Three Essays on Skill-Specific Labor Markets, Inequality and Consumption over the Business Cycle

D I S S E R T A T I O N

zur Erlangung des akademischen Grades
doctor rerum politicarum
(Dr. rer. pol.)
im Fach Volkswirtschaftslehre
ingereicht an der
Wirtschaftswissenschaftlichen Fakultät
der Humboldt-Universität zu Berlin

von
F r a u R u n l i X i e - U e b e l e , g e b . X i e , M . S c .
geboren am 29.06.1979 in Chengdu, China

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ingereicht am: 16.02.2011
Tag der mündlichen Prüfung: 15.04.2011
I would like to thank my supervisor Professor Michael C. Burda, Ph.D., for his advice and support. I am grateful for insightful comments from Lutz Weinke, Nikolaus Wolf, Alexandra Spitz-Oener and participants at conferences and workshops in Padua, Vigo, Strasbourg, Istanbul, Melbourne, Milan, Graz, Surrey, Ammersee, Magdeburg, Aarhus, Essen, Glasgow and Kiel, as well as at the Brown Bag Seminar at Humboldt University.

Special thanks are due to my colleagues who have always supported and encouraged me. I received excellent research assistance from Patrick Bunk, Daniel Neuhoff, Stefanie Seele and Felix Strobel. My dear friends Margarita Kalamova, Áron Kiss, Julian Yokoyama and Emily Sullivan Sanford provided brilliant help in correcting my English. All remaining errors are my own.

I appreciate the financial support from the Deutsche Forschungsgemeinschaft through the CRC 649 “Economic Risk”.

It remains for me to express my deep gratitude to my family and my friends, whose warm support and unshaken belief in me have been the best stimulus over the past few years.

There is one person I want to thank most and to whom I dedicate this dissertation: My husband.
Abstract

This dissertation addresses the labor market performance and consumption dynamics of different socioeconomic groups. The first part examines the connection between cyclical variations in skilled and unskilled labor markets. Using a business cycle model with search frictions in skill-specific markets, I find that imperfect substitution between skilled and unskilled labor creates an important channel for variations in the skill-specific markets. Together with a skill-neutral or -biased technology shock, the model generates downward-sloping Beveridge curves in aggregate and skill-specific labor markets.

I extend the study to allow for a dynamic link between the skill-specific labor markets. Human capital investment is determined endogenously and idiosyncratic shocks shift the skilled labor share and change tightness in both skilled and unskilled markets. Upon a neutral shock, the decrease of total unemployment is two-staged: Firstly with a reduction in unskilled unemployment, and then with a sharp decline of skilled unemployment when skill substitution dominates. A larger elasticity of substitution between the two types of labor leads to higher volatility of the model variables and higher correlation between unemployment and vacancies.

The second part studies the link between group-specific consumption growth and its volatility in a framework of heterogeneous agents, under the assumption of a consumption externality. Household preferences are related to the consumption growth volatility through asset holding decisions: The volatility decreases with groups’ patience, and increases with the eagerness to keep up with the group average. Moreover, consumption growth is expected to be positively related to its volatility. This last hypothesis is tested using household data imputed from the German Socio-Economic Panel and the German Income and Expenditure Survey, where a U-shaped relationship is found between nondurable consumption growth and its volatility.

Keywords: business cycle, search frictions, skill substitution, skill-specific labor markets, human capital investment, idiosyncratic shocks, consumption growth, within-group inequality, SOEP, EVS

JEL Codes: D31, D64, D91, E21, E24, E32, J24, J63


Schlüsselbegriffe: Konjunkturzyklen, Suchkosten, Qualifikationssubstitution, qualifikationsspezifische Arbeitsmärkte, Humankapitalinvestitionen, idiosynkratische Schocks, Konsumwachstum, gruppenspezifische Ungleichheit, SOEP, EVS
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1 Introduction

Unemployment is one of the key indicators of economic performance. It is a well-known business cycle stylized fact that unemployment is countercyclical and lagging behind output. Meanwhile, volatility of unemployment over the cycle is an important element in welfare analysis, especially when more detailed aspects of unemployment, such as the skill context, are explored. Empirical evidence shows that skilled and unskilled workers are exposed to different levels of unemployment risks and thus endure different durations of unemployment. In addition to a higher level of unemployment, unskilled workers are also subject to higher employment volatility than skilled workers (Kydland, 1984, Keane and Prasad, 1993). Such discrepancies are at the focus of this dissertation.

In particular, this dissertation studies the labor market performance and consumption dynamics of different socioeconomic groups. Chapter Two and Three are based on two papers on skill-specific labor markets where the effects of imperfect skill substitutability and skill training on unemployment are examined separately. Chapter Four analyses the consumption behavior of heterogeneous households in an environment where the well-being of households is affected not only by their own consumption but also by the consumption of others. There, household preferences and the degree of patience co-influence the equilibrium group consumption growth and volatility.

Previous studies on the long-run trend in the labor market identify the deteriorating position of unskilled workers due to permanent technological shifts. My dissertation complements this research by adding a business cycle perspective of skill-specific unemployment. Its results confirm the inferior situation of the unskilled workers in the labor market.
1 Introduction

1.1 Skills and the Labor Market

As a result of the 2008-2009 recession, the Beveridge curve in the U.S. labor market has shifted outwards, generating a policy debate as to whether it was driven by insufficient aggregate demand or structural problems related to the supply side (see, for example, Mulligan, 2009, 2010, Rogerson and Shimer, 2010, Sterk, 2010 and Farber, 2010). While the well-known negative correlation between unemployment and vacancies is captured by the downward-sloping Beveridge curve, changes in the structural characteristics of the economy can shift the curve inward or outward (summarized in Abraham and Katz, 1986). The unexpected outward shift at the lower end of the Beveridge curve, therefore, could potentially be accounted for by both demand and supply reasons:

- A stagnation of real aggregate labor demand because of insufficient product demand. Many companies posted vacancies in order to avoid net contraction, whereas new vacancies addressing external recruits were limited (Farber, 2010).

- The extended duration of unemployment benefits, which may have encouraged some of the unemployed to search longer (Farber, 2010).

- Weak geographical mobility of the workers. When posted jobs are located somewhere else, instead of directly moving to it, homeowners are bound to their current location because of the low home equity level and the consequent inability to take new mortgage loans for new accommodations (Sterk, 2010).

Can such reasoning also explain the differences between skilled and unskilled unemployment? Does the interaction between the two types of labor and capital investment, namely the skill substitutability, play a role in the diversity of the volatilities? If a channel exists between the skilled and unskilled labor force, how does the transition contribute to skill-specific and aggregate unemployment? Beyond the dynamics in the aggregate labor market, these questions are discussed in Chapter Two and Chapter Three of this dissertation, where the internal structure of the skill-specific markets and the connection between them are elaborated upon.
1.2 Cyclical Skill-Specific Unemployment

The connection between unemployment and skill substitution is worth studying, because skilled and unskilled workers appear to be imperfect substitutes in production. A good example is the work load allocation between a technician and a layman in the construction sector. The technician is proficient in the relevant skills and techniques and has a relatively practical understanding of the theoretical principles. He can operate highly complex equipment. In contrast, without systematic training, a layman fulfills his job mostly without sound technical or theoretical understanding. As there could be some overlapping work between the technician and layman, their different technical backgrounds ensure that they cannot fully substitute for each other in production. The measurement of the degree of substitutability, also known as the elasticity of substitution, has been a task for many empirical studies. Katz and Murphy (1992) find an elasticity about 1.41, while Angrist (1995) estimates this elasticity to be 2. The majority of empirical findings suggests that the elasticity of substitution between skills lies between 1 and 2.

Chapter Two analyzes the theoretical background and consequences of this imperfect substitutability. The constant-elasticity-of-substitution (CES)-nested Cobb-Douglas production function implies that, from the perspective of labor demand alone, skilled and unskilled employment depend on each other. Note that, at this stage, no transition between the skilled and unskilled labor force from the labor supply perspective is allowed for yet. Employing a stylized business cycle model with search frictions in the respective sub-markets, Chapter Two concludes that the imperfect skill substitution creates a channel for variations in the sub-markets. Upon a positive technology shock, demand increases for both skilled and unskilled workers. Initially, more vacancies are created for skilled labor rather than for the unskilled, due to the higher marginal productivity of skilled labor. At a given level of search frictions, the resulting change of skilled labor input is higher than that of the unskilled. Meanwhile, since the unemployment stocks stay unchanged from last period, the relative surplus of skilled vacancies leads to higher market tightness in the skilled market, creating a positive impact upon the skill premium. In total, this positive impact dominates the negative effect from rising relative skill supply, thus slightly increasing the equilibrium skill premium. After reaching the maximum point, the deviation of the skill premium returns slowly to the steady state as firms, being highly cost-sensitive,
are induced to return, at least partly, to unskilled labor. The substitution of skilled over unskilled goes further in recessions, when the skill premium is lower and skilled workers are easier to acquire than in booms. For given search frictions in skilled and unskilled labor markets and without applying the assumption of capital-skill complementarity (such as in Lindquist, 2004 or in Balleer and van Rens, 2009), the model reproduces counter-cyclical relative employment and can generate second-moments consistent with those of monthly data from the U.S. Census Bureau’s Current Population Survey. The substitution effect is even stronger at a larger elasticity of substitution, where unskilled workers suffer to a greater extent from the business cycle turbulence, showing higher volatility in unemployment.

1.3 Endogenous Human Capital Formation

The model in Chapter Two succeeds at generating downward-sloping Beveridge curves in skill-specific labor markets without introducing interaction between the skilled and unskilled labor on the labor supply side. The interaction, through endogenous decisions on human capital investment, is crucial because the skill share in the labor force is affected, and so are the participation, tightness and job finding rates in skill-specific markets. These issues are discussed in Chapter Three, where labor force skill composition is determined by the households endogenously.

The importance of the effect of labor supply can be analogously seen in the long-run labor market performance. Although according to a popular hypothesis, the increase of relative unemployment of the unskilled results from the demand shift toward skilled labor, this shift is too small to explain the unemployment and wage dynamics in Europe and the U.S. from the 1970s to the 1990s, especially when the trends in relative wages and relative employment are somewhat different between the U.S. (as well as the U.K.) and continental Europe.

As in the U.S. and the U.K., both employment and wage situations of the unskilled have been deteriorating during this period, while the trends are much less clear-cut in continental Europe. A consensus has been formed that labor and skill supply play at least as important a role as technology change in determining labor market outcomes. In the
1.3 Endogenous Human Capital Formation

case of the U.S., an acceleration in skill supply in the 1970s as a result of the expansive college enrollment seemed to create profit incentives and induce research and development investments. Skill-biased innovation created such excessive demand for skills that the trends in relative skill supply did not keep pace with relative demand shift, and college premiums rose again in the 1980s. A similar case during the same period in the U.K. shows that government policy can make the relative skill supply less elastic and affect the labor market performance and skill premium. Moreover, such acceleration in the skills bias, as a response to the rapid increase in skill supply, is assumed to be responsible for the rising unemployment of low-skilled workers.

While consensus regards technological progress as the potential cause for real wage differences in Europe, social welfare benefits and labor market institutions such as the wage compression and collective bargaining seem to play a more significant role in determining the long-run equilibrium unemployment rate (Bean, 1994 and Krugman, 1994). It is generally agreed that a more compressed wage structure leads to an increase in unskilled unemployment. However, an increase in the share of skilled workers is found to be negatively correlated with the unemployment of the more educated (Biagi and Lucifora, 2005), suggesting a high relevance of education and training policy. Most importantly, the incentives to acquire skills should be strong enough so that human capital investment and accumulation can be achieved. For many observers Germany presents a positive example because of its strong emphasis on the school system and a comprehensive vocational training system. As a result, the portion of the labor force with middle-level qualification is far higher in Germany than in the U.S., which forms a more flexible basis for the upcoming biased technology change and demand shift. This flexibility enables endogenous skill upgrade much more efficiently and thus, despite the wage compression, there is a much lower unskilled unemployment rate than in the U.S. and the U.K. (Nickell and Bell, 1996, Acemoglu and Pischke, 1999, Pischke, 2001).

As the evidence points to the important role of education and training systems in determining aggregate and skill-specific unemployment, especially in a demand-shift environment, it is gratifying to observe that training for the unemployed has become more prevalent in many industrialized countries and has been embedded in the framework of active labor market (ALM) policies. Moreover, different types of training vary in the timing of
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effects: Job search assistance programs appear to have relatively favorable short-run impacts, whereas classroom and on-the-job training programs tend to show better outcomes in the medium-run than in the short-run (Card, Kluve and Weber, 2010). Comparatively, subsidized public sector employment programs have the least favorable impact estimates – a finding confirming earlier studies from Boone and Van Ours (2004) and Kluve (2006).

Above all, training is key to human capital flows between the labor markets because of its effect upon the skilled and unskilled labor forces and market structure. Dellas and Sakellaris (2003) find that formal schooling is significantly countercyclical, while Sepulveda (2004) finds training after formal schooling (both on- and off-the-job) to be weakly countercyclical and leading the cycle. If training is countercyclical, as these studies indicate, there is a force pushing human capital flow into the skilled labor force; that is, the expected wage premium given current job finding rates in both markets. Together with the realization of shocks to human capital and the outcome of learning-by-doing on the job, the human capital flow impacts the market tightness of both skilled and unskilled, thus changing the equilibrium wage and relative unemployment of skilled and unskilled workers.

In previous human capital related studies unemployment is mostly assumed to be voluntary. For example, Ljungqvist and Sargent (2007a, 2007b, 2008) compare the impact of unemployment insurance and employment protection on the unemployment rate and duration in different model setups. However, in their framework, no endogenous decisions are made by the workers on their own skill accumulation. Observing ALM policies gaining popularity in Europe and the U.S. (starting from the 1980s in Scandinavian countries), one may inquire their potential effects on unemployment, given workers’ decision in attending training programs. Admittedly, recent data shows that unemployed workers are engaged far too little in training and hence suffer from severe loss of human capital. A more encompassing way to approach the problem is to allow for different degrees of human capital depreciation and examine workers’ investment in their own education. Even if there may be little change to the total labor force, the aggregation of single workers’ choices in human capital investment would change the shares of skilled and unskilled labor forces, and subsequently their market tightness. This is exactly the goal of Chapter Three: To explore the role of human capital formation and skill differences in explaining
unemployment. Households’ endogenous decisions on skill accumulation are realized through general training and learning-by-doing and, correspondingly, the volume of the skill-specific labor force and the respective market tightness vary. As the immediate reaction to a positive human capital shock includes a surge in skilled unemployment and a reduction in unskilled unemployment, the changes of relative wage and market tightnesses induce firms to post more skilled vacancies so that at the second stage of the positive human capital shock, skilled workers substitute unskilled workers.

1.4 Consumption Growth and its Volatility

While the incidence of unemployment is skill-relevant, household income and consumption are also affected by the skill level of households. This observation motivates Chapter Four, where the consumption dynamics of heterogeneous households are studied. Indeed, empirical evidence drawn from German data shows that not only the consumption distribution but also household consumption growth and its volatility are significantly related to household head occupation, as well as to household size, age, education level and nationality. This is because households in various socioeconomic groups, defined by the aforementioned characteristics, differ in consumption preferences, be it patience toward future consumption or attitudes toward the associative references. Compared to patience, only until the recent decade has the latter preference, sometimes called “consumption externality”, gained more attention in mainstream economics. Before that, neoclassical reasoning has been typically based on the hypothesis of self-interest; i.e., people are exclusively motivated by their material well-being.

The idea of relative consumption is not new in other scientific fields such as psychology and sociology. As early as 1899, Veblen discussed conspicuous consumption; i.e., lavish spending on goods and services acquired mainly for the purpose of displaying income or wealth. Such purchases are not any more limited to materialistic consumption, but serve as a means of attaining or maintaining social status. In a similar spirit, Duesenberry put forward the Relative Income Hypothesis (1949), which states that individual attitudes to consumption and saving are dictated more by income in relation to others than by the abstract standard of living. Again, the purpose of consumption goes beyond the original
utilization of goods and extends to the signaling of social status.

Psychological and economic studies often show that both absolute and relative consumption matter for individual well-being and behavior (see, e.g., Duesenberry, 1949, Diener et al., 1999, Luttmer, 2005). Individuals’ satisfaction derived from being better than their peers can be interpreted as envy, inequity aversion, aversion of relative deprivation, or a propensity to judge one’s achievement relative to that of others. The “others” here are reference groups, a concept brought about in social psychology early in the 1940s (Hyman, 1942). Depending on the situation, they can be coworkers, relatives, neighbors, or members of clubs and organizations. Moreover, they can also be people who are geographically distant and do not interact with the actor physically. According to Shibutani (1955), reference groups can be: (1) those serving as comparison points, (2) those to which individuals aspire, and (3) those sharing the same perspectives with the individuals. The last category requires common communication channels, each of which gives rise to a separate world or a socioeconomic group. The social worlds can be ethnic minorities, the social elite, a medical association, a theater audience, readers of certain periodicals, or, in today’s context, groups in online social networks such as Facebook. In a word, these associative reference groups realistically represent the individuals’ current equals or near-equals; i.e., they are from the same socioeconomic background. This will be the definition of groups in Chapter Four.

If reference groups matter, households regard the current average consumption of their group as the local norm to set realistic consumption goals. For example, consider inequity aversion: Inequity-averse persons want to achieve an equitable distribution of material resources; i.e., they want to neither surpass nor fall behind others in the reference groups. Therefore, the group mean becomes their benchmark. This setup is slightly different from the case wherein individuals would like to emulate the top households of the group, which coincides with the “aspiring” case in Shibutani’s (1955) definition and would result in a larger deviation from an externality-free economy.

While other peoples’ income can hardly be detected, households can relatively easily observe the lifestyles and infer the consumption levels of others with similar socioeconomic status. Their optimal security holding will adapt accordingly and their consumption smoothing path is different from that in an externality-free world. As a result, their
1.4 Consumption Growth and its Volatility

evaluation of other peoples’ consumption affects group consumption growth inequality. The direction of this effect depends on how exactly households react to their peers’ well-being. Alternatively, this reaction can be interpreted as individuals’ life satisfaction upon the change of their peers’ income. While such an attitude can hardly be identified in empirical data, happiness is often used as a proxy to capture an individual’s utility. Studies based on developed countries find that subjective welfare depends positively on one’s own consumption but negatively on the average consumption level of others nearby (Easterlin, 2001, Blanchflower et al., 2004, Luttmer, 2005). Knies (2010) finds comparable evidence that West Germans are significantly unhappier with their lives if their neighbors are getting richer, implying an urge of the West German households to avoid lagging behind their neighbors, or alternatively, the urge to keep up. This effect is slightly more marked in neighborhoods with presumably more social interactions, so that households may be able to assess more accurately the change of their neighbors’ financial position. Conversely, Fafchamps and Shilpi (2008) find that in Nepal, households in isolated areas care more about what their neighbors consume. Their reasoning is that neighbors in isolated communities can more accurately approximate the relevant reference group than in more mobile urban communities. These observations require economic models to take the social environment into account, whose effects on aggregate consumption dynamics and distribution are heterogeneous according to agents’ socioeconomic background.

The preference for relative consumption can be regarded as a special form of physical consumption or a conceptual consumption separate from physical consumption. Long discussed by sociologists and anthropologists in the field of consumer behavior, it is summarized in Ariely and Norton (2009a) that “physical consumption is used not just to satisfy basic needs but also to signal to ourselves and others our beliefs, attitudes, and social identities”. Therefore, conceptual consumption strongly influences physical consumption, and the possession of a BMW convertible is often only partly due to the need for transport. The concept consumed is the (relative) social status, which dates back to Veblen’s (1899) discussion of conspicuous consumption and Duesenberry’s Relative Income Hypothesis (1949), and accords with the “inequality aversion” in Fehr and Schmidt (1999) and the “social preferences” in Fehr and Fischbacher (2002).

The discussion and confirmation of the importance of relative consumption suggest
Chapter Four shows that one direct result from households’ optimization is that households deviate from their original consumption smoothing path because they are keen to consume the same as the majority in the group. In the case of keeping up with the Joneses, mean and median of group consumption are regarded as the same. The equilibrium result suggests that less patient groups and groups with stronger eagerness to keep up appear to have higher volatility in consumption growth. Moreover, a positive association between group consumption growth and volatility is found; i.e., fast growing groups may also experience high uncertainty. This latter hypothesis can be tested using micro data on household consumption.

Combining two supplementary German micro datasets, the Socio-economic Panel (SOEP) and the Sample Survey of Income and Expenditure (EVS), and exploiting the panel structure of the former and the rich consumption information in the latter, I impute consecutive household consumption and construct consumption growth and volatility. Non-durable consumption appears to have a more significant relationship between growth and volatility than durable consumption, which may be due to better data quality. Overall, household nondurable consumption growth appears to have a U-shaped relationship with volatility. Across the groups, young households experience both high growth and high volatility, while older, larger, and better educated households seem to be exposed to low uncertainty and limited growth. Those with relatively low growth and medium volatility are the older households and those whose heads are foreign-born. The results in Chapter Four therefore help to provide an insight to the divergent consumption patterns of various socioeconomic groups.

1.5 Policy Implications

As the studies in this dissertation outline the weak welfare position of certain social groups, the question for the social planner’s role and the function of welfare redistribution through effective taxation systems should be raised. In the end, aggregate economic indicators can only tell part of the story. As many studies have found, business cycle variation can have a far-reaching effect on some households, especially when they start from
1.5 Policy Implications

Likewise, the simulation experiments in Chapter Two and Three have also shown a variation of the vulnerability of unskilled workers over the business cycle. Skill substitution is persistent and especially strong during recessions, leading to a high unskilled unemployment both in level and volatility. The volatility of labor market variables are subject to the differentiation between the skilled and unskilled and the persistence of the human capital shock.

Training programs provide skill upgrade opportunities to the lower-skilled workers, and consequently preserve the average skill level of the total labor force. The positive effects implied are not only on the skill-specific, but also on the aggregate labor market variables. As found in Card, Kluve and Weber (2010), various ALM programs appear to need different time frames so as to come into effect, while their impacts are highly heterogeneous. The challenging tasks in reality therefore include at least two points: How to identify an effective combination of active labor market policies that help achieve the short-run and longer-run goals simultaneously? And how to set the correct incentive schemes so as to encourage workers, and especially unemployed workers, to participate in training?

Looking at the whole range of ALM policies from a skill-specific perspective enables me to draw additional conclusions as compared to an aggregate approach:

(1) Training marginal unskilled workers has obvious beneficial welfare effects for this special group. The increase of market tightness in the unskilled labor market raises the chance to find a job. In the medium- or long-run, the high-skilled share increases as a consequence, which reduces the tightness on the skilled labor market and accordingly the skill premium. Given the substitutability between skilled and unskilled, this induces a demand shift toward the skilled. Therefore, in aggregate, total unemployment would decline responding to an improvement in training programs.

(2) Investing in training for skilled workers offsets skill depreciation and therefore keeps up the human capital level in both skilled and aggregate labor markets. This can also maintain the human capital potential for innovation and the ability to apply new technologies in production.

(3) A relevant policy question would be: What is the return of a unit investment in train-
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ing in skilled workers compared to that in unskilled workers? While this question cannot be answered within the present framework, I have shown that the skill-specific approach matters, especially in a business cycle context. Future research may therefore extend the present work and compare the marginal returns to training investment in skilled and unskilled labor markets.

(4) My results deliver some hands-on recommendations for the policy maker. Considering the implementation of ALM policies in a skill-specific and business cycle context can improve policy design and performance. Business cycle effects are different for skilled and unskilled workers. Because unskilled workers are especially sensitive to business cycles and undergo more severe conditions in recessions, programs with medium-run effect such as classroom training can prepare them with necessary skills for the time when the labor market recovers. Programs like job search assistance, however, may play a more effective role at the end of recessions. If possible, on-the-job-training programs should be applied to the whole labor force and over the full business cycle, since such can most efficiently make up for the ever present human capital depreciation.

To conclude, coherent social policies are needed to help vulnerable groups to smooth their consumption and partially compensate the loss of permanent income due to long-term inactivity or unemployment. Nonetheless, unemployment and other social subsidies should not be too generous since they can run into the danger of discouraging workers from active job searching and lead to long-term unemployment. Helping the poor but not discouraging them from helping themselves is one of the fundamental tradeoffs of economic policy. There is no “one and only” optimal approach for every country, and successful policies stem mostly from years of experience and practice. Skill-specific, or in a broader context, socioeconomic-group-specific welfare conditions thus require more attention in future economic research and an extension of this dissertation can be studying the optimal taxation policies regarding various socioeconomic groups.
2 Cyclical Skill-Specific Unemployment with Imperfect Substitution of Skills

2.1 Introduction

Over the past three and a half decades, one of the defining characteristics of the U.S. labor market has been the inferior position of low-skilled workers. In addition to the long-run stagnation or even deterioration of real wages and unemployment, lower-skilled groups also seem to be more vulnerable to cyclical fluctuations. As shown in Kydland (1984) and Keane and Prasad (1993), employment of skilled workers is less cyclical than its counterpart for unskilled workers.

Figure 2.1 shows the unemployment rates of “college equivalents” and “high school equivalents” in the U.S. between 1977 and 2005. In line with Autor, Katz and Krueger (1998), college equivalents are defined as those with a college education plus half of those with some college. High school equivalents are those with twelve or fewer years of schooling (or high school diploma or less) plus half of those with some college. Here skill levels are proxied by educational attainment, since skills are difficult to measure. Unemployment rates by educational attainment are only available since 1977 in the census data. The upper panel of Figure 1 shows the persistently higher unemployment level of less educated workers. In the lower panel, where also GDP trend deviation is plotted, it can be seen that the unemployment rates of both groups are clearly countercyclical, while the unemployment rate of high school equivalents is much more volatile than that of college equivalents. The exact means, coefficient of variation and standard deviations of detrended data are reported in Table 2.1.

1 The unemployment rate series and log of real GDP were detrended with a Hodrick-Prescott filter with a lambda of 100.
It is commonly acknowledged that the lower skilled group has endured a worsening trend, with decreasing real wages and a consistently rising unemployment rate. Autor, Katz and Kearney (2005) report that after a slight increase in the 1970s, real wages of high school graduates fell by nearly 10 percent. Between 1979 and 1995 real wages of high school dropouts fell by an even more shocking 19 percent, with a modest recovery period between 1995 and 2003. The unemployment rate of males aged 25-64 with less than 4 years of high school (comparable to high school dropouts) was at 4 percent in 1970, peaked at 11 percent in the early 1990s, and was still 8 percent in 2003. The unemployment rate of high school graduates developed only slightly better, but overall similarly.

Studies on the difference between skill groups have mostly focused on the long-run trend. Different approaches to explain the inferior position of unskilled workers have been employed in the rich literature available on the subject. One focus is on the skill-premium. Acemoglu (1998) notes increasing skill supply as a reason for a change in the job composition. The larger supply of more skilled workers since the 1970s facilitated tech-
2.1 Introduction

Table 2.1: Level and Volatility of Education-Specific Unemployment, U.S., 1977-2005

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>College Equiv.</th>
<th>High School Equiv.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean*</td>
<td>5.53%</td>
<td>3.11%</td>
<td>6.98%</td>
</tr>
<tr>
<td>Std./mean**</td>
<td>0.271</td>
<td>0.204</td>
<td>0.266</td>
</tr>
<tr>
<td>Std. of detrended data***</td>
<td>0.90%</td>
<td>0.55%</td>
<td>1.12%</td>
</tr>
</tbody>
</table>

* Mean of annual unemployment rate in levels. ** Coefficient of variation. *** Standard deviation of HP(100)-detrended annual unemployment rate. Source: U.S. Census Bureau, Statistical Abstract.

Technological inventions favoring skills. Once such inventions come into production, skilled workers have a better position in the labor market than unskilled workers. Capital-skill complementarity is considered another possible explanation for the skill-premium evolution in the three decades after 1960. Arguing that capital equipment is more complementary to skilled workers than unskilled workers, Krusell et al. (2000) relates the increasing demand for skilled workers to increasing stock of capital equipment. Their study shows that the variation of the skill-premium during these three decades can be decomposed into relative skill supply effect and capital-skill complementarity effect. Another focus is on the variance of the unemployment rate. Based on a similar idea of Acemoglu that the endogenous technology change is a response to the increasing skill supply, Gautier (2002) and Pierrard and Sneessens (2003) emphasize the mismatch of skills and job types or the “crowding-out” effect as the reason of the increasing unemployment rate of unskilled workers. Supposing all workers can do simple jobs and only skilled workers are able to perform complex jobs, when the skill supply increases, more skilled workers enter the competition for simple jobs and unskilled workers are thus affected. Another approach is chosen by Cuadras-Morató and Mateos-Planas (2006) who assume imperfect skill-education correlation. Their model aims at examining both wage premium and unemployment rates. They find that a substantial share of the increases in these two in the U.S. between 1970 and 1990 can be explained by skill-biased technology shifts and labor market frictions.

While the studies focusing on skill-premium assume a Walrasian labor market and flexible labor supply, the latter two papers which analyze unemployment use the framework of the stylized Mortensen-Pissarides (MP) search and matching model (Mortensen and...
Pissarides, 1994). In this popular model agents are risk neutral and technology of pro-
duction is unspecified. Partly for these reasons the MP model is very practical for the
homogeneous workers setup. However, if there are different skill levels in the labor force,
relying on the MP model neglects the link between skilled and unskilled workers in the
production process. Indeed, they are proven empirically as imperfect substitutes to each
other (Katz and Murphy, 1992, Card and Lemieux, 2001). Moreover, as the firms in such
economies are single-worker firms and all produce with the same technology, capital is
not considered and thus it is impossible to examine the effects of investment on wages
and unemployment.

These two aspects may play important roles in the relative skill supply and demand as
well as skill-specific unemployment, and thus a micro-founded propagation mechanism
is needed where the households’ choice for education investment and the firms’ problem
are endogenized and specified.

Aiming at both unemployment rates and skill-premium, this chapter provides such a
mechanism by using a multi-worker firm setup and a nested CES production function
with two types of labor as imperfect substitutes, while physical capital joins as comple-
ment to produce. Compared to Lindquist (2004) who uses a capital-skill complementarity
assumption to examine wage inequality over the cycle, I use a standard labor economic
setup. While his model bears the Walrasian property which implies a perfect labor market,
I include search frictions so that both unemployment and skill-premium can be studied
and wage rigidities can be captured by this model.

Focusing on the “between-group” differences, this chapter makes the same assumption
as Greiner, Rubart and Semmler (2004) which states that skilled and unskilled workers
search and match within the skilled and unskilled markets respectively. This simplifica-
tion is supported by empirical evidence: The proportion of college-educated workers in
“non-college” occupations has been small and is declining (Gottschalk and Hansen, 2003),
and so is the proportion of less educated workers in white-collar jobs. Until 2001, only 13.6
percent of all managerial/professional jobs were taken by non-college workers.

The approach taken in the current chapter is based on the stylized RBCM models (real
at embedding labor market frictions in real business cycle models in order to improve
the cyclical properties. The ability of such models is later questioned (Shimer, 2005, Hall, 2005b) on generating the observed cyclical volatility in key variables (such as job vacancies, unemployment, market tightness and job finding probability) under common parameter values. Such critique is based on the assumption of low workers’ cost of working compared to their productivity, which leads to large match surplus and strong wage movements upon productivity shocks. A variable wage dampens the hiring incentive and eliminates the fluctuation in unemployment and vacancies in the model. Responding to Shimer (2005) and Hall’s (2005a, 2005b) critiques, Hagedorn and Manovskii (2008) employ another calibration strategy where the nonmarket returns are high and firm surplus is small. Consequently, firms react strongly in posting new vacancies upon a technology shock and both vacancies and unemployment are more volatile. Though innovative, their calibration strategy can not fundamentally solve the unemployment volatility puzzle, since if the nonmarket returns are high, the response of unemployment to labor-market policy (in particular unemployment insurance) is too large (Costain and Reiter, 2008). Further focus of the discussion on the effectiveness of the RBCM models is thus on reconciling the trade-off between policy effects and cyclical volatilities. Various solutions range from considering wage stickiness (Hall, 2005a), assuming match-specific productivity shock (Costain and Reiter, 2008), differentiating wage formation of new jobs and ongoing jobs (Pissarides, 2009), exploring the labor participation margin (Ebell, 2008), examining the role of payroll taxes and social insurance (Burda and Weder, 2010), to including labor heterogeneity and nonlinearities in the production function (Hagedorn, Manovskii and Stetsenko, 2008).

