Three essays on Efficiency and Incentives in Teams and Partnerships

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von
Frau M.A. Jianpei Li
geboren am 02.06.1975, Henan, V.R.China

Präsident der Humboldt-Universität zu Berlin:
Prof. Dr. Christoph Markschies

Dekan der Wirtschaftswissenschaften Fakultät:
Prof. Oliver Günther, Ph. D.

Gutachter:

1. Prof. Dr. Elmar Wolfstetter
2. Prof. Dr. Ulrich Kamecke

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Abstract

Classic economic theory claims that teams and partnerships are inflicted with free–riding incentives and are inferior to capitalist firms in eliciting proper actions from players. This contradicts the popularity of teams and partnerships as organization forms.

By accounting for several factors that may significantly affect effort provision (investment) but have been generally neglected in the literature: 1) tournament effect within teams; 2) other–regarding preferences; 3) possible dissolution of a partnership, this dissertation shows that teams and partnerships are not handicapped organization forms. Whether efficient efforts provision (investment) is achieved or not depends critically on the mechanisms employed in the allocation of team output and dissolution of a partnership.

If some noisy ranking of efforts is observable and contractible, efficient provision of efforts can always be achieved in symmetric equilibrium by using rank–order tournaments to allocate the team output. If the players are subjected to limited liability, achievement of efficiency requires sufficient accuracy of the ranking technology. If the players show other–regarding preferences, efficient provision of efforts can be achieved through a randomly punishing contract. If the partnership faces possible dissolution, efficient investment and dissolution can be achieved through an option contract. However, the most widely used and recommended dissolution rule "buy–sell provision" fails to achieve efficiency as it leads to either excessive investment combined with efficient dissolution or underinvestment combined with excessive dissolution.

Keywords:
teams and partnerships, moral hazard in teams, partnership dissolution, buy–sell provision

Zusammenfassung

Die klassische ökonomische Theorie behauptet, dass Teams und Partnerschaften (Teilhaberschaften) vom Trittbrettfahrerproblem betroffen sind und dass kapitalistische Unternehmen besser in der Lage sind, Spieler zu gewünschten Handlungen zu motivieren. Dies steht jedoch im Widerspruch zur Popularität von Teams und Partnerschaften als Organisationsform.

Diese Dissertation zeigt, dass Teams und Partnerschaften keine nachteiligen Organisationsformen sind. Sie trägt verschiedenen Faktoren Rechnung, die großen Einfluss auf die Anreize zur Anstrengung haben, aber in der Literatur weitgehend vernachlässigt wurden: 1) Turniereffekte in Teams; 2) soziale Präferenzen; 3) Auflösung einer Partnerschaft. Ob effiziente Anstrengung (Investition) erreicht werden kann, hängt entscheidend davon ab, welche Mechanismen angewandt werden um den Arbeitserfolg von Teams zu verteilen bzw. um eine Partnerschaft aufzulösen.

Wenn ein verrauschtes Signal der Rangfolge der Anstrengungen beobachtbar und kontrahierbar ist, dann kann effiziente Anstrengung als symmetrisches Gleichgewicht immer erreicht werden - durch Anwendung von Rangfolge-Turnieren zur Verteilung des Arbeitserfolges des Teams. Wenn die Spieler beschränkt haftbar sind, dann muss die Rangfolge-Technologie hinreichend genau sein um effiziente Anstrengung zu generieren. Wenn die Spieler soziale Präferenzen haben, kann effiziente Anstrengung erreicht werden durch Vereinbarung von zufälliger Bestrafung. Wenn eine Partnerschaft von möglicher Auflösung bedroht ist, dann kann effiziente Investition und Auflösung der Partnerschaft mit Hilfe eines Optionsvertrages erreicht werden. Am häufigsten verwendet und empfohlen ist die "Buy-sell-Regel". Sie ist jedoch ineffizient, da sie entweder zu Überinvestition in Verbindung mit effizienter Auflösung oder zu Unterinvestition kombiniert mit exzessiver Auflösung führt.

Schlagwörter:
Teams und Partnerschaften, Moral hazard in Teams, Auflösung von Partnerschaften, Buy-sell Regel

To my family.

Contents

Chapter 1

Introduction

Teams and partnerships are popular ways of organizing economic activities. Joinson [1999] and Strozniak [2000] reported that approximately 80% of Fortune 500 companies have half or more of their employees on teams. And 68% of small U.S. manufacturers are using teams in their production areas. In the U.S., according to the 1997 economic census report of U.S. Census Bureau, about 60% of CPA offices, 47.5% of lawyer offices, 45% of accounting and taxes services are organized as partnerships.

Team (partnership) production refers to production that several types of resources are used and the product is not a sum of separable outputs of each cooperating resource. In addition, not all resources used belong to one agent. A conspicuous feature is that marginal products of team members are not directly and separably observable. Consequently it is difficult to compensate the agents conditional on their marginal contributions, which creates a free–riding problem—-agents tend to shirk and underinvest as they have to share their marginal contributions with others but bear the marginal costs alone. Hence, the classical literature concludes that teams and partnerships are inefficient organizational forms and are inferior to capitalistic firms. (See, for example, Alchian and Demsetz [1972] and Holmström [1982]).

This traditional wisdom, however, contradicts the popularity of teams and partnerships as organization forms. In reality, team members are also reported to be more cooperative than predicted in the classic theory.

This dissertation consists of three self–contained essays that study efficiency and incentives in teams and partnerships. The first two essays make an endeavor to capture real life features of teams and partnerships in order to explore possible factors that eliminate the free–riding problems. In the first essay, it is assumed that some noisy ranking of individual efforts that contribute to the cooperative final output can be observed and documented. Given such information, an endogenous partnership formation game leads to full efficiency. In that game, one arbitrary partner initiates the formation of a partnership by proposing a sharing rule of the final output. Different from standard sharing scheme, allocation of final shares depends on the partners' places in effort ranking. The partner with highest rank receives the largest share. Under such allocation scheme, full efficiency is always achieved in symmetric equilibrium if the partners can bear unlimited liability, and can be achieved if the partners are subject to limited liability provided that the ranking technology is sufficiently accurate.

When shares are awarded in a contest, there are two effort–inducing effects, which operate in the same direction: increased output from which a partner receives a share, and increased probability of winning. The two effects reinforce each other and offset the partners' incentive to free ride.

The second essay deviates from standard assumption that agents are self–interested and assume instead that they exhibit other–regarding preferences and are inequity averse. It is shown that there exists a mechanism that elicits efficient action profiles, which punishes some randomly chosen agents by imposing a fine on them when final output falls short of the efficient level. When individuals are inequity averse, that mechanism threatens to create inequalities among the agents and reduces the attractiveness of free–riding as the agents suffer from inequalities between themselves and their fellow members.

Partnerships form and dissolve. Different from the first two essays, the third essay abstracts away the free–riding problem by assuming that investment is contractible and focuses instead on the interaction between *ex ante* investment incentive and *ex post* dissolution incentive. In that model, when agents form partnerships, they expect to benefit from their complementary skills. However, new opportunities may arise that make some partners' skills useless and hence trigger a request for dissolution. The anticipation of possible future break–up affects the joint investment into the partnership, and in turn, the choice of investment affects the dissolution decision.

The essay studies and evaluates a widely used dissolution rule: buy–sell provision. Such a mandatory dissolution rule has the advantage that the partnership is always dissolved whenever it is called for and hence can be used to avoid costly deadlocks, litigations and lengthy court battles. However, it always gives rise to inefficiencies, either in the form of excessive dissolution combined with underinvestment, or efficient dissolution combined with overinvestment. The essay also looks at different variations of the standard buy–sell provision: supplementing the rule with veto right, or leaving it open to renegotiations. Although these variations may restore efficiency in some cases, they are not remedies: adding veto right may block a dissolution when it is efficient to do so while adding the option of renegotiation creates an additional hold–up problem by giving the informed partner an opportunity to exploit the uninformed one.

Chapter 2

Efficient Tournaments in Teams

This chapter is based on Gershkov et al. [2007].

2.1 Introduction

We study teams and partnerships in which risk neutral partners jointly produce output which they share among themselves. It is generally accepted that such partnerships are inefficient if the partners' actions are not verifiable. The argument is that partners shirk because they must share their marginal benefit of effort with others but bear the cost alone. The question is, then, why are there well publicized examples of extremely profitable partnerships which seem to have very little trouble incentivating partners?

We provide an intuitive answer to this question by focusing on team compensation schemes which reward partners on the basis of the relative ranking of their efforts. We find that full efficiency can be obtained under the assumption that some noisy ranking of the partners' efforts is verifiable—which should be less costly to acquire than cardinal information on efforts. Our result thus helps explaining a phenomenon where economic theory seems to have previously been at odds with reality. In the motivating examples below, there are two main elements present on which we build our analysis: team output determines the pool from which prizes are taken and a relative performance ranking is used as the means of allocating these prizes.

Partnerships are the dominant organizational form in several fields of the professional services industry, especially in law, accounting and, until recently, investment banking.[1] The perceived advantages of partnerships are highly valued by dominant firms in these sectors. For instance, when Goldman Sachs was converted into a public company in 1999 it retained important elements of the partnership that it maintained for 130 years previously. Through the first nine months of 2006, the $13.9 billion that

[1] Greenwood and Empson [2003] list the percentage of partnerships as form of governance for the top 100 firms per industry as follows: Law 100%, Accounting 56%, Management consulting 17%, Architecture 18%.

Goldman Sachs set aside for salaries and bonuses was roughly 50% of its net revenue.[2] This amounts to bonuses averaging $542,000 for its 26,000 staff. New partners are elected every two years. As their share of this compensation pool is disproportionally larger than the associates' share, there is a fierce promotion tournament going on among the lower ranks.

Similarly, consider a partnership of lawyers. Rebitzer and Taylor [2007] argue that "these firms are typically structured as partnerships. Attorneys become partners via up-or-out promotion contests." Promotions are indeed lucrative, as "at Sullivan & Cromwell, for example, according to the American Lawyer, the average partner earned $2.35 million last year" while young lawyers at the same firm have to make do with a meagre $145,000.[3]

In these partnership examples, it is crucial that the tournament prizes are determined by the joint output. This distinguishes our setup from the fixed prizes which are usually studied in the tournaments literature. We capture this feature by using final output as the total sum of prizes awarded in a tournament. In our model, the sharing rule which specifies the percentage of output allocated to the winner, second, etc, is proposed by an arbitrary player and the partnership is only formed if all players agree. A (subgame perfect) equilibrium consists of two elements: a sharing rule which specifies the prizes in the compensation tournament and a set of efforts which determines the output and the probabilities of winning these prizes. There are two main incentive effects of a player increasing effort. On the one hand, additional effort increases the total output of the team of which the agent only receives a share. On the other hand, however, increased effort also raises the agent's chance of winning the tournament. Relative to the socially optimal level of effort, the first effect leads to under-investment while the second gives the leverage to counter this adverse effect. For the offered sharing rule, these two effects exactly cancel out in symmetric equilibrium and we obtain full efficiency. Moreover, for a sufficiently precise ranking technology, the players who are not ranked first in the tournament also get a positive prize in symmetric equilibrium.

If we allow for limited liability of partners, there is the additional caveat that the (marginal) chance of winning the tournament must be reasonably responsive to changes in effort.[4] Thus relative performance compensation schemes under limited liability are only useful if the ranking technology is not too inaccurate. This is arguably easier to achieve in partnerships where professionals share a certain specialization than in general corporations. Our model can therefore explain why partnerships emerge rather between similarly specialized professionals than between professionals with complementary skills.

The remainder of the paper is organized as follows. In section 2.2, we introduce the model and our team game. Section 2.3 illustrates the main result through example

[2] Reuters, 25-Oct-06.

[3] The Wall Street Journal, 1-April-06. We discuss further applications in the concluding section.

[4] Under limited liability, a partner cannot lose more than the amount invested, that is, his share of output is non-negative. Allowing for limited liability is important, because "since the introduction of legal forms such as the limited liability partnership and the limited liability company, unlimited liability partnerships are rarely seen in the professional services." Levin and Tadelis [2005, p162]

and in section 2.4, we prove the efficiency result. We provide an extension to teams and partnerships with more than two members in the appendix which also contains all proofs and technical details.

2.1.1 Related literature

Alchian and Demsetz [1972] and Holmström [1982] pose the original problem of unattainability of first best efforts in neoclassical partnerships when output is ex ante non-contractible and shared among partners. Legros and Matthews [1993] show that full efficiency can be obtained in some cases, for example, for partnerships with finite action spaces or with Leontief technologies. Nevertheless, they confirm and generalize Holmström [1982]'s result that full efficiency is unattainable for neoclassical partnerships, ie. the case which we study where the production and utility functions are smooth. They show that approximate efficiency can be achieved by mixed-strategy equilibria, where one partner takes an inefficient action with small probability. However, sustaining such equilibria depends crucially on the partners bearing *full* liability. If the partners are subject to limited liability, the mechanism does not work since it is impossible to impose the large fine on a partner which is necessary to prevent deviation. In our result, full efficiency is attainable even with limited liability, provided that the ranking technology gives a sufficiently high marginal probability of winning for symmetric efforts. Battaglini [2006] discusses the joint production of heterogenous goods. For multi-dimensional output he finds that implementing the efficient allocation is possible whenever the average dimensionality of the agents' strategy spaces is lower than the number of different goods produced. He confirms, however, that efficiency is unattainable in the standard case.

The classic reference on efficiency in tournaments is Lazear and Rosen [1981]. [5] They compare rank order wage schemes to wages based on individual output and find that, for risk-neutral agents, both allocate resources efficiently. In their setup, the fixed prizes—and thus their efficiency result—arise from perfectly competitive and centralized markets. However, influential studies such as Mortensen and Pissarides [1994] argue convincingly that, for example, the labour market can be viewed as neither centralized nor competitive. Yet any market imperfection leaving positive profits to a firm will render the efficiency result in Lazear and Rosen [1981] inapplicable. In contrast, by endogenizing contests, we do not require any market at all, and therefore, our model credibly lends itself to the analysis of incentive problems in a single partnership regardless of the industry market structure.

Moldovanu and Sela [2001] characterize the optimal prize structures in tournaments. They analyze an exogenously given, fixed budget for prizes and show that for convex cost functions, it is optimal to give positive prizes not only to the winner. Our analysis shows that convexity per se does not lead to the multiple prizes in partnerships. In order to get multiple prizes, the ranking technology has a crucial role as well. Cohen et al. [2004] also characterize optimal prize structures. In contrast to

[5] Kurshid and Sahai [1993] survey the measurements literature which lends support to the tournaments approach by arguing that ordinal statistics are inherently cheaper to produce than cardinal statistics.

Moldovanu and Sela [2001], they allow an agent's reward to depend on his effort. They do not, however, address the problem of efficiency in partnerships or teams. Galanter and Palay [1991] discuss compensation in elite law firms and argue that promotion-to-partner schemes indeed constitute tournaments. The contests literature has flourished recently and is surveyed by Konrad [2004].

Lazear and Kandel [1992] show that the existence of peer pressure can weaken the free rider problem in teams and partnerships. Their concept of peer pressure among partners captures factors such as guilt, norms, and mutual monitoring which all serve as disciplinary devices. The difference to our approach is that their compensation scheme is not a tournament but consists of constant shares of output. Miller [1997] shows that whenever a single partner can observe and report on at least one other's actions, efficient efforts can be implemented. Strausz [1999] shows that when agents choose their efforts sequentially and observe the actions taken by their predecessors, there exists a sharing rule which implements efficient efforts. This sharing rule induces players to reveal a shirking partner by influencing final output in a particular way.

2.2 The model

There are two identical, risk neutral agents exerting unobservable individual efforts $e_i \in [\delta, \infty)$, $i \in \{1, 2\}$, for some positive δ arbitrarily close to zero.[6] However, some noisy ranking of efforts of the partners is assumed to be observable and verifiable. The technology that translates a partner's effort into his place in the ranking is described, for $x := e_i/e_j$, by[7]

$$\Gamma(e_i, e_j) = [f_i(x), f_j(x)] \tag{2.1}$$

where $f_k(\cdot)$ is the probability that partner $k \in \{i, j\}$ gets the first place in the ranking given his effort e_i and the rival partner's effort e_j, and $f_i + f_j = 1$. We make the following assumptions on $f(\cdot)$

A1 Symmetry: $f_i(x) = f_j(1/x)$, for $x \in [\delta, \infty)$;

A2 Responsiveness: $\dfrac{df_i(x)}{dx} > 0$, $\dfrac{df_j(x)}{dx} < 0$; $\lim_{x \to \delta} f_i(x) = 0$ and $\lim_{x \to \infty} f_i(x) = 1$;

A3 $f(\cdot)$ is twice continuously differentiable.

Assumption **A1** captures the symmetry of the two partners. Assumption **A2** captures the idea that the probability that one partner is ranked first in effort is dependent upon the relative performance of the two partners, measured by $x = e_i/e_j$. In particular, partner i's winning probability is increasing in x, but partner j's winning probability is decreasing in x.

[6]The natural effort choice set would be $[0, \infty)$ but we avoid zero effort for technical reasons. We generalize our results to more than two players in the appendix but the full intuition can be understood from the two players case.

[7]This class includes the Tullock success function $\frac{e_i^r}{e_i^r + e_j^r}$ where $f_i(x) = \frac{1}{1+x^{-r}}$.

If a partnership is formed, the output of the partnership is a function of the total efforts of the partners. Denote the production function as $y := y(\sum_i e_i)$. The production function is smooth and twice continuously differentiable, with $y(2\delta) = 0$, $y'(\cdot) > 0$ and $y''(\cdot) \leq 0$. A partner who receives a share s of the final output, given his own effort e_i and the other partner's effort e_j, gets utility

$$u_i(e_i, e_j) = sy(e_i + e_j) - C(e_i)$$

where $C : [\delta, \infty) \to \mathbb{R}$ is a cost function with $C(\delta) = 0$, $C'(\cdot) > 0$ and $C''(\cdot) > 0$.

The objective of a partner is to maximize his own expected utility. Our goal is to analyze whether it is possible to induce the players to exert the efficient level of efforts using a rank order compensation scheme.

2.2.1 Timing

At the first stage, an arbitrary partner initiates the partnership formation by making a proposal to the other partner, offering a sharing rule $(s, 1 - s)$. Without loss of generality let partner 1 be the proposer. Partner 2 then decides whether to accept the proposal or not. If he accepts, the partnership is set up, and the game proceeds to the next stage. If he rejects, the game ends and each player obtains his reservation utility which we normalize to zero. At the second stage, conditional on the formation of the partnership, the partners choose their efforts simultaneously to maximize their own expected utility. Some noisy ranking of efforts is observed and the final output is realized. The final output is distributed between the two partners. The partner who ranks first in efforts obtains the share s of the final output, and the other partner gets $1 - s$.

