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Introduction

Competition plays a central role in the functioning of markets, as it promotes innovation, productivity, and growth. To maintain efficient competition and to prevent anticompetitive behavior, authorities, like the European Commission, established competition laws. The European Commission is continuously working on improving its antitrust and merger regulations, so that all firms in EU markets compete equally and fairly. Two aspects covered by these regulations, particularly relevant for this thesis, are related to the R&D agreements between competing firms and to the horizontal partial acquisitions.

In 2010, the European Commission issued "block exemptions" regulations which provide an automatic exemption from competition law for certain types of R&D agreements. The reason is that the cooperation between competitors to jointly undertake and exploit R&D may promote technological and economic progress. Hence, collaborations between competitors on the investment level should be treated differently to anticompetitive agreements, like cartels or market-sharing, which are prohibited by Article 101 of the Treaty on the Functioning of the EU (TFEU).

When it comes to horizontal partial acquisitions, the European Commission is seeking to extend its power to review the acquisitions of non-controlling minority shareholdings. Its concern is that such partial acquisitions, especially those among competitors, may substantially lessen competition and escape scrutiny. In its "White Paper: Towards more effective EU merger control", the European Commission proposed particular changes to the EC Merger Regulation with respect to partial acquisitions.

Chapters 1 and 2 of this thesis aim to evaluate some aspects of the aforementioned changes and proposals of modifications to the EU competition laws,

whereas Chapter 3 addresses much broader scope of problems. Specifically, Chapter 1 studies potential anticompetitive effects of partial acquisitions among competitors. Chapter 2 analyses collaborations between competitors on an investment level. Chapter 3 focuses on the information transmission in the presence of information asymmetries. Given that Chapter 1 considers a situation where the outside firm may not be able to observe whether the acquiring firm can influence decisions of the target firm, the asymmetric information is crucial for both Chapter 1 and Chapter 3. The chapters are described in more detail below.

Chapter 1 studies anticompetitive issues that arise from partial acquisition between competitors. Under current EC Merger Regulation (ECMR), the European Commission has authority to inspect partial acquisitions in which acquiring firm attains control over target firm's decisions. The European Commission argues that non-controlling shareholdings between competitors can as well give significant influence to the acquirer and that this influence could be difficult to assess in practice. Hence, the parties that do not participate in the acquisition, such as outside firms, may not be able to observe it.

We consider a situation in which an acquiring firm can influence a target firm's strategic decisions with some probability. The results suggest that no matter whether this influence is observable by an outside firm or not, the partial acquisition can create a greater harm to competition than a merger when the acquisition share and the probability of decisive influence are high. However, when the influence is not observable, all firms in the market suffer reduction of profits, provided that the prices are lower. As a consequence, this leads to a higher social welfare.

Chapter 2 is based on joint work with Juliane Fudickar (Fudickar and Rakić (2016)) and it analyzes cooperation between competitors at the investment level. Several studies[1] show that supporting firms to collaborate in R&D with spillovers boosts R&D investment and consequently increases the social welfare. Those studies considered a symmetric setup. However, new market environments, in which established goods are in competition with innovative goods, may lead to asymmetries between firms. Therefore, we consider R&D investment with spillovers in a market where a multi-product firm competes with a single-product firm. In this environment we analyze whether investment incentives are higher under R&D

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[1]See, among others, D'Aspremont and Jacquemin (1988) and Kamien et al. (1992).

cooperation or competition to evaluate the effect of the new "block exemptions" regulation by the European Commission.

We show that this depends on the technology spillover and also on the degree of product differentiation. Namely, when the established and the innovative products are close substitutes, total R&D investment under cooperation will be lower than under competition even if the spillover is substantial. More specifically, R&D investment of the single-product firm may be higher under competition than under cooperation even if the spillover is large. Moreover, for medium spillovers and high product substitutability the multi-product firm also invests less under R&D cooperation.

Chapter 3 is based on joint work with Martin Pollrich and it focuses on strategic information transmission between an informed expert and an uninformed decision maker. Examples of such communication abound: before implementing a policy, politicians consult their advisors; or before deciding on an investment strategy, investors consult stockbrokers. Such interactions are often accompanied by a conflict of interest. As a consequence, the expert may have the incentive to mislead the information. Previous literature shows that, due to the conflict of interest, the full revelation of information is impossible.[2] However, when the preferences of the sender and the receiver are not too far apart, some information transmission can occur. Additionally, communication via a mediator[3] (interested or disinterested) can improve upon direct talk. In this chapter, we address the question as to what extent these results rely on the availability of the mediator. Therefore, we study the robustness of strategic communication between an informed expert and an uninformed principal via a mediator.

We assume that with some probability the mediator is not available for communication. In this case the expert is forced to talk directly to the principal. We show that when the probability of absence is not too large, there exists an equilibrium of this perturbed game, that is outcome equivalent to the one with the mediator.

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[2]See, for example, Crawford and Sobel (1982).
[3]See Myerson (1986), Forges (1986), Goltsman et al. (2009), and Ivanov (2010a) among others.

Chapter 1

Horizontal partial acquisition with unobservable control

1.1 Introduction

When buying shares in a target firm, an acquiring firm obtains cash-flow rights whereby it incorporates profit of the target firm in proportion to the acquisition share. In addition to this financial interest, the acquiring firm may also obtain corporate rights so that it can control the strategic decisions of the target firm. A non-controlling partial acquisition is called a "structural link", whereas a partial acquisition where control is specifically granted to the acquirer is called a "concentration". While the European Commission has a jurisdiction to inspect concentrations, there is an ongoing debate whether structural links, at least those between competitors, should also belong to the scope of EC Merger Regulation (ECMR)[1]. One of the arguments is that partial acquisitions that do not qualify for concentration can still give significant influence to the acquirer and, more specifically, that this influence could be difficult to assess in practice. Hence, the parties that do not participate in the acquisition (e. g. outside firms and antitrust authorities) may not be able to observe it.

Such acquisitions among competitors are widespread in many different markets

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[1] In 2014, the European Commission has issued a "White Paper: Towards more effective EU merger control" where it proposed to broaden the scope of the EU Merger Regulation so it can also review the structural links between rivals.

and are seen even among historic rivals. Some examples include: the Irish low-cost airline Ryanair owns 29,8% of shares in its competitor Aer Lingus, Microsoft holds 7% of shares in its rival Apple Inc. and the biggest US producer of wet shaving razors, Gillette, owns 22,9% of shares in Wilkinson Sword.

To the best of our knowledge, the underlying assumption in the existing literature on the effects of the partial acquisition on competition is that the outside firms can observe the organizational structure between the firms involved in the acquisition. The goal of this chapter is to investigate the effects of partial acquisitions on competition and on the choice of an optimal acquisition share when the outside firm lacks such information. To address this question, we consider a differentiated market with three firms: an acquiring firm, a target firm and an outside firm. The acquiring firm owns a share in the target firm, whereas the outside firm neither owns shares in its competitors, nor is owned by either of them. Due to the difficulty to assess the level of influence of the acquiring firm on the target firm's strategic decisions, we assume that with some probability, the acquiring firm has control over the target firm and with the remaining probability it only has a financial interest. The firms involved in the acquisition know whether the acquirer controls the target or not, while the outside firm only observes the probability of decisive influence.

As a benchmark, we first analyze the case when the outside firm can observe whether the acquirer has corporate control over the target or not. We show that in this case, the partial acquisition can create greater harm to the competition than a total merger. Additionally, when the probability of total control is high and when the goods are not too close substitutes, the optimal acquisition share is smaller than 100%, i.e. total merger does not maximize the joint profit of the acquirer and the target. The reason is that the acquirer, when owning less than 100% of the target and having a total control over the target, sets the price of the target firm higher than the price of its own product, also higher than under merger, so that the demand for its own product increases. Given that the prices are strategic complements, an increase in the price of the target firm leads the outside firm to increase its price as well. Thus, partial acquisition in the case of total control serves as a strategic device to relax competition.

The situation is different when the outside firm cannot observe the partial

ownership arrangement between the firms taking part in the acquisition. In the case of total control, the acquiring firm cannot signal the outside firm that it would keep the price of the target firm high. As a result, the outside firm sets its price lower than in the case of complete information. The lack of transparency in the market, therefore, creates a reduction of profits to the firms, but it also lowers the anticompetitive concerns. Despite this positive welfare effect due to the asymmetric information, partial acquisition can still create greater harm to competition than the full merger. Additionally, when the probability of decisive influence and the product differentiation are high, the preferred acquisition share is again smaller than 100%.

The remainder of the chapter is organized as follows. Section 1.2 revises the related literature. The theoretical model is presented in Section 1.3. In Section 1.4 we analyze the benchmark case of complete information. In Section 1.5 we assume that the outside firm cannot observe whether the acquiring firm controls the decisions of the acquired firm or not and compute the optimal prices and acquisition share. Section 1.6 derives the welfare implications and Section 1.7 concludes. The proofs of all formal results are relegated to the Appendix.

1.2 Related literature

The importance of separation between the financial interest and corporate control in the context of horizontal partial acquisition is pointed out by Bresnahan and Salop (1986) and Salop and O'Brien (2000). They conduct a comprehensive analysis of different types of partial acquisitions in regard to the influence of the acquiring firm on the target firm's decisions. Bresnahan and Salop (1986) develop a Modified Herfindahl-Hirschman Index and Salop and O'Brien (2000) a Price Pressure Index to quantify anticompetitive effects arising from partial ownership. Missing from their studies is a formal analysis of presented acquisition scenarios on the competition.

Foros et al. (2011) study the effects of the partial acquisition on the product market outcomes and the optimal acquisition share, by assuming that the acquiring firm has the corporate control over the target firm. They use a Salop model with three firms and differentiated products. They, as we do, find that the partial

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ownership can lead to higher joint profits than the full merger. We extend their model by introducing a more general demand function. Moreover, we consider the anticompetitive effects of non-observability of the acquiring firm's influence on the target firm and derive the optimal acquisition share.

Reynolds and Snapp (1986) study the effects of pure financial interest on competition. They show that in a homogenous Cournot oligopoly such passive partial ownership can relax the competition between the rivals. Jovanovic and Wey (2014) show that a passive partial ownership can be used to get a full merger approved, which would be blocked in the first place. They call such a strategy a sneaky takeover.

Passive partial acquisition can also be multilateral. Malueg (1992) and Gilo et al. (2006) show that passive partial cross-ownership in rivals can trigger tacit collusion. Malueg (1992) considers a repeated Cournot duopoly with homogeneous good, while Gilo et al. (2006) examine a repeated Bertrand game with n symmetric firms that produce homogeneous goods. Both studies assume that the acquisition shares are exogenously given.

Karle et al. (2011) also allow for partial cross-ownership, but they focus on a static model and derive the optimal acquisition share endogenously. They consider a differentiated Bertrand competition with two firms and an investor that owns a stake in one firm and can acquire a stake in the second firm either directly or indirectly via the first firm. They show that the investor might not want to fully acquire both firms, hence a two product monopoly might not be preferred. In contrast, we do not allow for cross-ownerships but focus on the effects of market opacity on the product market outcomes and the optimal acquisition share.

There is also a growing literature on the competitive effects of the partial ownership agreements in vertically related markets. The passive acquisitions are considered by Hunold and Stahl (2016), Greenlee and Raskovich (2006), Fiocco (2016), Dasgupta and Tao (2000), Baumol and Ordover (1994), and Riordan (2008), while the acquisition of a controlling stake in a vertically related firm is studied by Gilo et al. (2016), Spiegel (2013) and Brito et al. (2016).

1.3 Model

Consider a market in which three firms, A, B and C, produce imperfect substitute goods. Demand for good $i \in I = \{A, B, C\}$ is given by the following function:

$$q_i(p_A, p_B, p_C) = \frac{1}{3}[1 - (1 + m)p_i + \frac{m}{3} \sum_{j \in I} p_j] \qquad (1.1)$$

where p_i and p_j are prices of goods produced by firms $i, j \in I = \{A, B, C\}$ and $m > 0$ represents the degree of product differentiation between the goods.[2]

Suppose firm A acquires a share α in the rival firm B. The outside firm C neither holds shares in its rival firms nor is owned by any of them. Thus, the profit functions are:

$$\Pi_A(p_A, p_B, p_C) = \pi_A(p_A, p_B, p_C) + \alpha \pi_B(p_A, p_B, p_C) \qquad (1.2)$$

$$\Pi_B(p_A, p_B, p_C) = (1 - \alpha)\pi_B(p_A, p_B, p_C) \qquad (1.3)$$

$$\Pi_C(p_A, p_B, p_C) = \pi_C(p_A, p_B, p_C) \qquad (1.4)$$

where $\pi_i(p_A, p_B, p_C) = p_i q_i(p_A, p_B, p_C)$ denotes the operating profit of firm $i \in \{A, B, C\}$. We assume that the unit cost of producing the goods is fixed and equalize it to zero.

After acquisition, with some probability the acquiring firm A can have a pure financial interest in the target firm B, so that it only incorporates the profits generated by firm B in proportion to the share acquired. With the remaining probability it can additionally have corporate rights, so that it influences firm B's strategic decisions. We distinguish between two cases, the first is a benchmark case wherein the outside firm C can observe whether the acquiring firm has an influence on the target firm's strategic decisions or not. Second, we assume that the partial ownership arrangement is private information to firms A and B.

We study the following two-stage game for both cases. First, firms A and B decide about the acquisition share α to maximize their joint profit. The firms then compete in the second stage à la Bertrand and set prices simultaneously. We solve

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[2]Derived from the utility function of a representative consumer (Shubik and Levitan (1980)): $U(q_A, q_B, q_C) = \sum_{i \in I} q_i - \frac{3}{2(1+m)}[\sum_{i \in I} q_i^2 + \frac{m}{3}(\sum_{i \in I} q_i)^2]$, where $I = \{A, B, C\}$ and $m > 0$.

for the equilibria by backward induction.

1.4 Benchmark case of complete information

We first assume that the outside firm can observe the organizational structure between the firms taking part in the acquisition.

1.4.1 Price competition

In the second stage, firms compete simultaneously in the product market given the acquisition share α. We first consider the situation when the acquiring firm has only a pure financial interest in the target firm, which we call the *silent control scenario*. Firm A only incorporates the profits generated by firm B in the proportion of the shares acquired and has no impact on the pricing decision. Hence, the firms maximize their profits in the usual way:

$$\max_{p_i} \Pi_i(p_A, p_B, p_C) \tag{1.5}$$

Under silent control, from the first-order conditions $\partial(\pi_A + \alpha\pi_B)/\partial p_A = \partial\pi_B/\partial p_B = \partial\pi_C/\partial p_C = 0$ we obtain the best response functions:

$$R_A^s(p_B, p_C) = \frac{3 + m((1 + \alpha)p_B + p_C)}{6 + 4m} \tag{1.6}$$

$$R_B^s(p_A, p_C) = \frac{3 + m(p_A + p_C)}{6 + 4m} \tag{1.7}$$

$$R_C^s(p_A, p_B) = \frac{3 + m(p_A + p_B)}{6 + 4m} \tag{1.8}$$

All reaction functions are upward sloping so that an increase in a competitor's price leads to a rise in one's own price.

By solving the best response functions, we calculate the equilibrium prices [3] as

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[3] The second-order conditions for a maximum are satisfied: $\partial^2(\pi_A + \alpha\pi_B)/\partial p_A^2 = \partial^2\pi_B/\partial p_B^2 = \partial^2\pi_C/\partial p_C^2 = -2(3 + 2m)/9 < 0$ for every $m > 0$.

functions of the acquisition share α:

$$p_A^s(\alpha) = \frac{3(6 + (5 + \alpha)m)}{36 + m(42 + (10 - \alpha)m)} \tag{1.9}$$

$$p_B^s(\alpha) = p_C^s(\alpha) = \frac{3(6 + 5m)}{36 + m(42 + (10 - \alpha)m)} \tag{1.10}$$

We next consider the case when firm A has a decisive influence over firm B, which we call the *total control scenario*. Hence, firm A decides not only about the price of its own product, but also about the price of the product of the target firm. Thus, the firms face the following maximization problems:

$$\max_{p_A, p_B} \Pi_A(p_A, p_B, p_C) \tag{1.11}$$

$$\max_{p_C} \Pi_C(p_A, p_B, p_C) \tag{1.12}$$

From the first-order conditions $\partial(\pi_A + \alpha\pi_B)/\partial p_A = \partial(\pi_A + \alpha\pi_B)/\partial p_B = \partial\pi_C/\partial p_C = 0$ we obtain the best response functions:

$$R_A^t(p_C) = \frac{\alpha(6 + (5 + \alpha)m)(3 + mp_C)}{36\alpha + 48\alpha m + ((14 - \alpha)\alpha - 1)m^2} \tag{1.13}$$

$$R_B^t(p_C) = \frac{(m + \alpha(6 + 5m))(3 + mp_C)}{36\alpha + 48\alpha m + ((14 - \alpha)\alpha - 1)m^2} \tag{1.14}$$

$$R_C^t(p_A, p_B) = \frac{3 + m(p_A + p_B)}{6 + 4m} \tag{1.15}$$

which are all upward sloping.

By solving the relevant best response functions, we obtain the equilibrium prices[4] as functions of α:

$$p_A^t(\alpha) = \frac{3\alpha(6 + 5m)(6 + (5 + \alpha)m)}{D_1(\alpha)} \tag{1.16}$$

$$p_B^t(\alpha) = \frac{3(6 + 5m)(m + \alpha(6 + 5m))}{D_1(\alpha)} \tag{1.17}$$

$$p_C^t(\alpha) = \frac{36\alpha(1 + m)(3 + 2m)}{D_1(\alpha)} \tag{1.18}$$

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[4]The second-order conditions for a maximum are satisfied: $\partial^2(\pi_A + \alpha\pi_B)/\partial p_A^2 = \partial^2\pi_C/\partial p_C^2 = -2(3 + 2m)/9 < 0$ and $\partial^2(\pi_A + \alpha\pi_B)/\partial p_B^2 = -2\alpha(3 + 2m)/9 < 0$ for every $m > 0$ and $0 < \alpha < 1$.

where $D_1(\alpha)$ is positive[5] for all $0 < \alpha < 1$ and $m > 0$.

If a merger between firms A and B occurs, we assume for convenience that firm A acquires all of the shares of firm B, hence, $\alpha = 1$, and call it a *merger scenario*. From (1.16) - (1.18) it is then straightforward to calculate the equilibrium prices under the merger scenario:

$$p_A^m = p_B^m = \frac{6 + 5m}{2(6 + m(6 + m))} \tag{1.19}$$

$$p_C^m = \frac{3 + 2m}{6 + m(6 + m)} \tag{1.20}$$

In addition, we also derive competitive prices before any acquisition takes place. By inserting $\alpha = 0$ into (1.9)-(1.10), we obtain:

$$p_A^c = p_B^c = p_C^c = 3/(6 + 2m). \tag{1.21}$$

Before deriving the optimal acquisition share α, we examine how the acquisition share α affects prices in the second stage by differentiating expressions (1.9)-(1.10) and (1.16)-(1.18) with respect to α. Under silent control, the prices always increase in α:

$$\frac{\partial p_i^s}{\partial \alpha} > 0, \, i \in \{A, B, C\}. \tag{1.22}$$

By acquiring a higher share in its rival, firm A has a higher financial interest in firm B. Hence, to make the product of firm B more attractive, it increases the price of its own good. As prices are strategic complements, an increase in the price of good A, leads firms B and C to increase their prices, too.

One would imagine that an increase in the acquisition share under the total control scenario would have a similar effect on prices. However, the analysis of this case is much more involved. When owning only a small share in firm B, firm A does not care much about the profit of firm B as it only has a small financial interest in it. Hence, firm A finds it profitable to set the price of good B very high because it leads to a boost in demand for its own good. By doing so, firm A sends a credible signal to firm C to also keep the price of its product high. However, as

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[5]The denominator of the prices under total control scenario is equal to: $D_1(\alpha) = 216\alpha + 432\alpha m + 6((44 - \alpha)\alpha - 1)m^2 + (\alpha(46 - 5\alpha) - 5)m^3 > 0$ for $0 < \alpha < 1$ and $m > 0$.

firm A increases the share in its rival, the financial interest of firm A in the firm B's profit increases and firm A cares more about firm B's profit and thus decreases the price of good B. Consequently, the prices of firms B and C are decreasing in α:

$$\frac{\partial p_i^t}{\partial \alpha} < 0, \, i \in \{B, C\}. \tag{1.23}$$

If firm A owns only a small share in the rival firm B and has total control over firm B's strategic decisions, it might completely shut down firm B. That is, firm A might find it profitable to set the price of firm B so high, such that firm B does not sell any of its good. To ensure positive quantities, we need to restrict our attention to $\alpha > m/(6 + 3m)$.

Lastly, the price of the good of firm A decreases for small α, as firm A collects a higher profit from selling its own good. However, as the financial interest in the rival firm B increases, firm A collects a higher share of firm B's operating profit. Thus, it is then beneficial for firm A that the profit of firm B is higher. Therefore, with higher α firm A increases its own price, so it makes good B more attractive. Thus:

$$\frac{\partial p_A^t}{\partial \alpha} = \begin{cases} < 0, & \text{if } \alpha < \bar{\alpha} \\ > 0, & \text{if } \alpha > \bar{\alpha} \end{cases} \tag{1.24}$$

We summarize the previous observations in the following proposition.

Proposition 1.1 *Under silent control:*

(i) $p_A^s(\alpha)$, $p_B^s(\alpha)$, $p_C^s(\alpha)$ increase in α for every $0 < \alpha < 1$

Under total control:

(ii) $p_A^t(\alpha)$ decreases in α, when $\alpha < \bar{\alpha}$ and increases in α otherwise[6]

(iii) $p_B^t(\alpha)$ and $p_C^t(\alpha)$ decrease in α for every $m/(6 + 3m) < \alpha < 1$.

We next compare obtained equilibrium prices and present them in Figure 1.1. If an acquiring firm has total control over the target firm, it increases the price of

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[6]The threshold $\bar{\alpha}$ is equal to: $\bar{\alpha} = ((6 + 5m)m^2 + 6(216m + 828m^2 + 1254m^3 + 937m^4 + 345m^5 + 50m^6)^{1/2})/(216 + 468m + 324m^2 + 71m^3)$
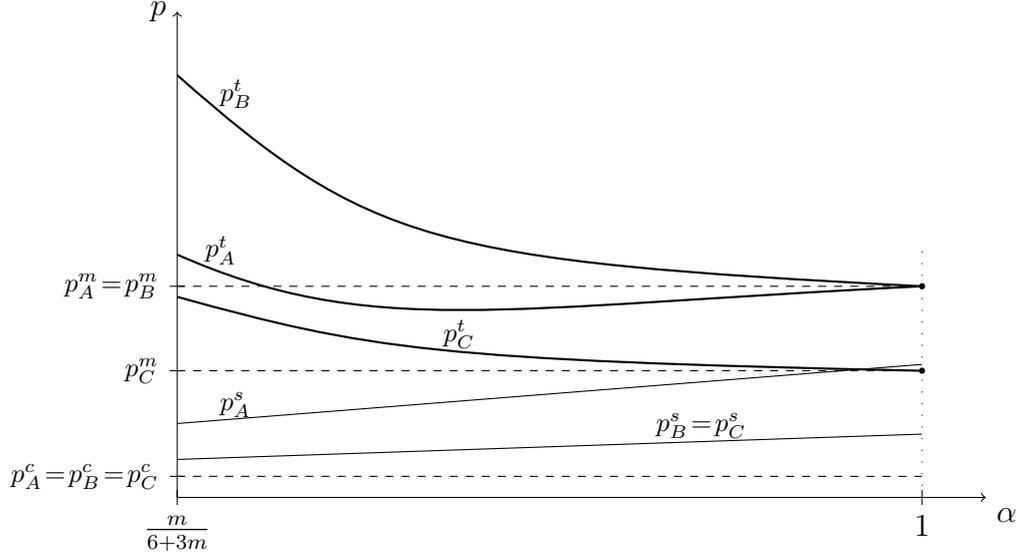
Figure 1.1: Comparison between prices for $m = 30$

the target's product above the merger price. In addition, it keeps its own price lower than the target's price and also lower than the merger price, so that it increases demand for its own product. The outside firm reacts to the acquisition by increasing its price, which is a positive strategic effect for the firms engaged in the acquisition. Under the silent control scenario, all three firms raise their prices higher than competitive prices, but lower than the merger prices. In contrary to the total control scenario, here the acquiring firm increases its own price higher than the target firm's price.

1.4.2 Acquisition

After solving the second stage of the game by deriving the equilibrium prices, we next solve the first stage by calculating the optimal acquisition share α.

In what follows, we assume that with probability $\beta_n(\alpha)$ the acquiring firm has a total control over the target firm and with probability $1 - \beta_n(\alpha)$ only the silent control. It is natural to assume that when $\alpha = 0$, firm A has no influence on firm B, hence $\beta_n(0) = 0$ and when $\alpha = 1$ the merger occurs and firm A fully controls firm B, hence $\beta_n(1) = 1$. We further assume that $\beta_n(\alpha)$ increases in α and that it

has the following functional form:

$$
\beta_n(\alpha) = \begin{cases} \frac{(2\alpha)^{2n}}{2}, & \text{if } \alpha \leq \frac{1}{2} \\ 1 - \frac{(2\alpha-2)^{2n}}{2}, & \text{if } \alpha \geq \frac{1}{2} \end{cases} \tag{1.25}
$$

Parameter $n \in \mathbb{N}$ controls the steepness of the curve. Specifically, the higher the parameter n, the steeper the curve at $1/2$ and the flatter at the ends of the interval. The curve has flat tangents at 0 and 1. Moreover, this function is continuous and differentiable on the whole support. We choose this functional form for $\beta_n(\alpha)$ because when $n \to \infty$, it converges to:

$$
\beta_\infty(\alpha) = \begin{cases} 0, & \text{if } \alpha < \frac{1}{2} \\ \frac{1}{2}, & \text{if } \alpha = \frac{1}{2} \\ 1, & \text{if } \alpha > \frac{1}{2} \end{cases} \tag{1.26}
$$

which represents the standard 50% rule that is typically assumed in the corporate finance literature. That is, when the acquiring firm buys less than 50% of the shares in its target, it has no corporate influence and when it buys more than 50% of the shares, it fully controls the target firm. Figure 1.2 illustrates function $\beta_n(\alpha)$ for $n = 1$ and $n \to \infty$.
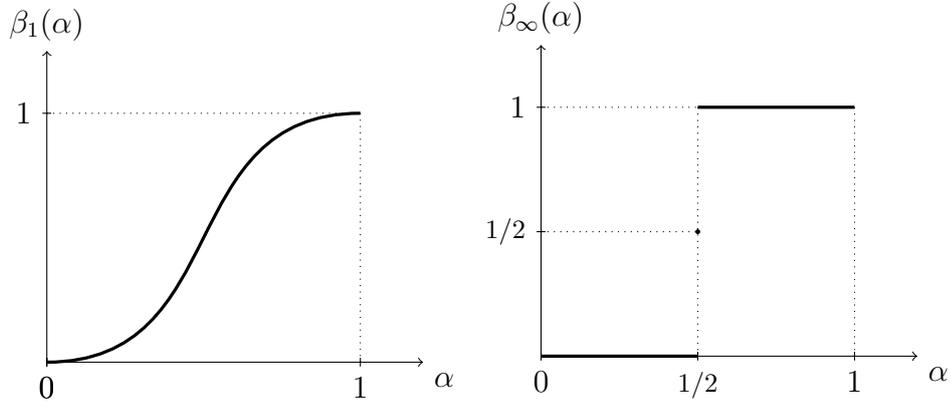

Figure 1.2: Probability of total control for $n = 1$ (left panel) and $n \to \infty$ (right panel)

Given the equilibrium prices, we next solve the first stage of the game by calculating the optimal acquisition share α such that the expected joint profit of

firms A and B is maximized:

$$\max_{\alpha} \Pi_{AB}^{b} = \beta_n(\alpha)(\pi_A^t + \pi_B^t) + (1 - \beta_n(\alpha))(\pi_A^s + \pi_B^s) \tag{1.27}$$

where $\pi_A^t = \pi_A(p_A^t(\alpha), p_B^t(\alpha), p_C^t(\alpha))$ and $\pi_B^s = \pi_B(p_A^s(\alpha), p_B^s(\alpha), p_C^s(\alpha))$.

