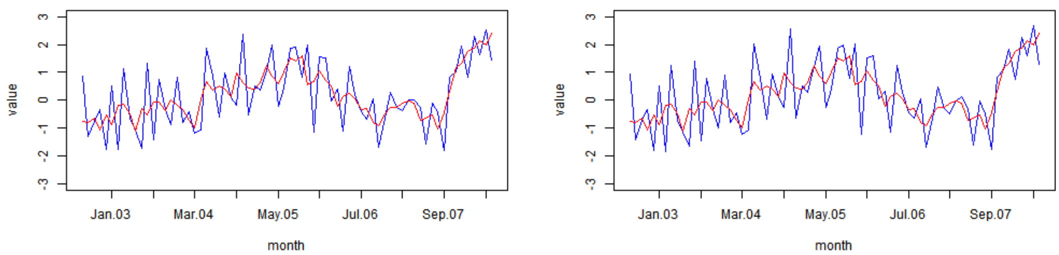


Figure 8.3: WCL Fitted transformed series and real transformed series Belgium (in-sample). Left is normal prediction and right is missing value prediction

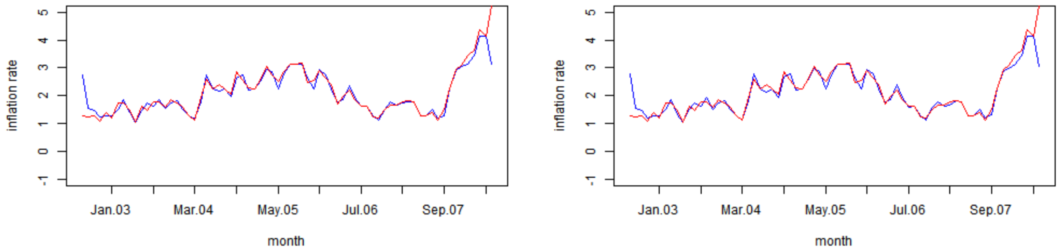


Figure 8.4: Smoothed WCL fitted inflation rate and real inflation rate of Belgium (in-sample). Left is normal prediction and right is missing value prediction

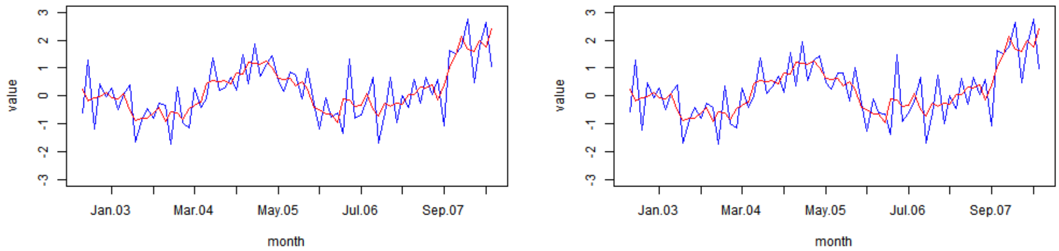


Figure 8.5: WCL Fitted transformed series and real transformed series of Austria (in-sample). Left is normal prediction and right is missing value prediction

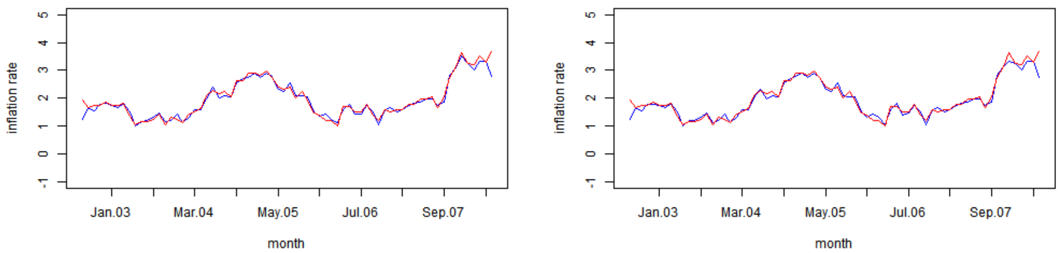


Figure 8.6: Smoothed WCL fitted inflation rate and real inflation rate of Austria (in-sample). Left is normal prediction and right is missing value prediction