2.2.2 (In-)Efficiency benchmark

Efficient actions are those which maximize the total welfare of the two partners absent of any incentive aspects

$$\max_{(e_i, e_j)} w(e_i, e_j) := y(e_i + e_j) - C(e_i) - C(e_j).$$

The first best effort level is determined by

$$y'(2e^*) = C'(e^*)$$

where $e_i^* = e_j^* = e^*$. Suppose the two partners fix the shares (s_i, s_j) ex ante, with $s_i + s_j = 1$. As shown by Holmström [1982], there is no sharing rule that achieves full efficiency and satisfies a balanced budget at the same time. Given the sharing rule (s_i, s_j), the partners choose their efforts to maximize

$$u_i(e_i, e_j) = s_i y(e_i + e_j) - C(e_i).$$

Conditional on the formation of the partnership, partner i's best response is given by

$$s_i y'(e_i + e_j) = C'(e_i),$$

where equilibrium efforts are dependent upon the share s_i received. The bigger the share received, the higher the effort. However, since $s_i + s_j = 1$, at least one of the partner always chooses suboptimal effort.

2.3 Example of efficient team production

In this section we use a specific example to illustrate that the proposed partnership tournament game achieves full efficiency. Let the production function be linear in total efforts

$$y = \alpha(e_i + e_j),\ \alpha > 0$$

and let costs functions be quadratic

$$C(e_i) = \frac{1}{2}e_i^2,\ i \in \{1, 2\}.$$

Let the technology which transforms partners' efforts into a ranking of efforts be described by the Tullock success function. Partner $i \in \{1, 2\}$ is ranked first with probability $f_i(\frac{e_i}{e_j}) = \frac{e_i}{e_i+e_j}$ if he exerts effort e_i and the other partner exerts effort e_j. The partner who is ranked first receives share s of the final outcome, and the partner who is ranked second receives share $1 - s$.

The efficient effort levels are given by $(e_1^*, e_2^*) = (\alpha, \alpha)$. In our tournament game, given the shares agreed on at the first stage, the partners choose their efforts non-cooperatively at the second stage. Thus partner $i \in \{1, 2\}$ chooses effort e_i to maximize

$$u_i(e_i, e_j) = \underbrace{\frac{e_i}{e_i + e_j}}_{pr\ \text{winning}} \underbrace{s\alpha(e_i + e_j)}_{\text{payoff if win}} + \underbrace{\frac{e_j}{e_i + e_j}}_{pr\ \text{losing}} \underbrace{(1 - s)\alpha(e_i + e_j)}_{\text{payoff if lose}} - \underbrace{\frac{1}{2}e_i^2}_{\text{cost}}. \tag{2.2}$$

The equilibrium efforts depend on s and are symmetric: $e_i(s) = e_j(s) = s\alpha$. We point out that equilibrium efforts are increasing in the share s. In the extreme case of $s = 1$, both partners exert the efficient level of efforts. The intuition is straightforward. As one partner increases effort, given the other partner's effort level, he increases the final output, and at the same time increases his probability of being ranked first. This implies that he has a higher probability of receiving the winning share of a bigger final outcome. The larger the winner's share s, the higher the incentive for a partner to exert high effort. This incentive reaches its maximum when s takes its maximum value. This result is similar to—but in our case stronger than—the standard tournaments result with fixed prizes where incentives increase with the spread between the prizes.

Comparing a partner's objective function (2.2) with that of a social welfare maximizer

$$w(e_1, e_2) = \alpha(e_1 + e_2) - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2$$

we see that in (2.2), each partner's incentive to exert effort consists of two parts. The first one is that exerted effort increases total output $\alpha(e_i + e_j)$, which increases a partner's payoff no matter whether he is ranked first or second. This motive also exists in the social welfare maximization problem. In a partnership, an agent expects to receive only part of the output and thus does not internalize the positive externality of higher effort on the other partner. This is the usual incentive to free-ride leading to under-investment of effort in partnerships. In our game, however, a tournament is used to allocate the shares. Therefore, partners have an additional motive to exert effort, because higher effort increases the probability of getting a bigger share of the output and decreases the probability of getting the smaller share. With a Tullock success function and $s = 1$, this extra incentive brought about by the tournament exactly offsets the disincentive from profit sharing.

To illustrate that full efficiency is achieved, we still need to show that it is optimal for partner 1 to propose the share $s = 1$ at the first stage and for partner 2 to accept. Given the equilibrium efforts $e(s)$, partner 1 chooses share s at the first stage to maximize

$$u_1(s) := u_1(e_1(s), e_2(s)) = \frac{1}{2}(2 - s)s\alpha^2 \tag{2.3}$$

subject to participation of the second player. Since choosing a minimal effort of δ generates nonnegative utility, this participation is ensured.[8] As $s = 1$ maximizes (2.3), the shares are chosen appropriately and the efficient equilibrium effort levels are implemented.

In this example, when a tournament is used as the share allocation mechanism, the optimal allocation rule is to give the entire outcome to the partner who ranks first in efforts. This is not a feature of the efficient mechanism in general. As we show in the next section, for a sufficiently precise ranking technology, the efficient mechanism shares output between the players such that each agent receives a positive prize.

2.4 Results

We now show that in the general setup, full efficiency is attainable for linear and concave production functions and a large class of ranking technologies. Recall the production technology

$$y = y(e_i + e_j), \quad \text{with} \quad y(2\delta) = 0, \quad y'(\cdot) > 0, \quad y''(\cdot) \leq 0.$$

Given the sharing rule s and partner j's effort of e_j, partner i's expected utility from exerting effort e_i is

$$u_i(e_i, e_j) = f_i\left(\frac{e_i}{e_j}\right) sy(e_i + e_j) + \left(1 - f_i\left(\frac{e_i}{e_j}\right)\right)(1 - s)y(e_i + e_j) - C(e_i).$$

Assuming the existence of interior solutions, this implies for $i = 1, 2$,

$$\begin{aligned} &f_i'\left(\frac{e_i}{e_j}\right)\frac{1}{e_j}(2s - 1)y(e_i + e_j) + \\ &\left(f_i\left(\frac{e_i}{e_j}\right)s + \left(1 - f_i\left(\frac{e_i}{e_j}\right)\right)(1 - s)\right)y'(e_i + e_j) - C'(e_i) = 0. \end{aligned} \tag{2.4}$$

[8] For the case of unlimited liability, the second player's participation constraint has to be examined separately.

Given j's effort e_j, (2.4) implies that marginally increasing effort e_i has three effects: 1) a marginal increase of final output, 2) a marginal increase of partner i's winning probability, and 3) a marginal increase of effort cost.

When the symmetric Nash solution exists, $e_i = e_j = e$ and $f_i(1) = \frac{1}{2}$. Substituting these, we obtain from (2.4) that

$$\frac{f_i'(1)}{e}(2s-1)y(2e) + \frac{1}{2}y'(2e) = C'(e). \tag{2.5}$$

As equilibrium effort e is a function of s we write effort as $e(s)$ and the associated output as $y = y(2e(s))$. Intuitively, $f_i'(1)$ relates to the precision of the tournament's ranking technology. A high value of $f_i'(1)$ corresponds to a highly precise ranking technology. A high-precision technology involves a drastic change of the winning probability as the ratio e_i/e_j approaches 1.[9] In the following lemma we begin the analysis of equilibrium effort choice.

Lemma 2.1. *Equilibrium effort e at the second stage is increasing in s.*

This corresponds to the standard tournament literature result that a partner's incentive is increasing in the spread between prizes. Here we have replaced the fixed prizes with a fixed sharing of the final output. It is natural that effort is increasing in the share s since a larger share means a bigger prize for the winner.

Lemma 2.2. *Denoting the first best, efficient efforts by e^*, the sharing rule s which satisfies*

$$f_i'(1)\frac{1}{e^*}(2s-1)y(2e^*) = \frac{1}{2}y'(2e^*) \tag{2.6}$$

elicits the efficient effort choice at the second stage.

Limited liability restricts the partners' possible shares to $s \in [0,1]$. The next lemma establishes a threshold precision for the ranking technology for efficiency to obtain under limited liability.

Lemma 2.3. *Under unlimited liability, there always exists a share s^* such that (2.6) is satisfied. Under limited liability with $s \in [0,1]$, there exists an s^* that satisfies (2.6) if $f_i'(1) \geq \frac{1}{4}$.*

We now show that with unlimited liability, first best can always be implemented.

Proposition 2.1. *Under unlimited liability, full efficiency is obtained. At the first stage, partner 1 proposes a sharing rule $(s^*, 1-s^*)$ and at the second stage, each partner exerts first best efforts.*

Notice that when $f_i'(1)$ is sufficiently low, the equilibrium share which induces efficient efforts may exceed 1. Denote by $\tilde{s}$ the solution to (2.6).

[9] The role of output variance in ensuring pure strategy equilibrium existence in Lazear and Rosen [1981] or Nalebuff and Stiglitz [1983] is in our case taken by the assumed differentiability of the ranking technology.

Proposition 2.2. *Under limited liability, if $\tilde{s} \in [0,1]$, then full efficiency can always be obtained. If $\tilde{s} \notin [0,1]$, then player 1 proposes shares $(s^*, 1-s^*) = (1,0)$ and the agents choose suboptimal efforts.*

Since for a sufficiently precise ranking technology it is always the case that $\tilde{s} \in [0,1]$, there is a threshold precision above which efficiency is guaranteed.

Corollary 2.1. *If the ranking technology is sufficiently precise, that is if $f_i'(1) \geq \frac{1}{4}$, then full efficiency can always be obtained.*

The above propositions show that for the class of production functions studied, as long as the ranking technology is such that the marginal winning probability for symmetric efforts is sufficiently large, full efficiency can always be achieved, even under limited liability. There is no necessity for a budget breaker. The only requirement is the observability of some noisy ranking of efforts. This result does not depend on whether or not one can deduce the other partner's effort after output is observed. The efficiency result is robust to production functions of other forms, as long as the concept of symmetric equilibrium can be applied.

The condition on the marginal winning probability for symmetric efforts is critical for limited liability. In the symmetric equilibrium we consider here, a partner is only willing to increase his efforts if doing so significantly increases his probability of winning a bigger share of the final output. We emphasize that $f'(1) < {}^1/_4$ is a necessary but not sufficient condition for inefficiency under limited liability. When inefficiency occurs, it depends on the curvature of the production function and the tournament ranking technology. In the example of section 2.3, if we replace the Tullock success function with the more general function (2.1) and leave the linear production function unchanged, $f'(1) = {}^1/_4$ is exactly the critical value between full efficiency and inefficiency. If the production function is strictly concave, the difference between $y(e)$, where $e = e_1 + e_2$, and its linear approximation $y'(e)e$ is positive and the required threshold on the marginal probability of winning decreases.

Finally it is worth pointing out that whenever the ranking technology is precise enough, the player who comes out second also receives a positive share. The exact precision threshold depends on the used production function.

Corollary 2.2. *If the ranking technology is sufficiently precise, that is if $f_i'(1) > \frac{1}{4}$, then $s^* < 1$, ie. both players receive a positive share. Moreover, the winner's share s^* exceeds ${}^1/_2$ and decreases with $f_i'(1)$.*

Although the previous discussion assumes homogenous agents, the existence of sharing rules leading to efficient efforts does not depend on this symmetry. In order to achieve efficiency, however, a mechanism will then generically need to resort to identity-dependent sharing rules specifying different rewards for different players. In the appendix we show that our main findings extend to the case of more than two partners.

2.5 Concluding remarks

The model's most direct application is to partnerships in the professional services. However, since our setup is applicable to any partnership or team structure as long as there exist performance related bonuses paid from the joint product, there is a much wider area of application in virtually any form of cooperation. Instances which match our model precisely are, for example, political contests where the partners in a coalition government work jointly on what may be viewed as maximising the countries' tax base. A follow-up election is a rank order tournament which may not confirm all coalition parties in office. Joint research among tenure track researchers at a university which may grant tenure based on the perceived quality of individual research is a further example. Apart from this promotion aspect, publishing itself can be viewed as a tournament: in the sciences and engineering, several researchers usually work together on one project. Although they share joint output, the most important contributor is typically made the first author of a resulting publication.

Warfare history and the vassal system are rich sources for anecdotes. In the Thirty Years' War, for instance, Albrecht von Wallenstein, a Bohemian nobleman, was rewarded for his services to the Catholic emperor Ferdinand II against the Danish King Christian: in 1628, Wallenstein received the duchy of Mecklenburg where combined forces of Wallenstein and Count Tilly had previously defeated the Danes.[10] Similarly, during the Napoleonic wars, a Royal Navy man-of-war capturing an enemy prize split the proceeds according to a fixed rule specified by the Cruziers and Convoy Act (1708). It granted three eighths to the ship's captain, one eighth each to the (increasingly numerous sets of) wardroom, principal warrant and petty officers, and the final two eighths to the crew.[11] Promotion, thus, was more than a source of pride.

Finally, a team or partnership in which information quality is the key in the selection of projects is a natural application. Partners spending more effort in collecting information increase the quality of information, hence the quality of project selection, and thus expected output. Therefore, a partner with good information plays a more important role in decision making than the ones with bad information who would rather rubber stamp the suggestion of the former. The partners, as a matter of fact, engage in a contest in the collection of information about projects. In order to provide incentives to exert effort, a larger share of output should be granted to better informed partners.[12]

For a sufficiently precise ranking technology, the players who are not ranked first in the tournament also get a positive prize in symmetric equilibrium. This is a property which, together with limited liability, corresponds to actual compensation schemes. If we were to compare two similar limited liability partnerships, one with a very precise ranking technology and one where rankings are rather imprecise, we could obtain two

[10] Friedrich Schiller gives a literary, historically accurate account in his 1792 *History of the Thirty Years' War*.

[11] Kert [1997] details related incentive systems existing in other navies and the private sector.

[12] Incidentally, many TV game shows—for example the CBS reality show *Survivor*—have this format. Players start out in teams but the final prizes are awarded to individuals based on their earlier team performance.

very different sharing rules which nevertheless both implement first best efforts. In the case of the precise ranking technology, all partners could get nearly the same share of output while for the imprecise effort ranking a sharing rule giving all output to the winner could be optimal. Thus, in our setting, decidedly egalitarian looking partnerships may actually arise from pure efficiency considerations.

2.6 Appendix

2.6.1 More than two partners

In this subsection we show that our full efficiency result is not an artifact of two member partnerships, where one can deduce the effort level of the other partner from observing the output. We prove that given the formation of a partnership of n members, the mechanism which allocates the entire final outcome among the partners achieves first best efforts.[13] Moreover, we show that this efficient sharing rule will be proposed at the first stage of the game.

At the first stage, partner 1 proposes a sharing rule of $(s_1, s_2, \cdots, s_n)$. Suppose that the production function takes the form

$$y(e) = \alpha \sum_i e_i \tag{2.7}$$

and the winning probability technology is described by the Tullock success function which specifies the probability of partner i coming out first in the ranking of effort with probability

$$p_i^1(e) = \frac{e_i}{e_1 + e_2 + \cdots + e_n}, \; i \in \{1, \cdots, n\},$$

which is also the probability that partner i wins the share s_1 of the final outcome. Denote $e_{-i} := (e_1, ..., e_{i-1}, e_{i+1}, ..., e_n)$. Then, partner i wins share s_2 with probability

$$p_i^2(e) = p_2^1(e) \cdot \frac{e_1}{e_{-2}} + \cdots + p_n^1(e) \cdot \frac{e_1}{e_{-n}}.$$

Probabilities $p_i^3, \cdots, p_i^n$ are given similarly.[14] Then, given that a partnership of n partners is set up, an allocation rule that assigns the entire output to the partner who is ranked first in effort elicits first best efforts.

Proposition 2.3. *Under the sharing rule* $(1, 0, \cdots, 0)$*, agents choose efforts efficiently.*

In the next proposition, we show that for production functions (2.7), if some sharing rules elicit first best effort levels at the second stage, they are among partner 1's choice

[13] Assume that, if any of the n partners fails to participate in the mechanism, the partnership is not formed and the game ends.

[14] Important are not so much the exact probabilities of coming out second, third etc, but the symmetry of the ranking probabilities between agents and the existence of a symmetric equilibrium for any sharing rule.

set of sharing rules in the first stage. This implies that the sharing rule $(1, 0, \cdots, 0)$ stipulated in proposition 2.3 is part of a subgame perfect equilibrium. That this sharing rule is indeed chosen is shown in the following proposition.[15]

Proposition 2.4. *Suppose the ranking technology is such that, in symmetric equilibrium, each partner is ranked at each place with equal probability $1/n$. If there exist shares $\hat{s} = (s_1, s_2, \cdots, s_n)$ which elicit first best efforts at the second stage, then such $\hat{s}$ also maximizes partner 1's expected payoff at the first stage.*

As in the case of $n = 2$, if the production function is strictly concave, full efficiency is easier to obtain than in the above linear case. A full characterization of the case of arbitrary ranking technologies is difficult for partnerships with more than two partners since the generalization of the ranking technology poses conceptual and technical problems.

2.6.2 Proofs

Proof of lemma 2.1. From the implicit function theorem, it follows that

$$\frac{de(s)}{ds} = \frac{\frac{2f_i'(1)}{e(s)} y(2e(s))}{C'' - y''(2e(s)) + f_i'(1)\frac{1}{e(s)}(2s-1)\left(\frac{y(2e(s))}{e(s)} - 2y'(2e)\right)}.$$

If (2.4) is the foc leading to an equilibrium, then an additional derivative wrt e_i must be negative. This derivative equals

$$\begin{aligned}\frac{d^2 u_i(e_i, e_j)}{de_i^2} =& f_i''\left(\frac{e_i}{e_j}\right)\frac{1}{e_j^2}(2s-1)y(e_i+e_j) + 2f_i'\left(\frac{e_i}{e_j}\right)\frac{1}{e_j}(2s-1)y'(e_i+e_j) \\ &+ \left(f_i\left(\frac{e_i}{e_j}\right)s + \left(1 - f_i\left(\frac{e_i}{e_j}\right)\right)(1-s)\right)y''(e_i+e_j) - C''(e_i)\end{aligned}$$

At the point of symmetric efforts $e = e_i = e_j$ we have

$$f_i''(1)\frac{1}{e^2}(2s-1)y(2e) + 2f_i'(1)\frac{1}{e}(2s-1)y'(2e) + \frac{1}{2}y''(2e) - C''(e) < 0 \tag{2.8}$$

We will now show that

$$f_i''(1) = -f_i'(1). \tag{2.9}$$

We know that $f_i(x) + f_j(x) = 1$ for any $x \in (0, \infty)$. Differentiating this expression wrt x gives

$$f_i'(x) + f_j'(x) = 0. \tag{2.10}$$

We know from assumption **A1**, that for any $x \in (0, \infty)$, $f_i(x) = f_j(^1/_x)$. Differentiating this expression gives

$$f_i'(x) = -\frac{1}{x^2}f_j'(\frac{1}{x}). \tag{2.11}$$

[15] As shown in the proof, proposition 2.4 holds for more general production functions than (2.7).

