Essays on Asset Pricing and the Macroeconomy

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Abstract

This thesis consists of three self-contained essays that investigate the interaction of asset prices and financial markets with the macroeconomy. All papers extend the existing literature in order to enhance the understanding of the strong degree of cross-linking between financial markets and the 'rest of the economy'. In particular, the thesis focuses on habitually formed preferences and Bayesian techniques to yield theoretical and empirical insights, which help to reduce the existing gap between asset pricing and macroeconomic literature.

The first essay examines and compares the ability of habitually formed preferences to explain the cross section of asset returns compared to successful factor models. Such consumption-based asset pricing models are based on micro-founded preferences, implying a linkage to individual and aggregate behavior. For this reason, the essay uses a Bayesian approach with a priori information derived from the empirical Business Cycle literature.

In the second essay which is joint work with Harald Uhlig, we use Bayesian techniques to estimate a DSGE model. Especially, we explore a way to include conditional second moments of asset returns into the estimation. Moreover, we constrain the estimation by a priori probabilities on the Sharpe ratio and the Frisch elasticity. By doing so, the estimated model can well jointly explain key business cycle facts, different volatilities of several asset returns, and the empirically observed equity premium.

The third essay presents a DSGE model, which covers the observed co-movements of stock market boom and bust episodes in the 1980's and 1990's and the economy. By including non-separable preferences and nominal rigidities, the model explains the simultaneous rise of consumption, output, investments, hours worked, and wages during a boom and the subsequent bust. Finally, the role of monetary policy during stock market booms is discussed, and optimal monetary policy rules are evaluated.

Keywords:
financial markets, asset pricing, Bayesian methods, second moments, habit formation, monetary policy

Zusammenfassung

Diese Dissertation beinhaltet drei eigenständige Aufsätze, die die Interaktionen von Bewertungsmodellen für Wertpapiere, Finanzmärkten und der Volkswirtschaft untersuchen. Alle drei Papiere tragen zu einem besseren Verständnis von Verknüpfungen zwischen Finanzmärkten und Realwirtschaft. Im Mittelpunkt dieser Arbeit stehen Gewohnheitspräferenzen und Bayesianische Schätzmethoden, um sowohl theoretische als auch empirische Erkenntnisse zu liefern, die helfen, die makroökonomische und die Finanzliteratur stärker zu verbinden.

Das erste Essay beschäftigt sich mit Gewohnheitspräferenzen und deren Fähigkeit, verschiedene Aktienrenditen in einem Portfolio zu erklären. Die zugrunde gelegten konsumbasierten Bewertungsmodelle basieren auf mikrofundierten Präferenzen und implizieren somit individuelles und aggregiertes Verhalten von Individuen. Aus diesem Grund werden Bayesianische Methoden genutzt, um diese a priori Information in die Schätzung einfließen zu lassen.

Im zweiten Essay, einer gemeinsamen Arbeit mit Harald Uhlig, schätzen wir ein DSGE-Modell. Hervorzuheben ist, dass wir sowohl die Momente zweiter Ordnung für Wertpapierrenditen berücksichtigen als auch die a priori Wahrscheinlichkeiten für stilisierte Fakten wie Frisch-Elastizität und Sharpe ratio. Dieses Vorgehen liefert eine Modellschätzung, die gleichzeitig Fakten der Konjunkturzyklen, Momente zweiter Ordnung von Wertpapierrenditen sowie Finanzmarktfakten besser erklären kann.

Das dritte Essay präsentiert ein DSGE-Modell, das die Interaktionen der Aktienmarktbooms zum Ende der 1980er und 1990er Jahre mit der Realwirtschaft erklären kann. Mit Hilfe nichtseparabler Präferenzen und nominaler Rigiditäten lässt sich der simultane Anstieg von BIP, Konsum, Investitionen, geleisteten Arbeitsstunden und Löhnen in dieser Zeit erklären. Abschließend wird die Rolle der Geldpolitik während Aktienmarktbooms diskutiert, und es werden optimale geldpolitische Regeln hergeleitet.

Schlagwörter:
Finanzmärkte, Wertpapierbewertung, Bayesianische Methoden, Momente zweiter Ordnung, Gerwohnheitspräferenzen, Geldpolitik

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1 Introduction

1.1 Scope of the Study

This dissertation investigates the interaction of asset prices and financial markets with the macroeconomy. The actual crisis which started as a crisis of the financial intermediaries and the subsequently encroaches on the 'real economy' and finally seems to tend to a meltdown of the whole global economy, illustrates the strong degree of cross-linking between financial markets and the 'rest of the economy'. Unfortunately, within the literature this strong relation is not reflected. In particular, for a long time financial and macroeconomic theory have developed independently from each other. Of course, some effort has already be undertaken to close the gap between both strands of literature, but the current crisis has visibly unfolded the deficits and emphasized the necessity to close this gap prospectively. During the last four years and even before this scientific gap was fascinating but also sometimes frustrating for me to deal with. This thesis reports the exploration of this research period and maybe sheds some light on the relation of asset prices and the macroeconomy and thus helps to reduce the gap.

The thesis is partitioned into three chapters, each deals with the gap between financial and macroeconomic theory by investigating different fields of interest. In the first chapter, a prominent example of the differences in both strands of literature is investigated; the problem to explain asset returns without neglecting implications of the macroeconomic theory. Mehra and Prescott (1985) have made one aspect of this problem prominent by referring to it as the equity premium puzzle. This puzzle illustrates the problem of the so-called representative agent models of asset returns as postulated by Lucas (1978) and Breeden (1979) to explain the different empirically observable returns of different asset classes, e.g. the differences between the return on equity and the risk-free return. This class of preferences is an important part of modern macroeconomics, and international economics and as discussed by

Kocherlakota (1996), any empirical defect in the representative agent model of asset returns also represents misspecifications within these other fields.

For this reason it seems necessary to combine both strands of literature. A vast literature has evolved which seeks to resolve this problem. But the phenomena have shown to be very robust under the assumptions of standard utilities, asset market completeness, and no transaction costs. There exists an extensive literature regarding each of these assumptions. In this thesis, I focus on alternative preferences, especially I highlight the strand of literature on habit formation. In this research I investigate different kinds of habit formation with respect to their ability to explain the cross section of asset returns. In contrast to most of the existing literature, which often simulates or estimates only a few asset returns, the present research estimates the preferences within a modern cross-sectional setup. Additionally, I use Bayesian techniques to evaluate the models with respect to their explanatory power regarding asset returns. This estimation approach is favourable to implement restrictions from the macroeconomic theory into the estimation. In more detail, I use a Bayesian approach based on a limited information likelihood, which is an extension of the common Generalized Methods of Moments approach often used in the empirical finance literature (Kim 2002). Unlike the recent literature, the success of a model is not only measured in terms of matching the data. Additionally, the success is measured in matching the data given a priori information about the parameters of the model. The results show that our a priori information from the business cycle literature can only be updated on a weak basis by the consumption-based asset pricing models (CBAPM). Even though the results disclose the common problems of CBAPMs, the proposed approach seems helpful to evaluate other preferences within a modern cross-sectional setup without neglecting stylized macroeconomic facts.

Another question which this thesis tries to answer is: How can we estimate DSGE models more accurately with resepect to asset prices? The estimation approaches for DGSE models have made a strong progress. Especially the usage of Bayesian techniques has become a favoured methodology to estimate these kind of models. In contrast to formerly used techniques this approach mimics the space of possible parameter estimates along a path which is reasonable from an economic perspective. Joint with Harald Uhlig, we investigate these techniques in more detail to find a way to estimate asset pricing implications within a DSGE model more correctly. By estimating DSGE models in general, the deviations of the variables from their steady state will be estimated. But especially asset pricing implications are determined by

steady state values and by second moments. The incorporation of both into the estimation seems necessary in order to estimate asset prices more correctly (see also Schmitt-Grohe and Uribe 2004). The approach used in this dissertation applies implied second moments of the model to shift the steady state values. By doing so, the final estimates do not only explain the deviations but also the mean of different asset returns correctly. Moreover, this approach allows to better estimate risk measures like the Sharpe ratio. By using different financial and macroeconomic time series and our a priori knowledge about the parameters, the model is able to better explain financial and macroeconomic stylized facts simultaneously.

As mentioned above, the current crisis highlights the necessity to understand the interactions of financial markets and the macroeconomy in more detail. This raises the question: Can policy agencies reduce or avoid distortions in the 'real economy' due to stock market booms and busts? The fourth chapter of this dissertation approaches this question. In particular in the case that stock market booms are triggered by overoptimistic expectations about the future technology and later on they bust because of the anticipated shift of technology does not occur, is investigated. The model presented in this essay is able to replicate the stylized facts of the boom and bust episodes in the 1980's and at the end of the 1990's. These episodes were characterized by a decreasing inflation and decreasing nominal interest rates during the boom of the stock market. In the literature this point is widely discussed that this could be a reason for an on-heating of the stock market booms due to established credit booms because of the reduced nominal interest rates (see e.g. Christiano, Ilut, Motto, and Rostagno 2007). I present a model which is better in line with the stylized facts of these episodes. In contrast to a large part of the literature I do not investigate, wether the central bank should directly react to asset prices or not. Moreover, the distortions of the economy are investigated under different optimized monetary policy regimes during the boom and bust episodes. The results suggest that, independently from the policy regime, if it is only focused on inflation or not, the central bank should respond negatively to past output and that in this case the monetary authority would rather increase than decrease the interest rates during such a boom episode. This confirms the findings of Cecchetti, Genberg, and Wadhwani (2002) that 'leaning against the wind' policy would reduce distortions. Additionally, a non-strict inflation targeting policy is doing better in reducing the distortion on the 'real economy' based on a stock market boom and bust.

1.2 Literature Review

In this section I overview the existing literature regarding the main topics investigated in the essays of this thesis. As mentioned before this thesis focuses on the joint explanation of asset pricing facts and on macroeconomic facts. For this reason, firstly, I investigate in more detail the recent developments in the literature of consumption-based asset pricing with a detailed view on habit formation and the impact of leisure demand of households. Furthermore, this section focuses on the use of Bayesian techniques to combine both strands of the literature. Especially, the second part of the section discusses the development of Bayesian techniques in the econometric literature. Here I focus on the application of this approach within moments estimation as well as within the field of DSGE model estimation. Finally, the last part of this section deals with the recent topic of asset market booms and busts and their consequences for the macroeconomy. In this subsection I also investigate the major developments in the literature regarding monetary policy during such episodes.

1.2.1 Consumption-Based Asset Pricing

For about 30 years Lucas (1978) and Breeden (1979) postulated the so-called representative agent models of asset returns. In this framework the consumption stream of an investor is perfectly correlated with per capita consumption. As summarized by Kocherlakota (1996) these kind of representative agent models are an important part of modern macroeconomics and international economics. To agree with Kocherlakota (1996), any empirical defect in the representative agent model of asset returns also represents misspecifications within these other areas.

For this reason, the interaction between consumption and asset prices in general and the stochastic discount factor in more detail are already well-investigated. Based on the work of Lucas and Breeden, especially standard CRRA utility models were investigated to resolve for common asset pricing facts. In particular, in their seminal paper Mehra and Prescott (1985) describe an empirical problem of the representative agent model. The authors show that under the model assumptions of Lucas, only a high degree of risk aversion explains the differences in covariances of risky returns and risk-free returns. As a result, because high values for risk aversion are rejected

by most of the macroeconomic literature, a phenomenon arises known as the *equity premium puzzle* (Mehra and Prescott 1985).

As argued by Cochrane (2001), the small values of relative risk aversion often used in the macroeconomic literature seems to be more a tradition than a fact. However, recent work by Constantinides (1990), Campbell and Cochrane (1999), or Epstein and Zin (1990, 1991) has shown that a high relative risk aversion is not necessarily needed to resolve for the differences in asset returns.

Unfortunately, also these findings suggest a high elasticity of innovations in consumption with the stochastic discount factor to resolve for stylized asset pricing facts. This elasticity is inversely related to with the elasticity of intertemporal consumption substitution (EIS) (see e.g. Lettau and Uhlig 2002). For most of the current prominent preferences this characteristic still holds. Consequently, a high elasticity of the pricing kernel with respect to innovation in consumption would imply a small EIS, which in turn implies a strong consumption smoothness by the consumers and seems implausible from a business cycle perspective (see Lucas 1990).

Besides that, the intensive investigation of CBAPMs reveals another prominent phenomenon in the literature. Solving for the risk-free rate implies that for common values of the discount factor, the risk-free rate must be high and volatile, which is both definitely not in line with the data. To generate a small and nonvolatile risk-free rate, a discount factor larger than unity is needed. But discount factors larger than one go along with negative time preferences, which is not impossible but unreasonable (see Cochrane 2001). The assumption of positive time preferences implies that people prefer early consumption, which is a cornerstone of the business cycle literature. This second phenomenon is postulated by Weil (1989) as the *risk-free rate puzzle*.[1]

These major puzzles within the macroeconomic theory have triggered a vast literature which seeks to resolve those. But the phenomena have been shown to be very robust under the assumptions of standard utilities, asset market completeness, and no transaction costs. There exists an extensive literature regarding each of these assumptions. Since, in this thesis I concentrate on alternative preferences, I want to pick this strand of literature here.

In my analysis throughout this thesis I focus one habit formation. This kind of preferences had great success during the last decades with respect to consumption-

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[1]Kocherlakota (1990) shows that a positive time preference is also guaranteed, with discount factors larger than one in a growing economy.

based asset pricing models. Moreover, the developments have found their ways
into other fields of macroeconomics. The main idea of these preferences is that
the consumption decision of an individual today also depends on past decisions or
perhaps on society levels. Especially the latter point has been already investigated
by Duesenberry (1949). This dependence was later modeled by Abel (1990) as
'Catching up with the Jones' and was also called external habit formation; and it
has became access to modern macroeconomics. Similarly the importance of former
individual consumption decisions on the decision of the individual today has been
already investigated before it becomes a substantial part of asset pricing and business
cycle literature (see e.g. Becker and Murphy 1988; Pollak 1970). However, at last the
recent work by Ferson and Constantinides (1991) and Heaton (1995) have evaluated
these preferences with respect to asset prices.

The success of these models within the asset pricing literature is at most based
on simulations (Campbell and Cochrane 1999) or the estimation of a few asset
returns (Heaton 1995). However, this kind of preferences also needs to be evaluated
within a modern cross-sectional setup. The investigation of how average returns
vary across stocks was mainly contributed to by Fama and French (1992, 1993,
1996). Nevertheless, the literature which evaluates consumption-based asset pricing
models with respect to these observations is still rare. Ferson and Constantinides
(1991) is one of the first papers that investigates a representative agent model of
asset returns with habit formation and includes the cross section of assets into their
framework. Parker and Julliard (2005), Jagannathan and Wang (2005), and Chen
and Ludvigson (2007) extended this subfield of empirical finance. Similarly, Parker
and Julliard (2005) and Jagannathan and Wang (2005), suggest that cross section
of asset returns can be explained by their exposure to 'long-run' consumption risk,
by investigating multiperiod or annual moment conditions. While the latter authors
have focused on the simple consumption based model, Chen and Ludvigson (2007)
evaluate a habit model using the Fama-French 25 size and book/market portfolio.
They investigate external as well as internal habitually formed preferences, where the
habit process is nonparametric. By comparing their results with several prominent
factor models, they conclude that their internal habit model outperforms the Fama-
French 3-factor model. The second chapter of this thesis extends this literature
in two ways. First, I use a different estimation approach as discussed in the next
subsection and more general preferences, which also include leisure.

The use of leisure or labor within the representative agent model of asset returns
is also rare, especially the estimation of such models. As highlighted by Lettau

(2003) including leisure nonseparable from consumption into the utility makes the stochastic pricing kernel less volatile than with consumption alone. But as stated by Cochrane (2005), the higher correlation of labor with asset returns may still make asset pricing work better. The implications of leisure on the equity premium and especially for the Sharpe ratio are investigated by Uhlig (2004a). The author investigated the macroeconomic consequences that occur with nonseparability of consumption and leisure if they have to be simultaneously in line with stylized asset pricing facts.

An empirical investigation of similar preferences is presented by Eichenbaum, Hansen, and Singleton (1988). The authors estimate preferences which take into account decisions about consumption and leisure one period backwards. Additionally, they also allow for durable consumption and durable leisure and not only for habit formation in consumption and leisure. Within their framework the ability of these preferences to explain the risk-free rate and wages simultaneously is tested. By taking additional macroeconomic friction like wages explicitly into account during the estimation, this paper is one of the first, which tries to combine asset returns and macroeconomics empirically.

1.2.2 Bayesian Estimation

As mentioned above, the representative agent model of asset returns is an important feature of several disciplines in modern macroeconomics. The estimation thereof cannot be done uncoupled from their implications for these fields. This was imposingly illustrated by the paper of Mehra and Prescott (1985). However, most of the empirical work mentioned above is based on the Generalized Methods of Moments (GMM) approach as postulated by Hansen (1982) and Hansen and Singleton (1982, 1983). Unfortunately, using this approach (but also by using maximum likelihood techniques) it is difficult to ensure estimates which are in line with other not explicitly modeled frictions. For this reason, using Bayesian techniques seems favorable to include a priori knowledge from other disciplines into the estimation. In contrast to a GMM or a standard maximum likelihood approach, the ability to add prior information into the estimation mimics the likelihood along an economic reasonable parameter space. Using a Bayesian approach allows to test consumption-based asset pricing models with respect to their ability to explain the cross section of asset returns without simultaneously neglecting stylized facts of recent macroeconomic research.

The Bayesian estimation approach used in the first essay follows the findings of Kim (2000, 2002). The author formulates a likelihood function based on the limited information available in the generalized method of moments (GMM) framework. The work of Kim (2002) extends the Bayesian method of moments (BMOM) approach of Zellner (1998) and Zellner and Tobias (2001) to the general situation of GMM and additionally formulates a specific likelihood function. Kim (2002) constructs a limited information likelihood (LIL) using the moments conditions of GMM while minimizing the entropy distance. The approach takes into account a set of LIL functions that fulfil the GMM moment conditions on the parameters. Afterwards, the LIL is chosen with its mode at the standard GMM estimator and closest to the true likelihood, based on the Kullback-Leibler information criterion or entropy distance. Given this LIL it is possible to implement a Bayesian inference framework, where a limited-information posterior (LIP) can be obtained by combining the LIL with a prior distribution. This approach is generalized by the work of Atkinson and Dorfman (2005) to the case of an unknown Covariance matrix using Gibbs sampling.

In addition to the mentioned techniques used in the empirical finance literature, the evaluation of DSGE models has become an important strand of literature within the macroeconomic research. During the last three decades of economic research this approach has become even more popular and also the attempt to verify it with data. Several formal and informal econometric procedures to parameterize and evaluate DSGE models have evolved, where especially the first contributions of this discipline, the quantitative evaluation was conducted without formal statistical methods (An and Schorfheide 2007).

Early, most of the literature has followed the informal calibration approach (Kydland and Prescott 1982, 1996). This was justified by the implicit model misspecifications of simple DSGE models, due to their strong restrictions to actual time series. However, the calibration approach was often criticized for several reasons.[2] One main aspect of discussion regarding the calibration methodology is the usual match of steady-state implications of the model to time series averages. By doing so the model is parameterized based on sample means by neglecting autocorrelation and cross correlations. Moreover, this framework assumes that sample means are robust to measurement errors and with respect to alternative specifications of the short-run dynamics of the model (see Hansen and Heckman 1996). This is criticized by Sargent (1989), because time series correlations and cross correlations, especially

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[2]See Hansen and Heckman (1996), Kydland and Prescott (1996), and Sims (1996) for the methodology debate about estimation and evaluation of DSGE models.

with measurement errors, can still provide more information in contrast to sample means. This latter argument is supported by Hansen and Heckman (1996), that for stochastic models, in general, it is not possible to calibrate all parameters only based on means of macroeconomic time series. Additionally, the authors argue that the usage of micro data as inputs for DSGE models can also be problematic. Because "[...] the implicit economic environments invoked to justify microeconomic estimations procedures seldom match the dynamic stochastic single-agent models [...]" (Hansen and Heckman 1996, p. 94). Finally, the authors conclude that the calibration framework has still not delivered a coherent framework for extracting parameters from microeconomic data.

As a reaction to the upcoming critique on the calibration approach and due to the developments and the improved structural models, more standard econometric techniques were used to parameterize DSGE models. The usage of econometric techniques was also a response to the formerly mentioned critique, that times series evidence is essential to determine many fundamentally aggregative parameters (Hansen and Heckman 1996). Early approaches have used Simulated Methods of Moments (SMM), e.g. Canova (1994), or a Generalized Method of Moments (GMM) approach, e.g. Christiano and Eichenbaum (1992). Both approaches were used to match moment characteristics in the model and in the data and often based on a subset of equilibrium relationships (e.g. Euler equation). Another strand of the literature uses minimum distance estimation to reduce the the differences of implied impulse responses of a Vector Autoregression and a DSGE model, like Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans (2005).

In contrast to these methods another strand of literature focuses on DSGE models as a full characterization of aggregate time series. This literature flow was contributed to by e.g. Altug (1989), McGrattan (1994), and Leeper and Sims (1994). In contrast, the full-information likelihood approach is system-based and fits the model to a vector of aggregate time series (see An and Schorfheide 2007).

Using likelihood methods goes along with a singularity problem because of a rank-deficit covariance matrix regarding the model variables. Such a misspecification occurs if the number of structural shocks in the model is smaller than the number of time series. For this reason one branch of the literature adds so-called measurement errors to the structural equations to estimate the model based on more time series (see e.g. Ireland 2004; Sargent 1989). Another branch of the literature concentrates on adding structural shocks to the system (DeJong, Ingram, and Whiteman 2000; Leeper and Sims 1994; Smets and Wouters 2003) and reducing the observed time

series used for estimation. Since within the second approach the estimation is based on a subset of time series in comparison to the involved variables of a DSGE model, this leaves some arbitrariness in the procedure of identifying parameters. On the other side, adding measurement errors to the structural equations is difficult to interpret economically.[3]

A further problem of pure maximum likelihood estimation is the so-called "dilemma of absurd parameter estimates" (see An and Schorfheide 2007). While maximum likelihood estimation is only based on the set of observations used, the final estimates of the structural parameters are often in contrast to the information that economic research has collected in the last decades. For this reason Bayesian techniques were introduced to parameterize DSGE models (e.g. DeJong et al. 2000; Smets and Wouters 2003). Within this approach the likelihood function is re-weighted with a priori information about the parameters. Of course, introducing prior information about the structural parameters may shift the peak of the posterior, which goes along with a reduced explanatory power regarding the time series of the DSGE model in comparison to a pure maximum likelihood approach. However, it increases the reasonability of the estimates itself. Recalling this fact, the prior choice during the estimation is vitally important (see Del Negro and Schorfheide 2008).

1.2.3 Monetary Policy and Financial Markets

In this thesis, I contribute to the literature on the interaction of financial markets and monetary policy. Not least because of the current crises; the question tends up: Should monetary policy react to stock market movements?

In the case the answer is yes, how exactly should central banks respond to asset price movements? The suggestions in the literature vary from preemptive approaches to reactive approaches (see Bean 2004; Greenspan 2002). The reason for the oppositional positions is the different quantification of extraordinary asset price movements as fundamental or not.

Beginning with the work of Bernanke and Gertler (2000, 2001) rapidly increasing asset prices were classified as non-fundamental movements. The literature assumes that based on an exogenous shock, the price of an asset differs from his fundamental price. The foregoing authors as well as Tetlow (2006) argue that in this context a strong inflation-targeting monetary policy would automatically reduce the distor-

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[3]See Canova (2007) for a discussion of both methodologies.

tion due to asset price movements. This conclusion is based on the simultaneously increasing inflation, due to increasing aggregate demand in the economy, which is followed by increasing marginal cost of the firm. Under this circumstance the former policy rule would be beneficial. The extension by Gilchrist and Leahy (2002) also suggests a "strict" inflation-targeting monetary authority if exogenous bubbles have a persistent effect on technology growth. Also Mishkin and White (2002) suggest that the central bank should only respond to a stock market crash in order to prevent financial instability. In this case the stock market crash is unlikely to result in changes of aggregate demand and the policy maker should not directly react to the stock market movements.

However, in a related model framework, Cecchetti, Genberg, Lipsky, and Wadhwani (2000) show that there may be some benefits to responding to asset prices and that a monetary policy can avoid an overshooting asset prices bubble. The contrasting results of the latter authors regarding Bernanke and Gertler (2000, 2001) within a similar model framework is due to the different assumption about what exactly can be observed by the policymaker (Cecchetti et al. 2002). Dupor (2002, 2005) finds similar results. He suggests that in response to inefficient shocks to investment demand, optimal policy reduces both price fluctuations and non-fundamental asset price movements. This raises the importance of both as targets of the monetary authority.

Furthermore, Cecchetti et al. (2002) have mentioned the apprehension of the possibility that a monetary policy could also heat-on the asset market boom, which could avoided by 'leaning against the wind' of interest rate changes, when disturbances originate in the money market. Various research has shown, that during the stock market booms in the 1980's and at the end of the 1990's inflation and nominal interest rates have decreased (e.g. Adalid and Detken 2007; Detken and Smets 2004; Lowe and Borio 2002). These findings suggest that reducing nominal interest rates by the monetary authority was followed by an additional boom of the credit market what could have heated on the stock market boom.

Christiano et al. (2007) show that their monetized DSGE model with a standard inflation-targeting monetary policy generates boom-bust cycles with simultaneously decreasing nominal interest rates and decreasing inflation. Their model is triggered by an over-optimistic anticipated shock about the technology level of the economy. The usage of news shock to explain stock market behavior is based on several work by Beaudry and Portier (2006).

However, their model fails to explain the simultaneous increase of wages during a stock market boom, while the authors also argue for the necessity of nominal wages rigidities to receive booms. Moreover, the model suggest an overshooting reaction of different variables that cannot be observed in the data. Christiano et al. (2007) argue that a 'leaning against the wind' policy would reduce the distortions to the stock market booms based on overoptimistic expectations about future technology. A similar approach is proposed by Gilchrist and Saito (2006). The authors argue that asset price booms occur because agents do not know the true state of technology growth but instead learn about it over time. Under these circumstances, there exists a motivation to respond to the gap between observed asset prices and their potential level, to reduce the distortions of resource allocations. However, the implied imperfect information in the economy also affects the policymaker's decision about the potential asset price, which is followed by a welfare-reducing monetary policy.

The fourth chapter in this thesis extends the research by Christiano et al. (2007). By presenting a DSGE model which is more in line with the stylized facts during the stock market booms in the 1980's and 1990's. Additionally, the analysis of different monetary policy regimes suggests that indeed 'leaning against the wind' would reduce the distortions to a comparable stock market boom.

1.3 Outline of the Thesis

The second chapter addresses the ability of consumption-based asset pricing models to explain the cross-section of asset returns. Specifically, I examine and compare the ability of habitually formed preferences in a cross-sectional setup and compares the results successful and prominent factor models within the literature. Such consumption-based asset pricing models based on micro-founded preferences imply a relation to individual and aggregate behavior. For this reason, the chapter incorporates these linkages by using a Bayesian approach with a priori information about the parameters extracted from the empirical business cycle literature. Moreover, the results are compared and discussed with respect to the estimates based on a Bayesian estimation with diffuse priors. Throughout the estimation I can identify plausible values for the elasticity of intertemporal consumption substitution as well as the Frisch elasticity. Finally, the chapter illustrates the reduced explanatory power of the investigated models with respect to asset returns, especially, to cross-

sectional returns and the Sharpe ratio, if a priori information about the parameters are incorporated.

The third chapter approaches the combination of asset pricing and the business cycle literature from a different point. The chapter uses Bayesian techniques to estimate the dynamic stochastic general equilibrium (DSGE) with macroeconomic and financial time series. In this joint work with Harald Uhlig, a way to include conditional second moments of asset returns into the estimation is explored. This approach allows to estimate the model around a more accurate specified steady state with respect to asset prices. Given the estimated model, we can explain key business cycle facts, different volatilities of several asset returns, and an equity premium more close to the observed one. Additionally, the model fits historical business cycle time series as well as the observed return on equity. This circumstance allows to discover prominent shocks of the last decades and to investigate the co-movements of asset prices and the macroeconomy in more detail.

The fourth chapter of this thesis examines a DSGE model which covers the observable co-movements of stock market boom and bust episodes in the 1980's and 1990's and the economy. The boom episodes within the model are triggered by news shocks about the future technology. By additionally including nonseparable preferences and nominal rigidities, the model explains the simultaneous rise of consumption, output, investments, hours worked, and wages during a boom and their later bust. Furthermore, featuring a standardized monetary authority, the model also replicates the observed fact of declining inflation during the boom episodes. As a result the model allows for a more fundamental discussion of central bank activism during stock market booms. The paper concludes that a monetary authority, which is not only "strict" inflation-targeting but also continuous and moderate, can reduce the welfare losses through stock market booms and busts.

2 Habit Preferences and the Cross Section of Asset Returns: A Bayesian Approach

This chapter examines and compares the ability of habitually formed preferences to explain the cross section of asset returns compared to successful factor models in the literature. Such consumption-based asset pricing models are based on micro-founded preferences implying a linkage to individual and aggregate behavior. The present chapter incorporates these linkages by using a Bayesian approach with a priori information about the parameters derived from the empirical Business Cycle literature. Throughout the estimation I identify plausible values for the elasticity of intertemporal consumption substitution as well as the Frisch elasticity. Finally, the chapter illustrates the reduced explanatory power of the proposed models with respect to asset returns, especially, to cross-sectional returns and the Sharpe ratio, if a priori information about the parameter are incorporated.

2.1 Introduction

This paper examines a general class of consumption-based asset pricing models (CBAPM) with respect to their ability to explain the historically observed asset returns and especially the cross section of asset returns. A central point of interest in this research is habit formation. This kind of preferences had become a prominent explanation theory of asset returns in the last decades (see e.g. Abel 1990; Campbell and Cochrane 1999; Constantinides 1990). Moreover, the theory has influenced the business cycle literature and it is a prominent feature to model individual preferences (Boldrin, Christiano, and Fisher 1997; Uhlig 2007).

The success of these model in the asset pricing literature is often based on simulations

(Campbell and Cochrane 1999) or the estimation of few asset returns (Heaton 1995). However, this kind of models also needs to be evaluated within a modern cross-sectional setup. A lead position in this field is captured by Ferson and Constantinides (1991). Similar to more recent work in this discipline, the special relationship of these models to the Business Cycle literature is often neglected, or this coherence is the central point of the investigation and necessary asset pricing facts are neglected (Eichenbaum et al. 1988). Motivated by the latter point, the intention of the present paper is to bring in line both strands of literature and their individual interests. The present paper estimates different prominent habit preferences and discusses them in a modern cross-sectional setup. Moreover, it introduces a technique to incorporate stylized Business cycle facts into the estimation due to the usage of a Bayesian inference framework as developed by Kim (2002).

The importance of an incorporation of business cycle facts into the estimation of CBAPMs is obvious and it is needed to judge the estimates not only on their explanatory power regarding asset returns. By investigating CBAPMs from an asset pricing perspective, it is well known that a high volatile stochastic discount factor is necessary in order to solve for observed excess returns, the Sharpe ratio, and to explain the cross-section of asset returns. As discussed by Lettau and Uhlig (2002), a general characteristic for a wide class of CBAPMs is that their success in explaining asset returns depends on the elasticity of the stochastic discount factor with respect to innovations in consumption.

The interaction between innovations in consumption and the stochastic discount factor are already well-investigated. For example in standard CRRA utility models (e.g. Lucas 1978; Mehra and Prescott 1985) or time-separable preferences (e.g. Abel 1990) a high elasticity is appropriate to resolve for common asset pricing facts. In these special classes of preferences the elasticity coincides with the relative risk aversion regarding consumption. Consequently, high values for the elasticity are rejected by most of the macroeconomic literature, and a phenomenon arises known as the *equity premium puzzle* (Mehra and Prescott 1985).

As argued by Cochrane (2001), the small values of relative risk aversion often used in the macroeconomic literature seem to be more a tradition than a fact. However, recent work by Constantinides (1990), Campbell and Cochrane (1999), or Epstein and Zin (1990, 1991) has verified that the linkage between this elasticity and the relative risk aversion can be broken up. This literature presents a possibility to resolve stylized asset pricing facts without high risk aversion.

However, there exists a factual relation between the elasticity of the stochastic discount factor with respect to consumption and the elasticity of intertemporal consumption substitution (EIS). Lettau and Uhlig (2002) show that both are inversely related to each other. For most of the current prominent preferences this characteristic holds. Furthermore, a high elasticity of the pricing kernel with respect to innovation in consumption implies a small EIS, which would in turn imply a strong consumption smoothness by the consumers and seems implausible from a business cycle perspective (see Lucas 1990).

Besides, the intensive investigation of CBAPMs reveals another prominent phenomenon. Solving for the risk-free rate implies that for common values of the discount factor, the risk-free rate must be high and volatile, which is both definitely not in line with the data. To generate a small and nonvolatile risk-free rate, a discount factor larger than unity is needed. But discount factors larger than unity go along with negative time preferences, which is not impossible but unreasonable (see Cochrane 2001). Positive time preferences imply that people prefer early consumption which is a cornerstone of the Business Cycle literature. This second phenomenon is postulated by Weil (1989) as the *risk-free rate puzzle.*

The foregoing prominent examples illustrate that any parameter of micro-founded preferences has a direct or indirect relation to observable aggregate or individual behavior. From this point of view it seems necessary to respect this also from an econometrically perspective. Exactly this is the motivation of the present chapter. My purpose is to investigate and compare the explanatory power of habitually formed preferences within CBAPMs to resolve for asset returns without neglecting findings from the macroeconomic literature. To do so, I use a set of intensively investigated variables from the literature, which all mainly depend on the preference parameters. These variables are the previously mentioned elasticity of intertemporal consumption substitution (EIS), the discount factor as well as the Frisch elasticity.

The present paper focuses on three prominent preference classes using habit formation. In particular, I use a more general class of preferences by allowing for leisure within the same. Of course, nonseparability of consumption and leisure might reduce the ability to resolve for stylized asset pricing facts. However, this kind of preferences has had success in the macroeconomic literature and should not be neglected per se. 1.) The first investigated preferences assume that the individual forms her decisions depending on past aggregate consumption and leisure. This external habit formation also known as "Catching up with the Joneses" is denoted *external-1l* in the following. 2.) In the second preferences the habits base on past

individual consumption and leisure. These preferences are denoted as *internal-1L*. Both models assume that the habits include only values one period backward. As shown by Campbell and Cochrane (1999), the differences between both types of habit formation will will reduce by including more periods. 3.) For the latter reason, I just investigate external habit formation with infinitely lags of consumption and leisure involved. The model, in the following referred as *external-AR*, is an extension of the successful preferences developed by Campbell and Cochrane (1999). In contrast to these authors, the preferences are nonseparable between consumption and leisure, and, moreover both habits are modeled using autoregressive processes.

The estimation approach uses a Bayesian inference framework with a priori information about the variables previously discussed. This procedure allows to compare the models with respect to their explanation power regarding asset returns, but also, it reduces the econometric investigation on a path that does not neglect well-known macroeconomic findings. Throughout this research, I identify plausible values for the EIS, the discount factor, the Frisch elasticity, and the Sharpe ratio. The EIS and the Frisch elasticity are estimated in a range between 0.2 and 0.5 across all models and parameter distributions.

However, there is no such thing as a free lunch. The Inclusion of these variables yields a nonvolatile stochastic discount factor. Obviously, this reduces the ability of a CBAPM to explain the observed asset pricing facts and especially the cross section of asset returns. I compare the different models' ability to explain the Fama-French 2x3 size/book-market returns portfolio by using posterior model probabilities. After that, I investigate how well the estimated models explain the cross section of this portfolio as well as the Fama-French 10 industry portfolio and the Fama-French 5x5 size/book-market return portfolio, by using the method of Hansen and Jagannathan (1997). Furthermore, I use successful linearized factor models as benchmark models. Finally, I compare the results of the Bayesian estimation with informative prior with estimates resulting by using non-informative (diffuse) priors.

By using diffuse priors for the estimation, most of the asset pricing facts can be resolved as well as the cross section of asset returns. Especially, the *external-AR* model and the *external-1L* model are quite successful. Introducing more restrictive a priori information about the parameters into the estimation reduces the explanatory power with respect to the cross section of asset returns dramatically. In particular, the *external-AR* seems to be deprived of its explanatory power if it has to be in line with the macroeconomic stylized facts. Finally, there exist only small differences between the different models in order to explain the cross section of asset returns.

In contrast to the pure likelihood estimation and the factor models this seems an advantage on a poor basis.

This paper is related to recent work of Ferson and Constantinides (1991) and Heaton (1995) with respect to the estimation of CBAPMs using preferences, that include habits based on past consumption decisions. An even closer relation exists to Chen and Ludvigson (2007) and Grishenko (2007). The latter investigates different kinds of habit formation and allows for a mixture of internal and external habit formation, which shows to be successful in explaining the mean returns of portfolios. However, Grishenko (2007) does not investigate the cross section of asset returns in detail. Another recent paper investigating the ability of habit preferences in order to explain the cross section of asset returns is the work by Chen and Ludvigson (2007). The authors use a nonparametric habit function and show that under this specification, an internal habit is preferable to an external habit. Furthermore, the authors postulate that the model beats the Fama-French 3-Factor model and the CAPM model based on the method derived by Hansen and Jagannathan (1997). However, both papers econometrically investigate the CBAPMs from a pure asset pricing point of view. Unfortunately, the parameter estimates are far away from our a prior information about aggregate and individual behavior.

The presenting chapter is further related to the work of Eichenbaum et al. (1988). The authors estimate preferences which take into account decisions about consumption and leisure. In contrast to the present paper, the authors also allow for durable consumption and durable leisure and not only for habit formation of consumption and leisure. They estimate the risk-free rate and take into account macroeconomic facts by a simultaneous estimation of wages. However, this paper does not consider the question of how good the model explains the cross-sections of asset returns. The present chapter has also a close relationship to the work of Uhlig (2004a). In this paper the author shows the relationship between leisure within nonseparable preferences and their importance for asset returns. The present paper picks up these point and tests these relationships empirically.

The estimation methodology in this paper follows the findings of Kim (2000, 2002). The author formulates a likelihood function based on the limited information available in the generalized method of moments (GMM) framework. Further, this limited information likelihood (LIL) allows to implement a Bayesian inference framework, where the posterior is obtained from a likelihood and a prior. The work by Kim (2002) extends the Bayesian method of moments (BMOM) approach of Zellner (1998) and Zellner and Tobias (2001) to the general situation of GMM and addi-

tionally formulates a specific likelihood function. In contrast to a GMM or a pure likelihood approach, the ability to add prior information into the estimation mimics the likelihood along an economic reasonable parameter space. Atkinson and Dorfman (2005) extend the findings of Kim (2002) to a framework, which allows for a nonconstant covariance matrix.

The chapter is organized as follows. Section 2.2 describes the preferences and the habit formations explicitly and reviews the main asset pricing implications of CBAPMs. After introducing the data used in this study in section 2.3, section 2.4 reviews the econometric methodology and in particular, the used Bayesian framework based on the limited likelihood framework postulated by Kim (2002). That section also discusses the prior choice and the posterior estimation approach used in the present chapter. The results of the Bayesian estimation with informative as well as with diffuse prior are presented in section 2.5. In the second part of that section I compare the models based on posterior model probabilities and with respect to their ability to explain the cross section of asset returns. Section 2.6 concludes the study.

2.2 Model

2.2.1 Preferences

The individuals are assumed to be identical and live infinitely long. The representative agent maximizes his expected discounted utility, conditional on the information available at time t:

$$\max \quad E_t \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right), \tag{2.1}$$

where β represents the individual discount factor. The utility function has the form

$$U\left(C_t, L_t\right) = \frac{\left(\left(C_t - H_t^c\right)^{\chi}\left(L_t - H_t^l\right)^{1-\chi}\right)^{1-\gamma} - 1}{1 - \gamma}. \tag{2.2}$$

The agent maximizes his utility by a choice of consumption an leisure at time t. The power parameter γ reflects concavity and the parameter χ denotes the substitution between consumption and leisure. Both, leisure and consumption decisions today are affected by past consumption and leisure. These habits follow autoregressive

processes of past consumption or leisure:

$$H_t^c = \nu \sum_{j=1}^{J} \rho^{j-1} C_{t-j} \qquad H_t^l = \psi \sum_{i=1}^{I} \phi^{i-1} L_{t-i}, \tag{2.3}$$

where J, respectively I, capture the past or the memory of former consumption or leisure decisions. The parameters ρ and ϕ measure the degree of each autoregressive process. Furthermore, the parameter ν (ψ) denotes the fraction of the aggregated lagged consumption (leisure), reflecting the habit subsistence level of today's consumption (leisure).