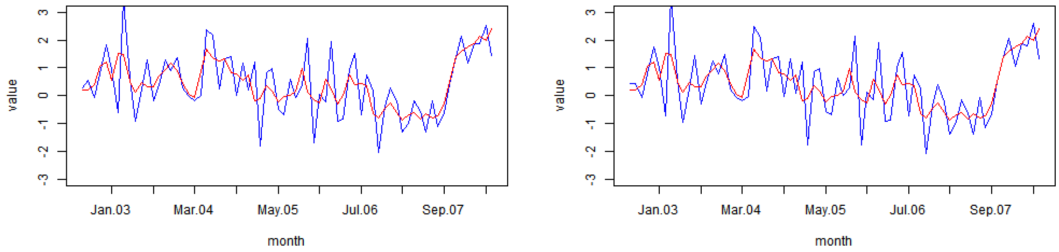


Figure 8.7: WCL Fitted transformed series and real transformed series of France (in-sample). Left is normal prediction and right is missing value prediction

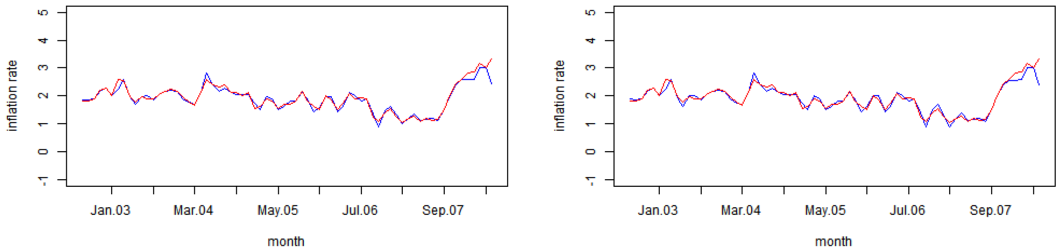


Figure 8.8: Smoothed WCL fitted inflation rate and real inflation rate of France (in-sample). Left is normal prediction and right is missing value prediction

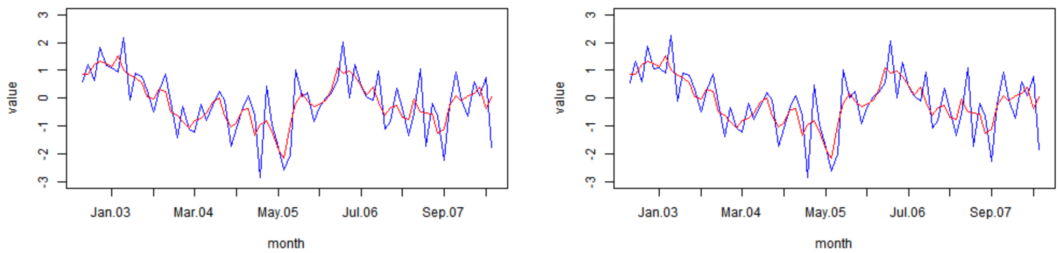


Figure 8.9: WCL Fitted transformed series and real transformed series of Portugal (in-sample). Left is normal prediction and right is missing value prediction