Plugging (2.11) into (2.10), we obtain

$$f_i'(x) - \frac{1}{x^2} f_i' \left(\frac{1}{x}\right) = 0.$$

Differentiating this identity wrt x, we get

$$f_i''(x) - \left((-2)x^{-3} f_i' \left(\frac{1}{x}\right) - \frac{1}{x^2} f_i'' \left(\frac{1}{x}\right)\right) = 0.$$

Therefore, for $x = 1$, we obtain the required equality (2.9). If we plug this identity back into (2.8), we get

$$C'' - \frac{1}{2}y''(2e(s)) + \frac{f_i'(1)}{e(s)}(2s-1)\left(\frac{y(2e(s))}{e(s)} - 2y'(2e)\right) > 0.$$

Since $C'' > 0$, $y''(\cdot) \leq 0$ and $y(x) \geq y'(x)x$, we are done. □

Proof of lemma 2.2. Given s that satisfies (2.6), at stage 2, partner i chooses effort such that (2.5) is satisfied. Substituting (2.6) into (2.5), one obtains

$$y'(2e) = C'(e)$$

which determines the fully efficient effort level. □

Proof of lemma 2.3. Rewrite equation (2.6) as

$$4f_i'(1)(2s-1)y(2e^*) = 2y'(2e^*)e^* \tag{2.12}$$

Since $y(\cdot)$ is concave function, for any $x \in [\delta, \infty)$ holds that $y(x) \geq y'(x)x$. Therefore, whenever $4f_i'(1) \geq 1$, there exists $s^* \in [0,1]$ that solves (2.12). □

Proof of proposition 2.1. Expecting the symmetric equilibrium effort levels $e_1(s) = e_2(s) = e(s)$ that are determined by equation (2.5), in choosing the optimal share s, partner 1's expected utility is

$$\begin{aligned} u_1(s) = u_1(e(s), e(s)) &= f_1(1)sy(2e(s)) + (1 - f_1(1))(1-s)y(2e(s)) - C(e(s)) \\ &= \frac{1}{2}y(2e(s)) - C(e(s)) \end{aligned}$$

subject to partner 2's participation constraint which we will verify later for the derived equilibrium. Solving partner 1's utility maximization problem gives us the following first order condition

$$\frac{d}{ds}\, u_1(s) = (y'(2e(s)) - C'(e(s)))\,\frac{de(s)}{ds} = 0.$$

Partner 1 chooses $\hat{s}$ such that

$$y'(2e(\hat{s})) = C'(e(\hat{s})).$$

Therefore the sharing rule which implements efficient efforts maximizes player 1's utility. It is now easily verified that partner 2's participation constraint holds because in symmetric equilibrium both players expect the same utilities and by offering $s = {}^1/_2$, the proposer can ensure non-negative utility. $\square$

Proof of proposition 2.2. If $\tilde{s} \in [0,1]$, then the proof is exactly as the proof of the previous proposition. If there is no $\tilde{s} \in [0,1]$ which solves (2.6), meaning $\tilde{s} > 1$. Since $de(s)/ds > 0$ limited liability equilibrium efforts are necessarily lower than the efficient levels e^*. Therefore we have, for the optimal sharing rule s^*,

$$\frac{d}{ds}\, u_1(s^*) = (y'(2e(s^*)) - C'(e(s^*)))\,\frac{de(s^*)}{ds} > 0,$$

where $de(s^*)/ds > 0$ by lemma 1 and $y'(2e) > C'(e)$ for any $e < e^*$ from our curvature assumptions on production and cost functions. This implies that the optimal $s^* = 1$. $\square$

Proof of proposition 2.3. Given the allocation rule, partner i chooses effort e_i receives share 1 with probability $p_i(e_i, e_{-i})$ and receives a share of 0 otherwise. He chooses his effort e_i to maximize

$$u_i(e_i, e_{-i}) = \frac{e_i}{\sum_{j=1}^n e_j} y\left(\sum_{i=1}^n e_i\right) - C(e_i) = \alpha e_i - C(e_i).$$

The optimal choice of e_i is determined by the first order condition

$$\alpha = C'(e_i)$$

which implies that the equilibrium effort level is equal to the first best effort level. $\square$

Proof of proposition 2.4. Since in symmetric equilibrium, each player expects a payoff of ${}^1/_n s_1 + {}^1/_n s_2 + \cdots {}^1/_n s_n$, partner 1 faces the following maximization problem at the first stage

$$u_1(s) = u_1(ne(s)) = \frac{1}{n} y(ne(s)) - C(e(s)).$$

Partner 1 chooses the s that satisfies the first order condition, which is

$$(y'(ne(s)) - C'(e(s)))\frac{de(s)}{ds} = 0$$

The s solving this first order condition elicits the first best effort level. $\square$

Chapter 3

Moral Hazard in Inequity–averse Teams

3.1 Introduction

Holmström [1982] showed in standard self-interest model that when agents are risk neutral, there exists no budget balancing sharing rule that elicits first best efforts in team production. However, reality provides many examples indicating that people are more cooperative than assumed in the standard self-interest model. In many teams, agents work hard even when the pecuniary incentives go in the opposite direction.

This paper investigates moral hazard problem in teams with inequity–averse agents. Agents are envious when their coworkers receive higher monetary payoff than themselves, and are sympathetic when they receive higher monetary payoff than their co-workers. The psychologic feeling of envy or sympathy affects the agents' utility. (Other–regarding preference.) We show that when agents exhibit other–regarding preference, a budget balancing, randomly punishing contract can elicit efficient efforts in equilibrium given that the agents are sufficiently inequity averse or the liabilities they can bear are sufficiently big. That contract distributes the output equally when the output is high and punishes some agents randomly when the output is low.

Such randomly punishing contract is implicit in many team structures. Usually, if a team is successful, it continues and everyone stays with the team and enjoys its success. If it is unsuccessful, some or most members have to leave, except for a few remaining to keep the team going.

When agents are inequity averse, given that everyone else exerts the efficient effort level, an agent's shirking leads to an output level below the efficient one. In that case, some randomly chosen agents get punished, and the associated unequal monetary payoffs among the agents reduce that agent's utility, hence reducing the attractiveness of shirking. If every agent is sufficiently inequity averse, or if the liability one can bear is sufficiently big, everyone chooses efficient action in equilibrium.

This paper is an application of the mechanism discussed in Rasmusen [1987] to an environment where agents care about their own payoffs relative to those of their co–workers. The difference is that he studies risk–aversion of the agents but this paper focuses on the other–regarding preference.

Bartling and von Siemens [2004] and Rey Biel [2003] also analyze the team production problem when the agents are inequity averse. However, their models have two important restrictions: efforts are binary and observable to all other agents (contractible in the latter paper). When the agents' action space is discrete, full efficiency is attainable when the agents are self–interested, as shown by Legros and Matthews [1993]. The attainability of full efficiency when agents exhibit other-regarding preferences may not be surprising anyway in those cases. The important question is the neoclassical teams with continuous action space and differentiable production and cost functions.

The remaining part of the paper is organized as follows. In the next section, we describe the model with agents homogeneous in their liability capacities. Section 3.3 presents the randomly punishing contract and prove the sustainability of full efficiency in equilibrium. Section 3.4 illustrates the result with an example. Section 3.5 concludes. In the appendix, we show that the efficiency result obtained in section 3.3 does not depend on the homogeneity of liability capacities.

3.2 The model

A team here consists of at least two agents, $N = \{1, \cdots, n\}$. Each agent, indexed by i, takes an unobservable action or effort level $e_i \in [0, +\infty)$. Write

$$e_{-i} = (e_1, \cdots, e_{i-1}, e_{i+1}, \cdots, e_n) \quad \text{and} \quad e = (e_i, e_{-i}) \tag{3.1}$$

Denote the production function as $y := y(e)$. The production function is strictly increasing, concave, and continuously differentiable. Output depends only on the effort levels, and there is no random error.

A compensation rule specifies $s_i(y)$ as agent i's monetary payoff if the output is y, which is observable and verifiable. Agent i has limited liability, and his compensation cannot be less than the liability bound $-\omega$, with $\omega \geq 0$. That is, $s_i \geq -\omega$.[1] Agent i's utility function is separable in his satisfaction from monetary payoff and effort cost:

$$u_i(s, e) = m_i(s_i(y(e)), s_{-i}(y(e))) - C_i(e_i) \tag{3.2}$$

The major difference between this model and that of Holmström [1982] is that here an agent's utility depends not only on his own monetary payoff, but also on his co–workers' monetary payoffs.

Exerting effort e_i is costly to agent i, with the cost function $C_i : [0, \infty) \to \mathbb{R}$ with $C_i(0) = 0$, $C_i'(\cdot) > 0$ and $C_i''(\cdot) > 0$. The first part of the agent's utility function, m_i,

[1] We assume that the agents are identical in their liability capacities to simplify notation and analysis. The result holds as well when agents have different ω. That discussion is relegated to the Appendix.

is defined in accordance with the utility function in Fehr and Schmidt [1999]: [2]

$$m_i(s_i, s_{-i}) = s_i - \left(\alpha_i \frac{1}{n-1} \sum_{j \neq i} \max\{s_j - s_i, 0\} + \beta_i \frac{1}{n-1} \sum_{j \neq i} \max\{s_i - s_j, 0\} \right) \quad (3.3)$$

The terms in the brackets capture the utility effects of disadvantageous and advantageous inequality respectively. We assume that $\alpha_i \geq 0$ and $\beta_i \geq 0$, for all i, with one inequality holding strict. $\alpha_i \geq 0$ implies that a worse off individual is willing to trade off some of his personal gain against a decrease in his co–workers' payoffs. This may denote envy or the disutility of being outdone. Positive β_i implies that a better off individual is fair-minded (or suffers from the envy of others) since he would trade off some decrease in his personal monetary gain against an increase in the co–workers' payoffs.

As agents' utility loss from unequal payoffs can be removed through equal monetary allocation, *efficient* actions are those which maximize the total welfare of the team

$$\max_{e} \; w(e) := y(e) - \sum_i C_i(e_i). \quad (3.4)$$

We assume the existence of unique equilibrium e^* and denote the corresponding output as $y^* = y(e^*)$. We concentrate on the sustainability of such first best action profile as Nash Equilibrium.

Definition 3.1. *Full efficiency is sustainable in equilibrium if $e_i = e_i^*$ (weakly) maximizes agent i's expected utility given $e_{-i} = e_{-i}^*$.*

Definition 3.2. *A mechanism is budget balancing if $\sum_i s_i(y) = y$.*

In a partnership, the agents can not credibly commit themselves to destroying some part of output ex post and there is no budget breaker available, as suggested by Holmström [1982]. Therefore, it is important to consider budget balancing mechanisms where the output is fully distributed among the agents.

In the standard self-interest model with $\alpha_i = \beta_i = 0$, for all i , the agents do not care about other agents' monetary payoffs, the utility function (3.2) becomes:

$$s_i(y(e)) - C_i(e_i), \quad i = 1, \cdots, n$$

As shown in Holmström [1982], there is no sharing rule that achieves full efficiency and balances the budget at the same time.

Proposition 3.1 (Holmström [1982])**.** *Full efficiency is not attainable with any budget - balancing mechanism if $\alpha_i = \beta_i = 0$.*

[2] Agents' inequity averse can also be defined on the basis of utility comparison instead of monetary payoff comparison with co–workers. The result remains unchanged but then one needs the extra assumption that effort is observable to all co–workers.

3.3 Efficient mechanism

We show in this section that if the agents exhibit other regarding preference as captured by functions (3.2) and (3.3), full efficiency can be obtained in equilibrium through a randomly punishing contract. Consider the following allocation rule:

$$s_i(y) = \begin{cases} \frac{y}{n} & \text{if } y \geq y(e^*), \\ \frac{y+l\omega}{n-l} \text{ with probability } \frac{n-l}{n} & \text{if } y < y(e^*) \\ -\omega \text{ with probability } \frac{l}{n} & \text{if } y < y(e^*) \end{cases} \tag{3.5}$$

where l is some integer in the range of $[1, n-1]$. This contract stipulates that if the observed final output is not lower than the efficient level $y(e^*)$, the entire output is evenly distributed among all the agents. If the final output is below the efficient level, l agents are chosen randomly, with each of them being charged a fine ω. The collected fine $l\omega$ and the output are distributed among the remaining $n-l$ agents. We shall show that if α_i or β_i is sufficiently big, given that every other agents choose the efficient effort level, agent i will not find it worthwhile to deviate from the efficient effort e_i^*, and hence first best action profile is implemented as a Nash equilibrium.

Lemma 3.1. *Allocation rule (3.5) is budget balancing.*

Proof. When $y \geq y(e^*)$, the total payments received by all the agents are equal to $n \cdot y/n = y$.

When $y < y(e^*)$, the total payments received by all the agents are

$$(n-l) \cdot \frac{y+l\omega}{n-l} + l \cdot (-\omega) = y$$

□

Given $e_{-i} = e_{-i}^*$, by exerting effort $e_i = e_i^*$, the final output is exactly y^*, which is evenly distributed among the agents. Agent i's utility is

$$u(e_i^*, e_{-i}^*) = u_i^*(s(y^*)) = \frac{y(e^*)}{n} - C_i(e_i^*)$$

Lemma 3.2. *Given action profile e_{-i}^*, agent i's optimal deviating effort $\hat{e}_i$ is lower than the efficient effort level e_i^* and is nonincreasing in α_i and β_i.*

Proof. If agent i deviates, the associated output is $y(\hat{e}_i, e_{-i}^*)$. Suppose $\hat{e}_i \geq e_i^*$, since y is an increasing function, $y(\hat{e}_i, e_{-i}^*) > y(e^*)$. According to allocation rule (3.5), agent i receives $y(\hat{e}_i, e_{-i}^*)/n$, optimal $\hat{e}_i$ is determined by:

$$\frac{\partial u_i(e_i, e_{-i}^*)}{\partial e_i}|_{e_i=\hat{e}} = \frac{1}{n}\frac{\partial y(e_i, e_{-i}^*)}{\partial e_i}|_{e_i=\hat{e}} - C_i'(e_i)|_{e_i=\hat{e}} = 0$$

However, such $\hat{e}_i$ can not exceed e_i^* that is determined by $\frac{\partial y(e_i^*, e_{-i}^*)}{\partial e_i} = C_i'(e_i^*)$, which leads to a contradiction. Hence we conclude that $\hat{e}_i < e_i^*$.

When $\hat{e}_i < e_i^*$, the output $y(\hat{e}_i, e_{-i}^*)$ is below the efficient one $y(e^*)$. According to allocation rule (3.5), with probability $1/n$, agent i is punished and has to pay a fine equal to the amount ω. Agent i then suffers from envy since $n-l$ among the other agents receive payoffs $\left(y(\hat{e}_i, e_{-i}^*) + l\omega\right)/(n-l)$, which is higher than his own monetary payoff $-\omega$. His utility in such a scenario is:

$$-\omega - \alpha_i \cdot \frac{1}{n-1} \cdot (n-l) \cdot \left(\frac{y(\hat{e}_i, e_{-i}^*) + l\omega}{n-l} + \omega\right) - C_i(\hat{e}_i) := A$$

With probability $(n-l)/n$, agent i is not punished, and he receives the monetary payoff equal to $\left(y(\hat{e}_i, e_{-i}^*) + l\omega\right)/(n-l)$. However, since l of the other team members receive a payoff equal to $-\omega$ and are worse off than him, he suffers from psychological loss due to sympathy. His utility in that scenario is:

$$\frac{y(\hat{e}_i, e_{-i}^*) + l\omega}{n-l} - \beta_i \cdot \frac{1}{n-1} \cdot l \cdot \left(\frac{y(\hat{e}_i, e_{-i}^*) + l\omega}{n-l} + \omega\right) - C_i(\hat{e}_i) := B$$

Therefore, given other agents choosing the efficient action, if agent i deviates from e_i^*, he chooses some effort e_i to maximize the following:

$$\max_{e_i} u_i(e_i, e_{-i}^*) = u_i(s(y(e_i, e_{-i}^*))) = \frac{l}{n}A + \frac{n-l}{n}B \tag{3.6}$$

$$= \left(\frac{1}{n} - \frac{l(\alpha_i + \beta_i)}{n(n-1)}\right) y(e_i, e_{-i}^*) - \frac{l(\alpha_i + \beta_i)}{n-1} \cdot \omega - C_i(e_i) \tag{3.7}$$

The optimal deviating $\hat{e}_i$ is determined by the following Kuhn-Tucker Conditions:

$$\frac{\partial}{\partial e_i} u_i(s(y(\hat{e}_i, e_{-i}^*))) = \left(\frac{1}{n} - \frac{l(\alpha_i + \beta_i)}{n(n-1)}\right) \frac{\partial y}{\partial e_i}(\hat{e}_i, e_{-i}^*) - C_i'(\hat{e}_i) \leq 0, \tag{3.8}$$

$$\hat{e}_i \frac{\partial}{\partial e_i} u_i(s(y(\hat{e}_i, e_{-i}^*))) = 0 \tag{3.9}$$

If $\alpha_i + \beta_i \leq (n-1)/l$, the above maximization problem has a unique interior solution due to the concavity of y and convexity of C_i, and the unique $\hat{e}_i$ can be derived from:

$$\left(\frac{1}{n} - \frac{l(\alpha_i + \beta_i)}{n(n-1)}\right) \frac{\partial y}{\partial e_i}(\hat{e}_i, e_{-i}^*) = C_i'(\hat{e}_i) \tag{3.10}$$

The associated second order condition is also satisfied since

$$\frac{\partial}{\partial e_i^2} u_i^2(s(y(\hat{e}_i, e_{-i}^*))) = \left(\frac{1}{n} - \frac{l(\alpha_i + \beta_i)}{n(n-1)}\right) y''(\hat{e}_i, e_{-i}^*) - C_i''(\hat{e}_i) < 0$$

By assumption $\alpha \geq 0$ and $\beta_i \geq 0$ (with one strict inequality), we have

$$\frac{1}{n} - \frac{l(\alpha_i + \beta_i)}{n(n-1)} < \frac{1}{n} < 1$$

This confirms that $\hat{e}_i$ is lower than the efficient effort level e_i^* which is determined by

$$\frac{\partial y}{\partial e_i}(e_i^*, e_{-i}^*) = C_i'(e_i^*)$$

By implicit function theorem and first order condition (3.10), we have

$$\frac{\partial \hat{e}_i}{\partial \alpha_i} = \frac{\partial \hat{e}_i}{\partial \beta_i} = -\frac{\frac{l}{n(n-1)}y'}{C_i'' - (\frac{1}{n} - \frac{l(\alpha_i+\beta_i)}{n(n-1)})y''} < 0$$

since $C_i'' > 0$ and $y'' < 0$. Hence, $\hat{e}_i$ is a decreasing function in α_i and β_i.

