Essays on Information Asymmetry and Vertical Relations

D I S S E R T A T I O N

zur Erlangung des akademischen Grades

doctor rerum politicarum

(Doktor der Wirtschaftswissenschaft)

eingereicht an der

Wirtschaftswissenschaftlichen Fakultät

der Humboldt-Universität zu Berlin

von

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Tag des Kolloquiums: 18.07.2016
Acknowledgements

For the continuous and highly valuable support during the writing of this dissertation I want to thank my advisors Georg Weizsäcker and Pio Baake; not only for their very profound advice concerning my research, but also for their help in sustaining the motivation and determination to complete it.

Further I want to thank my coauthors, which—in addition to Pio Baake—includes Vanessa von Schlippenbach and Friederike Heiny, for the very instructive discussions during the previous years.

I also want to thank my colleagues at DIW Berlin and the DIW Graduate Center, foremost Beatrice Pagel, Tobias Schmidt, Paul Viefers and Lilo Wagner, for helpful comments during seminars and in private discussions.

The financial support from the DIW Berlin and the DIW Graduate Center for the time of writing this thesis is highly appreciated.

Finally, I want to thank my friends and especially my family for the uninterrupted support, which made this dissertation possible.
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1 General Introduction

1.1 Motivation

The chapters of this dissertation discuss two different modelling variants in game theory and industrial economics: The first being asymmetric information (chapters 2 and 4) and the second being vertical relations (chapters 3 and 4). The chapters dealing with asymmetric information introduce different types of signalling models. In chapter 2 a model is presented, where an informed seller might influence the belief of uninformed buyers about him being a good or a bad type. In chapter 4 a supplier can signal his information about market demand to a retailer, but has to make sure that the signalled information is credible. The two chapters, which treat models with vertical relations, involve intricate contracts between up- and downstream firms. In chapter 3 suppliers can decide to supply one of two intermediaries, with the contracts offered by the intermediaries being conditional on the profits made in the downstream market. In chapter 4 the upstream monopoly can use two-part tariffs to avoid the problem of double marginalization (see Spengler, 1950), when contracting with a single downstream firm.

Asymmetric information and vertical relations apply to different aspects of a model: its information structure and its interaction structure respectively. However, both modelling variants are increasing the complexity of the model, to which they are applied. If, in general, models are simplifications of reality, i.e., reductions of the complexity of the latter, both these modelling variants can be regarded as bringing a model a bit closer to reality by increasing its complexity. Considering any form of transaction it seems highly unlikely that all parties involved share the same set of relevant information and modern supply chains consist of way more layers than just output producing firms and consumers. Unfortunately the increase in complexity comes at the price of reduced tractability. Thus, general statements about a model’s behavior can often only be made on a very fundamental level, while assumptions on functional forms or even parameters have to be made to shed light on the properties of the model. Still, comparative statics offer a possibility to analyze a model beyond specific parameter constellations, while attention is restricted to specific equilibria (or types thereof) in order to keep the models somewhat tractable.

In this dissertation, the primary interest is a methodological one: to understand how the models’ dynamics as well as the implications concerning efficiency differ, if certain assumptions or
parameters change. While the results of the other chapters still require their application in order to lead to prolific policy advice, in chapter 3 an implication for economic policy is already mentioned.

1.2 Methodological Discussion

The models, which are discussed in this dissertation, employ solution concepts from non-cooperative game theory. The models, which treat aspects of asymmetric information (chapters 2 and 4), use the concept of perfect Bayesian equilibrium (see Harsanyi, 1967) and backward induction (see Von Neumann and Morgenstern, 1944) to find subgame perfect equilibria (see Selten, 1978). In contrast, the model in chapter 3 is nested in a full information setting, which allows to solely use subgame perfection as a solution criterion.

In chapter 2, the model consists of a simple dynamic stage game, which is repeated a finite number of times. The agents’ action spaces are fairly limited, while the information dynamic is quite complex, allowing the involved parties to learn along the equilibrium path. Counterfactuals and working hypotheses are used to show that a specific route through the supergame’s game tree may be part of an equilibrium. The product, which is traded in this model is an experience good with buyers learning their utility after consumption. However, they can form an expectation about their utility from the seller’s reputation for being a "good" or a "bad" type. While consumers would always want to buy from a good seller, they would never want to buy from a bad seller. Although the reputation stochastically depends upon the seller’s effort, there is still some uncertainty about it from the seller’s perspective, as she cannot directly observe, whether a buyer actually liked the product or not. The seller is only able to form a believe about that, while the buyers know whether previous buyers liked the product. Thus, the model presents uncertainty in a couple of dimensions: a) for the buyers the seller’s type and the seller’s effort choice, b) for the seller the buyer’s utility from interaction and the seller’s reputation. Therefore, while the stage game being structurally very simple, the dynamic generated by the information structure restricts the analysis to focus on a specific type of equilibrium, in contrast to solving the game entirely. However, in each of the seller’s decision nodes an expected continuation value of each respective action can be computed in order to allow the seller to decide upon her effort. Thus, in an infinitely repeated game the seller’s decision problem could be solved as a stochastic dynamic programming problem with her effort choice as the control and her expected reputation as the state variable. While the dynamic
programming framework is retained, the equilibrium is analyzed by using backward induction, as
the effect can be discussed more concisely in a finitely repeated game.

Chapter 3 presents the only full information model, which is analyzed by using backward in-
duction. However, as the market structure involves multiple firms at each end of the supply chain,
the possibilities for interaction are a lot richer compared to the models in the other chapters. Fur-
thermore, this model includes different firm types on the downstream level in the sense that their
objective functions differ. The downstream firms may be "regular" investor-owned firms, which
maximize their profits and have to acquire an input good (or production capacity) from suppliers.
However, the suppliers might also join a cooperative, which maximizes its members' profits. Af-
ter suppliers have made their decisions, downstream firms compete by choosing quantities to be
supplied on a homogenous product market, given their capacity constraints. The need to acquire
production capacity links this model to the literature on limited capacity Duopoly models, but
unlike in the classical Bertrand models of Levitan and Shubik (1970) and Kreps and Scheinkmann
(1983) firms in our model can not freely choose to acquire capacity as they wish in a stage previous
to Cournot competition. Instead, there is a limited number of suppliers, i.e., a limited amount of
capacity, available to be contracted. Therefore, in our model firms may have incentives to increase
their own capacity beyond what they actually desire to provide to consumers, in order to reduce
the capacity available to their competitor. We vary the number of allocatable suppliers in order to
identify different endogenously formed market structures on the downstream part of the supply
chain.

In chapter 4, the model combines asymmetric information and vertical relations, analyzing a
supply chain with a single supplier and a single retailer as well as uncertainty about consumer
demand on a perfectly competitive market. The information asymmetry stems from the agents’
signal about the state of demand being private information. Thus, the model is again solved for
a perfect Bayesian equilibrium, also employing the intuitive criterion (see Cho and Kreps, 1987)
to reduce the number of potential equilibria by restricting off-equilibrium beliefs. The upstream
firm (supplier) offers a menu of contracts to the downstream firm (retailer) with each contract
specifying a combination of a quantity to be sold to consumers and a transfer to the supplier. The
contracts may depend on the supplier’s private signal about the state of demand. Additionally, the
agents are able to decide upon the precision of their private signals, given the technical limitations
to resolve the uncertainty. Varying this technical limitation allows to generate different types of equilibria with the seller choosing to have the private signal either as precise or as imprecise as possible.

Two of the models discussed in this dissertation present different types of asymmetric information (uncertainty about types of players in chapter 2 and states of nature in chapter 4 respectively). Despite that both models can be regarded as signalling models (see Spence, 1973). In chapter 2 the seller can signal her type by choosing actions, which allow buyers to better differentiate her from the other type. In chapter 4 the supplier may choose to offer contracts, which credibly signal the private information she has about the state of demand to the retailer. However, the supplier may also decide to be completely uninformed, which leads her to offer pooling contracts.

Two models also represent different types of vertical structures (downstream oligopoly with Cournot competition in chapter 3 and a single supplier/retailer relation with perfect downstream competition in chapter 4 respectively). However, both models deal with supplier/intermediary relations and allow for conditional contracts. As already mentioned, the contracts may depend on the supplier’s private information in chapter 4, but in addition a menu of contracts for the different realizations of the retailer’s signal is offered. The contracts between up- and downstream firms in chapter 3 may depend on the profits made by the downstream firms in the Cournot competition. The models also differ in that in chapter 3 the downstream firms make take it or leave it offers to the suppliers, while in chapter 4, the supplier designs the menu of contracts.

1.3 Main results

In chapter 2 a model is discussed where several short-run players (buyers) decide sequentially whether or not to interact with a long-run player (seller), who can choose her effort in order to increase the probability to provide a satisfying product to the buyers. As mentioned, whether the product has been satisfying influences the long-run player’s reputation for being a "good" or a "bad" type. The reputation then determines whether future short-run players want to buy the product. A common feature of such games is, that if one of the short-run players decides not to interact, all remaining ones will do so as well, as they are in an informational equivalent situation concerning the seller’s reputation. In such a game the buyers, despite being myopic and therefore not able to coordinate, are able to excercise pressure on the seller to maintain a high effort, in
order to prevent her reputation from turning bad. It is indicated in an example that this does not necessarily hold, if the long-run player’s action set is sufficiently rich. Although information transmission on one side of the market stops, the refusal to interact by the short-run players may be informative for the long-run player. This can change the long-run player’s behavior on the equilibrium path in such a way that short-run players again consider it favorable to interact. Thus, the severe grim-trigger like punishment (see Friedman, 1971) by the short-run players due to the long-run player’s bad reputation does not necessarily occur.

In chapter 3, which is a joint work with Vanessa von Schlippenbach, we analyze a vertical structure, in which intermediaries have to acquire an input from an exogenously given number of suppliers. The intermediaries then choose the quantities of a homogenous product to be sold to consumers. We find that in case one of the intermediaries is organized as a cooperative the cost of the input increases, leading to inefficiently low supply to consumers. The downstream supply may be even lower than in case a regular monopoly is the market’s sole intermediary. For suppliers the average profit is of course higher, if one of the intermediary firms is a cooperative. Thus, we can confirm the competitive yardstick effect of cooperatives that they improve the transfers paid to suppliers (see Rogers and Petraglia, 1994 or Hanisch et al., 2013), but show that in our model it comes at the expense of consumer surplus. Furthermore, we can show that a cooperative (or an identically acting investor-owend firm) may be a monopoly in the downstream market, if the upstream production capacity is sufficiently small, whereas we get a Cournot duopoly with one firm maximizing its profit and one firm maximizing average profit, if upstream capacity is large. In any case supply to consumers is equal or smaller than if both intermediaries are regular profit maximizing firms. However, if upstream capacity is very small, it may be more efficient to have a cooperative downstream, as the resulting monopoly incurs less cost than the duopoly of two investor-owned firms, but produces the same output.

In chapter 4, which is a joint work with Pio Baake and Friederike Heiny, we analyze a simple supply chain with one supplier, one retailer and uncertainty about market demand. Focusing on the incentives of the supplier and the retailer to enhance their private information about the actual market conditions, we show that choices on information acquisition are strategic complements. While the retailer’s incentives are mainly driven by the information rent that she can earn, the supplier will choose to acquire information only, if the retailer is rather well informed, even though
the information is free of charge. The reason is that, if the supplier receives a signal of a high state of demand and wants to appropriate some of the resulting information rent, the contracts, which she offers to the retailer, need to credibly signal the private information. This credibility can be achieved by reducing the quantity offered to a retailer with a signal for low demand, which is rather unlikely, if the supplier herself has received a "high signal". Thus, if the technical limitation on the precision of the signals is such that being informed does not reduce enough uncertainty, the potential information rent is small and the supplier may choose to stay uninformed in the first place and avoid the credibility problem by offering pooling contracts. This means that regardless, whether the supplier received a high or a low signal, the supplied quantity as well as the corresponding transfer are the same. If the agents can choose to be fully informed, the supplier will do so and extract all rent from the retailer. However, there is still some distortion required on the quantity for a retailer with a low signal, as the retailer might not be fully informed as well and thus the credibility problem may still be binding. If the technical limitation is such that full information is not possible, we get either the described pooling equilibrium or an equilibrium, where both agents decide almost always to be informed as much as possible. The latter type of equilibrium exists, if the maximum signal precision is sufficiently high, which means that the information rent, which can be gained by choosing a high signal precision, is high as well.
2 Information Asymmetry and Reentry

2.1 Introduction

This chapter analyzes an example of a repeated game in which players decide upon participation in an experience goods market. It is shown that although participation stops, it may start again at a later point in time.

The model presented here involves a series of short-run players, who each decide once in an exogenously given sequence whether or not to interact with a single long-run player. There is uncertainty about the long-run player's type, linking the model to incomplete information models like discussed in Harsanyi (1967). The short-run players might be able to acquire some information about the long-run player by observing signals, which are due to past interactions. This puts the model at hand in line with reputation models like the ones of Kreps and Wilson (1982) and Milgrom and Roberts (1982). Although their models include different stage games in fashion of the chain-store game in Selten (1978), the way in which reputation is formed is similar in the present model.

A feature of equilibrium behavior in similar models is that once a short-run player refuses to participate, all subsequent short-run players will do so as well. This happens as the refute to interact prevents the accumulation of additional information about the long-run player. In turn, this leads to grim-trigger like behavior, although short-run players are myopic and therefore would not be able to coordinate on such a strategy as a strategic punishment. Moreover, it gives the long-run player an incentive to invest highly in achieving signals, which induce short-run players to participate, as punishment can be pretty severe.

Similar to Ely and Välimäki (2003) the information, which is relevant to short-run players in order to decide upon participation, is the type of long-run player they are facing. In contrast to their paper our model does not allow the preferable type to identify as such as we assume imperfectly observed actions. Therefore, our model is close to the one in Fudenberg and Levine (1992), which also uses a similar stage game. But while their paper focuses on the payoff bounds for the long-run player, the model at hand rather stresses on the mechanic of information transmission and its implication for equilibrium behavior. 

1See for instance Ely and Välimäki (2003) or the examples in Fudenberg and Levine (1989).

2The main difference in modeling assumptions is that we assume the long-run player to infer the signals, which short-run players received in the past, from the short-run players' behavior, while Fudenberg and Levine (1992) assumes signals to be public information.
The main argument illustrated in this chapter is that even though the information transmission between short-run players may stop, non-participation might convey information for the long-run player. This may alter the long-run player’s behavior and in turn lead short-run players to interact again, although they have the same belief about the long-run player’s type as the short-run player, who chose not to participate.

The next section introduces a model for a game, in which reentry can happen in equilibrium. In section 2.3 it is illustrated that if the number of feasible actions for the preferable type of long-run player is sufficiently large, reentry can be part of an equilibrium. The last section of this chapter wraps up the discussion and introduces possible extensions.

2.2 Model

Let us consider a 4 period model with a long-run player \( L \) and a short-run player \( s_i \) for each period, with \( i \in \{1, \ldots, 4\} \). The long-run player is one of two types \( \Omega = \{\omega, \pi\} \).

Starting with \( s_1 \) each short-run player and the long-run player play the following stage game:

**Actions:** First \( s_i \) chooses whether to participate or not, which is denoted by \( a_i \in A = \{\text{in}, \text{out}\} \).

If \( a_i = \text{in} \), \( L \) chooses an unobservable effort level \( e_i \in E = \{e, \pi\} \) and therefore incurs a cost \( C(e) = 0, C(\pi) = c, c > 0 \). Then, each short-run player receives a benefit \( q_i \in Q = \{q, \overline{q}\} \) with \( \overline{q} = 1 \) and \( q = -1 \). The probabilities for realizing \( q \) are

\[
\begin{align*}
\Pr(\overline{q} | e, \omega) &= \Pr(\overline{q} | e, \pi) = \phi \\
\Pr(\overline{q} | e, \pi) &= \theta \\
\Pr(q | e, \omega) &= \bar{\theta}
\end{align*}
\]

Furthermore, \( 1 > \bar{\theta} \geq \theta > \frac{1}{2} > \phi > 0 \). The assumption \( \bar{\theta} \geq \theta > \frac{1}{2} \) makes it desirable to choose \( \text{in} \), if \( L \) is \( \pi \) regardless of the chosen effort. On the other hand, \( \frac{1}{2} > \phi \) makes it undesirable to interact with a long-run player of type \( \omega \). If \( a_i = \text{out} \), \( q_i = 0 \) with certainty and the period ends. Figure 2.1 shows the game tree for the stage game for a given type of long-run player.