Consumption-based asset pricing models are characterized by validity of the following condition,

$$1 = E_t \left[M_{t+1} R_{n,t+1} \right]. \tag{2.4}$$

The equation is widely known as the Lucas asset pricing formula or Euler equation. It implies that the expected benefit of holding an asset n for one period is equal to the marginal loss of consumption today, occurring due to the decision to collect this asset. The intertemporal substitution of consumption is reflected by the expression M_{t+1}, the pricing kernel or Stochastic Discount Factor (SDF):

$$M_{t+1} = \beta \frac{MU_{t+1}}{MU_t}, \tag{2.5}$$

where MU_t denotes the marginal utility regarding consumption. In the following, an additional parameter τ_c is used in order to distinguish between internal and external habit formation. In the case of internal habit formation, the marginal utility is not time-separable and $\tau_c = 1$, while for external habit formation $\tau_c = 0$ and the marginal utility reduces to the common time-separable form. Altogether, the marginal utility becomes:

$$MU_t = \chi \left(C_t - H_t^c \right)^{\chi(1-\gamma)-1} \left(L_t - H_t^l \right)^{(1-\chi)(1-\gamma)} - \tag{2.6}$$
$$\tau_c \chi E_t \left[\sum_{j=0}^{J} \beta^j \left(C_{t+j} - H_{t+j}^c \right)^{\chi(1-\gamma)-1} \left(L_{t+j} - H_{t+j}^l \right)^{(1-\chi)(1-\gamma)} \frac{\partial H_{t+j}^c}{\partial C_t} \right]$$

In the following, I capture the relationship between consumption (leisure) and their corresponding habits by convenient surplus ratios. The consumption surplus ratio, S_t^c, and the leisure surplus ratio, S_t^l, are given by:

$$S_t^c = \frac{C_t - H_t^c}{C_t} \quad \text{and} \quad S_t^l = \frac{L_t - H_t^l}{L_t} \tag{2.7}$$

Hence, the fraction of both habits of current consumption or leisure is given by,

$$\frac{H_t^c}{C_t} = 1 - S_t^c \quad \text{and} \quad \frac{H_t^l}{L_t} = 1 - S_t^l \, , \tag{2.8}$$

which obviously depend on the explicit habit formation used. For further convenience, I introduce the variables a and b,

$$a = \chi(1-\gamma) - 1 \quad \text{and} \quad b = (1-\chi)(1-\gamma) \, . \tag{2.9}$$

Given the above notation and inserting the derivation of the consumption habit process from (2.3), the marginal utility can be rewritten as,

$$MU_t = \chi (S_t^c C_t)^a \left(S_t^l L_t\right)^b - \tau_c \nu \chi E_t \left[\sum_{j=1}^{J} \beta^j \left(S_{t+j}^c C_{t+j}\right)^a \left(S_{t+j}^l L_{t+j}\right)^b\right] \, , \tag{2.10}$$

and the pricing kernel M_{t+1} becomes:

$$M_{t+1} = \beta \left(\frac{S_{t+1}^c}{S_t^c} \frac{C_{t+1}}{C_t}\right)^a \left(\frac{S_{t+1}^l}{S_t^l} \frac{L_{t+1}}{L_t}\right)^b \tag{2.11}$$

$$\left[\frac{1 - \tau_c \nu E_{t+1}\left[\sum_{j=1}^{J} \beta^j \left(\frac{S_{t+j+1}^c}{S_{t+1}^c} \frac{C_{t+j+1}}{C_{t+1}}\right)^a \left(\frac{S_{t+j+1}^l}{S_{t+1}^l} \frac{L_{t+j+1}}{L_{t+1}}\right)^b\right]}{1 - \tau_c \nu E_t\left[\sum_{j=1}^{J} \beta^j \left(\frac{S_{t+j}^c}{S_t^c} \frac{C_{t+j}}{C_t}\right)^a \left(\frac{S_{t+j}^l}{S_t^l} \frac{L_{t+j}}{L_t}\right)^b\right]}\right] \, .$$

This general specification encompasses various prominent and successful consumption based asset pricing models in the literature. In particular, for $\chi = 1$ the preferences only depend on consumption, which is reflecting separable utilities as often used in the literature. In this case, depending on the lags (J) involved to form the consumption habit and of the time-separability of the preferences (external habit formation or internal habit formation), the pricing kernel reduces to those proposed by e.g. Lucas (1978), Abel (1990), Ferson and Constantinides (1991), and Heaton (1995). Moreover, setting the parameter $J = \infty$, equation (2.11) is equivalent to the one used by Constantinides (1990) for internal habit formation or to Campbell and Cochrane (1999) for external habit formation.

Given that $\chi < 1$, the preferences are nonseparable between consumption and leisure. Eichenbaum et al. (1988) have investigated this case, by forming a time-separable utility, which accounts for consumption and leisure decisions one period backward ($J = I = 1$). In contrast to the work presented in this chapter, the

authors allow not only for habit formation but also for durable consumption and leisure ($H_t^c < 0$ and $H_t^l < 0$).

In the following, I investigate a subset of possible preferences with respect to their ability to explain cross-sections of asset returns. As shown by Campbell and Cochrane (1999), for these power-utility models with a linear habit formation process and a constant interest rate, there exists for $J \nearrow \infty$ no effect on allocations and asset prices (for similar results see also Hansen and Sargent 2005). Consequently, I investigate only external habit formation for $J = I = \infty$ and differentiate between internal and external habit formation only for limited periods covered by the habits, $I = J = 1$. Following the argumentation of Campbell and Cochrane (1999), the differences between internal and external habit formation should be most significant with $I = J = 1$ and continuously reducing by increasing J or I up to infinity.

To conclude, I investigate internal habit formation with one lag (*internal-1L*) and external habit formation with one lag (*external-1L*) to figure out, wether time-separable or time-nonseparable preferences performs better.

Additionally, I want to investigate if an inclusion of more lags into the habit formation improves the model in order to explain the cross-sections of asset returns. To do so, I use an extension of the Campbell-Cochrane preferences (*external AR*) to investigate the case of $I = J = \infty$. The evaluation of the pricing kernel and the necessary assumptions about the autoregressive processes for consumption an leisure, following Campbell and Cochrane (1999), can reviewed as follows.

The pricing kernel of the extended Campbell-Cochrane utility is equivalent to equation (2.11), with $\tau_c = 0$ and looks like:

$$M_{t+1} = \beta \left(\frac{S_{t+1}^c}{S_t^c} \frac{C_{t+1}}{C_t} \right)^a \left(\frac{S_{t+1}^l}{S_t^l} \frac{L_{t+1}}{L_t} \right)^b ,$$

with the consumption and leisure surplus ratios as described in (eq. 2.7). Furthermore, assume that log-consumption follows an i.i.d. process with trend[1],

$$c_{t+1} = g_c + c_t + \varepsilon_{c,t+1} \qquad \varepsilon_{c,t+1} \sim \text{i.i.d.} \mathcal{N} \left(0, \sigma_c^2 \right) , \qquad (2.12)$$

while leisure is a stationary i.i.d. process

$$l_{t+1} = l_t + \varepsilon_{l,t+1} \qquad \varepsilon_{l,t+1} \sim \text{i.i.d.} \mathcal{N} \left(0, \sigma_l^2 \right) . \qquad (2.13)$$

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[1]Note, up from here, lower case letters denote the logarithm of capital letters.

Both surplus ratios follow an auto-regressive heteroscedastic process. I follow Campbell and Cochrane (1999) and use the following AR(1) process for the consumption surplus,

$$s_{t+1}^c = (1 - \rho)\, \bar{s}^c + \rho s_t^c + \lambda \left(s_t^c\right) \varepsilon_{c,t+1} \tag{2.14}$$

and model a similar one for the leisure surplus ratio,

$$s_{t+1}^l = (1 - \phi)\, \bar{s}^l + \phi s_t^l + \lambda \left(s_t^l\right) \varepsilon_{l,t+1} \; . \tag{2.15}$$

An important feature by Campbell and Cochrane (1999) is the constant relative risk-free rate. To ensure this assumption the authors introduced the sensitivity function $\lambda\left(s_t^c\right)$ to control the response of the consumption surplus ratio to random changes in consumption growth. This sensitivity function also ensures that consumption is always above the corresponding habit, since $S = \exp(s) > 0$. The final feature of this function ensures that habit is predetermined near its steady state. In the present paper I also follow Campbell and Cochrane (1999) and use the same sensitivity function for the leisure surplus ratio.

$$\lambda\left(s_t^k\right) = \begin{cases} 1/\bar{S}^k \sqrt{a - 2\left(s_t^k - \bar{s}^k\right)} - 1 & s_t^k < s_{\max}^k \\ 0 & s_t^k \geq s_{\max}^k \end{cases} \; , \tag{2.16}$$

where $k \in \{c, l\}$ and

$$s_{\max}^k = \bar{s}^k + \frac{1}{2} - \frac{\left(\bar{S}^k\right)^2}{2} \; . \tag{2.17}$$

The steady-state values for both surplus ratios are given by:

$$\bar{S}^c = \sigma_c \sqrt{\frac{-a}{1 - \rho}} \quad \text{and} \quad \bar{S}^l = \sigma_l \sqrt{\frac{-b}{1 - \phi}} \tag{2.18}$$

Through out the estimation I use these steady-state values as starting values (s_0^k) to obtain the surplus ratio processes. Due to this, the remaining parameters $\theta = [\beta, \gamma, \chi, \rho, \phi]$ are obtained through the estimation.[2]

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[2]This approach is similar to Engsted and Møller (2008), a different approach is e.g. to extend the parameters to estimate by s_0, as done by Fillat and Gardunõ (2005). The parameters g_c, σ_c for log-consumption, as well as σ_l for log-leisure are calculated from the corresponding time series used in this chapter (see section 2.3).

2.2.2 Asset Pricing Implications

In this subsection I derive several spillover effects of the Business Cycle theory into the Asset Pricing theory, by using consumption-based preferences. The analyze is quite common and follows the main literature (Campbell 2003; Cochrane 2001). Major parts of this analyze is following, e.g. by Lettau and Uhlig (2002) and Uhlig (2004a). In the following, I review these findings and their derivations for all preferences used in this paper and figure out the importance of the respective preference by evaluating stylized business cycle facts like the Frisch elasticity and the EIS as well as asset pricing facts like the Sharpe ratio.

At first let me decompose the logarithmic returns and the logarithmic pricing kernel into their expected values and innovations:

$$r_{i,t+1} = E_t r_{i,t+1} + \varepsilon_{R,t+1} \quad \text{and} \quad m_{t+1} = E_t m_{t+1} + \varepsilon_{M,t+1}, \qquad (2.19)$$

where $r_i = \log(R_i)$, $m = \log(M)$, while the innovations of the log-returns of asset i and the logarithmic pricing kernel are assumed to be normally distributed with $\varepsilon_{R_i,t} \sim \mathcal{N}\left(0, \sigma^2_{R_i,t}\right)$ and $\varepsilon_{M,t} \sim \mathcal{N}\left(0, \sigma^2_{M,t}\right)$.

Assuming that the pricing kernel and asset returns are log-normal distributed and homoscedastic, the Euler equation (2.4) for any asset i can be written as:

$$0 = E_t r_{i,t+1} + E_t m_{t+1} + \frac{1}{2}\left(\sigma^2_{R_i} + \sigma^2_m + 2\sigma_{R_i m}\right), \qquad (2.20)$$

where $\sigma_{R_i m}$ is the unconditional covariance of the innovations in returns and the pricing kernel. Obviously, given this equation the expected return of a risky asset is given by,

$$E_t r_{i,t+1} + \frac{\sigma^2_{R_i}}{2} = -E_t m_{t+1} - \frac{1}{2}\left(\sigma^2_m + 2\sigma_{R_i m}\right), \qquad (2.21)$$

where the second term on the left hand side is the Jensen's inequality adjustment, because of using expectations of log returns. For risk-less asset the variance and covariance are zero and (2.21) reduces to:

$$E_t r_{f,t+1} = -E_t m_{t+1} - \frac{\sigma^2_m}{2} \qquad (2.22)$$

Using both, (2.21) and (2.22), its obviously that the risk premium of a risky asset i

over a riskless asset is given by:

$$E_t \left[r_{i,t+1} - r_{f,t+1} \right] + \frac{\sigma_{R_i}^2}{2} = -\sigma_{R_i m}. \tag{2.23}$$

Given this general derivation, it is easy to resolve these steps given the preference-based pricing kernel, which depends on leisure and consumption. Similarly, we can decompose log-consumption and log-leisure into,

$$c_{t+1} = E_t c_{t+1} + \varepsilon_{c,t+1} \quad \text{and} \quad l_{t+1} = E_t l_{t+1} + \varepsilon_{l,t+1}, \tag{2.24}$$

and the pricing kernel M can now be decomposed into:

$$m_{t+1} = E_t m_{t+1} + \eta_{mc} \varepsilon_{c,t+1} + \eta_{ml} \varepsilon_{l,t+1}, \tag{2.25}$$

where η_{mc} and η_{ml} are the elasticities of the pricing kernel with respect to innovations in log-consumption and log-leisure. The innovations in log-consumption and log-leisure are assumed to be independently normally distributed with $\varepsilon_{c,t} \sim \mathcal{N}\left(0, \sigma_{c,t}^2\right)$ and $\varepsilon_{l,t} \sim \mathcal{N}\left(0, \sigma_{l,t}^2\right)$.

Given these assumptions the corresponding risk premium can be calculated as:

$$E_t \left[r_{i,t+1} - r_{f,t+1} \right] + \frac{\sigma_{R_i}^2}{2} = -\left(\eta_{mc} \sigma_{R_i c} + \eta_{ml} \sigma_{R_i l} \right). \tag{2.26}$$

As shown by Hansen and Jagannathan (1991), the first-order condition for excess returns, $0 = E_t \left[M_{t+1} R_{t+1}^e \right]$, implies that the Sharpe ratio (SR) for any asset is given by:

$$SR_t = \max_{\{\text{all assets}\}} \frac{E_t \left(R_{t+1}^e \right)}{\sigma_t \left(R_{t+1}^e \right)} = -\rho_t \left(M_{t+1}, R_{t+1}^e \right) \frac{\sigma_t \left(M_{t+1} \right)}{E_t \left(M_{t+1} \right)} \leq \frac{\sigma_t \left(M_{t+1} \right)}{E_t \left(M_{t+1} \right)}, \tag{2.27}$$

where ρ_t is the conditional correlation between the excess return and the pricing kernel. Using the log-normality characteristics of M, the largest possible Sharpe ratio is given by (see Campbell and Cochrane 1999):

$$SR_t = \frac{\sigma_t \left(\exp \left(M_{t+1} \right) \right)}{E_t \left(\exp \left(M_{t+1} \right) \right)} = \left(\exp \left(\sigma_{m,t}^2 \right) - 1 \right)^{1/2} \tag{2.28}$$

For the nonseparable preferences used in this paper this implies that the highest

possible Sharpe ratio is given by:

$$SR_t = \left(\exp\left(\eta_{mc}^2 \sigma_{c,t}^2 + \eta_{ml}^2 \sigma_{l,t}^2\right) - 1\right)^{1/2} \tag{2.29}$$

This representation of the Sharpe ratio implies a perfect correlation between the pricing kernel and asset returns. Within CBAPM the pricing kernel is driven by consumption, and also leisure in my case, where consumption has an smaller correlation than one and next to that leisure is negative correlation with asset returns. A more accurate representation for the Sharpe ratio can be found by assuming that the risk premium is measured as $R_t^e = R_t/R_t^f$. Using (eq. 2.26), the Sharpe ratio is given by:

$$SR_t = -\eta_{mc}\sigma_{c,t}\rho_{cR} - \eta_{ml}\rho_{lR}\sigma_{l,t} , \tag{2.30}$$

where ρ_{cR}, ρ_{lR} are the correlation of consumption and leisure with asset returns.

By equations 2.26 2.29, the risk premium and the Sharpe ratio depend only on the elasticities η_{mc} and η_{ml}, which are both denoted by the preference parameters. To illustrate the spillover effects between the Business Cycle literature and asset pricing literature by using consumption-based asset pricing models, let investigate these parameter in more detail (e.g. Lettau and Uhlig 2002; Uhlig 2004a).

For all power utility models e.g. CRRA, habit formation, "Catching up with the Joneses", or Campbell-Cochrane as widely discussed in the literature, and also for the extensions used in this paper holds that the elasticity η_{mc} is related to the elasticity of intertemporal consumption substitution (EIS) since:

$$\eta_{mc} = -\frac{1}{\text{EIS}} . \tag{2.31}$$

To elicit the elasticities η_{mc} and η_{ml} for the preferences used in this paper, first, I investigate the internal and external habit model. Recall, that log-consumption and log-leisure each follow an auto-regressive processes described in (eq. 2.24) and note that the pricing kernel (eq. 2.11) depends on conditional expectations. As shown by Lettau and Uhlig (2002) the elasticity η_{mc} can be derived as:

$$\eta_{mc} = \frac{a}{1 - \dfrac{\bar{H}^c}{\bar{C}}} \cdot \frac{1 + \tau_c \dfrac{\bar{H}^c}{\bar{C}} \beta \nu e^{ag_c}}{1 - \tau_c \beta \nu e^{ag_c}} , \tag{2.32}$$

where g_c is the trend of log-consumption and $\bar{H}^c/\bar{C} = \nu e^{-g_c}$ the steady-state value

of the fraction of habit of total consumption. As mentioned before, τ_c controls for internal vs. external habit formation. In the case of external habit formation the formula reduces to the first factor of the right hand side of 2.32, represents the effect that the elasticity of changes in consumption depends on the effective consumption, which is a proportion of consumption net of the corresponding habit (Lettau and Uhlig 2002). The second factor of the right hand side of 2.32 only occur for internal habit formation ($\tau_c = 1$) and represents that within time-nonseparable preferences an increase in consumption today also has an effect on the marginal utility tomorrow. Recapturing the formulation for a in eq. 2.9, if the parameter χ decreases, the proportion of consumption with respect to leisure in the utility decreases, and the elasticity η_{mc} decreases as well, and finally the willingness to substitute consumption between today and tomorrow, as captured by the EIS increases. A similar result can be found for the second elasticity,

$$\eta_{ml} = \frac{b}{1 - \dfrac{\bar{H}^l}{\bar{\bar{L}}}} \cdot \frac{1 + \tau_c \beta \psi \nu e^{a g_c}}{1 - \tau_c \beta \nu e^{a g_c}} \ , \tag{2.33}$$

where the steady state value of the fraction of habit of total leisure is $\bar{H}^l/\bar{L} = \psi$ because leisure is modeled as a stationary AR(1)-process. As shown by Uhlig (2004a), this elasticity is the cross-derivative of consumption and leisure.

Note that the elasticities η_{mc} and η_{ml} are, up to here, both constant over time. Evaluating these elasticities for the preferences used in the *external-AR* model yields a time-varying elasticity:

$$\eta_{mc,t} = -\frac{1}{\text{EIS}_t} = a \cdot (1 + \lambda(s_t^c)). \tag{2.34}$$

The time-variation of η_{mc} is an important feature of the model by Campbell and Cochrane (1999), because it allows to model a time-varying Sharpe ratio as observed in the data. Therefore, the EIS is also changing over time. For η_{ml} the findings are similar:

$$\eta_{ml,t} = b \cdot \left(1 + \lambda\left(s_t^l\right)\right) \tag{2.35}$$

To compare the EISs and Sharpe ratios implied by the different models, I refer to averages over time for the elasticities, EIS, or the Sharpe ratio for these preferences.

As shown, a CBAPM prices assets by the elasticities η_{mc} and η_{ml}, which are denoted only by the preference parameters. These parameters also specify individual behavior as illustrated by EIS. Furthermore, the Business Cycle literature uses the

marginal utilities of leisure and consumption to resolve for wages, which the agents demand for a reduction of his leisure. In absence of any real or nominal rigidities, such frictionless wages are given by:

$$w_t^f = \frac{MU_t^l}{MU_t^c} \ ,$$

where MU_t^c is the marginal utility regarding consumption and MU_t^l the marginal utility regarding leisure. Obviously, the frictionless wage, w^f, is the pricein additional consumption units the agent demands for reducing leisure by one unit.

As motivated by Uhlig (2007), the Frisch elasticity (FE) seems to be a good measure to capture this additional feature of the preferences. Moreover, the Frisch elasticity is a well investigated measure in the empirical literature (see section 2.4.1). It is defined as the elasticity of labor supply with respect to frictionless wages, holding the marginal rate of consumption constant,

$$\text{FE} = \left. \frac{dn}{dw^f} \frac{w^f}{n} \right|_{U_c} . \tag{2.36}$$

For the preferences used in this paper the Frisch elasticity can be evaluated as:

$$\text{FE} = \frac{U_n}{\bar{n} \left[U_{nn} - \frac{U_{nc}^2}{U_{cc}} \right]} \tag{2.37}$$

Under the common assumption that in the steady state, leisure is given by two-third of the total time endowment ($\bar{l} = 2/3$), the Frisch elasticity can be written depending on preference parameters:

$$\text{FE} = \frac{(1-\psi)\,\bar{l}}{\left(1-\bar{l}\right)} \cdot \frac{(1-\chi\,(1-\gamma))\,(1-\tau\beta\psi)\,(1+\tau\beta\nu^2)}{\gamma + \tau\beta\,(1-\chi)\,\chi\left[(\nu-\psi)^2\,(1+\gamma^2) + 4\nu\psi + 2\psi^2 + 2 - \left[\frac{\nu^2+(1+\beta)\psi^2}{(1-\chi)\chi}\right]\right]} \tag{2.38}$$

The equation looks quite scary. This is due to the fact that both consumption and leisure decision today have an effect on the marginal utilities of leisure and of consumption tomorrow. Furthermore, the equation nicely illustrates, what happens if the number of former periods within the habits is increasing. For $I = J \nearrow \infty$ the Frisch elasticity reduces to the well-known solution for external habit formation, namely:

$$\text{FE} = (1-\psi)\,\frac{\bar{l}}{\left(1-\bar{l}\right)} \cdot \frac{1-\chi\,(1-\gamma)}{\gamma} \tag{2.39}$$

This confirms the discussion above, that including lags into the internal habit, removes the differences between internal and external habit formation.

Solving for the Frisch elasticity for the *external-AR* model, the extended Campbell-Cochrane preferences, the following equation obtains:

$$\text{FE}_t = S_t^l \frac{\bar{l}_t}{\left(1 - \bar{l}_t\right)} \cdot \frac{1 - \chi\left(1 - \gamma\right)}{\gamma} \ . \tag{2.40}$$

Similar to EIS and Sharpe ratio the Frisch elasticity is time-varying for this utility function. The second term of the right hand side of this equation is the Frisch elasticity if no habit in leisure would exist ($\phi = 0$). This part of the equation is constant over time. If the leisure of the individual is only a little bit larger than his habit of leisure, S_t^l is small and the Frisch elasticity decreases. This means that the individual is less willing to increase working time if wages are increasing. By contrast when S_t^l is large, the agent would increase his working time also for small increases in wages.

2.3 Data

This section briefly describes the time series used and if necessary, some of the calculations done. The time span of all time series covers 1965 until the end of 2007. A more detailed description of the data set and the source of the data can be found in appendix A.1.1

Monthly private consumption is measured as expenditures on nondurables and services. To receive real values, nominal private consumption is deflated by the personal consumption deflator of the Bureau of Economic Analysis. The deflator is a chain-type price index with 2000 = 100. Given this price deflator a monthly inflation rate is calculated. Finally, the real consumption per capita series is derived by using the Civilian Labor Force series of the U.S. Department of Labor: Bureau of Labor Statistics (BLS) for persons 16 years of age and older as a proxy for the population.

The monthly leisure growth time series used is based on an index series for aggregate working hours per week: total private industries, from the Bureau of leisure statistics. Monthly leisure growth is measured as the opposite of total working hours growth, corrected by changes of population as measured above.

In the first part of this paper I estimate a portfolio of asset returns. Later on I

investigate the ability of the estimated parameters to explain the cross-section of asset returns of this and two other portfolios. All three portfolios in this study contain next to stock returns also the return of the relative risk free rate.

As a proxy for the relative risk-free rate, I use the three-month Treasury Bill Rate available from the Federal Reserve Board of Governors.

The main portfolio used in both parts of this paper is the 6 size/book-market returns portfolio. It contains the return of six portfolios, which are the intersections of 2 portfolios formed on size (market equity, ME) and 3 portfolios formed on the ratio of book equity to market equity (BE/ME). *Portfolio I* contains, inclusive with the relative riskless return, 7 asset returns.

The second Portfolio contains the above-mentioned T-Bill rate and the 10 Industry Portfolios, where returns of each NYSE, AMEX, and NASDAQ stock are assigned to an industry portfolio at the end of June of year t based on its four-digit SIC code at that time. There are 11 asset returns in *Portfolio II*.

In the third portfolio there are 25 size/book-market value weighted returns for NYSE, AMEX, NASDAQ, where the returns are created using the CRSP database. It contains value-weighted returns for the intersections of 5 market equity categories and 5 book equity-market equity categories. The portfolios are constructed at the end of June in each year. *Portfolio III* is the most comprehensive portfolio used in this analysis and includes 26 asset returns.

These three portfolios of asset returns are similar to those of Chen and Ludvigson (2007) and based on data available on Kenneth French's website. All asset returns include dividends, measured on a monthly basis. Finally, all nominal asset returns as well as the T-Bill rate are deflated using the previously mentioned inflation time series.

		Mean	Standard deviation
Stock Return	r_t	7.88	16.79
T-Bill Return	r_t^f	1.67	0.79
Risk Premium	r_t^e	6.30	16.74
Consumption Growth	Δc_t	2.02	1.34
Leisure Growth	Δl_t	-0.22	1.78

Table 2.1: Selected stylized asset pricing facts between 1965:1 and 2007:12. All data are in logarithmic, annualized values. Stock return includes all value-weighted returns of the NYSE, AMEX, and NASDAQ. The risk premium is calculated as excess return, $\log\left(R_t/R_t^f\right)$.

The tables 2.1 and 2.2 list the main stylized facts of the time series used in this paper, especially, mean, standard deviations, and correlations are used later during the estimation.

	R_t	R_t^f	Δc_t	Δl_t
R_t	1	0.0902	0.1885	-0.0172
R_t^f		1	0.1777	0.0086
Δc_t			1	-0.1892
Δl_t				1

Table 2.2: Correlation of selected monthly time series between 1965:1 and 2007:12.

Recalling the asset pricing implications in the previous section and investigating these stylized facts in more detail, it is easy to recover the fact that standard CBAPMs need high values for η_{mc} to solve for e.g. the risk premium and the Sharpe ratio.

2.4   Estimation Technique

In the this section I describe the econometric technique used in this research. Here of particular importance is the transfer of the pure GMM estimation as introduced

by Hansen (1982) and Hansen and Singleton (1982) into a Bayesian framework. In this, I follow the work of Kim (2000, 2002) and later extended by Atkinson and Dorfman (2005).

Starting from the Euler equation, evaluated in equation 2.4, we can write a set of n moments, where n is the number of assets in a portfolio.

$$h\left(x_{t+1}, \theta\right) = \left[\begin{array}{c} M_{t+1} R_{1,t+1} - 1 \\ \vdots \\ M_{t+1} R_{N,t+1} - 1 \end{array}\right] \tag{2.41}$$

Denote $u_{t+1} = h\left(x_{t+1}, \theta_0\right)$ the standard first-moment condition for the estimator $\theta_0 \in \Theta$ and the data set x_{t+1} than,

$$E_t\left[u_{t+1}\right] = 0, \tag{2.42}$$

has to be fulfilled with the additional assumption that the m constitutes of u_{t+1} have finite second moments. A standard assumption in the GMM of Hansen (1982) is that u_{t+1} has to be stationary and ergodic. We know that the gross returns are stationary and can assume that leisure is also stationarily. Because of the trend in consumption, it is necessary to ensure the stationarity of the Euler equation throughout the estimation (see Hansen, Heaton, and Yaron 1996).

GMM estimations often use instruments. In this paper, however, I do not use instrumental variables. Nevertheless, I review the estimation technique with instruments to allow for a more general introduction of the method. Assume that a vector of instruments contains a constant and a set of other instrumental variables i:[3]

$$z_t = \left[1, z_{1,t}, \ldots, z_{i,t}\right]. \tag{2.43}$$

Let z_t denote a q-dimensional vector with finite second moments observable by the econometrician. Define

$$f\left(x_{t+1}, z_t, \theta\right) = h\left(x_{t+1}, \theta_0\right) \otimes z_t. \tag{2.44}$$

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[3]The related literature (e.g., Chen and Ludvigson 2007; Grishenko 2007) often use variables that are tested on their predictive power for asset returns. Common choices are the *real relative T-bill rate* (Campbell 1991; Hodrick 1992) and the log consumption-wealth ratio, *cay*, introduced by Lettau and Ludvigson (2001). Furthermore, the GMM literature includes different lagged explanatory variables and observation variables into the instrument vector (e.g., Hansen and Singleton 1982).

where f maps $R^k \times R^q \times R^l$ into R^r, with $r = m \cdot q$ and $\otimes$ is the Kronecker product of the moments and the instruments. Using this expression, the unconditional moment conditions can expressed by

$$E\left[f\left(x_{t+1}, z_t, \theta_0\right)\right] = 0. \tag{2.45}$$

This unconditional expectation represents a set of r population orthogonality conditions from which an estimator θ can be constructed, given that r is at least as a large as the number of unknown parameters l (Hansen and Singleton 1982).

Define the long-run covariance matrix of f as:

$$S = \sum_{j=-\infty}^{\infty} E_P \left[f\left(x_{t+1}, z_t, \theta\right) f\left(x_{t+1-j}, z_{t-j}, \theta\right)'\right] \tag{2.46}$$

and

$$g_T\left(x_T, \theta\right) = \frac{1}{T} \sum_{t=1}^{T} f\left(x_{t+1}, z_t, \hat{\theta}\right). \tag{2.47}$$

Equations 2.45 and 2.46 form conditions on the first and second moments implied by the probability measure P. The Matrix S is the asymptotic variance of $\sqrt{T} g_T\left(x_T, \theta\right)$:

$$S = \lim_{T \nearrow \infty} E_P\left[T g_T\left(x_T, \theta\right) g_T\left(x_T, \theta\right)'\right] \tag{2.48}$$

To ensure a heteroscedasticity- and autocorrelation-consistent estimation for S, I use a Bartlett estimate as postulated by Newey and West (1987) to ensure that the estimate is positive definite:

$$\hat{S}_T = \sum_{j=-k}^{k} \left(\frac{k - |j|}{k}\right) \frac{1}{T} \sum_{t=1}^{T} f\left(x_{t+1}, z_t, \theta\right) f\left(x_{t+1-j}, z_{t-j}, \theta\right)', \tag{2.49}$$

where k is the bandwidth of autocorrelations included in the estimation. This kind of estimator only respects autocorrelations up to kth order, where $k < T$, higher-order autocorrelations are downweighted. Here, the bandwidth is automatically calculated by following the approach of Newey and West (1994).[4]

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[4]For a more detailed discussion of the Generalized Methods of Moments estimation see Davidson and MacKinnon (2004) or Hamilton (1994). The paper of Andrews (1991) provides a detailed treatment of heteroscedasticity and autocorrelation consistent estimation (HAC) and alternatives to the Newey-West estimator.

Finally, the GMM estimator $\hat{\theta}$ is the value of θ that minimizes the objective function

$$g_T\left(x_T, \theta\right)' \hat{S}^{-1} g_T\left(x_T, \theta\right) . \tag{2.50}$$

As mentioned above, the motivation of this paper is to use a priori information about the parameters of the model, where Bayesian estimation seems to be a predestined framework to capture such a priori information. To do so, it is necessary to embed the moment estimator into a likelihood-based inference framework. Such a method was studied by Kim (2000, 2002). In the following paragraphs, I review the main findings and proofs of Kim (2002) in detail.

Recall the moment conditions from 2.45, 2.46, and 2.46. Based on these moment conditions, Kim (2002) constructs a kind of semi-parametric limited information likelihood which forms a set of limited information on the data generating process. The approach is based on maximum entropy theory to obtain a likelihood that is the closest to the (unknown) true likelihood in an information distance. An implication of the conditions 2.45-2.46 is:

$$\lim_{T \nearrow \infty} E_P\left[T g_T\left(x_T, \theta\right) S^{-1} g_T\left(x_T, \theta\right)'\right] = r . \tag{2.51}$$

Given the true probability measure P, we are interested in the probability measure Q that implies the same moment conditions. Following Kim (2002), let $\mathcal{Q}$ be a family of probability measures that are continuous w.r.t. P such that

$$\mathcal{Q}\left(\theta\right) = \left\{Q : \lim_{T \nearrow \infty} E_Q\left[T g_T\left(x_T, \theta\right) S^{-1} g_T\left(x_T, \theta\right)'\right] = r\right\} , \tag{2.52}$$

where $Q \in \mathcal{Q}$ implies the same moment conditions as P in (eq. 2.51). Of course, Q is not unique and we are interested in that $Q^* \in \mathcal{Q}$ that is the closest to the true probability measure P in the entropy distance, the Kullback-Leibler information criterion distance (White 1982), or the I-divergence distance (Csiszar 1975). As proposed by Kim (2002) the minimization problem yields:

$$Q^*\left(\theta\right) = \arg\min_{Q \in \mathcal{Q}(\theta)} I\left(Q||P\right) = \arg\min_{Q \in \mathcal{Q}(\theta)} \int \ln\left(dQ/dP\right) dQ , \tag{2.53}$$

where dQ/dP is the Radon-Nikodym derivative (or density) of Q with respect to P. Thus, Q^* is the solution of the constrained minimization where the constraint refers to the moments implied in the probability measure P. Following Csiszar (1975), Q^* can be called the I-projection of P on $\mathcal{Q}$. Kim (2002) denotes $q_P^*\left(\theta\right) = dQ^*/dP$ as

Radon-Nikodym derivative of Q^* with respect to the probability measure P and call it the limited information density or the I-projection density as by Csiszar (1975). Proven by Kim (2002), $q_P^*(\theta)$ is uniform in $\theta \in \Theta$ and satisfies the moment in (eq. 2.52). The foregoing author interprets $q_P^*(\theta)$ as likelihood of θ and calls it the limited information likelihood (LIL) or the I-projection likelihood. Under these conditions on Q, the minimization of (2.53) yields:

$$q_P^*(\theta) = p_P^*(X, \theta) = c_0 \exp\left\{\lim_{T \nearrow \infty} c_1 T g_T(\theta)' S^{-1} g_T(\theta)\right\}, \tag{2.54}$$

where c_0 and $c_1 (< 0)$ are constants to control the scale and shape of the LIL. This is a solution of the underlying GMM assumptions. By using the central limit theorem implies that 2.54 is essentially a normal density.[5] Further, Kim derives that $q_{P,T}^*(\theta)$ is a finite analogue limited information likelihood to (2.54),

$$q_{P,T}^*(X_T, \theta) = c_0 \exp\left[c_1 T g_T(\theta)' \hat{S}_T^{-1} g_T(\theta)\right], \tag{2.55}$$

where $\hat{S}_T$ is a consistent estimate of the moments covariance matrix S. As shown by Kim (2002) $c_1 = -1/2$ is a desirable choice. Because the LIL embeds the optimal GMM estimate in a likelihood-inference framework, the estimator $\hat{\theta}$ which maximizes the LIL is the same as the optimal GMM estimator of Hansen (1982) (see Kim 2002).

Given these findings, we can study a limited information Bayesian framework, by evaluating the limited information posterior LIP from the LIL and a prior. The LIP is given by:

$$p(\theta|X_T) \propto p(\theta) q_{P,T}^*(X_T|\theta) \tag{2.56}$$

A maximization of LIP yields a posterior mode by updating the prior information used in the estimation.

2.4.1 Prior Choice

The intention of this paper is to estimate the pricing kernel of several preferences without neglecting our knowledge about the parameters from the economic theory. This knowledge has thus to be our prior within the Bayesian estimation technique described above.

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[5]By assuming that a process of moments is stationary and ergodic and a positive semidefinite matrix S has finite probability limits, then the central limit theorem implies that $\sqrt{T/S} g_T(x_T|\theta) \xrightarrow{d} N(0, I)$.

From the economic literature we do not really know something about the habit subsistence levels or about the correct memory of past decisions. But we have a good intuition about the discount factor, β, the elasticity of intertemporal substitution of consumption (EIS), and the Frisch elasticity.

Next to these priors I have additional knowledge about each parameter. Due to ensure concavity I know that γ has to be positive. Furthermore, I want to investigate the case of habit formation, such that the parameters ν and ψ have to take values positive and smaller than unity. As mentioned before, negative values for these parameter would allow for durability in consumption and leisure. At last, the parameter χ has to be between the bounds 0 and 1. By assuming that these are my only a priori information about the parameters, I suggest that the parameter are uniformly distributed between in the corresponding bounds.

From the business cycle literature we know that the discount factor β is related to the steady state risk-free return and potential economy growth. A β larger than unity would often correspond to a negative rate of time preference, wknown as the risk-free rate puzzle by Weil (1989). However, within a growing economy this is not necessarily true as shown by Kocherlakota (1990). However, within the Business Cycle literature values for the discount factor of slightly smaller the unity for quarterly data are often assumed. To capture this fact, I assume that β is normally distributed with mean 0.98 and a standard deviation of 0.05.

By investigating the elasticity of intertemporal substitution (EIS), the differences between the macroeconomic and the econometric literature are easily observable.[6] While in the macroeconomic literature, it is often argued for an EIS similar to one because of the observations on growth and aggregate fluctuations, the econometricians argue that observable co-movements between consumption and interest rates would imply an EIS close to zero (see Guvenen 2006).[7] But such small EIS would suggest that individuals are extremely unwilling to adjust consumption, what results in a far too smooth consumption path (Lettau and Uhlig 2002). Lucas (1990) postulates that for the standard consumption-based asset pricing model an EIS below 0.5 seems implausible. Early empirical investigation of EIS can be found by Hansen and Singleton (1983), Hall (1988), and Campbell and Mankiw (1989, 1991) using instrumental variables (IV) regression approaches. While Hansen and Singleton

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[6]The EIS is defined as the inverse of $-\eta_m c$. A formal derivation for the different preferences can be found in section 2.2.2.

[7]Guvenen (2006) establishes a RBC model which captures both facts, a high as well as a small EIS for heterogenous agents.

(1983) find values for EIS similar to one, Hall (1988) emphasize that EIS seems to be unlikely larger than 0.1. Campbell and Mankiw (1989, 1991) confirms this view by finding small elasticities of substitution in of U.S. data as well as in international data.[8] Further empirical investigations show that the level of EIS depends on, (i) wether we analyze aggregate data or e.g. state-level or cohort data (Attanasio and Weber 1989, 1993; Beaudry and van Wincoop 1996), (ii) the observation time-span (Basu and Kimball 2002), (iii) or the limited market participation of the individuals (Vissing-Jørgensen 2002). This recent literature finds an EIS significantly different from zero. The span of estimates includes values between 0.35 and one. Furthermore, Bansal and Yaron (2004) and Vissing-Jørgensen and Attanasio (2003) also argue for elasticities above one. For the prior distribution within this paper I follow the recent empirical findings and the suggestions from the Business Cycle literature by forming a prior over the inverse of the EIS, $-\eta_{mc}$, as Gamma distribution with mean 2.0 and standard deviation 0.75. Finally, over 90% of the distribution covers the values for EIS from 0.3 to 1.5.

In the business cycle literature models often assume a relative high Frisch elasticity (FE) larger than two. Recent examples are Prescott (1986) or King, Plosser, and Rebelo (1988) using a Frisch elasticity up to four.[9] Within the recent Bayesian DSGE model estimation literature, for example Smets and Wouters (2003) and Del Negro, Schorfheide, Smets, and Wouters (2005), the Frisch elasticity is estimated between 0.4 and 0.5, while Justiniano and Primiceri (2006) argue for a Frisch elasticity between 0.26 and 0.41. Del Negro and Schorfheide (2008) show that for nominal rigidities the Frisch elasticity is increasing to values slightly above one.[10] These finding are in line with the corresponding microdata-based literature. While the work of Kimball and Shapiro (2008) and Chang and Kim (2006) argue for a Frisch elasticity slightly above one (1.004-1.15), Pistaferri (2003) argues for values between zero and 0.7. These boundaries include the findings of MaCurdy (1981), Lee (2001), and Ziliak and Kniesner (2005) who all estimate the Frisch elasticity up to a maximum of approximately 0.5.

In this paper I form the prior for Frisch elasticity following the recent Bayesian DSGE model literature and assume that the inverse of the Frisch elasticity is Gamma distributed with mean 2 and standard deviation 0.75.[11] This prior implies that

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[8]See Campbell (2003) for an excellent overview including estimates.

[9]The Frisch elasticity regarding any preference used in the paper is determined in section 2.2.2.

[10]DSGE models often use separable utility functions like, $\log c_t - \frac{\varepsilon_0}{1+\kappa} \left(1 - l_t\right)^{1+\kappa}$, where κ reflects the inverse of Frisch elasticity (e.g., Del Negro and Schorfheide 2008; Smets and Wouters 2003).

[11]See Del Negro and Schorfheide (2008), Smets and Wouters (2003), Justiniano and Primiceri (2006) and Del Negro and Schorfheide (2008) for an equivalent prior assumption.

approximately 92.5% of the prior distribution is between 0.3 and 1.3, which seems reasonable with respect to the empirical findings (Del Negro and Schorfheide 2008). Table 2.3 summarizes the assumed prior distributions for any parameter.