If $\alpha_i + \beta_i > (n-1)/l$, $\partial u_i/\partial e_i < 0$ for any positive e_i, the maximization problem has a corner solution, $\hat{e}_i = 0$, which is obviously below e_i^* and is independent of the magnitude of α_i and β_i. This completes the proof. □

From the above lemma, we notice that α_i and β_i have symmetric impact on agent i's optimal deviating effort. This allows us to concentrate on the total effect of the two parameters, which describes an agent's overall degree of inequity averse.[3] Denote $\alpha_i + \beta_i := \delta_i$.

Given allocation rule (3.5), agent i will find it not worthwhile to deviate from e_i^* if the following is nonnegative:

$$\begin{aligned}\Delta_i &= u_i(e_i^*, e_{-i}^*) - u_i(\hat{e}_i, e_{-i}^*) \\ &= \frac{y(e^*)}{n} - C_i(e_i^*) - \left(\frac{1}{n} - \frac{l\delta_i}{n(n-1)}\right) \cdot y(\hat{e}_i, e_{-i}^*) + \frac{l\delta_i}{n-1} \cdot \omega + C_i(\hat{e}_i)\end{aligned}$$

Proposition 3.2. *For given ω, there exists a $\tilde{\delta}_i$ such that if $\delta_i \geq \tilde{\delta}_i$, efficient action profile is sustained in equilibrium. For given δ_i, there exists a $\tilde{\omega}$ such that if $\omega > \tilde{\omega}$, efficient action profile is sustained in equilibrium.*

Proof. For given ω, we have

$$\begin{aligned}\frac{\partial \Delta_i}{\partial\ \delta_i} &= (-\frac{1}{n}y' + \frac{l\delta_i}{n(n-1)}y' + C_i')\frac{d\hat{e}_i}{d\delta_i} + \frac{l}{n(n-1)}(y(\hat{e}_i, e_{-i}^*) + n\omega) \\ &= \frac{l}{n(n-1)}(y(\hat{e}_i, e_{-i}^*) + n\omega) > 0\end{aligned}$$

The second line obtains because if $\delta_i \geq (n-1)/l$, $\hat{e}_i$ is interior and determined by $\frac{1}{n}y' - \frac{l\delta}{n(n-1)}y' = C_i'$. If $\delta_i < (n-1)/l$, $\hat{e}_i = 0$, and $d\hat{e}_i/d\delta_i = 0$. Δ_i is a monotone increasing function in δ_i implies that for given ω, there exists a threshold $\tilde{\delta}_i$ such that if $\delta_i \geq \tilde{\delta}_i$, $\Delta_i \geq 0$ and first best effort levels are sustained in equilibrium.

Notice that $\hat{e}_i$ is independent of ω. For given δ_i, we have

$$\frac{\partial \Delta_i}{\partial \omega} = \frac{l\delta_i}{n-1} > 0$$

which implies that, there exists a threshold $\tilde{\omega}$ such that if $\omega \geq \tilde{\omega}$, $\Delta_i \geq 0$ and first best effort levels are sustained in equilibrium. □

[3] This is no longer true when the agents differ in their liability capacities.

Proposition (3.2) indicates that if either δ_i or ω is sufficiently big, full efficiency is sustained in equilibrium for partnerships with smooth production function and continuous action space, in contrast to the impossibility result in Holmström [1982] and Legros and Matthews [1993].

The intuition behind is that when the agents exhibit other regarding preferences, a randomly punishing contract can be used as a threat to create inequality among the agents. If an agent shirks, he suffers not only from less final output, but also psychological loss due to the associated inequality between him and his co–workers. The double losses reduce the attractiveness of shirking and offset the agent's incentive to free ride on others.

As a result, the agents are willing to work hard in order to avoid the probability of being unequal with their co–workers. Here, envy and sympathy for others have the same effect. In contrast, a negative β_i (satisfaction from overdoing co–workers) is detrimental to the implementation of full efficiency.

The working mechanism is similar to risk aversion of the agents as analyzed by Rasmusen [1987]. When agents are risk averse, similar randomizing contract can be used to implement efficient actions because agents suffer from disutility when their payoffs fluctuate, their attitudes toward risk prevent them from shirking. The difference between risk aversion and inequity aversion is that risk attitude is concerning the fluctuation of one's own payoff, while inequity aversion is concerned with psychologic loss from comparison of one's own status with his reference group.

The working mechanism is also similar to peer pressure described by Lazear and Kandel [1992]. In their setup, peer pressure comes mainly from shame and feeling of guilty, the cultivating of which usually require past investment of a corporation or the building of team norms. The psychologic effect modelled in this paper, envy and sympathy, comes mostly from internal feelings, and do not need others taking actions.

Corollary 3.1. *If l becomes larger, the threshold values $\tilde{\delta}_i$ and $\tilde{\omega}$ required to sustain efficiency decrease.*

Proof. We have

$$\frac{\partial^2 \Delta_i}{\partial l \ \partial \delta_i} = \frac{1}{n(n-1)}(y(\hat{e}_i, e^*_{-i}) + n\omega) > 0$$

$$\frac{\partial^2 \Delta_i}{\partial l \ \partial \omega} = \frac{\delta_i}{n-1} > 0$$

This implies that if l increases, Δ_i increases faster with δ_i and ω, hence the threshold value $\tilde{\delta}_i$ and $\tilde{\omega}$ required to turn Δ_i into positive decreases. □

The above corollary implies that the easiness of sustaining full efficiency is increasing with l, a massacre contract where $n-1$ agents get punished in case total output falls below the efficient level is easiest to achieve full efficiency.

3.4 An example

Consider a partnership of two agents with production function

$$y = \gamma(e_i + e_j),\ \gamma > 0$$

and cost function

$$C_i(e_i) = \frac{1}{2}e_i^2,\ i \in \{1, 2\}.$$

In this two-player example, the agents' utility from monetary payoff, i.e. equation (3.3), simplifies to:

$$m_i(s_i, s_j) = s_i - \alpha_i \max\{s_j - s_i, 0\} - \beta_i \max\{s_i - s_j, 0\}$$

The efficient effort levels are given by $(e_1^*, e_2^*) = (\gamma, \gamma)$. When both agents stick to the efficient actions, the final output is equal to $y(e_1^*, e_2^*) = 2\gamma^2$.

The only available randomly punishing contract is $l = 1$. Given allocation rule (3.5), if the final output is above the efficient level $2\gamma^2$, the two agents share the output equally. If the final output is below $2\gamma^2$, one randomly chosen agent has to pay a fine equal to the amount ω while the other agent receives the fine and the entire output.

Given that agent j chooses the efficient action, by choosing e_i^*, partner i's monetary payoff is equal to γ^2 and he suffers no psychological loss since the output is equally distributed between him and the other agent. His expected utility is $u(e_i^*, e_j^*) = \gamma^2 - \frac{1}{2}\gamma^2 = \frac{1}{2}\gamma^2$.

If he deviates, agent i chooses some $\hat{e}_i$ that is lower than the efficient level γ. Then the associated final output is lower than $y(e^*)$ since y is a strictly increasing function. As a result, agent i receives $y(\hat{e}_i, e_j^*) + \omega$ with probability $\frac{1}{2}$ and $-\omega$ with probability $\frac{1}{2}$. He chooses effort e_i to maximize the following objective function:

$$\begin{aligned} u_i(e_i, e_j^*) =& \frac{1}{2}\left(y(e_i, e_j^*) + \omega - \beta_i\left(y(e_i, e_j^*) + 2\omega\right)\right) \\ &+ \frac{1}{2}\left(-\omega - \alpha_i\left(y(e_i, e_j^*) + 2\omega\right)\right) - \frac{1}{2}e_i^2 \\ =& \frac{1}{2}\left(y(e_i, e_j^*) - (\alpha_i + \beta_i)(y(e_i, e_j^*) + 2\omega) - e_i^2\right) \end{aligned}$$

The optimal deviating effort is given by

$$\hat{e}_i = \max\{\frac{1}{2}\gamma(1 - \alpha_i - \beta_i), 0\}$$

By assumption, $\alpha_i \geq 0$ and $\beta \geq 0$, it is confirmed that the optimal deviating effort is lower than the efficient effort γ. If $\alpha_i + \beta_i$ is sufficiently big, (i.e. exceeding 1,) the optimal deviating effort is equal to zero. Here we distinguish two cases.

Case 1: $\alpha_i + \beta_i \geq 1$. The optimal deviating effort is equal to zero. Given that agent j chooses $e_j^* = \gamma$, agent i's expected utility is

$$\begin{aligned} u_i(0, e_j^*) &= \frac{1}{2}\left(y(0,\gamma) + \omega - (\alpha_i + \beta_i)(y(0,\gamma) + 2\omega)\right) \\ &= \frac{1}{2}\left((1 - (\alpha_i + \beta_i))y(0,\gamma) + (1 - 2(\alpha_i + \beta_i))\omega\right) \\ &< 0 < u_i(e_i^*, e_j^*) \end{aligned}$$

Obviously, it is not worthwhile for agent i to deviate from e_i^*.

Case 2: $\alpha_i + \beta_i < 1$. The optimal deviating effort is $\hat{e}_i = \frac{1}{2}\gamma(1 - \alpha_i - \beta_i)$, which is lower than $e_i^* = \gamma$.

By choosing the effort level $\hat{e}_i$, agent i's expected utility is equal to

$$u_i(\hat{e}_i, e_j^*) = \frac{1}{8}(5 - \alpha_i - \beta_i)(1 - \alpha_i - \beta_i)\gamma^2 - (\alpha_i + \beta_i)\omega$$

Such deviation is not worthwhile for agent i if

$$\begin{aligned} \Delta_i &= u_i(e_i^*, e_j^*) - u_i(\hat{e}_i, e_j^*) \\ &= \frac{1}{2}\gamma^2 - \frac{1}{8}(5 - \alpha_i - \beta_i)(1 - \alpha_i - \beta_i)\gamma^2 + (\alpha_i + \beta_i)\omega \end{aligned}$$

is nonnegative. The sign of Δ_i depends on the parameters: α_i, β_i, and ω. When $\alpha_i = \beta_i = 0$, $\Delta_i = -\frac{1}{8}\gamma^2$, and full efficiency is not sustainable even if the agents can bear unlimited liabilities.

When $\alpha_i + \beta_i > 0$, if either $\alpha_i + \beta_i$ or ω is sufficiently big, full efficiency can be sustained in equilibrium. Suppose $\gamma = 1$. The total equilibrium output then is $y(e_1^*, e_2^*) = 2$. If $\omega = 0$, $\alpha_i + \beta_i > 3 - 2\sqrt{2}$ is required to sustain full efficiency. If $\omega = 1$, $\alpha_i + \beta_i > 7 - \sqrt{48}$ is required to sustain full efficiency. If $\alpha_i + \beta_i = 0.1$, then full efficiency is achieved if $\omega > 0.5$.

3.5 Conclusion

In this paper, we have shown that when agents exhibit other regarding preferences, full efficiency can be sustained through a budget-balancing mechanism that punish some agents randomly if output falls below efficient level, given that the agents are sufficiently inequity averse or the liabilities they can bear are sufficiently big. This result does not depend on the homogeneous liability capacity assumed in the model, as will be shown in the appendix. The model provides an explanation why teams and partnerships are popular organization forms in spite of the free riding problem.

The sustainability of full efficiency crucially depends on the assumption that all agents are inequity averse. When some agents pursue purely self-interest, full efficiency is not attainable with the given contract, as those agents will surely deviate from the efficient action profile if their marginal contributions to the team output are not fully compensated. In equilibrium the inequity averse agents may find it to their benefits to overwork to make up the loss of output due to shirking agents, the overall equilibrium action profile is however suboptimal, even though the total effort obtained may be higher than that from standard self-interest model.

As the assumption about agents' preferences is the driving force in the conclusion about efficiencies, it is important to find out the true preferences of the agents. In recent years, research in this direction has been flourishing, however, there has been no decisive conclusion. Which model truly captures the individual behavior in team production is still a question to be tested. Experimental and empirical work will be very helpful in this regard.

3.6 Appendix: Heterogenous liability capacities

In this appendix, we spell out the details concerning agents with different liability capacities. We show that the main result remain unchanged.

Each agent is now characterized by $(\alpha_i, \beta_i, \omega_i)$, The major difference is that when $\omega_i = \omega$, for all i, there is no uncertainty in each agent's expected payoff when he unilaterally deviates from the efficient action profile. When the agents differ in their liability capacities, an agent's expected payoff from unilateral deviation depends on the average liability of the other agents, which is a random variable.

To illustrate in the simplest manner, we consider here a scapegoat contract with $l = 1$, that is, if overall output falls below a certain level, one agent is selected randomly as the scapegoat and is punished. The scapegoat contract is as follows:

$$s_i(y) = \begin{cases} \frac{y}{n} & \text{if } y \geq y(e^*), \\ \frac{y+z_i}{n-1} \text{ with probability } \frac{n-1}{n} & \text{if } y < y(e^*) \\ -\omega_i \text{ with probability } \frac{1}{n} & \text{if } y < y(e^*) \end{cases} \tag{3.11}$$

where z_i is a random variable taking the value ω_j with probability $\frac{1}{n-1}$ for $j = 1, \cdots, n, j \neq i$.

Now suppose that agent i makes unilateral deviation from the efficient action profile, he will choose $\hat{e}_i < e_i^*$. Then with probability $1/n$ he pays a fine equal to $-\omega_i$, and with probability $(n-1)/n$ he receives a monetary payoff equal to $(y + z_i)/(n-1)$. Therefore, his expected utility in case of unilateral deviation is:

$$
\begin{aligned}
u(\hat{e}_i, e^*_{-i}) =& \frac{1}{n}\left(-\omega_i - \alpha_i\left(\frac{y(\hat{e}_i, e^*_{-i}) + \omega_i}{n-1} + \omega_i\right)\right) \\
&+ \frac{n-1}{n} E\left[\frac{y(\hat{e}_i, e^*_{-i}) + z_i}{n-1} - \beta_i \cdot \frac{1}{n-1}\left(\frac{y(\hat{e}_i, e^*_{-i}) + z_i}{n-1} + z_i\right)\right] \\
&- C_i(\hat{e}_i) \\
=& \frac{1}{n}\left(-\omega_i - \alpha_i\left(\frac{y(\hat{e}_i, e^*_{-i}) + \omega_i}{n-1} + \omega_i\right)\right) + \frac{1}{n} \cdot \frac{n-1-\beta_i}{n-1} \cdot y(\hat{e}_i, e^*_{-i}) \\
&+ \frac{1}{n} \cdot \frac{n-1-\beta_i n}{n-1} \cdot \frac{1}{n-1} \sum_{j \neq i} \omega_j - C_i(\hat{e}_i)
\end{aligned}
$$

with $\hat{e}_i$ determined by the following Kuhn-Tucker conditions

$$
\begin{aligned}
&\frac{\partial}{\partial e_i} u(\hat{e}_i, e^*_{-i}) = \frac{n-1-\alpha_i-\beta_i}{n(n-1)} \cdot \frac{\partial y(\hat{e}_i, e^*_{-i})}{\partial e_i} - \frac{\partial C_i(\hat{e}_i)}{\partial e_i} \leq 0 \\
&\hat{e}_i \frac{\partial}{\partial e_i} u(\hat{e}_i, e^*_{-i}) = 0
\end{aligned}
$$

If $\alpha_i + \beta_i \leq n-1$, $\hat{e}_i$ is interior and is a decreasing function in α_i and β_i. If $\alpha_i + \beta_i > n-1$, then $\hat{e}_i = 0$.

Again, agent i will not deviate if the following is true:

$$
\Delta_i = u(e^*_i, e^*_{-i}) - u(\hat{e}_i, e^*_{-i}) \geq 0
$$

where

$$
u(e^*_i, e^*_{-i}) = \frac{y(e^*_i, e^*_{-i})}{n} - C_i(e^*_i)
$$

As in the case with homogeneous liability capacities, one can easily show the following:

$$
\begin{aligned}
\frac{\partial \Delta_i}{\partial \alpha_i} &= \frac{y(\hat{e}_i, e^*_{-i}) + n\omega_i}{n(n-1)} > 0 \\
\frac{\partial \Delta_i}{\partial \beta_i} &= \frac{y(\hat{e}_i, e^*_{-i})}{n(n-1)} + \frac{1}{(n-1)^2} \sum_{j \neq i} \omega_j > 0 \\
\frac{\partial \Delta_i}{\partial \omega_i} &= \frac{n-1+\alpha_i n}{n(n-1)} > 0
\end{aligned}
$$

However, note that

$$
\frac{\partial \Delta_i}{\partial \omega_j} = -\frac{n-1-\beta_i n}{n(n-1)^2}
$$

the sign of which depends on β_i. If $\beta_i \geq \frac{n-1}{n}$, then Δ_i is also increasing in ω_j, which implies that a large average liability of the other agents facilitates the sustainability of efficiency. However, this is not true when β_i is small. In that case, agent i suffers little from downward psychological disadvantages. A large expected value of z_i increases the incentive of deviation, as the expected fine from other agents is big.

The above analysis leads to the following proposition.

Proposition 3.3. *If $\beta_i \geq \frac{n-1}{n}$, full efficiency is obtained if either of α_i, ω_i or $\omega_j, \forall j \neq i$ is sufficiently big. If $\beta_i < \frac{n-1}{n}$, given $\omega_i, \forall i$, full efficiency is obtainable if $\alpha_i, \forall i$ is sufficient big.*

If efficient actions can be sustained as an equilibrium in a contract with $l = 1$, then they can be sustained in any contract with $l \geq 2$. The formal discussion of that is omitted as the notations become much more messy and no new insight is obtained.

Chapter 4

Partnership Dissolution, Complementarity, and Investment Incentives

This chapter is based on Li and Wolfstetter [2007].

4.1 Introduction

Partnerships are a frequently observed form of business organization among professionals. Examples range from law firms, medical practices, to engineering, and business consulting. Typically, a partnership is an association of a few highly skilled professionals who have complementary skills and who are kept together by the promise of sharing the jointly earned profits and overhead expenditures. Indeed, roughly 80% of all partnerships have only two partners, and roughly two thirds of all two-partner partnerships exhibit equal share ownership (see Hauswald and Hege [2003]).

Partnerships form and dissolve. Frequently, one partner finds a new business opportunity which he may not want to share with his fellow partners, and therefore requests dissolution of the partnership. Other events that may trigger dissolution are offers from outsiders to purchase a partner's share, divorce settlements in which a partner's ex spouse receives a share in the firm, foreclosure of debt secured by a partner's share, personal bankruptcy, and the disability or death of a partner, to name just a few.

Rational agents foresee that a partnership may unexpectedly be dissolved, no matter how promising it may be at the time when it is formed. Therefore, they take precaution and include a dissolution rule into the partnership contract, already at the time when the partnership is formed.

