Hazard Functions and Macroeconomic Dynamics

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I dedicate this work
to my family in China and my friends in Germany

Abstract

The Calvo assumption (Calvo, 1983) is widely used in the macroeconomic literature to model market frictions that limit the ability of economic agents to reoptimize their control variables. In spite of its virtues, the Calvo assumption also implies singular adjustment behavior at the firm level as well as a restrictive aggregation mechanism for the whole economy. In this study, I examine implications of the Calvo assumption for macroeconomic dynamics. To do so, I extend the Calvo assumption to a more general case based on the concept of the statistical hazard function. Two applications of this approach are studied in the DSGE framework. In the first essay, I apply this approach to a New Keynesian model, and demonstrate that tractability gained from the Calvo pricing assumption is costly in terms of inflation dynamics. The second essay estimates aggregate price reset hazard function using the theoretical framework constructed in the first essay, and shows that the constant hazard function implied by the Calvo assumption is strongly rejected by the aggregate data. In the third essay, I further explore implications of the empirically based hazard function for inflation persistence and monetary policy. I find that the empirically plausible aggregate price reset hazard function can generate simulated data that are consistent with inflation gap persistence found in the US CPI data. Based on these results, I conclude that the price reset hazard function plays a crucial role for generating inflation dynamics. The last essay applies the same modeling approach to a RBC model with employment rigidity. I find that, when introducing a more general stochastic adjustment process, the employment dynamics vary with a parameter, which determines the monotonic property of the hazard function. In particular, the volatility of employment is increasing, but the persistence is decreasing in the value of the parameter.

Key words:

Bayesian estimation, hazard function, heterogeneous employment rigidity, inflation persistence, New Keynesian Phillips curve, nominal rigidity, trend inflation, Weibull distribution

Zusammenfassung

Die Calvo-Annahme (Calvo, 1983) wird in der makroökonomischen Literatur oft verwendet um jene Marktunvollkommenheiten zu modellieren, die Begrenzungen für die Möglichkeiten der Marktteilnehmer darstellen, ihre Kontrollvariablen anzupassen. Trotz ihrer zahlreichen Vorteile folgt aus der Calvo-Annahme ein unrealistisches Anpassungsverhalten von Firmen, sowie ein sehr restriktiver Aggregationsmechanismus auf gesamtwirtschaftlicher Ebene. In dieser Arbeit werden die Folgen der Calvo-Annahme in dynamischen makroökonomischen Modellen untersucht. Dafür wird die Calvo-Annahme unter Anwendung des Konzepts der statistischen Hazardfunktion verallgemeinert. Ich untersuche zwei mögliche Anwendungen dieses Ansatzes innerhalb von DSGE-Modellen. Im ersten Artikel zeige ich, dass der Zugewinn an Handhabbarkeit, der aus der Calvo-Annahme für Neu-Keynesianische Modelle folgt, mit unerwünschten Folgen in Bezug auf die Inflationsdynamiken einher geht. Der zweite Artikel schätzt die aggregierte Hazardfunktion unter Verwendung des theoretischen Rahmens des ersten Artikels. Es zeigt sich, dass die Annahme einer konstanten Hazardfunktion, die aus der Calvo-Annahme folgt, von den Daten eindeutig abgelehnt wird. Im dritten Artikel analysiere ich die Implikationen der empirisch geschätzten Hazardfunktion für die Persistenz von Inflation und die Geldpolitik. Die Untersuchungen zeigen, dass mittels der empirisch plausiblen aggregierten Hazardfunktion Zeitreihen simuliert werden können, die mit der Persistenz der inflatorischen Lücke im US Verbraucherpreisindex konsistent sind. Anhand dieser Ergebnisse komme ich zu dem Schluss, dass die Hazardfunktion eine entscheidende Rolle für die dynamischen Eigenschaften von Inflation spielt. Der letzte Artikel wendet den selben Modellierungsansatz auf ein Real-Business-Cycle Model mit rigidem Arbeitsmarkt an. Unter Verwendung eines allgemeineren stochastischen Anpassungsprozess stelle ich fest, dass die Arbeitsmarktdynamiken von einem Parameter beinflusst werden, der das Monotonieverhalten der Hazardfunktion bestimmt. Insbesondere steigt die Volatilität des Beschäftigungsniveaus, wohingegen dessen Persistenz mit zunehmendem Parameterwert abnimmt.

Schlüsselwörter:

Bayes-Schätzung, Hazardfunktion, Heterogene Arbeitsmarktrigidität, Inflationspersistenz, Neu-keysianische Phillipskurve, Nominale Rigidität, Inflationstrend, Weibull Verteilung

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1 Introduction and Literature Review

1.1 Introduction

The concept of sticky prices has a long tradition in the macroeconomic literature. As early as in the late 19th and early 20th century, economists recognized that sticky nominal prices or wages play an important role in explaining excess demand or supply in the goods market and unemployment in the labor market[1]. More recently, price stickiness has become one of the most important issues in the literature, because different perceptions of its significance for the business cycles distinct two major modern schools of macroeconomics, labeled as “New-classical” and “New-Keynesian”. As a result, modeling of sticky prices has received great attention in the macroeconomic research. Different to the traditional literature, which postulates sticky prices as an integral feature of the economy, modern macroeconomic models built its implications of sticky prices on the base of explicit modeling assumptions of sluggish nominal adjustment.

Since the time when the empirical observation of so-called "Phillips curve" (Phillips, 1958) came into the scene, economists have believed nominal rigidity to be the main economic source of trade-off between the rate of inflation and unemployment, at least in the short run[2]. However, it is until formal incorporation of sticky prices modeling into DSGE models that gives economists an analytical apparatus to quantitatively analyze positive and normative questions, such as, the nature of inflation persistence and the optimal monetary policy given sticky prices. Works by Fischer [1977] and Taylor [1980] introduced staggered wage contracts into a rational-expectations macro model to deliver short-run real effects of monetary policy. The most influential formulation of sticky price-setting is based on the work by Calvo [1983] which constructed a model of random price adjustment and summarized the dynamic relationship between inflation and real driving forces into a simple equation known as “New Keynesian Phillips

[1] Laidler [1992] provided a more detailed discussion on this early literature.

[2] Seminal contributions of monetarist economists, e.g. Friedman [1968], and rational-expectations analysts, such as Sargent [1971] and Lucas [1976], gave the insight of the role played by expectations of inflation in generating the long-run dynamics.

curve" (NKPC)[3].

Given the significance of the Calvo assumption in the macroeconomic literature, in this thesis, I want to shed light on the hidden implications of the Calvo pricing assumption for the dynamic behavior of aggregate variables. In particular, I extend the Calvo assumption to a general form based on the concept first proposed by Wolman [1999] and further developed by Whelan [2007]. I implement this modeling approach to two economic applications - staggered price-setting and staggered labor adjustment. In both cases, I find that the richer dynamic structure resulted from the extension is quantitatively important for explaining aggregate dynamics. More precisely, the following questions are addressed in the four main chapters of the thesis:

1. Whether is the dynamic relationship between inflation and real marginal cost sensitive to the shape of the price reset hazard function? If yes, what is missing in the NKPC based on the Calvo pricing assumption, and why?

2. How does the more general sticky price model fit to the data? Is the constant hazard function a good proxy for more empirically plausible hazard functions?

3. Can the generalized NKPC account for the inflation persistence observed in the data? If yes, what is the mechanism at work?

4. Whether explicit modeling of the micro lumpiness changes the model's implication for the aggregate dynamics? How does the labor adjustment hazard function affect employment dynamics?

The rest of this introductory chapter is organized as follows: I first review the literature related to the topics of this thesis. For each topic, I first discuss empirical evidence that motivates the study. Then I focus on the theoretical development which has been devoted to tackle the empirical challenge. In the last part, I summarize the main contributions and findings of each chapter.

1.2 literature Review

The purpose of this section is to survey the literature that is related to the topics of this thesis. I aim at describing the current research frontier with respect to questions of interest outlined in the previous section. In doing so, it enables us to position four main chapters of this thesis in the respective literature.

[3] The Calvo assumption is also widely used in modeling other partial adjustments in the DSGE models, e.g., nominal wage rigidity (Jeanne, 1997), lumpy investment (Sveen and Weinke, 2005) and sluggish information diffusion (Mankiw and Reis, 2002).

1.2.1 Nominal Price Rigidity

In this section, I offer a selective review of recent developments in the empirical and theoretical modeling of price stickiness. It is selective because I am not attempting to provide a comprehensive summary of the vast volume of research devoted to this topic in recent years. Rather, I would like to shed light on two key issues which are closely related to my theme: first, empirical evidence of the price reset hazard rate from micro and macro data; and second, the theoretical modeling of sticky prices in the DSGE model.

In recent years, detailed micro-level pricing data sets have become available. Empirical work using these data sets generally reach the consensus that, instead of having economy-wide uniform price stickiness, the frequency of price adjustments differs substantially within the economy[4]. On average, individual prices change at least once a year. The frequency of price changes is higher in the U.S. than in the EURO area. When looking closely into sectors, prices differ greatly in how often they change. In general, prices of durable goods are more flexible than non-durable goods and services. Posted prices for apparel, at one extreme, last on average only 3 months, while prices for medical care services, at the other extreme, have the average duration of more than 14 months.

Additional to the sectoral average price stickiness, the price reset hazard function turns out to be useful in evaluating theoretical models to identify relevant propagation mechanisms for monetary policy. For example, the Calvo assumption implies a constant hazard function, meaning that the probability of adjusting prices is independent of the length of the time since the last price revision, while the state-dependent pricing model implies an increasing hazard function (Dotsey et al., 1999). Micro price data set provides information about the price reset hazard function. Cecchetti [1986] used newsstand prices of magazines in the U.S. and Goette et al. [2005] apply Swiss restaurant prices. Both studies find strong support for increasing hazard functions. By contrast, recent studies using more comprehensive micro data find that hazard functions are first downward sloping and then mostly flat, interrupted by a spike at one year frequency (See, e.g.: Campbell and Eden, 2005, Alvarez, 2007 and Nakamura and Steinsson, 2008). Even though the evidence rejecting the constant hazard function was robust, micro data failed to reach a consensus about the shape of the hazard function. The reason is that, first, those data sets differ substantially in the range of goods included, the countries and time periods covered, and thereby make it difficult to compare results;

[4]See: e.g. Bils and Klenow (2004), Alvarez *et al.* (2006), Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008) Klenow and Malin (2010)

second, even though comprehensive micro data sets now become available, they are usually short compared to the aggregate time series. Most of the CPI or PPI data for the U.S. and Europe frequently begin in the late 80's and typically continue through the present[5]. As a result, it is arguable that the price reset hazard function depends on the underlying economic conditions, and therefore it changes over the time periods of the collected data. As a result, it is desirable to study the empirical shape of the hazard function by using the time series data as a complement to the micro-econometric studies.

Early empirical models using aggregate data were solely based on the standard Calvo NKPC (See, e.g. Gali and Gertler, 1999, Gali et al., 2001 and Sbordone, 2002). These authors estimated the NKPC with GMM, and found a considerable degree of price rigidity in the aggregate data. The empirical price reset hazard rate is around 20% per quarter for the U.S. and 10% for Europe. Due to the discrepancies between the macro and micro evidence, empirical models allowing for more flexible price durations or hazard functions have become popular in the recent literature. Jadresic [1999] presented a staggered pricing model featuring a flexible distribution over price durations and used VAR approach to demonstrate that the dynamic behavior of inflation and other macroeconomic variables provides information about the disaggregated price dynamics underlying the data. More recently, Sheedy [2007] constructed a generalized Calvo model and parameterized the hazard function in such a way that the resulting NKPC implied intrinsic inflation persistence when the hazard function was upward sloping. Based on this hazard function specification, he estimated the NKPC using GMM and found evidence of an upward-sloping hazard function. Coenen et al. [2007] developed a staggered nominal contracts model with both random and fixed durations, and estimated the generalized NKPC with an indirect inference method. Their results showed that price rigidity is characterized by a very high degree of real rigidity, as opposed to modest nominal rigidity with an average duration of about 2-3 quarters. Carvalho and Dam [2008] estimated a semi-structural multi-price-duration model with the Bayesian approach, and found that allowing for prices to last longer than 4 quarters is crucial to avoid underestimating the relative importance of nominal rigidity. Chapter 3 contributes to this literature by providing results of a Bayesian estimation based on the full-fledged DSGE model.

I turn now to the theoretical modeling of sticky prices. A large part of sticky price theory can be usefully classified on the basis of the price reset hazard function. In the literature, there are two kinds of hazard functions. One is the time-dependent hazard

[5] For more details see Table 2 in Alvarez [2007].

function, which is defined as the adjustment probability conditional on time since the last price adjustment[6]. The other hazard function is state-dependent, defining the hazard function as the probability of price adjustment depending on the deviation from the optimal target price[7]. While the state-dependent hazard function is arguably more theoretically attractive, the time-dependent hazard function is widely used in the literature due to its tractability. Despite these differences, one can argue that there is no sharp dichotomy between these two kinds of hazard functions, as far as only aggregate shocks are concerned. Dotsey et al. [1999] showed that a more general time-dependent specification is formally a first-order approximation to a richer state-dependent pricing model. Woodford [2008] constructs a more general model of state-dependent pricing motivated by the 'rational inattention' assumption[8], which nests both the standard state-dependent pricing model and the Calvo model as limiting cases. He finds that, given small shocks, the time-dependent model is a reasonably accurate approximation of the exact equilibrium dynamics.

In this thesis, I focus on the time-dependent hazard function and summarize theoretical models in this term. In the limiting cases, Calvo [1983] assumed probabilities of nominal price adjustment to be constant and independent of time-since-last-adjustment, while the hazard function in the staggered-contract model of Taylor [1980] are either zero within the spell of the contract or one at the end of the contract. Pioneer work by Wolman [1999] raised the issue that inflation dynamics should be sensitive to the hazard function underlying different pricing rules. Kiley [2002] compared the Calvo and Taylor staggered-pricing models and showed the dynamics of output following monetary shocks are both quantitatively and qualitatively different across the two pricing specifications unless one assumes a substantial level of real rigidity in the economy.

Recently, more general sticky price models have been developed, based on the view that the time profile of hazard function could be substantially different to those extreme cases. Mash [2004] derived the generalized NKPC under an increasing hazard function. He showed some general results that this version of the Phillips curve can replicate a large part of persistence in inflation and output gap dynamics. Similarly, Dixon and Kara [2005] generalized the Taylor-wage-contract model to account explicitly for the presence of varying contract lengths, and Carvalho [2006] constructed a sticky price model that allows for heterogeneous Calvo-sticky-price sectors. Both works found that

[6] See: e.g.Fischer [1977], Taylor [1980], Calvo [1983], Wolman [1999]

[7] See: e.g. Caplin and Spulber [1987], Dotsey et al. [1999], Caballero and Engel [2007] Golosov and Lucas [2007].

[8] See: Sims [1998] and Sims [2003]

the presence of a small portion of highly rigid sector leads to larger and more persistent real effects of monetary shocks than the benchmark New Keynesian model with a uniform nominal rigidity. Sheedy [2007] studied the general pricing hazard model on the topic of inflation persistence. He derived the generalized NKPC under a recursive formulation of hazard functions and showed that, under the recursive parameterization, the dependence of current and lagged inflation is determined by the slope of the hazard function. This result, however, seems contradict to Whelan [2007], who derived a general result showing that the NKPC under the general formulation of the hazard function has always a negative coefficient on lagged inflation. This difference results from their different formulation of the hazard function. In the Sheedy's model, the recursive formula of hazard functions transforms all past reset prices into lagged aggregate prices, so that it allows him to summarize all information of the past pricing behavior in the lagged inflations in the NKPC, while, in the Whelan's setup, he can not use past aggregate price to express past reset prices because of the general hazard function, as a result, terms of lagged expectations show up in the NKPC additional to the lagged inflations that enter directly into the NKPC. In Chapter 4, I argue that the Sheedy's result is analytically correct, but it is really a mixture of two counteracting effects from two channels, in addition, his result depends on the counterfactual assumption that hazard functions are with an infinite time horizon, i.e. $h(i)$ for $i = 1, 2, ...$ In the finite case, meaning that there is a finite maximum price duration J, his derivation does not work. In Chapter 4, I further show that price reset hazard functions play a crucial role in generating inflation persistence.

1.2.2 Labor Market Rigidity

The standard RBC model has been extended in various directions to enhance its internal propagation mechanism. One string of the literature emphasizes the role played by imperfect labor adjustment in propagating business cycles. In the early development of the literature, frictions in the labor market have been modeled by the labor adjustment costs of various functional forms. The quadratic adjustment cost model was popular, because they gave rise to an analytical solution (see e.g. Sargent, 1978). But this cost structure is strongly rejected by most empirical studies (Hamermesh and Pfann, 1996). Early responses to this challenge have been devoted to building more complex cost structures into the model. For instance, the piecewise linear cost function was studied by Bentolila and Bertola [1989], Kemp and Wan [1974], Nickell [1978], Nickell [1986] and Leban and Lesourne [1980], and Lump-Sum costs are implemented by Hamermesh [1993] and Abowd and Kramarz [2003]. Even though those cost structures are more empirically attractive, the fact that there is no analytical solution for these models deters further empirical study on the structural parameters of the model.

The further development provides more micro-founded theory to motivate frictions in the labor market. For example, the search and matching RBC model (Merz, 1995 and Andolfatto, 1996) generates persistence in labor dynamics by assuming matching frictions in the hiring process. The factor hoarding model (Burnside and Eichenbaum, 1996) assumes that extensive margins are predetermined, while the intensive margins can only be adjusted in a costly way; The habit formation model (Wen, 1998) emphasizes the role of the household's willingness to smooth the path of leisure; And the learning-by-doing model (Chang et al., 2002) is motivated by the assumption that current labor input affects future output through accumulation of worker's skills.

Most recently, an increasing number of empirical evidence has been accumulated, showing that labor demand by firms exhibits a lumpy and asynchronized manner. Employment adjustments are mostly discrete and followed by long periods of inactivity. Hamermesh [1989a], Caballero and Engel [1993b] and Caballero et al. [1997b] found strong evidence supporting the lumpy and asynchronous adjustments in the firm-level employment. More recently, Letterie et al. [2004a] investigated the complementarity between labor and capital demand using plant-level data for the Dutch manufacturing sector and observed lumpy adjustment for both factors and a strong degree of coordination between the two. Varejao and Portugal [2007] documented that large adjustments of more than 10% of the plant's labour force are accounted for about 66% of the total job turnover, and, on average, around 75% of all observed Portuguese employers do not change employment over an entire quarter.

In theoretical work, (S,s) models are widely used for studying lumpy factor adjustments[9]. Early partial equilibrium models of lumpy factor adjustment[10] found that increasing hazard models outperform constant-hazard-partial-adjustment models in describing aggregate dynamics. In particular, Caballero and Engel [1993a] used the concept of the adjustment hazard function to classify the existing literature, and their empirical study using U.S. manufacturing data suggested increasing hazard model has larger explanation power for aggregate employment changes. Caballero et al. [1997a] found that aggregate shocks account for about 90% of dynamics in the average employment growth rate and the micro-nonlinearity amplifies the effect of large aggregate shocks. Caballero and Engel [1999] constructed a more general (S,s) framework, in which firm's optimal adjustment policies become probabilistic and the adjustment

[9] Caplin and Spulber [1987] was the early work applying the (S,s) approach to macro models.

[10] See: e.g.: Caballero and Engel [1993a], Caballero et al. [1997a], Caballero and Engel [1999].

hazard rates grow smoothly with the discrepancy from the optimal target.

The later (S,s) models focus on the implication of lumpy adjustment in general equilibrium. Veracierto [2002] integrated the generalized (S,s) model into a neoclassical growth framework where heterogeneous establishments are subject to partially irreversible investment and found that in the general equilibrium framework large effect of lumpiness disappears. This result is confirmed by Thomas [2002], Khan and Thomas [2003], who showed that in general equilibrium household preference for smoothing consumption predominates effects caused by lumpiness in the micro-level. Furthermore, Cooper and Willis [2003] argued that the findings of nonlinearities in time series data used by Caballero et al. may reflect mismeasurement of the unobservable gaps between the target and current levels of employment rather than the aggregation of the plant-level nonlinearities. Responding to this challenge, Bachmann et al. [2006] argued that the reason why general equilibrium models by Thomas et al. failed to capture the effects of micro friction is that they had a decomposition of smoothing effects between partial equilibrium(PE) and general equilibrium(GE) forces similar to that of frictionless RBC model. As a result, their microeconomic lumpiness has almost no effect on the aggregate economy even in partial equilibrium. They suggested a different calibration strategy to reallocate smoothing effect from GE to PE by increasing the parameter of inverse of intertemporal elasticity of substitution to 10. They showed that, with this value, about 60% of smoothing can be explained by micro-level friction. King and Thomas [2006] constructed a generalized framework of discrete employment adjustment due to idiosyncratic shocks on the fix cost and find its result is 'observationally' equivalent at aggregate level to the standard partial adjustment model.

In Chapter 5, I tackle this issue with a novel modeling strategy. I simplify the (S,s) model into a more tractable framework and show that micro lumpy labor adjustment could play an important role at the macro level.

1.3 Overview

This thesis consists of four chapters which implement the new modeling of staggered adjustment in two separate economic applications. Chapters 2 to 4 address issues concerning sticky price setting and its implications for aggregate dynamics and monetary policy; Chapter 5 studies the effects of lumpy labor adjustment on business cycle fluctuations.

Chapter 2 is motivated by evidence of non-constant pricing hazards posted by the recent micro-econometric studies on pricing data. Empirical work using these data sets reach the consensus that the frequency of price adjustments differs substantially across sectors in the economy, and the price reset hazard function is not constant. These evidence strongly disagree with the implications of the popular Calvo approach. In light of these deficiencies, it is important to understand to what extent the conclusions of the sticky price models based on the Calvo assumption are robust to the implied constant hazard function.

I study this issue through the lens of a New Keynesian Phillips curve based on a general form of the hazard function. The generalized NKPC incorporates components, such as lagged inflation and lagged expectations, which is missing in the standard NKPC. I explain also the economic reason why these two new dynamic components should play a role in generating inflation dynamics. Additional to these analytical results, in the numerical experiments, I study the dynamic behavior of the general equilibrium model under increasing hazard functions. I show that the calibrated model accounts for both persistence of inflation and output gap, even without real rigidity. When introducing some degree of real rigidity and non-zero trend inflation, the generalized NKPC gives rise to hump-shaped impulse response of inflation to a nominal money growth shock. Last but not least, when the real effects of monetary policy shocks are measured by the accumulative impulse responses of the real output gap, models with an increasing hazard function generate real effects of monetary policy which are 2-3 times larger than those in the corresponding Calvo model.

To conclude the second chapter, I show that the non-constant hazard function could bring about significant changes in the dynamic behavior of the sticky price model. Even though similar results have been shown in the literature with some simple examples (Wolman, 1999 and Kiley, 2002), in this paper, I draw the conclusion under a more general setup, giving this result a more general relevance to macroeconomic theory.

Even though theoretical study in the chapter 2 sheds some light on the economic consequences of the non-constant hazard function, it leaves an interesting question open for the further investigation. In particular, since the numerical results of Chapter 2 are sensitive to the shape of hazard functions, it arises the question about the shape of the empirical hazard function. This is the theme of Chapter 3.

Chapter 3 uses the Bayesian approach to estimate a full-fledged DSGE model with the generalized NKPC derived in the chapter 2. The objective of this chapter is to estimate the price reset hazard function directly from the aggregate data. Identifying the hazard function from aggregate data is a useful exercise, because this method allows me to identify price reset hazard rates which are caused by the reactions to the aggregate shock. Micro hazard rates are typically higher than aggregate hazard rates, because individual prices react to both idiosyncratic and aggregate shocks. It is very difficult to disentangle them in micro data set. By contrast, my model identifies the hazard function from the aggregate data, in which effects of idiosyncratic shocks are naturally removed. As a result, this study delivers useful insights for macroeconomists to guide macro modeling.

I estimate the pricing hazard function using the U.S. quarterly data of inflation, the growth rate of real output and effective federal funds rate from 1955 to 2008. The identification of the aggregate hazard function is possible due to the fact that inflation rate can be decomposed into current and past reset prices and its composition is determined by the aggregate hazard function. The derivation of the generalized NKPC links those composition effects to the hazard function, so that only aggregate data is needed to extract information about the price reset hazard function. The empirical hazard function has a U-shape with a spike at the fourth quarter. one quarter and 4-quarter are the most important frequencies of the aggregate price adjustment. About 34.2% of prices hold for less than one quarter, while, 12.4% of prices have the mean duration of four quarters. The general shape of the empirical hazard function remarkably resembles those found in micro-econometric studies.

Equipped with the empirical pricing hazard function, the focus of Chapter 4 is to further explore the implication of the pricing hazard function for a more policy relevant question, namely, what is the nature of inflation persistence? It is an important question, because its answer has critical implications for effects of monetary policy. While the standard Calvo NKPC has the implication of no costly disinflationary monetary policy, models incorporating lagged inflation into the NKPC[11] implies that inflation persistence is mainly 'intrinsic' and disinflationary monetary policy have long painful consequences for the real economy. However, it is generally agreed that neither of these theories give us a satisfactory answer to the nature of inflation persistence because they either fail in the empirical test or are based on a theory based on a week micro-foundations.

In Chapter 4, I first document some stylized facts distinguishing inflation gap persistence from inflation level persistence. I find evidence that the inflation gap constitutes a large part of inflation persistence for the U.S. CPI data. Then, I investigate whether the stylized fact can be explained by the theoretical NKPC, and further identify the mechanism which is important for generating inflation gap persistence.

I analyze the dynamic structure of the generalized NKPC and shed light on the important role played by inertia of expectations in generating inflation dynamics. Ac-

[11] See: e.g. Gali and Gertler [1999] and Christiano et al. [2005].

cording to this model, instead of being intrinsic, inflation persistence is inherited from the additional moving-average terms of real driving forces through the lagged expectations. More importantly, since the coefficient on lagged inflation comes partly from the expectational channel, its magnitude depends on the whole model including the specification of monetary policy. If this theory is the origin of the NKPC, then the hybrid NKPC is be subject to the Lucas critique (Lucas, 1976), and thereby can be used in the monetary policy analysis only with great caution.

Whelan [2007], however, rejected this model empirically based on the evidence of the positive backward-dependence of inflation typically found in the empirical reduced-form Phillips curve. He argues that the general-pricing-hazard model fails to replicate this statistical regularity even in the general equilibrium setup. In the numerical analysis of Chapter 4, I first replicate his finding and check their robustness to alternative setups of the model. Especially, I test the result using the empirical hazard function I estimate in the chapter 3. The result shows that it is the 4-period-Taylor-contract hazard function used in the Whelan's paper that gave rise to the negative coefficient on lagged inflation. Under the empirically based pricing hazard function, the simulated data accounts quite well for the inflation gap persistence I find in the U.S. CPI data after the Volcker disinflation period.

Finally, in Chapter 5, I applied the same modeling strategy to a staggered labor adjustment RBC model. This study is motivated by the recent empirical evidence from the firm level data, showing that the firm's employment adjustment exhibits lumpy, asynchronous pattern. This evidence brings difficulty for many widely using models in the literature that implies either a smoothing or synchronous adjustment at the firm level.

The main theme running in the literature is whether explicit modeling of the micro lumpiness changes the model's implication for the aggregate dynamics. Contrast to the (S,s) models commonly used in the literature for this issue, I embed a general stochastic labor adjustment process in a prototypical RBC model. To formalize this idea, in the benchmark model I introduce the firm's stochastic labor adjustment in the spirit of Calvo [1983], which implies that the underlying labor adjustment process is characterized by a constant hazard function. Then I extend the baseline model to a more general case, in which I implement a Weibull-distributed labor adjustment process to capture features of increasing hazard rates corroborated by micro evidence documented by Varejao and Portugal [2007]. This extension has a significant impact on both the persistence and the magnitude of business cycles. When calibrating the model with the empirically plausible hazard function, adjustment probabilities vary across labor vintages. The longer a firm remains inactive, the more likely it adjusts its labor in the current period. As a result, heterogeneous labor dynamics emerge naturally from the underlying labor adjustment process, and as shown in the numerical results, the model matches several important aspects of the U.S. business cycles. In particular, the model can jointly account for persistent aggregate labor, smoothing real wages and features observed in both micro and macro labor adjustment data: i.e. at the micro level, labor adjustment exhibits a lumpy pattern in response to the technology shock, while the aggregate employment reacts smoothly and sluggishly. In addition, sensitivity analysis shows that aggregate dynamics vary with the extent of increasing hazard function, e.g., the volatility of aggregate labor is increasing, but the persistence is decreasing in degree of the increasing hazard of the labor adjustment. My result suggests that micro lumpy labor adjustment could play an important role at the macro level.