The respective first order condition of joint profit is given by:

$$
\begin{aligned}
\frac{d\Pi_{AB}^{b}}{d\alpha} =& \beta_n(\alpha) \sum_{i \in \{A,B\}} \sum_{j \in \{A,B,C\}} \frac{\partial \pi_i}{\partial p_j} \frac{\partial p_j^t}{\partial \alpha} + (1 - \beta_n(\alpha)) \sum_{i \in \{A,B\}} \sum_{j \in \{A,B,C\}} \frac{\partial \pi_i}{\partial p_j} \frac{\partial p_j^s}{\partial \alpha} \\
& + \frac{\partial \beta_n}{\partial \alpha} \left(\pi_A^t + \pi_B^t - \pi_A^s - \pi_B^s \right)
\end{aligned}
\tag{1.28}
$$

We first check whether a merger between firms A and B maximizes their joint profit. If the joint profit would be maximized when the merger occurs, the first order condition of the joint profit (1.28) would have to be equal to zero when evaluated at $\alpha = 1$ and the second order condition would have to be negative. Then $\alpha = 1$ implies $\beta_n(1) = 1$ and $1 - \beta_n(1) = 0$, moreover $\partial \beta_n(\alpha)/\partial \alpha = 0$ when evaluated at $\alpha = 1$. Hence, equation (1.28) simplifies to:

$$
\frac{d\Pi_{AB}^{b}}{d\alpha} = \frac{\partial \pi_A}{\partial p_A} \frac{\partial p_A^t}{\partial \alpha} + \frac{\partial \pi_A}{\partial p_B} \frac{\partial p_B^t}{\partial \alpha} + \frac{\partial \pi_A}{\partial p_C} \frac{\partial p_C^t}{\partial \alpha} + \frac{\partial \pi_B}{\partial p_A} \frac{\partial p_A^t}{\partial \alpha} + \frac{\partial \pi_B}{\partial p_B} \frac{\partial p_B^t}{\partial \alpha} + \frac{\partial \pi_B}{\partial p_C} \frac{\partial p_C^t}{\partial \alpha}
\tag{1.29}
$$

Recall that $\partial(\pi_A + \alpha \pi_B)/\partial p_A = 0$ when evaluated in the optimum, p_A^t, and $\partial(\pi_A + \alpha \pi_B)/\partial p_B = 0$ when evaluated in p_B^t, as a result of profit maximization in the second stage. Therefore, from equation (1.28) we obtain:

$$
\frac{d\Pi_{AB}^{b}}{d\alpha} = (1 - \alpha) \left(\frac{\partial \pi_B}{\partial p_A} \frac{\partial p_A^t}{\partial \alpha} + \frac{\partial \pi_B}{\partial p_B} \frac{\partial p_B^t}{\partial \alpha} \right) + \frac{\partial p_C^t}{\partial \alpha} \left(\frac{\partial \pi_A}{\partial p_C} + \frac{\partial \pi_B}{\partial p_C} \right)
\tag{1.30}
$$

Subsequently, when $\alpha = 1$, the first term in equation (1.30) is also equal to zero and for our demand specification, we have $\partial p_C^t/\partial \alpha = 0$. We, thus, conclude that at $\alpha = 1$

$$\frac{d\Pi_{AB}^{b}}{d\alpha} = 0.$$

After a routine check of the second order conditions, we show that $\alpha = 1$ is a local minimum when the goods are not too close substitutes and when the probability of total control is high. The following proposition summarizes the

result.

Proposition 1.2 *Partial acquisition is more profitable than a merger when $m > 3(2 + \sqrt{6})$ and $n > 1$.*
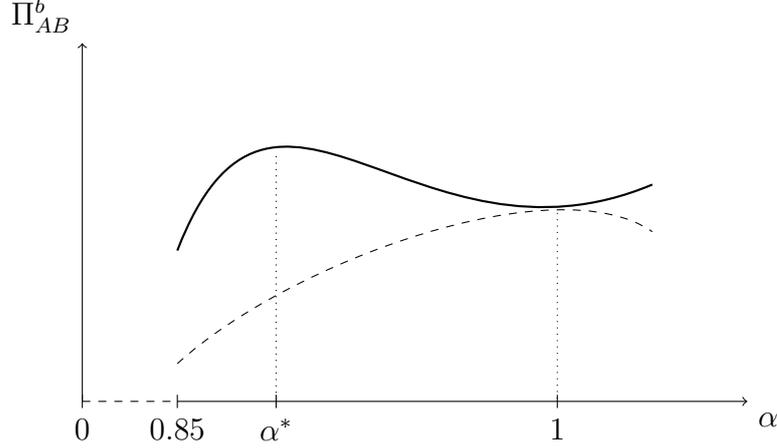
Figure 1.3: Firm A's and B's joint profit for $n = 1$ (dashed line), $n = 2$ (solid line) and $m = 100$

Figure 1.3 plots firm A's and B's joint profit function when $n = 1$ (dashed line) and $n = 2$ (solid line). Recall that, when $\alpha > 1/2$, the higher n indicates the higher probability of total control. We observe that when $n = 1$ the merger between firms A and B maximizes their joint profit, however, when $n \geq 2$, firms prefer partial ownership.

This result follows from the fact that when the acquiring firm owns less than 100% of the shares in the target firm and has total control over the target, the competition is even more relaxed than if the two firms would merge. Hence, if the probability of total control is high, the partial acquisition is preferred by the two firms in comparison to the merger.

Numerical example 1.1

Due to the analytical complexity, we calculate the optimal acquisition share α^* numerically. As known from Proposition 1.2 the merger is optimal when $n = 1$ and $m > 0$ and also when $n \geq 1$ and $m < m_1 = 3(2 + \sqrt{6})$. We, therefore, focus on $n > 1$ and $m > m_1$.

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In the first instance, we fix $m = 30$ and $n = 2$. Inserting $n = 2$ into equation (1.25), we obtain

$$\beta_2(\alpha) = \begin{cases} 8\alpha^4, & \text{if } \alpha \leq \frac{1}{2} \\ 1 - 8(\alpha - 1)^4, & \text{if } \alpha \geq \frac{1}{2} \end{cases} \tag{1.31}$$

By substituting $m = 30$ and $\beta_2(\alpha) = 1 - 8(\alpha - 1)^2$ into equation (1.28), we obtain the relevant first order condition of firm A's and B's joint profit that we next solve for α. Although we get 11 candidates for a solution, only two lie in the relevant interval $[1/2, 1]$. Those are $\alpha_1 = 1$ and $\alpha_2 \approx 0.928$. From Proposition 1.2, we know that $\alpha = 1$ is a local minimum, hence the optimal acquisition share is $\alpha^* \approx 0.928$. As the product differentiation m increases, the optimal α converges to 0.899.

Similarly, after inserting $m = 30$ and $\beta_2(\alpha) = 8\alpha^2$ into the first order condition (1.28) of firms A's and B's joint profit and solve for α, we again obtain 11 candidates for a solution, but none is in the relevant interval $[0, 1/2]$.
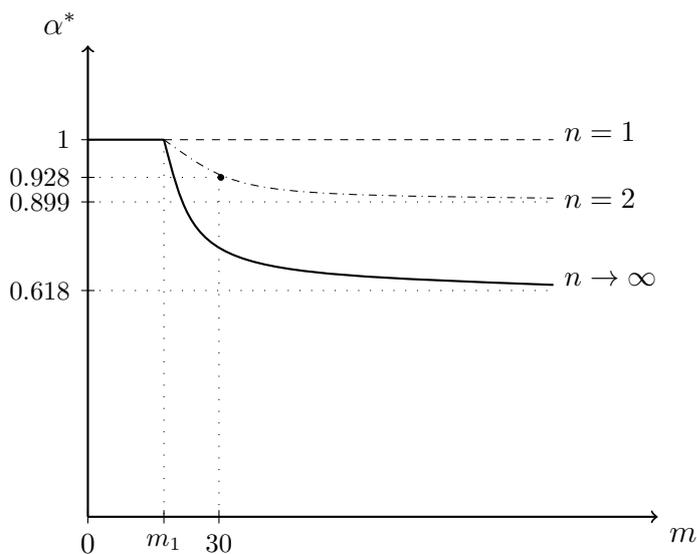
Figure 1.4: Optimal α under complete information

We repeat the previous analysis for different values of m and n and summarize the values of the optimal α in Figure 1.4. The dashed line represents the optimal α when $n = 1$, the dash-dotted line characterizes the optimal α when $n = 2$ and

the solid line when $n \to \infty$. We observe that the higher the probability of total control, n, and the higher the level of product differentiation, m, the lower the acquisition share α is necessary to maximize the joint profit.

1.5 Case of asymmetric information

We have so far focused on the case when the outside firm can observe whether the acquiring firm can control the corporate decision of the target firm. However, given that it may be difficult to assess the level of influence of the acquiring firm on the target, we next assume that the outside firm cannot observe if the firm A has a control over firm B's pricing decisions or not. In this section, we, therefore, investigate how asymmetric information influences the firms' market outcomes and the optimal acquisition share.

1.5.1 Price competition

Firm C cannot observe whether firm A has control over firm B's pricing decisions and knows only the probability $\beta_n(\alpha)$. With probability $\beta_n(\alpha)$ firm A has *total control* over pricing decisions of firm B and it chooses p_{A1} and p_{B1} to maximize

$$\max_{p_{A1}, p_{B1}} \pi_A(p_{A1}, p_{B1}, p_C) + \alpha \pi_B(p_{A1}, p_{B1}, p_C) \tag{1.32}$$

On the other hand, with probability $1 - \beta_n(\alpha)$ firm A has only a *silent control* over firm B, meaning, firm A chooses price p_{A2} and firm B price p_{B2} to maximize:

$$\max_{p_{A2}} \pi_A(p_{A2}, p_{B2}, p_C) + \alpha \pi_B(p_{A2}, p_{B2}, p_C) \tag{1.33}$$

$$\max_{p_{B2}} (1 - \alpha) \pi_B(p_{A2}, p_{B2}, p_C) \tag{1.34}$$

Firm C does not know if firm A has a total control over firm B and hence it chooses p_C to maximize its expected profit:

$$\max_{p_C} E\pi_C = \beta_n(\alpha) \pi_C(p_{A1}, p_{B1}, p_C) + (1 - \beta_n(\alpha)) \pi_C(p_{A2}, p_{B2}, p_C) \tag{1.35}$$

From the first order conditions $\partial(\pi_A + \alpha \pi_B)/\partial p_{A1} = \partial(\pi_A + \alpha \pi_B)/\partial p_{B1} =$

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$\partial(\pi_A + \alpha\pi_B)/\partial p_{A2} = \partial\pi_B/\partial p_{B2} = \partial E\pi_C/\partial p_C = 0$, we derive the following reaction functions:

$$R_{A1}(p_C) = \frac{\alpha(6 + (5 + \alpha)m)(3 + mp_C)}{F(\alpha)} \tag{1.36}$$

$$R_{B1}(p_C) = \frac{(m + \alpha(6 + 5m))(3 + mp_C)}{F(\alpha)} \tag{1.37}$$

$$R_{A2}(p_{B2}, p_C) = \frac{3 + m((1 + \alpha)p_{B2} + p_C)}{6 + 4m} \tag{1.38}$$

$$R_{B2}(p_{A2}, p_C) = \frac{3 + m(p_{A2} + p_C)}{6 + 4m} \tag{1.39}$$

$$R_C(p_{A1}, p_{B1}, p_{A2}, p_{B2}) = \frac{3 + m(\beta_n(\alpha)(p_{A1} + p_{B1}) + (1 - \beta_n(\alpha))(p_{A2} + p_{B2}))}{6 + 4m} \tag{1.40}$$

where $F(\alpha) = 36\alpha + 48\alpha m + ((14 - \alpha)\alpha - 1)m^2$ is positive for all $0 < \alpha < 1$ and $m > 0$. All reaction functions are upward sloping.

From the reaction functions (1.36) - (1.40) we obtain the equilibrium prices[7] as functions of the acquisition share α. With the following simplification $G(\alpha) = 36 + m(48 + (15 - \alpha)m)$, the equilibrium prices of firms A and B have the following forms:

$$p_{A1}^*(\alpha) = \frac{3\alpha(6 + 5m)(6 + (5 + \alpha)m)G(\alpha)}{D_2(\alpha)} \tag{1.41}$$

$$p_{B1}^*(\alpha) = \frac{3(6 + 5m)(m + \alpha(6 + 5m))G(\alpha)}{D_2(\alpha)} \tag{1.42}$$

$$p_{A2}^*(\alpha) = \frac{3(6 + 5m)(6 + (5 + \alpha)m)F(\alpha)}{D_2(\alpha)} \tag{1.43}$$

$$p_{B2}^*(\alpha) = \frac{3(6 + 5m)^2 F(\alpha)}{D_2(\alpha)} \tag{1.44}$$

$D_2(\alpha)$[8] is positive for all $0 < \alpha < 1$ and $m > 0$.

---

[7] The second-order conditions for a maximum are satisfied: $\partial^2(\pi_A + \alpha\pi_B)/\partial p_{A1}^2 = \partial^2(\pi_A + \alpha\pi_B)/\partial p_{A2}^2 = \partial^2 E\pi_C/\partial p_C^2 = -2(3 + 2m)/9 < 0$, $\partial^2(\pi_A + \alpha\pi_B)/\partial p_{B1}^2 = -2\alpha(3 + 2m)/9 < 0$ and $\partial^2\pi_B/\partial p_{B2}^2 = 2(-1 + \alpha)(3 + 2m)/9 < 0$ for every $m > 0$ and $0 < \alpha < 1$.

[8] The denominator of the optimal prices is $D_2(\alpha) = \alpha^3 m^4(6 + 5m) - m^2(6 + 5m)^2(6 + (2 + \beta_n(\alpha))m) - \alpha^2 m^2(432 + m(900 + m(594 + (120 + \beta_n(\alpha))m))) + \alpha(6 + 5m)(1296 + m(3240 + m(2880 + m(1068 + (141 - 2\beta_n(\alpha))m))))$.

While the equilibrium price of firm C is equal to:

$$p_C^*(\alpha) = (3(1 - \beta_n(\alpha))m^2(6 + 5m)^2 + 3\alpha^2 m^2(36 + m(60 + (25 - \beta_n(\alpha))m)$$
$$+ 6\alpha(6 + 5m)(108 + m(234 + m(162 + (35 + \beta_n(\alpha))m)))))/D_2(\alpha) \tag{1.45}$$

.

Before deriving the optimal share α, we first examine how α influences the equilibrium prices by differentiating expressions (1.41)-(1.45) with respect to α. Note that when the acquisition share is small and firm A has corporate control over firm B's decisions, firm A would completely shut down the production of firm B. Therefore, we restrict our attention on the area where, $q_{B1}^* > 0$, that is when $\alpha > m/(6 + 3m)$.

Under *total control* scenario, the responses of prices p_{A1}^* and p_{B1}^* to α are analogous to the responses of prices p_A^t and p_B^t in the benchmark case, namely:

$$\frac{\partial p_{A1}^*}{\partial \alpha} = \begin{cases} < 0, & \text{if } \alpha < \hat{\alpha} \\ > 0, & \text{if } \alpha > \hat{\alpha} \end{cases} \tag{1.46}$$

$$\frac{\partial p_{B1}^*}{\partial \alpha} < 0, \text{ for } \alpha > \frac{m}{6 + 3m} \tag{1.47}$$

Under *silent control* scenario, the responses of p_{A2}^* and p_{B2}^* to α are the same as the responses of p_A^s and p_B^s:

$$\frac{\partial p_{i2}^*}{\partial \alpha} > 0, \text{ for } \alpha > \frac{m}{6 + 3m}, i \in \{A, B\} \tag{1.48}$$

However, given the fact that firm C does not know if firm A has a decisive influence over firm B, it cannot engage in the collusive behavior as in the case of complete information. Therefore,

$$\frac{\partial p_C^*}{\partial \alpha} = \begin{cases} > 0, & \text{if } m/(6 + 3m) < \alpha < \tilde{\alpha}, n \geq 2 \text{ and } 0 < \alpha < 1, n = 1 \\ < 0, & \text{if } \alpha > \tilde{\alpha}, n \geq 2 \end{cases} \tag{1.49}$$

We summarize the previous observations in the following proposition.

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Proposition 1.3 *For every* $m > 0$:

(i) *there exists* $\hat{\alpha} \in (m/(6 + 3m), 1)$ *such that,* $p^*_{A1}(\alpha)$ *decreases in* α, *when* $\alpha < \hat{\alpha}$ *and* $n \geq 1$, *and increases in* α *when* $\alpha > \hat{\alpha}$ *and* $n \geq 1$

(ii) $p^*_{B1}(\alpha)$ *decreases in* α *when* $m/(6 + 3m) < \alpha < 1$ *and* $n \geq 1$

(iii) $p^*_{A2}(\alpha)$ *and* $p^*_{B2}(\alpha)$ *increase in* α *when* $m/(6 + 3m) < \alpha < 1$ *and* $n \geq 1$

(iv) *there exists* $\tilde{\alpha} \in (m/(6+3m), 1)$ *such that,* $p^*_C(\alpha)$ *increases in* α, *when* $\alpha < \tilde{\alpha}$ *and* $n \geq 2$ *and also when* $0 < \alpha < 1$ *and* $n = 1$, *and decreases in* α *when* $\alpha > \tilde{\alpha}$ *and* $n \geq 2$.

Figure 1.5 present the comparison between obtained equilibrium prices.
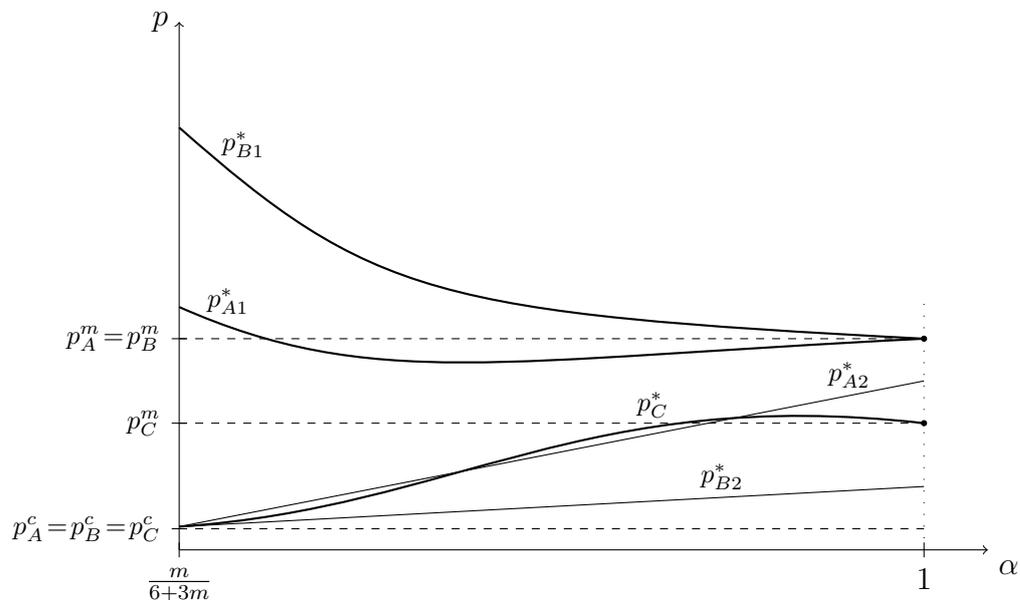
Figure 1.5: Comparison between prices under asymmetric information for $n = 2$ and $m = 100$

As in the case of complete information, the acquiring firm, when having a total control over the target, raises the price of the target product higher than its own price and also higher than the merger price. On the other hand, it also keeps its own price lower than the merger price. In comparison to the benchmark case,

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firm C sets its price lower than p_C^t and higher than p_C^s. As a response, firm A when having only a silent control over the target sets its price higher than in the benchmark case $(p_{A2}^* > p_A^s)$.

Just by looking at prices, we cannot assess the overall influence of the asymmetric information on the welfare and profits. Therefore, we address this question separately in Section 1.6.

1.5.2   Acquisition

Given the equilibrium prices, we next solve the first stage of the game by calculating the optimal acquisition share α such that the expected joint profit of firms A and B is maximized:

$$\max_{\alpha} \Pi_{AB}^a = \beta_n(\alpha)(\pi_{A1} + \pi_{B1}) + (1 - \beta_n(\alpha))(\pi_{A2} + \pi_{B2}) \qquad (1.50)$$

where, for simplicity, $\pi_{Ai} = \pi_A(p_{Ai}^*(\alpha), p_{Bi}^*(\alpha), p_C^*(\alpha))$ and $\pi_{Bi} = \pi_B(p_{Ai}^*(\alpha), p_{Bi}^*(\alpha), p_C^*(\alpha))$, $i \in \{1, 2\}$.

When the merger occurs the joint profit function under complete information, Π_{AB}^b, coincides with Π_{AB}^a. Therefore, the result of Proposition 1.2 holds for the asymmetric case as well.

Proposition 1.4 *Under asymmetric information, the partial acquisition is more profitable than the merger when $n > 1$ and $m > 3(2 + \sqrt{6})$.*

Consider Figure 1.6, which plots firm A's and B's joint profit function under asymmetric information versus their joint profit function under complete information. We observe that the optimal α under asymmetric information is always higher than the correspondent α under complete information. It means that firm A under asymmetric information scenario has to obtain a higher share in order to maximize the joint profit. We confirm this in the following numerical example.
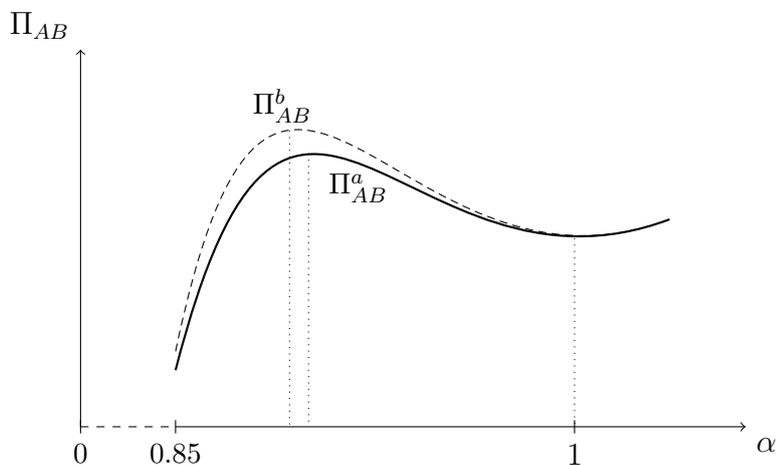
Figure 1.6: Firm A's and B's joint profit under benchmark case (dashed line) and the case of asymmetric information (solid line) for $n = 2$ and $m = 100$

Numerical example 1.2

As under complete information, we perform numerical calculations to obtain the optimal acquisition share α. When the goods are close substitutes, i.e. when $m < m_1$, the full acquisition $\alpha = 1$ maximizes the joint profit. While, when $m > m_1$ and $n \geq 2$ we have an interior solution.

We next calculate the optimal α for the same parameter constellation as in the Numerical example 1 which enable us to compare the optimal α under complete information versus asymmetric information. For $n = 2$ and $m = 30$, we have $\alpha^* \approx 0.932$. The following Figure 1.7 illustrates optimal level of α as a function of m, where the dashed line represents the optimal α under complete information and the solid line under the asymmetric information for $n = 2$. Under asymmetric information, firm A needs to buy a higher share in the rival firm B, in order to assure the joint profit maximization.
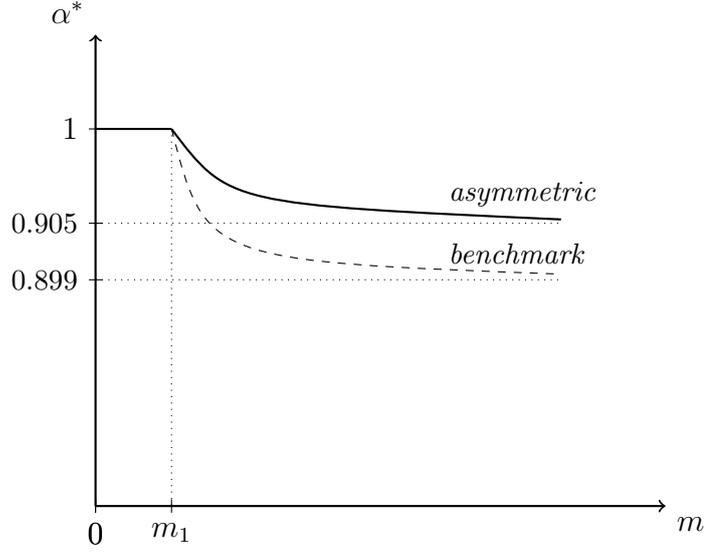
Figure 1.7: Optimal α under asymmetric information (solid line) and complete information (dashed line) for $n = 2$

1.6 Welfare effects

Antitrust authorities usually use a social welfare standard to evaluate the anti-competitive concerns arising from mergers and acquisitions. Therefore, in order to evaluate the welfare effects of the model at hand, we next compare the obtained social welfare under asymmetric information to the one under the benchmark case, and as well to the ones under merger and pre-acquisition.

Social welfare is equal to the utility function of the representative consumer:

$$SW = U(q_A, q_B, q_C) = \sum_{i \in I} q_i - \frac{3}{2(1+m)} \left[\sum_{i \in I} q_i^2 + \frac{m}{3} \left(\sum_{i \in I} q_i \right)^2 \right] \qquad (1.51)$$

where demand q_i, $i \in \{A, B, C\}$ is given by equation (1.1).

Before any acquisition takes place, the relevant demand is calculated by substituting competitive prices given by equation (1.21) into the equation (1.1):

$$q_i^c = q_i(p_A^c, p_B^c, p_C^c) = \frac{1}{3} \left[1 - (1+m)p_i^c + \frac{m}{3} \sum_{j \in I} p_j^c \right] = \frac{3 + 2m}{18 + 6m} \qquad (1.52)$$

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where $i \in \{A, B, C\}$. From the demand function, we can then easily calculate the relevant pre-acquisition social welfare:

$$SW^c = U(q_A^c, q_B^c, q_C^c) = \frac{1}{2} - \frac{9}{8(3+m)^2} \tag{1.53}$$

If firms A and B would merge, the resulting prices are given by equations (1.19) - (1.20). By substituting them into the equation (1.1), we obtain the following demands:

$$q_A^m = q_B^m = q_A(p_A^m, p_B^m, p_C^m) = \frac{(3+m)(6+5m)}{18(6+m(6+m))} \tag{1.54}$$

$$q_C^m = q_C(p_A^m, p_B^m, p_C^m) = \frac{(3+2m)^2}{9(6+m(6+m))} \tag{1.55}$$

The social welfare resulting from the merger between firms A and B is:

$$SW^m = U(q_A^m, q_B^m, q_C^m) = \frac{486 + m(1044 + m(765 + m(215 + 18m)))}{36(6 + m(6 + m))^2} \tag{1.56}$$

The closed forms of the relevant demands and social welfare for the benchmark case and the case of asymmetric information are rather complex, hence we present comparisons graphically for $n = 2$.[9]

Under the benchmark case of complete information, with probability $\beta_n(\alpha)$ the firms are in the total control scenario and with probability $1 - \beta_n(\alpha)$ in the silent control scenario. Therefore, the relevant demands are calculated by substituting optimal prices, given by equations (1.16)-(1.18) and (1.9)-(1.10), into the demand function (1.1):

$$q_i^b = \beta_n(\alpha)q_i(p_A^t, p_B^t, p_C^t) + (1 - \beta_n(\alpha))q_i(p_A^s, p_B^s, p_C^s) \tag{1.57}$$

The acquiring firm A, when having total control over firm B, may find it profitable to increase the price of good B so high, such that firm B does not sell any of its product. Hence, in order to have only positive quantities, we need to restrict our attention to $\alpha > 0.11$. The relevant consumer surplus is $SW^b = U(q_A^b, q_B^b, q_C^b)$.

