ESSAYS ON MACROECONOMIC CONSEQUENCES OF UNCERTAINTY

DISSERTATION

zur Erlangung des akademischen Grades
doctor rerum politicarum
(Doktor der Wirtschaftswissenschaft)

eingereicht an der

Wirtschaftswissenschaftlichen Fakultät
der Humboldt-Universität zu Berlin

von

Grzegorz Ryszard Długoszek

Präsidentin der Humboldt-Universität zu Berlin:
Prof. Dr.-Ing. Dr. Sabine Kunst
Dekan der Wirtschaftswissenschaftlichen Fakultät:
Prof. Dr. Daniel Klapper

Gutachter:	1.	Prof. Dr. h.c. Michael Burda, Ph.D.
	2.	Prof. Lutz Weinke, Ph.D.

Tag des Kolloquiums: 24 Mai 2019

To my wife and parents

Abstract

This thesis examines the macroeconomic implications of uncertainty which has recently attracted attention within both the academic literature and policy community.

The recent global financial crisis has highlighted the role of the financial sector as a source of economic fluctuations. For this reason, the first essay investigates the effects of uncertainty originating in financial markets. To this end, I first document empirical relevance of financial uncertainty using SVAR methods. Second, I employ the DSGE framework developed by Gertler and Karadi (2011) to uncover the underlying transmission mechanism. In line with the empirical evidence, the model generates a decline in economic activity in response to an increase in financial uncertainty. This outcome arises mainly because of tightening of leverage constraints which in turn triggers the financial accelerator mechanism.

In the second essay, I propose an asymptotic perturbation method to solve DSGE models with endogenous portfolio choice. In contrast to existing local techniques, it can be used to compute a higher-order approximation of gross asset holdings. Hence, it is suitable for investigating a variety of economic questions for which risk/uncertainty plays an important role. To facilitate comparison with other solution techniques, I abstract from second-moment shocks and evaluate the proposed method by solving a version of commonly used Lucas tree model with portfolio choice. The focus lies on implications of cross-country structural heterogeneity in economic uncertainty for international asset holdings. The proposed method accounts for these asymmetries and can consequently lead to an improvement in quality of the approximation.

Finally, the third essay examines the consequences of global uncertainty shocks for banking portfolios and macroeconomic aggregates. To this end, I employ a two-country DSGE model with balance-sheet constrained financial intermediaries and endogenous portfolio choice. Countries are assumed to be ex-ante asymmetric, which allows me to consider both developed and emerging economies. The model implies a home bias in banking assets that is consistent with the data. Moreover, an increase in financial uncertainty leads to a decline in cross-border portfolios and a worldwide reduction in economic activity, which is consistent with dynamics observed during the global financial crisis.

Zusammenfassung

Gegenstand dieser Dissertation sind die Auswirkungen von Unsicherheit, die in der letzten Zeit hohe Aufmerksamkeit unter Akademikern und Politikern erregt hat.

Die globale Finanzkrise hat gezeigt, dass die Wirtschaftsschwankungen ihren Ursprung im Finanzsektor haben können. Aus diesem Grund beschäftigt sich der erste Aufsatz mit den Folgen von Unsicherheit, die von Finanzmärkten ausgeht. Zu diesem Zweck belege ich zunächst die empirische Relevanz von Finanzmarktunsicherheit mithilfe von SVAR Methoden. Anschließend benutze ich das von Gertler and Karadi (2011) entwickelte DSGE-Modell, um den Transmissionsmechanismus aufzudecken. Im Einklang mit der empirischen Evidenz impliziert das Modell einen Rückgang der Wirtschaftsleistung als Reaktion auf einen Anstieg der Finanzmarktunsicherheit. Dieses Ergebnis entsteht hauptsächlich aufgrund einer Verschärfung der endogenen Leverage-Beschränkung, die den finanziellen Akzelerator auslöst.

Im zweiten Aufsatz schlage ich eine asymptotische Perturbationsmethode vor, um DSGE Modelle mit endogener Portfolioentscheidung zu lösen. Im Gegensatz zu existierenden Verfahren kann sie benutzt werden, um Approximationen höheren Grades von Bruttovermögenswerten zu ermitteln. Daher ist sie für die Analyse von einer Vielzahl an volkswirtschaftlichen Fragen nützlich, für die das Risiko bzw. Unsicherheit eine große Rolle spielt. Um den Vergleich mit anderen Lösungsmethoden zu erleichtern, abstrahiere ich in diesem Aufsatz von Volatilitätsschocks und evaluiere den vorgeschlagenen Lösungsalgorithmus, indem ich eine Version vom häufig verwendeten Lucas-Tree-Modell mit Portfolioentscheidung löse. Der Schwerpunkt liegt dabei auf den Folgen von struktureller Heterogenität in der wirtschaftlichen Unsicherheit zwischen den Ländern. Die vorgeschlagene Methode erfasst diese Asymmetrie und kann demzufolge zu einer Verbesserung von der Qualität der Approximation führen.

Der dritte Aufsatz untersucht schließlich die Folgen von globalen Unsicherheitsschocks für die Bankenportfolios und die makroökonomischen Aggregate. Zu diesem Zweck benutze ich ein Zwei-Länder DSGE Modell mit endogener Portfolioentscheidung und Bilanzrestriktionen im Bankensektor. Die Bankenportfolios sind charakterisiert durch einen Home Bias, der mit den Daten konsistent ist. Außerdem führt ein Anstieg der Finanzmarktunsicherheit zum Rückgang der grenzüberschreitenden Bruttoanlagen und der Wirtschaftsleistung weltweit. Dies entspricht den Entwicklungen während der globalen Finanzkrise.

Acknowledgements

In the course of writing this dissertation I have benefited from many people. First of all, I am highly indebted to my supervisor Michael Burda who provided guidance, encouragement, and support throughout my graduate studies. His numerous suggestions and comments have had a great impact of each chapter of this thesis. Furthermore, I am grateful to Lutz Weinke for valuable feedback and encouragement throughout the last years. I also thank Frank Heinemann who fueled my early excitement about macroeconomics and has been continuously supporting me since my undergraduate studies.

In addition, this thesis has been improved by comments by Michał Brzoza-Brzezina, Wouter Den Haan, Jordi Galí, Marcin Kolasa, Jesper Lindé, Johannes Pfeifer, Michael Reiter, Stephanie Schmitt-Grohé, Mathias Trabandt, Martin Uribe, and Pablo Winant. Furthermore, all chapters have benefited from comments of participants of the Brown Bag Seminar Macroeconomics at Humboldt-Universität zu Berlin and of several workshops and conferences.

Moreover, I am indebted to my colleagues Julien Albertini, Anna Almosova, Maren Brede, Thomas Dengler, Tobias König, Johanna Krenz, Hong Lan, Falk Mazelis, Alexander Meyer-Gohde, Daniel Neuhoff, Julia Otten, Mauricio Salgado Moreno, Stefanie Seele, Felix Strobel, Andreas Tryphonides, and Simon Voigts at Humboldt-Universität zu Berlin for their invaluable mental support, comments, discussions, and practical help in the course of writing this dissertation. Spending time with them broadened my mind - not only in economics. Moreover, I am also thankful to Claudia Keidel for her great help in administrative tasks and to our student research assistants, especially Carl Meyer, Tobias Bergmann and Daniel Jacob.

As ever, I am grateful to my family for their continuous support. I am truly thankful to my parents who provided me with unlimited opportunities to explore and to learn. Most of all, I am grateful to my wife Anna for her love and encouragement, for being by my side and sharing the ups and downs that went along with this thesis.

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Chapter 1

Introduction

What is the role of uncertainty for the real economy? This question has attracted attention within both the academic literature and policy community in the aftermath of the global financial crisis.[1] But, what is uncertainty? According to Bloom (2014), it is an "amorphous concept" and can be defined in different ways. For example, Knight (1921) proposes a distinction between risk and uncertainty. The former concept refers to events for which agents know the probability distribution, whereas in case of uncertainty no numerical probabilities are known. In contrast, this dissertation follows the modern macroeconomic literature and uses the term uncertainty as a catch-all term to cover both risk and uncertainty.

Interest in uncertainty has been sparked by the robust observation that it rises in recessions (see, e.g., Bloom, 2009, Jurado et al., 2015). The literature provides two contrasting explanations for this countercyclicality. First, a growing body of literature, led by the seminal paper by Bloom (2009), argues that uncertainty has detrimental effects on the real economy. On the other hand, various studies argue that fluctuations in uncertainty are an endogenous response to other fundamental shocks.[2] Policymakers seem to favor the first explanation, as many of them have suggested in various speeches that uncertainty plays an important role for both business cycle fluctuations and the policymaking process. One example is the introductory speech by Draghi (2017) at the ECB Forum on Central Banking in which he said that

> [...] *in the current context where global uncertainties remain elevated, there are strong grounds for prudence in the adjustment of monetary policy parameters, even when accompanying the recovery.*

This thesis abstracts from endogenous components of uncertainty and examines the macroeconomic effects of exogenous uncertainty shocks. The analysis is

[1] While the impact of uncertainty has been explored already by earlier studies such as Bernanke (1983), this literature has experienced a rapid growth after the global financial crisis.

[2] See, among others, Bachmann et al. (2011), Fostel and Geanakoplos (2012), He and Krishnamurthy (2014), Gomes (2016), and Fajgelbaum et al. (2017).

partitioned into three chapters. Chapter 2 investigates the implications of financial uncertainty, i.e, uncertainty originating in the financial sector. I focus on this type of uncertainty for two reasons. First, the recent global financial crisis has highlighted the role of the financial sector as a source of economic fluctuations. Second, Ludvigson et al. (2015) distinguish between financial and macroeconomic uncertainty, i.e., uncertainty about real economic fundamentals, and provide empirical evidence that movements in the former are an important source of economic fluctuations. On the other hand, time-varying macroeconomic uncertainty seems to be driven by other shocks. In contrast to these empirical findings, the DSGE literature focuses mainly on macroeconomic uncertainty. The objective of chapter 2 is to close this gap. My contribution is twofold. First, I estimate a Structural Vector Autoregressive (SVAR) model and provide evidence that an increase in financial uncertainty has an adverse effect on main macroeconomic aggregates, such as GDP, consumption, investment and hours worked. Second, I introduce time-varying volatility of financial disturbances to the DSGE model developed by Gertler and Karadi (2011) in order to uncover the transmission mechanism of financial uncertainty shocks. The model generates dynamics of macroeconomic variables that are consistent with the empirical evidence. In particular, output, investment, consumption and hours worked drop, while the risk premium rises in response to an increase in financial uncertainty. The key feature of the model responsible for this outcome is tightening of the endogenous leverage constraint which in turn triggers the financial accelerator mechanism. In addition, both nominal rigidities and internal habit formation are necessary to generate the empirically observed co-movement among macroeconomic aggregates.

Chapter 3 proposes an asymptotic perturbation method to solve DSGE models with endogenous portfolio choice. These models are most commonly employed to explain international asset allocation and its impact on the real economy. In contrast to existing solution algorithms, e.g., the workhorse routine by Devereux and Sutherland (2010, 2011), the proposed method can be used to obtain a higher-order approximation of gross asset holdings.[3] Hence, it can be used to address a variety of interesting questions in the area of macro-finance, such as the implications of uncertainty shocks for cross-country portfolios (see chapter 4). To facilitate comparison with other solution techniques, I abstract from time-varying uncertainty in this essay and evaluate the proposed method by solving a two-country Lucas tree model with incomplete markets and endogenous portfolio choice. Yet, uncertainty still plays an important role in this analysis. In particular, countries are assumed to be ex-ante asymmetric, as their endowments are characterized by different volatilities. This structural heterogeneity in economic uncertainty implies in turn

[3]Gross capital flows have attracted a lot of attention in the international finance literature, as they contain more information and are more volatile than net flows (see, e.g., Broner et al., 2013).

different hedging needs across countries and will be reflected in the optimal asset allocation. Chapter 3 examines whether capturing this asymmetry, which is possible because of the use of the proposed method, leads to an improvement in quality of the approximation. The answer is yes. Taking into account cross-country heterogeneity leads to more accurate solution in the example model, as reflected by lower Euler equation errors and ergodic moments that lie closer to their global solution counterparts.

Finally, chapter 4 builds upon the preceding essays and examines the implications of global uncertainty shocks for banks' cross-border claims and the world economy. The analysis is motivated by a decline in international asset holdings during the global financial crisis - a phenomenon labeled by Tille and Van Wincoop (2010) as *the great retrenchment*. As documented by the empirical literature, cross-border bank lending played an important role - particularly for advanced economies - during the retrenchment episode (see, e.g., Bertaut and Demarco, 2009). Moreover, various studies argue that *the great retrenchment* was driven by an increase in uncertainty which forced investors to take more cautious view of the investment opportunities.[4] The objective of this chapter is to determine conditions under which an adverse global uncertainty shock implies a reduction in external portfolios and induces a worldwide decline in economic activity. To this end, I employ a real two-country version of the model previously used in chapter 2. Countries are assumed to be ex-ante asymmetric, which allows me to distinguish between emerging market and developed economies. Following Ludvigson et al. (2015), I consider two types of uncertainty: financial and macroeconomic. Finally, since a higher-order approximation of gross asset holdings is required to investigate the effects of uncertainty shocks, I solve the model with the perturbation method proposed in chapter 3. The numerical analysis yields the following results. First, the model generates a home bias in banks' portfolios that is consistent with data. Second, an increase in financial uncertainty leads to a reduction in cross-border assets and induces a worldwide drop in GDP. In contrast, an adverse macroeconomic uncertainty shock incentivize financial intermediaries to increase their international exposure and results in non-synchronized dynamics of real variables across countries. These contrasting results reflect the fact that different types of uncertainty shocks induce different changes in banks' hedging needs.

All in all, this dissertation highlights that the origin of uncertainty plays a crucial role, as different types of uncertainty shocks can have different implications and are propagated via different channels. Furthermore, it provides the theoretical support for empirical findings of Ludvigson et al. (2015) and points to financial markets as the source of uncertainty being an important driver of economic fluctuations.

[4]See, among others, Tille and Van Wincoop (2010), Forbes and Warnock (2012), Fratzscher (2012), Nier et al. (2014) as well as Rey, 2015

Chapter 2

Macroeconomic Effects of Financial Uncertainty

How does uncertainty originating in the financial sector affect the real economy? To address this question, I first document empirical relevance of financial uncertainty using SVAR methods. Then, I employ the DSGE framework developed by Gertler and Karadi (2011) to uncover the underlying transmission mechanism. The model generates macroeconomic dynamics that are consistent with the SVAR evidence. In particular, an increase in financial uncertainty raises the risk premium and leads to a decline in output, consumption, investment and hours worked. This outcome arises mainly because of an endogenous tightening of the financial constraint which in turn triggers the financial accelerator mechanism. Finally, internal habit formation and nominal rigidities act as additional amplification mechanisms for financial uncertainty shocks.

Keywords: Stochastic Volatility, Financial Frictions, Financial Uncertainty, Third-Order Approximation
JEL Classification Numbers: E44, E32, E21

2.1 Introduction

There exists a rapidly growing literature on macroeconomic implications of uncertainty shocks. Interest in this topic has been sparked by the robust observation that uncertainty rises in recessions (see, e.g., Bloom, 2009, Jurado et al., 2015). The DSGE literature focuses on macroeconomic uncertainty, i.e., uncertainty surrounding real economic fundamentals, such as total factor productivity or economic policy.[1] This is also true for studies assessing the role of financial frictions as a

[1]The DSGE literature investigates a variety of real uncertainty shocks. Caldara et al. (2012) consider total factor productivity in a model with recursive preferences. Moreover, Mumtaz and Zanetti (2013) look at monetary policy uncertainty, while Born and Pfeifer (2014) investigate

propagator of economic uncertainty.[2] However, Ng and Wright (2013) document that all post-1982 U.S. recessions have origins in financial markets. In addition, Ludvigson et al. (2015) provide empirical evidence that movements in financial uncertainty, i.e., uncertainty originating in the financial sector, are an important source of economic fluctuations. The authors conclude their analysis with the following statement

> *These findings point to the need for a better understanding of how uncertainties in financial markets are transmitted to the macroeconomy* [...]

The aim of this study is to contribute to the existing literature by investigating macroeconomic implications and the transmission mechanism of financial uncertainty. My contribution is twofold. First, I estimate a Structural Vector Autoregressive (SVAR) model and provide evidence that an increase in financial uncertainty has an adverse effect on main macroeconomic aggregates, such as GDP, consumption, investment and hours worked. Second, I introduce time-varying volatility of financial disturbances to the DSGE model developed by Gertler and Karadi (2011) in order to uncover the transmission mechanism of financial uncertainty shocks. Since this framework embeds financial intermediaries operating under funding constraints, it is suitable for investigating the effects of uncertainty related to disturbances originating in the financial sector. Financial level shocks are introduced to the model by assuming that financial intermediaries are forced to exit the market with a stochastic probability in each period. An adverse shock to the survival rate induces a drop in net worth of the banking sector, as more intermediaries leave the market. As the result, aggregate investment falls and this leads ultimately to a recession.

The model generates dynamics of macroeconomic variables that are consistent with the empirical evidence. In particular, output, investment, consumption and hours worked drop, while the risk premium rises in response to an increase in financial uncertainty. The key feature of the model responsible for this outcome is tightening of the endogenous leverage constraint which in turn triggers the financial accelerator mechanism. Specifically, due to an increase in financial uncertainty, households provide less funding to financial intermediaries. This reduces aggregate investment and asset prices decline. As a consequence, financial position of intermediaries deteriorates even further forcing them to reduce their lending again. Simultaneously, under the assumption of internal habit formation

the contribution of monetary and fiscal uncertainty to economic fluctuations in the United States. Finally, Basu and Bundick (2017) investigate uncertainty associated with the aggregate demand and argue that nominal rigidities are key to generate co-movement among macroeconomic aggregates following a rise in uncertainty.

[2] See, e.g., Bonciani and Van Roye (2016).

and sticky prices, aggregate consumption falls. Thereby, these model features act as additional amplification mechanisms for financial uncertainty shocks.

Finally, I use the theoretical framework to compare the effects of financial uncertainty shocks with the consequences of macroeconomic uncertainty. While both types of uncertainty have qualitatively similar effects on economic activity, their key propagation mechanisms differ. In particular, macroeconomic uncertainty relies more extensively on nominal and real rigidities.

Related Literature This study is related to two recent strands of the literature. The first one is the growing literature (both empirical and theoretical) on macroeconomic implications of uncertainty shocks. The empirical literature investigates various types of uncertainty: macroeconomic (e.g., Jurado et al., 2015), economic policy (e.g., Baker et al., 2016), geopolitical (Caldara and Iacoviello, 2018), and financial (e.g., Ludvigson et al., 2015). Both my empirical and theoretical analysis are based on the study by Ludvigson et al. (2015). The authors construct a novel measure of financial uncertainty and conduct an empirical investigation by employing correlation and event constraints. They find evidence that financial uncertainty is a likely source of economic fluctuations, while movements in macroeconomic uncertainty seem to be an endogenous response to other economic disturbances. The empirical part of this paper differs from the study of Ludvigson et al. (2015) in two respects. First, as I focus solely on financial uncertainty, I identify uncertainty shocks by using a Cholesky decomposition. In contrast, Ludvigson et al. (2015) require an identification strategy that allows them to distinguish between different types of uncertainty. Second, the authors investigate only the effects of financial uncertainty on industrial production, whereas I include further macroeconomic aggregates, a measure of risk premium, and a measure of monetary policy stance in the SVAR model.

The theoretical literature on general equilibrium effects of uncertainty focuses mainly on macroeconomic uncertainty. However, some studies investigate shocks that can be (partly) related to financial markets. First, there exists literature analyzing macroeconomic implications of second-moment disturbances to (real or nominal) interest rates. For instance, Fernández-Villaverde et al. (2011) focus on economic effects of time-varying volatility of international interest rates on emerging market economies. Another example is a recent paper by Richter and Throckmorton (2018). The authors develop a new method to quantify the effects of different types of uncertainty using estimates from a nonlinear DSGE model. This approach allows them to distinguish between exogenous and endogenous sources of uncertainty. In their model, financial uncertainty is related to a second-moment shock to the return on a nominal bond. While time-varying volatility of interest rates may have its origin in financial markets, it can also reflect different factors such as political instability or news shocks (Fernández-Villaverde et al., 2011). In contrast, I use a framework with a micro-founded banking sector allowing me to investigate uncertainty originating in financial markets. Second, Christiano et al. (2014) introduced to the literature risk shocks, i.e., disturbances to the dispersion in the idiosyncratic productivity of entrepreneurs. As these shocks have a direct impact on the probability of firms' default, they affect funding conditions in the economy. However, risk shocks originate in the non-financial sector. Their relation to the real side of the economy is reflected by the fact that Cesa-Bianchi and Corugedo (2018) use the cross-sectional standard deviation of establishment-level TFP innovations as their measure. In addition, risk shocks represent micro-uncertainty (Cesa-Bianchi and Corugedo, 2018), having first-order effect, as opposed to the aggregate uncertainty considered in this paper.

Finally, this paper can be related to the vast literature on macroeconomic effects of financial shocks that emphasizes the importance of financial factors for business cycle fluctuations.[3] While existing studies investigate a variety of level shocks such as credit spread or net worth disturbances, I focus on the second-moment innovations, i.e., exogenous movements in volatility of financial factors.

The rest of the paper is organized as follows. In section 2.2, I provide empirical evidence on the macroeconomic effects of financial uncertainty. The theoretical model is outlined in section 2.3. In section 2.4, I present the chosen calibration and the method used to solve the model. Section 2.5 discusses implications of the theoretical framework. Section 2.6 concludes.

2.2 Empirical Evidence

To provide empirical evidence on the relevance of financial uncertainty for economic fluctuations, I estimate a VAR model using quarterly U.S. data for the period 1986:Q1-2016:Q4. The structural model is given by

$$A_0 y_t = A_1 y_{t-1} + ... + A_{t-p} y_{t-p} + \epsilon_t, \tag{2.1}$$

where y_t is an $n \times 1$ vector of variables of interest and $\epsilon_t \sim \mathcal{N}(0, I_n)$. The corresponding reduced-form VAR can then be written as

$$y_t = B_1 y_{t-1} + ... + B_p y_{t-p} + u_t, \tag{2.2}$$

with $B_j \equiv A_0^{-1} A_j$ and $u_t \equiv A_0^{-1} \epsilon_t \sim \mathcal{N}(0, \Sigma_u)$.

Eight variables comprise the system for estimation: 1) a measure of financial uncertainty, $FinUnc$, 2) per capita real GDP, Y, 3) per capita consumption, C,

[3] See, among others, Meh and Moran (2010), Meeks (2012), Jermann and Quadrini (2012), Gilchrist and Zakrajšek (2012), Brzoza-Brzezina et al. (2013), and Iacoviello (2015).

4) per capita investment, I, 5) hours worked, L, 6) inflation rate, π, measured as the percentage change of the GDP implicit price deflator, 7) risk premium, spr, measured by the difference between BAA corporate bond yield and 10 year treasury yield, 8) and finally the federal funds rate, $EFFR$. A detailed description of the data can be found in Appendix A.1. All variables except the inflation rate, the risk premium and the federal funds rate enter the VAR in log levels. Finally, all variables are detrended by applying the HP filter with a smoothing parameter of 1600.

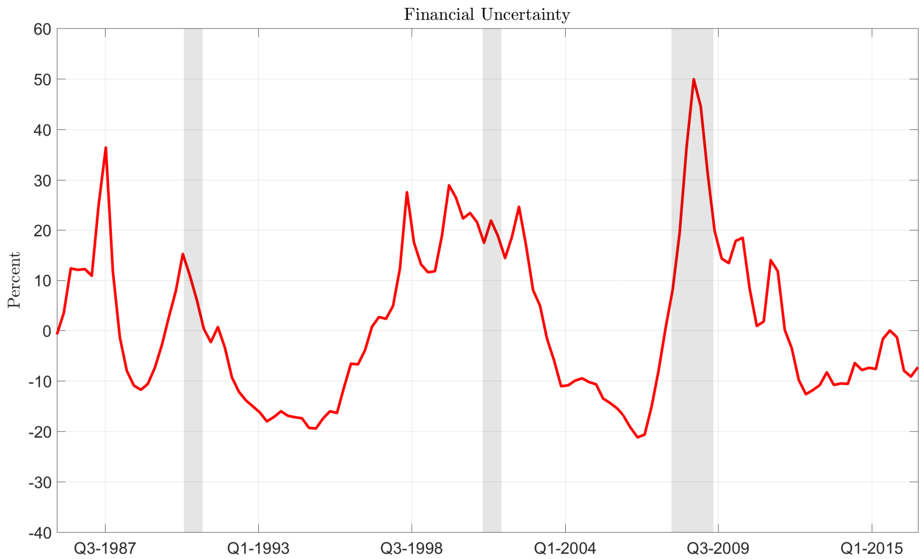


Figure 2.1: Measure of financial uncertainty by Ludvigson et al. (2015). *The time series is represented as percentage deviation from its pre-crisis mean. The gray bars represent periods of recession defined by the NBER.*

Figure 2.1 depicts the measure of financial uncertainty constructed by Ludvigson et al. (2015). The time series is represented as percentage deviation from its pre-crisis mean. Following Jurado et al. (2015), this proxy is based on factor augmented VAR methods. It aggregates a large number of estimated uncertainties obtained from a rich data set consisting of 148 series such as dividend-price ratios, yield-spreads as well as various measures of variation in the market risk premium. Uncertainty associated with an individual series x_t is defined as the volatility of its h-period ahead forecast error (Ludvigson et al., 2015)

$$U_x(h) \equiv \sqrt{E_t\left[(x_{t+h} - E_t\left[x_{t+h}\right])^2\right]}, \tag{2.3}$$

where E_t denotes the expectation operator given the information set in period t.[4] As argued by Jurado et al. (2015), the advantage of uncertainty measures based on forecast errors is that they can truly capture the degree to which the economy has become more or less predictable, i.e., uncertain. In contrast, other existing proxies reflect rather time-varying dispersion or volatility of economic indicators. Moreover, they can often provide misleading information. For example, stock market volatility can fluctuate even if uncertainty remains constant due to changes in investors' risk aversion or sentiment.