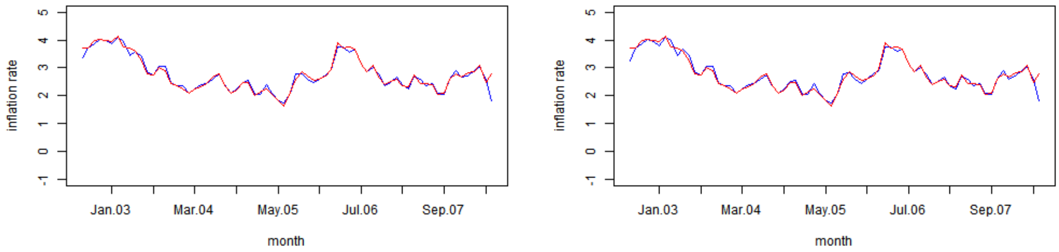


Figure 8.10: Smoothed WCL fitted inflation rate and real inflation rate of Portugal (in-sample). Left is normal prediction and right is missing value prediction

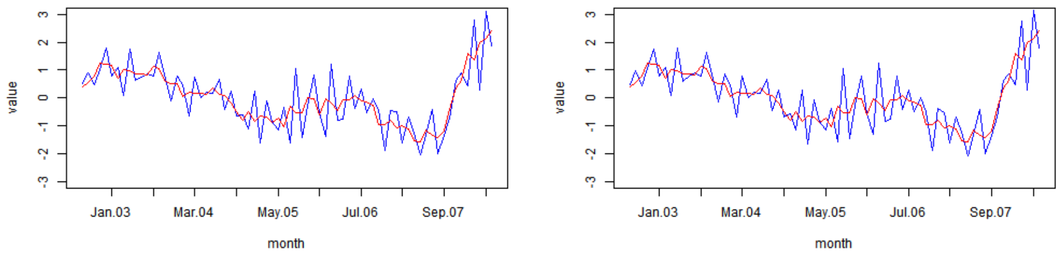


Figure 8.11: WCL Fitted transformed series and real transformed series of Italy (in-sample). Left is normal prediction and right is missing value prediction

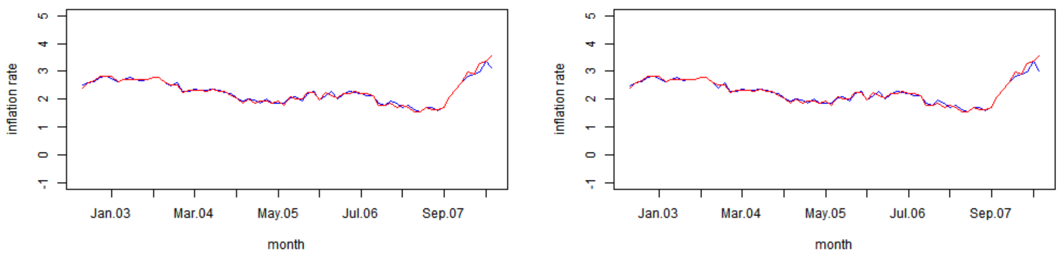


Figure 8.12: Smoothed WCL fitted inflation rate and real inflation rate of Italy (in-sample). Left is normal prediction and right is missing value prediction

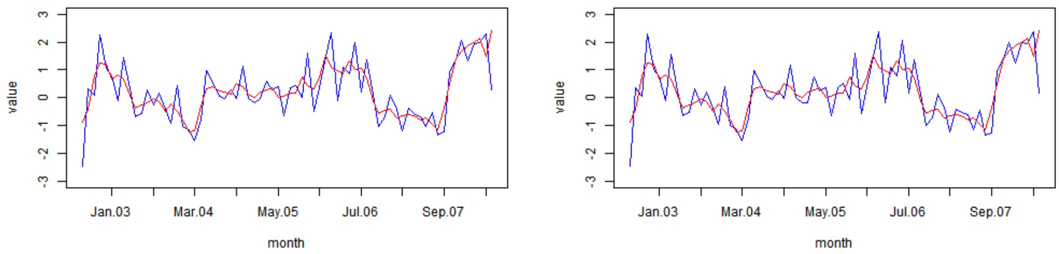


Figure 8.13: WCL Fitted transformed series and real transformed series of Spain (in-sample). Left is normal prediction and right is missing value prediction