Under asymmetric information, the optimal prices are given by equations (1.41)-

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[9]We obtain the same result for any $n \geq 2$.

(1.45) and the relevant quantities are:

$$q_i^a = \beta_n(\alpha)q_i(p_{A1}^*, p_{B1}^*, p_C^*) + (1 - \beta_n(\alpha))q_i(p_{A2}^*, p_{B2}^*, p_C^*) \qquad (1.58)$$

As in the case of complete information, we need to restrict our attention to $q_B^a > 0$ only, and that is satisfied when $\alpha > 0.0718$. The relevant consumer surplus is $SW^a = U(q_A^a, q_B^a, q_C^a)$.
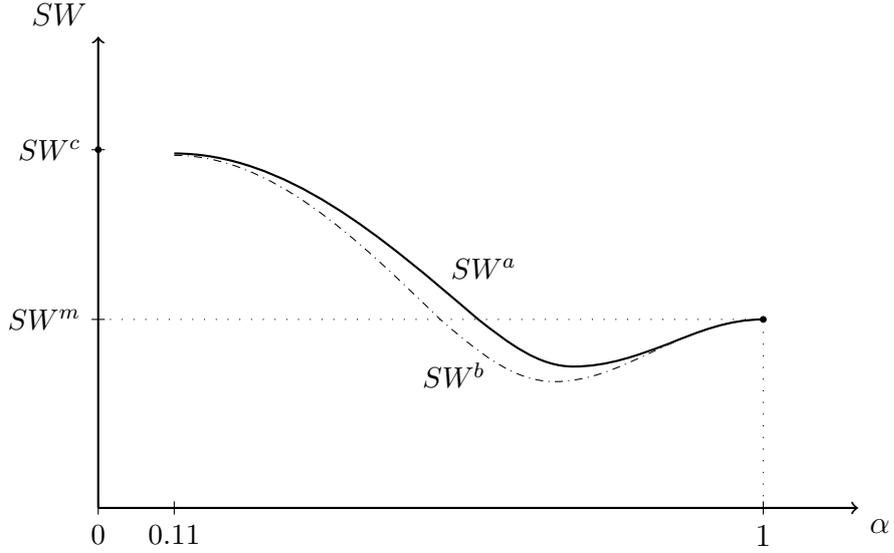
Figure 1.8: Social welfare under asymmetric information (solid line) and under benchmark case (dashed line) for $n = 2$

Figure 1.8 presents social welfare for both benchmark case and the case of asymmetric information. The partial acquisition can create greater consumer loss than a merger when the acquisition share and the probability of decisive influence are high. Additionally, we see that the social welfare under the benchmark case is lower than in the case of asymmetric information. This result follows from the fact that the partial acquisition under complete information serves as a strategic device to relax competition, as shown in section 1.4.1. However, the fact that the outside firm cannot observe the organizational structure of firms A and B, prevents firms of achieving as high collusive outcome. Therefore, under asymmetric information firms charge lower prices, which leads to higher social welfare.

On the other hand, when acquisition share is low, the partial acquisition does

not create substantial welfare concerns.

We also compare the relevant profits under the benchmark case and under the case of asymmetric information. The expected joint profit under the benchmark case, Π_{AB}^b, is given by equation (1.27), while the expected joint profit under asymmetric information is specified by (1.50). A direct comparison between Π_{AB}^b and Π_{AB}^a yields the following result:

Proposition 1.5 *For every $1/3 < \alpha < 1$, $m > 0$ and $n \geq 1$, the joint profit under complete information is larger than the joint profit under asymmetric information.*

The joint profit functions for the benchmark case and the case of asymmetric information are presented in the Figure 1.6.

We next calculate the profit of firm C under complete information by substituting optimal prices, given by equations (1.16)-(1.18) and (1.9)-(1.10), into:

$$\Pi_C^b = \beta_n(\alpha)\pi_C(p_A^t(\alpha), p_B^t(\alpha), p_C^t(\alpha)) + (1 - \beta_n(\alpha))\pi_C(p_A^s(\alpha), p_B^s(\alpha), p_C^s(\alpha))$$
(1.59)

The relevant profit of firm C under asymmetric information is given by:

$$\Pi_C^a = \beta_n(\alpha)\pi_C(p_{A1}^*(\alpha), p_{B1}^*(\alpha), p_C^*(\alpha)) + (1 - \beta_n(\alpha))\pi_C(p_{A2}^*(\alpha), p_{B2}^*(\alpha), p_C^*(\alpha))$$
(1.60)

where optimal prices p_{Ai}^*, p_{Bi}^* and p_C^*, $i \in \{1, 2\}$ are given by (1.41)-(1.45).

Again, a direct comparison between Π_C^b and Π_C^a yields the following result:

Proposition 1.6 *For every $0 < \alpha < 1$, $m > 0$ and $n \geq 1$, the profit of firm C under complete information is larger than it's profit under asymmetric information.*

We conclude that market opacity creates profit losses to all three firms. This, however, leads to the smaller anticompetitive concerns.

1.7 Conclusion

In this chapter we show that partial horizontal acquisition can impede competition more than a merger, even when an outside firm cannot observe the organizational

structure between the firms involved in the acquisition. However, this is the case only when the acquisition share and also the probability of total control are high. On the other hand, when the acquisition share is low, partial acquisition does not create significant welfare concerns. Therefore, our results suggest that the recent proposal of the European Commission to increase the power of EU antitrust authority to also review minority shareholders might not be necessary.

When the outside firm can observe whether the acquirer can influence the target's decisions, we show that the partial acquisition serves as a strategic device to relax competition. Yet, when the organizational structure between the firms taking part in the acquisition is their private information, the firms cannot commit to keep prices as high as in the case of complete information. Hence, they experience a reduction of profits. This, however, leads to higher social welfare and thus smaller anticompetitive concerns.

Throughout the article, we focused on a situation where the acquiring firm can have either control over the target firm or only financial interest in the acquired firm. However, in practice, the acquiring firm can have the whole range of different levels of influence on the acquired firm. We expect that relaxing the assumption of total control would create smaller anticompetitive concerns. A formal analysis is necessary to answer this question and we leave it for future research.

1.8 Appendix

Proof of Proposition 1.1:

(i) The first derivatives of the optimal prices under silent control (1.9)-(1.10) with respect to α are:

$$\frac{\partial p_A^s}{\partial \alpha} = \frac{9m(2+m)(6+5m)}{(36+m(42+(10-\alpha)m))^2} \tag{1.61}$$

$$\frac{\partial p_B^s}{\partial \alpha} = \frac{\partial p_C^s}{\partial \alpha} = \frac{3m^2(6+5m)}{(36+m(42+(10-\alpha)m))^2} \tag{1.62}$$

They are positive for all $0 < \alpha < 1$ and $m > 0$.

(ii) The first derivative of the firm A's optimal price under total control (1.16)

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with respect to α is:

$$\frac{\partial p_A^t}{\partial \alpha} = \frac{3m(6+5m)(\alpha^2(216+m(468+m(324+71m)))) - 2\alpha m^2(6+5m) - m(6+5m)^2)}{D_1^2(\alpha)}$$

where $D_1(\alpha) = 216\alpha + 432\alpha m + 6((44-\alpha)\alpha - 1)m^2 + (\alpha(46-5\alpha)-5)m^3$ is positive for all $0 < \alpha < 1$ and $m > 0$, hence the sign of $\partial p_A^t/\partial \alpha$ depends on the sign of the numerator. It is negative for

$$\alpha < \bar{\alpha} = \frac{(6m^2 + 5m^3) + 6(216m + 828m^2 + 1254m^3 + 937m^4 + 345m^5 + 50m^6)^{1/2}}{(216 + 468m + 324m^2 + 71m^3)^2}$$

and positive otherwise. For every $m > 0$, we have $0 < \bar{\alpha} < 1$.

(iii) The derivatives of the optimal prices of firms B and C under total control (1.17)-(1.18) with respect to α are:

$$\frac{\partial p_B^t}{\partial \alpha} = \frac{-3m(6+5m)(216+m(468-2\alpha m(6+5m)-\alpha^2(6+5m)^2+m(324+71m)))}{D_1^2(\alpha)}$$

(1.63)

$$\frac{\partial p_C^t}{\partial \alpha} = \frac{-36(1-\alpha^2)m^2(1+m)(3+2m)(6+5m)}{D_1^2(\alpha)}$$

(1.64)

They are negative for all $m/(6+3m) < \alpha < 1$ and $m > 0$. This concludes the proof of Proposition 1.1. ∎

Proof of Proposition 1.2:
If firm A's and firm B's joint profit would be maximized at $\alpha = 1$, the first order condition of the joint profit, given by (1.28), would be equal to zero and the second would be negative. Recall that $\partial(\pi_A + \alpha\pi_B)/\partial p_A = 0$ when evaluated in the optimum, p_A^t, and $\partial(\pi_A + \alpha\pi_B)/\partial p_B = 0$ when evaluated in p_B^t, as a result of profit maximization in the second stage. Therefore, from equation (1.28) we obtain:

$$\frac{d\Pi_{AB}^b}{d\alpha} = \beta_n(\alpha)\left[(1-\alpha)\left(\frac{\partial \pi_B}{\partial p_A}\frac{\partial p_A^t}{\partial \alpha} + \frac{\partial \pi_B}{\partial p_B}\frac{\partial p_B^t}{\partial \alpha}\right) + \frac{\partial p_C^t}{\partial \alpha}\left(\frac{\partial \pi_A}{\partial p_C} + \frac{\partial \pi_B}{\partial p_C}\right)\right]$$
$$+ (1-\beta_n(\alpha))\sum_{i\in\{A,B\}}\sum_{j\in\{A,B,C\}}\frac{\partial \pi_i}{\partial p_j}\frac{\partial p_j^s}{\partial \alpha} + \frac{\partial \beta_n}{\partial \alpha}\left(\pi_A^t + \pi_B^t - \pi_A^s - \pi_B^s\right) \quad (1.65)$$

When $\alpha = 1$, it follows $\beta_n(\alpha) = 1$, hence $1 - \beta_n(\alpha) = 0$. Moreover,

$$\frac{\partial \beta_n(\alpha)}{\partial \alpha} = \begin{cases} 2n(2\alpha)^{2n-1}, & \text{if } \alpha < \frac{1}{2} \\ -2n(2\alpha - 2)^{2n-1}, & \text{if } \alpha > \frac{1}{2} \end{cases} \tag{1.66}$$

which is equal to zero, when evaluated at $\alpha = 1$.

$$\left. \frac{\partial \beta_n(\alpha)}{\partial \alpha} \right|_{\alpha=1} = 0$$

Substituting these values into the first order condition, given by (1.65), the first order condition simplifies further to

$$\frac{d\Pi_{AB}^b}{d\alpha} = \frac{\partial p_C^t}{\partial \alpha} \left(\frac{\partial \pi_A}{\partial p_C} + \frac{\partial \pi_B}{\partial p_C} \right) \tag{1.67}$$

Furthermore, $\partial p_C^t / \partial \alpha$ is given by equation (1.64) and it is equal to zero when evaluated at $\alpha = 1$. We, thus, conclude that at $\alpha = 1$

$$\frac{d\Pi_{AB}^b}{d\alpha} = 0.$$

To check if $\alpha = 1$ is a local maximum, a saddle point or a local minimum, we do the second order conditions. For convenience, we define

$$\pi_{ij} = \frac{\partial \pi_i}{\partial p_j} , \ i \in \{A, B\} \ , \ j \in \{A, B, C\}. \tag{1.68}$$

In the following, from (1.65) we derive the second order conditions of the joint

profit with respect to α:

$$
\begin{aligned}
\frac{d^2\Pi_{AB}^b}{d\alpha^2} =& \frac{\partial\beta_n}{\partial\alpha}(1-\alpha)\left(\frac{\partial\pi_B}{\partial p_A}\frac{\partial p_A^t}{\partial\alpha} + \frac{\partial\pi_B}{\partial p_B}\frac{\partial p_B^t}{\partial\alpha}\right) + \frac{\partial\beta_n}{\partial\alpha}\frac{\partial p_C^t}{\partial\alpha}\left(\frac{\partial\pi_A}{\partial p_C} + \frac{\partial\pi_B}{\partial p_C}\right) \\
& - \beta_n(\alpha)\left(\frac{\partial\pi_B}{\partial p_A}\frac{\partial p_A^t}{\partial\alpha} + \frac{\partial\pi_B}{\partial p_B}\frac{\partial p_B^t}{\partial\alpha}\right) \\
& + \beta_n(\alpha)\left[\frac{\partial^2 p_C^t}{\partial\alpha^2}\left(\frac{\partial\pi_A}{\partial p_C} + \frac{\partial\pi_B}{\partial p_C}\right) + \frac{\partial p_C^t}{\partial\alpha}\left(\frac{\partial^2\pi_A}{\partial p_C^2} + \frac{\partial^2\pi_B}{\partial p_C^2}\right)\right] \\
& + \beta_n(\alpha)(1-\alpha)\left(\frac{\partial^2\pi_B}{\partial p_A^2}\frac{\partial p_A^t}{\partial\alpha} + \frac{\partial\pi_B}{\partial p_A}\frac{\partial^2 p_A^t}{\partial\alpha^2} + \frac{\partial^2\pi_B}{\partial p_B^2}\frac{\partial p_B^t}{\partial\alpha} + \frac{\partial\pi_B}{\partial p_B}\frac{\partial^2 p_B^t}{\partial\alpha^2}\right) \\
& - \frac{\partial\beta_n}{\partial\alpha}\sum_{i\in\{AB\}}\sum_{j\in\{A,B,C\}}\pi_{ij}\frac{\partial p_j^s}{\partial\alpha} + (1-\beta_n(\alpha))\sum_{i\in\{AB\}}\sum_{j\in\{A,B,C\}}\left(\frac{d\pi_{ij}}{dp_j}\frac{\partial p_j^s}{\partial\alpha} + \pi_{ij}\frac{\partial^2 p_j^s}{\partial\alpha^2}\right) \\
& + \frac{\partial^2\beta_n}{\partial\alpha^2}\left(\pi_A^t + \pi_B^t - \pi_A^s - \pi_B^s\right) + \frac{\partial\beta_n}{\partial\alpha}\left(\frac{\partial\pi_A^t}{\partial\alpha} + \frac{\partial\pi_B^t}{\partial\alpha} - \frac{\partial\pi_A^s}{\partial\alpha} - \frac{\partial\pi_B^s}{\partial\alpha}\right)
\end{aligned}
\tag{1.69}
$$

When $\alpha = 1$, then again $\beta_n(\alpha) = 1$, $1 - \beta_n(\alpha) = 0$ and $\partial\beta_n/\partial\alpha = 0$, therefore,

$$
\begin{aligned}
\frac{d^2\Pi_{AB}^b}{d\alpha^2} =& -\left(\frac{\partial\pi_B}{\partial p_A}\frac{\partial p_A^t}{\partial\alpha} + \frac{\partial\pi_B}{\partial p_B}\frac{\partial p_B^t}{\partial\alpha}\right) + \frac{\partial^2 p_C^t}{\partial\alpha^2}\left(\frac{\partial\pi_A}{\partial p_C} + \frac{\partial\pi_B}{\partial p_C}\right) \\
& + \frac{\partial^2\beta_n}{\partial\alpha^2}\left(\pi_A^t + \pi_B^t - \pi_A^s - \pi_B^s\right)
\end{aligned}
\tag{1.70}
$$

Additionally:

$$
\frac{\partial^2\beta_n}{\partial\alpha^2} = \begin{cases} 4n(2n-1)(2\alpha)^{2n-2}, & \text{if } \alpha \leq \frac{1}{2} \\ -4n(2n-1)(2\alpha-2)^{2n-2}, & \text{if } \alpha \geq \frac{1}{2} \end{cases}
\tag{1.71}
$$

We next split the analysis in two cases, $n = 1$ and $n \geq 2$. When $n = 1$:

$$
\frac{\partial^2\beta_n}{\partial\alpha^2} = \begin{cases} 4\alpha, & \text{if } \alpha \leq \frac{1}{2} \\ -4, & \text{if } \alpha \geq \frac{1}{2} \end{cases}
\tag{1.72}
$$

Hence, (1.70) simplifies to:

$$
\begin{aligned}
\frac{d^2\Pi^b_{AB}}{d\alpha^2} &= -\left(\frac{\partial \pi_B}{\partial p_A}\frac{\partial p_A^t}{\partial \alpha} + \frac{\partial \pi_B}{\partial p_B}\frac{\partial p_B^t}{\partial \alpha}\right) + \frac{\partial^2 p_C^t}{\partial \alpha^2}\left(\frac{\partial \pi_A}{\partial p_C} + \frac{\partial \pi_B}{\partial p_C}\right) \\
&\quad - 4\left(\pi_A^t + \pi_B^t - \pi_A^s - \pi_B^s\right) = \frac{U(m)}{V(m)} < 0
\end{aligned}
\tag{1.73}
$$

where, $U(m) = -m^2(186624 + m(995328 + m(2280960 + m(2919456 + m(2266416 + m(1081344 + m(304970 + 3m(15188 + 909m)))))))) < 0$ and $V(m) = 324(1+m)(6+m(6+m))^3(12+m(14+3m))^2) > 0$ for every $m > 0$. Thus, when $n = 1$, $\alpha = 1$ is a local maximum.

Next, when $n \geq 2$, (1.71) evaluated at $\alpha = 1$ equals:

$$
\left.\frac{\partial^2 \beta_n}{\partial \alpha^2}\right|_{\alpha=1} = 0
\tag{1.74}
$$

When $n = 2$, equation (1.70) leads to:

$$
\begin{aligned}
\frac{d^2\Pi^b_{AB}}{d\alpha^2} &= -\left(\frac{\partial \pi_B}{\partial p_A}\frac{\partial p_A^t}{\partial \alpha} + \frac{\partial \pi_B}{\partial p_B}\frac{\partial p_B^t}{\partial \alpha}\right) + \frac{\partial^2 p_C^t}{\partial \alpha^2}\left(\frac{\partial \pi_A}{\partial p_C} + \frac{\partial \pi_B}{\partial p_C}\right) \\
&= \frac{m^2(6+5m)^2((m-12)m - 18)}{324(1+m)(6+m(6+m))^3}
\end{aligned}
\tag{1.75}
$$

The last term is positive whenever $m > 3(2 + \sqrt{6}) \sim 13.4$. We can easily extend the analysis for every $n > 2$, but the condition for m does not change. Hence, we conclude that $\alpha = 1$ is a local minimum when $n \geq 1$ and $m > 3(2 + \sqrt{6})$. Therefore, for these parameter values, a total merger is never optimal.

To complete the proof, we need to analyze the case when $\alpha = 0$. For $\alpha = 0$, it is also $\beta(\alpha) = 0$, hence, the first order condition for joint profit of firms A and B (1.28) simplifies to:

$$
\frac{\partial \pi_B}{\partial p_A}\frac{\partial p_A^s}{\partial \alpha} + \frac{\partial \pi_A}{\partial p_B}\frac{\partial p_B^s}{\partial \alpha} + \frac{\partial p_C^s}{\partial \alpha}\left(\frac{\partial \pi_A}{\partial p_C} + \frac{\partial \pi_B}{\partial p_C}\right) = \frac{2m^2(6+5m)(18+m(30+13m))}{27(12+m(14+3m))^3}
\tag{1.76}
$$

which is positive for every $m > 0$.

From this we conclude, that when $m > 3(2 + \sqrt{6})$ and $n > 1$, the optimal acquisition share α is in the interval $(0, 1)$, hence partial ownership is preferred to

the merger. This concludes the proof. ∎

Proof of Proposition 1.3:

The equilibrium prices under asymmetric information are given by equations (1.41) - (1.45). Note that the denominator in those equations depends on the probability of total control, $\beta_n(\alpha)$, and it is equal to:

$$
\begin{aligned}
D_2(\alpha) =& \alpha^3 m^4(6+5m) - m^2(6+5m)^2(6+(2+\beta_n(\alpha))m) - \alpha^2 m^2(432 + m(900 \\
& + m(594 + (120 + \beta_n(\alpha))m))) + \alpha(6+5m)(1296 + m(3240 + m(2880 \\
& + m(1068 + (141 - 2\beta_n(\alpha))m))))
\end{aligned}
\tag{1.77}
$$

By plugging $\beta_n(\alpha)$ given by equation (1.25) into (1.77), we get

$$
\begin{aligned}
D_2(\alpha) =& \alpha^3 m^4(6+5m) - m^2(6+5m)^2(6+(2+2^{2n-1}\alpha^{2n})m) - \alpha^2 m^2(432 \\
& + m(900 + m(594 + (120 + 2^{2n-1}\alpha^{2n})m))) + \alpha(6+5m)(1296 \\
& + m(3240 + m(2880 + m(1068 + (141 - 2^{2n}\alpha^{2n})m))))
\end{aligned}
\tag{1.78}
$$

for $\alpha \leq 1/2$, and

$$
\begin{aligned}
\bar{D}_2(\alpha) =& \alpha^3 m^4(6+5m) - m^2(6+5m)^2(6+(3-\frac{1}{2}(-2+2\alpha)^{2n})m) - \alpha^2 m^2(432 \\
& + m(900 + m(594 + (121 - \frac{1}{2}(-2+2\alpha)^{2n})m))) + \alpha(6+5m)(1296 \\
& + m(3240 + m(2880 + m(1068 + (141 - 2(1 - \frac{1}{2}(-2+2\alpha)^{2n}))m))))
\end{aligned}
\tag{1.79}
$$

for $\alpha \geq 1/2$. Additionally,

$$
\begin{aligned}
D_2'(\alpha) =& 3\alpha^2 m^4(6+5m) - 2\alpha m^2(432 + m(900 + m(594 + (120 + 2^{-1+2n}\alpha^{2n})m))) \\
& - (6+5m)(-1296 + m(-3240 + m(-2880 + m(-1068 + (-141 \\
& + 2^{2n}\alpha^{2n})m)))) - 2^{2n}\alpha^{-1+2n}(\alpha^2 m^5 + 2\alpha m^4(6+5m) + m^3(6+5m)^2)n
\end{aligned}
\tag{1.80}
$$

is the first derivative of $\underline{D}_2(\alpha)$ with respect to α and

$$
\begin{aligned}
\bar{D}_2'(\alpha) = {} & 3\alpha^2 m^4(6+5m) - 2\alpha m^2(432 + m(900 + m(594 + (121 \\
& - 1/2(-2+2\alpha)^{2n})m))) - (6+5m)(-1296 + m(-3240 + m(-2880 \\
& + m(-1068 + (-141 + 2(1 - 1/2(-2+2\alpha)^{2n}))m)))) + 2(-2 \\
& + 2\alpha)^{-1+2n}(\alpha^2 m^5 + 2\alpha m^4(6+5m) + m^3(6+5m)^2)n
\end{aligned}
\tag{1.81}
$$

is the first derivative of $\bar{D}_2(\alpha)$ with respect to α.

To determine whether the equilibrium prices (1.41) - (1.45) are increasing or decreasing with respect to α, we take the first derivatives of prices with respect to α.

(i) The first derivative of the firm A's optimal price p_{A1}^* with respect to α is:

$$
\frac{\partial p_{A1}^*}{\partial \alpha} = \frac{f_{A1}' D_2 - f_{A1} D_2'}{(D_2)^2}
\tag{1.82}
$$

where $f_{A1}(\alpha) = (3\alpha(6+5m)(6+(5+\alpha)m)(36+m(48+(15-\alpha)m))$ is the numerator of price p_{A1}^* given by equation (1.41) and $f_{A1}'(\alpha) = -3(6+5m)(-216+m(3\alpha^2 m^2 - 4\alpha(3+m)(6+5m) - 3(156+5m(22+5m))))$ is its first derivative.

We first plug $D_2(\alpha) = \underline{D}_2(\alpha)$, given by equation (1.78), and $D_2'(\alpha) = \underline{D}_2'(\alpha)$, given by equation (1.80), into (1.82). We next evaluate the obtained first derivative (1.82) at the interval endpoints: $\alpha = m/(6+3m)$ and $\alpha = 1/2$.

$$
\begin{aligned}
\left.\frac{\partial p_{A1}^*}{\partial \alpha}\right|_{\alpha=\frac{m}{6+3m}} = {} & \frac{1}{D_3}(27(2+m)^2(6+5m)(-(3+m)(6+5m)(7776 + m(22032 \\
& + m(24192 + m(12780 + m(3210 + 299m)))))) + 4^n m(3+2m)^2 \\
& (m/(6+3m))^{2n}(-1296 - 3240m - 2952m^2 - 1152m^3 - 161m^4 \\
& + 8(3+2m)^2(36+m(42+11m))n)))
\end{aligned}
\tag{1.83}
$$

where $D_3 = 4(3+2m)(3888 + m(11016 + m(11988 + 6228m + 1539m^2 + 145m^3 - 3 \cdot 4^n(2+m)(3+2m)^2(m/(6+3m))^{2n})))^2 > 0$. The derivative is negative

$$
\left.\frac{\partial p_{A1}^*}{\partial \alpha}\right|_{\alpha=\frac{m}{6+3m}} < 0
\tag{1.84}
$$

for all $n \geq 1$ and $m > 0$.

Next, $\partial p^*_{A1}/\partial \alpha$ evaluated at $\alpha = 1/2$ is:

$$\left.\frac{\partial p^*_{A1}}{\partial \alpha}\right|_{\alpha=\frac{1}{2}} = \frac{1}{D_4}(m(13436928 + m(67184640 + m(139594752 + 153840384m$$
$$+ \; 93713760m^2 + 28102464m^3 + 1262088m^4 - 1363314m^5$$
$$- \; 243295m^6 + 2(6 + 5m)(12 + 11m)^3(72 + m(96 + 29m))n))))$$

$$(1.85)$$

where $D_4 = 6(5184 + m(17280 + m(21888 + m(12972 + m(3523 + 344m)))))^2 > 0$.
The derivative is positive

$$\left.\frac{\partial p^*_{A1}}{\partial \alpha}\right|_{\alpha=\frac{1}{2}} > 0 \qquad (1.86)$$

for all $n \geq 1$ and $m > 0$.

The second derivative $\partial^2 p^*_{A1}/\partial \alpha^2$ is positive for all $m/(6 + 3m) < \alpha < 1/2$, $n \geq 1$ and $m > 0$. Therefore, p^*_{A1} is convex in α for $m/(6 + 3m) < \alpha < 1/2$.