In business practice, the most commonly used dissolution rule is the "buy–sell provision" (BSP) or "Texas shoot-out", which is a variant of the well-known "I–cut–you–choose" cake cutting mechanism. There, the partner who requests a dissolution must propose a price at which the other partner may either sell his share or buy the proposer's share. That rule is widely used in practice and recommended by legal advisors (see for example Mancuso and Laurence [2003]). Indeed, as Brooks and Spier [2004] report: *"The importance of buy–sell agreements is now so broadly recognized that a lawyer's failure to recommend or include them in modern joint venture agreements is considered "malpractice" among legal scholars and practitioners."* Such mandatory provisions assure that a partnership is dissolved when it is called for and may avoid the cost of deadlocks, litigation, and lengthy court battles.

The present paper contributes to assess the efficiency properties of this common dissolution rule, by addressing the following questions:

1. If dissolution is requested, does BSP assure that ownership is awarded to the partner who values it most?

2. Does BSP assure that dissolution is requested if and only if single ownership creates more value?

3. Does BSP assure that the partnership chooses efficient investment, maximizing the present value of the sum of the partners' gains?

The literature on partnership dissolution using BSP generally focuses on the first of these three efficiency issues. McAfee [1992] shows that BSP assures efficient assignment of single ownership under complete information. De Frutos and Kittsteiner [2004] extend this to two-sided incomplete information provided the right to propose dissolution is auctioned among partners. Brooks and Spier [2004] show that a mandatory buy–sell provision ensures that all potential gains from dissolution are realized when the partnership has common value and partners have asymmetric information.[1]

One limitation of the partnership dissolution literature is that it takes the characteristics of the partnership and the dissolution decision as given and looks only at the issue of who shall be awarded single ownership. It thus ignores the question whether the partnership should be dissolved at all, and how dissolution rules shape the very formation of partnerships and investment incentives (see Wolfstetter [2002]).

The anticipation of a possible dissolution affects the joint investment into the partnership, and in turn, the choice of investment affects the dissolution decision. For example, if partners expect a dissolution to occur with high probability, they may attempt to minimize the losses of the partner who withdraws from the firm in the event of a break-up, by choosing a relatively low investment, which in turn contributes to make such a break-up even more likely. Alternatively, partners may decide to go the other extreme, and overinvest into assets that increase the gains from complementarity to such an extent that a dissolution is effectively precluded. Some degree of such overinvestment is often observed in business, and is a conspicuous feature in many marital partnerships.

[1] Another branch of the literature examines the existence of a dissolution rule that assures an efficient assignment of single ownership under various information structures. See Cramton et al. [1987], Fieseler et al. [2003], Jehiel and Pauzner [2006], Minehart and Neeman [1999], and Ornelas and Turner [2006].

The present paper attempts to extend the partnership dissolution literature by analyzing the interrelationship between investment and the dissolution decision. For this purpose we introduce a simple, explicit partnership model, in which partnerships form in order to take advantage of complementary skills, as in Farrell and Scotchmer [1988]. However, new opportunities may arise that make partners' skills useless, and hence trigger a request for dissolution. Anticipating that possibility of a break-up, the partnership contract includes a dissolution rule which is typically a buy-sell provision. Investment into the partnership increases the gains from complementarity, yet makes potentially efficient dissolutions less likely.

Our main result is that BSP gives rise to inefficiency, either in the form of excessive dissolutions, combined with underinvestment, or efficient dissolutions, combined with overinvestment,[2] although, in the present framework, it assures the efficient assignment of single ownership conditional on dissolution. Moreover, we show that efficiency may be restored if one supplements BSP with a right to veto, although the right to veto may also block efficient dissolution.

In the legal literature, BSP involves "partnership at will" that shall be dissolved if at least one partner calls for dissolution. Whereas BSP supplement with the right to veto involves a "term partnership" which no partner can lawful leave without the consent of fellow partners (UPA [1997]). However, one has to keep in mind that a contract can always be breached. Therefore, even a term partnership can be dissolved, although at the risk of litigation for damages.

We also examine whether renegotiations may restore efficiency, and find that it is not a remedy either. As in the case of adding a right to veto to BSP, renegotiations may achieve efficiency for some parameter sets; however, it gives rise to an additional hold–up problem, allowing the informed partner to further exploit the uninformed.

The plan of the paper is as follows. In Section 4.2 the model is presented. In Section 4.3, we characterize the efficient partnership contract, characterized by joint investment and a dissolution rule. This result is then used as a benchmark to assess the commonly used buy–sell provision in Sections 4.4. We show that it gives rise to either excessive dissolutions, combined with underinvestment, or efficient dissolution, combined with overinvestment. That inefficiency prevails even if one allows a partner to call for a preemptive breakup, as shown in Section 4.5. In Section 4.6 we add the right to veto, and show that this may restore efficiency, although it also gives rise to equilibria that block efficient dissolutions. In Section 4.7, we analyze how renegotiations affect the equilibrium. Section 4.8 concludes.

4.2 The model

Suppose two risk neutral agents set up an equal share partnership in order to take advantage of their complementary skills. Before they pool their resources, they sign

[2]This result relates to the literature on cooperative and selfish investment (see, for example, Che and Chung [1999], Che and Hausch [1999], and Rogerson [1992]). The driving force of that literature is the externality of investment, while ours is the buy–sell provision combined with one–sided asymmetric information.

a partnership contract, $\{I, D\}$ that prescribes the joint investment I and the BSP dissolution rule, D.

After the partnership has been put in place, one randomly chosen partner finds a new business opportunity that may be incompatible with his partner's skill. This triggers a reconsideration of the partnership which may lead to its dissolution.

The partnership is modelled as a sequential game, as follows:

In *stage one*, the two partners write the contract $\{I, D\}$, set up the equal share partnership, and share the irreversible investment expenditure $C(I) := \frac{1}{2}I^2$.

In *stage two*, nature draws one of the partners with equal probability, and endows him with a new business opportunity. The partner who has received the new business opportunity is referred to as partner 1; the other as partner 2. Partner 1 may request a break-up, which is then executed according to the rule D. Both partners know which role they have been assigned. And partner 2 is not allowed to call for a (preemptive) breakup.[3]

The new opportunity gives rise to a profit shock $\Pi \in \{0, \pi\}, \pi > 0$ and is either compatible or incompatible with the partners' skills. It is described by three states of world, $\Theta := \{th, nh, l\}$, which are drawn from the commonly known probability distribution $q_s := \Pr\{S = s\}, s \in \Theta$. There, h, l indicate that the profit shock is either high, $\Pi = \pi$, or low, $\Pi = 0$, and t, n (mnemonic for "team" resp. "no team") indicate that the innovation is compatible resp. incompatible with the partner's skill. The realization of $S \in \Theta$ is private information of partner 1.

If the partners stay together, each partner earns one half of the gross value of the firm, $V_p(I, s)$:[4]

$$V_p(I, s) := \begin{cases} (1+\alpha)I + \pi & \text{if} \quad s = th \\ (1+\alpha)I & \text{if} \quad s \neq th \end{cases} \tag{4.1}$$

where $\alpha > 0$ is a measure of the complementarity of partners' skills.

Whereas, if the partnership is dissolved, the benefit of complementarity is lost, and the firm's value is either $V_1(I, s)$ or $V_2(I, s)$, if partner 1, resp. partner 2, becomes single owner:

$$V_1(I, s) := \begin{cases} I + \pi & \text{if} \quad s \in \{th, nh\} \\ I & \text{if} \quad s \in \{l\} \end{cases} \tag{4.2}$$

$$V_2(I, s) := I, \quad \text{for all} \quad s \in \Theta. \tag{4.3}$$

Therefore, if partner 1 contemplates dissolution, he faces a trade-off between giving up the benefit of complementarity and pocketing his new business opportunity alone.

[3] Preemptive breakups are introduced in Section 4.5.

[4] The qualitative results do not change in the alternative specification, $V_p(I, th) = (1+\alpha)(I+\pi)$, i.e., in which the complementarity applies also to the new opportunity.

The parameters (π, α) are constrained as follows:

$$\pi \geq \alpha \left(1 + \alpha \left(1 - \frac{1}{2} q_{nh}\right)\right) =: \tilde{\pi}(\alpha) \tag{4.4}$$

$$\alpha \geq \frac{\sqrt{2} - 1}{1 - q_{nh} - q_{th}} =: \tilde{\alpha} \tag{4.5}$$

and $q_s : \Theta \to [0,1]$, $\sum_{s \in \Theta} q_s = 1$, has full support.

If (4.4) were not satisfied, it would never be efficient to break up. Constraint (4.5) is sufficient to assure formation of the partnership if the partnership is dissolved only either in state *nh* or *nh* and *th*. Partnership dissolution is a meaningful issue only if both constraints are satisfied (the detailed proof is in *Appendix* 4.9.1).

As an illustration of the profit shock Π, suppose each partner has two skills, say (a_1, b_1), and (a_2, b_2); skills a_1 and a_2 are complementary, which is why the partnership forms in the first place, whereas skills b_1 and b_2 are used only alone. Then state *th* would represent a positive demand shock for the products produced by combining the skills a_1 and a_2 and for the product produced with skill b_1 alone, whereas state *nh* represents a positive demand shock for the goods produced with skill b_1 alone.

The assumption that partners not only know the identity of the partner who has the new business opportunity, but can also exclude partner 2 from proposing dissolution, may seem to be critical. However, as we show in Section 4.5, the result is essentially unaffected if one allows partner 2 to call for a preemptive breakup.

The conclusions of the paper remain also unchanged if one assumes continuous instead of binary distribution of Π as long as that distribution is common knowledge.

4.3 Benchmark: Efficient investment and dissolution

At the outset we mention some basic facts about efficient dissolution. First, if the partnership is dissolved, single ownership should be awarded to partner 1, because $V_1(I, s) \geq V_2(I, s)$. Second, dissolution is never efficient in states *th* and *l*. Third, in state *nh* dissolution is efficient if and only if investment is lower than the level

$$\tilde{I} := \pi/\alpha, \tag{4.6}$$

because for $I = \tilde{I}$ the loss of complementarity gain, αI, that results from dissolution, is equal to the gain from the new opportunity π. This suggests that by choosing a sufficiently high investment ("too-big-to-fail" policy), the partnership can always prevent a future dissolution.

Let $\mu_i(s) := \Pr\{\text{partner i becomes single owner} \mid s\}, i \in \{1, 2\}$. As a benchmark, the first-best dissolution rule and investment are characterized in the following Lemma.

Lemma 4.1. *The efficient dissolution rule, $D^* := \{\mu_1^*(s), \mu_2^*(s)\}$, and joint investment, I^*, are*

$$\mu_1^*(nh) = 1, \quad \mu_1^*(s) = 0,\ \forall s \neq nh, \quad \mu_2^*(s) = 0,\ \forall s \in \Theta \tag{4.7}$$

$$I^* := 1 + \alpha(1 - q_{nh}) < \tilde{I}. \tag{4.8}$$

Proof. From the above explanation it follows immediately that $\mu_2^*(s) = 0, \forall s$, and

$$\mu_1(I, s) = \begin{cases} 1 & \text{if } s = nh \quad \text{and} \quad I \leq \tilde{I} \\ 0 & \text{otherwise.} \end{cases} \tag{4.9}$$

By assumption (4.4),

$$\tilde{I} = \frac{\pi}{\alpha} \geq \frac{\tilde{\pi}(\alpha)}{\alpha} = 1 + \alpha \left(1 - \frac{1}{2} q_{nh}\right) > 1 + \alpha\,(1 - q_{nh}) = I^*. \tag{4.10}$$

Therefore, (4.9) and (4.10) imply $\mu_1(I^*, s) = \mu_1^*(s)$.

Finally, we confirm that the efficient investment level is indeed equal to I^*. Given the dissolution rule (4.9), the *ex ante* net value of the firm is

$$\begin{aligned} V^*(I) :=& E_S\left[\mu_1(I,S)V_1(I,S) + (1 - \mu_1(I,S))\,V_p(I,S)\right] - C(I) \\ =& \begin{cases} (1+\alpha)I + q_{th}\pi - \frac{I^2}{2} =: \psi_1(I) & \text{if } I \geq \tilde{I} \\ (1+\alpha)I + (q_{th} + q_{nh})\pi - q_{nh}\alpha I - \frac{I^2}{2} =: \psi_2(I) & \text{if } I \leq \tilde{I}. \end{cases} \end{aligned} \tag{4.11}$$

Due to (4.10), (4.4), and the strict concavity of ψ_1, ψ_2, one has

$$\begin{aligned} I_1 :=& argmax_{I \geq \tilde{I}} \psi_1(I) = \max\{1+\alpha, \tilde{I}\} \geq \tilde{I} > I^* = argmax_{I \leq \tilde{I}} \psi_2(I) \\ \psi_1(I_1) \leq& \psi_1(1+\alpha) \\ =& \frac{(1+\alpha)^2}{2} + q_{th}\pi + q_{nh}\left(\tilde{\pi}(\alpha) - \alpha\left(1 + \alpha\left(1 - \frac{1}{2}q_{nh}\right)\right)\right) \\ \leq& \psi_2(I^*), \quad \text{since } \tilde{\pi}(\alpha) \leq \pi. \end{aligned}$$

Therefore, $I = I^*$ is the maximizer of $V^*(I)$, as asserted. □

We mention that the efficient allocation can be implemented by a call option, giving each partner the right to buy the other's share at a particular strike price, if partners can verify the new business opportunity in the event when they stay together.[5] However, it may not be possible to verify that new business opportunity. Moreover, agreeing on the strike price is difficult, especially if, at the time when the partnership is formed, partners are subject to uncertainty concerning the state space and the relevant probability distribution. These may be some of the reasons why partnerships tend to prefer using a price-finding rule like the BSP that uses information available at the time of a dissolution instead of relying on the more fuzzy information available at the time of contracting.

[5] The use of such options has been proposed in a somewhat different context in Grossman and Hart [1986] and Nöldeke and Schmidt [1998].

4.4 Buy–sell provision

The partnership contract includes a buy–sell provision. If a partner calls for dissolution, he must quote a price; then, the other partner has the option to either sell his share or buy the proposer's share at that stipulated price. We analyze the resulting partnership game, and find the perfect equilibrium partnership contract.

Following Samuelson [1984], we let the partner with more information be the proposer; this guarantees that the assignment of single ownership always maximizes the firm's value. This assumes that the new business opportunity can be documented with hard evidence. Since this may not always apply, we will also explore what happens if there is no evidence and partner 2 may preemptively call for dissolution (see Section 4.5).

4.4.1 Dissolution subgame equilibrium

After the joint investment has been made, and the state of the world $s \in \Theta$ has been realized and privately observed by partner 1, the two partners play the dissolution subgame. That game depends critically on the level of investment. As one would expect, if investment is very high, a dissolution is effective precluded ("too–big–to–fail" policy). Whereas a low level of investment gives rise to excessive dissolutions.

The strategies of partner 1 are denoted by $\sigma_1(I,s) := (\tau_1(I,s), p(I,s))$, where $\tau_1(I,s) := \Pr\{\text{propose} \mid I, S = s\}$, and $p(I,s)$ is the price quoted if a breakup is proposed in state s. In turn, the strategy of partner 2 is his buy–sell decision, contingent upon the quoted price p, denoted by $\sigma_2(p) := \Pr\{\text{sell} \mid p\}$, where it is understood that $1 - \sigma_2(p) = \Pr\{\text{buy} \mid p\}$.

The solution of the dissolution subgame is explained in the following Lemmas.

Lemma 4.2. *Partner 2 sells if and only if $p \geq p^* := I/2$, and partner 1 quotes the price $p(s) = p^*$, if he proposes dissolution.*

Proof. Suppose partner 1 has proposed dissolution and quoted the price p. If partner 2 buys, he earns the payoff $V_2(I,s) - p = I - p$, whereas if he sells he earns p. Therefore, he sells if and only if $p \geq I/2$. In turn, if partner 1 buys at price p^* he earns $V_1(I,s) - p^* \geq \frac{I}{2}$, whereas if he sells at price $p < p^*$ he earns only $p < \frac{I}{2}$. Therefore, if he proposes, he quotes the price $p = p^*$. □

We now show that the equilibrium of the dissolution subgame depends on the level of investment.

Lemma 4.3. *The dissolution subgame has the following equilibrium:*

$$\sigma_1(I,s) = (p^*, \tau_1(I,s)), \; \tau_1(I,s) = \begin{cases} 1 & \textit{if } s \in \Theta_1(I) \\ 0 & \textit{otherwise} \end{cases} \tag{4.12}$$

$$\sigma_2(p) = \begin{cases} 1 & \textit{if } p \geq p^* \\ 0 & \textit{otherwise.} \end{cases} \tag{4.13}$$

$$\Theta_1(I) = \begin{cases} \emptyset & \textit{if } I \geq 2\tilde{I} \\ \{nh\} & \textit{if } I \in [\tilde{I}, 2\tilde{I}) \\ \{nh, th\} & \textit{if } I \in [0, \tilde{I}) \end{cases} \tag{4.14}$$

Proof. The equilibrium strategy $\sigma_2(p)$ and equilibrium price p^* have already been established in Lemma 4.2. To confirm that $\tau_1(I,s)$ is part of partner 1's equilibrium strategy, note that

$$\frac{1}{2}V_P(I,th) \gtreqless V_1(I,th) - \frac{I}{2} \iff I \gtreqless \tilde{I}$$
$$\frac{1}{2}V_P(I,nh) \gtreqless V_1(I,nh) - \frac{I}{2} \iff I \gtreqless 2\tilde{I}$$

□

Therefore, $\tilde{I}$ can be interpreted as the smallest investment that deters dissolution in state $s = th$, and $2\tilde{I}$ as the smallest investment that deters dissolution in all states. In other words, $I \geq 2\tilde{I}$ is the "too-big-to-fail" policy.

4.4.2 Perfect Bayesian Nash equilibrium

To find the Perfect Bayesian Nash equilibria of the entire game, we compute the *ex ante* net value of the firm, for all choices of I, using the corresponding equilibrium of the above subgame (recall the definitions of ψ_1, ψ_2 in (4.11)):

$$\begin{aligned} V(I) :=& E_S\left[\tau_1(I,S)V_1(I,S) + (1-\tau_1(I,S))V_p(I,S)\right] - C(I) \\ =& \begin{cases} \psi_1(I) & \text{if} \quad I \geq 2\tilde{I} \\ \psi_2(I) & \text{if} \quad I \in [\tilde{I}, 2\tilde{I}) \\ \psi_3(I) := \psi_2(I) - q_{th}\alpha I & \text{if} \quad I \in [0, \tilde{I}) \end{cases} \end{aligned} \tag{4.15}$$

In a first step we show that equilibrium investment is equal to either one of two levels, either $\hat{I}$ or $\tilde{I}$, which also implies that neither efficient investment nor the "too-big-to-fail" investment level is part of the equilibrium.