2 Non-constant Price Reset Hazards and Inflation Dynamics

Abstract

This paper demonstrates that tractability gained from the Calvo pricing assumption is costly in terms of aggregate dynamics. Using a New Keynesian Phillips curve based on a general hazard function, I find that important dynamics in the NKPC are canceled out due to the restrictive Calvo assumption. The richer dynamic structure resulted from the non-constant hazards is quantitatively important for inflation dynamics and monetary policy. With plausible parameter values, the increasing hazard model generates hump-shaped impulse responses of inflation to the monetary shock, and the real effects of monetary shocks are 2-3 times higher than those in the Calvo model.

2.1 Introduction

The Calvo pricing assumption (Calvo, 1983) has become predominant in the world of applied monetary analysis under nominal rigidity. The main argument for using this approach, however, is mainly based on its tractability. In recent years, detailed micro-level data sets have become available for researchers. Empirical work using these data sets[1] generally reach the consensus that, instead of having economy-wide uniform price stickiness, the frequency of price adjustments differs substantially within the economy. In addition, the Calvo assumption also implies a constant hazard function of price setting, meaning that the probability of adjusting prices is independent of the length of the time since last adjustment. Unfortunately, constant hazard functions are also largely rejected by empirical evidence from the micro level data. Cecchetti [1986] used news-stand prices of magazines in the U.S. and Goette et al. [2005] apply Swiss restaurant prices. Both studies find strong support for increasing hazard functions. By contrast, recent studies using more comprehensive micro data find that hazard functions are first

[1]See: e.g. Bils and Klenow [2004], Alvarez et al. [2006], Midrigan [2007], Nakamura and Steinsson [2008] among others.

downward sloping and then mostly flat, interrupted periodically by spikes (See, e.g.: Campbell and Eden, 2005, Alvarez, 2007 and Nakamura and Steinsson, 2008).

Given the discrepancy between theory and empirical evidence, it is important to understand the consequences of a non-constant price reset hazard function for inflation dynamics and implications for the effect of monetary policy.

To tackle these questions, I study a New Keynesian Phillips curve (NKPC) featuring a general hazard function and real rigidity. The resulting NKPC incorporates components, such as lagged inflation, future and lagged expectations of inflation and real marginal costs. This version of the Phillips curve nests the Calvo case in the sense that, under a constant hazard function, effects of lagged inflation exactly cancel those of lagged expectations, so that, as in the Calvo NKPC, only current real marginal cost and expected future inflation remain in the expression. In the general case, however, expectations of future variables, lagged expectations and lagged inflation all should be presented in the dynamic structure of the Phillips curve.

The economic reason why those lagged dynamic components affect inflation dynamics is because past reset prices exert two opposing effects on current inflation through $\hat{p}_t$ and $\hat{p}_{t-1}$ respectively. On the one hand, lagged expectational terms represent influences of past reset prices on the current aggregate price $\hat{p}_t$ and hence on current inflation. On the other hand, past inflations reflect effects of past reset prices on the lagged aggregate price $\hat{p}_{t-1}$. The higher the past inflations prevail, higher the lagged aggregate price is, and thereby it deters current inflation to be high. The magnitudes of these two opposing effects depend on the shape of the price reset hazard function. In the general case, they are different to each other, so that both lagged expectations and lagged inflations appear in the generalized NKPC. Conversely, in the Calvo case, the constant hazard function leads past reset prices to exert the same amount of impact on both $\hat{p}_t$ and $\hat{p}_{t-1}$, and thereby causes them to be canceled out.

To illustrate the importance of the lagged expectations in general equilibrium, I simulate a full-fledged DSGE model by combining the generalized NKPC with a standard IS curve and the nominal money growth rule. The simulation results show that, even without real rigidity, the increasing hazard function helps to increase both persistence of inflation and output gap. When introducing some degree of real rigidity, the generalized NKPC gives rise to substantially different inflation dynamics, namely, the impulse response of inflation to a nominal money growth shock becomes hump-shaped. Moreover, non-zero trend inflation amplifies this effect even further. The economic intuition behind these results is that, on the one hand, increasing hazard function postpones the timing of the price adjustment. On the other hand, strategic complementary makes earlier adjusting firms choose a small size for the adjustment, while the later adjusting firms make a larger price adjustment. In another words, the increasing-hazard pricing together with some degree of real rigidity not only affect the timing of the price adjustment, but also the average magnitude of firms' adjustments, leading to a hump-shaped response. Trend inflation amplifies this effect even further, because high trend inflation causes relative prices to disperse quickly. Last but not least, when the real effects of monetary policy shocks are measured by the accumulative impulse responses of the real output gap, models with an increasing hazard function generate real effects of monetary policy which are 2-3 times larger than those in the corresponding Calvo model.

In the literature, the general-hazard-pricing model has been studied in different contexts. Wolman [1999] raised the issue that inflation dynamics should be sensitive to the hazard function underlying different pricing rules and thereby implications of the constant-hazard Calvo model is not robust to the shape of the hazard function. He showed this result in a partial equilibrium analysis. Kiley [2002] compared the Calvo and Taylor staggered-pricing models and showed the dynamics of output following monetary shocks are both quantitatively and qualitatively quite different across the two pricing specifications unless one assumes a substantial level of real rigidity in the economy. Mash [2003] constructed a general pricing model that nests both the Calvo and Taylor cases. He found that implications for optimal monetary policy based on those limiting cases are not robust to the change in the hazard function. Sheedy [2007] focused on the relationship between the shape of hazard functions and inflation persistence. He parameterized the hazard function in such a way that the resulting NKPC has a positive coefficient on lagged inflation given that the hazard function is upward sloping. The most closely related work is Whelan [2007], who derived the generalized NKPC under a general hazard function, but rejected this model based on the observation from the reduced-form Phillips curve regression that inflation is positively dependent on its lags. However, this argument is vulnerable in light of the evidence presented by Dotsey [2002], who shows that the positive reduced-form coefficients themselves could be spurious due to omitted variables in a misspecified regression model. This argument is supported by Cogley and Sbordone [2008], who find that when correctly accounting for the time-varying trend of inflation, the purely forward-looking model explains the persistence of the inflation deviation from its trend quite well.

The remainder of the paper is organized as follows: in section 2, I present the model with the generalized time-dependent pricing and derive the New Keynesian Phillips curve; section 3 shows analytical results regarding new insights gained from relaxing the constant hazard function underlying the Calvo assumption; in section 4, I simulate the complete DSGE model with some commonly used parameter values in the literature and then present the simulation results; section 5 contains some concluding remarks.

2.2 The Model

In this section, I present a DSGE model of sticky prices based on both nominal and real rigidities. The scheme of nominal rigidity in the model allows for a general shape of the hazard function. A hazard function of price setting is defined as the probabilities of price adjustment conditional on the spell of the time elapsed since the price was last set. Real rigidity is introduced similarly as in Sbordone [2002], who incorporates upward-sloping marginal cost as a source of strategic complementarity.

2.2.1 Representative Household

A representative, infinitely-lived household derives utility from the composite consumption good C_t, its labor supply and the real money holding M_t^d/P_t, and it maximizes a discounted sum of utilities of the form:

$$\max_{\{C_t, M_t^d, L_t, B_{t+1}\}} \quad E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi_H \frac{L_t^{1+\phi}}{1+\phi} + \chi_M \log\left(\frac{M_t^d}{P_t}\right) \right) \right]$$

Here C_t denotes an index of the household's consumption, which is produced by using individual goods $C_t(i)$ following a constant-elasticity-of-substitution technology (Dixit and Stiglitz, 1977).

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \tag{2.1}$$

where $\eta > 1$, and it follows that the corresponding cost-minimizing demand for $C_t(i)$ and the welfare based price index P_t are given by

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\eta} C_t \tag{2.2}$$

$$P_t = \left[\int_0^1 P_t(i)^{1-\eta} di \right]^{\frac{1}{1-\eta}} \tag{2.3}$$

For simplicity, I assume that household supplies homogeneous labor units (L_t) in an economy-wide competitive labor market.

The flow budget constraint of the household at the beginning of period t is

$$P_tC_t + M_t^d + \frac{B_t}{R_t} \leq M_{t-1}^d + W_tL_t + B_{t-1} + \int_0^1 \pi_t(i)di. \tag{2.4}$$

Where B_t is a one-period nominal bond and R_t denotes the gross nominal return on the bond. $\pi_t(i)$ represents the nominal profits of a firm that sells good i. I assume that each household owns an equal share of all firms. Finally this sequence of period budget constraints is supplemented with a transversality condition of the form $\lim_{T\to\infty} E_t\left[\frac{B_T}{\Pi_{s=1}^T R_s}\right] \geq 0$.

The solution to the household's optimization problem can be expressed in three first order necessary conditions. First, optimal labor supply is related to the real wage:

$$\chi_H L_t^{\phi} C_t^{\sigma} = \frac{W_t}{P_t}, \tag{2.5}$$

Second, the Euler equation gives the relationship between the optimal consumption path and asset prices:

$$1 = \beta E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \frac{R_tP_t}{P_{t+1}}\right], \tag{2.6}$$

Finally, the demand of real money balance is determined by weighting between the benefits and costs of holding money.

$$\chi_M \frac{M_t}{P_t} = \frac{C_t^{\sigma}}{1 - R_t^{-1}}, \tag{2.7}$$

2.2.2 Firms in the Economy

In the economy, there is a continuum of monopolistic competitive firms, who use labor as the single input to produce good i.

$$Y_t(i) = Z_tL_t(i)^{1-a} \tag{2.8}$$

where Z_t denotes an aggregate productivity shock. Log deviations of the shock $\hat{z}_t$ follow an exogenous AR(1) process $\hat{z}_t = \rho_z\hat{z}_{t-1} + \varepsilon_{z,t}$, where $\varepsilon_{z,t}$ is white noises and $\rho_z \in [0, 1)$. $L_t(i)$ is the demand of labor by firm i. Following equation (2.2), demand for intermediate goods is given by:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\eta} Y_t \tag{2.9}$$

Pricing Decisions under Real Rigidity

In Appendix (1), I derive the economy-wide optimal relative price, which is the ratio between the average optimal price chosen by the adjusting firms and aggregate price index. Note that even through the individual optimal prices are not the same due to the fact that marginal costs generally depend on the amount produced, we can still derive the aggregate optimal relative-price ratio at period t from the average marginal cost in the economy.

$$\frac{P_t^*}{P_t} = \left(\frac{\eta}{\eta-1}\frac{1}{1-a}\right)^{\frac{1-a}{1-a+\eta a}} Y_t^{\frac{\phi+\sigma(1-a)+a}{1-a+\eta a}} Z_t^{-\frac{1+\phi}{1-a+\eta a}} \tag{2.10}$$

To show how real rigidity affects price setting in this model, I log-linearize the relative price equation (2.10). Define $\hat{x}_t = logX_t - log\bar{X}$ as the log deviation from the steady state, up to a log linearization approximation, one can show that the log deviation of the relative price is equal to the log deviation of the economy-wide marginal cost, which in turn is a linear function of log deviations of output gap and the technology shock.

$$\begin{aligned} \widehat{rp}_t = & \quad \widehat{mc}_t = \gamma\left(\kappa_1 \hat{y}_t - \kappa_2 \hat{z}_t\right) \\ where: & \\ \gamma = & \quad \frac{1}{1-a+a\eta} \\ \kappa_1 = & \quad a+\phi+\sigma(1-a) \\ \kappa_2 = & \quad 1+\phi \end{aligned}$$

Parameters γ and κ_1 have the economic interpretation as the measure of real rigidity. γ is the elasticity of relative prices to the change in real marginal cost, while κ_1 measures the sensitivity of real marginal cost to the change in the output gap. Following Woodford [2003], price-setting decisions are called strategic complementarity when $\gamma\kappa_1 < 1$. When we assume that the monetary authority controls the growth rate of the nominal aggregate demand $\hat{d}_t$, then at equilibrium we have $\hat{y}_t = \hat{d}_t - \hat{p}_t$. In this case, price adjustments are "sticky" even under a flexible price setting, because relative price reacts less than one-to-one to a monetary shock. On the other hand, price setting decisions can be dubbed strategic substitutes when $\gamma\kappa_1 > 1$, so that relative price reacts strongly to monetary policy shocks.

Now we can discuss how changes in the labor share a affect the magnitude of real rigidity of price setting in the model. When setting a equal to zero, creating a linear production technology, then $\gamma = 1$ and $\kappa_1 = \delta + \phi$. Under the standard calibration values in the RBC literature ($\delta = 1$ and $\phi = 0.5$), the real rigidity parameter $\gamma\kappa_1$ is equal to 1.5 and price decisions are strategic substitutes. When the value of a rises, the real rigidity parameter becomes smaller, and price decisions turn into strategic complementarity.

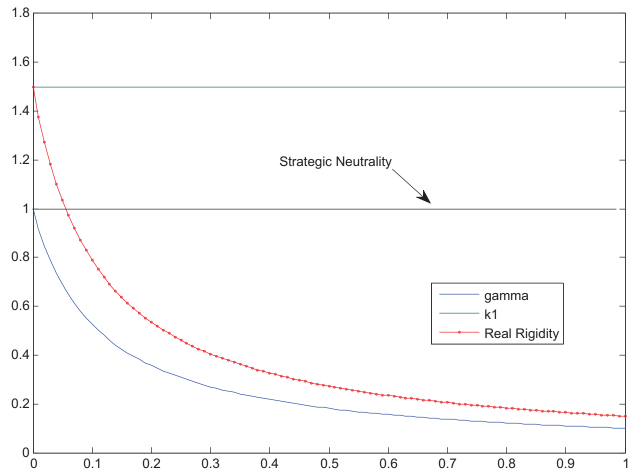

Figure 2.1: Real Rigidity, when $\sigma = 1, \phi = 0.5$ and $\eta = 10$

In Figure (2.1), I plot values of γ and κ_1 against values of a, while setting $\sigma = 1, \phi = 0.5$ and $\eta = 10$. In this special case, the sensitivity of real marginal cost to the change in the output gap κ_1 is not affected by the labor share, while γ decreases fairly quickly as a becomes larger. This means that, given the parameter values, real rigidity is mainly driven by the sensitivity of the relative price to changes in real marginal cost, and the degree of real rigidity is decreasing in a. Only with a modest value of the labor share (around 0.1), real rigidity drops below the strategic neutrality threshold.

Pricing Decisions under Nominal Rigidity

In this section, I introduce a general form of nominal rigidity, which is characterized by an arbitrary hazard function. Many well known price setting models in the literature can be shown to have the incorporation of a hazard function of one form or another. The hazard function in this price setting is defined as the probability of price adjustment conditional on the spell of time elapsed since the price was last set. I assume that monopolistic competitive firms cannot adjust their price whenever they want. Instead, opportunities for re-optimizing prices depend on the hazard function h_j, where j denotes the time-since-last-adjustment and $j \in \{0, J\}$. J is the maximum number of periods in which a firm's price can be fixed. To keep the model general, I do not parameterize the hazard function, so that the relative magnitudes of hazard rates are totally free. As a result, this model is able to nest a wide range of staggered pricing New Keynesian models.

Dynamics of the vintage distribution In the economy, firms' prices are heterogeneous with respect to the time since their last price adjustment. I call them price vintages, while the vintage label j indicates the age of each price group. Table (2.1) summarizes key notations concerning the dynamics of vintages.

Vintage j	Hazard Rate h_j	Non-adj. Rate α_j	Survival Rate S_j	Distribution $\theta(j)$
0	0	1	1	$\theta(0)$
1	h_1	$\alpha_1 = 1 - h_1$	$S_1 = \alpha_1$	$\theta(1)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
j	h_j	$\alpha_j = 1 - h_j$	$S_j = \prod_{i=0}^{j} \alpha_i$	$\theta(j)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
J	$h_J = 1$	$\alpha_J = 0$	$S_J = 0$	$\theta(J)$

Table 2.1: Notations of the dynamics of price-vintage-distribution.

Using the notation defined in Table (2.1), and also denoting the distribution of price durations at the beginning of each period by $\Theta_t = \{\theta_t(0), \theta_t(1) \cdots \theta_t(J)\}$, we can derive the ex post distribution of firms after price adjustments ($\tilde{\Theta}_t$)

$$\tilde{\theta}_t(j) = \begin{cases} \sum_{i=1}^{J} h_j \theta_t(i) \text{ , when } j = 0 \\ \alpha_j \theta_t(j) \text{ , when } j = 1 \cdots J \end{cases} \tag{2.11}$$

Intuitively, those firms that reoptimize their prices in period t are labeled as 'vintage 0', and the proportion of those firms is given by hazard rates from all vintages multiplied by their corresponding densities. The firm left in each vintage are the firms that do not adjust their prices. When period t is over, this ex post distribution $\tilde{\Theta}_t$ becomes the ex ante distribution for the new period Θ_{t+1}. All price vintages move to the next one, because all prices age by one period.

As long as the hazard rates lie between zero and one, dynamics of the price-duration distribution can be viewed as a Markov process with an invariant distribution, Θ, and is obtained by solving $\theta_t(j) = \theta_{t+1}(j)$. It yields the stationary price-duration distribution $\theta(j)$ as follows:

$$\theta(j) = \frac{S_j}{\sum\limits_{j=0}^{J-1} S_j}, \text{ for } j = 0, 1 \cdots J-1. \tag{2.12}$$

Here, I give a simple example. When $J = 3$, then the transition matrix of the price-duration-group Markov chain is illustrated as follows:

j	0	1	2	3
0	0	1	0	0
1	h_1	0	α_1	0
2	h_2	0	0	α_2
3	1	0	0	0

According to equation (2.12), this Markov chain eventually converges to the stationary price-duration distribution $\Theta = \left\{ \frac{1}{1+\alpha_1+\alpha_1\alpha_2}, \frac{\alpha_1}{1+\alpha_1+\alpha_1\alpha_2}, \frac{\alpha_1\alpha_2}{1+\alpha_1+\alpha_1\alpha_2} \right\}$.

Let's assume the economy converges to this invariant distribution quickly, so that regardless of the initial price-duration distribution, I only consider the economy with the invariant distribution of price durations.

The Optimal Pricing under Nominal Rigidity In a given period when a firm is allowed to reoptimize its price, the optimal price chosen should reflect the possibility that it will not be re-adjusted in the near future. Consequently, adjusting firms choose optimal prices that maximize the discounted sum of real profits over the time horizon during which the new price is expected to be fixed. The probability that the new price is fixed is given by the survival function, S_j, defined in Table (2.1).

Here I setup the maximization problem of an average adjuster as follows:

$$\max_{P(i)_t^*} E_t \sum_{j=0}^{J-1} S_j Q_{t,t+j} \left[Y_{t+j|t}^d \frac{P(i)_t^*}{P_{t+j}} - \frac{TC(i)_{t+j}}{P_{t+j}} \right]$$

Where E_t denotes the conditional expectation based on the information set in period t, and $Q_{t,t+j}$ is the stochastic discount factor appropriate for discounting real profits from t to $t+j$. Note that here $P(i)_t^*$ is defined as the average optimal price chosen by the average adjusting firm. Therefore $TC(i)_t$ denotes the average total costs of producing output $Y(i)^d_t$. The representative adjusting Firm maximizes profits subject to demand for intermediate goods in period $t+j$ given that the firm resets the price in period t, $(Y(i)^d_{t+j|t})$.

$$Y(i)^d_{t+j|t} = \left(\frac{P(i)^*_t}{P_{t+j}}\right)^{-\eta} Y_{t+j},$$

It yields the following first order necessary condition for the optimal price:

$$P(i)^*_t = \frac{\eta}{\eta-1} \frac{\sum_{j=0}^{J-1} S_j E_t[Q_{t,t+j} Y_{t+j} P^{\eta-1}_{t+j} MC(i)_{t+j}]}{\sum_{j=0}^{J-1} S_j E_t[Q_{t,t+j} Y_{t+j} P^{\eta-1}_{t+j}]} \tag{2.13}$$

MC_t denotes the average nominal marginal costs of adjusting firms. The optimal price is equal to the markup multiplied by a weighted sum of future marginal costs, where weights depend on the survival rates. In the Calvo case, where $S_j = \alpha^j$, this equation reduces to the Calvo optimal pricing condition.

Finally, given the stationary distribution $\theta(j)$, aggregate price can be written as a distributed sum of all vintage prices. I define the aggregate optimal price which was set j periods ago as P^*_{t-j}. Following the aggregate price index equation (2.3), the aggregate price is then obtained by:

$$P_t = \left(\sum_{j=0}^{J-1} \theta(j) P^{*1-\eta}_{t-j}\right)^{\frac{1}{1-\eta}} \tag{2.14}$$

Non-zero-inflation Steady State

If I assume that the gross growth rate of nominal money stock is g, then the steady state is characterized by constant real variables and a growing path of all nominal variables at the rate g. Because the aggregate price level increases with trend inflation in the steady state, firms need to adjust their prices so that the relative prices are close to the optimal ratio specified below. If we define $\overline{X}$ as the steady state value of variable X, then the optimality condition (2.13) can be rewritten as:

$$\bar{p}_t^* = \frac{\eta}{\eta-1}\frac{\sum\limits_{j=0}^{J}\beta^j S_j \bar{Y}\bar{P}_{t+j}^{\eta}}{\sum\limits_{j=0}^{J}\beta^j S_j \bar{Y}\bar{P}_{t+j}^{\eta-1}}mc = \frac{\eta}{\eta-1}\frac{\sum\limits_{j=0}^{J}\beta^j S_j \bar{Y}\bar{P}_{t}^{\eta}g^{\eta j}}{\sum\limits_{j=0}^{J}\beta^j S_j \bar{Y}\bar{P}_{t}^{\eta-1}g^{(\eta-1)j}}mc$$

$$\frac{\bar{p}_t^*}{\bar{P}_t} = \frac{\eta}{\eta-1}mc\left[\frac{\sum\limits_{j=0}^{J}\beta^j S_j g^{\eta j}}{\sum\limits_{j=0}^{J}\beta^j S_j g^{(\eta-1)j}}\right] \tag{2.15}$$

As seen in Equation (2.15), the optimal relative price ratio is equal to a markup multiplied by the real marginal cost along with an extra term, which reflects how fast trend inflation erodes the relative prices in the economy. When the gross inflation rate is equal to one, this term is also equal one. In this case, we have the standard static price setting equation. However, when trend inflation is greater than one, it follows that the extra term is also greater than one, meaning that the adjusting firms want to 'front-load' their price adjustments in order to hedge the risk that they may not adjust again in the near future. As a result, they adjust their prices more than those in the case of zero inflation. The higher relative price, in turn, leads to lower steady state output and hence, induces an additional welfare loss caused by the steady state inflation.

2.2.3 Derivation of the New Keynesian Phillips Curve

In this section, I derive the New Keynesian Phillips curve for this generalized model. To do that, I first log-linearize equation (2.13) around the steady state with the trend inflation ($\bar{\pi}$). This is motivated by King and Wolman [1996] and Ascari [2004], who show that trend inflation plays an important role in both the long-run and the short-run dynamics.

The log-linearized optimal price equations are obtained by

$$\hat{p}_t^* = E_t\left[\sum_{j=0}^{J-1}\frac{(\beta g^{\eta})^j S_j}{\Omega}\left(\widehat{mc}_{t+j}+\hat{p}_{t+j}\right)\right], \tag{2.16}$$

where :

$$\Omega = \sum_{j=0}^{J-1}(\beta g^{\eta})^j S_j \text{ and } \widehat{mc}_t = \frac{a+\phi+\sigma(1-a)}{1-a+a\eta}\hat{y}_t - \frac{1+\phi}{1-a+a\eta}\hat{z}_t$$

In a similar fashion, I derive the log deviation of the aggregate price by log linearizing equation (2.14).

$$\hat{p}_t = \sum_{k=0}^{J-1}\tau(k)\ \hat{p}_{t-k}^*, \quad where \quad \tau(k) = \frac{\theta(k)g^{(\eta-1)k}}{\sum_{k=0}^{J-1}\theta(k)g^{(\eta-1)k}} \tag{2.17}$$

New Keynesian Phillips Curve

To reveal implications of the general hazard function on the inflation dynamics, I derive the generalized NKPC from equations (2.16) and (2.17). To keep the equation as simple as possible, I first derive it without trend inflation, i.e. $g = 1$. After some tedious algebra, I obtain the New Keynesian Phillips curve as follows[2]:

$$\hat{\pi}_t = \sum_{k=0}^{J-1}\frac{\theta(k)}{1-\theta(0)}E_{t-k}\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i-k}\right)$$
$$- \sum_{k=2}^{J-1}\Phi(k)\hat{\pi}_{t-k+1}, \quad where\ \Phi(k) = \frac{\sum_{j=k}^{J-1}S_j}{\sum_{j=1}^{J-1}S_j}, \quad \Psi = \sum_{k=0}^{J-1}\beta^j S_j \tag{2.18}$$

The generalized NKPC differs from the standard NKPC in two aspects. First, the general-hazard NKPC has not only current and forward-looking terms but also lagged variables and lagged expectations. In addition, all coefficients in the new NKPC are nonlinear functions of price reset hazard rates $\left(\alpha_j = 1 - h_j\right)$ and the subjective discount factor β. Thereby, short-run dynamics of inflation gap are affected by both the shape and magnitude of the price reset hazard function. To see the dynamic structure more

[2]The detailed derivation of the NKPC can be found in the technical Appendix (6).

clearly, I write down a simple example of the NKPC with $J = 3$.

$$
\begin{aligned}
\hat{\pi}_t = \quad & \frac{1}{(\alpha_1+\alpha_1\alpha_2)\,\Psi}\widehat{mc}_t + \frac{\alpha_1}{(\alpha_1+\alpha_1\alpha_2)\,\Psi}\widehat{mc}_{t-1} + \frac{\alpha_1\alpha_2}{(\alpha_1+\alpha_1\alpha_2)\,\Psi}\widehat{mc}_{t-2} \\
& + \frac{1}{\alpha_1+\alpha_1\alpha_2}E_t\left(\frac{\beta\alpha_1}{\Psi}\widehat{mc}_{t+1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\widehat{mc}_{t+2} + \frac{\beta\alpha_1+\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t+1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t+2}\right) \\
& + \frac{\alpha_1}{\alpha_1+\alpha_1\alpha_2}E_{t-1}\left(\frac{\beta\alpha_1}{\Psi}\widehat{mc}_{t} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\widehat{mc}_{t+1} + \frac{\beta\alpha_1+\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t+1}\right) \\
& + \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_1\alpha_2}E_{t-2}\left(\frac{\beta\alpha_1}{\Psi}\widehat{mc}_{t-1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\widehat{mc}_{t} + \frac{\beta\alpha_1+\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t-1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t}\right) \\
& - \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_1\alpha_2}\hat{\pi}_{t-1}
\end{aligned}
$$

$$
\textit{where:} \qquad \Psi = 1+\beta\alpha_1+\beta^2\alpha_1\alpha_2
$$

The NKPC with Trend Inflation (g)

When I derive the NKPC by log-linearizing pricing equations around a steady state with non-zero trend inflation, it can be shown that the resulting Phillips curve has the exact same structure as the one without trend inflation. However, trend inflation affects the magnitude of all coefficients in the NKPC. Again, using the example where $J = 3$, we obtain

$$
\begin{aligned}
\hat{\pi}_t = \quad & \frac{1}{\Psi}mc_t + \frac{1}{\Psi}mc_{t-1} + \frac{1}{\Psi}mc_{t-2} \\
& + \gamma_1' E_t\left(\frac{\beta\alpha_1 g^{\eta}}{\Psi}mc_{t+1} + \frac{\beta^2\alpha_1\alpha_2 g^{2\eta}}{\Psi}mc_{t+2} + \frac{\Psi-1}{\Psi}\hat{\pi}_{t+1} + \frac{\beta^2\alpha_1\alpha_2 g^{2\eta}}{\Psi}\hat{\pi}_{t+2}\right) \\
& + \gamma_2' E_{t-1}\left(\frac{\beta\alpha_1 g^{\eta}}{\Psi}mc_{t} + \frac{\beta^2\alpha_1\alpha_2 g^{2\eta}}{\Psi}mc_{t+1} + \frac{\Psi-1}{\Psi}\hat{\pi}_{t} + \frac{\beta^2\alpha_1\alpha_2 g^{2\eta}}{\Psi}\hat{\pi}_{t+1}\right) \\
& + \gamma_3' E_{t-2}\left(\frac{\beta\alpha_1 g^{\eta}}{\Psi}mc_{t-1} + \frac{\beta^2\alpha_1\alpha_2 g^{2\eta}}{\Psi}mc_{t} + \frac{\Psi-1}{\Psi}\hat{\pi}_{t-1} + \frac{\beta^2\alpha_1\alpha_2 g^{2\eta}}{\Psi}\hat{\pi}_{t}\right) && (2.19) \\
& - \gamma_3'\hat{\pi}_{t-1} && (2.20) \\
& && (2.21)
\end{aligned}
$$

$$
\begin{aligned}
\gamma_1' = \quad & \frac{1}{\alpha_1 g^{\eta-1}+\alpha_2\alpha_1 g^{2\eta-2}}, \qquad \gamma_2' = \frac{\alpha_1 g^{\eta-1}}{\alpha_1 g^{\eta-1}+\alpha_2\alpha_1 g^{2\eta-2}} \\
\gamma_3' = \quad & \frac{\alpha_1\alpha_2 g^{2\eta-2}}{\alpha_1 g^{\eta-1}+\alpha_2\alpha_1 g^{2\eta-2}}, \qquad \Psi' = 1+\beta\alpha_1 g^{\eta}+\beta^2\alpha_1\alpha_2 g^{2\eta}
\end{aligned}
$$

In this case, trend inflation (g) enters every coefficient in the Phillips curve, and hence it has not only a significant impact on the steady state, but affects the inflation dynamics in a complex way as well. In general, γ_1 and γ_2 are decreasing in g, while γ_3 is increasing in g. So the changes in trend inflation alter the relative importance between the forward-looking and backward-looking terms in the Phillips curve.