We next plug $D_2(\alpha) = \bar{D}_2(\alpha)$, given by equation (1.79), and $D'_2(\alpha) = \bar{D}'_2(\alpha)$, given by equation (1.81), into (1.82) and evaluate it at $\alpha = 1$:

$$\left.\frac{\partial p^*_{A1}}{\partial \alpha}\right|_{\alpha=1} = \frac{4m(6 + 5m)(18 + m(24 + 7m))((6 + m(6 + m))(18 + m(24 + 7m))}{(3(1 + m)(4(6 + m(6 + m))(18 + m(24 + 7m)))^2}$$

$$(1.87)$$

which is positive for all $n \geq 1$ and $m > 0$. Additionally, $\partial p^*_{A1}/\partial \alpha$ is positive for all $\alpha \in (1/2, 1)$.

To summarize, p^*_{A1} has a negative slope at $\alpha = m/(6 + 3m)$, it is convex for $\alpha \in (m/(6 + 3m), 1/2)$, has a positive slope at $\alpha = 1/2$, it is increasing for $\alpha \in (1/2, 1)$ and has a positive slope at $\alpha = 1$.

We thus conclude that there exists some $\hat{\alpha} \in (m/(6 + 3m), 1/2)$, such that $\partial p^*_{A1}/\partial \alpha < 0$ for $\alpha < \hat{\alpha}$ and $\partial p^*_{A1}/\partial \alpha > 0$ for $\alpha > \hat{\alpha}$.

(ii) The first derivative of the firm B's optimal price p^*_{B1} with respect to α is:

$$\frac{\partial p^*_{B1}}{\partial \alpha} = \frac{f'_{B1}D_2 - f_{B1}D'_2}{(D_2)^2} \tag{1.88}$$

where $f_{B1}(\alpha) = 3(6+5m)(m+\alpha(6+5m))(36+m(48+(15-\alpha)m))$ is the numerator of price p^*_{B1} given by equation (1.42) and $f'_{B1} = 6(6+5m)(108+m(234+m(165+37m-\alpha(6+5m))))$.

We first plug $D_2(\alpha) = \underline{D}_2(\alpha)$, given by equation (1.78), and $D'_2(\alpha) = \underline{D}'_2(\alpha)$, given by equation (1.80), into (1.88). The computations reveal that $\partial p^*_{B1}/\partial \alpha$ is negative for all $\alpha \in [m/(6+3m), 1/2]$, $m > 0$ and $n \geq 1$.

We next plug $D_2(\alpha) = \bar{D}_2(\alpha)$, given by equation (1.79), and $D'_2(\alpha) = \bar{D}'_2(\alpha)$, given by equation (1.81), into (1.88) and get that $\partial p^*_{B1}/\partial \alpha$ is negative for $\alpha \in [1/2, 1]$, $m > 0$ and $n \geq 1$.

We thus conclude that p^*_{B1} decreases for all $\alpha \in [m/(6+3m), 1]$, $m > 0$ and $n \geq 1$.

(iii) The first derivative of the firm A's and B's optimal prices p^*_{A2} and p^*_{B2} with respect to α is:

$$\frac{\partial p^*_{i2}}{\partial \alpha} = \frac{f'_{i2}D_2 - f_{i2}D'_2}{(D_2)^2} \tag{1.89}$$

where $i = A, B$ and $f_{A2}(\alpha) = 3(6+5m)(6+(5+\alpha)m)(36\alpha+48\alpha m+((14-\alpha)\alpha-1)m^2)$ is the numerator of price p^*_{A2} given by equation (1.43) and $f_{B2}(\alpha) = 3(6+5m)^2(36\alpha+48\alpha m+((14-\alpha)\alpha-1)m^2)$ is the numerator of price p^*_{B2} given by equation (1.44). $f'_{A2} = -9(6+5m)(-72-12(13+2\alpha)m-4(27+7\alpha)m^2+(-23+(-6+\alpha)\alpha)m^3)$ and $f'_{B2} = -6(6+5m)^2(-18+m(-24+(-7+\alpha)m))$ are their first derivatives.

We first plug $D_2(\alpha) = \underline{D}_2(\alpha)$, given by equation (1.78), and $D'_2(\alpha) = \underline{D}'_2(\alpha)$, given by equation (1.80), into (1.89). The computations reveal that $\partial p^*_{i2}/\partial \alpha$ is positive for all $\alpha \in [m/(6+3m), 1/2]$, $i = A, B$, $m > 0$, and $n \geq 1$.

We next plug $D_2(\alpha) = \bar{D}_2(\alpha)$, given by equation (1.79), and $D'_2(\alpha) = \bar{D}'_2(\alpha)$, given by equation (1.81), into (1.89) and get that $\partial p^*_{i2}/\partial \alpha$ is positive for $\alpha \in [1/2, 1]$, $i = A, B$, $m > 0$, and $n \geq 1$.

We thus conclude that both p^*_{A2} and p^*_{B2} increase for all $\alpha \in [m/(6+3m), 1]$,

$m > 0$ and $n \geq 1$.

(iv) The optimal price of firm C, p_C^*, is more complex, as the numerator also depends on $\beta_n(\alpha)$. The first derivative of the firm C's optimal price with respect to α is:

$$\frac{\partial p_C^*}{\partial \alpha} = \begin{cases} (\underline{f}_C' \underline{D}_2 - \underline{f}_C \underline{D}_2')/(\underline{D}_2)^2, & \text{if } \alpha \leq \frac{1}{2} \\ (\bar{f}_C' \bar{D}_2 - \bar{f}_C \bar{D}_2')/(\bar{D}_2)^2, & \text{if } \alpha \geq \frac{1}{2} \end{cases} \tag{1.90}$$

where $\underline{f}_C(\alpha) = 1/2(3 \cdot 4^n \alpha^{2n} m^2 (6 + (5 + \alpha)m)^2 - 6(6 + 5m)^2(-36\alpha - 48\alpha m + (1 + (-14 + \alpha)\alpha)m^2))$ is the numerator of price p_C^* for $\alpha < 1/2$ and $\bar{f}_C(\alpha) = -(3/2)(-2 + 2\alpha)^{2n} m^2 (6 + 5m + \alpha m)^2 - 36\alpha(1 + m)(3 + 2m)(-36 + m(-48 + (-15 + \alpha)m))$ for $\alpha > 1/2$. Next, $\underline{f}_C' = -6(6 + 5m)^2(-18 + m(-24 + (-7 + \alpha)m)) + 34^n \alpha^{-1+2n} m^2 (6 + (5 + \alpha)m)(\alpha m + (6 + (5 + \alpha)m)n)$ and $\bar{f}_C' = -36(1 + m)(3 + 2m)(-36 + m(-48 + (-15 + 2\alpha)m)) - 34^n(-1 + \alpha)^{-1+2n} m^2 (6 + (5 + \alpha)m)(6n + m(-1 + \alpha + (5 + \alpha)n))$.

We first plug $\underline{D}_2(\alpha)$, given by equation (1.78), and $\underline{D}_2'(\alpha)$, given by equation (1.80), and also $\underline{f}_C(\alpha)$ and $\underline{f}_C'$ into (1.90). The computations reveal that $\partial p_C^*/\partial \alpha$ is positive for all $\alpha \in [m/(6 + 3m), 1/2]$, $m > 0$, and $n \geq 1$.

We next plug $\bar{D}_2(\alpha)$, given by equation (1.79), and $\bar{D}_2'(\alpha)$, given by equation (1.81), and also $\bar{f}_C(\alpha)$ and $\bar{f}_C'$ into (1.90). The computations reveal that $\partial p_C^*/\partial \alpha$ is positive for all $\alpha \in [1/2, 1]$, $m > 0$, and $n = 1$.

For $n = 2$, we evaluate $\partial p_C^*/\partial \alpha$ at $\alpha = 1/2$ and at $\alpha = 1$:

$$\left. \frac{\partial p_C^*}{\partial \alpha} \right|_{\alpha = \frac{1}{2}} = \frac{1}{D_5}(m^2(6 + 5m)(373248 + m(1700352 + m(3129408 + m(2958336$$

$$+ m(1501368 + m(383368 + 38029m))))))) \tag{1.91}$$

which is positive for all $m > 0$, where $D_5 = 2(5184 + m(17280 + m(21888 + m(12972 + m(3523 + 344m)))))^2$. At $\alpha = 1$, we have

$$\left. \frac{\partial p_C^*}{\partial \alpha} \right|_{\alpha = 1} = 0 \tag{1.92}$$

for $m > 0$.

The second derivative $\partial^2 p_C^*/\partial \alpha^2$ is negative for all $1/2 < \alpha < 4/5$, $n = 2$ and

$m > 0$. Therefore, p_C^* is concave in α for $1/2 < \alpha < 4/5$. Additionally, $\partial p_C^*/\partial \alpha$ is negative for all $\alpha \in [4/5, 1)$.

To summarize, for $n = 1$, p_C^* increases for all $\alpha \in [m/(6+3m), 1)$ and $m > 0$. For $n = 2$, p_C^* increases for $\alpha \in [m/(6+3m), 1/2]$, it is concave for $\alpha \in (1/2, 4/5)$, and it decreases for $\alpha \in [4/5, 1)$. We thus conclude that for $n = 2$, there exists some $\tilde{\alpha} \in (1/2, 4/5)$, such that $\partial p_C^*/\partial \alpha > 0$ for $\alpha < \hat{\alpha}$ and $\partial p_C^*/\partial \alpha > 0$ for $\alpha > \hat{\alpha}$. We can easily extend the analysis for every $n > 2$. ∎

Proof of Proposition 1.4:

We next derive the optimal α by using the Envelope Theorem:

$$\frac{d\Pi_{AB}^a}{d\alpha} = \beta_n(\alpha) \sum_{i \in \{AB\}} \sum_{j \in \{A,B,C\}} \frac{\partial \pi_i}{\partial p_j} \left(\frac{\partial p_{j1}^*}{\partial \alpha} + \frac{\partial \beta_n}{\partial \alpha} \frac{\partial p_{j1}^*}{\partial \beta_n} \right) + \frac{\partial \beta_n}{\partial \alpha} \left(\pi_{A1}^* + \pi_{B1}^* - \pi_{A2}^* - \pi_{B2}^* \right)$$
$$+ (1 - \beta_n(\alpha)) \sum_{i \in \{AB\}} \sum_{j \in \{A,B,C\}} \frac{\partial \pi_i}{\partial p_j} \left(\frac{\partial p_{j2}^*}{\partial \alpha} + \frac{\partial \beta_n}{\partial \alpha} \frac{\partial p_{j2}^*}{\partial \beta} \right) \tag{1.93}$$

We next assume that $\alpha = 1$, hence $\beta_n(\alpha) = 1$, $1 - \beta_n(\alpha) = 0$ and $\partial \beta_n / \partial \alpha = 0$

$$\frac{d\Pi_{AB}^a}{d\alpha} = \sum_{i \in \{AB\}} \sum_{j \in \{A,B,C\}} \frac{\partial \pi_i}{\partial p_j} \frac{\partial p_{j1}^*}{\partial \alpha} \tag{1.94}$$

Recall that $\partial(\pi_A + \alpha \pi_B)/\partial p_A = 0$ and $\partial(\pi_A + \alpha \pi_B)/\partial p_B = 0$ when evaluated in the optimum, p_{i1}^*, $i \in \{A, B\}$ as a result of profit maximization in the second stage.

Hence (1.93) simplifies to:

$$\frac{d\Pi_{AB}^a}{d\alpha} = (1 - \alpha) \left(\frac{\partial \pi_B}{\partial p_A} \frac{\partial p_{A1}^*}{\partial \alpha} + \frac{\partial \pi_{B1}^*}{\partial p_B} \frac{\partial p_{B1}^*}{\partial \alpha} \right) + \frac{\partial p_C^*}{\partial \alpha} \left(\frac{\partial \pi_{A1}}{\partial p_C} + \frac{\partial \pi_B^*}{\partial p_C} \right) \tag{1.95}$$

Further on, when $\alpha = 1$, the first term is equal to zero and for our demand specification, we have $\partial p_C^*/\partial \alpha = 0$. We conclude that in $\alpha = 1$,

$$\frac{d\Pi_{AB}^a}{d\alpha} = 0.$$

To check if $\alpha = 1$ is a local maximum, a saddle point or a local minimum, we must do the second order conditions. For convenience, we define $\pi_{ij} = \partial \pi_i / \partial p_j$, where, $i \in \{A, B\}$ and $i \in \{A, B, C\}$.

$$\frac{d^2\Pi_{AB}^a}{d\alpha^2} = \frac{\partial\beta_n}{\partial\alpha} \sum_{i\in\{AB\}}\sum_{j\in\{AB,C\}} \pi_{ij}\left(\frac{\partial p_{j1}^*}{\partial\alpha} + \frac{\partial\beta_n}{\partial\alpha}\frac{\partial p_{j1}^*}{\partial\beta_n}\right)$$

$$+ \beta_n(\alpha) \sum_{i\in\{AB\}}\sum_{j\in\{AB,C\}} \left(\frac{d\pi_{ij}}{dp_j}\left(\frac{\partial p_{j1}^*}{\partial\alpha} + \frac{\partial\beta_n}{\partial\alpha}\frac{\partial p_{j1}^*}{\partial\beta_n}\right)\right.$$

$$+ \pi_{ij}\left(\frac{\partial^2 p_{j1}^*}{\partial\alpha^2} + \frac{\partial^2\beta_n}{\partial\alpha^2}\frac{\partial p_{j1}^*}{\partial\beta_n} + \frac{\partial\beta_n}{\partial\alpha}\frac{\partial^2 p_{j1}^*}{\partial\beta^2}\right)\right)$$

$$- \frac{\partial\beta_n}{\partial\alpha} \sum_{i\in\{AB\}}\sum_{j\in\{AB,C\}} \pi_{ij}\left(\frac{\partial p_{j2}^*}{\partial\alpha} + \frac{\partial\beta_n}{\partial\alpha}\frac{\partial p_{j2}^*}{\partial\beta_n}\right)$$

$$+ (1-\beta_n(\alpha)) \sum_{i\in\{AB\}}\sum_{j\in\{AB,C\}} \left(\frac{d\pi_{ij}}{dp_j}\left(\frac{\partial p_{j2}^*}{\partial\alpha} + \frac{\partial\beta_n}{\partial\alpha}\frac{\partial p_{j2}^*}{\partial\beta_n}\right)\right.$$

$$+ \pi_{ij}\left(\frac{\partial^2 p_{j2}^*}{\partial\alpha^2} + \frac{\partial^2\beta_n}{\partial\alpha^2}\frac{\partial p_{j2}^*}{\partial\beta_n} + \frac{\partial\beta_n}{\partial\alpha}\frac{\partial^2 p_{j2}^*}{\partial\beta_n^2}\right)\right)$$

$$+ \frac{\partial^2\beta_n}{\partial\alpha^2}\left(\pi_{A1}^* + \pi_{B1}^* - \pi_{A2}^* - \pi_{B2}^*\right)$$

$$+ \frac{\partial\beta_n}{\partial\alpha}\left(\frac{\partial\pi_{A1}^*}{\partial\alpha} + \frac{\partial\pi_{B1}^*}{\partial\alpha} - \frac{\partial\pi_{A2}^*}{\partial\alpha} - \frac{\partial\pi_{B2}^*}{\partial\alpha}\right) \tag{1.96}$$

Again, when $\alpha = 1$, $\beta_n(\alpha) = 1$, $1 - \beta_n(\alpha) = 0$ and $\partial\beta_n/\partial\alpha = 0$. In addition $\partial^2\beta_n/\partial\alpha^2 = -4n(2n-1)(2\alpha-2)^{2n-2}$ so for $n = 1$, $\partial^2\beta_n/\partial\alpha^2 = -4$ and for every $n \geq 2$, $\partial^2\beta_n/\partial\alpha^2 = 0$.

Hence, when $n = 1$ (1.96) simplifies to:

$$\frac{d^2\Pi_{AB}^a}{d\alpha^2} = \sum_{i\in\{AB\}}\sum_{j\in\{AB,C\}} \left(\frac{d\pi_{ij}}{dp_j}\frac{\partial p_{j1}^*}{\partial\alpha} + \pi_{ij}\left(\frac{\partial^2 p_{j1}^*}{\partial\alpha^2} + \frac{\partial^2\beta}{\partial\alpha^2}\frac{\partial p_{j1}^*}{\partial\beta_n}\right)\right)$$

$$- 4\left(\pi_{A1}^* + \pi_{B1}^* - \pi_{A2}^* - \pi_{B2}^*\right) \tag{1.97}$$

which is negative for every $m > 0$. Thus, for $n = 1$, $\alpha = 1$ is a local maximum.

When $n = 2$, we have

$$\frac{d^2\Pi_{AB}^a}{d\alpha^2} = \sum_{i \in \{AB\}} \sum_{j \in \{AB,C\}} \left(\frac{d\pi_{ij}}{dp_j} \frac{\partial p_{j1}^*}{\partial \alpha} + \pi_{ij} \left(\frac{\partial^2 p_{j1}^*}{\partial \alpha^2} + \frac{\partial^2 \beta_n}{\partial \alpha^2} \frac{\partial p_{j1}^*}{\partial \beta_n} \right) \right)$$

$$= -1 \left(\frac{\partial \pi_B}{\partial p_A} \frac{\partial p_{A1}^*}{\partial \alpha} + \frac{\partial \pi_B}{\partial p_B} \frac{\partial p_{B1}^*}{\partial \alpha} \right) + \frac{\partial^2 p_C^*}{\partial \alpha^2} \left(\frac{\partial \pi_A}{\partial p_C} + \frac{\partial \pi_B}{\partial p_C} \right)$$

$$= \frac{m^2(6 + 5m)^2((m - 12)m - 18)}{324(1 + m)(6 + m(6 + m))^3} \tag{1.98}$$

The last equation is bigger than zero whenever $m > 3(2 + \sqrt{6}) \sim 13.4$. We can easily extend the analysis for every $n > 2$, but the condition for m does not change.

To complete the proof, we need to analyze the case when $\alpha = 0$. For $\alpha = 0$, it is also $\beta(\alpha) = 0$, hence, the equation (1.28) simplifies to

$$\frac{\partial \pi_B^s}{\partial p_A} \frac{\partial p_A^s}{\partial \alpha} + \frac{\partial \pi_A^s}{\partial p_B} \frac{\partial p_B^s}{\partial \alpha} + \frac{\partial p_C^s}{\partial \alpha} \left(\frac{\partial \pi_A^s}{\partial p_C} + \frac{\partial \pi_B^s}{\partial p_C} \right) = \frac{2m^2(6 + 5m)(18 + m(30 + 13m))}{27(12 + m(14 + 3m))^3} \tag{1.99}$$

which is positive for every $m > 0$.

From this we conclude, that the optimal α is in the interval $(0, 1)$, hence the partial ownership is preferred to the merger. This concludes the proof. ∎

Proof of Proposition 1.5:
After substituting optimal prices p_i^t, p_i^s, $i \in \{A, B, C\}$, (1.16)-(1.18) and (1.9)-(1.10) into the equation (1.27) of the firm A's and B's joint profit function under benchmark case, we obtain: :

$$\Pi_{AB}^b = \frac{1}{D_1}((3\beta_n(\alpha)(1 + m)(6 + 5m)^2(-72\alpha^2 - 96\alpha^2 m + (1 + \alpha(2 + \alpha(-30 + \alpha(2 + \alpha)))))m^2)) + \frac{1}{(-36 + m(-42 + (-10 + \alpha)m))^2}((-1 + \beta_n(\alpha))(-216 + m(-504 + 3(-130 + \alpha(2 + \alpha))m + (-5 + \alpha)(20 + 3\alpha)m^2))) \tag{1.100}$$

After substituting optimal prices p_{ij}^*, $i \in \{A, B\}$, $j \in \{1, 2\}$ and p_C^* (1.41)-(1.45) into the equation (1.50) of the joint profit function of firms A and B under asym-

metric information, we get:

$$\Pi_{AB}^a = \frac{1}{D_2}((6 + 5m)^2(3\beta_n(\alpha)(1 + m)(72\alpha^2 + 96\alpha^2 m - (1 + \alpha(2 + \alpha(-30 + \alpha(2$$
$$+ \alpha))))m^2)(-36 + m(-48 + (-15 + \alpha)m))^2 + (1 - \beta_n(\alpha))(36\alpha + 48\alpha m$$
$$- (1 + (-14 + \alpha)\alpha)m^2)^2(216 + m(504 - m(-10(39 + 10m) + \alpha(6 + 5m$$
$$+ 3\alpha(1 + m)))))))) \tag{1.101}$$

Direct comparison of the two profit functions completes the proof. ∎

Proof of Proposition 1.6:
After substituting optimal prices p_i^t, p_i^s, $i \in \{A, B, C\}$, (1.16)-(1.18) and (1.9)-(1.10) into the equation (1.59) of the expected profit of firm C under complete information, we obtain:

$$\Pi_C^b = \frac{144\alpha^2\beta_n(\alpha)(1 + m)^2(3 + 2m)^3}{D_1} + \frac{(1 - \beta_n(\alpha))(3 + 2m)(6 + 5m)^2}{(-36 + m(-42 + (-10 + \alpha)m))^2} \tag{1.102}$$

After substituting optimal prices p_{ij}^*, $i \in \{A, B\}$, $j \in \{1, 2\}$ and p_C^* (1.41)-(1.45) into the equation (1.60) of the expected profit of firm C under asymmetric information, we obtain:

$$\Pi_C^a = \big[(3 + 2m)(-(1 - \beta_n(\alpha))m^2(6 + 5m)^2 - \alpha^2 m^2(36 + m(60 + (25 - \beta_n(\alpha))m))$$
$$+ 2\alpha(6 + 5m)(108 + m(234 + m(162 + (35 + \beta_n(\alpha))m))))^2\big]/D_5 \tag{1.103}$$

where $D_5 = (-\alpha^3 m^4(6 + 5m) + m^2(6 + 5m)^2(6 + (2 + \beta_n(\alpha))m) + \alpha^2 m^2(432 + m(900 + m(594 + (120 + \beta_n(\alpha))m))) + \alpha(6 + 5m)(-1296 + m(-3240 + m(-2880 + m(-1068 + (-141 + 2\beta_n(\alpha))m)))))^2$

Direct comparison of the two profits leads to the result. ∎

Chapter 2

Cooperative and noncooperative R&D in an asymmetric multi-product duopoly with spillovers

This chapter is based on Fudickar and Rakić (2016).

2.1 Introduction

A firm investing in research and development (R&D) with spillovers usually imposes a positive externality on other companies which can then appropriate the results of this investment. D'Aspremont and Jacquemin (1988) show in a symmetric environment that encouraging firms to collaborate in R&D activities increases R&D investment and hence, social welfare by internalizing the externality. The European Commission has recognized these benefits of joint R&D and has thus issued revised "block exemption" regulations in 2010 that provide an automatic exemption from competition law for certain types of joint R&D agreements.

We study R&D investment in a market where a multi-product firm produces an established and an innovative product and a single-product firm only produces an innovative product. Thereby, we extend the model of D'Aspremont and Jacquemin

(1988) by incorporating two additional aspects. First, we consider an asymmetric market environment where a multi-product firm competes with a single-product firm. Second, the innovative and the established goods are substitutes so that R&D investment in the innovative product might come at the expense of the sales of the established product. It is often assumed that innovative products are independent of any other products that the firms are producing. Such an assumption seems, however, rather restrictive. Hence, firms have to consider the impact of their R&D investments not only on the output decision of the innovative product but also on the established product.

The two extensions enable us to study asymmetric competition between multi-product and single-product firms as commonly observed in situations where "dirty" products compete with "clean", environmentally friendly products. An example of such a market is the automobile industry. Traditional car manufacturers compete with firms that specialize in the production of electric vehicles. For example, Tesla Motors produces exclusively electric cars and competes with more traditional businesses that produce both electric and gasoline cars. The most challenging issue related to the future development of electric vehicles is the battery charging. Companies invest in R&D to improve the loading time and reduce the size and cost of these batteries. Firms often cooperate in R&D investments to benefit from each other's know-how. One example of such a strategic relationship is the cooperation between Daimler and Tesla Motors, which started in 2009.

Investments in R&D are strategic as they influence product market outcomes. Hence, when firms compete in R&D, in addition to the *direct effect* by which firms benefit from cost reductions, there are two potential strategic effects. Through a *within-product competition effect*, a firm's investment decision indirectly affects its profit by its influence on its competitor's output decision of the innovative good. Depending on the level of the spillover, this effect can be negative or positive. Particular to our asymmetric set-up is the second strategic effect, the *cross-product competition effect*. It states that, in addition to changes in the output of the same product, the multi-product firm also modifies its output of the established good, which in turn benefits the single-product firm as it is able to steal some business.

In contrast to R&D competition, we obtain three additional effects under co-operation. When choosing an investment level to maximize joint profit, firms

internalize the effect of their R&D investment on the competitor's profit. Because of the *spillover effect*, an increase in R&D investment benefits rival's profit by also decreasing its marginal cost; hence, R&D investment is stimulated. Moreover, through the *within-product cooperation effect*, by investing more, a firm gains a competitive advantage over its rival in the same product, which hurts the competitor. The third effect, *cross-product coordination effect*, is unique to our multi-product environment. When the multi-product firm increases its R&D expenditure, it reduces its output of the substitute good to mitigate within firm cannibalization. This output reduction has a positive impact on the single-product firm's profit and hence, increases investment incentives of the multi-product firm.

When the sum of these additional effects of cooperation is positive for a firm, its investment incentives under cooperation are higher than under competition because its investment then benefits the other firm. We find that the additional, positive, cross-product coordination effect of the multi-product firm together with the spillover effect counteracts the negative within-product coordination effect. Hence, the profit externality conferred on the profit of the single-product firm is positive for a greater range of values of the spillover level and the degree of product differentiation in comparison to the single-product firm.

Our central result states that when the established and the innovative products are close substitutes, total R&D investment under cooperation will be lower than under competition even if the spillover is substantial. More specifically, R&D investment of the single-product firm may be higher under competition than under cooperation even if the spillover is significant. Moreover, for medium spillovers and high product substitutability the multi-product firm also invests less under R&D cooperation. Thus, in contrast to standard results in D'Aspremont and Jacquemin (1988) which suggest that R&D investment under cooperation is higher than under competition when the spillover is high, we find that it not only depends on the technology spillover but also on the degree of product substitutability.

The remainder of this chapter is organized as follows. Section 2.2 revises the related literature. The theoretical model is presented in Section 2.3. In section 2.4 we analyze the retail market equilibrium. Sections 2.5 and 2.6 identify the investment incentives under competition and cooperation, respectively. In Section 2.7 we compare R&D investment under competition and coordination. Section 2.8

concludes. The proofs of all formal results are relegated to the Appendix.

2.2 Related literature

As mentioned before, the starting point of our analysis is the study by D'Aspremont and Jacquemin (1988), which also serves as our benchmark. They analyze firms' incentives to invest in R&D with spillovers under R&D competition and cooperation in a symmetric, homogeneous product duopoly. They show that cooperation increases R&D investment levels compared to competitive R&D only when the spillovers are sufficiently high. Kamien et al. (1992) extend their model by introducing heterogeneity among the firms. They show that the general results of D'Aspremont and Jacquemin (1988) still hold. The key intuition in that strand of the literature is that private incentives to conduct R&D are reduced when there are knowledge spillovers from one firm to another due to free-rider incentives.

Lin and Zhou (2013) analyze R&D investment incentives in a multi-product environment. They consider R&D investment in a two-product duopoly with differentiated goods, where each firm has an initial cost advantage in one of the products. They find that when a firm invests more in one particular good, its competitor will respond by investing more in the other good. When the goods become more substitutable, this effect will be stronger. Moreover, R&D coordination in R&D lowers investment. In contrast to Lin and Zhou (2013), we analyze an asymmetric setting without cost advantages, but instead, we allow for spillovers.