The last variation is also the perspective taken by the current chapter, whereas the main differences are that Hagedorn et al. (2008) emphasize the complementarity between capital and skilled labor, whereas I focus on the importance of substitutability between skilled and unskilled workers. Another minor difference is that I start with the assumption of variable search intensity by the unemployed, and analyze its effect in the equilibrium. Moreover, while in total the skilled and unskilled labor forces sum up to a fixed number, the shares of the skill-specific labor forces can vary in the future study, which allows skill transition through human capital investment and depreciation.

I study the separate effect of a skill-neutral and a skill-biased technology shock in the
2 Cyclical Skill-Specific Unemployment with Imperfect Substitution of Skills

model so as to compare their respective effects. Another possible shock could be a supply shock to skill, or a permanent structural change of the labor force. However, on the basis of the recent finding of Balleer and van Rens (2009) that such a shock has little effect on the impulse responses to technology shocks, I do not include the shock to supply of skill in this chapter. Calibrated to U.S. data from 1977 to 2004, my model replicates certain stylized facts: Wages are less volatile than labor productivity, and output is more persistent, while a skill-biased shock generates results of better cyclical properties. This finding implies the biasedness of the technology shocks in the last decades. Due to the time-consuming matching process, productivity leads employment over the cycle. The model is able to produce higher volatility of unskilled unemployment, as well as of unskilled vacancies. Downward-sloping Beveridge curves result in each sub-market although the resulting negativity is weaker than the data shows.

The remainder of this chapter is organized as follows: Section 2.2 presents the model and the equilibrium, while section 2.3 contains the calibration. Numerical results and discussions can be found in section 2.4 whereas Section 2.5 concludes.

2.2 The Model

In this section a decentralized equilibrium is derived. The large homogeneous households are composed of skilled and unskilled workers, and each type searches for jobs in the segmented skill-specific labor market $i$, where $i = s$ denotes the skilled market and $i = u$ the unskilled market. There is no mismatch of skills and job types. Households own the capital and rent it to the firms. Firms post vacancies to hire workers and produce with capital, where skilled and unskilled workers substitute for each other imperfectly. The structure of the model is shown in Figure 2.2.

![Figure 2.2: Structure of the Model](image-url)
$\theta_i$ is the market tightness, $v_i$ denotes the vacancies in the respective markets and $u_{i-1}$ the unemployment stocks. As is shown in Figure 2.2, it’s in firms’ production that skilled and unskilled labor interact with each other again. Firms produce with physical capital $k_{i-1}$, skilled labor $n_{i-1}^s$ and unskilled labor $n_{i-1}^u$. Exogenous technology shocks occur to the production process, one skill-neutral (via $A_t$) and the other skill-biased (via $B_t$).

Since the focus of this model is on the business cycle horizon, a balanced growth path is assumed. The labor force structure, which is subject to long-run educational investment, is assumed to be constant. Furthermore, the contemporary gain and loss of aggregate skills of the households are assumed to be equal, and so is the portion of skilled labor.

### 2.2.1 Labor Market: Search and Matching

The labor market is composed of two separate sub-markets for skilled and unskilled workers. Both sub-markets are characterized by the standard search and matching framework, and $i$ stands for $(s, u)$. In the sub-labor market $i$ aggregate stocks of unemployed skilled workers $u_{i-1}^s$ at search intensity $s_i^i$ match with vacancies $v_i^i$ for new jobs by a constant return to scale matching function $M_i^i = m_i^i (v_i^i)^{\theta_i} (s_i^i u_{i-1}^s)^{1-\theta_i}$, where $\theta_i$ is the matching elasticity and $0 < \theta_i < 1$. $m_i^i$ measures the efficiency of matching. Defining the respective labor market tightness as $\theta_i^i = \frac{v_i^i}{u_{i-1}^s}$, workers find jobs at rate $p_i^i = \frac{M_i^i}{s_i^i u_{i-1}^s} = m_i^i (\theta_i^i)^{\theta_i} (s_i^i)^{1-\theta_i}$, and vacancies are filled at rate $q_i^i = \frac{M_i^i}{v_i^i} = m_i^i (\theta_i^i)^{\theta_i-1} (s_i^i)^{1-\theta_i}$. Therefore, it holds that $p_i^i s_i^i = \theta_i^i q_i^i$.

Within the skilled and unskilled labor markets, respectively, a skilled worker earns wage $w_i^s$ when employed, and searches for a job when unemployed. In the next period, she can become unemployed because either her firm has exited the market with probability $\kappa$ or she loses her previous job in the firm with probability $\tilde{\chi}_i^s$. Suppose there is no correlation between these two sources of unemployment. Finally, workers lose their jobs and become unemployed at the rate $\chi_i = \kappa + \tilde{\chi}_i^s - \kappa \tilde{\chi}_i^s$. The skill-specific unemployment rates evolve as

$$\bar{u}_i^s = \chi_i^s \left(1 - \bar{u}_{i-1}^s\right) + \left(1 - s_i^i p_i^s\right) \bar{u}_{i-1}^s.$$  

### 2.2.2 Household

Assume there is a continuum of mass one of identical, infinitely-living households. Each household consists of a large number of individuals who pool their income so as to be in-
Cyclical Skill-Specific Unemployment with Imperfect Substitution of Skills

Supposing all members are able to provide labor, a representative household has a portion $\Delta$ of skilled labor force and $1 - \Delta$ of unskilled labor force.

Among the skilled members, $n_{t-1}^s$ ones work and earn a high wage $w^s_t$, while the rest $\Delta - n_{t-1}^s$ are unemployed and receive value from non-market activities $b^s$. Obviously the contemporary unemployment rate of skilled workers is then $1 - n_{t-1}^s / \Delta$. Similarly, among the $1 - \Delta$ unskilled labor force, $n_{t-1}^u$ work and earn a corresponding wage $w^u_t$, while the rest $1 - \Delta - n_{t-1}^u$ are unemployed and generate value from non-market activities $b^u$. The unemployment rate of unskilled workers is then $1 - n_{t-1}^u / (1 - \Delta)$. Households also own the capital and rent it out to firms at a market rate $r_t$.

The representative household chooses consumption $c_t$, capital investment $i_t$, labor supply and search intensity for both types of labor in order to maximize the sum of the discounted future utilities,

$$\max_{\{c_t, s_{t-1}^s, n_{t-1}^s, i_t, s_{t-1}^u, n_{t-1}^u\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ H(c_t) - G(n_{t-1}^s, n_{t-1}^u, s_{t-1}^s, n_{t-1}^s, i_t) \right]$$

where $c_t$ is consumption, $n_{t-1}^s$ and $n_{t-1}^u$ are skilled and unskilled labor supply respectively, $u_{t-1}^s$ and $u_{t-1}^u$ are unemployed stocks, $s_{t-1}^s$ and $s_{t-1}^u$ are search intensities, and $\beta$ is the common discount factor in the economy. $H$ is an increasing and concave function and $G$ is convex so that their difference is concave in labor inputs:

$$H(c_t) = \ln c_t,$$

$$G(n_{t-1}^s, n_{t-1}^u, s_{t-1}^s, n_{t-1}^s, i_t) = \frac{(n_{t-1}^s + n_{t-1}^u + s_{t-1}^s u_{t-1}^s + s_{t-1}^u u_{t-1}^u)^{1+\frac{1}{\psi}}}{1 + \frac{1}{\psi}}.$$  

The parameter $\psi$ roughly measures the Frisch elasticity of labor supply. Being unemployed alone does not harm agents’ utility, but once the unemployed searches intensively, it is similar to doing a job and thus causes disutility. Therefore the “effective” unemployment enters the utility function in the same way as working.
2.2 The Model

The period-to-period budget constraint of the household is given as

\[ w^s_t n^s_t - 1 + w^u_t n^u_t - 1 + b^s_t u^s_t - 1 + b^u_t u^u_t - 1 + r_i k_{i-1} = c_t + i_t. \] (2.2)

There is no government in this model. Instead of pecuniary unemployment compensation, the households receive \( b^i (i = s, u) \), non-tradable productivities from activities such as home production. The left-hand side is inflow to the households, including wages, capital rental income and the value of being unemployed. Meanwhile, the households consume and invest in physical capital. The assumption is that non-market production (household production) could also contribute to consumption.

Other constraints are

- Capital evolution
  \[ k_t = (1 - \tau)k_{t-1} + i_t, \] (2.3)

- Skilled labor stock
  \[ n^s_t = (1 - \chi^s) n^s_{t-1} + p^s_t s^i_t u^s_{t-1}, \] (2.4)

- Unskilled labor stock
  \[ n^u_t = (1 - \chi^u) n^u_{t-1} + p^u_t s^i_t u^u_{t-1}. \] (2.5)

\( \tau \) in constraint (2.3) is the capital depreciation rate. Constraints (2.4) and (2.5) display the intertemporal labor market transitions. While the existing job matches could be destroyed at rate \( \chi^i \), the unemployed search for jobs at intensity \( s^i \) and would be employed with probability \( p^i_t \). Note when deciding on the optimal search intensity, the household takes the corresponding probability as given. The remained matches and newly formed jobs make up the new labor employment stocks.

The representative household’s problem can be solved by setting up a Lagrangian, where the solutions are characterized by the following Euler equations: The first is the standard intertemporal condition to allocate physical capital investment optimally.

\[ H_{c_t} = \beta E_{t+1} H_{c_{t+1}} [r_{t+1} + (1 - \tau)]. \] (2.6)

The last two Euler equations reflect the households’ optimal searching decisions that
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equate the marginal cost of search to the expected payoff.

\[
G_{s_t} = \beta p^{it}u^{it-1}_{E_t} \{ \text{disutility from job search} \} + \frac{H_{c_{t+1}}}{p^{it}_{t+1}u^{it}_{t+1}} \{ \text{utility from net wage income} \} + \frac{G_{s_{t+1}}}{p^{it}_{t+1}u^{it}_{t+1}} \{ \text{avoided future disutility in searching} \}.
\]

(2.7)

Take equation (2.7) for example: The left-hand side represents the current disutility caused by searching, while the right-hand side shows the compound effect in the next period. With this optimal search intensity the skilled part of the household experiences an increase in employment, which leads to additional work in the next period and thus disutility from working, but also to increased utility from net wage surplus and saved future search effort. The expected payoff is conditioned on the job realization of the additional search effort, i.e., with probability \( p^{st}_{it} \).

The dynamic surpluses of current employment and unemployment are defined as \( \Omega^{E,i}_{it} \) and \( \Omega^{U,i}_{it} \), and evolve as the following unique Bellman equations show:

\[
\Omega^{E,i}_{it} = w^{it} + \tilde{\beta}_t E_t \left[ \chi^{it} \Omega^{U,i}_{t+1} + \left( 1 - \chi^{it} \right) \Omega^{E,i}_{t+1} \right]
\]

whereas \( \Omega^{U,i}_{it} \), the value of being unemployed is

\[
\Omega^{U,i}_{it} = b^{it} + \tilde{\beta}_t E_t \left[ p^{it}_{s_t} \Omega^{E,i}_{t+1} + \left( 1 - p^{it}_{s_t} \right) \Omega^{U,i}_{t+1} \right].
\]

\( \tilde{\beta}_t \) is the household’s stochastic discount factor and is defined as

\[
\tilde{\beta}_t = \beta E_t \frac{H_c(c_{t+1})}{H_c(c_t)}.
\]

The unemployed worker receives value from non-market productivities \( b^{it} \). In unit time she expects to move into employment with probability \( p^{it}_{s_t} \) if she searches with intensity \( s^{it}_t \).

Defining \( \Omega^{i}_{it} = \Omega^{E,i}_{it} - \Omega^{U,i}_{it} \) as the expected gain from change of the employment state, I reach the following recursive law of motion:

\[
\Omega^{i}_{it} = w^{it} - b^{it} + \left( 1 - \chi^{it} - p^{it}_{s_t} \right) \tilde{\beta}_t E_t \Omega^{i}_{t+1}.
\]

(2.8)

This difference between the current values of being employed and being unemployed is the surplus which the worker uses to bargain with the firm.
2.2 The Model

2.2.3 Products and Firms

There is a continuum of identical firms on the unit interval. Firms are perfectly competitive and have the following production function, where physical capital $k_{t-1}$ and labor $L_{t-1}$ enter in a constant return to scale Cobb-Douglas manner:

$$f(\cdot) = y_t = A_t^{a} k_{t-1}^{1-a} L_{t-1}^a.$$  

$L_{t-1}$ is a CES aggregate of two types of labor, the skilled $n^s_{t-1}$ and unskilled $n^u_{t-1}$, which are imperfect substitutes to each other and are augmented by a skill augmenting technology shock:

$$L_{t-1} = \left[ \alpha \left( B_t n^s_{t-1} \right)^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha) \left( n^u_{t-1} \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{1}{\sigma}}.$$  

Note that labor employment here is regarded as a result of last period’s search and matching and therefore counted as a state variable. Two shocks occur to the production process, one skill-neutral (via $A_t$) and the other skill-biased (via $B_t$). Parameters $\alpha$ and $1 - \alpha$ measure the specific productivity level of the skilled and unskilled workers whereas $\sigma$ is the elasticity of substitution between the two types of labor. This setup imposes a unit elasticity of substitution between capital and each type of labor, and allows later in the calibration to use different values of the elasticity of substitution between the skilled and unskilled labor.

In each period firms rent the capital from the households and pay the market rate. Meanwhile firms open as many vacancies $v^j_t$ as necessary in order to hire in expectation the desired number of workers for the next period, taking into account that the real cost to opening a vacancy is $\kappa^j_i$. Wages for both skilled and unskilled workers are the outcome of wage bargaining. Firms maximize the sum of discounted future profits by choosing physical capital and vacancies to be posted for skilled and unskilled labor:

$$\max \{ v^s_t, \{ v^u_t \}, \{ k_t \} \} E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t \Pi_t,$$

where the firm makes profit $\Pi_t$ from selling their output $y_t$ at a price that is normalized to one, less wages payment for both types of labor, the costs associated with new vacancies, as well as the rents for capital. As mentioned above, $\tilde{\beta}_t$ is the stochastic discount factor. It is imposed on the profit and capital utilization of the firm,

$$\Pi_t = y^s_t - \sum_i w^s_i n^s_{t-1}^i - \sum_i \kappa^s_i v^s_i - r_t k_{t-1}. $$

This maximization problem is subject to:
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\[ y_t = A_t^d k_{t-1}^{1-d} \left[ \alpha \left( n_{t-1}^s \right)^{\zeta_{n^s}} + (1 - \alpha) \left( n_{t-1}^u \right)^{\zeta_{n^u}} \right] \rho_{nt}^{\theta}, \tag{2.9} \]

\[ n_t^i = \left( 1 - \chi_t^i \right) n_{t-1}^i + q_t^i v_t^i, \tag{2.10} \]

\[ \ln A_t = \rho_A \ln A_{t-1} + \epsilon_t, \tag{2.11} \]

\[ \ln B_t = \rho_B \ln B_{t-1} + \omega_t. \tag{2.12} \]

Equation (2.10) captures the employment evolution for skilled and unskilled labor. Equations (2.11) and (2.12) show the autoregressive process for skill-neutral and skill-biased technology evolutions and the exogenous shocks \( \epsilon_t \sim i.i.d. (0, \sigma^2_{\epsilon}) \) and \( \omega_t \sim i.i.d. (0, \sigma^2_{\omega}) \). Here I also assume independence of the neutral and biased shocks in order to examine the separate effects of the two shocks. Firms maximize their profits taking the wage curves as it would be given from bargaining.

Note that for wage realization it matters what the firm perceives the wage to depend on. Once the firm takes into consideration that wage is based on the amount of labor and capital inputs, the firm would make a different decision of vacancy posting and capital employment, and as a result hire more workers and employ less capital.² Since in reality we observe neither frequent wage renegotiations at any time nor evidences for firms’ overhiring, I stay with the assumption that firms simply take wages as given from Nash bargaining.

Since capital is owned by the households, firms only have to decide on capital employment at each period, which is the standard first order condition for the capital market:

\[ \frac{\partial y_t}{\partial k_{t-1}} = r_t. \tag{2.13} \]

The Euler equations concerning labor demand are:

\[ \frac{\kappa_t^i}{q_t^i} = \beta_t E_t \left\{ \frac{\partial y_{t+1}}{\partial n_{t+1}^i} - \omega_{t+1}^i + \left( 1 - \chi_t^i \right) \frac{\kappa_t^i}{q_{t+1}^i} \right\}. \tag{2.14} \]

The cost of posting a vacancy would be compensated by discounted future profits conditioned on the vacancy filling probability. Once the job match succeeds, the firm profits from the marginal product of extra labor input net of the wage payment. Furthermore, if the match remains with probability \( (1 - \chi_t^e) \), the firm also saves the future cost to post a new vacancy.

Regarding the individual wage bargaining, what concerns the firm is the contribution of an extra

²More details about this overhiring effect can be found in the appendix.
worker to its value. The marginal value of a skilled/unskilled worker is  

\[
\frac{\partial V_i}{\partial n_{i-1}} = \frac{\partial y_i}{\partial n_{i-1}} - w_i + \left(1 - \chi^i\right) \frac{\kappa^i}{q^i}.
\]  

(2.15)

These marginal values are also the surpluses the firm uses in the bargaining.

The timing in the short-run is as follows: The representative firm treats each worker as a marginal worker and bargains with her for the wage; taking wages from bargaining, the households choose the search intensity, labor supply and capital investment, while firms choose the number of vacancies so as to maximize their discounted sum of future utilities.

### 2.2.4 Wage Setting

In this subsection the bargaining process is explained in detail. The representative firm treats each worker as a marginal worker and bargains with her for the wage. Nash bargaining is assumed where firm and worker choose wage in order to maximize the (log) geometric average of their dynamic surpluses from a successful job match, whereas employment is ex post chosen by the firm to maximize profits given the bargained wage (also known as the “right to manage” bargaining model). Free-entry condition on the product market drives firm’s outside option down to zero.

\[
\begin{align*}
\bar{w}_i^t &= \arg \max \left(1 - \eta \right) \ln \left( \frac{\partial V_i}{\partial n_{i-1}} \right) + \eta \ln \Omega_i^t, \\
&\text{subject to the firm’s surplus (2.15) and the respective worker’s surplus (2.8). The parameter} \\
&\text{\eta indicates the bargaining power of the worker, and 1 - \eta is the firm’s weight. Obviously, the} \\
&\text{higher \eta is, the more power the worker has when negotiating. The firm knows the skill level of} \\
&\text{the worker or can use the educational and experience background as proxy, thus always using the} \\
&\text{right marginal contribution of the worker when bargaining.} \\
&\text{The bargaining solutions take the following form:} \\
\bar{w}_i^t &= \eta \left[ \frac{\partial y_i}{\partial n_{i-1}} + \left(1 - \chi^i\right) \frac{\kappa^i}{q^i} \right] + \left(1 - \eta\right) \left[b^i - \left(\chi^i - p^i s^i\right) \tilde{\beta} t E t \Omega_i^t \right].
\end{align*}
\]  

(2.16)

where the future surplus of workers being employed is still included and can be further refined. Nonetheless, these intermediate wage equations can already help to refine the firm’s Euler equations. Differentiating equation (2.16) and substituting it into the firm’s Euler equation for labor demand yields a more explicit form:

\[
\frac{\kappa^i}{\tilde{\beta} t q^i_{t-1}} + \bar{w}_i^t - \left(1 - \chi^i\right) \frac{\kappa^i}{q^i} = \frac{\partial y_i}{\partial n_{i-1}}.
\]  

(2.17)
The left-hand side of equation (2.17) is the cost of the firm to employ an extra worker. Compared to a perfectly competitive labor market where wage as the only labor cost equals the marginal product of labor, in an imperfect labor market the firm also takes into consideration the posting costs incurred and future posting costs saved.

As more skilled labor is hired its marginal product declines due to the law of diminishing marginal returns, while the marginal product of unskilled worker increases, since skilled and unskilled labor enter the Cobb-Douglas-CES production function in a complementary manner. As shown in equation (2.16), wages contain a fraction of the corresponding marginal products of labor. Therefore the skilled wage decreases and unskilled wage increases with an extra unit of skilled labor.

In order to find the final form of the solution, I still need to combine the optimality condition and the bargaining result for wage. Plugging the semi-final wage equation (2.17) back into the bargaining result and combining it with equation (2.8) I can solve for the value of employment,

\[ \Omega^i_t = \frac{\eta}{1 - \eta} \frac{\kappa^i}{\beta^i q^i_{t-1}}. \] (2.18)

Take (2.18) one period ahead, and recall that in the labor market \( p^i s^i_t = \theta^i q^i_t \) holds,

\[ E_t \Omega^i_{t+1} = \frac{\eta}{1 - \eta} \frac{\kappa^i}{\beta^i q^i_t} = \frac{\eta}{1 - \eta} \frac{\kappa^i}{\beta^i p^i s^i_t}. \] (2.19)

Using this result with equation (2.16), I can obtain the final wage curves for skilled and unskilled labor:

\[ w^i_t = \eta \left[ \frac{\partial y_t}{\partial n^i_{t-1}} + \theta^i \kappa^i \right] + (1 - \eta) b^i. \] (2.20)

A fixed part of the wage comes from worker’s backup position, the value of non-market activities, and wages are more rigid than their counterparts in an RBC model. In the “flexible” part, only a certain portion of the wage reflects the marginal product of labor, while the worker also shares part of the rent generated from matching.

### 2.2.5 The Model Equilibrium

The model equilibrium consists of

- the representative households’ optimal intertemporal decisions (2.6) and (2.7)
- the firm’s capital choice (2.13) and labor demand (2.14)
- the wage curves (2.20)
2.3 Calibration

- as well as the characteristic equations from both labor markets.

The endogenous variables are

\[ \{ u_s^t, u_u^t, v_s^t, v_u^t, w_s^t, w_u^t, s_s^t, s_u^t, p_s^t, p_u^t, q_s^t, q_u^t, \theta_s^t, \theta_u^t, n_s^t, n_u^t, i_t, y_t, c_t, r_t, k_t \} \]

This complex system of nonlinear equations is solved by Dynare.

2.3 Calibration

As Merz (1995) proves, if search intensity were endogenized, the negative relationship between vacancies and unemployment would be blurred. Therefore she fixes the search intensity and examines the effect. Following her procedure, I calibrate the model in two cases. In the first case, I set \( \gamma \), the weight of search activity in utility, equal to 1 and let \( s_i \) be endogenously determined. In the second case, \( \gamma \) is set to zero. Not surprisingly, only if the search intensity is exogenously given I can expect the model to replicate business cycle properties and downward-sloping Beveridge curves.

2.3.1 Aggregate Economics

I use and target quarterly data from the U.S. economy between 1975 and 2003. The quarterly depreciation rate for capital is set as 2.6 percent so that the long-run investment-output ratio in the post-war era roughly equals to 0.25 (Francis and Ramey, 2005). The depreciation rate is about 10 percent annually. Based on this result, I can calculate the quarterly net rate of return on capital, which is 3.6 percent, and consequently \( \alpha \), which is approximately 0.64. Note that due to the non-Walrasian market structure wage is smaller than the marginal product of labor alone so that \( \alpha \) is not exactly the labor share. These macroeconomic variables and parameters are in line with the calibration by Krueger and Perri (2006).

2.3.2 Labor Market

The first question is if I am allowed to treat the separation rate as a constant parameter. Hall (2005a) estimates the separation rate for the past 50 years and finds it almost constant over the business cycle. I use this result and calibrate \( \chi_t \) targeting at a proper skill-specific unemployment rate. Together with an effective monthly job finding rate 0.47 for skilled worker and a slightly lower rate 0.45 for the unskilled workers, I can pin down the search intensity. Note that the unemployment rate used here is the expanded unemployment rate (Hall, 2005a), which is an alternative measure
Cyclical Skill-Specific Unemployment with Imperfect Substitution of Skills

and larger than the official unemployment rate. Table 2.2 shows the reason for including people in the expanded measure who are classified as out of the labor force but with high likelihood of job-seeking. The table gives the transition matrix in the CPS among the three states of “not in labor force”, “unemployed” and “working”.

Table 2.2: Transition from and into Unemployment

<table>
<thead>
<tr>
<th>To</th>
<th>From</th>
<th>Not in LF</th>
<th>Unemployed</th>
<th>Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not in LF</td>
<td>92.8</td>
<td>22.7</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td>Unemployed</td>
<td>2.5</td>
<td>49.6</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Working</td>
<td>4.7</td>
<td>27.6</td>
<td>95.4</td>
<td></td>
</tr>
</tbody>
</table>

Transaction matrix for the CPS, 1967-2004, percent per month, Shimer’s tabulations of raw data from the CPS, used by Hall (2005a).

Each month, 2.5 percent of the workers who were out of the labor force in the previous month enter unemployment this month, while almost twice so many become employed directly. This astonishing result shows that those out of the labor force do not enter labor force first as an unemployed, but rather start seeking for a job during the time when they are classified as “out of the labor force”. According to the U.S. Bureau of Labor Statistics, this group includes the discouraged workers and marginally attached workers who have been included in the expanded unemployment rate U-6 from 1994 onwards. I use U-6 as a basis for the “expanded unemployment rates”.

The number of workers who are out of the labor force is massive, especially in the lower skilled group. According to census data for the civilian noninstitutional population 25 to 64 years of age, while the college graduates’ participation rate increased slowly but steadily from 82.3 percent in 1970 to 88 percent in the middle 1980s and stood around this level until 2001, high school dropouts’ participation rates were much lower during the same period, oscillating between 60 – 63 percent. Consequently, the stock of out-of-labor-force workers who actually seek for jobs is especially large in the unskilled group. I take Hall’s approximation of the “expanded unemployment rates” between 1977 and 2004 and use it as my calculation basis. Together with the ratios between aggregate and education-specific unemployment rates, I can obtain 5 percent for the skilled and 9 percent for the unskilled as expanded education- or skill-specific unemployment rates.

Skilled and unskilled labor interact with each other in firm’s production, where they are imperfect substitutes. Parameter $\alpha$ represents their respective weight in the production and is closely related to the value of worker’s bargaining power. All parameter values are reported in Table 2.3.

3 Discouraged workers are those who want to work but believe no work is available for a variety of reasons. Marginally attached workers are those who give reasons such as transportation or child-care responsibilities. Both types choose to exclude themselves temporarily out of the labor force but have high likelihoods of return to the labor force in the near future.
2.4 Results and Discussion

Table 2.3: Calibration

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Case 1*</th>
<th>Case 2**</th>
<th>Symbol</th>
<th>Case 1*</th>
<th>Case 2**</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma)</td>
<td>1</td>
<td>0</td>
<td>(\Delta)</td>
<td>0.31</td>
<td>0.35</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.5</td>
<td>0.5</td>
<td>(\tau)</td>
<td>0.014</td>
<td>0.026</td>
</tr>
<tr>
<td>(\theta)</td>
<td>0.64</td>
<td>0.64</td>
<td>(\varrho)</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>(\eta)</td>
<td>0.14</td>
<td>0.14</td>
<td>(\theta)</td>
<td>−1.14</td>
<td>−1.25</td>
</tr>
<tr>
<td>(\chi_s)</td>
<td>0.10</td>
<td>0.10</td>
<td>(\chi_u)</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>(b_s)</td>
<td>0.55</td>
<td>0.5</td>
<td>(b_u)</td>
<td>0.4</td>
<td>0.55</td>
</tr>
<tr>
<td>(\kappa_s)</td>
<td>0.1</td>
<td>0.1</td>
<td>(\kappa_u)</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

* Endogenous search intensity, with \(\gamma = 1\). ** Exogenous search intensity, with \(\gamma = 0\).

2.4 Results and Discussion

Table 2.4: Correlation Coefficients for the U.S. Data and the Model

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Endo s*</th>
<th>Exog s_neu**</th>
<th>Exog s_bias***</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho(v_s, u_s))</td>
<td>0.87</td>
<td>-0.33</td>
<td>-0.47</td>
</tr>
<tr>
<td>(\rho(v_s, u_u))</td>
<td>0.86</td>
<td>0.58</td>
<td>0.54</td>
</tr>
<tr>
<td>(\rho(v_u, u_s))</td>
<td>0.84</td>
<td>0.05</td>
<td>0.22</td>
</tr>
<tr>
<td>(\rho(v_u, u_u))</td>
<td>0.93</td>
<td>-0.18</td>
<td>-0.40</td>
</tr>
<tr>
<td>(\rho(u_s, y))</td>
<td>-0.74</td>
<td>-0.23</td>
<td>-0.04</td>
</tr>
<tr>
<td>(\rho(u_u, y))</td>
<td>-0.57</td>
<td>-0.15</td>
<td>-0.10</td>
</tr>
<tr>
<td>(\rho(\theta_s, y))</td>
<td>0.90</td>
<td>0.96</td>
<td>0.92</td>
</tr>
<tr>
<td>(\rho(\theta_u, y))</td>
<td>0.62</td>
<td>0.97</td>
<td>0.96</td>
</tr>
</tbody>
</table>

* Endogenous search intensity with a skill-neutral shock. ** Exogenous search intensity with a skill-neutral shock. *** Exogenous search intensity with a skill-biased shock. The benchmark is U.S. data (CPS, 1951-2003): \(\rho(v_u) = -0.894\).

I summarize the results from the model simulation in Table 2.4 and Table 2.5. In the case of exogenous search intensity, a skill-neutral shock and a skill-biased shock are separately added, while the result of endogenous search intensity case is only reported upon a skill-neutral shock. Concerning the elasticity of substitution between two types of labor, Katz and Murphy (1992) find it about 1.41 while Angrist (1995) uses Palestinian data and estimates this elasticity as 2. The majority of empirical findings suggest that the elasticity of substitution between skills is between 1 and 2. I use different values (1.4, 1.7 and 2) to calibrate the elasticity of substitution. It turns out that even though correlation statistics are rather robust, the volatilities of the model vary regarding different elasticities (also captured in Figure 2.3, the impulse responses of relative labor input and relative wage). Table 2.4 reports the correlation statistics for endogenous \(s_i\) and exogenous \(s_j\) with
Table 2.5: Ratios between Standard Deviations of U.S. Data and the Model

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Endo s</th>
<th>Exog s_neu</th>
<th>Exog s_bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_c / \sigma_y$</td>
<td>0.61</td>
<td>1.13</td>
<td>0.72</td>
<td>0.67</td>
</tr>
<tr>
<td>$\sigma_i / \sigma_y$</td>
<td>3.79</td>
<td>0.96</td>
<td>5.51</td>
<td>7.7</td>
</tr>
<tr>
<td>$\sigma_{w_s} / \sigma_y$</td>
<td>0.42*</td>
<td>1.01</td>
<td>0.51</td>
<td>0.49</td>
</tr>
<tr>
<td>$\sigma_{w_u} / \sigma_y$</td>
<td>0.42*</td>
<td>0.82</td>
<td>0.46</td>
<td>0.43</td>
</tr>
<tr>
<td>$\sigma_{u_s} / \sigma_y$</td>
<td>6.1*</td>
<td>0.39</td>
<td>1.29</td>
<td>3.01</td>
</tr>
<tr>
<td>$\sigma_{u_u} / \sigma_y$</td>
<td>6.1*</td>
<td>0.67</td>
<td>1.69</td>
<td>4.16</td>
</tr>
<tr>
<td>$\sigma_{v_s} / \sigma_y$</td>
<td>7.31*</td>
<td>0.12</td>
<td>1.38</td>
<td>3.17</td>
</tr>
<tr>
<td>$\sigma_{v_u} / \sigma_y$</td>
<td>7.31*</td>
<td>0.49</td>
<td>1.83</td>
<td>4.21</td>
</tr>
</tbody>
</table>

* Due to the lack of skill-specific data, values of aggregate variables, taken from Ebell (2006) and Merz (1996), are presented here.

$\sigma = 1.4$. The correlations are similar when $\sigma$ takes the value of 1.7 or 2.