**Beliefs:** Short-run players do not know, which type of long-run player they are actually facing.

---

3While the intuition behind the mechanic is similar for \( i \in \{1, \ldots, \infty\} \), the formal arguments are much easier to illustrate in the finite case with 4 being the minimum number of periods for the mechanic to work.

4Assuming all probabilities to be strictly less than 1 and greater than 0, does not allow any sequence of signal realizations to fully reveal the long-run player’s type.
There exists a commonly known prior \( \mu_0 \in (0, 1) \) that the long-run player is the \( \omega \) type. Probability \( \mu_i \) denotes player \( s_i \)'s belief and \( \forall i > 0, \mu_i \) is a Bayesian' update of \( \mu_0 \), given the observed sequence \( (q_1, \ldots, q_{i-1}) \). This signals are not observable for \( L \). Therefore, the long-run player only has an expectation about \( \mu_i \) which is denoted by \( q_i = \mathbf{E}(\mu_i | q_{i-1}, a_i, e_{i-1}) \). The long-run player needs this expectation in order to assess the expected value of choosing either \( e \) or \( \bar{e} \) at each decision node. Given \( q_{i-1} \), \( e_{i-1} \) leads to a distribution over \( q_i \), which in turn can be used to derive an expected value for \( q_i \). Short-run player \( i \)'s actual decision \( a_i \) allows for an update of this expectation.

**Payoffs:** The long-run player has a discount factor of \( \delta \). Stage game payoffs are assumed as follows:

For player \( s_i \) : 
\[
 u_i = \begin{cases} 
 q_i, & \text{if } a_i = \text{in} \\
 0, & \text{otherwise}
\end{cases}
\]

For player \( L \) : 
\[
 \pi_i = \begin{cases} 
 1, & \text{if } a_i = \text{in} \text{ and } e_i = e \\
 1 - e, & \text{if } a_i = \text{in} \text{ and } e_i = \bar{e} \\
 0, & \text{otherwise}
\end{cases}
\]

At each of the long-run player's decision nodes a game has a continuation value \( V_i(q_i, e_i, \omega) \), which is the sum of discounted expected payoffs over all future periods, given the long-run player's type,

---

5While the effect is easier to show under this strong assumption, it is sufficient to assume that \( L \) cannot perfectly observe the signals.

6Note that as the short-run players' prior \( \mu_0 \) is common knowledge, it holds that \( q_0 = \mu_0 \).
the respective effort and expectation about the short-run players’ beliefs.

With the assumption that all players maximize their expected utility at each decision node, we turn to the equilibrium behavior.

### 2.3 Equilibrium and reentry

In equilibrium short-run players and both types of the long-run player optimize:

\[
a_i^* = \begin{cases} 
in, & \text{if } \psi_i \geq 0 \\
out, & \text{otherwise}
\end{cases}
\]  

\[
e_i^* = \arg \max V_i(g_i, e_i, \omega)
\]

$\psi_i$ denotes the expected payoff for short-run player $i$, when choosing $a_i = in$. Thus, simplifying (2.1) leads to the following conditions for all $L$ types and all $s_i$:

\[
\forall i : \arg \max V_i(g_i, e_i, \omega) = e
\]

Furthermore, $\overline{\omega}$ chooses $\overline{\omega}$ if:

\[
V_{i+1}(g_{i+1}(g_i, e, \omega), e_{i+1}^*(g_i, e, \omega), \omega) \\
\geq V_{i+1}(g_{i+1}(g_i, e, \omega), e_{i+1}^*(g_i, e, \omega), \omega)
\]

and $s_i$ chooses $a_i = in$, if:

\[
\mu_i \phi + (1 - \mu_i) \Pr(\overline{\omega} | e_i, \omega) > \frac{1}{2}
\]

**Lemma 2.1** *In any equilibrium agents behave according to equations (2.2), (2.3) and (2.4).*

It is straightforward to see that the strategies depicted in (2.2), (2.3) and (2.4) solve the respective agent’s maximization problem for given beliefs. Further, as $\mu_i$ is a Bayesian update of $\mu_0$, beliefs are consistent with equilibrium behavior. No off-equilibrium beliefs can be specified as off-equilibrium behavior cannot be detected.

We derive equilibrium conditions the following way.

2.2: As choosing $\overline{\omega}$ is costly, but the $\overline{\omega}$ type does not benefit from it, as it leads to the very same
distribution over outcomes as $e$, it is obvious that in equilibrium $e$ will play $e$ in all periods.

2.3. The $\pi$ type of long-run player faces a dynamic programming problem with the state variable $e_{t+1}$ and control variable $e_i$ and chooses $e_i$, when the following condition is met:

$$V_{i+1}(e_{i+1}(q_{i+1}, a_{i+1}^*, e_i) \cdot) + (1 - \theta)V_{i+1}(e_{i+1}(q_{i+1}, a_{i+1}^*, e_i) \cdot, \cdot) - \frac{c_i}{\delta}$$

$$\geq \theta V_{i+1}(e_{i+1}(q_{i+1}, a_{i+1}^*, e_i) \cdot) + (1 - \theta)V_{i+1}(e_{i+1}(q_{i+1}, a_{i+1}^*, e_i) \cdot, \cdot))$$

with (2.3) being a shorter notation for the same condition.

2.4. Now, the expected benefit for $s_i$ can be written as

$$\psi_i = \mu_i \sum_q \Pr(q_i | e_i, \omega) q_i + (1 - \mu_i) \sum_q \Pr(q_i | e_i^*, \omega) q_i$$

Combining this with (2.1) leads $s_i$ to choose $a_i = in$, if (2.4) holds.

Hence, we get the condition on $\mu_i$ for participation:

$$a_i^* = in | e_i^* = e, \text{ if } \mu_i < \mu = \frac{2\theta - 1}{2(\theta - \phi)}$$

$$a_i^* = in | e_i^* = \pi, \text{ if } \mu_i < \mu = \frac{2\phi - 1}{2(\phi - \phi)}$$

(2.5)

This means that if $\mu_i^* < \mu (\mu_i^* > \pi)$, $s_i$ will choose to participate (not to participate), regardless of $e_i^*$.

First, we indicate that reentry cannot happen, if $\pi$ has only one action available or, which is equivalent in our model, if $\pi = \theta$, as $q_i$ is then independent of $e_i$.

Corollary 2.1 Consider the game described above. Then, if $\bar{\theta} = \theta$ and for any $i$: $a_i^* = out$, $a_j^* = out, \forall j > i$.

As the long-run player cannot improve the probability of producing $\bar{q}$, it cannot be the case that once $\psi_i < 0, \psi_j \geq 0$ for any $j > i$ as $\psi_i = \psi_j$ for any $e_i^*$ and $e_j^*$.

Now, if $\pi$’s feasible actions lead to different, but strictly positive expected utilities for the short-run players, reentry can be part of an equilibrium. Therefore, we now assume that $\bar{\theta} > \theta$.

---

1. Thus, for the remainder of this section, payoffs, actions and strategies will refer to the $\pi$ type of long-run player, unless noted otherwise.
Proposition 2.1 Consider the game described above. Then, there exist technology parameters $\phi$, $\theta$ and $\overline{\theta}$ with $\overline{\theta} > \theta$ and a cost parameter $c$ such that $a^*_2 = \text{out}$ and $a^*_3 = \text{in}$ for some history of the game.

Proof. See Appendix. ■

For the intuition behind the possibility of reentry let us assume the following history:

\begin{align*}
a^*_1 &= \text{in, } e^*_1 = \varepsilon, \mu^*_1 = \mu_0 < \mu \\
a^*_2 &= \text{out, } \mu^*_2 \in [\mu, \overline{\mu}] \\
a^*_3 &= \text{in, } e^*_3 = \overline{\varepsilon}, \mu^*_3 = \mu^*_2 \\
a^*_4 &\in \{\text{in, out}\}, \ e^*_4 = \varepsilon, \mu^*_4 \in (0, 1)
\end{align*}

To get the game started, we assume $\mu_0 < \mu$. Let us further assume that $q_1 = \overline{q}$. This would lead $s_2$ to choose $a^*_2 = \text{out}$, if $e^*_2$ conditional on $a^*_2 = \text{in}$ would be $\varepsilon$. If $a^*_2 = \text{out}$, the long-run player learns that $q_1 = \overline{q}$, knowing she has to achieve $\overline{q}$ now, in order to make any profits in future. This disciplines $L$ to choose the high effort level in period 3 and makes this choice credible for short-run players. Therefore $s_3$ chooses to participate, despite sharing $s_2$’s belief about the long-run player. Now, if the second short-run player did participate, the long-run player might have reason to think that $q_1 = q$. This in turn makes it unfavorable for $s_2$ to participate as $\mu^*_2 \in [\mu, \overline{\mu}]$ due to $q_1 = \overline{q}$.

This example shows that it only needs little differences in the model assumptions in order to make reentry possible on the equilibrium path. This has a couple of implications for optimal effort investment, which are at the moment left for future research as it would need a different model in order to derive a benchmark scenario. However, it can be expected that it will lead to a lower average effort than in case reentry is not possible.

2.4 Conclusion

In this chapter, the aspect of the model, which is driving the mechanic, is the information structure, which allows both sides of the market to learn about each other along the equilibrium path. The short-run players are able to learn about the long-run player’s type and the long-run player is able

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8Assume for instance that $\mu_i > \overline{\mu}$ as soon as $\overline{q}$ has been realized twice.

9Remember that conditional on $a^*_2 = \text{in}$, $e^*_2 = \varepsilon$ which makes it optimal to choose $a^*_2 = \text{in}$ only for a short-run player, who received signal $q_1 = \overline{q}$.
to learn about short-run players’ beliefs about the long-run player’s type. Crucial for reentry, at least in the present model, is that short-run players’ decisions are informative about past signal realizations. A condition used above is that the long-run player needs to be able to increase the effort, once a short-run player refused to participate. Furthermore, this increase in effort has to be optimal along the equilibrium path, in order to be credible to short-run players and thereby induce reentry.

Possible applications include not only settings like the experience goods market in the example, but any kind of principal-agent model, satisfying a minimum degree of asymmetric information, which is yet to be specified.

\[\text{\footnotesize \cite{Mailath2007, Celentani1996}}\]

\[\text{\footnotesize For an assessment of difficulties in reputation games dealing with the classical principal-agent structure of two long-run players interacting repeatedly, see Mailath \cite{Mailath2007}. Celentani et al. \cite{Celentani1996} introduces a model dealing with such a situation.}\]
2.5 Appendix

Proof of Proposition 2.1. As noted in section 2.3 histories consistent with the proposition are as follows:

\[ a_1^* = \text{in, } e_1^* \in \{e, \overline{e}\}, \mu_1^* = \mu_0 < \mu \]
\[ a_2^* = \text{out, } \mu_2^* \in [\mu, \overline{\mu}] \]
\[ a_3^* = \text{in, } e_3^* = \overline{e}, \mu_3^* = \mu_2^* \]
\[ a_4^* \in \{\text{in, out}\}, e_4^* = e, \mu_4^* \in (0, 1) \]

Furthermore, assume the following beliefs:

\[ \mu_0 \leq \mu \]
\[ \mu_2(\mu_0, (q, \overline{e})) > \mu \]
\[ \mu_2(\mu_0, (q, e)) \leq \overline{\mu} \]
\[ \mu_3(\mu_0, (\overline{q}, e), (q, \overline{e})) \leq \mu \]
\[ \mu_4(\mu_0, (q, \overline{e}), (\overline{q}, \overline{e})) \leq \mu \]
\[ \mu_4(\mu_0, (\overline{q}, e), (q, e), (q, \overline{e})) > \mu \]

Let us start in \( i = 4 \) with backward induction.

In period 4 \( \overline{q} \) will choose \( e_4^* = q \). Thus \( s_4 \) will only choose \( a_4^* = \text{in} \), if \( \mu_4^* \leq \mu \). As a working hypothesis let us assume that the long-run player is able to infer \( q_1 \) from \( s_2 \)'s behavior in the following way:

\[ q_1 = \overline{q}, \text{ if } a_2 = \text{in} \]
\[ q_1 = q, \text{ if } a_2 = \text{out} \]

(2.7)

Regarding period 3, if \( q_1 = \overline{q} \) the long-run player knows that \( s_3 \)'s behavior is uninformative about \( q_2 \), as \( \mu_3(\mu_0, (\overline{q}, e_1), (q_2, e_2)) \leq \mu, \forall q_2 \in \{q, \overline{q}\} \) and \( e_1, e_2 \in \{e, \overline{e}\} \). This leads to the following
comparison for the expected realization of $q_2$:

- if $e_3 = \zeta : V_3 = \theta_2 \delta + (1 - \theta_2)\theta \delta$
- if $e_3 = \bar{\zeta} : V_3 = \theta_2 \delta + (1 - \theta_2)\theta \delta - c$

Note that $\theta_2 = \Pr(\bar{\theta} | e_2^*, \bar{\omega})$. This leads to $e_3^* = \bar{\zeta}$, if:

$$c \leq (1 - \theta_2)(\bar{\theta} - \bar{\theta})\delta$$  \hspace{1cm} (2.8)

Now in contrast, if $q_1 = \bar{q}$, $a_2 = out$ and hence, $q_2 = 0$. Therefore, $L$ knows that the payoff from period 4 will only be positive, if $q_3 = \bar{q}$. Hence, the comparison in period 3, if $q_1 = \bar{q}$, is $\bar{\theta} \delta$ versus $\bar{\theta} \delta - c$. This leads to $e_3^* = \bar{\zeta}$ if:

$$c \leq (\bar{\theta} - \bar{\theta})\delta$$  \hspace{1cm} (2.9)

Let us turn to $i = 2$. According to (2.7), $L$ assumes that if $a_2 = in$, $q_1 = \bar{q}$. Hence, conditional on $a_2 = in$, $\bar{\omega}$ compares:

- if $e_2 = \zeta : V_2 = \bar{\theta}(\delta + \delta^2) + (1 - \bar{\theta})(\delta + \theta_3 \delta^2)$
- if $e_2 = \bar{\zeta} : V_2 = \bar{\theta}(\delta + \delta^2) + (1 - \bar{\theta})(\delta + \theta_3 \delta^2) - c$

Note again $\theta_3 = \Pr(\bar{\theta} | e_3^*, \bar{\omega})$. This leads to choosing $e_3^* = \bar{\zeta}$, if:

$$c \leq (1 - \theta_3)(\bar{\theta} - \bar{\theta})\delta^2$$  \hspace{1cm} (2.10)

If (2.10) does not hold, $e_3^* = \zeta$. Then (2.7) is confirmed as $\mu_2(\mu_0, q_1 = \bar{q}) \in (\mu, \bar{\mu}), \forall e_1$. Moreover, one can see from (2.9) and (2.10) that it is possible to find a $c$, which leads (2.9) and $\neg$ (2.10) to hold. Thereby, as (2.9) leads to $a_3^* = in$ and $\neg$(2.10) leads to $a_3^* = out$, if $q_1 = \bar{q}$, reentry can happen in equilibrium.

What is left to be proved, is that technology parameters $\phi$, $\bar{\theta}$ and $\bar{\theta}$ can be found such that the sequence of Bayesian updated beliefs in (2.6) exists. Updates are derived for example as

$$\mu_2(\mu_0, (q_1, \bar{\theta})) = \frac{\mu_0 \phi}{\mu_0 \phi + (1 - \mu_0)\bar{\theta}}$$
If the system of Bayesian updates in (2.6) is computed for instance with the parametrization $\phi = 0.3, \theta = 0.6, \theta = 0.7$, the system is satisfied for $\mu_0 \in (0.246, 0.3]$. That equilibrium beliefs are consistent with short-run players' equilibrium behavior can be easily seen from (2.1).
3 Mixed Oligopoly in Vertical Relations: Cooperatives vs. Investor-Owned Firms

3.1 Introduction

In recent years there has been increasing awareness of the problems caused by buyer power in agricultural markets. As a remedy to the issues brought up by market dominating intermediaries, the organization of primary producers in cooperatives has been advocated by political institutions (see the EU milk package in Regulation (EU) No 1308/2013, as well as Regulation (EU) No 261/2012). In addition, there are exemptions from competition law to cooperatives in order to promote their formation and allow primary producers to improve their bargaining position vis-à-vis large intermediaries.