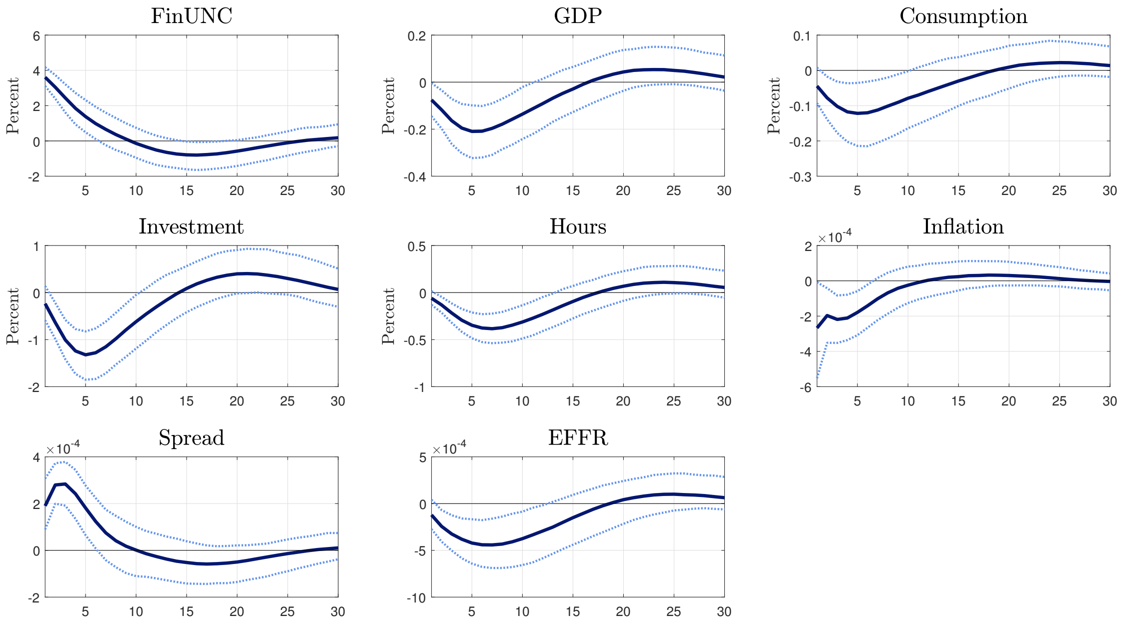


Figure 2.2: SVAR evidence: dynamic consequences of financial uncertainty. *Horizontal axes indicate quarters. The solid curve denotes the median response, whereas the dotted curves refer to the 95 % confidence interval. Responses of all variables except inflation, the risk premium and the federal funds rate are in percent.*

Following a large body of empirical literature, financial uncertainty shocks are identified using a Cholesky decomposition.[5] The measure of financial uncertainty is ordered first and is followed by the macroeconomic aggregates, the risk premium and the federal funds rate, i.e., $y_t = [FinUnc_t, Y_t, C_t, I_t, L_t, \pi_t, spr_t, EFFR_t]'$. This ordering implies that financial uncertainty affects the remaining variables of the system but is not contemporaneously influenced by them. Therefore, it is consistent with the theoretical model discussed in section 3 and the underlying intuition is in line with main findings of Ludvigson et al. (2015).[6]

[4] The data set constructed by Ludvigson et al. (2015) include three different forecast horizons: one month, one quarter and one year. In the following, I use quarterly forecasts to match the frequency of the time series used in the estimation.

[5] See, among others, Bloom (2009), Bachmann and Bayer (2013), Jurado et al. (2015), Baker et al. (2016), and Basu and Bundick (2017).

[6] Following Gilchrist and Zakrajšek (2012), the chosen identification strategy allows for *leaning against the wind*, i.e., a contemporaneous response of the monetary policy to credit spread shocks. Changing this assumption does not alter the results on the effects of financial uncertainty.

I estimate the model up to four lags and determine the appropriate specification by using the Bayesian information criterion, according to which the data prefers the model with one lag. Figure 2.2 plots the impulse responses to an identified financial uncertainty shock along with the 95 % confidence intervals. A one standard deviation increase in financial uncertainty leads to tighter financial conditions in the economy, as shown by the rise in the risk premium. It also leads to statistically significant declines in output, consumption, investment and hours worked. The peak response occurs after about a year and amounts in case of GDP to a drop of 0.21%. The subsequent recovery is followed by a rebound - a phenomenon labeled by Bloom (2009) as *volatility overshoot*. For example, in case of GDP the overshoot arises after about four years.

To assess the robustness of the VAR evidence, I modify the estimation exercise in several ways. First, I use an alternative ordering with the measure of financial uncertainty ordered last. Second, I replace the BBA spread by the credit spread indicator constructed by Gilchrist and Zakrajšek (2012). Moreover, as the estimation sample includes the zero lower bound episode, I use the shadow rate proposed by Wu and Xia (2016) to measure the stance of monetary policy. Finally, I extend the set of variables by including a measure of macroeconomic uncertainty constructed by Jurado et al. (2015). It is based on 132 time series of macroeconomic indicators ranging from real output and income to inventories and capacity utilization measures. As shown in Appendix A.2, results obtained under these alternative specifications do not substantially differ from the ones of the benchmark estimation.

2.3 The Model

To shed light on the transmission mechanism of uncertainty shocks originating in the financial sector, I employ the New Keynesian model with financial frictions developed by Gertler and Karadi (2011). One period corresponds to one quarter and there are six types of agents in the model: households, financial intermediaries, intermediate goods producers, monopolistically competitive retailers, capital producers and a central bank, whose actions are described by a standard Taylor-rule.

2.3.1 Households

There exists a continuum of identical households of unity mass. Within each household, there are $1 - f$ workers and f bankers. Workers supply labor and earn wages, whereas each banker manages a financial intermediary and accumulates funds ("net worth") which she transfers to the household upon exiting the business. To merge the within-household heterogeneity with the representative agent framework, I assume that there is perfect consumption sharing within each family.

Household's preferences are given by

$$max \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - hC_{t-1})^{1-\gamma} - 1}{1-\gamma} - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \right], \tag{2.4}$$

where C_t denotes consumption and L_t is labor supply. Moreover, $\beta \in (0,1)$ refers to the discount factor, $h \in (0,1)$ is parameter governing internal habit formation and γ represents the inverse of the intertemporal elasticity of substitution. Finally, φ is the inverse of the Frish elasticity of labor supply and χ denotes the weight of the disutility of labor supply.

Following Gertler and Karadi (2011), households do not have a direct access to capital stock. Rather, they save by depositing funds in financial intermediaries.[7] Bank deposits, denoted by D_t, are equivalent to one period real riskless bonds yielding gross real rate of return R_t from t to $t+1$. The budget constraint faced by the household is thus given by

$$C_t + D_t = W_t L_t + R_{t-1} D_{t-1} + T_t, \tag{2.5}$$

where W_t refers to the real wage and T_t are net profits from the ownership of both non-financial firms and financial intermediaries. Let U_{Ct} denote the marginal utility of consumption and $\Lambda_{t,t+1}$ the household's stochastic discount factor. Then maximizing the life-time utility with respect to consumption, labor and savings subject to the flow of funds constraint (2.5) yields the following first-order conditions

$$W_t U_{Ct} = \chi L_t^{\varphi}, \tag{2.6}$$

and

$$E_t \left[\Lambda_{t,t+1} \right] R_t = 1, \tag{2.7}$$

with $U_{Ct} = (C_t - hC_{t-1})^{-\gamma} - \beta h E_t \left[(C_{t+1} - hC_t)^{-\gamma} \right]$ and $\Lambda_{t,t+1} \equiv \beta \frac{U_{Ct+1}}{U_{Ct}}$.

2.3.2 Nonfinancial Firms

There are three types of nonfinancial firms: intermediate goods producers, monopolistically competitive retailers and capital producers.

[7] The implicit assumption is that households supply funds to banks other than the ones they own.

Intermediate Goods Producers

In period t competitive firms with identical constant returns to scale technology produce intermediate goods, Y_{mt}, by combining capital stock purchased at the end of period $t-1$, K_{t-1}, and labor, L_t, and by varying the utilization rate of capital U_t. This process is governed by the following Cobb-Douglas production function

$$Y_{mt} = A_t \left(\xi_t U_t K_{t-1}\right)^{\alpha} L_t^{1-\alpha}, \tag{2.8}$$

where $\alpha \in (0,1)$, A_t denotes an exogenously given technology level and ξ_t refers to a capital quality shock.[8] There are no adjustment costs at the firm level and thus the intermediate producer's maximization problem is static. In particular, at the end of each period, the firm replaces the depreciated capital, sells its entire capital stock and purchases capital that will be employed in the subsequent period. Following Gertler and Karadi (2011), I assume that the replacement price of used capital is equal to unity.[9] As a result, the decision with respect to capital utilization is independent of the price of capital.

To finance capital acquisition, the firm must obtain funds from financial intermediaries. To this end, it issues state contingent claims in the amount equal to the number of purchased units of capital. Thus, arbitrage requires that these claims are traded at the price of a unit of capital, Q_t. Given that R_k denotes the gross real interest rate paid on state contingent securities, the intermediate good producer chooses labor input and capital utilization to maximize her current profits

$$P_{mt} Y_{mt} + \left[Q_t - \delta(U_t)\right] \xi_t K_{t-1} - W_t L_t - R_{kt} Q_{t-1} K_{t-1}, \tag{2.9}$$

where P_{mt} denotes the price of intermediate goods relative to the final consumption basket and $\delta(U_t) = \delta_0 + \frac{\delta_1}{1+\delta_2} U_t^{1+\delta_2}$, with $\delta_0, \delta_1, \delta_2 > 0$, is the depreciation rate being a function of capital utilization. Solving this maximization problem yields the following first-order conditions

$$W_t = P_{mt} \left(1-\alpha\right) \frac{Y_{mt}}{L_t}, \tag{2.10}$$

and

$$\alpha P_{mt} \frac{Y_{mt}}{U_t} = \delta' \left(U_t\right) \xi_t K_{t-1}. \tag{2.11}$$

Note that under assumptions of competitive firms and constant returns to scale, the intermediate producers make zero profits in equilibrium. Thus, the ex-post

[8] See, among others, Gertler and Kiyotaki (2010), Gertler and Karadi (2011), and Dedola et al. (2013).

[9] This requires that the adjustment costs are on net investment. See the description of capital producers.

rate of return on state contingent assets is given by

$$R_{kt} = \frac{\left[\alpha P_{mt} \frac{Y_{mt}}{\xi_t K_{t-1}} + Q_t - \delta(U_t)\right] \xi_t}{Q_{t-1}}. \tag{2.12}$$

Retailers

A continuum of mass unity of monopolistically competitive retailers repackage intermediate output requiring one unit of intermediate good for each unit of retail output. Hence, the marginal cost of final good production is simply P_{mt}.

Final output, Y_t, is given by the CES aggregator of differentiated retailer goods, Y_{it},

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{\varrho-1}{\varrho}} di\right]^{\frac{\varrho}{\varrho-1}}, \tag{2.13}$$

where $\varrho > 1$ is the elasticity of substitution between different retailer goods. Cost minimization by the final output user yields

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varrho} Y_t, \tag{2.14}$$

and

$$P_t = \left[\int_0^1 P_{it}^{1-\varrho} di\right]^{\frac{1}{1-\varrho}}. \tag{2.15}$$

Retailers face nominal rigidities à la Calvo (1983).[10] In particular, each period a retailer is able to adjust her prices with probability $1 - \theta_{calvo}$. If she cannot freely update her prices, she is able to index them to the lagged rate of inflation. γ_p governs the degree of price indexation. A retailer, updating her price in period t, chooses the reset price, P_t^*, that maximizes the present value of profits generated while the price remains valid

$$max \quad E_t \left[\sum_{k=0}^{\infty} \theta_{calvo}^k \Lambda_{t,t+k} \left(\frac{P_t^*}{P_{t+k}} \prod_{j=1}^{k} (\Pi_{t+j-1})^{\gamma_p} - P_{mt+k}\right) Y_{it+k}\right]. \tag{2.16}$$

The corresponding first-order condition is given by

$$E_t \left[\sum_{k=0}^{\infty} \theta_{calvo}^k \Lambda_{t,t+k} \left(\frac{P_t^*}{P_{t+k}} \prod_{j=1}^{k} (\Pi_{t+j-1})^{\gamma_p} - \frac{\varrho - 1}{\varrho} P_{mt+k}\right) Y_{it+k}\right] = 0. \tag{2.17}$$

[10]Note that the heterogeneity introduced by the Calvo assumption may in general require tracking distributions when the model is solved with a higher-order perturbation (Born and Pfeifer, 2014). However, this is not the case in the underlying framework. The reason for this is that retailers only repackage goods and update their prices, whenever this is possible. They do not make any further decisions, especially regarding factors of production.

By rearranging (2.17) one can obtain the following relationship

$$\Pi_t^* = \frac{\varrho}{\varrho - 1} \frac{X_{1,t}}{X_{2,t}} \Pi_t, \tag{2.18}$$

with $\Pi_t^* \equiv \frac{P_t^*}{P_{t-1}}$ and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$. $X_{1,t}$ and $X_{2,t}$ are defined recursively as:

$$X_{1,t} = Y_t P_{mt} + \theta_{calvo} E_t \left[\Lambda_{t,t+1} \Pi_{t+1}^{\varrho} \Pi_t^{\gamma_p \varrho} X_{1,t+1} \right], \tag{2.19}$$

and

$$X_{2,t} = Y_t + \theta_{calvo} E_t \left[\Lambda_{t,t+1} \Pi_{t+1}^{\varrho - 1} \Pi_t^{-\gamma_p(\varrho - 1)} X_{2,t+1} \right]. \tag{2.20}$$

The relationship between aggregate final output and aggregate intermediate production can be written as

$$Y_t = Y_{mt} \Delta_{pt}, \tag{2.21}$$

where Δ_{pt} is the dispersion of individual prices. Its law of motion is given by

$$\Delta_{pt} = \theta_{calvo} \Delta_{pt-1} \Pi_t^{\varrho} \Pi_{t-1}^{-\gamma_p \varrho} + (1 - \theta_{calvo}) \left(\frac{\Pi_t^*}{\Pi_t} \right)^{-\varrho}. \tag{2.22}$$

Moreover, under the Calvo assumption and given the aggregate price index (2.15), the inflation rate can be expressed as

$$\Pi_t^{1-\varrho} = (1 - \theta_{calvo}) \left(\Pi_t^* \right)^{1-\varrho} + \theta_{calvo} \Pi_{t-1}^{\gamma_p (1-\varrho)}. \tag{2.23}$$

Finally, note that since P_{mt} represents the price of intermediate goods relative to the final output, the markup of monopolistic retailers, X_t, is its inverse

$$X_t = \frac{1}{P_{mt}}. \tag{2.24}$$

Capital Producers

Competitive capital producers replace the depreciated capital and produce new investment goods. Thus, the total investment is given by $I_t = I_{nt} + \delta(U_t)\xi_t K_{t-1}$, where I_{nt} denotes the net investment. To generate time-variation in the price of capital, I introduce investment adjustment cost into the model. Following Gertler and Karadi (2011), capital producers face adjustment costs associated only with producing new capital. In contrast, there are no such costs for refurbishing old capital stock. Consequently, costs of replacing depreciated capital stock are fixed to unity.

Capital producers choose I_{nt} that maximizes expected lifetime profits given by

$$E_t\left[\sum_{k=0}^{\infty}\Lambda_{t,t+k}\left((Q_{t+k}-1)I_{nt+k}-f_{inv}\left(\frac{I_{nt+k}+I_{SS}}{I_{nt+k-1}+I_{SS}}\right)(I_{nt+k}+I_{SS})\right)\right], \quad (2.25)$$

with I_{SS} denoting investment in the deterministic steady state and $f_{inv}\left(\frac{I_{nt}+I_{SS}}{I_{nt-1}+I_{SS}}\right) = \frac{\eta}{2}\left(\frac{I_{nt}+I_{SS}}{I_{nt-1}+I_{SS}}-1\right)^2$. The corresponding first-order condition determines the price of one unit of capital

$$\begin{aligned}Q_t &= 1+\frac{\eta}{2}\left(\frac{I_{nt}+I_{SS}}{I_{nt-1}+I_{SS}}-1\right)^2\\ &+\eta\left(\frac{I_{nt}+I_{SS}}{I_{nt-1}+I_{SS}}-1\right)\frac{I_{nt}+I_{SS}}{I_{nt-1}+I_{SS}}\\ &-\eta E_t\left[\Lambda_{t,t+1}\left(\frac{I_{nt+1}+I_{SS}}{I_{nt}+I_{SS}}-1\right)\left(\frac{I_{nt+1}+I_{SS}}{I_{nt}+I_{SS}}\right)^2\right].\end{aligned} \quad (2.26)$$

Finally, note that capital producers can earn non-zero profits outside of the steady state. These profits are assumed to be redistributed lump sum to households.

2.3.3 Financial Intermediaries

Financial intermediaries (or banks) provide funds to producers of intermediate goods. Their operations are financed by a combination of deposits, D_t, held by households, and their own net worth, N_t, which is accumulated from retained earnings. Hence, the balance sheet of a financial intermediary j is given by

$$Q_tK_{jt} = D_{jt}+N_{jt}. \quad (2.27)$$

As noted above, deposits made with banks at time $t-1$ pay the non-contingent real gross return R_{t-1} in the subsequent period. In contrast, assets held by intermediaries earn the stochastic return R_{kt} over the same period. Then, the law of motion for net worth of an intermediary j is given by

$$\begin{aligned}N_{jt} &= R_{kt}Q_{t-1}K_{jt-1}-R_{t-1}D_{jt-1}\\ &= (R_{kt}-R_{t-1})\,Q_{t-1}K_{jt-1}+R_{t-1}N_{jt-1},\end{aligned} \quad (2.28)$$

where the second equality follows from the balance sheet condition.

Intermediaries have an incentive to operate in period t only if the expected discounted rate of return on assets does not lie below the costs of borrowing. By applying household's discount factor, this condition can be written as

$$E_t\left[\Lambda_{t,t+1}\left(R_{kt+1}-R_t\right)\right] \geq 0. \quad (2.29)$$

Under frictionless capital markets, (2.29) holds always with equality. In contrast, the discounted spread between the two rates is positive in the presence of financial frictions, as they limit the ability of financial intermediaries to obtain funds. Thus, given financial constraints, a bank has an incentive to invest all its funds and retain all earnings until the time it exits the business. The event of exit occurs with time-varying probability $1-\theta_t$, where $\theta_t \equiv \theta\vartheta_t$, with ϑ_t being a disturbance to banks' survival probability.[11] Upon exiting, a banker transfers its terminal wealth to the household and becomes a worker.[12] Incorporating a finite horizon for financial intermediaries prevents them from accumulating enough net worth such that the financial constraint is no longer binding. Accordingly, a financial intermediary j determines optimal asset holdings and the amount of external funds to maximize its franchise value, given by

$$V_{jt} = max \quad E_t\left[\sum_{k=1}^{\infty}\Lambda_{t,t+k}\left(\prod_{i=t+1}^{t+k-1}\theta_i\right)(1-\theta_{t+k})\,N_{jt+k}\right], \tag{2.30}$$

with $\left(\prod_{i=t+1}^{t}\theta_i\right)\equiv 1$.

Following Gertler and Karadi (2011), I introduce a moral hazard problem to motivate a limited ability of obtaining funds by financial intermediaries. In particular, at the beginning of each period, a banker can divert a non-bank specific fraction, λ, of her assets and transfers it to her household. In this situation, depositors can force her into bankruptcy and recover the remaining fraction of assets, $1-\lambda$. Hence, households are willing to supply funds to an intermediary j only if the continuation value of its operations is greater (or equal) than the gain from diverting the assets, i.e.,

$$V_{jt} \geq \lambda Q_t K_{jt}. \tag{2.31}$$

To solve the model, I first write (2.30) recursively

$$V_{jt} = max \quad E_t\left[\Lambda_{t,t+1}\left((1-\theta_{t+1})\,N_{jt+1} + \theta_{t+1}V_{jt+1}\right)\right] \tag{2.32}$$

and conjecture that the solution is linear in the value of assets and deposits

$$\begin{aligned} V_{jt} &= v_t^k Q_t K_{jt} - v_t D_{jt} \\ &= \mu_t Q_t K_{jt} + v_t N_{jt}, \end{aligned} \tag{2.33}$$

[11] This shock can be interpreted as a net worth shock because it reduces the internal funds of the banking system. See, e.g., Afrin (2017) or Aoki and Sudo (2012). However, as it also directly affects the stochastic marginal value of net worth, I will rather refer to a negative realization of this shock as to a bank distress shock.

[12] By applying the law of large numbers, $f(1-\theta_t)$ bankers exit the business in period t. They are replaced by workers who randomly become bankers. As a result, the size of each group remains constant over time.

where the second equality follows from the balance sheet condition. υ_t^k is the marginal gain of holding assets, whereas υ_t is the marginal cost of deposits and can be also interpreted as marginal value of net worth, holding the assets constant.[13] Thus, $\mu_t \equiv \upsilon_t^k - \upsilon_t$ can be interpreted as the marginal gain of expanding assets by one unit financed via deposits.[14] The financial constraint can be written as

$$\mu_t Q_t K_{jt} + \upsilon_t N_{jt} \geq \lambda Q_t K_{jt}. \tag{2.34}$$

Maximizing (2.33) subject to (2.34), under the assumption that the financial constraint always binds, yields the following conditions

$$\mu_t (1 + \psi_{jt}) = \lambda \psi_{jt}, \tag{2.35}$$

and

$$Q_t K_{jt} = \phi_t N_{jt}, \tag{2.36}$$

where ψ_t is the Lagrange multiplier on the incentive constraint. Furthermore, ϕ_t denotes the leverage ratio and is given by

$$\phi_t \equiv \frac{\upsilon_t}{\lambda - \mu_t}. \tag{2.37}$$

Note that holding net worth constant, the constraint binds more tightly, when the intermediary can divert a higher fraction of assets, λ, and the excess value of bank assets is low. With low excess value, the franchise value of the intermediary is lower and the managing banker has a strong incentive to divert funds.

To determine expressions for shadow values of assets and deposits, i.e., time-varying coefficients in the value function, I insert the law of motion of net worth into the Bellman equation, (2.32), and verify that the initial guess for the value function is correct for

$$\upsilon_t^k = E_t \left[\Lambda_{t,t+1} \Omega_{t+1} R_{kt+1} \right], \tag{2.38}$$

$$\upsilon_t = E_t \left[\Lambda_{t,t+1} \Omega_{t+1} \right] R_t, \tag{2.39}$$

and

$$\mu_t \equiv \upsilon_t^k - \upsilon_t = E_t \left[\Lambda_{t,t+1} \Omega_{t+1} \left(R_{kt+1} - R_t \right) \right], \tag{2.40}$$

where Ω_{t+1} is the stochastic marginal value of net worth in period $t+1$, defined in the following way

$$\Omega_{t+1} \equiv 1 - \theta_{t+1} + \theta_{t+1} \left(\upsilon_{t+1} + \phi_{t+1} \mu_{t+1} \right). \tag{2.41}$$

[13]Given bank's asset holdings, an additional unit of net worth leads to savings in borrowing costs.

[14]Note that the marginal values are not bank specific. The underlying assumption is that there are no structural differences across financial intermediaries.

Due to the presence of financial frictions, bankers do not only care about consumption fluctuations of their households (reflected by $\Lambda_{t,t+1}$), but they also consider their funding conditions (reflected by Ω_{t+1}).

Since the leverage ratio does not depend on bank specific factors (see 2.37), we can sum across all individual banks to obtain the aggregate leverage constraint

$$Q_t K_t = \phi_t N_t. \tag{2.42}$$

To obtain the law of motion for net worth of the entire banking system, one has to recognize that it is the sum of net worth of surviving intermediaries, N_{ot}, and net worth of new bankers, N_{nt}

$$N_t = N_{ot} + N_{nt}. \tag{2.43}$$

As already discussed, a fraction $1-\theta_t$ of financial intermediaries exit the market in period t and are replaced by workers who randomly become bankers. New bankers require a start-up capital to be able to attract funds from depositors. Similarly to Gertler and Karadi (2011), I assume that the household transfers a fraction, $\frac{\omega}{1-\theta_t}$, of the value of assets of exiting intermediaries. Hence,

$$N_{nt} = \omega Q_t K_{t-1}. \tag{2.44}$$

The net worth of the remaining θ_t bankers is given by

$$N_{ot} = \theta_t \left[\left(R_{kt} - R_{t-1} \right) \phi_{t-1} + R_{t-1} \right] N_{t-1}. \tag{2.45}$$

2.3.4 Aggregate Resource Constraint and Monetary Policy

Final output is divided between consumption and investment

$$Y_t = C_t + I_t + \frac{\eta}{2} \left(\frac{I_{nt} + I_{SS}}{I_{nt-1} + I_{SS}} - 1 \right)^2 \left(I_{nt} + I_{SS} \right). \tag{2.46}$$

The law of motion for capital is given by

$$K_t = \xi_t K_{t-1} + I_{nt}. \tag{2.47}$$

Following Gertler and Karadi (2011), I assume that the monetary policy is described by the following Taylor-rule

$$1 + i_t = (1 + i_{t-1})^{\rho_i} \left[(1 + i_{SS}) \Pi_t^{\kappa_\pi} \left(\frac{X_t}{X} \right)^{\kappa_x} \right]^{1-\rho_i}, \tag{2.48}$$

where i_t denotes the net nominal interest rate with a deterministic steady state value of i_{SS}. $\rho_i \in (0,1)$ is the smoothing parameter, and parameters κ_π and κ_x

capture the responsiveness of nominal interest rate to movements in inflation and markup, respectively. Markup fluctuations serve as a proxy for movements in output gap, defined as a deviation of actual output from its flexible-price level.