2.3 Analytical Result

In this section, I explore the dynamic structure of the generalized NKPC (2.18) to show which new insights we can learn from relaxing the constant hazard function underlying the Calvo assumption.

2.3.1 Economic Intuition behind the Generalized NKPC

Proposition 1 : *When assuming the hazard function is constant over the infinite horizon, the generalized NKPC (2.18) reduces to the standard Calvo NKPC:*

$$\hat{\pi}_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} mc_t + \beta E_t \hat{\pi}_{t+1} \tag{2.22}$$

Proof : *see Appendix (3).*

By iterating equation (2.22) backwards, the following equations hold

$$\hat{\pi}_{t-1} = E_{t-1}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i-1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-1}\right)$$

$$\hat{\pi}_{t-2} = E_{t-2}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i-1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-1}\right)$$

$$\vdots$$

In light of these analytical results, we learn that the generalized NKPC nests the Calvo Phillips curve in the sense that, given the constant hazard function, the effects of lagged inflation terms exactly equal the effects of lagged expectations. Moreover, lagged inflation and lagged expectations are not extrinsic to the time-dependent nominal rigidity model. They are missing in the Calvo setup only because the constant hazard assumption causes them to be canceled out.

In the generalized NKPC (2.18), there are three dynamic components affecting inflation. First, because the optimal price decision in this model is based on the sum of all current and future real marginal costs over the spell of time during which the reset price is fixed, the current and forward-looking terms reflect the influence of the current reset price on the current aggregate price. This is the channel highlighted in the Calvo model. Second, due to nominal rigidity, some fraction of past reset prices continue to affect the current aggregate price. Lagged expectational terms represent those influences of past reset prices on the current aggregate price and hence on current inflation. The higher past reset price is, the higher the current aggregate price is and hence the current inflation. Last, past inflations reflect the influence of past price decisions on the lagged aggregate price $\hat{p}_{t-1}$. The higher the past inflations prevail, higher the lagged aggregate price would be, and thereby it deters current inflation to be high. Because magnitudes of these dynamic components depend on the price reset hazard function, the general-hazard NKPC could give rise to very different inflation dynamics compared to the standard Calvo NKPC. In the next section, I use a numerical example to illustrate this point.

2.4 Numerical Results

2.4.1 The General Equilibrium Model

In the numerical experiment, I study the behavior of inflation dynamics in a general equilibrium setup. For this purpose, I close the model by adding a nominal money stock growth rule. The log-linearized equilibrium equations are summarized here:

$$
\begin{aligned}
\hat{\pi}_t = &\quad \sum_{k=0}^{J-1} W_1(k) E_{t-k}\left(\sum_{j=0}^{J-1} W_2(j)\widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} W_3(i)\hat{\pi}_{t+i-k}\right) - \sum_{k=2}^{J-1} W_4(k)\hat{\pi}_{t-k+1} \\
\widehat{mc}_t = &\quad \frac{\phi+\sigma+a}{1+\eta\phi+\eta a}\hat{y}_t - \frac{1+\phi}{1+\eta\phi+\eta a}\hat{z}_t \\
\sigma E_t\left[\hat{y}_{t+1}\right] = &\quad \sigma\hat{y}_t + \left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right]\right) \\
\hat{m}_t = &\quad \sigma\hat{y}_t - \frac{\beta}{1-\beta}\hat{\imath}_t \\
\hat{m}_t = &\quad \hat{m}_{t-1} - \hat{\pi}_t + g_t \quad where \;\; g_t \sim N(0, 0.0025^2) \\
\hat{z}_t = &\quad \rho_z * \hat{z}_{t-1} + \epsilon_t \quad where \;\; \epsilon_t \sim N(0, 0.007^2)
\end{aligned}
$$

Where all variable are expressed in terms of log deviations from the non-stochastic steady state. The weights (W_1, W_2, W_3, W_4) in the NKPC are defined in the equation (2.18). $\hat{m}_t$ is the real money balance, and g_t denotes the growth rate of the nominal money stock, which consists of a constant g and a white-noise shock u_t, representing the regular and irregular parts of the standing monetary policy.

2.4.2 Calibration

In the calibration, instead of referring to any micro-econometric evidence on the hazard function, I parameterize the hazard function in a parsimonious way. The reason is that, until now, there is not yet consensus on the shape of hazard functions in the empirical literature. As discussed in the introduction, it is evident that the shape of hazard functions is changing over time with the underlying economic conditions. Since the main purpose of the paper is to demonstrate the impact of varying hazard rates on the inflation dynamics, I choose to calibrate it based on the statistical theory of duration analysis. In particular, the functional form I apply is the hazard function of the Weibull distribution, which has two parameters:

$$h(j) = \frac{\tau}{\lambda}\left(\frac{j}{\lambda}\right)^{\tau-1} \tag{2.23}$$

λ is the scale parameter, which controls the average duration of the price adjustment, while τ is the shape parameter to determine the monotonic property of the hazard function. It enables the incorporation of a wide range of hazard functions by using various values for the shape parameter. In fact, any value of the shape parameter that is greater than one corresponds to an increasing hazard function, while values ranging between zero and one lead to a decreasing hazard function. By setting the shape parameter to one, we can retrieve the Poisson process from the Weibull distribution.

In this numerical experiment, I choose λ, such that it implies an average price duration of 3 quarters, which is largely consistent with the median price durations of 7-9 months documented by Nakamura and Steinsson (2008). The shape parameter is set in the interval between one and three, which covers a

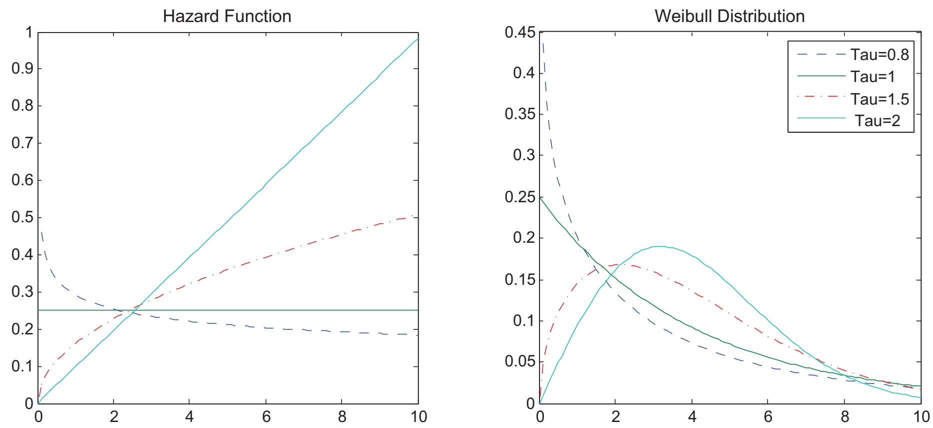

Figure 2.2: The Weibull Distribution

wide range of shapes of the hazard function[3]. As for the rest of the structural parameters, I use some common values in the literature to facilitate comparison the results. In the calibration of the preference parameters, I assume $\beta = 0.9902$, which implies a steady state real return on financial assets of about four percent per annum. I also assume the intertemporal elasticity of substitution $\sigma = 1$, implying log utility of consumption. I choose the Frisch elasticity of the labor supply to equal 0.5, a value that is motivated by using balanced-growth-path considerations in the macro literature. As for the technology parameters, I set labor's share $(1 - a)$ to be either 1 or 0.64 to show the effect of real rigidity. The elasticity of substitution between intermediate goods $\eta = 10$, which implies the desired markup over marginal cost should be about 11%. Finally, I set the standard deviation of the innovation to the nominal money growth rate to be 25 basic points per quarter. For the aggregate technology shock, I choose $\rho_z = 0.95$ and the standard deviation of 0.007, in line with commonly used values in the RBC literature, for example King and Rebelo [2000].

2.4.3 Simulation Results

To evaluate the quantitative implication for the aggregate dynamics, I apply the standard algorithm to solve for the log-linearized rational expectation model.

Effects of Increasing Hazard Functions

In the first experiment, I study the effects of varying the shape parameter on the equilibrium dynamics without any real rigidity and the trend inflation. In Table (2.2), I report second moments generated by the theoretical models, which are different with respect to the shape of the hazard function. Because I use the Weibull hazard function to calibrate the model, I can change the shape of the hazard function by varying the value of the shape parameter τ. In this experiment, I focus on the comparison between the baseline Calvo case, with a corresponding shape parameter of $\tau = 1$, and the increasing hazard models, where τ falls in the range between 1.6 and 3. In all cases, the moments are for a Hodrick-Prescott filtered time series.

[3]This range only covers increasing hazard functions because it makes the maximum number of price duration J well defined.

For each of these hazard functions, two sets of statistics are reported: first, the first-order autocorrelation coefficient of deviations on inflation, real marginal cost and output; and second, contemporaneous correlation coefficients between inflation and real marginal cost. In all models, I use a persistent technology shock and a transitory monetary shock, whose stochastic properties are specified above.

	Calvo Model	Increasing Hazard Models				
τ	1	1.6	1.8	2	2.5	3
$AR(1)\ \hat{\pi}$	0.166	0.524	0.537	0.549	0.567	0.576
$AR(1)\ \hat{y}$	0.811	0.876	0.874	0.873	0.870	0.868
$AR(1)\ \widehat{mc}$	0.169	0.362	0.338	0.318	0.280	0.264
$Corr(\hat{\pi}, \widehat{mc})$	0.998	0.977	0.965	0.950	0.915	0.891

Table 2.2: Second moments of the simulated data (HP filtered, lambda=1600)

The first noteworthy result from the table is that models with increasing hazard rates generate much higher persistence in inflation than in the Calvo model, *ceteris paribus*. Secondly, increases in the shape parameter reduces the persistence of real marginal cost and output. In the Calvo case, because inflation persistence is solely determined by the dynamics of real marginal cost, inflation persistence cannot exceed persistence of real marginal cost. In the increasing hazard model, however, the autoregressive terms of real marginal cost are brought into the Phillips curve through lagged expectations, and thus, in comparison to the Calvo model, this new transmission mechanism propagates more inflation persistence. Fuhrer [2006] presented empirical evidence showing that it is difficult to have a sizable coefficient on the driving process in the Calvo NKPC and that a reduced form shock in the NKPC explains a significant portion of the inflation persistence. We can understand this evidence through the lens of the generalized NKPC. The problem of the conventional NKPC is essentially caused by ignoring terms like lagged inflations and lagged expectations. As I show in the analytical result, this is not the case in the more general time-dependent pricing model. The misspecified Phillips curve fails to explain inflation persistence with its limited structure. Consequently, we either need to introduce the ad hoc backward-looking behavior or a persistent reduced-form shock to achieve a good fit to the data. Last but not least, as shown in the final row of the table, the increasing hazard pricing model also helps to reduce the correlation between inflation and current real marginal cost, a rather robust feature of the data (See: e.g. Hornstein, 2007).

Figure 2.3 shows the impulse responses of the Calvo model compared to the increasing hazard model with the shape parameter of 2. The left panel depicts the impulse responses of inflation while the right panel shows those of the output gap to a 1% increase in the annual nominal money growth rate. Without real rigidity and trend inflation, we observe that, even though the impulse response function of the increasing-hazard model is somewhat more persistent, the general pattern of the impulse responses are the same in both cases, namely, they drop monotonically back to the steady state.

Effects of Real Rigidity

As influentially argued in Woodford [2003], real price rigidity plays an important role in inflation dynamics in addition to nominal rigidity. In this model I introduce real rigidity in a parsimonious way, following Sbordone [2002]. I now set the labor share parameter $(1-a)$ equal to 0.64. Combining this with other

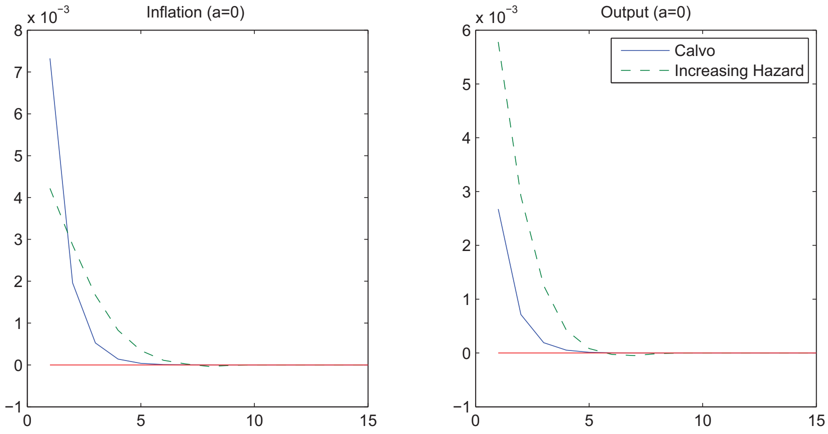

Figure 2.3: Comparing impulse responses functions

parameter values in the model, it implies that the real rigidity parameter ($\gamma\kappa_1 = \frac{a+\phi+\sigma(1-a)}{1-a+a\eta}$) equals 0.35, representing a modest level of strategic complementarity.

In Figure (2.4), I compare the impulse responses of inflation to a transitory money growth shock with and without real rigidity. The left panel shows the comparison in the Calvo model. Incorporation of real rigidity makes the impulse responses more long-lasting, but still monotonic. By contrast, in the right panel, impulse responses of inflation in the increasing hazard model change substantially with real rigidity. One can see that not only the persistence of the impulse response function gets improved, but, more importantly, the shape of it as well. In this case, the IRF becomes hump-shaped with a peak at around the second quarter.

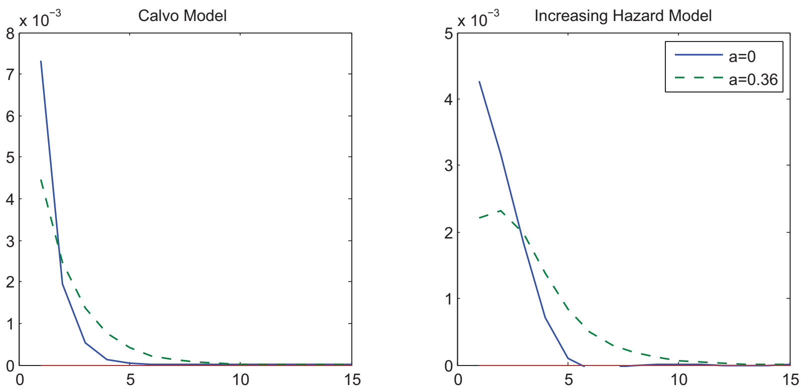

Figure 2.4: Impulse responses of inflation with real rigidity

The economic intuition behind this result is that, on the one hand, increasing hazard function postpones the timing of the price adjustment, i.e. only a few firms adjust their prices immediately after a shock, and more and more adjust later on. On the other hand, real rigidity helps to amplify this postponing effect even further. Because price decisions are strategic complementary, when fewer firms adjust their prices at the beginning phase of the IRF, even the adjusting firms choose a small size of the adjustment. Afterwards, however, when more firms reset their prices, the size of the price adjustment becomes also larger. In another words, the increasing-hazard pricing together with some degree of real rigidity not only affect the timing of the price adjustment, but also the average magnitude of firms' adjustments, leading to a hump-shaped response.

Effects of Trend Inflation

In his seminal paper, Ascari [2004] has shown that trend inflation has important implications for the model's dynamics when the Calvo pricing model is log-linearized around non-zero trend inflation. Here I analyze the dynamic effects of trend inflation in the increasing hazard pricing model. Combining these features is an interesting exercise, because, as I have shown in the previous section, introducing trend inflation affects all coefficients in the generalized NKPC (See Equation 2.19), and hence it changes the relative importance between the forward-looking and backward-looking terms in the Phillips curve. As a result, trend inflation exerts a larger impact on the dynamics of inflation in the increasing-hazard pricing model than in the Calvo case.

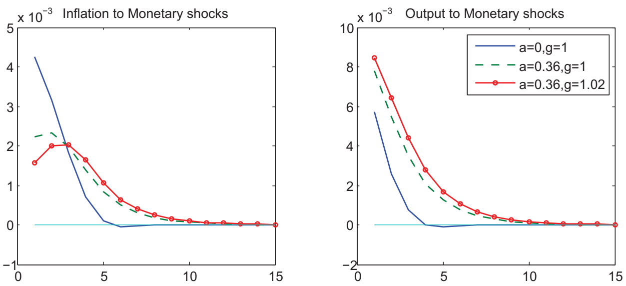

Figure 2.5: Impulse response functions with real rigidity and trend inflation

In Figure (2.5), I show the impulse responses of inflation and of the output gap to a transitory money growth shock in the increasing hazard model. In the left panel, inflation without real rigidity and trend inflation reacts to monetary shock monotonically (solid blue line), while the dashed green line depicts the impulse response of inflation when real rigidity is present. As shown earlier, this line becomes hump-shaped. Furthermore, when I add a non-zero trend inflation into the dynamic structure, the hump becomes even more salient and peaks later (red circled line). On the right panel, impulse responses of the output gap show that the real effect of the monetary shock is more persistent in the case when real rigidity and trend inflation are presented.

The reason why high trend inflation amplifies the effect of increasing hazard functions is, for one, that firms in the increasing hazard model are more likely to adjust when their prices are old. When presenting trend inflation, relative prices disperse quickly over time and, as a result, high trend inflation causes the size of a firm's first adjustment is increasing in the time since the shock occurred.

Real Effects of the Monetary Shock

In the previous sections, I have informally shown that the real effects of the monetary shock is larger in the increasing hazard model than in the Calvo case. Here I introduce a quantitative measure of the real effects of money. In Table (2.3), I report the accumulative IRF of the real output gap to a transitory 1% increase in the annual nominal money growth rate. The accumulative IRF is the area below the impulse response function over the whole horizon, and it is in the unit of percentage of the steady state level of real output.

Real Effects	Calvo Model		Increasing Hazard Model ($\tau = 2$)		
	a=0	a=0.36	a=0, g=1	a=0.36, g=1	a=0.36, g=1.02
$Acc.IRF$ (%)	0.09	0.26	0.22	0.48	0.56

Table 2.3: Real Effects of A Transitory Monetary Shock) with varying trend inflation

In the Calvo model without any real rigidity, the real effect of money is only about 0.09% of real output in the steady state, while this figure rises by a factor of 3 when a modest level of real rigidity is present. On the other hand, the increasing hazard model can generate this level of real effects of the monetary shock even without any helping features. When adding real rigidity into the increasing hazard model, however, real effects rise to 0.48% of steady state real output, and presenting trend inflation reinforces real effects even further. All in all, the increasing hazard model implies 2-3 times more real effects of the monetary shock than the constant-hazard Calvo model.

2.5 Conclusion

The central theme of this study is to show that non-constant hazard functions underlying a pricing assumption implies very different aggregate dynamics. To illustrate this point, I derive a general New Keynesian Phillips curve, reflecting an arbitrary hazard function, trend inflation and real rigidity.

My main analytical results show that, first, the generalized NKPC involves components including lagged inflation, forward-looking and lagged expectations of inflations and real marginal cost, which nests the standard Calvo Phillips curve as a limiting case. When the hazard function is constant, the effect of lagged inflation exactly cancels the effects of the lagged expectation terms, so that only current variables and forward-looking expectations remain in the expression.

In the numerical exercise, I show that inflation and output are more persistent in the increasing hazard model than in the Calvo case. Introducing real rigidity and trend inflation strengthens the dynamic effects of the increasing hazard function on inflation even further. The model can account for hump-shaped impulse responses of inflation to the monetary shock. The real effects of the monetary shock implied by the increasing hazard model are 2-3 times higher than those in the Calvo model. However, due to the calibration strategy I choose in my paper, the numerical results are limited in the class of the monotonic shape of hazard functions. For future research, empirically based hazard functions is clearly a favorable extension for further exploration of the topic.

3 Aggregate Hazard Function in Price-Setting: A Bayesian Analysis

Abstract

This paper presents an approach to identify price reset hazard rates from the joint dynamic behavior of inflation and real macroeconomic aggregates. The identification is possible due to the fact that inflation is composed of current and past reset prices and that the composition depends on the price reset hazard function. The derivation of the generalized NKPC links those composition effects to the hazard function, so that only aggregate data is needed to extract information about the price reset hazard function. The empirical hazard function is generally increasing with the age of prices, but with spikes at the 1st and 4th quarters. This finding reveals that the pricing decision has both time- and state-dependent aspects.

3.1 Introduction

In the current generation of monetary models, effects of monetary policy are closely related to the speed of the aggregate price level reacting to a nominal disturbance. The adjustment of aggregate price in turn depends on two factors. One is the optimal reset price an adjusting firm chooses, and the other is the fraction of firms changing their prices. With the exception of a few micro-founded state-dependent models[1], the majority of research on sticky prices is limited to addressing the optimal reset price decision, but leaving the adjustment timing to be exogenously given by some simplified assumptions, e.g. models incorporating the Calvo [1983] or Taylor (1980) approaches. Put in more technical terms, it amounts to restricting the price reset hazard function to a specific shape and studying other issues on the basis of this assumption.

Until recently, the aggregate price reset hazard function remains a largely ignored topic in the macro literature. It begins, however, to draw more attention, because the competing theoretical models of sticky prices deliver clear mappings between specific aggregate hazard functions and implications for macro dynamics and monetary policy. Pioneer work by Wolman [1999] and Kiley [2002] demonstrated that aggregate dynamics should be sensitive to the hazard function underlying different pricing rules. For this reason, the aggregate hazard function provides a new metric to select theoretical models and identify most relevant propagation mechanisms for monetary policy shocks.

Despite its uses, empirical studies of the aggregate hazard function are rare in the macro literature. By contrast, fast growing evidence from micro data sets becomes available in the recent years[2]. However, I

[1] See: e.g. Caplin and Spulber [1987], Dotsey et al. [1999], Caballero and Engel [2007], Golosov and Lucas (2007). The strength of those models is to endogenize both the optimal reset price decision and the adjustment timing decision in the same framework. However, due to the complexity of this approach, few of them are actually applied in the policy analysis.

[2] See: e.g. Bils and Klenow [2004], Alvarez et al. [2006] Midrigan [2007] and Nakamura and Steinsson [2008] among others.

want to argue that it is the aggregate hazard function that of great interest to macroeconomists, and it is important to distinguish between the macro and micro hazard function. The aggregate hazard is defined as the probability of the price adjustment reacting to aggregate shocks. In the theoretical macro models, those hazard rates can be clearly mapped into impulse responses of aggregate variables. By contrast, mapping between micro hazard functions and macro dynamics is much trickier[3]. For example, Caplin and Spulber [1987] demonstrated that, when the selection effect is present, the aggregate economy is completely immune to price stickiness at the micro level, and thereby has no real effect of monetary policy. Hazard functions estimated from the micro data are therefore not a perfect substitute for the aggregate hazard function defined in the theoretical models. Besides this theoretical consideration, there are also empirical pitfalls that cause for attention in interpreting micro hazard rates. First, micro hazard rates are typically higher than aggregate hazard rates, because individual prices react to both idiosyncratic and aggregate shocks. It is very difficult to disentangle them with a micro data set. Second, evidence of the shape of the hazard function from micro-econometric studies is not conclusive[4]. Micro data sets differ substantially in the range of goods included, the countries and time periods covered, and thereby make it difficult to compare their results; and, even though comprehensive micro data sets have now become available, they are usually short compared to aggregate time series data. Most of the CPI or PPI data sets for the U.S. or Europe are only available from the late 80's[5]. It is reasonable to think that the shape of hazard functions could depend on the underlying economic conditions, and would therefore change over the time periods of the collected data.

The objective of this paper is to estimate the aggregate price reset hazard function directly from the time series data. To do that, I first construct a fully-specified DSGE model featuring nominal rigidity that allows for a flexible hazard function of price setting. I derive the generalized New Keynesian Phillips curve (NKPC hereafter) and then estimate this model with the Bayesian approach. The identification of the aggregate hazard function is possible due to the fact that inflation rate can be decomposed into current and past reset prices and its composition is determined by the aggregate hazard function. The derivation of the generalized NKPC links those composition effects to the hazard function, so that only aggregate data is needed to extract information about the price reset hazard function. The advantages of this identification method is that, first, it is based on a generic assumption of the firm's level pricing behavior, making the mapping between the hazard function and aggregate dynamics robust to the modeling of sticky prices. In addition, this method identifies aggregate hazard function from the fluctuations of aggregate price level, so that effects of idiosyncratic shocks are removed. However, this method is not free from other typical identification problems which prevail in the estimated New Keynesian models[6], e.g. observational equivalence of the labor supply elasticity. For those poorly identified structural parameters, I conduct various sensitivity tests to check the robustness of hazard function estimates.

I estimate the hazard function using the U.S. quarterly data of inflation, the growth rate of real output and effective federal funds rate from 1955 to 2008. The empirical aggregate hazard function has a U-shape with a spike at the fourth quarter. The interpretation of this finding is that price setting is characterized by both time- and state-dependent aspects. For the time-dependent pricing aspect, one quarter and 4-quarter seem to be the most important frequencies of the aggregate price adjustment. About 34.2% of

[3] See: Mackowiak and Smets [2008] for elaboration on this point.

[4] Some find strong support for increasing hazard functions (e.g.: Cecchetti, 1986 and Fougere et al., 2005), while others find evidence in favor of decreasing hazards (e.g.: Campbell and Eden, 2005, Alvarez, 2007 and Nakamura and Steinsson, 2008).

[5] For more details see Table 2 in Alvarez [2007]

[6] For the recent discussion on this topic, see Canova and Sala [2009] and Rios-Rull et al. [2009].

prices hold for less than one quarter, while, 12.4% of prices have the mean duration of four quarters. Besides the time-dependent pricing pattern, the upward-sloping hazard function indicates that the state-dependent pricing also plays an important role in price decisions, especially when a price becomes more outdated. In fact, this generalized time-dependent model can be viewed as a tractable approximation for the more microfounded state-dependent model, when we consider a relative stable economy. The hazard function of the deviation from the optimum largely coincides with the hazard function of time-since-last-adjustment. The longer a price is fixed, the more likely it deviates significantly from the optimum, and hence its probability of being adjusted rises. Since the annual inflation rates in my data set stay under 2% for most of the sample periods except for 1970's, it is arguable that the time elapsed since last adjustment is a good proxy for the deviation from the optimum, therefore the increasing part of the hazard function gives the pricing decision a state-dependent aspect.

This paper is broadly related to progress in developing empirical models of sticky prices based on the New Keynesian framework. The early empirical model of sticky prices was solely based on the NKPC under the Calvo pricing assumption (See, e.g. Gali and Gertler, 1999, Gali et al., 2001 and Sbordone, 2002). These authors estimated the NKPC with GMM, and found a considerable degree of price rigidity in the aggregate data. The empirical price reset hazard rate is around 20% per quarter for the U.S. and 10% for Europe. These results, however, are at odds with micro evidence in two ways. First, recent micro studies generally conclude that the average frequency of price adjustments at the firm's level is not only higher, but also differs substantially across sectors in the economy[7]. Second, the Calvo assumption implies a constant hazard function, meaning that the probability of adjusting prices is independent of the length of the time since the last price revision, and the flat hazard function has been largely rejected by empirical evidence from micro level data (See, e.g.: Cecchetti, 1986, Campbell and Eden, 2005 and Nakamura and Steinsson, 2008). Due to the discrepancy between the macro and micro evidence, empirical models allowing for more flexible price durations or hazard functions have become popular in the recent literature. Jadresic [1999] presented a staggered pricing model featuring a flexible distribution over price durations and used VAR approach to demonstrate that the dynamic behavior of inflation and other macroeconomic variables provides information about the disaggregated price dynamics underlying the data. More recently, Sheedy [2007] constructed a generalized Calvo model and parameterized the hazard function in such a way that the resulting NKPC implied intrinsic inflation persistence when the hazard function was upward sloping. Based on this hazard function specification, he estimated the NKPC using GMM and found evidence of an upward-sloping hazard function. Coenen et al. [2007] developed a staggered nominal contracts model with both random and fixed durations, and estimated the generalized NKPC with an indirect inference method. Their results showed that price rigidity is characterized by a very high degree of real rigidity, as opposed to modest nominal rigidity with an average duration of about 2-3 quarters. Carvalho and Dam [2008] estimated a semi-structural multi-price-duration model with the Bayesian approach, and found that allowing for prices to last longer than 4 quarters is crucial to avoid underestimating the relative importance of nominal rigidity.