Kawasaki et al. (2014) also consider a multi-product model, in which firms engage in R&D investment. A multi-product firm has a monopoly in one market and competes with potential entrants in a second market. Contrary to our set-up, demands for the two products are independent and R&D efforts by the multi-market firm simultaneously reduce the marginal cost of both goods. They show that entry can stimulate investment in cost-reducing R&D.

None of those studies, however, considers the interaction between an asymmetric multi-product environment and the dynamics of R&D cooperation. Bulow et al. (1985) investigate strategic interaction in an asymmetric multi-market oligopoly. They find that a shock to a firm in one market also affects its competitor's strategy in a second market. This finding can be translated into our set-up as we consider

R&D investment as a strategic interaction. If a firm invests in the new technology product, the competitiveness of the substitute good is reduced. Therefore, the incumbent reduces its output of the established good.

2.3 Model

We consider a market with two firms A and B. Firm A is the single producer of an established product (good 1), while both firms produce a substitute product (good 2), which is based on a new technology. The prices of the two products are given by the following linear inverse demand functions[1]:

$$p_1(q_{A1}, q_{A2}, q_{B2}) = a - q_{A1} - g(q_{A2} + q_{B2}) \tag{2.1}$$

$$p_2(q_{A1}, q_{A2}, q_{B2}) = a - (q_{A2} + q_{B2}) - gq_{A1} \tag{2.2}$$

where $a > 0$, quantity q_{ji} is the output of good $i \in \{1, 2\}$ produced by firm $j \in \{A, B\}$ and $g \in [0, 1)$ represents the degree of product substitutability between goods 1 and 2. Therefore, the two products in the market are imperfect substitutes, while good 2 is homogeneous. This asymmetric market structure exists, for example, in the automobile industry, where traditional car manufacturers, producing gasoline and electric cars, compete with electric car manufacturers.

Focusing on R&D for the new technology good, we assume that the unit cost of producing the established good is fixed and equalize it to zero. Hence, only R&D investment in the new technology good is possible. The unit cost of producing the new technology good is $c > 0$, but each firm can invest some $x_j > 0$ in process R&D to reduce its unit cost[2]:

$$c_{j2} = c - x_j - \beta x_{-j}, \tag{2.3}$$

where the amount x_j is the R&D investment of firm j, the amount x_{-j} is the R&D investment of the rival, and $\beta \in [0, 1]$ is the spillover of the rival's R&D investment on firm j. Hence, firms benefit from their rival's R&D activity. We assume that

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[1] Derived from the utility function of a representative consumer (Dixit, 1979): $U(q_{A1}, q_{A2}, q_{B2}) = a(q_{A1} + q_{A2} + q_{B2}) - 1/2(q_{A1}^2 + 2gq_{A1}(q_{A2} + q_{B2}) + (q_{A2} + q_{B2})^2)$

[2] We assume that c is high enough, so the new technology is costlier than the established one.

the R&D cost is quadratic and given by γx_j^2, where $\gamma > 0$. Thus, the profit of the multi-product firm A is

$$\Pi_A = p_1(q_{A1}, q_{A2}, q_{B2})q_{A1} + [p_2(q_{A1}, q_{A2}, q_{B2}) - (c - x_A - \beta x_B)]\, q_{A2} - \gamma x_A^2$$
$$= \pi_A(q_{A1}, q_{A2}, q_{B2}, x_A, x_B) - \gamma x_A^2 \tag{2.4}$$

and the profit of the single-product firm B is

$$\Pi_B = [p_2(q_{A1}, q_{A2}, q_{B2}) - (c - x_B - \beta x_A)]\, q_{B2} - \gamma x_B^2$$
$$= \pi_B(q_{A1}, q_{A2}, q_{B2}, x_A, x_B) - \gamma x_B^2 \tag{2.5}$$

where $\pi_j(q_{A1}, q_{A2}, q_{B2}, x_A, x_B)$, $j = A, B$, denotes the profit gross of R&D investment cost.

We consider the following two-stage game. In the first stage firms simultaneously choose their level of R&D investment (x_A, x_B) to reduce marginal costs. We then examine R&D competition and cooperation. Based on their R&D choice, the firms compete in the second stage à la Cournot and set their production quantities simultaneously. We solve for the equilibria by backward induction.

2.4   Retail market outcomes

In the second stage, firms compete simultaneously in the product market given the R&D investment levels for the new technology good, x_A and x_B. Firm A maximizes its profit π_A by choosing quantities q_{A1} and q_{A2}, while firm B maximizes its profit π_B by only choosing quantity q_{B2}. From the first-order conditions $\partial \pi_A / \partial q_{A1} = \partial \pi_A / \partial q_{A2} = \partial \pi_B / \partial q_{B2} = 0$ we obtain the equilibrium quantities as functions of the R&D investments[3]:

$$q_{A1}^*(x_A, x_B) = \frac{a(1 - g) + g(c - x_A - \beta x_B)}{2(1 - g^2)} \tag{2.6}$$

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[3]The second-order conditions for a maximum are satisfied: $D(q_{A1}, q_{A2}) = (\partial^2 \pi_A / \partial q_{A1}^2)(\partial^2 \pi_A / \partial q_{A2}^2) - (\partial^2 \pi_A / (\partial q_{A2} \partial q_{A1}))^2 = 4 - 4g > 0$, $\partial^2 \pi_A / \partial q_{A1}^2 = -2 < 0$ and $\partial^2 \pi_B / \partial q_{B2}^2 = -2 < 0$.

$$q_{A2}^*(x_A, x_B) = \frac{1}{6(1-g^2)}(a(2-3g+g^2) - c(2+g^2) + (4+(2\beta-1)g^2$$

$$-2\beta)x_A - (2-(2-\beta)g^2 - 4\beta)x_B) \tag{2.7}$$

$$q_{B2}^*(x_A, x_B) = \frac{a-c+(2\beta-1)x_A + (2-\beta)x_B}{3} \tag{2.8}$$

To ensure positive output levels in the absence of R&D investments, we assume that $a > c(2+g^2)/(2-g(3-g))$. When there is no investment in R&D ($x_A = x_B = 0$), the multi-product firm produces more of its established good than of its new technology good because the established good has smaller marginal costs; hence, obtaining a competitive advantage. Moreover, firm B produces more of good 2 than firm A.

In the first stage, firms choose their R&D investment. We first examine how firms' R&D investments affect the market outcomes in the second stage by differentiating expressions (2.6)-(2.8) with respect to x_A and x_B. By increasing its investment in the innovative product, a firm reduces its marginal cost of that product. Thereby, it always reacts with an increase in its own quantity of the innovative product,

$$\frac{\partial q_{i2}^*}{\partial x_i} > 0. \tag{2.9}$$

Due to the technology spillovers, an increase in a firm's R&D investment not only reduces its own marginal cost of the new technology good but also reduces its competitor's marginal cost of the same product. When β is large, the spillover effect becomes strong so that the competitor also reacts with an increase in its quantity of the innovative product,

$$\frac{\partial q_{A2}^*}{\partial x_B} = -\frac{1}{3} + \frac{\beta(4-g^2)}{6(1-g^2)} = \begin{cases} > 0, & \text{if } \beta > \hat{\beta}_B \equiv \frac{2(1-g^2)}{4-g^2} \\ < 0, & \text{if } \beta < \hat{\beta}_B \equiv \frac{2(1-g^2)}{4-g^2} \end{cases} \tag{2.10}$$

and

$$\frac{\partial q_{B2}^*}{\partial x_A} = \frac{1}{3}(2\beta-1) = \begin{cases} > 0, & \text{if } \beta > \hat{\beta}_A \equiv \frac{1}{2} \\ < 0, & \text{if } \beta < \hat{\beta}_A \equiv \frac{1}{2} \end{cases} \tag{2.11}$$

As the new technology products compete directly with the established product,

there are also effects of R&D investment in the output of good 1. When R&D activity in the innovative product by either firm increases, firm A responds with a reduction in its output quantity of the traditional good:

$$\frac{\partial q_{A1}^*}{\partial x_i} < 0, \text{ for } g > 0. \tag{2.12}$$

Moreover, by taking the derivative of (2.12) with respect to g, it can be seen that the closer substitutes the goods are the more firm A suffers in the sales of its traditional product from investment in the new technology.

2.5   R&D competition

In the first stage, firms invest in R&D taking into account the optimal output strategy in stage two. Both firms decide on their R&D investments to maximize their respective profit. Firm i chooses x_i to maximize its total profit

$$\Pi_i = \pi_i(x_i, x_j, q_{i1}^*(x_i, x_j), q_{i2}^*(x_i, x_j), q_{j2}^*(x_i, x_j)) - \gamma x_i^2 \tag{2.13}$$

where $i \in \{A, B\}$ and $j \neq i$. The first-order condition for maximizing firm i's profit in expression (2.13) is

$$\frac{d\Pi_i}{dx_i} = \frac{d\pi_i}{dx_i} - 2\gamma x_i = 0. \tag{2.14}$$

In what follows we assume that γ is large enough so that all second-order conditions are satisfied in order to have interior solutions.

Assumption 1

$$\gamma > \gamma_{min} \equiv \frac{11g^2 - 20 + \beta(1 - g^2)(32 - 20\beta)(g^2 - 1)}{36(1 - g^2)}$$

The relevant first-order conditions then lead to the R&D best-response functions $R_A^N(x_B)$ and $R_B^N(x_A)$ for firm A and B, respectively[4]. The reaction functions are downward sloping for low values of β and upward sloping for higher values of β, as illustrated in Figure 2.1.

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[4]The closed forms are provided in the appendix by formulas (2.26) and (2.27).
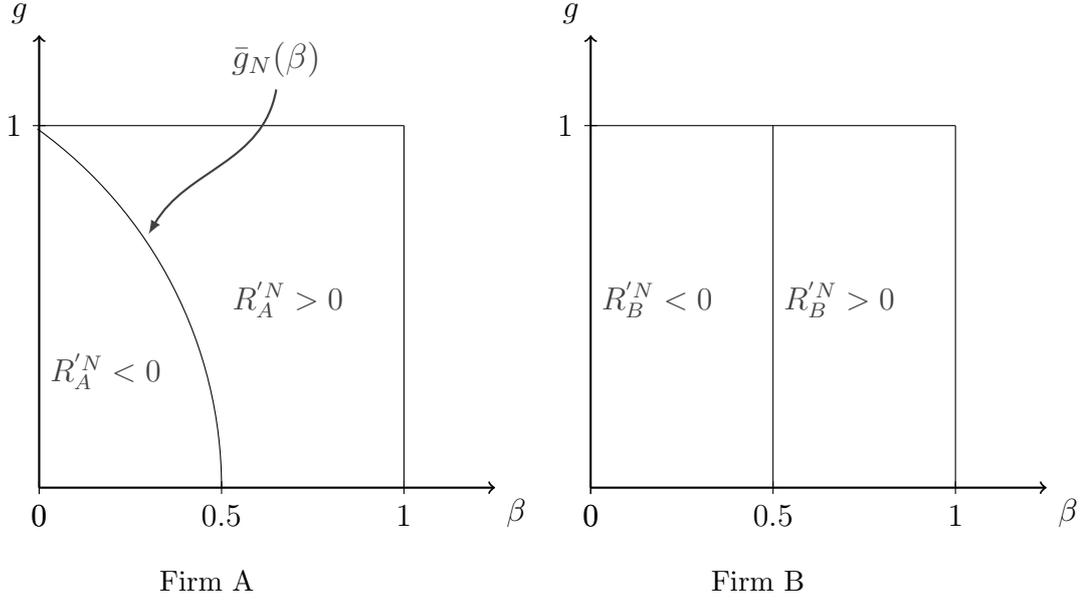
Figure 2.1: Signs of slopes of reaction functions

Lemma 2.1 *The slope of*

(i) $R_A^N(x_B)$ *is negative if* $0 < \beta < 1/2$ *and* $0 < g < \bar{g}_N(\beta)$ *and positive elsewhere, where* $\bar{g}_N(\beta) \equiv \sqrt{4(\beta - 2)(2\beta - 1)/(8 + \beta(8\beta - 11))}$

(ii) $R_B^N(x_A)$ *is negative if* $0 < \beta < 1/2$ *and* $0 < g < 1$ *and positive elsewhere.*

Lemma 2.1 shows that R&D investments are strategic substitutes when spillovers are low. Intuitively, an increase in R&D investment by one firm leads to a decrease in the output of the competitor. R&D investments turn into strategic complements when spillovers intensify and products are more substitutable. An increase in R&D by one firm leads to a decrease in the competitor's marginal cost due to the technology spillover. This reduction in marginal cost has a positive impact on competitor's output decision and thereby increases its incentives to invest in R&D. To stay competitive with firm B, firm A may also increase its R&D investment when spillovers are low. This case happens when the products are closer substitutes.

We next derive the marginal benefit of investment for each firm and identify different strategic effects that arise under R&D competition.

By applying the Envelope Theorem to $d\pi_A/dx_A$ in (2.14) we obtain

$$\frac{d\pi_A}{dx_A} = \underbrace{\frac{\partial \pi_A}{\partial x_A}}_{\substack{+ \\ \text{direct effect}}} + \underbrace{\frac{\partial \pi_A}{\partial q_{A1}}\frac{\partial q_{A1}^*}{\partial x_A}}_{\substack{=0 \\ \text{(envelope theorem)}}} + \underbrace{\frac{\partial \pi_A}{\partial q_{A2}}\frac{\partial q_{A2}^*}{\partial x_A}}_{\substack{=0 \\ \text{(envelope theorem)}}} + \underbrace{\frac{\partial \pi_A}{\partial q_{B2}}\frac{\partial q_{B2}^*}{\partial x_A}}_{\substack{- \quad -/+ \\ \text{within-product} \\ \text{competition effect}}} \qquad (2.15)$$

Recall that $\partial \pi_A/\partial q_{Ai} = 0$ when evaluated in the optimum q_{Ai}^*, $i = 1, 2$, as a result of profit maximization in the second stage. By applying the Envelope Theorem to $d\pi_B/dx_B$ in (2.14) we obtain

$$\frac{d\pi_B}{dx_B} = \underbrace{\frac{\partial \pi_B}{\partial x_B}}_{\substack{+ \\ \text{direct effect}}} + \underbrace{\frac{\partial \pi_B}{\partial q_{B2}}\frac{\partial q_{B2}^*}{\partial x_B}}_{\substack{=0 \\ \text{(envelope theorem)}}} + \underbrace{\frac{\partial \pi_B}{\partial q_{A2}}\frac{\partial q_{A2}^*}{\partial x_B}}_{\substack{- \quad -/+ \\ \text{within-product} \\ \text{competition effect}}} + \underbrace{\frac{\partial \pi_B}{\partial q_{A1}}\frac{\partial q_{A1}^*}{\partial x_B}}_{\substack{- \quad - \\ \text{cross-product} \\ \text{competition effect}}} \qquad (2.16)$$

We also recall that $\partial \pi_B/\partial q_{B2} = 0$ when evaluated in the optimum q_{B2}^* as a result of profit maximization in the second stage.

Each firm invests in R&D to reduce the costs of its innovative product. Those investments affect firms' profits in different ways. First of all, there is a *direct effect* of R&D investment, but additionally we identify two types of strategic effects:

(i) *within-product competition effect.* - A firm's investment decision indirectly affects its own profit through its influence on its competitor's output decision of the same product.

(ii) *cross-product competition effect.* - A firm's investment decision indirectly affects its own profit through its influence on its competitor's output decision of the substitute product.

We summarize the effects of R&D investments under R&D competition on the firms' gross profits (i.e. excluding R&D costs) in the following proposition.

Proposition 2.1 *An increase in firm j's R&D investment x_j affects its profits*
(i) *positively through the direct effect*
(ii) *positively for $0 \leq \beta < \hat{\beta}_j$ and negatively otherwise through the within-product competition effect.* [5]

---

[5]The critical spillover $\hat{\beta}_j$, $j \in \{A, B\}$ is defined by equations (2.10) and (2.11)

Additionally,

(iii) *an increase in firm B's R&D investment also influences its own profit posi-
tively through the cross-product competition effect.*

Intuitively, the direct effect is always positive. This direct effect results from the
fact that an increase in the level of R&D investment leads to a reduction in firm's
marginal cost of the new technology good, which in turn leads to an increase in
its profit.

Due to knowledge spillovers, an increase in a firm's R&D investment also re-
duces its rival's marginal cost. However, only when spillovers are significant, the
competitor's marginal cost is reduced substantially so that it also reacts more
aggressively and increases its output level. Then profit of the investing firm is
reduced. Hence, the *within-product competition effect* increases R&D incentives of
a firm when spillovers are low and decreases them for large spillovers. Only small
spillovers create a real competitive advantage for an investing firm because large
spillovers create greater potential for free-riding.

The *cross-product competition effect* is specific to the single-product firm. It
only prevails in our asymmetric environment. As the multi-product firm produces
two substitute goods, it influences the single-product firm's optimal R&D deci-
sion not only through its response regarding its output decision of the new tech-
nology product but also regarding its output decision of the traditional product.
Clearly, the multi-product firm lowers its output of the traditional product because
that good loses its competitive advantage. The single-product firm then benefits
from that output reduction as its competitiveness towards the established good
increases. The *cross-product competition effect*, therefore, always raises investment
incentives for firm B.

The marginal benefit of firm A's cost-reducing investment depends only on the
relative magnitudes of the direct effect and the within-product competition effect.
Formally,

$$\frac{d\pi_A}{dx_A} = \frac{1}{3}g(1-2\beta)q_{A1} + \frac{2}{3}(2-\beta)q_{A2} \tag{2.17}$$

The overall marginal benefit of firm B's cost-reducing investment under R&D
competition additionally depends on the cross-product competition effect. Hence,

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we obtain

$$\frac{d\pi_B}{dx_B} = \frac{2}{3}(2 - \beta)q_{B2}. \tag{2.18}$$

As the latter is positive, firm B always has an incentive to invest in R&D. On the contrary, firm A's investment incentives can be negative if $\beta > 1/2$ and g is very high. Under such a parameter constellation, firm A would not invest at all so that then $x_A = 0$.

2.6 R&D cooperation

We next consider cooperation in R&D investment while the second stage remains competitive. In the first stage firms choose investment levels x_A and x_B by maximizing their joint profits given by (2.13):

$$\begin{aligned}
\max_{x_A, x_B} \Pi_A + \Pi_B \\
= \pi_A(x_A, x_B, q_{1A}^*(x_A, x_B), q_{2A}^*(x_A, x_B), q_{2B}^*(x_A, x_B)) \\
+ \pi_B(x_A, x_B, q_{1A}^*(x_A, x_B), q_{2A}^*(x_A, x_B), q_{2B}^*(x_A, x_B)) \\
- \gamma x_A^2 - \gamma x_B^2
\end{aligned} \tag{2.19}$$

The first-order condition for investment under joint profit maximization for firm i in expression (2.19) is

$$\frac{d(\Pi_A + \Pi_B)}{dx_i} = \frac{d(\pi_A + \pi_B)}{dx_i} - 2\gamma x_i = 0 \tag{2.20}$$

where $d(\pi_A + \pi_B)/dx_A$ and $d(\pi_A + \pi_B)/dx_B$ are net marginal increases in the firms' joint profit. This then leads to

$$R_i^C(x_j) = arg\max_{x_i} [\Pi_A + \Pi_B]. \tag{2.21}$$

For convenience and abusing somewhat usual conventions we call $R_i^C(x_j)$, in what follows, reaction functions under cooperation[6].

---

[6]The closed forms are given in the Appendix by formulas (2.43) and (2.44).

Lemma 2.2 *The slope of $R_i^C(x_j)$ under R&D cooperation is negative if $0 \leq \beta < 1/2$ and $0 \leq g \leq \bar{g}_C(\beta) \equiv \sqrt{8(\beta - 2)(2\beta - 1)/(16 + \beta(16\beta - 31))}$ and positive otherwise.*

Specifically, for the R&D investment level of firm A, by applying the Envelope Theorem, we obtain the following marginal benefit of joint profit maximization:

$$
\begin{aligned}
\frac{d(\pi_A + \pi_B)}{dx_A} &= \frac{d\pi_A}{dx_A} + \frac{d\pi_B}{dx_A} \\
&= \underbrace{\frac{d\pi_A}{dx_A}}_{\text{Prop. 2.1}} + \underbrace{\frac{\partial \pi_B}{\partial x_A}}_{\substack{+ \\ \text{spillover effect}}} + \underbrace{\frac{\partial \pi_B}{\partial q_{A2}} \frac{\partial q_{A2}^*}{\partial x_A}}_{\substack{- \quad + \\ \text{within-product} \\ \text{coordination effect}}} + \underbrace{\frac{\partial \pi_B}{\partial q_{A1}} \frac{\partial q_{A1}^*}{\partial x_A}}_{\substack{- \quad - \\ \text{cross-product} \\ \text{coordination effect}}}
\end{aligned}
\tag{2.22}
$$

Similarly, we obtain the following result for firm B's investment:

$$
\begin{aligned}
\frac{d(\pi_A + \pi_B)}{dx_B} &= \frac{d\pi_B}{dx_B} + \frac{d\pi_A}{dx_B} \\
&= \underbrace{\frac{d\pi_B}{dx_B}}_{\text{Prop. 2.1}} + \underbrace{\frac{\partial \pi_A}{\partial x_B}}_{\substack{+ \\ \text{spillover effect}}} + \underbrace{\frac{\partial \pi_A}{\partial q_{B2}} \frac{\partial q_{B2}^*}{\partial x_B}}_{\substack{- \quad + \\ \text{within-product} \\ \text{coordination effect}}}
\end{aligned}
\tag{2.23}
$$

Cooperation may increase the incentive to conduct R&D by internalizing spillovers across the firms. However, R&D investment makes firms tougher competitors; hence, the effect of cooperation may be to reduce the incentive to conduct R&D.

The (strategic) effects of the first term, $d\pi_i/dx_i$, $i = A, B$, are derived and analyzed under R&D competition in Section 2.5 above. Under cooperation, each firm, also cares about how its choice of R&D investment affects the profit of its competitor. Hence, we identify a *spillover effect* and two further strategic effects under R&D cooperation:

(i) *within-product coordination effect.* - A firm's investment decision is influenced by the effect on its competitor's profit through changes in its own output decision of the same product.

(ii) *cross-product coordination effect.* - A firm's investment decision is influenced

by the effect on its competitor's profit through its own output decision of the substitute product.

We summarize the additional effects of R&D investment due to cooperation on the firms' joint gross profit in the following proposition.

Proposition 2.2 *When x_i increases, firm j's profit is influenced*
(i) *positively through the spillover effect*

(ii) *negatively through the within-product coordination effect.*
Additionally,
(iii) *when firm A increases its R&D investment, it also influences firm B's profit positively through the cross-product cooperation effect.*

When R&D by one firm spills over to the other firm, private incentives to conduct R&D are reduced due to potential free-riding. If firms choose R&D investment levels cooperatively, then these spillover externalities are internalized, and R&D investment is stimulated. An increase in one firm's R&D investment reduces the other firm's cost due to the technology spillover; hence, lower costs increase rivals profit.

The *within-product coordination effect* decreases a firm's investment incentives as it also cares about its rival's profit under cooperation. When a firm invests more in the new technology, it increases its output of the innovative product, as seen in expression (2.9). As a result, the competitor faces a decline in its market share and suffers from a loss of its profit.

The *cross-product coordination effect* only exists for the multi-product firm in this asymmetric set-up as it internalizes its positive impact on the single-product firm's profit when it reduces its output of the traditional good to mitigate within-firm cannibalization. The *cross-product coordination effect* is always positive because the new technology good becomes more competitive towards the established good by approaching the cost level of the established good. This effect is strengthened if the products are closer substitutes because then the multi-product firm will lose a significant competitive advantage in the established good.

Within-product competition and cooperation effects indicate how R&D investment influence profits through the new technology good, whereas the cross-product

competition and coordination effects indicated how R&D investment influence profits through the traditional good.

The overall marginal benefit of a firm's cost-reducing investment under R&D cooperation depends on the relative magnitudes of the direct and spillover effects and three strategic effects, where some of the strategic effects differ for the multi-product and the single-product firm.

2.7 R&D competition vs. cooperation

We next analyze the equilibrium R&D investment levels of each firm under competition and cooperation. In order to do so, we compare the reaction functions $R_i^C(x_j)$ and $R_i^N(x_j)$ in the $x_A - x_B$-diagram. Whether $R_i^C(x_j)$ under cooperation lies above or below $R_i^N(x_j)$ under competition depends only on the sign of the profit externality, $d\pi_j/dx_i$, in equations (2.22) and (2.23). If $R_i^C(x_j)$ is above (below) $R_i^N(x_j)$, a firm will respond with a higher (lower) investment level under cooperation than under competition.

By adding the spillover-, within-product coordination- and cross-product coordination-effects of expression (2.22), we obtain the profit externality conferred by A's R&D investment on the profit of firm B:

$$\frac{d\pi_B}{dx_A} = \frac{2}{3}(2\beta - 1)q_{B2} \tag{2.24}$$

Since q_{B2} is always positive, the position of $R_A^C(x_B)$ depends only on the level of the spillovers.

Lemma 2.3 *For all $0 \leq g < 1$, $R_A^C(x_B)$ lies below $R_A^N(x_B)$ if $0 \leq \beta < 1/2$ and above $R_A^N(x_B)$ if $1/2 < \beta \leq 1$.*

The result follows from the fact that the negative within-product coordination effect outweighs the sum of the positive spillover- and cross-product cooperation effects when the spillover is small, as illustrated in Figure 2.2. However, when the spillover is large, the opposite is true.

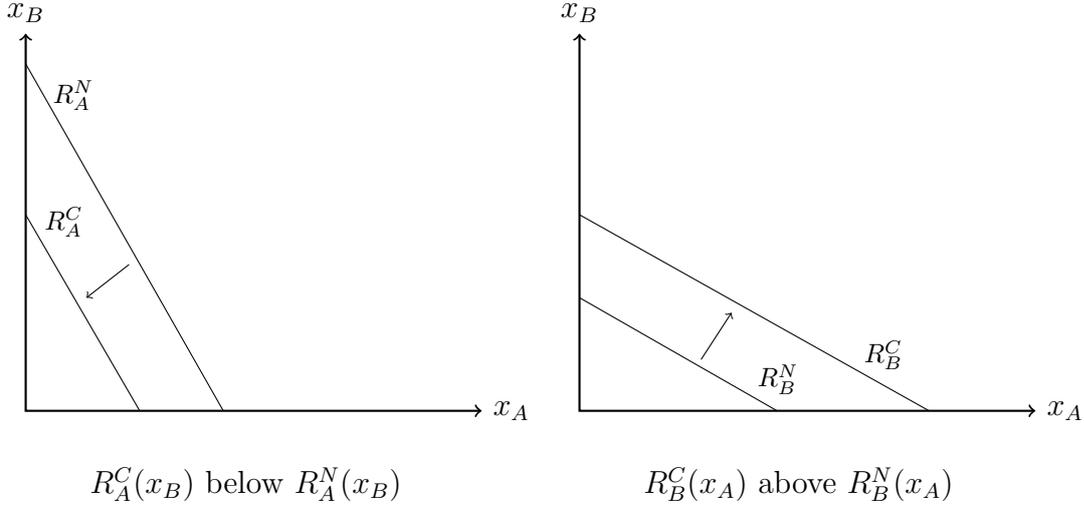$R_A^C(x_B)$ below $R_A^N(x_B)$ $R_B^C(x_A)$ above $R_B^N(x_A)$

Figure 2.2: Positions of reaction functions

From expression (2.23) we derive the profit externality of firm B's R&D investment on the profit of firm A:

$$\frac{d\pi_A}{dx_B} = \frac{2}{3}(2\beta - 1)q_{A2} - \frac{1}{3}g(2 - \beta)q_{A1} \qquad (2.25)$$

The first term, $2(2\beta - 1)q_{A2}/3$, is positive if and only if $\beta > 1/2$, whereas the second term, $-g(2 - \beta)q_{A1}/3$, is always negative. Hence, the position of $R_B^C(x_A)$ of firm B under cooperation depends not only on the knowledge spillover but also on the degree of product differentiation between goods 1 and 2. The reaction function $R_B^C(x_A)$ lies above $R_B^N(x_A)$ whenever the second term is negligible; that is if g approaches zero.