The following analysis wants to add to the discussion about cooperatives by not only looking at the relationship of intermediaries and suppliers, but by including the effects the presence of a supplier-cooperative in a certain market may have on consumers.

Cooperatives, while often not market dominating corporate organizations, are present in a wide range of different types of industries. Farmer-owned cooperatives play an important role in agriculture around the world. But also in other sectors firms have created cooperatives like for instance the Visa credit card network, which is collectively owned by its franchisee banks. Likewise, collective ownership of a franchisor by its franchisees such as for hardware and trucking firms (see Hansmann, 1996) is quite common. One of the main arguments for the advocacy of cooperatives is the competitive yardstick effect, which holds that the presence of cooperatives in the market forces investor-owned firms to offer higher procurement prices for suppliers’ products. Empirical support for the competitive yardstick effect has been observed in the food manufacturing industry in the US (see Rogers and Petraglia, 1994), the wheat market in Canada (see Zhang et al., 2007), the coffee market in Chiapas, Mexico (see Milford, 2012), and the European dairy industry (see Hanisch et al., 2013).

The model in this chapter reproduces the competitive yardstick effect of cooperatives to improve the procurement prices received by suppliers. However, it also shows that this improvement for

\[^{11}\text{For Germany see for instance §28 of the GWB (Gesetz gegen Wettbewerbsbeschränkungen: Act Against Restraints of Competition).}\]
suppliers may come at a cost for consumers. In our model higher procurement prices lead to a
decrease in the quantities supplied to consumers.

We analyze a situation, in which suppliers of primary products can either join a cooperative or
decide to offer their input production capacity to a regular profit maximizing, i.e., investor-owned
firm (IOF). Therefore, the cooperative competes with the profit maximizing firm not only in the
final goods market like the one for dairy products, but they also receive their input goods, e.g.,
raw milk, from the same pool of suppliers. Thus, regarded as a vertical relation with suppliers
on top, final goods producers in the middle and demand at the bottom, we can model the middle
or intermediary section of these markets as a mixed oligopoly with Bertrand competition towards
the top and Cournot competition towards the bottom. For comparison a scenario is analyzed, in
which two IOFs compete for the supply of the primary good and the demand in the final goods
market.

Furthermore, we discuss how the scarcity of production capacity determines its distribution
among intermediaries and the payments received by suppliers. As mentioned above, we also look
at the effect of a cooperative on the provision of the final good. We can show that the possibility to
join a cooperative—disregarding whether the cooperative is active or not—acts as a rent shifting
mechanism from the profit maximizing firms and consumers to the primary producers. Actually,
provision of the final good may be higher, if there is only a monopoly IOF without the threat of
suppliers joining a cooperative, than in case both types of firms may be active.

In our model, a cooperative is assumed to share its profits equally among its members, i.e.,
the suppliers, while the IOF can choose to pay a transfer to its suppliers, which maximizes its
profit. However, the IOF can not always benefit from the possibility to choose its transfer. As
the cooperative is assumed to maximize the suppliers’ profit and pays all its profit to the suppliers,
the IOF can only compete for capacity, if it does so as well. Thus, whenever capacity is so scarce
that one of the intermediaries would want to acquire all available capacity, either intermediary
may become a monopoly by shifting the downstream monopoly profit to the suppliers. The IOF
is only able to acquire production capacity, either if—in case capacity is scarce—it mimics the

---

12 Whether the suppliers’ good is an input good, which is processed by another firm, or already a final good, which
is only distributed by an intermediary, does not make a difference in our model. Hence, we use the terms input and
capacity interchangeably.

13 The respective sector inquiry of the German Bundeskartellamt (Bundeskartellamt 2012) indicates that these
different payment models between intermediaries and suppliers are present for instance in the German milk and
dairy sector.
cooperative, or if—in case capacity is abundant—it takes, whatever the cooperative does not want to acquire.

Although a frequent recommendation to suppliers to improve their bargaining position against large intermediaries, cooperatives have a rather mixed standing in the economics literature. One of the main arguments holds that cooperatives often have problems to act efficiently (see the literature following Porter and Scully, 1987 or Hansmann, 1988 and 1996). These inefficiencies are often due to members of a cooperative having lower incentives to reduce their individual output, when facing Cournot competition. The members do not internalize the cost imposed on other members, when total output is increased. A cooperative may still be able to benefit its members compared to supplying an IOF, as the cooperative’s members’ overproduction serves as a commitment device to supply a high quantity in Cournot competition (see Albaek and Schultz, 1998). In markets with uncertainty about the quality of the primary good, a cooperative’s revenue pooling may also insure members against risks of quality realization (see Saitone and Sexton, 2009), which is a major concern in agricultural production especially among small scale farmers. However, Pennerstorfer and Weiss (2012) show that in terms of quality cooperatives may also fare badly compared to an IOF as cooperative members tend to freeride on other members’ investments in a high quality product. It has also been pointed out that cooperative members—as small agricultural producers in general—have little interest in long-run investments, but rather focus on immediate payments (see Iliopoulus and Cook, 1999). When intermediaries are competing for input goods, this may put pressure on the transfers paid to primary producers, but at the same time reduce incentives to invest in long-run efficiency.

Karantininis and Zago (2001) present a model, which is close to ours, as producers of a raw commodity can decide to deliver to a cooperative or an IOF. While they assume Cournot competition for the raw commodity and perfect competition in the final goods market, our intermediaries post a price to be paid per unit for the input and compete à la Cournot in the downstream market. This means that as competition for the input is of a Bertrand type, in some scenarios rents are shifted from the intermediary to the suppliers. Furthermore, intermediaries in our model have an incentive to prevent their rival from acquiring any input, in order to improve their own position in the Cournot competition.

Concerning our benchmark scenario, the paper of Esö et al. (2010) is similar in that firms
compete for a scarce input and have to pay a per unit price. They show that the endogenously
determined market structure allows for asymmetry in the sense that there is one large capacity-
hoarding firm. However, while the transfers paid are uniform take-it-or-leave-it type of offers by
the IOFs in our model, capacity is auctioned at its marginal valuation in their model.

The remainder of this chapter is structured as follows: section 3.2 discusses the model as-
sumptions. Section 3.3 develops the benchmark scenario of two competing IOFs, while section 3.4
follows with the IOF/Coop game. Section 3.5 discusses some of our assumptions, while the final
section concludes.

3.2 Model

Consider a mass of \( N \) upstream firms (e.g., farmers or suppliers). Each supplier, indexed \( k \),
produces one unit of a homogenous input at marginal cost normalized to 0. Further, there are
two downstream firms \( i \in I = \{A, B\} \) (e.g., processing firms or intermediaries). Each upstream
firm exclusively supplies one of the downstream firms, which transform the input one-to-one into
a homogeneous final good for distribution to consumers.\(^\text{14}\) The downstream firm \( A \) is an investor-
owned firm (IOF), while the downstream firm \( B \) is either an investor-owned firm or a supplier-owned
cooperative. In contrast to the investor-owned firm, the cooperative intends to maximize the profit
of its members, i.e., its profit divided by its number of suppliers.

The downstream firms face an inverse demand function \( P(X) \) with \( P'(X) < 0 \), where \( X \) denotes
the total quantity supplied in the downstream market and \( x_i \) the quantity supplied by firm \( i \): \( X = \sum_{i \in I} x_i \). The downstream firms each incur processing cost \( C(x_i) \) with \( C'(x_i) > 0 \) and \( C''(x_i) > 0 \)
as well as a fixed cost \( f > 0 \). To avoid the endogenous foundation of various small cooperatives
and guarantee a pure mixed duopoly, it has to hold that \( f > f = C(x^M) - 2C(x^M/2) \), with \( x^M \)
being the quantity produced by a regular monopoly, which faces demand \( P(x_i) \) and production
cost \( C(x_i) \).

The downstream firms compete for the supply by the upstream firms. That is, they offer
a non-discriminatory transfer \( t_i \) for each unit of input and announce the maximum number of

\(^{14}\)The described industry structure corresponds, for example, to the structure of the dairy and meat-packing
industry in Europe and North America. In Germany for instance, both investor-owned and cooperative dairy firms
oblige their suppliers to sell their entire capacity to them (see Bundeskartellamt, 2009, p. 74). In other countries,
this holds at least for cooperatives (Regulation (EU) No. 261/2012, para. 7).
suppliers $\pi_i \leq N$ they are willing to accept for delivery. Transfer $t_i$ may be conditional on firm $i$’s profit in the downstream market. Given the contract offers, the upstream firms decide, which downstream firm they prefer to supply. Each downstream firm then obtains $n_i \leq \pi_i$ units of input with $n_i \in [0, N]$ and $\sum_{i \in I} n_i \leq N$, from a market maker, which distributes suppliers according to their preferences.

We model the interactions between the players as a three-stage game, where all information is public and all assumptions are common knowledge:

- First, the downstream firms simultaneously offer a correspondence $t_i$ for each unit of input delivered by the upstream firms and announce the maximum number of suppliers $\pi_i \leq N$ they will accept for delivery.

- Second, upstream firms decide, which downstream firm they prefer to supply. Each downstream firm $i$ acquires capacity $n_i \leq \pi_i$ from the market maker.

- In the last stage of the game, the downstream firms compete in quantities, given their capacity constraints, i.e., choosing a quantity $x_i \leq n_i$.

Following our assumptions, the profit of each downstream firm is given by

$$\Pi_i(x_i, x_j, \cdot) = P(X)x_i - C(x_i) - n_i t_i - f$$  \hspace{1cm} (3.1)

For later reference, we denote the profits net of fixed cost and cost of capacity as follows:

$$\tilde{\Pi}_i(x_i, x_j) = P(X)x_i - C(x_i)$$  \hspace{1cm} (3.2)

Due to $f > 0$, we get a minimum capacity, which is required for a firm to make zero profits. In order to denote this capacity for firm $j \neq i$, given firm $i$’s supply choice, we write

$$n_j \mid x_i := \min \left\{ n_j \mid \tilde{\Pi}_j(x_i, x_j = n_j) = f \right\}$$  \hspace{1cm} (3.3)

15 This assumption reflects the usual trading conditions in agricultural markets, where the processors and the suppliers agree on a basic input price per unit, which can be adjusted by quality parameters and other factors like transportation costs.

16 Note that we omit arguments, where it does not lead to any confusion.

17 The minimum capacity is the same for a cooperative and for an IOF, despite different objective functions.
Finally, upstream firm $k$’s profit, when delivering to downstream firm $i$, is given by:

$$
\pi_{ki}(x_i, x_j, n_i) = t_i(x_i, x_j, n_i)
$$

(3.4)

Using subgame perfection as our solution concept, we solve the game by backward induction. We first analyze the benchmark case of two investor-owned firms competing with each other. We then turn to the case of a mixed duopoly, where a cooperative competes with an investor-owned firm. We use the comparison of these scenarios to determine the effects a cooperative has on (i) the supply in the downstream market and (ii) the profits of suppliers. We illustrate these effects in an example using a linear demand and quadratic cost functions.

### 3.3 Benchmark: IOF vs. IOF

**Downstream Competition.** In the last stage of the game, the intermediaries choose their quantities to be supplied to consumers. Each firm plays its best-response to the other firm’s quantity decision. Hence, we get the standard Cournot result whenever the firms are not capacity constrained. We denote these standard Cournot quantities by $x^C$ \(^{18}\). The intermediaries may also choose their output such that their respective capacity constraint is binding, while they decide to supply nothing, if their production capacity does not exceed $n_i$:

$$
X_i^*(n_i, x_j) = \begin{cases}
\min\{\tilde{X}_i, n_i\}, & \text{if } n_i \geq \frac{n_i}{x_j} \\
0, & \text{else}
\end{cases}
$$

(3.5)

with:

$$
\tilde{X}_i := \arg\max_{x_i} \tilde{\pi}_i(x_i, x_j)
$$

$\tilde{X}_i$ denotes firm $i$’s best response function, if it did not face a capacity constraint. The quantity of firm $i$, if it were to act as a monopoly, can be written as:

$$
X^M_i(n_i) := X^*_i(n_i, 0)
$$

(3.6)

\(^{18}\)Thus, $x^C$ is each intermediary’s equilibrium quantity, if both play their Cournot best-responses, given demand $P(X)$ and cost function $C(x_i)$. 

24
As mentioned, we use $x^M$ to denote the standard monopoly quantity:

$$x^M := \arg \max_{x_i} \tilde{\Pi}_i(x_i, 0)$$

**Delivery Decisions.** When deciding upon which firm to contract with, the suppliers simply maximize their profit. Given $\forall i : t_i \geq 0$, they file a preference for one of the intermediaries, while willing to supply the other intermediary, in case the preferred one has reached its desired capacity:

$$\begin{aligned}
\text{deliver to} & \quad \begin{cases} 
i & \text{if } t_i > t_j \text{ and } n_i < \pi_i \\
j & \text{if } t_i > t_j \text{ and } n_i = \pi_i \\
 & \text{to either firm with } \Pr = \frac{1}{2} \text{ if } t_A = t_B \text{ and } \exists i : n_i < \pi_i
\end{cases} \\
\end{aligned}$$

(3.7)

A market maker then distributes the suppliers to firms $i$ and $j$ according to the suppliers’ preferences and the downstream firms’ offers from the previous stage, anticipating the equilibrium strategies in downstream competition. The decision described in (3.7) simply states a dominant strategy, which imposes that suppliers prefer to contract with the firm, which pays the higher transfer. If both firms’ transfers are the same, i.e., suppliers being indifferent, the market maker chooses either firm with probability $1/2$.

**Contract Offers.** In the first stage of the game both downstream firms make their contract offers $(t_i, \pi_i)$. We impose the following strategy to be part of an equilibrium:

$$\begin{aligned}
t_i^* &= \begin{cases} 
\frac{\tilde{\Pi}_i(X_i^*, x_j)}{n_i}, & \text{if } \tilde{\Pi}_i(X_i^*, x_j) < \Pi_i^C|N \\
0, & \text{otherwise}
\end{cases} \\
\pi_i^* &= N
\end{aligned}$$

(3.8)

with $\Pi_i^C|N$ being the Cournot profit for a given market capacity $N$ defined as

$$\Pi_i^C|N := \begin{cases} 
\tilde{\Pi}_i \left( \frac{N}{2}, \frac{N}{2} \right), & \text{if } N < 2x^C \\
\tilde{\Pi}_i \left( x^C, x^C \right), & \text{otherwise}
\end{cases}$$

(3.9)

This transfer payment is conditioned on the firms’ profits in the downstream market. Whenever

---

\(^{19}\)Note that there exist multiple capacity combinations for an equilibrium. However, we are only interested in aggregate choices, as these determine the intermediaries’ capacities.
firm $i$ earns less than the Cournot profit for a given capacity $N$, all of its downstream profit is shared equally among firm $i$’s suppliers. If the downstream profit matches the Cournot profit, the suppliers get a transfer of 0, which is equal to their marginal cost.

This strategy is only feasible, if it holds that $N$ is sufficiently large such that $\Pi_i^C|_N = f$, which we denote by

$$N_C = \min \left\{ N \mid \Pi_i \left( \frac{N}{2}, \frac{N}{2} \right) = f \right\}$$

For smaller $N$, we get that firms compete for upstream capacity, transferring all downstream profits to the suppliers. Thus, if $N < N_C$, firms make a profit of 0 and suppliers get

$$t_i = \frac{\Pi_i (N, 0) - f}{N}$$

However, in such a case it still holds that $X = N$.

**Proposition 3.1** If $N \geq N_C$ and $f$ sufficiently small, the actions specified in (3.5), (3.7) and (3.8) constitute an equilibrium of the IOF/IOF game.