The remainder of the paper is organized as follows: in section 2, I present the model with generalized time-dependent pricing and derive the New Keynesian Phillips curve; section 3 introduces the empirical method and the data I use to estimate the model. At the end, results regarding the hazard function are presented and discussed; section 4 contains some concluding remarks.

[7] See: e.g. Bils and Klenow [2004], Alvarez et al. [2006] Midrigan [2007] and Nakamura and Steinsson [2008] among others.

3.2 The Model

In this section, I present a DSGE model of sticky prices due to nominal rigidity. I introduce nominal rigidity by means of a general form of hazard functions[8]. A hazard function of price setting is defined as the probabilities of price adjustment conditional on the spell of time elapsed since the last price change. In this model, the hazard function is a discrete function taking values between zero and one on its time domain. Many well known models of price setting in the literature can be shown to imply hazard functions of one form or another. For example, the most prominent pricing assumption of Calvo [1983] implies a constant hazard function over the infinite horizon.

3.2.1 Representative Household

A representative, infinitely-lived household derives utility from the composite consumption good C_t, and its labor supply L_t, and it maximizes a discounted sum of utility of the form:

$$\max_{\{C_t,L_t,B_t\}} \quad E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\delta}}{1-\delta}-\chi_H\frac{L_t^{1+\phi}}{1+\phi}\right)\right],$$

Where C_t is an index of household's consumption produced using individual goods $C_t(i)$,.

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\eta-1}{\eta}}di\right]^{\frac{\eta}{\eta-1}}, \tag{3.1}$$

where $\eta > 1$, and it follows that the corresponding cost-minimizing demand for $C_t(i)$ and the welfare-based price index, P_t, are given by

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\eta} C_t \tag{3.2}$$

$$P_t = \left[\int_0^1 P_t(i)^{1-\eta}di\right]^{\frac{1}{1-\eta}}. \tag{3.3}$$

For simplicity, I assume that households supply homogeneous labor units (L_t) in an economy-wide competitive labor market.

The flow budget constraint of the household at the beginning of period t is

$$P_tC_t + \frac{B_t}{R_t} \leq W_tL_t + B_{t-1} + \int_0^1 \pi_t(i)di. \tag{3.4}$$

Where B_t is a one-period nominal bond and R_t denotes the gross nominal return on the bond. $\pi_t(i)$ represents the nominal profits of a firm that sells good i. I assume that each household owns an equal share of all firms. Finally this sequence of period budget constraints is supplemented with a transversality condition of the form $\lim_{T\to\infty} E_t\left[\frac{B_T}{\Pi_{s=1}^T R_s}\right] \geq 0$.

The solution to the household's optimization problem can be expressed in two first order necessary

[8] In the theoretical literature, the general time-dependent pricing model has been first outlined in Wolman [1999], who studied some simple examples and found that inflation dynamics are sensitive to different pricing rules. Similar models have also been studied by Mash [2004] and Yao [2009].

conditions. First, optimal labor supply is related to the real wage:

$$\chi_H L_t^{\phi} C_t^{\delta} = \frac{W_t}{P_t}. \tag{3.5}$$

Second, the Euler equation gives the relationship between the optimal consumption path and asset prices:

$$1 = \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\delta} \frac{R_t P_t}{P_{t+1}} \right]. \tag{3.6}$$

3.2.2 Firms in the Economy

Real Marginal Cost

The production side of the economy is composed of a continuum of monopolistic competitive firms, each producing one variety of product i by using labor. Each firm maximizes real profits, subject to the production function

$$Y_t(i) = Z_t L_t(i) \tag{3.7}$$

where Z_t denotes an aggregate productivity shock. Log deviations of the shock, $\hat{z}_t$, follow an exogenous AR(1) process $\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t}$, and $\varepsilon_{z,t}$ is white noise with $\rho_z \in [0,1)$. $L_t(i)$ is the demand of labor by firm i.

Following equation (3.2), demand for intermediate goods is given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\eta} Y_t. \tag{3.8}$$

In each period, firms choose optimal demands for labor inputs to maximize their real profits given nominal wage, market demand (3.8) and the production technology (3.7):

$$\max_{L_t(i)} \Pi_t(i) = \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} L_t(i) \tag{3.9}$$

And real marginal cost can be derived from this maximization problem in the following form:

$$mc_t = \frac{W_t / P_t}{(1-a) Z_t}.$$

Furthermore, using the production function (3.7), output demand equation (3.8), the labor supply condition (3.5) and the fact that at the equilibrium $C_t = Y_t$, I can express real marginal cost only in terms of aggregate output and technology shock.

$$mc_t = Y_t^{\phi+\delta} Z_t^{-(1+\phi)}. \tag{3.10}$$

Pricing Decisions under Nominal Rigidity

In this section, I introduce a general form of nominal rigidity, which is characterized by a set of hazard rates depending on the spell of the time since last price adjustment. I assume that monopolistic competitive firms cannot adjust their price whenever they want. Instead, opportunities for re-optimizing prices are dictated by the hazard rates, h_j, where j denotes the time-since-last-adjustment and $j \in \{0, J\}$. J is the maximum number of periods in which a firm's price can be fixed.

Dynamics of the Price-duration Distribution In the economy, firms' prices are heterogeneous with respect to the time since their last price adjustment.

Using the notation defined in Table 2.1 of Chapter 2, we can write the ex-post distribution of firms after price adjustments ($\tilde{\Theta}_t$) as

$$\tilde{\theta}_t(j) = \begin{cases} \sum_{i=1}^{J} h_i \theta_t(i) \text{ , when } j = 0 \\ \alpha_j \theta_t(j) \text{ , when } j = 1 \cdots J. \end{cases} \tag{3.11}$$

Firms reoptimizing their prices in period t are labeled with 'Duration 0', and the proportion of those firms is given by hazard rates of all duration groups multiplied by their corresponding densities. The firms left in each duration group are the firms that do not adjust their prices. When the period t is over, this ex-post distribution, $\tilde{\Theta}_t$, becomes the ex-ante distribution for the new period, Θ_{t+1}. All price duration groups move to the next one, because all prices age by one period.

It yields the stationary price-duration distribution $\theta(j)$:

$$\theta(j) = \frac{S_j}{\sum_{j=0}^{J-1} S_j}, \text{ for } j = 0, 1 \cdots J-1. \tag{3.12}$$

The Optimal Pricing under the General Form of Nominal Rigidity Given the general form of nominal rigidity introduced above, the only heterogeneity among firms is the time when they last reset their prices, j. Firms in price duration group j share the same probability of adjusting their prices, h_j, and the distribution of firms across durations is given by $\theta(j)$.

In a given period when a firm is allowed to reoptimize its price, the optimal price chosen should reflect the possibility that it will not be re-adjusted in the near future. Consequently, adjusting firms choose optimal prices that maximize the discounted sum of real profits over the time horizon in which the new price is expected to be fixed. The probability that a new price will be fixed at least for j periods is given by the survival function, S_j, defined in the table (2.1).

I setup the maximization problem of an adjuster as follows:

$$\max_{P_t^*} E_t \sum_{j=0}^{J-1} S_j Q_{t,t+j} \left[Y_{t+j|t}^d \frac{P_t^*}{P_{t+j}} - \frac{TC_{t+j}}{P_{t+j}} \right].$$

Where E_t denotes the conditional expectation based on the information set in period t, and $Q_{t,t+j}$ is the stochastic discount factor appropriate for discounting real profits from t to $t+j$. An adjusting firm maximizes the profits subject to the demand for its intermediate good in period $t+j$ given that the firm resets the price in period t and can be expressed as.

$$Y_{t+j|t}^d = \left(\frac{P_t^*}{P_{t+j}} \right)^{-\eta} Y_{t+j}.$$

This yields the following first order necessary condition for the optimal price:

$$P_t^* = \frac{\eta}{\eta - 1} \frac{\sum_{j=0}^{J-1} S_j E_t [Q_{t,t+j} Y_{t+j} P_{t+j}^{\eta-1} MC_{t+j}]}{\sum_{j=0}^{J-1} S_j E_t [Q_{t,t+j} Y_{t+j} P_{t+j}^{\eta-1}]}, \tag{3.13}$$

where MC_t denotes nominal marginal cost. The optimal price is equal to the markup multiplied by a weighted sum of future marginal costs, whose weights depend on the survival rates. In the Calvo case, where $S_j = \alpha^j$, this equation reduces to the Calvo optimal pricing condition.

Finally, given the stationary distribution, $\theta(j)$, aggregate price can be written as a distributed sum of all optimal prices. I define the optimal price which was set j periods ago as P^*_{t-j}. Following the aggregate price index from equation (3.3), the aggregate price is then obtained by:

$$P_t = \left(\sum_{j=0}^{J-1} \theta(j) P^{*1-\eta}_{t-j} \right)^{\frac{1}{1-\eta}}. \tag{3.14}$$

3.2.3 New Keynesian Phillips Curve

In this section, I derive the New Keynesian Phillips curve for this generalized model. To do that, I first log-linearize equation (3.13) around the flexible price steady state. The log-linearized optimal price equations are obtained by

$$\hat{p}^*_t = E_t \left[\sum_{j=0}^{J-1} \frac{\beta^j S(j)}{\Omega} \left(\widehat{mc}_{t+j} + \hat{p}_{t+j} \right) \right], \tag{3.15}$$

where :

$$\Omega = \sum_{j=0}^{J-1} \beta^j S(j) \text{ and } \widehat{mc}_t = (\delta + \phi)\hat{y}_t - (1+\phi)\hat{z}_t.$$

In a similar fashion, I derive the log deviation of the aggregate price by log linearizing equation (3.14).

$$\hat{p}_t = \sum_{k=0}^{J-1} \theta(k) \ \ \hat{p}^*_{t-k}. \tag{3.16}$$

After some algebraic manipulations on equations (3.15) and (3.16), I obtain the New Keynesian Phillips curve as follows:

$$\begin{aligned} \hat{\pi}_t = & \sum_{k=0}^{J-1} \frac{\theta(k)}{1-\theta(0)} E_{t-k} \left(\sum_{j=0}^{J-1} \frac{\beta^j S(j)}{\Psi} \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} \sum_{j=i}^{J-1} \frac{\beta^j S(j)}{\Psi} \hat{\pi}_{t+i-k} \right) \\ & - \sum_{k=2}^{J-1} \Phi(k) \hat{\pi}_{t-k+1}, \quad \textit{where } \Phi(k) = \frac{\sum_{j=k}^{J-1} S(j)}{\sum_{j=1}^{J-1} S(j)}, \ \ \Psi = \sum_{k=0}^{J-1} \beta^j S(j). \end{aligned} \tag{3.17}$$

As we can observe that all coefficients in equation (3.17) are expressed in terms of non-adjustment rates ($\alpha_j = 1 - h_j$) and the subjective discount factor, β, thereby the coefficients in the generalized NKPC link dynamic effects of reset prices on inflation to the hazard function. As a result, information about the price reset hazard rates can be extracted from the aggregate data through the dynamic structure of the Phillips curve.

3.2.4 The Final System of Equations

The general equilibrium system consists of equilibrium conditions derived from the optimization problems of economic agents, market clearing conditions and a monetary policy equation. Market clearing conditions require real prices clear the factor and good markets, while monetary policy determines nominal value of the real economy. I choose a Taylor rule to close the model.

$$I_t = I_{t-1}^{\rho_i} \left[\left(\frac{P_t}{P_{t-1}\bar{\pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_y} \right]^{1-\rho_i} e^{q_t}. \tag{3.18}$$

Equation (3.18) is motivated by the interest rate smoothing specification for the Taylor rule[9], which specifies a policy rule that the central bank uses to determine the nominal interest rate in the economy, where ϕ_π and ϕ_y denote short-run responses of the monetary authority to log deviations of inflation and the output growth rate, and q_t is a sequence of *i.i.d.* white noise with zero mean and a finite variance $(0, \sigma_q^2)$.

After log-linearizing equilibrium equations around the flexible-price steady state, the general equilibrium system consists of the generalized NKPC (3.19), real marginal cost (3.20), the household's intertemporal optimization condition (3.21), the Taylor rule (3.18) and exogenous stochastic processes. In the IS curve, I add an exogenous shock, d_t, to represent real aggregate demand disturbances[10]

$$\hat{\pi}_t = \sum_{k=0}^{J-1} W_1(k) E_{t-k} \left(\sum_{j=0}^{J-1} W_2(j) \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} W_3(i) \hat{\pi}_{t+i-k} \right) - \sum_{k=2}^{J-1} W_4(k) \hat{\pi}_{t-k+1}, \tag{3.19}$$

$$\widehat{mc}_t = (\delta + \phi)\hat{y}_t - (1+\phi)\,\hat{z}_t, \tag{3.20}$$

$$\delta E_t\left[\hat{y}_{t+1}\right] = \delta \hat{y}_t + \left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right]\right) + d_t, \tag{3.21}$$

$$\hat{\imath}_t = (1-\rho_i)\left(\phi_\pi \hat{\pi}_t + \phi_y\left(\hat{y}_t - \hat{y}_{t-1}\right)\right) + \rho_i \hat{\imath}_{t-1} + q_t, \tag{3.22}$$

$$\hat{z}_t = \rho_z * \hat{z}_{t-1} + \epsilon_t \quad where \ \ \epsilon_t \sim N(0, \sigma_z^2), \tag{3.23}$$

$$d_t = \rho_d * d_{t-1} + \varepsilon_t \quad where \ \ \varepsilon_t \sim N(0, \sigma_d^2), \tag{3.24}$$

$$q_t \sim N(0, \sigma_q^2), \tag{3.25}$$

where weights (W_1, W_2, W_3, W_4) in the generalized NKPC are defined in equation (3.17). I collect the structure parameters into a vector $\mu = (\beta, \delta, \phi, \eta, \phi_\pi, \phi_y, \rho_i, \alpha_j s, \rho_z, \rho_d, \rho_i, \sigma_z^2, \sigma_d^2, \sigma_q^2)$. In the empirical study, I am interested in estimating values for those structural parameters by using the Bayesian approach.

3.3 Estimation

In this section, I solve and estimate the New Keynesian model described above by using the Bayesian approach. The full information Bayesian method has some appealing features in comparison to methods employed in the literature. As pointed out by An and Schorfheide [2007], this method is system-based, meaning that it fits the DSGE model to a vector of aggregate time series. Through a full characterization of the data generating process, it provides a formal framework for evaluating misspecified models on the

[9] See: the empirical work by Clarida et al. [2000]

[10] Introducing this shock is not necessary for the theoretical model, but, in the Bayesian estimation, due to the singularity problem I need three shocks for three observables.

basis of the data density. In addition, the Bayesian approach also provides a consistent method for dealing with rational expectations – one of the central elements in the DSGE models.

3.3.1 Bayesian Inference

I apply the Bayesian approach, set forth by DeJong et al. [2000], Schorfheide [2000] among others, to estimate the structural parameters of the DSGE model. The Bayesian estimation is based on combining information gained from maximizing likelihood of the data and additional information about the parameters (the prior distribution). The main steps of this approach are as follows:

First, the linear rational expectation model is solved by using standard numerical methods (See: e.g. Uhlig, 1998 and Sims, 2002) to obtain the reduced form equations in its predetermined and exogenous variables.

For example, the linearized DSGE model can be written as a rational expectations system in the form

$$\Upsilon_0(\mu)S_t = \Upsilon_1(\mu)S_{t-1} + \Upsilon_\epsilon(\mu)\epsilon_t + \Upsilon_\omega(\mu)\omega_t. \tag{3.26}$$

Here, S_t is a vector of all endogenous variables in the model, such as $\hat{y}_t$, $\hat{\pi}_t$, $\hat{\imath}_t, etc$. The vector ϵ_t stacks the innovations of the exogenous processes and ω_t is composed of one-period-ahead rational expectations forecast errors. Entries of $\Upsilon(\mu)$ matrices are functions of structural parameters in the model. The solution to (3.26) can be expressed as

$$S_t = \Psi_1(\mu)S_{t-1} + \Psi_\epsilon(\mu)\epsilon_t. \tag{3.27}$$

The second step involves writing the model in state space form. This is to augment the solution equation (3.27) with a measurement equation, which relates the theoretical variables to a vector of observables Y_obs_t.

$$Y\_obs_t = A(\mu) + BS_t + CV_t. \tag{3.28}$$

Where $A(\mu)$ is a vector of constants, capturing the mean of S_t, and V_t is a set of shocks to the observables, including measurement errors.

Third, when we assume that all shocks in the state space form are normally distributed, we can use the Kalman filter (Sargent, 1989) to evaluate the likelihood function of the observables $(\mu|Y\_obs^T)$. In contrast to other maximum likelihood methods, the Bayesian approach combines the likelihood function with prior densities $p(\mu)$, which includes all extra information about the parameters of interest. The posterior density $p(\mu|Y\_obs^T)$ can be obtained by applying Bayes' theorem

$$p(\mu|Y\_obs^T)(\mu|Y\_obs^T)\ p(\mu). \tag{3.29}$$

In the last step, μ is estimated by maximizing the likelihood function given data $(\mu|Y\_obs^T)$ reweighed by the prior density $p(\mu)$, in that numerical optimization methods are used to find the posterior mode for μ and the inverse Hessian matrix. Finally, the posterior distribution is generated by using a random-walk Metropolis sampling algorithm[11].

[11] I implement the Bayesian estimation procedure discussed above by using the MATLAB based package DYNARE, which is available at: `http://www.cepremap.cnrs.fr/dynare/`

3.3.2 Data and Priors

According to the empirical framework and research questions to be addressed in this paper, I choose the following three observables: growth rate of real GDP per capita, annualized inflation rate calculated from the consumer price index (CPI) and nominal interest rate for the U.S. over the period 1955.Q1 - 2008.Q4[12]. The output growth rate and inflation are detrended by the Hodrick-Prescott filter. Based on the definition of the model's variables and the observables, the measurement equations are defined as follows:

$$\begin{aligned} y\_obs &= \hat{y}_t - \hat{y}_{t-1} \\ \pi\_obs &= \hat{\pi}_t \\ i\_obs &= \hat{\imath}_t. \end{aligned}$$

The priors I choose are in line with the mainstream values used in the Bayesian literature[13]. They are centered around the average value of estimates of micro and macro data with fairly loose standard deviations.

I fix two parameters in advance. The discount factor β is equal to 0.99, implying an annual steady state real interest rate of 4%. The elasticity of substitution between intermediate goods is set to be 10, implying an average mark-up of around 11%. Both values are common in the literature.

The key structural parameters in this model are the non-adjustment rates, α_j. I choose the priors for the these parameters based on micro evidence on the mean frequency of price adjustments, reported by Bils and Klenow [2004]. They find that the U.S. sectoral prices on average last only 2 quarters, which implies the hazard rate is equal to 0.5. Because the main goal of this study is to find out what shape of hazard function fits best to the macro data, I set all non-adjustment rate with the same mean of 2 quarters and a very loose standard deviation of 0.28. This prior leads to a 95% inter-quantile-range basically covering the whole interval between 0 and 1 quite evenly, except for the two extremes. In addition, same priors for all α_j reflect the view of a pricing model using a constant-hazard assumption. By choosing a large standard deviation for the prior, I allow the data to speak out quite freely about the shape of hazard rates over the time horizon, so that I can evaluate theoretical models from the point of view of a hazard function.

Moving to the other structural parameters, the prior for the relative risk aversion, δ, is set to follow a gamma distribution with mean 1.5 and a standard error of 0.375. This prior covers a wide range of values from the experimental and macro literature. The inverse elasticity of labor supply, ϕ, is difficult to calibrate, because there is a wide range of evidence in the literature. I choose the prior for this parameter to be normally distributed around the mean of 1.5. A mean of 1.5 is commonly estimated in the micro-labor studies (See: e.g. Blundell and Macurdy, 1999). I set a large standard error of 1.0. In the sensitivity analysis, I also check the robustness of my result to the other values of the prior mean.

Proceeding with priors for parameters in the Taylor rule, the priors for ϕ_π and ϕ_y are centered at the values commonly associated with a Taylor rule. I set a prior for the response coefficient to deviation of annualized inflation ϕ_π to be centered around 1.5 with a standard error of 0.1, and a prior for response coefficient to output growth rate ϕ_y to be centered around 0.5 with a standard error of 0.1. This rule also allows for interest rate smoothing with a prior mean of 0.5 and a standard deviation of 0.1.

Finally, I assume that the standard errors of the innovations follow an inverse-gamma distribution with a mean of 0.1 and two degrees of freedom. The persistence of the AR(1) process of the productivity shock

[12] Details on the construction of the data set are provided in Appendix (4).
[13] See: e.g. Smets and Wouters, 2007 and Lubik and Schorfheide, 2005

is beta-distributed with a mean of 0.8 and the standard deviation of 0.1, and the persistence of the AR(1) process of the aggregate demand shock is beta-distributed with a mean of 0.5 and the standard deviation of 0.1.

3.3.3 Results

By applying the methodology described above, I proceed to gauge the degree of nominal rigidity in terms of the estimated structural parameters based on those prior distributions discussed above. The posterior modes of parameters are calculated by maximizing the log likelihood function of the data, and then the posterior distributions are simulated using the "Metropolis-Hastings" algorithm. The results presented here are based on 500,000 out of 1 million total draws and the average acceptance rate is around 0.31. With the simulation, I obtain convergence and relative stability in all measures of the parameter moments. The posterior mode, mean and 5%, 95% quantiles of the 17 estimated parameters are reported in Table 3.1 and the prior-posterior distributions are plotted in the figure appendix (5). The data provides strong information about most of the structural parameters, except for the inverse of the elasticity of labor supply and one of Taylor rule parameters. In those cases, I conduct various sensitivity tests to check the robustness of estimates of hazard function to changes in those poorly identified structural parameters.

Parameters	Prior			Posterior (M-H 500,000)			
	Dist.	Mean	S.D.	Mode	Mean	5%	95%
δ	gamma	1.5	0.375	4.311	4.149	3.379	4.994
ϕ	normal	1.5	1.0	1.459	1.282	- 0.072	2.553
ϕ_π	normal	1.5	0.1	1.914	1.938	1.796	2.083
ϕ_y	normal	0.5	0.1	0.745	0.740	0.579	0.899
ρ_i	beta	0.5	0.1	0.634	0.622	0.572	0.674
α_1	beta	0.5	0.28	0.403	0.454	0.334	0.571
α_2	beta	0.5	0.28	0.941	0.855	0.717	0.998
α_3	beta	0.5	0.28	0.991	0.931	0.848	0.999
α_4	beta	0.5	0.28	0.663	0.676	0.431	0.962
α_5	beta	0.5	0.28	0.978	0.833	0.648	0.995
α_6	beta	0.5	0.28	0.975	0.801	0.620	0.983
α_7	beta	0.5	0.28	0.641	0.590	0.247	0.994
ρ_z	beta	0.8	0.1	0.992	0.988	0.978	0.998
ρ_d	beta	0.5	0.1	0.850	0.849	0.811	0.887
σ_z	invgam	0.1	2.0	1.663	1.859	1.142	2.602
σ_m	invgam	0.1	2.0	1.176	1.211	1.069	1.347
σ_d	invgam	0.1	2.0	0.721	0.735	0.591	0.873

Table 3.1: Posterior Distributions of Parameters (U.S.83-08)

Estimate for the relative risk aversion is high (4.149), but well in line with the benchmark values for macroeconomic studies, while the inverse of the elasticity of labor supply is not well identified in this model. Prior and posterior distributions are very close to each other, indicating that data does not provide information on this parameter under the current identification scheme.

The estimated monetary policy reaction function is consistent with the common view of the Taylor rule. Monetary policy responds strongly to the deviation of inflation (1.938), but not as much as to the output growth rate (0.74). There is a considerable degree of interest rate smoothing, as the posterior mean of ρ_i is around 0.62. But the response of the nominal interest rate to output growth rate is also not well identified.

I turn now to the nominal rigidity represented by the estimates of non-adjustment rates, α_j. Contrary to the prior distributions, which are motivated by the Calvo model where all hazard rates are constant over time, the estimates reveal that the hazard function changes shape over time and the data strongly advocates a non-constant hazard function. The mean frequency of price adjustment is 32% per quarter which implies a mean price duration of 9.2 months. This result is consist with findings by Nakamura and Steinsson (2008) using micro data. More importantly, price reset hazards vary substantially around this mean, depending on how long the price has been fixed. I will discuss the hazard function in more detail after the sensitivity analysis.

3.3.4 Robustness Tests

In this section, I test the robustness of the structural parameter estimates, especially those for the hazard function, to alternative priors, different model setups and data using different detrending methods. Table (3.2) and (3.3) report results of the posterior modes. In Table (3.2), I summarize results using the Hodrick-Prescott filter detrended data. I conduct the sensitivity analysis in the following steps: first, I check the prior sensitivity by altering the prior mean of ϕ, from 1.5 to 0.5. I choose to check this parameter because the estimation result shows that the inverse of labor supply elasticity is poorly identified. In addition, there is no consensus about the calibration value for this parameter in the literature. The first three columns in the table compare the results from the three alternative priors. 0.5 is a typical value motivated in the macro literature, while $\phi = 1.5$ is commonly estimated in the micro-labor studies (See: e.g. Blundell and Macurdy, 1999). I also check the value $\phi = 1$, which can be often found in the RBC literature. I find that changing the prior for ϕ mainly affects the posterior mode of ϕ itself, leaving estimates of other parameters for preference and monetary policy qualitatively unchanged. These results manifest an observational equivalence problem commonly found in estimating DSGE models. The log likelihood function is mostly flat on the choices of priors for ϕ. Posterior estimates are mainly driven by the prior instead of data. As for the non-adjustment rates, the choice of the prior for labor elasticity affects magnitudes of non-adjustment rates at all frequencies. Interestingly, it shows that making labor supply more elastic, decreasing in the value of ϕ, leads to more frequent price adjustments estimated. Non-adjustment rates are significantly lower at all frequencies except for the 7th. Despite changes in the magnitude, the general pattern of the hazard function remains the same.

In the next two columns, I change the model setup. When adopting the standard Taylor rule with output gap instead of output growth rate, it results in a large change in the estimated ϕ_y, which becomes very small, indicating that central bank reacts less to output gap in the monetary policy decision making, because it is an unobservable in the economy. Estimates for the non-adjustment rates, however, are almost identical as in the benchmark case. I also change the model setup by fixing the hazard rates to the value of 0.5, implying an average duration of 2 quarters[14]. As seen in the last column, fixing the hazard rates has implications for the estimates of other structural parameter. For example, it leads to a much lower

[14] I call it the pseudo-Calvo model because, in this case, I truncate the hazard function at the 7th quarter. As a result, it is not exactly equivalent to the Calvo model, which implies an infinite horizon for the hazard function. This pseudo-Calvo can be view as an approximation of the real Calvo model. I estimate also the pseudo-Calvo model with longer horizons, but it does not change the main conclusion.