Lemma 4.4. *The equilibrium investment is $I \in \{\hat{I}, \tilde{I}\}$, where*

$$\hat{I} := argmax_{I \in [0,\tilde{I})} \psi_3(I) = 1 + \alpha - (q_{nh} + q_{th})\alpha < I^* < \tilde{I}. \tag{4.16}$$

Proof. The equilibrium investment is the maximizer of either ψ_3 on $[0, \tilde{I})$ or ψ_2 on $[\tilde{I}, 2\tilde{I})$ or ψ_1 on $[2\tilde{I}, +\infty)$. All three functions, ψ_3, ψ_2, ψ_1 are strictly concave.

First, note that $\hat{I}$ is the maximizer of ψ_3 on its domain, because $\psi_3'(\hat{I}) = 0$, and (using (4.10))

$$\hat{I} = 1 + \alpha - (q_{nh} + q_{th})\alpha < 1 + \alpha - q_{nh}\alpha = I^* < \tilde{I}.$$

Second, $\tilde{I}$ is the maximizer of ψ_2 on its domain, since $\psi_2'(\tilde{I}) < 0$.

Third, $2\tilde{I}$ is the maximizer of ψ_1 on its domain, since $\psi_1'(2\tilde{I}) < 0$.

Finally, observe that:

$$\begin{aligned}\psi_3(\hat{I}) - \psi_1(2\tilde{I}) =& \frac{4\pi}{2\alpha^2}\left(\pi - \alpha(1 + \alpha - \frac{1}{2}q_{nh}\alpha)\right) + \frac{1}{2}(1 + q_l\alpha)^2 \\ \geq& \frac{1}{2}\left(1 + q_l\alpha\right)^2 > 0, \quad \text{by (4.4)}\end{aligned}$$

Therefore, $I = 2\tilde{I}$ is dominated by $I = \hat{I}$. □

Lemma 4.5. *The equilibrium investment is*

$$\begin{aligned} I = \hat{I} &\iff (\pi, \alpha) \in \mathcal{P}_+ := \{(\pi, \alpha) \mid \pi \geq \max\{\pi_0(\alpha), \tilde{\pi}(\alpha)\}\} \\ I = \tilde{I} &\iff (\pi, \alpha) \in \mathcal{P}_- := \{(\pi, \alpha) \mid \tilde{\pi}(\alpha) \leq \pi < \max\{\pi_0(\alpha), \tilde{\pi}(\alpha)\}\}. \end{aligned}$$

Proof. To determine whether $\hat{I}$ or $\tilde{I}$ is optimal, compute the payoff difference

$$\begin{aligned}\xi(\pi) :=& \psi_3(\hat{I}) - \psi_2(\tilde{I}) \\ =& \frac{1}{2\alpha^2}\left(\pi^2 - 2\pi\alpha(1 + (1 - q_{nh})\alpha) + \alpha^2(1 + (1 - q_{th} - q_{nh})\alpha)^2\right)\end{aligned}$$

The following equation implicitly defines the set of parameters (π, α) for which $\xi(\pi) = 0$:

$$\pi_0(\alpha) := \tilde{\pi}(\alpha) - \frac{1}{2}q_{nh}\alpha^2 + \sqrt{q_{th}\alpha^3\left(2 + 2\alpha - 2q_{nh}\alpha - q_{th}\alpha\right)}. \tag{4.17}$$

Since ξ is increasing in π for all feasible parameters, $\pi \geq \tilde{\pi}(\alpha)$, it follows immediately that $\hat{I}$ is optimal if and only if $\pi \geq \pi_0(\alpha)$ and $\pi \geq \tilde{\pi}(\alpha)$. □

The two parameter sets, $\mathcal{P}_+, \mathcal{P}_-$ are illustrated in Figure 4.1. There, the area under the dotted curve and to the left of the vertical line $\alpha = \tilde{\alpha}$ is the set of parameters that are not feasible (due to the constraints (4.4) and (4.5)), the area below the solid and at or above the dotted curve is the parameter set $\mathcal{P}_-$, and the area at and above the solid curve is $\mathcal{P}_+$.

Lemma 4.5 is intuitively appealing. For given α, if π is sufficiently big, becoming a single owner is attractive to partner 1, which implies that the partnership is dissolved with high probability. This leads to a low level of investment ($\hat{I} < I^*$). In contrast, if π is relatively small, retaining the partnership is more attractive to partner 1; as a result the equilibrium investment level is high ($\tilde{I} > I^*$).

Combining Lemmas 4.3–4.5, and recalling from Lemma 4.1 that efficiency means $I = I^*$ and dissolution if and only if $s = nh$, we conclude:

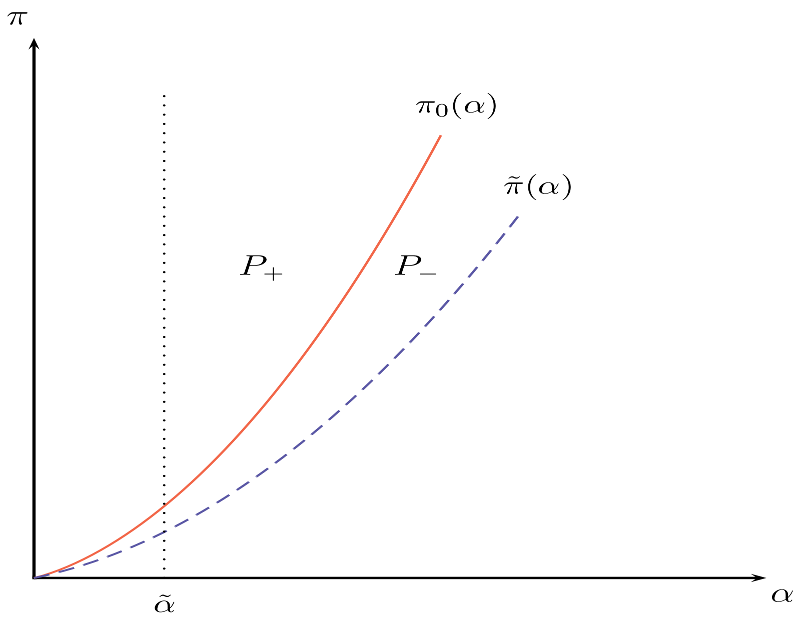

Figure 4.1: $\mathcal{P}_+, \mathcal{P}_-$ for $(q_{th}, q_{nh}) = (0.5, 0.25)$

Proposition 4.1. *The perfect equilibrium exhibits:*

1. *Excessive dissolution (in $s \in \{nh, th\}$) and underinvestment, $I = \hat{I} < I^*$, $\forall(\pi, \alpha) \in \mathcal{P}_+$.*

2. *Efficient dissolution and overinvestment, $I = \tilde{I} > I^*$, $\forall(\pi, \alpha) \in \mathcal{P}_-$.*

We close this section with two examples:

Example 4.1. *Let $\{\alpha = 1,5,\ \pi = 6.1,\ q_{nh} = 0,5,\ q_{th} = 0,25\}$. This leads to excessive dissolution, combined with underinvestment (illustrated in Figure 4.2).*

Example 4.2. *Let $\{\alpha = 3,\ \pi = 12,\ q_{nh} = 0,5,\ q_{th} = 0,25\}$. This leads to efficient dissolution, combined with overinvestment (illustrated in Figure 4.3).*

4.5 What if the uninformed partner may preempt?

We assumed so far that partners not only know the identity of the partner who has the new business opportunity, but they can also exclude partner 2 from proposing dissolution. We now digress and show how the analysis changes if partner 2 cannot be excluded, for example because the evidence that indicates who has the new opportunity is not verifiable in court.

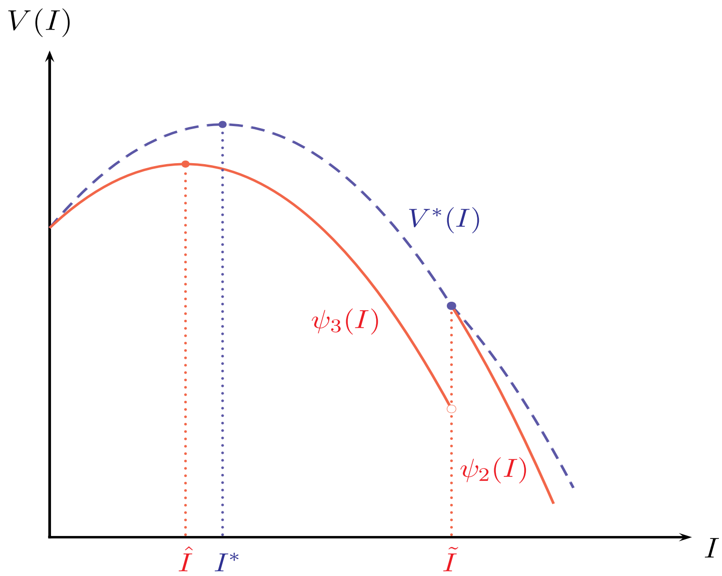

Figure 4.2: Excessive dissolution and underinvestment

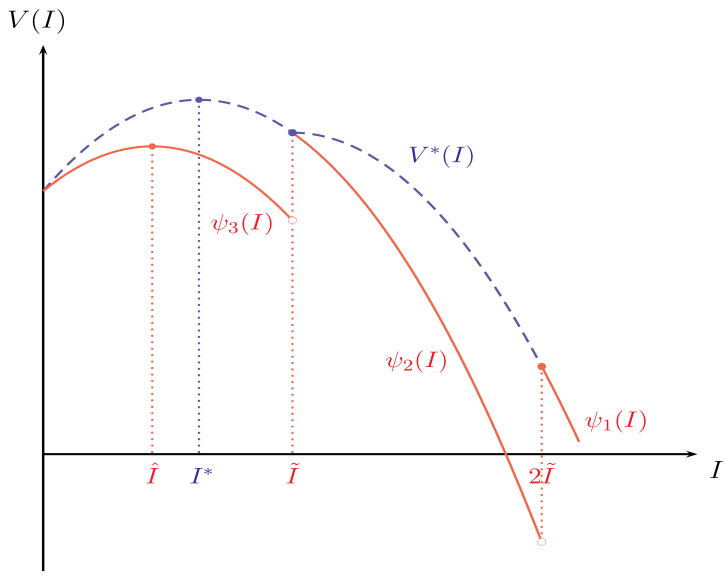

Figure 4.3: Efficient dissolution and overinvestment

The main conclusion will be that the parameter set for which partnerships are feasible become more restrictive; however, if a partnership is feasible and generates higher payoffs than going alone, the uninformed partner generally does not preempt and the solution of the game is as in Proposition 4.1.

The rules of the dissolution subgame are now specified as follows:

1. Both partners simultaneously choose their dissolution strategy (τ_i, p_i).
2. If at least one partner has proposed, the partnership is dissolved.
3. If both partners propose, i.e. if $\tau_1 = \tau_2 = 1$, the higher price offer is declared the strike price of the buy-sell option, $p = \max\{p_1, p_2\}$, and the partner who proposed the lower price is granted the buy-sell option; if they tie, the option is assigned at random.
4. If only one partner proposes, one proceeds as in the previous section.

Lemma 4.6. *If partner 2 proposes dissolution, then* $p_2 = \frac{I+\pi}{2}$.

Proof. Evidently, $p_2 = \frac{I+\pi}{2}$ is the highest price at which partner 1 is ready to buy in the events $s \in \{th, nh\}$. In order to induce partner 1 to sell, the price would have to exceed $\frac{I+\pi}{2}$, and buying at such a price is less profitable for partner 2 than selling at $p_2 = \frac{I+\pi}{2}$. □

Lemma 4.7. *If $I \geq 2\tilde{I}$ ("too-big-to-fail" policy), then in equilibrium no partner will call for dissolution, i.e.* $\tau_1(s) = 0, \forall s$ *and* $\tau_2 = 0$.

Proof. We have already shown that it is not in the interest of partner 1 to call for dissolution in this case (see Lemma 4.3). Suppose partner 2 unilaterally calls for dissolution with $p_2 = \frac{I+\pi}{2}$ (by Lemma 4.6). Call the associated payoff $u_2(\tau_2 = 1)$ and the payoff from not calling for dissolution $u_2(\tau_2 = 0)$. But then,

$$u_2(\tau_2 = 0) - u_2(\tau_2 = 1) \geq \frac{\pi}{2}(1 + q_{th} + 2q_l) > 0, \tag{4.18}$$

which is contradiction. □

Lemma 4.8. *If, in equilibrium, either 1) $\tau_2 = 1$ and $I < 2\tilde{I}$, or 2) $I \geq 2\tilde{I}$, then going alone payoff dominates joining the partnership, in which case the partnership will not form.*

Proof. If $\tau_2 = 1$, the partnership dissolves with certainty. In this case it is obviously better to not join the partnership, because the investment cost is subadditive and thus joining the partnership does not pay if the complementarity benefit is never achieved.

$I > 2\tilde{I}$ is payoff dominated by $I = 2\tilde{I}$, and therefore does not occur in equilibrium.

If $I = 2\tilde{I}$, the partnership will never dissolve, by Lemma 4.7. Denote the expected gain from joining the partnership that chooses $I = 2\tilde{I}$ by u_p and that from going alone and choosing the associated optimal investment (which is $I = 1$) by u_a, one finds:

$$
\begin{aligned}
u_a - u_p =& \frac{1}{2}(1 + (q_{nh} + q_{th})\pi) - \frac{1}{2}\left((1+\alpha)2\tilde{I} + q_{th}\pi - \frac{1}{2}(2\tilde{I})^2\right) \\
=& \frac{2\pi^2 + \alpha^2 - 2\pi\alpha - 2\pi\alpha^2 + \pi q_{nh}\alpha^2}{2\alpha^2}, \quad \text{by } \tilde{I} = \frac{\pi}{\alpha} \\
\geq& \frac{2\pi^2 + \alpha^2 - 2\pi\alpha - 2\pi\alpha^2 + \pi(2\alpha(1+\alpha) - 2\pi)}{2\alpha^2}, \quad \text{by (4.4)} \\
=& \frac{1}{2} > 0.
\end{aligned}
$$

We conclude that going alone payoff dominates joining the partnership. □

Proposition 4.2. *If, in equilibrium, the partnership forms, partner 2 never calls for dissolution ($\tau_2 = 0$) and the equilibrium solution is the same as in the game where only partner 1 is permitted to propose. A sufficient condition is* (4.4) *combined with* $q_l \geq 1/2$.

Proof. From the above Lemmas it follows immediately that $\tau_2 = 0$ is a necessary condition for joining a partnership. In *Appendix* 4.9.2 we show in detail for which parameters $\tau_2 = 0$ is indeed part of the equilibrium. □

4.6 Can the right to veto restore efficiency?

We now return to the setup where hard evidence of the new opportunity is available and consider the buy–sell provision modified by granting partner 2 the right to veto a proposed dissolution. This modification transforms the dissolution subgame into a signalling game in which the quoted price serves as a signal of partner 1's private information, and partner 2 uses that signal to update his prior beliefs concerning the value of the partnership, in order to assess whether he should either sell his share or veto and thus keep the partnership going.

In the following we employ the concept of a sequential equilibrium, characterized by strategies, and beliefs that are consistent with those strategies. With slight abuse of language, we will refer to the game played after a buy–sell provision has been offered as the dissolution "subgame".

In the dissolution subgame with right to veto, the action set of partner 2 has three elements: "buy", "sell", and "veto". And partner 1 chooses between "propose" a buy–sell provision and "don't propose". However,

Lemma 4.9. *The dissolution subgame can be reduced to one where partner 1 always proposes, and quotes a price* $p \geq \frac{I}{2}$*; and partner 2 only chooses between "sell" and "veto" (and never contemplates to "buy").*

Proof. 1) Observe that partner 2 will always veto, if partner 1 offers a price $p \in [\frac{I}{2}, \frac{I}{2}(1+\alpha)]$, because veto gives him a payoff equal to $\frac{I}{2}(1+\alpha)$ or more. Therefore, "don't propose" is payoff equivalent to proposing a price $p \in [\frac{I}{2}, \frac{I}{2}(1+\alpha)]$. We conclude that we can represent "don't propose" by "propose" a price from that interval.

2) Observe that if partner 1 proposes a price $p < \frac{I}{2}$, partner 2 will either buy or veto, since buying is better than selling in that case. Instead of selling at such a price, partner 1 prefers to maintain the partnership. Therefore, in the light of 1), proposing a price $p < \frac{I}{2}$ is inferior to proposing a price $p \in [\frac{I}{2}, \frac{I}{2}(1+\alpha)]$. We conclude that partner 1 will always propose and quote a price $p \geq \frac{I}{2}$.[6] □

4.6.1 A partial separating equilibrium that may restore efficiency

As in the game of BSP without right to veto, the equilibrium of the dissolution subgame depends on the level of investment. Four intervals of investment must be distinguished:

$$I_1 := \left[0, \frac{q_{nh}\tilde{I}}{2(q_{th}+q_{nh})}\right], \quad I_2 := \left(\frac{q_{nh}\tilde{I}}{2(q_{th}+q_{nh})}, \frac{\tilde{I}}{2}\right)$$
$$I_3 := \left[\frac{\tilde{I}}{2}, \tilde{I}\right), \quad I_4 := \left[\tilde{I}, +\infty\right). \tag{4.19}$$

Lemma 4.10. *The dissolution subgame has a "partial separating equilibrium". There, dissolution occurs:*
1) never if I is "high": $I \in I_4$,
2) only in state nh if $I \in I_3$,
3) in state nh and with positive probability also in state th if $I \in I_2$,
4) in states nh and th if I is "low": $I \in I_1$.

A detailed formulation and proof of Lemma 4.10 is in *Appendix* 4.9.3.

Notice that in such a separating equilibrium the critical investment required to deter dissolution becomes smaller than in the case without right to veto. Now, the minimum investment required to deter dissolution in all states is equal to $\tilde{I}$, and the minimum investment to deter dissolution in state *th* only is equal to $\frac{\tilde{I}}{2}$.

We now show that, when partner 2 has the right to veto, the overall game has a perfect equilibrium, for some subset of the feasible parameters, that implements the efficient investment and dissolution rule.

Proposition 4.3. *The partial separating equilibrium implements the efficient investment and dissolution rule if and only if $\pi \leq 2\tilde{\pi}(\alpha) - \alpha^2 q_{nh}$.*[7]

Proof. Suppose $\pi \leq 2\tilde{\pi}(\alpha) - \alpha^2 q_{nh}$. For those parameters, one has $I^* \in I_3$. By definition of I^*, the *ex ante* net value of the firm is maximized, provided dissolution occurs if and only if $s = nh$. In the partial separating equilibrium, that condition is satisfied for all $I \in I_3$, and therefore, for $I = I^*$.