**Proof.** See Appendix. □

In this equilibrium downstream firms are able to use their contracts with the suppliers to prevent any profit from being shifted upstream. Hence, the contracts enforce a collusive agreement of the downstream firms vis-à-vis the suppliers. Deviation from the equilibrium would only be desirable for firm $j$, if it allowed to increase its capacity, leading to a decrease of firm $i$’s capacity. This induces a decrease in $i$’s profit such that, according to (3.8) firm $i$ would pay all of its remaining profit to its suppliers. In order to receive additional capacity firm $j$ has to at least match firm $i$’s transfer. Thus, it would become very expensive for the deviating firm, to acquire additional capacity. The most profitable way to deviate from this equilibrium is by trying to become the monopoly in the downstream market. However, if $f$ is sufficiently small, deviation from (3.8) is not profitable, as the required transfer would have to be paid to many suppliers, in order for firm $j$ to become a monopoly.

While (3.8) leads to zero profits for the suppliers, supply in the downstream market is at least as high, as in the corresponding Cournot game:

$$X = \min \{ N, 2x_C \}$$

(3.10)
According to the indifference condition specified in \((3.7)\), production capacity is split equally between the intermediaries, as is the final good’s output.

There exist multiple equilibria for the competition between two investor-owned firms. We focus on the equilibrium presented above as there exists no equilibrium, where the downstream firms earn higher profits for given \(N\). Note that intermediaries are not able to collude vis-à-vis the consumers. For instance an equilibrium with half of the "monopoly profit" \(\Pi_i\left(\frac{x^M}{2}, \frac{x^M}{2}\right)\) instead of \(\Pi^C\) as the benchmark for the conditional payment, is not an equilibrium. As soon as \(N > x^M\), in the last stage of the game, where the capacities are already set, each intermediary would be playing a best response to a competitor playing \(\frac{x^M}{2}\), such that \(\tilde{X}_i\left(\frac{x^M}{2}\right) > \frac{x^M}{2}\). For \(N \leq x^M\), intermediaries make half of this "monopoly profit" anyway. Thus, the best they can do is the Cournot profit specified in \((3.9)\) at 0 cost of capacity.

We take this payoff dominant symmetric equilibrium as our benchmark case for the comparison with the equilibrium outcome of the case where an IOF competes with a cooperative.

### 3.4 Mixed Oligopoly: IOF vs. Coop

As in the previous section, firm \(A\) is an IOF, maximizing its profit according to \((3.1)\). However, firm \(B\) is now assumed to be a supplier-owned cooperative, which distributes its profit net of transfers, i.e., \(P(X)x_B - C(x_B) - f\), equally among its suppliers. Accordingly, \(t_B\) corresponds to the individual supplier’s share of the cooperative’s profit:

\[
t_B(n_B, \cdot) = \frac{P(X)x_B - C(x_B) - f}{n_B} \tag{3.11}
\]

While its transfer is fixed, the cooperative may still decide on its maximum number of suppliers \(n_B\) in the first stage of the game. As before, we analyze the game by using backward induction, starting with the quantity decisions of the final stage.

**Downstream Competition.** The supply decisions are analogous to \((3.5)\). Again, intermediaries either supply all the capacity, which they have bought from the upstream firms or they hold excess capacity by selling a quantity \(\tilde{X}_i < n_i\), with \(\tilde{X}_i\) being the Cournot best response to the decision of the other intermediary.

**Delivery Decisions.** Based on the contract offers by the downstream firms, the upstream firms decide, which downstream firm they prefer to supply according to \((3.7)\). Again, this determines
the distribution of capacity to the downstream firms, i.e., the realizations of $n_A$ and $n_B$.

**Contract Offers.** In the first stage of the game, the downstream firms make their contract offers to the suppliers. The cooperative’s transfer scheme is exogenously given: all members of the cooperative get an equal share of the cooperative’s net profit, i.e., $t_B$ as noted in (3.11). The IOF’s offer, however, depends essentially on the availability of upstream capacity, $N$. If $N$ is sufficiently large, the IOF may choose to acquire the capacity, which is not demanded by the cooperative, at a price of 0. If instead capacity is rather scarce, the cooperative might want to acquire all available capacity, giving the IOF an incentive to compete for capacity. In this case the IOF needs to offer the suppliers at least the average profit of the cooperative. As the cooperative by definition shifts all profit from the downstream market to the suppliers, the IOF has to do so as well in order to be able to compete for capacity. However, using the following results, we can restrict the types of equilibria for this section.

**Lemma 3.1** There exists no equilibrium of the IOF/Coop game with $0 < X_i^* < n_i$, $\forall i \in I$.

**Proof.** See Appendix. ■

The intuition to this is as follows: Assume $X_i^* < n_i$. Then, an increase of $n_i$ has no impact on the firm’s own output choice since $X_i^* = \bar{X}_i < n_i$ is the best response to $X_j^*$. Additionally, by increasing its capacity, firm $i$ cannot affect the competitor’s quantity $X_j^*$, if the competitor is also holding excess capacity, i.e., $X_j^* < n_j$. At least for cooperative members, whose profit is determined by profit sharing, excess capacity is always costly. Therefore, the cooperative has no incentive to hold excess capacity, whenever the investor owned firm does.

**Lemma 3.2** There exists no equilibrium of the IOF/Coop game with $X_A^* = n_A > 0$ and $n_B \geq X_B^* > 0$.

**Proof.** See Appendix. ■

Assume that the cooperative chooses a best reply to the IOF’s quantity. If the cooperative then decides, how much capacity to leave to the IOF, it has to weight the reduction in revenue from allowing the IOF to supply a strictly positive quantity downstream and the reduction in cost from reducing its own output and capacity. As long as the demand function is sufficiently steep and not too concave, the reduction in revenue is larger than the cost benefit. Then, the cooperative would rather keep the IOF from entering the market by choosing $\pi_B \geq N - \overline{N}_A$. 28
Considering the best responses in (3.5), we are left with two types of equilibria:

i) \( 0 < X^*_A < n_A \) and \( 0 < X^*_B = n_B \)

or

ii) \( X^*_i = 0 \) and \( 0 < X^*_j \leq n_j \)

The first type of equilibrium is the Cournot-type equilibrium of the IOF/Coop game, while the second type is the monopoly equilibrium.

To discuss the different market structures for different capacities \( N \) and the corresponding contract offers, we start at \( N = \frac{n_j}{x_i = 0} \) and increase \( N \) until the Cournot-type equilibrium of the IOF/Coop game. Therefore, we define the following critical capacities:

In order to cover the fixed cost \( f \), any firm \( j \) needs to acquire a minimum capacity, given firm \( i \) chooses a quantity of 0. We denote this minimum capacity by \( N = \frac{n_j}{x_i = 0} \).

If the cooperative is a monopoly in the downstream market, the IOF supplies a quantity of 0. Then, the number of suppliers that maximizes the profit of the cooperative’s members is given by:

\[
X^*_B = \arg \max_{n_B} \frac{\tilde{H}(n_B, 0) - f}{n_B} \quad (3.12)
\]

We denote the first critical \( N \) by \( N^{c1} = X^*_B \).

It is straightforward that an unconstrained monopoly would choose a higher supply, as soon as \( N > X^*_B \), as due to the different objective functions it holds that:

\[
x^M > X^*_B \quad (3.13)
\]

We denote the second critical \( N \) by \( N^{c2} = x^M + \frac{n_j}{x_i = x^M} \). For the remainder of this section, we assume that \( f \) is sufficiently small, such that \( x^M > X^*_B + \frac{n_A}{x_B = X^*_B} \).

In a duopoly of the IOF/Coop game, we get that the cooperative chooses its optimal capacity,

\footnote{A discussion of this assumption is presented in part C of section 3.5}
given the IOF will play a Cournot best-reply to that choice:

$$\pi_B(\cdot) = \arg \max_{n_B} \frac{\tilde{\Pi}(n_B, \tilde{X}_A(n_B)) - f}{n_B}$$  \hspace{1cm} (3.14)$$

Such a Cournot duopoly is possible as soon as $N$ is sufficiently large, which requires $N \geq \pi_B + \tilde{X}_A(\pi_B)$. However, to be part of an equilibrium the average profit from maintaining the monopoly position must be smaller or equal than in the Cournot duopoly:

$$t_B(x^M, 0, N - \frac{\pi_B}{X_M} |_{x_B=x^M})_N \leq t_B \left( \pi_B, \tilde{X}_A(\pi_B), \pi_B \right)_N \hspace{1cm} (3.15)$$

As $t_B(x^M, 0, N - \frac{\pi_B}{X_M} |_{x_B=x^M})$ decreases, while $t_B \left( \pi_B, \tilde{X}_A(\pi_B), \pi_B \right)$ is constant in $N$, we can guarantee that there exists an $N$, such that these transfers are equal.

We denote the final critical $N$ by $N_{c3} = \max \left\{ N \middle| t_B(x^M, 0, N - \frac{\pi_B}{X_M} |_{x_B=x^M}) = t_B \left( \pi_B, \tilde{X}_A(\pi_B), \pi_B \right) \right\}$.

**Lemma 3.3** If $N \in \left[ N_{c1}, N_{c2} \right)$, the intermediaries compete for a monopoly position with total supply in the downstream market $X = N$.

The cooperative allows at least as many suppliers to be members as are necessary in order to maximize the average profit. Supply is equal to its capacity, i.e., $X^*_B = N < N_{c1}$, as

$$\frac{\partial t_B(N, 0, N)}{\partial N} \bigg|_{N < N_{c1}} > 0$$

Lemma 3.3 implies that the IOF has to offer a transfer to the suppliers, which equals at least the cooperative’s average profit. Otherwise, all suppliers will supply the cooperative. Hence, there exist two equilibria with either the cooperative or the IOF supplying $N_{c1}$.

**Lemma 3.4** If $N \in \left[ N_{c1}, N_{c2} \right]$, the intermediaries compete for a monopoly position with total supply in the downstream market $X = N - \tilde{n}$ with $\tilde{n} = \min \{ N - X^*_B, \pi_B \}$.

If $N$ increases to $N \in \left[ N_{c1}, X^*_B + \frac{\pi_B}{X_M} |_{x_B=x^*_B} \right)$, the cooperative can still prevent market entry of the IOF by acquiring its optimal quantity $X^*_B$. This is due to the residual capacity in the

---

As noted in the introduction, it is quite frequently stated in the literature that IOFs are more efficient than cooperatives. In our model, if the IOF is more efficient than the cooperative, there exists only one equilibrium with the IOF acquiring capacity from all $N$ suppliers at a price equal to the cooperative’s maximized average profit, i.e., $t_B(\pi_B)$. In this case, the IOF would still be able to make positive profits, despite matching the cooperative’s offer for the transfer.
market, i.e., \( N - X^M_B \), being smaller than the minimum capacity the IOF needs to cover its fixed cost. Again, the IOF can mimic the cooperative’s behavior, which implies that both firms compete for the monopoly position in the market.

For \( N \in \left[ X^M_B + n_A|_{x_B=X^M_B}, N^{c2} \right) \), the downstream firms also compete for a monopoly. The cooperative is willing to accept a quantity of \( N - n_A \) to prevent market entry by the IOF, although this exceeds the quantity, which maximizes the average profit, i.e., \( X^M_B \). Assume \( N = X^M_B + n_A|_{x_B=X^M_B} + \varepsilon \). If the cooperative would allow the IOF to enter the market, the IOF’s quantity would be \( x_A = n_A|_{x_B} + \varepsilon \). Due to the effect on the price the reduction of \( t_B \) would therefore be larger then in case \( X^M_B + \varepsilon \) suppliers were allowed to be part of the cooperative. As the average profit is decreasing in the number of suppliers, if \( n_B > X^M_B \), the cooperative does not accept more than \( N - n_A \). The offer by the IOF has to be at least as good as the cooperative’s in order to attract suppliers.

The following corollary to lemma [3.4] holds for any \( N \in (N^{c1}, N^{c2}) \).

**Corollary 3.1** If \( N \in (N^{c1}, N^{c2}) \), total output in the IOF/Coop game is lower than the quantity supplied by a regular monopoly.

**Proof.** See Appendix. 

The intuition is that as the regular monopoly has its maximum at \( x^M \), it would supply as much as possible until \( N = x^M \). The firms in the IOF/Coop game have to maximize the average profit and thus have their maximum at \( X^M_B < N \). Therefore, they want to acquire only as much capacity, as is needed, to keep the other firm out of the market, and thus \( \pi_i = N - \bar{n} \) with \( \bar{n} = \min\{N - X^M_B, n_j|_{x_i=x^M} \} > 0 \). Therefore, as \( X < N < X^M + n_j|_{x_i=x^M} \), total supply in the IOF/Coop game is lower than the supply of a regular monopoly, if \( N \in (N^{c1}, N^{c2}) \).

If \( N \) increases beyond \( N^{c2} \), there is more capacity available than any type of monopoly would want to acquire.

**Lemma 3.5** If \( N \in [N^{c2}, N^{c3}] \), the intermediaries compete for a monopoly position with total supply in the downstream market \( X = x^M \).

Lemma [3.5] represents another inefficiency compared to the IOF/IOF game in section [3.3] where \( X = N \) until \( N = 2x^c \). In the IOF/Coop game a monopoly can still hold its position by acquiring \( N - n_j|_{x_i=x^M} \) units of capacity. As before, \( n_j|_{x_i=x^M} \) are uncontracted, but in addition
\( N - \pi_j \big|_{x_i = x_M} - x^M \) units of capacity are acquired by the monopoly, but not used for production. If \( N \) increases, this implies that while the profit in the downstream market stays constant, it has to be distributed among a larger number of suppliers. Thus, the suppliers’ transfer decreases until it is beneficial for the suppliers to give up the monopoly position and allow competition in the downstream market.

**Lemma 3.6** If \( N \geq N^{\text{eq}} \), the intermediaries do not compete for the delivery by the suppliers. Total supply is constant at \( X = \pi_B + \bar{X}_A(\pi_B) \).

If \( N \) is sufficiently large, the cooperative acquires a number of suppliers that maximizes its members’ profits, given the IOF will play a best response to that in the Cournot game. In the final stage of the game, the cooperative will sell exactly what it purchases from the upstream suppliers, i.e., \( X_B^* = \pi_B \) as it holds that

\[
\frac{\partial t_B(X_B^*, X_A^*, n_B)}{\partial n_B} \big|_{X_B^* < n_B \leq x_M} < 0
\]

The number of suppliers that maximizes the cooperative’s average profit and, thus, the profit of the cooperative’s members is now given by (3.14). The IOF wants to acquire enough capacity in order to play its best-response to the capacity choice of the cooperative. Due to suppliers’ marginal cost being equal to 0, the IOF can acquire as much capacity as is left by the cooperative \((N - \pi_B)\) for a price of \( t_A = 0 \). Hence, the IOF’s capacity decision for large \( N \) is \( n_A = N \), with \( N - \pi_B \) suppliers providing their capacity to the IOF.

**Proposition 3.2** The actions specified in (3.5), (3.7) and lemma 3.3 to 3.6 constitute an equilibrium of the IOF/Coop game, with total supply being smaller or equal than in the IOF/IOF game.

To better illustrate this result, we apply an example with a linear inverse demand function \( P(X) = 1 - X \) and quadratic cost functions for the downstream firms \( C(x_i) = x_i^2 / 10 \). As mentioned, the suppliers’ marginal cost of production are normalized to zero. The fixed cost of downstream firms are assumed to be \( f = 0.02 > f \approx 0.01 \), which also satisfies that \( x^M > X_B^M + \pi_A \big|_{n_B = X_B^M} \).

Figure 3.1 shows total supply \( X \) as a function of available capacity \( N \) in both scenarios. The upper (red or dashed) function is the total supply in the benchmark case of two IOFs competing,
Figure 3.1: Total supply \( X(N) \) in the IOF/IOF (dashed) and the IOF/Coop (dotted) game.

while the lower (blue or dotted) function shows total supply in case one of the intermediaries is a cooperative.

\( N \in [N^1, N^c1] \): Both scenarios lead to the same total supply \( X = N \). Note that in this interval the cooperative is more efficient than two IOFs, as the monopoly only has to bear the fix cost once, while the duopoly structure of the IOF/IOF game has to incur these costs twice.