Finally, the nominal interest rate affects the real economy via the Fisher equation

$$1 + i_t = R_t E_t \left[\Pi_{t+1}\right]. \tag{2.49}$$

2.3.5 Shock Processes

There are three first-moment shock processes present in the model: technology, A_t, capital quality, ξ_t, and a disturbance to the survival probability of financial intermediaries, ϑ_t:

$$A_t = (1 - \rho_A) + \rho_A A_{t-1} + e^{\bar{\sigma}^A} \epsilon_t^A, \tag{2.50}$$

$$\xi_t = (1 - \rho_\xi) + \rho_\xi \xi_{t-1} + e^{\bar{\sigma}^\xi} \epsilon_t^\xi, \tag{2.51}$$

and

$$\vartheta_t = (1 - \rho_\theta) + \rho_\theta \vartheta_{t-1} + e^{\sigma_{t-1}^\theta} \epsilon_t^\theta, \tag{2.52}$$

with ρ_j and σ^j, $j = \{A, \xi, \theta\}$, referring to autocorrelation coefficient and log standard deviation of the corresponding stochastic disturbance, respectively.

Financial uncertainty is introduced into the model by assuming that the volatility of shocks to the survival probability of bankers varies over time. The corresponding second-moment process is given by

$$\sigma_t^\theta = (1 - \rho_{\sigma^\theta})\bar{\sigma}^\theta + \rho_{\sigma^\theta} \sigma_{t-1}^\theta + \tau_{\sigma^\theta} \epsilon_t^{\sigma^\theta}, \tag{2.53}$$

where $\bar{\sigma}^\theta$ refers to the unconditional mean level of σ_t^θ, ρ_{σ^θ} is again the persistence parameter, and τ_{σ^θ} is the standard deviation of volatility innovations. Standard deviations of the remaining two level shocks are assumed to be constant in the baseline model.[15]

All innovations are independent and follow a symmetric distribution with bounded support, zero mean and unit variance. The first-moment processes are specified in levels, rather than logs to prevent changes in volatility from affecting their mean values through a Jensen's inequality effect.

[15] In section 2.5.3, I extend the model and discuss responses to stochastic volatility shocks associated with total factor productivity. The goal of this exercise is to detect differences in propagation mechanisms of different types of uncertainty. Moreover, it enables a comparison with the literature investigating the propagation of real uncertainty shocks under financial frictions (e.g., Bonciani and Van Roye, 2016).

2.4 Solution Method and Calibration

Due to nonlinearities present in the model, an exact solution is not feasible and thus one must rely on approximation methods. This section describes the technique used to solve the model and discusses the calibration underlying the analysis conducted in this paper.

2.4.1 Perturbation Methods

As shown by Fernández-Villaverde et al. (2011), at least a third-order approximation is necessary to investigate impulse responses to volatility shocks. I use the nonlinear moving average perturbation developed by Lan and Meyer-Gohde (2013). This technique has three advantages in a setup with time-varying volatility. First, it provides cumulative uncertainty correction, contrary to the state space methods, providing one-step ahead correction. Second, it starts the approximation at the stochastic steady state.[16] Finally, it delivers stable nonlinear impulse responses and simulations and thus no pruning algorithm (Kim et al., 2008; Andreasen et al., 2017) is necessary.

To explain the method, I will cast the underlying model into a general form

$$E_t \left[f \left(y_{t+1}, y_t, y_{t-1}, \epsilon_t \right) \right] = 0, \tag{2.54}$$

where $f : \mathbb{R}^{ny} \times \mathbb{R}^{ny} \times \mathbb{R}^{ny} \times \mathbb{R}^{ne} \rightarrow \mathbb{R}^{ny}$ is assumed to be analytic, $y_t \in \mathbb{R}^{ny}$ stands for the vector containing both endogenous and exogenous variables, and $\epsilon_t \in \mathbb{R}^{ne}$ is a vector of zero-mean iid shocks. The nonlinear moving average represents a solution to (2.54) as a direct mapping of the history of shocks to model variables, i.e.,

$$y_t = y(\sigma, \epsilon_t, \epsilon_{t-1}, ..), \tag{2.55}$$

where σ is the perturbation parameter, governing the size of uncertainty in the model. $\sigma = 0$ implies a deterministic setup, whereas $\sigma = 1$ refers to the fully stochastic world. The third-order Taylor approximation of this policy function, given a symmetric distribution of shocks and $\sigma = 1$, is given by

$$\begin{aligned} y_t^{(3)} = {} & y_{SS} + \frac{1}{2} y_{\sigma^2} + \sum_{i=0}^{\infty} \left(y_i + \frac{1}{2} y_{\sigma^2 i} \right) \epsilon_{t-i} + \frac{1}{2} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} y_{i_1,i_2} \left(\epsilon_{t-i_1} \otimes \epsilon_{t-i_2} \right) \\ & + \frac{1}{6} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \sum_{i_3=0}^{\infty} y_{i_1,i_2,i_3} (\epsilon_{t-i_1} \otimes \epsilon_{t-i_2} \otimes \epsilon_{t-i_3}), \end{aligned} \tag{2.56}$$

[16] The stochastic or risky steady state is defined as a fixed point in the absence of past and present shocks but taking into account the likelihood of shocks in the future. See, e.g., Coeurdacier et al. (2011), Juillard (2011) and Meyer-Gohde (2014).

where y_{SS} denotes the deterministic steady state of the model and y_i, y_{i_1,i_2}, y_{i_1,i_2,i_3}, y_{σ^2}, $y_{\sigma^2,i}$ refer to partial derivatives of the policy function evaluated at the deterministic steady state. The expression $y_{SS}+\frac{1}{2}y_{\sigma^2}$ corresponds to the third-order accurate stochastic steady state.[17] Moreover, $y_{\sigma^2,i}$ adjusts the approximate responses of endogenous variables to shock realizations for the risk of future disturbances.

2.4.2 Calibration

My aim is to use the model to provide a general qualitative assessment of the empirical evidence presented above. Therefore, for all parameters I choose values previously used in the literature, with Gertler and Karadi (2011) being the major source. Table 2.1 reports the benchmark calibration.

The inverse of the Frish elasticity of labor supply, φ, is set to 0.276, whereas parameter governing habit formation, h, is 0.815. The choice of the value for χ ensures that labor supply in the deterministic steady state equals 0.33.

The effective capital share, α, is 0.33. The elasticity of marginal depreciation with respect to the utilization rate, δ_2, is set to 7.2. The remaining parameters of the depreciation function, δ_0 and δ_1, are chosen such that the depreciation rate and the utilization of capital are respectively equal to 0.025 and 1 in the deterministic steady state. Following Born and Pfeifer (2014) I set the elasticity of substitution between intermediate goods, ϱ, to 10, implying a markup of 11 % in the deterministic steady state. Moreover, the price rigidity parameter, θ_{calvo}, takes the value of 0.779, resulting in an average price duration of four and a half quarters.

θ is set to 0.955 implying an average horizon of bankers of almost 6 years. Following Gertler and Karadi (2011), λ and ω and are chosen to hit the following two targets: an interest rate spread of one hundred basis points per year and banks' leverage ratio of four in the deterministic steady state.

The autocorrelation parameters of level shocks are set in accordance with Gertler and Karadi (2011) and Afrin (2017). Moreover, the unconditional mean of their respective log standard deviations is normalized to ln(0.01). To parametrize the second-moment process for the survival probability of bankers, I assume, in accordance with Ludvigson et al. (2015), that fluctuations in financial uncertainty are exogenous. Given this assumption, the financial uncertainty measure corresponds directly to the stochastic volatility process (2.53). To see this, consider one-step ahead forecast error

$$U_\theta \equiv \sqrt{E_t\left[\left(\theta_{t+1} - E_t\left[\theta_{t+1}\right]\right)^2\right]} = e^{\sigma_t^\theta}. \tag{2.57}$$

[17] As shown by Andreasen (2012), the third-order constant term, y_{σ^3}, corrects the approximation for the skewness of the shocks. Since I assume symmetric distributions, it is equal to zero and thus omitted from (2.56).

Parameter		Value	Justification
Household			
Discount factor	β	0.99	Gertler & Karadi (2011)
Habit parameter	h	0.815	Gertler & Karadi (2011)
Inverse of intertemporal elasticity of substitution	γ	1	Gertler & Karadi (2011)
Inverse Frish elasticity of labor supply	φ	0.276	Gertler & Karadi (2011)
Relative utility weight of labor	χ	3.1870	$L_{SS} = \frac{1}{3}$
Nonfinancial Firms			
Effective capital share	α	0.33	Gertler & Karadi (2011)
Inverse elasticity of net investment w.r.t price of capital	η	1.5	Gertler & Kiyotaki (2010)
Elasticity of marginal depreciation w.r.t. U_t	δ_2	7.2	Gertler & Karadi (2011)
Depreciation rate parameter 1	δ_1	0.0376	$U_{SS} = 1$
Depreciation rate parameter 2	δ_0	0.0204	$\delta_{SS} = 0.025$
Elasticity of substitution	ϱ	10	Born & Pfeifer (2014)
Calvo parameter	θ_{calvo}	0.779	Gertler & Karadi (2011)
Price indexation	γ_p	0.241	Gertler & Karadi (2011)
Financial Sector			
Survival rate of bankers	θ	0.955	Assumed
Divertable fraction	λ	0.3196	$\phi_{SS} = 4$ & $R_{k,SS} - R_{SS} = 0.0025$
Starting-up transfer	ω	0.0065	$\phi_{SS} = 4$ & $R_{k,SS} - R_{SS} = 0.0025$
Taylor Rule			
Interest Rate Smoothing Parameter	ρ_i	0.8	Gertler & Karadi (2011)
Inflation coefficient in Taylor rule	κ_π	1.5	Gertler & Karadi (2011)
Output coefficient in Taylor rule	κ_x	-0.125	Gertler & Karadi (2011)
Shock Processes			
Persistence - TFP	ρ_A	0.95	Gertler & Karadi (2011)
Persistence - Capital quality	ρ_ξ	0.66	Gertler & Karadi (2011)
Persistence - Survival probability	ρ_θ	0.9	Afrin (2016)
Persistence - Stochastic volatility	ρ_{σ^θ}	0.9	Estimated
Unconditional mean of log-S.D.	$\bar{\sigma}^i$	ln(0.01)	Normalization
S.D. - Stochastic volatility	τ_{σ^θ}	0.045	Estimated

Table 2.1: Baseline Calibration

Taking the natural logarithm of (2.57) yields

$$ln(U_\theta) = \sigma_t^\theta. \tag{2.58}$$

Therefore, I can use the financial uncertainty series to directly estimate the parameters of the stochastic volatility process for the survival probability of banks. The estimated autocorrelation parameter is 0.9, whereas the implied standard deviation is 0.045.

2.5 Results

In this section, I trace out aggregate effects of financial uncertainty shocks in the underlying framework. Then, I conduct sensitivity analysis by modifying the calibration in several ways to assess the importance of individual features of the model. Finally, I discuss differences in transmission mechanisms of financial and macroeconomic uncertainty. To this end, I extend the model by introducing time-varying volatility of total factor productivity shocks serving as a proxy for macroeconomic uncertainty.

2.5.1 Financial Uncertainty Shocks

Figure 2.3 depicts the response of the model economy to one standard deviation increase in financial uncertainty. The responses represent third-order accurate (percentage) deviations of model variables from their respective stochastic steady states.

An adverse financial uncertainty shock implies an increase in the expected stochastic marginal value of net worth in the subsequent period. This leads to a rise in shadow costs of deposits today which in turn reduces the franchise value of intermediaries and thus diminishes households' demand for riskless bonds, i.e., deposits. As a consequence, the real return on deposits increases. Simultaneously, current consumption declines given that expected future consumption affects today's marginal utility via the internal habit formation. Tighter funding conditions force banks to reduce their lending and as a consequence investment declines. Lower investment leads to a reduction in the price of capital which deteriorates the financial position of banks even further (*financial accelerator*). In particular, a lower price of capital translates into a lower rate of return on bank investment and net worth of the banking system diminishes. In addition, higher uncertainty leads to an increase in markups (see Born and Pfeifer, 2014 and Fernández-Villaverde et al., 2015). This precautionary pricing behavior contributes to the decline in the aggregate demand. The (nominal) price of intermediate goods, i.e., marginal costs of retailers, falls and thus the inflation rate drops despite a higher average markup.

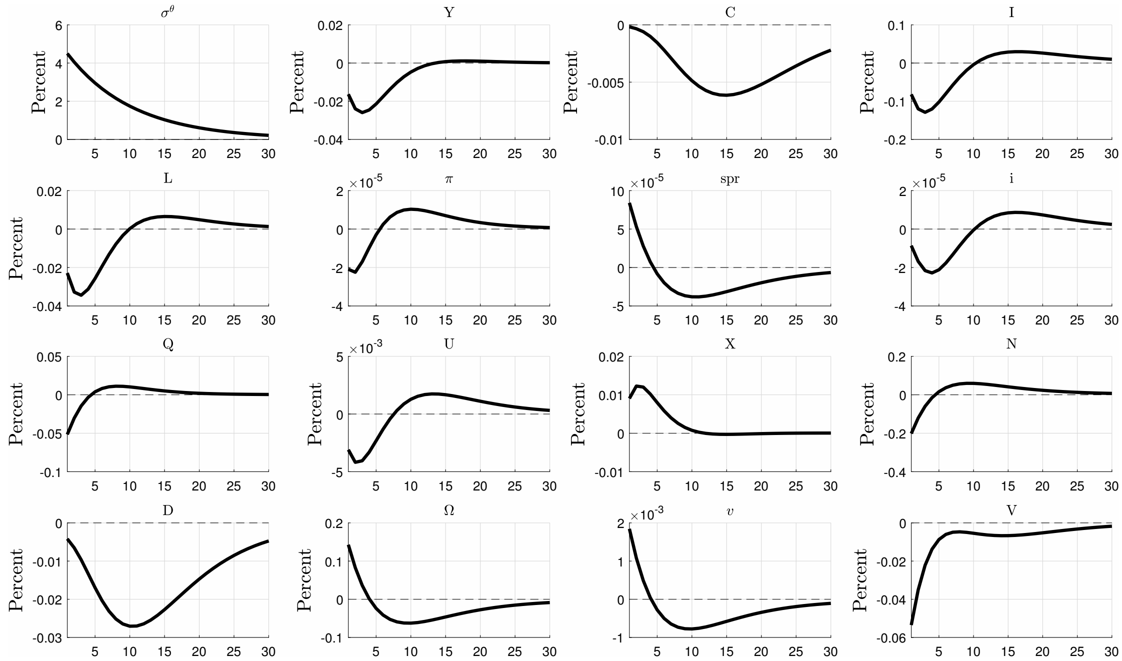


Figure 2.3: Dynamic consequences of financial uncertainty shocks. *Level shocks are held constant. Horizontal axes indicate quarters. All responses are in percent, except for inflation (π), the nominal interest rate (i), the shadow price of deposits (v), and the risk premium (spr), defined as the expected future rate of return on bank assets relative to the rate on deposits.*

Finally, falling aggregate demand for goods implies a reduction of hours worked and a decline in production level. The peak response of output occurs after three quarters and amounts to a drop of about 0.026 % of production in the stochastic steady state. The subsequent recovery is followed by a rebound, which is caused by the fact that capital stock needs to be replenished when the shock dies out. Hence, the model can generate the *volatility overshoot* that can be found in the data.

Finally, note that I consider only a 4.5 % increase in financial uncertainty. However, we can observe much stronger fluctuations in the chosen sample (see figure 2.1). In 2009, the measure by (Ludvigson et al., 2015) reached the maximum value, corresponding to 150 % of its pre-crisis mean. If we view this large positive deviation as an outcome of a single realization of the financial uncertainty shock, the underlying model implies a reduction in GDP of roughly 0.3 %.

2.5.2 Dissecting the Transmission Channels of Financial Uncertainty

This section assesses the importance of different model features for the transmission mechanism of financial uncertainty. In particular, I discuss the role of nominal rigidities, internal habit formation, variable capital utilization and monetary policy.

Nominal Rigidity

To understand the role of price rigidities for the transmission of uncertainty shocks, it is useful to inspect the clearing condition for the labor market (Basu and Bundick, 2017; Born and Pfeifer, 2014)

$$\frac{1}{X_t} A_t \left(1-\alpha\right) \left(\xi_t K_{t-1}\right)^{\alpha} U_{Ct} = \chi L_t^{\varphi+\alpha}. \tag{2.59}$$

A rise in volatility results in an increase in markups which in turn diminishes the demand for goods and consequently hours worked and output.

As explained by Fernández-Villaverde et al. (2015) and Born and Pfeifer (2017), a rise in markups following an uncertainty shock is caused by the precautionary pricing behavior of retailers. More specifically, a firm updating its price in a more uncertain environment has an incentive to charge a higher markup because higher prices partly compensate for a low quantity sold. On the other hand, lower prices imply a higher demand but the revenue per unit sold is lower. This diminishes retailers' profits. Because of the nonlinear nature of the pricing behavior, firms prefer higher prices.

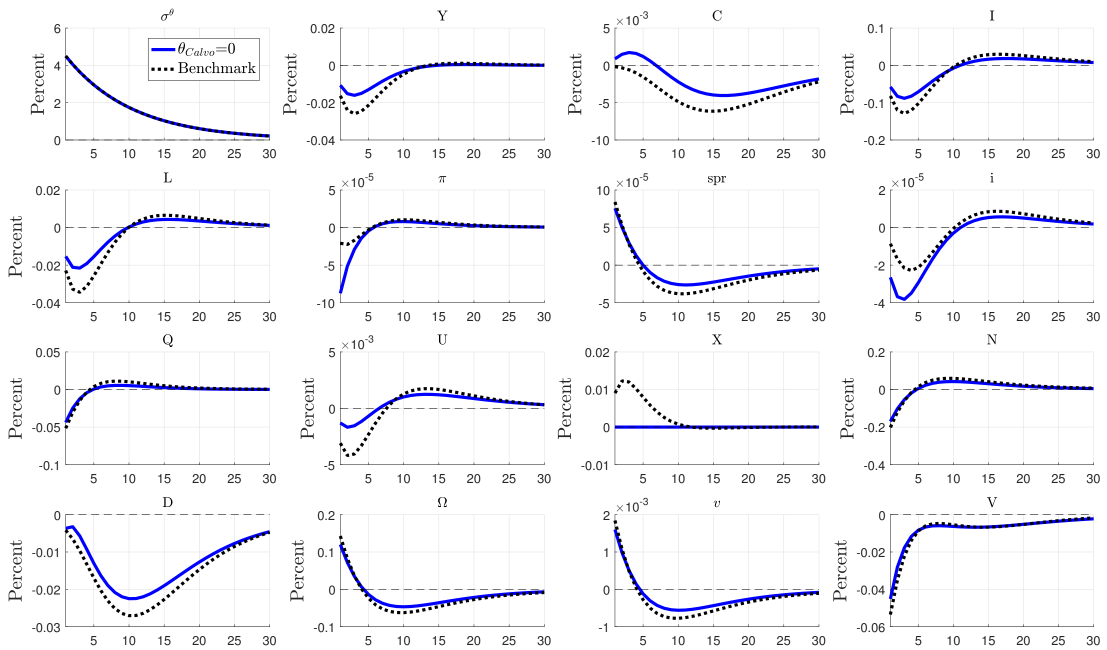


Figure 2.4: Assessing the importance of nominal rigidities. *Dynamic consequences of a one standard deviation increase in the volatility of shocks to the survival probability of banks. Level shocks are held constant. Horizontal axes indicate quarters. All responses are in percent, except for inflation (π), the nominal interest rate (i), the shadow price of deposits (v), and the risk premium (spr), defined as the expected future rate of return on bank assets relative to the rate on deposits.*

Figure 2.4 compares impulse responses to a financial uncertainty shock under the benchmark calibration with their counterparts under flexible prices. The precautionary pricing motive amplifies the effects of an increase in the volatility of shocks to the survival probability of bankers. In addition, similar to the model employed by Basu and Bundick (2017), nominal rigidities are necessary to replicate empirically observed co-movement among aggregate quantities in the underlying framework. However, the next section shows that sticky prices alone are not sufficient to generate this outcome. They have to be accompanied by the internal habit formation.

Internal Habit Formation

To quantify the importance of habit formation, I remove this feature by setting $h = 0$ and compare the implied model responses to the benchmark calibration. Figure 2.5 presents the results of this exercise. Without habit formation, the macroeconomic effects of an adverse uncertainty shock are weaker. More importantly, however, the model can no longer generate the co-movement among macroeconomic aggregates. To understand this outcome, note that internal habit formation implies that the marginal utility of consumption today depends on the expected future consumption stream, i.e., $U_{Ct} = (C_t - hC_{t-1})^{-\gamma} - \beta h E_t \left[(C_{t+1} - hC_t)^{-\gamma}\right]$. As shown by figure 2.5, consumption has to fall eventually due to lower production, even if h is set to zero. Hence, under internal habit formation, the household internalizes the habitual nature of consumption and starts reducing its stock of habit already in the current period to avoid large jumps in its marginal utility. Furthermore, note that a rise in uncertainty depresses, *ceteris paribus*, the marginal utility of today's consumption via the Jensen's inequality. Therefore, the household has an incentive to reduce its labor supply, as it derives less utility from a given wage. The implied reduction in hours worked is consistent with lower aggregate demand in the New Keynesian setup. In contrast, without internal habit formation, the household simply consumes additional resources that become available due to the reduction in bank deposits. Thus, aggregate consumption rises on impact despite a lower production level. As a result, the negative effect of higher financial uncertainty on economic activity is smaller.

Why are both sticky prices and internal habit formation necessary to generate a fall in consumption in response to an adverse financial uncertainty shock? A rise in financial uncertainty is transmitted to the real economy via tighter financial conditions. The resulting drop in deposits stimulates consumption, *ceteris paribus*. Under benchmark calibration, two conditions must be fulfilled to prevent an increase in the aggregate consumption. First, the decline in production must be

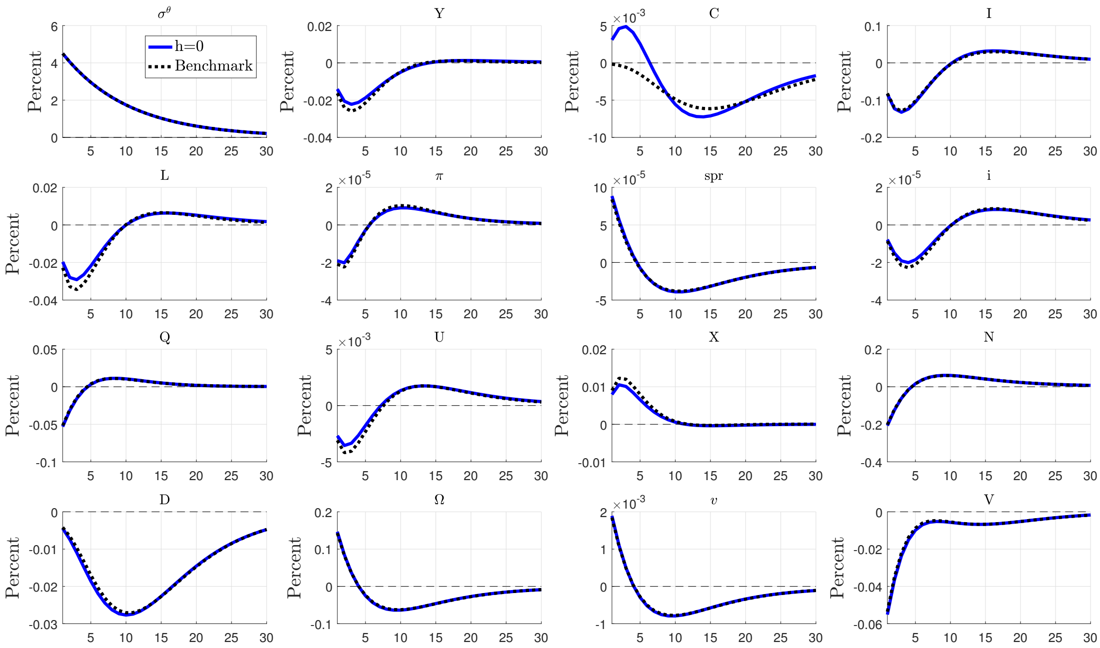


Figure 2.5: Assessing the importance of internal habit formation. *Dynamic consequences of a one standard deviation increase in the volatility of shocks to the survival probability of banks. Level shocks are held constant. Horizontal axes indicate quarters. All responses are in percent, except for inflation (π), the nominal interest rate (i), the shadow price of deposits (υ), and the risk premium (spr), defined as the expected future rate of return on bank assets relative to the rate on deposits.*

sufficiently large (due to nominal rigidities). Second, households must strongly dislike consumption fluctuations (due to habit formation).[18]

The role of internal habit formation as a propagator of macroeconomic uncertainty has been documented by Leduc and Liu (2016). The authors show that internal habit formation amplifies the effects of uncertainty shocks on unemployment in a framework with search frictions in the labor market. The reason for this is that the presence of habit induces a larger drop in the present value of a job match. This finding stands in contrast to the results of Born and Pfeifer (2014) who employ a New Keynesian model without search and financial frictions. In their framework, habit formation dampens the effects of uncertainty because adjustment in consumption is more costly in terms of utility. My results are in line with Leduc and Liu (2016) and confirm that internal habit formation can amplify uncertainty shocks in the presence of other frictions. In the underlying framework, this additional friction is the agency problem in the financial sector. In other

[18]In contrast, internal habit formation is not necessary to generate a drop in consumption in case of macroeconomic uncertainty modeled as the stochastic volatility of total factor productivity. The reason is that nominal rigidities are relatively more important for transmitting macroeconomic uncertainty shocks. See section 2.5.3.

words, endogenous leverage constraint is (indirectly) an important model feature for generating a decline in consumption in response to an increase in financial uncertainty.