Parameter	Domain	Density	Para (1)	Para (2)
β	$\mathbb{R}^+$	Normal	0.98	0.05
$1/EIS\ (-\eta_{mc})$	$\mathbb{R}^+$	Gamma	2.00	0.750
$1/FE$	$\mathbb{R}^+$	Gamma	2.00	0.750
γ	$\mathbb{R}^+$	Uniform	0	∞
χ	$\mathbb{R}^+$	Uniform	0	1
ν	$\mathbb{R}^+$	Uniform	0	1
ψ	$\mathbb{R}^+$	Uniform	0	1
ρ	$\mathbb{R}^+$	Uniform	0	1
ϕ	$\mathbb{R}^+$	Uniform	0	1

Table 2.3: Prior distribution for preference parameters and additional economic implications. Para (1) and Para (2) correspond to means and standard deviations for the Normal and Gamma distribution while for the Uniform distribution these values correspond to the lower and upper bounds.

2.4.2 Posterior Estimation

The posterior estimation is done in two steps. At first I estimate the posterior mode by maximizing $p\left(\theta|X\right)$ (eq. 2.56). For this I use an efficient GMM estimator.[12] On the first stage I use an identity weighting matrix W to estimate the parameter combination that maximizes equation (2.56). As described above, these parameter are used to estimate heteroscedasticity and autocorrelation consistent matrix $\hat{S}$. Afterwards, I use the inverse $\hat{S}$ as weighting matrix and maximize $p\left(\theta|X\right)$ again. This iteration is done until the maximization converges to a unique solution. The maximization of the LIP is done by using a BFGS quasi-Newton algorithm.

After that, I investigate the posterior distributions around the posterior mode. For this I conduct a Metropolis-Hastings algorithm (MH) to generate draws from the posterior density. The Metropolis-Hastings algorithm is an Markov Chain Monte

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[12]See Hansen et al. (1996) for a discussion of different GMM estimation algorithms.

Carlo (MCMC) simulation, which creates a Markov process with a stationary distribution similar to the posterior distribution of interest. Using a scaled version of the asymptotic covariance matrix as the covariance matrix of the proposal distribution, the MH algorithm has a normal jumping (proposal) distribution centered around the current point $\mathcal{N}\left(\theta^*|\theta^{(t-1)}, c^2\tilde{\Sigma}\right)$. Since that the covariance matrix of the proposal distribution is calculated as the inverse of the Hessian at the posterior mode, the MH algorithm constructs a Gaussian approximation around the posterior mode. In the following I present the implementation of the Metropolis-Hastings algorithm with N independent Markov chains.[13]

Random-Walk Metropolis-Hastings Algorithm:

(i) Calculate the posterior mode, $\tilde{\theta}$, by maximizing $p(\theta|X)$.

(ii) Let $\tilde{\Sigma}$ be the inverse of the Hessian computed at the posterior mode $\tilde{\theta}$.

(iii) Draw a starting value $\theta^{(0)}$ from $\mathcal{N}\left(\tilde{\theta}, c_0^2\tilde{\Sigma}\right)$ for the i-th chain

(iv) For $t = 1, \ldots, T$, draw θ^* from the proposal (jumping) distribution $\mathcal{N}\left(\theta^{(t-1)}, c^2\tilde{\Sigma}\right)$:

   1. Calculate the ratio of posterior densities:

$$r\left(\theta^{(t-1)}, \theta^*|X\right) = \frac{p(\theta^*|X)}{p(\theta^{(t-1)}|X)}$$

   2. Accept or reject the jump from $\theta^{(t-1)}$ following:

$$\theta^{(t)} = \begin{cases} \theta^* & \text{with probability} \quad \min\left(r\left(\theta^{(t-1)}, \theta^*|X\right), 1\right) \\ \theta^{(t-1)} & \text{otherwise} \end{cases}$$

(v) Go back to **(iii)** if $i < N$

In this paper I use the MH algorithm along two Markov chains. The starting value is drawn from the proposal distribution centered around the posterior mode using an high scaling factor (c_0). I then draw 50,000 times from the proposal distribution of θ, where I discard draws that fall outside the trust region. Within the MH algorithm I use a scaling factor that ensures an optimal convergence of the Markov chain. As mentioned above the proposal distribution has the same shape as the target distribution, in this case Gelman et al. (2004) mentioned that the optimal jumping

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[13]See e.g. Gelman, Carlin, Stern, and Rubin (2004) for an excellent discussion and more details about MCMC simulation and especially the MH algorithm.

rule has a acceptance rate of 0.44 for one-dimensional models, which is decreasing to 0.23 for higher dimensions.[14] The scaling factor c within the algorithm is set to obtain a similar value.

Following the method described in Gelman et al. (2004, pp. 296–298), the convergence of the two chains for each scalar estimate as well as for the entire distribution is monitored. Finally, I withdraw the first 80% of every chain and use this sequence of draws to approximate the posterior moments.

2.5 Estimation Results

In the following section I present and discuss the results of the estimation. To illustrate the inclusion of a priori information ont the parameters in the Bayesian framework, especially in the context of consumption-based asset pricing models, the first subsection shows the results of a Bayesian estimation with a diffuse or noninformative prior. The second subsection then focuses on the Bayesian approach, by using informative prior as described in the foregoing section. Finally, the last subsection compares the estimation results based on marginal data densities and investigates the ability of the different preferences and estimation procedures to explain the cross-section of asset return. For the comparison I use the method developed by Hansen and Jagannathan (1997).

2.5.1 Diffuse Prior

In this subsection I confirm the motivation by presenting the estimation results for the proposed CBAPM using a diffuse prior throughout the Bayesian estimation. The parameters were only bounded within their theoretical domains (see table 2.3). Within these bounds a uniform distribution is assumed to ensure a diffuse Jeffrey's prior. As discussed before, this kind of Bayesian estimation is equivalent to a GMM estimation. The results for each model can be found in the upper part of the corresponding tables 2.4, 2.5, and 2.6. The tables show the posterior mode with the corresponding standard errors in parentheses as well as the corresponding implication for the 1/FE, the 1/EIS, and the Sharpe ratio. The Sharpe ratio in this subsection is calculated as in equation (2.30). Furthermore, the tables show

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[14]See Roberts, Gelman, and Gilks (1997) for a more detailed analysis.

the results of the posterior simulations. A graphical illustration of the posterior distributions can be found in appendix A.1.2.

The results for internal and external habit formation with each one lag in consumption and leisure habit have some similarities. The habit parameters and the power utility parameter imply a similar high elasticity of the pricing kernel with respect to innovations in consumption. The inverse of the EIS is around 53, which implies an EIS of 0.02. The habit persistence in both preferences is high, similar to the existing literature. Especially, the external habit formation delivers a high habit in consumption slightly below one. Also the habit in leisure is high, but in difference to the consumption habit less well identified. For both preferences the parameter χ is different from one. However, if we consider the standard errors of the estimates only for the *internal-1L* model the null-hypotheses of $\chi = 1$ can be rejected at an acceptable level. An investigation of standard errors for the estimate of the power utility parameter γ illustrates the weak identification of this parameter in the *internal-1L* and *external-1L* model. Finally, the estimates for the discount factor β show to be small for the *external-1L* and *internal-1L* model.

In contrast to the former preferences, the estimation of *external-AR* yields different results, in particular regarding the role of leisure. The parameter χ is estimated slightly below one, which implies that the null-hypotheses cannot rejected. The other results are in line with the main literature. The discount factor is smaller (≈ 0.72) as for the other preferences and the autoregressive parameters ρ and ϕ are high and suggest high average for the fraction of habits (see Campbell and Cochrane 1999).

These results nicely illustrate the discussed problems with CBAPM. Of course, the usage of habit formation, implies smaller values for the power utility parameter γ as reported by Mehra and Prescott (1985), but all models imply small elasticities of intertemporal consumption substitution (EIS), between 0.006 and 0.02, especially the *external-AR* model. As discussed in the literature (Cochrane 2001; Lettau and Uhlig 2002, e.g.) and in this paper, such small values are implausible. However, these small values are necessary to obtain a high volatility of the pricing kernel M_{t+1}. With a high volatility the models are able to increase the predictability of the risk premium and the Sharpe ratio. Also the estimates of the Frisch elasticity, between 0.1 and 0.5, are rather small in contrast to the findings of the empirical literature. Moreover, the weak identification of the Frisch elasticity is not satisfying but with respect to the observation variables used not surprising.

2.5.2 Informative Prior

In contrast to the preceding subsection, this subsection presents the results of the Bayesian estimation using informative priors as described in section 2.4.1. The lower parts of the tables 2.4, 2.5, and 2.6 show the estimates at the posterior mode (column 2), at the posterior mean (column 3), both with corresponding standard error in parenthesis. The fourth and fifth column refer to the 10% and 90% confidence interval. The corresponding figures A.2, A.4, and A.6 can be found in the appendix and illustrate the posterior distribution, and the posterior mode of the estimates as well as their corresponding prior distribution.

Table 2.4 presents the results for the *internal-1L* model. A major fact is the reduced fraction of habits of current consumption and leisure. Both values for ν and ψ are approximately 0.23 and 0.25 at the posterior mean, in contrast to about 0.85 (ν) or 0.57 (ψ) within the diffuse prior estimation. This demonstrates the effect of habit formation. A higher habit reduces the willingness of the individual to abdicate consumption or leisure today. The elasticities of the pricing kernel with respect to consumption or leisure rise as well, and finally the volatility of the pricing kernel with respect to changes in consumption or leisure increases. However, such high elasticities are rejected by the empirical literature. Obviously, the estimation is not able to update the prior information to suitable high values for η_{mc} and η_{ml}.

Figure A.1 in the appendix illustrates the identification of all parameters in the estimation. The habit parameters and also the parameter χ indicates illustrate the weakness of the identification. Interestingly, the model can also identify the discount factor β. Moreover, the posterior mean of 1.001 is higher as the mean of the prior distribution, 0.98. Remembering the far smaller values of the pure likelihood estimation, this parameter is not simply shifted due to the prior.

Investigating table 2.5 and figure A.3 in the appendix for the results of the *external-1L* model, arrive similar conclusions for the *internal-1L* model. The main differences occur with respect to the parameters ν and ψ. Of course, both are much smaller than with diffuse prior, but above the Bayesian estimates for the *internal-1L* model. First, especially ν is higher but the identification is worse, than in the *internal-1L* model. However, this finding illustrates another interesting feature of external vs. internal habit formation, the time-nonseparability of internal habit formation. It is easy to see that both models imply similar values for $-\eta_{mc}$. Recalling eq. 2.32 and 2.33, for internal habit formation the elasticity $-\eta_{mc}$ and $-\eta_{ml}$ increase, because

diffuse prior

	Posterior mode (s.d.)	Posterior mean (s.d.)	Confidence Interval HPD inf	HPD sup
β	0.9102 (.0063)	0.9097 (.0058)	0.9008	0.9190
γ	2.0432 (.1288)	2.0522 (.1395)	1.8201	2.2731
χ	0.3279 (.0160)	0.3286 (.0181)	0.2969	0.3550
ν	0.8501 (.0003)	0.8501 (.0004)	0.8493	0.8508
ψ	0.5701 (.0560)	0.5733 (.0795)	0.4471	0.7083
1/EIS	52.921	52.970 (.9301)	51.463	54.398
1/FE	1.9803	2.1372 (.6416)	1.2317	3.1035
SR	0.1320	0.1319 (.0022)	0.1285	0.1354

informative prior

	Posterior mode (s.d.)	Posterior mean (s.d.)	Confidence Interval HPD inf	HPD sup
β	1.0016 (.0448)	1.0010 (.0014)	0.9987	1.0033
γ	4.9850 (34.78)	4.4306 (1.823)	1.2970	7.3399
χ	0.3286 (4.072)	0.2846 (.1418)	0.0168	0.4789
ν	0.1560 (4.423)	0.2279 (.1469)	0.0002	0.4311
ψ	0.2520 (1.141)	0.2558 (.0946)	0.1043	0.4191
1/EIS	3.2767	3.5740 (1.246)	1.9434	5.0605
1/FE	2.0373	20540 (.5893)	1.1970	2.9931
SR	0.0071	0.0078 (.0031)	0.0042	0.0116

Table 2.4: Results of the posterior mode estimation and the Metropolis-Hastings algorithm for internal habit formation with one lag (*internal-1L*).

decisions today also effect the marginal utility tomorrow. Finally, within external habit formation individuals take higher fractions of habits into account, to obtain equivalent intertemporal elasticities of substitutions. This also explain the different small EIS values obtained by the estimation with diffuse priors. Moreover, higher values for $-\eta_{mc}$ due to equivalent preference parameters, also explain why internal habits can easier describe the risk premium or Sharpe ratio.

The estimates for the EIS, the Frisch elasticity (FE), or the Sharpe ratio (SR) are similar across both kinds of habit formation. The estimates for the EIS within the 10% and 90% percentile vary between 0.19 and 0.51 for the *internal-1L* model and between 0.2 and 0.62 for *external-1L* model. Similar results obtain for the Frisch elasticity; for the *internal-1L* the values vary between 0.33 and 0.84, while for the *external-1L* model the Frisch elasticity is slightly higher in lays with 0.33 and 1.19. All these results are comparable to the existing empirical findings (see section 2.4.1), but in particulare. these results are arguments for an EIS and a Frisch elasticity at the lower bound of these literature.

Finally, the estimates for the Sharpe ratio suggest the same conclusion for both kind of models. The estimated Sharpe ratio is between 0.0042 and 0.012 for *internal-1L* and slightly smaller, between 0.034 and 0.012, for the *external-1L* model. These estimates take into account the exact correlation of asset returns with respect to consumption and leisure. By doing so, the Sharpe ratio is only slightly different from zero.

A stronger result for the decreasing Sharpe ratio by using informative prior appears during the estimation of the *external-AR* model. The average Sharpe ratio varies between 0.0016 and 0.0021 within the parameter distribution. The reason is - for this as well as for the models above - that incorporating business cycle facts disposes the pricing kernels of the models of their volatility. As table 2.6 and figure A.5 in the appendix show, the estimates for ρ and ϕ are nearly one and there exists no distribution for these parameters. Of course, such estimation results are not really optimal, but they nicely illustrates the features which drive the *external-AR* as well as the model by Campbell and Cochrane (1999). The parameters ρ and ϕ are responsible for the volatility of the auto-regressive processes for S_t^c and S_t^l. For example, the model of Campbell and Cochrane (1999) is able the resolve the Sharpe ratio and risk premium, because of the high volatility of the surplus ratio which increases the volatility of the pricing kernel without the necessity to increase the volatility of consumption due to a high power utility parameter. However, the empirical findings for the EIS and the Frisch elasticity do not support such an high

diffuse prior

	Posterior mode (s.d.)	Posterior mean (s.d.)	Confidence Interval HPD inf	HPD sup
β	0.8254 (.0719)	0.8289 (.0298)	0.7786	0.8759
γ	5.1046 (7.9892)	5.3092 (1.2653)	2.9785	7.0065
χ	0.3648 (.68204)	0.3450 (.1074)	0.1705	0.5180
ν	0.9730 (.00185)	0.9730 (.0009)	0.9716	0.9746
ψ	0.9109 (.2209)	0.8978 (.0371)	0.8454	0.9649
1/EIS	53.702	51.270 (7.433)	38.848	62.5660
1/FE	11.474	11.8361 (3.211)	7.205	16.879
SR	0.1281	0.1220 (.0184)	0.0935	0.1519

informative prior

	Posterior mode (s.d.)	Posterior mean (s.d.)	Confidence Interval HPD inf	HPD sup
β	1.0017 (.0348)	1.0009 (.0010)	0.9992	1.0026
γ	5.0415 (69.947)	3.8070 (1.8803)	0.5477	6.6103
χ	0.3557 (2.165)	0.3042 (.0997)	0.1264	0.4556
ν	0.1528 (10.631)	0.3964 (.2474)	0.0004	0.7533
ψ	0.5115 (.9323)	0.4954 (.0641)	0.3913	0.6037
1/EIS	2.8669	3.4120 (1.330)	1.5934	5.0299
1/FE	2.1168	1.9362 (.6284)	0.8422	2.9696
SR	0.0057	0.0034 (.0035)	0.0034	0.0121

Table 2.5: Results of the posterior mode estimation and the Metropolis-Hastings algorithm for external habit formation with one lag (*external-1L*).

diffuse prior				
	Posterior mode (s.d.)	Posterior mean (s.d.)	Confidence Interval HPD inf	HPD sup
β	0.7991 (.0287)	0.8015 (.0261)	0.7582	0.8436
γ	2.2352 (.1575)	2.2045 (.1927)	1.8924	2.5254
χ	0.9779 (.0472)	0.9439 (.0407)	0.8878	0.9997
ρ	0.7535 (.0085)	0.7524 (.0116)	0.7327	0.7712
ϕ	0.9016 (.0415)	0.8858 (.0488)	0.8090	0.9690
1/EIS	158.467	155.5382 (10.463)	138.512	172.414
1/FE	6.1161	14.880 (20.751)	0.0000	37.121
SR	0.1165	0.1143 (.0077)	0.1017	0.1267

informative prior				
	Posterior mode (s.d.)	Posterior mean (s.d.)	Confidence Interval HPD inf	HPD sup
β	0.9998 (.0025)	0.9995 (.0005)	0.9988	1.0003
γ	1.9727 (1.3487)	1.7680 (.4606)	1.0084	2.4349
χ	0.2129 (1.3483)	0.4773 (.2765)	0.0124	0.8608
ρ	0.9999 (.0000)	0.9999 (.0000)	0.9999	0.9999
ϕ	0.9999 (.0000)	0.9999 (.0000)	0.9999	0.9999
1/EIS	2.7122	2.6499 (.2068)	2.3283	2.9352
1/FE	1.7860	2.6719 (1.2665)	1.6026	3.9687
SR	0.0019	0.0016 (.0002)	0.0016	0.0021

Table 2.6: Results of the posterior mode estimation and the Metropolis-Hastings algorithm for external habit formation with ∞-lags (*external-AR*).

volatility. Furthermore, the parameter ρ and ϕ are chosen to reduce the volatility of the auto-regressive processes to obtain smaller values for the elasticities $-\eta_{mc}$ and $-\eta_{ml}$.

Moreover, the results for the discount factor β and the substitution parameter χ are similar to those of the *internal-1L* and *external-1L* model. Additionally, the estimates for EIS and Frisch elasticities are not quite different to the estimates before. The average Frisch elasticity is estimated within the range of 0.25 and 0.63, while the EIS is estimated between 0.34 and 0.43. Both variables are slightly smaller as before but also at the lower bound of the empirical findings. Finally, the fraction of habit of total current consumption is much smaller as in the estimation with diffuse prior. The mean fraction of habit of consumption reduces from over 0.95 to 0.08, while the mean fraction of habit of leisure decreases not so strong from over 0.95 to 0.2789. These are comparable to the *external-1L*, where the fractions reduces to similar values and also the fraction of habit of leisure is quite higher than that of consumption.

2.5.3   Model Comparison

In this section I compare the models estimated above. First, I investigate the estimation results with respect to their ability to explain the observation variables used (Portfolio I), by using posterior model probabilities. Secondly, I investigate the estimated model parameters with respect to their ability to explain the cross-section of asset returns of the observation variables (Portfolio I) as well as of the two other portfolios (Portfolio II and III). For this second comparison I use the method of Hansen and Jagannathan (1997). Based on this method I present the results of three prominent factor models. These factor models are helpful benchmarks to illustrate the performance of the CBAPMs.

I begin with the calculation of the posterior model probabilities:

$$\pi_i = \frac{\pi_{i,0} p\left(X|M_i\right)}{\sum_{i=1}^{10} \pi_{i,0} p\left(X_T|M_i\right)}, \tag{2.57}$$

where $\pi_{i,0}$ is the prior of model M_i and $p\left(X|M_i\right)$ is the marginal data density,

$$p\left(X|M_i\right) = \int p\left(X|\theta_i, M_i\right) p\left(\theta_i|M_i\right) d\theta_i \tag{2.58}$$

of Model M_i. The marginal log data density is calculated by using a Laplace ap-

proximation at the posterior mode as well as Geweke's modified harmonic mean estimator (see Geweke 1999a). Of course, this measure depends on the applied set of priors, so I compare the models only within any estimation method and not across them. As described before, using informative priors forces the parameters to another path. This implies that the ability to explain the observation data is higher for diffuse priors as with informative priors. But exactly this is the motivation of this paper, to force the parameter on the economically plausible path by taking into account a loss of explanation power regarding the observation variables. Furthermore, it is necessary to ensure the same prior distribution for any Bayesian model estimation. Table 2.7 shows the marginal log data density and the log posterior mode probability of each model, for Bayesian estimation with diffuse as well as with informative prior.

Model	*Laplace*	$\log \pi_i$	*GMHM*	$\log \pi_i$
diffuse prior				
internal-1L	-49.4617	80.2346	-46.5524	83.0078
external-1L	-35.4655	94.2308	-39.6952	89.8668
external-AR	-42.5719	87.1244	-41.1154	88.4466
informative prior				
internal-1L	-35.0926	107.0652	-54.7109	132.5232
external-1L	-34.7882	107.3696	-54.7311	132.5030
external-AR	-70.0798	72.0780	-75.5949	111.6392

Table 2.7: Estimated marginal log data densities using Laplace approximation and Geweke's modified harmonic mean estimator (GMHM) and corresponding posterior model probabilities.

Table 2.7 suggests that the *external-AR* has the highest model probability for diffuse prior estimation followed by the *external-AR* model and the *internal-1L* model. Interestingly, the internal habit model (*internal-1L*) generates the smallest model probability, while with informative priors these preferences generates the highest model probability.

The table shows that the differences between internal and external habit formation with one lag reduce with informative priors. In contrast to Chen and Ludvigson (2007), it cannot be confirmed that internal habit formation is to prefer for diffuse

priors. Moreover, these preferences perform better than the others by incorporating informative prior into the estimation. Another interesting fact is the poor performance of the *external-AR* model, by using informative priors. As shown before the models need an high volatility of the pricing kernel to match the data. Obviously, the use of informative priors will lead to a reduction of the elasticities η_{mc} and η_{ml}. Because high values for both are necessary to receive a Sharpe ratio as observed in the data, this variable will decrease dramatically in contrast to the estimation with diffuse priors. As mentioned before it is difficult to measure gain or loss of explanatory power regarding the prior used, because by taking prior information seriously, the estimation result is the best given the prior. However, to get an idea of the explanatory power only regarding the data, we can investigate the limited information likelihood in more detail. The likelihood ratio λ_i/λ_u (informative/uninformative) for the *internal-1L* and *external-1L* models is 0.73 vs. 0.72, and for the *external-AR* model the likelihood ratio is 0.91. These numbers nicely illustrate how the choice of the prior affects the posterior. A further illustration is given in table 2.8.

prior	*internal-1L*		*external-1L*		*external-AR*	
	1/EIS	SR	1/EIS	SR	1/EIS	SR
informative	3.28	0.007	2.87	0.006	2.7122	0.002
$\mu = 5, \sigma = 2.5$	8.25	0.017	6.56	0.015	7.153	0.0051
$\mu = 10, \sigma = 5$	14.19	0.032	11.82	0.029	17.90	0.013
$\mu = 20, \sigma = 10$	18.68	0.043	22.11	0.055	119.92	0.088
diffuse	52.92	0.132	53.702	0.1281	158.47	0.117

Table 2.8: Sensitivity analysis towards the prior on 1/EIS.

The table shows a sensitivity analysis regarding the prior on the inverse of the EIS. The first and the last row of the table show the known results for each preference with diffuse and informative prior as discussed above. Additionally, I present the results for EIS and Sharpe ratio based on the different priors on EIS while keeping the other priors constant like in the informative case. The table nicely illustrates how the ability to explain the Sharpe ratio reduces by increasing the degree of information.

However, this paper is also interested in the ability of the models to explain the cross-section of asset returns. For this reason, I use a second comparison method. Hansen and Jagannathan (1997) developed a method to compare asset pricing models when

the implied stochastic discount factors do not price all portfolios correctly. The procedure is also used in Chen and Ludvigson (2007) and is established as a common measure of how well SDFs are pricing a portfolio of N assets. This measure is called Hansen-Jagannathan distance (HJ distance). The procedure can be shortly explain as follows. For any parameter θ I calculate the criterion function:

$$g_T^{HJ}(\theta) = w_T(\theta)' G_T^{-1} w_T(\theta),$$ (2.59)

where G_T is the second moment matrix of N asset returns and w_T is the vector sample average pricing errors

$$w_T(\theta) = [w_{1,T}(\theta) \ldots w_{N,T}(\theta)]'$$ (2.60)

with

$$w_{n,T}(\theta) = 1/T \sum_{t=1}^{T} M_t R_{n,t} - 1.$$ (2.61)

The HJ distance is finally calculated as square root of g_T^{HJ}:

$$\delta_{HJ} = \sqrt{g_T^{HJ}(\theta)}$$ (2.62)

Within the Bayesian estimation approach, I also calculate the HJ distance along the Monte-Carlo Markov Chain algorithm for any not discarded draw of θ. This allows not only an investigation at the posterior mean but, furthermore I get an idea about the HJ distance under parameter uncertainty.

Additionally, I compare the specification errors of the described models with three prominent alternative asset pricing models. These benchmark models are linearized factor models, where the pricing kernel takes the form:

$$M_{t+1} = \theta_0 + \sum_{i=1}^{k} \theta_i F_{i,t+1},$$ (2.63)

where $F_{i,t+1}$ are the factors and θ_0 and θ_i are the factor loadings to be estimated. The choice of the factor models is comparable to Chen and Ludvigson (2007). I choose the three-factor, portfolio based asset pricing model of Fama and French (1993, 1996), where the factors are related to market capitalization, book equity-to-market equity, and the aggregate stock market. The factors are the "small-minus-big" portfolio return, the "high-minus-low" portfolio return, and finally the stock market return ($k = 3$). In addition to this model, I use a linearized version of the

standard CCAPM introduced by Breeden (1979), Breeden and Litzenberger (1978), and Breeden, Gibbons, and Litzenberger (1989). In this model consumption growth is the only factor ($k = 1$). Finally the classical CAPM developed by Sharpe (1963, 1964) and Lintner (1965) is chosen, in which the market return $R_{m,t+1}$ is the single variable factor. For a more detailed description and source of the data need for these factor models, have a look into the appendix A.1.1.

The estimates for these three models are obtained by minimizing each corresponding HJ distance. In contrast to the habit models investigated in this paper, this is, of course an advantage. However, the goal of this research is to illustrate how CBAPMs work as well as how additional a priori information influences the ability to explain the cross section of asset returns. The point of interest is not to find parameter combinations to beat one or all of these factor models. Table 2.9 shows the HJ distance for each model and each portfolio. The results for the factor models are the minimized HJ distance. The results for the Bayesian estimation show the 50%, 10% and 90% deciles of δ_{HJ} calculated along the Monte-Carlo Markov-Chains.

If we compare the estimates based on diffuse priors, it is apparent that in contrast to the comparison method before the *external-AR* model outperforms the other models. The *external-1L* model is still better than the *internal-1l* for portfolio I, but worse with respect to other the two portfolios. In comparison with the factor models the *external-AR* model also outperforms the CAPM model and is approximately comparable to the CCAPM model. However, it has less explanatory power in relation to the Fama-French three-factor model. For the other two portfolios the explanation power is smaller. This is obviously due to the fact that the estimates of the CBAPM are based on the estimation of Portfolio I. Because the differences for Portfolio I seem not so large, one could suspect that other CBAPMs are able to beat the factor models as argued by Chen and Ludvigson (2007). However, including informative priors into the estimation, the results for Portfolio I show that the HJ distance increases. Furthermore, the values in brackets illustrate that the variation of the HJ distance over the parameter distribution is very small. Surprisingly, the Bayesian estimates have greater success in explaining Portfolio II and III. Finally, the *external-AR* model loose its explanation power compared to the other models and performs similarly or worse.

Unfortunately, the HJ distance not really discovers the poor performance of the models. To illustrate the results, I also investigate wether the estimated average returns are in line with the realized average returns of the different portfolios. To do so, let recapitulate the given Euler equation (2.4). Using conditional expectations,

	HJ dist Portfolio I	HJ dist Portfolio II	HJ dist Portfolio III
DIFFUSE PRIOR			
internal-1L	0.2903 [.2871;.2931]	0.1940 [.1766;.2179]	0.4757 [.4679;.4877]
external-1L	0.2842 [.2806;.2955]	0.2205 [.1751;.3124]	0.5135 [.4791;.5870]
external-AR	0.2740 [.2705;.2798]	0.1674 [.1638;.1726]	0.4164 [.4142;.4198]
INFORMATIVE PRIOR			
internal-1L	0.3396 [.3388;.3402]	0.1756 [.1747;.1761]	0.4659 [.4654;.4663]
external-1L	0.3396 [.3388;.3402]	0.1751 [.1739;.1762]	0.4679 [.4669;.4692]
external-AR	0.3401 [.3400;.3403]	0.1762 [.1760;.1763]	0.4658 [.4656;.4660]
LINEAR FACTOR MODELS			
Fama-French	0.2365	0.1199	0.3971
CCAPM	0.2635	0.1481	0.4447
CAPM	0.3240	0.1316	0.4547

Table 2.9: HJ distance for each Model and Portfolio. For Bayesian estimation the table reports the HJ distance at the 50% decile and for 10% and 90% deciles in brackets. HJ distance for Factor Models based on minimizing estimates.

it is quite common to re-write the expectation of the product as the product of expectations plus the covariance (see, e.g., Campbell 2003),

$$E_t \left[M_{t+1} R_{i,t+1} \right] = E_t \left[M_{t+1} \right] \cdot E_t \left[R_{i,t+1} \right] + Cov_t \left[M_{t+1}, R_{i,t+1} \right]. \tag{2.64}$$

Substituting this evaluation into the Euler equation, we derive,

$$E_t \left[R_{i,t+1} \right] = \frac{1 - Cov_t \left[M_{t+1}, R_{i,t+1} \right]}{E_t \left[M_{t+1} \right]}, \tag{2.65}$$

which has to hold for any asset i. Given this equation we can evaluate, how good the estimated pricing kernel M_{t+1} prices different assets.
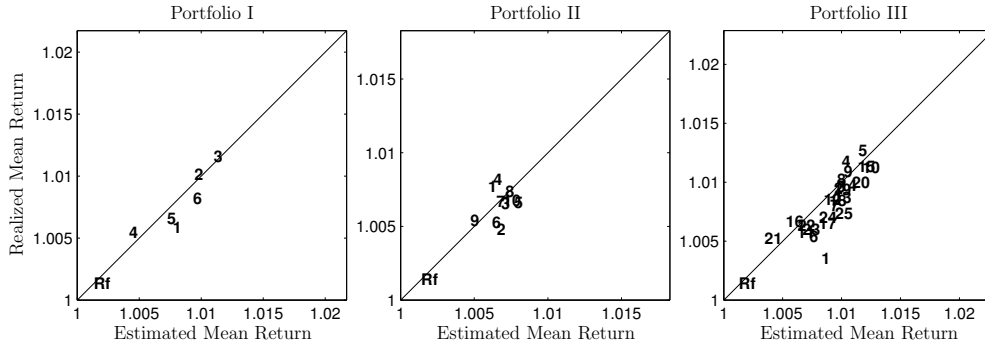
(a) Fama-French 3 Factor Model
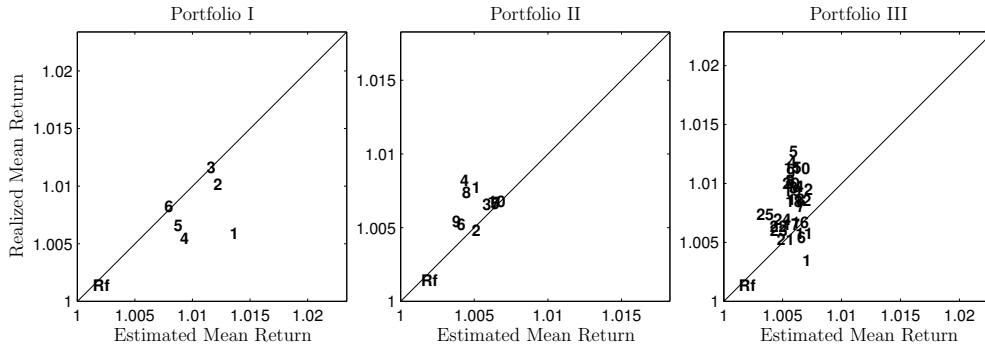
(b) CCAPM
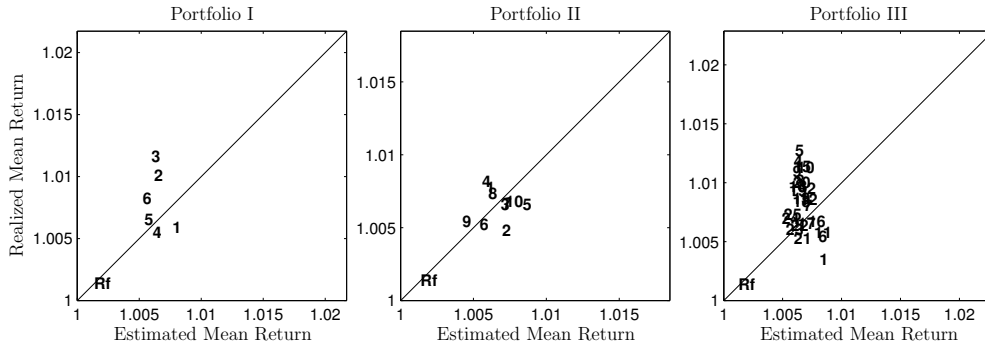
(c) CAPM

Figure 2.1: Estimated vs. realized mean returns between 1965-3 and 2006-11 for each portfolio, based on estimates for Linear Factor Models.

The figures 2.1, 2.2, and 2.3 illustrate the results, based on the investigated habit preferences as well as the factor models. Figure 2.1 show the plot of the mean estimated return vs. the mean realized return of each asset in the Portfolio for each of the factor models. The figure nicely confirms the success of the Fama-Franch three-factor-model to explain the returns of the different assets within the portfolio,

especially for portfolio I. Having this in mind, consider figure 2.2. By using diffuse priors, the CBAPMs are able to explain different asset returns, where each of the models overestimates the risk-free rate (Rf).

(a) internal-1L

(b) external-1L

(c) external-AR
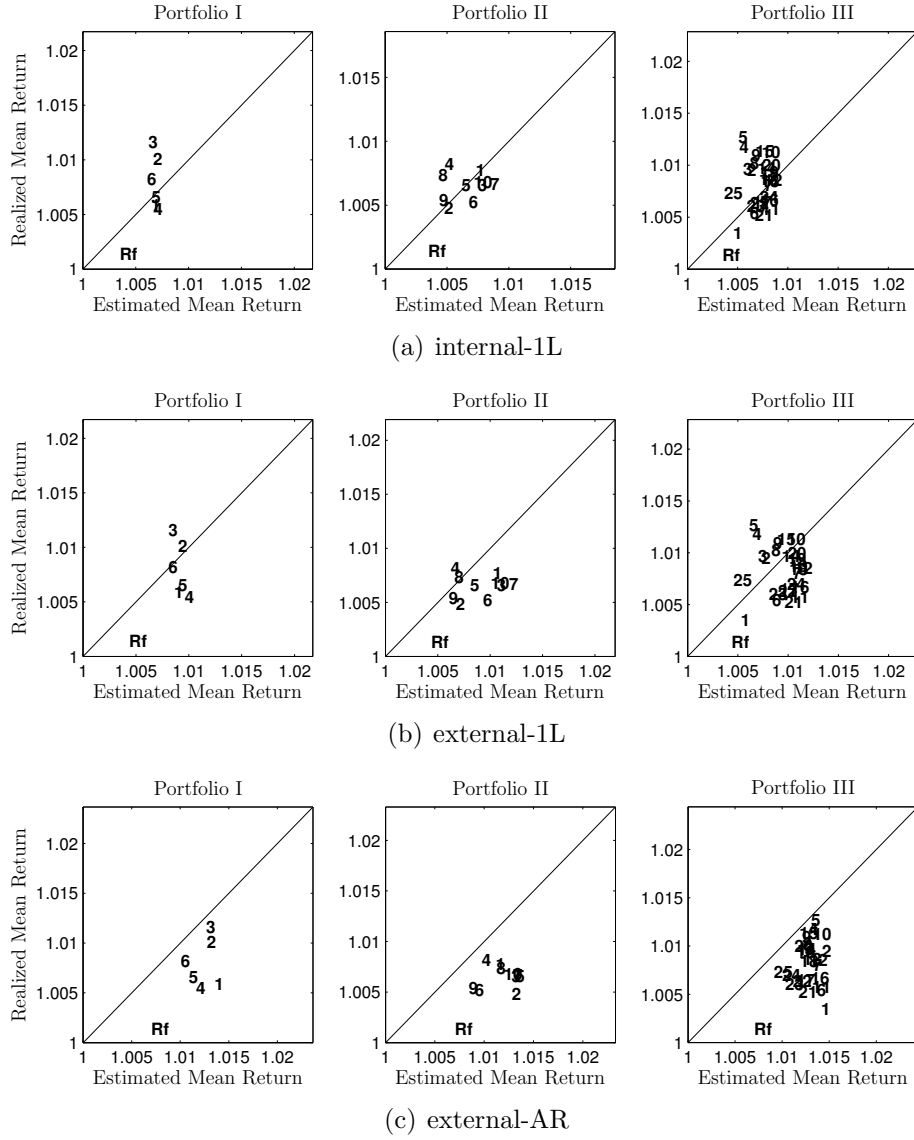
Figure 2.2: Estimated vs. realized mean returns between 1965-3 and 2006-11 for each portfolio, based on the posterior mode estimation with diffuse prior.

Now let's have a look on figure 2.3. Incorporating informative priors, now there exists no differentiation between any asset in the portfolio. Neglecting the results from the HJ distance, it is difficult to identify an outperforming model. All models underestimate the high returns observed in the data, only the risk-free rate can be estimataed. The solution of this is already discussed above, CBAPMs that are in line with empirical findings from the Business Cycle literature have a nonvolatile pricing kernel M_{t+1}, what results in similar covariances between the pricing kernel

and different asset returns. The figures also illustrate that the HJ distance can yield some misleading results.

(a) internal-1L

(b) external-1L

(c) external-AR
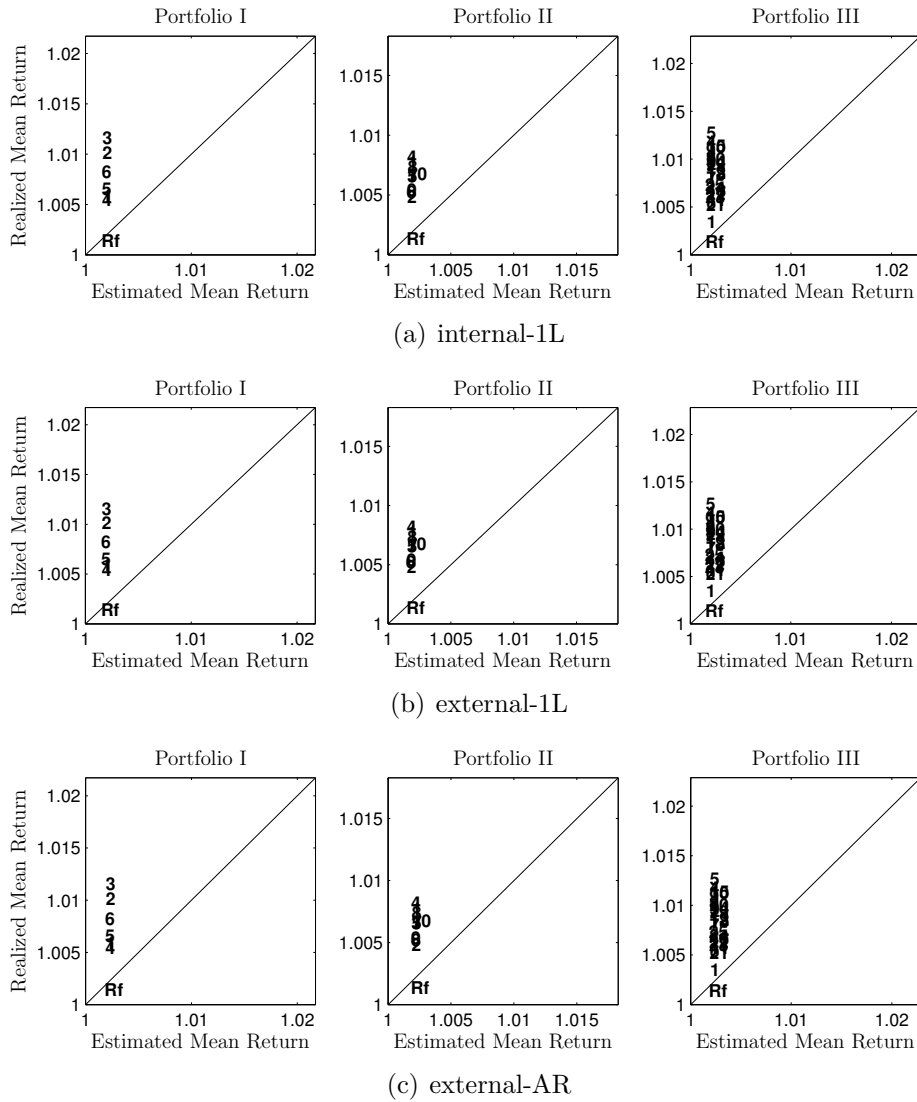
Figure 2.3: Estimated vs. realized mean returns between 1965-3 and 2006-11 for each portfolio, based on the posterior mode estimation with informative prior.