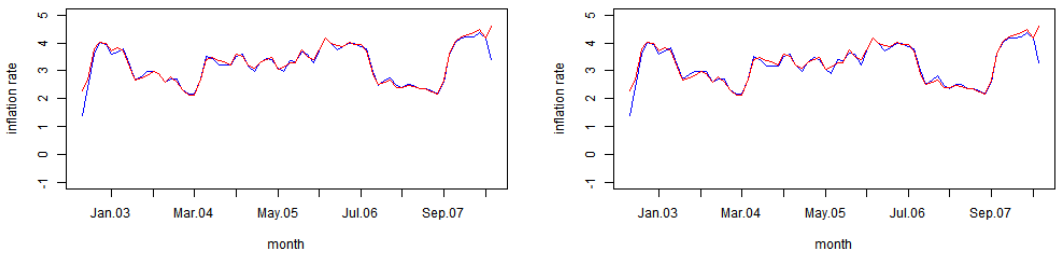


Figure 8.14: Smoothed WCL fitted inflation rate and real inflation rate of Spain (in-sample). Left is normal prediction and right is missing value prediction

Out-of-Sample Performance

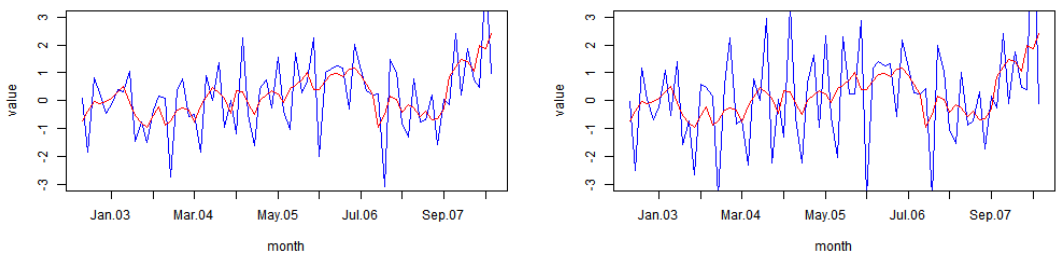


Figure 8.15: WCL Fitted transformed series and real transformed series Luxembourg (out-of-sample). Left is normal prediction and right is missing value prediction

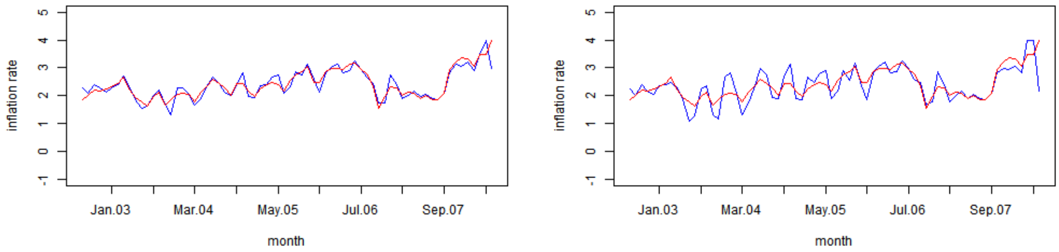


Figure 8.16: Smoothed WCL fitted inflation rate and real inflation rate of Luxembourg (out-of-sample). Left is normal prediction and right is missing value prediction

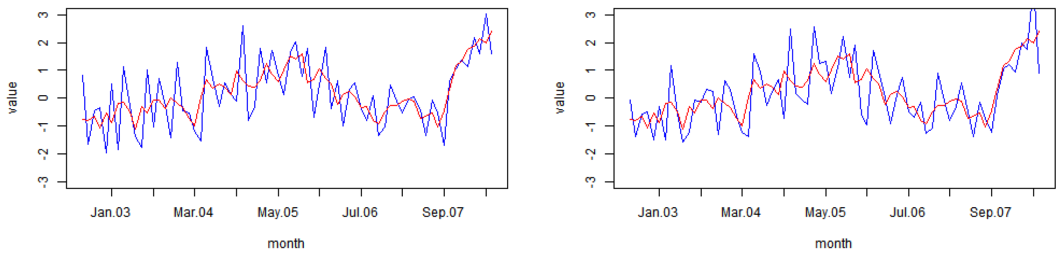


Figure 8.17: WCL Fitted transformed series and real transformed series Belgium (out-of-sample). Left is normal prediction and right is missing value prediction