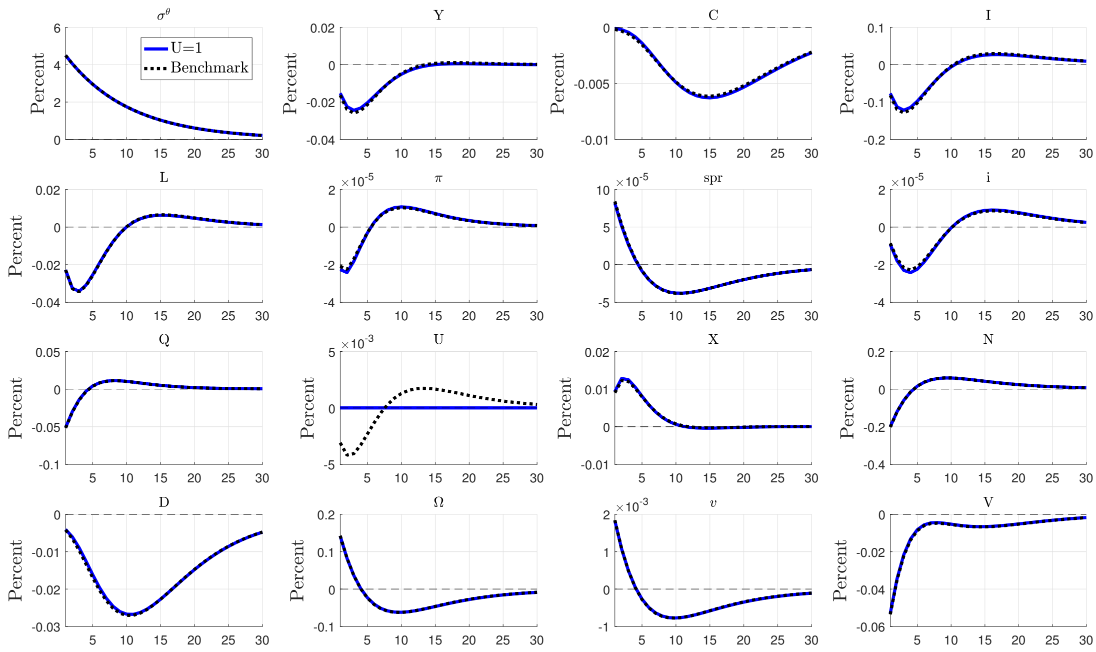


Figure 2.6: Assessing the importance of variable capital utilization. *Dynamic consequences of a one standard deviation increase in the volatility of shocks to the survival probability of banks. Level shocks are held constant. Horizontal axes indicate quarters. All responses are in percent, except for inflation (π), the nominal interest rate (i), the shadow price of deposits (v), and the risk premium (spr), defined as the expected future rate of return on bank assets relative to the rate on deposits.*

Variable Capital Utilization

To assess the importance of variable capital utilization, I compare the baseline framework with the model, where capital utilization is fixed at its deterministic steady-state level, i.e., $U_t = 1 \quad \forall t$. Figure 2.6 presents the results of this exercise. The effects of financial uncertainty shocks are stronger if we allow for variable capital utilization. To understand this outcome, consider equation (2.11) which, using the specification of the depreciation function, can be rewritten as

$$\alpha \frac{1}{X_t} A_t \left(\frac{L_t}{\xi_t K_{t-1}} \right)^{1-\alpha} = \delta_1 U_t^{\delta_2 + 1 - \alpha}. \tag{2.60}$$

An adverse financial uncertainty shock implies a higher economy-wide markup and lower labor supply which in turn depresses the marginal product of capital utilization, as reflected by a lower left-hand side of (2.60). Firms respond by lowering the capital utilization to reduce the (marginal) depreciation of capital. Finally, lower U_t leads to a decline in income and thereby depresses the aggregate demand, which ultimately results in a larger drop in production. Note, however, that the amplification effect of variable capital utilization is small. With fixed capital utilization, the peak response of GDP amounts to a reduction of 0.024% of the stochastic steady state production rather than 0.026 % realized in the benchmark model.

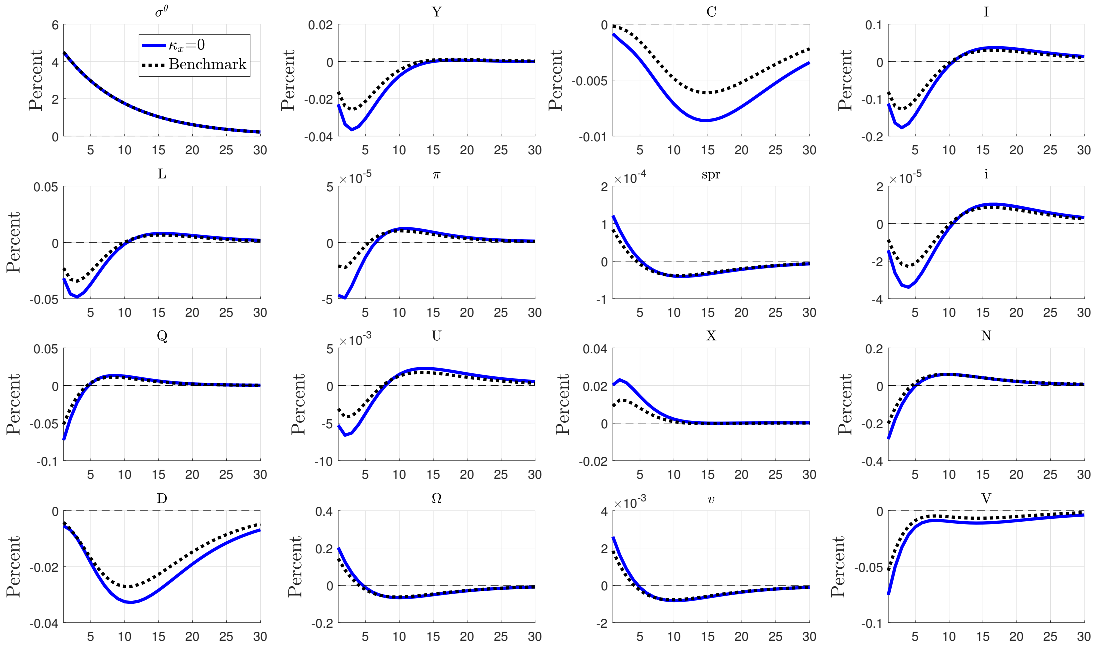


Figure 2.7: Assessing the role of monetary policy. *Dynamic consequences of a one standard deviation increase in the volatility of shocks to the survival probability of banks. Level shocks are held constant. Horizontal axes indicate quarters. All responses are in percent, except for inflation (π), the nominal interest rate (i), the shadow price of deposits (v), and the risk premium (spr), defined as the expected future rate of return on bank assets relative to the rate on deposits.*

Monetary Policy

An adverse financial uncertainty shock implies lower inflation rate and output gap, proxied by higher economy-wide markup. The central bank responds by lowering the interest rate, which mitigates the negative effect of financial uncertainty. To assess the effectiveness of monetary policy, I shut off the output feedback of the Taylor rule, i.e., $\kappa_x = 0$ (see figure 2.7).[19] Since the monetary authority no longer counteracts the fall in output gap, the cut in the interest rate is smaller, *ceteris*

[19] Varying other parameters of the Taylor rule, κ_π and ρ_i, yields similar results. In particular, both stronger inflation response of the monetary authority and weaker interest smoothing dampen the negative effects of financial uncertainty.

paribus. This implies a larger increase in the shadow cost of deposits and leads eventually to a stronger fall in investment. Simultaneously, less aggressive monetary policy strengthens the precautionary pricing motive of retailers because even higher markups are necessary to compensate for a potentially larger decline in demand. As a result, the output drop caused by an adverse financial uncertainty shock is twice as large, compared to the benchmark calibration. Finally, note that, due to general equilibrium effects and the associated fall in marginal costs of retailers, the decrease of inflation rate and nominal interest rate is more pronounced.

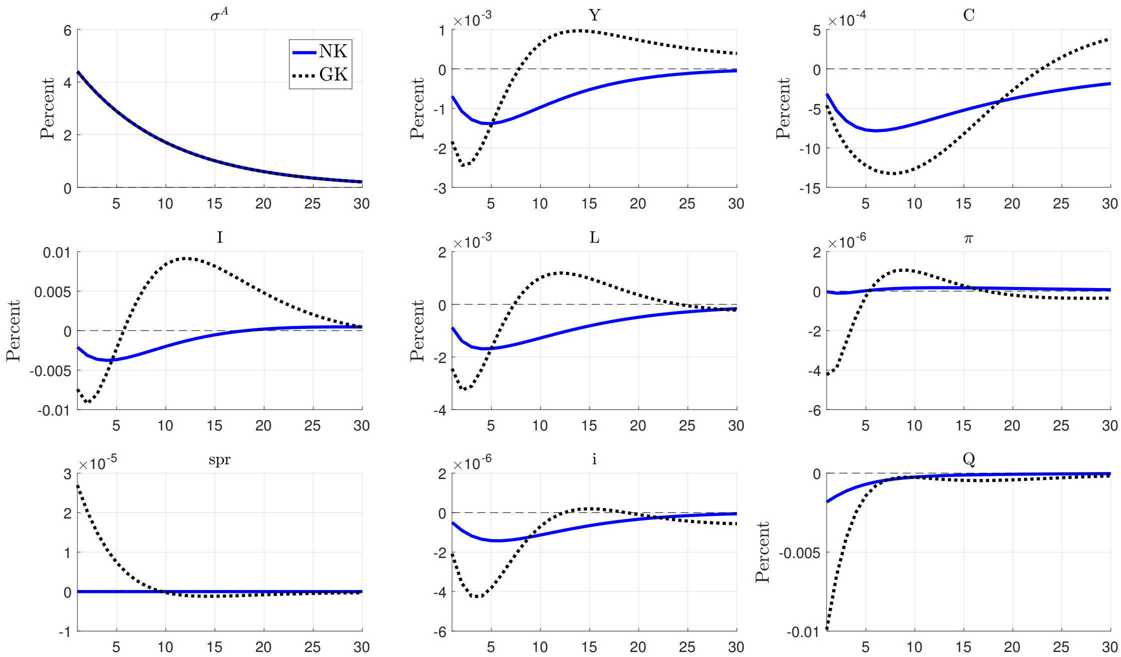


Figure 2.8: Dynamic consequences of macroeconomic uncertainty. *GK corresponds to the baseline model, whereas NK denotes the New Keynesian model without financial frictions. Level shocks are held constant. Horizontal axes indicate quarters. All responses are in percent, except for inflation (π), the nominal interest rate (i), and the risk premium (spr), defined as the expected future rate of return on bank assets relative to the rate on deposits.*

2.5.3 Comparison with Macroeconomic Uncertainty

In this section, I extend the model by introducing time-varying volatility of total factor productivity shocks serving as a proxy for macroeconomic uncertainty. For the sake of comparison, I calibrate the corresponding second-moment process by choosing the same parameter values as in the case of financial uncertainty. Another reason for following this calibration strategy is the fact that the literature provides mixed evidence on whether fluctuations in macroeconomic uncertainty represent exogenous shocks or an endogenous response to fundamentals. In particular, Ludvigson et al. (2015) provide evidence that movements in macroeconomic uncertainty are rather a consequence of changes in fundamentals and financial uncertainty. On the other hand, Caldara et al. (2016), exploiting a different identification strategy, find that uncertainty associated with real variables has significant effects on economic activity.

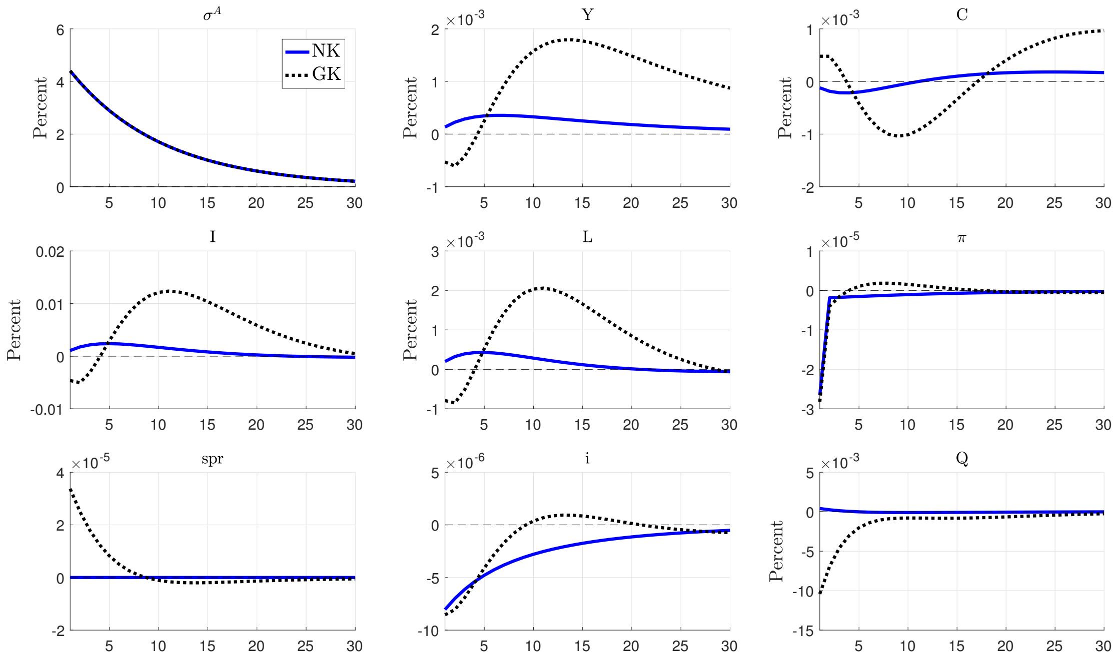


Figure 2.9: Dynamic consequences of macroeconomic uncertainty in the absence of nominal rigidities, habit formation and variable capital utilization. *GK corresponds to the baseline model, whereas NK denotes the New Keynesian model without financial frictions. Level shocks are held constant. Horizontal axes indicate quarters. All responses are in percent, except for inflation (π), the nominal interest rate (i), and the risk premium (spr), defined as the expected future rate of return on bank assets relative to the rate on deposits.*

Figure 2.8 depicts the response of the model economy to an adverse realization of the macroeconomic uncertainty shock and compares it to the dynamics in a framework without financial frictions.[20] First, fluctuations in macroeconomic and financial uncertainty have qualitatively similar effects on economic activity. Second, similar to the models used by Alfaro et al. (2018) and Bonciani and Van Roye (2016), the Gertler-Karadi framework employed in this study exhibits the "finance-uncertainty multiplier". Note, however, that the effects of macroeconomic uncertainty are amplified only in the first year and the economy recovers much faster, compared to the New Keynesian model without financial intermediaries.[21]

[20]Note that I cannot repeat this exercise for financial uncertainty shocks, as financial uncertainty has no effects in the absence of financial frictions.

[21]This outcome is consistent with the results of Gertler and Karadi (2011) for the technology level shock.

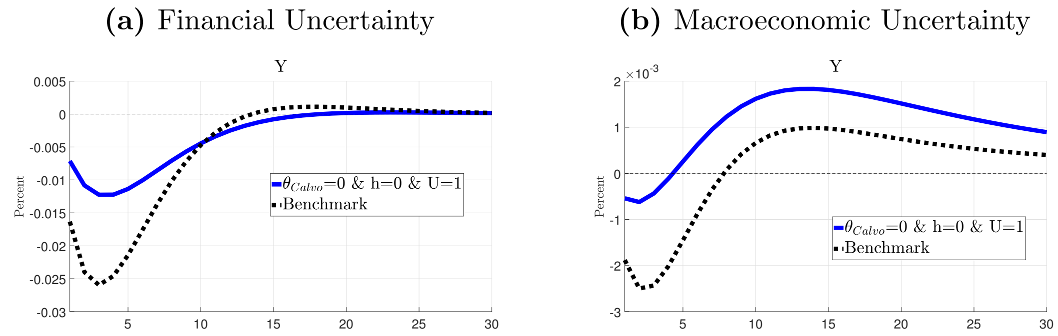


Figure 2.10: Macroeconomic versus financial uncertainty. *Level shocks are held constant. Horizontal axes indicate quarters. All responses are in percent.*

The reason for this fast recovery is a quick accumulation of net worth of the banking sector combined with cheaper investment goods.

Figure 2.9 assesses the role of nominal rigidities, internal habit formation and variable capital utilization for the transmission mechanism of macroeconomic uncertainty. If these features are eliminated from the model, the negative effect of macroeconomic uncertainty becomes much smaller and short-lived. GDP recovers already after one year and the economy experiences a persistent boom. The initial decline in the production is caused by tightening of the financial constraint which prevents households from making deposits with banks. In the absence of financial frictions, a rise in macroeconomic uncertainty stimulates precautionary saving (Carroll and Kimball, 2006) and thereby generates an immediate increase in the production level.

Finally, figure 2.10 compares the importance of nominal rigidities, internal habit formation and variable capital utilization for dynamic consequences of financial and macroeconomic uncertainty. First, note that, even in the presence of these features, economic fluctuations caused by a second-moment TFP shock are less pronounced, compared to the effects of financial uncertainty. In particular, the peak decline in GDP under macroeconomic uncertainty is more than ten times smaller. Second, in contrast to financial uncertainty, shocks to the volatility of technology disturbances are propagated mainly through nominal and real rigidities. If these features are eliminated from the model, the peak drop of GDP in response to the macroeconomic uncertainty shock is reduced by roughly 75 %. In contrast, the peak effect of financial uncertainty falls by 50 %.

2.6 Conclusion

This paper fills the gap in the DSGE literature on uncertainty shocks by shedding more light on the macroeconomic effects of uncertainty originating in financial markets. The goal is to determine the channels through which financial uncertainty shocks affect the real economy. To this end, I provide empirical evidence that financial uncertainty has a significant impact on the real activity. To explain empirical findings, I extend the DSGE framework developed by Gertler and Karadi (2011) by introducing time-varying volatility of financial shocks. The dynamics generated by the model are in line with their empirical counterparts. In particular, a rise in financial uncertainty leads to an increase in the risk premium and to a reduction in aggregate quantities. Finally, I conduct a series of experiments to uncover the main propagators of financial uncertainty. In the underlying setup, the key role is played by the endogenous leverage constraint faced by bankers. Specifically, due to an increase in financial uncertainty, households provide less funding to financial intermediaries triggering the financial accelerator mechanism. Finally, nominal rigidities and internal habit formation act as additional amplification mechanisms for financial uncertainty shocks and, more importantly, are necessary to generate co-movement among macroeconomic aggregates.

The analysis conducted in this paper provides evidence that financial uncertainty dampens economic activity. This finding raises the question as to what extent economic policy and/or macroprudential regulation can mitigate the adverse effects of uncertainty shocks. While the current study briefly discusses the role of monetary policy for the transmission mechanism, a rigorous normative analysis is missing. In addition, as financial uncertainty is propagated mainly through endogenous tightening of leverage constraint, financial regulation may be exceptionally effective in mitigating its negative effects. As these questions require a quantitative evaluation of the model, they cannot be addressed in this paper and are left to future research.

Another extension of my analysis would be to introduce an occasionally binding zero lower bound to (a simplified version of) the underlying model. As monetary policy is constrained in this case, the effects of financial uncertainty are likely to be more significant. Hence, the goal of this exercise would be to assess the importance of nonlinearities arising from the interaction between the leverage constraint and the zero lower bound.

Chapter 3

Solving DSGE Portfolio Choice Models with Asymmetric Countries

In this essay, I combine bifurcation theory and the nonlinear moving average approximation to solve asymmetric DSGE models with endogenous portfolio choice. The proposed method can be viewed as a generalization of the workhorse routine developed by Devereux and Sutherland (2010, 2011). Contrary to their approach, it can be used to obtain higher-order approximation of gross asset holdings capturing the direct effect of the presence of uncertainty on agents' portfolios. The risk-adjusted net and gross asset positions are shown to be in line with the global solution. Hence, the proposed method is able to account for asymmetries, which may lead to an accuracy improvement in terms of Euler equation errors relative to the Devereux-Sutherland procedure.

Keywords: Country Portfolios, Solution Method, Asymmetric Countries
JEL Classification Numbers: E44, F41, G11

3.1 Introduction

The explosion of cross-border gross asset positions over the last two decades, documented by Lane and Milesi-Ferretti (2001, 2007), has drawn researchers' attention to international portfolios. Obstfeld (2007) writes

> ***3. Wanted: A general-equilibrium portfolio-balance model***
> *In light of these important implications of international portfolios, it is imperative to understand how investors make asset allocation decisions for different asset classes across countries and currencies. [...] the need*

for such an approach [i.e. general equilibrium approach] has become acute as asset trade has expanded.

Investigating portfolio choice in a general equilibrium model under the assumption of incomplete markets is challenging, as such models are associated with indeterminacy in a certainty equivalent environment. As a consequence, standard local approximation methods cannot be applied. Furthermore, global techniques suffer from the curse of dimensionality and cannot be employed in models with a richer state space. In response to these problems, new solution methods have been developed.[1]

The workhorse routine to solve a DGSE model with portfolio choice is a perturbation-based method developed by Devereux and Sutherland (2010, 2011), henceforth *DS*. It is fast, easy to implement and can be applied to a variety of models. Rabitsch et al. (2015) show that *DS* performs well in comparison to global solution methods, but they also find some scope for improvement in a setup with asymmetric countries. In particular, they document that *DS* 1) does not capture the direct effect of uncertainty on portfolio holdings[2] and 2) approximates the policy function around net foreign positions equal to zero, even in the presence of cross-country differences. Moreover, Rabitsch et al. (2015) show that iterative procedure proposed by Devereux and Sutherland (2009) to update net foreign position deteriorates the accuracy of the approximation. As a result, applying *DS* may yield unsatisfactory results if, for instance, the focus lies on gross capital flows between developed and emerging market countries.

The aim of this paper is to improve upon the two shortcomings of *DS*. To this end, it combines bifurcation theory and the nonlinear moving average approximation (Lan and Meyer-Gohde, 2013, 2014a). The use of bifurcation methods overcomes the problem of indeterminacy of portfolio holdings, whereas policy functions approximated with the nonlinear moving average include a cumulative risk correction that can be related to the stochastic steady state. The proposed technique can be viewed as a generalization of *DS*. That is, it yields the same results up to first order of accuracy but can also be used to compute higher-order approximations accounting for the presence of uncertainty.

To evaluate the proposed solution method, I solve a real two-country endowment economy with portfolio choice. Countries are characterized by different degrees of economic uncertainty, i.e., volatility of endowment shocks differs across countries. The technique implies an uncertainty correction of gross and net asset positions that is in line with the solution provided by global methods. This indicates that asymmetries present in the model are captured already at the starting point of

[1]See, among others, Judd et al. (2002), Devereux and Sutherland (2010, 2011), Tille and Van Wincoop (2010), Evans and Hnatkovska (2012), Stepanchuk and Tsyrennikov (2015), and Reiter (2015).

[2]See also Rabitsch and Stepanchuk (2014).

approximation. Moreover, the proposed technique is fast and can handle models with a richer state space. The time necessary to compute a solution of the model considered in this paper amounts to 1.846683 seconds.[3] Finally, the procedure can be easily incorporated in Dynare, a popular software platform for solving DSGE and OLG models.[4]

Including second-order uncertainty correction under the proposed method is shown to improve quality of the approximation. First, the ergodic mean of gross asset holdings lies closer to its global solution counterpart with the largest discrepancy among available assets amounting to 3.63 %. By contrast, this figure is nearly twice as large for DS. Second, accounting for the direct effect of risk has the potential to improve the accuracy of the approximation measured by Euler equation errors. The largest documented average accuracy gain is one order of magnitude, whereas the maximum improvement amounts to five orders.

This paper builds mostly on Judd and Guu (2001) who discuss theoretical foundations of bifurcation methods and employ them to solve a partial equilibrium model with portfolio choice. I aim at extending their methods to general equilibrium models. In this regard, my work is closely related to Winant (2014). He independently developed a bifurcation-based solution method for DSGE models with portfolio choice. The main difference between this paper and Winant (2014) is the use of nonlinear moving average. In particular, I show that using standard state space methods instead can lead to highly volatile portfolios.

Implementation of the proposed methodology is based on root-finding algorithms and fixed point iteration techniques. Therefore, this work is also related to the paper by Tille and Van Wincoop (2010) who utilize iterative procedures to obtain an approximation to portfolio holdings. However, as their method is virtually the same as DS (the only difference being the way of implementing), it suffers from the two aforementioned drawbacks.

The rest of the paper is organized as follows. Section 3.2 presents the model which is used to explain and evaluate the proposed methodology. Section 3.3 discusses the key elements of the proposed method and the main steps of the solution algorithm. All results are discussed in section 3.4. Finally, section 3.5 concludes.

3.2 Motivating Example

This section presents the model used in the following to evaluate the proposed local approximation method. It is a version of a real two-country Lucas tree

[3] All experiments are conducted on a desktop computer with Intel® Core™ i5-4690 CPU (3.5 GHz).

[4] The algorithm has been implemented in Dynare/MATLAB. The codes are available upon request.

model with portfolio choice employed by Rabitsch et al. (2015). The choice of this particular model enables a direct comparison to the literature and thus speeds up the assessment of proposed method's potential to improve on existing techniques.