2.6   Conclusion

In this study, I have investigated the ability of different power utility models with different kinds of habit formation to explain the cross-section of asset returns. The research uses a limited likelihood approach and a Bayesian framework by using a priori information from the macroeconomic literature.

The paper is motivated by the manifold implications that map into such CBAPMs. Most of the asset pricing implications are driven by the preference parameters, which are also related to aggregate or individual behavior. Thus, it seems necessary to investigate the CBAPM not only with respect to their explanatory power of asset returns but also with respect to their ability to be simultaneously in line with these relations.

I identify estimates for the Frisch elasticity, the elasticity of intertemporal consumption substitution, the discount factor as well as the Sharpe ratio that are in line with the empirical observations. Throughout the Bayesian estimation, I also identify the additional preference parameters, in particular the fraction of habits of consumption and leisure. In contrast to the estimation with diffuse priors, these fractions are substantially smaller. Further, the volatility of the estimated pricing kernel is substantially smaller with informative priors. Because a high volatile pricing kernel is needed to explain stylized asset pricing facts as well as the cross-section of asset returns, a simultaneous explanation of macroeconomic facts reduces this ability. This relationship is already well-known. Altogether, this comes at the cost of poor performance of each of these models in order to explain the cross-sections of asset returns.

However, the methodology used in this paper has disclosed a way that allows to estimate CBAPMs along a path of economically logical parameters.

3 Bayesian Estimation of a DSGE Model with Asset Prices

with Harald Uhlig

This paper combines two strands of literature, namely the asset pricing and the business cycle literature. We use Bayesian techniques to estimate the dynamic stochastic general equilibrium (DSGE) with macroeconomic and financial time series. Moreover, we explore a way to include conditional second moments of asset returns into the estimation. Given the estimated model, we can better jointly explain key business cycle facts, different volatilities of several asset returns, and the empirically observed equity premium. Additionally, the model fits historical business cycle time series as well as the observed return on equity. This allows to discover prominent shocks of the last decades and to investigate the co-movements of asset prices and the macroeconomy more correctly.

3.1 Introduction

This paper presents an estimation of a dynamic stochastic general equilibrium model that jointly explains business cycle and asset pricing implications. The work makes a first step to analyze the relationships of both strands of literature by estimating macroeconomic and financial time series. In order to ensure that the main facts of both strands are not violated we introduce a set of restrictions. We pick key facts from each strand of literature, namely the Sharpe ratio and the Frisch elasticity of labor supply. These key facts combined with widely used priors about the model parameters ensure that the model is restricted to economically meaningful parameter combinations.

It has been shown in the literature that labor frictions seem to play an important role

when combining asset pricing facts and macroeconomic facts within one model.[1] On the one hand, endogenous labor supply decisions in a frictionless market would insure the agent against fluctuations of consumption. On the other hand, such smoothing of the consumption path makes it impossible for consumption-based asset pricing models to explain a high risk premium and the Sharpe ratio.

To resolve this, the present model includes frictions like inelasticity of leisure demand and wage rigidities. In addition to an external habit regarding consumption, the model also involves an external habit regarding leisure choice. This is motivated by the argument, that it seems not convincing that agents would make differences in their "Catching up with the Joneses" (see Abel 1990). The resulting inelasticity of labor supply, the smaller elasticity of leisure substitution, and the wage rigidities (see e.g. Blanchard and Galí 2005) help to explain the risk premium (Uhlig 2007). Our paper investigates these facts regarding their empirical validity. The model underlying the estimation is an extension of Uhlig (2007). We introduce three additional structural shocks. Additionally to the original labor force productivity shock, we add a preference shock to habit formation respectively leisure, a capital adjustment cost shock, and a government expenditure shock. The model is estimated based on U.S. macroeconomic time series (real GDP, real consumption, hours worked, government expenditures) and real value-weighted excess returns during 1951 and 2007.

The results of the paper show that the estimated model is consistent with a bunch of business cycle facts. Simultaneously, the model explains a high Sharpe ratio, different expected means of asset returns, and different volatilities of several asset returns. In particular, the model simulates quite successful a high volatile return on equity and a contemporaneous less volatile risk-free return.

Under the assumption that the model has to be the data generating process, we show, that in the context of high demand of labor, inelasticity of labor supply, and wage rigidities, the possibilities of the representative agent to assure himself against consumption fluctuations are so small that he accepts a higher risk premium. It is interesting that, in contrast to the common beliefs, not the elasticity of consumption substitution is the only driving force of asset returns (and so have to be extremely small). Moreover, the elasticity of leisure substitution seems to be important, too.

Finally, the smoothed variables of the model fit the data, asset returns and common macroeconomic time series quite successful. In the model, we can identify prominent

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[1]For such frictions on labor markets see Lettau and Uhlig (2000), Boldrin, Christiano, and Fisher (2001), Guvenen (2003), and Uhlig (2004a, 2007).

historical shocks like the oil price shocks and the crashes on the stock market of the last century. Moreover, these fluctuations seems to be driven rather by preference and investment shocks than by productivity shocks.

The estimation procedure in this paper follows Bayesian estimation techniques as used in the seminal papers of Geweke (1999a) and Schorfheide (2000). This kind of empirical investigation of Asset Pricing implications was introduced by Canova (1994) and Geweke (1999b) who added Bayesian techniques to calibration. Both papers have shown the empirical applications to assess business cycle and asset pricing implications, but not jointly. Furthermore, the work of Geweke (1999b) discloses the advantages and disadvantages of calibration and the different kinds of estimation procedures towards DSGE models.

Geweke (1999b) has distinguished between strong and weak econometric interpretation. The weak interpretation is close to the widely used calibration of a DSGE model. Nevertheless, there is a wide agreement in the literature that, by doing so a model economy only "mimic(s) the world along a carefully specified set of dimensions", as postulated by Kydland and Prescott (1996). Otherwise, the strong interpretation seeks to provide a full characterization of the observed data series. An pure maximum likelihood estimation can finish up in the "dilemma of absurd parameter estimates", as mentioned by An and Schorfheide (2007). This means that the estimates of the model are outside of every economic plausibility. To resolve this fact, modern likelihood estimation of DSGE models re-weight the objective function by a chosen prior density. Using priors to bear additional information into the estimation sample covers a certain amount of risk, because - as mentioned by Del Negro and Schorfheide (2008) - "priors often add curvature to a likelihood function that is (nearly) flat in some dimensions of the parameter space and therefore strongly influence the shape of the posterior distribution".

The present essay extensively discusses the prior choice within models with more complex preferences and demonstrates a way to incorporate affected priors into the estimation. Usually, standard solution techniques of DSGE models ignore second moments. But asset returns and in particular many stylized asset pricing facts are characterized by their second moments. For example, second moments of asset returns determine the difference of their expectations and their degree of risk (Sharpe 1963). Hence, it is important to resolve a DSGE model not only consistent with respect to first moments but also to second moments.[2] To account for this fact,

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[2]For the importance of second moments for DSGE models see also Schmitt-Grohe and Uribe (2004).

we estimate the model conditional on its second moments of asset returns. The estimation algorithm is an extension of the solution technique described by Canton (2002) for the method of undetermined coefficients.

The essay is organized as follows. Section two introduces the model and investigates necessary asset pricing implication of DSGE models. The third section presents the algorithm used to incorporate second moments into the likelihood. Moreover, this section characterizes the data and describes the prior choice in more detail. Section five presents the estimation results and reports the characteristics of the model by illustrating and discussing impulse response functions and smoothed variables. The sixth section concludes the essay.

3.2   Model

The present analysis bases upon the discounted stochastic growth economy proposed by Uhlig (2007). In the following subsections we describe the model in detail.

3.2.1   Firm

The production is given by the following Cobb–Douglas production function,

$$y_t = k_{t-1}^\theta \left(e^{z_{T,t}} n_t\right)^{1-\theta}. \tag{3.1}$$

It depends on the capital k_{t-1} accumulated in the former period as well as on the hired amount of labor n_t in the current period. Furthermore, in period t the production depends on the technology level $z_{T,t}$ the firm can implement. We assume the technology to follow a random walk with drift

$$z_{T,t} = \gamma + z_{T,t-1} + \epsilon_{T,t}, \tag{3.2}$$

with γ reflecting the trend of the technology. In the linearized model we assume that $\epsilon_{T,t}$ is normally i.i.d. with standard error σ_T.

Under the assumption of competitive markets firm profit is equal to zero. Hence, the first-order conditions of the firm's maximization problem yields the necessary conditions for the cost of hiring labor (market wages w_t) and borrowing capital

(dividends d_t):

$$w_t = \frac{(1 - \theta) y_t}{n_t} \quad \text{and} \tag{3.3}$$

$$d_t = \frac{\theta y_t}{k_{t-1}}. \tag{3.4}$$

3.2.2 Households

Households are characterized by a representative agent with preferences characterized by the following utility function:

$$U = E \left[\sum_{t=0}^{\infty} \beta^t \frac{\left((c_t - \chi_t C_{t-1}) \left(A + (l_t - z_{P,t} \psi_t L_{t-1})^v \right) \right)^{1-\eta} - 1}{1 - \eta} \right], \tag{3.5}$$

where c_t and l_t denote individual consumption and leisure. The latter leisure is given by total time endowment minus labor supply n_t of the agent. For simplicity the total time endowment is scaled to unity,

$$n_t + l_t = 1.$$

In addition to the discount factor β there exist the parameters A, ν, and the power utility parameter η. Because of the monotonicity and concavity constraints, the parameters have to satisfy $\nu > 0$ and $\eta > \nu / (\nu + 1)$.

Further, the utility of the representative agent depends on the economy-wide average level of consumption and leisure. These levels of habit for consumption C_t and for leisure L_t are represented by the parameters χ and ψ. Furthermore, we assume that there is the possibility of an exogenous preference shock, $z_{P,t}$, to the habit persistence level of leisure. The log-linearized shock can be expressed by a stochastic AR(1) process,

$$\hat{z}_{P,t} = \pi_P \hat{z}_{P,t-1} + \epsilon_{P,t}, \tag{3.6}$$

where ϵ_P is a normally i.i.d. shock variable with standard deviation σ_P.

The agent maximizes his utility by choosing leisure, consumption, and investments (x_t), taking as given the exogenous habits, real wages w_t, and dividends d_t for providing capital to firms. Furthermore he holds an initial endowment of capital

k_{-1} and one unit of time. Hence, the budget constraint of the agent is

$$c_t + x_t = d_t k_{t-1} + w_t n_t. \tag{3.7}$$

In this economy, capital accumulation is affected by a depreciation rate δ and capital adjustment costs $g(\cdot)$,

$$k_t = \left(1 - \delta + g\left(z_{I,t}\frac{x_t}{k_{t-1}}\right)\right)k_{t-1}. \tag{3.8}$$

We may assume the adjustment cost function $g(\cdot)$ to satisfy the following steady state conditions (see Jermann 1998):

$$g(\cdot) = \delta + e^\gamma - 1, \quad g'(\cdot) = 1, \quad g''(\cdot) = -\frac{1}{\zeta\frac{\bar{x}}{\bar{k}}} \quad \forall \quad \zeta > 0. \tag{3.9}$$

Besides, we assume that the adjustment costs are affected by the shock $z_{I,t}$ which is also defined as AR(1) process. The log-linearized counterpart can be written as:

$$\hat{z}_{I,t} = \pi_I \hat{z}_{I,t-1} + \epsilon_{I,t}, \tag{3.10}$$

where ϵ_I is a normally i.i.d. shock variable with standard deviation σ_I.

In the economy exist real wage rigidities as postulated by e.g. Hall (2005) and Shimer (2005). Under this assumption, the agent's first-order condition for labor supply, U_l/U_c, yields the frictionless wage w_t^f or the marginal rate of substitution. In a demand-constrained labor market, it is possible to specify the market wage w_t as,

$$w_t = (e^\gamma w_{t-1})^\mu \left(e^{\varepsilon_w} w_t^f\right)^{1-\mu}. \tag{3.11}$$

This specification is similar to Blanchard and Galí (2005) and assumes that real wages respond sluggishly to labor market conditions as a result of some frictions on the labor market. The parameter $\varepsilon_w \geq 0$ represents the steady-state wage markup to ensure that $w \geq w^f$ at all times. This specification also ensures that the workers receive more than their reservation wage if they decide to work. The parameter μ reflects the degree of frictions on the labor market. In the special case of $\mu = \varepsilon_w = 0$ there exist no frictions and the wages are fully flexible.

3.2.3 Government

For the government sector we assume that the fiscal policy is Ricardian, with a budget balanced period by period through lump-sum taxes and with an initial stock of government bonds of zero. The government budget constraint is:

$$g_t = T_t \tag{3.12}$$

Furthermore we model government expenditures exogenously, which can be expressed in linear terms as

$$\hat{g}_t = \pi_G \hat{g}_{t-1} + \epsilon_{G,t}, \tag{3.13}$$

where ϵ_G is a normally i.i.d. shock variable with standard deviation σ_G and π_G the corresponding autoregressive parameter.

3.2.4 Asset Pricing Implications & Business Cycle Facts

In order to better understand the relationship between asset pricing and business cycles, we investigate these relation in this subsection in more detail. Given the model described above, we can derive implications for the price of capital as well as for different asset returns.

Let μ_t be the Lagrangian multiplier of the capital accumulation formula and similarly λ_t be the multiplier of the resource constraint of the household. The first-order condition with respect to investment is given by:

$$\lambda_t = \mu_t g'(\cdot) z_{I,t} \tag{3.14}$$

Using our knowledge that λ_t is also the marginal utility of consumption and μ_t the marginal utility of capital, we can write the price of capital, which is defined as consumption cost of an additional unit of capital, as:

$$q_t = \frac{\frac{\partial U_t}{\partial k_t}}{\frac{\partial U_t}{\partial c_t}} = \frac{\mu_t}{\lambda_t}, \tag{3.15}$$

at the same time q_t also illustrates the Tobin's q relationship,

$$q_t = \frac{1}{g'\left(z_{I,t}\frac{x_t}{k_{t-1}}\right) z_{I,t}}. \tag{3.16}$$

Furthermore, the first-order condition with respect to capital k_t is:

$$\mu_t = \beta E_t \left[\mu_{t+1} \left(1 - \delta + g\left(\cdot\right) - g'\left(\cdot\right) z_{I,t+1} \frac{x_{t+1}}{k_t} \right) + \lambda_{t+1} d_{t+1} \right] . \qquad (3.17)$$

With the previous definition of the price of capital this formula can be re-written as:

$$q_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left[\left(1 - \delta + g\left(\cdot\right)\right) q_{t+1} - \frac{x_{t+1}}{k_t} + d_{t+1} \right] \right] . \qquad (3.18)$$

Defining the stochastic discount factor (SDF):

$$M_t = \beta \frac{\lambda_t}{\lambda_{t-1}} , \qquad (3.19)$$

and the return on total capital,

$$R_t^c = \frac{\theta \frac{y_t}{k_{t-1}} + \left(1 - \delta + g\left(\cdot\right)\right) q_t - \frac{x_t}{k_{t-1}}}{q_{t-1}}. \qquad (3.20)$$

It is easy to see that the Euler equation has to hold:

$$1 = E_t \left[M_{t+1} R_{t+1}^c \right]. \qquad (3.21)$$

The equation must also hold for any other kind of asset return, e.g. the risk-free return

$$1 = E_t \left[M_{t+1} \right] R_t^n. \qquad (3.22)$$

Because, later on we want to investigate the Sharpe ratio in more detail, we assume that the economy-wide total return on capital can be split into the risk-less return on and the return on equity, by assuming a leverage ratio ω. This suggests a further equilibrium condition,

$$R_t^c = \omega \cdot R_{t-1}^n + \left(1 - \omega\right) R_t^{\mathrm{eq}}. \qquad (3.23)$$

If we investigate the previous Euler equations in more detail, we can find that the differences between asset returns depends on the second moments. By assuming that asset returns are log-normally distributed we can rewrite the Euler equation for the risky return on total capital as:

$$E_t \left[\hat{r}_{t+1}^c \right] + \frac{\sigma_{R^c}^2}{2} = -E_t \left[\hat{m}_{t+1} \right] - \frac{\sigma_m^2}{2} - \rho_{mR^c} \sigma_m \sigma_{R^c}, \qquad (3.24)$$

where the variance term on the left-hand side reflects the Jensen's inequality condition. In the following we neglect this term, because it drops out by rewriting the return in levels. Moreover, we assume that the conditional moments are homoscedastic and not time-varying. The parameter σ and ρ on the right-hand side of eq. 3.24 refer to the conditional standard deviation and correlation of R^c respectively M. For a risk-less asset the same approach yields the following condition:

$$\hat{r}_t^n = -E_t\left[\hat{m}_{t+1}\right] - \frac{\sigma_m^2}{2}. \tag{3.25}$$

Both equations 3.24 and 3.25 illustrate the strong relationship of asset returns and conditional second moments. Especially the differences between different asset returns as well as the risk-measure Sharpe ratio depend only on second moments.

For example, let's investigate the risk premium of the return on equity over the risk-free return. If we measure this premium as $E_t\left[R_{t+1}^{eq}\right]/R_t^n$ we obtain that the equity premium depends only on the covariance between the log pricing kernel and the log return on equity:

$$E_t\left[\hat{r}_{t+1}^{eq}\right] - \hat{r}_t^n = -\rho_{mR^{eq}}\sigma_m\sigma_{R^{eq}} . \tag{3.26}$$

A similar result can be obtained for the Sharpe ratio:

$$SR = \frac{E_t\left[\hat{r}_{t+1}^{eq}\right] - \hat{r}_t^n}{\sigma_{R^{eq}}} = -\rho_{mR^{eq}}\sigma_m . \tag{3.27}$$

Following Hansen and Jagannathan (1997) and Campbell and Cochrane (2000) the highest possible Sharpe ratio is equal to σ_m, by assuming a correlation between the pricing kernel and the return of $\rho_{mR^{eq}} = -1$. Of course, it is unlikely that this restriction is satisfied by the data, but it illustrates the weakest assumption regarding the Sharpe ratio. The latter equations are convenient, as they show how the Sharpe ratio and excess returns can be obtained from second moments. Both exercises show the necessity of second moments for the evaluation of asset returns. To include these findings in our model we rewrite the Euler equations as follows for the risk-free return:

$$\frac{1}{R_t^n} = E_t\left[\exp\left(\log\left(\hat{m}_{t+1}\right) + \frac{\sigma_m^2}{2}\right)\right], \tag{3.28}$$

and comparably for the return and capital:

$$\frac{1}{E_t\left[R^c_{t+1}\right]} = E_t\left[\exp\left(\log\left(\hat{m}_{t+1}\right) + \frac{\sigma^2_m}{2} + \rho_{mR^c}\sigma_m\sigma_{R^c}\right)\right]. \qquad (3.29)$$

3.2.5 Equilibrium

Finally, note that the variables k_t, y_t, c_t, w_t, w^f_t, x_t, λ_t, and g_t have to be productivity detrended to solve the model. That is done by dividing each variable by $\exp(z_{T,t-1})$, except capital k_t, which is detrended with $\exp(z_{T,t})$ and λ_t which is detrended by $\exp(-\eta z_{T,t-1})$. Beside this, we assume l_t, n_t, q_t, R^n_t, R^c_t, R^{eq}_t, M_t, and d_t to be stationary. In the following all detrended variables are marked with $\sim$.

Given the initial values for $\tilde{k}_{-1} > 0$, $\tilde{c}_{-1} > 0$, $l_{-1} > 0$, $\tilde{w}_{-1} > 0$, and $R^n_{-1} > 0$ and a Ricardian fiscal authority; a rational expectations equilibrium is a set of sequences $\left\{\tilde{y}_s, \tilde{c}_s, \tilde{k}_s, \tilde{w}_s, \tilde{w}^f_s, M_s, \tilde{x}_s, q_s, l_s, n_s, d_s, R^n_s, R^c_s, R^{eq}_s\right\}^{\infty}_{s=t}$, which is satisfying the firms' first order condition, the households' first order condition, and the aggregate resource constraint, by clearing the labor market, clearing the market of capital, clearing the bond market, $B_t = 0$, and clearing the final goods market, $\tilde{y}_t = \tilde{c}_t + \tilde{x}_t + \tilde{g}_t$, and the transversality condition, for $\left\{z_{T,s}, z_{I,s}, z_{P,s}, \tilde{g}_s\right\}^{\infty}_{s=t}$.

3.3 Estimation Methodology

In this section we briefly describe the estimation procedure that has been conducted by using the software package DYNARE (see Juillard 2001). In particular, the inclusion of second moments into the estimation, the prior choices for e.g. the Sharpe ratio and the Frisch elasticity as well as the problems that occurred during the estimation of the structural parameters and the four stochastic processes are reported.

3.3.1 Method of undetermined coefficients

As previously discussed one aim of this research is to include second moments of asset returns into the estimation. The benefits of doing this are a more correct determination of the corresponding steady-state values and, of course, the better estimation of the historical time series including their means. As shown in the section

before, a accurate estimation of second moments is needed to explain observed risk measures like the Sharpe ratio, which also allows for a more detailed analysis of the underlying preferences.

The equilibrium of the model is determined by the standard deviation of the pricing kernel, σ_m, which also defines the maximal possible Sharpe ratio, the covariance of this pricing kernel, and the return on total capital, $\sigma_{R^c m}$. Unfortunately, we have to solve the model first, before we obtain its second moments which are needed to solve for the steady state. Altogether, we get a fixed point problem by solving our model accurately. To resolve this fixed point problem we use a similar approach as Canton (2002).

Following Lettau and Uhlig (2002) we can decompose the log pricing kernel and the log return of an asset into their conditional expectations and their innovations:

$$\hat{m}_{t+1} = E_t\left[\hat{m}_{t+1}\right] + \upsilon_t \tag{3.30}$$

$$\hat{r}_{t+1} = E_t\left[\hat{r}_{t+1}\right] + \varsigma_t \tag{3.31}$$

In our DSGE model innovations are introduced through the exogenous stochastic processes $\{z_{T,t}, z_{I,t}, z_{P,t}, \tilde{g}_t\}$, where all innovations,$\{\epsilon_{T,t}, \epsilon_{I,t}, \epsilon_{P,t}, \epsilon_{G,t}\}$, of these processes are assumed to be independent and normally distributed with the corresponding variances $\{\sigma_T^2, \sigma_I^2, \sigma_P^2, \sigma_G^2\}$. With this information we can rewrite the above equations as:

$$\hat{m}_{t+1} = E_t\left[\hat{m}_{t+1}\right] + \left[\epsilon_{T,t}\ \epsilon_{I,t}\ \epsilon_{P,t}\ \epsilon_{G,t}\right] \begin{bmatrix} \eta_{m z_T} \\ \eta_{m z_I} \\ \eta_{m z_P} \\ \eta_{mg} \end{bmatrix} \tag{3.32}$$

$$\hat{r}_{t+1} = E_t\left[\hat{r}_{t+1}\right] + \left[\epsilon_{T,t}\ \epsilon_{I,t}\ \epsilon_{P,t}\ \epsilon_{G,t}\right] \begin{bmatrix} \eta_{r z_T} \\ \eta_{r z_I} \\ \eta_{r z_P} \\ \eta_{rg} \end{bmatrix}, \tag{3.33}$$

where η_{xy} refers to the elasticity of the variable x with respect to the variable y. The conditional second moments of the return and the pricing kernel are the same as these of their corresponding innovation, which allows us to solve for the standard

deviation of the pricing kernel,

$$\sigma_m = \sqrt{[\sigma_T^2 \ \sigma_I^2 \ \sigma_P^2 \ \sigma_G^2] \cdot \begin{bmatrix} \eta_{mz_T}^2 \\ \eta_{mz_I}^2 \\ \eta_{mz_P}^2 \\ \eta_{mg}^2 \end{bmatrix}} \tag{3.34}$$

and similarly for the conditional covariance between the log pricing kernel and the log return of an asset,

$$\sigma_{rm} = \sqrt{[\sigma_T^2 \ \sigma_I^2 \ \sigma_P^2 \ \sigma_G^2] \cdot \begin{bmatrix} \eta_{rz_T}\eta_{mz_T} \\ \eta_{rz_I}\eta_{mz_I} \\ \eta_{rz_P}\eta_{mz_P} \\ \eta_{rg}\eta_{mg} \end{bmatrix}} \ . \tag{3.35}$$

Following Uhlig (1999) and Campbell (1994) we solve the model by using the method of undetermined coefficients.[3] We log-linearize the variables around their steady-state values to receive a linear approximation of the model. This solution technique yields the following recursive law of motion in the form:[4]

$$\hat{y}_t = A\hat{h}_{t-1} + Bu_t \ , \tag{3.36}$$

where $\hat{y}_t$ is a vector containing all log-linearized model variables and $\hat{h}_t = \log(h_t) - \log(h^{ss})$ is the vector containing all log-linearized state variables of the model, with h^{ss} their corresponding steady state values. The matrices A and B contain the partial elasticities we are interested in to solve for the conditional second moments as presented above.

To resolve the fixed point problem, we start with a set of second moments to solve for the steady state and to obtain the recursive law of motion with partial elasticities. Afterwards, we use the implicit second moments of this solution as new starting values and solve the model again. As mentioned by Canton (2002) usually few iterations suffice to achieve convergence and to resolve the fixed point problem. Figure 3.1 illustrates this iteration algorithm.

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[3]See Taylor and Uhlig (1990) for an overview of different methods to solve nonlinear stochastic models.

[4]The notation here differs from the one used by the formerly cited authors but is equivalent to the software package (Dynare) used.

$$\theta(\ldots, \sigma_{R^c m}, \sigma_m) \longrightarrow y_t(\theta) \longrightarrow [\tilde{\sigma}_{R^c m}, \tilde{\sigma}_m] \xrightarrow{\quad \tilde{\sigma} = \sigma \quad} \mathcal{L}(\theta|X)$$

$$\tilde{\sigma} \neq \sigma$$

$$\sigma_{R^c m} = \tilde{\sigma}_{R^c m} \text{ and } \sigma_m = \tilde{\sigma}_m$$
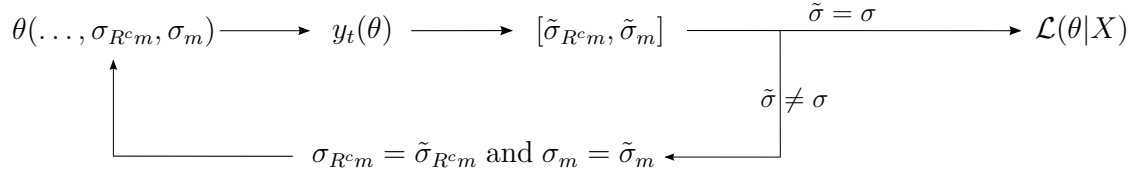
Figure 3.1: Calculation algorithm to resolve the fixed point problem.

Finally, based on the converged solution we can go ahead and solve for the corresponding likelihood by using a Kalman filter. Afterwards we can use our prior to build up the posterior for this set of structural parameters. Of course, the iteration algorithm has to be done for any set of structural parameters, which implies a significant increase of computational time.

3.3.2 Data

The following paragraphs describe the time series, some calculations, and finally the implementation of the observed data into the model. Because we are mainly interested in describing business cycles and thereby not neglecting the stock market and its characteristics, it is necessary to use related time series. For this reason we use data for hours worked, consumption, government expenditures, output and excess returns. To ensure that the model is not misspecified, we used them to create four time series (as much as shocks in the model), used as observations during the estimation.

We use the quarterly real Gross Domestic Produc as measure for output.[5] Additionally we use the Civilian Labor Force series - of the U.S. Department of Labor: Bureau of Labor Statistics (BLS) for persons 16 years of age and older - as a proxy for population to calculate a per capita time series. Because we want to estimate the productivity trend of the economy, the series for output is not detrended. By estimating the model with an unit root in productivity, it is necessary to keep in mind that the final state space model is not stationary. To resolve this circumstance we use a diffuse initialization of the Kalman filter as postulated by De Jong (1991). The implementation of the real logarithmic output per capita time series, $\hat{y}_t^{obs}$, into the model is done as follows:

$$\hat{y}_t^{obs} - \hat{y}_{t-1}^{obs} = \hat{\tilde{y}}_t - \hat{\tilde{y}}_{t-1} + \epsilon_{T,t-1} \tag{3.37}$$

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[5] See appendix A.2.2 for details about source and description of any data used in this research.

For consumption we use the expenditures on non-durables and services, where the quarterly data are seasonally adjusted at annual rates, in billions of current prices. The real consumption per capita series is derived by using the GDP price deflator and population series described above. The time series for government expenditures are provided by the Bureau of Economic Analysis (BEA), which is also transformed into real and per capita terms. Because we assume that within the model, all variables follow the same trend, we do not detrend the time series with log-differences or HP-filter techniques. Moreover, we calculate a government expenditures–consumption ratio and use the quarterly changes of this as a observable variable during the estimation.

The investigation of the persistence of leisure as well as of the existence of real wage rigidities plays an important role in the model. To capture this, we also use historical hours worked as observable variable. The corresponding time series is calculated based on total hours worked of the non-farm business sector and the perviously described proxy for population. The finally implemented stationary data are the log-differences of this time series.

To incorporate a financial time series into the estimation, we have decided for excess returns on equity over the risk-free rate. The return on equity is measured as the quarterly logarithmic return of the CRSP NYSE/AMEX/Nasdaq value-weighted market index. As a proxy for the risk-free return, we use quarterly returns calculated based on the monthly returns of the CRSP 90-Day T-Bill returns. The returns are calculated in real terms, too, by using the implicit inflation given by the price deflator mentioned above. Furthermore, the final excess return series is demeaned.

Recapitulating, the estimation of the model is based on four time series within the period from 1951:qII to 2007:qIV. All data are quarterly and in real terms. Moreover, the Business Cycle data are measured in per capita terms. All series are stationary with exception of output, which contains an unit root to achieve an estimation of the productivity trend.

3.3.3   Prior Choice

The prior choice in the Bayesian estimation is an important point. A priori information about parameters are added to the likelihood function to receive parameter estimates which are economically plausible. Priors have to reflect empirical observation about parameter, by using observations that are concurrent to the estimation

sample but excluded from the likelihood (Del Negro and Schorfheide 2008). Estimating a DSGE model which is in line with Business cycle facts as well as with asset pricing implications requires to think carefully about the economic denotation of each parameter. Especially, we have to recognize possible opposing implications of parameter values for observed business cycle facts and observed asset pricing facts.

An important role in determination of asset prices play the assumed preferences of the households. This common fact is widely discussed in the literature (see e.g. Lettau and Uhlig 2002). This strand of the literature has established some prominent puzzles, namely the *equity premium puzzle* and the *risk-free rate puzzle*.

Both puzzles illustrate frictions for preference parameters, by explaining asset prices and business cycle facts simultaneously. Prominent examples with opposing implications are the intertemporal elasticity of consumption substitution (EIS) and the time preference of consumers (for an excellent overview see e.g. Cochrane 2001).

Investigating the parameters of our model in more detail, we figure out that we have only a poor knowledge about the empirical determination of the parameters itself. In contrast to more simple preferences the parameters of our model cannot be related directly to the EIS or the Frisch elasticity. Moreover, the variables we have indeed empirical observations, are implicitly given by a bunch of preference parameters.

In this research, the Frisch elasticity is defined as the elasticity of labor supply to frictionless wages by holding the marginal rate of consumption constant,

$$\text{FE} = \frac{dn}{dw^f} \frac{w^f}{n} \bigg|_{\bar{U}_c} . \tag{3.38}$$

For the preferences used in this paper the Frisch elasticity can be evaluated as:

$$\text{FE} = \frac{U_n}{\bar{n} \left[U_{nn} - \frac{U_{nc}^2}{U_{cc}} \right]} , \tag{3.39}$$

finally given by:

$$\text{FE} = \frac{1 - \bar{n}}{\bar{n}} \cdot \frac{\eta (1 + \alpha) (1 - \psi)}{\eta (\alpha (1 - \nu) + 1 + \nu) - \nu} , \tag{3.40}$$

where $\alpha = A (1 - \psi)^{-\nu} \bar{l}^{-\nu}$. The former Business cycle literature often models a relatively high Frisch elasticity of two or more (King et al. 1988; Prescott 1986), while more recent papers of Bayesian DSGE model estimation found far smaller

values for the Frisch elasticity. For example Smets and Wouters (2003), Del Negro et al. (2005), or Justiniano and Primiceri (2006) argue for values between 0.25 and 0.5. These findings are in line with some micro-data based studies, which also argue for small values in a range between 0 and 0.7.[6] Similarly, we have good knowledge about the intertemporal elasticity of consumption substitution. For the preferences used in this paper the EIS is given as

$$EIS = \frac{1 - e^{-\gamma}\chi}{\eta} \ , \tag{3.41}$$

which is the inverse of the relative risk aversion for these preferences. This relation between EIS and relative risk aversion does not hold for any preferences, e.g. internal habit formation or Epstein-Zin preferences. Because of this reason as well as due to the controversial discussion regarding correct values for the relative risk aversion (see Cochrane 2001), we refer rather to the EIS than to the relative risk aversion. Former empirical research for the EIS often argues for values of slightly above zero (Campbell and Mankiw 1989, 1991; Hall 1988) while more recent research finds an EIS significant different from zero in the range between 0.35 and above unity.[7]

These examples illustrate, that we do not have empirical observations about the preference parameter itself. Moreover, we have some a priori knowledge about implicit variables. It is necessary to ensure that the informative prior used for e.g. the Frisch elasticity dominates the implicit one arising from any involved parameter. Because any prior on the involved parameter maps into e.g. the Frisch elasticity, what implies also a prior for the same. For this reason, we decided to use diffuse priors for the parameter itself within their domains.

A Business cycle econometrician would use a prior for the Frisch elasticity as well as for the EIS to ensure economically reasonable estimates for the preference parameters. However, we know that an a priori high EIS would reduce the ability of the model to match empirical asset pricing facts (see Lettau and Uhlig 2002). We build priors for the Sharpe ratio and the Frisch elasticity that ensure parameter combinations that explain Business Cycle facts and the high equity premium. Additionally, to judge the model and the solutions of the estimation, we calculate the EIS, but do not include an informative prior into the posterior. Recalling equations 3.40 and

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[6]Especially Pistaferri (2003) argues for this range, but also the findings of MaCurdy (1981), Lee (2001), and Ziliak and Kniesner (2005) postulate a small Frisch elasticity within these bounds.

[7]See for example Attanasio and Weber (1989, 1993), Beaudry and van Wincoop (1996), Vissing-Jørgensen (2002), Vissing-Jørgensen and Attanasio (2003), and Bansal and Yaron (2004).

3.41, we have priors about each preference parameter except for the consumption habit χ.

The prior for the quarterly maximal Sharpe ratio is assumed to be normally distributed with mean 0.18 and standard deviation of 0.03. These assumptions cover most of the postulated Sharpe ratios as well as the one of our data. The prior for the Frisch elasticity is chosen to capture most of the empirical findings mentioned above. For this reason we follow Del Negro and Schorfheide (2008) and use a prior about the inverse of the Frisch elasticity, which is Gamma distributed with mean 2.0 and a standard deviation of 0.75. This assumption implies that over 90% of the prior distribution cover values for the EIS between 0.3 and 1.5.

In explaining business cycle facts and asset pricing facts simultaneously, also the discount factor β plays an important role. The Business cycle literature often uses values for the discount factor slightly smaller than one to ensure a positive time preference of the representative agent and steady-state risk-free returns comparable to observed returns. However, from an asset pricing perspective discount factors with much smaller values or values greater than one are postulated.[8] These opposing assumptions are known as the *risk-free rate puzzle* (see Weil 1989). This parameter especially illustrates the difficulty of a proper combination of both strands of literature. We decided to follow the main business cycle literature and to ensure positive time preference by using a Beta distributed prior for β, which has a mean slightly smaller than one.

The prior for the remaining deep model parameters are chosen according to the recent literature.[9]. We intend not to introduce too much curvature or too much downweighting of some parameter spaces into the likelihood function by using wide priors for non-observable parameters, and more restrictive priors for well-documented or observable parameters like depreciation rate and growth rate. The standard deviations of all shocks are assumed to be Inverted-gamma distributed at a level of 0.02, with a degree of freedom equal to 4. We assume for every autoregressive parameter to be beta distributed, with mean 0.85 and standard deviation 0.1. An overview of the priors used for the parameters is given in table 3.1.

We also use a prior to control for the steady state value of leisure. Given the real wage rigidities and the nonseparable preferences used in this paper, the steady

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[8]Kocherlakota (1990) has shown that values for the discount factor above unity can be in line with positive time preference if the economy is growing.

[9]See e.g. Del Negro and Schorfheide (2008) and Smets and Wouters (2003)

Parameter	Domain	Density	Para(1)	Para(2)
Model parameter				
β	$[0,1)$	Beta	0.95	0.025
γ	$\mathbb{R}$	Normal	0.007	0.0005
ω	$[0,1)$	Beta	0.75	0.1
A	$\mathbb{R}^+$	InvGam	0.01	4
η	$\mathbb{R}^+$	Uniform	1	20
ν	$\mathbb{R}^+$	Gamma	2.5	1.75
χ	$[0,1)$	Beta	0.5	0.23
ψ	$[0,1)$	Beta	0.5	0.23
θ	$[0,1)$	Beta	0.33	0.05
δ	$[0,1)$	Beta	0.02	0.005
ζ	$\mathbb{R}$	Normal	4.0	1.0
μ	$[0,1)$	Beta	0.75	0.15
Implicit model parameter				
$1/FE$	$\mathbb{R}^+$	Gamma	2.00	0.750
SR	$\mathbb{R}$	Normal	0.18	0.03
$\bar{l}$	$[0,1)$	Beta	2/3	0.2
Autoregressive parameter and s.d. of shocks				
π_G	$[0,1)$	Beta	0.85	0.1
π_I	$[0,1)$	Beta	0.85	0.1
π_P	$[0,1)$	Beta	0.85	0.1
ϵ_T	$\mathbb{R}^+$	InvGam	0.02	4.0
ϵ_I	$\mathbb{R}^+$	InvGam	0.02	4.0
ϵ_P	$\mathbb{R}^+$	InvGam	0.02	4.0
ϵ_G	$\mathbb{R}^+$	InvGam	0.02	4.0

Table 3.1: Prior distribution for model parameter and additional parameter. Para(1) and Para(2) correspond to means and standard deviations for the Beta, Gamma, Inverted Gamma, and Normal distribution, while for the Uniform distribution these values correspond to the lower and upper bounds.

state value for leisure varies with changes in the preference parameters.[10] By using a prior for steady state leisure which is Beta distributed with mean 2/3 and a standard deviation of 0.2 we ensure a stable reasonable steady state and include our a priori information that leisure is probably two times as high as labor.

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[10]See appendix A.2.1 for more details about the calculation of the steady state.

Concluding, it is worth to pointing out that we estimate all parameters except of the wage mark-up ε_w. This parameter is set to 5%.

3.3.4 Estimation

As mentioned above we use the software package Dynare for the estimation. We had to add some tools to estimate the implicit variables and to ensure the convergence of the second moments for each draw of the deep model parameters. Furthermore, we ensure monotonicity and concavity of the utility for each draw of η and ν.

For the subsequent Metropolis Hastings algorithm around the posterior mode, we assume that proposal density and target density are the same. The proposal density is assumed to be a scaled version of the inverse Hessian calculated at the posterior mode (see Schorfheide 2000). For this reason it is necessary to ensure that the posterior maximization yields a global solution. Obviously, the estimation of 19 parameter is a highly dimensional problem. However, to obtain a global maximum, we introduce a random draw mechanism to choose initial values for the maximization. Finally, we used 1,000 random initial values to achieve the posterior mode.

For the posterior distribution estimation around the posterior mode, we conduct a Metropolis Hastings algorithm along two chains with one million draws each. As postulated by Roberts et al. (1997) a Metropolis Hastings algorithm of high-dimensional models converges optimally by an acceptance rate of 0.2431. We scale the inverse Hessian to receive a proposal density with a similar acceptance rate. Afterwards we investigate the convergence of both chains by using convergency diagnostics like Brooks and Gelman (1998). Because the model converges after about 800 thousand draws, we decided to discard any draw before. Finally, we use the last 200 thousands draws from each chain of the estimation for further calculations.

3.4 Estimation Results

The parameter estimates of the posterior mode maximization as well as the posterior mean and the higher probability densities (10% and 90% interval) are presented in table 3.2. The figures 3.2 and 3.4 illustrate the differences of the posterior estimation from the prior distribution used for structural deep model parameters, the

autoregressive parameters, and the standard deviations of the shocks. Figure 3.3 illustrates the posterior estimates of the additional parameters used during the estimation, i.e. the Frisch elasticity, the Sharpe ratio, and the steady-state leisure demand as well as the posteriors for selected second moments and the EIS.