\( N \in [N^c1, N^c2] \): We still get \( X = N \) in the IOF/IOF game, while \( X = N - \bar{n} \) in case firm \( B \) is a cooperative. Thus, in this range some capacity remains unsupplied due to the cooperative maximizing its members’ profits and the IOF being at best able to mimic this behavior by the cooperative.

\( N \in [N^c2, N^c3] \): Supply in the IOF/Coop game is constant at \( X = x^M \), while in the IOF/IOF game \( X = N \) until intermediaries change to regular Cournot quantities with \( X = 2x^c \) at \( N < N^c3 \).

Thus, in the IOF/Coop game the active intermediary is longer able to maintain a monopoly position. As \( N - \bar{n} \mid_{x_i = x^M} > x^M \), some capacity is bought by the monopoly, but remains unused.

\( N \geq N^c3 \): Intermediaries in the IOF/Coop game play a duopoly solution with \( X = \pi_B^* + \tilde{X}_A(\pi_B^*) \), which is less than supply in the Cournot solution of the IOF/IOF game, where \( X = 2x^C \).

Thus, in our model consumers are better off in the scenario of two IOFs competing with each other as supply is less or equal, if one of the intermediaries is a cooperative.

Figure 3.2 shows the transfer received by suppliers. As in the IOF/IOF game transfers are
equal to 0 except for very small \( N \), upstream firms are of course better off, if one of the firms is a cooperative. Thus, we can confirm the cooperative to have a competitive yardstick effect for prices of primary products. However, the benefit of being a member of the cooperative depends on the number of suppliers \( N \).

\( N \in [N, N^{c_1}) \): The transfer paid to suppliers is the industry profit of a monopoly offering all available capacity, divided by the number of suppliers.

\( N \in [N^{c_1}, N^{c_2}) \): Up to \( N^{c_2} \) the individual transfer is reduced, as the additional contribution to the cooperatives profit diminishes, but it has to be shared equally.

\( N \in [N^{c_2}, N^{c_3}) \): As the regular monopoly quantity \( x^M \) is reached, the output quantity and the downstream profit stay constant, but more suppliers have to be contracted, in order for the active intermediary to remain the monopoly. Hence, each supplier’s share of the profit is further reduced.

\( N \geq N^{c_3} \): If the intermediaries compete à la Cournot, the cooperative distributes its profit among its suppliers, while the IOF pays a transfer of 0. As the Cournot quantities and corresponding production capacities are constant, the transfers are constant as well.

### 3.5 Discussion

Our results are based on two main assumptions concerning the structure of the model. We first discuss these assumptions and then turn our attention to the hitherto assumed restriction on the fixed cost parameter \( f \).

**A) Separated downstream markets:** We assume that both the input as well as the final products are homogenous. However, if we allow final products to be differentiated, the changes in the model’s dynamics are straight forward. The more the final goods’ markets are separated,
the less the intermediaries’ incentives to keep the rival out of the market, as firm i’s output has less impact on firm j’s demand. For illustrative purposes, consider the most extreme case of fully differentiated downstream markets. Then intermediaries will simply try to acquire enough capacity to supply their monopoly quantity.

In the IOF/IOF game the equilibrium proposed in proposition 3.1 still holds. The supply in each market is

\[ X = \min \left\{ \frac{N}{2}, x^M \right\} \]

which is of course less than the supply in a single and homogenous product market, but total supply of both IOFs is equal or larger than that.

However, in case one of the firms is a cooperative, the inefficiency depicted in corollary 3.1 vanishes, if markets are fully differentiated.\(^{22}\) If \( N \leq X_B^M \), intermediaries have to compete for capacity such that the whole industry profit will be transferred to the upstream firms. If \( N > X_B^M \), the IOF can either take the residual capacity \( n_A = N - X_B^M \) at cost of 0 or compete for additional capacity. The IOF would have to pay the maximized average profit to all suppliers to get additional capacity, and thereby at best make a profit of 0. Thus, any \( n_A > n_A \) at cost of 0 leaves the IOF better off and we get one market with \( X = \min \{ N, X_B^M \} \) and one with \( X = \min \{ N - X_B^*, x^M \} \). An inefficiency still created by the cooperative, compared to the IOF/IOF game, is that its market gets less supply than in case of a regular monopoly, as due to its profit sharing rule \( X_B^M < x^M \).

Furthermore, as only one of the markets gets any supply until \( N = X_B^M \), the loss of consumer surplus is larger than if the supply is split equally like in the IOF/IOF game.

**B) Discriminatory transfers to suppliers:** The second assumption, which is to be discussed, is that transfers, which are paid to suppliers, are non-discriminatory. This assumption makes it as expensive as possible for the IOF to become a monopoly, as the cooperative’s transfer would have to be paid to all contracted suppliers; even to those, who are not able to join the cooperative and have therefore opportunity cost of 0 to supply the IOF.

As we follow the literature in having the cooperative share all its profits equally among its members, we do not vary this assumption for the cooperative. If the IOF can make bilateral agreements with suppliers, some form of sequential procedure is required instead of the simultaneous offers in the model above. The way, in which the sequentiality is structured, may have an important impact

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\(^{22}\) For intermediate ranges of product differentiation this inefficiency result still holds, but to a lesser extent.
on market outcomes. An effect, which can be generally expected from relaxing this assumption, is
that the cost of capacity for the IOF decreases. This can lead our inefficiency result from corollary
3.1 to be weakened or even vanish. As—compared to non-discriminatory transfers—the total cost
of producing an additional unit (production plus capacity cost) decreases, the IOF might be able
to contract all suppliers in order to choose a higher output.
However, for large \( N \), we still get an inefficiency in the Cournot-type equilibrium due to the lower
supply by the cooperative compared to an IOF.

C) Increasing fixed cost \( f \): For the exposition in section 3.4 we have assumed \( f \) to be
sufficiently small to guarantee
\[
x^M > X^M_B + \frac{n_A|n_B=X^M_B}{2}.
\] (3.16)
To satisfy condition (3.16) with linear demand \( P(X) = a - bX \) and a quadratic cost function
\( C(x_i) = cx_i^2 \), we get that
\[
f \leq \hat{f} = \frac{a^2 (3b^2 + 6bc + 4c^2 - b\sqrt{5b^2 + 12bc + 8c^2})}{8(b + c)(b + 2c)^2},
\] (3.17)
which is increasing in \( a \) and decreasing in \( b \) and \( c \).
However, if (3.17) is violated, we get that \( x^M < X^M_B + \frac{n_A|n_B=X^M_B}{2} \). Then, it holds that
\[
X^M_B \leq x^M < X^M_B + \frac{n_A|n_B=X^M_B}{2}.
\] (3.18)
Avoiding negative profits leads to a maximum \( f \) of
\[
\bar{f} = \frac{a^2}{4(b + c)}.
\] (3.19)
Clearly \( \bar{f} > \hat{f} \) and thus, if
\[
\hat{f} < f \leq \bar{f}
\]
we get that in equilibrium a monopoly cooperative would choose the same output as a regular
monopoly \( x^M \). Up until \( N = x^M \), it is also the same output as total output in section 3.3. If
\( N > x^M \) two competing IOFs would still choose \( X = N \), while in the IOF/Coop game the single
monopoly leaves its output at \( X = x^M \). Hence, the inefficiency in corollary 3.1 vanishes. However, consumers are still worse off, as for \( N > x^M \) total output remains lower than in the case of two IOFs.

### 3.6 Conclusion

In this model, we have shown that the possibility of suppliers to join a cooperative can lead to an inefficiently low supply by downstream firms. This is due to the fact that the contracts offered by the cooperative raise the cost for the input good or production capacity in such a way that total supply can be even less than the quantity supplied by a regular monopoly. Such an effect is not present in our benchmark case of two IOFs competing. Thus, the presence of a cooperative, while increasing the average transfer received by the suppliers, can reduce consumer surplus due to lower supply. While the literature on cooperatives and mixed oligopolies tends to highlight the first aspect of increased rents for suppliers, we claim that when the effects of cooperatives are evaluated—also in respect to exemptions made in competition law—the negative effect on consumer surplus needs to be taken into account as well.
3.7 Appendix

Proof of Proposition 3.1. It is straightforward that (3.5) and (3.7) are part of an equilibrium. With (3.8), the intermediaries are able to acquire their full Cournot profits, even if the number of suppliers $N$ is small. For this proof it holds that $N \in \left[N^C, 2x^C\right]$. If $N > 2x^C$, intermediaries could acquire some additional capacity for free, but would not want to use it anyway. Hence, the incentives to acquire additional capacity are larger, if $N \leq 2x^C$. If $N < N^C$ only one of the intermediaries may be active and they compete for the available capacity.

Assume that firm $j$ wants to deviate by acquiring a capacity $\hat{n} > \frac{N}{2}$. For the deviation profit $\hat{\Pi}(\cdot)$ it has to hold that

$$\hat{\Pi}(x^*_i, x_j) - f - \hat{n} \times \hat{t} > \Pi^C - f$$

to be profitable. Rearranging for $\hat{t}$ leads to

$$\hat{t} < \frac{\hat{\Pi}(x^*_i, x_j) - \Pi^C}{\hat{n}}. \quad (3.20)$$

As $\hat{n} > \frac{N}{2}$ leads firm $i$ to make a downstream profit $\hat{\Pi}_i(x^*_i, x_j) < \Pi^C$, firm $i$’s transfer is

$$\hat{t}_i = \frac{\hat{\Pi}_i(x^*_i, x_j)}{n_i}$$

as it sticks to the equilibrium strategy in (3.8). As the market maker can distribute suppliers to the firm, which pays a higher transfer, firm $j$ has to match firm $i$’s offer, in order to acquire a capacity $\hat{n} > \frac{N}{2}$. Thus, we get that

$$\hat{t} \geq \frac{\Pi^C}{N/2} \quad (3.21)$$

Combining (3.20) and (3.21), we get that deviation is only profitable, if

$$\frac{\hat{\Pi}(x_i, x_j)}{\Pi^C} > 1 + 2 \frac{\hat{n}}{N}.$$ 

Therefore, a deviation to increase capacity such that $\hat{n} = \frac{N}{2} + \varepsilon$ and $\varepsilon \to 0$ is clearly not profitable. However, if firm $j$ wants to be the monopoly in the downstream market, it needs $\hat{n} = N - \frac{2}{n_j|x_j=x^M}$, which leads to

$$\frac{\Pi^M}{\Pi^C} > 3 - \frac{\frac{2}{n_j}|_{x_j=x^M}}{N}. \quad 38$$
This cannot be the case, as long as \( f \) is sufficiently small. For example, to allow deviation with a linear demand function \( P(X) = a - bX \) and strictly convex cost function \( C(x_i) = cx^2 \), we get that

\[
f > \frac{a^2(b + 2c)^2}{16(b^3 + c + 3bc(b + c))}.
\]

\[
\text{Proof of Lemma 3.1.}
\]
Assume that \( X_i < n_i, \forall i \), then \( \partial X_i^* / \partial n_B = 0 \). As \( n_B > X_B^* \) leads to \( \partial X_B^* / \partial n_B = 0 \), we get \( \partial t_B / \partial n_B < 0 \). Thus, we cannot have an equilibrium, in which both firms hold excess capacity.

\[
\text{Proof of Lemma 3.2.}
\]
According to (3.5), there is a best response for the cooperative \( X_B^* = \tilde{X}_B(n_A) < x^M \) for any \( X_A^* = n_A \geq \frac{\pi_B}{P_0(X)} \). Given the IOF will choose its capacity as its output and the cooperative will play a best response to that, we can find the optimal quantity left to the IOF from the cooperative’s perspective. We will show in two steps that, given the IOF will choose \( X_A^* = n_A \), the cooperative does not want the IOF to enter the market.

While the cooperative maximizes \( t_B \), we use that as \( N = n_A + n_B \), \( t_B \) can also be maximized with respect to \( n_A \), by choosing the appropriate \( \pi_B \). Further, we use that (i) the first order condition of \( t_B \) with respect to \( n_A \) is negative at \( n_A = \frac{\pi_B}{P_0(X)} \) and (ii) that, if we can identify a solution of the first order condition, the second order condition of \( t_B \) with respect to \( n_A \) shows that this solution is not a maximum.

(i) Evaluated at \( n_A = \frac{\pi_B}{P_0(X)} \), the cooperative’s first order condition, with respect to \( n_A \) is

\[
\frac{\partial t_B(X_B^*, n_A, N - n_A)}{\partial n_A} \bigg|_{n_A = \frac{\pi_B}{P_0(X)}}, \quad \frac{\partial t_B(X_B^*, n_A, N - n_A)}{\partial n_A} \bigg|_{n_A = \frac{\pi_B}{P_0(X)}} = \frac{\tilde{\pi}_B(X_B^*, 0) - f + \left( N - \frac{\pi_B}{P_0(X)} \right) \times X_B^* P'(X)}{(N - \frac{\pi_B}{P_0(X)})^2}.
\]

Thus, in order for it to be negative, it has to hold that

\[
X_B^* \left( P(X_B^*) + P'(X) \left( N - \frac{\pi_B}{P_0(X)} \right) \right) - C(X_B^*) - f < 0,
\]

which is clearly negative, if

\[
\frac{P(X_B^*)}{N - \frac{\pi_B}{P_0(X)}} < -P'(X),
\]

39
which requires the demand function to be sufficiently steep.

(ii) Given \( t_B \) is maximized with respect to \( x_B \) and \( n_A \), the second order condition for the latter is

\[
\frac{d^2 t_B(X_B^n, n_A, N - n_A)}{d n_A^2} \bigg|_{n_A = n_A^*} = \left( \frac{P(X) - C'(\bar{X}_B(n_A^*))}{(N - n_A^*)^2 \bar{X}_B(n_A^*)} \right)^2 + \frac{C''(\bar{X}_B(n_A^*)) P''(X)}{(N - n_A^*)^2 \bar{X}_B(n_A^*)^2}
\]

In case of linear demand \( P''(X) = 0 \), we get that \( \frac{d^2 t_B}{d n_A^2} > 0 \), and \( n_A^* \) not being a maximum. Thus, the maximum of \( t_B \) with respect to \( n_A \) has to be a corner solution. As \( n_A = N \) leads to \( t_B = 0 \), the maximum has to be at \( n_A^* = 0 \). If \( P''(X) < 0 \), it just needs to be sufficiently large.

Thus, whenever \( P'(X) \) is sufficiently small and \( P''(X) \) is sufficiently large, we cannot have an equilibrium, with \( X_A^* = n_A > 0 \) and \( X_B^* = \bar{X}_B(n_A) > 0 \). This also holds for the case that \( X_B^* = n_B > 0 \), where the cooperative has even less of an incentive to leave any capacity to the IOF. ■

**Proof of Corollary 3.1** As for the regular monopoly it holds that

\[ \Pi'(x)|_{x<x^M} > 0 \quad \text{and} \quad \Pi''(x) < 0 \]

the monopoly would choose \( X = N \), if \( N \in \left(N^{c1}, N^{c2}\right) \). However, as firms in the IOF/Coop game have to maximize the average profit, they need to take into account that

\[
\frac{\partial t_B(X_B^*, 0, n_B)}{\partial n_B} \bigg|_{n_B > X_B^M} < 0 \quad \text{and} \quad \frac{\partial t_B(x_B, 0, n_B)}{\partial x_B} \bigg|_{x_B < x_B^M} > 0.
\]

Thus, while supplying all the quantity that they acquired, downstream firms would choose the capacity as low as possible, which is in this case \( n_B = X = N - \bar{n} \) with \( \bar{n} = \min\{N - X_B^M, n\}|_{X=x} \) > 0. ■
4 Information Acquisition in Vertical Relations

4.1 Introduction

Uncertainty about demand is a general phenomenon in markets for new products or in markets, where consumer preferences vary over time, e.g., markets for fashion goods. Similarly, exogenous demand shocks can lead to price fluctuations and thus expose firms to high economic risks. Information obtained from market research can reduce the firms’ uncertainty. In supply chains the question arises whether suppliers and/or retailers want to acquire information about market demand. There are two main strategic issues involved: First, suppliers can use the contracts they offer to signal their private information. Second, retailers can increase their information rent, if they reduce their uncertainty.