Now suppose in equilibrium $I = I^*$ and dissolution takes place if and only if $s = nh$. Then $\pi \leq 2\tilde{\pi}(\alpha) - \alpha^2 q_{nh}$ must hold. Suppose otherwise, that is, $\pi > 2\tilde{\pi}(\alpha) - \alpha^2 q_{nh}$.

[6] If $\alpha > 1$, this argument can be simplified, because in that case "veto" dominates "buy".
[7] Recall, the set of feasible (π, α) is $\{(\pi, \alpha) \mid \pi \geq \tilde{\pi}(\alpha)\}$.

It follows that $I^* < \frac{\tilde{I}}{2}$, which implies $I^* \in \{I_1, I_2\}$. If the efficient investment level I^* is chosen, the partnership is dissolved in state *nh* and with a positive probability in state *th* by lemma 4.10, which is a contradiction.

□

Proposition 4.3 states a necessary and sufficient condition for an efficient equilibrium. If that condition is not satisfied, the partial separating equilibrium results in inefficiencies, in the sense that investment or dissolution efficiency are violated. Nevertheless, the buy-sell provision with veto right still performs better than the one without veto right since it leads to a higher expected firm value.

Corollary 4.1. *Under the buy-sell provision with veto right the expected firm value is never lower than under the buy-sell provision without veto right.*

When $\pi \leq 2\tilde{\pi}(\alpha) - \alpha^2 q_{nh}$, the superiority of BSP with veto right is obvious. The proof of the remaining case is in *Appendix* 4.9.4.

4.6.2 An inefficient partial pooling equilibrium

As in other signalling games, the dissolution subgame has also other equilibria, with different properties.

Lemma 4.11. *The dissolution subgame has a "partial pooling equilibrium". There, for all investment levels $I \in I_3 \cup I_4$ and all states s partner 1 quotes the same price at which partner 2 vetoes, and no dissolution occurs.*

The detailed formulation and proof of Lemma 4.11 is also in the *Appendix* 4.9.3.

For those parameters for which the partial separating equilibrium of the dissolution subgame assures efficiency, the above partial pooling equilibrium of that subgame entails inefficiency.

Proposition 4.4. *If $\pi \leq 2\tilde{\pi}(\alpha) - \alpha^2 q_{nh}$, the partial pooling equilibrium implies inefficiency.*

Proof. Suppose the partners play partial pooling equilibrium of the dissolution subgame, and choose the efficient investment level $I = I^*$. Then, by Lemma 4.11, in all states s partner 1 quotes the same price at which partner 2 vetoes; therefore, no dissolution occurs, which is inefficient. □

As we have seen, adding the right to veto to BSP may restore efficiency for some parameters. However, it gives rise to multiple equilibria. If the inefficient equilibrium stated in Proposition 4.4 is played, under some parameters, dissolution is blocked when it is efficient. When partners expect that equilibrium to be played, they choose an investment level equal to I_1 which is higher than I^* ("too–big–to–fail" policy). In such cases, both buy–sell provision with and without veto right are inefficient. No veto right assures that a dissolution always occurs when it is called for while granting the veto right has the advantage of providing better investment incentives.

4.7 Can renegotiations restore efficiency?

Can partners restore efficiency through renegotiations? The presence of inefficient dissolutions gives the partners an incentive to renege the BSP rule. However, as in other asymmetric information problems, renegotiations can only restore efficiency in some cases (see, for example, Matthews [1995]).

After partner 1 has requested dissolution one partner may propose to renege the default BSP agreement in order to keep the partnership together in exchange for some transfer. This may then trigger a bargaining process. For simplicity, we assume the simplest possible bargaining process, where one party makes an ultimatum offer. If that offer is accepted, the partnership is maintained and a stipulated transfer is paid. Otherwise it is dissolved according to the default BSP agreement at the strike price initially proposed by partner 1. Two cases must be distinguished: 1) the informed party, partner 1, makes the renegotiation offer, and 2) partner 2 takes the initiative. The first case gives rise to a signalling and the second to a screening problem.

4.7.1 Signalling

In the signalling case, for $I \in \Lambda := [\frac{q_l \tilde{I}}{2(q_{th}+q_l)}, \frac{(q_l+2q_{th})\tilde{I}}{2(q_{th}+q_l)}] \subset [0, \tilde{I}]$, the dissolution / renegotiation subgame has a partial separating equilibrium that the partnership is only dissolved in state *nh*. In that subgame equilibrium, partner 1 proposes dissolution in all states and offers to renegotiate in states $\{th, l\}$ by asking for a positive transfer $t = \frac{\alpha I}{2} + \frac{q_{th}\pi}{2(q_{th}+q_l)}$ from partner 2. The renegotiation proposal by partner 1 perfectly differentiates state *nh* from other states, and partner 2 takes the signal and accepts the renegotiation offer when it is proposed. One can further show that if the parameters satisfy

$$\pi(\alpha) \in [\frac{2(q_l + q_{th})}{q_l + 2q_{th}} \left(\tilde{\pi}(\alpha) - \frac{1}{2} q_{nh}\alpha^2\right), \left(\frac{q_{th} + q_l}{q_l}\right) \left(2\tilde{\pi}(\alpha) - q_{nh}\alpha^2\right)],$$

the above subgame equilibrium forms a perfect equilibrium of the entire game and the equilibrium investment is $I = I^*$, which implies that efficiency is achieved. (Detailed statement of the strategies and beliefs is in *Appendix* 4.9.5.)

However, as in other signalling games, the dissolution/renegotiation subgame has also pooling equilibria for some parameters that, by definition, cannot implement efficiency. Moreover, as the above equilibrium illustrates, renegotiation creates an additional hold-up problem by granting partner 1 more opportunity to extract surplus from the weaker partner.

4.7.2 Screening

Instead of letting the informed party propose renegotiation one can, of course, let the uninformed partner 2 be the proposer. Interestingly, in this case one finds a pooling equilibrium that implements full efficiency for a small set of parameters. That equilibrium is characterized as follows.

The partnership chooses the efficient investment $I = I^*$. Partner 1 proposes dissolution in all states with price $p = p^* = I/2$, partner 2 renegotiates and offers the transfer $\bar{t} = (\pi - \alpha I)/2$ in exchange for keeping the partnership, and partner 1 accepts iff $t \in [\bar{t}, \bar{t}_+)$ and $s \in \{th, l\}$, or $t \in [\bar{t}_-, \bar{t})$ and $s = l$ or in all states if $t \geq \bar{t}_+$, where $\bar{t}_+$ denotes the smallest transfer that assures acceptance in all states and $\bar{t}_-$ the highest transfer that assures acceptance in state l. The beliefs of partner 2 are such that priors are confirmed if dissolution is requested. One can show that there exists a small parameter set for which these strategies are mutual best replies and the beliefs are consistent with them. As a result, the partnership is dissolved only in state $s = nh$, which is efficient for the given investment I^*.

However, the above equilibrium survives only for some fairly small parameter set. For example, it fails to exist if $q_l + q_{nh} > 1/2$ or if $q_{nh} > q_{th}$. Moreover, it gives rise to the same hold-up problem as when partner 1 is given the option of requesting for renegotiation.

4.8 Conclusion

In this paper, we extended the partnership dissolution literature, initiated by Cramton et al. [1987], by setting up an explicit model of partnerships that may explain why partnerships form and yet dissolve in the face of new business opportunities. We analyzed the effect of the commonly advised and frequently used dissolution rule, known as buy–sell provision. That rule assures that, conditional on dissolution, the assignment of single ownership is efficient. However, it always entails an efficiency loss, either in the form of excessive dissolutions, combined with underinvestment, or efficient dissolution, combined with overinvestment. When a right to veto is added, efficiency may be restored, but it gives rise to an equilibrium selection problem. Similarly, when renegotiation is allowed, efficiency may be restored in some cases; however, it gives rise to an additional hold–up problem where the informed partner extracts rent from the uninformed partner.

One testable hypothesis is that the buy–sell provision with veto right provides better investment incentives. Therefore, when it is important to protect specific investments, one might expect to see more term partnerships which partners cannot leave without the consent of their fellow partners (or one has to compensate for the damages in case of breach of contract).

Finally, although our model focuses on buy–sell provision, it should be regarded as a starting point for a more general analysis of the interaction between investment and dissolution incentives. For example, it would be interesting to see what happens if investments are intangible services that may not be contractible. We conjecture that this makes the inefficiency problem caused by BSP more severe, since it gives rise to an additional free riding problem. It would also be interesting to see how other dissolution rules perform, such as winner's bid auction, loser's bid auction, and bargaining with alternating offers.

4.9 Appendix

4.9.1 Proof of the parameter constraints (4.4) (4.5)

Here we show that partnership dissolution is a meaningful problem only if both parameter constraints are satisfied, which is why impose them in the paper.

First, suppose constraint (4.4) is violated, i.e.

$$\pi < \alpha(1 + \alpha - \frac{1}{2}q_{nh}\alpha). \tag{4.20}$$

We show that then it is never efficient to dissolve.

Recall the value function (4.11) of the firm under full efficiency. The assumption implies that the maximizer over the first branch, defined by $I \geq \tilde{I}$, is $I_1 = 1 + \alpha$ and that $I_1 > \tilde{I}$. Therefore, the assertion follows if I_1 is the maximizer of $V^*(I)$. Now suppose the maximizer of $V^*(I)$ is not equal to I_1. Then the optimal investment must be the maximizer over the second branch of the V^* function, defined for $I \leq \tilde{I}$, which is equal to $I_2 := \min\{I^*, \tilde{I}\}$, where $I^* = \arg\max_{I \geq 0} \psi_2(I) = 1 + \alpha(1 - q_{nh})$ (ignoring the constraint $I \leq \tilde{I}$). However,

$$\begin{aligned} \psi_1(I_1) - \psi_2(I_2) &\geq \psi_1(I_1) - \psi_2(I^*) \quad \text{by definition of } I^* \\ &= q_{nh}\left((1 + \alpha - \frac{1}{2}q_{nh}\alpha)\alpha - \pi\right) > 0 \quad \text{by assumption (4.20)} \end{aligned}$$

which contradicts the assumption that I_1 is not the maximizer of $V^*(I)$.

Second, we show that constraint (4.5) is sufficient to assure that a partnership is set up.

We have shown that if the partnership dissolves in all states, it will not be set up in the first place. Now we want to prove that if condition (4.5) is satisfied, the partnership is always set up if the partnership only dissolves in states $s \in \{th, nh\}$.

Recall that if the partnership is formed, each partner's expected payoff is equal to $\frac{1}{2}V(I)$, with $V(I)$ bounded from below by:

$$V(I) \geq \psi_3(\hat{I}) = \frac{1}{2}(1 + \alpha - q_{nh}\alpha - q_{th}\alpha)^2 + (q_{nh} + q_{th})\pi$$

Whereas if each partner goes alone, his expected payoff is equal to

$$V_i(I) := I + \frac{1}{2}(q_{nh} + q_{th})\pi - \frac{1}{2}I^2.$$

whose maximizer is $I_a = 1$.

The partnership is always set up if:

$$\begin{aligned} \frac{1}{2}\psi_3(\hat{I}) \geq V_i(I) &\Leftrightarrow \frac{1}{2}(1 + \alpha - q_{nh}\alpha - q_{th}\alpha)^2 + (q_{nh} + q_{th})\pi \geq 1 + (q_{nh} + q_{th})\pi \\ &\Leftrightarrow \alpha \geq \frac{\sqrt{2} - 1}{1 - q_{nh} - q_{th}} \end{aligned}$$

4.9.2 Proof of Proposition 4.2

Here we solve the dissolution subgames assuming that both partners may call for dissolution (as in Section 4.5) and prove the sufficient condition stated in Proposition 4.2.

We have already shown in Lemma 4.6–4.8 that if partner 2 proposes, he proposes the price $p_2 = \frac{I+\pi}{2}$; furthermore, in equilibrium $I < 2\tilde{I}$. Therefore, we only need to solve the dissolution subgames for 1) $I \in [0, \tilde{I})$, 2) $I \in [\tilde{I}, 2\tilde{I})$.

Lemma 4.12. *Suppose $I < \tilde{I}$. Then, the equilibrium dissolution strategies are: Partner 1 calls for dissolution and sets $p_1 = \frac{I}{2}$, if and only if $s \in \{th, nh\}$; and if he gets the buy-sell option, "buys" if and only if the strike price $p \leq \frac{I+\pi}{2}$ and $s \in \{th, nh\}$. Partner 2 calls for dissolution, i.e. $\tau_2(I) = 1$, sets $p_2 = \frac{I+\pi}{2}$, if and only if $(q_l, I) \in S_1 := \{(q_l, I) \mid q_l \leq \frac{\pi}{2\pi+\alpha I}\}$; if he gets the buy-sell option, he sells if and only if the strike price is $p \geq \frac{I}{2}$.*

Proof. Suppose partner 1 plays the asserted equilibrium strategy. We determine for which parameters it is a best reply of partner 2 to play $\tau_2(I) = 1$, $p_2 = \frac{I+\pi}{2}$.

Denote the payoff of partner 2 if he plays $\tau_2(I) = 1$ by u_2 and that if he plays $\tau_2(I) = 0$ by u_2'. Then,

$$\begin{aligned} u_2 - u_2' &= \frac{I+\pi}{2}(1-2q_l) + q_l I - \left((1-q_l)\frac{I}{2} + q_l \frac{I(1+\alpha)}{2}\right) \\ &\geq 0 \iff (q_l, I) \in S_1. \end{aligned}$$

Next, suppose partner 2 plays the asserted equilibrium strategy. If that strategy prescribes $\tau_2(I) = 0$, we are back in the game where only partner 1 proposes (see Lemma 4.3). If $\tau_2(I) = 1$, any price below p_2 (including the above stated price p_1) is a best reply of partner 1. Because then partner 1 buys at p_2 if and only if $s \in \{th, nh\}$ and thus earns the payoff $u_1 = \frac{I+\pi}{2}$, regardless of which state occurred; whereas if he quotes a higher price than p_2, partner 2 will sell, which leads to the payoff

$$u_1' = \begin{cases} I + \pi - p_1 & \text{if} \quad s \in \{th, nh\} \\ I - p_1 & \text{otherwise,} \end{cases}$$

which is obviously smaller than u_1. Hence, the asserted strategies are mutual best replies.

□

Lemma 4.13. *Suppose $I \in [\tilde{I}, 2\tilde{I})$. Then, the equilibrium dissolution strategies are: Partner 1 calls for dissolution and sets $p_1 = \frac{I}{2}$ if and only if $s = nh$; and if he gets the buy-sell option, "buys" if and only if the strike price $p \leq \frac{I+\pi}{2}$ and $s \in \{th, nh\}$. Partner 2 calls for dissolution, i.e. $\tau_2(I) = 1$ and $p_2 = \frac{I+\pi}{2}$, if and only if $(q_l, q_{th}, I) \in S_2 := \{(q_l, q_{th}, I) \mid q_l \leq \frac{\pi(1-q_{th}-\alpha I q_{th}}{2\pi+\alpha I}\}$; if he gets the buy-sell option, he sells if and only if the strike price is $p \geq \frac{I}{2}$.*

Proof. The proof is similar to the proof of Lemma 4.12 and hence omitted. □

From these two Lemmas we conclude that in equilibrium partner 2 never calls for dissolution if $q_l \geq \frac{1}{2}$, as asserted in Proposition 4.2.

4.9.3 Proof of Lemmas 4.10, 4.11

In the reduced game under BSP with veto right, the strategy of partner 1 is his probability of quoting a price p, denoted by $\sigma_1(p; I, s) := \Pr\{P = p \mid S = s\}$, with some support $\mathcal{P}$. The strategy of partner 2 is $\sigma_2(p; I) = \Pr\{\text{sell} \mid p\}$ and $1 - \sigma_2(p; I) = \Pr\{\text{veto} \mid p\}$. And the beliefs of partner 2 are denoted by $\delta_s(p, I) := \Pr\{S = s \mid p\}$.

Here we give detailed statements and proofs of Lemmas 4.10 and 4.11.

Lemma 4.10 *The equilibrium strategies and beliefs of the "partial separating equilibrium" are:*
Strategies:

$$\sigma_1(\hat{p}(I); th, I) := \eta(I), \quad \sigma_1(p_1'; th, I) := 1 - \eta(I) \tag{4.21}$$

$$\sigma_1(\hat{p}(I); nh, I) := 1, \quad \sigma_1(p_1; l, I) := 1 \tag{4.22}$$

$$\frac{I}{2} \leq p_1 < p_1' < \frac{(1+\alpha)I}{2} \leq \hat{p}(I) := \frac{1}{2}\left(I(1+\alpha) + \delta_{th}(\hat{p}(I), I)\pi\right) \tag{4.23}$$

$$\sigma_2(p; I) = \begin{cases} 1 & \text{if} \quad p > \hat{p}(I) \quad \text{or} \quad \left(p = \hat{p}(I) \quad \text{and} \quad I < \tilde{I}\right) \\ 0 & \text{otherwise} \end{cases} \tag{4.24}$$

$$\eta(I) := \begin{cases} 0 & \text{if} \quad I \in I_3 \cup I_4 \\ \frac{q_{nh}(\pi - 2\alpha I)}{2q_{th}\alpha I} & \text{if} \quad I \in I_2 \\ 1 & \text{if} \quad I \in I_1 \end{cases} \tag{4.25}$$

Beliefs:

$$\delta_{th}(p, I) := \begin{cases} 1 & \text{if} \quad p \in [p_1', \hat{p}(I)) \\ \frac{q_{th}\sigma_1(\hat{p}(I); th, I)}{q_{nh} + q_{th}\sigma_1(\hat{p}(I); th, I)} & \text{if} \quad p = \hat{p}(I) \\ 0 & \text{if} \quad p < p_1' \quad \text{or} \quad p > \hat{p}(I) \end{cases} \tag{4.26}$$

$$\delta_{nh}(p, I) := \begin{cases} 1 & \text{if} \quad p > \hat{p}(I) \\ \frac{q_{nh}}{q_{nh} + q_{th}\sigma_1(\hat{p}(I); th, I)} & \text{if} \quad p = \hat{p}(I) \\ 0 & \text{if} \quad p < \hat{p}(I) \end{cases} \tag{4.27}$$

$$\delta_l(p, I) := \begin{cases} 1 & \text{if} \quad p < p_1' \\ 0 & \text{otherwise} \end{cases} \tag{4.28}$$

Proof. The beliefs are obviously consistent with the stated strategies, using Bayes' rule, when it applies. Also, partner 2's strategy is evidently a best reply, given his beliefs. It remains to be shown that partner 1's strategies are best replies, given the beliefs $\delta(p, I)$, for all investment levels.

1) Suppose $I \in I_4$. Then, $\eta(I) = 0$, $\sigma_1(\hat{p}(I); nh, I) = \sigma_1(p_1'; th, I) = \sigma_1(p_1; l, I) = 1$, $\sigma_2(p) = 1$ if $p > \hat{p}(I)$ and $\sigma_2(p; I) = 0$ for all other p, $\delta_{th}(\hat{p}(I); I) = 0$, $\delta_{nh}(\hat{p}(I); I) = 1$, and $\hat{p}(I) = \frac{1}{2}V_p(I, nh)$.