Firstly, the simulation results confirm those of Merz (1995); i.e., the model with endogenous search intensities fails to generate stylized facts, while once $s_i$ is fixed the model can replicate the real economy in the aggregate level adequately well and generate negative Beveridge curves in the respective sub-markets. Market tightness is always strongly procyclical while unemployment is countercyclical (even though the countercyclicality is not very strong). When the scales of negativity of the vacancy-unemployment ratio are compared, the skill-biased shock can generate results closer to the benchmark ratio $0.863^4$. Skilled vacancies correlate positively to unskilled unemployment and vice versa, which could be a result of the substitution effects between the skilled and unskilled in production.

As is shown in Table 2.5, the model with exogenous $s_i$ and biased shock can generate higher volatilities in the labor markets which are observed from the data and are the main reason for Shimer’s (2005) critique on the MP model. Wages are less volatile than output as a result of the search frictions, and the volatility of wages matches the data better under a biased shock. Concerning the skill-specific labor market, the result shows more volatility of unemployment on the unskilled market whether the shock is skill-neutral or skill-biased. This confirms the observation in Figure 2.1 and Table 2.1. What’s more, the result of relative volatility here shows again that a skill-biased shock leads to results that better mirror the data.

Interestingly, a skill-neutral technology shock does not have neutral effects on skilled and unskilled labors. Furthermore, even though relative skill supply increases after the shock, skill-premium is increased, especially given a larger elasticity of substitution between the two types of labor. Impulse responses of the relative wage deviation and relative labor input deviation to a skill-neutral shock are shown in Figure 2.3.

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4I have not found data for vacancy-unemployment ratio for different skill levels and therefore use this general ratio as benchmark.
2.4 Results and Discussion

Figure 2.3 shows the impulse responses of relative wage and labor deviation, where relative wage deviation is the difference between the log-linearized skilled wage and unskilled wage ($\hat{w}_s - \hat{w}_u$), and relative labor deviation is the difference between the log-linearized skilled and unskilled labor input ($\hat{n}_s - \hat{n}_u$). A positive neutral shock induces demand for both skilled and unskilled workers, while more vacancies are created for skilled labor at the first moment than for the unskilled due to the higher marginal productivity of skilled labor. By given search frictions the resulting change of skilled labor input is higher than that of the unskilled. This relative surplus peaks immediately after the shock and declines slowly within 6 quarters.

We know from the available literature on trend of skill-premium that a relative increase of skilled labor reduces the skill-premium. This negative effect on wage also holds here. However, relative surplus of skilled vacancies also leads to relative higher market tightness in the skilled market (while unemployment stocks stay unchanged since last period). Wage equation 2.20 shows that wage is a weighted average of marginal product of labor, value of non-market activities, and the matching rent which includes the market tightness. Relative higher market tightness in the skilled market thus casts a positive impact on the skill premium, which dominates the negative effect from

---

Hats over variables mean deviations from steady state.
relative skill supply and causes skill-premium to increase. This increase will be slowly adjusted back since this relative factor price change induces firms to use unskilled labor to substitute the skilled. Labor structure converges slowly back to the initial level.

Note that this dominating result becomes especially strong when $\sigma$ increases. Therefore in booms, when skilled workers become relatively more expensive, unskilled labor substitutes the skilled to a higher degree given a larger $\sigma$, while during recessions, relatively cheaper skilled labor would be hired to substitute unskilled labor given a larger $\sigma$. As a result, the volatility of the unskilled unemployment increases with the elasticity of substitution.

In the case of a skill-biased shock, the biased nature of the technology shock contributes to additional positiveness to the relative marginal product of labor and thus the wage premium increases as well. Following the same argument on volatility of unemployment above, we can conclude that unskilled workers are more sensitive than the skilled to a skill-neutral business cycle shock. This higher volatility in the unskilled market also corresponds to the observation that the duration of lower-paid unskilled jobs is notably shorter than that of skilled jobs and thus more new unskilled vacancies are opened. While unskilled jobs are technically less demanding, skilled jobs require more job-specific human capital, providing an incentive for employers to keep skilled workers for a longer period of time.

2.5 Conclusion

The key idea of this chapter is to examine the effect of substitutability between skilled and unskilled workers on skill-specific unemployment rates over the business cycle. I use a stylized business cycle model with search frictions, and set up a decentralized economy with risk-averse agents. The large households include two types of labor, which is convenient for future research if I would like to include the household’s investment in education in order to endogenize the skill structure of the total labor force. Firms produce with two types of labor substituting each other which creates an additional channel between the (un)employment of differently skilled workers. In the equilibrium, households and firms meet in two skill-specific labor markets, where search and matching occur and wages are determined.

As a general labor augmenting neutral shock occurs, my model is able to capture certain business cycle properties: Rigid wages and volatile unemployment rates and vacancies. However, a skill-biased shock can generate even better results than a skill-neutral shock when the model is calibrated to U.S. data, which offers some evidence of the skill-biased nature of the technology shock in the past decades. My simulation also generates downward-sloping Beveridge curves between vacancies and unemployment in their respective sub-markets. I examine the elasticity
of substitution between skilled and unskilled workers and find that unemployed are more sensitive to business cycle shocks once $\sigma$ is larger because of firm’s decision on vacancies and layoffs. While some cyclical properties of the model are weaker compared to the data (such as the weak countercyclicality of unemployment), one future area of work would be to employ capital-skill complementarity which allows for different degree of substitution between capital equipment and both types of labor. Another future task would be to endogenize the household’s investment in education and add labor supply shocks to the model.
3 Human Capital Formation in Skill-Specific Labor Markets

3.1 Introduction

The cyclicality of aggregate labor market variables and the related performance of Mortensen-Pissarides (MP) type search and matching models came into heavy debate in recent years. Early papers by Merz (1995) and Andolfatto (1996) aimed at embedding labor market frictions in real business cycle models in order to improve the cyclical properties. Shimer’s (2005) seminal paper points out, that a standard MP model generates relatively low volatility of unemployment and vacancies compared to post-war U.S. data. Many research efforts afterwards focus mainly on how to fix this problem. Among others, Hall (2005a) and Costain and Reiter (2008) propose setting wages sticky as a modification, Hagedorn and Manovskii (2008) focus more on the calibration strategy and suggest a combination of low bargaining power and high home production value as the possible numerical solution, while Ebell (2008) emphasizes the participation margin and inelastic labor force participation to improve the model results.

Observing that wages in new matches are more volatile than the ongoing jobs, Pissarides (2009) examines a setup where the Nash sharing rule only holds in new jobs. While keeping the wage elasticities, he proposes to add a fixed component to the matching cost. Such modification can deliver more volatility in the job finding rate, unemployment and vacancies. Other efforts to provide an adequate explanation for observed volatility in labor-market aggregates include supplementing the model with payroll taxes and social insurance (Burda and Weder, 2010), resurrecting Calvo’s (1983) staggered multiperiod price setting model (Gertler and Trigari, 2009), incorporating separation rate shocks and adjusting the value of the elasticity of the matching function (Mortensen and Nagypál, 2007), and modeling a strategic wage bargaining process where the relative costs of delays to the bargaining parties are taken into account (Hall and Milgrom, 2008).

The current chapter goes beyond the dynamics in aggregate labor market by exploring the relationship between skilled and unskilled labor market variables. There are two channels connecting them, one on the labor demand and the other on the labor supply side. Similar to my Chapter Two
Human Capital Formation in Skill-Specific Labor Markets

and Hagedorn, Manovskii and Stetsenko (2008), demand for skilled and unskilled labor is incorporated in the production function with heterogeneous labor inputs\(^1\). Hagedorn et al. (2008) argue that the reasons for high volatility differ in skilled and unskilled unemployment. While unskilled workers experience volatility because of the small difference between their productivity and home production value, skilled workers are subject to high volatility due to capital investment shocks and the consequent changes in their productivity. They do not allow endogenous skill transition between the two markets, shutting down one important mobility channel between the skilled and unskilled labor force. If one aims to study the important variance of unemployment caused by the inter-market movement, it is necessary to allow for human capital investment and skill acquisition. This is not only highly relevant for current labor market policies, but can also help investigate aggregate unemployment from more specific angles, namely the short- and long-run effects bolstered by changes in skilled and unskilled unemployment separately.

Skilled and unskilled workers are subject to different costs over the business cycle. Krusell and Smith (1999), Mukoyama and Sahin (2006) report on considerable heterogeneity in the welfare cost of cycles among agents with different levels of wealth. The differences result from the higher unemployment risk among the unskilled and their lower ability to self-insure due to less wealth. Because this normative study aims at evaluating the welfare cost of business cycles on skill-specific workers, unemployment is modelled for simplicity as an exogenous random process and so is the flow between workers’ skill status.

In the long-run, the relative unemployment rate of the unskilled increases, which, according to many, results from a demand shift toward skilled labor. This relative demand shift, however, is not adequate to explain unemployment and wage dynamics in Europe and the U.S. in the 1970s-1980s. It explains only a modest but significant part of the large rise in unemployment in some European countries. Germany presents a good example of the European training system; i.e., a strong emphasis in the schooling system and a comprehensive vocational training system. The portion of labor force with middle-level qualification is far higher in German than in the U.S., which forms a more flexible basis for the upcoming biased technology change and demand shift. This flexibility enables endogenous skill upgrade much more easily and thus even given the wage compression, there was much a smaller unskilled unemployment rate (Acemoglu and Pischke, 1999, Pischke, 2001). Pischke’s (2001) study also reveals that most workplace training seems to be general and free for the workers to participate.

Training for the unemployed is prevalent in many industrialized countries and is embedded in the framework of active labor market (ALM) policies. Different types of trainings vary in the tim-

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\(^1\)One difference is that Hagedorn et al. (2008) emphasizes the capital-skill complementarity while I take the perspective of a conventional CES-nested Cobb-Douglas function where the elasticity of substitution between capital and unskilled labor is equal to that between capital and skilled labor.
3.1 Introduction

ing of the effects. A recent meta-analysis by Card, Kluve and Weber (2010) evaluating ALM poli-
cies finds that job search assistance programs have relatively favorable short-run impacts, whereas
classroom and on-the-job training programs tend to show better outcomes in the medium-run than
the short-run. Across the countries, short-term program impacts appear to be relatively unfavor-
able in the German-speaking countries, but relatively favorable in the English-speaking countries.
In the medium term the differences across country groups are smaller, and in the long term the
relative position of the German-speaking and English-speaking countries is reversed. Moreover,
subsidized public sector employment programs have the least favorable impact estimates - a find-
ing that confirms earlier studies from Heckman et al. (1999), Boone and Van Ours (2004) and

In all, training is key to human capital flows between the skill-specific labor forces by affecting
the skilled and unskilled labor market structure. Several papers contribute to exploring the cyclical
behavior of skill acquisition. DeJong and Ingram (2001) model training time as the representa-
houses’ endogenous decision so as to boost subsequent labor productivity. As aggregate data
other than training is used to estimate the parameters of the model, the simulation results suggest
skill acquisition activities to be distinctly countercyclical. Perli and Sakellaris (1998) introduce
human capital into RBC type models to improve the model’s ability to produce persistent output.
It is due to the human capital accumulation process that labor input continues to increase after a
positive technology shock, resulting in persistent output growth. A similar smoothing effect can be
achieved by assuming labor adjustment costs in order to propagate shocks over time (Sepulveda,
2004). Krebs (2003a, 2003b) developed an incomplete asset market model to examine uninsurable
idiosyncratic labor income risk on capital investment decisions, growth and welfare.

More recent attempts explore the role of human capital formation and skill difference in explain-
ing labor market institution and unemployment. While in previous human capital related studies
unemployment is mostly assumed voluntary, Ljungqvist and Sargent (2007a, 2007b, 2008) com-
pare the impact of unemployment insurance and employment protection on unemployment and
duration in different model setups. However, no endogenous decisions are made by the workers
on their own skill accumulation. Observing ALM policies gaining popularity in Europe and the
U.S. (starting from the 1980s in Scandinavian countries), one may inquire their possible effects on
unemployment given workers’ decision in attending training programs. Admittedly, data shows
that unemployed workers are engaged much too little in skill trainings and hence suffer from se-
vere loss of their net human capital. This can raise questions on the effectiveness of existing ALM
policies but does not rule out the potentials of optimal policies. The key point, in fact, is how to de-
sign good training programs, communicate the message to the unemployed workers and motivate
them to take part.
Human Capital Formation in Skill-Specific Labor Markets

A broader way to set up the problem is to allow for different degrees of human capital depreciation and examine workers’ corresponding decision in human capital investment. Even if there may be little change in total labor force, the aggregation of single workers’ choice in human capital investment would change the shares of skilled and unskilled labor forces, and subsequently their market tightness. This is exactly the idea of the current chapter: Households’ endogenous decisions regarding skill accumulation are implemented through general training and learning-by-doing, and the volume of skill-specific labor force and the respective market tightness vary correspondingly.

The theoretical model shares similarity with Krebs (2003a, 2003b) in the sense that households own and invest in two types of capital, namely physical and human capital, and human capital is subject to idiosyncratic shocks. As in Krebs (2003a, 2003b), households’ labor supply is regarded in the same manner as human capital, consequently the quantity and quality of labor can not be disentangled. In the current chapter these two concepts are separated into labor supply (as quantity) and skill share (as quality of labor).

My results confirm the effect of relative price and skill substitution on aggregate unemployment. Model simulation shows that aggregate unemployment is countercyclical. Given a positive human capital shock, firstly the unskilled unemployment declines, and then due to skill substitution skilled unemployment decreases dominantly. Total vacancies also experience a two-stage response toward the shock, which sink firstly due to the dominant deduction of the unskilled vacancies, and recover shortly afterward because of the strong skilled vacancy creation and unskilled vacancy recovery. Further parameter variation shows that the elasticity of substitution plays an important role in the model dynamics. When skilled and unskilled workers are more likely to replace each other, the impulse responses upon the shock are enhanced and subsequently the volatilities of the variables increase. Unemployment-vacancy correlation also increases in absolute value and approaches the correlation in U.S. data.

In both skilled and unskilled labor markets, technology shocks induce changes similar to that in a single type labor model. Vacancies respond more strongly to the positive productivity shock than unemployment, and consequently market tightness and job-finding rates increase. The immediate positive impact response of unemployment is quickly reversed so that on average both skilled and unskilled unemployment remain countercyclical. Comparatively, skilled vacancies and unemployment react more intensively than their unskilled counterparts, suggesting higher sensitivity in the skilled labor market to a technology shock. Directly after the shock, human capital investment reacts positively, just as physical capital investment. Physical capital builds up gradually as a result of increasing output, which is consistent with the result in standard RBC models. The initial response of human capital, represented by the share of skilled population, is also positive but at a
3.2 The Model

In this section a decentralized equilibrium is derived. Households are ex ante homogeneous until the idiosyncratic human capital shock occurs. The large household assumption applies and each household is composed of skilled and unskilled members, with each type searching for jobs in the segmented skill-specific labor market \(i (i = s\) denotes the skilled market and \(i = u\) the unskilled market). Through skill depreciation and households’ investment in human capital, relative skill share changes. There is no mismatch of skills and job types. Households own the capital and rent it to the firms. Firms post vacancies to hire workers and produce with capital, where skilled and unskilled workers substitute each other imperfectly. The structure of the model is shown in Figure 3.1.

\[\theta_i^t = \frac{v_i^t}{u_i^t}\]

\[y_i = \exp(z_i)k_{i-1}^{1-\alpha}n_{is}^{\sigma-1}(1-\alpha)(n_{iu}^{\sigma-1})^{\alpha\sigma-1}\]

\(\theta_i^t\) is skill-specific market tightness, \(v_i^t\) denotes vacancies in the respective markets and \(u_i^t\) the unemployment stocks. As shown in Figure 3.1, it’s in firms’ production that skilled and unskilled workers interact with each other again. Firms produce with physical capital \(k_{i-1}\), skilled labor \(n_{is}^{i-1}\) and unskilled labor \(n_{iu}^{i-1}\). Exogenous technology shocks occur to the production process. More details of this CES nested Cobb-Douglas production function will be discussed in subsection 3.2.3.
3 Human Capital Formation in Skill-Specific Labor Markets

3.2.1 Search and Matching in the Labor Markets

Skilled and unskilled workers look for jobs in separate labor markets. Firms can observe the exact skill level of the worker and workers only look for vacancies within their own skill level. Therefore there is no mismatch of skills and job types. Both skilled and unskilled labor markets follow the standard search and matching structure. With $i = s, u$, vacancies $v_i$ and stock of unemployed workers $u_i$ jointly form new job matches through a constant return to scale matching function $m(u_i, v_i) = m^i (u_i)^{1-\varrho} (v_i)\varrho$. $m^i$ is the scaling parameter in the matching functions and can be interpreted as the efficiency of matching. $\varrho$ is the matching elasticity. Labor market tightness is defined as

$$\theta_i = \frac{v_i}{u_i},$$

the probabilities that firms meet proper unemployed workers are

$$q_i = m^i \left( \theta_i \right)^{e-1},$$

while the unemployed meet proper vacancies at rates

$$p_i = m^i \left( \theta_i \right)^e.$$

Within the skilled and unskilled labor markets, respectively, an employed worker can become unemployed in the next period because either her firm has exited the market with probability $\kappa$ or she loses her current job in the firm with probability $\tilde{\chi}$. Suppose there is no correlation between these two sources of unemployment. Thus, workers lose their jobs and become unemployed at the rate $\chi = \kappa + \tilde{\chi} - \kappa \tilde{\chi}$.

3.2.2 Households

A large household is composed of skilled and unskilled members. When the total household is normalized as 1, the share of skilled population is $\Delta_{t-1}$, and $1 - \Delta_{t-1}$ is the unskilled. The structure of the labor force can change over time through natural skill depreciation and skill upgrading from training. Both skilled and unskilled workers can have three statuses: Working, being unemployed (but search for jobs) and enjoying leisure. The time constraints are summarized in the following equations:

$$\Delta_{t-1} = n_{t-1}^s + u_{t-1}^s + l_t^s,$$

$$1 - \Delta_{t-1} = n_{t-1}^u + u_{t-1}^u + l_t^u.$$
where \( n_{t-1}^i \) is labor supply, \( u_t^i \) denotes unemployment and \( l_{t-1}^i \) stands for leisure for type ‘i’ household members.

Under such an assumption, there is a natural limit of human capital investment, and accordingly, a genuine difference to Krebs (2003a, 2003b). In Krebs’ setup, households have a portfolio of risk-free physical capital and risky human capital investment. When the uninsurable idiosyncratic labor income risk declines, households (in the steady state) are induced to possess more human capital and less physical capital. As a result, the return on human capital decreases and that on physical capital rises. The total investment interest, however, increases as the expected return on risky human capital investment exceeds the return on the risk-free physical capital investment. In comparison, in the current model labor is a hybrid of conventional labor form and human capital, in the sense that labor complements capital but due to the comparatively small amount, “human capital premium” is even more substantial. Still the quantity and quality of labor can be taken apart, with labor supply representing the former, and skill share embodying the latter.

The representative household chooses consumption, human capital investment, labor supplies and search intensity for both types of labor, in order to maximize the sum of the discounted future utilities,

\[
\max_{\{c_t, l_t^s, l_t^u, \Delta t\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[ H(c_t) + G(l_t^s, l_t^u) \right] \tag{3.6}
\]

where \( c_t \) is consumption, \( l_t^s \) and \( l_t^u \) are skilled and unskilled leisure respectively, and \( \beta \) is the common discounting factor in the economy. Both \( H \) and \( G \) are increasing and concave functions:

\[
H(c_t) = \ln c_t,
\]

\[
G(l_t^s, l_t^u) = \zeta^s \left( l_t^s \right)^{\psi^s} \frac{1}{1 + \psi^s} + \zeta^u \left( l_t^u \right)^{\psi^u} \frac{1}{1 + \psi^u}.
\]

Parameters \( \psi^s \) and \( \psi^u \) are rough measures of the Frisch elasticity of labor supply. \( \zeta^s \) and \( \zeta^u \) represent weights of utility gained from leisure. The period-to-period budget constraint is given as

\[
w_t^s n_t^s + w_t^u n_t^u = c_t + x_t. \tag{3.7}
\]

The left-hand side is households’ income, including wages of both labor types. Meanwhile,
households consume $c_t$ and invest in human capital $x_t$. Other constraints are:

\[
\text{skill evolution } \Delta_t = (1 - \delta + \xi_t) \Delta_{t-1} + x_t + F \left( n_{t-1}^s \right), \quad (3.8)
\]

\[
\text{skilled labor transition } n_t^s = (1 - \chi^s) n_{t-1}^s + p_t^s u_t^s, \quad (3.9)
\]

\[
\text{unskilled labor transition } n_t^u = (1 - \chi^u) n_{t-1}^u + p_t^u u_t^u. \quad (3.10)
\]

Equations (3.9) and (3.10) summarize the intertemporal transitions in skilled and unskilled labor markets separately. Equation (3.8) captures how human capital evolves. Skill loss happens to the skilled population at a constant rate $\delta$. $\xi_t$ is the shift in human capital level. The current share of skilled population, which can be interpreted as the human capital level of the household, stems from the undepreciated previous skilled share, human capital investment $x_t$ and new human capital formation from skilled labor activities. Note that the difference between $x_t$ and $F \left( n_{t-1}^s \right)$ is that the former investment is valid for the whole population, while the latter, “learning by doing”, is assumed to be particular for the skilled worker and differentiates them from the unskilled. This is also found in Burdett, Carrillo-Tudela and Coles (2009)\(^2\). $x_t$ induces new skills gained from on-the-job training and compulsory training for the unemployed, and hence lumping together specific human capital (on-the-job training) and general human capital (from unemployment training as part of the active labor market policy). Similarly, although $F \left( n_{t-1}^s \right)$ depicts the skill accumulation on the job, the skill gained can also be both general and specific. This assumption is theoretically and empirically justified. Acemoglu and Pischke (1999) argue that wage compression due to market imperfection provides firms the incentive to invest in general human capital. Pischke (2001) finds no evidence in SOEP data on how firm-specific the trainings are in Germany, while Loewenstein and Spletzer (1999) find surprising information from U.S. data (NLSY) that a large part of training paid by the employers are general.

As Burdett et al. (2009) argue, learning-by-doing is relevant to the wage distribution in equilibrium. Similar to DeJong and Ingram (2001), I assume that $F \left( n_{t-1}^s \right) = \mu \left( n_{t-1}^s \right)^\theta$. The parameter $\theta$ can be either greater or smaller than 1, implying convexity or concavity of $F \left( \cdot \right)$ respectively. $\xi_t$ can be interpreted as either a change in the population or a household-specific shift in human capital stock à la Krebs (2003a, 2003b). A positive shift could be improvement of agent’s health condition or having a good teacher in the training course, helping human capital stock formation so that the

\(^2\)Using a non-competitive labor market model with search frictions, Burdett et al. (2009) study the impact of human capital accumulation on equilibrium market outcomes. Their model emphasizes the importance of experience, and reveals that learning-by-doing increases equilibrium wage dispersion. Moreover, their numerical simulation shows that the equilibrium sorting implied by their model, namely more experienced workers also tend to find and quit to better paid jobs, may more than double the impact of learning-by-doing on measured wage inequality.
3.2 The Model

A household has a better chance to be upgraded to a higher-skilled job market in next period. In contrast, a negative shift, such as a sudden loss of firm-specific human capital due to job termination, can downgrade the household to a less-skilled job market. Following Krebs (2003a, 2003b), I assume that $\xi_t$ follows an AR(1) process:

$$\xi_t = \rho \xi_{t-1} + e_t$$ (3.11)

where the unpredictable residual $e_t$ is i.i.d. distributed across households and across time. The coefficient $\rho$ can be understood as the persistence of the human capital shock. One can find the counterpart of this idiosyncratic income shock in the micro studies on labor income, and the setup here mirrors the permanent income shock, in the sense that agents can effectively self-insure against transitory shocks through borrowing or their own savings, and the welfare effects of such shocks are quite small (Heaton and Lucas, 1996, Levine and Zame, 2002), while permanent income variances are hardly insurable (Meghir and Pistaferri, 2004).

Under this assumption human capital shocks can accumulate to a permanent labor income shock. Because equation (3.8) is the core to the structure of the labor market, how human capital exactly evolves, affects not only the steady state value but also the second moments. More detailed discussion on this issue can be found in section 3.4.2.

The first Euler equation resembles the standard intertemporal condition to allocate human capital investment optimally

$$\frac{1}{c_t} = \beta E_t \left[ (1 - \delta + \xi_{t+1}) \frac{1}{c_{t+1}} + \xi^u \left( l^u_{t+1} \right)^{-1} \psi - \xi^u \left( l^u_{t+1} \right)^{-1} \psi \right].$$ (3.12)

The utility forgone today for human capital investment is compensated by the additional human capital gain minus the difference between future utility in skilled leisure and unskilled leisure, since a few unskilled workers have been upskilled into the skilled labor share.

The Euler equations for skilled and unskilled labor participation are:

$$\xi^u \left( l^u_t \right)^{-1} \psi \frac{1}{p^u_t} = \beta E_t \left\{ \frac{u^u_{t+1}}{c_{t+1}} + \xi^u \left( l^u_{t+1} \right)^{-1} \psi \frac{1 - \lambda^u - p^u_{t+1}}{p^u_{t+1}} \right\},$$ (3.13)

$$\xi^s \left( l^s_t \right)^{-1} \psi \frac{1}{p^s_t} = \beta E_t \left\{ \left[ \mu \theta \left( n^s_t \right)^{d-1} + w^s_{t+1} \right] \frac{1}{c_{t+1}} + \xi^s \left( l^s_{t+1} \right)^{-1} \psi \frac{1 - \lambda^s - p^s_{t+1}}{p^s_{t+1}} \right\}. $$ (3.14)

Current leisure forgone for the worker imposes a compound effect in the next period, where the expected payoff is conditioned on the job realization of the additional search effort; i.e., with probability $p^s_t$. With this optimal labor participation the corresponding part of the household experiences an increase in employment and thus sacrificing leisure but gaining extra wage income.
3 Human Capital Formation in Skill-Specific Labor Markets

The last part of the marginal benefit of employment is the saved search cost once the match survives. What’s special of skilled workers is that through “learning by doing”, they can accumulate and utilize new human capital in the next period. The wage gain therefore reflects this late skill accumulation.

The values of current employment and unemployment are defined as $\Omega^E_i$ and $\Omega^U_i$, and evolve as the following Bellman equations show:

$$\Omega^E_i = w_i + \tilde{\beta}_i E_i \left[ \chi^i \Omega^U_{i+1} + \left( 1 - \chi^i \right) \Omega^E_{i+1} \right],$$

whereas $\Omega^U_i$, the value of being unemployed is

$$\Omega^U_i = b^i + \tilde{\beta}_i E_i \left[ p^i \Omega^E_{i+1} + \left( 1 - p^i \right) \Omega^U_{i+1} \right].$$

Though there is no direct pecuniary unemployment compensation, unemployed workers can carry out more home production $b^i$, such as gardening work or cooking, which de facto creates value to be unemployed and relaxes households’ budget constraint. $\tilde{\beta}_i$ is household’s stochastic discount factor and is defined as

$$\tilde{\beta}_i = \beta \frac{E_i H(c_i)}{H(c_i)}.$$

Defining $\Omega_i = \Omega^E_i - \Omega^U_i$ as the expected gain from change in the employment state, we reach the following recursive law of motion

$$\Omega_i = w_i - b^i + \left( 1 - \chi^i - p^i \right) \tilde{\beta}_i E_i \Omega_{i+1}$$

(3.15)

With this surplus, worker $i$ will enter the wage bargaining with the firm.

3.2.3 Products and Firms

There is a continuum of identical firms on the unit interval. Firms are perfectly competitive and produce with physical capital, skilled and unskilled labor. All factors enter production in a CES-nested Cobb-Douglas manner:

$$f(\cdot) = y_t = \exp(z_t) k_{t-1}^{1-a} \left[ \alpha \left( n_{s,t-1}^{a-1} \right)^{\frac{1}{1-a}} + \left( 1 - \alpha \right) \left( n_{u,t-1}^{a-1} \right)^{\frac{1}{1-a}} \right]^{\frac{a}{\sigma-1}},$$

$n_{s,t-1}$ and $n_{u,t-1}$ are imperfect substitutes to each other and are augmented by a technology shock $z_t$.

Within the compound labor input, parameters $\alpha$ and $1 - \alpha$ measure the specific productivity level of the skilled and unskilled workers whereas $\sigma$ is the elasticity of substitution between the two types of labor, and $a$ is the output elasticity of labor.
3.2 The Model

In each period firms open as many vacancies $v^t_i$ as necessary in order to hire in expectation the desired number of workers for the next period, taking into account that the real cost to opening a vacancy is $\kappa^t_i$. Wages for both skilled and unskilled workers are the outcome of wage bargaining. Firms own capital and maximize the sum of discounted future profits by choosing optimal capital investment and vacancy posts for skilled and unskilled labor:

$$\max \left\{ v^t_s, v^t_u, k^t \right\} \ E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \Pi^t$$

where the firm makes profit $\Pi^t$ from selling their output $y^t$ at a price that is normalized to one, less new capital investment and wage payment for both types of workers, as well as the costs associated with new vacancies. As mentioned above, $\tilde{\beta}^t$ is the stochastic discount factor. It is imposed on the profit and capital utilization of the firm.

$$\Pi^t = y^t_i - i_t - \sum_i w^i_t n^i_{t-1} - \sum_i \kappa^i v^i_t.$$ 

This maximization problem is subject to:

$$y^t = \exp(z^t) k^{1-a}_{t-1} \left[ (n^s_{t-1})^{1-\alpha} + (1-\alpha) (n^u_{t-1})^{1-\sigma} \right]^{\alpha \sigma}, \quad (3.16)$$

$$k^t = (1-\tau) k^{t-1} + i_t, \quad (3.17)$$

$$n^i_t = (1-\chi^i) n^i_{t-1} + q^i_t v^i_t, \quad (3.18)$$

$$z^t = \rho z^{t-1} + \epsilon^t. \quad (3.19)$$

Capital stock evolution follows (3.17) where $\tau$ is the capital depreciation rate. Employment for skilled and unskilled labor develops as shown in equation (3.18), and the technology evolution is summarized by equation (3.19). The exogenous shock to technology is $\epsilon^t \sim i.i.d. (0, \sigma^2_\epsilon)$. Firms maximize their profits taking the wage curves as it is given from wage bargaining.

The Euler equation for capital investment is

$$1 = \tilde{\beta}^t \left[ (1-a) \frac{y^t_{t+1}}{k^t} + (1-\tau) \right]. \quad (3.20)$$

The ones concerning labor demand are:

$$\frac{\kappa^i}{q^i_t} = \tilde{\beta}^t E_t \left\{ \frac{\partial y^t_{t+1}}{\partial n^i_t} - w^i_{t+1} + (1-\chi^i) \frac{\kappa^i}{q^i_{t+1}} \right\}. \quad (3.21)$$
The cost of posting a vacancy would be compensated by discounted future profits conditioned on
the vacancy filling probability. Once the job match succeeds, the firm profits from the marginal
product of extra labor input net of the wage payment; furthermore, if the match remains with
probability \((1 - \chi^s)\), the firm also saves the future cost to post a new vacancy.

Regarding the individual wage bargaining, what concerns the firm is the contribution of an
extra worker to its value. For a vacancy of total value \(V^i_t\), the marginal value of a skilled/unskilled
worker is

\[
\frac{\partial V^i_t}{\partial n^i_{t-1}} = \frac{\partial y^i_t}{\partial q^i_{t-1}} - w^i_t + \left(1 - \chi^i\right) \frac{\kappa^i}{q^i_t}.
\] (3.22)

These marginal values are also the surpluses the firm uses in the bargaining.

### 3.2.4 Wage Setting

In this subsection the bargaining process is explained in detail. The representative firm treats each
worker as a marginal worker and bargains with her for the wage. Nash bargaining is assumed
where firm and worker choose wage together in order to maximize the (log) geometric average
of their surpluses from a successful job match, whereas employment is ex post chosen by the firm
to maximize profits given the bargained wage (also known as the “right to manage” bargaining
model). Free-entry condition on the product market drives the firm’s outside option down to zero.

\[
w^i_t = \arg \max \left(1 - \eta\right) \ln\left(\frac{\partial V^i_t}{\partial n^i_{t-1}}\right) + \eta \ln \Omega^i_t,
\]

subject to the firm’s surplus (3.22) and the respective worker’s surplus (3.15). The parameter
\(\eta\) indicates the bargaining power of the worker, and \(1 - \eta\) is the firm’s weight. The firm knows
the skill level of the worker or can use the educational and experience background as proxy, thus
always using the right marginal contribution of the very worker when bargaining.