Lemma 2.4 *There is an upward sloping function $g_B^C(\beta) : [1/2, 1] \to (0, 1)$ such that $R_B^C(x_A)$ lies above $R_B^N(x_A)$ if $1/2 < \beta \leq 1$ and $0 \leq g \leq g_B^C(\beta)$, and below otherwise.*

Firm B also internalizes the impact of its R&D on the other firm through cooperation. However, as firm B only produces the innovative good, the positive cross-product coordination effect on firm A's established product does not exist. Hence, there are only two opposing effects of R&D cooperation on the investment

incentives of firm B. On one hand, if firms choose R&D investment levels coop-
eratively, then the spillover externalities are internalized, and R&D investment is
stimulated. On the other hand, the within-product coordination effect counteracts
this positive effect on R&D investment incentives and may even dominate it when
spillovers are high and product substitutability is high. This case happens when
firm A's loss in profit due to a decline in its market share in both products will
not be offset by an increase in its profit due to reduced marginal cost. If the prod-
ucts are close substitutes firm A will lose significant market power in product 1
following R&D investment by firm B which cannot be offset by the spillover effect.

Having analyzed the effect of cooperation on the incentives to invest in R&D
we are now able to determine the equilibrium R&D investment levels of both firms.
The directions of the slopes of the reaction functions under R&D competition and
under R&D cooperation are known from Lemmas 2.1 and 2.2, respectively. In
addition, Lemmas 2.3 and 2.4 describe the positions of the respective reaction
functions under R&D cooperation compared to R&D competition. We note that
these functions are linear[7].

If spillovers are sufficiently low (i.e. $0 \leq \beta < 1/2$), we know from Lemmas
2.3 and 2.4 that both $R_A^C(x_B)$ and $R_B^C(x_A)$ are below the reaction functions under
competition for all degrees of product substitutability. Hence, it follows directly
that, in this case, R&D investment levels under cooperation are lower than under
R&D competition. Figure 2.3 illustrates this in the $x_A - x_B$-diagram.

Similarly, if spillovers are sufficiently high (i.e. $1/2 \leq \beta \leq 1$) and at the same
time, the degree of product substitutability is low (i.e. $0 \leq g < g_B^C(\beta)$), then
both $R_A^C(x_B)$ and $R_B^C(x_A)$ lie above the reaction functions under competition. It
is thus straightforward to see that then R&D investment levels under cooperation
are higher for both firms.

When the positions are in the same direction, it is simple to determine the effect
of cooperation on R&D levels. However, when this is not the case, it becomes more
cumbersome. We illustrate all different cases subsequently.

Let us consider the case when $1/2 \leq \beta \leq 1$ and $g_B^C(\beta) < g < 1$, which is
illustrated in Figure 2.4. By Lemmas 2.1 and 2.2, slopes of all reaction functions
are upward sloping. By Lemma 2.3 $R_A^C(x_B)$ lies above $R_A^N(x_B)$, while by Lemma

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[7]This is easily seen from equations (2.26) - (2.27) and (2.43) - (2.44).
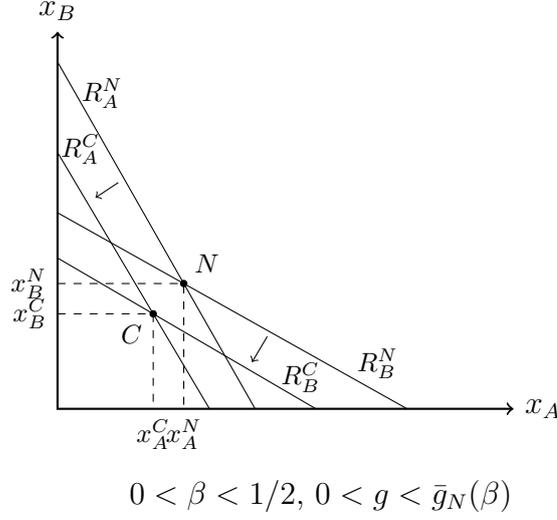
$$0 < \beta < 1/2,\ 0 < g < \bar{g}_N(\beta)$$

Figure 2.3: Optimal R&D investment levels under competition and cooperation

2.4 $R_B^C(x_A)$ lies below $R_B^C(x_A)$.

We observe two possible scenarios. First, the difference between $R_i^C(x_j)$ and $R_i^N(x_j)$ is of similar size and second, the difference between $R_B^C(x_A)$ and $R_B^N(x_A)$ is significantly larger than the difference between $R_A^C(x_B)$ and $R_A^N(x_B)$. The latter one occurs when the spillover approaches $1/2$ and the degree of substitutability is positive. Then, we observe from equation (2.24) that the difference $(R_A^C(x_B) - R_A^N(x_B))$ is approaching zero. Additionally, the difference $(R_B^C(x_A) - R_B^N(x_A))$ is significant as the first term in equation (2.25) is negligible, while the second term increases as g increases.

From the two diagrams in Figure 2.4 we can see that, in the observed interval, the R&D investment level of firm B is lower under cooperation than under competition. Additionally, the R&D investment level of firm A could also be lower under cooperation than under competition when $\beta \to 1/2$ and $g \neq 0$.

In the following Proposition, we summarize the effects of R&D cooperation on the investment levels of both firms when all parameter constellations are taken into account. Figure 2.5 illustrates these results graphically.

$$1/2 < \beta < 1, \ g_B^C(\beta) < g < 1 \qquad\qquad \beta \to 1/2, \ g_B^C(\beta) < g < 1$$
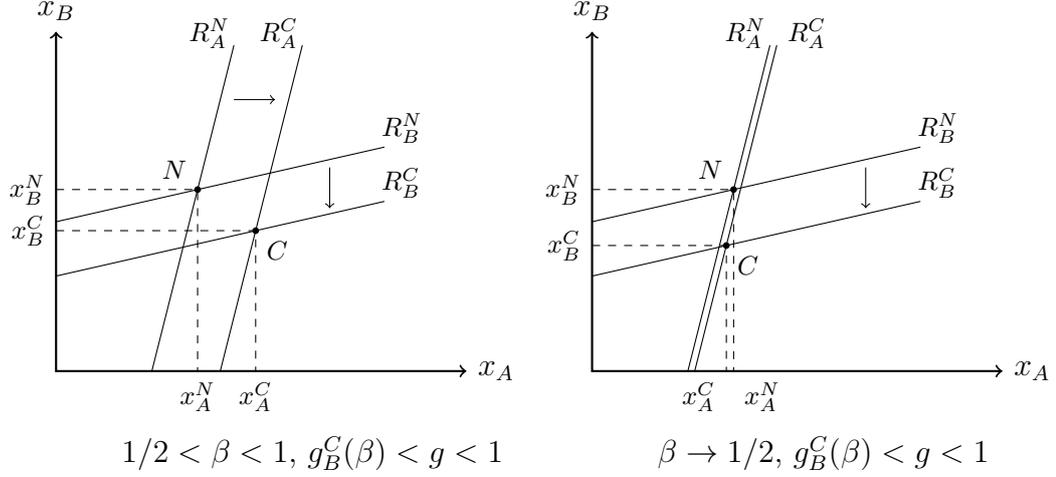
Figure 2.4: Optimal R&D investment levels under competition and cooperation

Proposition 2.3 *There is an upward sloping function $g_A^C(\beta) : [0,1] \to (0,1)$ such that:*

(i) $x_A^C < x_A^N$ *if* $0 < \beta \leq 1/2$ *and* $0 < g \leq 1$ *and* $1/2 < \beta < 1$ *and* $g_A^C(\beta) < g < 1$; *otherwise* $x_A^C > x_A^N$.

(ii) $x_B^C < x_B^N$ *if* $0 < \beta \leq 1/2$ *and* $0 < g \leq 1$ *and* $1/2 < \beta < 1$ *and* $g_B^C(\beta) < g < 1$; *otherwise* $x_B^C > x_B^N$.

We find that whether the level of R&D investment, x_i, increases or decreases following cooperation depends not only on the technological spillover but also on the degree of substitution between the two products. If spillovers are sufficiently high and the degree of substitution is relatively low, R&D investment levels under cooperation exceed those of competition. The internalization leads to an increase in R&D because the positive effect of the spillover on firm $j's$ profit is higher than the adverse effect of the reduction in the marginal cost on firm $j's$ profit.

If the products are independent ($g = 0$), our result replicates the standard result by D'Aspremont and Jacquemin (1988), as seen in Figure 2.5. Due to the positive product differentiation in our analysis, we find that the R&D investment of firm B is higher under competition than under cooperation even when the spillover is high.
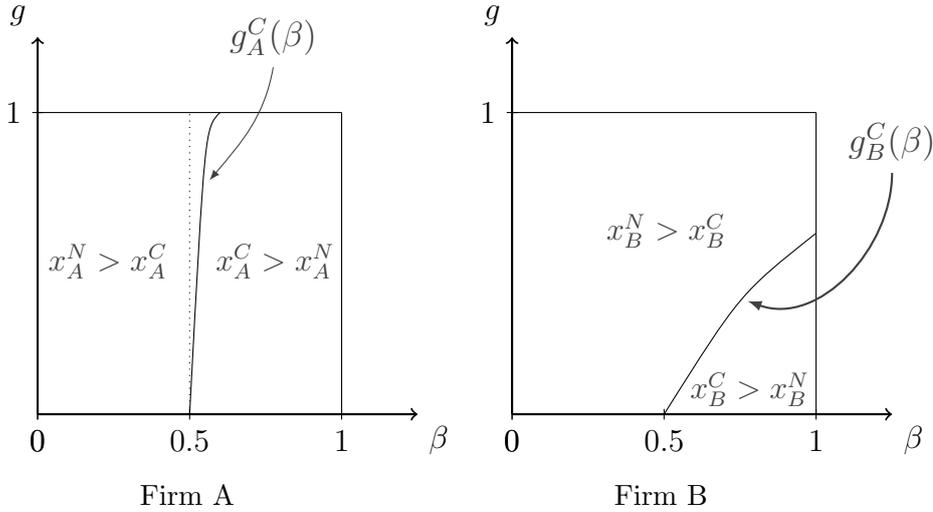
Figure 2.5: Comparison of R&D investments under competition and cooperation

From a policy perspective, it is important to determine the overall effect of cooperation on total R&D due to the marginal cost reduction. Total R&D depends on the sum of the changes in x_A and x_B. Let total competitive R&D investment be $x^N \equiv x_A^N + x_B^N$ and total cooperative R&D investment be $x^C \equiv x_A^C + x_B^C$. The next proposition directly follows from Proposition 2.3. Figure 2.6 illustrates the result.

Proposition 2.4 *Total investment under cooperation is lower than under competition if $0 < \beta \leq 1/2$ and $g > 0$ and if $1/2 < \beta < 1$ and g high enough.*

When $\beta > 1/2$ and $g_B^C(\beta) < g < g_A^C(\beta)$, cooperation will reduce x_B but increase x_A. It is, therefore, unclear whether R&D competition or R&D cooperation lead to a higher overall investment level. The net effect on total R&D will depend on the magnitudes of these changes.

We simulate the overall effect of cooperation on total R&D. We use the following specification: $a = 1000$, $c = 50$ and $\gamma = 60$. The resulting Figure 2.6 shows that, indeed, there exists a function $g^C(\beta)$, such that for for every $g > g^C(\beta)$ and $\beta > 1/2$ the overall investment in R&D under competition is greater than under R&D cooperation.
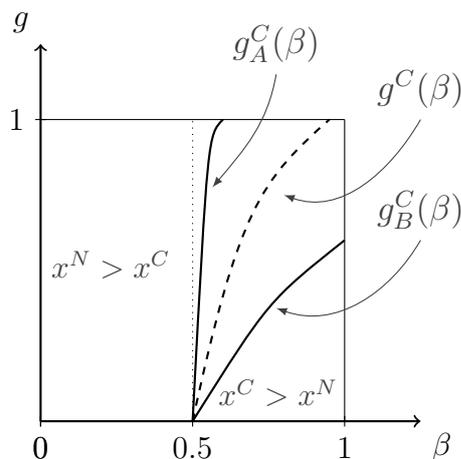
Figure 2.6: Comparison of total R&D investment

2.8 Conclusion

In this chapter, we study strategic R&D investment between a multi-product firm and a single-product firm. Investigating whether such asymmetric firms should be allowed to coordinate their decisions at the R&D stage, as in D'Aspremont and Jacquemin (1988), we find that R&D investment levels under cooperation are lower when the established and the innovative product are close substitutes even if the spillover is substantial. Hence, the asymmetry between the firms leads to higher R&D investment levels under competition than under cooperation for many values of the technology spillovers and degrees of product substitution. Our results, therefore, indicate that regulators need to be more cautious about allowing R&D joint ventures in an asymmetric context.

Besides, we also identify several strategic effects that are incorporated under R&D cooperation. For the multi-product firm, investment incentives are lower under cooperation when spillovers are low because the negative *within-product coordination effect* then dominates the positive spillover and *cross-product coordination* effects. For the single-product firm, if product substitutability is high, investment incentives are also lower under cooperation even when spillovers are significant. Following R&D investment by the single-product firm, the multi-product firm would lose significant market share in the established good if the products

are close substitutes. Then this loss cannot be offset by the spillover effect.

It would seem natural to assume that the spillovers of the two firms are not identical, given that large multi-product firms can protect their patents better than smaller firms. Hence, we can extend our analysis for asymmetric spillovers, namely $\beta_A < \beta_B$. However, our main result (Proposition 2.4) still holds.

2.9   Appendix

Proof of Lemma 2.1:

From the optimal quantities (2.6)-(2.8) in the second stage and profit maximization of (2.13) with respect to x_i the relevant first-order conditions lead to the following R&D best-response functions

$$
\begin{aligned}
R_A^N(x_B) = &\frac{1}{K_1}(a(g-1)(g-8+4\beta(1+g)) - c(8+g^2+4\beta(g^2-1)) \\
&+ (4(2-\beta)(2\beta-1) + (8+\beta(8\beta-11))g^2)x_B)
\end{aligned}
\tag{2.26}
$$

$$
R_B^N(x_A) = \frac{(2-\beta)(a-c-(1-2\beta)x_A)}{9\gamma - (2-\beta)^2}
\tag{2.27}
$$

where $K_1 = 36\gamma(1-g^2) - 16 + 7g^2 + 4\beta(\beta-4)(g^2-1)$. The derivative with respect to the strategic variable of the competitor, x_j, yields the slope of the reaction function. For firm A, we obtain from (2.26)

$$
\frac{dR_A^N(x_B)}{dx_B} = \frac{(8+\beta(8\beta-11))g^2 - 4(\beta-2)(2\beta-1)}{36\gamma(1-g^2) - 16 + 7g^2 + 4\beta(\beta-4)(g^2-1)}
\tag{2.28}
$$

To determine the slope of $R_A^N(x_B)$ we need the sign of (2.28). By assumption 1 the denominator is always positive. Hence, it remains to show the sign of the numerator of (2.28), which has two components. The first component, $(8+\beta(8\beta-11))g^2$ is positive for all $0 < \beta < 1$ and $0 < g < 1$. The second component, $-4(\beta-2)(2\beta-1)$, is, for all $g \geq 0$, positive if $1/2 < \beta < 1$ and negative if $0 < \beta < 1/2$.

Therefore, the derivative in expression (2.28) is always positive if $1/2 \leq \beta < 1$.

Moreover, if $0 < \beta < 1/2$, it is also positive if substitutability is high:

$$g > \bar{g}_N(\beta) \equiv \sqrt{\frac{4(\beta - 2)(2\beta - 1)}{8 + \beta(8\beta - 11)}} \tag{2.29}$$

For $0 < \beta < 1/2$ and $0 < g < \bar{g}_N(\beta)$, the derivative is negative.

For firm B, we obtain from (2.27)

$$\frac{dR_B^N(x_A)}{dx_A} = \frac{(2 - \beta)(2\beta - 1)}{9\gamma - (2 - \beta)^2} \tag{2.30}$$

The sign of sign of (2.30) determines the slope of $R_B^N(x_A)$. Substituting $\gamma > \gamma_{min}$ (assumption 1) into the denominator we find that the denominator is always positive. Then it is easy to see that for all $g > 0$ the derivative in (2.30) is positive if $1/2 < \beta < 1$ and negative if $0 < \beta < 1/2$.

Hence, this concludes the proof of Lemma 2.1. ∎

Proof of Proposition 2.1:

First, we derive each effect in (2.15) for firm A separately:

(i) From (2.1), (2.2) and (2.4), the direct effect

$$\frac{\partial \pi_A}{\partial x_A} = q_{A2} > 0 \tag{2.31}$$

is always positive.

(ii) The within-product competition effect consists of two components. From (2.1), (2.2) and (2.4), the first component

$$\frac{\partial \pi_A}{\partial q_{B2}} = -(gq_{A1} + q_{A2}) < 0 \tag{2.32}$$

is always negative. The second component derived from (2.8)

$$\frac{\partial q_{B2}^*}{\partial x_A} = \frac{2\beta - 1}{3} \tag{2.33}$$

is positive if $1/2 < \beta \leq 1$, zero if $\beta = 1/2$ and negative otherwise. Hence, the within-product competition effect

$$\frac{\partial \pi_A}{\partial q_{B2}} \frac{\partial q_{B2}^*}{\partial x_A} = -\frac{2\beta - 1}{3}(gq_{A1} + q_{A2}) \tag{2.34}$$

is negative if $1/2 < \beta \leq 1$, zero if $\beta = 1/2$ and positive otherwise.

Second, we derive each effect in (2.16) for firm B:

(i) From (2.1), (2.2) and (2.5), the direct effect

$$\frac{\partial \pi_B}{\partial x_B} = q_{B2} > 0 \qquad (2.35)$$

is always positive.

(ii) The within-product competition effect consists of two components. Also from (2.1), (2.2) and (2.5), the first component

$$\frac{\partial \pi_B}{\partial q_{A2}} = -q_{B2} < 0 \qquad (2.36)$$

is always negative. The second component derived from (2.7) is given by

$$\frac{\partial q_{A2}^*}{\partial x_B} = \frac{4\beta + (2 - \beta)g^2 - 2}{6(1 - g^2)} \qquad (2.37)$$

The denominator is always positive for $0 < g < 1$. Hence, the whole term is positive if

$$\beta > \frac{2g^2 - 2}{g^2 - 4} \equiv \bar{\beta} \qquad (2.38)$$

Hence, the within-product competition effect

$$\frac{\partial \pi_B}{\partial q_{A2}} \frac{\partial q_{A2}^*}{\partial x_B} = -\frac{4\beta + (2 - \beta)g^2 - 2}{6(1 - g^2)} q_{B2} \qquad (2.39)$$

is negative if $\bar{\beta} < \beta < 1$ and $0 \leq g < 1$ and positive otherwise.

(iii) The cross-product competition effect consists of two components. Also derived from (2.1), (2.2) and (2.5), the first component

$$\frac{\partial \pi_B}{\partial q_{A1}} = -gq_{B2} < 0 \qquad (2.40)$$

is negative. The second component derived from (2.6)

$$\frac{\partial q_{A1}^*}{\partial x_B} = -\frac{\beta g}{2(1 - g^2)} < 0 \qquad (2.41)$$

is also negative. Hence, the cross-product competition effect

$$\frac{\partial \pi_B}{\partial q_{A1}} \frac{\partial q_{A1}^*}{\partial x_B} = \frac{\beta g^2}{2(1-g^2)} q_{B2} > 0 \tag{2.42}$$

is positive $\forall \beta, g$. ∎

Proof of Lemma 2.2:

From optimal second stage quantities (2.6)-(2.8) and joint profit maximization of (2.19) with respect to x_i the relevant first-order conditions yield x_i as a function of x_j:

$$R_A^C(x_B) = \frac{1}{K_2}(-4(1+\beta)c + (4\beta - 5)cg^2 - a(g-1)(4 - 5g + 4\beta(1+g))$$
$$+ (-8(\beta - 2)(2\beta - 1) + (16 + \beta(16\beta - 31))g^2)x_B) \tag{2.43}$$

$$R_B^C(x_A) = \frac{1}{K_3}(-c(4(1+\beta) + (5\beta - 4)g^2) + a(g-1)(-4(1+g) + \beta(5g - 4))$$
$$+ (-8(\beta - 2)(2\beta - 1) + (16 + \beta(16\beta - 31))g^2)x_A) \tag{2.44}$$

where $K_2 = 36\gamma(1 - g^2) - 20 + 11g^2 + 4\beta(5\beta - 8)(g^2 - 1)$ and $K_3 = -32\beta(g^2 - 1) + \beta^2(11g^2 - 20) - 4(g^2 - 1)(9\gamma - 5)$. The derivative of $R_i^C(x_j)$ with respect to the strategic variable of the competitor, x_j, yields the slope of $R_i^C(x_j)$. For firm A, we obtain from (2.43)

$$\frac{dR_A^C(x_B)}{dx_B} = \frac{(16 + \beta(16\beta - 31))g^2 - 8(\beta - 2)(2\beta - 1)}{36\gamma(1 - g^2) - 20 + 11g^2 + 4\beta(5\beta - 8)(g^2 - 1)} \tag{2.45}$$

In order to determine the slope, we need the sign of (2.45). By assumption 1 the denominator is always positive. It remains to show the sign of the numerator of (2.45), which has two components. The first component, $(16 + \beta(16\beta - 31)g^2)$ is positive for all $0 \le \beta \le 1$ and $0 \le g < 1$. The second component, $-8(\beta - 2)(2\beta - 1)$, is positive if $1/2 < \beta \le 1$ and negative if $0 \le \beta < 1/2$. Therefore, the derivative in expression (2.45) is always positive if $1/2 \le \beta \le 1$. Moreover, if $0 \le \beta < 1/2$, it is also positive if substitutability is high:

$$g > \bar{g}_C(\beta) \equiv \sqrt{\frac{8(\beta - 2)(2\beta - 1)}{16 + \beta(16\beta - 31)}} \tag{2.46}$$

CHAPTER 2

For $0 \leq \beta \leq 1/2$ and $0 \leq g \leq \bar{g}_C(\beta)$, the derivative is negative.

For firm B, we obtain from (2.44)

$$\frac{dR_B(x_A)}{dx_A} = \frac{(16 + \beta(16\beta - 31))g^2 - 8(\beta - 2)(2\beta - 1)}{-32\beta(g^2 - 1) + \beta^2(11g^2 - 20) - 4(g^2 - 1)(9\gamma - 5)} \tag{2.47}$$

In order to determine the slope of $R_B^C(x_A)$, we need to determine the sign of (2.47). By assumption 1 we find that the denominator is always positive. It remains to show the sign of the numerator of (2.47), which is equivalent to the numerator of (2.45). Hence, for $0 \leq \beta < 1/2$ and $0 \leq g \leq \bar{g}_C(\beta)$, the derivative is negative. This concludes the proof of Lemma 2.2. ∎

Proof of Proposition 2.2:

First, we derive each part in (2.22) for firm A. The first term, $d\pi_A/dx_A$ is derived in proposition 2.1. We use (2.1), (2.2) and (2.5) to determine some of the following components.

(i) The spillover effect given by

$$\frac{\partial \pi_B}{\partial x_A} = \beta q_{B2} > 0 \tag{2.48}$$

is positive.

(ii) The within-product coordination effect consists of two components. The first component

$$\frac{\partial \pi_B}{\partial q_{A2}} = -q_{B2} < 0 \tag{2.49}$$

is always negative. The second component derived from (2.7) is given by

$$\frac{\partial q_{A2}^*}{\partial x_A} = \frac{4 + (2\beta - 1)g^2 - 2\beta}{6(1 - g^2)} \tag{2.50}$$

The term is always positive for all $\beta, g \in [0, 1]$. Hence, the within-product coordination effect

$$\frac{\partial \pi_B}{\partial q_{A2}} \frac{\partial q_{A2}^*}{\partial x_A} = -\frac{4 + (2\beta - 1)g^2 - 2\beta}{6(1 - g^2)} q_{B2} \tag{2.51}$$

is always negative.

(iii) The cross-product coordination effect consists of two components. The

first component

$$\frac{\partial \pi_B}{\partial q_{A1}} = -g q_{B2} < 0 \tag{2.52}$$

is negative. The second component derived from (2.6)

$$\frac{\partial q_{A1}^*}{\partial x_A} = -\frac{g}{2(1-g^2)} < 0 \tag{2.53}$$

is also negative. Hence, the cross-product coordination effect

$$\frac{\partial \pi_B}{\partial q_{A1}} \frac{\partial q_{A1}^*}{\partial x_A} = \frac{g^2}{2(1-g^2)} q_{B2} > 0 \tag{2.54}$$

is positive $\forall \beta, g$.

Second, we derive each effect of firm B in (2.23). The first term, $d\pi_B/dx_B$ is derived in proposition 2.1. For the following components, we use (2.1), (2.2) and (2.4).

(i) The spillover effect given by

$$\frac{\partial \pi_A}{\partial x_B} = \beta q_{A2} > 0 \tag{2.55}$$

is positive.

(ii) The within-product coordination effect consists of two components. The first component

$$\frac{\partial \pi_A}{\partial q_{B2}} = -(g q_{A1} + q_{A2}) < 0 \tag{2.56}$$

is always negative. The second component derived from (2.8) is given by

$$\frac{\partial q_{B2}^*}{\partial x_B} = \frac{2-\beta}{3} > 0 \tag{2.57}$$

is always positive. Hence, the within-product coordination effect

$$\frac{\partial \pi_A}{\partial q_{B2}} \frac{\partial q_{B2}^*}{\partial x_B} = -\frac{2-\beta}{3}(g q_{A1} + q_{A2}) \tag{2.58}$$

is always negative. ■

Proof of Lemma 2.3:

As seen in (2.22), the marginal benefit for firm A under cooperation can be de-

composed into two components, where $d\pi_B/dx_A$ is the additional component under cooperation. Whether $R_A^C(x_B)$ lies above or below $R_A^N(x_B)$ under competition depends only on the sign of this additional component. If the additional component is positive, then $R_A^C(x_B) > R_A^N(x_B)$. When substituting (2.48), (2.51) and (2.54) into the additional component of (2.22) we obtain (2.24).

It is easy to see that whenever $\beta > 1/2$, then $d\pi_B/dx_A > 0$. Moreover, if $\beta < 1/2$, then $d\pi_B/dx_A < 0$ and if $\beta = 1/2$, then $d\pi_B/dx_A = 0$. This concludes the proof. ∎

Proof of Lemma 2.4:
In (2.23) the marginal benefit under cooperation for firm B can be decomposed into two components, where $d\pi_A/dx_B$ is an additional component under cooperation. Whether $R_B^C(x_A)$ lies above or below $R_B^N(x_A)$ under competition depends only on the sign of the additional component. If the additional component is positive, then $R_B^C(x_A) > R_B^N(x_A)$. When substituting (2.55) and (2.58) into the additional component of (2.23) we obtain (2.25). It is easy to show that if $0 \leq \beta < 1/2$ and any $g \geq 0$, then equation (2.25) will be negative.