We analyze a simple model with one supplier, one retailer and uncertainty about demand in a perfectly competitive market. The uncertainty is modelled as uncertainty about the price, at which the retailer can sell the supplier’s product and which might be either high or low. The supplier and the retailer receive private signals about the price of the product. The precision of these signals, i.e., the probability with which they signal the actual price, can be chosen by each of the players. Although the signals themselves are private, we assume that the choices on signal precision are observable. The supplier offers contracts to the retailer, which each specify a certain quantity and transfer. The contracts may depend on the supplier’s private information, which has a signalling effect towards the retailer. The retailer chooses one of the offered contracts, given his own private signal, and sells the respective quantity at the actual market price.

In this model we focus on the analysis of the agents’ incentives to acquire information. We show that there are two kind of equilibria: one where only the retailer decides to get informed and one where both the supplier and the retailer choose to increase the precision of their signals. The first type of equilibrium is due to the fact that contracts, which credibly signal the supplier’s private information to be a high price signal, are costly in the sense that the supplier has to distort the quantities he offers. Comparing the implied signalling cost with the potential gains from appropriating some of the retailer’s information rent, the supplier decides to remain uninformed as long as the retailer’s signal precision and thus his information rent is sufficiently small. By contrast and related to the second type of equilibrium, the supplier will choose to be informed, if
the retailer’s information rent is sufficiently high, i.e., if the retailer’s signal precision is sufficiently high. Furthermore we show that the information decisions of supplier and retailer are strategic complements such that they reinforce each other.

There is a vast literature on principal-agent models with endogenous information acquisition. Kessler (1998) analyzes an agent’s incentive to get informed before the contracting stage. This paper shows that it is not optimal for the agent to become perfectly informed as this would reduce the expected information rent. Considering the principal Bedard (2013), Kaya (2010) and Nosal (2006) show that acquiring information may not be valuable for the principal as it leads to signalling costs in the contracting stage. Our model replicates this result as we show that the incentives of the principal to get informed depend on the information the agent has. The better the agent is informed the higher are the incentives of the principal to get informed as well as this allows him to appropriate the agent’s information rent. Crémer et al (1998) as well as Crémer and Khalil (1994) examine the incentives of a principal to induce an agent to acquire additional information. In contrast to our model they assume that information gathering takes place after the principal has designed the contracts he offers, but before the agent decides, which contract to sign. We follow Kessler (1998) and Kaya (2010) by assuming that decisions about information acquisition are made prior to the contracting stage and that these decisions are observable. However, we allow both players to gather information and focus on the strategic interdependencies of these decisions.

The literature on information gathering within a supply chain can be classified in terms of who acquires the information and how the information is used within the supply chain. Guo and Iyer (2010) analyze the case, in which an upstream manufacturer can gather information on consumers’ perceived product fit. They observe that the manufacturer does not have an incentive to be fully informed even if it is free of charge. This correlates with one of our equilibria where the supplier does not want to be informed. However, they model the information stage in another way. The information acquisition takes place after contracting with a retailer and is affected by different information sharing mechanisms.

In contrast to this, there are several papers, which consider a downstream retailer/buyer investing in forecasting of demand or other uncertain parameters. Shin and Tunca (2010) show that if there is competition between retailers, the incentives of these retailers to invest in information acquisition are such that overinvestment occurs. If the investment is secret the overinvestment
can be resolved by market based contracts, while in case of observable investments an uniform-price auction is required to solve that issue. Closer to our model is the structure in Fu and Zhu (2010), where a single retailer acquires costly demand information after contract negotiations, but before ordering quantities. In their model, the added information might lead the informed party to improve its profits at the supplier’s expense. They suggest a sharing mechanism for the cost and the information in order for the added information to lead to a Pareto-improvement. Guo (2009) addresses the question of forced versus voluntary disclosure of information and the effects of the disclosure rules on the firms’ profits. It is shown that while forced disclosure actually harms the informed retailer and benefits the uninformed supplier, voluntary disclosure might restore the retailer’s incentives to get informed in the first place.

Treating a similar problem, Kurtulus et al (2011) allow both the supplier and the retailer to invest in more accurate information about demand. They focus on potential profit losses implied by production decisions of the supplier and either too low or high orderings of the retailer. The aim of their paper is to analyze the benefits of information sharing, i.e., of common demand forecasts by the supplier and the retailer. For this purpose Kurtulus et al (2011) consider risk sharing based on buy back clauses.

The papers above do not analyze whether the decisions on information acquisition are strategic complements or substitutes. Barlevy and Veronesi (2000) look at a financial market and show that learning can be a strategic complement. This contradicts Grossman and Stiglitz (1980), who show that learning is a strategic substitute. However, both papers only take a horizontal relation into account. The model in this chapter contributes to the literature, as it considers a comparative static analysis of the information decision in a supply chain and explains the existence of two different types of equilibria.

This chapter is structured as follows: Section 4.2 is concerned with the model framework. In section 4.3 we characterize the equilibrium contracts offered by the supplier for any given combination of private signals and precision choices. The decisions on information acquisition are analyzed in section 4.4. Section 4.5 presents a discussion of some of the model’s structural assumptions. Finally, section 4.6 concludes.
4.2 Model

We consider a three stage game with one supplier, one retailer and uncertain market conditions. In the first stage both players $i \in \{S, R\}$ can simultaneously choose the level of precision of the private signals about the actual market condition. While the selected signal precisions are observable, the signals themselves are private information. In the second stage, the supplier designs a menu of contracts, which he offers to the retailer. Then, the retailer chooses one of the contracts and offers the respective quantity on the market. The profit of the supplier is the transfer paid by the retailer minus the cost of producing the quantity specified in the contract, which has been chosen by the retailer. The retailer’s profit is this quantity times the market price for the good, minus the transfer paid to the supplier.

The retailer faces a perfectly competitive market. To capture uncertainty about market conditions, we assume that the market price for the product is $p \in \{p_L, p_H\}$ with $p_H = 1$, $p_L = \alpha$ and $1/2 < \alpha < 1$.\footnote{Note that our results also hold in case of a strictly concave industry profit and the uncertainty relating to a shift in the demand curve.} We further assume that the commonly known probabilities for the high and the low price are

$$\Pr\{p = 1\} = \Pr\{p = \alpha\} = \frac{1}{2}$$

Allowing for different prior probabilities would complicate the analysis without leading to qualitatively different results.

At the end of the first stage both players get private signals $\sigma_i \in \{H, L\}$ about the actual market price. Each player $i$ may choose his signal precision $\nu_i \in [1/2, \pi]$ with $1/2 < \pi \leq 1$, without any cost.\footnote{\[\pi < 1\] corresponds to assuming a convex cost function for information acquisition with $\lim_{\nu \to 1} C(\nu) = \infty$.} The probabilities for getting a correct signal are given by

$$\nu_i = \Pr\{\sigma_i = H | p = 1\} = \Pr\{\sigma_i = L | p = \alpha\}$$

Correspondingly, the probabilities for getting a wrong signal are given by

$$1 - \nu_i = \Pr\{\sigma_i = H | p = \alpha\} = \Pr\{\sigma_i = L | p = 1\}$$

Note that $\nu_i = 1/2$ implies that firm $i$ is completely uninformed. As mentioned, we assume that
\(\nu_S\) and \(\nu_R\) are observable while the signals themselves are private information.

Turning to the second stage, the supplier chooses the menu of contracts that he offers. Each contract entails a fixed payment \(T\) and a quantity \(x\), which is produced by the supplier at a cost of

\[
C(x) = \frac{1}{2}x^2
\]

Since we assume that this decision takes place after the supplier has received his private signal, the offered menu can be conditioned on the supplier’s signal \(\sigma_S\)

\[
C_{\sigma_S} = (C_{H_{\sigma_S}}, C_{L_{\sigma_S}}) = ((T_{H_{\sigma_S}, x_{H_{\sigma_S}}}), (T_{L_{\sigma_S}, x_{L_{\sigma_S}}})) \text{ with } \sigma_S \in \{H, L\}
\]

If \(\nu_S = 1/2\), the supplier’s signal is not informative. Then the contracts he offers do not depend on \(\sigma_S\) and correspond to pooling contracts, which do not signal any private information. On the other hand, \(\nu_S > 1/2\) leads to separating contracts \(C_H \neq C_L\) which in turn allows the retailer to update his beliefs about the state of demand. More precisely, given signal \(\sigma_S\) and the corresponding contract offer \(C_{\sigma_S}\), the conditional probability that signal \(\sigma_R = k\) reflects the true state of the world \(k \in \{H, L\}\) can be written as

\[
\mu_{\sigma_R|\sigma_S} = \Pr(p = p_k|\sigma_R, \sigma_S) = \begin{cases} \frac{\nu_R \nu_S}{(1-\nu_R)(1-\nu_S)+\nu_R \nu_S}, & \text{if } \sigma_R = \sigma_S \\ \frac{(1-\nu_S)\nu_R}{\nu_S (1-\nu_R)+(1-\nu_S)\nu_R}, & \text{if } \sigma_R \neq \sigma_S \end{cases}
\]

Our solution concept is the Perfect Bayesian Nash Equilibrium. To decrease the potentially high number of equilibria we make the following additional assumptions: Off equilibrium the retailer assumes that the supplier’s signal is \(\sigma_S = L\). Following the intuitive criterion we further assume that menu offer \(C_H\) induces the retailer to belief \(\sigma_S = H\), only if the supplier would not be better off by offering \(C_H\) than \(C_L\), if his signal was \(L\).

In the next section, we characterize the equilibrium contracts. We then turn to the supplier’s and retailer’s decisions on signal precision in the first stage of the game.

### 4.3 Equilibrium Contracts

We start with the case in which the signal of the supplier is \(\sigma_S = L\) and then turn to \(\sigma_S = H\).

If \(\sigma_S = L\), the supplier offers \(C_L = (C_{HL}, C_{LL})\). The expected profit of the retailer, if he chooses
contract \((T_{kL}, x_{kL})\) with \(k \in \{H, L\}\), can be written as

\[
E\Pi^R_{LL}(C_{kL}) = (1 - \mu_{LL})x_{kL} + \mu_{LL}\alpha x_{kL} - T_{kL}, \text{ if } \sigma_R = L
\]

\[
E\Pi^R_{HL}(C_{kL}) = \mu_{HL}x_{kL} + (1 - \mu_{HL})\alpha x_{kL} - T_{kL}, \text{ if } \sigma_R = H
\]

Since \(E\Pi^R_{HL}(C_{HL})\) and \(E\Pi^R_{LL}(C_{LL})\) satisfy the single crossing property, the binding constraints for the optimal incentive compatible contracts \(C_{HL}\) and \(C_{LL}\) are given by

\[
E\Pi^R_{HL}(C_{HL}) = E\Pi^R_{HL}(C_{LL}) \text{ and } E\Pi^R_{LL}(C_{LL}) = 0 \tag{4.1}
\]

The expected profit of the supplier given his signal was \(\sigma_S = L\) can be written as

\[
E\Pi^S_L(C_{L}) = (1 - \nu_S) \left( \nu_R \left( T_{HL} - \frac{1}{2}x^2_{HL} \right) + (1 - \nu_R) \left( T_{LL} - \frac{1}{2}x^2_{LL} \right) \right) + \nu_S \left( (1 - \nu_R) \left( T_{HL} - \frac{1}{2}x^2_{HL} \right) + \nu_R \left( T_{LL} - \frac{1}{2}x^2_{LL} \right) \right) \tag{4.2}
\]

Maximizing \(4.2\) subject to \(4.1\) we get:

**Lemma 4.1** The optimal menu \(C^*_L = (C^*_HL, C^*_LL)\) is characterized by

\[
x^*_{HL} = \alpha + \mu_{HL}(1 - \alpha)
\]

\[
x^*_{LL} = \alpha + \left[ 1 - \mu_{LL} - \frac{\nu_S((1 - \nu_S)\nu_R - (1 - \nu_S)(1 - \nu_R))}{\nu_S\nu_R + (1 - \nu_R)(1 - \nu_S)} \right](1 - \alpha)
\]

and

\[
E\Pi^R_{HL}(C^*_HL) = (\mu_{HL} - (1 - \mu_{LL}))(1 - \alpha)x^*_{LL}
\]

**Proof.** See appendix. ■

Note that quantities \(x^*_{HL}\) and \(x^*_{LL}\) entail the standard result of an optimal quantity for the "high type", i.e., \(\sigma_R = H\), and a downward distorted quantity for the "low type", i.e., \(\sigma_R = L\).

While a retailer with \(\sigma_R = H\) is able to make profit \(E\Pi^R_{HL}(C_{HL}) \geq 0\), a retailer with \(\sigma_R = L\) is left with 0 profit as it is depicted in \(4.1\).

To simplify notation, we omit the exogenous parameter \(\alpha\), as well as endogenous variables, which can be taken as given at the respective stage of the game.
With $\sigma_S = H$, the supplier offers $C_H = (C_{HH}, C_{LH})$ and the retailer’s expected profits are given by

$$E\Pi^R_{HH}(C_{HH}) = (1 - \mu_{LH})x_{kH} + \mu_{LH}\alpha x_{kH} - T_{kL}, \text{ if } \sigma_R = L$$
$$E\Pi^R_{HH}(C_{HH}) = \mu_{HH}x_{kH} + (1 - \mu_{HH})\alpha x_{kH} - T_{kL}, \text{ if } \sigma_R = H$$

Again, the single crossing property holds and the binding constraints are given by

$$E\Pi^R_{HH}(C_{HH}) = E\Pi^R_{HH}(C_{LH}) \text{ and } E\Pi^R_{HH}(C_{LH}) = 0 \quad (4.3)$$

The expected profit of a supplier with $\sigma_S = H$ can be written as

$$E\Pi^S_{HH}(C_{HH}) = \nu_S \left( \nu_R \left( T_{HH} - \frac{1}{2}x^2_{HH} \right) + (1 - \nu_R) \left( T_{LH} - \frac{1}{2}x^2_{LH} \right) \right)$$
$$+ (1 - \nu_S) \left( (1 - \nu_R) \left( T_{HH} - \frac{1}{2}x^2_{HH} \right) + \nu_R \left( T_{LH} - \frac{1}{2}x^2_{LH} \right) \right) \quad (4.4)$$

In contrast to the case with $\sigma_S = L$, offering $C_H$ has to be credible, i.e., observing $C_H$ the retailer must be convinced that the supplier’s signal was $\sigma_S = H$. Credibly signalling $\sigma_S = H$ increases the retailer’s belief that $p = 1$ and thus his willingness-to-pay. To ensure credibility the menu $C_H$ has to satisfy the following constraint

$$\Delta(C_H, C_L) := E\Pi^S_{HH}(C_L) - E\Pi^S_{HH}(C_H) \geq 0 \quad (4.5)$$

Comparing (4.3) and (4.5) shows that there exists a unique $\nu_R(\nu_S, \alpha) \in (0, 1]$ such that (4.5) is binding only for $\nu_R \leq \nu_R(\nu_S, \alpha)$. Thus, we have:

**Lemma 4.2** The optimal contracts $C^*_H = (C^*_{HH}, C^*_{LH})$ are characterized by

$$x^*_{HH} = \alpha + \mu_{HH}(1 - \alpha) \text{ and } x^*_{LH} = \begin{cases} \hat{x}^*_{LH}, & \text{if } \nu_R \geq \nu_R(\nu_S, \alpha) \\ \hat{x}^*_{LH} + \Delta^*_{LH}, & \text{if } \nu_R \leq \nu_R(\nu_S, \alpha) \end{cases}$$
where $\tilde{x}_{LH}^*$ and $\Delta_{LH}^*$ are given by

$$\tilde{x}_{LH}^* = \alpha + \left[ 1 - \mu_{LH} - \frac{\nu_S \left( (1 - \nu_S)\nu_R - (1 - \nu_S)(1 - \nu_R) \right)}{(1 - \nu_R)\nu_S + \nu_R(1 - \nu_S)} \right] (1 - \alpha)$$

$$\Delta_{LH}^* = \min \left\{ \Delta_{LH} \middle| \Delta(C_H, C_L^*)_{x_{HH} = x_{HH}, x_{LH} = \tilde{x}_{LH}^* + \Delta_{LH}} = 0 \right\} \leq 0.$$

**Proof.** See appendix. ■

Note that $x_{HH}^*$ and $\tilde{x}_{LH}^*$ correspond to the quantities for regular screening contracts $C_{HH}$ and $C_{LH}$ respectively. While $x_{HH}^*$ is undistorted, there is some distortion in $\tilde{x}_{LH}^*$ for any $\nu_s < 1$. In addition, $\Delta_{LH}^* < 0$ specifies the smallest additional downward distortion required for (4.5) to hold, if $\nu_R \leq \tilde{\nu}_R(\nu_S, \alpha)$.