By investigating the results for the structural parameters, we can obviously identify the preference parameters η, ν, the habit parameters χ and ψ as well as the discount factor β. These posteriors are significantly different from their corresponding prior distributions, while the preference parameter A seems not to be identified. The high posterior habit level for consumption ($\chi = 0.78$) coincides with other estimates in the literature. However, this result implements a small elasticity of consumption substitution (EIS) (0.05) which is in contrast to the main Business cycle literature. The habit level for leisure shows to be similarly high; $\psi \sim 0.7$ which indicates a high degree of persistence with respect to the leisure demand of the households.

The degree of real wage rigidities, μ, is significantly smaller than its prior ($\mu = 0.19$). This result must be considered in combination with the high persistence of leisure demand and the small Frisch elasticity. Because of the high habit level of leisure, the households are unwilling to substitute leisure over time (small elasticity of leisure substitution). This implies a small Frisch elasticity and eventually results in a kind of "natural" rigid wages. The estimate of the Frisch elasticity ($FE \sim 0.3$) is smaller than often assumed in the Business cycle literature but in line with estimates of Justiniano and Primiceri (2006).

Furthermore, we can identify significantly different results for the growth rate γ and the capital share θ. The estimated capital share is much smaller than usually assumed in the literature ($\theta = 0.22$). The deterministic trend within the economy is measured with 0.005, which implements an annual growth rate of approximately 2%. Unfortunately, we do not obtain significantly different values of the posterior from the prior for the nominal depreciation rate δ, the leverage ratio ω, and the elasticity of the price of capital $1/\zeta$. All posteriors are only slightly different from their assumed prior distribution. The leverage ratio ω is estimated slightly smaller, while the elasticity of the price of capital is estimated slightly above its prior. The nominal depreciation rate of 0.017 corresponds to a real depreciation rate of $\tilde{\delta} = 0.022$, and is thus similar to the usually assumed depreciation rate in the literature.

The implicit maximal quarterly Sharpe ratio within the model is not very different from its prior (SR=0.16). This result is comparable to an annual Sharpe ratio of 0.32, which is rather at the lower bound of the observable values. Moreover,

this also implies a standard deviation of the pricing kernel of 0.16 per quarter. This illustrates the well-known problem of consumption-based asset pricing models (CBAPM), where a high volatile pricing kernel is needed to explain asset pricing facts. This high volatility goes along with the previously mentioned small elasticity of consumption substitution and implements a smooth consumption path.

However, the model resolves the observable standard deviation of equity returns. The estimated conditional standard deviation of 0.085 per quarter is comparable to the stylized facts of equity returns with an annual standard deviation of approximately 0.16. Finally, the main benefit of the present model and the estimation technique is the ability to estimate different steady state values and different second moments of assets more accurately than with standard techniques. The standard deviation of the return of capital is 0.03 and the standard deviation of the risk-free return becomes approximately 2.0% per year which is only slightly above the stylized fact.

The parameter estimates of the structural shocks are illustrated in figure 3.4. The standard deviations can be identified significantly different from their prior distributions. The standard deviation of the technology shock ($\sigma_T = 0.008$) meets the findings in the literature. The standard deviation of the capital adjustment cost shock, $\sigma_I = 0.03$, is relatively high in comparison to the estimates of the other shocks. This suggests a high importance of this shock to explain the fluctuation of asset prices. Moreover, the degree of persistence of this shock cannot be identified significantly different from its prior while the other shocks are identified as high persistent shocks.

Of course there are several parameters, which cannot be identified at all or only on a poor basis, for example, the preference parameter A or the persistence parameters of the AR(1) processes. These parameters cannot be updated by the estimation, either because of the used time series or of the model specification itself. Since within the estimation approach used in the present paper, the estimation is based on a subset of time series in comparison to the involved variables of a DSGE model. Consequently, this leaves some arbitrariness in the procedure of identifying parameters (see e.g. Leeper and Sims 1994). However, adding more time series into the estimation, would require assumptions about additional structural shocks or measurement errors. In particular the economically justification of such measurement errors seems to be difficult.

Figure 3.2: Posterior (black) and prior (grey) density of deep model parameters (the dashed line is the mode of the posterior maximization)
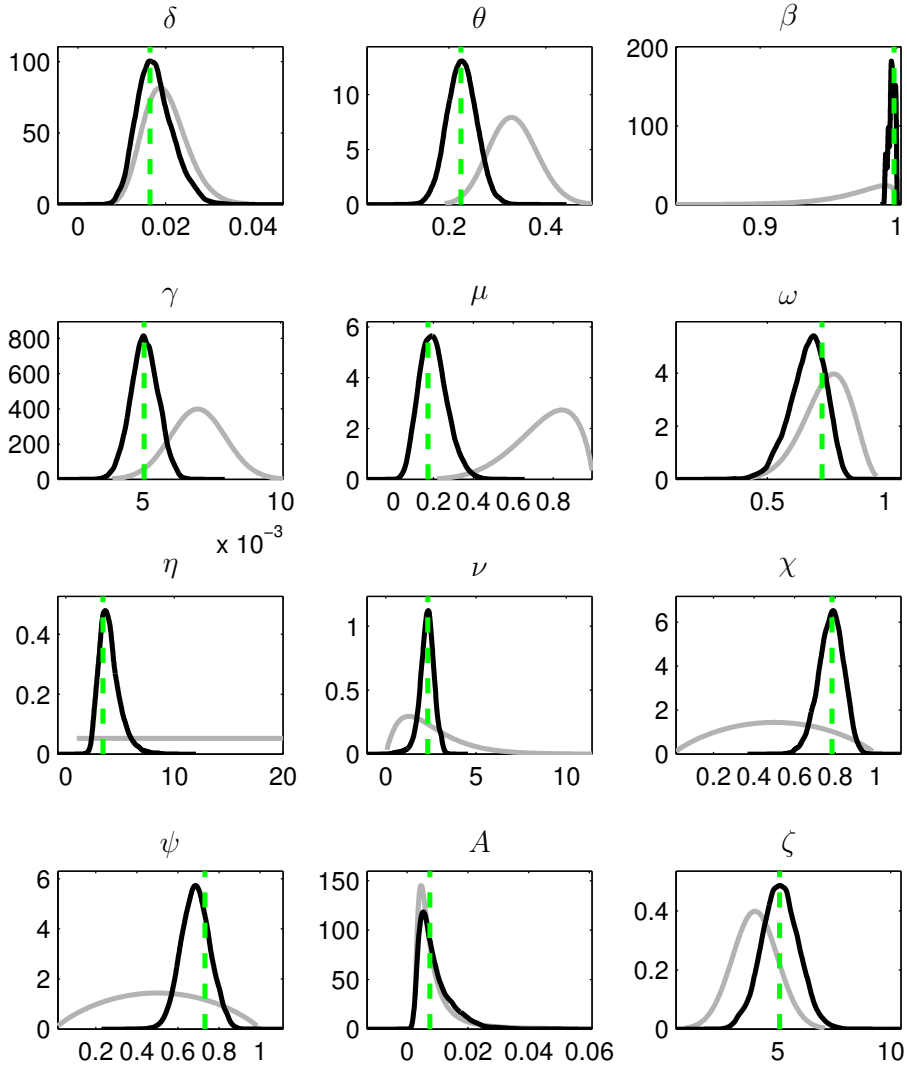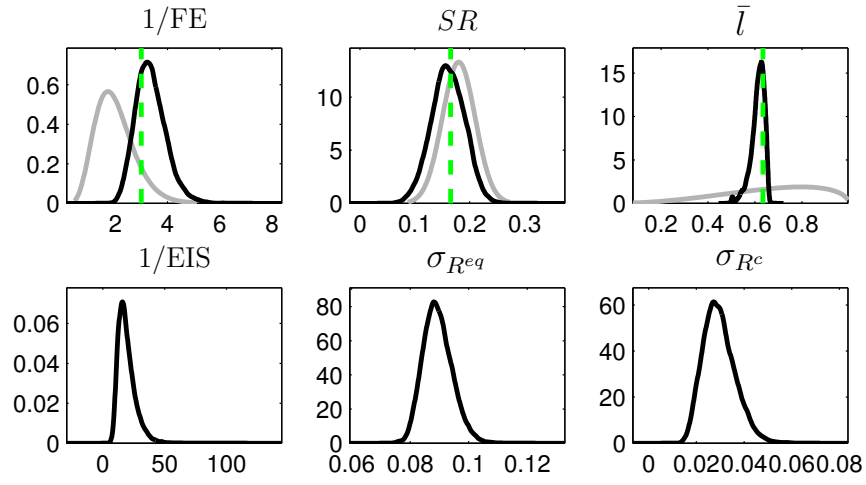
Figure 3.3: Posterior (black) and prior (grey) density of additional implicit model parameters (the dashed line is the mode of the posterior maximization)
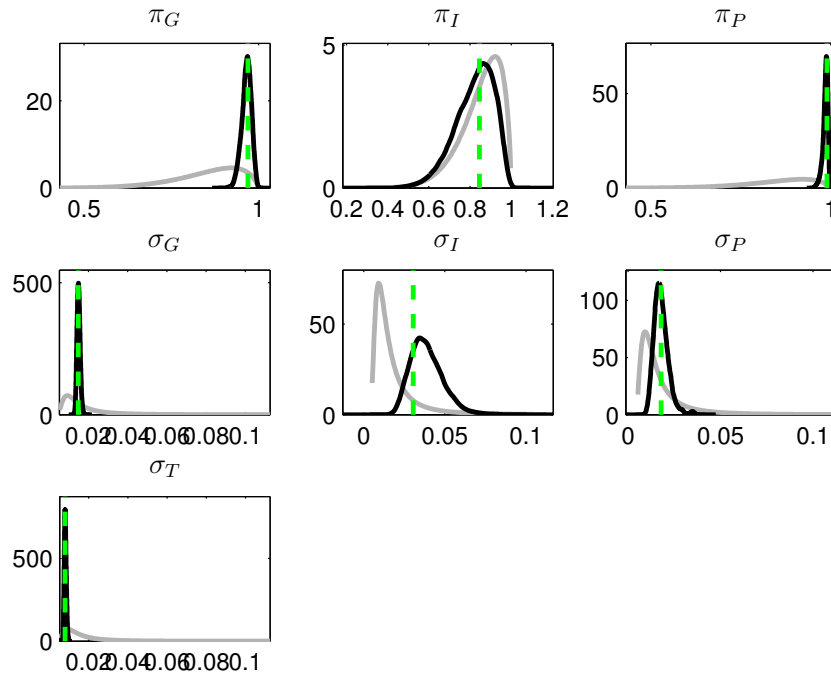

Figure 3.4: Posterior (black) and prior (grey) density of autoregressive parameters and standard deviation of shocks (the dashed line is the mode of the posterior maximization)

Parameter	Posterior mode	s.d.	Posterior mean	HPDinf	HPDsup
Model parameter					
β	0.9980	0.0002	0.9963	0.9929	0.9999
γ	0.0051	0.0005	0.0050	0.0042	0.0059
ω	0.7326	0.0943	0.6701	0.5480	0.7923
A	0.0075	0.0066	0.0099	0.0025	0.0170
η	3.4221	0.9138	4.1110	2.5443	5.4923
ν	2.3261	0.8327	2.1991	1.6077	2.9040
χ	0.7845	0.0631	0.7779	0.6774	0.8777
ψ	0.7317	0.4045	0.6794	0.5739	0.8019
θ	0.2251	0.0813	0.2259	0.1779	0.2759
δ	0.0164	0.0041	0.0176	0.0111	0.0244
ζ	5.0846	0.9830	5.1756	3.8312	6.4927
μ	0.1727	0.0902	0.1989	0.0837	0.3086
Implicit model parameter					
$1/FE$	2.9985	-	3.3870	2.4072	4.2538
SR	0.1652	-	0.1583	0.1106	0.2091
$\bar{l}$	0.6341	-	0.6074	0.5718	0.6556
$1/EIS$	-	-	19.6050	9.2106	29.6375
$\sigma^{R^{eq}}$	-	-	0.0892	0.0812	0.0972
σ^{R^c}	-	-	0.0295	0.0186	0.0402
Autoregressive parameter and s.d. of shocks					
π_G	0.9695	0.0141	0.9653	0.9431	0.9883
π_I	0.8453	0.1105	0.8192	0.6806	0.9705
π_P	0.9897	0.0233	0.9848	0.9740	0.9954
σ_T	0.0079	0.0007	0.0080	0.0072	0.0089
σ_I	0.0305	0.0119	0.0385	0.0224	0.0538
σ_P	0.0178	0.0143	0.0179	0.0117	0.0235
σ_G	0.0148	0.0008	0.0150	0.0137	0.0163

Table 3.2: Results from the Metropolis Hastings algorithm

3.4.1 Bayesian Impulse Response Functions

In order to understand the properties of the model, in this section, we investigate the impulse responses to the different shocks, based on the results of the posterior estimation. Figures 3.5 to 3.8 show the responses to the four structural shocks over the next 40 quarters. The figures include the response at the posterior mean and the 90% HPD interval. All variables' responses are plotted as percentage deviation from their steady-state values, due to a one percent increase of the respective shock.
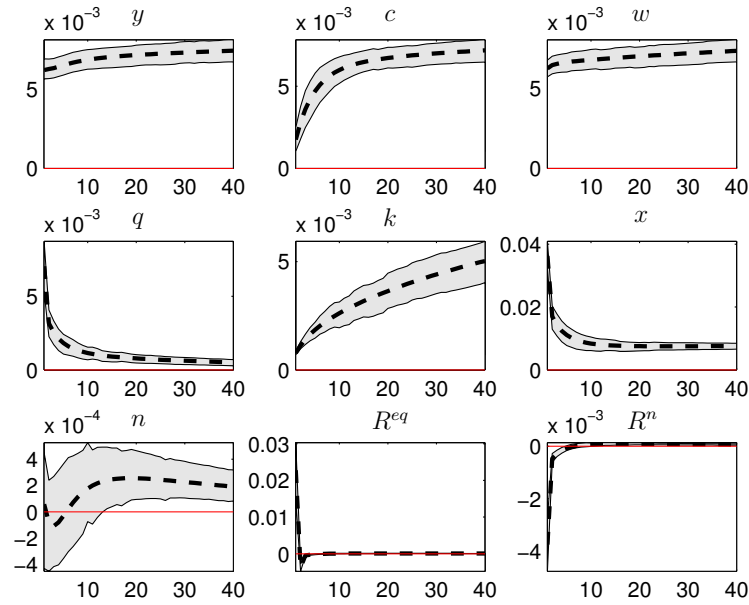

Figure 3.5: Bayesian IRF (DSGE model) to orthogonalized shock to ϵ_T.

Figure 3.5 reports the impulse responses to a permanent increase in productivity. While output and real wages quickly increase to the new productivity level, there is only a slow adjustment in consumption and capital. These slow increases are caused by the high level of consumption habit and the high adjustment costs, respectively. The large increase of investment in the first period, goes along with a strong increase of the price of capital, based on a high expectation for equity returns. The most interesting effect shows the impulse response of hours worked. There is no significant short-run effect of technology on this this variable. The small elasticities of consumption and leisure substitution go along with a reduction of hours worked in the short run. As pointed out by Gali (1999) and Francis and Ramey (2002), this effect would imply a negative correlation between hours worked and output.[11] As

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[11] See also Uhlig (2004b) for the controversial discussion of the role of productivity shocks.

mentioned above, the amplitude and the duration of the negative response depends on the level and relation of the elasticities of substitution. However, we cannot identify a significant sign for the short-run effect, but a significant positive response of hours worked in the mid- and long-run after a productivity shock.
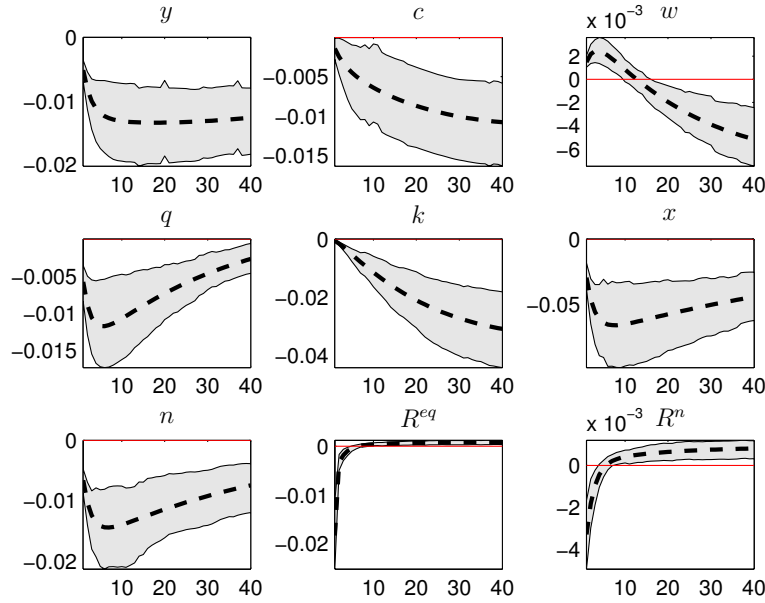

Figure 3.6: Bayesian IRF (DSGE model) to orthogonalized shock to ϵ_P.

Figure 3.6 shows the respond to a shock that increases the habit level of leisure. In this case, the representative agent is increasing her leisure as well. This goes along with an short-run increase of real wages. Because of a the slow reduction of consumption, the output is reduced, which makes labor input more expensive. However, in the mid-run and in the log-run there is an overall declining effect on the economy.

Figure 3.7 presents the impulse responses to an investment shock. This goes along with a declining price of capital and a strong increase of private investment in the short-run. To finance the higher investments in the short-run and because of the habitual formation of consumption, consumption is reduced for a few quarters. Additionally, the high capital adjustment costs increase the supply of labor, which reduces the wages in the short-run. In the long-run, the increasing capital stock and the increasing wages increase private consumption. The wide error bands illustrate, that we cannot identify the degree of persistence of these shocks very accurately.

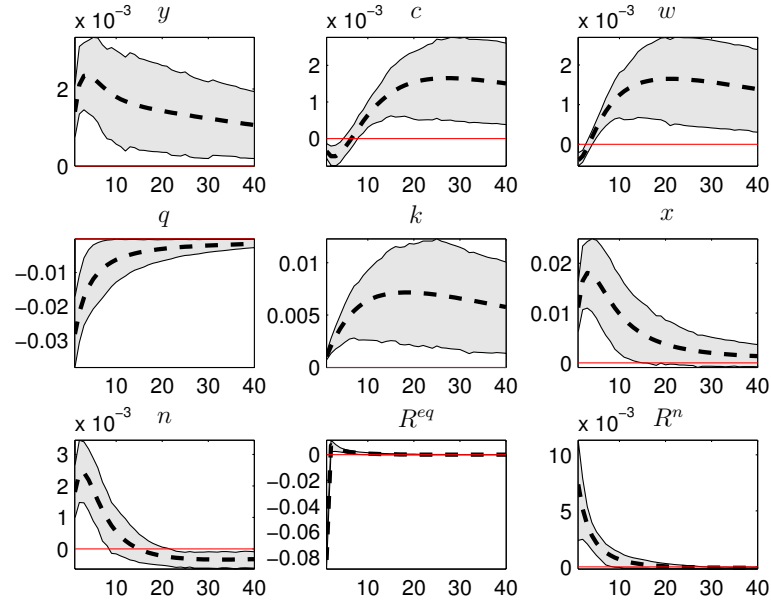Finally, figure 3.8 reports the responses of the endogenous variables to an unex-

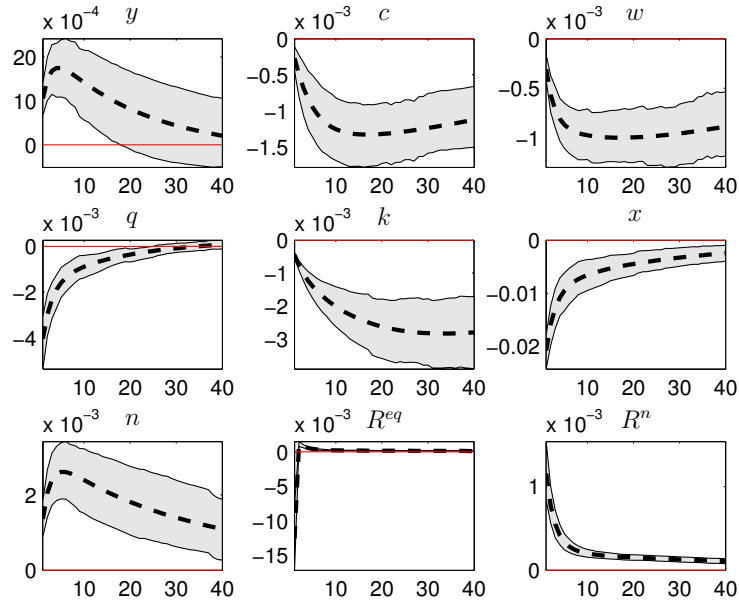Figure 3.7: Bayesian IRF (DSGE model) to orthogonalized shock to ϵ_I.

Figure 3.8: Bayesian IRF (DSGE model) to orthogonalized shock to ϵ_G.

pected increase in government expenditures. This shock is highly persistent and expansionary which increases the output of the economy as well as hours worked. This reduces the real wages, and because of the increased interest rate, private consumption is crowded out. These effects are accompanied by a falling price of capital, a reduction of private investment, and a declining capital stock.

3.4.2 Smoothed Variables

In this section we investigate how well the estimated model simulates historical variables. For this exercise, we calculate the smoothed variables, using a two-sided Kalman smoother, which includes all information available up to today.[12] All plots are evaluated at the posterior mode.

Figure 3.9 reports the smoothed observable variables as estimated by the model and the corresponding historical time series used to estimate the model. The figure shows that all time series introduced into the estimated can be replicated by the estimated solution. In this case the existing measurement error is approximately zero.
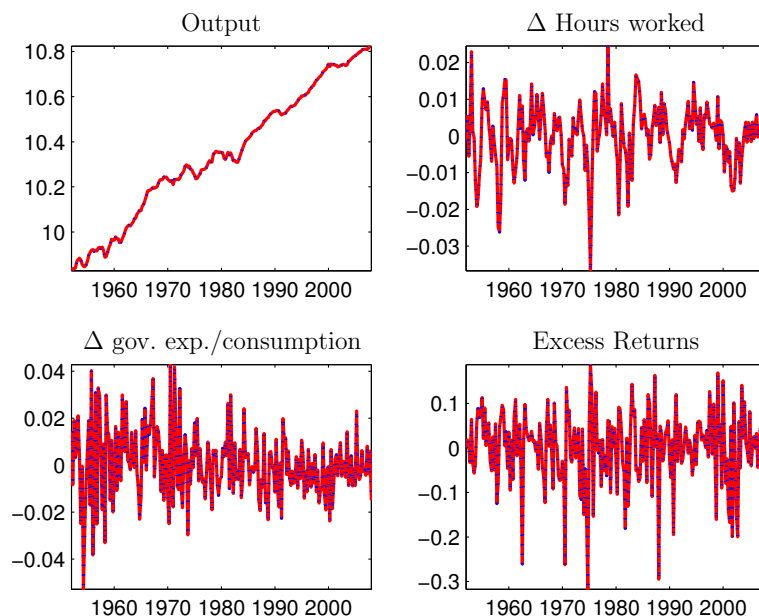

Figure 3.9: Smoothed observable variables of the model (solid line) and historical time series (dotted line).

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[12]The results can be directly received from Dynare.

In order to further check I test how well the estimated model fits the data of other endogenous variables that are not directly introduced into the estimation. Figure 3.10 reports selected smoothed variables (solid line) which indicate how well we can fit the historical business cycle time series. The smoothed variables for private consumption and government expenditures are plotted together with their historical observable counterparts as described in section 3.3.2. For real wages we use total compensation as comparable historical data, and private investment is measured as sum of private investment and durable consumption.[13] The historical data are designated by the dotted line. The figure illustrates that that the model successfully replicates the values for consumption and government expenditures up to the beginning 1980's. In addition to the fluctuations, which benefits from using the ratio of both as observable variable within the estimation, especially, the estimated trend of the productivity corresponds to the these variables. With beginning of the 1980's the gap between the smoothed variables and the historical data increases. This misfit of the model can be explained by its simplicity. For example with the beginning 1980's the U.S. budget deficit as well as the U.S. trade balance deficit increased substantially, but both, government deficit and international trade are not modeled.

Moreover, the model also replicates the trend in real wages as observable in the data. However, it looses explanatory power with respect to the explanation of the fluctuation of this time series. Especially, the strong increases in the end of the 1980's and the end of the 1990' cannot be sufficiently explained. Within these episodes of booming stock markets, wage bonus schemes got an greater importance which could explain the failure of the model. The simulation of private investment is more volatile. The smoothed variable explains historical fluctuations well but fails by explaining the trend.

The remaining subfigures of figure 3.10 are plotted without corresponding historical time series. Nevertheless, the plot of hours worked is identical to the historical series. Since we can simulate its fluctuations perfectly a shown before. More interesting is the simulated time series of the price of capital, the Tobin's q relation, of the model as well as the stock of capital. The price of capital has to be stationary by definition. We can identify prominent historical shocks within the smoothed variable. For example we can identify the oil price shocks in the mid of the 1970's and the beginning 1980's as well as the boom and bust episodes on the equity market at the end of the 1980's and 1990's. However, these shocks do not heavily affect the

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[13]See appendix A.2.2 for detailed information on the data.

capital stock. The fluctuations of capital reduce at the beginning of the 1980's, and the stock itself strongly increase.

Investigating the estimated productivity path, we find that the identified shocks in the price of capital are not related to productivity. Thus, there is more evidence for that these shocks are driven by preference and investment shocks.
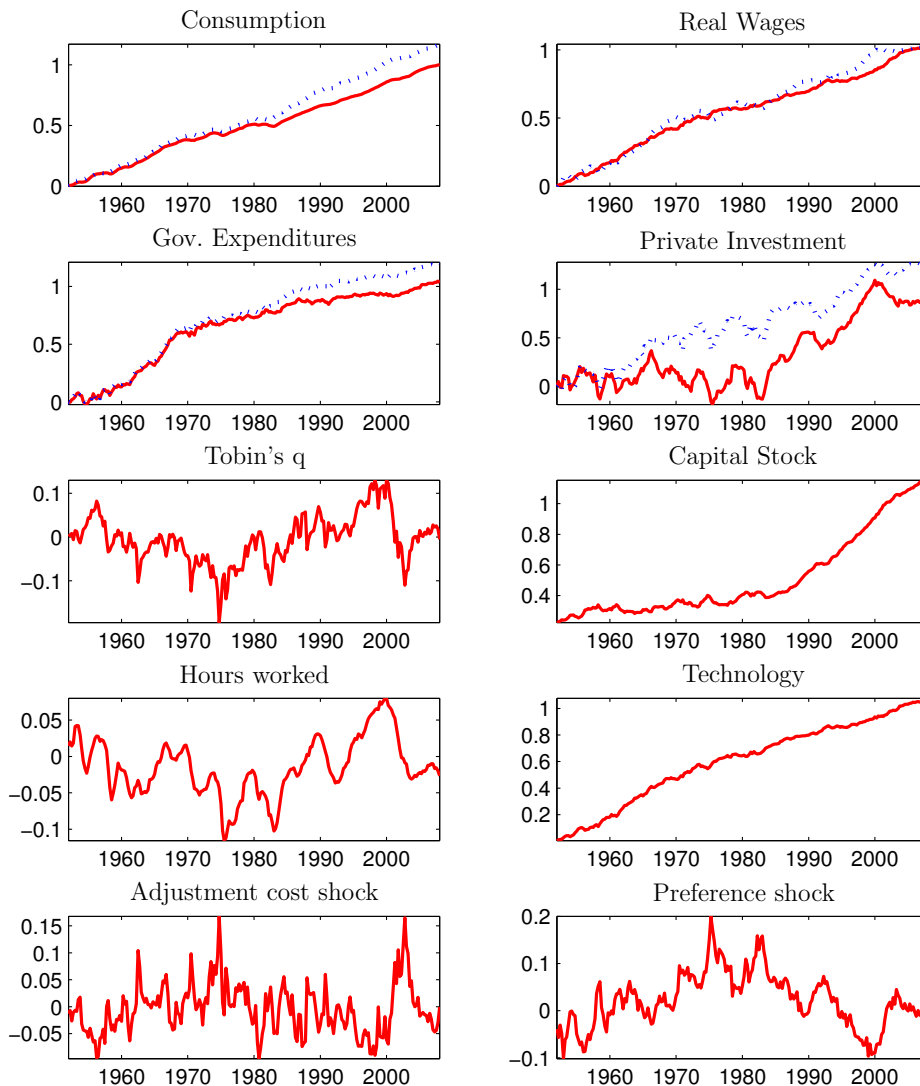

Figure 3.10: Smoothed Business cycle variables of the model (solid line) and historical time series (dotted line).

Up to this point, we can conclude that the model, with all its limitations, is quite successful in explaining business cycle movements. Additionally, our intention is

to explain asset pricing facts simultaneously. As previously discussed, the model replicates the different volatilities of different class of assets as well as the Sharpe ratio more precisely than standard business cycle models. Figure 3.11 presents the results of the smoothed return on equity and the risk-free rate as well as their corresponding historical time series. In contrast to the foregoing figures, we here plot the smoothed variables based on their steady-state values.

The figures illustrate the advantage of the estimation technique described in this paper. Because the different steady-state values for the different asset returns can better estimated, we can also fit the data more accurately. We have great success in explaining the return on equity, regarding its fluctuations as well as its level. Unfortunately, we still overestimate the volatility of the risk-free rate while fitting the level quite well. Of course, by definition, the smoothed return on total capital is closely related to the smoothed return on equity. We can estimate different levels as well as a smaller volatility of this variable. Finally, the plot of the pricing kernel illustrates that an high volatile pricing kernel is needed to resolve stylized asset pricing facts, especially the Sharpe ratio.
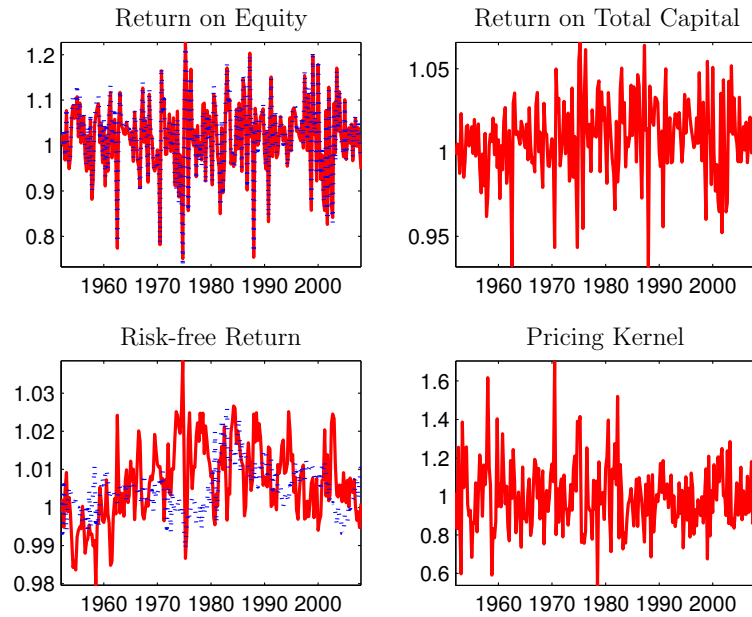
Figure 3.11: Smoothed asset returns of the model (solid line) and historical time series (dotted line).

3.5   Conclusion

In this paper, we have presented an estimated DSGE model, which simultaneously explains several macroeconomic and financial market facts. We found point estimates for macroeconomic facts like e.g. the Frisch elasticity, which are in line with the recent literature. Within the estimation we have accounted for second moments, which is necessary, especially, to explain asset returns more accurately. Due to this, we are able to estimate different asset return levels and second moments that match the historically observe that is closer to the data compared to estimates from standard "pure business cycle" DSGE models.

Additionally, the smoothed variables of the model are quite successful in fitting macroeconomic time series as well as the return on equity. Of course, there are still some misspecifications, like e.g. a to volatile risk-free return. However, the estimation technique as well as the properties of the model shed some light on the relationship of business cycles and asset prices.

Specifically, we obtain empirical evidence, that in a DSGE model, external habits in consumption and leisure play an important role for the simultaneous explanation of macroeconomic facts and asset market facts. As shown by Uhlig (2004a), the relation of the different elasticities of intertemporal substitution are an driving force, not only with respect to stylized asset pricing facts but also to understand the dynamics of the different macroeconomic variables better.

4 Modeling Stock Market Booms

This paper examines a DSGE model which covers the observed co-movements of stock market boom and bust episodes in the 1980's and 1990's and the economy. The boom episodes within the model are triggered by news shocks about the future technology. By including nonseparable preferences and nominal rigidities, the model explains the simultaneous rise of consumption, output, investments, hours worked, and wages during a boom and the subsequent bust. Furthermore, featuring a standardized monetary authority, the model also replicates the observed fact of a declining inflation during the boom episodes. As a result the model allows for a more fundamental discussion of central bank activism during stock market booms. The paper concludes that a monetary authority which is not only "strict" inflation-targeting can reduce the welfare losses through stock market booms.

4.1   Introduction

During the last decades a strand of monetary policy research tends to question, whether the monetary authority should respond to asset pricing movements. This interest seems obvious with respect to the recent history, but in the case the answer is yes, how exactly should central banks respond to asset price movements? To answer this question it is necessary to have a model at hand which helps to understand the co-movements of stock market booms and busts and the real economy in more detail. Given such a model the investigation and evaluation of policy instruments can help to resolve the aforementioned question.

The derivation of such a model and the investigation of monetary policy are the purposes of the present paper. First, the paper evaluates a New-Keynesian DSGE model which can replicate the movements of some key macro variables during the asset market boom at the end of the 1980's and 1990's. Afterwards different monetary policy regimes are investigated, and optimized rules are used to illustrate the pos-

sibilities of the monetary authority to reduce the distortions during the boom-bust episode.

For the boom and bust episodes of the 1980's and 1990's it can be empirically disclosed that during an asset market boom output, investments, consumption, and hours worked are all rising, followed by an overall reduction during the bust episode. Additionally, real wages are rising during the boom and later fall. The present model's ability to recapitulate this additional fact makes the model more applicable to detailed policy investigations than similar models in the literatures. The investigated episodes of booming equity markets have gone along with decreasing interest rates and decreasing inflation. These stylized fact are widely discussed in the recent literature (e.g., Adalid and Detken 2007) and contradict the model findings of Bernanke and Gertler (2000) that inflation tends to rise during asset market booms. Furthermore, the conclusion of Bernanke and Gertler (2000) that an inflation-targeting monetary authority automatically stabilizes the stock market seems not longer obvious. Moreover, as mentioned by e.g. Cecchetti et al. (2000) a 'leaning against the wind' monetary policy could prevent an additional heating-up of the boom due to a reduction of interest rates.

In order to investigate policy decisions it is necessary to discuss the source of the rapidly increasing stock market. The problematic identification of a boom and its source is the basis for further policy suggestions and the reason for the variaty of suggestions, namely from preemptive approaches to a reactive approaches (see Bean 2004; Greenspan 2002). Within the literature rapidly increasing asset prices are usually classified into fundamental or non-fundamental, into a boom or a bubble. The approaches solving for the appearance of huge asset prices movements vary from irrational exogenous shocks (see e.g. Bernanke and Gertler 2000, 2001; Tetlow 2006) to rational but wrong expectations about the future (Beaudry and Portier 2006; Christiano et al. 2007; Gilchrist and Saito 2006). To shed light on the debate, it seems essential to investigate the interdependencies between asset market booms and the rest of the economy in more detail.

The model presented in this paper is an extension of Christiano et al. (2007). In contrast to the authors, the model in this paper can also simulate the simultaneous increase of wages and hours worked during asset market booms due to the use of nonseparable preferences between consumption and leisure. Furthermore, the representative agent has habitually formed preferences with respect to her former level of consumption and leisure. This makes the agent more unwilling to change her leisure over time. Moreover, the individual concurrently demands a higher wage for

an increase of her hours on the job due to a small Frisch elasticity. The model shows that the nominal wages are slightly increasing as a reaction of the overoptimistic anticipated shift of technology, which is in line with the data. Of course, this effect is also supported by nominal wage rigidities and is a necessary fact in the model, but cannot soley resolve the simultaneous increase of real wages and hours worked (Christiano et al. 2007). Finally, the increase of real wages depends on the decrease in inflation. As mentioned by Christiano et al. (2007) the interaction of real wages and inflation targeting in the form of a standard Taylor rule can trigger a boom episode. By capturing this fact more accurate, the model is more accurate to the observed boom and bust episodes, which increases the ability of the model to investigate policy activities.

In the contrast to most of the literature, the present paper does neither investigate additional features of the monetary policy rule nor argues for optimal monetary policy rules. Instead, the main interest is to investigate the reactions of a standardized monetary policy rule during asset market booms and busts. Especially, the ability of this monetary policy rule to stabilize the economy under different monetary policy regimes is focused. For example, as previous discussed, in the present model with an anticipated increase of technology, the increasing real wages tend to down-shift inflation due to the nominal rigidities in the economy. An inflation-targeting regime would cut the nominal interest rate followed by a credit boom which in turn is heating-up the boom episode (see Christiano et al. 2007).

In order to investigate the consequences of different regimes from "strict" inflation-targeting to a more "flexible" inflation-targeting regime, I assume that the monetary authority is interested in stabilizing the economy with respect to fluctuations in inflation, output gap, and changes of the nominal interest rate. For a comparison I calculate optimized monetary policy rules based on the loss function of the central bank (e.g. Levin and Williams 2003). Under these optimized rules only small differences between the regimes are discovered. However, it can be concluded that a monetary authority should increase the nominal interest rates during the boom. This finding confirms the 'leaning-against the wind' policy as suggested by Cecchetti et al. (2002). Additionally, a monetary policy regime which accounts more for a steady interest rate and small output fluctuations is welfare-enhancing. With respect to the debate about central bank activism this finding suggests that a continuous and moderate monetary policy is favorable.

As mentioned above, the paper is closely related to Christiano et al. (2007). Comparable to their approach, my model is triggered by an over-optimized anticipated

future technology and motivated by the findings of Beaudry and Portier (2006). A similar approach is proposed by Gilchrist and Saito (2006). The authors argue that asset price booms occur because agents do not know the true state of technology growth but learn about it over time instead. Under these circumstances, there exists a motivation to respond to the gap between observed asset prices and their potential level, in order to reduce the distortions of resource allocations. However, the imperfect information in the economy also affects the policymaker's decision about the potential asset price, which results in a welfare-reducing monetary policy.

Another strand of the literature investigates stock market booms as non-fundamental bubbles and studies the effects of allowing monetary policy to respond to asset price movements. Bernanke and Gertler (2000, 2001) and Tetlow (2006) show that an irrational exogenous shock to the asset price increases the aggregate demand within the economy. They conclude that a strong inflation-targeting regime is sufficient. The extension by Gilchrist and Leahy (2002) also suggests a "strict" inflation-targeting monetary authority if exogenous bubbles have a persistent effect on technology growth. However, in a similar model framework, Cecchetti et al. (2000) show that there may be some benefits to responding to asset prices and that a monetary policy can avoid an overshooting asset prices bubble. The contrasting results within similar model frameworks are due to different assumptions about what exactly can be observed by the policymaker (Cecchetti et al. 2002). Dupor (2002, 2005) finds similar results and he suggests that in response to inefficient shocks to investment demand, optimal policy reduces both price fluctuations as well as non-fundamental asset price movements. This raises the importance of both as targets of the monetary authority. Furthermore, Mishkin and White (2002) suggest that the central bank should only respond to a stock market crash in order to prevent financial instability. In this case the stock market crash is unlikely to result in changes of aggregate demand and the policy maker should not directly react to stock market movements.

The paper is organized as follows. Section two presents the stylized facts of the identified boom and bust episodes during the last decades. The third section introduces the model including financial frictions and nominal rigidities. In section four, the benchmark simulation of the model is presented and the responses to different shocks within the economy are discussed. The ensuing section compares the benchmark solution to the data of the known shocks of the 1980's and 1990's. Afterwards, section six investigates different monetary policy regimes and compares these regimes based on optimized rules with respect to their ability to stabilize the economy throughout

boom episodes and their unexpected busts. Section seven concludes the paper and discusses implications.

4.2   Stylized Facts

To investigate the relationship between asset price booms and busts and key macroeconomic variables, I identify boom episodes on the US equity market. I use real equity prices of the S&P 500 in quarterly frequency, starting at 1948.[1] That method is similar to the one used by Detken and Smets (2004) or Lowe and Borio (2002) for annual data and Adalid and Detken (2007) for quarterly data. Following Adalid and Detken (2007), I define an asset price boom as a period, in which real asset prices differ from their trend by more than ten percent for a minimum of four quarters. Because of the end-point problem of a standard HP-filter, which occurs due to the fact that such a filter also has a forward-looking part, I decide to follow the cited literature and use a one-sided HP-filter to estimate the trend of real equity prices. The one-sided HP filter is estimated recursively by taking into account only data available at that time.[2] Furthermore, I use an observation period of 40 quarters to estimate the first trend. Finally, I calculate the trend for equity prices during 1958 and 2007. The one-sided HP filter is implemented using a $\lambda = 10000$; this implies the trend to adjust slowly and allows to identify episodes of deviations. The chosen value for λ is smaller compared to the related literature (e.g. Adalid and Detken 2007, uses a value of $\lambda = 100000$), but still larger than the usually used value for quarterly data of $\lambda = 1600$ as postulated by Hodrick and Prescott (1997).