Economic environment. It is assumed that the world consists of two countries: Home (H) and Foreign (F). Each country is endowed with two types of income. They are labeled as "capital income" (Y^K) and "labor income" (Y^L) for convenience. Total GDP is thus simply the sum of both types of income, i.e. $Y_{it} = Y_{it}^K + Y_{it}^L$, with $i = \{H, F\}$ being the country index.
The logarithm of country $i's$ income streams follows an autoregressive process of order one

$$\log(Y_{it}^K) = \rho_K \log(Y_{it-1}^K) + \epsilon_{it}^K, \tag{3.1}$$

and

$$\log(Y_{it}^L) = \rho_L \log(Y_{it-1}^L) + \epsilon_{it}^L. \tag{3.2}$$

Innovations are assumed to be normally distributed and independent across countries but correlation between shocks within a country is allowed to be non-zero, i.e., $\epsilon_{it}^j \sim N(0, \sigma_{i\epsilon}^2)$ and $corr(\epsilon_{Ht}^j, \epsilon_{Ft}^j) = 0$, with $j \in \{K, L\}$. Moreover, I introduce asymmetries into the model, by assuming that foreign income stream is twice as volatile as the endowment in home country. This assumption should capture the empirical observation that emerging market countries are characterized by higher macroeconomic uncertainty (Aguiar and Gopinath, 2007). Thus, foreign economy can be viewed as a developing country.

Following Lan and Meyer-Gohde (2013), the model is perturbed via future shocks. Hence, all future disturbances are scaled by the perturbation parameter σ which governs the size of uncertainty in the model. $\sigma = 0$ implies a deterministic setup, whereas $\sigma = 1$ refers to fully stochastic world.[5]

Household. Country i is populated by a representative household, whose preferences are described by the following lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \phi_{it} \frac{C_{it}^{1-\gamma}}{1-\gamma}. \tag{3.3}$$

C_{it} stands for a single good consumption and ϕ_{it} is the endogenous discount factor, one of the mechanisms proposed by Schmitt-Grohé and Uribe (2003) to ensure stationarity of the approximate solution under the assumption of incomplete markets. Following Devereux and Sutherland (2011), the endogenous discount factor is given by

[5] One should distinguish between σ measuring the size of uncertainty, $\sigma_{i\epsilon}$ being the standard deviation of shocks in country i, and σ_{iY} denoting the resulting standard deviation of the income.

$$\phi_{it} = \bar{\beta} C_{Ait}{}^{-\eta} \phi_{it-1}, \qquad \phi_{i0} = 1, \tag{3.4}$$

with $\bar{\beta}$ denoting the discount factor in the deterministic steady state. Note that endogenous discount factor does not depend on the individual consumption but on the economy average (C_{Ait}). This assumption prevents the agent from internalizing the effects of her savings choice on the discount factor and thus avoids further complications. In equilibrium, the individual consumption is equal to the aggregate level, as there exists one representative household in each country.

The representative household allocates its wealth between two internationally traded assets which represent claims on "capital income" of the respective country. Because of their definition, assets can be interpreted as equity shares. The resulting budget constraint of the agent in country i can be written as

$$C_{it} + Q_{Ht}\theta_{it}^{H} + Q_{Ft}\theta_{it}^{F} = (Q_{Ht} + Y_{Ht}^{K})\theta_{it-1}^{H} + (Q_{Ft} + Y_{Ft}^{K})\theta_{it-1}^{F} + Y_{it}^{L}, \tag{3.5}$$

where Q_{it} denotes the price of claims on country $i's$ "capital income", whereas θ_{it}^{H} and θ_{it}^{F} stand for holdings of home and foreign assets, respectively.

The household in country i maximizes its lifetime utility subject to the budget restriction. Solving this maximization problem yields the following Euler equations

$$Q_{Ht} = E_t \left[\beta \frac{C_{it}^{\gamma-\eta}}{C_{it+1}^{\gamma}} (Q_{Ht+1} + Y_{Ht+1}^{K}) \right], \tag{3.6}$$

and

$$Q_{Ft} = E_t \left[\beta \frac{C_{it}^{\gamma-\eta}}{C_{it+1}^{\gamma}} (Q_{Ft+1} + Y_{Ft+1}^{K}) \right]. \tag{3.7}$$

Market clearing. The goods market clears when

$$Y_{Ht} + Y_{Ft} = C_{Ht} + C_{Ft}. \tag{3.8}$$

The supply of each asset is normalized to unity, so that financial markets clear if

$$\theta_{Ht}^{H} + \theta_{Ft}^{H} = 1, \tag{3.9}$$

and

$$\theta_{Ht}^{F} + \theta_{Ft}^{F} = 1. \tag{3.10}$$

Note that, because of the normalization of asset supply to one, θ_{Ht}^{H} can be interpreted as the share of home equity held by home country.

3.3 Solution Methods

3.3.1 Preliminaries

Rewriting the model This section discusses methods that are employed to solve our example model. To apply local approximation techniques, it is helpful to rewrite the model such that gross asset positions are in zero net supply.[6] To this end, I follow Rabitsch et al. (2015), and define $\alpha_{Ht}^{H} \equiv \left(\theta_{Ht}^{H} - 1\right) Q_{Ht}$ and $\alpha_{Ht}^{F} \equiv \theta_{Ht}^{F} Q_{Ft}$ as "net funds" invested in home and foreign assets by home country.[7] With these definitions, the budget constraint of the home agent can be written as

$$C_{Ht} + \alpha_{Ht}^{H} + \alpha_{Ht}^{F} = R_{Ht}\alpha_{Ht-1}^{H} + R_{Ft}\alpha_{Ht-1}^{F} + Y_{Ht}, \tag{3.11}$$

where

$$R_{it} = \frac{Q_{it} + Y_{it}^{K}}{Q_{it-1}} \tag{3.12}$$

is the rate of return on equity issued by country i. Similarly, the market clearing conditions for financial markets are given by

$$\alpha_{Ht}^{H} = -\alpha_{Ft}^{H} \tag{3.13}$$

$$\alpha_{Ht}^{F} = -\alpha_{Ft}^{F}. \tag{3.14}$$

According to (3.11), consumption in the deterministic steady state depends on steady-state portfolio holdings. However, as explained below, the latter cannot be pinned down in a non-stochastic environment. This problem will be solved by applying a sequential procedure, where the Nth-order approximation of nonportfolio variables will be computed together with the (N-1)th-order approximation of asset holdings (Samuelson, 1970; Devereux and Sutherland, 2010, 2011). To this end, the budget constraint will be rewritten in terms of net foreign assets (NFA_{Ht})

$$C_{Ht} + NFA_{Ht} = R_{xt}\alpha_{Ht-1}^{H} + R_{Ft}NFA_{Ht-1} + Y_{Ht}, \tag{3.15}$$

with

$$NFA_{Ht} = \alpha_{Ht}^{H} + \alpha_{Ht}^{F}, \tag{3.16}$$

and $R_{xt} \equiv R_{Ht} - R_{Ft}$ denoting the excess rate of return on home equity. Market clearing conditions (3.13) and (3.14) imply that $NFA_{Ht} = -NFA_{Ft}$.

The main focus of this paper lies on portfolio holdings reflected by α's. It is sufficient to obtain a solution for α_{Ht}^{H} to determine the entire asset allocation in

[6] See Devereux and Yetman (2010).

[7] "Net" signifies that $\alpha's$ are expressed relative to full home bias. If $\alpha_{Ht}^{H} < 0$, less than 100 % of home assets is owned by home household.

the model.[8] For this reason, I simplify the notation and denote α^H_{Ht} as α_t in what follows.

Equilibrium The full equilibrium of the rewritten model is described by equations (3.6)-(3.7), (3.12), (3.15)-(3.16) for both home and foreign country, and market clearing conditions (3.8), (3.13)-(3.14). This gives 13 equations and 12 endogenous variables: $\alpha^H_H, \alpha^F_H, \alpha^H_F, \alpha^F_F, NFA_H, NFA_F, Q_H, Q_F, R_H, R_F, C_H, C_F$, with one equation being redundant by the Walras' law.

Model Solution The model solution can be represented either as a set of state space policy functions (see e.g., Jin and Judd, 2002; Schmitt-Grohé and Uribe, 2004) or as nonlinear moving average policy functions introduced by Lan and Meyer-Gohde (2013). The former approach uses a time-invariant mapping of state variables (y^{state}) and a vector of shocks (ϵ) to model variables (y)

$$y_t = g(\sigma, z_t), \tag{3.17}$$

where

$$z_t = [y^{state}_{t-1}, \epsilon_t]^{\mathrm{T}},$$

with "T" denoting a transpose.

By contrast, the nonlinear moving average represents a direct mapping of the history of shocks to model variables, i.e.,

$$y_t = y(\sigma, \epsilon_t, \epsilon_{t-1}, ..). \tag{3.18}$$

Note that size of uncertainty enters as a separate argument in both cases because it has a direct effect on the policy function.

Due to nonlinearities present in the model, an exact solution is not feasible and thus one must rely on approximation methods. Following Lan and Meyer-Gohde (2014a), the Mth-order Taylor approximation of the state space policy function around the deterministic steady state can be written as

$$y_t^{(M)} = \sum_{j=0}^{M} \frac{1}{j!} \left[\sum_{i=0}^{M-j} \frac{1}{i!} g_{z^j \sigma^i} \sigma^i \right] (z_t - \bar{z})^{\otimes[j]}, \tag{3.19}$$

where $(z_t - \bar{z})^{\otimes[j]}$ denotes the jth fold Kronecker product of $(z_t - \bar{z})$ with itself. Furthermore, $g_{z^j\sigma^i}$ is the partial derivative of the state-space policy function with respect to z_t j times and with respect to the perturbation parameter i times, evaluated

[8] All other α's can be computed via clearing conditions for financial markets and the definition of home net foreign assets.

at the deterministic steady state. Due to the rewritten form of the underlying model the state space is reduced to $y_t^{state} = [Y_{Ht}^K, Y_{Ht}^L, Y_{Ft}^K, Y_{Ft}^L, NFA_{Ht}, Q_{Ht}, Q_{Ft}, \alpha_t]^{\mathrm{T}}$.

On the other hand, taking the Mth-order Taylor approximation of the nonlinear moving average policy function yields

$$y_t^{(M)} = \sum_{m=0}^{M} \frac{1}{m!} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} ... \sum_{i_m=0}^{\infty} \left[\sum_{n=0}^{M-m} \frac{1}{n!} y_{\sigma^n i_1 i_2 ... i_m} \sigma^n \right] (\epsilon_{t-i_1} \otimes \epsilon_{t-i_2} ... \otimes \epsilon_{t-i_m}). \quad (3.20)$$

$y_{\sigma^n i_1 i_2 ... i_m}$ refers to the partial derivative of y_t with respect to the mth fold Kronecker product of disturbances from i_1,i_2, ... and i_m periods ago, and with respect to the perturbation parameter n times, evaluated at the deterministic steady state. To facilitate the comparison with state space methods, I will exploit the recursive representation of the nonlinear moving average approximation. As argued by Lan and Meyer-Gohde (2014a), (3.20) can be rewritten as follows

$$y^{(M)} = \sum_{m=0}^{M} \frac{1}{m!} y_{\sigma^m} + \sum_{m=1}^{M} dy_t^{(m)}, \quad (3.21)$$

where $dy_t^{(m)} \equiv y_t^{(m)} - y_t^{(m-1)} - \frac{1}{m!} y_{\sigma^m}$, $m = 1, 2, ...$ denotes the mth-order increment in the nonlinear moving average approximation. Lan and Meyer-Gohde (2014a) derive a recursive representation for these increments and show that deterministic coefficients in (3.21) match the ones in the state space policy function. However, there exist differences in uncertainty correction terms. First, the constant risk adjustment in the nonlinear moving average approximation can be directly used to compute an approximation to the stochastic steady state, defined as a fixed point in the presence of uncertainty ($\sigma = 1$), but in absence of shocks $\epsilon_t = 0$ (Meyer-Gohde, 2014). In particular, setting the history of shocks to zero yields the following expression for the stochastic steady state

$$\bar{y}^{stoch} \approx \sum_{m=0}^{M} \frac{1}{m!} y_{\sigma^m}, \quad (3.22)$$

with $\bar{y} \equiv y_{\sigma^0}$. By contrast, standard state space methods deliver a one-step ahead uncertainty correction. Therefore, computing an approximation of the stochastic steady by using the state space representation is not as straightforward as in the case of the nonlinear moving average and involves iterative numerical procedures (see Juillard, 2011 and Coeurdacier et al., 2011).

Going beyond constant risk adjustment, Meyer-Gohde (2014) shows that the time-varying uncertainty correction of the nonlinear moving average can be related to first-order derivatives of underlying policy functions evaluated at the stochastic steady state.

These properties allow the nonlinear moving average approximation to account for risk characteristics of the model, as reflected by the starting point of the approximation. As a result, it is suitable to solve DSGE models with portfolio choice because risk considerations play a major role in this setup. The reason for this is that optimal asset holdings are determined by agents' hedging motives.

In the course of this paper, I discuss how one can pin down coefficients in the approximate solution for portfolio holdings. Moreover, I show that it matters for the solution whether the state space approach or the nonlinear moving average is being used.

3.3.2 Failure of Regular Perturbation Techniques

Solving the example model with perturbation methods involves two difficulties. First, uncertainty is completely eliminated in the deterministic steady state. As the two assets differ only in their risk characteristics, they become then perfect substitutes and yield the same rate of return. This can be seen by investigating the Euler equations (3.6) and (3.7). They imply that $\bar{R}_H = \bar{R}_F$, with a bar over a variable standing for its steady state value. As a consequence, countries' gross asset positions cannot be uniquely pinned down in the non-stochastic steady state.

Second, even if indeterminacy of the approximation point is somehow resolved, a first-order approximation is not sufficient to determine the dynamics of portfolio holdings. A first-order approximation of Euler equations implies

$$E_t\left[\hat{R}^{(1)}_{Ht+1}\right] = E_t\left[\hat{R}^{(1)}_{Ft+1}\right], \tag{3.23}$$

where hats denote log-deviations from the deterministic steady state. Thus, up to a first order of accuracy, all assets have the same expected rate of return and portfolio holdings are again indeterminate.[9] Consequently, higher-order perturbations are necessary to obtain approximate dynamics of portfolio holdings.

To explain general implications of the existence of portfolio choice for the perturbation approach, I will cast our example model in a more general form. In particular, as a member of a family of discrete time rational expectations models, it can be written as

$$E_t\left[f\left(y_{t+1}, y_t, y_{t-1}, \epsilon_t\right)\right] = 0, \tag{3.24}$$

where $f : \mathbb{R}^{ny} \times \mathbb{R}^{ny} \times \mathbb{R}^{ny} \times \mathbb{R}^{ne} \to \mathbb{R}^{ny}$ is assumed to be analytic, $y_t \in \mathbb{R}^{ny}$ stands for the vector containing both endogenous and exogenous variables, and $\epsilon_t \in \mathbb{R}^{ne}$ is a vector of zero-mean iid shocks.

Standard local approximation methods are based on the Taylor Approximation and the Implicit Function Theorem (Judd, 1998). The idea is to insert policy

[9] This is an implication of the certainty equivalence of first-order approximation (see Schmitt-Grohé and Uribe, 2004).

functions (state space or nonlinear moving average) into (3.24) and apply successive differentiation, where each derivative is evaluated at the non-stochastic steady state. Applying this procedure to find first-order coefficients in the state space policy function and postmultiplying the result with z_y yields[10]

$$f_{y^+}(g_z z_y)^2 + f_y(g_z z_y) + f_z z_y = 0. \tag{3.25}$$

(3.25) is a matrix quadratic equation in $g_z z_y$ measuring the dependence of y on state variables. Note that $g_z z_y$ can be interpreted as a lead operator (henceforth, F) in the absence of shocks. Thus, multiplying (3.25) by $\hat{y}_{t-1}$ yields a second-order difference equation which in turn can be converted to a first-order system

$$\begin{aligned} (DF - E)\hat{x}_t = 0 \quad with \quad D &\equiv \begin{bmatrix} 0_{ny\times ny} & I_{ny} \\ f_{y^+} & 0_{ny\times ny} \end{bmatrix}, \\ E &\equiv \begin{bmatrix} I_{ny} & 0_{ny\times ny} \\ -f_y & -f_z z_y \end{bmatrix} \quad and \quad \hat{x}_t \equiv \begin{bmatrix} \hat{y}_t \\ \hat{y}_{t-1} \end{bmatrix}. \end{aligned} \tag{3.26}$$

A unique solution to (3.26) can be obtained by using the generalized Schur decomposition of D and E, if the pencil defined by those matrices, $P(z) = Dz - E$, is regular (Klein, 2000), i.e.,

$$\exists a \quad z: \quad |Dx - E| \neq 0.$$

However, DSGE portoflio choice models are characterized by a collinear relationship among the Euler equations up to first order of accuracy. As a consequence, the above regularity condition is violated and there exists a matrix polynomial $\varphi(z)$, such that $\varphi(z)\,(Dz - E) = 0$ (King and Watson, 1998). Multiplying (3.26) by $\varphi(F)$ implies $0 = 0$. Thus, there exists infinitely many solutions to (3.25) with the result that standard perturbation methods cannot be applied to DSGE models embedding a portfolio choice problem.

3.3.3 Devereux Sutherland Method

Devereux and Sutherland (2010, 2011) aim to obtain the following approximation of the portfolio solution

$$\alpha_t^{(1)} = \bar{\alpha} + \tilde{g}^{\alpha}_{y^{state}} \hat{y}_t^{(1),state}, \tag{3.27}$$

with '∼' reflecting the fact that the coefficient measures the dependence on the current values of state variables and a hat denoting again the log-deviation from

[10] Shifting the state space policy function (3.17) one period into the future yields $y_{t+1} = g^+(\sigma, z_{t+1})$ with $z_{t+1} = [y_t, \sigma\epsilon_{t+1}]^{\mathrm{T}}$ (Lan and Meyer-Gohde, 2014b).

the deterministic steady state.[11] To this end, the authors decompose the model into a *portfolio equation* and a *macroeconomic part* (i.e., the remaining equations). The *portfolio equation* can be obtained by combining the Euler equations

$$E_t\left[\left(C_{Ht+1}^{-\gamma}-C_{Ft+1}^{-\gamma}\right)\left(R_{Ht+1}-R_{Ft+1}\right)\right]=0. \tag{3.28}$$

In the *macroeconomic part* of the model, portfolio holdings appear only in the budget constraint (3.15) and are multiplied by the excess return. As mentioned above, this fact allows for a sequential solution strategy. To see this, consider the log-linearized version of the budget constraint

$$N\hat{F}A_{Ht}^{(1)}=\frac{1}{\beta}N\hat{F}A_{Ht-1}^{(1)}+\frac{1}{\beta\bar{Y}_H}\bar{\alpha}\hat{R}_{xt}^{(1)}-\hat{C}_{Ht}^{(1)}+\hat{Y}_{Ht}^{(1)}, \tag{3.29}$$

where $N\hat{F}A_{Ht}=\frac{NFA_{Ht}}{\bar{Y}_H}$ and $\hat{R}_{xt}=\hat{R}_{Ht}-\hat{R}_{Ft}$. Note that (3.29) does not include $\hat{\alpha}_t$ so that first-order nonportfolio variables depend only on the steady-state value of α. Moreover, since the expected excess return is zero up to a first-order accuracy, one can eliminate the expression $\frac{1}{\beta\bar{Y}_H}\bar{\alpha}\hat{R}_{xt}^{(1)}$ by introducing an auxiliary wealth shock ($\zeta_t\equiv\frac{1}{\beta\bar{Y}_H}\bar{\alpha}\hat{R}_{xt}^{(1)}$). The *macroeconomic part* can be then solved conditional on this shock. This approximate solution is in turn used to compute zero-order portfolio holdings. Devereux and Sutherland (2010) show that this procedure can be extended to determine first-order portfolio dynamics. In general, portfolio equation needs to be approximated up to the order N+2, whereas the *macroeconomic part* to the (N+1)-th order, to be able to pin down the Nth-order component of portfolio holdings (Samuelson, 1970).

3.3.4 Bifurcation Methods

Standard perturbation techniques cannot be employed to solve DSGE models with portfolio choice as there are infinitely many optimal portfolio holdings when uncertainty is eliminated (i.e. $\sigma=0$). However, as long as some risk is present, there exists a unique solution, given that standard regulatory conditions are fulfilled (concavity of the objective function etc.). This change in the number of solutions, as the perturbation parameter varies, is an example of a *bifurcation.*

Definition (Bifurcation, Judd and Guu, 2001). *Suppose that $H(\alpha,\sigma)$ is analytic and $\alpha(\sigma)$ is implicitly defined by $H(\alpha(\sigma),\sigma)=0$. One way to view equation $H(\alpha,\sigma)=0$ is that for each σ it defines a collection of α that solves it. Bifurcation occurs if number of such α changes as we change σ.*

[11] It does not matter whether the approximate solution links portfolio holdings to the current or past values of states, as both representations are equivalent. (3.27) follows the convention of *DS*.

Bifurcation problems can be tackled by employing bifurcation theory. In the following, I present two concepts that are crucial for solving a DSGE model with portfolio choice: a bifurcation point and a bifurcation theorem applicable to the problem at hand.

Definition (Bifurcation Point, Zeidler, 1986). *(α_0, σ_0) is a bifurcation point of H iff the number of solutions α to $H(\alpha, \sigma) = 0$ changes as σ passes through σ_0, and there are at least two distinct parametric paths $(\alpha_{A,n}, \sigma_{A,n})$ and $(\alpha_{B,n}, \sigma_{B,n})$ which converge to (α_0, σ_0) as $n \to \infty$.*

Theorem (Bifurcation Theorem for $\mathbb{R}^n$). *Suppose H: $\mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$, H is analytic for (α, σ) in a neighborhood of (α_0, σ_0), and $H(\alpha, \sigma_0) = 0$ for all $\alpha \in \mathbb{R}^n$. Furthermore suppose that*

$$i)\ H_\alpha(\alpha_0, \sigma_0) = 0$$

$$ii)\ H_{\sigma\sigma}(\alpha_0, \sigma_0) = 0$$

$$iii)\ det\,(H_{\sigma\sigma\alpha}(\alpha_0, \sigma_0)) \neq 0.$$

Then (α_0, σ_0) is a bifurcation point and there is an open neighborhood N of (α_0, σ_0) and a function $h(\sigma)$: $\mathbb{R} \to \mathbb{R}^n$, such that h is analytic and $H(h(\sigma), \sigma) = 0$ for $(h(\sigma), \sigma) \in N$.

Proof. See Appendix B.1. □

The intuition behind the above theorem can be understood as follows. The original function H, characterized by a first-order singularity in a non-stochastic environment, is replaced by some other function, $\tilde{H}$. Given that $\sigma = 0$, this new function has a zero at the bifurcation point of H. Moreover, since the indeterminacy issue does not apply to $\tilde{H}$, the successive differentiation can be employed again. In the context of a DSGE model, it can be shown that

$$\tilde{H}(\alpha, \sigma) = \begin{cases} \frac{H(\alpha,\sigma)}{\sigma^2} & if \quad \sigma \neq 0 \\ \frac{\partial^2 H(\alpha,\sigma)}{(\partial\sigma)^2} & if \quad \sigma = 0. \end{cases}$$

In the following, I will demonstrate the practical implementation of the bifurcation theory. To this end, I use firstly state-space methods and then discuss how the approach changes if we the use the nonlinear moving average instead.

State Space Approach

The bifurcation theorem cannot be directly applied to DSGE models with portfolio choice because it requires that all endogenous variables are indeterminate at the approximation point.[12] This is true only for portfolio holdings, whereas all non-portfolio variables are pinned uniquely in the deteministic steady state. To overcome

[12]See condition i) of the bifurcation theorem.

this problem, I follow Devereux and Sutherland (2010, 2011) and decompose the model into a *portfolio equation* and a *macroeconomic part*.[13] To this end, I will distinguish between several types of variables and rewrite the model (3.24) as follows

$$E_t\left[n\left(y_{t+1}^{fwd}, y_t^{fwd}, y_t^{state}, y_{t-1}^{state}, \alpha_t, \alpha_{t-1,}\epsilon_t\right)\right] = 0, \tag{3.30}$$

$$E_t\left[m(\mu_{t+1}) \otimes b(r_{t+1})\right] = 0, \tag{3.31}$$

where n and b are vector-valued functions with dimensions $ny \times 1$ and $na \times 1$ respectively, whereas $\mathbb{R}$ is the range of m . All functions are assumed to be analytic in the neighborhood of the bifurcation point. Moreover, $y^{fwd} \in \mathbb{R}^{nyfwd}$ denotes forward looking variables, $y^{state} \in \mathbb{R}^{nystate}$ contains both endogenous and exogenous non-portfolio state variables and $\alpha_t \in \mathbb{R}^{na}$ represents gross asset holdings. Note the following link to (3.24): $ny = nyfwd + nystate + na$. Finally, (3.31) decomposes y^{fwd} into a vector of rates of returns, $r \in \mathbb{R}^{nr}$, and the remaining forward looking variables, $\mu \in \mathbb{R}^{nmu}$. Thus, $nyfwd = nr + nmu = na + 1 + nmu$. Given a guess for the policy function for α, a unique approximate solution to the real part (3.30) of the model can be obtained. The approximate solution can then be exploited to express the *portfolio equation* (3.31) in terms of portfolio holdings, perturbation parameter, future shocks and state variables of the model

$$\begin{aligned}
&H\left(\sigma, \alpha_t, y_t^{state}, \epsilon_{t+1} | g_{guess}^{\alpha}\right) \equiv \\
&E_t\left[m\left(g^{\mu}\left(\sigma, \alpha_t, y_t^{state}, \epsilon_{t+1}\right| g_{guess}^{\alpha})\right) \otimes b\left(g^{r}\left(\sigma, \alpha_t, y_t^{state}, \epsilon_{t+1} | g_{guess}^{\alpha}\right)\right)\right],
\end{aligned} \tag{3.32}$$

where "$|g_{guess}^{\alpha}$" indicates that policy functions for non-portfolio variables have been approximated given a guess for the policy function governing gross asset holdings. The function H defined in (3.32) fulfills all requirements stated by the bifurcation theorem.