Tests	H-P filter Detrended Data				
Parameter	$\phi = 1.5$	$\phi = 1$	$\phi = 0.5$	*TaylorRule*	*Calvo*
δ	4.311 (0.322)	4.012 (0.062)	4.097 (0.063)	4.790 (0.332)	2.949 (0.247)
ϕ	1.46 (0.867)	1.063 (0.647)	0.618 (0.918)	1.687 (0.433)	0.315 (0.309)
ϕ_π	1.914 (0.091)	1.864 (0.087)	1.859 (0.087)	1.956 (0.084)	2.052 (0.083)
ϕ_y	0.745 (0.097)	0.794 (0.097)	0.793 (0.097)	0.111 (0.069)	0.704 (0.097)
α_1	0.403 (0.076)	0.050 (0.034)	0.051 (0.034)	0.469 (0.085)	0.5
α_2	0.941 (0.138)	0.772 (0.148)	0.781 (0.148)	0.918 (0.126)	0.5
α_3	0.991 (0.028)	0.968 (0.153)	0.968 (0.152)	0.992 (0.072)	0.5
α_4	0.663 (0.151)	0.408 (0.157)	0.407 (0.156)	0.624 (0.145)	0.5
α_5	0.980 (0.067)	0.914 (0.232)	0.916 (0.231)	0.973 (0.147)	0.5
α_6	0.980 (0.064)	0.926 (0.242)	0.926 (0.241)	0.978 (0.137)	0.5
α_7	0.641 (0.262)	0.737 (0.388)	0.741 (0.386)	0.586 (0.252)	0.5
Log Margin. Likeli.	−907.32	−914.91	−912.63	−935.57	−918.04

Table 3.2: Sensitivity Check for H-P Detrended Data

Tests	Linearly detrended Data				
Parameter	$\phi = 1.5$	$\phi = 1$	$\phi = 0.5$	*TaylorRule*	*Calvo*
δ	4.352 (0.640)	4.357 (0.627)	4.364 (0.621)	5.226 (0.473)	3.671 (0.489)
ϕ	2.10 (0.886)	1.72 (0.854)	1.373 (0.816)	2.305 (0.878)	1.942 (0.797)
ϕ_π	1.912 (0.081)	1.911 (0.081)	1.911 (0.081)	1.908 (0.097)	1.952 (0.077)
ϕ_y	0.669 (0.099)	0.669 (0.099)	0.670 (0.099)	0.258 (0.395)	0.662 (0.098)
α_1	0.504 (0.075)	0.493 (0.075)	0.482 (0.075)	0.5632 (0.077)	0.5
α_2	0.716 (0.126)	0.710 (0.126)	0.705 (0.128)	0.789 (0.138)	0.5
α_3	0.980 (0.063)	0.979 (0.064)	0.979 (0.065)	0.969 (0.187)	0.5
α_4	0.143 (0.218)	0.124 (0.216)	0.106 (0.215)	0.397 (0.135)	0.5
α_5	0.193 (0.645)	0.182 (0.604)	0.180 (0.608)	0.601 (0.389)	0.5
α_6	0.462 (1.216)	0.446 (1.180)	0.437 (1.163)	0.785 (0.415)	0.5
α_7	0.329 (1.149)	0.359 (1.147)	0.379 (1.147)	0.221 (0.460)	0.5
Log Margin. Likeli.	−792.12	−792.44	−792.79	−837.09	−797.88

Table 3.3: Sensitivity Check for Linearly Detrended Data

estimate for the intertemporal elasticity of substitution and inverse of labor elasticity. In addition, in terms of log marginal likelihood, both the output-gap-Taylor-rule model and the fixed-hazard setup are clearly less favored by the data. In the last row of the table, I report the log marginal likelihood of the data for each model. It shows that changing priors of ϕ only marginally affects the data density, but the data gives strong support of the flexible hazard model as opposed to the fixed-hazard model and the output gap Taylor rule. The Bayes factor in favor of the flexible hazard model is approximately in the order of 10^5.

I conduct the same tests by using linearly detrended data again, which is reported in Table (3.3). All results from previous exercises are confirmed, but the drawback of using linearly detrended data is that they do not deliver accurate information about the hazard function. As seen in the table, non-adjustment rates are much different to what we have from the HP-detrended data and those after the 3rd quarter are all statistically insignificant. The reason for this could be that the linearly detrending mixes macro dynamics at the business cycle frequencies with those from other frequencies, so that it biases the estimates and reduces the efficiency of the estimation too.

3.3.5 Aggregate Hazard Function and Implications for Macro Modeling

In this part, I evaluate new evidence on the aggregate price reset hazard rates obtained from my empirical analysis and also discuss its implication for macro modeling of sticky prices. I plot the estimates of hazard

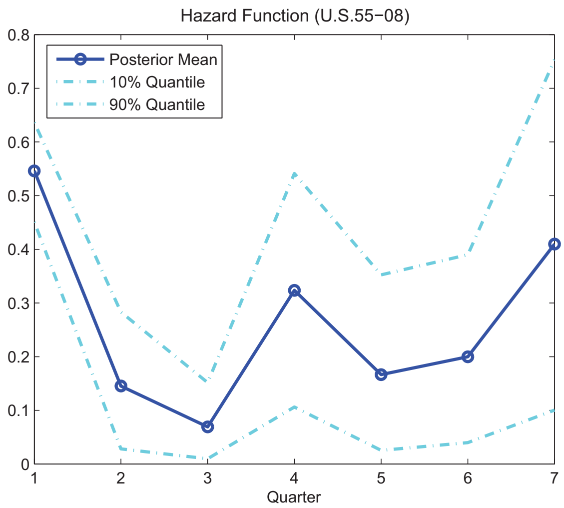

Figure 3.1: The Empirical Price Reset Hazard Function

rates in Figure (3.1). The hazard rate is high one quarter after the last price adjustment (55%), and drops in the next two consecutive quarters to around 10% and rise again in the 4th quarter. Afterwards, hazard rates are largely increasing with the age of the price. Overall, the hazard function has a U-shape with a spike at the fourth quarter. I also calculate the distribution of price durations from the estimated hazard rates by using formula (3.12). It yields that around 34.2% of prices last for less than one quarter. From micro data studies, we learn that prices of apparel, unprocessed food, energy and travel are the most frequently adjusted prices, whose median durations last less than three months. Moreover, 12.4% of prices have the mean duration of four quarters. Examples for those prices are services, such as hairdressers or public transportation.

This finding has important bearing on the macro modeling of sticky prices. Overall, I find new evidence can not be explained by any single theory of sticky prices. For the first half of the hazard function (from the first to the fourth quarters), it appears that the pricing decision is mainly characterized by either the flexible price setting or by a time-dependent aspect (e.g. Taylor, 1980). The survey evidence has shown that many firms conduct yearly price revisions due to costly information. This kind of behavior can also be motivated by the theory of customer markets, which indicates that long-term customer relationships are an important consideration in pricing decisions (See: e.g. Rotemberg, 2005). On the other hand, the upward-sloping part of the hazard function indicates that the state-dependent pricing also plays an important role in the pricing decision, the more outdated a price becomes. In fact, this generalized time-dependent model can be viewed as a proxy of the more microfounded state-dependent model. More microfounded pricing models, such as Dotsey et al. [1999], show that the state-dependent pricing behavior implies an increasing hazard function. If we consider a relative stable economy, the hazard function of the deviation from the optimum largely coincides with the hazard function of time-since-last-adjustment. The longer a price is fixed, the more likely it deviates significantly from the optimum, and hence its probability of being adjusted rises. Since in my data set, the annual inflation rates stay under 2% for most of the sample periods except for the turbulent decade between early 1970s and early 1980s, it is arguable that the time elapsed since last adjustment is a good proxy for the deviation from the optimum, therefore the increasing part of the hazard function gives the pricing decision a state-dependent aspect.

To summarize these results, the new evidence of the aggregate hazard function reveals that, for the less sticky prices ranging in duration from one to four quarters, time-depend pricing plays a major role, while, for stickier prices with a duration longer than 4 quarters, the state-dependent pricing dominates.

3.4 Conclusion

In this paper, I document new evidence on the shape of the aggregate hazard function. I construct a DSGE model featuring nominal rigidity that allows for a flexible hazard function of price setting. The generalized NKPC possesses a richer dynamic structure, with which I can infer the shape of the hazard function underlying aggregate dynamics. Identifying the hazard function from aggregate data is a useful exercise, because, first, estimating hazard rates directly from a DSGE model provides the most consistent way to compare the theoretical concept with the empirical evidence. Second, it overcomes some weaknesses of estimates using micro data, such as contamination by the idiosyncratic effects and the limited availability of the long time series data. As a result, this study delivers some useful insights for macroeconomists, which can be readily used to guide macro modeling. At last, a caveat should be note that the identification method relies on scrutinizing effects of past reset prices on current inflation, and those effects decay over time. Consequently, the information contents of aggregate data for hazard rates deteriorate fast with the length of the time since last adjustment. After the fourth quarter, estimated hazard rates become imprecise.

4 Can the New Keynesian Phillips Curve Explain Inflation Gap Persistence?

Abstract

In this paper, I use a generalization of the Calvo NKPC to study implications of the price reset hazard function for inflation persistence. I first replicate the Whelan [2007]'s finding that the generalized NKPC fails to account for the positive coefficient of lagged inflation in a typical reduced-form Phillips curve regression, and then show that it is the 4-period-Taylor-contract hazard function that gives rise to this result. In contrast, an empirically-based price reset hazard function can generate simulated data that are consistent with inflation gap persistence found in the US CPI data. I conclude that a price reset hazard plays a crucial role for generating realistic inflation dynamics.

4.1 Introduction

The nature of inflation persistence is a complex phenomenon because it is influenced by many aspects of the economy. For example, Cogley and Sbordone [2008] argue that it is important to distinguish between inflation-trend persistence and inflation-gap persistence, since they arise from different economic sources. While the dynamics of trend inflation result largely from shifts in the long-run target of the monetary policy rule, inflation-gap persistence is influenced primarily by pricing behavior at the firm level and the price aggregation mechanism.

The focus of this paper is the dynamics of the inflation gap — the difference between the actual inflation and trend inflation. I first document some stylized facts distinguishing inflation gap persistence from inflation level persistence. I find evidence from the U.S. CPI data that the inflation gap constitutes a large part of inflation persistence. Second, I investigate whether the stylized fact can be explained by the theoretical New Keynesian Phillips curve (hereafter: NKPC), and further identify which mechanism of the model is most important for generating inflation gap persistence.

The purely forward-looking NKPC is often criticized for generating too little inflation persistence (See: e.g. Fuhrer and Moore, 1995). To overcome this weakness, various generalizations of the basic NKPC have been developed in the literature, they offer, however, different interpretations on the nature of inflation gap persistence. The hybrid NKPC incorporates lagged inflation into the standard NKPC motivated by the positive backward-dependence of inflation in the empirical reduced-form Phillips curve[1]. According to this line of literature, inflation gap persistence should be interpreted as 'intrinsic' (Fuhrer, 2006) and the dependency between current and lagged inflation should be treated as a fixed primitive relationship, which is independent of monetary policy. By contrast, the more micro-founded general-pricing-hazard models[2] shed new lights on the important role played by inertia of expectations in generating inflation gap

[1]See: e.g. Gali and Gertler [1999] and Christiano et al. [2005].

[2]See: e.g. Carvalho [2006], Sheedy [2007], Coenen et al. [2007] and Whelan [2007].

persistence. According to this class of models, inflation gap persistence is inherited from the additional moving-average terms of real driving forces through the lagged expectations. More importantly, since the coefficient on lagged inflation depends on the whole model including the specification of monetary policy, it implies that, if this theory is the origin of the NKPC, then the hybrid NKPC is subject to the Lucas critique (Lucas, 1976), and thereby can be only used in the monetary policy analysis with great caution.

Despite the theoretical solidity of the general-pricing-hazard model, Whelan [2007] rejected it empirically. He showed that the general-pricing-hazard model fails to replicate the positive backward-dependence of inflation typically found in the empirical reduced-form Phillips curve. In partial equilibrium, Whelan proved that the coefficient on the lagged inflation is always negative, regardless of the form of the price reset hazard function. Furthermore, he used a simple DSGE model to show that, even in general equilibrium, this model still generates negative coefficients on inflation lags.

In this paper, I first replicate his findings and check their robustness to alternative setups of the model. In particular, I test the result using different price reset hazard functions, aggregate demand conditions and monetary policy rules. I find that it is the 4-period-Taylor-contract hazard function used in the Whelan's setup that gave rise to the result. Under an empirically based pricing hazard function estimated by Yao [2010], the simulated data accounts quite well for the inflation gap persistence I find in the U.S. CPI data after the Volcker disinflation period. The reason why the hazard function greatly affects inflation gap persistence is that backward-dependence of inflation in the model is determined by two counteracting channels. The "front-loading channel" always weakens inflation gap persistence, because lagged inflation enters the NKPC with negative coefficients, magnitudes of which are purely determined by the price reset hazard function. By contrast, the second channel works through the expectational terms in the NKPC. In this channel, lagged inflations have positive coefficients when lagged inflations act as leading indicator of other variables. As a result, the magnitude of the "expectation channel" is not only affected by the price reset hazard function, but also by the other general equilibrium forces, such as aggregate demand side of the economy and monetary policy. Overall, inflation gap persistence in this framework results from a more complex propagation mechanism, in which the price reset hazard function exerts crucial effects through various channels.

The general-pricing-hazard models have been studied in the macro literature to understand consequences of different price reset hazard functions for macro dynamics. It is important, because, in recent years, empirical studies using detailed micro-level price data sets[3] generally reach the consensus that, instead of having economy-wide uniform price stickiness, the frequency of price adjustments differs substantially across sections. This new evidence issues a serious challenge to the Calvo pricing assumption (Calvo, 1983). In addition, micro empirical evidence largely rejects the constant hazard function, implied by the Calvo model (See, e.g.: Cecchetti, 1986, Alvarez, 2007 and Nakamura and Steinsson, 2008). In response to this challenge, theoretical work by Wolman [1999] raised the issue that inflation dynamics should be sensitive to the hazard function underlying different pricing rules. He showed this result in a partial equilibrium analysis. Kiley [2002] compared the Calvo and Taylor staggered-pricing models and showed the dynamics of output following monetary shocks are both quantitatively and qualitatively different across the two pricing specifications unless one assumes a substantial level of real rigidity in the economy. Carvalho [2006] constructed a sticky price model that allows for heterogeneous Calvo-sticky-price sectors. He found that existence of heterogeneity in price stickiness generates large and persistent real effects of monetary policy, which can be replicated by a constant-hazard-pricing model only when it is calibrated with an unrealistic low frequency of price adjustments. Sheedy [2007] derived the generalized

[3]See: e.g. Bils and Klenow [2004] and Alvarez et al. [2006] among others.

NKPC under a recursive formulation of the hazard function and showed that, under this parameterization, the dependence of current and lagged inflation is determined by the slope of the hazard function. In a more general setup, Whelan [2007] showed that backward-dependence of inflation in this structural Phillips curve is mostly negative. Based on this finding he drew the conclusion that this class of models can not explain the observation from the reduced-form Phillips curve regression that inflation is positively dependent on its lags.

It is noteworthy that non-zero trend inflation is also important for the short-run inflation dynamics(See: Ascari, 2004). Furthermore, Cogley and Sbordone [2008] extend the Calvo NKPC by allowing for time-drifting trend inflation and they show that changing trend inflation affects coefficients of the NKPC and hence the short-run inflation dynamics. Even though the general-hazard NKPC does not incorporate this feature, this limitation does not prohibit the general-price-hazard model from standing as a useful analytical tool for inflation dynamics. Empirical evidence shows that, while non-constant hazard function is a robust feature of the pricing behavior in the data, the time-varying trend inflation is not always equally important all the time. During the oil crises in the 1970's, volatile inflation trend maybe predominated inflation dynamics, but, after early 1980's, U.S. trend inflation became moderate and stable in the data. These two versions of the generalized NKPC complement each other, combining them, however, gives an interesting perspective for future work.

The remainder of the paper is organized as follows: Section 1 documents stylized fact of inflation gap persistence in the U.S. data. In section 2, I present the model with the generalized time-dependent pricing and derive the New Keynesian Phillips curve; section 3 shows analytical results regarding new insights gained from relaxing the constant hazard function underlying the Calvo assumption and implications for inflation gap persistence is also discussed; in section 4, I simulate the DSGE model with different setups and identify the most important feature in generating inflation gap persistence; section 5 contains some concluding remarks.

4.2 Inflation Persistence in the Data

Whelan [2007] has documented that U.S. inflation in the post-WWII periods is highly persistent when measured by the sum of autocorrelation coefficients of inflation level and the coefficient of lagged inflation in the reduced-form Phillips curve. Based on this evidence, he rejected the general-pricing hazard model as a valid model for inflation dynamics. However, it is important to distinguish the inflation gap persistence from the inflation trend persistence, because sticky price models are really designed to explain the short-run dynamics of inflation gap which are caused by the collective pricing behavior of firms in the economy, instead of the dynamics of trend inflation which are mainly determined by the central bank's monetary policy targets.

Recently, there are a growing number of studies on inflation persistence controlling a drifting trend inflation. Authors[4] document using both U.S. and European data that, when correctly accounting for the time-varying trend inflation, various measures of inflation gap persistence fall significantly. Here I present evidence on inflation gap persistence using the U.S. CPI data. In addition, I report results controlling different measures for trend inflation.

I estimate two measures of inflation persistence using the U.S. time series data from 1960 Q1 to 2007

[4] See: e.g.Levin and Piger [2003], Altissimo et al. [2006], Cogley and Sbordone [2008] and Cogley et al. [2008]

Q4[5]. First, following Levin and Piger [2003], I calculate the sum of AR coefficients as a measure of overall inflation persistence (Andrews and Chen, 1992). Second, I estimate a reduced-form Phillips curve by including real driving forces into the regression. This reduced-form inflation regression distinguishes inflation persistence driven by its own lags[6] from those imparted by persistent real driving forces. The reduced-form inflation regression is specified in the following form and I report the coefficient ρ as the measure of inflation persistence

$$\pi_t = \eta + \rho\pi_{t-1} + \sum_{i=1}^{3} \beta_i \Delta\pi_{t-i} + \sum_{i=0}^{3} \eta_i y_{t-i} + u_t. \tag{4.1}$$

To construct inflation gap, we need to first calculate measures of the inflation trend. Since there is no standard way to do it in the literature, I first choose a naive method to detrend inflation by the Hodrick-Prescott (H-P) filter. The biggest limitation of this method, however, is that the H-P filter is only based on the univariate process. As argued by Cogley and Sbordone [2008], when the trend inflation is nonzero and drifting over time, then it should also depend on other real variables, such as the trend of real marginal cost. To account for this feature of the data, they proposed to estimate a VAR model with drifting parameters and stochastic volatility for four variables - output growth rate, the log of unit labor cost, inflation and the nominal discount factor. After that, they calculate an approximation of trend inflation by defining it as the level to which inflation expectation settles in the long run. Following the same methodology, I construct CPI inflation trend for the periods between 1960 Q1 to 2007 Q4[7]. In Figure (4.1), I plot the two measures of trend inflation. In the left panel, we observe that the two trends differ substantially. While the H-P trend (dashed line) follows closely to actual inflation, the Cogley-Sbordone trend (hereafter: C-S trend) is much more moderate. The median estimate of trend inflation rose by roughly 1% at the annual rate during 1970's and fell back to around 1.3% in the early 80's, then stayed relative stable until 2007. On the right panel, I compare the two trends more closely. As portrayed by the two dash lines, the 90% confident interval of estimated C-S inflation trend is quite wide, especially during the volatile periods in 1970's. It indicates a great deal of uncertainty about trend inflation associating with the C-S method. Even through the H-P trend is substantially different to the C-S trend, it lies within the confident interval for the most of sample periods. Due to this reason, in Table (4.1), I report measures of inflation gap persistence for both H-P and C-S trend inflation.

The first row of the table indicates which definition of inflation is used to calculate the measures of

[5] I download data from the database FRED maintained by the Federal Reserve Bank of St. Louis. I calculate the inflation rate by using the Consumer Price Index data for all urban consumers: all items and seasonally adjusted (Series: CPIAUCSL). The monthly data is first converted into quarterly frequency by arithmetic averaging and then the annualized Inflation rate is defined as $400 \times \ln\left(P_t/P_{t-1}\right)$. Furthermore, to measure the real inflationary pressures, I first construct data of real output gap per capita, which is based on the Real GDP (Series: GDPC1). They are in the unit of billions of chained 2005 dollars, quarterly frequency and seasonally adjusted. To calculate real GDP per capita, I use the Civilian Non-institutional Population (Series: CNP16OV) from the Bureau of Labor Statistics. The monthly data in the unit of thousands is first converted into quarterly frequency by arithmetic averaging. The real GDP per capita is defined as: $\ln\left(GDP_t \times 1,000,000/POP_t\right)$. Finally real output gap per capita is obtained by detrending the data by the Hodrick-Prescott filter. In addition, I download the unit labor share for non-farm business sector (Series: PRS85006173) from the U.S. Bureau of Labor Statistics as a measure of real marginal cost.

[6] It is denoted as the intrinsic inflation persistence by some authors, e.g.: Sheedy [2007]

[7] For calculating this inflation trend, I implement the MATLAB codes provided by Timothy Cogley and Argia M. Sbordone on their website.

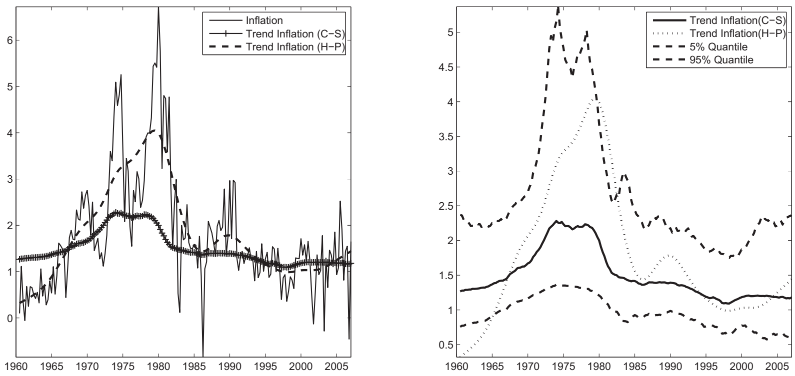

Figure 4.1: Measures of Trend Inflation

	Inflation level			Inflation Gap (H-P)			Inflation Gap (C-S)		
Sample	AR	$\rho(\hat{y})$	$\rho(LS)$	AR	$\rho(\hat{y})$	$\rho(LS)$	AR	$\rho(\hat{y})$	$\rho(LS)$
$1960-2007$	0.887 (0.041)	0.897 (0.041)	0.882 (0.046)	0.559 (0.082)	0.479 (0.095)	0.548 (0.084)	0.825 (0.051)	0.849 (0.053)	0.807 (0.055)
$1960-1985$	0.902 (0.048)	0.895 (0.047)	0.906 (0.051)	0.659 (0.094)	0.574 (0.109)	0.642 (0.103)	0.858 (0.056)	0.873 (0.058)	0.850 (0.063)
$1986-2007$	0.491 (0.145)	0.494 (0.155)	0.475 (0.153)	0.064 (0.185)	0.013 (0.200)	0.062 (0.187)	0.376 (0.16)	0.364 (0.172)	0.378 (0.165)

Note: Numbers in the parenthesis are the standard deviations.

Table 4.1: Empirical Results based on the Inflation Data

persistence. I report results for inflation level, inflation gap detrended by the H-P filter and inflation gap detrended by the Cogley and Sbordone method. Under each label, three measures of inflation persistence are presented, i.e. the sum of autocorrelation coefficients AR, the coefficient of lagged inflation in the reduced-form Phillips curve when the real driving force is measured by H-P detrended real output per capita $\rho(\hat{y})$, and the coefficient of lagged inflation in the reduced-form Phillips curve when the real driving force is measured by the unit labor share $\rho(LS)$. The first noteworthy result from the table is that the CPI inflation was indeed highly persistent over the subsample from 1960 to 1985. It fell dramatically, however, after the Volcker disinflation of 1980's. This finding is consistent with what is found in the literature. Second, the magnitude of inflation gap persistence crucially depends on the measure of trend inflation. When the H-P trend is used, inflation gap persistence is significantly lower than that in the inflation level. It becomes even insignificant from zero during the second subsample. By contrast, when the C-S trend is used, inflation gap persistence is lower, but much closer to the measured inflation level persistence. It is instructive to compare the C-S trend with two extreme cases of inflation detrending, namely the mean detrending and the detrending by the H-P filter. While the mean detrending does not change the inflation persistence at all, the H-P detrending reduces it to the greatest extent. The multivariable-based C-S method gives values between these two extreme cases. Even through it is not very accurate, one can still draw conclusion from this evidence that the true inflation gap persistence is significant and positive and inflation gap persistence constitutes a large part of inflation persistence. In the later section, I will use the C-S measure of inflation gap persistence as the benchmark for evaluating the performance of the theoretical model.

In the light of these results, we can sum up some stylized facts of inflation gap persistence. 1. Inflation gap persistence constitutes a large part of inflation persistence in the U.S. CPI data. 2. CPI inflation gap is highly persistent during periods between 1960 to 1985. The sum of coefficients on lagged inflation lies in the range around 0.85 with the standard deviation of 0.06. 3. inflation gap persistence reduces significantly after the Volcker disinflation period. The sum of coefficients on lagged inflation reduces to around 0.37 with the standard deviation of 0.16.

4.3 The Model

In this section, I use the same model developed in the previous chapters to analyze the persistence of inflation gap found in the U.S. data. The main feature of the model is the incorporation of a general price reset hazard function into an otherwise standard New Keynesian model.

4.3.1 New Keynesian Phillips Curve

In Chapter 2, I derived the general-hazard NKPC as follows:

$$\begin{aligned} \hat{\pi}_t = & \sum_{k=0}^{J-1} \frac{\theta(k)}{1-\theta(0)} E_{t-k} \left(\sum_{j=0}^{J-1} \frac{\beta^j S(j)}{\Psi} \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} \sum_{j=i}^{J-1} \frac{\beta^j S(j)}{\Psi} \hat{\pi}_{t+i-k} \right) \\ & - \sum_{k=2}^{J-1} \Phi(k) \hat{\pi}_{t-k+1}, \quad where\ \Phi(k) = \frac{\sum_{j=k}^{J-1} S(j)}{\sum_{j=1}^{J-1} S(j)}, \ \ \Psi = \sum_{k=0}^{J-1} \beta^j S(j). \end{aligned} \tag{4.2}$$

The general-hazard NKPC differs from the standard NKPC in two aspects. First, the general-hazard NKPC has not only current and forward-looking terms but also lagged variables and lagged expectations. In addition, all coefficients in the new NKPC are nonlinear functions of price reset hazard rates $\left(\alpha_j = 1 - h_j\right)$ and the subjective discount factor β . Thereby, short-run dynamics of inflation gap are affected by both the shape and magnitude of the price reset hazard function.

Even though the general-hazard NKPC looks very different compared to the Calvo NKPC, they share the same economic intuition. However, only because of the more general form of the hazard function, the general-hazard NKPC has a richer dynamic structure. To understand the economic intuition of this class of models, we need to categorize its dynamic components and exam the effect of each component on inflation. The general-hazard NKPC can be decomposed into three parts: 1) all forward-looking and current terms, 2) Lagged expectations and 3) lagged inflations. In the following analysis, I represent these three parts with short-hand symbols $E_t(.)$, $E_{t-j}(.)$ and $\hat{\pi}_{t-k}$ respectively and $W_x(h_j)$ denotes coefficients of those terms. Furthermore, by definition, inflation is equal to the log difference between two consecutive aggregate prices and the aggregate price in the period t can be further written as the distributed sum of current and past optimal reset prices. As illustrated in the following expressions (4.3), these three dynamic components of the general-hazard NKPC affect inflation through current reset price, past reset prices and past aggregate price respectively.

$$
\begin{array}{lccccc}
\hat{\pi}_t = & & & \hat{p}_t & - & \hat{p}_{t-1} \\
\hat{\pi}_t = & \overbrace{\boxed{\theta(0)\hat{p}^*_t} + \boxed{\theta(1)\hat{p}^*_{t-1} + \cdots + \theta(J-1)\hat{p}^*_{t-J-1}}} & & & - & \hat{p}_{t-1} \\
 & \Uparrow & & \Uparrow & & \Uparrow \\
\hat{\pi}_t = & W_1(h_j)E_t(.) & + & W_2(h_j)E_{t-j}(.) & - & W_3(h_j)\hat{\pi}_{t-k}
\end{array}
\tag{4.3}
$$

The economic reasons why those three components should show up in the general-hazard NKPC is that: first, the current and forward-looking terms - $E_t(.)$ - enter the Phillips curve through their influence on the current reset price. As same as in the Calvo sticky price model, the price setting in this model is forward-looking. The optimal price decision is based on the sum of current and future real marginal costs over the time span the reset price is fixed. The only difference now is that the time horizon of the pricing decision is not infinite, but depends on the hazard function. Second, due to price stickiness, some fraction of past reset prices continue to affect the current aggregate price. Lagged expectational terms -$E_{t-j}(.)$- represent influences of past reset prices on current inflation. Last, past inflations enter the NKPC, because they affect the lagged aggregate price $\hat{p}_{t-1}$. The higher the past inflations prevail, higher the lagged aggregate price would be, and thereby it deters current inflation to be high.

The new insights gained from this analysis is that the two new dynamic components have opposing effects on inflation through $\hat{p}_t$ and $\hat{p}_{t-1}$ respectively. The magnitudes of these effects depend on the price reset hazard function. In the general case, they should be different to each other. Conversely, in the Calvo case, the constant hazard function causes reset prices to exert the same amount of impact on both $\hat{p}_t$ and $\hat{p}_{t-1}$, and thereby causes lagged expectations and lagged inflation to be canceled out.