Figure 4.1 shows the real price of the S&P 500, its trend (red line) and the identified boom episodes (shaded areas) for the last 50 years. Moreover, Figure 4.1 shows identified short boom episodes at the beginning of the 80s' and during 2007. These episodes ended all after a duration of four or six quarters. Additionally, I identify two longer boom episodes from 1984-Q4 to 1987-Q3 and between 1995-Q3 and 2000-Q2. The first boom ended after twelve quarters while the second one has a duration

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[1]The time series for equity prices bases on the data collection of Robert J. Shiller. I am very thankful to him for making the data available on his website. The finally used real prices of the S&P 500 are calculated from these nominal values and a corresponding consumption deflator. For more details see appendix A.3.5.

[2]For a discussion and an alternative approach to calculate an one-sided HP-filter see Stock and Watson (1999). For completeness, I should mention that there, of course, exits different approaches to identify asset market gaps or booms (see, e.g., Bordo and Jeanne 2003).
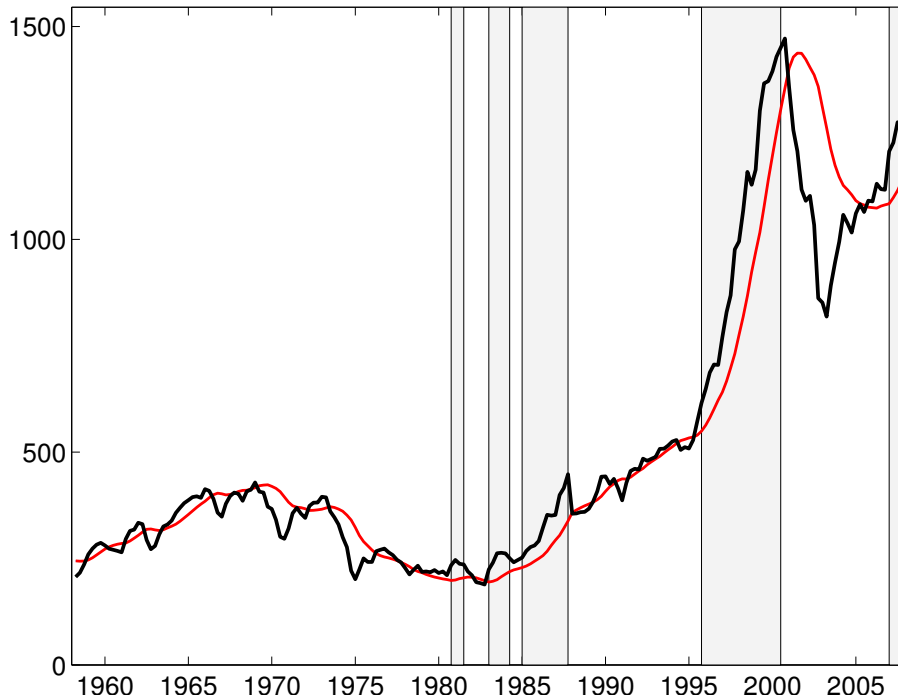
Figure 4.1: Real price of equity based on S&P 500 (black line), its estimated HP-trend (red line), and identified boom episodes (shaded areas).

of 20 quarters. These results are not surprising and in line with the literature (see Adalid and Detken 2007).

In order to investigate the association with macroeconomic variables, I have a more detailed view on the two longer boom episodes. I use the ending dates of both booms (1987-Q3 vs. 2000-Q2) as reference points and analyze how the business cycle components of the macroeconomic time series have changed during the four years before and after the reference point.[3]

The investigated time series include real GDP, real consumption, real private invest-ment, hours worked, and real wages. All these data are per capita and in current dollars using the same consumption deflator as mentioned above.[4] Additionally, the consumption deflator as well the return of the treasury bill are also investigated during and after the boom episodes. Moreover, I detrend each time series with its corresponding trend estimated with a one-sided HP-filter by using $\lambda = 1600$. Addi-

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[3]The related literature - Detken and Smets (2004), Lowe and Borio (2002), or Adalid and Detken (2007) - often defines the peak within a boom episode, at the point where the deviation from the trend is the highest. For the model investigated in this paper, it is more interesting to use the end of a boom as peak or reference point.

[4]For detailed information on the used data (e.g. source and adjustments) see appendix A.3.5.

tionally, I calculate the averages over both booms for 16 quarters before and after the reference point.

Figure 4.2 shows the resulting Burns-Mitchell diagram, which plots the detrended data as percentage deviations from their trends, except for hours worked. Because hours worked are assumed to be stationary, I plot the deviation of hours worked from its average over the investigated quarters.
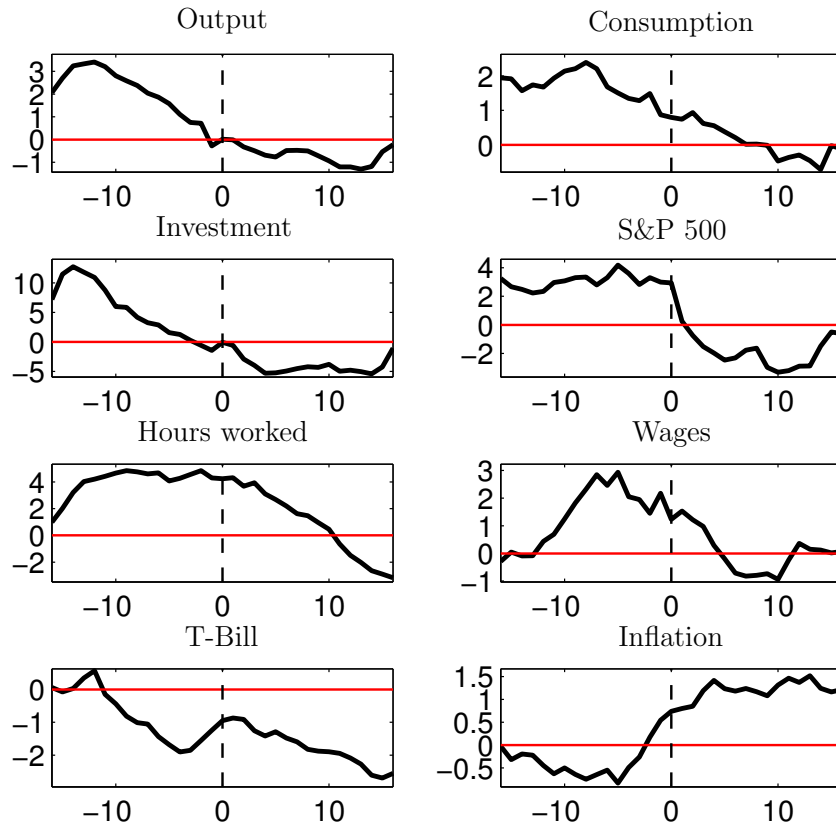
Figure 4.2: Percentage deviation of macroeconomic time series from their trend during and after asset market booms.

The figure 4.2 confirms that during an asset market boom output, investments, consumption, hours worked, and real wages rise and afterwards begin to decrease. Both episodes of a booming equity market go along with a decreasing inflation. This fact is widely discussed in the recent literature (e.g., Adalid and Detken 2007) and contradicts the model findings of Bernanke and Gertler (2000) that inflation tends to rise during asset market booms. Consequently, the conclusion of Bernanke and Gertler (2000) that an inflation-targeting monetary authority automatically stabilizes the stock market seems no longer obvious. The main point the present

paper addresses is to simulate the co-movements of these macroeconomic variables during asset market booms. Especially, the simultaneous increase of real wages per capita together and hours worked seems hard to fix (Christiano et al. 2007). However, to start an educated discussion about central bank activism with respect to asset price movements it is necessary to have a model at hand which is able to simulate stock market booms and their association with macroeconomic variables as good as possible.

Figure 4.3 illustrates the time series with trend by normalizing all time series to the same starting point of unity. The Burns-Mitchell diagram illustrates that the boom and especially the bust have not decreased the levels dramatically. However, neither the boom nor the bust have influence an impact on the long-run, but the distortions in the short run around their trends are obvious and generates welfare distortions.
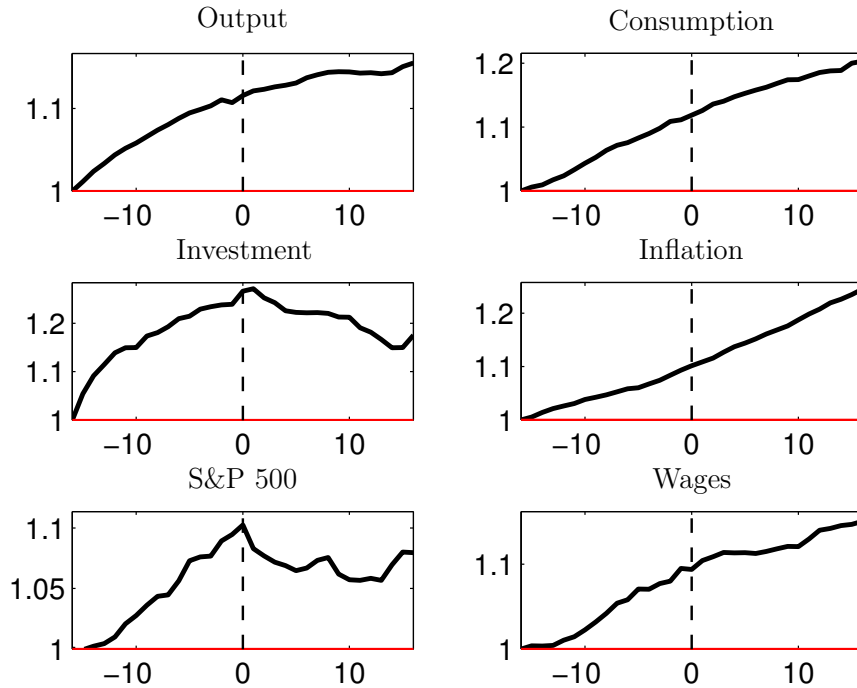

Figure 4.3: Normalized macroeconomic time series during and after asset market booms.

4.3 Model

In this section, I describe the economy investigated in this paper. The economy as a whole is similar to the one in Christiano et al. (2007) and Gilchrist and Saito (2006)

it includes financial frictions modeled through a "financial accelerator" mechanism as postulated by Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999). Following Uhlig (2007) the preferences of the households are modeled in a more general way than in the foregoing papers. Basu and Kimball (2002) show that a nonseparability of consumption and leisure explains the data much better than the standard separable utilities. Separable preferences a often used in the literature can only be a proxy because these are rigid assumptions, which investigate only subsequences.

4.3.1   Households

In the economy exists a continuum of households indexed by $i \in (0,1)$. Each household i consumes or holds one-period riskless deposits with a nominal return known at the time of purchase. Furthermore, each household purchases securities with payments contingent upon whether it can reoptimize its wage decision. Additionally, the wage rate is set after learning about if it is allowed to optimize wages. Finally, the household decides about the fraction of money balances held in form of currency. The preferences of the representative household are similar to the specification in Uhlig (2007):

$$E_0 \sum_{t=0}^{\infty} \left[\beta^t \frac{\left(\left(C_t\left(i\right) - \chi C_{t-1}\right)\left(A + \left(L_t\left(i\right) - \psi L_{t-1}\right)^{\nu}\right)\right)^{1-\eta} - 1}{1 - \eta} + \upsilon \left(\frac{M_t\left(i\right)}{P_t}\right) \right], \quad (4.1)$$

where $C_t\left(i\right)$ is the individual consumption of the household i in period t, which is chosen in each period to maximize the households utility. Leisure $L_t\left(i\right)$ is given by the total time endowment of the household minus working hours $H_t\left(i\right)$ offered to the entrepreneurs. For simplicity the total time endowment is scaled up to unity, which implies that the leisure of the household in period t is given by:

$$L_t\left(i\right) = 1 - H_t\left(i\right) \qquad (4.2)$$

Furthermore, the preferences are characterized by the discount factor β, the power utility parameter η, and ν the impact of leisure on the utility. The parameters χ and ψ measure the habit persistence regarding consumption or leisure respectively. Both habits are assumed to be externally formed and depend either on the aggregate past level of consumption or on the past level of leisure. Because of monotonicity and

concavity constraints the preference parameter have to fulfil the following conditions:

$$\eta > 0 \,, \ \nu > 0 \qquad \text{and} \qquad \eta > \frac{\nu}{\nu + 1} \tag{4.3}$$

In each period, the households consume and invest a part of their income into a nominal one-period riskless deposit, D_t. They receive a nominal labor income, $W_t(i) \cdot H_t(i)$ and receive the deposit invested in period $t-1$ in addition to the interest rate for this riskless deposit. Moreover, they also receive $S_t(i)$ the net cash flow from the insurance market. Additionally, they obtain real dividends Π_t from the retail firms and pay lump-sum taxes T_t to the government. Finally, the budget constraint of the household is characterized by

$$C_t(i) + \frac{D_t(i)}{P_t} + T_t = \frac{W_t(i)}{P_t} H_t(i) + \frac{R_{t-1}^N D_{t-1}(i)}{P_t} + \Pi_t + S_t(i) - \frac{M_t(i) - M_{t-1}(i)}{P_t}. \tag{4.4}$$

The first-order condition regarding consumption can be expressed as:

$$\lambda_t = (C_t(i) - \chi C_{t-1})\left(A + (L_t(i) - \psi L_{t-1})^\nu\right)^{1-\eta}, \tag{4.5}$$

where λ is the multiplier of the budget constraint in the Lagrangian representation of the household's problem. The first-order condition with respect to D_t is given by the Euler equation:

$$1 = E_t\left[\beta \frac{\lambda_{t+1}}{\lambda_t} R_t^N \frac{P_t}{P_{t+1}}\right]. \tag{4.6}$$

Following Erceg, Henderson, and Levin (2000), I model the wage setting analogously to staggered price setting introduced by Calvo (1983). Each household supplies a differentiated type of labor service, $h_t(i)$, which is aggregated into a homogenous labor good by a representative competitive firm. This firm uses the following technology:

$$H_t = \left[\int_0^1 H_t(i)^{\frac{\varepsilon_w - 1}{\varepsilon_w}}\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}},$$

where $\varepsilon_w > 1$ is the elasticity of substitution. Finally, the demand for labor of type i is given by,

$$H_t(i) = \left[\frac{W_t(i)}{W_t}\right]^{-\varepsilon_w} H_t, \tag{4.7}$$

where $W_t(i)$ is the nominal wage demanded by labor of type i and W_t is the wage

index defined as

$$W_t = \left[\int_0^1 W_t\left(i\right)^{\varepsilon_w - 1} \right]^{\frac{1}{\varepsilon_w - 1}}.$$

Given the demand curve of labor, each household supplies as many labor services as demanded at this wage. The household has to set his wage. In each period the household can optimize his wage with probability $1 - \theta_w$ and with probability θ_w he cannot. If the household can not optimize its wage, the wage rate in t is given by:

$$W_t(i) = \bar{\pi} W_{t-1}(i), \tag{4.8}$$

where $\bar{\pi}$ is the steady-state inflation rate of the economy. The household optimizes its wage $W_t(i)$ by maximizing the following objective function:

$$E_t \left[\sum_{j=0}^{\infty} \left(\theta_w \beta \right)^j \left[\lambda_{t+j} \bar{\pi}^j \frac{W_t\left(i\right)}{P_{t+j}} H_{t+j}\left(i\right) - U\left(H_{t+j}\left(i\right), C_{t+j}\left(i\right) \right) \right] \right] \tag{4.9}$$

The corresponding first order condition of the household is given by:

$$E_t \left[\sum_{j=0}^{\infty} \left(\theta_w \beta \right)^j \left[\bar{\pi}^j \frac{W_t\left(i\right)}{P_{t+j}}\left(i\right) - \frac{\varepsilon_w - 1}{\varepsilon_w} MRS_{t+j}\left(H_{t+j}\left(i\right), C_{t+j}\left(i\right) \right) \right] \right] = 0 \tag{4.10}$$

In the following I restrict the analysis of the influences of the nonseparability between consumption and leisure to the individual consumption and wage decision. As stated by Christiano et al. (2005), the uncertainty of the household whether it can reoptimize its wages or not is idiosyncratic because each household supplies a different amount of labor and earns a different wage rate. As a consequence, the households are also heterogenous in consumption and asset holdings. A widely used argument in the staggered-wage-setting literature (see Erceg et al. 2000; Woodford 2003) is that the existence of an insurance market implies the equalization of the marginal utility of wealth across households. Using separable preferences, this assumption allows to assume that the households are homogenous with respect to consumption and asset holdings, but heterogenous with respect to wages and labor supply (Christiano et al. 2005).

Since, this paper uses a preference structure which is nonseparable in consumption and leisure, I want to illustrate the effects of nonseparability in more detail. The presentation follows Guerron-Quintana (2007). In particular, I reproduce the results for the preferences applied in this present paper.

In general, the assumption of an insurance market because of the complete market hypothesis imposes that the following condition has to hold:

$$U_c\left(C_t\left(i\right),H_t\left(i\right)\right)=U_c\left(C_t\left(i'\right),H_t\left(i'\right)\right)\qquad\forall\,i,i'\in(0,1)\qquad(4.11)$$

This condition implies that the households differ in their ex-post consumption levels because of their labor schedules, resulting from different wage schedules. Guerron-Quintana (2007) conclude, that the relative consumption of the household is a linear function of the relative wage. This can be illustrated by the following function,

$$\hat{c}_{R,t}\left(i\right)=\Upsilon\hat{w}_{R,t}\left(i\right),\qquad(4.12)$$

where Υ is a constant, $\hat{c}_{R,t}\left(i\right)$, and $\left(\hat{w}_{R,t}\left(i\right)\right)$ are the log-linear approximation of the individual consumption or individual wage relative to aggregate consumption or economy-wide wage respectively. Of course, the log-linear approximation requires that this relation only holds for small changes around the steady state.

To specify Υ, I evaluate the log-linear approximation of equation (4.11). The evaluation can be written in terms conditioning on deep parameters, the relative consumption, and the individual labor supply of a household:

$$\Theta_1\hat{c}_{R,t}\left(i\right)+\Theta_2\hat{h}_t\left(i\right)=\Theta_1\hat{c}_{R,t}\left(i'\right)+\Theta_2\hat{h}_t\left(i'\right).\qquad(4.13)$$

Given the labor demand function (4.7) and equation (4.12), it is easily verified that the complete market condition of equal marginal utilities across households only holds for

$$\Upsilon=\varepsilon_w\cdot\frac{\Theta_2}{\Theta_1}=\varepsilon_w\cdot\frac{\bar{H}}{1-\bar{H}}\frac{\nu\left(1-\eta\right)\left(1-\chi\right)}{\eta\left(1+\Gamma\right)\left(1-\psi\right)},\qquad(4.14)$$

where $\bar{H}$ is the steady state labor supply and $\Gamma=A\left(1-\psi\right)^{-\nu}\left(1-\bar{H}\right)^{-\nu}$ is another helpful steady state condition.[5] Because the aggregate nominal wage is given by $\hat{w}_t=\theta_w\hat{w}_{t-1}+\left(1-\theta_w\right)\hat{w}_t\left(i\right)$ and the wage inflation is evaluated as $\hat{\pi}_t^w=\hat{w}_t-\hat{w}_{t-1}$, it is obvious that

$$\hat{w}_{R,t}=\frac{\theta_w}{1-\theta_w}\hat{\pi}_t^w.$$

Finally, it can be stated that the individual consumption level can be written in

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[5]For more details about the evaluation of this condition and for the discussion of the non-positiveness of Υ see Guerron-Quintana (2007).

logarithmic terms as [6]

$$\hat{c}_t(i) = \hat{c}_t + \Upsilon \frac{\theta_w}{1 - \theta_w} \hat{\pi}_t^w(i), \tag{4.15}$$

4.3.2 Entrepreneurs

Entrepreneurs manage the production of the wholesale good and are risk neutral. Following Bernanke et al. (1999) the entrepreneurs have a finite lifetime. In particular, with probability κ each entrepreneur survives to the next period. Each of those who have left are replaced by new entrepreneurs in next period. The entrepreneurs use the following production process to produce the wholesale good Y_t:

$$Y_t = \varepsilon_t K_{t-1}^\alpha (Z_{T,t} N_t)^{1-\alpha}, \tag{4.16}$$

where capital K_{t-1} is purchased at the end of period $t-1$ for the production of the wholesale goods in period t. The parameter α refers to the capital share used for production. The variable Z_t reflects the exogenous technology common to all entrepreneurs and is modeled as AR(1) process with drift, which can be written in log-linearized terms as:

$$\hat{z}_{T,t} = \mu + \hat{z}_{T,t-1} + \epsilon_{T,t}, \tag{4.17}$$

where $\epsilon_{T,t}$ is i.i.d. normally distributed with standard deviation σ_T and μ is the technology growth path.

The variable ε_t captures an anticipated shock, equivalent to Christiano et al. (2007)[7], which is modeled in log-linearized terms as:

$$\hat{\varepsilon}_t = \rho_\varepsilon \hat{\varepsilon}_{t-1} + \epsilon_{\varepsilon,t-p} + \epsilon_{\varepsilon,t}^\star, \tag{4.18}$$

where $\epsilon_{\varepsilon,t}$ and $\epsilon_{\varepsilon,t}^\star$ are uncorrelated over time and with each other. The intuition of this shock process is that an impulse $t = 1$ suggests an increase in $\hat{\varepsilon}$ in $t = 1 + p$ periods, a modeling of $\epsilon_{\varepsilon,t+p}^\star = -\epsilon_{\varepsilon,t}$ implies that the shock in period $t = 1 + p$ is not realized and the boom episode will bust after p periods. Finally I assume that $\epsilon_{\varepsilon,t} \sim i.i.d.\ N(0, \sigma_\varepsilon^2)$.

The entrepreneurs demand a level of labor, N_t, for the production. The total level of

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[6]Note that, for the moment the analysis ignores any balanced growth path requirements. For the finally used equations see appendix A.

[7]This kind of anticipated shock was introduced by Beaudry and Portier (2006) and their former articles. Because of the similarities of the present research to Christiano et al. (2007), I, however mainly refer to these authors.

labor is given by the households H_t and entrepreneurial hours worked H_t^e in period t, and can be written as:

$$N_t = H_t^{1-\Omega} (H_t^e)^{\Omega} \ , \tag{4.19}$$

where H^e is assumed to be inelastic and thus equal to one. This assumption is needed to ensure that new entrepreneurs have some funds available when starting out with production(see Gilchrist and Saito 2006).

Let $P_{w,t}$ denote the nominal price of the wholesale goods and Q_t the price of capital at the stock market relative to the aggregate price P_t. Then, the entrepreneurs' real revenues can be written as the sum of production revenues and the real value of depreciated capital,

$$\frac{P_{w,t}}{P_t} K_{t-1}^\alpha (Z_{T,t} N_t)^{1-\alpha} + Q_t (1 - \delta) K_{t-1},$$

where δ is the physical depreciation rate of capital.

At the end of period t, the entrepreneurs purchase capital K_t from the capital producers at the asset market price Q_t. The new amount of capital for production in $t+1$ is financed partly with net worth of the entrepreneurs and partly with debt borrowed from the households:

$$Q_t K_t = NW_t + \frac{D_t}{P_t} \tag{4.20}$$

Given the amount of capital available for production in period t, entrepreneurs demand households' and entrepreneurial labor. The first order conditions regarding labor choice are given by:

$$(1 - \Omega)(1 - \alpha) \frac{Y_t}{H_t} = \frac{W_t}{Pw,t} \tag{4.21}$$

and

$$\Omega (1 - \alpha) \frac{Y_t}{H_t^e} = \frac{W_t^e}{Pw,t} \tag{4.22}$$

The optimal first-order condition for capital purchase is given, such that the marginal revenues and the marginal costs of capital are equalized:

$$E_t \left[R_{t+1}^s Q_t \right] = E_t \left[\frac{P_{w,t+1}}{P_{t+1}} \alpha \frac{Y_{t+1}}{K_t} + (1 - \delta) Q_{t+1} \right] \tag{4.23}$$

Bernanke et al. (1999) define the external finance premium F_t as the ratio of costs of external funds to costs of internal funds:

$$F_t = \frac{E_t \left[R_{t+1}^s \right]}{E_t \left[R_t^N \frac{P_t}{P_{t+1}} \right]} \tag{4.24}$$

In the absence of financial market imperfections, the external finance premium does not exist ($F_t = 1$). The external finance premium is affected moreover, by the balance-sheet conditions of the entrepreneur. It increases if the ratio of capital expenditures to entrepreneurial net worth increases:

$$F_t = F \left(\frac{Q_t K_t}{NW_t} \right) = \left(\frac{Q_t K_t}{NW_t} \right)^\sigma \tag{4.25}$$

Following the approach of Gilchrist and Saito (2006) this parametric function is assumed; it is increasing for $NW_t < Q_t K_t$.

The aggregate net worth of an entrepreneur, NW_t, at the end of period t is defined as:

$$NW_t = \kappa \left(R_t^s Q_{t-1} K_{t-1} - E_{t-1} \left[R_t^s \right] \frac{D_{t-1}}{P_{t-1}} \right) + \frac{W^e}{P_t}, \tag{4.26}$$

where κ is the probability that an entrepreneur survived from period $t-1$ to t. Moreover, the aggregate net worth is defined as the sum of the equity held by entrepreneurs who have survived and the entrepreneurial real wage. The fraction of entrepreneurs who leave the business in period t consumes the residual equity:

$$C_t^e = (1 - \kappa) \left(R_t^s S_{t-1} K_{t-1} - E_{t-1} \left[R_t^s \right] \frac{D_{t-1}}{P_{t-1}} \right), \tag{4.27}$$

where C_t^e refers to the consumption of the entrepreneur and $1 - \kappa$ obviously captures the fraction of entrepreneurs who have left the business.

4.3.3 Capital Producers

The capital used by the entrepreneurs for the production of the wholesale good is produced with existing capital K_{t-1} and the investments in period t. The production process of new capital is characterized by the function

$$\Phi \left(Z_{I,t} \frac{I_t}{K_{t-1}} \right) K_{t-1},$$

where $\Phi(\cdot)$ is an increasing and concave function that satisfies the following steady-state conditions (see also Jermann 1998)

$$\Phi(\cdot) = \delta, \quad \Phi'(\cdot) = 1, \text{ and } \quad \Phi''(\cdot) = -\frac{1}{\zeta \frac{\bar{i}}{\bar{k}}} \quad \forall \quad \zeta > 0$$

The variable $Z_{I,t}$ refers to a cost push shock in the production process of capital and is given as an autoregressive process in the log-linearized form,

$$\hat{z}_{I,t} = \rho_I \hat{z}_{I,t-1} + \epsilon_{I,t} \tag{4.28}$$

with ρ_I the AR(1) parameter and $\epsilon_{I,t}$ the exogenous normally i.i.d. distributed shock parameter with standard deviation σ_I.

The aggregate capital accumulation is given by

$$K_t = \left(1 - \delta + \Phi\left(Z_{I,t}\frac{I_t}{K_{t-1}}\right)\right) K_{t-1} . \tag{4.29}$$

Finally, the first-order condition for capital producers is finally given by,

$$Q_t = \frac{1}{\Phi'\left(Z_{I,t}\frac{I_t}{K_{t-1}}\right)}, \tag{4.30}$$

what implies that investments and the quantity of new capital increases when the market price of capital, Q_t, increases.

4.3.4 Staggered Prices

The wholesale goods are purchased by an existing continuum of monopolistically competitive firms (retailers), who produce the final good at zero resource costs (see also Gilchrist and Saito 2006). The final good, Y_t, is produced under the constant-return-to-scale production function:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon_p-1}{\varepsilon_p}}\right]^{\frac{\varepsilon_p}{\varepsilon_p-1}},$$

where $Y_t(i)$ is the retail good and let $P_t(i)$ be its nominal price, such that the corresponding price index, P_t is given by:

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon_p} \right]^{\frac{1}{1-\varepsilon_p}}.$$

It is assumed that consumers, capital producers, and the government demand the final good.

The demand curve of the retail good is given by the first-order condition of the monopolistically competitive firm:

$$Y_t(i) = \left[\frac{P_t}{P_t(i)} \right]^{\varepsilon_p} Y_t \tag{4.31}$$

As postulated by Calvo (1983) I assume that the prices are staggered. This means that a retailer can adjust his prices, $P_t^\star$, with probability $1 - \theta_p$, independently from other retailers and independently of the subsequent price setting. Thus, a fraction of $1 - \theta_p$ retailers adjust their prices in period t, while the rest of the retailers θ_p cannot adjust their prices and set $P_t(i) = \bar{\pi} P_{t-1}$. These assumption ca be written as aggregate price index in form of:

$$P_t = \left[\theta_p \left(\bar{\pi} P_{t-1} \right)^{1-\varepsilon_p} + (1 - \theta_p) \left(P_t^\star \right)^{1-\varepsilon_p} \right]^{\frac{1}{1-\varepsilon_p}} \tag{4.32}$$

The real marginal costs for each retailer are given by the price ratio of the wholesale good and the final good, $P_{w,t}/P_t$. Furthermore, each retailer takes the demand curve and the wholesale price as given and set $P_t(i)$. Under these circumstances, the profit maximization of the retailer becomes

$$\max_{P_t(i)} \quad E_t \sum_{j=0}^{\infty} \theta_p^j m_{t+j} \left[\bar{\pi}^j P_t(i) Y_{t+j}(i) - MC_{t+j} Y_{t+i}(i) \right]. \tag{4.33}$$

MC_t denotes the nominal marginal cost of the retailer and m_t is the real stochastic discount factor given as $m_{t+j} = \beta^j \frac{\lambda_{t+j} P_t}{\lambda_t P_{t+j}}$. The first-order condition of this maximization problem implies that retailers set their prices in period t according to:

$$\frac{P_t(i)}{P_t} = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{E_t \left[\sum_{j=0}^{\infty} \theta_p^j m_{t+j} mc_{t+j} Y_{t+j}(i) \frac{P_{t+j}}{P_t} \right]}{E_t \left[\sum_{j=0}^{\infty} \theta_p^j m_{t+j} \bar{\pi}^j Y_{t+j}(i) \right]}, \tag{4.34}$$

where the mc_{t+j} refers to real marginal costs.

4.3.5 Government & Aggregate Resource Constraint

The general resource constraint of the economy is given by

$$Y_t = C_t + C_t^e + I_t + G_t \ . \tag{4.35}$$

In contrast to Bernanke et al. (1999), this research assumes that resource costs with respect to bankruptcy are negligible. This assumption is comparable to the assumption made by Gilchrist and Saito (2006).

Government expenditures are exogenous and financed by lump-sum taxes and money creation:

$$G_t = \frac{M_t - M_{t-1}}{P_t} + T_t \tag{4.36}$$

The government expenditures, G_t, are modeled as an exogenous process and can be written as AR(1) process in the log-linearized form as

$$\hat{g}_t = \rho_G \hat{g}_{t-1} + \epsilon_{G,t}, \tag{4.37}$$

where ρ_G is the autoregressive parameter and the noise $\epsilon_{G,t}$ is normally i.i.d. with standard deviation σ_G.

4.3.6 Monetary Policy Rules

The policy maker uses interest rate to lead monetary policy. As a benchmark policy rule I assume that the monetary policy has only information on past inflation and past output in the economy and sets the interest rate in log-linearized terms as follows:

$$\hat{r}_t^N = \gamma_R \hat{r}_{t-1}^N + (1 - \gamma_R) \left[\gamma_\pi \hat{\pi}_{t-1} + \gamma_Y \hat{y}_{t-1} \right] \ . \tag{4.38}$$

The different γ-parameters refer to different weights within the interest setting rule. The benchmark rule does not recognize any activities of the asset market directly.

4.4 Simulation

I simulate the model using a standard calibration following the recent literature. The financial frictions are modeled according to Bernanke et al. (1999) and Gilchrist and

Saito (2006). As suggested by Gilchrist and Saito (2006) the steady-state leverage ratio ρ, the ratio of the market value of capital stock to the entrepreneurs' net worth, is 80%. Since one period in the model is a quarter, the elasticity of the finance premium or the risk spread is chosen to be 0.05. This implies a steady-state risk spread of 2.98%. The values of the price elasticity and the wage elasticity are set to be 11 and 5 and the probabilities to adjust prices or wages are calibrated to 0.75 and 0.65 respectively. The steady state growth path is chosen modestly with $\mu = .005$, which corresponds to an annual growth rate of 2.02%. Furthermore, with the discount factor $\beta = 0.995$ the steady-state quarterly risk-free rate is 1.26%. The probability κ that an entrepreneur survives the period is implied by the previously presented parameters.[8] The resulting probability $\kappa = .9604$ corresponds to the recent literature (Bernanke et al. 1999). The preference parameters are chosen to receive plausible steady-state values for the Frisch elasticity and the labor supply. The resulting Frisch elasticity of 0.535 is small but in line with recent findings of Justiniano and Primiceri (2006), while the steady state labor supply evaluated at 0.23 is at the lower end of the conventional wisdom. Table 4.1 sums up the deep parameters of the simulated model.

The standard deviation of all shocks is set to 1%. The autoregressive parameters for shocks to capital adjustment costs and to government spending are chosen to be 0.95, while I follow Christiano et al. (2007) with respect to the technology shock and use $\rho_\varepsilon = 0.83$. Given the provided parameter γ_R in the monetary policy rule is equal to 0.8, figure 4.4 shows possible parameter combinations for past inflation and past output in order to obtain a stable equilibrium. The choice of $\gamma_\pi = 1.8$ and $\gamma_y = 0.15$ is similar to the rule used by Christiano et al. (2007). This parameter setting rule is used to achieve a benchmark calibration which illustrates the interactions within the economy. In the following, different monetary policy regimes will be discussed in more detail. All, above mentioned parameters can be found in table 4.2.

Figure 4.5 presents the responses of selected variables to a anticipated shock to technology, which finally not occur. Recalling the stylized facts, the observed boom/bust episodes of the last decades had an average duration of six or seven years. Following Christiano et al. (2007) I simulate that the individuals in period t assume a level-shift of technology in sixteen periods (equal to four years).

As observed in the stylized facts, output, investments, hours worked, and real wages increase during the boom episode. At the point, the individuals recognize that the

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[8]For more details about the calculation of the steady state see appendix A.3.3.

Parameter	Description	Value
β	discount factor	.995
A	preference parameter	.0075
χ	habit persistence in consumption	.4
ψ	habit persistence in leisure	.8
ν	preference parameter	1
η	power utility parameter	1.5
δ	depreciation rate	.025
α	capital share	.35
Ω	discount factor	.0154
μ	steady state growth rate	.005
$1/\zeta$	elasticity of the price of capital	.20
σ	elasticity of external finance premium	.05
ε_p	price elasticity	11
ε_w	wage elasticity	5
θ_p	Calvo parameter for prices	.75
θ_w	Calvo parameter for wages	.65
ρ	leverage	.8
κ	probability to survive	.9604

Table 4.1: Calibration of deep model parameters

Parameter	Description	Value
ρ_I	AR(1) parameter capital adjustment cost process	.95
ρ_ε	AR(1) parameter technology process	0.83
ρ_G	AR(1) parameter government spending	.95
$\sigma_{T,t}$	standard deviation of anticipated technology shock	.01
$\sigma_{j,t}$	standard deviation of any other shock	.01
γ_R	weight on past nominal interest rate	0.8
γ_π	weight on past inflation	1.8
γ_y	weight on past output gap	.15

Table 4.2: Calibration of exogenous parameters and monetary policy parameters
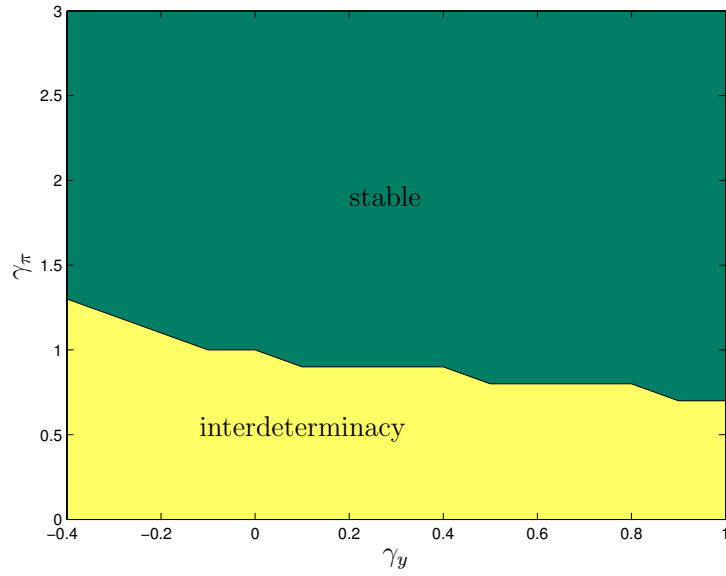
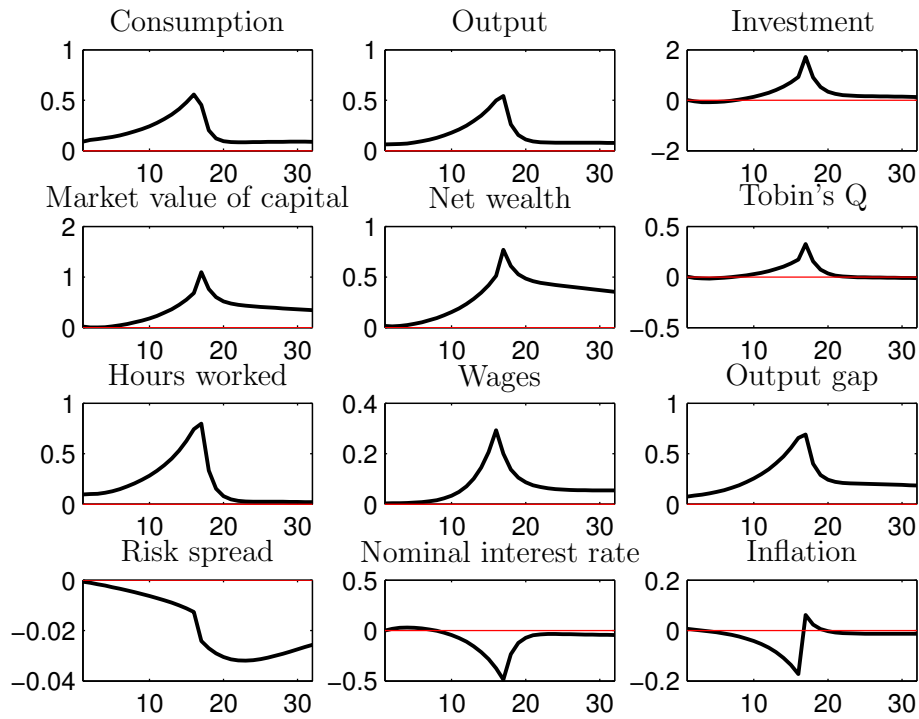Figure 4.4: Interdeterminacy region for the simple monetary policy rule

Figure 4.5: Impulse responses to a news shock which finally not occurs

shock does not occur, all these variables decrease. Furthermore, the inflation is declining during the boom period, which also in line with the stylized facts.

After the anticipation of the shock the entrepreneurs start to adjust their productivity. This increases the capital stock, investments, and hours worked. However, due to the nominal rigidities and a small Frisch elasticity the households are unwilling to reduce their nominal wages. This limits the increase of additional employment and explains the comparatively higher investments. The increasing net wealth of the entrepreneur reduces the external finance premium and makes credits more attractive for firms. Along with the decreasing costs of debt, the marginal costs reduce and inflation decreases, too. This is followed by increasing real wages. The inflation-targeting central bank recognizes that and starts to reduce the nominal interest rates. Unfortunately, due to a decreasing risk spread this additionally triggers a credit boom, what extends the stock market boom. Finally, it is worth pointing out that in the investigated economy, the magnitude of the boom essentially depends on the monetary policy, the nominal wage rigidities, and the Frisch elasticity of the households.

4.5 Theory and Data

In this section, the impulse responses of the investigated model are compared to selected variables. As discussed in section 4.2 I investigate the fluctuations of several business cycle facts around their trend. First, I compare the average fluctuations during the identified boom and bust episodes at the end of the 1980's and 1990's. Afterwards, I investigate the model with the stock market boom and bust at the end of the 1990's in more detail.