Intuitively, to ensure credibility of having received $\sigma_S = H$, the supplier has to decrease the quantity $x_{LH}$ as this lowers his profits, if the retailer gets signal $\sigma_R = L$ and therefore chooses $C_{LH}$—an event which is more unlikely with $\sigma_S = H$ as compared to $\sigma_S = L$. Thus, the increase in distortion of $x_{LH}^*$ allows the supplier to credibly signal $\sigma_S = H$, as the expected profit of $C_H^*$ would be reduced significantly, if $\sigma_S = L$.

Figure 4.1 shows $\tilde{\nu}_R(\nu_S, \alpha)$ for $\alpha = 1/2$ and $\alpha = 2/3$. Note that $\lim_{\nu_S \searrow 0.5} \tilde{\nu}_R(\nu_S, \alpha) = \lim_{\nu_S \nearrow 1} \tilde{\nu}_R(\nu_S, \alpha) = 1$. Thus, for $\nu_S$ being either rather high or low, additional distortion is required even for relatively high values of $\nu_R$.

![Diagram](image_url)

**Figure 4.1:** Critical values $\tilde{\nu}_R(\nu_S, 1/2)$ and $\tilde{\nu}_R(\nu_S, 2/3)$. 

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Note further that we have \( C^*_H \neq C^*_L \), if \( \nu_S \neq 1/2 \) and \( \lim_{\nu_S \to 1/2} (C^*_H - C^*_L) = 0 \). Thus, \( \nu_S = 1/2 \) leads to pooling contracts, which do not require the additional distortion to solve the supplier’s credibility problem. On the other hand \( \nu_S = 1 \) resolves all uncertainty for the supplier, but still requires additional distortion to credibly signal \( \sigma_S = H \) to a retailer, who is less then perfectly informed.

4.4 Choice of Signal Precision

Turning to the first stage of the game, the expected profits of the supplier \( \Pi^S \) and the retailer \( \Pi^R \) can be written as

\[
\Pi^S(\nu_S, \nu_R) = \frac{1}{2} \Pi^S_H(C^*_H) + \frac{1}{2} \Pi^S_L(C^*_L)
\]

(4.6)

\[
\Pi^R(\nu_R, \nu_S) = \frac{1}{2} ((1 - \nu_R)(1 - \nu_S) + \nu_R \nu_S) \Pi^R_H(C^*_HH) + \frac{1}{2} ((1 - \nu_S) \nu_R + \nu_S (1 - \nu_R)) \Pi^R_H(C^*_HL)
\]

(4.7)

Regarding the shape of these profit functions, we get the following result for the retailer:

Lemma 4.3 A solution \( \nu^*_R(\alpha, \nu_S) \) such that

\[
\frac{\partial \Pi^R(\nu^*_R(\alpha, \nu_S), \nu_S)}{\partial \nu_R} = 0 \quad \text{and} \quad \nu^*_R(\alpha, \nu_S) < \tilde{\nu}_R(\nu_S, \alpha)
\]

only exists, if \( \nu_S \) is sufficiently small. Furthermore, the retailer’s marginal incentives for information acquisition have a downward kink at \( \nu_R = \tilde{\nu}_R(\nu_S, \alpha) \), i.e.,

\[
\lim_{\nu_R \to \tilde{\nu}_R(\nu_S, \alpha)} \frac{\partial \Pi^R(\nu_R, \nu_S)}{\partial \nu_R} > \lim_{\nu_R \to \tilde{\nu}_R(\nu_S, \alpha)} \frac{\partial \Pi^R(\nu_R, \nu_S)}{\partial \nu_R}
\]

(4.8)

Finally, we have \( \partial \Pi^R(\nu_R, \nu_S) / \partial \nu_R < 0 \), if \( \alpha, \nu_R, \nu_S \) are such that \( \tilde{x}_{LH}^* = \tilde{x}_{LH}^* = \varepsilon \) with \( \varepsilon > 0 \) but small.

Proof. See appendix. ■

The first part of the lemma implies that for \( \nu_S = 1/2 \), the retailer’s best response is not to be as informed as possible. This is simply due to the fact that the downward distortions of the quantities \( x_{LL}^* \) and \( x_{LH}^* \) are increasing in \( \nu_R \) which impacts the retailer’s expected profit negatively. The
second part of lemma 4.3 indicates that $\tilde{\nu}_R(\nu_S, \alpha)$ may be part of an equilibrium, if the kink of $E\Pi^R(\nu_R, \nu_S)$ is such that its left side is upward, while the right side is downward sloping. The final part guarantees that the retailer’s best response never leads to $x_{LH}^* = 0$, as the retailer would rather reduce the precision of his signal in order to prevent such an extreme distortion.

Turning to the supplier, we get:

**Lemma 4.4** The supplier’s incentives for information acquisition are characterized by

$$\frac{\partial E\Pi^S(\nu_S, \nu_R)}{\partial \nu_S} \bigg|_{\nu_S = 1/2} < 0 \text{ for } \nu_R < 1 \text{ and }$$

$$\frac{\partial E\Pi^S(\nu_S, \nu_R)}{\partial \nu_S} > 0 \text{ for } \nu_S \text{ and } \nu_R \text{ sufficiently high.}$$

Furthermore, $E\Pi^S(\nu_S, \nu_R)$ is strictly convex in $\nu_S$ for all $\nu_R, \nu_S$ such that $x_{LH}^* > 0$.

**Proof.** See appendix. ■

Lemma 4.4 guarantees by convexity of $E\Pi^S(\nu_S, \nu_R)$ that the supplier either chooses to get not informed at all or as much as possible. Intuitively, starting with $\nu_S = 1/2$ an increase in $\nu_S$ forces the supplier to reduce $x_{LH}^*$ in order to satisfy the credibility constraint, which offsets any potential gains from being better informed. If $\nu_S$ and $\nu_R$ are sufficiently high, the credibility constraint is not binding and the supplier can fully benefit from signalling his information. Thus, if $\nu_R > \tilde{\nu}_R(\nu_S, \alpha)$, the supplier has an incentive to be as informed as possible.

Combining lemma 4.3 and 4.4 we can formulate the following proposition:

**Proposition 4.1** Suppose $\overline{\nu} = 1$. Then, there exist multiple equilibria with $\nu_S^* = 1$ and $\nu_R^*$ sufficiently high.

**Proof.** See appendix. ■

If $\overline{\nu} = 1$, i.e., if players can choose to be perfectly informed, the supplier is able to extract the entire rent from the retailer by choosing $\nu_S^* = 1$ and the appropriate contracts. Thus, $E\Pi^R(\nu_R, 1) = 0$, while for the supplier it holds that

$$E\Pi^S(1, \nu_R) = \frac{2\nu_R - 1 + \alpha^2}{4\nu_R} > 0$$

If we restrict $\overline{\nu} < 1$, the analysis of the equilibrium choices $\nu_S^*$ and $\nu_R^*$ is more involved since we have to distinguish whether $\nu_R$ is lower or higher than $\tilde{\nu}_R(\nu_S, \alpha)$. Furthermore, we now get that
two types of equilibria may coexist.

**Proposition 4.2** Assume \( \nu < 1 \). Then, there exist \( \nu_1(\alpha), \nu_2(\alpha) \in (1/2, 1) \) and two types of equilibria characterized by:

i) If \( \nu \leq \nu_1(\alpha) \),

\[
\nu_S^* = \frac{1}{2} \quad \text{and} \quad \nu_R^* = \min \{ \nu'_R, \nu \}
\]

with \( \nu'_R \) being the solution to \( \partial \Pi^R(\nu_R, 1/2) / \partial \nu_R = 0 \).

ii) If \( \nu \geq \nu_2(\alpha) \),

\[
\nu_S^{**} = \nu \quad \text{and} \quad \nu_R^{**} = \min \{ \max \{ \nu'_R(\nu, \nu), \nu_H \}, \nu \}
\]

with \( \nu'_R \) being the solution to \( \partial \Pi^R(\nu_R, \nu) / \partial \nu_R = 0 \) for \( \nu_{LH}^* = \hat{x}_{LH}^* \).

**Proof.** See appendix. ■

The proposition states that—depending on the maximum precision \( \nu \)—we either get an equilibrium, where the supplier decides to stay uninformed, or an equilibrium, where the supplier wants to be informed as much as possible. The two types of equilibria show two different strategies for the supplier to cope with the credibility problem discussed above. When the retailer is not able to get well informed (\( \nu \) low), the supplier can refuse to get informed at all, as the distortion from the retailer’s information is low and the credibility problem can thus be avoided by offering pooling contracts. When the retailer can get well informed (\( \nu \) high), the information rent is relatively high. Therefore, it is beneficial for the supplier to offer separating contracts, despite the required additional distortion of \( x_{LH}^* \).

The graphs of \( \nu_1(\alpha) \) and \( \nu_2(\alpha) \) are shown in Figure 4.2. Note that we have \( \nu_1(\alpha) > \nu_2(\alpha) \) for \( \alpha \) sufficiently small, while \( \Pi^S(1/2, \nu^*_H) = \Pi^S(\nu, \nu_H^*) \) and \( \nu_R^* = \nu \) lead to \( \nu_1(\alpha) = \nu_2(\alpha) \) for \( \alpha \) sufficiently high. Note also that for sufficiently small \( \alpha \), both types of equilibria exist. This is due to the complementarity of the decisions on signal precision. Consider for example \( \nu = 0.79 \) and two different values of \( \alpha \). With \( \alpha = 1/2 \) we get \( \nu'_R = 3/4 < \nu^{**}_R = \nu \): As \( \alpha \) is small, the effect of \( \nu_R \) on the quantities is large and a high \( \nu_R \) would thus imply highly distorted quantities \( x_{Lk}^*, k \in \{ H, L \} \). This leads the retailer to choose a low \( \nu_R \). If the retailer chooses \( \nu_R = \nu'_R \), the
supplier’s best response is to stay uninformed, which represents the first type of equilibrium. On the other hand, with \( \nu^*_R = \nu \) the retailer’s information rent is already high enough to induce the supplier to get informed as well. Hence, the two types of equilibria coexist. With \( \alpha = 2/3 \) we have \( \nu'_R = 11/12 > \nu^*_R = \nu \): As \( \alpha \) is large, the effect of \( \nu_R \) on the quantities is small and a high \( \nu_R \) leads to less distortion and the retailer has a stronger incentive to choose a high \( \nu_R \). This induces the supplier to choose \( \nu_S = \nu \). Thus, the first type of equilibrium does not exist.

To illustrate different types of equilibria, assume that \( \alpha = 2/3 \). Then, starting with \( \nu = 1/2 + \varepsilon \) as \( \nu \leq \nu_1(2/3) \), we have an equilibrium, with \( \nu^*_S = 1/2 \) and \( \nu^*_R = \nu \) until \( \nu = \nu_1(2/3) = \nu_2(2/3) \). If \( \nu_2(2/3) \leq \nu < \tilde{\nu}_R(\nu, 2/3) \), we have the equilibrium with both players acquiring as much information as possible, i.e., \( \nu^*_S = \nu \) and \( \nu^*_R = \nu \). For \( \tilde{\nu}_R(\nu, 2/3) \leq \nu < 1 \), we get that \( \nu^*_S = \nu \) and \( \nu^*_R = \tilde{\nu}_R(\nu, 2/3) \). Finally, if \( \nu = 1 \), we get the equilibria specified in proposition 4.1. The corresponding equilibrium values \( \nu^*_R, \nu^*_S \) as well as \( \nu^*_R, \nu^*_S \) are shown in Figure 4.3.

![Figure 4.2](image_url)

**Figure 4.2:** Equilibrium with \( \nu^*_S = 1/2 \) (\( \nu^*_R = \nu \)) below \( \nu_1(\alpha) \) (above \( \nu_2(\alpha) \)).

![Figure 4.3](image_url)

**Figure 4.3:** Equilibrium decisions for \( \alpha = 2/3 \) and \( \nu \in [2/3, 1) \).
4.5 Discussion

In this section, we discuss variations of three structural assumptions of our model. First, we allow the supplier to get his signal verified and address the question, whether the supplier wants to disclose his signal to the verification process. Second, we modify the game such that the supplier designs the contracts before learning his private signal. Afterwards, the supplier receives his private signal, which he may report to the retailer before the retailer chooses one of the contracts. Third, we analyze the game when either the retailer or the supplier decides first on the precision of their signals. We retain our main assumptions about the observability of these decisions.

A) Verifiable Signals: Assume that in contrast to the model above the supplier’s signal is verifiable. Then—as in the case of signals being public information—the supplier does not face the credibility problem, which is discussed above. This would lead to standard adverse selection results in our model. Now further assume that there is a chance that the supplier did not receive any signal and it is thus credible for him to pretend not to have received private information, even if he did.

The credibility problem is now a different one. A supplier with an $H$ signal would always want to disclose his private information. As the signal can be verified a supplier with an $H$ signal can offer standard adverse selection contracts, as there is no additional credibility condition to be satisfied. However, a supplier with an $L$ signal would want to pretend to have not received any signal, as the regular pooling contracts offer a higher profit, than the separating contracts, given an $L$ signal is revealed. This requires the pooling contracts, which are offered by a supplier, who really did not receive any signal to distort the quantity for a retailer with an $L$ signal such that it is no longer favourable for a supplier with an $L$ signal to pretend that he did not receive any private information. This additional distortion makes it credible for the supplier without a signal that he did not receive an $L$ signal. Thus, the supplier without a signal faces a similar problem as the supplier with an $H$ signal in our main model.

Concerning the supplier’s decision to get informed, we find that—compared to the main model—he decides to stay uninformed for relatively high $\overline{\tau}$. This allows the supplier to avoid the risk of receiving no signal, which would require him to offer distorted pooling contracts, while he can offer regular pooling contracts, if he does not acquire any information.
B) Contract Predesign: Assume that the supplier is allowed to design the contracts before he learns his signal and can thereby commit to a specific menu of contracts. After the contracts are fixed and the supplier has learned his signal, he may report it to the retailer. We now have to distinguish two cases.

First, assume that \( \overline{\pi} \) is sufficiently small such that the information rent from being informed is so small that the supplier does not have an incentive to get informed. Then, the supplier will choose \( \nu_s^* = \frac{1}{2} \) and design pooling contracts as before. As the supplier’s signal is not informative, it does not matter, what he reports to the retailer.

Second, assume that \( \overline{\pi} \) is sufficiently large such that the supplier wants to offer separating contracts. To guarantee incentive compatibility, the supplier has to design 4 different contracts, one for each possible combination of \( \sigma_s \) and \( \sigma_r \). In order for the report of his signal to be credible, the contracts have to be incentive compatible to the supplier in the same way, as in the model above. However, as now the retailer has the possibility to choose from 4 instead of 2 contracts, incentive compatibility for the retailer is harder to achieve. As the highest quantity contract \( C_{HH} \) remains undistorted, the supplier can only guarantee incentive compatibility by further distorting the remaining contracts. Thus, he can never benefit from designing the contracts before his signal realization.

If, however, the supplier for some reason needs to do so, the range of \( \overline{\pi} \) for which the supplier prefers to offer pooling contracts is again larger than in the main model, as the incentive compatibility is more difficult to achieve with separating contracts. For example, if \( \alpha = 2/3 \), this leads the supplier to offer only pooling contracts regardless of \( \overline{\pi} \).