This cancellation can be also seen in the derivation of the Calvo NKPC:

$$\begin{aligned}
\hat{p}_t &= (1-\alpha)\sum_{j=0}^{\infty}\alpha^j \hat{p}^*_{t-j} \\
&= (1-\alpha)\left[\hat{p}^*_t + \alpha\hat{p}^*_{t-1} + \alpha^2\hat{p}^*_{t-2} + \cdots\right] \\
&= (1-\alpha)\hat{p}^*_t + \underbrace{(1-\alpha)\left[\alpha\hat{p}^*_{t-1} + \alpha^2\hat{p}^*_{t-2} + \cdots\right]}_{=\alpha\hat{p}_{t-1}} \\
\hat{p}_t &= (1-\alpha)\hat{p}^*_t + \alpha\hat{p}_{t-1} \\
&\vdots \\
\hat{\pi}_t &= \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\widehat{mc}_t + \beta E_t(\hat{\pi}_{t+1}).
\end{aligned}$$

The crucial substitution from line (3) to line (4) is only possible, when the distribution of price durations takes the form of a power function. In conclusion, we learn that, lagged inflation and lagged expectations are not extrinsic to the time-dependent sticky price model. They are missing in the Calvo setup only because of the restrictive constant-hazard assumption.

4.3.2 Implications for inflation gap persistence

The purely forward-looking NKPC is often criticized for generating too little inflation gap persistence(See: e.g. Fuhrer and Moore, 1995). In response to this challenge, the hybrid NKPC has been developed to capture the positive dependence of inflation on its lags (See:e.g. Gali and Gertler, 1999 and Christiano et al., 2005). According to this strand of the literature, inflation persistence should be interpreted as 'intrinsic' and the dependency between current and lagged inflation is mechanically modeled as a fixed primitive relationship, which is independent of changes in monetary policy. By contrast, the generalized Calvo sticky price model, such as the one introduced in the previous section, captures this backward-dependency of inflation in a more micro-founded way. Unlike the hybrid models, inflation gap persistence in this framework is the result of two counteracting channels. The first channel gives lagged inflation a direct role, which works through the past aggregate price. I call it the "front-loading channel" because it weakens inflation gap persistence, and its magnitude is purely determined by the price reset hazard function. By contrast, the second channel is an indirect one, where lagged inflation affects current inflation only through the expectational terms in the NKPC, I name it the "expectation channel". In this channel, lagged inflations have positive coefficients when lagged inflations act as the leading indicator of other variables. Because, in the general equilibrium, the expectation formulation is determined by the whole setup of the model, the magnitude of the "expectation channel" is not only affected by the price reset hazard function, but also by the other general equilibrium forces, such as aggregate demand side of the economy and monetary policy.

$$\begin{aligned}
\hat{\pi}_t &= \underbrace{W_1(h_j)E_t(.) + W_2(h_j)E_{t-j}(.)}_{Expectation\ Channel} \quad - \quad \underbrace{W_3(h_j)\hat{\pi}_{t-k}}_{Front-loading\ Channel} \\
& \qquad\qquad\qquad\qquad\qquad \downarrow \qquad\quad \searrow \qquad \downarrow \\
\pi_t &= \sum_{i=0}^{I}\gamma_i mc_{t-i} \quad + \quad \sum_{i=1}^{I}\rho_i\pi_{t-i} \quad + \epsilon_t.
\end{aligned}$$

In the light of these results, the general-hazard NKPC preserves the economic rationale of the standard Calvo NKPC for inflation gap persistence, which is in stark contrast to those from the hybrid NKPC. First of all, inflation gap persistence can not be interpreted as 'intrinsic'. Instead, more persistence is inherited from the additional moving-average terms of real driving forces introduced by the expectations. The positive coefficient on lagged inflation in the reduced-form Phillips curve results from the correlation between lagged inflation and other variables in the general equilibrium, and therefore it is not a real economic behavioral relation, but a "statistical illusion". More importantly, since the coefficient on lagged inflation depends on the whole model, changes in any part of the general equilibrium setup ultimately affects its value. Therefore, if this theory is the origin of the NKPC, then the hybrid NKPC is subject to the Lucas critique (Lucas, 1976), and thereby can be only used in the monetary policy analysis with great caution.

Overall, inflation gap persistence in this framework is the result of these two counteracting channels. Whelan [2007] has proved that, in the partial equilibrium setting, the net effect of these two opposing forces is always negative, regardless of the form of the hazard function. He further showed that, even in the general equilibrium, the general-hazard sticky price model fails to replicate the positive backward-dependence of inflation. My numerical analysis reveals, however, that it is the 4-period-Taylor-contract hazard function that gave rise to this result. When I use an empirically based hazard function, the simulated data can account well for the inflation gap persistence I find in the U.S. aggregate data after the Volcker disinflation period.

4.3.3 The General Equilibrium Analysis

In this section, I study the behavior of inflation dynamics in the general equilibrium setup. For this purpose, I close the model by adding the aggregate demand side of the economy and a monetary policy rule. The log-linearized equilibrium equations are summarized in the following table:

Aggregate Supply:

$$\hat{\pi}_t = \sum_{k=0}^{J-1} W_1(k) E_{t-k} \left(\sum_{j=0}^{J-1} W_2(j) \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} W_3(i) \hat{\pi}_{t+i-k} \right) - \sum_{k=2}^{J-1} W_4(k) \hat{\pi}_{t-k+1}$$

$$\widehat{mc}_t = (\phi + \delta)\, \hat{y}_t - (1 + \phi)\, \hat{z}_t$$

$$\hat{z}_t = \rho_z * \hat{z}_{t-1} + \epsilon_t \quad where \;\; \epsilon_t \sim N(0, \sigma_z^2)$$

Aggregate Demand:

$$E_t \left[\hat{y}_{t+1}\right] = \hat{y}_t + \tfrac{1}{\delta} \left(\hat{\imath}_t - E_t \left[\hat{\pi}_{t+1}\right]\right)$$

or:

$$\hat{y}_t = \hat{m}_t - \hat{p}_t \quad and \quad \hat{m}_t = \delta \hat{y}_t - \tfrac{\beta}{1-\beta} \hat{\imath}_t$$

Monetary Policy:

$$\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + q_t, \;\; q_t \sim N(0, \sigma_q^2)$$

or:

$$\hat{m}_t = \hat{m}_{t-1} - \hat{\pi}_t + g_t \quad where \;\; g_t \sim N(0, \sigma_g^2)$$

Where all variable are expressed in terms of log deviations from the non-stochastic steady state. The weights (W_1, W_2, W_3, W_4) in the general-hazard NKPC are defined in the equation (4.2). $\hat{m}_t$ is the real money balance, and g_t denotes the growth rate of the nominal money stock. The aggregate demand block is motivated either by the standard household intertemporal optimization problem outlined in the model section or by the quantity equation of money[8]. The monetary policy is specified in terms of either a nominal money growth rule[9] or a simple Taylor rule.

Calibration

In the calibration of the general equilibrium model, I choose some common values for the standard structural parameters. For the preference parameters, I assume $\beta = 0.9902$, which implies a steady state real return on financial assets of about four percent per annum. I also assume the intertemporal elasticity of substitution $\delta = 1$, implying log utility of consumption. The Frisch elasticity of the labor supply is set to be 0.5, a value that is motivated by using balanced-growth-path considerations in the macro literature. In addition, I choose the elasticity of substitution between intermediate goods $\eta = 10$, which implies the desired markup over marginal cost should be about 11%.

Since the main purpose of the paper is to study the impact of the hazard function on inflation gap persistence, I calibrate the hazard function as follows: My first hazard function takes the form of $\{0, 0, 0, 1\}$, which is motivated by the 4-period-Taylor-contract theory. This hazard function is used in the general equilibrium analysis of Whelan [2007]. Alternatively, I refer to the empirical finding by Yao [2010], who

[8] In this case, model has not enough structure to pin down the relationship between real marginal cost and output gap. To make the results quantitatively comparable, I assume, in this case, that real marginal cost holds the same relationship to output gap as in the complete model $\widehat{mc}_t = (\phi + \delta)\, \hat{y}_t - (1 + \phi)\, \hat{z}_t$.

[9] In this case, the money demand equation is derived in the chapter 2, see Equation (2.7).

estimates the aggregate hazard function using the same framework and the same aggregate data set applied in this paper. As seen in the table (4.2) and the figure (4.2), the empirical hazard function differs sharply to the hazard function used in Whelan [2007]. Overall, the aggregate hazard function is first decreasing and then increases slowly with the age of the price. In comparison to the Taylor hazard function, where firms only adjust their prices after 4 quarters, the empirical hazard function highlights two important frequencies of the price adjustment. Additional to the yearly frequency, it is also evidence of a large flexible price setting sector in the economy.

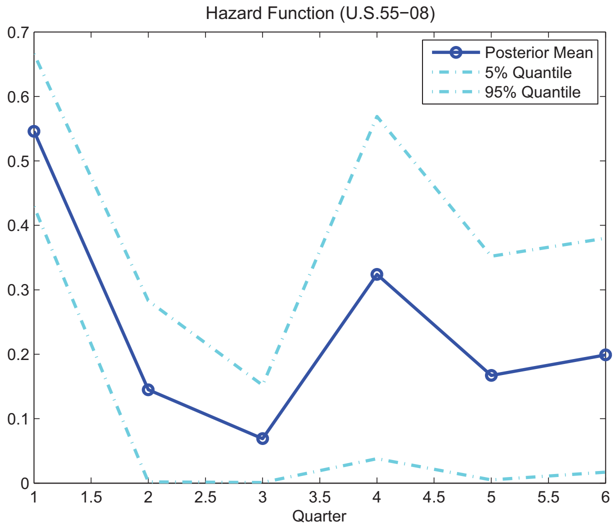

Figure 4.2: Empirical Hazard Function

Hazard function	h_1	h_2	h_3	h_4	h_5	h_6
4-period-Taylor-contract	0	0	0	1	-	-
Yao (2010)	0.55	0.15	0.07	0.33	0.17	0.20

Table 4.2: Hazard Function Calibration

Proceeding with monetary policy parameters, the responses of nominal interest rate to inflation and output gap (ϕ_π and ϕ_y) are chosen at the values commonly associated with the simple Taylor rule. Following Taylor [1993], I set ϕ_π to be 1.5, and the response coefficient to output gap ϕ_y to be 0.5. Finally, I set the standard deviation of the innovation to monetary policy shock to be 25 basic points per quarter.

Numerical Results

To evaluate the quantitative implications of the hazard function for inflation gap persistence, I simulate different setups of the general-pricing-hazard model, then estimate the reduced-form Phillips curve using the artificial data. This Phillips curve regression is specified in the following form

$$\pi_t = \eta + \rho\pi_{t-1} + \sum_{i=1}^{3}\beta_i\Delta\pi_{t-i} + \sum_{i=0}^{3}\gamma_i mc_{t-i} + \sum_{i=0}^{3}\eta_i y_{t-i} + \epsilon_t.$$

I include both output gap and real marginal cost in the Phillips curve regression, because in the theoretical model real marginal cost is the true driving force of inflation and output gap also affect the inflation dynamics through the monetary feedback rules.

	Model Setup			**Sum of AR Coef.**
Model	Hazard Function	Monetary Police	Agg. Demand	ρ
1	4-period-contract	Money growth rule	$\hat{y} = \hat{m} - \hat{p}$	-0.538
2	4-period-contract	Money growth rule	IS curve	-1.068
3	4-period-contract	Taylor rule (1.5,0.5)	IS curve	-0.805
4	Yao (2010)	Money growth rule	$\hat{y} = \hat{m} - \hat{p}$	0.286
5	Yao (2010)	Money growth rule	IS curve	0.242
6	Yao (2010)	Taylor rule (1.5,0.5)	IS curve	0.308
7	Yao (2010)	Taylor rule (2,0)	IS curve	0.217

Table 4.3: Simulation based Empirical Results

In Table (4.3), I report the sum of AR coefficients of lagged inflations (ρ) generated by the simulated data of different theoretical setups. The first three rows are models applying the 4-period-Taylor-contract hazard function. All these models produce negative coefficient on inflation lag, implying no inflation persistence. The benchmark case (Model 1) has the same setup as in Whelan [2007], combining 4-period-Taylor-contract hazard function with the nominal money growth rule and simple aggregate demand equation. In this model, the reduced-form lagged inflation coefficient is negative (-0.538). Model 2 replaces the simple aggregate demand equation with the intertemporal IS curve derived from the household problem. This setup generates a even more negative coefficient on inflation lag than Model 1. In Model 3, I replace the money growth rule with the simple Taylor rule for monetary policy. inflation gap persistence in this case becomes a little stronger than that in Model 2. By contrast, setups using the empirical hazard function (Model 4 to 7) generate realistic inflation gap persistence as we observe in the data from 1986 to 2007. This comparison reveals that it is the unrealistic hazard function that drives the result that leads Whelan to reject the general-pricing-hazard model. From the analysis in the previous section, we know that the hazard function has direct influence on both propagation channels in the general-hazard NKPC. When the magnitude of the second channel is large enough to compensate the negative coefficients introduced by the first channel, the reduced-form Phillips curve reveals a positive backward-dependence of inflation. From the numerical results, it turns out that the hazard function is the most important factor in the complex propagation mechanism of inflation dynamics.

Moreover, other parts of the general equilibrium model plays also a role in determining the magnitude of inflation gap persistence. In contrast to the hazard function, this general equilibrium influence mainly occurs through the expectation channel. Similar to the pattern revealed by the model 1 2 and 3, Model 4, 5, 6 conduct the same numerical experiments under the empirically based hazard function. In the model 4, the reduced-form lagged inflation coefficient is positive (0.286). Model 5 replaces the simple demand equation with the IS curve and generates a slightly less inflation gap persistence than Model 4. The reason why inflation becomes even less persistent is that, with the intertemporal optimizing IS curve, demand shocks are not propagated completely to output gap and inflation dynamics, but they are partially dampened by the rise of real interest rate. So that expectational channel becomes less powerful than the previous case. In Model 6, I replace the money growth rule with the simple Taylor rule. inflation gap persistence in this case becomes a little stronger than that in Model 4. The Taylor rule changes inflation gap persistence, because it introduces an extra channel, through which inflation and real forces feedback to the economy, so that the expectation channel is strengthened. In addition, in Model 7, I apply another Taylor rule with a stronger inflation response parameter and a zero response parameter to output gap. Shutting down the feedback of output gap to the interest rate rule makes the Taylor rule less powerful, so that it performs similar to the money growth rule.

In conclusion, both monetary policy rule and demand side of economy are important in propagating inflation dynamics, but the fundamentally important factor in this mechanism is the hazard function. Using the empirically based hazard function along with the Taylor rule and IS curve (Model 6), the general-pricing-hazard model preforms best in replicating the stylized fact of inflation gap persistence found in the U.S. CPI data from 1986 to 2007. It is not a surprising result, because most macroeconomists agree that monetary policy is well approximated by the simple Taylor rule with coefficients conforming to the Taylor principle during this period of time. In addition, this time span is also characterized by low and stable trend inflation. This character of data validates the use of the general-pricing-hazard model.

4.4 Conclusion

In this paper, I investigate whether the general-hazard NKPC is capable of accounting for the inflation gap persistence. In the empirical part, I find that, after detrending inflation by the Cogley-Sbordone method, inflation gap persistence is still significant and large in the U.S. CPI data. In the theoretical part, I redo the general equilibrium analysis by Whelan [2007], and check robustness of the result to different setups of the model. I find that the general-pricing-hazard model with empirically based price reset hazard function can account quite will for inflation gap persistence found in the data of post Volcker's disinflation periods. The key mechanism at work in this model is the expectational channel in the generalized NKPC, which depends on the setup of the whole model, therefore inflation gap persistence is also not independent of monetary policy. This result directly implies that the hybrid sticky price model is subject to the Lucas critique, and thereby can be only used in the monetary policy analysis with great caution.

5 Lumpy Labor Adjustment as a Propagation Mechanism of Business Cycles

Abstract

This paper explores aggregate effects of micro lumpy labor adjustment in a prototypical RBC model. I first model lumpy labor adjustment in the spirit of Calvo(1983) and extend it by introducing a Weibull-distributed stochastic labor adjustment process to capture the increasing hazard function corroborated by the micro data. My principal findings are: The aggregate labor demand equation derived from the baseline Calvo-style model corresponds to the same reduced form as the quadratic-adjustment-cost model and deep parameters have a one-to-one mapping. However, this result does not hold in general. When introducing the Weibull labor adjustment, aggregate dynamics vary with the slope of the hazard function. In particular, volatility of employment is increasing, but persistence is decreasing in the shape parameter of the Weibull distribution.

5.1 Introduction

Recent evidence from the firm level data shows that firms adjust their labor input discretely at infrequent intervals of stochastic length. Put into other words, labor adjustment exhibits a lumpy and asynchronous pattern. Earlier evidence has been presented by Hamermesh [1989b], Caballero and Engel [1993b] and Caballero et al. [1997b]. More recently, Letterie et al. [2004b] investigate the dynamic interrelation between factor demand with plant-level data for the Dutch manufacturing sector. They find that both adjustments of capital and labor are lumpy, and they are coordinated with each other in time. In addition, Varejao and Portugal [2007] find that large employment adjustments (larger than 10% of the plant's labor force) account for about 66% of the total job turnover, and on average around 75% of all observed Portuguese employer do not change employment over an entire quarter.

This evidence brings difficulty for many widely using models in the RBC literature, which imply either a smoothing or synchronous adjustment at the firm level[1]. In light of this discrepancy, the main question arose is whether modeling the micro lumpiness explicitly changes the model's implication for the aggregate dynamics. To address this issue, (S,s) models are widely used in the literature[2]. The earlier partial equilibrium (S,s) models of labor adjustment (See: e.g. Caballero and Engel, 1993a, Caballero et al., 1997a) found that employment growth depends on the cross-sectional distribution of the employment deviation

[1] For example, the convex-adjustment-cost model (see e.g. Sargent, 1978) implies a smoothing adjustment at the firm level, and the search and matching RBC models (Merz, 1995 and Andolfatto, 1996) give rise to synchronous adjustment behavior.

[2] Caplin and Spulber [1987] was the early work applying the (S,s) approach to macro models.

from optimal target. In particular, Caballero et al. [1997a] found that the adjustment hazard rises with large shocks and thus amplifies the shock's effect in aggregate adjustment. These findings were taken as evidence that lumpy adjustment pattern at firm's level matters for aggregate economy. However, the recent development of the general equilibrium (S,s) models show that this considerable effect of lumpiness at the plant level disappears with changes in the equilibrium prices. King and Thomas [2006] construct a general equilibrium (S,s) model of discrete employment adjustment and find that simulation results are 'observationally' equivalent to the quadratic-adjustment-cost model[3].

In this paper, I pursue the business cycle implications of the lumpy labor adjustment from a new perspective, which is motivated by the recent evidence of the empirical hazard function of labor adjustment[4]. Varejao and Portugal [2007] estimated parameters of a Weibull hazard function with the Portuguese employer survey data, they found that the shape parameter lies in the range between 1.174 and 1.309, indicating an increasing hazard function in the elapsed time since last adjustment. Motivated by this evidence, my question in this chapter is whether the aggregate dynamics are affected by the shape of the labor adjustment hazard function?

To see the connection between the statistical hazard function and lumpy labor adjustment, I embed a general stochastic labor adjustment process in a prototypical RBC model. The essence of the model is that, when making labor adjustment probabilistic on the time since the last adjustment, at the firm level, reoptimizing labor input becomes forward-looking and heterogeneous across different vintage groups. In these circumstances, aggregation mechanism(the distribution of labor vintages), which is also affected by the hazard function, matters for the aggregate behavior, so that the propagation mechanism of the model is significantly enriched.

To formalize this idea, in the benchmark model I introduce the firm's stochastic labor adjustment in the spirit of Calvo [1983], which implies that the underlying labor adjustment process is characterized by a constant hazard function. As a result, even though the 'front-loading' effect helps amplify the volatility of labor at the micro level, the large labor adjustment is neutralized by the restrictive aggregation mechanism implied by the Calvo-style labor adjustment. To this end, I show analytically that the aggregate labor demand equations derived from the Calvo-adjustment model and the quadratic-adjustment-cost model correspond to the same reduced form, and deep parameters have a one-to-one mapping of each other. With these results I confirm the finding by King and Thomas [2006] discussed above.

In the second part of the paper, I extend the baseline model to a more general case, in which a Weibull-distributed labor adjustment process is implemented to capture the increasing hazard function. This extension has an impact on both the persistence and the magnitude of business cycles. When calibrating the model with the empirically plausible hazard function, adjustment probabilities vary across labor vintages. The longer a firm remains inactive, the more likely it adjusts its labor in the current period. As a result, heterogeneous labor dynamics emerge naturally from the underlying labor adjustment process, and as shown in the numerical results, the model matches several important aspects of the U.S. business cycles. In particular, the model can jointly account for features observed in both micro and macro labor adjustment data: i.e. at the micro level, labor adjustment exhibits a lumpy pattern in response to the technology shock, while the aggregate employment reacts smoothly and sluggishly. In addition, sensitivity analysis shows that aggregate dynamics vary with the extent of increasing hazard function, e.g., the volatility of

[3] Similar results have been also found in the capital adjustment context. See, e.g., Veracierto [2002] and Thomas [2002].

[4] To quantify the concept of lumpy labor adjustment, Caballero et al. [1997a] used a hazard function in terms of economic deviations from optimal targets.

aggregate labor is increasing, but the persistence is decreasing in degree of the increasing hazard of the labor adjustment.

My model is intrinsically related to the (S,s) approach with respect to many modeling concepts, it contributes to the literature, however, in the sense that it uses a more tractable framework to generate the findings of the general equilibrium (S,s) models, and then it extends the approach in an empirically plausible direction, and show that the micro lumpy labor adjustment could play an important role in propagating business cycles.

The remainder of the paper is organized as follows: Section 1 introduces the baseline model with a staggered employment adjustment at the firm's level ; In section 2, I show some analytical results to reveal the key mechanism underlying the model; Section 3 extends the basic model to the Weibull-adjustment model; and in section 4 I introduce the calibration of model parameters and present simulation results; Section 5 contains some concluding remarks.

5.2 The Baseline Model

In this section, I set up the baseline model in a RBC framework. The main feature of this basic model is to introduce the lumpy labor adjustment in the spirit of Calvo(1983). Even though this modeling idea has been existing for a long time and it is familiar to most researchers in macroeconomics, I formulate it here formally in the context of the statistical duration model, which also serves as the solid theoretical base for the extension in the next section.

5.2.1 Household

There is a continuum of identical households, who are endowed with K_0 units of capital at $t = 0$ and then with one additional unit for each subsequent period of time, which can be spent on either working or leisure. The infinitely-lived representative household chooses consumption, labor supply and investment to maximize the expected discounted utility:

$$U = \max_{\{C_t, L_t, I_t\}} \quad E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(U(C_t) - V(L_t) \right) \right\}. \tag{5.1}$$

The instantaneous utility $U(.)$ and $V(.)$ are bounded, continuously differentiable, increasing and strictly concave in consumption and leisure. I take the following function form for instantaneous utility:

$$U(C_t) - V(L_t) = \frac{C_t^{1-\eta}}{1-\eta} - \chi \frac{L_t^{1+\phi}}{1+\phi} \tag{5.2}$$

In each period, households receive wage income, rental payment for their capital stock and a lump-sum transfer of net profits resulting from firm ownership, which can be spent on consumption and investment in capital stocks. Due to the assumption of complete financial markets, all households can perfectly share their idiosyncratic income risk, so that they consume and invest the same amount. Consequently, the sequence of aggregate budget constraints is given by:

$$C_t + I_t \leq W_t L_t + R_t K_t + T_t \tag{5.3}$$

The capital stock evolves according to the following law of motion:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{5.4}$$

Finally, I impose the transversality condition for the capital stock:

$$\lim_{T\to\infty} E_0 \left[\prod_{t=0}^{T} R_{t,t+1}^{-1}\right] K_{T+1} = 0, \tag{5.5}$$

Based on this setup, the following first order conditions must hold in an equilibrium:

$$\chi L_t^{\phi} C_t^{\eta} = W_t, \tag{5.6}$$

$$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\eta} (r_{t+1} + 1 - \delta)\right], \tag{5.7}$$

5.2.2 Firms

The economy is populated by a continuum of firms, which is normalized to one. Firms operate in a rigid labor market, where some unspecified frictions cause a fixed ratio of firms not to adjust their labor input in each period. In effect, the more rigid the labor market is, the lower the adjustment ratio is, as expected by agents in the market. Due to this rigid labor adjustment process, firms are differentiated with respect to the amount of time that has elapsed since the last adjustment and hence by their stocks of labor force. I index firms by j, corresponding to the "time-since-last-adjustment". I call them hereafter “labor vintages”. Furthermore, given the complete financial market, adjusting firms choose a common target labor adjustment at each period. Firms in any labor vintage share an equal amount of employment, and hence the state of the economy can be summarized by the vintage index j with the corresponding labor stock ($l_{j,t}$).

Stochastic Labor Adjustment Process and Distribution of Firms

Now I formally introduce the staggered labor adjustment process in the context of the statistical duration model.

Here I consider a process in which the firm’s employment adjustment occurs randomly over time. It turns out that under some basic assumptions with respect to independence and uniformity in time, this random process is governed by the Poisson process[5]. This assumption simplifies the real-world continuous factor adjustment decisions in terms of a sequence of generic trials that satisfy the following assumptions:

- Each trial has two possible outcomes, called adjustment and non-adjustment.
- The trials are memoryless, i.e. the outcome of one trial has no influence over the outcome of another trial.
- For every firm, the probability of adjusting is $1-\alpha$ and the probability of non-adjusting is α.

[5] In this paper, as I write the model in the discrete-time, the discretized adjustment process follows the Bernoulli trials process, which is the discrete version of the Poisson process.

Formally I define the labor adjustment process as a Bernoulli process as follows:

Given the factor adjustment process follows the Bernoulli process, the probability of receiving zero adjusting signal in an interval of j periods is:

$$Pr(0) = \begin{pmatrix} j \\ 0 \end{pmatrix} (1-\alpha)^0 \alpha^j = \alpha^j \qquad \text{for } j = 0, 1, 2, ... \tag{5.8}$$

And, the probability that a duration spell terminates at the period j is

$$Pr(j) = (1-\alpha)\alpha^{j-1} \qquad \text{for } j = 0, 1, 2, ... \tag{5.9}$$

Define $\Theta = \{\theta(j)\}_{j=0}^{\infty}$ as the distribution of firm over labor vintages. It can be easily shown that $\theta(j) = (1-\alpha)\alpha^j$ for $j = 0, 1, 2, ...$[6].

The hazard function corresponding to the Bernoulli process is:

$$H(j) = \frac{\theta(j)}{1 - F(j)} = 1 - \alpha \tag{5.10}$$

The hazard function embedded in the Bernoulli distribution is constant. It implies that the probability of adjusting is independent of the period time elapsed. The aggregate stock of labor can be summed up with respect to the distribution of firms over labor vintages, i.e. the aggregate labor is the weighted sum of all past optimal labor demands, and weights are equal to the probability density function over vintages j.

Finally aggregate labor is obtained by[7]:

$$L_t = \sum_{j=0}^{\infty} \theta(j) l_{j,t} = \sum_{j=0}^{\infty} (1-\alpha)\alpha^j l_{j,t} \tag{5.12}$$

Since the fraction of firms that adjust their employment is randomly drawn across the population, it follows that the recursive law for aggregate employment is obtained by:

$$L_t = (1-\alpha) l_{0,t} + \alpha L_{t-1} \tag{5.13}$$

or equivalently,

$$\Delta L_t = L_t - L_{t-1} = (1-\alpha)(l_{0,t} - L_{t-1}) \tag{5.14}$$

This equation reveals the partial adjustment nature of this model, that the actual job turnover is only a fraction of the optimal adjustment. The speed of adjustment depends on the extent of market rigidity

[6] Because, by assumption there is $1-\alpha$ fraction of firm in the group zero, and α percent of them goes to group one, this gives the density of group one to be $(1-\alpha)\alpha$. Similarly, α percent of untis in group one goes to group two, so the density of group two is $(1-\alpha)\alpha^2$, and so on.

[7] Note that equation 5.18 implies that firms in the vintage j group must also use same amount of capital. Thus the distribution of plants over labor is the same as over capital stocks. As a result, we can aggregate capital in the same way.

$$K_t = \sum_{j=0}^{\infty} (1-\alpha)\alpha^j k_{j,t} \tag{5.11}$$

$(1-\alpha)$. When no friction exists in the labor market ($\alpha = 0$), all firms re-optimize their labor by $l_{0,t}$, then this model is reduced to the standard RBC case.

Capital Market and Technology

Furthermore I assume that firms can access an instantaneous rental market for capital, which is supplied by households in any given period. This assumption is desirable because the firm's first order condition requires the capital and labor ratio to be identical in the entire economy[8], the instantaneous capital market makes possible for those firms that can not change their employment to fulfill this requirement. The aggregate capital stock, however, is still predetermined by the household.