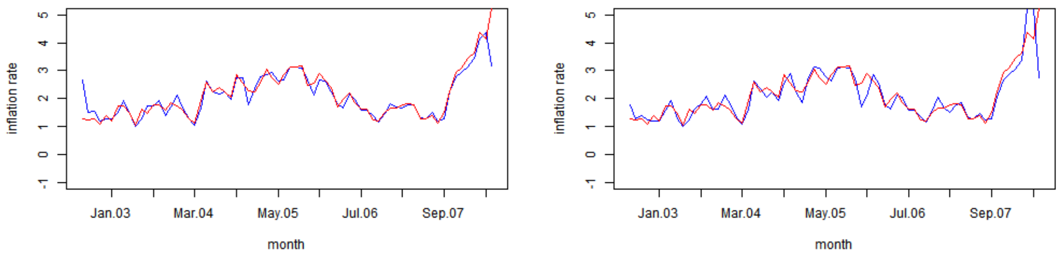


Figure 8.18: Smoothed WCL fitted inflation rate and real inflation rate of Belgium (out-of-sample). Left is normal prediction and right is missing value prediction

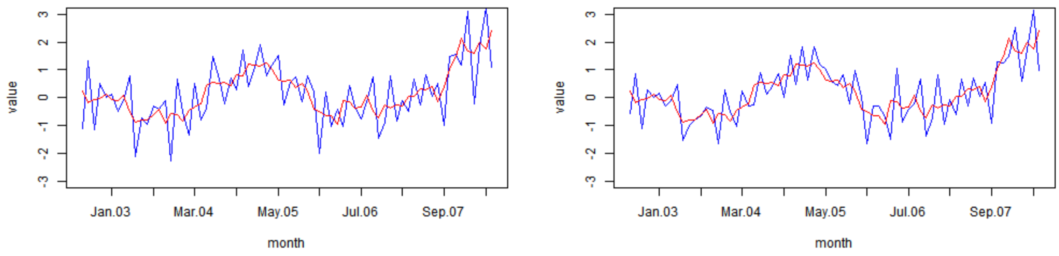


Figure 8.19: WCL Fitted transformed series and real transformed series of Austria (out-of-sample). Left is normal prediction and right is missing value prediction

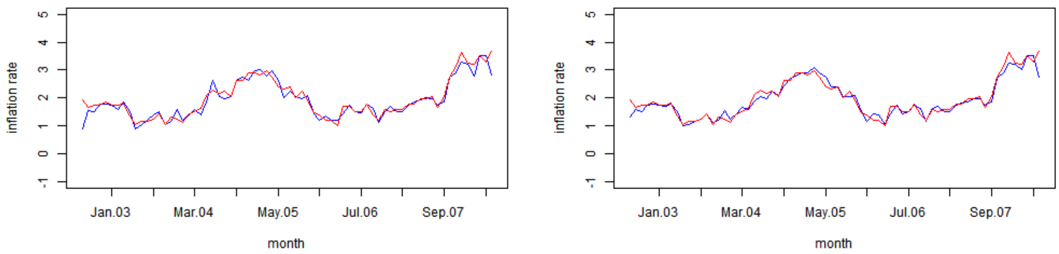


Figure 8.20: Smoothed WCL fitted inflation rate and real inflation rate of Austria (out-of-sample). Left is normal prediction and right is missing value prediction