To solve a decomposed version of a DSGE portfolio choice model, a fixed point needs to be found, i.e. $g^{\alpha} = g_{guess}^{\alpha}$. This can be done relatively easily by applying a recursive procedure given that Nth-order components of non-portfolio variables depend only on the (N-1)th-order component of gross asset holdings. As already explained, this is the case in the rewritten version of the example model. Likewise, I will assume throughout the general exposition that this condition is fulfilled.

Assumption (Recursiveness). *Nth-order components of non-portfolio variables depend only on the (N-1)th-order component of portfolio holdings.*

[13] This is also the approach adopted by Winant (2014).

In the following, I will explain how to use bifurcation theory to compute a second-order approximation of the optimal portfolio[14]

$$\alpha_t^{(2)} = \bar{\alpha} + \tilde{g}_\sigma^\alpha + \frac{1}{2}\tilde{g}_\sigma^\alpha + \tilde{g}_{y^{state}}^\alpha \hat{y}_t^{(2),state} + \frac{1}{2}\tilde{g}_{y^{state}y^{state}}^\alpha \left(\hat{y}_t^{(2),state} \otimes \hat{y}_t^{(2),state}\right). \quad (3.33)$$

Computing the bifurcation portfolio According to the bifurcation theorem, zero-order portfolio holdings ($\bar{\alpha}$) satisfy the following condition[15]

$$\bar{H}_{\sigma\sigma} \equiv H_{\sigma\sigma}|_{\sigma=0,\hat{y}_t^{states}=0} = -2b_r g_\epsilon^r \Sigma g_\epsilon^{\mu\top} m_\mu^\top = 0, \quad (3.34)$$

where g_ϵ denotes vector of coefficients measuring the dependence on shock realizations and Σ stands for variance-covariance matrix of the underlying shock process. To evaluate (3.34), one requires a first-order approximation of non-portfolio variables which in turn depends on the zero-order portfolio holdings. To solve the resulting root-finding problem standard nonlinear solvers can be applied. The iterative procedure can be summarized as follows

Algorithm 1. *Computing the Bifurcation Portfolio*

1. *Select an error tolerance δ for the stopping criterion and an initial guess for $\bar{\alpha}$.*

2. *Solve the macroeconomic part of the model conditional on the guess.*

3. *Use results from step 2 to evaluate (3.34).*

4. *Check stopping criterion: if $|\bar{H}_{\sigma\sigma}| < \delta$, the guessed value of $\bar{\alpha}$ represents the bifurcation portfolio. Otherwise, update the guess (according to the numerical procedure used) and go back to step 2.*

Equation (3.34) coincides with the condition characterizing steady state portfolio holdings computed with DS.[16] Thus, I provide a formal proof that DS always yields the bifurcation point as steady-state portfolio holdings.

Computing first-order coefficients. Given $\bar{\alpha}$, the bifurcation theorem enables implicit differentiation to pin down first-order coefficients of the approximated policy function

$$\tilde{g}_\sigma^\alpha = -\frac{1}{3}\bar{H}_{\sigma\sigma\alpha}^{-1}\bar{H}_{\sigma\sigma\sigma}, \quad (3.35)$$

[14]In general, a second-order approximation (3.33) also includes $\tilde{g}_{\sigma y^{state}}^\alpha \hat{y}_t^{(2),state}$. However, due to symmetrically distributed shocks, $\tilde{g}_{\sigma y^{state}}^\alpha$ is a zero vector (see Appendix B.2.3). For that reason, I simplify the analysis by omitting this term. In contrast, $\tilde{g}_\sigma^\alpha$ is still included, despite being equal to zero, because I want to explain why Devereux and Sutherland (2010) consider Taylor expression only with respect to state variables.

[15]See Appendix B.1.

[16]See Devereux and Sutherland (2010), p. 1331, equation (21).

and

$$\tilde{g}^{\alpha}_{y^{state}} = -\bar{H}^{-1}_{\sigma\sigma\alpha}\bar{H}_{\sigma\sigma y^{state}}. \tag{3.36}$$

The first-order dynamics of gross asset holdings is driven by time-varying risk components which are reflected by third derivatives of the portfolio equation.

To evaluate (3.35) and (3.36), a second-order approximation of non-portfolio variables is necessary.[17] It depends in turn on the first-order dynamics of portfolio holdings. Therefore, the problem at hand takes again the form of a fixed point search and can be solved by applying the following algorithm:

Algorithm 2. *Computing First-Order Components of Portfolio Holdings*

1. *Select an error tolerance δ for the stopping criterion and an initial guess for $\tilde{g}^{\alpha}_{\sigma}$ and $\tilde{g}^{\alpha}_{y^{state}}$.*

2. *Solve the macroeconomic part of the model conditional on the guess $\tilde{g}^{\alpha}_{\sigma}(k)$ and $\tilde{g}^{\alpha}_{y^{state}}(k)$, where k is the iteration index.*

3. *Use results from step 2 to compute (3.35) and (3.36): $\tilde{g}^{\alpha}_{\sigma}(k+1)$ and $\tilde{g}^{\alpha}_{y^{state}}(k+1)$.*

4. *Check the stopping criterion: if $\|p(k+1) - p(k)\| < \delta\,(1 + \|p(k)\|)$ with $p \equiv [\tilde{g}^{\alpha}_{\sigma}, \tilde{g}^{\alpha}_{y^{state}}]^{\mathrm{T}}$, a fixed point has been reached. Otherwise, update the guess and go back to step 2.*

As shown in Appendix B.2.2, first-order uncertainty correction can be expressed as

$$\tilde{g}^{\alpha}_{\sigma} = \bar{\tau}\Sigma_3, \tag{3.37}$$

with $\bar{\tau}$ denoting the skew tolerance at the bifurcation point, as in Judd and Guu (2001), and $\Sigma_3 \equiv E_t\left[\epsilon_t^{\otimes[3]}\right]$ referring to the matrix of third moments of the underlying shock structure. Note that (3.37) implies that $\tilde{g}^{\alpha}_{\sigma}$ is equal to zero under symmetrically distributed shocks. This result can be seen as an extension of the certainty equivalence of the first-order approximation documented by Schmitt-Grohé and Uribe (2004). Furthermore, it explains why Devereux and Sutherland (2010) consider only state variables in their first-order approximation, given that they assume a symmetric distribution.

Computing Second-Order Risk Correction Term The above procedure can be easily extended to pin down coefficients of higher-order approximations of portfolio holdings. The focus of this paper lies on the heterogeneity across countries implied by differences in economic uncertainty. This asymmetry results in different magnitudes of agents' precautionary motives which will be reflected by the

[17]See Appendix B for expressions of the respective derivatives of the portfolio equation.

uncertainty correction. Therefore, I will also discuss how to obtain the second-order risk adjustment for gross asset holdings. Note that it has to be computed together with second-order deterministic coefficients, i.e., coefficients with respect to states, $\tilde{g}^{\alpha}_{y^{state}y^{state}}$. Implicit differentiation yields the following expressions under normally distributed shocks

$$\tilde{g}^{\alpha}_{\sigma\sigma} = -\frac{1}{6}\bar{H}^{-1}_{\sigma\sigma\alpha}\bar{H}_{\sigma\sigma\sigma\sigma}, \tag{3.38}$$

and

$$\tilde{g}^{\alpha}_{y^{state}y^{state}} = -\bar{H}^{-1}_{\sigma\sigma\alpha}\Gamma, \tag{3.39}$$

with

$$\begin{aligned}\Gamma \equiv\ & \bar{H}_{\sigma\sigma\alpha\alpha}\left(\tilde{g}^{\alpha}_{y^{state}} \otimes \tilde{g}^{\alpha}_{y^{state}}\right) \\ & + \bar{H}_{\sigma\sigma\alpha y^{state}}\left(I_{nystate} \otimes \tilde{g}^{\alpha}_{y^{state}}\right)\left(I_{nystate^2} + K_{nystate,nystate}\right) + \bar{H}_{\sigma\sigma y^{state}y^{state}}.\end{aligned} \tag{3.40}$$

I_n denotes an $n \times n$ identity matrix and $K_{n,n}$ is a commutation matrix with dimension $n^2 \times n^2$ (Magnus and Neudecker, 1979). Second-order coefficients of portfolio holdings are thus driven by forth-order accurate interaction between non-portfolio variables of the model.

Expressions (3.38) and (3.39) can be evaluated with the help of the third-order approximation of non-portfolio variables. Their third-order components depend in turn on second-order asset holdings. Therefore, we face again a fixed point problem which can be solved by employing the following iterative routine:

Algorithm 3. *Computing Second-Order Components of Portfolio Holdings*

1. *Select an error tolerance δ for the stopping criterion and an initial guess for $\tilde{g}^{\alpha}_{\sigma\sigma}$ and $\tilde{g}^{\alpha}_{y^{state}y^{state}}$.*

2. *Solve the macroeconomic part of the model conditional on the guess $\tilde{g}^{\alpha}_{\sigma\sigma}(k)$ and $\tilde{g}^{\alpha}_{y^{state}y^{state}}(k)$, where k is the iteration index.*

3. *Use results from step 2 to compute (3.38) and (3.39): $\tilde{g}^{\alpha}_{\sigma\sigma}(k+1)$ and $\tilde{g}^{\alpha}_{y^{state}y^{state}}(k+1)$.*

4. *Check the stopping criterion: if $\|p(k+1) - p(k)\| < \delta\,(1 + \|p(k)\|)$ with $p \equiv [\tilde{g}^{\alpha}_{\sigma\sigma}, \tilde{g}^{\alpha}_{y^{state}y^{state}}]^{\mathrm{T}}$, a fixed point has been reached. Otherwise, update the guess and go back to step 2.*

Nonlinear Moving Average

If the nonlinear moving average is used instead of state space methods, the second-order approximation of portfolio holdings is given by[18]

$$\alpha^{(2)} = \bar{\alpha} + \alpha_\sigma + \frac{1}{2}\alpha_{\sigma\sigma} + \tilde{g}^{\alpha,nlma}_{y^{state}}\left(dy_t^{(1),state} + dy_t^{(2),state}\right) + \frac{1}{2}\tilde{g}^{\alpha,nlma}_{y^{state}y^{state}}\left(dy_t^{(1),state} \otimes dy_t^{(1),state}\right). \tag{3.41}$$

To combine bifurcation theory with the nonlinear moving average approximation, one can exploit ideas presented in the previous section. In particular, the H function needs to be replaced by H^{nlma} defined as

$$H^{nlma}\left(\sigma, \alpha_t, \epsilon_{t+1}, \epsilon_t, \epsilon_{t-1}, \ldots | \alpha_{guess}\right) = H\left[\sigma, \alpha_t, y_t^{state}\left(\sigma, \epsilon_t, \epsilon_{t-1}, \ldots\right), \epsilon_{t+1} | \alpha_{guess}\right]. \tag{3.42}$$

Implicit differentiation of (3.42) yields the following results. First, it does not matter for the bifurcation portfolio, whether the nonlinear moving average or state space methods are being used. Since only the first-order approximation is necessary to compute zero-order asset holdings, it holds, due to certainty equivalence, that $\bar{H}^{nlma}_{\sigma\sigma} = \bar{H}_{\sigma\sigma}$. A similar result applies to the first-order uncertainty correction which is always zero, when shocks follow a symmetric distribution, no matter which representation of the policy function is being used. On the other hand, state space methods and the nonlinear moving average will imply different first-order dynamics, as reflected by $\tilde{g}^{\alpha}_{y^{state}}$. This can be seen by inspecting the following relationship: $\bar{H}^{nlma}_{\sigma\sigma y^{state}} = \bar{H}_{\sigma\sigma y^{state}} + \bar{H}_{y^{state}y^{state}}\left(I_{nystate} \otimes y^{state}_{\sigma\sigma}\right)$, with $I_{nystate}$ denoting the identity matrix with dimension $nystate \times nystate$. If we want to go beyond the first-order approximation, it can be shown that the second-order risk adjustment implied by the nonlinear moving average is given by

$$\alpha_{\sigma\sigma} = g^{\alpha,nlma}_{\sigma\sigma} + g^{\alpha,nlma}_{y^{state}} y^{state}_{\sigma\sigma} = g^{\alpha}_{\sigma\sigma} + \Delta + g^{\alpha,nlma}_{y^{state}} y^{state}_{\sigma\sigma}. \tag{3.43}$$

Note that the one-step-ahead uncertainty correction ($g^{\alpha,nlma}_{\sigma\sigma}$) differs from its state space counterpart as it includes the factor Δ. The reason for this is that the excess return (R_x) does not depend on state variables up to a first-order accuracy along the equilibrium path. However, this is no longer the case for higher-order approximations. Thus, Δ reflects the transition to the second order of accuracy. Finally, second-order coefficients with respect to state variables implied by the nonlinear moving average are given by

$$\tilde{g}^{\alpha,nlma}_{y^{state}y^{state}} = -\bar{H}^{-1}_{\sigma\sigma\alpha}\Gamma^{nlma}, \tag{3.44}$$

[18] Second-order terms being equal to zero are omitted once again.

with

$$\begin{aligned}\Gamma^{nlma} \equiv\ & \bar{H}_{\sigma\sigma\alpha\alpha}\left(\tilde{g}^{\alpha,nlma}_{y^{state}} \otimes \tilde{g}^{\alpha,nlma}_{y^{state}}\right) \\ & + \bar{H}_{\sigma\sigma\alpha y^{state}}\left(I_{nystate} \otimes \tilde{g}^{\alpha,nlma}_{y^{state}}\right)\left(I_{nystate^2} + K_{nystate,nystate}\right) \\ & + \bar{H}_{\sigma\sigma y^{state} y^{state}} + \bar{H}_{y^{state} y^{state} y^{state}}\left(I_{nystate} \otimes y^{state}_{\sigma\sigma}\right).\end{aligned}$$

Note that $\Gamma \neq \Gamma^{nlma}$, as the latter is adjusted for the presence of uncertainty, i.e., it includes $y^{state}_{\sigma\sigma}$. As a result, state space methods and the nonlinear moving average imply different second-order deterministic coefficients.

3.4 Numerical Results

This section evaluates three perturbation methods: *DS*, bifurcation theory used together with the state space approach (henceforth: *BIF*), and a combination of bifurcation methods and the nonlinear moving average (henceforth: *BIFN*). As the nonlinear moving average approximation is automatically pruned, solutions obtained with *DS* and *BIF* are pruned as well for the sake of comparability. The second-order approximation will be pruned with the Kim et al. (2008) algorithm, whereas the procedure developed by Andreasen et al. (2017) will be used for the third-order approximation. An additional advantage of pruning the solution of BIF and DS is the possibility to represent all three methods in a unified state space. In particular, Lan and Meyer-Gohde (2014a) express the above pruning algorithms recursively in terms of approximation increments, exactly as in the case of the nonlinear moving average.

3.4.1 Calibration

In all numerical exercises, I employ calibration used by Rabitsch et al. (2015). This allows me to use their global solution[19] as a benchmark for evaluating accuracy of the proposed technique. Table 3.1 reports the chosen parameter values. Almost all of them are commonly used in the macroeconomic literature. The only exception is the consumption elasticity of the endogenous discount factor which is set to 0.001, whereas the standard choice is 0.022 (Mendoza, 1991; Schmitt-Grohé and Uribe, 2003). A small value of η aims at minimizing the effect of this stationarity-inducing device on the predictions of the model.[20] Note that this assumption implies a high persistence of net foreign assets, as the corresponding eigenvalue is close to unity.

[19] Rabitsch et al. (2015) use time iteration spline collocation algorithm to solve the model globally.

[20] See Rabitsch et al. (2015) for a more detailed discussion.

This property of the model will make lengthy simulations necessary to compute underlying ergodic distributions.

Parameter		Value
Discount factor in deterministic steady state	$\bar{\beta}$	0.95
Elasticity of the endogenous discount factor	η	0.001
Risk aversion	γ	2
Capital income share	$\frac{\bar{Y}^K}{\bar{Y}}$	0.3
Persistence	ρ	0.8
Volatility of endowment in Home	$\sigma_{Y_H^K}, \sigma_{Y_H^L}$	0.02
Volatility of endowment in Foreign	$\sigma_{Y_F^K}, \sigma_{Y_F^L}$	0.04
Correlation	$corr(Y^K, Y^L)$	0.2

Table 3.1: Calibration

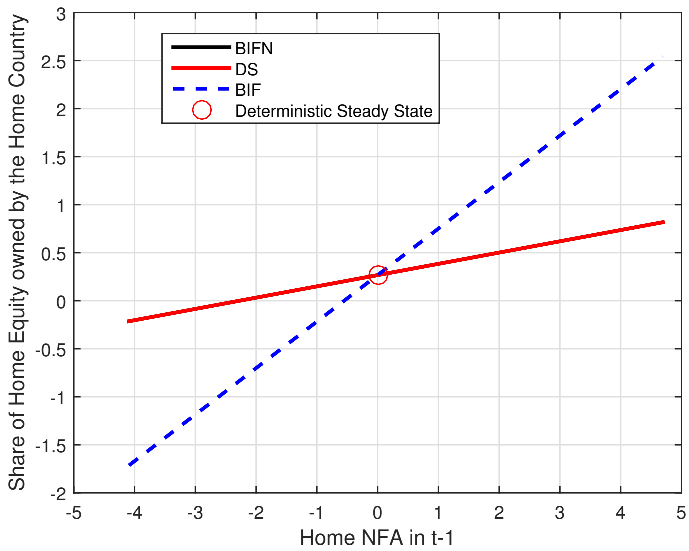


Figure 3.1: First-order accurate share of home equity owned by home country. *Policy functions are depicted in an interval based on the ergodic set of the home NFA implied by DS. The ergodic set is defined as an interval covering 95 % of the probability mass of the underlying distribution. It is determined by simulating 10 million periods and subsequently discretized by 1001 equidistant grid points.*

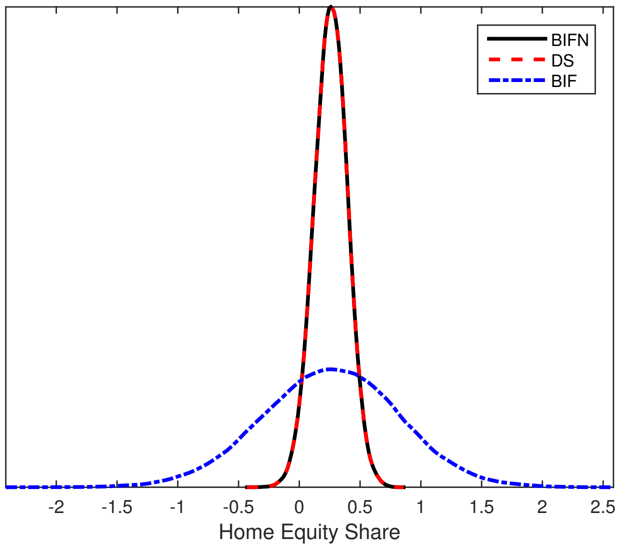


Figure 3.2: Ergodic distribution of the share of home equity owned by home country. *A proxy for the ergodic distribution is obtained by simulating 10 million of periods.*

3.4.2 First-Order Accurate Dynamics

Before considering the role of uncertainty correction, I evaluate first-order differences between *BIF* and *BIFN* documented in the previous section. The latter technique yields the same first-order dynamics as *DS*. On the other hand, portfolio holdings implied by BIF are more volatile.[21]

Figure 3.1 reports the first-order accurate share of home equity held by the domestic agent (θ_H^H) in an interval based on the ergodic set for home net foreign assets. All other state variables take their respective steady state values. Policy function obtained with *DS* and *BIFN* are indistinguishable, whereas *BIF* yields more variation of asset holdings.

Figure 3.2 shows that higher volatility implied by *BIF* is not only a short-run outcome but is also reflected by the ergodic distribution of θ_H^H which is obtained by simulating 10 million periods. Compared to the other two methods, *BIF* implies a standard deviation that is roughly four times larger: 0.56 in contrast to 0.138. To understand this outcome, consider the respective derivative of H and H^{nlma} in the

[21] In a symmetric setup, all three methods imply the same dynamics of portfolio holdings. The reason for this are identical precautionary motives of economic agents that offset each other.

example model:[22]

$$
\begin{aligned}
\bar{H}_{\sigma\sigma y^{state}} = & \boxed{-2\gamma\left(g^{C_h}_{{y^{state}}_{\epsilon}} - g^{C_f}_{{y^{state}}_{\epsilon}}\right)\left(I_{nystate} \otimes \Sigma\left(g^{R_h}_{\epsilon} - g^{R_f}_{\epsilon}\right)^{\mathrm{T}}\right)} \\
& \boxed{-2\gamma\left(g^{R_h}_{{y^{state}}_{\epsilon}} - g^{R_f}_{{y^{state}}_{\epsilon}}\right)\left(I_{nystate} \otimes \Sigma\left(g^{C_h}_{\epsilon} - g^{C_f}_{\epsilon}\right)^{\mathrm{T}}\right)} \\
& + 2\gamma^2 g^{C_h}_{y^{state}}\left(g^{C_h}_{\epsilon}\Sigma\left(g^{R_h}_{\epsilon} - g^{R_f}_{\epsilon}\right)^{\mathrm{T}}\right) \\
& - 2\gamma^2 g^{C_f}_{y^{state}}\left(g^{C_f}_{\epsilon}\Sigma\left(g^{R_h}_{\epsilon} - g^{R_f}_{\epsilon}\right)^{\mathrm{T}}\right) \\
& - \gamma\left(g^{C_h}_{y^{state}} - g^{C_f}_{y^{state}}\right)\left(g^{R_h}_{\epsilon}\Sigma g^{R_h\,\mathrm{T}}_{\epsilon} - g^{R_f}_{\epsilon}\Sigma g^{R_f\,\mathrm{T}}_{\epsilon}\right) \\
& - \gamma\left(g^{C_h}_{y^{state}} - g^{C_f}_{y^{state}}\right)\left(\boxed{g^{R_h}_{\sigma\sigma} - g^{R_f}_{\sigma\sigma}}\right) \\
& - \gamma\left(g^{C_h}_{y^{state}} - g^{C_f}_{y^{state}}\right)\left(g^{R_h}_{\epsilon\epsilon} - g^{R_f}_{\epsilon\epsilon}\right) vec(\Sigma)
\end{aligned}
$$

$$
\begin{aligned}
\bar{H}^{nlma}_{\sigma\sigma y^{state}} = & \boxed{-2\gamma\left(g^{C_h}_{{y^{state}}_{\epsilon}} - g^{C_f}_{x\epsilon}\right)\left(I_{nystate} \otimes \Sigma\left(g^{R_h}_{\epsilon} - g^{R_f}_{\epsilon}\right)^{\mathrm{T}}\right)} \\
& \boxed{-2\gamma\left(g^{R_h}_{{y^{state}}_{\epsilon}} - g^{R_f}_{{y^{state}}_{\epsilon}}\right)\left(I_{nystate} \otimes \Sigma\left(g^{C_h}_{\epsilon} - g^{C_f}_{\epsilon}\right)^{\mathrm{T}}\right)} \\
& + 2\gamma^2 g^{C_h}_{y^{state}}\left(g^{C_h}_{\epsilon}\Sigma\left(g^{R_h}_{\epsilon} - g^{R_f}_{\epsilon}\right)^{\mathrm{T}}\right) \\
& - 2\gamma^2 g^{C_f}_{y^{state}}\left(g^{C_f}_{\epsilon}\Sigma\left(g^{R_h}_{\epsilon} - g^{R_f}_{\epsilon}\right)^{\mathrm{T}}\right) \\
& - \gamma\left(g^{C_h}_{y^{state}} - g^{C_f}_{y^{state}}\right)\left(g^{R_h}_{\epsilon}\Sigma g^{R_h\,\mathrm{T}}_{\epsilon} - g^{R_f}_{\epsilon}\Sigma g^{R_f\,\mathrm{T}}_{\epsilon}\right) \\
& - \gamma\left(g^{C_h}_{y^{state}} - g^{C_f}_{y^{state}}\right)\left(\boxed{R_{h,\sigma\sigma} - R_{f,\sigma\sigma}}\right) \\
& - \gamma\left(g^{C_h}_{y^{state}} - g^{C_f}_{y^{state}}\right)\left(g^{R_h}_{\epsilon\epsilon} - g^{R_f}_{\epsilon\epsilon}\right) vec(\Sigma)
\end{aligned}
$$

The green box points to the difference across the bifurcation methods. *BIF* is presented first, whereas the second equation shows the corresponding derivative under *BIFN*. The only difference between the two approaches is the second-order uncertainty adjustment term of the excess rate of return on home assets. *BIF* includes a one-step-ahead risk adjustment given by state space methods. By contrast, *BIFN* considers a cumulative uncertainty correction that can be linked to the stochastic steady state.

[22]Note that derivatives of a policy function, that has been shifted one period into the future, with respect to σ do not only include the risk correction term but also the coefficients on ϵ_{t+1} as the latter is scaled by σ. However, I will slightly abuse the notation by letting g_{σ^j} and y_{σ^j} denote only the risk correction in order to simplify the exposition.