Next, if $g = 0$ (as in D'Aspremont and Jacquemin (1988)) and $\beta = 1/2$, we have

$$\frac{d\pi_A}{dx_B} = 0. \tag{2.59}$$

This implies that then $R_A^C(x_B) = R_A^N(x_B)$.

If we now keep $\beta = 1/2$ and increase g, we have

$$\frac{d\pi_A}{dx_B} = -\frac{gq_{A1}}{2} < 0. \tag{2.60}$$

This means that for $\beta = 1/2$ and every $g > 0$ the reaction function under coordination is below the one under competition.

It remains to show what happens if $\beta > 1/2$ and $g > 0$. Expression (2.25) is positive if

$$g < g_B^C(\beta) \equiv \frac{2(2\beta - 1)q_{A2}}{(2 - \beta)q_{A1}} \tag{2.61}$$

and negative otherwise.

Given that the quantities q_{A1} and q_{A2} depend on the parameter g themselves, we cannot obtain the closed form for $g_B^C(\beta)$. However, when $1/2 < \beta \leq 1$, it is $2\beta - 1 > 0$ and also $q_{A1} > 0$ and $q_{A2} > 0$, $g_B^C(\beta) \in (0, 1)$. $\blacksquare$

Proof of Proposition 2.3:

To compare R&D investment levels under competition and cooperation, we need to analyze the positions of $R_A^C(x_B)$ and $R_B^C(x_A)$ under R&D cooperation compared to $R_A^N(x_B)$ and $R_B^N(x_A)$ under R&D competition.

We observe that all relevant reaction functions, (2.26), (2.27), (2.43) and (2.44), are linear. Hence, it is convenient to compare R&D investment levels under cooperation and cooperation graphically in a $x_A - x_B$-diagram.

Case 1: $0 \leq \beta < 1/2$. According to Lemmas 2.3 and 2.4, both $R_i^C(x_j)$ under R&D cooperation lie below $R_i^N(x_j)$, hence R&D investment levels under cooperation are lower than under competition. Figure 2.3 illustrates the case when $0 < g < \bar{g}_N(\beta)$. For this parameter range, from Lemmas 2.1 and 2.2, we know that the slopes of $R_i^k(x_j)$ for $k \in C, N$ are negative. The cases $\bar{g}_N(\beta) < g < \bar{g}_C(\beta)$ and $\bar{g}_C(\beta) < g < 1$ are depicted in Figure 2.7.

$$0 < \beta < 1/2, \bar{g}_N(\beta) < g < \bar{g}_C(\beta) \qquad 0 < \beta < 1/2, \bar{g}_C(\beta) < g < 1$$
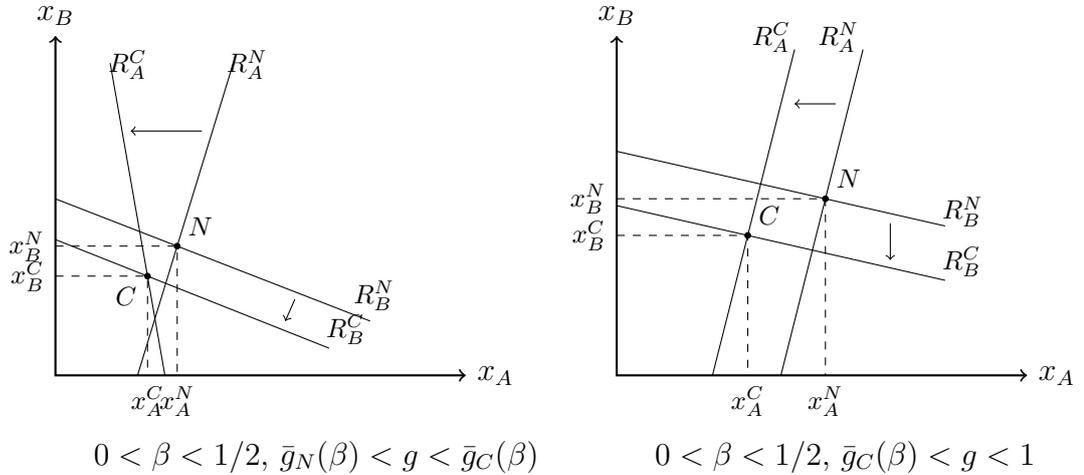
Figure 2.7: Optimal R&D investment levels under competition and cooperation

We conclude that when $0 \leq \beta < 1/2$ and $0 \leq g < 1$, both firms invest more under R&D competition than under R&D cooperation, hence $x_A^N > x_A^C$ and $x_B^N > x_B^C$.

Case 2: $1/2 \leq \beta < 1$. When $g_B^C(\beta) < g < 1$, by Lemma 2.3, $R_A^C(x_B) > R_A^N(x_B)$, while by Lemma 2.4, $R_B^C(x_A) < R_B^N(x_A)$. According to Lemmas 2.1 and 2.2 the slopes of the reaction functions under both regimes are positive. This is illustrated in Figure 2.4. The R&D investment level of firm B is lower under cooperation than under competition and the R&D investment level of firm A could also be lower under cooperation than under competition when the difference $|R_B^C(x_A) - R_B^N(x_A)|$ is significantly greater than the difference $|R_A^C(x_B) - R_A^N(x_B)|$. This scenario happens when $\beta \to 1/2$ and $g \neq 0$. There exists a function $g^A(\beta) : [1/2, 1] \to (0, 1)$ with $g_B^C(\beta) \leq g_A^C(\beta) \leq 1$ such that $x_A^N > x_A^C$, when $g > g_A^C(\beta)$ and $x_A^N < x_A^C$, when $g < g_A^C(\beta)$.
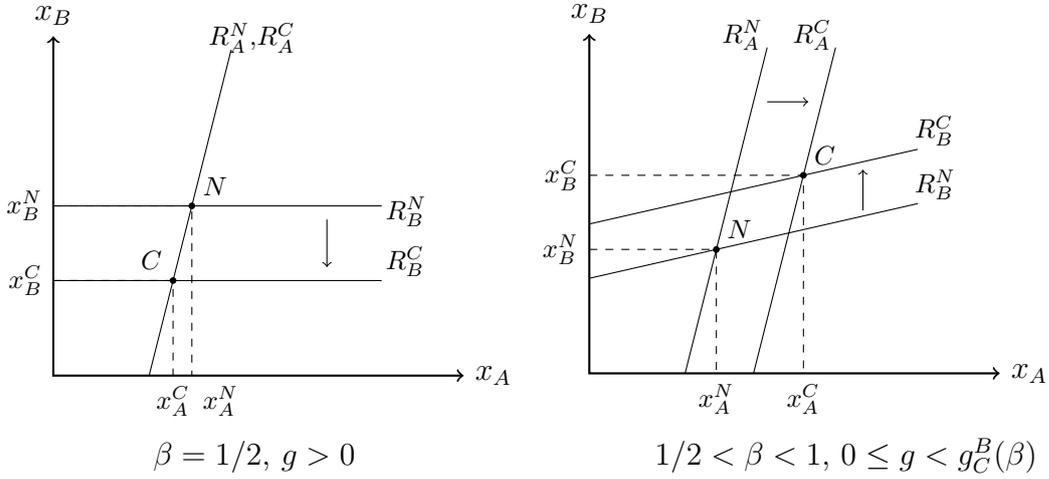
Figure 2.8: Optimal R&D investment levels under competition and cooperation

Moreover, when $\beta = 1/2$ and $g \in (0, 1)$ by Lemma 2.3, $R_A^C(x_B) = R_A^N(x_B)$, while, by Lemma 2.4, $R_B^C(x_A) < R_B^N(x_A)$. This case is depicted in the left diagram of Figure 2.8. It is easy to see that $x_A^N > x_A^C$ and $x_B^N > x_B^C$. Only if $g = 0$, both (2.24) and (2.25) are the same as under competition such that $x_A^N = x_A^C$ and $x_B^N = x_B^C$.

It remains to analyze the case when $0 < g < g_B^C(\beta)$. According to Lemmas 2.3 and 2.4, $R_i^C(x_j)$ of both firms under cooperation lie above those under competition. According to Lemmas 2.1 and 2.2, the slopes are all positive. This is illustrated in the right diagram of Figure 2.8. It is easy to see that $x_A^N < x_A^C$ and $x_B^N < x_B^C$. ∎

Proof of Proposition 2.4:

From proposition 2.3, it follows directly that when $0 < \beta \leq 1/2$ and $0 < g < 1$ and when $\beta > 1/2$ and $g_C^A(\beta) < g < 1$:

$$x_A^C < x_A^N \tag{2.62}$$

$$x_B^C < x_B^N \tag{2.63}$$

Hence, for these parameter values, $x^C = x_A^C + x_B^C < x_A^N + x_B^N = x^N$.

Further on, in the area $1/2 < \beta < 1$ and $0 < g < g_B^C(\beta)$ the following holds:

$$x_A^C > x_A^N \tag{2.64}$$

$$x_B^C > x_B^N \tag{2.65}$$

Thus, for these parameter values, $x^C = x_A^C + x_B^C > x_A^N + x_B^N = x^N$.

In the remaining area, i.e. $1/2 < \beta < 1$ and $g_B^C(\beta) < g < g_A^C(\beta)$, we have

$$x_A^C > x_A^N \tag{2.66}$$

$$x_B^C < x_B^N \tag{2.67}$$

hence, there exists a function $g^C(\beta) : [0,1] \rightarrow (0,1)$ with $g_B^C(\beta) \leq g^C(\beta) \leq g_A^C(\beta)$, such that $x^C > x^N$, when $g > g^C(\beta)$ and $x^C < x^N$, when $g < g^C(\beta)$. We, however, do a numerical analysis for this special case. See Figure 2.6 for clarification. ∎

CHAPTER 2

Chapter 3

Robustness of strategic mediation

This chapter is based on the joint work with Martin Pollrich.

3.1 Introduction

There are many situations in which decision makers lack relevant information concerning a decision to make and seek advice from experts who have such information. For example, before implementing a policy, politicians consult their advisors; or prior to deciding on a new product, CEOs of big companies ask engineers about technical feasibility to manufacture the product. Such interactions are often accompanied by a conflict of interest, i.e. given the information, each party may prefer a different decision to take. Previous literature has addressed the problem of strategic information transmission along several lines.[1] In particular, it reveals the insight that indirect communication via a mediator (interested[2] or disinterested[3]) improves upon direct talk. In this chapter, we address the question as to what extend these results rely on the availability of the mediator.

We thus perform a robustness test where we allow for a possibility of small perturbation that the mediator is not available. With such a test we want to check whether such equilibrium is robust to some real-life deviations. For instance,

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[1]See Crawford and Sobel (1982), Board et al. (2007), Goltsman et al. (2009), Ivanov (2010a), Ambrus et al. (2013), Krishna and Morgan (2004) to name a few.

[2]See Ivanov (2010a).

[3]See Myerson (1986), Forges (1986), Goltsman et al. (2009) among othrers.

whether it is robust in a technological sense, e.g. when the communication device breaks down, or in a practical sense, i.e. when the mediator is sick. In other words, we study the practical implementability of mediated equilibrium.

We show that, unless the failure gets too likely, there exist an equilibrium of this perturbed game, that is outcome equivalent to the one with the mediator. That is, the equilibrium is robust to the possibility of the mediator being absent.

Our analysis is based on the uniform-quadratic model by Crawford and Sobel (1982) (CS hereafter). CS study direct communication and show that the full revelation of information is impossible when the interests of the sender and receiver are not totally aligned. However, when the preferences of the expert and the decision maker are not too far apart, some information transmission can occur. Ivanov (2010a) adds a biased mediator into CS model, i.e. a strategic player that is interested in influencing the receiver's decision. He shows that when the sender and the mediator have opposing biases, the mediated communication between sender and receiver can be more informative than the direct talk. In their model, the expert is committed to talking to the receiver in one specific way. Namely, the sender first learns the state of the world, then sends a message to the mediator, the mediator, consequently, gives a recommendation to the receiver. A natural question arises: can we obtain such outcome even when the expert can decide on the mode of communication (to talk the receiver either directly or via a mediator) or when the mediator is not present.

To address that question, we consider the following modification of mediated communication. With some probability $\epsilon \in [0, 1)$ the mediator is not available for communication. In this case, the sender can only send a message directly to the receiver. Otherwise, if the mediator is available, the sender can either send a signal to the mediator or send a message directly to the receiver. The receiver only observes the message and channel from which the message comes, but does not know which channel is available.

We first argue that when both channels are available ($\epsilon = 0$) it is always possible to implement the most informative outcome of either of the two channels. Such an equilibrium replicates the equilibrium if only the respective channel was available for communication. Additionally, it adds beliefs on the off-equilibrium path for any message that is sent through the other channel in a specific way: these

beliefs should be consistent with the receiver taking an action which the sender can ensure by sending some message through the intended channel.

When the probability of mediator being absent is positive but not too high, it is still possible to implement the most informative mediated outcome. In one such equilibrium, on the direct channel, every sender type sends the same message and the receiver interprets it as babbling. When the mediator is present, the lowest sender's types randomize between sending a message directly to the receiver or sending a signal to the mediator. The rest of the equilibrium path on the mediated channel is equivalent to the equilibrium in Ivanov (2010a).

The remainder of this chapter is organized as follows. Section 3.2 gives the preview of the related literature. Section 3.3 presents the theoretical model and the equilibrium concept. In Section 3.4 we review the direct communication as studied by CS and also the mediated communication studied by Goltsman et al. (2009) and Ivanov (2010a). Section 3.5 presents our main result. Section 3.6 concludes.

3.2 Related literature

The seminal work on cheap talk is by Crawford and Sobel (1982). CS study strategic communication between an informed sender and an uninformed receiver. The sender can send a single message only. CS show that communication is not fully informative, as long as sender's and receiver's preferences are not fully aligned, and characterize the set of perfect Bayesian equilibria of the communication game.

While CS restrict communication to a single round of message exchange from sender to receiver, Krishna and Morgan (2004) allow for multiple rounds of bilateral message exchange. In their model, the receiver thus has an active role. They use the uniform-quadratic framework of the CS-model and show that such multi-stage communication typically improves upon one stage communication, both in terms of revealed information and players' payoffs.

Board et al. (2007) consider noisy communication in the spirit of Rubinstein (1989) and Myerson (1991). One could think of their communication as a talk through a mechanical device that with some probability passes on an unchanged message from the sender to the receiver, and with the remaining probability it sends

a random message. They show that adding a proper amount of technological noise to the communication may improve the informativeness of the talk in comparison to the CS model.

Goltsman et al. (2009) study mediation in the uniform-quadratic framework of the CS-model. They determine the highest ex-ante expected payoff that can be reached by the receiver (and sender) and show that the threshold is achieved by the specific noise structure analyzed by Board et al. (2007). The communication structure by Krishna and Morgan (2004) also attain the highest expected payoff to the receiver, however only when the conflict of interest between the sender and receiver is much smaller.

Ivanov (2010a) introduce a strategic mediator that can have its own agenda, i.e. it has its own preference. He also considers the uniform-quadratic framework of the CS-model and shows that the mechanical mediator studied by Goltsman et al. (2009) can be replaced by a strategic player who has similar preferences as the sender and receiver but must have a specific bias (in relation to sender's bias).

The fact that the mediation can improve communication was shown much earlier. Forges (1986) and Myerson (1986) studied general communication devices or impartial mediators. In their models, all players have private information. The mediator collects reports from all players and then calculates recommended action for each player based on these reports. In CS setup, however, only one player has private information and sends a message to the uninformed player who then takes an action.

In all aforementioned extensions of the model by CS, there is a commitment to the specific channel of communication. That is, it is implicitly assumed that all communication between the sender and the receiver can only pass through this channel and, for instance, the sender cannot bypass the channel and send a message directly to the receiver. In this article, we weaken this assumption.

This article also contributes to the literature on cheap talk in organizations. Dessein (2002), Ivanov (2010b), Alonso et al. (2008), and McGee and Yang (2013) investigate when is delegation of authority to the informed expert preferred to centralized decision-making. Dessein (2002) and Ivanov (2010b) consider a situation with a single sender, while Alonso et al. (2008) and McGee and Yang (2013)

allow for multiple senders[4] where the senders communicate with the receiver via cheap talk. Delegation due to informational asymmetries is also studied by Alonso and Matouschek (2007) where they considered an infinitely repeated game with a long-lived principal and a many of short-lived agents. Similar to these papers, we also study the practical implementations of achieved equilibria.

3.3 Model

3.3.1 Setup

We study the uniform-quadratic specification of the CS model of strategic communication. There are two players, sender and receiver. The sender, called S, privately observes the state of nature $\theta \in [0,1]$. The receiver, called R, takes an action $a \in \mathbb{R}$. The payoffs of both players depend on the sender's private information θ and the receiver's action a. Specifically,

$$U_S(a, \theta, b) = -(a - (\theta + b))^2 \tag{3.1}$$

$$U_R(a, \theta) = -(a - \theta)^2 \tag{3.2}$$

where $b > 0$ denotes the sender's bias. Intuitively, R's optimal action conditional on the state θ is $a^R(\theta) = \theta$, i.e. R seeks to match the state. On the other hand, S's optimal action is $a^S(\theta) = \theta + b$.

3.3.2 Mediated talk

Building on the insights obtained by CS, several authors studied alternative modes of communication[5]. This chapter focuses on mediation as one such channel. The sender first reports her type to the mediator and the mediator then gives a recommendation to the receiver about what action to take. Subsequently, the receiver chooses an action.

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[4]Cheap talk with multiple senders was also studied by Krishna and Morgan (2001), Ambrus and Takahashi (2008), Battaglini (2002), Milgrom and Roberts (1986), Austen-Smith (1993), Levy and Razin (2007), among others.

[5]See for example, Board et al. (2007), Ivanov (2010a), Krishna and Morgan (2004), Ambrus et al. (2013).

To find an optimal mediation scheme for the uniform-quadratic case, Goltsman et al. (2009) maximize the ex-ante expected utility of the receiver. It is sufficient to rank the equilibria from the receiver's point of view because $V_S = V_R - b^2$, where V_R is receiver's and V_S sender's ex-ante expected utility.

Goltsman et al. (2009) determine an upper bound of the receiver's expected utility for the case of a mechanical mediator. Ivanov (2010a) examines a mediator that is a strategic actor, i.e. it has its own utility,

$$U_M(a, \theta, \beta) = -(a - (\theta + \beta))^2 \tag{3.3}$$

where $\beta \in \mathbb{R}$ is the bias of the mediator. Ivanov (2010a) shows that for some specific β the same upper bound can be reached by strategic mediator.

3.3.3 Imperfect mediation

This chapter addresses the practical implementability of Ivanov's finding. Consider the following alteration of mediated communication. With probability $\epsilon \in [0, 1)$ the mediator is not available. Non-availability may be for technical reasons—e.g. failure of electronic devices—or for other reasons—e.g. sickness of the mediator. In this case, the sender is forced to send a message $m \in M$ directly to the receiver. Otherwise, if the mediator is available, the sender can either send a signal to the mediator or send a message directly to the receiver. The receiver does not know which channel is available, but only from which channel does the message come.

The timing of this game is the following, nature first decides whether the strategic mediator is available for the communication and then chooses sender's type. The sender then privately observes the value of θ. If the mediator is not present, the sender can only send message $m^0 \in M^0$ directly to the receiver, where M^0 is a message space available for the direct talk. However, if the mediator is available, the sender decides on sending a message m^0 directly to the receiver or a signal $s \in S$ to the mediator. The mediator, if received the signal, makes a recommendation $m^1 \in M^1$ to the receiver. Upon receiving m^0 or m^1, the receiver chooses an action a.

3.3.4 Equilibrium

We next characterize strategies for all three players in game Γ. Sender's strategy is a vector $\rho = (\rho^0, \rho^1)$ with $\rho^0 : \Theta \longrightarrow M^0$ and $\rho^1 : \Theta \longrightarrow M^0 \times S$ specifying for each θ a probability distribution on the sets of messages and signals. A mediator's strategy is a mapping $\sigma : S \longrightarrow \Delta(M^1)$, specifying for any received signal a distribution over the set of messages M^1. Finally, the receiver's strategy is a vector $a = (a^0(m^0), a^1(m^1))$ with $a^i : M^i \longrightarrow \mathbb{R}$, $i \in \{0, 1\}$, where $a^i(m^i)$ is an action taken by the receiver after receiving message m^i.

A perfect Bayesian equilibrium of the game Γ consists of a signaling strategy for the expert $\rho = (\rho^0(m^0|\theta), \rho^1(m^0, s|\theta))$, a messaging strategy for the mediator $\sigma(m^1|s)$, an action rule for the receiver $a = (a^0(m^0), a^1(m^1))$, a belief function for the mediator $F(\theta|s)$, which assigns probability distributions over states θ for every s, and a belief functions of the receiver $G(\theta|m^0)$ and $G(\theta|m^1)$, which assign probability distributions over states θ to m^0 and m^1 respectively.

To form an equilibrium, it is required, that for any message m^0 or m^1, the receiver maximizes his expected utility given his beliefs $G(\theta|m^0)$ and $G(\theta|m^1)$. Also, for any signal s, the mediator maximizes her expected utility given her belief $F(\theta|s)$ and $a(m^1)$. The expert maximizes her expected utility given $\sigma(m^1|s)$ and $a = (a^0(m^0), a^1(m^1))$. Finally, any $F(\theta|s)$ and $G(\theta|m^1)$ are derived from $\rho = (\rho_0, \rho_1)$ and $\sigma(m^1|s)$ using Bayes' rule wherever possible.

3.4 Review of existing results

To study game Γ that consists of direct and mediated channel, we first recall the relevant existing results on direct and mediated communication. We start with the seminal work by Crawford and Sobel (1982) on cheap talk with one round of face-to-face communication which is followed by Goltsman et al. (2009) on communication via an impartial mediator. In the end, we discuss results by Ivanov (2010a) on communication via a strategic mediator.

3.4.1 One round face-to-face communication

CS analyze a game similar to ours, only that there is no third party involved. Formally, the sender S first privately observes the state θ and then sends a single message $m \in M$ to the receiver, where M is some message space. After observing m, but not the state θ, the receiver R takes an action $a \in \mathbb{R}$.

The equilibrium notion for this game called Γ^D, are the following, sender's strategy is a mapping $\rho^D : \Theta \longrightarrow M^D$ and a receiver's strategy is a mapping $a^D : M^D \longrightarrow \mathbb{R}$. The belief function of the receiver is $G^D(\theta|m^D)$. For the uniform-quadratic specification of utilities, their results can be summarized in the following proposition.

Proposition 3.1 (Crawford and Sobel (1982)) *For every $b > 0$, there exists an integer[6] $N^D(b) = \lceil -1/2 + 1/2(1+2/b)^{1/2} \rceil$ such that, for every N with $1 \leq N \leq N^D(b)$, there exists exactly one equilibrium of game Γ^D with N induced actions and there is no equilibrium which induces more than $N^D(b)$ actions. Equilibria are described by a partition $(\theta_0^D, ..., \theta_N^D)$, such that*

$$0 = \theta_0^D < \theta_1^D < ... < \theta_N^D = 1 \tag{3.4}$$

$$\theta_i^D = \frac{i}{N} + 2bi(i - N) \ , \ i \in \{0, ..., N\} \tag{3.5}$$

$$a_i^D = \frac{\theta_{i-1}^D + \theta_i^D}{2}. \tag{3.6}$$

Proof of Proposition 3.1: See Crawford and Sobel (1982). ∎

The sender's type space is partitioned into N intervals and each induced action is associated with one interval. That is, for every $i \in \{1, 2, ..., N\}$, all types from interval $[\theta_{i-1}^D, \theta_i^D]$ pool together and send the same message. The receiver's best response is than simply the average of the types from the relevant interval, as given by (3.6). Additionally, a type between two adjacent intervals $[\theta_{i-1}^D, \theta_i^D]$ and $[\theta_i^D, \theta_{i+1}^D]$ is indifferent between inducing actions a_i^D and a_{i+1}^D.

As a consequence of Proposition 3.1, already for $b > 1/4$, the only equilibrium of game Γ^D is the babbling equilibrium. That is, every sender type sends the same signal which is thus uninformative and the receiver takes the uninformed action

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[6] $\lceil x \rceil$ is the smallest integer greater than or equal to x.
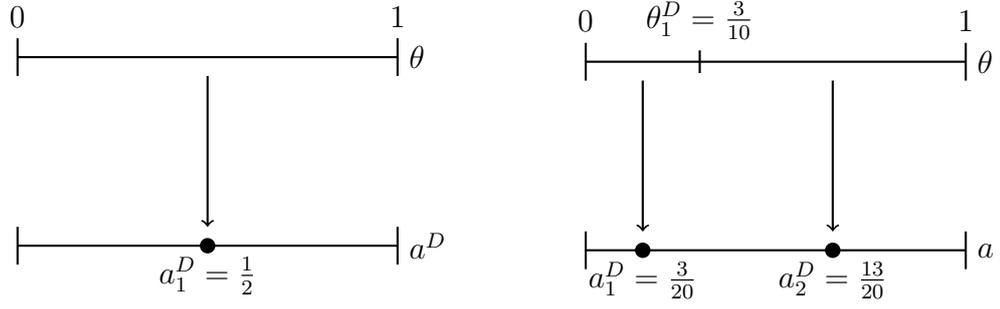
Figure 3.1: Most informative equilibria with one-round of face-to-face communication for large bias, $b = 1/3$ (left panel), and intermediate bias, $b = 1/10$ (right panel).

$a = 1/2$. Only for small biases, $b < 1/4$, there exist equilibria in which the sender reveals some information to the receiver.

The partitioning equilibira established above yield the following comparative statics. For any given bias $b > 0$, equilibria that partition the sender's type space into finer intervals yield higher ex-ante expected utility for both the sender and the receiver. In particular, among the partitioning equilibria for a particular bias $b > 0$, the one with $N^D(b)$ partitioning elements Pareto dominates all others. The maximum number of induced actions, $N^D(b)$, is decreasing in b, i.e. the lower the conflict of interest, the more information can be revealed in equilibrium. To illustrate this, we consider the following simple examples and present them in Figure 3.1.

Example 3.1 *Consider the sender's bias $b = 1/3$. The maximum number of partitions is $N^D(1/3) = 1$, thus the only equilibrium is babbling, i.e. no information is revealed. The receiver's optimal action is thus $a_1^D = 1/2$.*

Example 3.2 *Consider the sender's bias $b = 1/10$. The maximum number of partitions is $N^D(1/10) = 2$ and the 2-step equilibrium involves a cut-off type $\theta_1^D = 3/10$ and induced actions $a_1^D = 3/20$ and $a_2^D = 13/20$.*

3.4.2 Mediated communication

In principle, sender and receiver can follow more sophisticated communication protocols, such as multiple rounds of message exchange, communication via third

parties, via noisy channels, etc. A convenient short-cut for analyzing such complex communication structures lies in applying the revelation principle. Invoking the latter yields the following tractable structure. The sender first reports her type to the mediator. Next, the mediator recommends the receiver an action to take. Finally, the receiver takes an action, which may differ from the mediator's recommendation. The revelation principle further implies that the analysis can be restricted to equlibria where the sender reports her type truthfully and the receiver follows the mediator's recommendation. This structure has been analyzed by Goltsman et al. (2009). The following proposition summarizes their results.