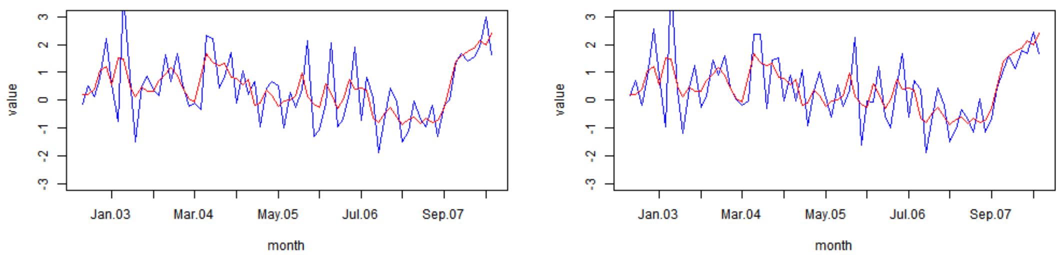


Figure 8.21: WCL Fitted transformed series and real transformed series of France (out-of-sample). Left is normal prediction and right is missing value prediction

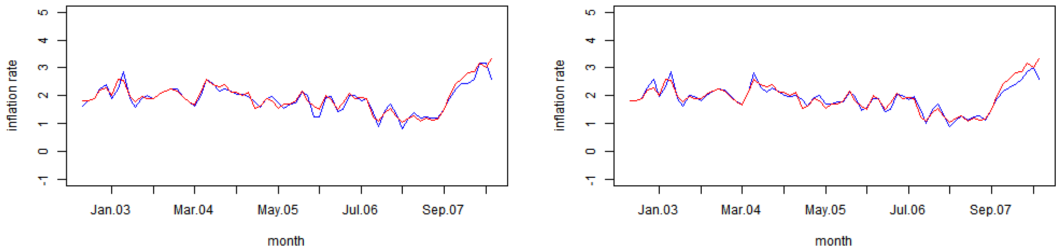


Figure 8.22: Smoothed WCL fitted inflation rate and real inflation rate of France (out-of-sample). Left is normal prediction and right is missing value prediction

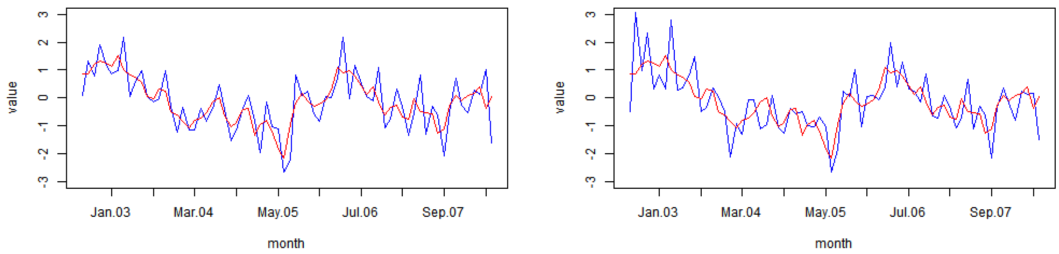


Figure 8.23: WCL Fitted transformed series and real transformed series of Portugal (out-of-sample). Left is normal prediction and right is missing value prediction

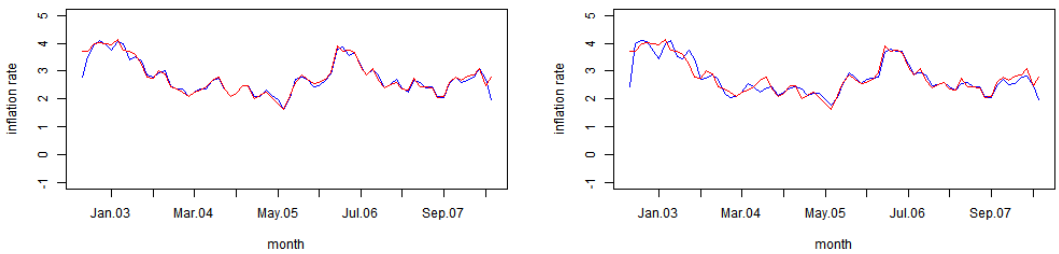


Figure 8.24: Smoothed WCL fitted inflation rate and real inflation rate of Portugal (out-of-sample). Left is normal prediction and right is missing value prediction