Figure 4.6 illustrates the response to a two percent anticipated technology shock, which finally not occurs and additionally the corresponding business cycle variables. As mentioned above, the model is able to replicate the signs of the variables during the boom as well as the bust episode. Of course, there exists a problem to replicate the correct timing of each variable wit respect to the data. The model also underestimates the impact of the bust on the variables. Additionally, the model cannot explain the strong increase of hours worked and wages compared to consumption and output. However, given the simplicity of the model as well as the standard parameterizations, the model is successful.
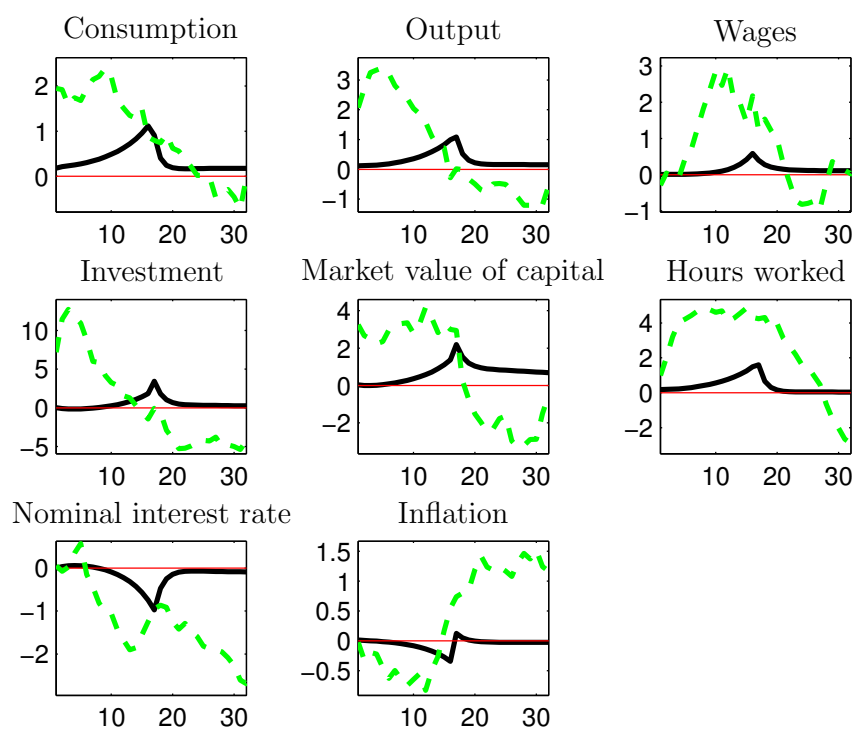
Figure 4.6: Impulse responses (solid line) and data (dashed line) for stock market booms in the 1980's and late 1990's
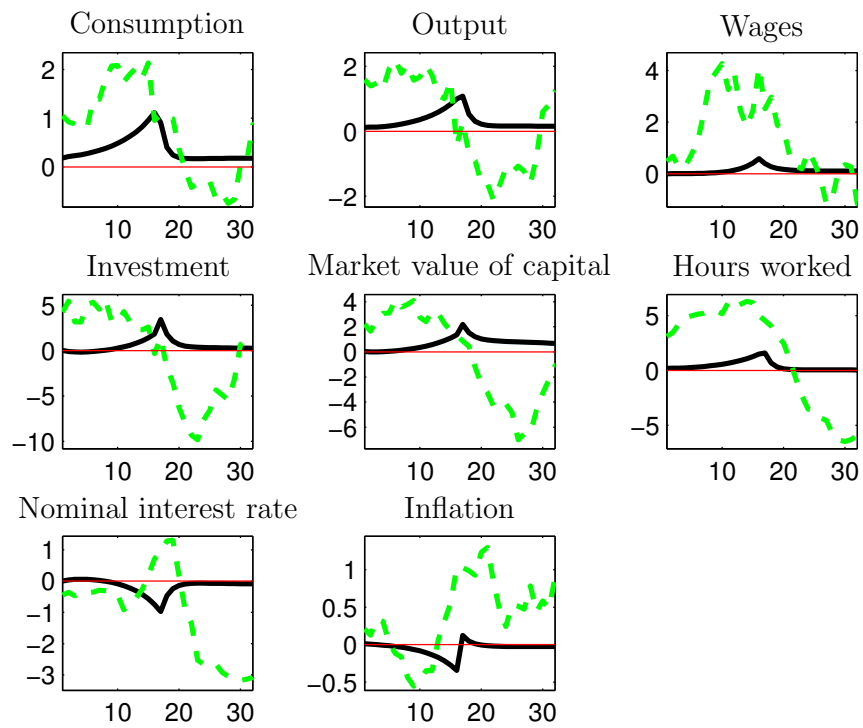
Figure 4.7: Impulse responses (solid line) and data (dashed line) for the stock market boom in the late 1990's

As a proper example of an anticipated technology shock, which finally not occurs often the 'new economy boom' is mentioned. For this reason figure 4.7 compares just the data of this episode with the model. Considering the nominal interest rates as well as the inflation behavior before the peak of this boom episode, the model is able to replicate the interaction of the monetary policy regime and the real economy in these days.

4.6   Monetary Policy

This section investigates the stabilization performance of the given simple policy rule. Therefore I investigate its performance regarding the respond coefficients under different policy regimes. I assume that the monetary authority has a standard loss function, equal to the weighted sum of unconditional variances of inflation, output gap, and changes in the nominal interest rate:

$$\mathcal{L} = Var\left(\pi_t\right) + \lambda_y Var\left(\Delta y_t\right) + \lambda_r Var\left(\Delta r_t^n\right) \ . \tag{4.39}$$

The weight $\lambda_y \geq 0$ measures the policy-maker's preference to reduce output gap variability and $\lambda_r \geq 0$ the preference to reduce nominal interest rate variability, $\Delta r_t^n = r_t^n - r_{t-1}^n$, relative to inflation variability. The loss function used in this paper corresponds to this used by Küster and Wieland (2005), Coenen (2007), Levin and Williams (2003), or Levin, Wieland, and Williams (1999). Of course, it would be beneficial to use an micro-founded loss function derived from a second-order-approximation of the representative agent's utility following Rotemberg and Woodford (1997) or based on a linear-quadratic approximation as postulated by Benigno and Woodford (2006). These welfare criterions would suggest weights that are functions of the model parameter. Unfortunately, the model previously presented is only accurate up to first-order. Due to this fact, I use this standard quadratic loss function, not to suggest optimal policy, moreover, to get an idea about the stabilization performance of the optimized monetary policy rule under different regimes.

Therefore, I investigate different sets of weights, which refer to different policy regimes. For $\lambda_y = \lambda_r = 0.1$ the monetary policy corresponds to a "strict" inflation targeting monetary authority, while $\lambda_y, \lambda_r > 0$ characterizes a more "flexible" inflation targeting authority (Küster and Wieland 2005). The analyzed weights are $\lambda_y = \{0, 0.5, 1\}$ and $\lambda_r = \{0.1, 0.5, 1\}$, which are similar to those studied by Levin and Williams (2003) and Küster and Wieland (2005).

	$\lambda_y = 0$			$\lambda_y = 0.5$			$\lambda_y = 1.0$		
	γ_r	γ_π	γ_y	γ_r	γ_π	γ_y	γ_r	γ_π	γ_y
$\lambda_r = 0.1$	0.7539	3.0450	-0.0665	0.7096	2.4918	-0.0619	0.9000	8.8436	0.1431
$\lambda_r = 0.5$	0.8220	4.4857	-0.0732	0.7797	3.5086	-0.0669	0.7751	3.7497	-0.0393
$\lambda_r = 1.0$	0.8136	4.2486	-0.0724	0.8096	4.3033	-0.0567	0.8421	5.8670	-0.0032

Table 4.3: Optimized monetary policy rules

I calculate the optimized monetary policies, due to minimizing the loss-function. During the minimization it is ensured that the Blanchard-Kahn conditions for a stable unique solution are satisfied. I neglect the anticipated news shock during the minimization. Obviously, the finally obtained rules depend only on the five different shocks to technology, to a labor augmented technology, to adjustment costs, to monetary policy, and to government spending. This implies a degree of uncertainty by the central bank because the different regimes do not take into account the possibility of asset market booms triggered by overoptimistic expectations. This simplification allows to investigate how well the standardized monetary policy rules would work during asset market boom and bust episodes. Table 4.3 presents the optimized monetary policy rule coefficients under different regimes. Each of the rules is optimal regarding its loss function, which makes it impossible to compare the losses with each other. The results, especially the negative response to past output is a known phenomenon for money in the utility (Woodford 2003).

Because the differences are small over the different regimes, in the following, I investigate the two most extreme regimes. At first, the strictly inflation-targeting monetary authority, $\lambda_y = 0$ and $\lambda_r = 0.1$ and, secondly, the more flexible inflation-targeting regime with modest changes of the nominal interest rate, $\lambda_r = 1$ and $\lambda_y = 1$.

Figure 4.8 illustrates the response of the economy to an anticipated technology shock, which finally not occurs, under "strict" inflation-targeting monetary policy (solid line) as well as under the mentioned "flexible" inflation targeting optimized policy rule (dashed line). It is obvious, that the differences between both are small. Both policies reduces the fluctuation during the boom and bust episode accordingly to the benchmark solution previously presented. Under both regimes, it is optimal to continuously increase the nominal interest rates during the boom. Due to the fact that the central bank does not reduce the nominal interest rate with respect to
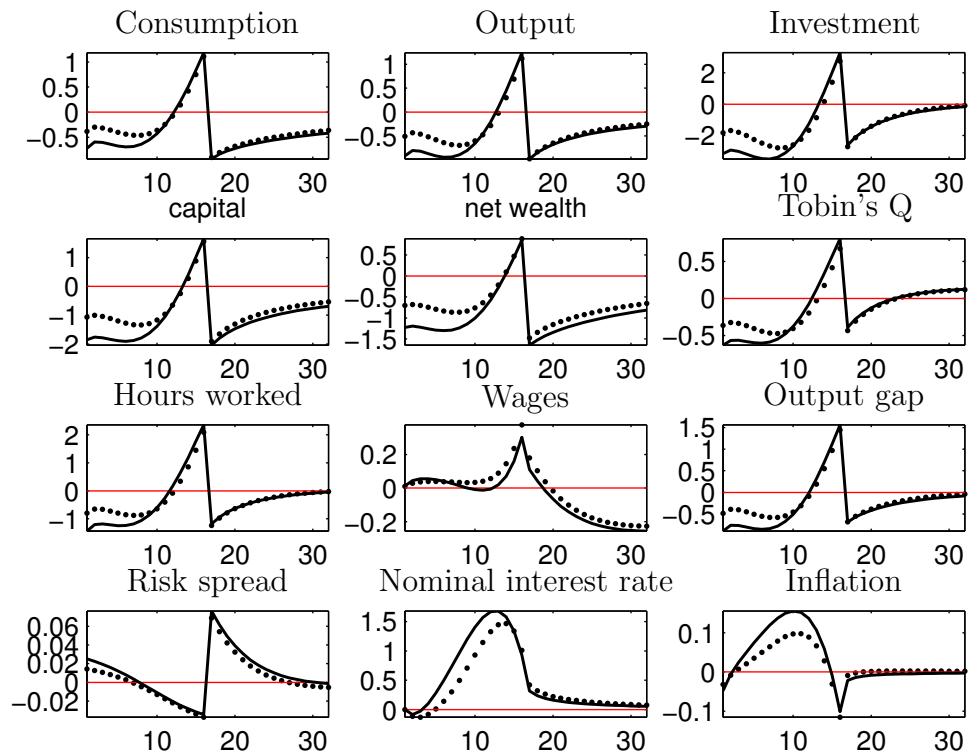
Figure 4.8: Relative response to an anticipated technology shock which finally not occurs using optimized policy rules. The solid line denotes the responses within a "strict" inflation-targeting regime while the dashed line denotes the responses in a "flexible" inflation targeting regime.

the decreasing inflation, the boom is not further fueled. This "leaning against the wind" policy avoids a credit expansion.

Moreover, it can be figured out that the more flexible monetary policy rule has an additional advantage because the central bank cares more about the changes of the nominal interest rate, thus the interest rate has a more slightly increase. Driven by this fact, the fluctuations are not as large, when the expectations about the future technology are disappointed.

To conclude the investigation of the different monetary policy regimes, it can be adhered that under any policy regime it is optimal to increase the nominal interest rate during a boom episode. Also for strictly inflation-targeting regimes it is advantageous to incorporate output into the monetary policy rule. However, such a strict policy would raise the nominal interest rate too much, in the hope to avoid an overshooting boom. Within the given model framework a more flexible monetary policy is welfare-increasing because it stabilizes the economy during the boom and during the bust more effectively.

4.7   Conclusion

In this paper, I have presented a DSGE model that is an extension of the model presented by Christiano et al. (2007). The extended model allows to simulate the joint rise of consumption, investment, output, hours worked, and especially real wages during an asset market boom and the overall fall during the bust. Furthermore, the standardized monetary policy rule yields a declining inflation during such a boom episode. All these are stylized facts of the observed boom and bust episodes in the 1980's and late 1990's.

The main contribution of explaining the simultaneous rise of hours worked and real wages is a necessary point to allow for a more detailed discussion regarding monetary policy during asset market booms. The use of nonseparable preferences between consumption and leisure, which are both habitually formed is necessary in order to obtain this simultaneous increase. Both features result in a small Frisch elasticity with respect to the conventional wisdom. Combined with nominal wage rigidities, the individuals are less willing to reduce their wages as a consequence of a technology shock. Obviously, this is not efficient with respect to potential

employment. However, together with the decreasing inflation this allows to recover the observed increase of real wages in the stylized facts.

The investigation of different monetary policy regimes in this paper suggests that for any regime it is necessary to continuously increase the nominal interest rate throughout a stock market boom. This avoids an overshooting of the stock market boom and stabilizes the economy with respect to an unexpected bust of the stock market. Finally, the paper proposes that a "flexible" inflation-targeting monetary policy which is preferable to a "strict" inflation-targeting policy rule.

A Appendix

A.1 Appendix to chapter 2

A.1.1 Data

Within this paper I use several macro and financial time series. This appendix describes some modifications and especially the source of the raw data.

Private Consumption: Nominal consumption expenditures for non-durables and services is the sum of the respective values of the series *PCND, Personal Consumption Expenditures: Nondurable Goods* and *PCESV, Personal Consumption Expenditures: Services* from the U.S. Department of Commerce: Bureau of Economic Analysis. Both series are measured in billions of dollars.
Source: *http://research.stlouisfed.org/fred2/*

Consumption Deflator: This is measured by the series *Personal Consumption Expenditures: Chain-type Price Index, PCEPI*, Index 2000=100 from the U.S. Department of Commerce: Bureau of Economic Analysis.
Source: *http://research.stlouisfed.org/fred2/*.

Hours worked: This index series (2002=100) is measured as *Aggregate Weekly Hours Index: Total Private Industries, AWHI* by the U.S. Department of Labor: Bureau of Labor Statistics.
Source: *http://research.stlouisfed.org/fred2/*

Civilian Population: This is a measure for the population given by the the monthly values of the series *CNP16OV, Civilian Noninstitutional Population over 16 years* from the U.S. Department of Labor: Bureau of Labor Statistics. The numbers have been converted from thousands to billions.
Source: *http://research.stlouisfed.org/fred2/*

Risk-Free Rate: The measure of the risk-free rate is based on the series *3-Month Treasury Bill: Secondary Market Rate, TB3MS* from the Board of Governors of the Federal Reserve System, Release: H.15 Selected Interest Rates .
Source: *http://research.stlouisfed.org/fred2/.*

S&P 500: The nominal returns of the S&P 500 are calculated by the monthly prices and dividends of the S&P 500. The monthly values are averages of daily closing prices calculated by Robert J. Shiller and provided on his website.
Source: *http://www.econ.yale.edu/ shiller/data.htm*

2x3 Portfolios: Thes portfolios are based on the calculations by Kenneth R. French and includes NYSE, AMEX, and NASDAQ stocks. The portfolios, which are constructed at the end of each June, are the intersections of 2 portfolios formed on size (market equity, ME) and 3 portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoint for year t is the median NYSE market equity at the end of June of year t. BE/ME for June of year t is the book equity for the last fiscal year end in t-1 divided by ME for December of t-1. The BE/ME breakpoints are the 30th and 70th NYSE percentiles. For more details see the website of Kenneth R. French.
Source: *http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/*

5x5 Portfolios: Thes portfolios are based on the calculations by Kenneth R. French and includes NYSE, AMEX, and NASDAQ stocks. The portfolios, which are constructed at the end of each June, are the intersections of 5 portfolios formed on size (market equity, ME) and 5 portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoints for year t are the NYSE market equity quintiles at the end of June of t. BE/ME for June of year t is the book equity for the last fiscal year end in t-1 divided by ME for December of t-1. The BE/ME breakpoints are NYSE quintiles. For more details see the website of Kenneth R. French.
Source: *http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/*

10 Industry Portfolios: These portfolios are based on the calculations by Kenneth R. French. Each NYSE, AMEX, and NASDAQ stock is assigned to an industry portfolio at the end of June of year t based on its four-digit SIC code at that time. For more details see the website of Kenneth R. French.
Source: *http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/*

Fama-French Factors: The Fama/French factors are constructed using the 6 value-weight portfolios formed on size and book-to-market. (See the descrip-

tion of the 2x3 size/book-to-market portfolios.) SMB (Small Minus Big) is the average return on the three small portfolios minus the average return on the three big portfolios, HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios, Rm, is the value-weight return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) (for more details see Fama and French 1993).

Source: *http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/*

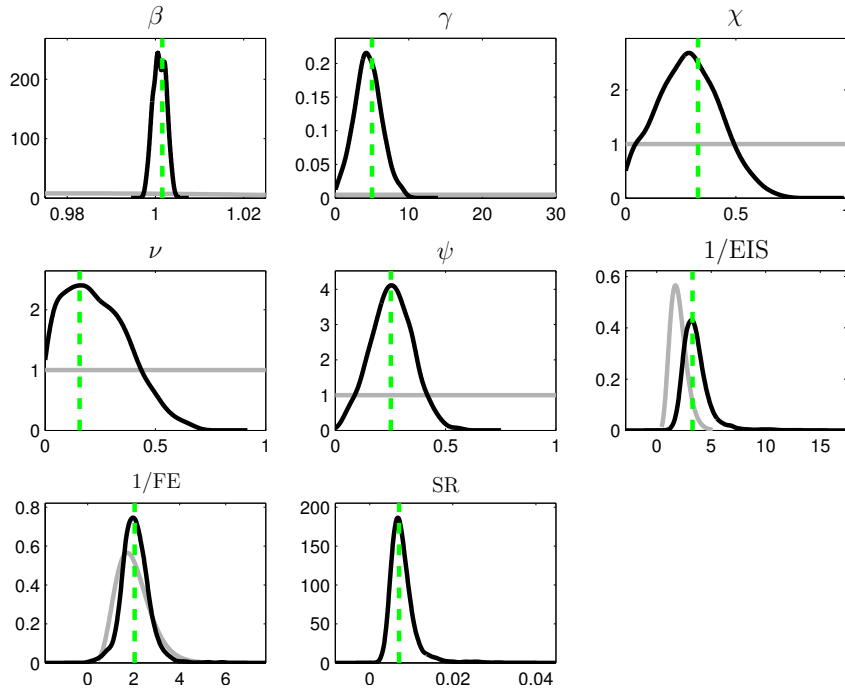### A.1.2 Posterior Distributions

Figure A.1: Posterior and prior distribution for internal habit formation with one lag (*internal-1L*) with informative prior. The grey line denotes the prior distribution, the black line the posterior distribution. Finally, the green dashed line denotes the posterior mode.
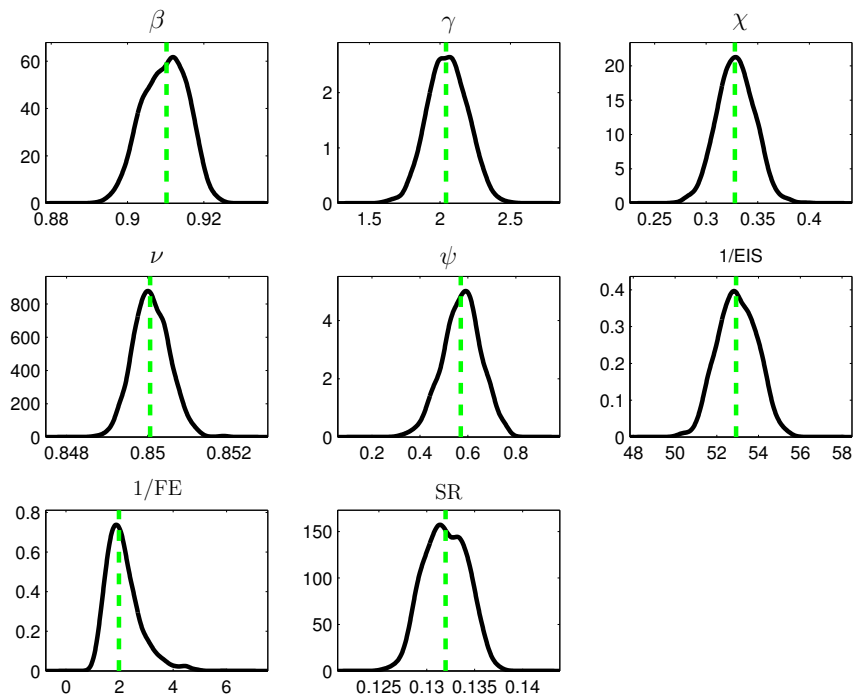
Figure A.2: Posterior distribution for internal habit formation with one lag (*internal-1L*) with diffuse prior. The black line the posterior distribution. Finally, the green dashed line denotes the posterior mode.
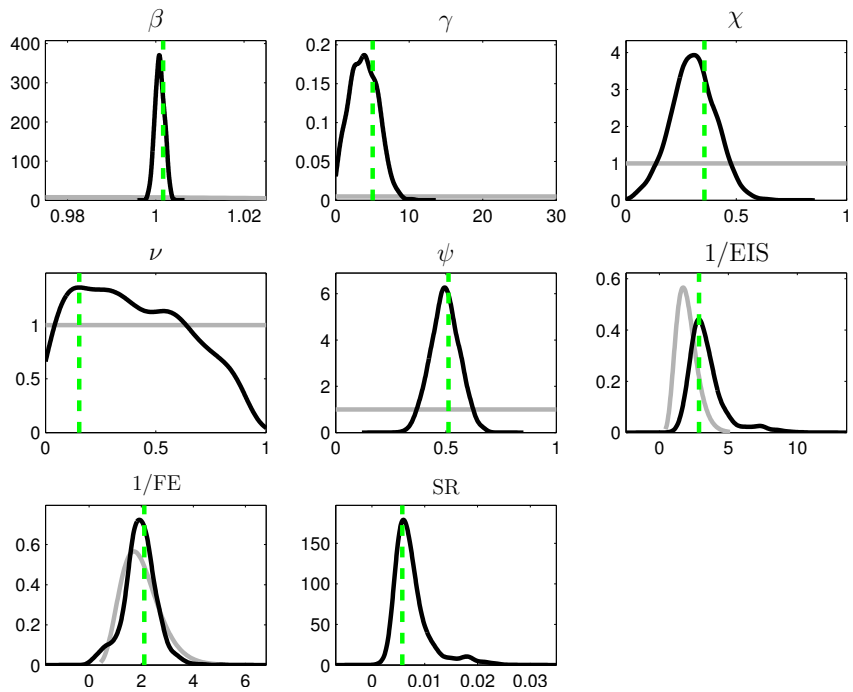

Figure A.3: Posterior and prior distribution for external habit formation with one lag (*external-1L*) with informative prior. The grey line denotes the prior distribution, the black line the posterior distribution. Finally, the green dashed line denotes the posterior mode.
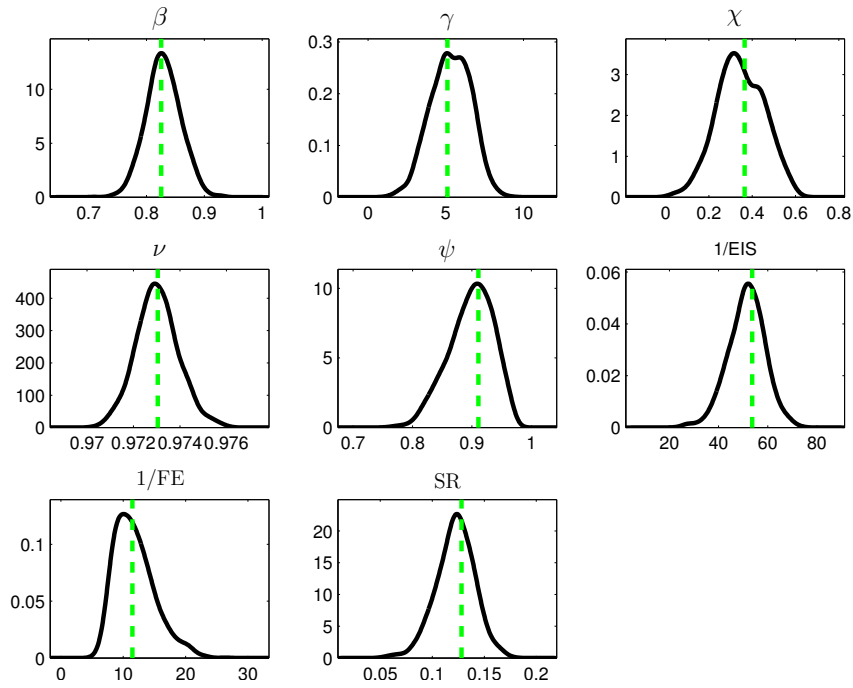
Figure A.4: Posterior distribution for external habit formation with one lag (*external-1L*) with diffuse prior. The black line the posterior distribution. Finally, the green dashed line denotes the posterior mode.
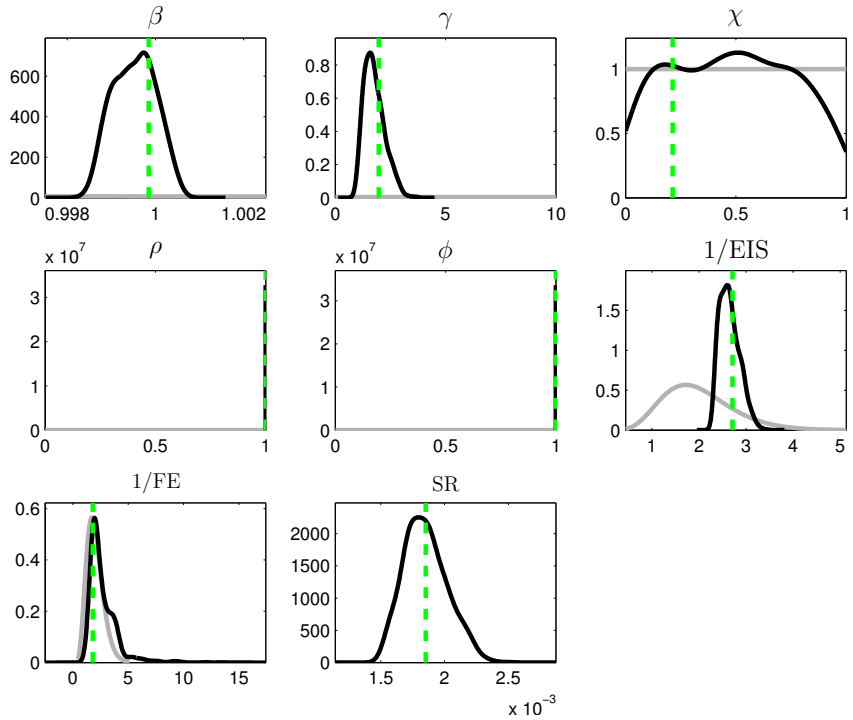

Figure A.5: Posterior and prior distribution for external habit formation with ∞-lags (*external-AR*) with informative prior. The grey line denotes the prior distribution, the black line the posterior distribution. Finally, the green dashed line denotes the posterior mode.
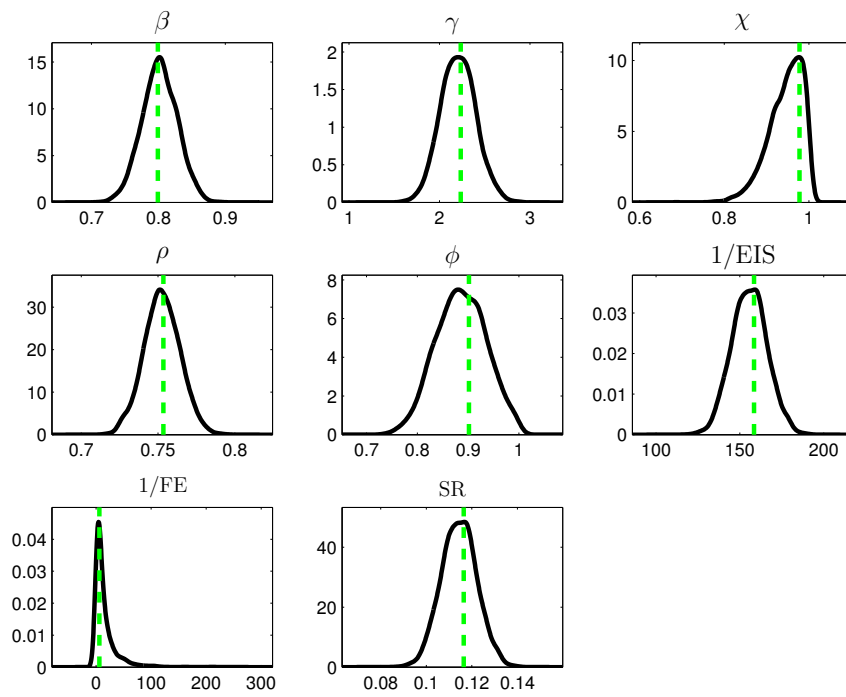
Figure A.6: Posterior distribution for external habit formation with ∞-lags (*external-AR*) with diffuse prior. The black line the posterior distribution. Finally, the green dashed line denotes the posterior mode.

A.2 Appendix to chapter 3

A.2.1 Calculations

FONC

The economy described in the paper follows the trend γ. To write the equilibrium conditions in stationary terms, the set of variables has to be detrended by z_{t-1} as follows:

$$\tilde{c}_t = \frac{c_t}{e^{z_{T,t-1}}}, \qquad \tilde{y}_t = \frac{y_t}{e^{z_{T,t-1}}}, \qquad \tilde{w}_t = \frac{w_t}{e^{z_{T,t-1}}}, \qquad \tilde{w}_t^f = \frac{w_t^f}{e^{z_{T,t-1}}} \qquad \text{(A.2.-1)}$$

$$\tilde{k}_{t-1} = \frac{K_{t-1}}{e^{z_{T,t-1}}}, \qquad \tilde{x}_{t-1} = \frac{x_{t-1}}{e^{z_{T,t-1}}}, \qquad \tilde{\lambda}_t = \frac{\lambda_t}{e^{-\eta z_{T,t-1}}}$$

Following, the set of the stationary first order necessary conditions of the equilibrium can be rewritten as:

$$n_t = 1 - l_t \qquad \text{(A.2.-2)}$$

$$R_t^c q_{t-1} = \frac{\theta \tilde{y}_t}{\tilde{k}_{t-1}} + \left(1 - \delta + g\left(z_{I,t}\frac{\tilde{x}_t}{\tilde{k}_{t-1}}\right)\right) q_t - \frac{\tilde{x}_t}{\tilde{k}_{t-1}} \qquad \text{(A.2.-3)}$$

$$q_t z_{I,t} = \frac{1}{g'\left(z_{I,t}\frac{\tilde{x}_t}{\tilde{k}_{t-1}}\right)} \qquad \text{(A.2.-4)}$$

$$\frac{1}{R_t^n} = E_t\left[\exp\left(\log\left(M_{t+1}\right) + \frac{\sigma_m^2}{2}\right)\right] \qquad \text{(A.2.-5)}$$

$$\frac{1}{E_t\left[R_t^c\right]} = E_t\left[\exp\left(\log\left(M_{t+1}\right) + \frac{\sigma_m^2}{2} + \sigma_{R^{eq}m}\right)\right] \qquad \text{(A.2.-6)}$$

$$M_t = \beta\frac{\tilde{\lambda}_t}{\tilde{\lambda}_{t-1}}\exp\left(-\eta\left(\gamma + \epsilon_{T,t-1}\right)\right) \qquad \text{(A.2.-7)}$$

$$R_t^c = \omega R_{t-1}^n + (1 - \omega) R_t^{eq} \tag{A.2.-8}$$

$$\tilde{\lambda}_t = \left(\tilde{c}_t - \chi \frac{\tilde{c}_{t-1}}{\exp{(\gamma + \epsilon_{T,t-1})}} \right)^{-\eta} \left(A + (l_t - z_{P,t} \psi l_{t-1})^\upsilon \right)^{1-\eta} \tag{A.2.-9}$$

$$\tilde{w}_t^f = \frac{\upsilon \left(\tilde{c}_t - \chi \frac{\tilde{c}_{t-1}}{\exp{(\gamma + \epsilon_{T,t-1})}} \right)}{A \left(l_t - z_{P,t} \psi l_{t-1} \right)^{1-\upsilon} + l_t - z_{P,t} \psi l_{t-1}} \tag{A.2.-10}$$

$$\tilde{w}_t = \frac{(1 - \theta) \tilde{y}_t}{n_t} \tag{A.2.-11}$$

$$\exp{(\mu \epsilon_{T,t-1})} \tilde{w}_t = (\tilde{w}_{t-1})^\mu \left(\tilde{w}_t^f \right)^{1-\mu} \tag{A.2.-12}$$

$$\tilde{y}_t = \left(\tilde{k}_{t-1} \right)^\theta \left(\exp{(\gamma + \epsilon_{T,t})} n_t \right)^{1-\theta} \tag{A.2.-13}$$

$$\tilde{k}_t = \left(1 - \delta + g \left(z_{I,t} \frac{\tilde{x}_t}{\tilde{k}_{t-1}} \right) \right) \tilde{k}_{t-1} \tag{A.2.-14}$$

$$\tilde{c}_t + \tilde{x}_t + \tilde{g}_t = \tilde{y}_t \tag{A.2.-15}$$

The equilibrium is defined together with the exogenous variables $z_{P,t}$, $z_{I,t}$, and $\tilde{g}_t$.

Steady-state

To calculate the steady state we take the following as given:

$$\bar{z}_P = \bar{z}_I = 1 \quad \text{and} \quad \bar{q} = 1 \,, \tag{A.2.-16}$$

as well as that the steady-state ratio of government expenditures to output is 28%:

$$\frac{\bar{\bar{g}}}{\bar{\bar{y}}} = 0.28 \tag{A.2.-17}$$

Furthermore, we use the established real depreciation rate:

$$\tilde{\delta} = e^{\gamma} + \delta - 1$$

Remembering the previous discussion about the asset pricing implications, we know that the Euler equation has to hold for any asset. This implies that (eq. A.2.-5) is equal to (eq. A.2.-6). Following, the steady state risk-less return is given by,

$$\bar{R}^{n} = \frac{\exp\left(\eta\gamma\right)}{\beta} \cdot \exp\left(-\frac{\sigma_{m}^{2}}{2}\right) \tag{A.2.-18}$$

the return on capital is equal to:

$$\bar{R}^{c} = \exp\left(\log\left(\bar{R}^{n}\right) - \sigma_{R^{c}m}\right) . \tag{A.2.-19}$$

The return on equity is finally determined by:

$$\bar{R}^{eq} = \frac{1}{1-\omega}\bar{R}^{c} - \frac{\omega}{1-\omega}\bar{R}^{n} \tag{A.2.-20}$$

With the determination of the steady-state value of the return on capital we can further solve:

$$\frac{\bar{\tilde{y}}}{\bar{\tilde{k}}} = \frac{\bar{R}^{c} + \delta - 1}{\theta} \tag{A.2.-21}$$

$$\frac{\bar{\tilde{x}}}{\bar{\tilde{y}}} = \frac{\tilde{\delta}\theta}{\bar{R} + \delta - 1} \tag{A.2.-22}$$

and

$$\frac{\bar{\tilde{c}}}{\bar{\tilde{y}}} = 1 - \frac{\bar{\tilde{x}}}{\bar{\tilde{y}}} - \frac{\bar{\tilde{g}}}{\bar{\tilde{y}}}. \tag{A.2.-23}$$

In the case of wage rigidities, the following steady-state relationship between the market wage and the frictionless wage (marginal rate of substitution) has to hold:

$$\bar{\tilde{w}} = \tilde{w}^{f}e^{\varepsilon_{w}} , \tag{A.2.-24}$$

where the market wage is determined by the condition:

$$\bar{\tilde{w}} = (1-\theta)\frac{\bar{\tilde{y}}}{\bar{n}} \tag{A.2.-25}$$

and the frictionless wage is given through:

$$\bar{\bar{w}}^f = \frac{1-\chi}{1-\psi}\frac{\upsilon}{1+\alpha}\frac{\bar{\bar{c}}}{\bar{l}} \tag{A.2.-26}$$

where

$$\alpha = A\left(1-\psi\right)^{-\upsilon}\bar{l}^{-\upsilon} \ . \tag{A.2.-27}$$

Given the solution of (eq. A.2.-23) the steady-state leisure has to fulfil the following condition:

$$\frac{(1-\theta)\left(1-\psi\right)}{\left(1-\chi\right)\upsilon e^{\varepsilon_w}}\frac{\bar{\bar{y}}}{\bar{\bar{c}}} = \frac{1}{1+A\left(1-\psi\right)^{-\upsilon}\bar{l}^{-\upsilon}}\cdot\frac{1-\bar{l}}{\bar{l}} \tag{A.2.-28}$$

After resolving this equation numerically we are able to solve for:

$$\bar{n} = 1 - \bar{l} \tag{A.2.-29}$$

and

$$\bar{\bar{k}} = \left[\frac{\bar{\bar{y}}}{\bar{\bar{k}}}\right]^{\frac{1}{\theta-1}}\bar{n}e^{\gamma} \tag{A.2.-30}$$

Afterwards we can use $\bar{\bar{k}}$ to solve for any remaining steady-state value.

Log-linearization

$$\hat{l}_t = -\frac{\bar{n}}{1-\bar{n}}\hat{n}_t \tag{A.2.-31}$$

$$\hat{r}_t^c + \hat{q}_{t-1} = \left[\frac{\bar{R}^c - 1 + \delta}{\bar{R}^c}\right]\left(\hat{\hat{y}}_t - \hat{\hat{k}}_{t-1}\right) + \frac{e^{\gamma}}{\bar{R}^c}\hat{q}_t + \frac{\tilde{\delta}}{\bar{R}^c}\hat{z}_{I,t} \tag{A.2.-32}$$

$$\hat{q}_t = \frac{1}{\zeta}\hat{\hat{x}}_t + \left(\frac{1}{\zeta}-1\right)\hat{z}_{I,t} - \frac{1}{\zeta}\hat{\hat{k}}_{t-1} \tag{A.2.-33}$$

$$\hat{\hat{w}}_t = \hat{\hat{y}}_t - \hat{n}_t \tag{A.2.-34}$$

$$\hat{\hat{w}}_t^f = \hat{c}_t^d + \left[\frac{\upsilon\alpha}{1+\alpha}-1\right]\hat{l}_t^d \tag{A.2.-35}$$

$$\hat{\bar{\lambda}}_t = -\eta \hat{c}_t^d + \left[\frac{\upsilon (1 - \eta)}{1 + \alpha} \right] \hat{l}_t^d \tag{A.2.-36}$$

$$\left(1 - \frac{\chi}{\exp{(\gamma)}} \right) \hat{c}_t^d = \hat{\bar{c}}_t - \frac{\chi}{\exp{(\gamma)}} \hat{\bar{c}}_{t-1} + \frac{\chi}{\exp{(\gamma)}} \epsilon_{T,t-1} \tag{A.2.-37}$$

$$(1 - \psi) \hat{l}_t^d = \hat{l}_t - \psi \hat{l}_{t-1} - \bar{\psi} \hat{z}_{P,t} \tag{A.2.-38}$$

$$0 = E_t \left[\hat{r}_{t+1}^c + \hat{m}_{t+1} \right] \tag{A.2.-39}$$

$$0 = E_t \left[\hat{m}_{t+1} \right] + \hat{r}_t^n \tag{A.2.-40}$$

$$\bar{R}^c \hat{r}_t^c = \omega \bar{R}^n \hat{r}_t^n + (1 - \omega) \bar{R}^{eq} \hat{r}_t^{eq} \tag{A.2.-41}$$

$$\hat{m}_t = \hat{\bar{\lambda}}_t + \hat{\bar{\lambda}}_{t-1} - \eta \epsilon_{T,t-1} \tag{A.2.-42}$$

$$\hat{\bar{y}}_t = \theta \hat{\bar{k}}_{t-1} + (1 - \theta) \hat{n}_t + (1 - \theta) \epsilon_{T,t} \tag{A.2.-43}$$

$$\hat{\bar{w}}_t = \mu \hat{\bar{w}}_{t-1} + (1 - \mu) \hat{\bar{w}}_t^f - \mu \epsilon_{T,t-1} \tag{A.2.-44}$$

$$e^\gamma \hat{\bar{k}}_t = (1 - \delta) \hat{\bar{k}}_{t-1} + \tilde{\delta} \hat{\bar{x}}_t + \tilde{\delta} \hat{z}_{I,t} - e^\gamma \epsilon_{T,t} \tag{A.2.-45}$$

$$\bar{\bar{y}} \hat{\bar{y}}_t = \bar{\bar{c}} \hat{\bar{c}}_t + \bar{\tilde{x}} \hat{\bar{x}}_t + \bar{\bar{g}} \hat{\bar{g}}_t \tag{A.2.-46}$$

Shocks:

$$\hat{z}_{P,t} = \pi_P \hat{z}_{P,t-1} + \epsilon_{P,t} \tag{A.2.-47}$$

$$\hat{z}_{I,t} = \pi_I \hat{z}_{I,t-1} + \epsilon_{I,t} \qquad \text{(A.2.-48)}$$

$$\hat{\bar{g}}_{I,t} = \pi_G \hat{\bar{g}}_{t-1} + \epsilon_{G,t} \qquad \text{(A.2.-49)}$$

A.2.2 Data

Within this paper I use several macro and financial time series. This appendix describes some modifications and especially the source of the raw data. The finally used frequency of all data is quarterly.