Consider type $s = nh$. In the asserted equilibrium, he shall quote the price $\hat{p}(I)$ with certainty. If he deviates, he can only change the outcome if he quotes a higher price, p. However, this does not pay, since the gain from that deviation is negative:

$$\begin{aligned} I + \pi - p - \frac{1}{2}V_p(I, nh) <& I + \pi - V_p(I, nh) \\ =& I + \pi - (1+\alpha)I \\ =& \pi - \alpha I \\ \leq& \pi - \alpha\tilde{I} \quad (\text{since } I \geq \tilde{I}) \\ =& 0 \quad (\text{by definition of } \tilde{I}). \end{aligned}$$

Consider $s = th$. In the asserted equilibrium, partner 1 proposes the price p_1', and partner 2 vetoes. If partner 1 deviates, he can only change the outcome by proposing a price $p > \hat{p}$, at which partner 2 sells, just like in the above case $s = nh$. Evidently, maintaining the partnership is more profitable than in the event $s = nh$. Therefore, such a deviation is even less profitable than in the case $s = nh$, described above.

Consider $s = l$. In the asserted equilibrium, partner 1 proposes the price p_1, and partner 2 vetoes. Again, partner 1 can only make a difference if he quotes a price $p > \hat{p}(I)$, which pays even less for him than in the cases described above.

2) Suppose $I \in I_3$. Then, $\eta(I) = 0$, $\sigma_1(\hat{p}(I); nh, I) = \sigma_1(p_1'; th, I) = \sigma_1(p_1; l, I) = 1$, $\sigma_2(p) = 1$ if $p \geq \hat{p}(I)$ and $\sigma_2(p; I) = 0$ for all other p, $\delta_{th}(\hat{p}(I); I) = 0$, $\delta_{nh}(\hat{p}(I); I) = 1$, and $\hat{p}(I) = \frac{1}{2}V_p(I, nh)$.

Consider type $s = nh$. In the asserted equilibrium, he shall quote the price $\hat{p}(I)$, at which partner 2 sells. If partner 1 deviates, he can only change the outcome by quoting a lower price, $p < \hat{p}(I)$. However, this does not pay, since the gain from that deviation is negative:

$$\begin{aligned} \frac{1}{2}V_p(I, nh) - (I + \pi - \frac{1}{2}V_p(I, nh)) =& V_p(I, nh) - (I + \pi) \\ =& \alpha I - \pi \\ <& \alpha\tilde{I} - \pi \quad (\text{since } I < \tilde{I}) \\ =& 0 \quad (\text{by definition of } \tilde{I}). \end{aligned}$$

Consider $s = th$. In the asserted equilibrium, partner 1 proposes the price p_1', and partner 2 vetoes. If partner 1 deviates, he can only change the outcome by proposing a price $p \geq \hat{p}$, at which partner 2 sells. However, the gain from such a deviation is negative, since

$$\begin{aligned}(I+\pi-p)-\frac{1}{2}V_p(I,th) \leq & \frac{\pi}{2}-\alpha I \\ \leq & \frac{1}{2}\left(\pi-\alpha\tilde{I}\right) \quad (\text{since } I \geq \frac{\tilde{I}}{2}) \\ = & 0 \quad (\text{by definition of } \tilde{I}).\end{aligned}$$

Consider $s = l$. In the asserted equilibrium, partner 1 proposes the price p_1, and partner 2 vetoes. Again, partner 1 can only make a difference if he quotes a price $p = \hat{p}(I)$, which pays even less for him than in the previous case.

3) Suppose $I \in I_2$. Then, $\eta(I) = \frac{q_{nh}(\pi-2\alpha I)}{2q_{th}\alpha I}$, $\sigma_1(\hat{p}(I); nh, I) = \sigma_1(p_1; l, I) = 1$, $\sigma_1(\hat{p}(I); th, I) = \eta$, $\sigma_1(p_1'; th, I) = 1-\eta$, $\sigma_2(p) = 1$ if $p \geq \hat{p}(I)$ and $\sigma_2(p; I) = 0$ for all other p, $\delta_{th}(\hat{p}(I); I) = 1 - \frac{2\alpha I}{\pi}$, $\delta_{nh}(\hat{p}(I); I) = \frac{2\alpha I}{\pi}$, and $\hat{p}(I) = \frac{1}{2}(I(1-\alpha)+\pi)$.

Consider type $s = nh$. In the asserted equilibrium, he shall quote the price $\hat{p}(I)$, at which partner 2 sells. If partner 1 deviates, he can only change the outcome by quoting a lower price, $p < \hat{p}(I)$. However, this does not pay, since the gain from that deviation is negative:

$$\frac{I(1+\alpha)}{2} - (I+\pi-\hat{p}(I)) = -\frac{\pi}{2} < 0.$$

Consider $s = th$. In the asserted equilibrium, partner 1 randomizes between the prices p_1' and $\hat{p}$, partner 2 vetoes if $p = p_1'$ and sells if $p = \hat{p}(I)$. For that to be an equilibrium, partner 1 must be indifferent between these two actions, which confirms:

$$\frac{1}{2}\left((1+\alpha)I+\pi\right) - (I+\pi-\hat{p}(I)) = 0.$$

If he deviates, that can only make a difference if he quotes either a price lower than p_1 (but those prices are dominated and were already eliminated in Lemma 4.9) or a price above $\hat{p}(I)$, which is obviously not an improvement either.

Consider $s = l$. In the asserted equilibrium, partner 1 proposes the price p_1, and partner 2 vetoes. Partner 1 can only make a difference if he quotes a price $p = \hat{p}(I)$. However, the gain from that deviation is negative:

$$I - \hat{p}(I) - \frac{I(1+\alpha)}{2} = -\frac{\pi}{2} < 0.$$

4) Suppose $I \in I_1$. Then, $\eta(I) = 1$,

$$\sigma_1(\hat{p}(I); nh, I) = \sigma_1(p_1; l, I) = \sigma_1(\hat{p}(I); th, I) = 1$$

, $\sigma_2(p) = 1$ if $p \geq \hat{p}(I)$ and $\sigma_2(p; I) = 0$ for all other p, $\delta_{th}(\hat{p}(I); I) = \frac{q_{th}}{q_{nh}+q_{th}}$, $\delta_{nh}(\hat{p}(I); I) = \frac{q_{nh}}{q_{nh}+q_{th}}$, and $\hat{p}(I) = \frac{1}{2}(I(1+\alpha) + \frac{q_{th}}{q_{nh}+q_{th}}\pi)$.

Consider type $s = th$. In the asserted equilibrium, he shall quote the price $\hat{p}(I)$, at which partner 2 sells. If partner 1 deviates, he can only change the outcome by quoting a lower price, $p < \hat{p}(I)$. However, this does not pay, since the gain from that deviation is negative:

$$\frac{I(1+\alpha)+\pi}{2} - (I+\pi-\hat{p}(I)) = \alpha I - \frac{\pi}{2} + \frac{\pi q_{th}}{2(q_{nh}+q_{th})} < 0.$$

Consider $s = nh$. In the asserted equilibrium, he shall quote the price $\hat{p}(I)$, at which partner 2 sells. If partner 1 deviates, he can only change the outcome by quoting a lower price, $p < \hat{p}(I)$. However, this does not pay, since the gain from that deviation is obviously even smaller than the gain from the same deviation for type *th*, which was already shown to be negative.

Consider $s = l$. In the asserted equilibrium, partner 1 proposes the price p_1, and partner 2 vetoes. Partner 1 can only make a difference if he quotes a price $p = \hat{p}(I)$. However, the gain from that deviation is negative:

$$I - \hat{p}(I) - \frac{I(1+\alpha)}{2} = -\frac{\pi q_{th}}{2(q_{nh}+q_{th})} - \alpha I < 0.$$

□

Lemma 4.11 *The equilibrium strategies and beliefs of the "partial pooling equilibrium" for $I \in I_3 \cup I_4$ are:*
Strategies:

$$\sigma_1(p_1; s, I) := 1, \text{for all } s \in \Theta \tag{4.29}$$

$$\frac{I}{2} \leq p_1 < \hat{p}(I) := \frac{1}{2}\left(I(1+\alpha)+\pi\right) \tag{4.30}$$

$$\sigma_2(p; I) = \begin{cases} 1 & \text{if} \quad p \geq \hat{p}(I) \\ 0 & \text{otherwise} \end{cases} \tag{4.31}$$

Beliefs:

$$\delta_{th}(p, I) := \begin{cases} 1 & \text{if} \quad p \geq \hat{p}(I) \\ q_{th} & \text{if} \quad p \in [p_1, \hat{p}(I)) \\ 0 & \text{otherwise} \end{cases} \tag{4.32}$$

$$\delta_{nh}(p, I) := \begin{cases} q_{nh} & \text{if} \quad p \in [p_1, \hat{p}(I)) \\ 0 & \text{otherwise} \end{cases} \tag{4.33}$$

$$\delta_{l}(p, I) := \begin{cases} q_{l} & \text{if} \quad p \in [p_1, \hat{p}(I)) \\ 1 & \text{if} \quad p < p_1 \\ 0 & \text{otherwise} \end{cases} \tag{4.34}$$

Proof. The beliefs are obviously consistent with the stated strategies, using Bayes' rule, when it applies. Also, partner 2's strategy is evidently a best reply, given his beliefs. Partner 1 could only make a difference if he deviates and quotes a price $p \geq \hat{p}(I)$, at which partner 2 sells for sure. However, that never pays. $\square$

4.9.4 Proof of Corollary 4.1

Proof. Suppose $\pi > 2\tilde{\pi}(\alpha) - \alpha^2 q_{nh}$, we want to show that the buy-sell provision with veto right never leads to a lower expected firm value than the buy-sell provision without veto right. The *ex ante* net value of the firm for all choices of I, using the subgame equilibrium in Lemma 4.10 is:

$$V(I) := \begin{cases} \psi_1(I) & \text{if} \quad I \in I_4 \\ \psi_2(I) & \text{if} \quad I \in I_3 \\ \psi_2(I) - q_{th}\alpha I \eta(I) := \psi_{2a}(I) & \text{if} \quad I \in I_2 \\ \psi_3(I) & \text{if} \quad I \in I_1 \end{cases} \tag{4.35}$$

The maximizer of the first branch over I_4 of the above value function is $\min\{\tilde{I}, 1+\alpha\}$; the maximizer over I_3 is $\frac{\tilde{I}}{2}$; that over I_2 is $1+\alpha$ if $\pi \in [2\alpha(1+\alpha), 2(1+\frac{q_{th}}{q_{nh}})\alpha(1+\alpha)]$, is $\frac{\tilde{I}}{2}$ if $\pi < 2\alpha(1+\alpha)$, and is equal to $\frac{q_{nh}\tilde{I}}{2(q_{nh}+q_{th})}$ if $\pi > 2(1+\frac{q_{th}}{q_{nh}})\alpha(1+\alpha)$; that over I_1 is $\min\{\hat{I}, \frac{q_{nh}\tilde{I}}{2(q_{nh}+q_{th})}\}$.

Recall from Lemma 4.5, the optimal investment level under buy-sell provision without veto right is $I \in \{\tilde{I}, \hat{I}\}$. Denote the equilibrium firm value under BSP without veto right as V_{nv} and that under BSP with veto right as V_v. We distinguish two cases.

1) Suppose $\tilde{\pi}(\alpha) \leq \pi \leq \max\{\pi_0(\alpha), \tilde{\pi}(\alpha)\}$. Then $V_{nv} = \psi_2(\tilde{I})$. Since $\frac{\tilde{I}}{2}$ is a local maximizer of value function (4.35), we have $V_v \geq \psi_2(\frac{\tilde{I}}{2})$. Then

$$V_v - V_{nv} \geq \psi_2(\frac{\tilde{I}}{2}) - \psi_2(\tilde{I}) = \frac{\pi}{8\alpha^2}(3\pi - 4\alpha(1+\alpha-q_{nh}\alpha)) > 0$$

by assumption $\pi > 2\tilde{\pi}(\alpha) - \alpha^2 q_{nh}$.

2) Suppose $\pi > \max\{\pi_0(\alpha), \tilde{\pi}(\alpha)\}$. Then $V_{nv} = \psi_3(\hat{I})$. If $I = \hat{I}$ is a local maximizer over I_1 of value function (4.35), obviously $V_v \geq V_{nv}$. In the following we show that $V_v \geq V_{nv}$ when $\hat{I}$ is not the local maximizer on I_1 of (4.35).

Suppose $\pi \leq (1+\frac{q_{th}}{q_{nh}})(2\tilde{\pi}(\alpha) - \alpha^2 q_{nh} - 2\alpha^2 q_{th})$ holds. Then the local maximizer of (4.35) over I_1 is equal to $\frac{q_{nh}\tilde{I}}{2(q_{nh}+q_{th})}$, instead of $I = \hat{I}$.

Since $\frac{\tilde{I}}{2}$ is the maximizer of (4.35) over I_3, we have $V_v \geq \psi_2(\frac{\tilde{I}}{2})$. Suppose the firm value without the right to veto is higher, that is, $V_v < V_{nv}$. Then:

$$\begin{aligned} \psi_3(\hat{I}) - \psi_2(\frac{\tilde{I}}{2}) &= \frac{1}{8\alpha^2}\left(\pi^2 - 4\pi\alpha(1+\alpha-q_{nh}\alpha) - 4\alpha^2(1+\alpha-q_{nh}\alpha-q_{th}\alpha)^2\right) \\ &\geq V_{nv} - V_v > 0 \end{aligned}$$

which implies $\pi > 2\pi_0(\alpha)$. However, that contradicts the assumption $\pi \leq (1 + \frac{q_{th}}{q_{nh}})(2\tilde{\pi}(\alpha) - \alpha^2 q_{nh} - 2\alpha^2 q_{th})$ since

$$(1 + \frac{q_{th}}{q_{nh}})(2\tilde{\pi}(\alpha) - \alpha^2 q_{nh} - 2\alpha^2 q_{th}) < 2\pi_0(\alpha)$$

since $q_{nh} < \frac{1}{\alpha}(1 + \alpha - q_{th}\alpha)$. □

4.9.5 Efficient equilibrium if renegotiation is allowed

In this appendix, we spell out the strategies and belief systems of the partial separating equilibrium described in section 7.1.

Recall that we are considering the case that partner 1 proposes renegotiation, after he has called for dissolution; full efficiency means $I = I^*$ and dissolution if and only if $s = nh$.

We proceed as follows: First, we show that efficient dissolution is established through renegotiation only if $I \in \Lambda := [\frac{q_l \tilde{I}}{2(q_{th}+q_l)}, \frac{(q_l+2q_{th})\tilde{I}}{2(q_{th}+q_l)}] \subset [0, \tilde{I}]$. Since $I^* \in [0, \tilde{I}]$, we conclude that full efficiency is restored if $I^* \in \Lambda$.

1) The following beliefs and strategies are a perfect equilibrium of the dissolution/renegotiation subgame if $I \in \Lambda$.

1a) Partner 1 requests dissolution with a price $p = p^* = I/2$ in all states and offers renegotiation if and only if $s \in \{l, th\}$, in which case he requests a transfer $t = \bar{t} := \frac{\alpha I}{2} + \frac{q_{th}\pi}{2(q_{th}+q_l)}$.

1b) Partner 2 has the following beliefs: $\Pr\{S = nh \mid t\} = 0$ if renegotiation is offered and the transfer offered is $t \leq \bar{t}$; if renegotiation is offered and the $t > \bar{t}$ or if no renegotiation is offered, $\Pr\{S = nh \mid t\} = 1$.

1c) Partner 2 accepts a renegotiation offer if and only if $t \leq \bar{t}$; if he is not offered renegotiation or rejects a renegotiation offer, he sells if $p \geq p^*$ and buys otherwise.

The associated equilibrium outcome is that the partnership is dissolved if and only if $s = nh$ and partner 1 earns the transfer $\bar{t}$ in exchange for having revoked his request for dissolution in all other states.

Given that belief system, partner 2 updates his beliefs to $\Pr\{S = l\} = q_l/(q_l + q_{th}), \Pr\{S = th\} = q_{th}/(q_l + q_{th})$ if he is offered renegotiation with $t \leq \bar{t}$ and to $\Pr\{S = nh\} = 1$ if he is not offered renegotiation. Based on these beliefs, the above strategies are mutually best replies, and the assumed beliefs are consistent with the stated strategies.

2) One can easily confirm that no (partially) separating equilibrium exists that implements efficiency if $I^* \notin \Lambda$.

3) We conclude that full efficiency may be restored only if $I^* \in \Lambda$, which occurs if and only if the parameters satisfy

$$\pi(\alpha) \in [\frac{2(q_l + q_{th})}{q_l + 2q_{th}} \left(\tilde{\pi}(\alpha) - \frac{1}{2} q_{nh}\alpha^2\right), \left(\frac{q_{th} + q_l}{q_l}\right) \left(2\tilde{\pi}(\alpha) - q_{nh}\alpha^2\right)]$$

in addition to constraints (4.4) and (4.5). If $I^* \notin \Lambda$ the above partial separating equilibrium no longer implements efficiency.

Chapter 5

Final Remarks

Although the essays included in this dissertation are self–contained, they are interrelated as all are on efficiencies and incentives in teams and partnerships. The first two essays asked the question of whether and how efficient investments can be achieved in teams, despite the compelling free–riding problem, while the third one checked the agents' investment incentives if there is possible future dissolution. Besides, all the three models were based on well–observed real life phenomena. The first one was built on the observation that there is a *de facto* tournament going on in teams and partnerships, the second on the observation that agents care about monetary payoffs of their reference group, and the third on the observation that investment and dissolution incentives have nontrivial mutual impacts.

The first two essays provide better understanding that different sharing schemes can be used to achieve efficient investments. The third essay extends the partnership dissolution literature, initiated by Cramton et al. [1987], by endogenizing the partners' dissolution decisions and accounting for investment incentives.

Nevertheless, there are still many unresolved issues. The efficiency results in the first two essays depend critically on the assumption about agents' social preferences. In the first essay, the agents are assumed to be self–interested and risk neutral, while assumed to be inequity averse in the second essay. However, in many teams, agents may have mixed social preferences, and whether full efficiency is still attainable remains an intriguing future research agenda.

The third essay should be regarded as the starting point of a more general analysis of the interrelationship between investment and dissolution incentives. The essay focuses on the widely used dissolution rule: buy–sell provision. However, it is also interesting to see how other dissolution rules, for example, winner's bid auction, loser's bid auction and bargaining with alternative offers, perform in that environment, and why auctions are rarely used although it is proclaimed to be efficient in classic partnership dissolution literature.

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Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung meiner Dissertation benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

Berlin, den 24 April 2007

Jianpei Li