The bargaining solutions take the following form:

\[
w^i_t = \eta \left(\frac{\partial y^i_t}{\partial q^i_{t-1}} + (1 - \chi^i) \frac{\kappa^i}{q^i_t}\right) + (1 - \eta) \left[b^i_t - \left(1 - \chi^i - p^i_t\right) \tilde{\beta} E_t \Omega^i_{t+1}\right]
\] (3.23)

where the future surplus of workers being employed is still included and can be further simplified. Nonetheless, these intermediate wage equations can already help to refine the firm’s Euler
equations. Differentiating equation (3.23) and substituting it into the firm’s Euler equation for
labor demand help express the marginal product of labor more explicitly:

\[
\frac{\kappa^i}{\tilde{\beta} q^i_{t-1}} + w^i_t - \left(1 - \chi^i\right) \frac{\kappa^i}{q^i_t} = \frac{\partial y^i_t}{\partial n^i_{t-1}}.
\] (3.24)
3.2 The Model

The left-hand side of equation (3.24) is the cost of the firm to employ an extra worker. Compared to a perfectly competitive labor market where wage as the only labor cost equals the marginal product of labor in an imperfect labor market the firm also takes into consideration the posting costs incurred and future posting costs saved.

As more skilled labor is hired its marginal product declines due to the law of diminishing marginal returns, while the marginal product of unskilled worker increases, since skilled and unskilled labor enter the CES production function in a complementary manner. As shown in equation (3.23), wages contain a fraction of the corresponding marginal products of labor. Therefore the skilled wage decreases and unskilled wage increases with an extra unit of skilled labor.

In order to find the final form of the solution, we still need to combine the optimality condition and the bargaining result for the wage. Plugging the semi-final wage equation (3.24) back into the bargaining result and combining it with equation (3.15) we can solve for the value of employment,

$$\Omega^i_t = \frac{\eta}{1 - \eta} \frac{\kappa^i}{\beta^i q_{t-1}^i}.$$  \hspace{1cm} (3.25)

Take (3.25) one period ahead, and recall that in the labor market $p^i_t = \theta^i_q q_t^i$ holds,

$$E_t\Omega^i_{t+1} = \frac{\eta}{1 - \eta} \frac{\kappa^i}{\beta^i q_t^i} = \frac{\eta}{1 - \eta} \frac{\kappa^i q_{t-1}^i}{\beta^i q_t^i}.$$  \hspace{1cm} (3.26)

Using this result with equation (3.23), we can attain the final wage curves for skilled and unskilled labor:

$$w^i_t = \eta \left( \frac{\partial y_t}{\partial n^i_{t-1}} + \kappa^i p_t^i q_t^i \right) + (1 - \eta) b^i.$$  \hspace{1cm} (3.27)

These two wage curves enter the model equilibrium, which is defined as sequences of prices and labor market tightness which solve the firm’s, the household’s and the bargaining problems and clear the capital and labor markets. Other equilibrium equations include households’ Euler equations (equations (3.12), (3.13) and (3.14)), human capital evolution ((3.8) and (3.11)), labor transition equations ((3.9) and (3.10)), time constraint ((3.4) and (3.5)), labor market transitions ((3.1)-(3.3)), firms’ Euler equations ((3.20) and (3.21)), production function (3.16), capital evolution (3.17) and technology evolution (3.19), as well as the aggregate budget constraint:

$$y_t = c_t + k_t - (1 - \tau) k_{t-1} + x_t + \kappa^n v_t^i + \kappa^u v_t^u.$$  \hspace{1cm} (3.28)

The equilibrium is a system of 24 equations in 24 unknowns ($\Delta_t; x_t; n_t^i; n_t^u; l_t^i; l_t^u; u_t^i; u_t^u; v_t^i; v_t^u; \theta_t^i; \theta_t^u; q_t^i; q_t^u; p_t^i; p_t^u; w_t^i; w_t^u; y_t; c_t; k_t; i_t; z_t; \xi_t$). With the help of Dynare, this non-linear system can be simulated around given steady state values, which will be the task of the next section.
3 Human Capital Formation in Skill-Specific Labor Markets

3.3 Calibration

I choose the model period to be one quarter, and as a robustness test I also use monthly data to calibrate and then aggregate the results to a quarterly frequency. The results are not exactly the same due to the specific persistence of technology shocks at different time frequencies and minor changes in steady state values. In order to keep the results comparable to available data and avoid the possible imprecision from time aggregation, the simulation results at quarterly frequency are reported.

The parameters related to the aggregate economy are set to match post-war quarterly U.S. data, except some alterations due to the model structure. Exploiting the steady state equation of (3.20), the discount factor \( \beta \) is chosen to match an annual risk free rate of 4 percent. Francis and Ramey (2005) report that the investment share of income in the post-war data, \( \frac{i}{y} \), is 0.25. To match this and a labor share of 70 percent\(^3\), the quarterly physical capital depreciation rate is about 4 percent (15 percent annually).

According to the 2004 Current Population Survey (CPS), the labor hours performed by workers with 12 years of education or less had fallen to less than 45 percent. I correspondingly use it as the steady state value of \( \Delta \). The weight of non-participation or leisure is set as 0.6 for all workers, as main time use data shows that on average people spend more than 60 percent of their time for leisure and home production, and this ratio even increased in the last decades (Aguiar and Hurst, 2007). The technology parameter in production function \( \alpha \) is set as 0.5 for neutrality\(^4\).

The parameterization of labor market variables follows Ebell (2008), even though her model is calibrated to a weekly frequency. As Shimer (2005) has estimated an average monthly separation rate as 0.026, I choose 0.07 and 0.09 for skilled and unskilled workers respectively, which leads to the monthly \( \chi_s \) and \( \chi_u \) to be 0.024 and 0.031 separately. Targeting a skilled unemployment rate of 0.07 and unskilled unemployment rate of 0.1, I set the job finding rates (\( p_s \) and \( p_u \)) as 0.875 and 0.833 for skilled and unskilled respectively. Again, their monthly value, 0.5 and 0.45, are based on Shimer’s estimation from monthly data, 0.45. The job-filling rate \( \hat{q}_s \) and \( \hat{q}_u \) are set to 0.976 (or 0.71 monthly), which are in line with Den Haan, Ramey and Watson’s (2000) finding. Consequently, tightness for the skilled and unskilled labor markets (\( \theta_s \) and \( \theta_u \)) are 0.897 and 0.855 respectively. The scaling parameters of the matching functions \( m^s \) and \( m^u \) can also be pinned down as 0.92 and 0.9 each.

The next pair of parameters to fix are the vacancy posting costs, \( \kappa_s \) and \( \kappa_u \). Combining the wage

\(^3\)In this numerical exercise, the choice of the labor share, which is higher than what’s often used in RBC literature (see Kydland and Prescott, 1982, who estimated the capital share to be around 0.36, and labor share is 0.64), is to mitigate the human capital depreciation problem in the human capital formation equation.

\(^4\)This renders skilled-unskilled wage ratio to be 1.2. This number is relatively small compared to Card and DiNardo’s (2002) estimate using average hourly earnings data from the March CPS.
curves and firms’ Euler equations, a relationship between the $\kappa^i$, workers’ bargaining power $\eta^i$, and the value of non-market activity $b^i$ can be found. As Hagedorn and Manovskii’s (2008) calibration strategy aims at and succeeds in generating large fluctuations of vacancies and market tightness, there is hardly any empirical evidence to support the extremely high value of non-market activity, or the little bargaining power of workers. Furthermore, Cheron (2005) has shown that, if hiring costs are merely borne to the firms and workers’ quasi-rents are protected by contract so that the hold-up problem is avoided, the Hosios condition delivers efficiency when workers’ bargaining power equals elasticity of the matching function. A conventional choice is to set both $\eta$ and $\varphi$ as 0.5 (Blanchard and Diamond, 1989). $\frac{b_i}{w_i}$, the ratio of non-market activity to wage, is chosen to be 0.6, as a compromise of the extremely high values in Hagedorn and Manovskii (2008) and the small value in Shimer (2005). I show later that the variation of these ratios does not appear to change the final result to a large extent. The $\kappa$s take the values 0.28 and 0.23.

The key parameters left to be decided are the two in the human capital transition equation: The human capital depreciation rate $\delta$ and the coefficient $\mu$ in new human capital formation through on-the-job-training (learning by doing). The two equations concerned are (3.12), the Euler equation in optimal human capital investment, and (3.8), the human capital formation equation. The elasticity of human capital formation, $\vartheta$, revealing how fast human capital is accumulated during work, can lie between 0 and 1, so that $F(n_{t-1})$ is an upward-sloping concave curve with a relatively small slope. The choice of $\vartheta$ within this range is not so strict, unless one targets at a reasonable value of $\delta$. The proper target of $\delta$ is often under discussion.

In reality, human capital can depreciate due to either voluntary reasons (mostly family-related) or involuntary (unemployment, sick leave) career interruptions. The depreciation in the former case, mostly occurs to workers still on the jobs and is rather difficult to observe due to wage rigidity and the lack of proper measurement of productivity. As a result, the wage depreciation rate after unemployment is estimated as a proxy for human capital depreciation rate. For example, Keane and Wolpin (1997) use NLSY data and (structurally) estimate an annual wage depreciation rate for white U.S. males during unemployment of between 9.6 percent (for blue collars) and 36.5 percent (for white collars). Jacobson, LaLonde and Sullivan (1993) use plant closing data and find wage depreciation rates between 10 percent and 25 percent. However, wage is more rigid than human capital, in the sense that due to contract issues the wage does not correspondingly decrease as an immediate response to human capital declining. Consequently, the aforementioned estimation results turn to underestimate the human capital depreciation. By setting $\vartheta = 0.1$, $\delta$ takes the value of 0.065, corresponding to an annual depreciation rate (0.23). This is still within the range mentioned above. This depreciation rate is almost the same as that of physical capital, making both types of capital stock more comparable.
The elasticities of leisure in households’ utility function are key for the participation volatility. The assumption of flexible labor force is introduced into the RBC version of the MP model in Tripier (2004) and Veracierto (2008). Their model specifications fail to reproduce, most importantly, the countercyclical unemployment rate observed in U.S. data. Moreover, the resulting unemployment-vacancy correlation is strongly positive (Tripier, 2004) and unemployment fluctuates as much as output (Veracierto, 2008). One of the possible reasons for the poor performance of the models is how they parameterize the participation elasticity. Both papers choose the parameter value to reproduce the observed standard deviation of employment, implying relatively high elasticity (Tripier’s choice implies a value of about 3). Comparatively, Ebell (2008) novelly uses the relative volatility of the participation rate to pin down this elasticity. Since the data shows low relative volatility of the participation, the elasticity pinned down is small, which, consequently, discourages worker’s entering search from non-participation in response to a positive technology shock. Therefore, \( \psi_s \) and \( \psi_u \) in the current model are assumed to be identical and are set following Ebell (2008), aiming at forming a very inelastic labor supply and thus market entering of the inactive workers.

The resulting labor supply elasticities are smaller than unity, which is consistent with many microeconomic studies. I choose the elasticity parameter value as 0.05, yielding the relative volatilities of participation as \( \frac{\sigma_{ps}}{\sigma_y} = 0.07 \) and \( \frac{\sigma_{pu}}{\sigma_y} = 0.08 \). The model’s recursive law of motions further reveal that a 1 percent increase in total factor productivity leads to a 5 percent increase in skill labor participation, and a slightly less than 4.7 percent increase of unskilled labor participation. Exploiting the steady state Euler equations of households concerning labor participation, I obtain the values of the utility parameters representing the weights of leisure.

The model structure in the current chapter deviates from that in Krebs (2003a), but the concept of human capital being substitute of physical capital investment stays the same. However, due to the natural constraint that human capital lies within range of (0, 1), its absolute level is much smaller than that of capital stock. Therefore I cannot directly use Krebs’ (2003a) method to pin down the parameters of labor income risks. Instead, I borrow the result from microeconometric studies on labor income (Meghir and Pistaferri, 2004 and Krishna and Senses, 2009), and set the estimated variance of the permanent income shock to 0.008 (annualized to 0.031). The standard deviation of the idiosyncratic shock is thus 0.089, which seems to be extremely high for the current model and does not fit the model structure. Instead, I calibrate the \( \sigma_\eta \) to be half of the standard deviation of

---

5See Krebs (2003a), equation (12). There the optimal choice is expressed as \( 1 - \theta = \frac{(r_h - r_k)}{\sigma_\eta^2} \), whereas \( \theta = \frac{1}{k + h} \). After simple alteration, it becomes the following equation:

\[
\sigma_\eta^2 = \frac{(k + h) (r_h - r_k)}{h}.
\]
Concerning the persistence of the human capital shock \( \rho_\xi \) in equation (3.11), Krebs (2003a) chooses this coefficient to be one and allows human capital shocks to amount to permanent labor income shocks, whereas the latter is often empirically found highly autoregressive (and perhaps having even a unit root). The result of this specification is that the individual labor income process in equilibrium follows (approximately) a logarithmic random walk, which would lead to overestimated cross-sectional dispersion and variance of labor income. Meanwhile, a recent study by Huggett and Kaplan (2010) using data on male annual labor earnings from the PSID finds the persistence of the permanent component of labor income risk to be 0.934 (0.835 for high school equivalents and 0.915 for college equivalents separately). The magnitude of these estimates, according to the authors, could be overstated because much of the large rise in the log-earning variance observed over the working lifetime can actually be accounted for by learning ability differences across individuals (Huggett, Ventura and Yaron, 2006).

I thus calibrate \( \rho_\xi \), the persistence of the human capital shock at a high (0.95) and a low value (0.85) and study the effect of the human capital shock persistence. For the technology shock, I follow what’s widely used in the RBC literature (e.g. Hansen, 1985), to set the coefficient \( \rho = 0.95 \), and the standard deviation of the residual as 0.01.

Finally, what’s also important for the heterogeneous skills story, the choice of elasticity of substitution comes into sight. As summarized by Acemoglu (2002), the majority of micro studies estimate this elasticity, through the behavior of skill premium, to be between 1 and 2. Autor, Katz and Krueger (1998) argue that a consensus estimate is a value around 1.5, when the two skill groups are college and high school workers (e.g. 1.4 by Katz and Murphy, 1992 for the 1963-87 period using March CPS). Consequently I set \( \sigma = 1.4 \) in the benchmark model, and further examine the effect of a high elasticity of 2, the value implied in Angrist (1995). All simulation results will be compared with the second moments of U.S. data summarized by Shimer (2005) in Table 3.1.

### 3.4 Simulation and Impulse Response Analysis

#### 3.4.1 Simulated Results of the Benchmark Model \((\sigma = 1.4, \rho_\xi = 0.85)\)

Due to the participation margin and low participation elasticity, the model can generate negative Beverage curves in both skilled and unskilled markets. The reasoning is well-argued in Ebell (2008). Under a positive productivity shock, workers respond latently in participating in the labor force (exiting non-participation) and start searching. The impact on unemployment is small compared to that on vacancy creation, thus tightness and job-finding rates increase strongly. The strong increase of job-finding rates speeds up workers’ leaving unemployment and thus unem-
Table 3.1: Shimer’s Summary Statistics, Quarterly U.S. Data, 1951-2003

<table>
<thead>
<tr>
<th>Variable</th>
<th>U</th>
<th>V</th>
<th>V/U</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>0.190</td>
<td>0.202</td>
<td>0.382</td>
<td>0.020</td>
</tr>
<tr>
<td>Relative std. deviation ( \frac{v}{u} )</td>
<td>9.5</td>
<td>10.1</td>
<td>19.1</td>
<td>1.0</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.936</td>
<td>0.940</td>
<td>0.941</td>
<td>0.878</td>
</tr>
</tbody>
</table>

Correlation

\[ \begin{array}{lccccc}
\text{Autocorrelation} & u & v & v/u & z \\
\hline
\text{Correlation} & u & 1 & -0.894 & -0.971 & -0.408 \\
v & - & 1 & 0.975 & 0.364 & \\
v/u & - & - & 1 & 0.396 & \\
z & - & - & - & 1 & 
\end{array} \]

Source: Shimer (2005, Table 1); Relative standard deviation \( \frac{v}{u} \) is own calculation.

Employment decreases soon after the shock. As shown in Table 3.2, skilled and unskilled unemployment has a positive correlation of 0.314, as a result of unbiased technology shocks and biased human capital shocks. The correlations between unemployment and vacancies are negative in both markets (−0.22 and −0.46 for skilled and unskilled respectively), while the cross-correlations are even higher (\( \rho_{v,u,v} = -0.87, \rho_{v,u,u} = -0.83 \)). In total, the correlation between total unemployment and vacancy is −0.83, which is quite a good match compared to Shimer’s (2005) data summary (−0.894).

Table 3.2: Baseline Results: Cyclicality of Labor Market Variables in Skill and Unskilled (\( \sigma = 1.4, \rho_\eta = 0.85 \))

<table>
<thead>
<tr>
<th>Variable</th>
<th>U</th>
<th>U</th>
<th>V</th>
<th>V</th>
<th>V/U</th>
<th>U</th>
<th>V</th>
<th>V/U</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative std. dev. ( \frac{v}{u} )</td>
<td>0.539</td>
<td>0.708</td>
<td>0.617</td>
<td>0.722</td>
<td>0.975</td>
<td>0.977</td>
<td>0.948</td>
<td>0.965</td>
<td>0.977</td>
</tr>
<tr>
<td>Autocor.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( u_u )</td>
<td>1</td>
<td>0.314</td>
<td>-0.219</td>
<td>-0.832</td>
<td>-0.743</td>
<td>-0.690</td>
<td>0.764</td>
<td>-0.625</td>
<td>-0.717</td>
</tr>
<tr>
<td>( u_v )</td>
<td></td>
<td>-1</td>
<td>-0.867</td>
<td>-0.458</td>
<td>-0.781</td>
<td>-0.833</td>
<td>0.852</td>
<td>-0.72</td>
<td>-0.813</td>
</tr>
<tr>
<td>( v_v )</td>
<td></td>
<td>-1</td>
<td>1</td>
<td>0.586</td>
<td>0.816</td>
<td>0.839</td>
<td>0.866</td>
<td>0.832</td>
<td>0.828</td>
</tr>
<tr>
<td>( v_u )</td>
<td></td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>0.895</td>
<td>0.873</td>
<td>-0.77</td>
<td>0.913</td>
<td>0.887</td>
</tr>
<tr>
<td>( v_u/v_u )</td>
<td></td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>0.985</td>
<td>-0.94</td>
<td>0.964</td>
<td>0.995</td>
</tr>
<tr>
<td>( v_v/v_u )</td>
<td></td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>-0.946</td>
<td>0.963</td>
<td>0.997</td>
</tr>
<tr>
<td>( u )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( v )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( v/v_u )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( z )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

All variables reported are log deviations from an HP trend.

Another quantitative benchmark usually discussed in related literature is the correlation between unemployment and output. As the data shows this correlation to be −0.88, the model generates even higher (more negative) correlation, −0.95. The skill-specific unemployment rates are less correlated with output (−0.72 and −0.81 respectively). As Hagedorn and Manovskii (2010) re-
Recently find, exploiting the Current Population Survey (CPS), the total labor productivity, defined as output over total labor, is strongly correlated with employment (correlation: $\rho_{p,n} = 0.719$), unemployment ($\rho_{p,u} = -0.633$), vacancies ($\rho_{p,v} = 0.719$), and market tightness ($\rho_{p,\theta} = 0.703$). My model generates much higher values (subsequent correlations: 0.99, −0.94, 0.96, 0.99), which is common in related literature, and as Hagedorn and Manovskii (2010) point out, this discrepancy can be alleviated by adding two new model features, namely “time to build” (lags in vacancy posting) and a stochastic value of home production. Finally, training investment ($x_t$) responds positively to the technology shock, but negatively to the human capital shock. In total, the negative response dominates and $x_t$ negatively correlates with output, even though the correlation is small (−0.03). This result matches well with several empirical findings on the cyclicality of training (Sepulveda, 2004, Bassanini and Brunello, 2007).

The model can generate volatile standard deviations of labor market variables. Relative to the standard deviation of the productivity shock, the standard deviation of the total market tightness is 10.87 times higher. Total unemployment is 5 times and total vacancy is 6.32 times more volatile than productivity. Specifically, relative standard deviation of the skilled unemployment (6.45) is slightly higher than the unskilled (5.98), while that of the skilled vacancy creation (7.48) is higher than its unskilled counterpart (6.8). The standard deviation of human capital investment is very large compared to output (33 times), which is in line with Sepulveda’s (2004) finding, that training is highly volatile over the cycle.

### 3.4.2 Impulse Responses

**The Effect of a Technology Shock ($\epsilon_t$)**

Both skilled and unskilled unemployment reacts first positively to the technology shock, and decreases strongly after two periods (Figure 3.2).

The immediate positive responses of both types of vacancies contribute to their high procyclicality. The response of skilled vacancies slightly exceeds that of unskilled labor, which corresponds to the difference of their coefficients in the policy functions (skilled vacancies react 7 percent more strongly than unskilled vacancies). Consequently, the skilled tightness also exceeds the unskilled tightness by 4.5 percent in the policy function and compared to $\theta^u$, $\theta^s$ is slightly higher correlated with $y$. Small as these differences are, they indicate that the neutral technology shock is in fact skill-biased, in the sense that the skilled labor market situation improves more due to the technology shock. The underlying quantitative reason is the smaller share (less than 50 percent) of skilled workers in the total labor force. Skilled labor productivity (average and marginal) are accordingly higher than their unskilled counterpart. A positive technology shock, therefore, benefits
Figure 3.2: Impulse Responses to a Technology Shock, Skilled and Unskilled ($\sigma = 1.4$)
skilled labor more than unskilled. In contrast, a negative shock induces a larger proportional loss for a marginal skilled worker than for a marginal unskilled worker, resulting in a declining skill premium. The firms would naturally prefer an additional skilled worker to an unskilled worker under such circumstances, and therefore the unskilled workers suffer even more from losing the market power and being replaced in recessions.

Figure 3.3 shows that aggregate labor market variables behave similarly to Ebell (2008). Due to small participation elasticity, the uptick in unemployment is more modest than vacancies so that tightness increases strongly. The sharp drop of unemployment after the initial moment insures its countercyclicality and the negative correlation between unemployment and vacancies.

![Figure 3.3: Impulse Responses to a Technology Shock, Aggregate Variables](image)

**The Effect of a Human Capital Shock ($e_t$)**

The inclusion of human capital formation and transition allows for examining the crucial contribution of active labor market policy to decreasing aggregate unemployment upon a positive technology shock. The effect of a human capital shock is in both short-run and long-run. As a
prompt reaction to a positive shock, there is a surge of supply of skilled workers. Unable to find jobs immediately, these workers flow into skilled unemployment, whereas their shift away from unskilled labor market mitigates unskilled unemployment. Therefore we observe a positive immediate response of skilled unemployment and a negative reaction of the unskilled unemployment.

Skilled vacancies also react positively to the shock, and the amplitude is smaller than that of skilled unemployment. Comparatively, as unskilled vacancies react negatively to the shock just like unskilled unemployment, the percentage deviation of vacancies is smaller than that of unemployment. On the one hand, market tightness and job finding rates decrease in the skilled labor market and increase in the unskilled market. The vacancy filling rate, on the opposite, responds positively in the skilled market and negatively in the unskilled market. Since it becomes relatively easier to recruit skilled workers, and more difficult to hire unskilled, firms post fewer skilled vacancies and more unskilled vacancies. This explains the change in direction of the impulse response of vacancies. On the other hand, the wage difference between the skilled and unskilled declines, meaning that skilled workers become relatively cheaper. The natural reaction of the firms is to adjust their labor input share to the change in the labor market structure, using more productive workers to replace less productive ones. In equilibrium, skilled workers’ participation increases more than that of the unskilled workers, because of the larger margin between participation and non-participation.

Figures 3.4 and 3.5 report the effect of a positive human capital shock, which contemporaneously increases the skilled and decreases the unskilled labor force. As an example, one can consider one effective training course in the scheme of active labor market policy, through which a small share of unskilled workers are upgraded into the skilled labor force. As the skilled labor participation extends, the newly-trained skilled workers cannot find jobs instantly but enter skilled unemployment directly, thus at once the skilled unemployment responds positively. The reaction of skilled vacancy posting is also very prompt, even though its percentage deviation is slightly lower than that of skilled unemployment. Thus the skilled labor market tightness reacts negatively to the shock. Declining market tightness pushes the firm into a better position, since for every posted vacancy there are more applicants. This instant over-supply and under-price of skilled labor induce firms to use skilled labor to substitute the unskilled, so that the deviation of the skilled unemployment soon returns to the steady state and becomes negative.

There is a small drop of unskilled participation due to those up-skilled workers. Meanwhile, unemployment of the unskilled workers decreases first due to the sudden contraction of labor force, which is accompanied by a smaller reduction of unskilled vacancies. Unskilled labor market tightness increases accordingly, and afterwards returns slowly to the steady state. As vacancies per searching worker increase, unskilled wage also rises, and the relative price of unskilled worker
Figure 3.4: Impulse Responses to a Human Capital Shock, Skilled ($\sigma = 1.4$)
Figure 3.5: Impulse Responses to a Human Capital Shock, Unskilled ($\sigma = 1.4$)
becomes higher. As discussed above, unskilled workers are partly replaced by the skilled, and unskilled unemployment converges quickly toward the initial level.

In aggregate, total unemployment reacts at first negatively to the human capital shock (Figure 3.6). This is mainly due to the reduction of unskilled unemployment. The response returns to the steady state quickly, but experiences a second negative impulse. This second unemployment reduction is mostly fuelled by the decreasing skilled unemployment because of the skill substitution. Total vacancies also observe a two-stage response toward the shock, which sink firstly due to the dominant deduction of the unskilled vacancies, and recover shortly afterward because of the strong skilled vacancy creation and unskilled vacancy recovery. The response of aggregate market tightness, which is positive and shows a smooth hump, can also explain the second-stage decline of total unemployment, since on average workers can find jobs more easily.

Figure 3.6: Impulse Responses to a Human Capital Shock, Aggregate ($\sigma = 1.4$)

An increase in $\rho_z$ increases the persistence of human capital shock, and therefore multiplies the shock effect on the variables. Relative standard deviations of the skilled and unskilled variables all increase, while, at an aggregate level, unemployment, employment, vacancy and market tightness
increase too, only to a smaller extent. Because the initial response of total employment is more negative and that of capital stock does not increase sufficient, total output decreases in the first two periods, and the response becomes positive from the third quarter on.

Skill-specific vacancies and unemployment increase by the same amount, so that correlation within- and across markets declines. The aggregate $u-v$ correlation, nonetheless, does not vary much.

### 3.4.3 The Importance of the Elasticity of Substitution

The elasticity of substitution indicates how well the two labor factors can substitute each other. For a given change in relative prices, a higher $\sigma$ implies a larger change in the labor inputs. The statistics of the simulation results are summarized in Table 3.3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$u_s$</th>
<th>$u_u$</th>
<th>$v_s$</th>
<th>$v_u$</th>
<th>$v_s/u_s$</th>
<th>$v_u/u_u$</th>
<th>$u$</th>
<th>$v$</th>
<th>$v/u$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative std. dev.</td>
<td>7.77</td>
<td>6.99</td>
<td>8.74</td>
<td>7.57</td>
<td>11.18</td>
<td>11.69</td>
<td>5.3</td>
<td>6.6</td>
<td>11.44</td>
<td>5.09</td>
</tr>
<tr>
<td>Autocor.</td>
<td>0.349</td>
<td>0.619</td>
<td>0.475</td>
<td>0.639</td>
<td>0.977</td>
<td>0.979</td>
<td>0.954</td>
<td>0.876</td>
<td>0.979</td>
<td>0.981</td>
</tr>
<tr>
<td>Corr.</td>
<td>$u_s$</td>
<td>1</td>
<td>0.036</td>
<td>0.088</td>
<td>-0.847</td>
<td>-0.627</td>
<td>-0.57</td>
<td>0.633</td>
<td>-0.51</td>
<td>-0.587</td>
</tr>
<tr>
<td>$u_u$</td>
<td>-1</td>
<td>1</td>
<td>-0.900</td>
<td>-0.288</td>
<td>-0.729</td>
<td>-0.785</td>
<td>0.796</td>
<td>-0.697</td>
<td>-0.771</td>
<td>-0.759</td>
</tr>
<tr>
<td>$v_s$</td>
<td>-1</td>
<td>-0.333</td>
<td>0.721</td>
<td>0.753</td>
<td>-0.644</td>
<td>0.783</td>
<td>0.749</td>
<td>0.738</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_u$</td>
<td>-1</td>
<td>-0.849</td>
<td>0.82</td>
<td>-0.736</td>
<td>0.847</td>
<td>0.83</td>
<td>0.836</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$v_s/u_s$</td>
<td>-1</td>
<td>1</td>
<td>0.986</td>
<td>-0.944</td>
<td>0.967</td>
<td>0.995</td>
<td>0.997</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_u/u_u$</td>
<td>-1</td>
<td>-0.953</td>
<td>0.965</td>
<td>0.998</td>
<td>0.995</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_s/v_u$</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-0.848</td>
<td>-0.952</td>
<td>-0.953</td>
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</tr>
<tr>
<td>$v/u$</td>
<td>-1</td>
<td>0.97</td>
<td>1</td>
<td>0.999</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$v/u$</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>1</td>
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<td></td>
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</tr>
<tr>
<td>$y$</td>
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<td>-0.848</td>
<td>-0.952</td>
<td>-0.953</td>
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</tr>
</tbody>
</table>

All variables reported are log deviations from an HP trend.

The relative standard deviations increase for all variables. The aggregate correlation becomes more negative to $-0.85$. As the $u-v$ correlation across the markets rise to $\rho_{v_s,u_s} = -0.90$ and $\rho_{v_u,u_u} = -0.85$, the within-market correlation decreases. $\rho_{v_s,u_s}$ declines to $-0.29$, and $\rho_{v_u,u_u}$ even becomes positive (0.09).

Skilled participation, on the opposite, becomes more procyclical, implying more workers entering the labor market searching for skilled jobs. The uptick of unemployment becomes larger and on average the quarterly countercyclicality of unemployment is now weaker ($\rho_{u_s,y} = -0.60$ v.s. $-0.72$ in the benchmark case). Meanwhile, as skilled vacancies becomes less pro-cyclical ($\rho_{v_s,y} = 0.74$) than before (0.83), the correlation between skilled unemployment and vacancies becomes positive.

On the unskilled market, even though the cyclicality of both unemployment and vacancies become weaker than the benchmark, the changes are to a similar degree (Figure 3.7). The impulse response of unskilled participation shows that the initial positive reaction is lower, and the re-
versed further reaction is larger than the baseline. As a result, on average fewer inactive unskilled workers’ enter search and unskilled participation becomes countercyclical. In aggregate, the variables react more intensively to the shock (Figure 3.8).

![Figure 3.7: Impulse Responses to a Technology Shock, Skill and Unskilled (σ = 2)](image-url)

Given a high $\sigma$, the substitution between two types of labor becomes larger upon a change of the relative factor price than the baseline. Key labor market variables react more strongly to human capital shocks. In the impulse response graphic of the total unemployment to a positive human capital shock, even though the first-stage downtick is similar, the second-stage downtick is much more prominent than the baseline, which contributes to the higher correlation (in absolute value) between total unemployment and output (Figure 3.9). On the opposite, aggregate vacancies react to a smaller extent to the same shock than the baseline. As aggregate unemployment becomes much more countercyclical, and total vacancies experience smaller changes, the $u - v$ correlation becomes higher in absolute value and thus closer to the data.
Figure 3.8: Impulse Responses to a Technology Shock, Aggregate ($\sigma = 2$)
3.4 Simulation and Impulse Response Analysis

Figure 3.9: Impulse Responses to a Human Capital Shock, Aggregate ($\sigma = 2$)
3 Human Capital Formation in Skill-Specific Labor Markets

3.5 Policy Implication

The simulation experiments have shown that the weak position of unskilled workers varies over the business cycle. Skill substitution is persistent and especially strong during recessions, leading to high unskilled unemployment both in level and volatility. The previous section has shown that skill-specific labor markets become more volatile and the vulnerability of the unskilled workers rises with the differentiation between the skilled and unskilled (an increase in $\sigma$), as well as with the persistence of the human capital shock.