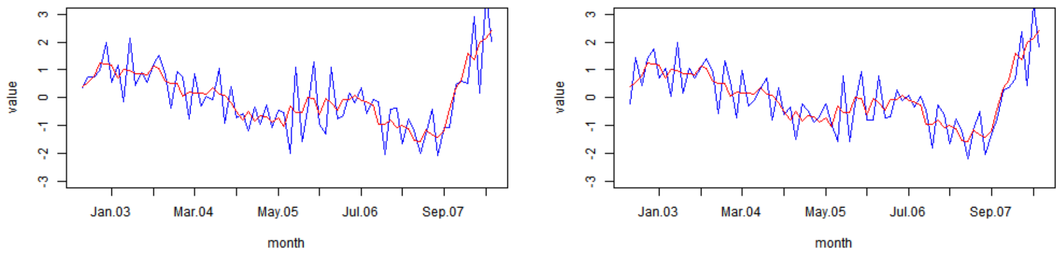


Figure 8.25: WCL Fitted transformed series and real transformed series of Italy (out-of-sample). Left is normal prediction and right is missing value prediction

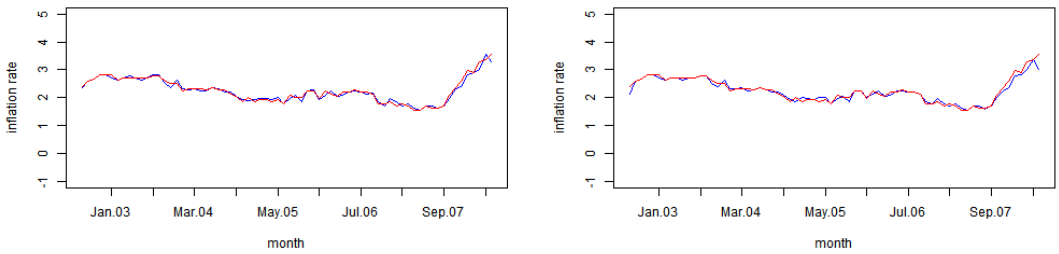


Figure 8.26: Smoothed WCL fitted inflation rate and real inflation rate of Italy (out-of-sample). Left is normal prediction and right is missing value prediction

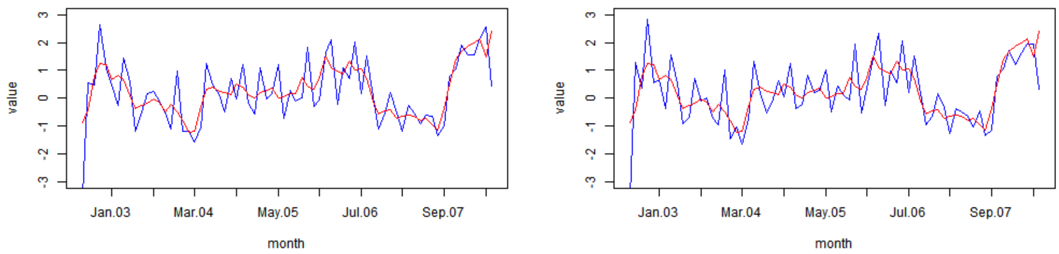


Figure 8.27: WCL Fitted transformed series and real transformed series of Spain (out-of-sample). Left is normal prediction and right is missing value prediction