Firms use a decreasing-return-to-scale technology to produce output[9].

$$y_t = Z_t l_t^a k_t^b - \iota \qquad \text{and} \qquad a+b<1 \tag{5.15}$$

Where (ι) denotes the fixed cost of operation, which is equal to the profits earned in the steady state. Consequently, firms expect zero profit and thus the number of firms is constant in the long run.

Z_t summarizes the aggregate productivity shock, which consists of a trend component $\bar{Z}_t$ and a realization of a stochastic process z_t. The trend component $\bar{Z}_t$ evolves at a constant growth rate g, while z_t follows an AR(1) process in logs:

$$Z_t = \bar{Z}_t z_t, \tag{5.16}$$

$$\text{where} \qquad z_t = z_{t-1}^{\varsigma} e^{v_t}, \qquad \text{and} \qquad v_t \sim i.i.d.N(0;\sigma^2)$$

Firm's optimization Problem

In spite of heterogeneous nature of the problem, the firms' maximization problem can be written in a representative fashion: a typical firm maximizes the expected discounted real value of all future profits by choosing nonnegative values for current optimal labor $l_{0,t}$ and a sequence of optimal capital stocks $\left\{k_{j,t}\right\}_{j=0}^{\infty}$, taking the real wage w_t and real rental rate r_t as given.

$$\max_{l_{0,t},\left\{k_{j,t}\right\}_{j=0}^{\infty}} \quad V_t = \sum_{j=0}^{\infty} E_t\{\tilde{\beta}_{t,t+j}\, \alpha^j [F\left(l_{0,t}, k_{j,t}\right) - w_{t+j} l_{0,t} - r_{t+j} k_{j,t+j}]\} \tag{5.17}$$

where $\tilde{\beta}_{t,t+j}$ is the stochastic discount factor, which is defined according to equation (5.7).

Since, at the steady state, all real variables except for labor grow at rate g along the balanced growth path, I will work with detrended variables without changing the notions from now on.

[8] This is the case when the production function is constant-return-to-scale, however, when assuming decreasing-return-to-scale, as shown in equation(5.18), a power function of labor and capital depends only on the rental rate and aggregate shocks, hence it should be identical for all firms in the economy.

[9] In the equation (5.21), it shows that some extent of DRTS is needed to show the lumpy effect at the firm level. However, my main numerical results do not crucially depend on this assumption.

First order conditions for the firm's optimization problem are:

$$r_t = f_k(j,t) = b\, z_t \frac{l_{j,t}^a}{k_{j,t}^{1-b}} \tag{5.18}$$

$$\sum_{j=0}^{\infty} \alpha^j E_t[\tilde{\beta}_{t,t+j}(a z_{t+j} l_{0,t}^{a-1} k_{j,t+j}^b)] = \sum_{j=0}^{\infty} \alpha^j E_t[\tilde{\beta}_{t,t+j}(w_{t+j})] \tag{5.19}$$

Eqation (5.19) shows that the optimal labor demand is determined by balancing all future discounted marginal benefits of adding one more worker (marginal product of labor) and the marginal costs of having a worker (real wage). This condition contrasts to the standard RBC case, where real wage is equal to labor productivity period by period. Due to the labor adjustment friction the optimal labor demand in this model becomes forward-looking.

To reveal the model's implication for the optimal labor demand at the firm level, I derive the firm's optimal employment demand by combining first order conditions and solving for the plant's optimal labor demand $l_{0,t}$ at period t:

$$l_{0,t}^{\frac{1-a-b}{1-b}} = ab^{b/1-b} \frac{\sum_{j=0}^{\infty} \alpha^j E_t[\tilde{\beta}_{t,t+i} z_{t+j}^{1/1-b} / r_{t+j}^{b/1-b}]}{\sum_{j=0}^{\infty} \alpha^j E_t[\tilde{\beta}_{t,t+i} w_{t+j}]}, \tag{5.20}$$

Equation(5.20) shows that, at the firm level, the optimal labor demand reacts to all future shocks and the equilibrium prices. In the case of the first-order-approximation, it is increasing in all expected future shocks z_{t+j} and decreasing in all expected future prices w_{t+j} and r_{t+j}. In the partial equilibrium, where prices are constant, it is easy to show that a positive persistent shock will make the individual labor adjustment higher than that in the frictionless economy. Firms hire more labor than they currently need to hedge the risk they might not be able to re-optimize it in the near future and vice verse for the negative shocks. It is the 'front-loading' effect of the labor demand under the uncertainty in the labor adjustment process. Note that the magnitude of the front-loading effect is dependent on the labor rigidity parameter α. The larger the value of α is, the higher weight is attached on the expectations of future variables. In another words, when frictions in the labor market are more severe, the labor demand is more sensitive to the future economic state.

5.2.3 Equilibrium

Given an exogenous stochastic process for aggregate technology shocks and the common knowledge of the firms' distribution across vintage groups Θ, I define the competitive equilibrium as a set of stochastic processes of endogenous variables $\{Y_t, C_t, L_t, l_{j,t}, k_{j,t}, I_t, K_t, w_t, r_t\}_{t=0}^{\infty}$ such that:

1. Given K_t and the market prices $\{w_t, r_t\}_{t=0}^{\infty}$, the sequences $\{C_t^s, L_t^s, I_t^s\}_{t=0}^{\infty}$ [10]solve the representative household's problem (5.1) subject to (2)-(5).
2. Given $\{w_t, r_t\}_{t=0}^{\infty}$, $\left\{l_{j,t}, k_{j,t}\right\}_{t=0}^{\infty}$ solve the Firms' problem (5.17) subject to production technology (5.15) and exogenous technology shock process (5.16).

[10] Here, superscript s denotes "supply"; Similar notation d for "demand"

3. Aggregate demands for employment L_t^d and capital K_t^d are determined by (5.12) and (5.11) respectively.
4. Markets clear: $L_t^s = L_t^d = L_t$ in labor market, $K_t^s = K_t^d = K_t$ in capital market and $C_t + I_t = Y_t$ in the goods market.
5. Finally, market's equilibrium determines the equilibrium real wage and rental rate $\{w_t, r_t\}_{t=0}^{\infty}$

5.3 Analysis

5.3.1 Dynamic Labor Demand Equations

To gain further intuition of the firm's behavior, I log-linearizing the FOCs (5.18) and (5.19) around the non-stochastic steady state[11]. In contrast to the other partial adjustment model, the Calvo-adjustment model implies different labor demand behaviors at different aggregation levels.

$$\hat{l}_{0,t} = \alpha\beta E_t[\hat{l}_{0,t+1}] - b\frac{(1-\alpha\beta)}{1-a-b}\hat{r}_t - (1-b)\frac{(1-\alpha\beta)}{1-a-b}\hat{w}_t + \frac{1-\alpha\beta}{1-a-b}z_t \tag{5.21}$$

Equation (5.21) reveals that at the firm level optimal adjustment is forward-looking and a trade-off exists between the weights assigned to the current shock and future shocks. When α is large, firms put more weight on future shocks than on current shocks.

Together with equation (5.13), the aggregate labor demand equation is obtained by:

$$\alpha\beta\kappa E_t[\hat{l}_{t+1}] - (1+\alpha^2\beta)\kappa\hat{l}_t + \alpha\kappa\hat{l}_{t-1} - b\,\hat{r}_t - (1-b)\hat{w}_t + z_t = 0 \tag{5.22}$$

where $\kappa = \frac{(1-a-b)}{(1-\alpha)(1-\alpha\beta)}$. The aggregate labor demand (5.22) exhibits more complex dynamics, which are not only dependent on the forward-looking component, but also on the lagged labor. Moreover, it demonstrates that equilibrium prices work here as a counter factor to the technology shock. In this equation, one can explicitly see that, when the aggregate technology shock, real wage and interest rate all rise by 1%, then the total effect of those changes on the aggregate labor are exactly cancelled.

Note that both equations require some degree of decreasing-returns-to-scale ($1 - a - b > 0$) to ensure that the size of labor demand is determined.

5.3.2 Equivalence to the Quadratic-adjustment-cost Models

The quadratic-adjustment-cost model has lost footing in macroeconomic literature because economists have grown disenchanted with its smoothing and synchronous implication relating to the firm-level factor adjustment. As discussed in the introduction, mounting micro evidence shows that firms adjust their labor in a discrete and asynchronous fashion. Despite this fact, the quadratic adjustment cost model has been used widely in theoretical and empirical work, because they are easily solved and produce aggregate equations in a form suitable for estimation. By contrast, as I have shown in the equation (5.20), the Calvo-adjustment model can capture lumpy and asynchronous features in firm's labor adjustment, while

[11] Variables with hat are denoted as log deviation from the non-stochastic steady state, such as $\hat{x}_t = logX_t - log\bar{X}$; and the derivation is shown in a technical appendix, which is available upon request.

aggregate labor demand in this model is characterized by a smoothing AR(2) dynamic process (see: Equation 5.22). The key question addressed in this subsection is whether the quadratic-adjustment-cost model is equivalent to the Calvo-adjustment model concerning the aggregate dynamics. If this is true, it can be treated as a reduced form model and is still valid in the empirical work using aggregate data.

In Appendix (6), I derive the aggregate labor demand equation from a textbook quadratic-adjustment-cost model (See e.g. Hamermesh, 1993). As Rotemberg [1987] has shown that the equivalence between the Calvo model and the quadratic cost model in the price adjustment context, it can also be shown analytically that aggregate labor demand equations derived from both models conform to the same reduced form. In addition, the deep parameters of the two models have a one-to-one mapping of each other.

Equation (44) is the dynamic labor demand equation derived from the quadratic adjustment cost model:

$$\gamma\beta E_t[\hat{l}_{t+1}] - [(1-a-b)+\gamma(1+\beta)]\,\hat{l}_t + \gamma\hat{l}_{t-1} - b\,\hat{r}_t - (1-b)\hat{w}_t + z_t = 0$$

where I denote $\gamma = \frac{d\bar{n}}{\bar{w}}(1-b)$.

And it is the dynamic labor demand equation derived from the Calvo-adjustment model:

$$\alpha\beta\kappa E_t[\hat{l}_{t+1}] - (1+\alpha^2\beta)\kappa\hat{l}_t + \alpha\kappa\hat{l}_{t-1} - b\,\hat{r}_t - (1-b)\hat{w}_t + z_t = 0$$

where $\kappa = \frac{(1-a-b)}{(1-\alpha)(1-\alpha\beta)}$

Comparing these two equations, I find that these two equations can be put into the following reduced form equation, so that the aggregate data alone can not differentiate between them.

$$\varphi_1 E_t[\hat{l}_{t+1}] + \varphi_2\hat{l}_t + \varphi_3\hat{l}_{t-1} - b\hat{r}_t - (1-b)\hat{w}_t + z_t = 0$$

When I set $\alpha\kappa = \gamma$, For example, the correspondence among parameters in both models is expressed by equation (5.23). Then the Calvo-adjustment model is equivalent to the quadratic-adjustment-cost model with respect to the aggregation relations and they consequently generate the exact same aggregate dynamics, given that all other aspects of both models are equal.

$$\frac{d\bar{n}}{\bar{w}} = \frac{\alpha(1-a-b)}{(1-\alpha)(1-\alpha\beta)(1-b)} \tag{5.23}$$

Note that both parameters d and α govern the rigidity of the labor adjustment process in both models and this equation gives the exact mapping between these two rigidity parameters.

5.4 Extension

In this section, I extend the baseline model to a more general case in which the labor adjustment process is characterized by an increasing hazard function. In particular, I apply the Weibull distribution[12] to model the firm's labor adjustment process. Because of its flexibility, the Weibull distribution is frequently used in statistical analysis of duration phenomena. In fact, it enables the incorporation of a wide range of hazard functions by using various values of the shape parameter.

[12] For detailed discussion on Weibull distribution, see technical appendix (7)

5.4.1 The Weibull-adjustment Model

To integrate the Weibull-labor-adjustment into the RBC framework, I only have to modify the firm's problem, while keeping the household's optimal conditions (5.6) and (5.7) as they are in the baseline model.

I consider an economy with a continuum of perfectly competitive firms, which are differentiated with respect to the time elapsed since their last labor adjustments, indexed by $j \in \{0, J\}$[13]. I assume that the stochastic labor duration follows a Weibull distribution. According to the statistical duration theory, the distribution of firms with respect to time-since-last-adjustment (vintage groups) is summarized by the density function of the Weibull distribution, and the hazard rate in the vintage group j is obtained by:

$$h(j) = \frac{\tau}{\lambda}\left(\frac{j}{\lambda}\right)^{\tau-1} \qquad \forall j \leq J \tag{5.24}$$

where τ and λ are the parameters of the Weibull distribution and j is the amount of time that has elapsed since the last adjustment. Note that this hazard function is increasing when τ is greater than one, thereby the adjustment probability in each vintage is dependent on the vintage index j. The longer a firm remains inactive, the more likely it adjusts its labor in the current period.

When resetting its labor $l_{0,t}^*$ at time t, a firm uses the survival function of the Weibull distribution to access the probabilities that its reseted labor input will remain fixed in the future, and the firm chooses an optimal labor adjustment to maximize:

$$\max_{l_{0,t}^*, \{k_{j,t+i}\}_{j=0}^{J}} \quad V_t = \sum_{i=0}^{J} S(i) E_t\{\tilde{\beta}_{t,t+i}[f\left(l_{0,t}^*, k_{j,t+i}\right) - W_{t+i} l_{0,t} - R_{t+i} k_{j,t+i}]\}$$

where $S(i)$ denotes the probability that firm's newly adjusted labor force will survive for i periods in the future, which is obtained by the formula $S(i) = 1 - F(i) = exp\left(-\left(\frac{i}{\lambda}\right)^{\tau}\right)$.

The first order necessary condition gives us the optimal labor demand:

$$l_{0,t}^{*\,\frac{1-a-b}{1-b}} = ab^{b/1-b} \frac{\sum_{i=0}^{J} S(i) E_t[\tilde{\beta}_{t,t+i} z_{t+i}^{1/1-b} / r_{t+i}^{b/1-b}] \sum_{i=0}^{J} S(i) E_t[\tilde{\beta}_{t,t+i} w_{t+i}]}{,} \tag{5.25}$$

Equation(5.25) has the same form as in the baseline model, except that the survival function is now a more complex function of the elapsed inactive time. This change enriches the labor dynamics of the model, but as the same time it also puts a challenge to the computation of the solution.

I log-linearize equation(5.25) for the labor demand as follows[14]:

$$\hat{l}_{0,t}^* = \tilde{\alpha}\beta E_t[\hat{l}_{0,t+1}^*] - \frac{b}{\Psi(1-a-b)}\hat{r}_t - \frac{1-b}{\Psi(1-a-b)}\hat{w}_t + \frac{1}{\Psi(1-a-b)} z_t \tag{5.26}$$

Analog to the Calvo-adjustment model, $\tilde{\alpha}$ governs dynamic properties of the labor demand. Given my calibration values of the model's parameters, $\tilde{\alpha}$ is equal to 0.75, which is slightly less than its counterpart (0.77) in the Calvo-adjustment model. As in the baseline model, the optimal labor adjustment is increasing in all expected future shocks z_{t+j} and decreasing in all expected future prices w_{t+j} and r_{t+j}, and thus the

---

[13] $J = \lambda\left(\frac{\lambda}{\tau}\right)^{1/\tau-1}$ is the maximum number of vintage groups, which is obtained through equaling the hazard rate of the group J to one ($H(J) = 1$).

[14] The derivation of this equation is shown in the Appendix (8).

'front-loading' effect is also at work here. It is important to note that the parameters in this equation nest those in the corresponding equation (5.21), where the hazard function is constant.

To aggregate the labor demand, I use a two-stage aggregation scheme. First I define a dummy sectoral labor demand as $\hat{l}_{j,t}$, which is the sum of labor demand in a labor vintage before reshuffling firms into the new vintage groups, and let $\alpha_j = 1 - h(j)$ denote the probability of non-adjusting.

$$\hat{l}_{j,t} = (1 - \alpha(j))\, \hat{l}^{*}_{0,t} + \alpha(j)\, \hat{l}_{j,t-1} \tag{5.27}$$

In equation (5.27), we can see that the heterogeneous sectoral labor demands arise as a result of the non-constant hazard function. Because the hazard rates $\alpha(j)$ are disparate across vintages due to the increasing-hazard function, each vintage labor group is composed of the optimal labor adjustments ($\hat{l}^{*}_{0,t}$) and the lagged sectoral labor demand with different compositions. As a result, heterogeneity in labor emerges naturally from the underlying labor adjustment process in this economy. Given the increasing hazard rate in the time-since-last-adjustment, the labor demand in the younger labor vintage is more persistent, but less volatile than those in the older labor vintage.

At last, the aggregate labor demand can be derived by using the sectoral labor demand and the Weibull density function:

$$\hat{l}_t = \int_0^J \theta(j) \hat{l}_{j,t}\, dj. \tag{5.28}$$

Equation (5.28) reveals that, given the heterogeneous nature of the economy, the aggregation mechanism plays an important role in forming aggregate dynamics. In this model dynamics properties in the different labor vintages are divergent, and their contributions to the aggregate behavior depend on their weights that are given by the distribution of labor vintages $\theta(j)$.

5.5 Calibration and Simulation Results

In this paper, I investigate quantitative significance of lumpy labor adjustment as a propagation mechanism for business cycles. In order to address this question properly, I follow the tradition of RBC literature and calibrate my optimal growth model such that it is consistent with long-run growth facts in U.S. data, and then study its short-run dynamics by investigating the statistical properties of simulated time series and impulse responses functions. In the following sections, I address the calibration method for this model and then present the quantitative results and impulse response functions.

5.5.1 Calibration

For most parameters in the model, I take the standard values in the RBC literature. As for special parameters of the Weibull distribution, I refer to evidence of empirical studies using micro employment data.

For the quarterly discount rate β I use 0.9902 to reflect that the real rate of interest in the U.S. economy is around 4% per annum. The depreciation rate δ is 0.025, indicating an annual rate of 10%. Given these two values, I select the capital share b to be 0.329 to match the average capital-output ratio of 2.353 (Thomas and Khan, 2004), and the labor share of output a is set to be 0.58, which is consistent with direct estimates for the U.S. economy. (King et al., 1988).

As to the preference parameters, I choose $\phi = 0.25$ implying that the average household allocates one quarter of the time to productive activities (Benhabib and Farmer, 1992), and $\sigma = 1$, which gives rise to a log utility function for consumption.

The labor adjustment parameter is calibrated according to empirical work estimating the hazard function using aggregate net flow data. Caballero and Engel [1993a] used U.S. manufacturing employment and job flow data (1972:1-1986:4) to estimate the constant hazard function. Their results suggest that on average, 22.9% of firms in the U.S. adjust their employment per quarter. As a result, I choose 0.77 as the value for α in the baseline model, which implies that the mean duration of employment is 4.35 quarters.

The Weibull parameters are set as follows: In the standard case, I set the shape parameter τ to be 1.2, implying an increasing hazard function. This value is based on Varejao and Portugal [2007], in which they found that the shape parameter is in the range between 1.174 to 1.309[15]. Since there is yet no standard value for this parameter in the literature, I will test the sensitivity of my results to the value of τ in the later part of this section. To calibrate the scale parameter λ, I apply the equation (47), implying that the characteristic life of the Weibull distribution is equal to 4.62 quarters, given $\tau = 1.2$ and the average duration of 4.35 quarters.

Finally, I select the values of ς and σ_ϵ for aggregate technology shocks. I choose $\varsigma = 0.95$ and a standard deviation of 0.007, which are estimated parameters of Solow residuals that are commonly used in the RBC literature (King and Rebelo, 2000).

Parameters	Values	Interpretation
β	0.9902	Annual real rate 4%
δ	0.025	Annual depreciation rate 10%
b	0.329	Capital to output ratio of 2.35(Thomas and Khan [2004])
a	0.58	Labor's share of output (King et al. [1988])
η	1	$logC_t$, common in the literature
ϕ	0.25	On average, one quarter of the time are allocated to productive activities(Benhabib & Farmer,1992)
λ	4.62	Average duration of employment of 4.35 quarters ($\alpha = 0.77$)
τ	1.2	Increasing hazard function Varejao and Portugal [2007]
ς	0.95	Solow residual estimate,
σ^2	0.007	Solow residual estimate,

Table 5.1: Calibration Values

5.5.2 Simulation Results

To evaluate the quantitative performance of the Weibull-adjustment model, I apply the log-linear approximation method of King et al. [1988], which produces linear decision rules depending on the state variables, and then solve the rational expectation equilibrium by using the standard algorithm[16].

[15] Since Portuguese labor market emerges as the most regulated in Europe in all existing rankings of indexes of employment protection (OECD,1999), this evidence may be thought of as lower-bounds for the slope of the hazard function.

[16] See: e.g. Blanchard and Kahn [1980]

In table (4)-(6), I report the second moments of U.S. data and those generated by the theoretical models. In all cases, the moments are for HP-filtered time series. For each of these models, three sets of statistics are reported: first, absolute and relative standard deviation; second, contemporaneous correlation coefficients relative to output; and third, the cross correlations with respect to output.

In Table (5.2), I summarize some results regarding variables for the labor market.

Labor	Relative	Cross Correlation with output				
	S.D.	-2	-1	0	1	2
U.S. data(Hours)	0.98	0.54	0.78	0.92	0.90	0.78
U.S. data(Employment)	0.82	0.47	0.72	0.89	0.92	0.86
RBC model	0.47	0.47	0.70	0.98	0.61	0.32
Weibull model	0.45	0.55	0.76	0.96	0.88	0.67
Real wage		-2	-1	0	1	2
U.S. data	0.44	0.58	0.66	0.68	0.59	0.46
RBC model	0.54	0.37	0.64	0.99	0.72	0.49
Weibull model	0.44	0.43	0.69	0.96	0.83	0.68

Table 5.2: Statistics for labor and output

It is well documented in the RBC literature that the standard RBC model fails to match some important aspects of the U.S. business cycle facts. Cogley and Nason [1995] has shown that the standard RBC models fail to account for the observed positive serial correlation in the output growth rate and aggregate labor, and its persistent dynamics rely on the high autocorrelation of the productivity shocks. Introducing stickiness in the labor adjustment improves the model's performance with regard to the persistence of aggregate labor and output. As shown in the table, when propagating the same aggregate technology shocks, the standard RBC model generates low volatile and nonpersistent aggregate labor and output. By contract, with empirically plausible labor rigidity, the Weibull-adjustment model replicate procyclical and persistent labor dynamics.

Moreover, the Weibull-adjustment model can also replicate the stylized facts regarding real wage. As seen in the lower panel of the table (5.2), it implies smoothing real wage even in a Walrasian labor market setting. As discussed in the search and matching literature (e.g.,Shimer, 2005 and Hall, 2005), the real wage rigidity plays an important role in propagating business cycles in the labor market. Because this mechanism is missing in my model, it is not able to replicate highly volatile labor and acyclical real wage. The new insight revealed by this model, however, is that there is a smoothing effect of labor rigidity on real wage. The reason is that in this model the direct link between productivity and real wage is weakened by the forward-looking labor adjustment behavior. As seen in Equation (5.19), all future real wages appear to be the cost for the current labor adjustment, therefore firms have incentive to smooth real wage by their labor demand decisions. These results reveal that stickiness in the labor adjustment can be a source of real wage rigidity.

5.5.3 Impulse Responses

In the figure (5.1), I compares the responses of the Weibull model to a one percent increase in the aggregate technology shock.

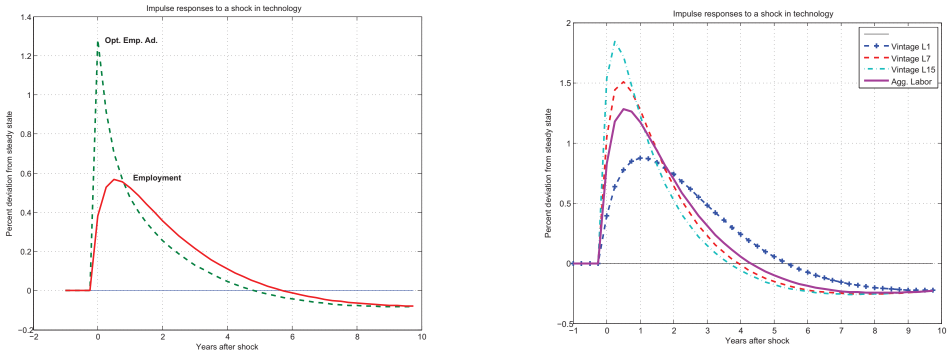

Figure 5.1: Impulse responses comparison

First, in the left panel we can observe that the individual firm's labor adjustment and the aggregate labor respond to the aggregate technology shock differently. While the impulse response of aggregate labor is humped-shaped (The solid line), the labor input at the firm's level reacts to the shock immediately and by a large amount (The dash line). These results illustrate that the lumpy-adjustment models are able to reconcile features observed in both micro and macro labor adjustment data: i.e. at the micro level, labor adjustment exhibits a lumpy pattern in response to the technology shock, while the aggregate employment reacts smoothly and sluggishly.

These results manifest the unique feature of the lumpy adjustment model in propagating business cycles. Different to other partial adjustment models, the Calvo-type assumption does not necessarily lead to the dampened volatility of labor dynamics. At the firm level, it generates strong lumpy labor adjustment through the 'front-loading' effect. The intuition is as follows. The firm's optimal demand depends on expectations of all future prices and shocks. Suppose that in some period t firms experience a positive productivity shock, some firms are labor-adjustment constrained, so they have to increase their demand of capital in the rental market, while, on the supply side, the household's capital stock is predetermined. This leads to an increase in interest rates for the whole economy and rises household savings. On the other hand, those labor-unconstrained firms will adjust labor more than they currently need in order to hedge the adjustment-risk in the future. This in turn drives real wage up. Put them together, all those rises in productivity and prices can be expected by rational agents, so that the adjusting firms will, in addition to their risk-hedging motive, demand even more workers. Moreover, if labor supply is elastic, rise in the interest rate triggers the intertemporal substitution effect in the labor supply side, because real wage is higher today and wage tomorrow is discounted at a higher rate, the household is willing to enjoy less leisure today thus supply more labor. Consequently, both labor and investment rise sharply at the micro level. However, at the aggregate level, this strong effect is to a large extent neutralized by the underlying aggregation mechanism.

To further illustrate the important role played by the heterogeneous labor and the aggregation mechanism in this model, I show in the right panel of the figure (5.1) the impulse response functions of aggregate labor along with the responses of labor in different vintage groups. Recalling the aggregate labor demand equation (5.28), the aggregate labor is a weighted average of vintage labor demands, where the weights correspond to the probability density function of the Weibull distribution. This can be visualized in this figure. The aggregate labor (the solid thick line) is composed of the sectoral labor from different vintages (Dashed lines). As discussed in the previous section, given the increasing hazard rates, the labor demand in the younger labor vintage is more persistent, but less volatile than those in the older labor vintage. IRFs of the sectoral labor vary from the persistent but less volatile younger vintage labor (The vintage $L1$) to the volatile but less persistent older vintage labor (e.g. the vintage $L15$).

5.5.4 Sensitivity Analysis

Now I use numerical results to test how sensitive my results are in response to the key parameter τ, which measures the shape of the Weibull distribution. In Table (5.3), I report the relative volatility of aggregate labor to output and the first-order autocorrelations of aggregate labor that are generated by a wide range of values of the shape parameter[17].

The shape parameter τ	1.1	1.3	1.5	1.7	1.9	2.2
Relative S.D. to y_t	0.446	0.455	0.476	0.484	0.496	0.50
Autocorr. $Corr(L_t, L_{t-1})$	0.89	0.88	0.87	0.86	0.83	0.81

Table 5.3: Sensitivity Analysis for τ

In general, I find that the value of the shape parameter exerts an important influence on the aggregate labor dynamics. As the shape parameter increases, the relative volatility of labor to output rises, while the persistence of labor decreases. These results confirm the intuition of the model, in which the higher is τ, the less likely firms sustain a fix amount of labor for a long period of time, and hence the labor market is less rigid. On the other hand, with the increasing value of τ, the economy becomes more heterogeneous with respect to the labor adjustment risk. In Figure(5.2), we can see that as the value of τ increases, the hazard function becomes steeper, which implies a trade-off between the probability of adjusting today and the probability of adjusting later. If I use the extent of changes in the hazard rates to measure the economic risk in the labor market, then the economic risk associated to the high value of τ is higher than that in the Calvo case, where the probabilities of adjusting are equal. In another words, economic risk is high in the sense that, for a given time horizon, the volatility of hazard rates is larger. Consequently, firms will adjust more to hedge the higher risk in the labor market, and hence the aggregate labor also becomes more volatile. This mechanism serves as an example, in which the aggregation mechanism plays an important role in forming aggregate dynamics when the economy is featured by heterogeneous labor demand.