Associated with such observations, what can be done to alleviate the inferior situation of the unskilled workers? Let’s return to the active labor market policy discussions at the beginning of this chapter. As found in Card, Kluve and Weber (2010), classroom and on-the-job training programs appear to be particularly likely to yield more favorable medium-term than short-term impact estimates. This coincides with the observation above, that aggregate unemployment experiences a two-stage decline: Firstly with a reduction in unskilled unemployment due to the skill upgrade of marginal workers through active training, and then with a sharp decline of skilled unemployment when skill substitution dominates. Training programs provide skill upgrade opportunities to the lower-skilled workers, and consequently preserve the average skill level of the total labor force. The positive effects implied are not only on the skill-specific, but also on the aggregate labor market variables. Therefore, the challenging tasks in the real world include at least two points: How to set the correct incentive schemes so as to encourage workers, and especially unemployed workers, to participate training, and how to identify an effective combination of active labor market policies that help achieve the short-run and longer-run goals simultaneously.

3.6 Conclusion

This chapter studies the rationale of skill-specific labor market variables in a framework where labor force structure is endogenized and the flow between skill types is allowed through training decisions. Skilled and unskilled workers are not only connected due to their substitutability in production, but also through the skill-training system. As labor, also interpreted as human capital here, can experience skill-downgrade due to human capital depreciation, it can also be accumulated and upgraded through sufficient training, be it on the job or general training.

By modeling the transmission between skilled and unskilled labor force, this framework allows the study of the effect of human capital shock on the dynamics in skilled and unskilled markets. This trait is important not only due to its direct relevance to the highly debated active labor market policy, but also because of the decomposition of aggregate unemployment into skill specific and term specific. As a consequence, idiosyncratic shocks in human capital formation, in conjunc-
tion with the technology shocks, produce high volatility and downward-sloping the Beveridge curves in skill-specific labor markets. Inelastic labor participation also contributes to these results. Moreover, in aggregate, unemployment and vacancies display data-resembling high negative correlation.

In the current setup allowing for skill substitution and transition between two types of labor, the negative slope of the Beveridge curve is a result of two-staged effects on unemployment on vacancies. Particularly, upon the human capital shock, total unemployment reacts negatively due to the reduction of unskilled unemployment; a second unemployment reduction results from the overwhelmingly decreasing skilled unemployment, since skilled labor substitutes out the unskilled. Active labor market policy can therefore reinforce the unemployment reduction caused by technology shocks, especially at higher elasticity of substitution between the two types of labor. Total vacancies also observe a two-stage response toward the shock, which sink firstly due to the dominant deduction of the unskilled vacancies, and recover shortly afterward because of the strong skilled vacancy creation and unskilled vacancy recovery.

This model setup can be used to explore the skill-specific market dynamics and cross-connections between the key variables. What’s worth studying further includes the exact form of the human capital accumulation. A more specified setup can help evaluate the effects of specific training program on unemployment and other labor market indicators.
4 Consumption Growth and Volatility with Consumption Externalities

4.1 Introduction

Consumption inequality is a direct measure for the well-being of a population, while consumption growth and volatility are alternative welfare measures at higher orders. Various socioeconomic groups, defined by age, household size, occupation etc., have diverse preferences and are subject to heterogeneous shocks. The modification of the trade-off between consumption and saving differs, which further affects consumption fluctuations and the growth trend to different extents. Consequently, not only consumption patterns but also their growth and fluctuation are divergent across groups. Groups subject to large shocks and lacking a smoothing possibility appear to have on average lower growth and higher fluctuations, indicating that they are at disadvantageous welfare positions. For example, income and consumption growth inequality for different age groups can vary widely (Figure 4.1). As younger groups have higher consumption growth, their consumption volatility is also higher.
4 Consumption Growth and Volatility with Consumption Externalities

Figure 4.1: Income and Consumption Growth Inequality, at Different Age

The contribution of this chapter is three-fold: (1) providing a theoretical framework of heterogeneous agents with consumption externality in order to examine the link between group-specific consumption growth and volatility, (2) finding empirical evidence on the aforementioned relationship using matched household data from the German Socio-Economic Panel (SOEP) and the German Income and Expenditure Survey (Einkommens- und Verbrauchsstichprobe, or EVS in later text), and (3) examining the empirical relationship between growth and within-group inequality.

Aiming at examining the relationship between consumption growth and volatility, I use a framework stemming from the literature studying income shocks and consumption inequality. Complete market hypothesis is not used here for two reasons. First, the perfect insurance against idiosyncratic shocks implied by the complete market theory is rejected by empirical evidence (Attanasio and Davis, 1996, Attanasio and Pavoni, 2007). Moreover, the complete market assumption, often resembled by a complete set of Arrow-Debreu security for each state, suggests that, given identical preferences, there should be no consumption volatility because everyone is insured similarly. This, however, is also strongly rejected by the data (Fisher and Johnson, 2006, Jappelli and Pistaferri, 2000). According to Lucas (1992), “if the children of Noah had been able and willing to pool risks, Arrow-Debreu style, among themselves and their descendants, then the vast inequality we see today, within and across societies, would not exist.”

In contrast, incomplete market models are generally adopted to study the diverse evolution of
4.1 Introduction

Income and consumption inequality (Blundell and Preston, 1998, Blundell, Pistaferri and Preston, 2008). Be the reason of market incompleteness limited enforcement of contracts (Krueger and Perri, 2006) or private information problems (Attanasio and Pavoni, 2007), risk-sharing exists but is not perfect. In fact, a model with one single asset and heterogeneous household preferences can offer partial but relatively good insurance against income shocks (Krusell and Smith, 1999), whereas under certain assumptions it can match the real-world wealth distribution relatively well. More discussion and literature review on incomplete markets model can be found in Heathcote et al. (2009). For simplicity, while not losing generality, the theoretical framework of the current chapter is reduced to a “standard incomplete market” model in an endowment economy, where a large number of agents draw idiosyncratic realizations of endowment, and make independent choices for consumption and asset holding. Their choices determine, in aggregate, the total amount of capital for production and the equilibrium rental rate for capital.

Households from various socioeconomic groups differ in patience and attitude toward their reference; i.e., the group average consumption. This is different from the neoclassical economic reasoning, which is typically based on the self-interest hypothesis; i.e., people are exclusively motivated by their material self-interest. Indeed, both absolute and relative consumption matter for households in the current model, whereas the idea of relative consumption associates with conceptual consumption (Ariely and Norton, 2009a, 2009b) and “social preferences” (Fehr and Fischbacher, 2002), and can go back to Veblen’s (1899) discussion of conspicuous consumption and Duesenberry’s Relative Income Hypothesis (1949).

Acknowledging consumption growth inequality as a result of income uncertainties (permanent and transitory) and consumption innovation, I approximate the Euler equation of heterogeneous households with group externalities in general equilibrium to study the link between two key features of consumption evolution: Growth and volatility. Comparative statics show that volatility decreases with groups’ degree of patience, and increases with household eagerness to keep up with the group average. The strength of the effects varies over the business cycle. Moreover, the correlation between the group average growth and volatility indicated by the model is positive once parameters take consensus values. Due to data limitation, only the last proposition is able to be examined empirically for distinct socioeconomics groups.

The grouping method forming the heterogeneous preferences, indeed, is crucial for studying the link between growth and volatility of the economy. Cross-country estimates using aggregate data and cross-sector studies using sector level data can generate opposite results. For example, in the case of output growth, as Ramey and Ramey (1995) find higher volatility accompanied by lower growth in two samples of countries, Imbs (2007) re-examines the issue at sector level and presents evidence of positive correlation.
The procedure taken in this chapter is adjusted to the availability and structure of the data. Studies on consumption inequality in Germany are less prevalent than on income inequality due to the limited availability of survey data. Recently, Fuchs-Schündeln, Krueger and Sommer (2010) look into both income and consumption inequality in Germany. They document an upward inequality trend of wage income after reunification, and finds a more modest rise of consumption inequality over the same period\(^1\). The analysis of the current chapter focuses on consumption growth and volatility, and complements a number of studies that use micro data to document the evolution of income or wage inequality in Germany in the last 25 years (among others, Biewen, 2000, Dustmann, Ludsteck and Schönberg, 2009).

Two data sets are under investigation, the German Income and Expenditure Survey (EVS) and an imputed sample from EVS and the German Socio-Economic Panel (SOEP). There are two approaches to impute consumption: One by using the estimated coefficients, and the other through matching cells in EVS and SOEP. The imputed consumption is used to construct consumption growth, volatility and within-group variance. The resulting consumption measure incorporates the well-documented consumption and income information in EVS and the panel structure in SOEP.

Although it is impossible to identify the direction of households’ attitude toward peers’ well-being with the current data, the finding of Knies’ (2010) using income and life satisfaction data appears to support the “relative income” hypothesis in West Germany. The empirical focus of the current chapter lies in identifying the correlation between growth and volatility, which is found positive and significant in fixed-effect estimates using EVS data. A more complex nonlinear relationship is found when the data sets are matched so as to explore the panel structure. Moreover, group growth also appears to be positively linked to within-group variances, implying higher inequality as the welfare cost for faster growing groups regardless of the driving factors of growth. Household size, age and nationality of the household head turn out to be significantly relevant to individual consumption growth and volatility, whereas community size and heads’ occupation are only related to volatility. Heads’ education appears irrelevant. Figure 4.1 shows in detail how strong the age effect is not only in growth, but also in volatility.

The rest of the chapter is organized as follows: Section 4.2 presents the theoretical model and derives four propositions; Section 4.3 introduces the data and specifies the grouping strategy; in Section 4.4 proposition four is tested and the estimate results are discussed; Section 4.5 concludes.

\(^1\)Evidences on the trend of consumption inequality are mixed for other developed countries. Blundell and Preston (1998) document substantial differences in inequality growth over the 1980s across birth cohorts in the U.K., while Crossley and Pendakur (2002) notice that overall consumption inequality in Canada has fallen slightly over the period 1969 to 1999. Barrett et al. (2000) find much lower inequality in consumption than in income in Australia. The disjuncture between income and consumption inequality, also found in the U.S. over the 1980s, can be explained by changes in the persistence of income shocks (Blundell et al., 2008) or by predictable income shocks (Primiceri and van Rees, 2009).
4.2 Consumption Growth and Volatility

4.2.1 Social Interaction and Relative Consumption

Among the extensions added to the incomplete market setup in the asset pricing literature, one special aspect is to include relative consumption into household utility as a consumption externality.

Psychological and economic studies often show that both absolute and relative consumption matter for individual well-being and behavior (see, e.g. Duesenberry, 1949, Diener et al., 1999, Luttmer, 2005). Individuals’ satisfaction derived from being better than their peers can be interpreted as envy, inequity aversion, relative deprivation or a human propensity to judge one’s achievement relative to that of others. The “others” here are the reference groups of actors, a concept brought about in social psychology early in the 1940s (Hyman, 1942). Depending on the situation, they can be coworkers, relatives, neighbors or members of clubs and organizations. Moreover, they can also be people who are geographically away and do not interact with the actor physically. According to Shibutani (1955), reference groups can be: (1) those who serve as comparison points, (2) those to which men aspire and (3) those sharing the same perspectives with the individuals. The last category requires common communication channels, each of which gives rise to a separate world, or, a socioeconomic group. The social worlds can be ethnic minorities, the social elite, medicine association, theater audience, readers of certain periodicals, or, in today’s context, groups on facebook. In a word, these associative reference groups realistically represent the individuals’ current equals or near-equals; i.e., they are from the same socioeconomic background, which is the definition for groups in the current chapter.

While others’ income can hardly be detected, households can relatively easily observe the life styles and infer the consumption levels of others with similar socioeconomic status. Their optimal security holding will adapt accordingly and their consumption smoothing path is different from an externality-free world. As a result, their evaluation of others’ consumption affects the group consumption growth inequality. The direction of this effect depends on how exactly households react to their peers’ well-being (whether they are “altruistic” or meant to “keep up with the Joneses”). Alternatively, this reaction can be interpreted as individuals’ life satisfaction upon the change of their peers’ income. While such attitude can be barely identified in empirical data, happiness is often used as proxy to capture individual’s utility.

Studies based on developed countries find that subjective welfare depends positively on one’s own consumption but negatively on the average consumption level of others nearby (Easterlin, 2001, Blanchflower et al., 2004, Luttmer, 2005). Knies (2010) finds comparable evidence in West Germany where West Germans are significantly unhappier with their lives if their neighbors are
4 Consumption Growth and Volatility with Consumption Externalities

going richer, implying an urge of the West German households to avoid being lagging back from their neighbors, or alternatively, the urge to keep up. This effect is slightly more marked in neighborhoods with presumably more social interactions, so that households may be able to assess more accurately the change of their neighbors’ financial position. On the opposite, Fafchamps and Shilpi (2008) find that in Nepal, households in isolated areas care more about what their neighbors consume. Their reasoning is that in isolated communities, neighbors can more accurately approximate the relevant reference group than in more mobile urban communities. These observations require economic models to take social environment into account, whose effects are heterogeneous according to agents’ socioeconomic background.

The preference on relative consumption can be regarded as a special form of physical consumption or a conceptual consumption besides the physical consumption. Long discussed by sociologists and anthropologists in the field of consumer behavior, it is summarized in Ariely and Norton (2009a) that “physical consumption is used not just to satisfy basic needs but also to signal to ourselves and others our beliefs, attitudes, and social identities”. Therefore conceptual consumption strongly influences physical consumption, and the possession of a BMW convertible is often only partly due to the need for transport. The concept consumed is the (relative) social status, which dates back to Veblen’s (1899) discussion of conspicuous consumption and Duesenberry’s Relative Income Hypothesis (1949), and accords with the “inequality aversion” in Fehr and Schmidt (1999) and “social preferences” in Fehr and Fischbacher (2002).

As a special type of consumption externalities, relative consumption serves as powerful non-pecuniary motives. The model setup of the current chapter borrows the spirit of Galí (1994). How this externality exactly matters for individuals can be captured in individuals’ utility in relative well-being comparing to their reference groups, which, as stressed in sociological literature, tends to consist of others who are similar in terms of background variables such as age, education and household size (see, for example, Merton and Kitt, 1950, and Festinger, 1954). Household preferences are assumed to be heterogeneous accordingly. As Shibutani (1955) emphasizes, culture, a perspective that is shared by those in a particular group, may also constitute the frame of the reference and matter for the direction of their preference. This is indeed documented in Knies (2010), where compared to West Germans’ becoming unhappier on their neighbors’ increasing wealth, East Germans’ life satisfaction positively, though insignificantly, correlates with neighborhoods’ income.

As previous sociologists and psychologists emphasize the role of positional goods (a similar concept to the aforementioned conspicuous consumption) in relative consumption, it was assumed that higher income groups care more about it since a larger part of their consumption is composed of positional goods. However, relative consumption is also found to be important for vacations
4.2 Consumption Growth and Volatility

and insurance, which are typically seen as non-positional goods (Alpizar et al., 2005). Besides, evidence shows that poorer groups care no less about the relative consumption than their richer counterparts do (Falchamps and Shilpi, 2008). It seems that the effect of relative consumption prevails over the economy.

What to keep up with are the associative references, or, the group mean. In a world of uncertainty, the current group mean serves as the local norm for households to set realistic goals, which is the third type of reference summarized by Shibutani (1955). As mentioned above, the incentive to keep up can be interpreted as envy, inequity aversion, relative deprivation, or a human propensity to judge one’s achievement relative to that of others. Take inequity aversion for instance, inequity averse persons want to achieve an equitable distribution of material resources; i.e., they want to neither surpass nor fall behind others in the reference groups, but keeping up with those above them and staying the same with those below them. Therefore, the group mean becomes their benchmark. This setup is slightly different from the case when individuals would like to emulate the top households of the group, which coincides with the “aspiring” case in Shibutani’s (1955) definition and would cause more deviation from an externality-free economy.

There is a subtle difference if agents take past or current average consumption as benchmark. The former, which is a variation of the habit formation setup, is the case of “catching up with the Joneses” (Mehra and Prescott, 1985, Abel, 1990, Campbell and Cochrane, 1999) and the latter “keeping up with the Joneses” (Galí, 1994). While the former involves the interdependence between the agents’ past, present and future well-being, the latter setup emphasizes contemporaneous trade-offs and generates simpler results. Since the true task is to study contemporaneous consumption distribution in a cross-sectional panel setting, the current chapter imposes “keeping up with the Joneses” assumption so as to avoid more complex intertemporal considerations.

4.2.2 A Heterogeneous Agent Model

The setup follows Galí (1994) where households regard contemporaneous group average consumption as an external benchmark (“keeping up with the Joneses”). While Galí’s (1994) model describes the homogeneous households in the whole economy, the current chapter takes the perspective of each group, and the “keeping up” mechanism bounds the agents within the group. The heterogeneity of agents between groups is captured as the different preferences, namely patience and attitude toward the benchmark. Using a heterogeneous agent model enables the contemporaneous examination of consumption growth inequality within the group, while still allowing for comparison in

\footnote{In fact, Ljungqvist and Uhlig (2000) discuss optimal tax policies using these two differentiated cases and find procyclical taxes for the former and a flat tax rate for the latter to be optimal. Guo (2004) elaborates the latter case by adding capital accumulation and imperfect competition in the goods market and finds a similar result.}
the time dimension and/or group-to-group dimension.

There is a continuum of households of measure 1 in this economy. Households belong to different groups \( i \in \{1, \ldots, M\} \), where the level of patience \( (\beta_i) \) and the attitude toward group average consumption \( (\kappa_i) \) differ. These differences capture the socioeconomic heterogeneity in the population. In the empirical part of the chapter later groups are defined according to household size, community size, household heads’ nationality, age, education level and job type. One can also intuitively interpret a group as a highly similar neighborhood. \( p_i \) denotes the number of households in each group. Households belong to certain groups because of the aforementioned features but are still subject to small idiosyncratic shocks, either from income or consumption innovation. Although households in a given group do not observe the exact income of other group members, they can observe their consumption patterns. If they would like to be identical with the others in a similar socioeconomic class, it is the case of “keeping up with the Joneses”. Otherwise, if they also benefit when others are doing well, we have “altruistic” households. I label the result of this additional externality a group effect on household consumption decisions.

Households receive idiosyncratic endowment every period\(^3\). One household in group \( i \) has a stochastic endowment process \( \{Y_t, Y_{i,t}, Y_t\} \), where \( Y_t \) and \( Y_{i,t} \) are the stochastic economy-wide and group-specific income endowment respectively, and \( \{y_t\} \) is the idiosyncratic component for each household in the economy. This implies that, within one group, households’ endowments share a common group-specific element while differing in being subject to idiosyncratic shocks in each period. \( \{y_t\} \) follows a Markov process with initial probability distribution \( \Pi_0 (\cdot) \) and transition probabilities \( \pi_t (y' | y) \). \( y^t = (y_0, y_1, \ldots, y_t) \) captures the history of endowment shocks, such that the compound probability of a history \( y^t \) given an initial endowment \( y_0 \) is \( \pi_t (y^t | y_0) = \pi_{t-1} (y_t | y_{t-1}) \pi_{t-2} (y_{t-1} | y_{t-2}) \ldots \pi_0 (y_1 | y_0) \). At date \( t \) households are distinguished jointly by their group \( i \), their initial asset holdings \( \alpha_{ij,t} \), and their initial endowment shock \( y_t \). Intertemporally, households transfer their resources by trading one single asset economy wide. The borrowing, however, is subject to a household-specific debt limit \( A_{ij,t} (Y_t, Y_{i,t}, Y^t) \); i.e., a pre-specified credit line is contingent on the economy, group and household-specific endowment histories up to period \( t \).

For simplicity it is assumed that households have zero mobility across groups at a point in time. The reason is two-fold. On the one hand, SOEP data shows that mobility is not the dominant issue, since more than half of the households in the samples (56.8 percent) between 1984 and 2005 have never changed their groups, while among the group switchers over half of them (51.8 percent) have changed only once, among which over half happened due to aging. In a word, these heterogeneous households appeared to stay relatively persistently in their group. On the other hand, the later

\(^3\)This is a simplified version of a model with stochastic labor endowment, such as in Krueger and Perri (2006). Inclusion of labor supply in the current model is possible but not crucial.
4.2 Consumption Growth and Volatility

use of panel data is to examine consumption growth in sequential years, where cross-sectional comparison is the final aim.

Define $c_{ij,t}$ as the time $t$ consumption of the $j$th household in $i$th group, with group average consumption $X_{i,t}$. Since the purpose of this chapter is on the consumption dynamics, the model is reduced to an endowment economy and the household problem is boiled down to consumption and asset holding decisions. With a group-specific discount factor $\beta_i$, which implies that groups are different in patience, a household from group $i$ of type $(Y_t, Y_{i,t}, y_t)$ chooses a consumption stream and asset holding plans for one single asset to solve the following maximization problem:

$$\max \left\{ c_{ij,t} \right\}_{t=0}^{\infty} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \left[ u(c_{ij,t}, X_{i,t}) \right]$$

subject to

$$c_{ij,t} + q_t \alpha_{ij,t+1} \leq Y_t Y_{i,t} y_t + \alpha_{ij,t}. \quad (4.1)$$

One unit asset is priced $q_t$ in period $t$ and pays one unit of consumption good in period $t+1$. In econometric studies on consumption, household’s consumption $c_{ij,t}$ are sometimes decomposed of a principal part and an exogenous idiosyncratic shock which captures small consumption innovation of the household (such as Blundell and Preston, 1998, and Blundell et al., 2008). Parker and Preston’s (2005) estimate shows that such change in consumption preference is crucial for the variance of household consumption growth. Initial asset holding $\alpha_{ij,0}$ is given and the borrowing constraints hold in order to rule out Ponzi schemes:

$$-\alpha_{ij,t+1} \leq A_{ij,t} \left( Y_t, Y_{i,t}, y_t \right).$$

The utility function has the following isoelastic form:

$$u(c_{ij,t}, X_{i,t}) = \frac{c_{ij,t}^{1-\gamma} X_{i,t}^{-(1-\gamma)\kappa_i} - 1}{1 - \gamma}. \quad (4.2)$$

$\gamma$ is the risk aversion parameter and is usually larger than 1\(^4\). Note that in absence of household-specific idiosyncratic shock, $c_{ij,t}$ equals $X_{i,t}$. (4.2) can be rewritten as

$$u(c_{ij,t}, X_{i,t}) = \frac{c_{ij,t}^{1-\gamma_i} - 1}{1 - \gamma_i}.$$  

\(^4\)Alternatively, to elaborate elastic labor supply, the utility function could take the form

$$u(c_t, X_{i,t}, l_t) = \frac{c_t^{1-\gamma} X_{i,t}^{-(1-\gamma)\kappa_i} - 1}{1 - \gamma_i} - \chi_{l_t}^{1-\psi} \frac{l_t^{1-\psi}}{1-\psi},$$

where $l_t = Y_t Y_{i,t} y_t$. 

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where $\gamma - (\gamma - 1) \kappa_i = \gamma_i$. This transformation implies that the combination of economy-wide identical risk aversion and group-specific attitude toward consumption externality is equivalent to a neoclassical economy with no consumption externality but heterogeneous risk aversion. Both cases lead to the same Euler equation, though.

Group consumption serves as an external benchmark, and $\kappa_i < \frac{\gamma}{\gamma - 1}$ as the attitude of group $i$ households toward this benchmark can be interpreted as “how important is my neighbors’ consumption for me”. Taking log of the core of the utility function yields:

$$
(1 - \gamma) \ln c_{i,t} - (1 - \gamma) \kappa_i \ln X_{i,t} = (1 - \gamma) \left[ (1 - \kappa_i) \ln c_{i,t} + \kappa_i \ln \frac{c_{i,t}}{X_{i,t}} \right].
$$

Scaled by parameter $\kappa_i$, the household’s consumption preference is a weighted average of the absolute and relative consumption (compared to group average). There is no restriction on $\kappa_i$ to be positive or negative, which allows us to examine three cases considering the group effect in consumption:

1. When $0 < \kappa_i < \frac{\gamma}{\gamma - 1}$, the household would like to “keep up with the Joneses”. Average consumption decreases the household’s utility level but increases household’s marginal utility of an additional unit of consumption. This reflects exactly the economic implication of “keeping up with the Joneses”, since “any given addition to his current level of consumption becomes more valuable”\(^5\). In the later part of the chapter, it will become clear that such partial preferences, keeping up with the Joneses, could reduce contemporaneous consumption growth inequality but drive up consumption volatility over the business cycle further from a model without consumption externals.

2. When $\kappa_i < 0$, households do not take the group mean as benchmark, but rather gain utility once the others in the group are doing well. For philanthropists this could be interpreted as altruism. However, a more economic intuition is that the group-mean welfare acts as “substitute” for the household’s own welfare. In the absence of government in the current model, one can imagine the public good as good weather or air quality. Knies (2010) interprets it in another cultural context. Comparing West and East Germany and being in line with the result of Senik (2004, 2008), she conjectures that in East Germany this post-transition economy, positive changes in others’ circumstances can serve as a positive signal for possible improvements in one’s own financial situation. As a result, a positive association is expected between neighborhood income and life satisfaction.

3. When $\kappa_i = 0$, the utility function is reduced to a typical self insurance version, where agents are only concerned with their own consumption.

\(^5\)Gali (1994).
The resulting Euler equation is:

\[
q_t = \beta_i E_t \left[ \left( \frac{c_{ij,t+1}}{c_{ij,t}} \right)^{-\gamma} \left( \frac{X_{ij,t+1}}{X_{ij,t}} \right)^{-\gamma} \right].
\]

Since all households in group \(i\) have the identical optimization problem, through aggregation, it holds for group \(i\) in general equilibrium:

\[
q_t = \beta_i E_t \left[ \left( \frac{X_{ij,t+1}}{X_{ij,t}} \right)^{-\gamma} \right], 
\tag{4.3}
\]

where \(q_t\) is determined by demand and supply in the financial market and is exogenous for single households. The aggregated Euler equation (4.3) implies that the degree of risk aversion, and the group-specific discount factor as well as the attitude to neighbors’ consumption determine the group consumption growth together. Group consumption growth is slow when households in the group are less patient (small \(\beta_i\)) and prefer current to future consumption, or when they put more value on their current relative position in the group (\(\kappa_i\) is positive and increases) and would rather “keep up” consumption than buying security (a similar effect to households’ being “impatient”).

If for most households the idiosyncratic shock \(y_t\) turns out to be negative, implying a negative income shock in the aggregate, net borrowing demand (sales of the security) increases and ceteris paribus, the asset price \(q_t\) will decrease, and the return for those households purchasing the security increases. Needless to say, in a general equilibrium \(q_t\) is also subject to the distribution of \(\beta_i, \kappa_i\) and \(\gamma\).

### 4.2.3 Implication for Consumption Dynamics

The permanent income hypothesis states that periodical consumption is subject to lifetime resources, instead of each period’s income. Household wealth is thus a better candidate as a consumption constraint. However, while the change of household consumption is additionally triggered by consumption innovations, the main shocks occurring to household consumption are often identified as contemporaneous income shocks in the related literature\(^7\).

\(^6\)In Abel’s (1990) model households compare themselves with the previous consumption of the group members, so as to “catch up with the Joneses”. Households still buy one unit of risk-free bond at price \(q_t\)

\[
q_t \left( \frac{X_{ij,t}}{X_{ij,t-1}} \right)^{-(1-\gamma)x} = \beta E_t \left[ \left( \frac{c_{ij,t+1}}{c_{ij,t}} \right)^{-\gamma} \right].
\]

Taking logs gives the same result as above, since the growth rate of \(X_{ij,t}\) is time invariant. This picture, however, can be totally different if consumption growth is time-variant.

\(^7\)According to Meghir and Pistaferri (2004), among others, the log of income growth is subject to permanent and transitory income shocks. Once good panel data is available on income and consumption, one can
Another way to look at the sources of consumption growth is to track the causes in a group and individual level. This helps to bridge the individual level and group level variables, and approximate equation (4.3). The decomposition is analogous to that in a macroeconomic study on sectoral output growth and volatility in Imbs (2007), who disentangles the origin of sectoral output growth into three orthogonal shocks: A global, a country-specific and a residual shock. The consumption growth rate of household \( j \) in group \( i \) is therefore given by

\[
g_{ij,t} = \theta_{ij} + \eta_t + \eta_{ij,t}. \tag{4.4}
\]

Household consumption growth can deviate from an average constant \( \theta_{ij} \) because of three orthogonal zero-mean, independent shocks: An economy-wide shock \( \eta_t \) affecting all households in all groups (think about a common technology shock to the economy-wide endowment \( \Upsilon_t \) in equation (4.1)), a group-specific shock \( \eta_{ij,t} \) which is related to the stochastic group-specific endowment \( \Upsilon_{i,t} \), as well as a residual specific to household \( j \) in group \( i \), \( \eta_{ij,t} \). This last household-specific residual contains the idiosyncratic endowment \( y_t \) and the consumption innovation shock. \( g_{ij,t} \) is thus distributed i.i.d. \(~ (\theta_{ij}, \theta_t + \theta_{i,t} + \theta_{ij,t}) \) where \( \theta_t = E_t \left[ (\eta_t)^2 \right], \theta_{i,t} = E_t \left[ (\eta_{i,t})^2 \right], \theta_{ij,t} = E_t \left[ (\eta_{ij,t})^2 \right]. \)

The average consumption growth for group \( i \) is thus

\[
g_{i,t} = \frac{1}{J} \sum_{j} g_{ij,t} = \frac{1}{J} \sum_{j} \theta_{ij} + \eta_t + \frac{1}{J} \sum_{j} \eta_{ij,t}. \tag{4.5}
\]

with the mean and variance given by

\[
E_t \left( \frac{1}{J} \sum_{j} g_{ij,t} \right) = \frac{1}{J} \sum_{j} \theta_{ij} = \bar{g}_t,
\]

\[
V_t \left( \frac{1}{J} \sum_{j} g_{ij,t} \right) = \theta_t + \theta_{i,t} + \frac{1}{J} \sum \theta_{ij,t} = \sigma_{\bar{g}}^2.
\]

The group average consumption growth rate is assumed to be stationary and (conditionally and unconditionally) log-normally distributed \( g_{i,t+1} \sim (\bar{g}_t, \sigma_{\bar{g}}^2)^8 \). With this information and the help of a second order Taylor approximation, equation (4.3) turns out to be\(^9\):

\[
q_{t+1} \approx \beta_t \exp \left[ (-\gamma - (1 - \gamma) \kappa_t) g_t + \frac{(-\gamma - (1 - \gamma) \kappa_t)^2}{2} \sigma_{\bar{g}}^2 \right]. \tag{4.8}
\]

The security price \( q_t \) is determined in the general equilibrium as a product of the state of the

\(^8\)Once define \( G_{i,t+1} = 1 + g_{i,t+1} = \frac{X_{i,t+1}}{X_{i,t}} \), \( \ln \frac{X_{i,t+1}}{X_{i,t}} = \ln G_{i,t+1} \approx g_{i,t+1} \).

\(^9\)See Appendix I for a detailed derivation.
4.2 Consumption Growth and Volatility

The economy, and the aggregation of all groups’ saving and borrowing decisions, which in turn depend on the group-specific endowment and the distribution of the idiosyncratic income shocks. As consumption growth and its variance are also conditional on the aggregate economic condition (business cycle properties), the following arguments are first valid for cross-sectional comparison within one period. That is, holding $q_t$ unchanged.