Proposition 3.2 (Goltsman et al. (2009)) *For every* $0 < b < 1/2$:

a) *The upper bound of the receiver's ex-ante expected utility is*
$\bar{V}_R(b) = -b(1 - b)/3$.

b) *The following partitioning equilibrium reaches this bound: For every* $\theta^M \in [\theta_i^M, \theta_{i+1}^M]$, $i \in \{0, ..., N^M - 1\}$, *the mediator recommends action b with probability* μ *and action* a_{i+1} *with probability* $1 - \mu$, *where*

$$\theta_0^M = 0 \tag{3.7}$$

$$\theta_i^M = 2bi^2 - (2b(N^M)^2 - 1)\frac{2i - 1}{2N^M - 1} \ , \ i \in \{1, ..., N^M\} \tag{3.8}$$

$$a_i^M = b(i + 1) - 2bi(N^M - i) + \frac{(2 - b)i}{2N^M - 1} \tag{3.9}$$

$$\mu = 1 - \frac{1 - 2b}{4(1 - b)}\Big(\frac{1}{N^M - 1} - \frac{1}{N^M} - \frac{2 - b}{bN^M - 1} + \frac{2 - b}{bN^M - b + 1}\Big). \tag{3.10}$$

with $N^M = \lceil 1/\sqrt{2b} \rceil$.

Proof of Proposition 3.2: See Goltsman et al. (2009). ∎

Goltsman et al. (2009) derive an upper bound on the receiver's payoff, depending on the bias. This bound is reached when the lowest sender's type $\theta = 0$ gets its preferred action b.[7] They further give an explicit equilibrium that achieves this upper bound. The equilibrium construction is taken from results in Board et al.

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[7] The payoff of the sender of type 0 in the proposed equilibrium is $U(b, 0, b) = 0$.
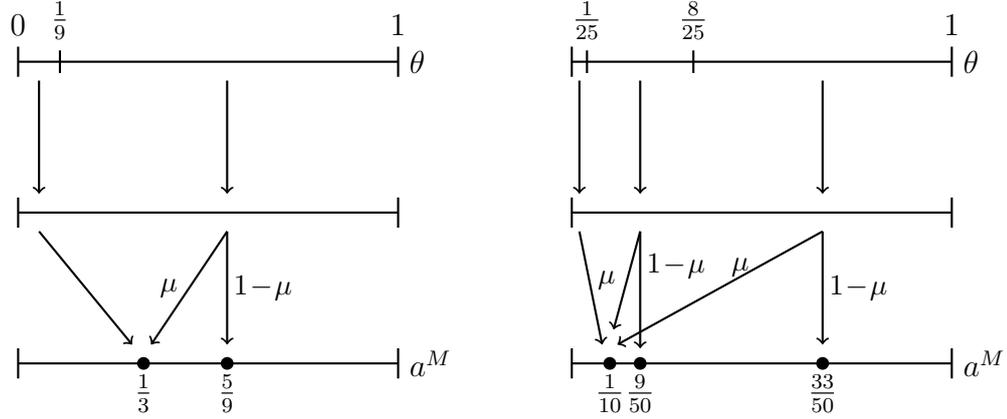
Figure 3.2: Most informative equilibria with mediated communication for large bias, $b = 1/3$ (left panel), and intermediate bias, $b = 1/10$ (right panel).

(2007) on noisy talk, where the sender sends messages to the receiver, but these messages may get lost with probability μ.

In order to compare this equilibrium with the one achieved by the one-round face-to-face communication, we consider examples with the same biases as in Section 3.4.1.

Example 3.1 (continued) *Consider the sender's bias $b = 1/3$. The maximum number of partitions is $N^M(1/10) = 2$ and the 2-step equilibrium involves a cut-off type $\theta_1^M = 1/9$ with induced actions $a_1^M = 1/3$ and $a_2^M = 5/9$. If the sender reports type $\theta < 1/9$, the mediator always recommends a_1^M. If the sender reports type $\theta > 1/9$ he recommends a_1^M with probability $\mu = 5/32$ and a_2^M otherwise.*

Example 3.2 (continued) *Consider the sender's bias $b = 1/10$. The maximum number of partitions is $N^M(1/10) = 3$ and the 3-step equilibrium involves cut-off types $\theta_1^M = 1/25$ and $\theta_2^M = 8/25$ with induced actions $a_1^M = 1/10$, $a_2^M = 9/50$ and $a_3^M = 33/50$. If the sender reports type $\theta < 1/25$, the mediator always recommends a_1^M. If $1/25 < \theta < 8/25$, the mediator recommends a_1^M with probability $\mu = 5/32$ and a_2^M otherwise. Similarly, if $8/25 < \theta < 1$, the mediator recommends a_1^M with probability $\mu = 5/32$ and a_3^M otherwise.*

The equilibria from the previous examples are illustrated in Figure 3.2.

In contrast to the CS, where the sender reveals some information only for $b < 1/4$, the equilibrium described in Proposition 3.2 is informative already for $b < 1/2$. Additionally, for almost all biases the mediated talk is more informative and generates higher payoff.

3.4.3 Communication via a strategic mediator

The results established in Goltsman et al. (2009) rely on a mechanical mediator. This mediator is assumed to be impartial and trustworthy. In particular, the mediator should not have its own agenda. That is, it should not have its own interest in the receiver's action. Ivanov (2010a) studies the case where the mediator has a preference over the receiver's actions and is thus not impartial. The mediator's utility function is given in (3.3).

A talk via a strategic mediator proceeds as follows. First S privately observes the state θ and sends a signal $s \in \mathcal{S}^S$ to the mediator. The mediator privately observes signal s^S and sends message $m^S \in M^S$ to R. R observes m^S and takes action a^S.

The sender's strategy is a mapping $\rho^S : \Theta \longrightarrow S$, the mediator's strategy is a mapping $\sigma^S : S \longrightarrow \Delta(M^S)$ and the receiver's strategy is a mapping $a^S : M \longrightarrow \mathbb{R}$. The belief functions for the mediator and the receiver are the following $F^S(\theta|s^S)$ and $G^S(\theta|m^S)$. We call this game Γ^S.

Ivanov (2010a) shows that the highest payoff identified in Goltsman et al. (2009) can also be achieved with a strategic mediator, as long as the mediator's bias is appropriately chosen. An equilibrium characterized in the following proposition reaches this bound.

Proposition 3.3 (Ivanov (2010a)) *For every $b > 0$, there exists $\beta \in [-2b, 0]$ such that game Γ^S has a partitioning equilibrium with $N^S(b) = \lfloor 1/\sqrt{2b} \rfloor$ induced actions. For every $\theta^S \in [\theta_1^S, \theta_2^S]$, the mediator recommends action a_2^S with probability σ, and action b with probability $1 - \sigma$, moreover*

$$\theta_0^S = 0 \tag{3.11}$$

$$\theta_1^S = \frac{1}{3}(2b((N^S - 2)^2 - 1) - 2^{3/2}b^{1/2}(N^S - 2) + 1) \tag{3.12}$$

$$\theta_2^S = 3\theta_1^S + 2b \tag{3.13}$$

$$\theta_i^S = \theta_2^S + (i-2)(2bi + \frac{1}{N^S - 2}(1 - \theta_2 - 2b(N^S - 2)(N^S - 3)) - 6b), \ 3 \le i \le N^S \tag{3.14}$$

$$a_1^S = b, \ a_i^S = \frac{\theta_{i-1}^S + \theta_i^S}{2}, \ i = 2, ..., N^S \tag{3.15}$$

$$\sigma^S = \frac{3}{8} \frac{3\theta_1^S + 2b}{\theta_1^S + b} \tag{3.16}$$

Proof of Proposition 3.3: See Ivanov (2010a). ∎

The optimal mediator's bias has a negative sign, hence it counterbalances the positive bias of the sender. That is, the sender always tends to overstate the information, while the mediator tries to understate it. Such mediator, when receiving a signal from the second lowest interval, randomizes between the two lowest actions. When the mediator gets a signal from any other interval, it simply recommends the corresponding action.

In order to compare this equilibrium with the one presented in Proposition 3.2, we consider examples with the same biases as in Section 3.4.2.

Example 3.1 (continued) *Consider the sender's bias $b = 1/3$. The maximum number of partitions is $N^S(1/3) = 2$ and the 2-step equilibrium involves a cut-off type $\theta_1^S = 1/9$ and induced actions $a_1^S = 1/3$ and $a_2^S = 5/9$. If the sender reports type $\theta < 1/9$, the mediator always recommends a_1^S. If the sender reports $\theta > 1/9$, the mediator recommends a_1^S with probability $\sigma = 5/32$ and a_2^S otherwise.*

Example 3.2 (continued) *Consider the sender's bias $b = 1/10$. The maximum number of partitions is $N^S(1/10) = 3$ and the 3-step equilibrium involves cut-off types $\theta_1^S = 0.035$ and $\theta_2^S = 0.3$ with induced actions $a_1^S = 0.1$, $a_2^S = 0.17$ and $a_3^S = 0.65$. If the sender reports type $\theta < 0.035$, the mediator always recommends a_1^M and if the sender reports type $\theta > 0.3$, the mediator always recommends a_3^M. If $0.035 < \theta < 0.3$, the mediator recommends a_1^M with probability $\sigma = 0.15$ and a_2^M otherwise.*

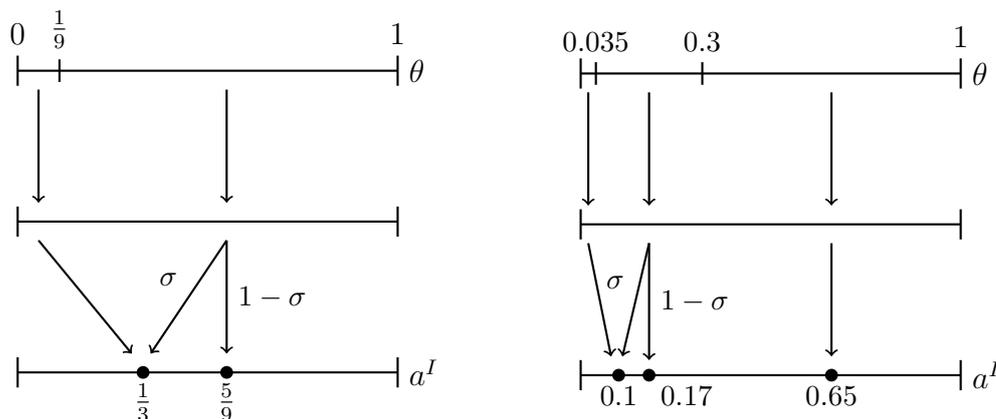Figure 3.3 illustrates those two equilibria.

Figure 3.3: Most informative equilibria with communication via a strategic mediator for $b = 1/3$ (left panel) and $b = 1/10$ (right panel).

The equilibrium constructions of the communication via an impartial mediator and via the strategic one are very similar. In both, low types are sometimes mixed up with the higher types. However, the impartial mediator randomizes in each state, while the strategic mediator mixes only when receiving a signal from the second lowest interval.

3.5 Robustness of strategic mediation

After recalling the equilibrium characterizations of the direct talk and the talk via a strategic mediator, we now analyze game Γ. The equilibrium notion of game Γ is given in Section 3.3.4. In this game with probability $\epsilon \in [0, 1)$ the mediator is not available for communication. In this case, the sender is forced to communicate to the receiver directly. Otherwise, if the mediator is available, the sender can either send a signal to the mediator or send a message directly to the receiver. When receiving a message directly from the sender, the receiver does not know whether it is because the mediator is not present or because the sender simply decided to approach receiver directly.

We first consider a case when the probability of the mediator being absent is equal to zero, hence, both channels are available. The sender can, thus, choose whether to talk directly or indirectly to the receiver.

Lemma 3.1 *When $\epsilon = 0$, for any equilibrium outcome of game Γ^D or Γ^S, there is an equilibrium of game Γ which achieves the same outcome.*

Proof of Lemma 3.1: Consider the following equilibrium outcome of game Γ^i, $i \in \{D, S\}$: $(\theta_0^i, \theta_1^i, ..., \theta_{N^i+1}^i)$ is the equilibrium partition of the sender's type space and $a_1^i, ..., a_{N^i}^i$ are the induced actions.

Let $\bar{a}^i$ be an action induced with certainty by some sender's report $\theta^i \in [\theta_{k-1}^i, \theta_k^i]$, $k \in \{1, ..., N^i\}$.[8] An equilibrium of game Γ achieves the same outcome as some equilibrium of game Γ^i if the receiver after observing message m_n^j, $j \in \{D, S\}$, $\jmath \neq i$, $n \in \{1, ..., N^j\}$, believes that the true state lays in the interval $[\theta_{k-1}^i, \theta_k^i]$. Hence, the receiver after receiving message m_n^j, takes an action $\bar{a}^i$.

The equilibrium partition of game Γ on direct channel is:

$$\theta_n^0 = \theta_n^D, n \in \{0, ..., N^0\}, N^0 = N^D \tag{3.17}$$

and on the mediated channel:

$$\theta_m^1 = \theta_m^S, m \in \{0, ..., N^1\}, N^1 = N^S \tag{3.18}$$

For $i = D$, the actions induced in equilibrium are

$$a_n^0 = a_n^D, n \in \{0, ..., N^0\}, N^0 = N^D \tag{3.19}$$
$$a_m^1 = \bar{a}^D, m \in \{0, ..., N^1\}, N^1 = N^S \tag{3.20}$$

Otherwise, for $i = S$, the equilibrium actions are

$$a_n^0 = \bar{a}^S, n \in \{0, ..., N^0\}, N^0 = N^D \tag{3.21}$$
$$a_m^1 = a_m^S, m \in \{0, ..., N^1\}, N^1 = N^S \tag{3.22}$$
$$\sigma = \sigma^S \tag{3.23}$$

where σ is the probability that the mediator recommends action a_2^1 and with the remaining probability the action a_1^1. ∎

The previous lemma shows that when both channels are available, there is an equilibrium of game Γ that replicates either of the equilibria of games Γ^D or Γ^S.

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[8] All actions in game Γ^D and all but a_2^S in game Γ^S are induced with certainty.

CHAPTER 3

For example, in order to attain the equilibrium outcome of game Γ^S (Γ^D), when the receiver gets a message from the direct (mediated) channel he takes an action which the sender can ensure by sending a message through the mediated (direct) channel. The off-equilibrium beliefs are consistent with such receiver's strategy.

This makes perfect sense from a purely theoretical perspective, as the perfect Bayesian equilibrium places no restrictions on the receiver's beliefs at the off-equilibrium path, hence we can set all out-off-equilibrium beliefs equal to some equilibrium beliefs. However, it is hard to imagine that the decision maker would completely ignore suggestions from either of the two channels. Yet, the existing criteria for the equilibrium selection are not applicable in this case[9].

We next assume that the probability of mediator being absent is positive, $\epsilon > 0$. The receiver does not know which channel is available, but only from which channel does the message come from. In this case, the receiver cannot ignore the messages coming from the unexpected channel.

Proposition 3.4 *If $0 < \epsilon < \bar{\epsilon}$, game Γ has an equilibrium that satisfies $V_R = \bar{V}_R(b)$.*[10]

Proof of Proposition 3.4: Consider the following strategy profile:

$$\theta_0^0 = 0, \theta_1^0 = 1 \tag{3.24}$$

$$\theta_i^1 = \theta_i^S, i \in \{0, ..., N^1\}, N^1 = N^S \tag{3.25}$$

$$a_1^0 = a_1^1 = b \tag{3.26}$$

$$a_i^1 = \frac{\theta_{i-1}^1 + \theta_i^1}{2}, i \in \{2, ..., N-1\} \tag{3.27}$$

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[9]The standard refinement concepts for signaling games (e.g. the intuitive criterion by Cho and Kreps (1987) and divinity criterion by Banks and Sobel (1987)) have no selection power for cheap-talk games. The refinement concepts specially designed for cheap-talk games, like neologism-proofness by Farrell (1993), announcement proofness by Matthews et al. (1991) or credible message rationalizability by Rabin (1990), can select most informative equilibria in some classes of cheap-talk games, they are however not applicable under CS uniform-quadratic framework. Chen et al. (2008) propose the NITS condition that uniquely selects the most informative equilibrium of the uniform-quadratic CS model. Even though NITS works well for the CS direct talk, it cannot be applied to mediated communication.

[10]Where $\bar{\epsilon} = (2\sqrt{2b}(N^1-2) - 1 - 2b(N^1-4)N^1)(1 - 2\sqrt{2b}(N^1-2) + 2b(3 - 4N^1 + (N^1)^2))/4(2 + \sqrt{2b}(N^1-2) + b(12N^1 - 3(N^1)^2 - 14) - b^2 N^1(19N^1 - 8(N^1)^2 + (N^1)^3 - 12) + \sqrt{2}b^{3/2}(2(N^1)^3 - 12(N^1)^2 + 19(N^1) - 6))$ with $N^1 = N^S$.

$$\alpha = \frac{(2b - \theta_1^1)\theta_1^1 - \epsilon(1 - 2b + \theta_1^1)(1 - \theta_1^1)}{(1 - \epsilon)(2b - \theta_1^1)\theta_1^1} \tag{3.28}$$

$$\sigma = \frac{(1 - 2b)\epsilon}{8(1 - \epsilon)\theta_1^1(\theta_1^1 + b)} + \frac{3(3\theta_1^1 + 2b)}{8(\theta_1^1 + b)} \tag{3.29}$$

When $\theta \in [\theta_1^1, \theta_2^1]$, the mediator recommends action a_1^1 with probability σ, and action a_2^1 with probability $1 - \sigma$. Also, when the mediator is available, the sender sends a message directly to the receiver with probability α, otherwise with probability $1 - \alpha$, he reports his type to the mediator.

The only action that can be obtained on the direct channel is $a_1^0 = b$. That action is equivalent to the lowest action that can be achieved on the mediated channel $a_1^1 = b$. Therefore, when the mediator is available, the sender's types from the lowest interval $[0, \theta_1^1]$ are indifferent between taking directly or indirectly to the receiver, as they always get action b.

When the mediator is not available all sender types send message m_1^0, otherwise (when the mediator is available) only low types, $\theta \in [0, \theta_1^1]$, send message m_1^0, hence the receiver's best response is:

$$a_1^0 = \frac{\epsilon\frac{1}{2} + (1 - \epsilon)\theta_1^1\alpha\frac{1}{2}\theta_1^1}{\epsilon + (1 - \epsilon)\theta_1^1\alpha} \tag{3.30}$$

Plugging $a_1^0 = b$ into (3.30) and solving for α, results in:

$$\alpha = \frac{(2b - \theta_1^1)\theta_1^1 - \epsilon(1 - 2b + \theta_1^1)(1 - \theta_1^1)}{(1 - \epsilon)(2b - \theta_1^1)\theta_1^1} \tag{3.31}$$

given that $b, \epsilon, \theta_1^1 \in (0, 1)$, it follows that $\alpha \in (0, 1)$.

When the mediator is available, the action a_1^1 is induced by the two lowest sender's partitions. The receiver's best response is

$$a_1^1 = \frac{\theta_1^1(1 - \alpha)\frac{1}{2}\theta_1^1 + (1 - \sigma)(\theta_2^1 - \theta_1^1)\frac{1}{2}(\theta_2^1 + \theta_1^1)}{\theta_1^1(1 - \alpha) + (1 - \sigma)(\theta_2^1 - \theta_1^1)} \tag{3.32}$$

Plugging $a_1^1 = b$ into (3.32) and solving for σ results in :

$$\sigma = \frac{(1 - 2b)\epsilon}{8(1 - \epsilon)\theta_1^1(\theta_1^1 + b)} + \frac{3(3\theta_1^1 + 2b)}{8(\theta_1^1 + b)} \tag{3.33}$$

and $\sigma \in (0, 1)$, whenever $\epsilon < \bar{\epsilon} = (2\sqrt{2b}(N^1 - 2) - 1 - 2b(N^1 - 4)N^1)(1 - 2\sqrt{2b}(N^1 -$
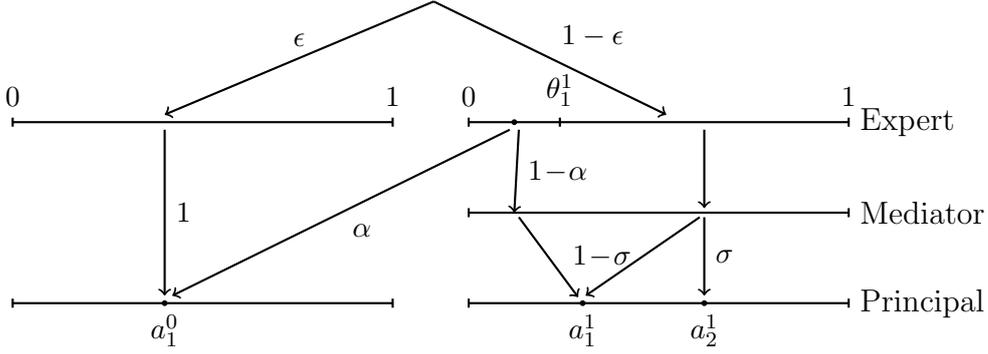
Figure 3.4: Most informative equilibrium of game Γ for $b = 1/3$ and $\epsilon = 1/9$

$2) + 2b(3 - 4N^1 + (N^1)^2))/4(2 + \sqrt{2b}(N^1 - 2) + b(12N^1 - 3(N^1)^2 - 14) - b^2N^1(19N^1 - 8(N^1)^2 + (N^1)^3 - 12) + \sqrt{2}b^{3/2}(2(N^1)^3 - 12(N^1)^2 + 19(N^1) - 6)) \in (0,1)$.

The remaining strategies are identical to the equilibrium strategies in game Γ^S and according to Proposition 3.3 they constitute the equilibrium.

Goltsman et al. (2009) show that the upper bound for the receiver's ex-ante expected payoff $(\bar{V}_R(b))$ is achieved if and only if the sender's expected payoff of the lowest type $\theta = 0$ is 0. Given that $a_1^0 = a_1^1 = b$, we have $U_S(a, \theta, b) = \epsilon U_S(a_1^0, \theta, b) + (1 - \epsilon)U_S(a_1^1, \theta, b) = 0$. This concludes the proof. ■

The previous proposition shows that when the probability of the mediator being absent is not too high, the optimal mediated outcome is achievable. In such an equilibrium, the receiver puts some reasonable beliefs on the messages coming directly from the sender.

The construction of the equilibrium is as follows. When the mediator is not available everyone sends message m_1^0. The receiver then implements action $a_1^0 = b$. On the other hand, when the mediator is present, types from the lowest sender's partition implement action $a_1^1 = b$. Hence, those lowest sender's types are indifferent between talking directly or indirectly to the receiver and they mix over those two channels. The mediator when receiving the signal from the second lowest partition of the sender's type space, randomizes between recommending action a_1^1 and a_2^1. After receiving signals from other sender's types, it simply recommends the corresponding action. Hence, the equilibrium path on the mediated channel is equivalent to equilibrium in Proposition 3.3.

Therefore, Ivanov's strategic mediator is robust to the possibility of direct talk and also to a small perturbation that the mediator is not present.

Example 3.1 (continued) *Consider the sender's bias $b = 1/3$ and the probability of mediator being absent $\epsilon = 1/9$. The maximum number of partitions on the mediated channel is $N^1(1/3) = 2$ and the sender's partition of the type space on the mediated channel involves a cut-off type $\theta_1^1 = 1/9$. There are two induced actions $a_1^1 = 1/3$ and $a_2^1 = 5/9$ on the mediated channel, while we have only one action, $a_1^0 = 1/3$ on the direct channel. When the mediator is available, the lowest sender's types randomize between a_1^0 and a_1^1 with probabilities $\alpha = 13/40$ and $1 - \alpha = 27/40$. If the sender reports type $\theta < 1/9$ to the mediator, the mediator always recommends a_1^1. If the sender reports type $\theta > 1/9$, the mediator recommends a_1^1 with probability $\sigma = 0.004$ and a_2^1 otherwise. This equilibrium is illustrated in Figure 3.4.*

Example 3.2 (continued) *Consider the sender's bias $b = 1/10$ and the probability of mediator being absent $\epsilon = 7/1000$. The maximum number of partitions on the mediated channel is $N^1(1/3) = 3$ and the sender's partition of the type space on the mediated channel involves cut-off types $\theta_1^1 = 0.035$ and $\theta_2^1 = 0.3$. There are three induced actions $a_1^1 = 0.1$, $a_2^1 = 0.17$ and $a_3^1 = 0.65$ on the mediated channel, while we have only one action, $a_1^0 = 0.1$, on the direct channel. When the mediator is available, the lowest sender's types randomize between a_1^0 and a_1^1 with probabilities $\alpha = 0.03$ and $1 - \alpha = 0.97$. If the sender reports type $\theta < 0.035$ to the mediator, the mediator always recommends a_1^1 and if the sender reports $\theta > 0.3$, the mediator always recommends a_3^1. If $0.035 < \theta < 0.3$, the mediator recommends a_1^1 with probability $\sigma = 0.99$ and a_2^1 otherwise. This equilibrium is illustrated in Figure 3.5.*

It is easy to see that the equilibrium outcomes of this game and a game where all conversion has to go through the strategic mediator (Γ^S) are identical. However, in game Γ, the mediator, when receiving a signal from the second lowest interval, recommends action a_1^1 with a lower probability then the mediator in game Γ^S.
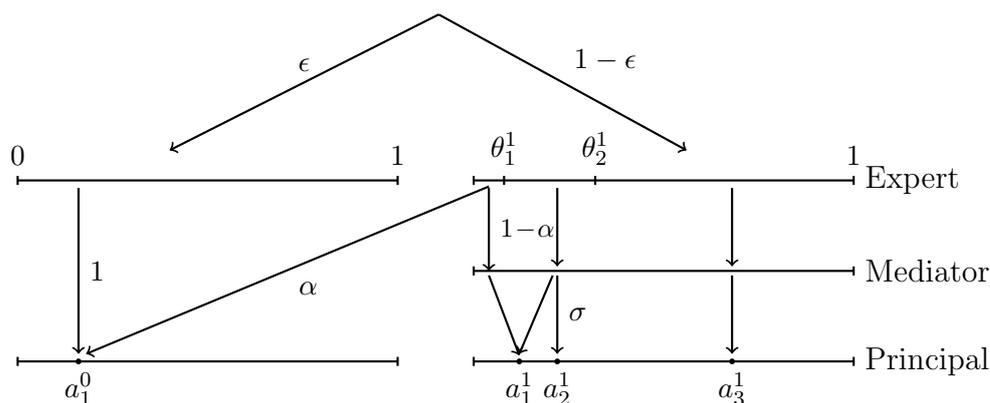
Figure 3.5: Most informative equilibrium of game Γ for $b = 1/10$ and $\epsilon = 7/1000$

3.6 Conclusion

In the standard cheap-talk models with a mediator, it is usually assumed that all communication goes through one specific channel, either through a neutral mediator, or through a strategic one, or through a chain of mediators, etc. The situations when the sender simply wants to talk directly to the receiver and hence bypass the mediator or when the mediator is not available for communication are ignored. In this chapter, we address the question as to what extent such alternations affect the mediated equilibria. We assume that with some probability the mediator is not available for communication, hence in this case the communication has to go through the direct channel. We thus perform a robustness test on the mediated equilibrium, but rather than perturbing strategies, we perturb the game itself. We show that when the probability of absence is not too large, there exist an equilibrium of this perturbed game, that is outcome equivalent to the one with the biased mediator. Hence, the equilibrium designed by Ivanov (2010a) is robust to the small perturbation that the mediator is not present.

The same logic can be applied for the mechanical mediator given that the most informative equilibrium of Ivanov (2010a) can be implemented with an impartial mediator.

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Selbständigkeitserklärung

Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung meiner Dissertation benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

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