5.6 Concluding Remarks

In this paper, I embed a stochastic labor adjustment process into a protypical RBC model. The innovation of the model is to apply the statistical duration analysis to extend the well-established time-dependent adjustment scheme in the spirit of Calvo [1983] in a DSGE framework. Using the increasing-hazard Weibull

[17] Here I check the range in which the hazard function of the Weibull distribution is increasing and 2.2 is the maximum value that guarantees an unique stable solution of this dynamic system, given other parameters' value that I specify in the calibration section.

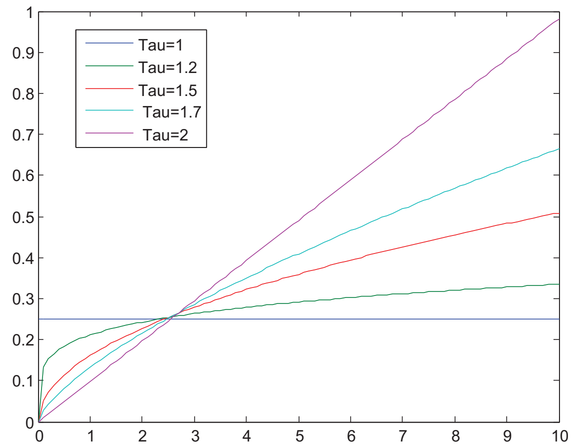

Figure 5.2: Hazard Function with different shape parameters

distribution, the model generates heterogeneous labor vintages, which are different not only in the time of adjustment, but also in terms of the volatility and the persistence of dynamics.

The key message conveyed in this paper is that impediment in the labor adjustment process induces firms to make precautionary labor adjustments, and non-constant hazard adjustment process brings about heterogeneity in the economy. In addition, given the heterogeneous nature of the economy, the underlying aggregation mechanism play a crucial role in forming the aggregate dynamics. My model is an endeavor to illustrate how this mechanism works in propagating realistic business cycle fluctuations.

Appendix of Chapter 2

1 A.1 Deviation of Marginal Cost

I assume that there is an economy-wide competitive labor market, and hence intermediate firms are price takers in this market. In each period, firms choose optimal demands for labor inputs to maximize their real profits given wage and the production technology (2.8).

$$\max_{L_t(i)} \Pi_t(i) = \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} L_t(i) \tag{29}$$

Real marginal cost can be derived from this maximization problem in the form:

$$mc_t(i) = \frac{W_t / P_t}{(1-a)\, Z_t L_t(i)^{-a}}$$

Using the production function (2.8), output demand equation (2.9), the labor supply condition (2.5) and the fact that at the equilibrium $C_t = Y_t$, we obtain the real marginal cost as follows:

$$mc_t(i) = \frac{1}{1-a} Y_t^{\frac{\phi+\sigma(1-a)+a}{1-a}} Z_t^{-\frac{1+\phi}{1-a}} \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\eta a}{1-a}} \tag{30}$$

Because marginal costs depend on the demand of the individual good, the price set by the firm also affects the marginal costs of the firm. Next, firms determine their optimal prices given marginal costs and the market demand for their goods (2.9)

$$\max_{P_t(i)} \Pi_t(i) = Y_t(i) \left(\frac{P_t(i)}{P_t} - mc_t(i)\right)$$

The first order condition for $P_t(i)$ yields:

$$\frac{P_t^*(i)}{P_t} = \frac{\eta}{\eta - 1} mc_t(i)$$

The optimal relative price is equal to the markup multiplied by real marginal cost. By substituting the real marginal cost with equation (30), we get the economy-wide average relative price in the form:

$$\frac{P_t^*}{P_t} = \left(\frac{\eta}{\eta-1}\frac{1}{1-a}\right)^{\frac{1-a}{1-a+\eta a}} Y_t^{\frac{\phi+\sigma(1-a)+a}{1-a+\eta a}} Z_t^{-\frac{1+\phi}{1-a+\eta a}} \tag{31}$$

2 Deviation of the New Keynesian Phillips Curve

Here I derive the NKPC for $g = 1$[18], starting from 2.16

[18] For the case of $g > 1$, derivation is similar.

$$\hat{p}_t^* = E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\left(\widehat{mc}_{t+j}+\hat{p}_{t+j}\right)\right] \tag{32}$$

$$= E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j}\right] + E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{p}_{t+j}\right] \tag{33}$$

The last term can be further expressed in terms of future rates of inflation

$$\begin{aligned}
\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{p}_{t+j} = & \quad \frac{1}{\Psi}\hat{p}_t + \frac{\beta S_1}{\Psi}\hat{p}_{t+1} + \cdots + \frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{p}_{t+J-1} \\
= & \quad \frac{1}{\Psi}\hat{p}_t + \frac{\beta S_1}{\Psi}\hat{p}_t + \frac{\beta S_1}{\Psi}\left(\hat{p}_{t+1}-\hat{p}_t\right) + \cdots + \frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{p}_{t+J-1} \\
= & \quad \left(\frac{1}{\Psi}+\frac{\beta S_1}{\Psi}\right)\hat{p}_t + \sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j} + \frac{\beta^2 S_2}{\Psi}\hat{p}_{t+1} + \cdots + \frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{p}_{t+J-2} \\
= & \quad \left(\frac{1}{\Psi}+\frac{\beta S_1}{\Psi}+\frac{\beta^2 S_2}{\Psi}\right)\hat{p}_t + \sum_{j=1}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j} + \sum_{j=2}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j-1} \\
& + \frac{\beta^3 S_3}{\Psi}\hat{p}_{t+1} + \cdots + \frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{p}_{t+J-2} \\
& \vdots \\
= & \quad \left(\frac{1}{\Psi}+\frac{\beta S_1}{\Psi}+\cdots+\frac{\beta^{J-1}S_{J-1}}{\Psi}\right)\hat{p}_t + \sum_{j=1}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j} \\
& + \sum_{j=2}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j-1} + \ldots + \frac{\beta^{J-1}S_{J-1}}{\Psi}\hat{\pi}_{t+1} \\
= & \quad \hat{p}_t + \sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i}
\end{aligned}$$

The optimal price can be expressed in terms of inflation rates, real marginal cost and aggregate prices.

$$\hat{p}_t^* = \hat{p}_t + E_t\left[\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j}\right] + E_t\left[\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i}\right] \tag{34}$$

Next, I derive the aggregate price equation as the sum of past optimal prices. I lag Equation 34 and substitute it for each $\hat{p}^*_{t-j}$ into Equation 2.17

$$
\begin{aligned}
\hat{p}_t = & \quad \theta(0)\ \hat{p}^*_t + \theta(1)\ \hat{p}^*_{t-1} + \cdots + \theta(J-1)\hat{p}^*_{t-J+1} \\
= & \quad \theta(0)\left[\hat{p}_t + E_t\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j}\right) + E_t\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i}\right)\right] \\
+ & \quad \theta(1)\left[\hat{p}_{t-1} + E_{t-1}\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j-1}\right) + E_{t-1}\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i-1}\right)\right] \\
& \quad \vdots \\
+ & \quad \theta(J-1)\left[\hat{p}_{t-J+1} + E_{t-J+1}\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j-J+1}\right) + E_{t-J+1}\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i-J+1}\right)\right] \\
\hat{p}_t = & \quad \sum_{k=0}^{J-1}\theta(k)\left[\hat{p}_{t-k} + \underbrace{E_{t-k}\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i-k}\right)}_{F_{t-k}}\right] \qquad (35)
\end{aligned}
$$

Where F_t summarizes all current and lagged expectations formed at period t.
Finally, we derive the New Keynesian Phillips curve from Equation 35.

$$
\hat{p}_t = \sum_{k=0}^{J-1}\theta(k)\ \hat{p}_{t-k} + \underbrace{\sum_{k=0}^{J-1}\theta(k)F_{t-k}}_{Q_t}
$$

$$
\begin{aligned}
\hat{\pi}_t = & \quad \sum_{k=0}^{J-1}\theta(k)\ \hat{p}_{t-k} - \hat{p}_{t-1} + Q_t \\
= & \quad \theta(0)\ (\hat{p}_t - \hat{p}_{t-1}) + \theta(0)\hat{p}_{t-1} + \theta(1)\hat{p}_{t-1} + \cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
= & \quad \theta(0)\ (\hat{p}_t - \hat{p}_{t-1}) + (\theta(0)+\theta(1))\ \hat{p}_{t-1} + \theta(2)\hat{p}_{t-2} + \cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
= & \quad \underbrace{\theta(0)}_{W(0)}\hat{\pi}_t + \underbrace{(\theta(0)+\theta(1))}_{W(1)}\hat{\pi}_{t-1} + (\theta(0)+\theta(1)+\theta(2))\ \hat{p}_{t-2}\cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
& \quad \vdots \\
= & \quad W(0)\ \hat{\pi}_t + W(1)\hat{\pi}_{t-1} + \cdots + W(J-2)\hat{\pi}_{t-J+2} + \underbrace{W(J-1)}_{=1}\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
= & \quad W(0)\ \hat{\pi}_t + \cdots + W(J-2)\hat{\pi}_{t-J+2} + \underbrace{\hat{p}_{t-J+1} - \hat{p}_{t-J+2}}_{-\hat{\pi}_{t-J+2}} + \hat{p}_{t-J+2} - \cdots + \underbrace{\hat{p}_{t-2} - \hat{p}_{t-1}}_{-\hat{\pi}_{t-1}} + Q_t
\end{aligned}
$$

$$
\begin{aligned}
(1 - W(0))\hat{\pi}_t = & \quad -(1 - W(2))\hat{\pi}_{t-1} - \cdots - (1 - W(J-1))\hat{\pi}_{t-J+2} + Q_t \\
\hat{\pi}_t = & \quad -\sum_{k=2}^{J-1} \frac{1 - W(k)}{1 - \theta(0)} \hat{\pi}_{t-k+1} + \sum_{k=0}^{J-1} \frac{\theta(k)}{1 - \theta(0)} F_{t-k}
\end{aligned}
$$

The generalized New Keynesian Phillips curve is:

$$
\begin{aligned}
\hat{\pi}_t = & \quad \sum_{k=0}^{J-1} \frac{\theta(k)}{1 - \theta(0)} E_{t-k} \left(\sum_{j=0}^{J-1} \frac{\beta^j S_j}{\Psi} \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} \sum_{j=i}^{J-1} \frac{\beta^j S_j}{\Psi} \hat{\pi}_{t+i-k} \right) \\
& - \sum_{k=2}^{J-1} \Phi(k) \hat{\pi}_{t-k+1}, \quad where\ \Phi(k) = \frac{\sum_{j=k}^{J-1} S_j}{\sum_{j=1}^{J-1} S_j}, \quad \Psi = \sum_{j=0}^{J-1} \beta^j S_j
\end{aligned} \tag{36}
$$

3 Proof for Proposition 1

In the Calvo pricing case, all hazards are equal to a constant between zero and one. Denote the constant hazard as $h = 1 - \alpha$, and substitute it into the NKPC (2.18):

$$
\begin{aligned}
\hat{\pi}_t + \sum_{k=1}^{\infty} \alpha^k \hat{\pi}_{t-k} = & \quad (1-\alpha) \sum_{k=0}^{\infty} \alpha^k E_{t-k} \left((1 - \alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-k} + \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-k} \right) \\
\hat{\pi}_t + \alpha\hat{\pi}_{t-1} + \alpha^2\hat{\pi}_{t-2} + \cdots = & \quad E_t \left((1-\alpha)(1 - \alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
+ & \quad \alpha E_{t-1} \left((1-\alpha)(1 - \alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right) \\
+ & \quad \alpha^2 E_{t-2} \left((1-\alpha)(1 - \alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-2} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-2} \right) \\
& \quad \vdots
\end{aligned} \tag{37}
$$

Iterate this equation one period forward, I obtain

$$
\begin{aligned}
\hat{\pi}_{t+1} + \alpha\hat{\pi}_t + \alpha^2\hat{\pi}_{t-1} + \alpha^3\hat{\pi}_{t-2} \cdots = & \quad E_{t+1} \left((1-\alpha)(1 - \alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i+1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right) \\
+ & \quad \alpha E_t \left((1-\alpha)(1 - \alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
+ & \quad \alpha^2 E_{t-1} \left((1-\alpha)(1 - \alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right) \\
& \quad \vdots
\end{aligned}
$$

Using Equation (37) to substitute $\hat{\pi}_t, \hat{\pi}_{t-1}, \hat{\pi}_{t-2} \cdots$ in the left hand side, I get

$$
\begin{aligned}
&\hat{\pi}_{t+1} + \alpha E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
&+\alpha^2 E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right) \\
&+\alpha^3 E_{t-2} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-2} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-2} \right) \\
&\vdots \\
&= E_{t+1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i+1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right) \\
&+\alpha E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
&+\alpha^2 E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right). \\
&\vdots
\end{aligned}
$$

After canceling out equal terms from both sides of the equation, It yields the following equation:

$$
\hat{\pi}_{t+1} = E_{t+1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i+1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right)
$$

Lag this equation and rearrange it, I obtain the NKPC of the Calvo model.

$$
\begin{aligned}
\hat{\pi}_t &= E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \qquad (38) \\
\hat{\pi}_t &= (1-\alpha)(1-\alpha\beta) mc_t + (1-\alpha)\hat{\pi}_t + \alpha\beta E_t \left(\hat{\pi}_{t+1} \right) \\
\hat{\pi}_t &= \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} mc_t + \beta E_t \left(\hat{\pi}_{t+1} \right)
\end{aligned}
$$

Proof done

Appendix of Chapter 3

4 Data Description

The data used in this paper is taken from the FRED (Federal Reserve Economic Data) maintained by the Federal Reserve Bank of St. Louis.

- Growth rate of real GDP per capita: is based on the Real Gross Domestic Product (Series: GDPC1). They are in the unit of billions of chained 2005 dollars, quarterly frequency and seasonally adjusted. To construct per capita GDP, I use the Civilian Noninstitutional Population (Series: CNP16OV) from the Bureau of Labor Statistics, U.S. Department of Labor. The monthly data is converted into quarterly frequency by arithmetic averaging. Per capita real output growth is defined as: $100 \times [\ln(GDP_t/POP_t) - \ln(GDP_{t-1}/POP_{t-1})]$. Finally the data is detrended by the Hodrick-Prescott filter.
- Inflation rate: is calculated by using Consumer Price Index for all urban consumers: all items (Series: CPIAUCSL), seasonally adjusted. The monthly data is converted into quarterly frequency by arithmetic averaging. Annualized Inflation rate is defined as $400 \times \ln(P_t/P_{t-1})$. Finally the data is detrended by the Hodrick-Prescott filter.
- Nominal interest rate: is the Effective Federal Funds Rate (Series: FEDFUNDS). The monthly data is converted into quarterly frequency by arithmetic averaging. The data is detrended with the trend inflation calculated by using the Hodrick-Prescott filter and then mean adjusted.

5 Figure

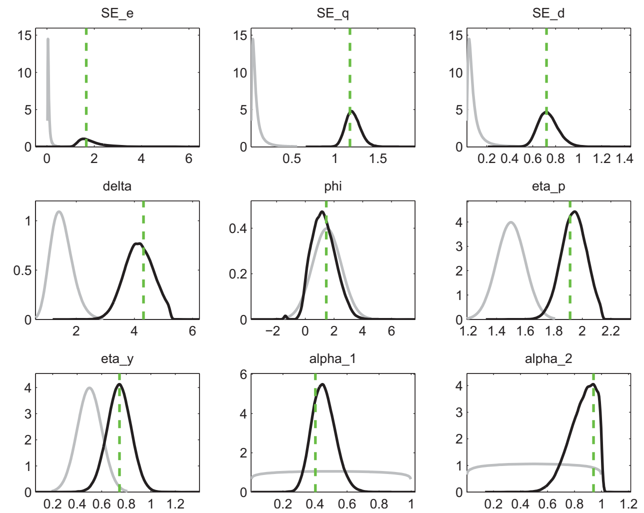

Figure 3: Prior and Posterior Distributions for data 1955-2008 (1)

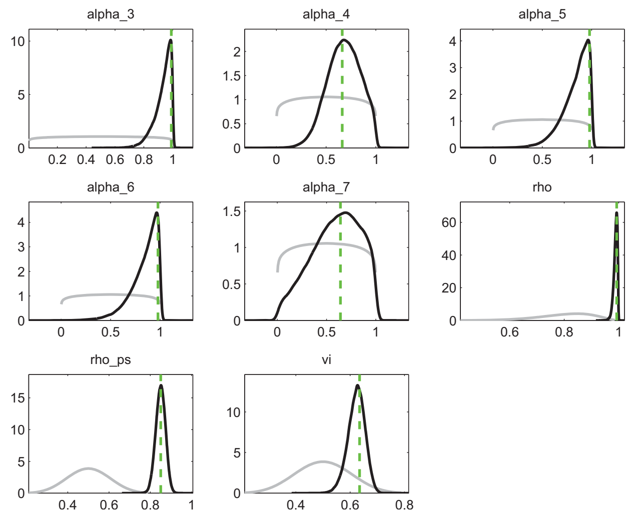

Figure 4: Prior and Posterior Distributions for data 1955-2008 (2)

Appendix of Chapter 5

6 Equivalence of the Partial Adjustment Models

I first derive the aggregate labor demand equation from a textbook quadratic adjustment cost model(See e.g. Hamermesh, 1993).

In this economy, each firm is assumed to maximize the expected discounted real value of all future profits by choosing nonnegative values for optimal sequence of labors l_{t+i} and optimal sequence of capital stocks k_{t+i}, subject to the quadratic labor adjustment costs.

The objective function of firm is:

$$\max_{l_{t+i},k_{t+i}} \quad V_t = \sum_{i=0}^{\infty} E_t\{\tilde{\beta}_{t+i}[F(l_{t+i}, k_{t+i}) - w_{t+i}l_{t+i} - r_{t+i}k_{t+i} - \frac{d}{2}(l_{t+i} - l_{t+i-1})^2]\} \tag{39}$$

where d is denoted as the adjustment cost parameter.

subject to

$$y_t = Z_t l_t^a k_t^b \tag{40}$$

and the total productivity shock Z_t and the household's problem are the same as in the Calvo adjustment model.

The first order conditions are:

$$r_{t+i} = F_K(t+i) = bZ_{\frac{t+i}{l^a_{t+i}}} k_{t+i}^{1-b} \tag{41}$$

$$\tilde{\beta}_{t+i}[F_L(t+i) - w_{t+i} - d(l_{t+i} - l_{t+i-1})] + E_t[\tilde{\beta}_{t+i+1}d(l_{t+i} - l_{t+i-1})] = 0 \tag{42}$$

It follows:

$$a\, Z_{t+i} l_{t+i}^{a-1} k_{t+i}^b - w_{t+i} + \beta\, d\, l_{t+i+1} - d(1+\beta) l_{t+i} + d\, l_{t+i-1} = 0 \tag{43}$$

If I log-linearize these FOCs around the steady state, I get the following dynamic labor demand equation:

$$\gamma\beta E_t[\hat{l}_{t+1}] - [(1-a-b) + \gamma(1+\beta)]\, \hat{l}_t + \gamma\hat{l}_{t-1} - \frac{b\bar{R}}{\bar{r}}\, \hat{R}_t - (1-b)\hat{w}_t + z_t = 0 \tag{44}$$

Where I denote $\gamma = \frac{d\bar{n}}{\bar{w}}(1-b)$.

7 Weibull Distribution

The PDF of Weibull distribution is given by the following expression:

$$Pr(j) = \frac{\tau}{\lambda}\left(\frac{j}{\lambda}\right)^{\tau-1} exp\left(-\left(\frac{j}{\lambda}\right)^{\tau}\right) \tag{45}$$

and the cumulative probability function is:

$$F(j) = 1 - exp\left(-\left(\frac{j}{\lambda}\right)^{\tau}\right) \tag{46}$$

The parameters that characterize the Weibull distribution are the scale parameter λ and the shape parameter τ. The shape parameter determines the shape of the Weibull's pdf function, e.g. when $\tau = 1$, it reduces to an exponential case; while $\tau = 3.4$, the Weibull amounts to the normal distribution. The scale parameter defines the characteristic life of the random process that amounts to the time, at which 63.2% of the firm will adjust their labor. This can be seen with the evaluation of the cdf function of the Weibull distribution at j equaling the scale parameterλ. Then we have, $F(\lambda) = 1 - e^{(-1)} = 0.632$.

Note that it relates to the mean duration $\bar{j}$ according to the following equation:

$$\bar{j} = \frac{1}{\bar{\alpha}} = \lambda\Gamma(\frac{1}{\tau} + 1), \tag{47}$$

where $\Gamma()$ is the Gamma function.

It follows that the hazard function of Weibull distribution is:

$$h(j) = \frac{\tau}{\lambda}\left(\frac{j}{\lambda}\right)^{\tau-1} \tag{48}$$

Note that this hazard is constant when the shape parameter τ equals one, and increasing when τ is greater than one.

8 Derivation of the Dynamic Labor Demand Equation

First, log-linearize Equation 5.25:

$$\sum_{i=0}^{J} S(i)\,\beta^i E_t[(a-1)\hat{l}^*_{0,t} + b\hat{k}_{j,t+i} - \hat{w}_{t+i} + z_{t+i}] = 0 \tag{49}$$

$$\sum_{i=0}^{J} S(i)\,\beta^i E_t[b\hat{k}_{j,t+i} - \hat{w}_{t+i} + z_{t+i}] + (a-1)\,\hat{l}^*_{0,t}\sum_{i=0}^{J} S(i)\,\beta^i = 0$$

Let $\Psi = \sum_{i=0}^{J} S(i)\beta^i$ and rearrange this equation:

$$(1-a)\Psi\,\hat{l}^*_{0,t} = \sum_{i=0}^{J} S(i)\,\beta^i E_t[b\hat{k}_{j,t+i} - \hat{w}_{t+i} + z_{t+i}]$$

Then, substitute out $\hat{k}_{j,t+i}$ with the log-linearized Equation (5.18):

$$(1-a)\Psi\,\hat{l}^*_{0,t} = \sum_{i=0}^{J} S(i)\,\beta^i E_t[\frac{ab}{1-b}\hat{l}_{j,t+i} - \frac{b}{1-b}\hat{r}_{t+i} - \hat{w}_{t+i} + \frac{1}{1-b}z_{t+i}]$$

Note that $\hat{l}^*_{0,t} = \hat{l}_{j,t+i} \quad \forall\, j \in (0, J)$, we obtain:

$$\frac{1-a-b}{1-b}\Psi\,\hat{l}^*_{0,t} = \sum_{i=0}^{J} S(i)\,\beta^i \underbrace{E_t[\frac{1}{1-b}z_{t+i} - \frac{b}{1-b}\hat{r}_{t+i} - \hat{w}_{t+i}]}_{X_{t+i}} \tag{50}$$

$$= S(0)X_t + S(1)\,\beta X_{t+1} + S(2)\,\beta^2 X_{t+2} + S(3)\,\beta^3 X_{t+3} + ...$$

And, iterate Equation 50 one period forward, we obtain:

$$\frac{1-a-b}{1-b}\Psi\,\hat{l}^*_{0,t+1} = \sum_{i=0}^{J} S(i)\,\beta^i E_t[\frac{1}{1-b}z_{t+i+1} - \frac{b}{1-b}\hat{r}_{t+i+1} - \hat{w}_{t+i+1}]$$

$$= S(0)X_{t+1} + S(1)\,\beta X_{t+2} + S(2)\,\beta^2 X_{t+3} + ...$$

$$= \frac{S(0)}{S(1)}S(1)\,X_{t+1} + \frac{S(1)}{S(2)}S(2)\,\beta X_{t+2} + \frac{S(2)}{S(3)}S(3)\,\beta^2 X_{t+3} + ...$$

Multiply both sides of this equation by β:

$$\beta\frac{1-a-b}{1-b}\Psi\,\hat{l}^*_{0,t+1} = \frac{S(0)}{S(1)}S(1)\beta\,X_{t+1} + \frac{S(1)}{S(2)}S(2)\,\beta^2 X_{t+2} + \frac{S(2)}{S(3)}S(3)\,\beta^3 X_{t+3} + ...$$

where $\frac{S(i)}{S(i+1)} = exp\left(\frac{(i+1)^\tau - i^\tau}{\lambda^\tau}\right)$. Given my calibration values of the Weibull parameters, these values can be approximated to be a constant ($\tilde{\alpha}$).

$$\beta\frac{1-a-b}{1-b}\Psi\,\hat{l}^*_{0,t+1} = \frac{1}{\tilde{\alpha}}S(1)\beta\,X_{t+1} + \frac{1}{\tilde{\alpha}}S(2)\,\beta^2 X_{t+2} + \frac{1}{\tilde{\alpha}}S(3)\,\beta^3 X_{t+3} + ...$$

$$\tilde{\alpha}\beta\frac{1-a-b}{1-b}\Psi\,\hat{l}^*_{0,t+1} = S(1)\beta\,X_{t+1} + S(2)\,\beta^2 X_{t+2} + S(3)\,\beta^3 X_{t+3} + ... \tag{51}$$

Substitute (51) into (50), we obtain:

$$\frac{1-a-b}{1-b}\Psi\,\hat{l}^*_{0,t} = X_t + \tilde{\alpha}\beta\frac{1-a-b}{1-b}\Psi\,\hat{l}^*_{0,t+1} \tag{52}$$

And, it follows the equation 5.26, which is introduced in the text.

$$\hat{l}^*_{0,t} = \tilde{\alpha}\beta E_t[\hat{l}^*_{0,t+1}] - \frac{b}{\Psi(1-a-b)}\,\hat{r}_t - \frac{1-b}{\Psi(1-a-b)}\,\hat{w}_t + \frac{1}{\Psi(1-a-b)}\,z_t$$

where $\tilde{\alpha}^{-1} = \frac{1}{J+1}\sum_{i=0}^{J} exp\left(\frac{(i+1)^\tau - i^\tau}{\lambda^\tau}\right)$

Variables	S.D.(%)	Relative S.D.	Cross Correlation with output -3	-2	-1	0	1	2	3
Hours*	1.69	0.98	0.38	0.54	0.78	0.92	0.90	0.78	0.63
Employment*	1.41	0.82	0.22	0.47	0.72	0.89	0.92	0.86	0.73
Real wage	0.76	0.44	0.47	0.58	0.66	0.68	0.59	0.46	0.29
Consumption	1.27	0.74	0.57	0.72	0.82	0.83	0.67	0.46	0.22
Output	1.72	1.00	0.38	0.63	0.85	1.00	0.85	0.63	0.38
Investment	5.34	3.10	0.43	0.63	0.82	0.90	0.81	0.60	0.35
productivity	0.73	0.42	0.44	0.45	0.34	0.34	0.10	-0.09	-0.30

Notes: all statistics are reported in Cooley [1995] Table(1.1)

*: Based on establishment survey.

Table 4: Business Cycle Statistics for the U.S. Economy

Variables	S.D.(%)	Relative S.D.	Cross Correlation with output -3	-2	-1	0	1	2	3
Hours	0.59	0.47	0.29	0.47	0.70	0.98	0.61	0.32	0.10
Capital	0.32	0.26	-0.31	-0.16	0.06	0.36	0.54	0.63	0.65
Real wage	0.67	0.54	0.15	0.37	0.64	0.99	0.72	0.49	0.31
Consumption	0.38	0.31	0.02	0.24	0.53	0.90	0.75	0.60	0.46
Output	1.24	1.00	0.22	0.42	0.68	1.00	0.68	0.42	0.22
Interest rate	0.04	0.04	0.32	0.49	0.70	0.96	0.57	0.27	0.05
Investment	3.84	3.10	0.27	0.46	0.70	0.99	0.63	0.35	0.14
productivity	0.67	0.54	0.15	0.37	0.64	0.99	0.72	0.49	0.31

Table 5: Business Cycle Statistics for the Standard RBC Model

Variables	S.D.(%)	Relative S.D.	Cross Correlation with output						
			-3	-2	-1	0	1	2	3
Labor	0.55	0.45	0.36	0.55	0.76	0.96	0.88	0.67	0.43
Capital	0.37	0.30	-0.29	-0.13	0.09	0.35	0.54	0.66	0.71
Real wage	0.54	0.44	0.20	0.43	0.69	0.96	0.83	0.68	0.51
Consumption	0.42	0.34	0.14	0.37	0.64	0.92	0.79	0.66	0.53
Output	1.22	1.00	0.34	0.55	0.78	1.00	0.78	0.55	0.34
Interest rate	0.04	0.03	0.45	0.62	0.80	0.96	0.66	0.38	0.14
Investment	3.95	3.24	0.40	0.59	0.79	0.99	0.74	0.49	0.26
productivity	0.70	0.57	0.31	0.52	0.75	0.98	0.65	0.42	0.26

Table 6: Business Cycle Statistics for the Weibull-Adjustment RBC Model

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Selbständigkeitserklärung

Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung meiner Dissertation benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

Fang Yao
New York,USA, den 11.10.2010