A none-zero $\kappa_i$ leads to the deviation from an externality-free case where the household optimization problem is independent of others’ consumption behavior. This deviation could be one way to mitigate the equity premium puzzle in asset pricing. Rearranging equation (4.8) gives:

$$\sigma^2_{gi} = \frac{2 [\gamma + (1 - \gamma) \kappa_i] g_i + \ln q_t - \ln \beta_i}{[\gamma + (1 - \gamma) \kappa_i]^2}.$$  (4.9)

It yields a relationship between the group average consumption growth and volatility. Note that once the group average plays no role for single households ($\kappa_i = 0$), the equation is reduced to the externality-free model:

$$\sigma^2_{gi} = \frac{2 \gamma g_i + \ln q_t - \ln \beta_i}{\gamma^2} > 0.$$  (4.10)

Comparing these two equations tells the effect of the externality. Frank (1989) argues that, given this externality, market conditions for Pareto optimal are violated because “each person’s consumption imposes negative externalities on others”. The magnitude of these external effects is often very large because if any one person increases his consumption, he also raises the consumption standard for others unintentionally. Consequently, the efficient outcome based on independent decisions of self-seeking may not hold any longer. In an economy where goods vary in the degree of being positional, there would be excessive resources devoted to the production and acquisition of positional goods, insufficient resources devoted to non-positional goods (Frank, 1985a, 1985b). Moreover, agents will consume more and save less than in an externality-free world (see more discussion in Proposition 2). For a reasonable value of risk aversion, i.e., $\gamma > 1^{10}$, the following propositions hold:

**Proposition 1** For a given consumption growth rate, more patient groups have smaller volatility.

Proof: Taking partial derivatives of $\sigma^2_{gi}$ in equation (4.9) according to group-specific discount factor $\beta_i$ yields:

$$\frac{\partial \sigma^2_{gi}}{\partial \beta_i} = -\frac{2}{[\gamma + (1 - \gamma) \kappa_i]^2} \beta_i < 0.$$  

---

10 Other than assuming the values of the key parameters, one can use maximum likelihood (MLE) to estimate them, which will be the next step of the research. The further task of the current chapter is to examine the empirical relationship between group average consumption growth and volatility.
The implication is straightforward. Patient households tend to have a higher propensity to save, which insures the households against income shocks in next period to a higher degree. As a result, the volatility of growth is smaller.

This proposition is well shown in the data. The empirical study in the later part of the chapter shows that consumption volatility is significantly related to age: Older households appear to have smaller volatility. One of the possible reasons of such finding lies on the link between income growth and degree of patience. Carroll (2001) has argued that, “positive income growth makes consumers more impatient (in the sense of wanting to spend more than current income) because forward-looking consumers with positive income growth will want to spend some of their higher future income today”. On the opposite, older populations, with expected lower future income growth, are thus more patient and have a weaker wish to discount future consumption, which, consequently, leads to smaller consumption volatility.

Proposition 2 In presence of precautionary saving, volatility increases with household eagerness to keep up.

Proof: Taking the partial derivative of $\sigma^2_{g_i}$ with respect to $\kappa_i$ yields:

$$\frac{\partial \sigma^2_{g_i}}{\partial \kappa_i} = \frac{2 (\gamma - 1)}{[\gamma + (1 - \gamma) \kappa_i]^2} \left[ g_i + 2 \frac{(\ln q_t - \ln \beta_i)}{\gamma + (1 - \gamma) \kappa_i} \right].$$

Using the steady state value of $g_i$, which is derivable from equation (4.3), the equation above can be written as

$$\frac{\partial \sigma^2_{g_i}}{\partial \kappa_i} = \frac{2 (\gamma - 1)}{[\gamma + (1 - \gamma) \kappa_i]^2} \frac{\ln q_t - \ln \beta_i}{\gamma + (1 - \gamma) \kappa_i}.$$ (4.11)

Rearranging equation (4.9) delivers

$$\ln q_t - \ln \beta_i = \frac{[\gamma + (1 - \gamma) \kappa_i]^2}{2} \sigma^2_{g_i} - [\gamma + (1 - \gamma) \kappa_i] g_i.$$ (4.12)

Under precautionary saving, i.e., agents attempt to ‘self-insure’ against consumption fluctuations, prudent agents increase savings (here demand for the single asset) when growth is more volatile. Greater demand of assets puts downward pressure on interest rates, and return of the security is slightly below the discount rate of patient agents. Accordingly, security price $q_t$ is larger than the discount factor $\beta_i$, so that $\ln q_t > \ln \beta_i$. Meanwhile, because $\frac{1}{\gamma+1}$ is the upper bound to $\kappa_i$, $\gamma + (1 - \gamma) \kappa_i > 0$. Hence in equation (4.11), $\frac{\partial \sigma^2_{g_i}}{\partial \kappa_i} > 0$.

Household preferences show a dislike of deviation from the group average. The faster the others in your group are upgrading than you are, the larger is the “punishment” of not being able to keep up with them. At a high degree of such dislike (the case of “keeping up with the Joneses”, with a positive $\kappa_i$ approaching 1), households prefer current consumption to security purchases, which
leads to low insurance against future shock and higher volatility in consumption growth. Following the same argument, volatility is lower in the case when households weigh group average well-being more heavily (regarding it as a public good) or lack the incentive to keep up.

**Proposition 3** The effect of households’ eagerness to keep up on consumption volatility is strengthened (weakened) in booms (recessions).

Proof: Recall the partial derivative $\frac{\partial \sigma^2_{gi}}{\partial \kappa_i}$ in (4.11), taking derivative according to the security price $q_t$ leads to:

$$\frac{\partial \sigma^2_{gi}}{\partial \kappa_i} \frac{\partial \kappa_i}{\partial q_t} = \frac{2 (\gamma - 1)}{[\gamma + (1 - \gamma) \kappa_i]^2} \frac{1}{\gamma + (1 - \gamma) \kappa_i} \frac{1}{q_t}.$$  

As discussed above, $\gamma > 1$ and $\gamma + (1 - \gamma) \kappa_i > 0$, therefore $\frac{\partial \sigma^2_{gi}}{\partial \kappa_i} \frac{\partial \kappa_i}{\partial q_t} > 0$, implying that the effect of households’ eagerness on consumption volatility increases in security price $q_t$.

Comparing to the first two propositions with a particular group’s perspective, the business cycle effects are general and apply to all groups (all $\kappa_i$). Because the economy-wide endowment $Y_t$ is subject to a positive shock, most agents expect to experience income growth in booms and are willing to lend out their resources (through buying more securities). Higher demand of securities drives up the unit price $q_t$ in general equilibrium, which further intensifies the effect of household preferences (degree of patience and households’ attitude toward external benchmark). In contrast, when most agents are subject to negative income shocks in recessions, an overwhelming borrowing wish leads to a decline of the security price and dampens the preference effect.

**Proposition 4** There is a positive relationship between growth and volatility, unless agents have extremely high desire to “keep up with the Joneses” ($\kappa_i > \frac{\gamma}{1 - \gamma}$).

Proof: In equation (4.9), taking partial derivative of $\sigma^2_{gi}$ with respect to $g_i$ shows

$$\frac{\partial \sigma^2_{gi}}{\partial g_i} = \frac{2}{\gamma + (1 - \gamma) \kappa_i}.$$  

Under condition that $\kappa_i$ is bounded by $\frac{\gamma}{1 - \gamma}$, there is a positive relationship between $\sigma^2_{gi}$ and $g_i$, which suggests that groups with higher consumption growth also have to bear the welfare cost of larger volatility. Nonetheless, for a large $\kappa_i$, i.e., when it’s extremely important for agents to keep up, they would short sell securities up to the liquidity constraints. By doing so, they indirectly insure their consumption next period, achieving a small volatility at a given consumption growth rate.

The current chapter does not aim at empirically identifying the direction of households’ attitude toward group mean, whereas the “keeping up with the Joneses” hypothesis is indirectly confirmed by Knies’ (2010) finding about West Germany; i.e., a negative neighborhood income effect on individual life satisfaction. In the following sections, the correlation between group consumption
growth and volatility (Proposition 4) is the key hypothesis to be tested.

4.3 Bringing the Model to the Data

The partial equilibrium derived from the theoretical model suggests a relationship between average consumption growth and volatility for different socioeconomic groups, which can be examined cross-sectionally using micro data. Micro data with panel structure such as the Panel Study of Income Dynamics (PSID) or the British Family Expenditure Survey data would be ideal for this study purpose. In a social democratic country like Germany, where conventional measures show that inequality grows in recent years but is still lower than the Anglo-Saxon countries, the study on consumption is rather scarce due to data limitation. An exploration of two main micro data sets on households' income and consumption, nonetheless, can help to reveal part of the story on consumption inequality. These are the German Income and Expenditure Survey (Einkommens- und Verbrauchsstichprobe, EVS) and the German Socio-Economic Panel (SOEP).

Both EVS and SOEP are related to the Micro Census. EVS is a quota sample with voluntary participation to the annual Micro Census, while SOEP is annual longitudinal survey with stratified random samples where Micro Census serves as weighting benchmark. EVS takes continuous bookkeeping approach to record income and consumption in detail, whereas SOEP household income is imputed from monthly household income on the survey month ("screener"), major gross income components in the month of interview and the retrospective income data for previous year. EVS recorded tax payment and deduction apart from the tax benefit, while SOEP estimates tax payment based on households' account on the previous year tax payment, and the possible tax benefit is not included. More differences between EVS and SOEP are summarized in Becker et al. (2002), and can be found in Table 4.1.

Before entering the discussion about group consumption patterns, the crucial question would be, how to define groups so that it makes sense. Factor analysis using principal components is used to distill the various household characteristics into the most informative ones in both data sets. Regressions of the consumption growth and volatility on household demographics can further reveal those significantly associated characteristics (Table 4.2).

With variables such as federal states discarded, the variables contributing most to group the households in EVS are age, gender, and occupation of the household heads, as well as household size, whereas the best grouping criteria for the imputed data are age, education, occupation and nationality of household heads, and household size (see Table 4.3 and 4.4). Even though community size does not account much for consumption difference between households, the theoretical model implies an indirect impact of the comparison and attitude of group members on group av-
Table 4.1: Methodological Characteristics of Household Income Surveys in EVS and SOEP

<table>
<thead>
<tr>
<th></th>
<th>EVS</th>
<th>SOEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survey</td>
<td>repeated cross-section</td>
<td>panel</td>
</tr>
<tr>
<td>Sampling method</td>
<td>quota sample based on the mandatory random Micro Census</td>
<td>stratified random sample</td>
</tr>
<tr>
<td>Sample size</td>
<td>1998: app. 60,000 households</td>
<td>1984-2000: app. 6,000 households; since 2001: app. 12,000 households</td>
</tr>
<tr>
<td>Collection of income data</td>
<td>continuous bookkeeping by the participants</td>
<td>monthly (net) household income; major gross income components in the interview month; retrospective income data for the previous year</td>
</tr>
<tr>
<td>Foreign household head</td>
<td>Coverage since 1993</td>
<td>explicit over-sampling</td>
</tr>
<tr>
<td>Coverage of upper and lower end</td>
<td>no homeless; non-coverage of households with monthly net income over 35,000 DM (1998)</td>
<td>no homeless; starting 2002 SOEP includes an additional sample of the very rich</td>
</tr>
<tr>
<td>Tax and social security contribution</td>
<td>payments during the response period included in survey, but no allowance for final tax assessment</td>
<td>imputation based on basic tax routines and flat deduction for employees, provisional lump sums, tax exemptions for capital income, and child allowances</td>
</tr>
</tbody>
</table>

Table 4.2: Imputed Data: Nondurable v.s. Durable Consumption Growth and Volatility

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Community size</td>
<td>0.001</td>
<td>-0.002</td>
<td>0.002</td>
<td>0.003</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Household size</td>
<td>0.002*</td>
<td>-0.002**</td>
<td>0.000</td>
<td>-0.007***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age of hh head</td>
<td>-0.000***</td>
<td>-0.001***</td>
<td>-0.001***</td>
<td>-0.003***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non German</td>
<td>-0.011***</td>
<td>0.010**</td>
<td>-0.014**</td>
<td>0.032***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Higher education</td>
<td>0.005*</td>
<td>-0.002</td>
<td>0.007</td>
<td>-0.003</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skilled jobs</td>
<td>0.004</td>
<td>-0.015***</td>
<td>0.009*</td>
<td>-0.042***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.013</td>
<td>0.119***</td>
<td>0.000</td>
<td>0.326***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>10842</td>
<td>10842</td>
<td>10841</td>
<td>10841</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.007</td>
<td>0.010</td>
<td>0.006</td>
<td>0.026</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$t$ statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

All regressions use imputed data, where consumption is adjusted with equivalent scale.
4 Consumption Growth and Volatility with Consumption Externalities

erage consumption growth and volatility. A reasonable deduction is that community size affects
the extent to which households can observe others with similar socioeconomic backgrounds, and
therefore community size is added as one grouping condition for both EVS and SOEP.

Table 4.3: EVS Grouping Criteria

<table>
<thead>
<tr>
<th>EVS</th>
<th>Community Size</th>
<th>Household Size</th>
<th>Age of Household Head</th>
<th>Gender of Household Head</th>
<th>Occupation of Household Head</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Categories</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Definition</td>
<td>1 if less than 20000 residents, 2 if between 20000-100000 residents, 3 if more than 100000 residents</td>
<td>1 if fewer than 3, 2 if 3 or more than 3</td>
<td>1 if no older than 35, 2 if between 35-55, 3 if older than 55</td>
<td>1 if male, 2 if female</td>
<td>1 if self-employed farmer, 2 if other self-employed, 3 if civil servants, 4 if dependent employee, 5 if worker, 6 if unemployed or inactive</td>
</tr>
</tbody>
</table>

Table 4.4: Imputed Data Grouping Criteria

<table>
<thead>
<tr>
<th>Matched sample</th>
<th>Community Size</th>
<th>Household Size</th>
<th>Age of Household Head</th>
<th>Education of Household Head</th>
<th>Occupation of Household Head</th>
<th>Nationality of Household Head</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Categories</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Definition</td>
<td>1 if less than 20000 residents, 2 if between 20000-100000 residents, 3 if more than 100000 residents</td>
<td>1 if fewer than 3, 2 if 3 or more than 3</td>
<td>1 if no older than 35, 2 if between 35-55, 3 if older than 55</td>
<td>1 if one has at least post-secondary non-tertiary education (higher educated), 2 if otherwise</td>
<td>1 if more skilled (high/low level service, routine non-manual, self-employed, manual supervision, and skilled manual jobs), 2 if otherwise</td>
<td>1 if German native, 2 if not</td>
</tr>
</tbody>
</table>

4.3.1 EVS: Data and Methodology

EVS is one of the major surveys containing personal and households’ income and consumption distributions in Germany\textsuperscript{11}. The Federal Statistical Office delivers a cross-sectional survey every

\textsuperscript{11}EVS is not a random sample but a quota sample with voluntary participation. However, it takes as benchmark for recruiting participants the annual Current Population Survey of Germany (Mikrozensus), which

The EVS data has several advantages. Besides the rich information on consumption and income it contains, it includes a large number of households (defined as consumer units), and even more observations when individuals are concerned. While individual samples are comparatively easy to be extracted from the household observations, they contain dependent employees, self-employed, unemployed as well as citizens who are out of labor force. This large variety of occupational status enriches the objects of the study to the general population and makes it possible to examine consumption and welfare effects over time.

As Cutler and Katz (1991) take a “top-down” approach to construct nondurable consumption out of total expenditure, data structure in EVS allows for constructing nondurable, durable and total consumption (the sum of nondurable, durable consumption plus rent) in a “bottom-up” manner. In all, this chapter takes the same point as Cutler and Katz (1991) to exclude housing costs, vehicle purchases, spending on major appliances, insurance premia and expenditures for financial services from nondurable consumption. Specifically, I construct nondurable consumption of households using the existing detailed account on Classification of Individual Consumption by Purpose (COICOP), in line with Fuchs-Schündeln, Krueger and Sommer (2010). What are included in the nondurable consumption are expenditures for food, clothes, energy, health, body care, travel, communication, education, rent, and household services, while part of leisure and miscellaneous also belong to nondurables. Exceptions such as electric appliances, photo cameras, sport equipment or other high-valued durable goods join furniture, car repairs, garage rental fees, and large electric device maintenance to be counted as durable consumption. Summing up the durable, nondurable consumption as well as the rent, yields the total consumption. One should note that the every-five-year data collection in EVS may cause a little bias to nondurable consumption due to its smooth feature. However, since durable consumption is much more sensitive to the business cycle than the nondurables and may vary much from year to year (Mankiw, 1985), the reported durable consumption in EVS sample years may not be representative over the study years. The imputation of durable consumption is thus less justified than the nondurables. This may be one of the reasons why the later estimations concerning the imputed data are significant for nondurables but insignificant for durable consumption (Tables 4.5 and 4.6).

is a mandated random survey of large size. Consequently, the household net income brackets in the EVS are defined identically to those in the Mikrozensus.
Table 4.5: Imputed Data: Nondurable Consumption Growth and Volatility / Inequality

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatility</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{nd}$</td>
<td>-0.366***</td>
<td>-0.349***</td>
<td>-0.331***</td>
</tr>
<tr>
<td></td>
<td>(-4.93)</td>
<td>(-4.32)</td>
<td>(-4.08)</td>
</tr>
<tr>
<td>$\sigma_{nd}^2$</td>
<td>0.815***</td>
<td>0.764***</td>
<td>0.757***</td>
</tr>
<tr>
<td></td>
<td>(5.13)</td>
<td>(4.60)</td>
<td>(4.54)</td>
</tr>
<tr>
<td>Lag nond. consumption</td>
<td>-0.182***</td>
<td>-0.194***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-6.56)</td>
<td>(-6.55)</td>
<td></td>
</tr>
<tr>
<td>Time effect</td>
<td></td>
<td></td>
<td>significant**</td>
</tr>
<tr>
<td>Constant</td>
<td>0.026***</td>
<td>1.633***</td>
<td>1.716***</td>
</tr>
<tr>
<td></td>
<td>(6.25)</td>
<td>(6.67)</td>
<td>(6.56)</td>
</tr>
<tr>
<td>Observations</td>
<td>2567</td>
<td>2423</td>
<td>2423</td>
</tr>
<tr>
<td>F</td>
<td>14.113</td>
<td>27.840</td>
<td>8.586</td>
</tr>
<tr>
<td>$R^2_{within}$</td>
<td>0.063</td>
<td>0.110</td>
<td>0.130</td>
</tr>
<tr>
<td>$R^2_{between}$</td>
<td>0.074</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>$R^2_{overall}$</td>
<td>0.064</td>
<td>0.055</td>
<td>0.068</td>
</tr>
<tr>
<td><strong>Inequality</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta_{nd}$</td>
<td>-0.217***</td>
<td>-0.240***</td>
<td>-0.237***</td>
</tr>
<tr>
<td></td>
<td>(-3.91)</td>
<td>(-4.03)</td>
<td>(-3.86)</td>
</tr>
<tr>
<td>$\Delta_{nd}^2$</td>
<td>0.300***</td>
<td>0.311***</td>
<td>0.317***</td>
</tr>
<tr>
<td></td>
<td>(3.64)</td>
<td>(3.83)</td>
<td>(3.82)</td>
</tr>
<tr>
<td>Lag nond. consumption</td>
<td>-0.162***</td>
<td>-0.175***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-6.95)</td>
<td>(-7.22)</td>
<td></td>
</tr>
<tr>
<td>Time effect</td>
<td></td>
<td></td>
<td>significant**</td>
</tr>
<tr>
<td>Constant</td>
<td>0.040***</td>
<td>1.478***</td>
<td>1.578***</td>
</tr>
<tr>
<td></td>
<td>(4.40)</td>
<td>(7.13)</td>
<td>(7.40)</td>
</tr>
<tr>
<td>Observations</td>
<td>2346</td>
<td>2226</td>
<td>2226</td>
</tr>
<tr>
<td>F</td>
<td>7.839</td>
<td>19.941</td>
<td>8.322</td>
</tr>
<tr>
<td>$R^2_{within}$</td>
<td>0.017</td>
<td>0.065</td>
<td>0.096</td>
</tr>
<tr>
<td>$R^2_{between}$</td>
<td>0.001</td>
<td>0.016</td>
<td>0.011</td>
</tr>
<tr>
<td>$R^2_{overall}$</td>
<td>0.014</td>
<td>0.016</td>
<td>0.031</td>
</tr>
</tbody>
</table>

$t$ statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Group nondurable consumption growth rate is the dependent variable.
### 4.3 Bringing the Model to the Data

#### Table 4.6: Imputed Data: Durable Consumption Growth and Volatility / Inequality

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatility</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_d )</td>
<td>0.094</td>
<td>0.068</td>
<td>0.088</td>
</tr>
<tr>
<td>(0.54)</td>
<td>(0.37)</td>
<td>(0.49)</td>
<td></td>
</tr>
<tr>
<td>( \sigma_d^2 )</td>
<td>-0.190</td>
<td>-0.168</td>
<td>-0.172</td>
</tr>
<tr>
<td>(-0.93)</td>
<td>(-0.80)</td>
<td>(-0.84)</td>
<td></td>
</tr>
<tr>
<td>Lag dur. consumption</td>
<td>-0.104***</td>
<td>-0.115***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-3.96)</td>
<td>(-3.90)</td>
<td></td>
</tr>
<tr>
<td>Time effect</td>
<td></td>
<td></td>
<td>significant**</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.015</td>
<td>0.784***</td>
<td>0.912***</td>
</tr>
<tr>
<td></td>
<td>(-1.07)</td>
<td>(3.82)</td>
<td>(4.04)</td>
</tr>
<tr>
<td>Observations</td>
<td>2567</td>
<td>2423</td>
<td>2423</td>
</tr>
<tr>
<td>F</td>
<td>0.892</td>
<td>6.262</td>
<td>8.261</td>
</tr>
<tr>
<td>( R^2_{within} )</td>
<td>0.025</td>
<td>0.039</td>
<td>0.081</td>
</tr>
<tr>
<td>( R^2_{between} )</td>
<td>0.006</td>
<td>0.241</td>
<td>0.250</td>
</tr>
<tr>
<td>( R^2_{overall} )</td>
<td>0.020</td>
<td>0.010</td>
<td>0.039</td>
</tr>
<tr>
<td><strong>Inequality</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta_d )</td>
<td>0.108</td>
<td>0.112</td>
<td>0.140</td>
</tr>
<tr>
<td>(0.59)</td>
<td>(0.61)</td>
<td>(0.77)</td>
<td></td>
</tr>
<tr>
<td>( \Delta_d^2 )</td>
<td>-0.240</td>
<td>-0.245</td>
<td>-0.247</td>
</tr>
<tr>
<td>(-1.31)</td>
<td>(-1.34)</td>
<td>(-1.35)</td>
<td></td>
</tr>
<tr>
<td>Lag dur. consumption</td>
<td>-0.089***</td>
<td>-0.091***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-4.15)</td>
<td>(-3.86)</td>
<td></td>
</tr>
<tr>
<td>Time effect</td>
<td></td>
<td></td>
<td>significant**</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.011</td>
<td>0.674***</td>
<td>0.727***</td>
</tr>
<tr>
<td></td>
<td>(-0.30)</td>
<td>(3.98)</td>
<td>(4.00)</td>
</tr>
<tr>
<td>Observations</td>
<td>2345</td>
<td>2225</td>
<td>2225</td>
</tr>
<tr>
<td>F</td>
<td>6.478</td>
<td>10.575</td>
<td>9.804</td>
</tr>
<tr>
<td>( R^2_{within} )</td>
<td>0.051</td>
<td>0.067</td>
<td>0.113</td>
</tr>
<tr>
<td>( R^2_{between} )</td>
<td>0.043</td>
<td>0.050</td>
<td>0.053</td>
</tr>
<tr>
<td>( R^2_{overall} )</td>
<td>0.047</td>
<td>0.027</td>
<td>0.063</td>
</tr>
</tbody>
</table>

* \( t \) statistics in parentheses  
* \( * \) \( p < 0.1 \), \( ** \) \( p < 0.05 \), \( *** \) \( p < 0.01 \)

Group durable consumption growth rate is the dependent variable.

The groups are defined by households characteristics available for all waves; i.e., household size, age of household head, occupation of household head, community size and the gender of household head. The information on the nationality of household head only starts from 1988 and the education (professional training) level starts from 1993, therefore they are not used for dividing the groups. The exact grouping criteria are summarized in Table 4.3.

Though there is no direct micro information on households’ consumption growth, we can use the difference of group average log consumption to approximate the average group consumption.
Consumption Growth and Volatility with Consumption Externalities

growth, because

\[ \frac{1}{J} \sum_{j} g_{i,j,t} \approx \frac{1}{J} \sum_{j} (c_{ij,t} - c_{ij,t-1}) = \frac{1}{J} \sum_{j} c_{ij,t} - \frac{1}{J} \sum_{j} c_{ij,t-1} = c_{i,t} - c_{i,t-1}. \]

The econometric framework would be:

\[
c_{i,t} - c_{i,t-1} = a_0 + a_1 SD_t (\Delta c_{i,t}) + W_{i,t}^f a_2 + \mu_i + \delta_t + \epsilon_{i,t} \tag{4.14}
\]

where \( SD_t (\Delta c_{i,t}) \) denotes the standard deviation of consumption growth from mean\(^{12}\), and \( a_1 \) and \( a_2 \) are vectors of coefficients assumed common across groups. \( \mu_i \) captures the time-invariant group characteristics which are used to group the samples (fixed effect), \( \delta_t \) is a time dummy and the residual \( \epsilon_{i,t} \) represents the deviation of growth from its predicted value. \( W_{i,t} \) is a vector of controls for the group, a unique combination of which determines the group-specific parameters \( \beta_i \) and \( \kappa_i \) in the theoretical model.

Meanwhile, the data allows us to explore the relationship between consumption growth and the change of within-group inequality. In the following regression equation, the main difference from (4.14) is the \( \Delta SD_t (c_{i,t}) \) term, representing the change of within-group standard deviation across household observations at time \( t \) along the group-mean consumption growth. This serves as an additional examination of the welfare effect of the consumption growth.

\[
c_{i,t} - c_{i,t-1} = a_0 + a_1 \Delta SD_t (c_{i,t}) + W_{i,t}^f a_2 + \mu_i + \delta_t + \epsilon_{i,t}. \tag{4.15}
\]

Table 4.7 provides some summary statistics for EVS on the cross-section and over time of consumption growth \( g_i \), volatility \( \sigma_i \) and within-group inequality \( \Delta_i \). The size of all groups over time varies between 1 and 4864, with 210.7 as mean and 53 as median, showing a large variation between the groups. Unconditional correlation between group growth and its standard deviation for each time period is negative while that between group growth and change in within-group standard deviation is positive. These correlations between aggregated variables cannot tell much since no group or time effect is taken into consideration.

The EVS, nonetheless, can only provide an approximation of the consumption growth due to lack of panel structure. Since only limited household characteristics are available, sampled households in one group in different time period can bear large consumption variation due to unobservable features, implying a time-variant household-specific residual \( (\theta_{ij,t}) \) and thus a varying \( \sigma_{g_i}^2 \) in

\(^{12}\)Instead of variance, using standard deviation as control variable helps to interpret the result of the point estimation as percentage to percentage change.
4.3 Bringing the Model to the Data

### Table 4.7: EVS: Summary Statistics of Per Capita Consumption

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nondurable consumption</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_i$</td>
<td>0.87%</td>
<td>0.74%</td>
<td>-0.98</td>
<td>1.05</td>
<td>200</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>0.13</td>
<td>0.09</td>
<td>0</td>
<td>0.62</td>
<td>200</td>
</tr>
<tr>
<td>$\Delta_i$</td>
<td>0.011</td>
<td>0.008</td>
<td>-0.22</td>
<td>0.42</td>
<td>200</td>
</tr>
<tr>
<td>Correlation($g_i, \sigma_i$)</td>
<td>$-0.09$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation($g_i, \Delta_i$)</td>
<td>0.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Durable consumption</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_i$</td>
<td>-0.093%</td>
<td>-0.062%</td>
<td>-2.589</td>
<td>1.531</td>
<td>200</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>0.474</td>
<td>0.317</td>
<td>0</td>
<td>2.959</td>
<td>200</td>
</tr>
<tr>
<td>$\Delta_i$</td>
<td>0.003</td>
<td>0</td>
<td>-0.832</td>
<td>0.386</td>
<td>200</td>
</tr>
<tr>
<td>Correlation($g_i, \sigma_i$)</td>
<td>$-0.19$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation($g_i, \Delta_i$)</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total consumption</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_i$</td>
<td>0.023%</td>
<td>0.017%</td>
<td>-0.714</td>
<td>0.955</td>
<td>200</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>0.137</td>
<td>0.095</td>
<td>0</td>
<td>0.62</td>
<td>200</td>
</tr>
<tr>
<td>$\Delta_i$</td>
<td>0.005</td>
<td>0.005</td>
<td>-0.234</td>
<td>0.312</td>
<td>200</td>
</tr>
<tr>
<td>Correlation($g_i, \sigma_i$)</td>
<td>$-0.12$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation($g_i, \Delta_i$)</td>
<td>0.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Equation (4.7). Moreover, the inclusion of various households in every wave naturally increases the dispersion of the residuals, suggesting an overestimation of the $\sigma^2_{g_i}$ and thus $\sigma_{i,t}$ in (4.14). Additionally, since the EVS survey was carried out every five years, possibly each wave is at a similar time point of the business cycle, say, in the extreme case, all above or all below the long-run trend of output. The direct result, compared to a panel-structured study over the years, would be an underestimation of the variation of $\theta_t$. The impact on $\theta_{i,t}$ is more difficult to tell, which depends on the distribution of the group-specific shocks. In all, the use of EVS data can only provide a rough picture.

#### 4.3.2 Imputation Using the SOEP

An alternative strategy is to borrow the panel structure from the SOEP and to match the two data sets so that household consumption growth can be derived. Starting from 1984, SOEP data is based on household interviews, and contains crucial questions on living and income. The sample used in this chapter includes all West Germans from 1984, whereas immigrant households are added starting in 1995. Considering the lower end of the income distribution, both EVS and SOEP do not cover homeless households, while SOEP better covers households receiving social benefits. From 2002 onwards SOEP includes a subsample of high income households whose monthly income exceeds 4,500 euro. But because EVS does not include high income households, I exclude these high income household samples in SOEP for years 2002 and 2003.
SOEP does not offer much information on consumption, and it is also unfeasible to construct consumption from the available information on financial inflows and outflows because there is little information on yearly credit or any other form of borrowing the households may have taken. The forcible imputation of consumption would bear a large bias, which is especially serious for low-income households who compose the fat left tail of the imputed consumption distribution.

Serving as basis for calculating group-specific consumption growth and volatility, household consumption can be imputed in two ways from EVS and SOEP. The first method follows Skinner (1987) and Fisher and Johnson (2006), and involves imputing consumption using EVS information on household consumption, net income, and various household demographics for the available six waves, namely 1978, 1983, 1988, 1993, 1998 and 2003. It shows that,

$$c_{i,t} = \alpha_0 + \alpha_1 \cdot \text{inc} + W_{i,t}^r \alpha_2 + \epsilon_{i,t}.$$  (4.16)

Interpolating the estimated coefficients for the intermediate years and applying the results to the comparable SOEP samples (multiplying the household net income and demographics with respective coefficients) yield the imputed household consumption.