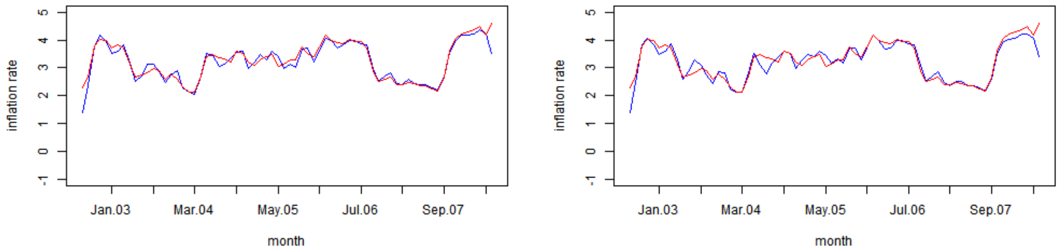


Figure 8.28: Smoothed WCL fitted inflation rate and real inflation rate of Spain (out-of-sample). Left is normal prediction and right is missing value prediction

Mean Square Error

MSEs of WCL fitted inflation rates of all eight countries are given in Table 8.4, 8.5 and 8.6. And MSEs of AR fitted inflation rates are given in Table 8.7:

	normal prediction		missing value prediction	
	in-sample	out-of-sample	in-sample	out-of-sample
Luxembourg	0.49103	1.08502	0.51972	2.45009
Belgium	0.74211	0.77789	0.81023	0.68913
Austria	0.45269	0.66073	0.48174	0.44167
Germany	0.78343	0.60415	0.83258	0.70258
France	0.61433	0.67717	0.67353	0.71424
Portugal	0.44152	0.37152	0.45358	0.52570
Italy	0.45856	0.56280	0.46493	0.42693
Spain	0.31920	0.50702	0.34682	0.55954

Table 8.4: MSE of WCL fitted transformed inflation rates

	normal prediction		missing value prediction	
	in-sample	out-of-sample	in-sample	out-of-sample
Luxembourg	0.02907	0.05738	0.03573	0.18674
Belgium	0.05944	0.07863	0.06687	0.09183
Austria	0.02285	0.05935	0.02266	0.03664
Germany	0.02140	0.04050	0.02310	0.07280
France	0.01750	0.03988	0.02084	0.03839
Portugal	0.01348	0.02661	0.01442	0.09650
Italy	0.01337	0.02078	0.01330	0.02887
Spain	0.04793	0.12350	0.05037	0.13598

Table 8.5: MSE of WCL fitted smoothed transformed inflation rates

	normal prediction		missing value prediction	
	in-sample	out-of-sample	in-sample	out-of-sample
Luxembourg	0.01361	0.03250	0.01766	0.09779
Belgium	0.04886	0.06034	0.05311	0.08785
Austria	0.01756	0.04344	0.01858	0.02587
Germany	0.00996	0.02151	0.01048	0.03705
France	0.00955	0.02111	0.01097	0.02067
Portugal	0.00880	0.02177	0.01098	0.06022
Italy	0.00447	0.00706	0.00463	0.00829
Spain	0.01779	0.03137	0.01996	0.03990

Table 8.6: MSE of WCL fitted smoothed untransformed inflation rates (in percentage)

	unsmoothed		smoothed	
	in-sample	out-of-sample	in-sample	out-of-sample
Luxembourg	0.07716	0.08326	0.05116	0.05785
Belgium	0.11943	0.12641	0.05631	0.06587
Austria	0.07232	0.07645	0.04359	0.04916
Germany	0.07688	0.08217	0.04384	0.04423
France	0.06267	0.06790	0.03559	0.04164
Portugal	0.06959	0.07278	0.03422	0.03073
Italy	0.02084	0.02221	0.01088	0.01308
Spain	0.07394	0.07967	0.03579	0.04033

Table 8.7: MSE of AR fitted inflation rates

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Declaration of Authorship

I hereby confirm that I have authored this master thesis independently and without use of others than the indicated sources. Where I have consulted the published work of others, in any form (e.g. ideas, equations, figures, text, tables), this is always explicitly attributed.

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Mengshan Xu