C) Sequential Decisions: Assume that the retailer selects the precision of his signal first and that the supplier observes this decision. Since the supplier’s profit function is convex in \( \nu_s \) (see lemma 4.4), he either chooses \( \nu_s = 1/2 \) or \( \nu_s = \overline{\pi} \). Comparing the retailer’s profits for \( \nu_s = 1/2 \) and \( \nu_s = \overline{\pi} \) reveals

\[
E\Pi^R(\nu_R, 1/2) > E\Pi^R(\nu_R, \overline{\pi}) \text{ for all } \overline{\pi} > 1/2, \tag{4.8}
\]

which implies that the retailer wants to choose \( \nu_R \) such that it induces the supplier to choose \( \nu_s = \frac{1}{2} \). We can define a critical \( \nu_R^c \), with \( \nu_R \leq \nu_R^c \) inducing \( \nu_s = \frac{1}{2} \), whenever feasible:

\[
\nu_R^c = \min \left\{ \nu_R \mid E\Pi^S(\overline{\pi}, \nu_R) - E\Pi^S(1/2, \nu_R) \geq 0, \ \overline{\pi} \right\}.
\]
Using the definition of $\nu^R$ in proposition 4.2 we therefore get that in the sequential move game, where the retailer decides first, the equilibrium is characterized by $\nu^R_F = 1/2$ and $\nu^R_R = \min(\nu^R_R, \nu^R_S)$. Thus, the retailer prefers the pooling contracts, whenever feasible, as the supplier can extract more rent, if he offers separating contracts. In addition, the downward distortion required to solve the supplier’s credibility problem is avoided, if pooling contracts are used.

Now assume that the supplier decides first on his signal precision. Using the retailer’s best response $\nu^S_F \in \{\nu^*_F, \nu^{**}_F\}$ and analyzing the supplier’s maximization problem, we get that in the sequential move game, where the supplier decides first, the equilibrium is characterized by $\nu^S_F = 1/2$ for $\alpha$ and $\theta$ sufficiently small; otherwise the supplier chooses $\nu^S_F = \theta$. The intuition is analogous to the main model, where the supplier has to decide, if he wants to avoid the credibility problem by choosing $\nu^S_F = 1/2$ or if he wants to solve it by distorting $x^L_H$. In the latter case, the supplier chooses $\nu^S_F = \theta$. However, if $\alpha$ is sufficiently small such that $\nu_1(\alpha) > \nu_2(\alpha)$ (as in Figure 4.2), the supplier is now able to select the equilibrium, which he prefers. In our model this leads the supplier to choose $\nu^S_F = 1/2$. Thus, if the supplier decides first, pooling contracts are offered more often.

### 4.6 Conclusion

We have analyzed the incentives of a supplier and a retailer to acquire more accurate information about actual market conditions. Using a simple model we show that choices on information acquisition are strategic complements and that the supplier chooses to get informed, only if the information of the retailer is sufficiently precise even though the cost of information acquisition is 0.

If the maximum signal precision is sufficiently high, the retailer’s information rent from acquiring additional information is relatively high as well. Thus, the supplier has stronger incentives to get informed as well and appropriate some of the information rent. If the maximum signal precision is sufficiently small, the retailer’s information rent is rather small too. Thus, it becomes more attractive for the supplier to solve the credibility problem, which he faces, if he receives a signal for a high price, by not being informed and choosing pooling contracts.
4.7 Appendix

Proof of Lemma 4.1 Solving \( E_{HL}(C_{HL}) = E_{HL}(C_{LL}) \) and \( E_{LL}(C_{LL}) = 0 \) for \( T_{HL} \) and \( T_{LL} \), substituting in \( E_{L}(C_{HL}, C_{LL}) \) and maximizing with respect to \( x_{HL} \) and \( x_{LL} \) leads to the stated results.

Proof of Lemma 4.2 Solving \( E_{HH}(C_{HH}) = E_{HH}(C_{LH}) \) and \( E_{LH}(C_{LH}) = 0 \) for \( T_{HH} \) and \( T_{LH} \) and substituting in \( E_{H}(C_{HH}, C_{LH}) \) we get

\[
\frac{\partial E_{HH}}{\partial x_{HH}} = \frac{1 - \nu_{R} - (1 - 2\nu_{R})\nu_{S}}{\nu_{R} + (1 - 2\nu_{R})\nu_{S}} \frac{\partial}{\partial x_{HH}} \Delta(C_{H}, C_{L}^{*})
\]

Hence, the optimal quantity \( x_{HH}^{*} \) is given by

\[
x_{HH}^{*} = \alpha + \mu_{HH}(1 - \alpha)
\]

Turning to \( x_{LH} \) assume first that with \( x_{HH} = x_{HH}^{*} \) the constraint \( \Delta(C_{H}, C_{L}^{*}) \geq 0 \) is not binding. Then, \( \partial E_{HH} / \partial x_{LH} = 0 \) leads to \( \hat{x}_{LH}^{*} \).

Turning to the question whether \( \Delta(C_{H}, C_{L}^{*}) \geq 0 \) is binding, we use \( C_{L}^{*}, x_{HH}^{*} \) and define \( x_{LH}^{*} := \hat{x}_{LH}^{*} + \Delta_{LH}^{*} \). This allows us to write \( \Delta(C_{H}, C_{L}^{*}) \) as

\[
\Delta(C_{H}, C_{L}^{*})|_{x_{HH}=x_{HH}^{*}} = \Phi(\nu_{S}, \nu_{R}, \alpha) - \frac{(1 - 2\nu_{R})\nu_{S}(1 - \nu_{S}(3 - 2\nu_{S}))(1 - \alpha)}{(\nu_{R} + \nu_{S} - 2\nu_{R}\nu_{S})^{2}(1 - \nu_{S} - \nu_{R}(1 - 2\nu_{S}))} \Delta_{LH}
\]

\[-\frac{1}{2} [1 - \nu_{S} - \nu_{R}(1 - 2\nu_{S})] \Delta_{LH}^{2}
\]

where \( \Phi(\nu_{S}, \nu_{R}, \alpha) \) is given by

\[
\Phi(\nu_{S}, \nu_{R}, \alpha) = E_{H}(C_{H})|_{x_{HH}^{*}, x_{LH}=\hat{x}_{LH}^{*}} - \frac{(2\nu_{R} - 1)^{3}(1 - \nu_{S})^{2}\nu_{S}(1 - 2\nu_{S})(1 - \alpha)^{2}}{2(\nu_{R} + \nu_{S} - 2\nu_{R}\nu_{S})^{4}(1 - \nu_{S} - \nu_{R}(1 - 2\nu_{S}))^{2}}.
\]

Solving \( \Phi(\nu_{S}, \nu_{R}, \alpha) = 0 \) for \( \nu_{R} \) shows that there exists a unique solution \( \tilde{\nu}_{R}(\nu_{S}, \alpha) \in [1/2, 1] \) and that

\[
\Phi(\nu_{S}, \nu_{R}, \alpha) \leq 0 \iff \nu_{R} \leq \tilde{\nu}_{R}(\nu_{S}, \alpha).
\]
To determine the optimal quantity $x_{LH}$ for $\nu_R < \tilde{\nu}_R(\nu_S, \alpha)$, note that

\[
\frac{\partial}{\partial x_{LH}} \left[ E\Pi^S_L(C_L) - E\Pi^S_H(C_H) \right]_{x_{LH} = \tilde{x}_{LH}} < 0 \quad \text{and} \quad \frac{\partial^2}{\partial x_{LH}^2} \left[ E\Pi^S_L(C_L) - E\Pi^S_H(C_H) \right] = (1 - 2\nu_R)(1 - 2\nu_S) > 0
\]

Furthermore, note that

\[
\frac{\partial^2 E\Pi^S_H(C_H)}{\partial x_{LH}^2} = -\nu_S - \nu_R(1 - 2\nu_S) < 0
\]

Therefore we have that $E\Pi^S_H(C_H)$ as well as $\Delta(C_H, C^*_L)$ are symmetric around their maximum and minimum respectively. While there are two values for $\Delta_{LH}$, which satisfy $\Delta(C_H, C^*_L) = 0$, we choose the one, which puts less distortion on $x^*_LH$. As it can be shown that

\[
\arg\min_{x_{LH}} \Delta(C_H, C^*_L) > \arg\max_{x_{LH}} E\Pi^S_H(C_H),
\]

we know by symmetry of the functions that the optimal distortion $\Delta^*_{LH}$ is given by

\[
\Delta^*_{LH} = \min \left\{ \Delta_{LH} \mid \Delta(C_H, C^*_L)_{x_{LH} = \tilde{x}_{LH}^*_H, x_{LH} = \tilde{x}_{LH}^*_H + \Delta^*_{LH} = 0} \right\}
\]

Hence, it holds that $x^*_LH < \tilde{x}^*_LH$, whenever $\nu_R < \tilde{\nu}_R(\nu_S, \alpha)$.

**Proof of Lemma 4.3** The first part of the lemma follows from solving $\partial E\Pi^R(\nu_R, \nu_S) / \partial \nu_R = 0$ with $x^*_LH = \tilde{x}^*_LH + \Delta^*_{LH}$ for $\nu_S$ as a function $\nu^*_S(\nu_R, \alpha)$. Numerical calculations show that $\nu^*_S(\nu_R, \alpha)$ attains its maximum at $\nu_R \approx 0.83$ and $\alpha = 1/2$ and that this maximum is given by $\nu^*_S(0.83, 1/2) \approx 0.625$. Hence, for $\nu_S > 0.625$ there exists no solution $\nu^*_R(\alpha, \nu_S)$ such that $\partial E\Pi^R(\nu^*_R(\alpha, \nu_S), \nu_S) / \partial \nu_R = 0$ and $\nu^*_S(\alpha, \nu_S) \leq \tilde{\nu}_R(\nu_S, \alpha)$.

The second results follow from evaluating $\partial E\Pi^R(\nu_R, \nu_S) / \partial \nu_R$ for $\nu_R \neq \tilde{\nu}_R(\nu_S, \alpha)$ using $x^*_LH = \tilde{x}^*_LH + \Delta^*_{LH}$ and for $\nu_R \neq \tilde{\nu}_R(\nu_S, \alpha)$ using $x^*_LH = \tilde{x}^*_LH$. This leads to

\[
\lim_{\nu_R \neq \tilde{\nu}_R(\nu_S, \alpha)} \frac{\partial E\Pi^R(\nu_R, \nu_S)}{\partial \nu_R} > 0 \quad \text{and} \quad \lim_{\nu_R \neq \tilde{\nu}_R(\nu_S, \alpha)} \frac{\partial E\Pi^R(\nu_R, \nu_S)}{\partial \nu_R} > \lim_{\nu_R \neq \tilde{\nu}_R(\nu_S, \alpha)} \frac{\partial E\Pi^R(\nu_R, \nu_S)}{\partial \nu_R}.
\]
The final part of the lemma can be proved by solving $\tilde{x}^*_{LH} = 0$ for $\nu_R$ which leads to

$$\nu'_R = \frac{\nu_S(1 + \nu_S - 3(1 + \nu_S)\alpha) - \sqrt{(1 - \nu_S)\nu_S(1 - \alpha)(\nu_S(5 - \nu_S(1 - \alpha) + 3\alpha) - 4\alpha)}}{2(-1 + 2\nu_S)(\nu_S - (1 - \nu_S)\alpha)}.$$ 

Substituting into $\partial \Pi^R(\nu_R, \nu_S) / \partial \nu_R$ reveals

$$\lim_{\nu_R / \nu'_R} \frac{\partial \Pi^R(\nu_R, \nu_S)}{\partial \nu_R} < 0.$$

**Proof of Lemma 4.4**  The first two results follow from evaluating $\partial \Pi^S(\nu_R, \nu_S) / \partial \nu_S$ and $x^*_{LH} = \tilde{x}^*_{LH} + \Delta^*_{LH}$ for $\nu_S = 1/2$ and $x^*_{LH} = \tilde{x}^*_{LH}$ for $\nu_R > \tilde{\nu}_R(\nu_S, \alpha)$. Numerical calculations show that $\Pi^S(\nu_R, \nu_S)$ is strictly convex in $\nu_S$ as long as $x^*_{LH} > 0$.

**Proof of Proposition 4.1**  Starting with the retailer, note that

$$x^*_{LH} = \tilde{x}^*_{LH} + \Delta^*_{LH} \text{ for } \nu_S = 1 \text{ and } \Pi^R(\nu_R, 1) = 0.$$ 

Considering the supplier, we get

$$\Pi^S(1, \nu_R) > \Pi^S(\nu_S, \nu_R) \text{ for all } \nu_S \in [1/2, 1)$$

as long as $\nu_R$ is such that

$$\alpha^2 + 4\nu_R^2(3 - \alpha(5 - 2\alpha)) - (2\nu_R + 10\nu_R^3)(1 - \alpha)^2 \geq 1,$$

which can be guaranteed by $\nu_R > 0.708$ for all values of $\alpha$. According to lemma 4.1 and 4.2, the quantities in an equilibrium with $\nu'_S = 1$ are $x^*_{LL} = x^*_{HL} = \alpha$ and $x^*_{HH} = 1$ and are undistorted, while the credibility problem still leads to distortion in $x^*_{LH}$, which is

$$x^*_{LH}|_{\nu'_S = 1, \nu_R < 1} = 1 - \frac{\sqrt{\nu_R - \alpha^2}}{\nu_R}.$$ 

Still, if $\nu'_R = 1$ as well, it also holds that $x^*_{HH} = x^*_{LH}$.

**Proof of Proposition 4.2**  Using lemma 4.4, we can prove the first part of the proposition by comparing the supplier’s profits for $\nu_S = 1/2$ and for $\nu_S = \tau$ respectively—taking into account
the optimal decision of the retailer. Consider first $\nu_S = 1/2$. Then, $\partial E\Pi^R(\nu_R, 1/2)/\partial \nu_R = 0$ leads to

$$\nu'_R = \frac{5}{12} + \frac{1}{6(1 - \alpha)}$$

and

$$\nu''_R = \min \{\nu'_R, \overline{\nu}\} ,$$

as well as

$$E\Pi^S(1/2, \nu''_R) = \frac{1}{2}(2 + 5\nu''^2_R(1 - \alpha)^2 - (2 - \alpha)\alpha - 2\nu'_R(3 - \alpha(5 + 2\alpha))).$$

Solving $E\Pi^S(1/2, \nu''_R) = E\Pi^S(\overline{\nu}, \nu''_R)$ for $\overline{\nu}$ leads to $\nu_1(\alpha)$ and the graph shown in Figure 4.2.

Turning to second part of the proposition, the retailer’s best response $\nu^*_R = \min\{\overline{\nu}, \max\{\tilde{\nu}_R(\overline{\nu}, \alpha), \nu''_R\}\}$ is determined by whether $\tilde{\nu}_R(\overline{\nu}, \alpha)$ or $\nu''_R$ are feasible and by

$$\lim_{\nu_R \to \tilde{\nu}_R(\overline{\nu}, \alpha)} \frac{\partial E\Pi^R(\nu_R, \overline{\nu})}{\partial \nu_R} \begin{cases} < 0 & \Rightarrow \nu^{**}_R = \tilde{\nu}_R(\overline{\nu}, \alpha) \\ > 0 & \Rightarrow \nu^{**}_R = \nu''_R \end{cases}$$

Numerical calculations show that $E\Pi^R(\nu_R, \nu_S)$ is strictly concave in $\nu_R$ as long as $\nu_R \geq \tilde{\nu}_R(\overline{\nu}, \alpha)$ and that $\partial E\Pi^R(\nu_R, \overline{\nu})/\partial \nu_R$ is strictly positive for all $\nu_R \leq \tilde{\nu}_R(\overline{\nu}, \alpha)$ and $\nu_S \geq \nu_2(\alpha)$ (see also the proof of lemma 4.3). Hence, the best response of the retailer is given by $\nu^*_R$.

Considering the supplier, lemma 4.4 implies that the supplier’s best response is either $\nu_S^* = 1/2$ or $\nu_S^* = \overline{\nu}$. Solving

$$E\Pi^S\left(\frac{1}{2}, \nu^*_R\right) = E\Pi^S(\overline{\nu}, \nu^*_R)$$

for $\overline{\nu}$ leads to $\nu_2(\alpha)$ shown in Figure 4.2.
References