Real GDP: This series is *BEA NIPA table 1.1.6 line 1 (A191RX1)*.

Nominal GDP: This is a measure for the nominal GDP given by the series *GDP, Gross Domestic Product* at the Federal Reserve Board of St. Louis. It is measured in billions of dollars.
Source: *http://research.stlouisfed.org/fred2/*.

Implicit GDP Deflator: The implicit GDP deflator is calculated as the ratio of **N**ominal GDP to **R**eal GDP.

Private Consumption: Nominal consumption expenditures for non-durables and services is the sum of the respective values of the series *PCND, Personal Consumption Expenditures: Nondurable Goods* and *PCESV, Personal Consumption Expenditures: Services* at the Federal Reserve Board of St. Louis. Both series are measured in billions of dollars.
Source: *http://research.stlouisfed.org/fred2/*

Private Investment: Total real private investment is the sum of the respective nominal values of the series *BEA NIPA table 1.1.6 line 6 (A006RX1)* and *PCDG, Personal Consumption Expenditures: Durable Goods* at the Federal Reserve Board of St. Louis and finally deflated by the deflator mentioned above (billions of dollars).
Source: *http://research.stlouisfed.org/fred2/*

Government Expenditures: Current government expenditures is the series in *BEA NIPA table 3.1 line 15 (W022RC1)*.
Source: *http://www.bls.gov/data*

Hours worked: This index series (1992=100) is measured as hours worked in non-farm business sectors by the Bureau of Labor Statistics. The series' identification number is: *PRS85006033.*
Source: *http://www.bls.gov/data*

Wage: Wages are measured as total compensation of employees *BEA NIPA table 2.1.*
Source: *http://www.bea.gov/fred2/*

Civilian Population: This is a quarterly measure for the population given by the respective average of the monthly values of the series *CNP16OV, Civilian Noninstitutional Population* at the Federal Reserve Board of St. Louis. The numbers have been converted from thousands to billions.
Source: *http://research.stlouisfed.org/fred2/*

Return on Equity: The quarterly return on equity is calculated from monthly returns of the CRSP NYSE/AMEX/Nasdaq value-weighted market index including dividends (*INDNO:1000080*).
Source: ©12/2007 CRSP ®, Center for Research in Security Prices. Graduate School of Business, The University of Chicago. Used with permission. All rights reserved. www.crsp.uchicago.edu, accessed [10/2008].

Risk-free Rate: The quarterly risk-free return is calculated from monthly returns of the CRSP 90-Day Bill returns (*INDNO:1000707*).
Source: ©12/2007 CRSP ®, Center for Research in Security Prices. Graduate School of Business, The University of Chicago. Used with permission. All rights reserved. www.crsp.uchicago.edu, accessed [10/2008].

A.3 Appendix to chapter 4

A.3.1 Calculations

The economy described in the paper follows the trend μ_t. Additionally, I add a the variable z_t:

$$s_t = \frac{z_t}{z_{t-1}}, \tag{A.3.-1}$$

what implies that shocks to labor augmented technology processes can be expressed as:

$$s_t = \exp\left(\mu + \epsilon_{\mu,t}\right). \tag{A.3.-2}$$

To write the equilibrium conditions in stationary terms, the set of variables has to be detrended by z_{t-1} as follows:

$$\tilde{c}_t = \frac{C_t}{z_{t-1}}, \qquad \tilde{y}_t = \frac{Y_t}{z_{t-1}}, \qquad \tilde{c}_t^e = \frac{C_t^e}{z_{t-1}}, \qquad \tilde{w}_t = \frac{W_t}{z_{t-1}}, \qquad \tilde{w}_t^e = \frac{W_t^e}{z_{t-1}}$$

$$\widetilde{mrs}_t = \frac{MRS_t}{z_{t-1}} \quad \tilde{k}_{t-1} = \frac{K_{t-1}}{z_{t-1}}, \quad \tilde{d}_{t-1} = \frac{D_{t-1}}{z_{t-1}}, \quad \widetilde{nw}_{t-1} = \frac{NW_{t-1}}{z_{t-1}}, \quad \widetilde{\lambda}_t = \frac{\lambda_t}{z_{t-1}^{-\eta}}$$

A.3.2 FONCs

Because of the transformation of the equilibrium conditions into stationary equation all necessary conditions will be rewritten in this subsection.

$$H_t = 1 - L_t \tag{A.3.-3}$$

$$\widetilde{\lambda}_t = \left(\tilde{c}_t - \chi\frac{\tilde{c}_{t-1}}{s_{t-1}}\right)^{-\eta} \left(A + (L_t - \psi L_{t-1})^\nu\right)^{1-\eta} \tag{A.3.-4}$$

$$\widetilde{mrs}_t = \frac{\nu\left(\tilde{c}_t - \chi\frac{\tilde{c}_{t-1}}{s_{t-1}}\right)}{A\left(L_t - \psi L_{t-1}\right)^{1-\nu} + L_t - \psi L_{t-1}} \tag{A.3.-5}$$

$$1 = E_t\left[\beta\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t}s_t^{-\eta}R_t^N\frac{P_t}{P_{t+1}}\right] \tag{A.3.-6}$$

$$\tilde{y}_t = \varepsilon_t\tilde{K}_{t-1}^\alpha N_t^{1-\alpha}s_t^{1-\alpha} \tag{A.3.-7}$$

$$N_t = H_t\left(i\right)^{1-\Omega}\left(H_t^e\right)^\Omega \tag{A.3.-8}$$

$$Q_t \tilde{k}_t = \widetilde{nw}_t + \tilde{d}_t \tag{A.3.-9}$$

$$(1 - \Omega)(1 - \alpha)\frac{\tilde{y}_t}{H_t}mc_t = \tilde{w}_t \tag{A.3.-10}$$

$$\Omega(1 - \alpha)\frac{\tilde{y}_t}{H_t^e}mc_t = \tilde{w}_t^e \tag{A.3.-11}$$

$$E_t\left[R_{t+1}^q Q_t\right] = E_t\left[\frac{P_{w,t+1}}{P_{t+1}}\alpha\frac{\tilde{y}_{t+1}}{\tilde{k}_t} + (1 - \delta)Q_{t+1}\right] \tag{A.3.-12}$$

$$F_t = \frac{E_t\left[R_{t+1}^q\right]}{E_t\left[R_t^N \frac{P_t}{P_{t+1}}\right]} \tag{A.3.-13}$$

$$F_t = \left(\frac{Q_t \tilde{k}_t}{\widetilde{nw}_t}\right)^\sigma \tag{A.3.-14}$$

$$\widetilde{nw}_t s_t = \kappa\left(R_t^q Q_{t-1}\tilde{k}_{t-1} - E_{t-1}\left[R_t^q\right]\tilde{d}_{t-1}\right) + \tilde{w}^e \tag{A.3.-15}$$

$$\tilde{c}_t^e = (1 - \kappa)\left(R_t^q Q_{t-1}\tilde{k}_{t-1} - E_{t-1}\left[R_t^q\right]\tilde{d}_{t-1}\right) \tag{A.3.-16}$$

$$\tilde{k}_t s_t = \left(1 - \delta + \Phi\left(Z_{I,t}\frac{\tilde{i}_t}{\tilde{k}_{t-1}}\right)\right)\tilde{k}_{t-1} \tag{A.3.-17}$$

$$Q_t = \frac{1}{\Phi'\left(Z_{I,t}\frac{\tilde{i}_t}{\tilde{k}_{t-1}}\right)} \tag{A.3.-18}$$

The necessary first order conditions of the households to set wages:

$$E_t\left[\sum_{j=0}^\infty (\theta_w\beta)^j\left[\bar{\pi}^j\frac{W_t(i)}{P_{t+j}} - \frac{\varepsilon_w}{\varepsilon_w - 1}MRS_{t+j}\left(H_{t+j}(i), C_{t+j}(i)\right)\right]\right] = 0 \tag{A.3.-19}$$

with the corresponding aggregate wage index:

$$W_t = \left[\theta_w\left(\bar{\pi}W_{t-1}\right)^{1-\varepsilon_w} + (1 - \theta_w)\left(W_t(i)\right)^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}} \tag{A.3.-20}$$

Similar, the first order condition of the monopolistic firms to set their prices:

$$\frac{P_t(i)}{P_t} = \frac{\varepsilon_p}{\varepsilon_p - 1}\frac{E_t\left[\sum_{j=0}^\infty \theta_p^j m_{t+j} mc_{t+j} Y_{t+j}(i)\frac{P_{t+j}}{P_t}\right]}{E_t\left[\sum_{j=0}^\infty \theta_p^j m_{t+j}\bar{\pi}^j Y_{t+j}(i)\right]}, \tag{A.3.-21}$$

and also the corresponding aggregate price index:

$$P_t = \left[\theta_p\left(\bar{\pi}P_{t-1}\right)^{1-\varepsilon_p} + (1 - \theta_p)\left(P_t(i)\right)^{1-\varepsilon_p}\right]^{\frac{1}{1-\varepsilon_p}} \tag{A.3.-22}$$

The wage inflation is described by the following equation:

$$\frac{\pi_{w,t}}{\pi_t} = \frac{\tilde{w}_t}{\tilde{w}_{t-1}} s_{t-1} \qquad \text{(A.3.-23)}$$

Finally, the aggregate resource constraint of the economy:

$$\tilde{y}_t = \tilde{c}_t + \tilde{c}_t^e + \tilde{i}_t + \tilde{g}_t \qquad \text{(A.3.-24)}$$

The economy is closed by the remaining structural shock equations and the monetary policy rule described in the corresponding section.

A.3.3 Steady State

To calculate the steady state of the model, lets take the following as given:

The steady state values of technology and cost-function for producing capital are 1 and the growth path is given through $\exp(\mu)$:

$$\bar{z}_T = \bar{z}_I = 1 \quad \text{and} \quad \bar{s} = e^\mu; \qquad \text{(A.3.-25)}$$

The government spending is approximately 20% of the total output of the economy:

$$\frac{\bar{g}}{\bar{y}} = 0.2 \qquad \text{(A.3.-26)}$$

Further, there exists no inflation, $\bar{\pi} = 1$, and the real marginal costs are:

$$\overline{mc} = \frac{\varepsilon_p - 1}{\varepsilon_p} \qquad \text{(A.3.-27)}$$

The steady state price of capital is:

$$\bar{q} = 1 \qquad \text{(A.3.-28)}$$

Given these assumption and the parameter φ for leverage of the entrepreneurs, it holds:

$$\frac{\bar{k}}{nw} = 1 + \varphi \qquad \text{(A.3.-29)}$$

Given these assumption, its easy to solve for:

$$\bar{r}^N = \frac{e^\mu}{\beta} \qquad \bar{f} = \left[\frac{\bar{k}}{nw}\right]^\sigma \qquad \bar{r}^S = \bar{f}\bar{r}^N \qquad\qquad \text{(A.3.-30)}$$

From equation (A.3.-17) and (A.3.-12) we can solve for:

$$\frac{\bar{i}}{\bar{k}} = e^\mu - 1 + \delta \qquad \frac{\bar{y}}{\bar{k}} = \frac{\bar{r}^q - 1 + \delta}{\alpha \cdot \overline{mc}} \qquad\qquad \text{(A.3.-31)}$$

Given (eq. A.3.-11) and (eq. A.3.-15),it is easy to see that for $\bar{r}^k$ it has also to hold that:

$$0 = \kappa\bar{r}^q + \frac{\Omega(1-\alpha)}{\alpha}\frac{\bar{k}}{nw}\left(\bar{r}^q - 1 + \delta\right) \qquad\qquad \text{(A.3.-32)}$$

To capture this condition the parameter κ is set to solve this equation.

Now it is easy to solve for:

$$\frac{\bar{w}^e}{\bar{k}} = \Omega(1-\alpha)\frac{\bar{y}}{\bar{k}}\overline{mc} \qquad\qquad \text{(A.3.-33)}$$

$$\frac{\bar{c}^e}{\bar{k}} = (1-\kappa)\bar{r}^q\left(\frac{\bar{k}}{nw}\right)^{-1} \qquad\qquad \text{(A.3.-34)}$$

Following the market clearing condition implies that:

$$\frac{\bar{c}}{\bar{y}} = 1 - \frac{\bar{i}}{\bar{k}}\frac{\bar{k}}{\bar{y}} - \frac{\bar{c}^e}{\bar{k}}\frac{\bar{k}}{\bar{y}} - \frac{\bar{g}}{\bar{y}} \qquad\qquad \text{(A.3.-35)}$$

Given the information that $\bar{w} = \frac{\varepsilon_w}{\varepsilon_w - 1}\overline{mrs}$ and using equation (A.3.-5) and (A.3.-10)we can find:

$$\left(\frac{\bar{c}}{\bar{y}}\right)^{-1} \cdot \frac{\varepsilon_w - 1}{\varepsilon_w}\frac{(1-\Omega)(1-\alpha)(1-\psi)\overline{mc}}{\left(1 - \frac{\chi}{\exp(\mu)}\right)\nu} = \frac{1-\bar{l}}{\bar{l}(1-\Gamma)}, \qquad\qquad \text{(A.3.-36)}$$

where

$$\bar{l} = 1 - \bar{h} \qquad \text{and} \qquad \Gamma = A(1-\psi)^{-\nu}\bar{l}^{-\nu}. \qquad\qquad \text{(A.3.-37)}$$

Given the equations A.3.-36 and A.3.-37 we can now determine the steady state values for h and l.

Under the assumption that the entrepreneur's steady state supply of hours worked is constant and 1 its also holds that,

$$\bar{n} = \bar{h}^{1-\Omega} \cdot 1^\Omega \qquad\qquad \text{(A.3.-38)}$$

and

$$\bar{k} = \left(\frac{\bar{y}}{\bar{k}}\right)^{\frac{1}{\alpha-1}} \bar{n} e^{\mu}. \qquad (\text{A.3.-39})$$

Given all these results, we can now solve for:

$$\bar{c}, \bar{c}^e, \bar{g}, \bar{i}, \bar{w}^e, \overline{nw}, \bar{\lambda}, \overline{mrs}, \text{ and } \bar{w}$$

A.3.4   Log-Linearization

The fluctuation of leisure around its steady state are given by:

$$\hat{l}_t = -\frac{\bar{h}}{1-\bar{h}}\hat{h}_t \qquad (\text{A.3.-40})$$

Given the first order condition for the wages (A.3.-19) and extend the equation with W_t/W_t and W_{t+j}/W_{t+j}. Afterwards, define real wages, $w_t = W_t/P_t$, relative wages, $W_{R,t} = W_t(i)/W_t$, and finally the nominal wage inflation, $\pi_t^w = s_{t-1} \cdot W_t/(W_{t-1}\bar{\pi})$; the corresponding FONC can rewritten as:

$$E_t\left[\sum_{j=0}^{\infty}(\theta_w\beta)^j\left[W_{R,t}\left(\prod_{s=1}^{j}\frac{1}{\pi_{t+s}^w}\right)\tilde{w}_{t+j} - \frac{\varepsilon_w}{\varepsilon_w-1}\widetilde{MRS}_{t+j}\left(H_{t+j}(i), C_{t+j}(i)\right)\right]\right] = 0$$
$$(\text{A.3.-41})$$

Doing log-linearize the equation and respect that for steady state holds $\bar{\pi}^w = 1$ and $\bar{w} = \frac{\varepsilon_w}{\varepsilon_w-1}\overline{MRS}$, it is easy to show that the following holds:

$$\frac{1}{1-\beta\theta_w}\hat{w}_{R,t} + \sum_{j=0}^{\infty}(\beta\theta_w)^j\hat{\tilde{w}}_{t+j} - \sum_{j=0}^{\infty}(\beta\theta_w)^j\sum_{s=1}^{j}\hat{\pi}_{t+s}^w = \sum_{j=0}^{\infty}(\beta\theta_w)^j\widehat{mrs}_t(i) \quad (\text{A.3.-42})$$

The individual marginal substitution rate can written in log-linear terms as:

$$\widehat{mrs}_t(i) = \left(\frac{1}{1-\frac{\chi}{\mu}}\right)\left(\hat{\tilde{c}}_t(i) - \frac{\chi}{\mu}\left(\hat{\tilde{c}}_{t-1} - \hat{s}_{t-1}\right)\right) - \qquad (\text{A.3.-43})$$
$$\left[\frac{\Gamma(1-\upsilon)+1}{1+\Gamma}\right]\frac{\bar{h}}{1-\bar{h}}\frac{1}{1-\psi}\left(\hat{h}_t(i) - \psi\hat{h}_{t-1}\right)$$

Using know the knowledge about the aggregate labor demand and the evaluated aggregate consumption level, the corresponding individual levels in logarithmic terms

are:

$$\hat{h}_{t+j}\left(i\right) = \hat{h}_{t+j} - \varepsilon_w \left(\hat{w}_{R,t} - \sum_{s=1}^{j} \hat{\pi}_{t+s}^w\right) \tag{A.3.-44}$$

$$\hat{c}_{t+j}\left(i\right) = \hat{c}_{t+j} + \Upsilon \left(\hat{w}_{R,t} - \sum_{s=1}^{j} \hat{\pi}_{t+s}^w\right) \tag{A.3.-45}$$

For simplicity, define:

$$\hat{\Theta}_t = \left(\frac{1}{1-\frac{\chi}{\mu}}\right)\left(\hat{\tilde{c}}_t - \frac{\chi}{\mu}\left(\hat{\tilde{c}}_{t-1} - \hat{s}_{t-1}\right)\right) - \left[\frac{\Gamma\left(1-\upsilon\right)+1}{1+\Gamma}\right]\frac{\bar{h}}{1-\bar{h}}\frac{1}{1-\psi}\left(\hat{h}_t - \psi\hat{h}_{t-1}\right) \tag{A.3.-46}$$

and

$$\Xi = \frac{\Upsilon}{1-\frac{\chi}{\mu}} + \left[\frac{\Gamma\left(1-\upsilon\right)+1}{1+\Gamma}\right]\frac{\bar{h}}{1-\bar{h}}\frac{\varepsilon_w}{1-\psi}, \tag{A.3.-47}$$

additionally use equations (A.3.-43), (A.3.-44), and (A.3.-45) to rewrite (A.3.-42) as:

$$\frac{1-\Xi}{1-\beta\theta_w}\hat{w}_{R,t}+\sum_{j=0}^{\infty}\left(\beta\theta_w\right)^j\hat{\tilde{w}}_{t+j}-\left(1-\Xi\right)\sum_{j=0}^{\infty}\left(\beta\theta_w\right)^j\sum_{s=1}^{j}\hat{\pi}_{t+s}^w = \sum_{j=0}^{\infty}\left(\beta\theta_w\right)^j\hat{\Theta}_t \tag{A.3.-48}$$

After some algebra and re-arranging the equation can also be expressed as:

$$\frac{1-\Xi}{1-\beta\theta_w}\hat{w}_{R,t} = \frac{\left(1-\Xi\right)\beta\theta_w}{1-\beta\theta_w}\hat{w}_{R,t+1} + \frac{\left(1-\Xi\right)\beta\theta_w}{1-\beta\theta_w}\hat{\pi}_{t+1}^w + \hat{\Theta}_t - \hat{\tilde{w}}_t \tag{A.3.-49}$$

As written at the end of subsection 4.3.1, it is known that $\hat{w}_{R,t} = \frac{\theta_w}{1-\theta_w}\hat{\pi}_t^w$, which allows to rewrite the equation above in the following familiar way

$$\hat{\pi}_t^w = \beta\hat{\pi}_{t+1}^w + \frac{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)}{\left(1-\Xi\right)\theta_w}\left(\hat{\Theta}_t - \hat{\tilde{w}}_t\right), \tag{A.3.-50}$$

as shown by Woodford (2003), Ξ depends on the inverse of the Frisch elasticity (FE) of labor supply like $\Xi = -\varepsilon_w/\text{FE}$.

From the complete capital market assumption it is known that the marginal utility across households has to be constant. Taking into account the findings from the previous sections, the log-linear formulation is is given by:

$$\hat{\lambda}_t = \left(\frac{-\eta}{1-\frac{\chi}{\mu}}\right)\left(\hat{\tilde{c}}_t - \frac{\chi}{\mu}\left(\hat{\tilde{c}}_{t-1} - \hat{s}_{t-1}\right)\right) - \left[\frac{\upsilon\left(1-\eta\right)}{1+\Gamma}\right]\frac{\bar{h}}{1-\bar{h}}\frac{1}{1-\psi}\left(\hat{h}_t - \psi\hat{h}_{t-1}\right). \tag{A.3.-51}$$

Finally, the euler equation completes the households first order conditions and can be written in log-linear terms as:

$$\hat{\bar{\lambda}}_t + \eta \hat{s}_t = E_t \left[\hat{r}_t^N + \hat{\bar{\lambda}}_{t+1} - \hat{\pi}_{t+1} \right] \tag{A.3.-52}$$

For the entrepreneurial sector the log-linearized equations are:

Technology:

$$\hat{\bar{y}}_t = \hat{\varepsilon}_t + \alpha \hat{\bar{k}}_{t-1} + (1 - \alpha) \, \hat{n}_t + (1 - \alpha) \, \hat{s}_t \tag{A.3.-53}$$

with

$$\hat{n}_t = (1 - \Omega) \, \hat{h}_t. \tag{A.3.-54}$$

Real marginal cost:

$$\hat{mc}_t = \hat{\bar{w}}_t - \hat{\bar{y}}_t + \hat{h}_t \tag{A.3.-55}$$

Entrepreneurial wage:

$$\hat{w}_t^e = \hat{\bar{y}}_t + \hat{mc}_t \tag{A.3.-56}$$

The net worth of the entrepreneur is given through:

$$\widehat{nw}_t + \hat{s}_t = \kappa \frac{\bar{R}^q}{\bar{s}} \frac{\bar{k}}{\overline{nw}} \left(\hat{r}_t^q - E_{t-1} \left[\hat{r}_t^q \right] \right) + \kappa \frac{\bar{R}^q}{\bar{s}} \left(\widehat{nw}_{t-1} + E_{t-1} \left[\hat{r}_t^q \right] \right) + \frac{\bar{w}^e}{\overline{nw}\,\bar{s}} \hat{w}_t^e \tag{A.3.-57}$$

Following, the corresponding consumption of the entrepreneur is given by:

$$\hat{\bar{c}}_t^e = (1 - \kappa) \frac{\bar{R}^q}{\bar{c}^e} \bar{k} \left(\hat{r}_t^q - E_{t-1} \left[\hat{r}_t^q \right] \right) + (1 - \kappa) \frac{\bar{R}^q}{\bar{c}^e} \overline{nw} \left(\widehat{nw}_{t-1} + E_{t-1} \left[\hat{r}_t^q \right] \right) \tag{A.3.-58}$$

The term spread $\hat{f}$ implies the following equation on the capital market side

$$\hat{f}_t = E_t \left[\hat{r}_{t+1}^q \right] + E_t \left[\hat{\pi}_{t+1} \right] - \hat{r}_t^N \tag{A.3.-59}$$

and on firm-level

$$\hat{f}_t = \sigma \left(\hat{\bar{k}}_t + \hat{q}_t - \widehat{nw}_t \right). \tag{A.3.-60}$$

For the expected return on capital the following equation can established

$$E_{t-1} \left[\hat{r}_t^q \right] = \frac{\bar{R}^q - (1 - \delta)}{\bar{R}^q} \left(\hat{mc}_t + \hat{\bar{y}}_t - \hat{\bar{k}}_{t-1} \right) + \frac{1 - \delta}{\bar{R}^q} \hat{q}_t - \hat{q}_{t-1} \tag{A.3.-61}$$

For the capital producers the necessary log-linearized equations are:

Capital accumulation:

$$\bar{s}\hat{s}_t + \bar{s}\hat{\bar{k}}_t = (1 - \delta)\hat{\bar{k}}_{t-1} + (\bar{s} - 1 + \delta)\left(\hat{z}_{I,t} + \hat{\bar{i}}_t\right) \tag{A.3.-62}$$

The cost of capital are given through:

$$\hat{q}_t = \frac{1}{\zeta}\hat{\bar{i}}_t - \frac{1}{\zeta}\hat{\bar{k}}_{t-1} + \frac{1}{\zeta}\hat{z}_{I,t} \tag{A.3.-63}$$

Similar to the evaluation of the wage inflation equation above, I use first order condition for the optimal price (A.3.-21) and define relative prices, $P_{R,t} = P_t(i)/P_t$. Additionally, using the nominal price inflation is given by $\pi_t = P_t/(P_{t-1}\bar{\pi})$, the elasticity condition,

$$\tilde{y}_{t+j}(i) = \tilde{y}_{t+j}\left(\frac{P_t(i)}{P_t}\frac{P_t\bar{\pi}^j}{P_{t+j}}\right)^{-\varepsilon_p},$$

and the equation for the pricing kernel

$$\tilde{m}_{t+j} = \beta^j\frac{\tilde{\lambda}_{t+j}P_t}{\tilde{\lambda}_t P_{t+j}}\left(\frac{z_{t+j}}{z_t}\right)^{-\eta}.$$

The transformed FONC for the optimal price is then:

$$P_{R,t}E_t\left[\sum_{j=0}^{\infty}(\theta_p\beta)^j\frac{\tilde{\lambda}_{t+j}}{\tilde{\lambda}_t}\left(\prod_{s=1}^{j}\frac{1}{\pi_{t+s}}\right)\left(\frac{z_{t+j}}{z_t}\right)^{-\eta}\tilde{y}_{t+j}\left(P_{R,t}\frac{P_t\bar{\pi}^j}{P_{t+j}}\right)^{-\varepsilon_p}\right] =$$
$$\frac{\varepsilon_p}{\varepsilon_p - 1}E_t\left[\sum_{j=0}^{\infty}(\theta_p\beta)^j\frac{\tilde{\lambda}_{t+j}}{\tilde{\lambda}_t}\left(\frac{z_{t+j}}{z_t}\right)^{-\eta}mc_{t+j}\tilde{y}_{t+j}\left(P_{R,t}\frac{P_t\bar{\pi}^j}{P_{t+j}}\right)^{-\varepsilon_p}\right]. \tag{A.3.-64}$$

Doing log-linearize the equation and respect that for steady state holds $\bar{\pi} = 1$ and $\frac{\varepsilon_p - 1}{\varepsilon_p} = \overline{mc}$, it is easy to show that the following holds:

$$\frac{1}{1 - \beta\theta_p}\hat{p}_{R,t} - \sum_{j=0}^{\infty}(\beta\theta_p)^j\sum_{s=1}^{j}\hat{\pi}_{t+s} = \sum_{j=0}^{\infty}(\beta\theta_p)^j\widehat{mc} \tag{A.3.-65}$$

Employing that $\hat{p}_{R,t} = \frac{\theta_p}{\theta_p - 1}\hat{\pi}_t$ and some algebra the finally log-linearized inflation equation is given by:

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \frac{(1 - \beta\theta_p)(1 - \theta_p)}{\theta_p}\hat{mc}_t \tag{A.3.-66}$$

With the knowledge about the inflation, its easy to figure out the interaction between price inflation and nominal wage inflation.

$$\hat{\pi}_t^w - \hat{\pi}_t = \hat{\tilde{w}}_t - \hat{\tilde{w}}_{t-1} + \hat{s}_{t-1} \qquad \text{(A.3.-67)}$$

The aggregate resource constraint is characterized by:

$$\bar{y}\hat{\tilde{y}}_t = \bar{c}\hat{c}_t + \bar{c}^e\hat{\tilde{c}}_t^e + \bar{\tilde{i}}\hat{\tilde{i}}_t + \bar{g}\hat{\tilde{g}}_t \qquad \text{(A.3.-68)}$$

The number of exogenous state variables within the model will be extended by the different shock process, which introduces different impulses into the economy.

Labor augmenting technology shock:

$$\hat{s}_t = \epsilon_{T,t} \qquad \text{(A.3.-69)}$$

Anticipated technology shock:

$$\hat{\varepsilon}_t = \rho_\varepsilon \hat{\varepsilon}_{t-1} + \epsilon_{\varepsilon,t-12}^1 + \epsilon_{\varepsilon,t}^2 \qquad \text{(A.3.-70)}$$

Government spending shock:

$$\hat{\tilde{g}}_t = \rho_G \hat{\tilde{g}}_{t-1} + \epsilon_{G,t} \qquad \text{(A.3.-71)}$$

Capital adjustment cost shock:

$$\hat{z}_{I,t} = \rho_I \hat{z}_{I,t-1} + \epsilon_{I,t}, \qquad \text{(A.3.-72)}$$

Finally, the monetary policy, which closes the economy is given by:

$$\hat{r}_t^N = \gamma_R \hat{r}_{t-1}^N + (1 - \gamma_R)\left[\gamma_\pi \hat{\pi}_{t-1} + \gamma_Y \hat{\tilde{y}}_{t-1} + \gamma_Q \hat{\tilde{q}}_{t-1}\right] \qquad \text{(A.3.-73)}$$

Up to this point the model is closed and can be solved. Additional corresponding interesting variables in levels values are:
Consumption:

$$\hat{c}_t = \hat{\tilde{c}}_t + \hat{z}_{t-1} \qquad \text{(A.3.-74)}$$

Investment:

$$\hat{i}_t = \hat{\tilde{i}}_t + \hat{z}_{t-1} \tag{A.3.-75}$$

Capital:

$$\hat{k}_t = \hat{\tilde{k}}_t + \hat{z}_t \tag{A.3.-76}$$

Output:

$$\hat{y}_t = \hat{\tilde{y}}_t + \hat{z}_{t-1} \tag{A.3.-77}$$

Wages:

$$\hat{w}_t = \hat{\tilde{w}}_t + \hat{z}_{t-1} \tag{A.3.-78}$$

A.3.5   Data

Within this paper I use several macro and financial time series. This appendix describes some modifications and especially the source of the raw data. The finally used frequency of all data is quarterly.

Nominal GDP: This is a measure for the nominal GDP given by the series *GDP, Gross Domestic Product* at the Federal Reserve Board of St. Louis. It is measured in billions of dollars.
Source: *http://research.stlouisfed.org/fred2/.*

Private Consumption: Nominal consumption expenditures for non-durables and services is the sum of the respective values of the series *PCND, Personal Consumption Expenditures: Nondurable Goods* and *PCESV, Personal Consumption Expenditures: Services* at the Federal Reserve Board of St. Louis. Both series are measured in billions of dollars.
Source: *http://research.stlouisfed.org/fred2/*

Implicit Consumption Deflator This series is *BEA: NIPA table 1.1.5 line 2* divided by *BEA: NIPA table 1.1.6 line 2.*

Private Investment: Total real private investment is the sum of the respective nominal values of the series *BEA NIPA table 1.1.6 line 6 (A006RX1)* and *PCDG, Personal Consumption Expenditures: Durable Goods* at the Federal Reserve Board of St. Louis and finally deflated by the consumption deflator mentioned above (billions of dollars).
Source: *http://research.stlouisfed.org/fred2/*

Hours worked: This index series (1992=100) is measured as hours worked in non-farm business sectors by the Bureau of Labor Statistics. The series' identification number is: *PRS85006033.*
Source: *http://www.bls.gov/data.*

Wage: The wage rate is the series *COMPNFB, Nonfarm Business Sector: Compensation Per Hour* at the Federal Reserve Board of St. Louis.
Source: *http://research.stlouisfed.org/fred2/*

Civilian Population: This is a quarterly measure for the population given by the respective average of the monthly values of the series *CNP16OV, Civilian Noninstitutional Population* at the Federal Reserve Board of St. Louis. The numbers have been converted from thousands to billions.
Source: *http://research.stlouisfed.org/fred2/*

S&P 500: The quarterly nominal price index of the S&P 500 is calculated by the quarterly average of monthly values of this series. The monthly values are averages of daily closing prices calculated by Robert J. Shiller and provided on his website.
Source: *http://www.econ.yale.edu/ shiller/data.htm*
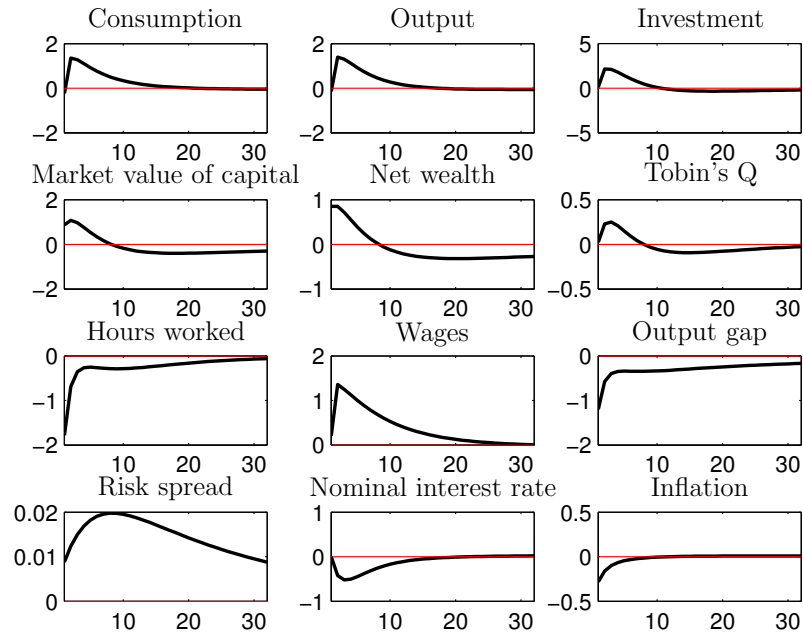
A.3.6   Impulse Responses

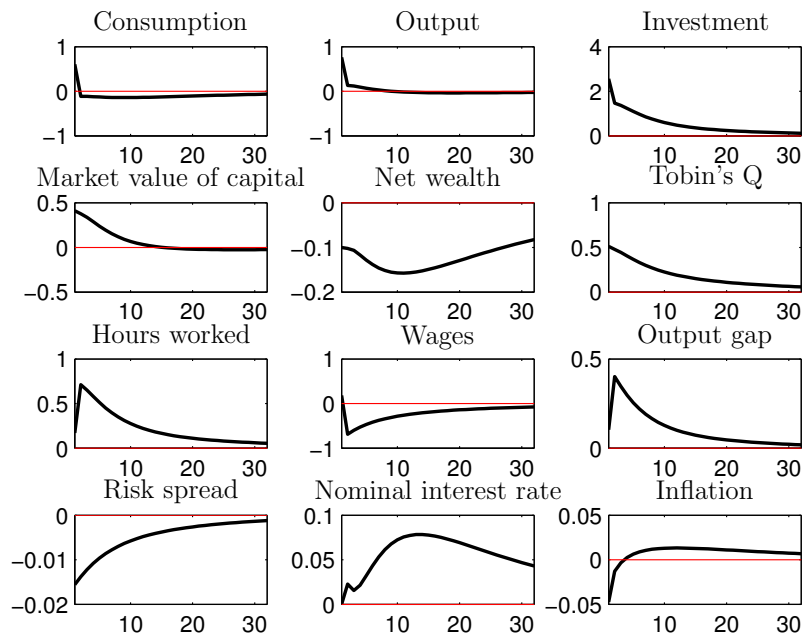Figure A.7: Impulse Responses to productivity shock

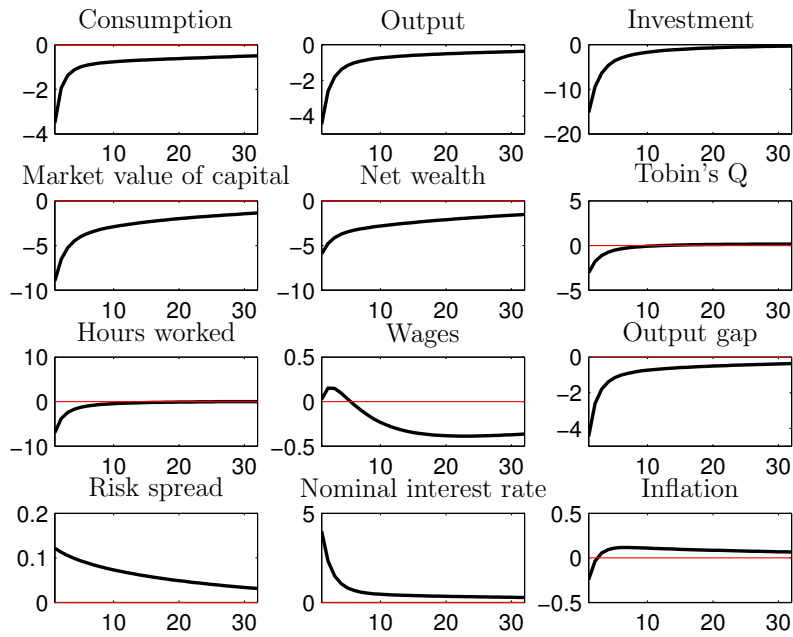Figure A.8: Impulse Responses to a labor augmented productivity shock

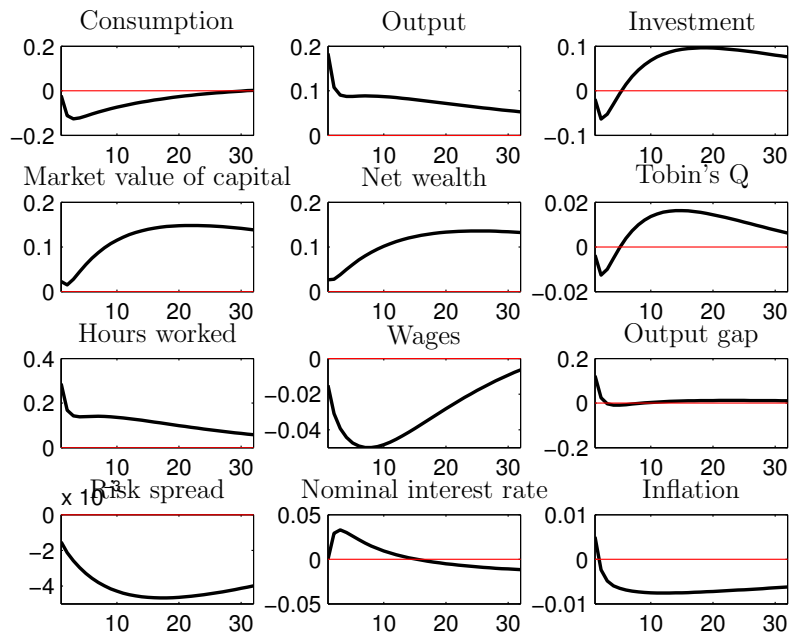Figure A.9: Impulse Responses to a monetary policy shock

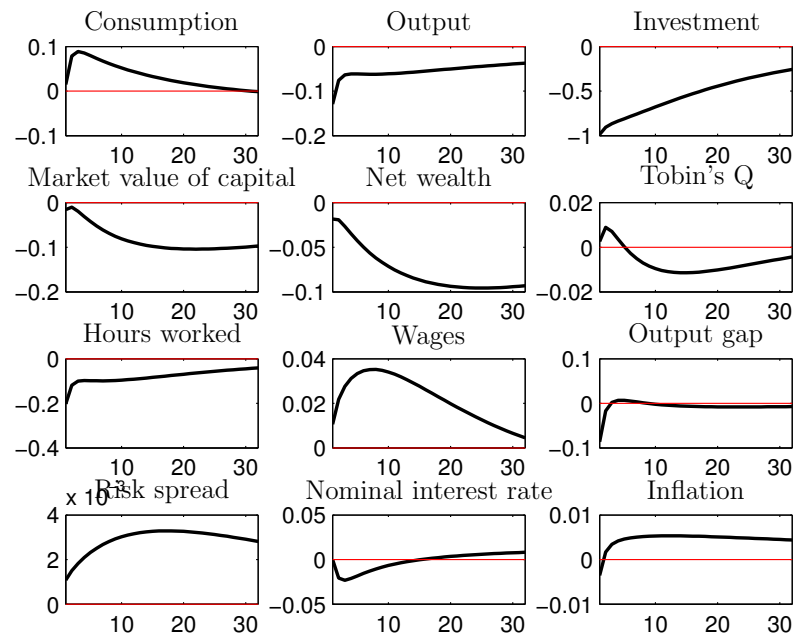Figure A.10: Impulse Responses to a government spending shock

Figure A.11: Impulse Responses to a adjustment cost shock

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Selbständigkeitserklärung

Ich erkläre hiermit, dass ich die vorliegende Dissertation mit dem Titel "Essays on Asset Pricing and the Macroeconomy" selbständig und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe; die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sind als solche kenntlich gemacht.

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Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung meiner Dissertation benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

Berlin, den 3. April 2009   Martin Kliem