In addition, the red boxes highlight terms included also by DS. Why does the difference exist? Devereux and Sutherland (2010) eliminate the remaining terms by exploiting the second-order approximation of the expected future excess return

$$E_t\left[\hat{R}^{(2)}_{xt+1}\right] = -\frac{1}{2}\left(g_\epsilon^{R_h}\Sigma g_\epsilon^{R_h\,\mathrm{T}} - g_\epsilon^{R_f}\Sigma g_\epsilon^{R_f\,\mathrm{T}}\right) + \frac{1}{2}\gamma\left(g_\epsilon^{C_h} + g_\epsilon^{C_f}\right)\Sigma\left(g_\epsilon^{R_h} - g_\epsilon^{R_f}\right)^{\mathrm{T}}. \tag{3.45}$$

As shown by (3.45), this substitution requires that $E_t\left[\hat{R}_{xt+1}\right]$ is a constant up to a second-order accuracy. In general, this condition is not fulfilled by the state space approximate solution. To see this consider the pruned second-order approximation of $\hat{R}_x$:

$$\begin{aligned}\hat{R}^{(2)}_{xt+1} = {} & \left(g^{R_h}_{y^{state}} - g^{R_f}_{y^{state}}\right)\hat{y}_t^{(2),state} + \left(g_\epsilon^{R_h} - g_\epsilon^{R_f}\right)\sigma\epsilon_{t+1} + \frac{1}{2}\left(g_{\sigma\sigma}^{R_h} - g_{\sigma\sigma}^{R_f}\right)\sigma^2 \\ & + \frac{1}{2}\left(g^{R_h}_{y^{state}y^{state}} - g^{R_f}_{y^{state}y^{state}}\right)\hat{y}_t^{(1),state}\otimes\hat{y}_t^{(1),state} + \frac{1}{2}\left(g_{\epsilon\epsilon}^{R_h} - g_{\epsilon\epsilon}^{R_f}\right)\sigma^2\epsilon_{t+1}\otimes\epsilon_{t+1} \\ & + \left(g^{R_h}_{y^{state}\epsilon} - g^{R_f}_{y^{state}\epsilon}\right)\sigma\hat{y}_t^{(1),state}\otimes\epsilon_{t+1}.\end{aligned} \tag{3.46}$$

(3.46) can be also rewritten in terms of approximation increments (Lan and Meyer-Gohde, 2014a)

$$\hat{R}^{(2)}_{xt+1} = d\hat{R}^{(1)}_{xt+1} + d\hat{R}^{(2)}_{xt+1}, \tag{3.47}$$

$$d\hat{R}^{(1)}_{xt+1} = \left(g_\epsilon^{R_h} - g_\epsilon^{R_f}\right)\sigma\epsilon_{t+1}, \tag{3.48}$$

and

$$\begin{aligned}d\hat{R}^{(2)}_{xt+1} = {} & \left(g^{R_h}_{y^{state}} - g^{R_f}_{y^{state}}\right)d\hat{y}_t^{(2),state} + \frac{1}{2}\left(g_{\sigma\sigma}^{R_h} - g_{\sigma\sigma}^{R_f}\right)\sigma^2 \\ & + \frac{1}{2}\left(g^{R_h}_{y^{state}y^{state}} - g^{R_f}_{y^{state}y^{state}}\right)d\hat{y}_t^{(1),state}\otimes d\hat{y}_t^{(1),state} \\ & + \frac{1}{2}\left(g_{\epsilon\epsilon}^{R_h} - g_{\epsilon\epsilon}^{R_f}\right)\sigma^2\epsilon_{t+1}\otimes\epsilon_{t+1} \\ & + \left(g^{R_h}_{y^{state}\epsilon} - g^{R_f}_{y^{state}\epsilon}\right)\sigma d\hat{y}_t^{(1),state}\otimes\epsilon_{t+1}.\end{aligned} \tag{3.49}$$

Suppose now that we start in period $t = 0$ with $d\hat{y}_0^{(1),state} = d\hat{y}_0^{(2),state} = 0$. Then, because the uncertainty correction is included in the recursion (see 3.49), it holds that $E_0\left[\hat{R}^{(2)}_{x1}\right] \neq E_1\left[\hat{R}^{(2)}_{x2}\right]$. On the other hand, $E_t\left[\hat{R}^{(2)}_{xt+1}\right]$ implied by the nonlinear moving average is constant for all t.

As pointed by Lan and Meyer-Gohde (2014a), the state space approximation and the nonlinear moving approximation are asymptotically identical up to a second order of accuracy. Thus, the expected difference in log rates of return implied by the former is asymptotically constant. Moreover, if we initialize the state space approximation at this asymptotic point, then it will yield exactly the

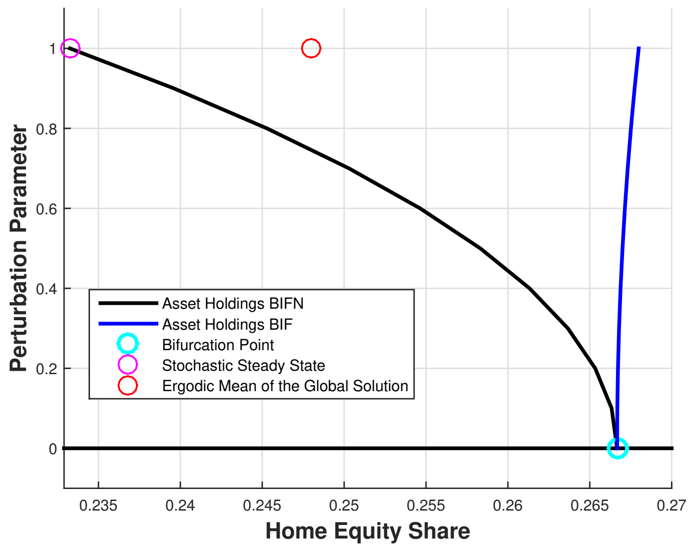


Figure 3.3: Risk-adjusted portfolio holdings. $\sigma = 0$ *corresponds to the deterministic steady state, whereas* $\sigma = 1$ *denotes fully stochastic environment. The ergodic mean of the global solution is taken from Rabitsch et al. (2015).*

same predictions as the nonlinear moving average at every point in time. However, this approach requires ex-ante knowledge of the stochastic steady state.

3.4.3 The Direct Effect of Uncertainty on Portfolio Holdings

One of the drawbacks of DS highlighted by Rabitsch et al. (2015) is the fact that it fails to capture the direct effect of the presence of uncertainty on gross asset positions. Thus, the question arises whether higher-order risk correction may affect model implications in a significant way and thereby improve quality of the local approximation. To tackle this question, I extend the first-order approximations of portfolio holdings by including the second-order uncertainty correction

$$\alpha_t = \bar{\alpha} + \tilde{g}^{\alpha}_{y^{state}} \hat{y}_t^{(1),state} + \frac{1}{2}\tilde{g}^{\alpha}_{\sigma\sigma}, \tag{3.50}$$

and

$$\alpha_t = \bar{\alpha} + \tilde{g}^{\alpha,nlma}_{y^{state}} d\hat{y}_t^{(1),state} + \frac{1}{2}\alpha_{\sigma\sigma}. \tag{3.51}$$

In contrast to the first-order risk adjustment, the second-order term is in general not equal to zero, even under the normality assumption, and its value depends on method being used. Figure 3.3 compares risk-adjusted portfolio holdings for a particular size of uncertainty, given that all state variables take their steady

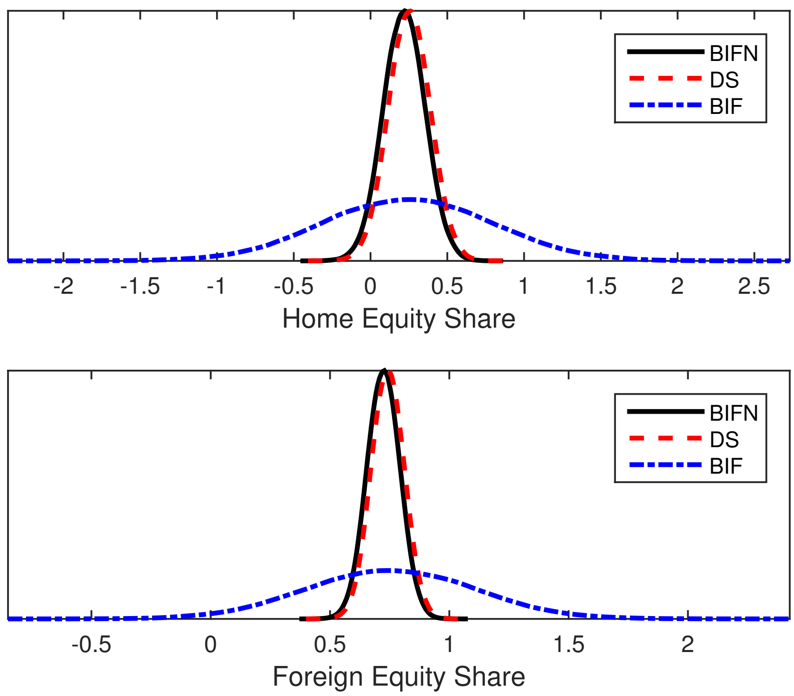


Figure 3.4: Ergodic distributions of portfolio holdings (home and foreign assets) owned by home country. *A proxy for the ergodic distribution is obtained by simulating 10 million of periods.*

state values.[23] The ergodic mean of the global solution, reported by Rabitsch et al. (2015), is used as a benchmark.[24] As the size of uncertainty goes asymptotically to zero and the bifurcation point is reached, home representative agent holds 26.7 % of home equity. This foreign equity bias is caused by the positive correlation between domestic "labor" and "capital income". According to Figure 3.3, *BIFN* takes into account the cross-country heterogeneity in hedging needs, caused by different degrees of economic uncertainty. Since the foreign country is subject to more volatile shocks, its precautionary motive is stronger and thus its long position in the home equity becomes larger as σ increases. By contrast, *BIF* fails to account for this effect and predicts that home country raises its holdings of the domestic equity.

The second-order accurate effect of uncertainty on the ergodic distribution of asset holdings is visualized by Figure 3.4. Due to the stronger precautionary motive in the foreign country, the distribution under *BIFN* is slightly shifted to the left, compared to *DS*.

[23]Given σ, risk-adjusted portfolio holdings are given by $\bar{\alpha} + \frac{1}{2}\tilde{g}^{\alpha}_{\sigma\sigma}\sigma^2$ (BIF) or $\bar{\alpha} + \frac{1}{2}\alpha_{\sigma\sigma}\sigma^2$ ($BIFN$).

[24]Although ergodic mean and the risk-adjusted value are two distinct concepts, this comparison can determine whether heterogeneous precautionary motives, reflected by the ergodic mean, are also accounted for at the starting point of the approximation.

Ergodic Mean of GS	BIFN	BIF	DS plus Updating
-0.168	-0.2857	1.3021e-4	-6.19

Table 3.2: Risk-adjusted net foreign assets. *The ergodic mean of the global solution (GS) is taken from Rabitsch et al. (2015). The value for NFA implied by the iterative DS procedure is taken from its working paper version. Entries for BIF and BIFN represent second-order risk correction terms.*

3.4.4 Non-zero Net Foreign Asset Positions

Another issue raised by Rabitsch et al. (2015) refers to the fact, that the approximation of an asymmetric two-country model is still computed at zero net foreign assets, although the presence of asymmetries implies most likely non-zero positions. Alternatively, Devereux and Sutherland (2009) propose an iterative procedure to update the value for net foreign assets at the approximation point. However, Rabitsch et al. (2015) show that this procedure reduces the accuracy of the local approximation. Constructing the approximation around a point with non-zero net foreign assets (e.g. stochastic steady state) is beyond the scope of this paper. Nevertheless, it is of interest to investigate whether *BIFN* can mitigate the problem by yielding correctly risk-adjusted net foreign assets. In particular, I propose to start with net position equal to zero and let model's risk characteristics endogenously determine the risk-adjusted net foreign assets that are used as a starting point for the approximation.

Table 3.2 gives risk-adjusted net asset positions implied by different methods. The ergodic mean of the global solution reported by Rabitsch et al. (2015) is used again as a benchmark. Mean net foreign liabilities of home country under the global solution represent 16.8 % of the steady state domestic output.[25] *BIFN* correctly captures the effect of uncertainty and predicts a negative home net foreign asset position caused by a stronger precautionary motive in foreign country. On the other hand, *BIF* yields slightly positive net assets. Though it is important to note that the net position reported for *BIF*, consistently with the state space approach, accounts only for one-step-ahead constant uncertainty correction and transits deterministically to the second-order accurate stochastic steady state (see Lan and Meyer-Gohde, 2014a). In general, differences in the implied gross assets between *BIF* and *BIFN* may lead to different values of net positions in the stochastic steady state. Yet, numerical exercises show that this difference can be neglected in the case of our example model.[26] Nevertheless, in short-run simulations, constant risk correction terms still differ.

[25]Steady state output is normalized to 1.

[26]Still, *BIF* and *BIFN* yield different ergodic moments for net foreign position as presented in the next section.

	GS		BIFN		BIF		DS	
	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.
NFA	-0.168	1.11	-0.17	1.157	-0.169	1.233	-0.17	1.157
θ_h^h	0.248	0.13	0.239	0.138	0.266	0.562	0.265	0.138
θ_h^f	0.723	0.066	0.728	0.07	0.735	0.36	0.736	0.07

Table 3.3: Ergodic moments. *Mean and standard deviation of the global solution are taken from Rabitsch et al. (2015). To obtain moments of local approximation methods, the model is simulated ten times. Each simulation consists of 10 million periods.*

According to *DS* with an updating procedure, home country's debt adds up to 619 % of the steady state output. Thus, the iterative algorithm overestimates the precautionary motive of the Foreign country and yields net foreign positions that differ greatly from the implications of the global solution.

3.4.5 Performance Evaluation

The analysis so far shows that *BIFN* can account for direct effects of uncertainty on both net and gross asset positions. In the following, I investigate whether capturing these effects improves the quality of the approximation. To this end, I compare ergodic moments implied by the different methods and conduct the Euler equation error test to measure their accuracy.

Simulated Moments

This section reports the ergodic moments of gross and net asset holdings implied by the three perturbation methods. As in the case of previous sections, the global solution, reported by Rabitsch et al. (2015), is used as a benchmark.

To obtain moments of local approximation techniques, the model is simulated 10 times. Each simulation contains 10 million observations. Table 3.3 reports the results of this exercise. First, as already discussed, (both home and foreign) equity holdings of home country implied by *BIF* are characterized by high volatility. The standard deviation of home equity share is more than four times greater than predicted by the global solution. This translates also into a higher volatility of net foreign asset positions. On the other hand, *DS* and *BIFN* yield second moments for both gross positions and the implied net foreign assets that are more in line with the global solution. Second, among local approximation methods considered in this study, the mean of portfolio holdings implied by *BIFN* is closest to its global solution counterpart. The largest discrepancy among the available assets amounts to 3.63 %. By contrast, this figure is nearly twice as large for *DS*.

Euler Equation Errors

The focus of this investigation lies on the importance of uncertainty adjustment of portfolio holdings. In the underlying model, there is no Euler equation embedding asset positions explicitly. Therefore, I use pseudo Euler equation errors, proposed by Kazimov (2012), to measure the accuracy of local approximations. In particular, I directly introduce assets into Euler equations as follows

$$NFA_{Ht} = E_t \left[\left(\beta(C_{Ht}) \left(\frac{C_{Ht}}{C_{Ht+1}} \right)^{\gamma} \right) (R_{Ht+1}\alpha_t + R_{Ft+1}(NFA_{Ht} - \alpha_t)) \right], \quad (3.52)$$

and

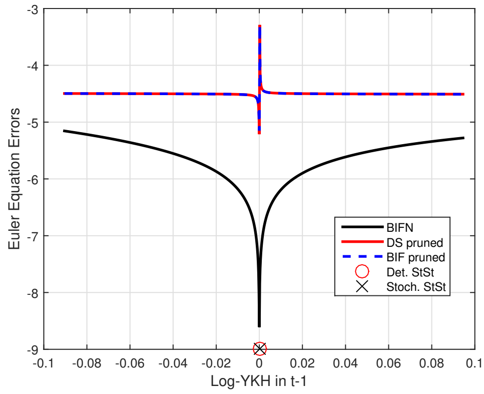
$$NFA_{Ht} = E_t \left[\left(\beta(C_{Ft}) \left(\frac{C_{Ft}}{C_{Ft+1}} \right)^{\gamma} \right) (R_{Ht+1}\alpha_t + R_{Ft+1}(NFA_{Ht} - \alpha_t)) \right]. \quad (3.53)$$

Equations (3.52) and (3.53) can be interpreted as home and foreign agent's portfolio Euler equation, respectively. The underlying idea is that the rate of return on an optimally constructed portfolio must obey similar restrictions as individual asset returns. In the following, I use the common logarithm of the absolute value of approximation errors as a measure of accuracy. According to this definition an Euler equation error of -3 implies one dollar error for every thousand dollars spent. To obtain a scalar measure of accuracy, I average the errors associated with (3.52) and (3.53). Figure 3.5 evaluates the performance of local approximations

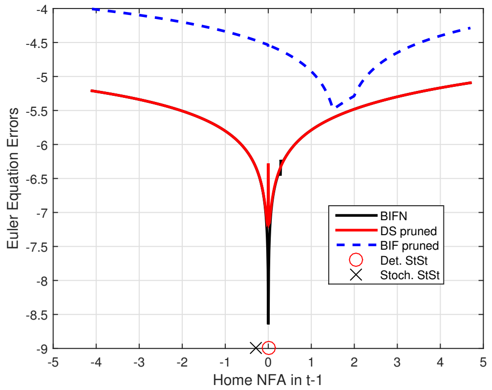
	Average Errors			Maximum Errors		
	DS	BIF	BIFN	DS	BIF	BIFN
YKH	-4.5018	-4.5024	-5.6531	-3.2887	-3.2896	-5.1536
NFA	-5.5762	-4.5120	-5.5829	-5.0926	-4.0066	-5.0926
QH	-6.4219	-4.5417	-6.4699	-5.8697	-4.4380	-5.3270

Table 3.4: Euler equation errors

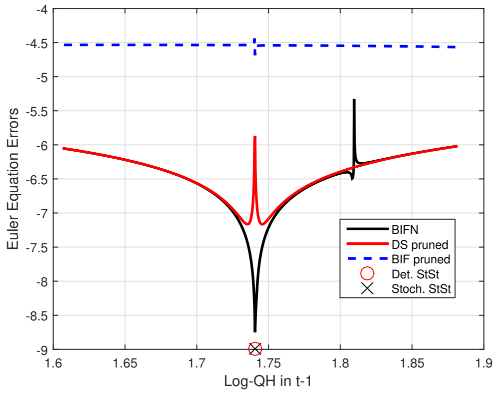
within an interval based on the ergodic set (under *DS*) of home "capital income", home net foreign asset and price of home equity, respectively. All other state variables always take their steady state values. According to the figure, *BIFN* performs uniformly better over the entire ergodic set of home "capital income", with maximal improvement being roughly five orders of magnitude. On the other hand, the advantage of BIFN is less pronounced for net foreign assets and price of home equity. Compared to *DS*, *BIFN* performs significantly better in the immediate neighborhood of the deterministic steady state. However, there exists also a small subset where *DS* is associated with lower approximation errors. Table 3.4 shows



(a) Home 'Capital Income'



(b) Home NFA



(c) Price of Home Equity

Figure 3.5: Euler equation errors. *Euler equation errors are computed within intervals based on the respective ergodic sets under DS. All other state variables always take their steady state values. Each ergodic set is determined by simulating 10 million periods and subsequently discretized by 1001 equidistant grid points.*

that in case of net foreign assets and price of home equity, *BIFN* and *DS* perform similarly on average. In the case of the home "capital income", *BIFN* leads to a significant improvement.

On the other hand, it is apparent from Figure 3.5 and Table 3.4 that *BIF* is outperformed by its competitors. This result reflects the excessive volatility of portfolio holdings implied by this method. Therefore, the bifurcation theory should be applied together with an approximation including a cumulative risk correction, e.g., the nonlinear moving average approximation.

3.5 Conclusion

I propose a combination of bifurcation methods and the nonlinear moving average approximation (*BIFN*) as a technique to solve asymmetric DSGE models with portfolio choice. The use of bifurcation theory overcomes the problem of indeterminacy of portfolio holdings, whereas the nonlinear moving average accounts for uncertainty correction at the stochastic steady state.

The main advantage of the proposed method is the fact that it can be used to compute higher-order approximations of portfolio holdings. Thereby, it can account for the direct effect of uncertainty on both, gross and net asset holdings. This is reflected by the starting point of the approximation as well as by the moments of the implied ergodic distribution. Moreover, *BIFN* improves accuracy of the approximation measured by Euler equation errors relative to the workhorse routine developed by Devereux and Sutherland (2010, 2011). The biggest documented average accuracy gain is of one order of magnitude, whereas the maximum improvement amounts to five orders.

As a local approximation method, *BIFN* can be applied to investigate a variety of issues in macro-finance within the DSGE framework with a large state space. In particular, it can be to tackle economic questions that require at least third-order approximation of the non-portfolio variables. One example is the analysis of channels through which time-varying uncertainty affects country portfolios (see chapter 4).

Chapter 4

Uncertainty Shocks and the Great Retrenchment: A DSGE Perspective

This essay investigates the implications of global uncertainty shocks for international banking portfolios and macroeconomic aggregates. To this end, I employ a two-country DSGE framework with financial frictions à la Gertler and Karadi (2011) and endogenous portfolio choice. Countries are assumed to be ex-ante asymmetric, which allows me to consider both developed and emerging market economies. The model implies a home bias in banking assets that is consistent with the data. Furthermore, an increase in uncertainty originating in the financial sector leads to a reduction in banks' cross-country assets and results in a worldwide decline in economic activity. These findings support the empirical evidence that uncertainty shocks played an important role for the great retrenchment in capital flows during the global financial crisis. Finally, they point to financial markets as the relevant source of uncertainty.

Keywords: Asymmetric Countries, Portfolio Choice, International Capital Flows, Stochastic Volatility, Financial Frictions, Financial Uncertainty, Third-Order Approximation
JEL Classification Numbers: F34, F41, F44, G11

4.1 Introduction

The years preceding the global financial crisis saw an explosion in cross-country asset holdings as documented by Lane and Milesi-Ferretti (2001, 2007). However, international financial flows evaporated during the crisis - a phenomenon labeled by Tille and Van Wincoop (2010) as *the great retrenchment*. This resulted in a reduction in external portfolios and led consequently to a stronger home bias in assets.

Various empirical studies have documented that cross-border bank lending played an important role - particularly for advanced economies - during the retrenchment episode.[1] Furthermore, literature on capital flows argues that global uncertainty is an important push factor driving international capital flows. In particular, there exists empirical evidence that an increase in uncertainty during the crisis forced investors to take more cautious view of the investment opportunities and led ultimately to *the great retrenchment*.[2]

Figure C.1 provides some insights on the negative co-movement of uncertainty and cross-border banking assets and liabilities in the USA, Germany and Brazil during the global crisis. Following Ludvigson et al. (2015), I consider two types of uncertainty: "macroeconomic", i.e., uncertainty about real economic fundamentals, and "financial", i.e., uncertainty originating in the financial sector. In all three countries, outstanding claims and liabilities fell during the crisis. At the same time, global macroeconomic and financial uncertainty, proxied, respectively, by measures of Jurado et al. (2015) and Ludvigson et al. (2015), rose sharply.

The goal of this paper is to shed more light on these empirical findings by identifying factors that can justify the reduction in cross-country banking assets in response to global uncertainty shocks. Moreover, it investigates the macroeconomic implications of such shocks and uncovers their transmission mechanism. To this end, I introduce time-varying volatility of TFP and financial disturbances into a two-country model with endogenous portfolio choice employed previously by Dedola et al. (2013). Since this framework embeds financial intermediaries operating under leverage constraints à la Gertler and Karadi (2011), it is suitable to investigate the effects of uncertainty on banks' portfolio holdings. In contrast to Dedola et al. (2013), I assume that countries are ex-ante asymmetric, which allows me to distinguish between developed countries and emerging market economies.[3]

The model implies a home bias in asset holdings, representing a starting point of the analysis, which is consistent with the data.[4] Moreover, it generates a fall in banks' external assets in response to an adverse financial uncertainty shock, modeled as an increase in the volatility of shocks to the survival probability of financial intermediaries. In contrast, an increase in macroeconomic uncertainty

[1] See, among others, Bertaut and Demarco (2009), Hoggarth et al. (2010), as well as Tille and Van Wincoop (2010)

[2] See, among others, Tille and Van Wincoop (2010), Forbes and Warnock (2012), Fratzscher (2012), Rey (2015), and Nier et al. (2014)

[3] Another rationale for this modeling choice is the focus on global shocks. In a symmetric setup, such shocks would generate identical responses of the macroeconomic variables in both countries.

[4] Coeurdacier and Rey (2013) document a significant home bias in banking assets for various emerging and advanced economies. The degree of home bias, measured as $BHB = 1 - \frac{Share\ of\ Foreign\ Banking\ Assets\ in\ Country's\ Banking\ Assets}{Share\ of\ Foreign\ Banking\ Assets\ in\ Total\ Foreign\ Outstanding\ Loans}$, ranges from 60 to 90 %.

leads to an expansion of banks' external portfolios. In both cases, capital flows are driven by a change in hedging needs of financial intermediaries. Besides inducing a change in external assets that is consistent with the empirical evidence, an adverse global financial uncertainty shock leads to a decline in economic activity in both economies. The key feature of the model responsible for this outcome is tightening of the endogenous leverage constraint which in turn triggers the financial accelerator mechanism. On the other hand, an increase in TFP volatility leads to non-synchronized dynamics of macroeconomic variables across countries. This result is driven by heterogeneous precautionary motives implied by the asymmetry assumption. All in all, the analysis provides a theoretical rationale for a fall in external banking assets in response to a rise in uncertainty. It also points to financial markets as the source of uncertainty that caused *the great retrenchment* and contributed to the worldwide recession.