Table 4.8 compares mean and median household income and consumption in EVS and the imputed data, where the imputed consumption appears to be lower than the EVS level, and the imputation basis, net income, is substantially lower in SOEP than in the EVS.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Income</td>
<td>24114</td>
<td>33569</td>
<td>25745</td>
<td>36057</td>
<td>24395</td>
<td>38568</td>
<td>26240</td>
<td>38724</td>
</tr>
<tr>
<td></td>
<td>(22642)</td>
<td>(31693)</td>
<td>(24213)</td>
<td>(33229)</td>
<td>(22986)</td>
<td>(35314)</td>
<td>(24179)</td>
<td>(35379)</td>
</tr>
<tr>
<td>Non. consumption</td>
<td>11898</td>
<td>14695</td>
<td>12572</td>
<td>15149</td>
<td>12351</td>
<td>15878</td>
<td>13446</td>
<td>16281</td>
</tr>
<tr>
<td></td>
<td>(11688)</td>
<td>(14388)</td>
<td>(12551)</td>
<td>(14743)</td>
<td>(12323)</td>
<td>(15467)</td>
<td>(13546)</td>
<td>(16042)</td>
</tr>
<tr>
<td>(Original EVS)</td>
<td>- 14721</td>
<td>- 15184</td>
<td>- 15895</td>
<td>- 16292</td>
<td>- 16292</td>
<td>- 16292</td>
<td>- 16292</td>
<td>- 16292</td>
</tr>
<tr>
<td></td>
<td>- (13851)</td>
<td>- (14000)</td>
<td>- (14545)</td>
<td>- (14929)</td>
<td>- (14929)</td>
<td>- (14929)</td>
<td>- (14929)</td>
<td>- (14929)</td>
</tr>
<tr>
<td>Dur. consumption</td>
<td>4212</td>
<td>5596</td>
<td>4614</td>
<td>6000</td>
<td>3584</td>
<td>5024</td>
<td>3376</td>
<td>4475</td>
</tr>
<tr>
<td></td>
<td>(4202)</td>
<td>(5665)</td>
<td>(4605)</td>
<td>(5895)</td>
<td>(3657)</td>
<td>(4993)</td>
<td>(3356)</td>
<td>(4394)</td>
</tr>
<tr>
<td>(Original EVS)</td>
<td>- 5616</td>
<td>- 6022</td>
<td>- 5061</td>
<td>- 4518</td>
<td>- 4518</td>
<td>- 4518</td>
<td>- 4518</td>
<td>- 4518</td>
</tr>
<tr>
<td></td>
<td>- (3580)</td>
<td>- (4017)</td>
<td>- (2609)</td>
<td>- (2590)</td>
<td>- (2590)</td>
<td>- (2590)</td>
<td>- (2590)</td>
<td>- (2590)</td>
</tr>
<tr>
<td>Tot. consumption</td>
<td>20459</td>
<td>25498</td>
<td>21791</td>
<td>26620</td>
<td>22224</td>
<td>28829</td>
<td>22971</td>
<td>28282</td>
</tr>
<tr>
<td></td>
<td>(20129)</td>
<td>(25062)</td>
<td>(21594)</td>
<td>(25851)</td>
<td>(22269)</td>
<td>(28079)</td>
<td>(22920)</td>
<td>(27607)</td>
</tr>
<tr>
<td>(Original EVS)</td>
<td>- 25540</td>
<td>- 26663</td>
<td>- 28968</td>
<td>- 28407</td>
<td>- 28407</td>
<td>- 28407</td>
<td>- 28407</td>
<td>- 28407</td>
</tr>
<tr>
<td></td>
<td>- (23813)</td>
<td>- (24535)</td>
<td>- (26005)</td>
<td>- (25643)</td>
<td>- (25643)</td>
<td>- (25643)</td>
<td>- (25643)</td>
<td>- (25643)</td>
</tr>
</tbody>
</table>

Note: in euros, 1995 prices.

Households in EVS and SOEP with the same demographics are compared, and households with insufficient information are not included in the matching process.
4.3 Bringing the Model to the Data

This observation is in line with Becker et al. (2002). The reasoning is manifold: (1) SOEP covers slightly more households receiving social benefits and many more households with foreign heads. (2) Compared to EVS’ detailed recorded income and expenditure in diary, income information in SOEP is an imputation of current monthly income and a rough estimation of income from the previous year, therefore SOEP income is subject to underestimation. (3) Concerning the tax issue, SOEP tax estimates are based on households’ account on the previous year tax payment and exclude possible tax benefits; Therefore SOEP possibly overestimates tax payments and underestimates household net income. (4) Concerning the demographics of the households in the overlapping sample years (1988, 1993, 1998 and 2003), the EVS and SOEP bear strong similarities in most characteristics, except the occupation distribution of the household heads. EVS includes a much higher share of civil servants and the dependently employed, while the SOEP samples include a larger portion of self-employed, workers, unemployed and inactive ones (Table 4.9).

This result is similar for the second imputation method, whose focus is on the consumption-income ratio of each specific group in five waves (1983-2003). Small cells are formed according to common households’ characteristics in the EVS and SOEP, including the residing federate state, community size, type of household, the age and the occupation of household head. Average consumption-income ratios are calculated for EVS for available waves and linear interpolation helps to fill in the gaps between the waves. Needless to say, in this data matching process the more precise the criteria, the smaller the cells, and the better the match. This ideal match would be that each single household in EVS can be matched to its SOEP counterpart, which is, however, impossible given the heterogeneity of the two data sets. Aggregation of the consumption-income ratio for households sharing the same characteristics results in less variance among the households when consumption growth is derived in SOEP, and reduces \( \theta_{ij,t} \) due to elimination of the household-specific shocks. Consequently, volatility of the group consumption growth across time would be underestimated. Such limitation requires that the results relating to the imputed data should be very carefully interpreted. For the data matching purpose, I choose a relatively detailed definition of the group (Table 4.10), which leads to altogether 43,200 cells.

Interpolating this ratio between the observation years using a year trend and applying the estimated propensities to those SOEP households in the same cells, one can impute the consumption for SOEP samples between 1984 and 2003 and further calculate the corresponding consumption growth rate. As a result, the imputed consumption growth rate would both reflect consumption, income information in the EVS and pick up the income and time structure in the SOEP. Table 4.11 reports the average consumption-income ratios of all groups in each wave, where the consumption is either nondurable, durable or total, and income is the net household income.

---

14 Some other household characteristics such as education level or years are available either in the SOEP or in the EVS but not simultaneously, thus they cannot be used to construct the cells.
Table 4.9: Descriptive Statistics: Comparison of SOEP and EVS Demographics by Year

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Household size (number)</strong></td>
<td>2.710</td>
<td>2.696</td>
<td>2.630</td>
<td>2.601</td>
<td>2.519</td>
<td>2.597</td>
<td>2.407</td>
<td>2.441</td>
</tr>
<tr>
<td><strong>Age of hh head (years)</strong></td>
<td>48.159</td>
<td>48.369</td>
<td>48.488</td>
<td>48.347</td>
<td>48.999</td>
<td>48.209</td>
<td>51.966</td>
<td>50.080</td>
</tr>
<tr>
<td><strong>Male head</strong></td>
<td>75.5%</td>
<td>72.4%</td>
<td>71.3%</td>
<td>68.1%</td>
<td>66.2%</td>
<td>67.5%</td>
<td>64.0%</td>
<td>64.4%</td>
</tr>
<tr>
<td><strong>Female head</strong></td>
<td>24.5%</td>
<td>27.6%</td>
<td>28.7%</td>
<td>31.9%</td>
<td>33.8%</td>
<td>32.5%</td>
<td>36.0%</td>
<td>35.6%</td>
</tr>
<tr>
<td><strong>Berlin west</strong></td>
<td>3.9%</td>
<td>4.0%</td>
<td>3.6%</td>
<td>3.3%</td>
<td>3.4%</td>
<td>2.3%</td>
<td>3.1%</td>
<td>0.6%</td>
</tr>
<tr>
<td><strong>Schl.-Holstein</strong></td>
<td>3.4%</td>
<td>5.0%</td>
<td>3.3%</td>
<td>5.7%</td>
<td>3.3%</td>
<td>5.1%</td>
<td>3.8%</td>
<td>5.0%</td>
</tr>
<tr>
<td><strong>Hamburg</strong></td>
<td>2.5%</td>
<td>2.9%</td>
<td>1.9%</td>
<td>3.2%</td>
<td>1.8%</td>
<td>3.2%</td>
<td>2.0%</td>
<td>2.9%</td>
</tr>
<tr>
<td><strong>Niedersachsen</strong></td>
<td>10.3%</td>
<td>9.9%</td>
<td>10.7%</td>
<td>9.5%</td>
<td>11.5%</td>
<td>10.2%</td>
<td>11.2%</td>
<td>9.5%</td>
</tr>
<tr>
<td><strong>Bremen</strong></td>
<td>1.2%</td>
<td>1.2%</td>
<td>1.3%</td>
<td>1.4%</td>
<td>1.2%</td>
<td>1.5%</td>
<td>1.1%</td>
<td>1.5%</td>
</tr>
<tr>
<td><strong>Nord.-Westfalen</strong></td>
<td>27.0%</td>
<td>27.5%</td>
<td>27.0%</td>
<td>30.0%</td>
<td>27.0%</td>
<td>27.2%</td>
<td>28.3%</td>
<td>27.3%</td>
</tr>
<tr>
<td><strong>Hessen</strong></td>
<td>9.8%</td>
<td>8.8%</td>
<td>9.9%</td>
<td>8.6%</td>
<td>9.2%</td>
<td>8.5%</td>
<td>8.5%</td>
<td>9.9%</td>
</tr>
<tr>
<td><strong>Rhein.-Pfalz</strong></td>
<td>7.1%</td>
<td>7.9%</td>
<td>6.9%</td>
<td>8.3%</td>
<td>7.7%</td>
<td>7.7%</td>
<td>8.4%</td>
<td>8.0%</td>
</tr>
<tr>
<td><strong>Baden-Württemberg</strong></td>
<td>18.2%</td>
<td>14.5%</td>
<td>17.8%</td>
<td>13.9%</td>
<td>16.9%</td>
<td>15.4%</td>
<td>15.4%</td>
<td>15.7%</td>
</tr>
<tr>
<td><strong>Bayern</strong></td>
<td>16.7%</td>
<td>18.3%</td>
<td>17.5%</td>
<td>16.2%</td>
<td>18.0%</td>
<td>18.8%</td>
<td>18.2%</td>
<td>19.6%</td>
</tr>
<tr>
<td><strong>Below 20,000 pop.</strong></td>
<td>34.8%</td>
<td>40.2%</td>
<td>34.6%</td>
<td>38.6%</td>
<td>38.4%</td>
<td>40.3%</td>
<td>38.8%</td>
<td>42.3%</td>
</tr>
<tr>
<td><strong>20,000-100,000</strong></td>
<td>27.3%</td>
<td>25.9%</td>
<td>27.4%</td>
<td>25.7%</td>
<td>27.1%</td>
<td>25.4%</td>
<td>28.1%</td>
<td>26.0%</td>
</tr>
<tr>
<td><strong>Over 100,000</strong></td>
<td>38.0%</td>
<td>34.0%</td>
<td>37.9%</td>
<td>35.7%</td>
<td>34.5%</td>
<td>34.3%</td>
<td>33.1%</td>
<td>31.7%</td>
</tr>
<tr>
<td><strong>Sing. women</strong></td>
<td>13.4%</td>
<td>13.3%</td>
<td>14.1%</td>
<td>15.3%</td>
<td>15.2%</td>
<td>14.1%</td>
<td>16.1%</td>
<td>15.7%</td>
</tr>
<tr>
<td><strong>Sing. men</strong></td>
<td>9.1%</td>
<td>6.3%</td>
<td>9.3%</td>
<td>8.9%</td>
<td>9.4%</td>
<td>8.3%</td>
<td>11.1%</td>
<td>9.2%</td>
</tr>
<tr>
<td><strong>Sing. par+1 kid</strong></td>
<td>3.5%</td>
<td>2.6%</td>
<td>3.5%</td>
<td>2.8%</td>
<td>3.6%</td>
<td>2.8%</td>
<td>3.8%</td>
<td>3.0%</td>
</tr>
<tr>
<td><strong>Sing. par+more kids</strong></td>
<td>2.1%</td>
<td>1.3%</td>
<td>1.4%</td>
<td>1.5%</td>
<td>2.0%</td>
<td>2.1%</td>
<td>2.1%</td>
<td>1.8%</td>
</tr>
<tr>
<td><strong>Couple no kid</strong></td>
<td>24.4%</td>
<td>26.0%</td>
<td>26.4%</td>
<td>28.5%</td>
<td>28.9%</td>
<td>29.8%</td>
<td>31.2%</td>
<td>33.6%</td>
</tr>
<tr>
<td><strong>Couple+1 kid</strong></td>
<td>17.5%</td>
<td>18.5%</td>
<td>17.7%</td>
<td>15.3%</td>
<td>16.1%</td>
<td>13.1%</td>
<td>13.7%</td>
<td>11.9%</td>
</tr>
<tr>
<td><strong>Couple+more kids</strong></td>
<td>25.8%</td>
<td>26.7%</td>
<td>22.8%</td>
<td>25.1%</td>
<td>21.2%</td>
<td>25.6%</td>
<td>19.9%</td>
<td>21.1%</td>
</tr>
<tr>
<td><strong>Others</strong></td>
<td>4.2%</td>
<td>5.3%</td>
<td>4.8%</td>
<td>2.7%</td>
<td>3.5%</td>
<td>4.1%</td>
<td>2.1%</td>
<td>3.2%</td>
</tr>
<tr>
<td><strong>Farmer</strong></td>
<td>0.8%</td>
<td>0.7%</td>
<td>0.6%</td>
<td>0.6%</td>
<td>0.4%</td>
<td>0.5%</td>
<td>0.0%</td>
<td>0.4%</td>
</tr>
<tr>
<td><strong>Self-employed</strong></td>
<td>6.0%</td>
<td>1.6%</td>
<td>6.3%</td>
<td>2.5%</td>
<td>2.1%</td>
<td>2.6%</td>
<td>0.5%</td>
<td>2.6%</td>
</tr>
<tr>
<td><strong>Civil servant</strong></td>
<td>6.4%</td>
<td>16.8%</td>
<td>6.1%</td>
<td>16.3%</td>
<td>5.6%</td>
<td>14.0%</td>
<td>5.6%</td>
<td>11.3%</td>
</tr>
<tr>
<td><strong>Employed</strong></td>
<td>22.4%</td>
<td>36.4%</td>
<td>20.3%</td>
<td>35.7%</td>
<td>24.1%</td>
<td>42.7%</td>
<td>26.7%</td>
<td>41.1%</td>
</tr>
<tr>
<td><strong>Worker</strong></td>
<td>32.4%</td>
<td>15.7%</td>
<td>29.9%</td>
<td>14.8%</td>
<td>24.3%</td>
<td>11.8%</td>
<td>20.4%</td>
<td>11.9%</td>
</tr>
<tr>
<td><strong>Unempl./inactive</strong></td>
<td>32.0%</td>
<td>28.9%</td>
<td>36.8%</td>
<td>30.2%</td>
<td>43.5%</td>
<td>28.3%</td>
<td>46.8%</td>
<td>32.6%</td>
</tr>
<tr>
<td><strong>Sample size</strong></td>
<td>4793</td>
<td>43803</td>
<td>4419</td>
<td>31497</td>
<td>5123</td>
<td>39060</td>
<td>7310</td>
<td>33818</td>
</tr>
</tbody>
</table>
### Table 4.10: Data Matching

<table>
<thead>
<tr>
<th>Matching Criteria</th>
<th>Community Size</th>
<th>Federal States</th>
<th>Age of Household Head</th>
<th>Occupation of Household Head</th>
<th>Household Type</th>
<th>Survey Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Categories</td>
<td>3</td>
<td>10</td>
<td>6</td>
<td>6</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>Definition</td>
<td>1 if less than 20000 residents, 2 if between 20000-100000 residents, 3 if more than 100000 residents</td>
<td>“Old” federal states incl. West Berlin and excl. Saarland</td>
<td>1 if no older than 25, 2 if between 25-35, 3 if between 35-45, 4 if between 45-55, 5 if between 55-65, and 6 if older than 65</td>
<td>1 if self-employed farmer, 2 if other self-employed, 3 if civil servants, 4 if dependent employee, 5 if worker and 6 if unemployed or inactive</td>
<td>1 if single women, 2 if single men, 3 if single parent with 1 child, 4 if single parent with 2 or more children, 5 if pair with no child, 6 if pair with 1 child, 7 if pair with 2 or more children and 8 if others, including multi-generation households</td>
<td>1983, 1988, 1993, 1998, 2003</td>
</tr>
</tbody>
</table>
Table 4.11: Consumption-Income Ratios over the Years

<table>
<thead>
<tr>
<th>Year</th>
<th>Mean ($c_{nd}$)</th>
<th>S. Dev. ($c_{nd}$)</th>
<th>Mean ($c_d$)</th>
<th>S. Dev. ($c_d$)</th>
<th>Mean ($c_t$)</th>
<th>S. Dev. ($c_t$)</th>
<th>Nr. of Obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>0.492</td>
<td>0.086</td>
<td>0.162</td>
<td>0.059</td>
<td>0.820</td>
<td>0.121</td>
<td>3773</td>
</tr>
<tr>
<td>1986</td>
<td>0.486</td>
<td>0.079</td>
<td>0.163</td>
<td>0.058</td>
<td>0.817</td>
<td>0.114</td>
<td>3441</td>
</tr>
<tr>
<td>1987</td>
<td>0.481</td>
<td>0.082</td>
<td>0.164</td>
<td>0.068</td>
<td>0.817</td>
<td>0.128</td>
<td>3425</td>
</tr>
<tr>
<td>1988</td>
<td>0.476</td>
<td>0.096</td>
<td>0.164</td>
<td>0.078</td>
<td>0.817</td>
<td>0.147</td>
<td>3372</td>
</tr>
<tr>
<td>1989</td>
<td>0.474</td>
<td>0.085</td>
<td>0.167</td>
<td>0.065</td>
<td>0.816</td>
<td>0.132</td>
<td>3666</td>
</tr>
<tr>
<td>1990</td>
<td>0.473</td>
<td>0.085</td>
<td>0.169</td>
<td>0.064</td>
<td>0.817</td>
<td>0.134</td>
<td>3439</td>
</tr>
<tr>
<td>1991</td>
<td>0.474</td>
<td>0.084</td>
<td>0.169</td>
<td>0.062</td>
<td>0.817</td>
<td>0.129</td>
<td>3439</td>
</tr>
<tr>
<td>1992</td>
<td>0.471</td>
<td>0.083</td>
<td>0.171</td>
<td>0.064</td>
<td>0.816</td>
<td>0.126</td>
<td>3218</td>
</tr>
<tr>
<td>1993</td>
<td>0.471</td>
<td>0.097</td>
<td>0.174</td>
<td>0.084</td>
<td>0.820</td>
<td>0.154</td>
<td>3359</td>
</tr>
<tr>
<td>1994</td>
<td>0.471</td>
<td>0.091</td>
<td>0.171</td>
<td>0.084</td>
<td>0.829</td>
<td>0.150</td>
<td>3125</td>
</tr>
<tr>
<td>1995</td>
<td>0.470</td>
<td>0.090</td>
<td>0.165</td>
<td>0.073</td>
<td>0.834</td>
<td>0.144</td>
<td>3374</td>
</tr>
<tr>
<td>1996</td>
<td>0.473</td>
<td>0.097</td>
<td>0.160</td>
<td>0.086</td>
<td>0.847</td>
<td>0.158</td>
<td>3426</td>
</tr>
<tr>
<td>1997</td>
<td>0.476</td>
<td>0.109</td>
<td>0.156</td>
<td>0.096</td>
<td>0.860</td>
<td>0.185</td>
<td>3399</td>
</tr>
<tr>
<td>1998</td>
<td>0.473</td>
<td>0.109</td>
<td>0.151</td>
<td>0.114</td>
<td>0.864</td>
<td>0.183</td>
<td>3305</td>
</tr>
<tr>
<td>1999</td>
<td>0.478</td>
<td>0.100</td>
<td>0.148</td>
<td>0.095</td>
<td>0.866</td>
<td>0.176</td>
<td>3465</td>
</tr>
<tr>
<td>2000</td>
<td>0.478</td>
<td>0.087</td>
<td>0.147</td>
<td>0.091</td>
<td>0.860</td>
<td>0.158</td>
<td>3415</td>
</tr>
<tr>
<td>2001</td>
<td>0.483</td>
<td>0.097</td>
<td>0.140</td>
<td>0.078</td>
<td>0.858</td>
<td>0.159</td>
<td>5549</td>
</tr>
<tr>
<td>2002</td>
<td>0.484</td>
<td>0.111</td>
<td>0.137</td>
<td>0.085</td>
<td>0.853</td>
<td>0.176</td>
<td>5512</td>
</tr>
<tr>
<td>2003</td>
<td>0.485</td>
<td>0.089</td>
<td>0.134</td>
<td>0.091</td>
<td>0.845</td>
<td>0.154</td>
<td>5906</td>
</tr>
</tbody>
</table>

Over the waves, nondurable consumption is slightly less than half of the net income, and durable consumption varies between 13.4 – 17.4 percent of the income, indicating that nondurable consumption is dominant and about three times of durable consumption. This is reasonable in the sense that durable goods consumption, such as the purchase of TV sets and cars, is much less frequent than nondurable consumption. Therefore reported durable consumption for the EVS sample years is less representative than reported nondurable consumption. Examining the ranks of the groups in various consumption definitions displays that, compared to durable consumption, groups’ positions in nondurable consumption distribution resemble their positions in total consumption to a greater extent\(^\text{15}\). Consequently, the behavior and properties of total consumption is more similar to nondurable consumption.

Both imputation methods have their advantages and disadvantages. In all, because the estimated and imputed coefficients in the first method are the average of all EVS households in each wave, the heterogeneity in the imputed SOEP consumption is even more underrepresented than in the second method. Therefore, in the following I report consumption growth and volatility based on imputed consumption with the second method.

Net income and nondurable consumption\(^\text{16}\) in the EVS are used to calculate the ratio, which

\(^{15}\) In about 70 percent of the cases, group rank in total consumption is closer to its rank in nondurable consumption than that in durable consumption.

\(^{16}\) The inclusion of durable goods, especially real estate and automobiles, requires much information and complex imputation. Neither the EVS nor the SOEP provides sufficient information for a sound imputa-
can be understood as the average propensity to consume. Since nondurable consumption is calculated as above, net income is defined as the household gross income\textsuperscript{17} net of health insurance, pension insurance, unemployment insurance, various income taxes, church tax as well as other social contribution.

Complementary to the EVS data, the SOEP survey data includes important information on the household members’ education level both in schooling years and according to the International Standard Classification of Education (ISCED-1997). Moreover, occupation profiles are also recorded in detail according to the Erikson Goldthorpe Classification (EGP) and the occupational position (Stellung im Beruf, coded by Statistisches Bundesamt). I use the ISCED\textsuperscript{18} and EGP\textsuperscript{19} to be in line with related literature when grouping the samples, even though an alternative estimation using schooling years and occupational position does not show a significant difference. A household is counted as higher educated if one has at least post-secondary non-tertiary education, otherwise the household is classified as lower educated. Finally, I use the EGP to label the occupation as of higher level if the index is less or equal to 8 (including high and low level service, routine non-manual, self-employed, manual supervision, and skilled manual jobs), otherwise it is considered as lower level.

As is shown in Table 4.4, other household characteristics used to group the households include household size, community size, the age and nationality of household heads. The division of household size, community size and age of household heads follow the same rule to that of EVS. Regarding age particularly, suppose on average one person can work 40 years (between 25 and 65 years old), then the first 10 years (25-35) would be the phase of trying out and stabilizing, and the last 10 years is the adjusting period before retirement, whereas the intermediate 20 years is the most stable period regarding income and social status. Therefore, I consider the household head to be young if she or he is under 35, middle aged if between 35 and 55, and old if older than 55. Finally, the households can be “German” or “Non-German” according to the nationality of the household head. Altogether, these classifications divide the sample into 144 groups. Note that the criteria and classifications used to group households are different from those in the data matching process because they serve different purposes.

\textsuperscript{17} Including wage income, freelancing income, financial income, public and non-public transfer and real estate leasing income.

\textsuperscript{18} Dividing levels of education into: Pre-Primary Education, Primary Education or First Stage of Basic Education, Lower Secondary or Secondary Stage of Basic Education, (Upper) Secondary Education, Post-Secondary Non-Tertiary Education, First Stage of Tertiary Education, and Second Stage of Tertiary Education.

The regression equation is similar to equation (4.14) and the main difference is how group average consumption growth is calculated. Because the imputed data allows calculating per capita consumption growth directly, which avoids the missing link between the EVS households over time, group average consumption growth is a mean of all group members’ consumption growth. Since the imputed data presents a nonlinear relationship between group consumption growth and volatility, a quadratic term $\sigma^2_{i,t}$ is added to the right hand side of the regression equation. Moreover, the relevant household characteristics are also included, according to which households are included in certain socioeconomic groups and bear group-specific preferences such as patience and attitude toward the consumption benchmark in the theoretical model.

$$g_{i,t} = \alpha_0 + \alpha_1 SD_t (g_{i,t}) + a_2 \sigma^2_{i,t} + W_i' a_3 + \mu_t + \delta_t + \epsilon_{i,t}$$  \hspace{1cm} (4.17)

where

$$g_{i,t} = \frac{1}{J} \sum_{j} g_{ij,t}.$$

The data also allows studying the relationship between consumption growth and within-group dispersion. The question is, do groups with average higher consumption growth rates also see higher within-group differences? Again standard deviation and variance of consumption growth within-group are included on the right-hand side to account for nonlinearity:

$$g_{i,t} = a_0 + a_1 \Delta_t + a_2 \Delta^2_t + W_i' a_3 + \mu_t + \delta_t + \epsilon_{i,t}$$  \hspace{1cm} (4.18)

where

$$\Delta^2_t = \frac{1}{J} \sum (g_{ij,t} - g_{i,t})^2.$$

Table 4.12 summarizes the key variables and correlations for the imputed data. Group-mean growth rate, volatility and the unconditional correlations bear differences from those in the EVS (Table 4.7). While the low group growth in EVS is due to the approximation method aiming at constructing consumption growth by taking difference of the aggregated consumption, the higher cross-sectional group average consumption growth rate and volatility in the imputed data may result from both differences in household income (from SOEP) and the variation in consumption-income ratio (from EVS). Regarding durable consumption, the unconditional correlations between growth and volatility and between growth and standard deviation are shown as positive, while these correlations concerning nondurable consumption are negative. These unconditional correlations, however, cannot tell us much since many important issues such as group-specific effects and time effects are not considered yet.
### Table 4.12: Imputed Data: Summary Statistics of Per Capita Consumption

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
<th>Nr. of HHs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nondurable consumption</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_{ij}$</td>
<td>0.98%</td>
<td>1.11%</td>
<td>-2.19</td>
<td>2.96</td>
<td>11062</td>
</tr>
<tr>
<td>$\sigma_{ij}$</td>
<td>0.17</td>
<td>0.14</td>
<td>0</td>
<td>2.25</td>
<td>11062</td>
</tr>
<tr>
<td>Correlation($g_{ij}$, $\sigma_{ij}$)</td>
<td>0.008</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_i$</td>
<td>0.01</td>
<td>0.012</td>
<td>-0.134</td>
<td>0.165</td>
<td>144</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>0.116</td>
<td>0.086</td>
<td>0.021</td>
<td>0.490</td>
<td>144</td>
</tr>
<tr>
<td>$\Delta_i$</td>
<td>0.242</td>
<td>0.241</td>
<td>0.003</td>
<td>0.566</td>
<td>144</td>
</tr>
<tr>
<td>Correlation($g_i$, $\sigma_i$)</td>
<td>0.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation($g_i$, $\Delta_i$)</td>
<td>0.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Durable consumption</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_{ij}$</td>
<td>-0.51%</td>
<td>-0.38%</td>
<td>-3.52</td>
<td>4.22</td>
<td>11061</td>
</tr>
<tr>
<td>$\sigma_{ij}$</td>
<td>0.248</td>
<td>0.194</td>
<td>0</td>
<td>2.88</td>
<td>11062</td>
</tr>
<tr>
<td>Correlation($g_{ij}$, $\sigma_{ij}$)</td>
<td>0.009</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_i$</td>
<td>-0.014</td>
<td>-0.017</td>
<td>-0.375</td>
<td>0.25</td>
<td>144</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>0.202</td>
<td>0.140</td>
<td>0.034</td>
<td>1.00</td>
<td>144</td>
</tr>
<tr>
<td>$\Delta_i$</td>
<td>0.385</td>
<td>0.392</td>
<td>0.032</td>
<td>0.809</td>
<td>144</td>
</tr>
<tr>
<td>Correlation($g_i$, $\sigma_i$)</td>
<td>-0.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation($g_i$, $\Delta_i$)</td>
<td>-0.11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total consumption</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_{ij}$</td>
<td>1.04%</td>
<td>1.13%</td>
<td>-2.19</td>
<td>2.9</td>
<td>11062</td>
</tr>
<tr>
<td>$\sigma_{ij}$</td>
<td>0.169</td>
<td>0.143</td>
<td>0</td>
<td>2.25</td>
<td>11062</td>
</tr>
<tr>
<td>Correlation($g_{ij}$, $\sigma_{ij}$)</td>
<td>0.015</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_i$</td>
<td>0.013</td>
<td>0.012</td>
<td>-0.112</td>
<td>0.186</td>
<td>144</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>0.116</td>
<td>0.085</td>
<td>0.013</td>
<td>0.538</td>
<td>144</td>
</tr>
<tr>
<td>$\Delta_i$</td>
<td>0.241</td>
<td>0.236</td>
<td>0.01</td>
<td>0.56</td>
<td>144</td>
</tr>
<tr>
<td>Correlation($g_i$, $\sigma_i$)</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation($g_i$, $\Delta_i$)</td>
<td>-0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Due to the panel structure of the imputed consumption, it’s possible to obtain the direct relationship between individual consumption growth and its volatility, which share a slightly positive unconditional correlation of 0.02. For a more direct view, Figure 4.2 plots the consumption growth against its volatility for the 144 groups in the imputed data set, which are defined by household size, community size, age, education level, occupational background and nationality of household heads. The unconditional correlation is captured by the slightly non-linear curve, even though when outliers are excluded the fitted line is not any more upward-sloping. A more sensible analysis would go beyond the rough unconditional correlation, and explore the time structure and panel structure of the data.

![Figure 4.2: Group Nondurable Consumption Growth and Volatility, Imputed Data](image)

**4.4 Estimation Results**

**4.4.1 Different Patterns of the Groups**

The positive and significant link suggested by the regression results can be interpreted as the welfare price the groups have to pay when they experience high group average consumption growth. And what are these fast-growing groups? In another word, what would be the important explanatory variables for per capita consumption growth? This question can be answered by the following
4.4 Estimation Results

regression equation:

\[ g_{ij,t} = \alpha_0 + W_{ij,t}\alpha_1 + \epsilon_{ij,t} \]  

(4.19)

with \( W_{ij,t} \) denoting household-specific control variables including community size, household size, household heads’ age, nationality, education level and job type. The OLS regression using the imputed data controlling for heteroskedacity shows that these household characteristics associate similarly to the growth and volatility of nondurable and durable goods consumption. As Table 4.2 shows, household size links negatively to the volatility of growth, indicating that larger households turn to experience less volatility. This possibly results from better insurance among the members in large households with more diverse income resources. The age of household heads seem to relate significantly and slightly negatively to both growth and volatility, implying slower growth and smaller volatility for older households. When the household heads are non-German, the members turn to have slower growth and more volatility, suggesting an inferior position for non-German households in welfare measures compared to their German counterparts. As higher education and more skilled jobs appear to have positive though insignificant link to growth, they are negatively associated with volatility, suggesting possible insurance from income associated with higher education and more skilled jobs. Community size seems to be irrelevant to household consumption growth and volatility.

More precisely, what are the groups with high consumption growth and high volatility? Among all 144 groups in Figure 4.2, these are small foreign households with higher education and skilled jobs. It is surprising to see that the groups at the weakest position from the welfare perspective (low growth and high volatility) are households, be there foreign or native German, with high education but unskilled jobs. Moreover, young and small families with higher vocational education tend to have higher consumption growth.

Figure 4.1 shows in more detail the age effect of consumption growth inequality, where income and consumption growth of the young, middle and old groups are compared. Just as shown in Figure 4.1, young households appear to have the highest and most volatile income and consumption growth inequality, and old households the lowest and flattest growth inequality over the years. Again, since a large part of the young population is still out of the labor force and has thus limited income, consumption differences between them and young professionals are big. However, once they start working, the sudden relaxation of their financial constraint boosts their consumption to such a degree that the consumption growth of the young groups is higher than the growth of the older groups. Contrary to the level case, where income variance dominates consumption variance, consumption growth sometimes presents a higher variance than income growth, especially from the late 1980s until the mid 1990s. Middle-aged and old households appear to have a much lower
income and consumption growth variance than the younger ones, whereas it is almost always the case that their consumption growth variance surpasses their income growth variance. This is also the case for old households. This may reflect the different saving habits of households when they are young or different credit constraints for older households which are based on their existing wealth and credit history.

4.4.2 EVS

Table 4.13: EVS: Nondurable Consumption Growth and Volatility / Inequality

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatility</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{nd}$</td>
<td>-0.072</td>
<td>0.099</td>
<td>0.106</td>
</tr>
<tr>
<td></td>
<td>(-0.65)</td>
<td>(1.24)</td>
<td>(1.48)</td>
</tr>
<tr>
<td>Lag nond. consumption</td>
<td>-1.180***</td>
<td>-1.211***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-21.09)</td>
<td>(-20.65)</td>
<td></td>
</tr>
<tr>
<td>Time effect</td>
<td>0.013</td>
<td>10.691***</td>
<td>10.993***</td>
</tr>
<tr>
<td>Constant</td>
<td>(1.07)</td>
<td>(21.27)</td>
<td>(20.76)</td>
</tr>
<tr>
<td>Observations</td>
<td>935</td>
<td>935</td>
<td>935</td>
</tr>
<tr>
<td>F</td>
<td>0.421</td>
<td>287.443</td>
<td>140.395</td>
</tr>
<tr>
<td>$R^2_{within}$</td>
<td>0.002</td>
<td>0.576</td>
<td>0.628</td>
</tr>
<tr>
<td>$R^2_{between}$</td>
<td>0.009</td>
<td>0.006</td>
<td>0.007</td>
</tr>
<tr>
<td>$R^2_{overall}$</td>
<td>0.003</td>
<td>0.091</td>
<td>0.103</td>
</tr>
<tr>
<td><strong>Inequality</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta_{nd}$</td>
<td>0.061</td>
<td>0.079</td>
<td>0.101</td>
</tr>
<tr>
<td></td>
<td>(0.48)</td>
<td>(1.22)</td>
<td>(1.64)</td>
</tr>
<tr>
<td>Lag nond. consumption</td>
<td>-1.170***</td>
<td>-1.205***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-22.52)</td>
<td>(-21.13)</td>
<td></td>
</tr>
<tr>
<td>Time effect</td>
<td>0.004***</td>
<td>10.614***</td>
<td>10.948***</td>
</tr>
<tr>
<td>Constant</td>
<td>(2.79)</td>
<td>(22.53)</td>
<td>(21.13)</td>
</tr>
<tr>
<td>Observations</td>
<td>935</td>
<td>935</td>
<td>935</td>
</tr>
<tr>
<td>F</td>
<td>0.234</td>
<td>253.687</td>
<td>135.238</td>
</tr>
<tr>
<td>$R^2_{within}$</td>
<td>0.001</td>
<td>0.575</td>
<td>0.628</td>
</tr>
<tr>
<td>$R^2_{between}$</td>
<td>0.096</td>
<td>0.007</td>
<td>0.008</td>
</tr>
<tr>
<td>$R^2_{overall}$</td>
<td>0.004</td>
<td>0.094</td>
<td>0.106</td>
</tr>
</tbody>
</table>

$t$ statistics in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

The dependent variable is $g_{ij}$, and the time period is from 1984-2003. (1) is the result of a cross-sectional regression of group nondurable consumption growth on its volatility. The lagged group nondurable consumption is added in (2) and (3), while time dummies are included in (3).
### 4.4 Estimation Results

Table 4.14: EVS: Durable Consumption Growth and Volatility / Inequality

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatility</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>0.229</td>
<td>-0.087</td>
<td>-0.155*</td>
</tr>
<tr>
<td></td>
<td>(0.57)</td>
<td>(-1.07)</td>
<td>(-1.73)</td>
</tr>
<tr>
<td>Lag dur. consumption</td>
<td>-1.324***</td>
<td>-1.389***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-10.85)</td>
<td>(-10.24)</td>
<td></td>
</tr>
<tr>
<td>Time effect</td>
<td>-0.162</td>
<td>9.999***</td>
<td>10.216***</td>
</tr>
<tr>
<td>Constant</td>
<td>(-1.05)</td>
<td>(10.86)</td>
<td>(10.13)</td>
</tr>
<tr>
<td>Observations</td>
<td>935</td>
<td>935</td>
<td>935</td>
</tr>
<tr>
<td>F</td>
<td>0.324</td>
<td>62.920</td>
<td>185.051</td>
</tr>
<tr>
<td>$R^2_{within}$</td>
<td>0.011</td>
<td>0.660</td>
<td>0.742</td>
</tr>
<tr>
<td>$R^2_{between}$</td>
<td>0.041</td>
<td>0.102</td>
<td>0.124</td>
</tr>
<tr>
<td>$R^2_{overall}$</td>
<td>0.000</td>
<td>0.333</td>
<td>0.383</td>
</tr>
</tbody>
</table>

| **Inequality**           |      |      |      |
| $\Delta_d$               | -0.545*** | -0.183*** | -0.106** |
|                          | (-7.54) | (-3.37) | (-2.23) |
| Lag dur. consumption     | -1.272*** | -1.336*** |      |
|                          | (-9.63) | (-11.04) |      |
| Time effect              | -0.068*** | 9.575*** | 9.771*** |
| Constant                 | (-85.18) | (9.56) | (10.98) |
| Observations             | 935  | 935  | 935  |
| F                        | 56.864 | 170.629 | 223.625 |
| $R^2_{within}$           | 0.088 | 0.668 | 0.741 |
| $R^2_{between}$          | 0.001 | 0.086 | 0.098 |
| $R^2_{overall}$          | 0.081 | 0.339 | 0.376 |

\( t \) statistics in parentheses

* \( p < 0.1 \), ** \( p < 0.05 \), *** \( p < 0.01 \)

The dependent variable is \( g_{i,t} \), and the time period is from 1984-2003. (1) is the result of a cross-sectional regression of group durable consumption growth on its volatility. The lagged group durable consumption is added in (2) and (3), while time dummies are included in (3).
The dependent variable is $g_{ijt}$, and the time period is from 1984-2003. (1) is the result of a cross-sectional regression of group total consumption growth on its volatility. The lagged group total consumption is added in (2) and (3), while time dummies are included in (3).

The upper panels of Table 4.13, 4.14 and 4.15 present cross-sectional fixed effect estimations of group nondurable consumption growth on volatility (equation (4.14) and (4.17)) for nondurable, durable and total consumption, respectively. The estimators are cluster robust. To exclude the effect of household size, per capita consumption growth is the key variable in both the current and next subsection, where OECD defined equivalent scale is employed. Group fixed effects are considered for all regressions and one extreme outlier is excluded. Column (1) is the result of a cross-sectional regression of group consumption growth on the volatility. The lagged group consumption is added in (2) and (3), while time dummies are included in (3). Neither durable, nondurable or total consumption shows a significant relation between consumption growth and volatility, and the signs of the estimated coefficient concerning volatility are also mixed.
The data also allows studying the welfare effect of consumption growth. The key question lies in the evolution of the within-group standard deviation along with the group average growth, as are summarized by equation (4.15) and (4.18). Regression results are summarized in the lower panels of Tables 4.13-4.15. The relationship between growth and inequality is positive but insignificant for nondurable consumption, while durable consumption growth appears to negatively relate to the within-group inequality. This implies that groups with higher durable goods consumption also appear to be more equal. In the “keeping up with the Joneses” context and especially regarding the conventional hypothesis on positional goods, this makes sense since groups with high durable consumption growth may be those signaling strongly with the purchase of positional goods, where the average will to keep up with others is also strong. The growth-inequality relationship reverses when it comes to total consumption. Also significant is that, groups with high total consumption growth also appear to have within-group inequality. This result is not controversial to the previous one because the high-growing groups here are not the same as in the durable consumption case. They can be described as rather young, small households who are at the start of their careers and subject to more diverse income and other shocks. Nondurable consumption grows fast while durables still pick up slowly due to their budget constraint. The positive regression results thus reflect the dominant nondurable share in total consumption.

**4.4.3 Imputed Consumption**

Figures 4.2 and 4.3 vaguely display nonlinear growth-volatility associations for durable and nondurable consumption with a 95 percent confidence interval. The U-shaped relationships for nondurable and total consumption (negative coefficient for standard deviation and positive coefficient for its square) are confirmed in cross-sectional fixed effect regressions (Table 4.5 and 4.16). Similar to the regressions for EVS consumption, group fixed effect and time effect are considered, whereas lagged consumption and time effect are gradually added to the right hand side of the equation. Perhaps due to the unrepresentative information on durable consumption, there is no significant growth-volatility relation reported.

Let’s focus on the nondurable and total consumption. For groups with consumption growth under a threshold growth rate \( \bar{g} \), the growth is accompanied by diminishing volatility; Above \( \bar{g} \), faster growing groups witness higher volatility. This threshold growth rate can be calculated using the estimates of the coefficients \((-\frac{\hat{\alpha}_1}{2\hat{\alpha}_2})\). Recall the results presented in Table 4.2. The groups with high nondurable consumption growth and high volatility are the young households, who are mapped at the upper right area in Figure 4.2. Those with relatively low growth and medium volatility are the older and/or foreign head households, located at the middle of the fitted line. A large number of groups gather at the left end of Figure 4.2. This majority of households with
Table 4.16: Imputed Data: Total Consumption Growth and Volatility / Inequality

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatility</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-0.378***</td>
<td>-0.362***</td>
<td>-0.349***</td>
</tr>
<tr>
<td></td>
<td>(-5.07)</td>
<td>(-4.93)</td>
<td>(-4.73)</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.765***</td>
<td>0.751***</td>
<td>0.749***</td>
</tr>
<tr>
<td></td>
<td>(4.61)</td>
<td>(4.76)</td>
<td>(4.74)</td>
</tr>
<tr>
<td>Lag total cons.</td>
<td>-0.198***</td>
<td>-0.215***</td>
<td>-0.215***</td>
</tr>
<tr>
<td></td>
<td>(-7.03)</td>
<td>(-6.58)</td>
<td>(-6.58)</td>
</tr>
<tr>
<td>Time effect</td>
<td>significant**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.49)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.031***</td>
<td>1.887***</td>
<td>2.021***</td>
</tr>
<tr>
<td></td>
<td>(6.70)</td>
<td>(7.16)</td>
<td>(6.61)</td>
</tr>
<tr>
<td>Observations</td>
<td>2567</td>
<td>2423</td>
<td>2423</td>
</tr>
<tr>
<td>F</td>
<td>13.901</td>
<td>36.348</td>
<td>11.649</td>
</tr>
<tr>
<td>$R^2_{within}$</td>
<td>0.074</td>
<td>0.136</td>
<td>0.150</td>
</tr>
<tr>
<td>$R^2_{between}$</td>
<td>0.096</td>
<td>0.014</td>
<td>0.011</td>
</tr>
<tr>
<td>$R^2_{overall}$</td>
<td>0.074</td>
<td>0.065</td>
<td>0.072</td>
</tr>
<tr>
<td><strong>Inequality</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta$</td>
<td>-0.070</td>
<td>-0.096</td>
<td>-0.092</td>
</tr>
<tr>
<td></td>
<td>(-1.12)</td>
<td>(-1.46)</td>
<td>(-1.41)</td>
</tr>
<tr>
<td>$\Delta^2$</td>
<td>0.156</td>
<td>0.177*</td>
<td>0.179*</td>
</tr>
<tr>
<td></td>
<td>(1.48)</td>
<td>(1.70)</td>
<td>(1.73)</td>
</tr>
<tr>
<td>Lag total cons.</td>
<td>-0.159***</td>
<td>-0.174***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-6.53)</td>
<td>(-6.35)</td>
<td></td>
</tr>
<tr>
<td>Time effect</td>
<td>significant**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.017*</td>
<td>1.521***</td>
<td>1.652***</td>
</tr>
<tr>
<td></td>
<td>(1.84)</td>
<td>(6.60)</td>
<td>(6.44)</td>
</tr>
<tr>
<td>Observations</td>
<td>2346</td>
<td>2226</td>
<td>2226</td>
</tr>
<tr>
<td>F</td>
<td>1.167</td>
<td>14.431</td>
<td>4.828</td>
</tr>
<tr>
<td>$R^2_{within}$</td>
<td>0.008</td>
<td>0.056</td>
<td>0.076</td>
</tr>
<tr>
<td>$R^2_{between}$</td>
<td>0.000</td>
<td>0.028</td>
<td>0.028</td>
</tr>
<tr>
<td>$R^2_{overall}$</td>
<td>0.008</td>
<td>0.009</td>
<td>0.016</td>
</tr>
</tbody>
</table>

$t$ statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Group total consumption growth rate is the dependent variable.
medium growth and low volatility may be described as, among others, older, large in number, well educated and as having skilled jobs. Nonetheless, as mentioned before, the interpretation of the imputed data should be very careful due to the over- and underestimation problem of the volatility. In the process of matching, the underestimation of personal shocks $\theta_{ij}$ may differ at a group consumption level and is possibly particularly serious in the lower end of the distribution (assuming poorer households usually have higher consumption volatility due to a tighter credit constraint). Consequently, the difference in growth-volatility relationships for households with faster and slower consumption growth may partly result from the data imputation process.

Similar to the nondurable growth-volatility distribution, a large number of the groups are accumulated at the left end where both nondurable consumption growth and inequality are low (Figure 4.4). The distribution of durable consumption inequality is more dispersed, with the concentration of the groups spanning a wider area than the nondurable case. The lower panels of Tables 4.5-4.16 report the regression results of group growth on within-group inequality, where the U-shaped relationship also holds but is only significant for nondurable consumption. Groups with very low inequality can be those with older age and with stable occupation, who are rather subject to similar income and preference shocks. Such stability consequently allows for stable consumption growth. At the other end, highly unequal groups are subject to remarkably varied idiosyncratic shocks, where the high group growth rate could be a result of those households with
Figure 4.4: Group Nondurable Consumption Growth and Inequality, Imputed Data

extremely large positive shocks.

4.5 Conclusion

This chapter explores the link between household consumption growth and volatility from both theoretical and empirical perspectives. Heterogeneous households transfer resources intertemporally via trading one type of asset, which helps to store value and insure against income shocks. Contrary to typical neoclassical models, households incorporate group average consumption as reference, so that the relative standard of living becomes relevant besides the absolute level. The degree of (im)patience is another important group-specific parameter influencing households’ decision on security holding vs. current consumption.

The incomplete market setup, among other traits, offers partial insurance against income shocks and contributes to consumption smoothing. Still, the general model equilibrium predicts a positive link between consumption growth and volatility, implying unstable growth over time. Moreover, consumption dynamics vary among households with different preferences, especially when group average consumption serves as external benchmark. While more patient groups experience smaller volatility, household eagerness to keep up with group mean intensifies the volatility of the whole group. In a business cycle context, dominant positive income shocks and preference on con-
4.5 Conclusion

Consumption smoothing drive up the security price, and further strengthens the power of household preferences.

I further use German data to construct household consumption growth in order to test the hypothesis on the positive link between growth and volatility of durable, nondurable and total consumption. A look at individual level consumption growth, volatility and the households’ characteristics helps to identify the controls that have an important economic impact. Household size, the age and nationality of household heads are relevant to growth and volatility, whereas heads’ education levels do not seem important. Community size and heads’ job profiles are positively associated with consumption volatility. The most unfavorable households in the imputed data are those have high education but unskilled jobs, whose low growth and high volatility may come from low income and frequent job changing. Households with foreign heads also often find themselves in the category of low consumption growth and high volatility.

As EVS cannot provide significant evidence on the link, the imputed data reveals a U-shaped relationship in nondurable consumption growth and volatility. At the right end are those young households who experience both high growth and high volatility, at the left end are the households with older age, large in number, well educated and/or with skilled jobs, whereas those with relatively low growth and medium volatility are the older and/or foreign head households. From another perspective of welfare cost, also in the EVS, the link between group growth and within-group inequality is found positive for nondurable consumption but negative for durable consumption. The results suggest that lower income households, mostly young and small-sized, are experiencing higher growth in nondurable consumption and subject to more diverse shocks; Higher income households with faster growing purchase of positional goods are more similar in the case of durable consumption.
Appendix to Chapter 2

If firms foresee that wages are dependent on labor and capital employment, firms’ decisions for job opening and capital employment are slightly different. The profit maximization is additionally subject to the wage curves, which are functions of other input choices of the firms and are formed through bargaining:

\[ w_i^t = w^t \left(n_{t-1}^s, n_{t-1}^u, k_{t-1}, A_t, B_t \right). \]

As capital is concerned, the perceivable firms would make the following choice:

\[ \frac{\partial y_t}{\partial k_{t-1}} = r_t + \frac{\partial w^u_t}{\partial k_{t-1}} n_{t-1}^u + \frac{\partial w^s_t}{\partial k_{t-1}} n_{t-1}^s, \] (20)

additional payment to workers

The right hand side is the price the firm has to pay: Market rent for capital as well as the other parts paid out as wages through wage bargaining. This is because households have double roles as both capital holders and workers. As a result, the firm finds it optimal to take less capital than what would be efficient.

The Euler equations concerning the labor demand are:

\[ \frac{\kappa^s}{q^s_i} = \delta E_t \left\{ \frac{\partial y_{t+1}}{\partial n^s_{t+1}} - w^s_{t+1} - \frac{\partial w^s_{t+1}}{\partial n^s_{t+1}} n^s_{t+1} - \frac{\partial w^u_{t+1}}{\partial n^u_{t+1}} n^u_{t+1} + (1 - \chi^s) \frac{\kappa^s}{q^s_{t+1}} \right\}, \] (21)

\[ \frac{\kappa^u}{q^u_i} = \delta E_t \left\{ \frac{\partial y_{t+1}}{\partial n^u_{t+1}} - w^u_{t+1} - \frac{\partial w^u_{t+1}}{\partial n^u_{t+1}} n^u_{t+1} - \frac{\partial w^s_{t+1}}{\partial n^s_{t+1}} n^s_{t+1} + (1 - \chi^u) \frac{\kappa^u}{q^u_{t+1}} \right\}. \] (22)

Consequently the marginal value of a skilled worker is

\[ \frac{\partial V_t}{\partial n^s_{t-1}} = \frac{\partial y_t}{\partial n^s_{t-1}} - w^s_{t} - \left( \frac{\partial w^s_{t}}{\partial n^s_{t-1}} n^s_{t-1} + \frac{\partial w^u_{t}}{\partial n^u_{t-1}} n^u_{t-1} \right) + (1 - \chi^s) \frac{\kappa^s}{q^s_t}, \] (23)
and that of an unskilled worker is

$$\frac{\partial V_i}{\partial n_{t-1}^u} = \frac{\partial y_t}{\partial n_{t-1}^s} - w_t^s \frac{\partial w_t^s}{\partial n_{t-1}^s} n_{t-1}^s - \frac{\partial w_t^u}{\partial n_{t-1}^u} n_{t-1}^u + (1 - \chi_u) \frac{\kappa^u}{q_t^u}. \tag{24}$$

Note that the marginal value created by a worker is different from equation (23) in the way that both types of wages are affected by the amount of labor input.

The solutions to wage bargaining are

$$w_t^s = \eta \left[ \frac{\partial y_t}{\partial n_{t-1}^s} - \frac{\partial w_t^s}{\partial n_{t-1}^s} n_{t-1}^s - \frac{\partial w_t^u}{\partial n_{t-1}^u} n_{t-1}^u + (1 - \chi_u) \frac{\kappa^s}{q_t^s} \right] + (1 - \eta) \left[ - (1 - \chi^s - p_t^s \delta) \delta E_t \Omega_{t+1}^s \right],$$

$$w_t^u = \eta \left[ \frac{\partial y_t}{\partial n_{t-1}^u} - \frac{\partial w_t^s}{\partial n_{t-1}^s} n_{t-1}^s - \frac{\partial w_t^u}{\partial n_{t-1}^u} n_{t-1}^u + (1 - \chi_u) \frac{\kappa^u}{q_t^u} \right] + (1 - \eta) \left[ - (1 - \chi^u - p_t^u \delta) \delta E_t \Omega_{t+1}^u \right].$$

I can use the method of undetermined coefficients to solve the system of partial differential equations in order to obtain the bargained wages. The “constant terms” that do not contain $w_t^s$ or $w_t^u$ are excluded first and will be added back later. Therefore, the critical system I am solving becomes

$$w_t^s = \eta \left[ \frac{\partial y_t}{\partial n_{t-1}^s} - \frac{\partial w_t^s}{\partial n_{t-1}^s} n_{t-1}^s - \frac{\partial w_t^u}{\partial n_{t-1}^u} n_{t-1}^u \right],$$

$$w_t^u = \eta \left[ \frac{\partial y_t}{\partial n_{t-1}^u} - \frac{\partial w_t^u}{\partial n_{t-1}^u} n_{t-1}^u - \frac{\partial w_t^s}{\partial n_{t-1}^s} n_{t-1}^s \right].$$

From the model setup I can guess that the wages are proportional to the respective marginal products of labor, where the portions of skilled and unskilled are $X$ and $Y$, separately:

$$w_t^s = X \cdot A_t^X k_{t-1}^{-a} \left[ \alpha \left( n_{t-1}^s \right)^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha) \left( n_{t-1}^u \right)^{\frac{\sigma - 1}{\sigma}} \right] \frac{T_t}{T_t - 1} \left( n_{t-1}^s \right)^{-\frac{1}{\sigma}},$$

$$w_t^u = Y \cdot A_t^Y k_{t-1}^{-a} \left[ \alpha \left( n_{t-1}^s \right)^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha) \left( n_{t-1}^u \right)^{\frac{\sigma - 1}{\sigma}} \right] \frac{T_t}{T_t - 1} \left( n_{t-1}^u \right)^{-\frac{1}{\sigma}}.$$

Taking derivatives of them both and plugging them into the critical system yield:
\[
\frac{X}{\eta} \alpha (B_1) \frac{\sigma}{\alpha} (n_1^\prime) \frac{\sigma}{\alpha} + \frac{X}{\eta} (1 - \alpha) (n_1^\prime) \frac{\sigma}{\alpha}
\]
\[
= \left[ a a (B_1) \frac{\sigma}{\alpha} (B_1) \frac{\sigma}{\alpha} - X (a - 1) \right] a (B_1) \frac{\sigma}{\alpha} (n_1^\prime) \frac{\sigma}{\alpha}
\]
\[
+ \left[ (1 - \alpha) a a (B_1) \frac{\sigma}{\alpha} - \alpha \left( a - 1 + \frac{1}{\sigma} \right) (B_1) \frac{\sigma}{\alpha} Y + X \frac{1}{\sigma} (1 - \alpha) \right] (n_1^\prime) \frac{\sigma}{\alpha},
\]
and as well
\[
\frac{Y}{\eta} a (B_1) \frac{\sigma}{\alpha} (n_1^\prime) \frac{\sigma}{\alpha} + \frac{Y}{\eta} (1 - \alpha) (n_1^\prime) \frac{\sigma}{\alpha}
\]
\[
= \left[ a (1 - \alpha) a (B_1) \frac{\sigma}{\alpha} + Y \frac{1}{\sigma} a (B_1) \frac{\sigma}{\alpha} - X \left( a - 1 + \frac{1}{\sigma} \right) (1 - \alpha) \right] (n_1^\prime) \frac{\sigma}{\alpha}
\]
\[
+ \left[ a (1 - \alpha) (1 - \alpha) - Y (a - 1) (1 - \alpha) \right] (n_1^\prime) \frac{\sigma}{\alpha}.
\]

By comparing the parameters of left- and right-hand sides of the equations I can solve for \(X\) and \(Y\):
\[
X = \frac{a a \eta (B_1) \frac{\sigma}{\alpha}}{1 + \eta a - \eta'},
\]
\[
Y = \frac{(1 - \alpha) a \eta}{1 + \eta a - \eta'}
\]
and thus
\[
\omega_s^\prime = \frac{a a \eta (B_1) \frac{\sigma}{\alpha}}{1 + \eta a - \eta} A_f k_s^{1 - a} \left[ \alpha (B_1 n_1^\prime) \frac{\sigma}{\alpha} + (1 - \alpha) (n_1^\prime) \frac{\sigma}{\alpha} \right] \frac{\partial y_t}{\partial n_s} (n_1^\prime)^{\frac{\sigma}{\alpha} - \frac{1}{\sigma}},
\]
\[
\omega_t^\prime = \frac{(1 - \alpha) a \eta}{1 + \eta a - \eta} A_f k_t^{1 - a} \left[ \alpha (B_1 n_1^\prime) \frac{\sigma}{\alpha} + (1 - \alpha) (n_1^\prime) \frac{\sigma}{\alpha} \right] \frac{\partial y_t}{\partial n_t} (n_1^\prime)^{\frac{\sigma}{\alpha} - \frac{1}{\sigma}}.
\]

Adding back the constant terms yields
\[
\omega_s^\prime = \eta \left[ \frac{1}{1 - \eta (1 - a)} \frac{\partial y_t}{\partial n_s} + \eta' \kappa_s^\prime \right] + (1 - \eta) b_s^\prime,
\]
\[
\omega_t^\prime = \eta \left[ \frac{1}{1 - \eta (1 - a)} \frac{\partial y_t}{\partial n_t} + \eta' \kappa_t^\prime \right] + (1 - \eta) b_t^\prime.
\]
Appendix to Chapter 4

Taylor Approximation of the Group Average Euler Equation

According to second order Taylor approximation,

$$\ln G_{i,t+1} \approx \ln G_i + \frac{1}{G_i} (G_{i,t+1} - G_i) - \frac{1}{2G_i^2} (G_{i,t+1} - G_i)^2.$$

Since $EG_{i,t+1} = G_i$, taking unconditional mean of both sides yields

$$E \ln G_{i,t+1} = \ln EG_{i,t+1} + \frac{1}{G_i} E (G_{i,t+1} - G_i) - \frac{1}{2G_i^2} E (G_{i,t+1} - G_i)^2.$$

Rearrange it, we have

$$\ln EG_{i,t+1} = E \ln G_{i,t+1} + \frac{1}{2} E \left[ \left( \frac{G_{i,t+1} - G_i}{G_i} \right)^2 \right],$$

or,

$$E [G_{i,t+1}] \approx \exp \left\{ E \ln G_{i,t+1} + \frac{1}{2} E \left[ \left( \frac{G_{i,t+1} - G_i}{G_i} \right)^2 \right] \right\}.$$

Similarly according to Taylor approximation,

$$G_{i,t+1}^{(-\gamma-(1-\gamma)\kappa_i)} \approx G_i^{(-\gamma-(1-\gamma)\kappa_i)} + (-\gamma - (1 - \gamma) \kappa_i) G_i^{(-\gamma-(1-\gamma)\kappa_i-1)} (G_{i,t+1} - G_i)$$
$$+ \frac{(-\gamma - (1 - \gamma) \kappa_i) (-\gamma - (1 - \gamma) \kappa_i - 1)}{2} G_i^{(-\gamma-(1-\gamma)\kappa_i-2)} (G_{i,t+1} - G_i)^2,$$
and

\[ EG_{i,t+1}^{(-\gamma - (1-\gamma)\kappa_i)} \]
\[ \approx G_i^{(-\gamma - (1-\gamma)\kappa_i)} + (-\gamma - (1-\gamma)\kappa_i) G_i^{(-\gamma - (1-\gamma)\kappa_i-1)} \left(\frac{E(G_{i,t+1} - G_i)}{0}\right) \]
\[ + \frac{(-\gamma - (1-\gamma)\kappa_i) (-\gamma - (1-\gamma)\kappa_i - 1)}{2} G_i^{(-\gamma - (1-\gamma)\kappa_i-2)} E(G_{i,t+1} - G_i)^2, \]

\[ EG_{i,t+1}^{(-\gamma - (1-\gamma)\kappa_i)} \]
\[ \approx G_i^{(-\gamma - (1-\gamma)\kappa_i)} \left[ 1 + \frac{(-\gamma - (1-\gamma)\kappa_i) (-\gamma - (1-\gamma)\kappa_i - 1)}{2} E\left(\frac{G_{i,t+1} - G_i}{G_i}\right)^2 \right]. \]

Taking log of both sides yields

\[ \ln EG_{i,t+1}^{(-\gamma - (1-\gamma)\kappa_i)} \]
\[ \approx \ln G_i^{(-\gamma - (1-\gamma)\kappa_i)} \]
\[ + \ln \left[ 1 + \frac{(-\gamma - (1-\gamma)\kappa_i) (-\gamma - (1-\gamma)\kappa_i - 1)}{2} E\left(\frac{G_{i,t+1} - G_i}{G_i}\right)^2 \right]. \]

Since \( E\left(\frac{G_{i,t+1} - G_i}{G_i}\right)^2 \) is very small,

\[ \ln \left[ 1 + \frac{(-\gamma - (1-\gamma)\kappa_i) (-\gamma - (1-\gamma)\kappa_i - 1)}{2} E\left(\frac{G_{i,t+1} - G_i}{G_i}\right)^2 \right] \]
\[ \approx \frac{(-\gamma - (1-\gamma)\kappa_i) (-\gamma - (1-\gamma)\kappa_i - 1)}{2} E\left(\frac{G_{i,t+1} - G_i}{G_i}\right)^2. \]

Using the result from (25),

\[ \ln EG_{i,t+1}^{(-\gamma - (1-\gamma)\kappa_i)} \approx (-\gamma - (1-\gamma)\kappa_i) \left[ E \ln G_{i,t+1} + \frac{1}{2} E\left(\frac{G_{i,t+1} - G_i}{G_i}\right)^2 \right] \]
\[ + \frac{(-\gamma - (1-\gamma)\kappa_i) (-\gamma - (1-\gamma)\kappa_i - 1)}{2} E\left(\frac{G_{i,t+1} - G_i}{G_i}\right)^2 \]
\[ = (-\gamma - (1-\gamma)\kappa_i) E \ln G_{i,t+1} + \frac{(-\gamma - (1-\gamma)\kappa_i)^2}{2} E\left(\frac{G_{i,t+1} - G_i}{G_i}\right)^2, \]
where \( E \ln G_{i,t+1} = E \ln (1 + g_{i,t+1}) \approx E g_{i,t+1} = g_i \), and

\[
E \left[ \left( \frac{G_{i,t+1} - G_i}{G_i} \right)^2 \right] \approx E \left[ \ln \left( \frac{G_{i,t+1} - G_i}{G_i} + 1 \right) \right]^2 = E \left[ \ln \left( \frac{G_{i,t+1}}{G_i} \right) \right]^2 \\
= E [\ln G_{i,t+1} - \ln G_i]^2 = E [\ln (1 + g_{i,t+1}) - \ln (1 + g_i)]^2 \approx E [(g_{i,t+1} - g_i)^2] \\
= \sigma_{g_i}^2.
\]

Therefore

\[
\ln E G_{i,t+1}^{(-\gamma-(1-\gamma)\kappa_i)} = (-\gamma - (1 - \gamma) \kappa_i) g_i + \frac{(-\gamma - (1 - \gamma) \kappa_i)^2}{2} \sigma_{g_i}^2
\]

or,

\[
E G_{i,t+1}^{(-\gamma-(1-\gamma)\kappa_i)} = \exp \left( (-\gamma - (1 - \gamma) \kappa_i) g_i + \frac{(-\gamma - (1 - \gamma) \kappa_i)^2}{2} \sigma_{g_i}^2 \right).
\]
Definition of Variables
### Definition of Variables

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<td>$Y_{i,t}$</td>
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<td>history of endowment shock</td>
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<tr>
<td>$q_t$</td>
<td>asset price</td>
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<td>$a_{ij,t}$</td>
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<td>$c_{ij,t}$</td>
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<td>$X_{i,t}$</td>
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<td>$g_{ij,t}$</td>
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<td>$\beta_i$</td>
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<tr>
<td>$\kappa_i$</td>
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<tr>
<td>$p_i$</td>
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<tr>
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<td>$\phi_{ij}$</td>
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<td>$SD_t(\Delta c_{i,t})$</td>
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<td>$W_{ij,t}$</td>
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<td>$\Delta_i$</td>
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Bibliography


Bibliography


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J. Kluve. The Effectiveness of European Active Labor Market Policy. RWI Discussion Papers 37, 2006.


Bibliography


Bibliography


Selbständigkeitserklärung

Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung meiner Dissertation benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

Berlin, den 16.02.2011