Related Literature This paper brings together two strands of literature. First, it is related to the DSGE literature on the macroeconomic effects of stochastic volatility shocks.[5] Second, it is linked to the DSGE literature on cross-country portfolios.[6] To my knowledge, there exists only one other study making the same connection. In particular, Blengini (2012) employs a two-country endowment DSGE model to investigate how uncertainty surrounding demand and supply factors affect households' international portfolios. Besides methodology applied to solve the model, this paper differs from the work of Blengini (2012) by employing a DSGE framework with two ex-ante asymmetric production economies including an explicitly modeled financial sector. This setup is suitable to analyze international activities of financial intermediaries and allows for direct effects of portfolio composition on macroeconomic variables.

Another paper closely related to this study is a work conducted by Gourio et al. (2015). The authors analyze the effects of expropriation risk faced by foreign investors on real economy and international capital flows. In contrast to my work, their "risk" or "uncertainty" shock does not correspond to a stochastic volatility process but is modeled as a time-varying probability of expropriation. Another difference is the fact that their framework, similar to Blengini (2012), is a two-country endowment economy.

The rest of the paper is organized as follows. The theoretical model is outlined in section 4.2. In section 4.3, I discuss the chosen calibration and the method used to solve the model. Section 4.4 presents the results. Finally, section 4.5 concludes.

[5]See, e.g., Caldara et al. (2012), Fernández-Villaverde et al. (2011), Mumtaz and Zanetti (2013), Born and Pfeifer (2014), Basu and Bundick (2017), and Bonciani and Van Roye (2016).

[6]See, e.g., Devereux and Sutherland (2009, 2010, 2011), Devereux and Yetman (2010), Yao (2012), Coeurdacier (2009), Coeurdacier et al. (2011), Coeurdacier and Rey (2013), and Dedola et al. (2013).

4.2 The Model

To investigate the economic consequences of global uncertainty shocks, I employ an open economy real DSGE framework with endogenous portfolio choice and financial frictions à la Gertler and Karadi (2011). The world consists of two ex-ante asymmetric countries: home and foreign. I introduce asymmetries into the model by assuming that that foreign first-moment TFP shock is twice as volatile as its counterpart in home country. This assumption is based on the empirical evidence that emerging market countries are characterized by higher macroeconomic uncertainty (Aguiar and Gopinath, 2007). Therefore, foreign country can be interpreted as an emerging market economy. Finally, it should be highlighted that the presence of asymmetries is crucial for the analysis conducted in this paper. It generates heterogeneous precautionary motives across countries and thereby implies non-identical responses to a global uncertainty shock. In contrast, both countries would react in exactly the same way in a symmetric setup.

One period in the model corresponds to one quarter and each country is populated by four types of agents: households, financial intermediaries, goods producers and capital producers. In the following, I will present equations only for home country. Foreign variables will be denoted with an asterisk.

4.2.1 Households

There exists a continuum of identical households of unity mass. Within each household, there are $1 - f$ workers and f bankers. Workers supply labor and earn wages, whereas each banker manages a financial intermediary and accumulates funds ("net worth") that she transfers to the household upon exiting the business. To merge the within-household heterogeneity with the representative agent framework, I assume that there is perfect consumption sharing within each family.

Household's preferences are given by

$$max \quad E_0 \sum_{t=0}^{\infty} \Xi_t \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \right], \tag{4.1}$$

where C_t denotes consumption and L_t is labor supply. Moreover, γ represents the inverse of intertemporal elasticity of substitution, φ is the inverse of Frish elasticity of labor supply and χ denotes the weight of the disutility of labor supply. Finally, Ξ_t is the endogenous discount factor, one of the mechanisms proposed by Schmitt-Grohé and Uribe (2003) to ensure stationarity of the approximate solution under the assumption of incomplete markets. Following Devereux and Yetman (2010), the endogenous discount factor is given by

$$\Xi_t = \beta \left(C_{At} \right) \Xi_{t-1}, \tag{4.2}$$

with $\Xi_0 = 1$ and $\beta(C_{At}) = \omega_\beta(1 + C_{At})^{-\eta_\beta}$. Note that endogenous discount factor does not depend on the individual consumption but on the economy average (C_{At}). This assumption prevents the household from internalizing the effects of its savings choice on the discount factor and thereby simplifies the analysis. However it does not introduce an additional variable into the model, as household's individual and aggregate consumption are equalized along the equilibrium path.

Following Gertler and Karadi (2011), households do not have a direct access to capital stock. Rather, they save by depositing funds with domestic financial intermediaries.[7] The implicit assumption is that households supply to banks other than the ones they own. Bank deposits, denoted by D_t, are one period real riskless bonds paying the gross real return R_t from t to $t+1$. The budget constraint faced by the household is thus given by

$$C_t + D_t = W_t L_t + R_{t-1} D_{t-1} + T_t, \tag{4.3}$$

where W_t refers to the real wage and T_t are net profits from the ownership of both non-financial firms and financial intermediaries. Let U_{Ct} denote the marginal utility of consumption and $\Lambda_{t,t+1}$ the household's stochastic discount factor. Then maximizing the life-time utility with respect to consumption, labor and savings subject to the flow of funds constraint (4.3) yields the following first-order conditions

$$W_t U_{Ct} = \chi L_t^{\varphi}, \tag{4.4}$$

and

$$E_t\left[\Lambda_{t,t+1}\right] R_t = 1, \tag{4.5}$$

with $\Lambda_{t,t+1} \equiv \beta(C_t)\frac{U_{Ct+1}}{U_{Ct}}$ and $U_{Ct} = C_t^{-\gamma}$.

4.2.2 Nonfinancial Firms

There exist two types of nonfinancial firms in each country: goods producers and capital producers.

Goods Producers

In period t competitive firms with identical constant returns to scale technology combine capital stock purchased at the end of period $t-1$, K_{t-1} and labor, L_t, to produce final goods, Y_t. This process is governed by the following Cobb-Douglas production function

$$Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}, \tag{4.6}$$

[7] The assumption of country-specific deposit markets reflects the fact that private funds enter the global financial system through domestic institutions. See also Maggiori (2017).

where $\alpha \in (0,1)$ and A_t denotes an exogenously given technology level. There are no adjustment costs at the firm level and thus the intermediate capital producers' maximization problem is static. In particular, at the end of each period the firm sells the remaining capital stock and purchases new capital that will be employed in the subsequent period. To finance capital acquisition, the firm must obtain funds from financial intermediaries. To this end, it issues state contingent claims in the amount equal to the number of purchased capital units. Thus, arbitrage requires that these claims are traded at the price of a unit of capital, Q_t. Given that R_k denotes the gross real interest rate paid on state contingent securities, the representative intermediate good producer chooses labor input to maximize her current profits

$$Y_t + (1-\delta) K_{t-1} - W_t L_t - R_{kt} Q_{t-1} K_{t-1}, \tag{4.7}$$

where δ is the depreciation rate. Solving this maximization problem yields the following first-order condition

$$W_t = (1-\alpha)(K_{t-1})^{\alpha} L_t^{-\alpha}. \tag{4.8}$$

Note that under assumptions of competitive firms and constant returns to scale, the intermediate producers make zero profits in equilibrium. Thus, the ex-post rate of return on state contingent assets is given by

$$R_{kt} = \frac{\alpha \frac{Y_t}{K_{t-1}} + Q_t (1-\delta)}{Q_{t-1}} \tag{4.9}$$

Capital Producers

Competitive capital producers transform the final output into new capital. To produce I_t units of new capital, i.e. investment, the firm needs to purchase $(1 + f_{inv}(\cdot)) I_t$ units of the final good. $f_{inv}(\cdot)$ denotes investment adjustment costs introduced to generate time-variation in the price of capital. Following Dedola et al. (2013), I assume the following functional form for the adjustment costs

$$f_{inv}(I_t, K_{t-1}) = \frac{\kappa}{2}\left(\frac{I_t}{\delta K_{t-1}} - 1\right)^2 \frac{\delta K_{t-1}}{I_t}, \tag{4.10}$$

with $\kappa > 0$. The capital producers choose I_t that maximizes expected lifetime profits given by

$$E_t\left[\sum_{k=0}^{\infty} \Lambda_{t,t+k}\left(Q_{t+k} I_{t+k} - \left[1 + f_{inv}(I_{t+k}, K_{t+k-1})\right] I_{t+k}\right)\right]. \tag{4.11}$$

The corresponding first-order condition determines the price of one unit of capital

$$Q_t = 1 + \kappa \left(\frac{I_t}{\delta K_{t-1}} - 1 \right). \tag{4.12}$$

Finally, note that capital producers can earn non-zero profits outside of the steady state. These profits are assumed to be redistributed lump sum to households.

4.2.3 Financial Intermediaries

In contrast to deposit markets, markets for banks' assets are assumed to be integrated across countries. In other words, financial intermediaries can provide funds to goods producers in both countries. Their operations are financed by a combination of deposits, D_t, held by domestic households, and net worth, N_t, which is accumulated from retained earnings. Hence, the balance sheet of a financial intermediary j in the home country is given by

$$Q_t S_{jht} + Q_t^* S_{jft} = D_{jt} + N_{jt}, \tag{4.13}$$

where S_{jht} and S_{jft} denote bank j's holdings of home and foreign assets, respectively. As noted above, deposits made with banks at time $t-1$ pay the non-contingent real gross return R_{t-1} in the subsequent period. In contrast, assets held by intermediaries earn the stochastic return R_{kt} or R_{kt}^* over the same period. Then, the law of motion for net worth of an intermediary j is given by

$$\begin{aligned} N_{jt} &= R_{kt} Q_{t-1} S_{jht-1} + R_{kt}^* Q_{t-1}^* S_{jft-1} - R_{t-1} D_{jt-1} \\ &= \left(R_{kt} - R_{t-1}\right) Q_{t-1} S_{jht-1} + \left(R_{kt}^* - R_{t-1}\right) Q_{t-1}^* S_{jft-1} + R_{t-1} N_{jt-1}, \end{aligned} \tag{4.14}$$

where the second equality follows from the balance sheet condition.

Intermediaries have an incentive to operate in period t only if expected discounted rates of return on assets do not lie below costs of borrowing. By applying the household's discount factor, this condition can be written as

$$E_t \left[\Lambda_{t,t+1} \left(R_{kt+1} - R_t \right) \right] \geq 0 \tag{4.15}$$

and

$$E_t \left[\Lambda_{t,t+1} \left(R_{kt+1}^* - R_t \right) \right] \geq 0. \tag{4.16}$$

Under frictionless capital markets, (4.15) and (4.16) always hold with equality. In contrast, the discounted spreads are positive in presence of financial frictions, as they limit the ability of financial intermediaries to obtain funds. Thus, given financial constraints, a bank has an incentive to invest all its funds and retain all earnings until the time it exits the business. The event of exit occurs with time-varying probability $1-\theta_t$, where $\theta_t \equiv \theta\vartheta_t$, with ϑ_t being the disturbance to the banks' survival probability.[8] Upon exiting, a banker transfers its terminal wealth to the household and becomes a worker.[9] Accordingly, financial intermediary j determines optimal asset holdings and the amount of external funds to maximize its franchise value, given by

$$V_{jt} = max \quad E_t \left[\sum_{k=1}^{\infty} \Lambda_{t,t+k} \left(\prod_{i=t+1}^{t+k-1} \theta_i \right) (1-\theta_{t+k}) N_{jt+k} \right], \tag{4.17}$$

with $\left(\prod_{i=t+1}^{t} \theta_i\right) \equiv 1$. Incorporating a finite horizon for financial intermediaries prevents them from accumulating enough net worth such that the financial constraint is no longer binding.

Following Gertler and Karadi (2011), I introduce a moral hazard problem to motivate a limited ability of obtaining funds by financial intermediaries. In particular, at the beginning of each period, a banker can divert a non-bank specific fraction, λ, of her assets and transfers it to her household. In this situation, depositors can force her into bankruptcy and recover the remaining fraction of assets, $1-\lambda$. Hence, households are willing to supply funds to intermediary j only if the continuation value of its operations is greater (or equal) than the gain from diverting the assets, i.e.,

$$V_{jt} \geq \lambda \left(Q_t S_{jht} + Q_t^* S_{jft} \right). \tag{4.18}$$

To solve the model, I firstly write (4.17) recursively

$$V_{jt} = max \quad E_t \left[\Lambda_{t,t+1} \left((1-\theta_{t+1}) N_{jt+1} + \theta_{t+1} V_{jt+1} \right) \right], \tag{4.19}$$

and conjecture that the solution is linear in value of assets and deposits

$$\begin{aligned} V_{jt} &= v_{ht}^k Q_t S_{jht} + v_{ft}^k Q_t^* S_{jft} - v_t D_{jt} \\ &= \mu_{ht} Q_t S_{jht} + \mu_{ft} Q_t^* S_{jft} + v_t N_{jt}, \end{aligned} \tag{4.20}$$

where the second equality follows from the balance sheet condition. v_{it}^k is the marginal gain of holding country i's assets, whereas v_t is the marginal cost of deposits and can be also interpreted as marginal value of net worth, holding the

[8] This shock can be interpreted as a net worth shock since it reduces the internal funds of the banking system. See, e.g., Afrin (2017) or Aoki and Sudo (2012). However, as it also directly affect the stochastic marginal value of the net worth, I will rather refer to a negative realization of this shock as to a bank distress shock.

[9] By applying the law of large numbers, $f(1-\theta_t)$ bankers exit the business in period t. They are replaced by workers who randomly become bankers. As a result, the size of each group remains constant over time.

assets constant.[10] Thus, $\mu_{it} \equiv v^k_{it} - v_t$ can be interpreted as the marginal gain of increasing holdings of country i's assets by one unit financed via deposits.[11] Then, the financial constraint can be written as

$$\mu_{ht} Q_t S_{jht} + \mu_{ft} Q^*_t S_{jft} + v_t N_{jt} \geq \lambda \left(Q_t S_{jht} + Q^*_t S_{jft} \right). \tag{4.21}$$

Maximizing (4.20) subject to (4.21), under the assumption that the financial constraint always binds, yields the following conditions

$$\mu_{ht}(1 + \psi_{jt}) = \lambda \psi_{jt}, \tag{4.22}$$

$$\mu_{ft}(1 + \psi_{jt}) = \lambda \psi_{jt}, \tag{4.23}$$

and

$$Q_t S_{jht} + Q^*_t S_{jft} = \phi_t N_{jt}, \tag{4.24}$$

where ψ_t is the Lagrange multiplier on the incentive constraint. According to (4.22) and (4.23), marginal gains of asset holdings are equalized in an equilibrium, i.e., $\mu_{ht} = \mu_{ft} \equiv \mu_t$, or alternatively, $v^k_{ht} = v^k_{ft} \equiv v^k_t$. Furthermore, ϕ_t denotes the leverage ratio and is given by

$$\phi_t \equiv \frac{v_t}{\lambda - \mu_t}. \tag{4.25}$$

Note that, holding net worth constant, the constraint binds more tightly, when the intermediary can divert a higher fraction of assets, λ, and the excess value of bank assets is low. With low excess value, the franchise value of the intermediary is lower and the managing banker has a strong incentive to divert funds.

To determine expressions for shadow values of assets and deposits, i.e., time-varying coefficients in the value function, I insert the law of motion of net worth into the Bellman equation, (4.19) and verify that the initial guess for the value function is correct for

$$v^k_{ht} = E_t \left[\Lambda_{t,t+1} \Omega_{t+1} R_{kt+1} \right], \tag{4.26}$$

$$v^k_{ft} = E_t \left[\Lambda_{t,t+1} \Omega_{t+1} R^*_{kt+1} \right], \tag{4.27}$$

$$v_t = E_t \left[\Lambda_{t,t+1} \Omega_{t+1} \right] R_t, \tag{4.28}$$

$$\mu_{ht} \equiv v^k_{ht} - v_t = E_t \left[\Lambda_{t,t+1} \Omega_{t+1} \left(R_{kt+1} - R_t \right) \right], \tag{4.29}$$

$$\mu_{ft} \equiv v^k_{ft} - v_t = E_t \left[\Lambda_{t,t+1} \Omega_{t+1} \left(R^*_{kt+1} - R_t \right) \right], \tag{4.30}$$

[10] Given bank's asset holdings, an additional unit of net worth leads to savings in borrowing costs.

[11] Note that the marginal values are not bank specific. The underlying assumption is that there are no structural differences across financial intermediaries.

where Ω_{t+1} is the stochastic marginal value of net worth in period $t+1$, defined in the following way

$$\Omega_{t+1} \equiv 1 - \theta_{t+1} + \theta_{t+1} \left(v_{t+1} + \phi_{t+1} \mu_{t+1} \right). \tag{4.31}$$

Due to the presence of financial frictions, bankers do not only care about consumption fluctuations of their households (reflected by $\Lambda_{t,t+1}$) but they also consider their funding conditions (reflected by Ω_{t+1}).

Since the leverage ratio does not depend on bank specific factors (see 4.25), we can sum across all individual banks to obtain the aggregate leverage constraint

$$Q_t S_{ht} + Q_t^* S_{ft} = \phi_t N_t. \tag{4.32}$$

To obtain the law of motion for net worth of the entire banking system, one has to recognize that it is the sum of net worth of surviving intermediaries, N_{ot}, and net worth of new bankers, N_{nt}

$$N_t = N_{ot} + N_{nt}. \tag{4.33}$$

As already discussed, a fraction $1 - \theta_t$ of financial intermediaries exit the market in period t and are replaced by workers who randomly become bankers. New bankers require a start-up capital to be able to attract funds from depositors. Following Dedola et al. (2013), I assume that the household transfers a fraction, ω, of the portfolio of the representative incumbent. Hence,

$$N_{nt} = \omega \left(Q_{t-1} S_{ht-1} + Q_{t-1}^* S_{ft-1} \right). \tag{4.34}$$

The net worth of the remaining θ_t bankers is given by

$$N_{ot} = \theta_t \left[\left(R_{kt} - R_{t-1} \right) Q_{t-1} S_{ht-1} + \left(R_{kt}^* - R_{t-1} \right) Q_{t-1}^* S_{ft-1} + R_{t-1} N_{t-1} \right]. \tag{4.35}$$

4.2.4 Aggregate Resource Constraint and Remaining Equations

The world output is divided between consumption and investment

$$Y_t + Y_t^* = C_t + C_t^* + \left[1 + f_{inv} \left(I_t, K_{t-1} \right) \right] I_t + \left[1 + f_{inv} \left(I_t^*, K_{t-1}^* \right) \right] I_t^*, \tag{4.36}$$

Capital stocks of the two countries evolve according to following laws of motion:

$$K_t = \left(1 - \delta \right) K_{t-1} + I_t, \tag{4.37}$$

and

$$K_t^* = \left(1 - \delta \right) K_{t-1}^* + I_t^*. \tag{4.38}$$

Clearing conditions for international asset markets are given by

$$K_t = S_{ht} + S_{ht}^* \tag{4.39}$$

and

$$K_t^* = S_{ft} + S_{ft}^*. \tag{4.40}$$

Finally, let $\zeta_{ht} \equiv Q_t (S_{ht} - K_t)$ and $\zeta_{ft} \equiv Q_t^* S_{ft}$. Then, net foreign asset position of home country is defined as[12]

$$NFA_t \equiv \zeta_{ht} + \zeta_{ft}. \tag{4.41}$$

These definitions will prove to be useful when solving the model. In particular, after rewriting model equations in terms of NFA, solving for international portfolios boils down to finding the policy function for ζ_{ht}. The remaining asset holdings can be retrieved from the definition of net foreign assets.

4.2.5 Shock Processes

There are two types of country-specific first-moment shock processes present in the model: technology, A_t and a disturbance to the survival probability of financial intermediaries, ϑ_t

$$A_t = (1 - \rho_A) + \rho_A A_{t-1} + \upsilon_A \sigma_{t-1}^A \epsilon_t^A, \tag{4.42}$$

$$\vartheta_t = (1 - \rho_\theta) + \rho_\theta \vartheta_{t-1} + \upsilon_\theta \sigma_{t-1}^\theta \epsilon_t^\theta. \tag{4.43}$$

ρ_j and σ^j, $j = \{A, \theta\}$, refer to autocorrelation coefficient and standard deviation of the corresponding stochastic disturbance, respectively. υ_i, with $i \in \{A, \theta\}$, is a country-specific parameter, governing the relative size of volatility. Uncertainty shocks are introduced into the model by assuming that standard deviations of shocks vary over time. The corresponding second-moment processes are given by

$$\sigma_t^A = (1 - \rho_{\sigma^A})\bar{\sigma}^A + \rho_{\sigma^A}\sigma_{t-1}^A + \tau_{\sigma^A}\epsilon_t^{\sigma^A}, \tag{4.44}$$

$$\sigma_t^\theta = (1 - \rho_{\sigma^\theta})\bar{\sigma}^\theta + \rho_{\sigma^\theta}\sigma_{t-1}^\theta + \tau_{\sigma^\theta}\epsilon_t^{\sigma^\theta}, \tag{4.45}$$

where $\bar{\sigma}^i$, with $i \in \{A, \theta\}$, refers to the unconditional mean level of σ_t^i. ρ_{σ^i} is again the persistence parameter and τ_{σ^i} is the standard deviation of volatility innovations. Note that second-moment shocks are not country-specific and affect both countries simultaneously.

All innovations are independent and follow a symmetric distribution with bounded support, zero mean and unit variance. The processes are specified in

[12]Since the world consists of two countries only, the following holds for the net asset position of the foreign economy: $NFA_t^* = -NFA_t$

levels, rather than logs to prevent changes in volatility from affecting their mean values through a Jensen's inequality effect.

4.3 Solution Method and Calibration

Due to nonlinearities present in the model, an exact solution is not feasible and thus one must rely on approximation methods. This section firstly describes the technique used to solve the model and then discusses the calibration underlying the analysis conducted in this paper.

4.3.1 Bifurcation Methods

Solving the underlying model is challenging. First, as global solution methods suffer from the curse of dimensionality, they cannot be applied to a framework with a richer state space. Second, standard local approximation methods, i.e., approximating policy functions around a deterministic steady state, cannot be used either. To see this, combine expressions for shadow values of asset holdings, (4.29) and (4.30), with the first-order conditions for banks' maximization problem, (4.22) and (4.23). The resulting equation is

$$E_t\left[\tilde{\Omega}_{t+1}\left(R_{kt+1} - R^*_{kt+1}\right)\right] = 0, \tag{4.46}$$

with $\tilde{\Omega}_t \equiv \Omega_t U_{Ct}$ denoting marginal value of net worth expressed in terms of household's utility (or simply adjusted marginal utility). Note that uncertainty is completely eliminated in the deterministic steady state. As a result, the two assets become perfect substitutes because they differ only in their risk characteristics. Thus, they yield the same rate of return, i.e., $\bar{R}^*_k = \bar{R}^*_k$, with a bar over a variable standing for its value in the deterministic steady state. For this reason, countries' gross asset positions cannot be uniquely pinned down in the non-stochastic steady state.

Furthermore, even if indeterminacy of the approximation point is somehow resolved, the first-order approximation is not sufficient to determine the dynamics of portfolio holdings. The first-order approximation of (4.46) reads

$$E_t\left[\hat{R}_{kt+1}\right] \approx E_t\left[\hat{R}^*_{kt+1}\right], \tag{4.47}$$

where hats denote log-deviations from the deterministic steady state. Thus, up to a first order of accuracy, all assets have the same expected rate of return and portfolio holdings are again indeterminate.[13] Consequently, higher-order perturbations are necessary to obtain approximate dynamics of portfolio holdings.

In response to these problems, new solution methods have been developed. A prominent example is the method developed by Devereux and Sutherland (2010, 2011).[14] These authors provide readily applicable expressions for steady-state and first-order portfolio holdings. The idea underlying their technique is to combine different orders of approximation of the portfolio selection equation and the remaining part of the model. In general, the Nth-order component of the optimal asset allocation makes portfolio selection condition hold true up to the (N+2)th order of approximation. In our framework, the portfolio selection equation can be obtained by taking the difference between (4.46) and its counterpart for foreign country

$$E_t\left[\left(\tilde{\Omega}_{t+1}-\tilde{\Omega}^*_{t+1}\right)R_{xt+1}\right]=0, \tag{4.48}$$

where $R_{xt} \equiv R_{kt} - R^*_{kt}$.

As shown by Fernández-Villaverde et al. (2011), at least third-order approximation is necessary to investigate impulse responses to volatility shocks. Moreover, it can be proven that a third-order approximation of non-portfolio variables depend on the second-order approximation of gross asset holdings. For that reason, I solve the model by employing the asymptotic perturbation method evaluated in chapter 3. Recall that the chosen solution technique is a combination of bifurcation theory, discussed in a static framework by Judd and Guu (2001), and the nonlinear moving approximation developed by Lan and Meyer-Gohde (2013). In the following, I briefly repeat the intuition behind the method by describing its two elements.

Intuition behind Bifurcation Methods

To explain the idea behind bifurcation methods, I use a simplified illustration. Consider a static setup with a portfolio choice problem and suppose that we look for the optimal share of a risky asset in our portfolio, ζ. The solution will depend on σ which is a perturbation parameter, governing the size of uncertainty in the model. $\sigma = 0$ implies a deterministic setup, whereas $\sigma = 1$ refers to fully stochastic world. Then, the solution of this problem may take the form depicted in red in figure 4.1. Given some degree of uncertainty, there exists a unique solution to the portfolio problem, under some regularity conditions, e.g., concavity of the objective function. However, if $\sigma = 0$, the portfolio composition does not matter and there exist infinitely many solutions.[15] Bifurcation methods allow us to select a portfolio,

[13] This is an implication of the certainty equivalence of first-order approximation (see Schmitt-Grohé and Uribe, 2004).

[14] See also Samuelson (1970), Tille and Van Wincoop (2010), and Evans and Hnatkovska (2012).

[15] The change in the number of solution caused by varying the perturbation parameter is called *bifurcation*.

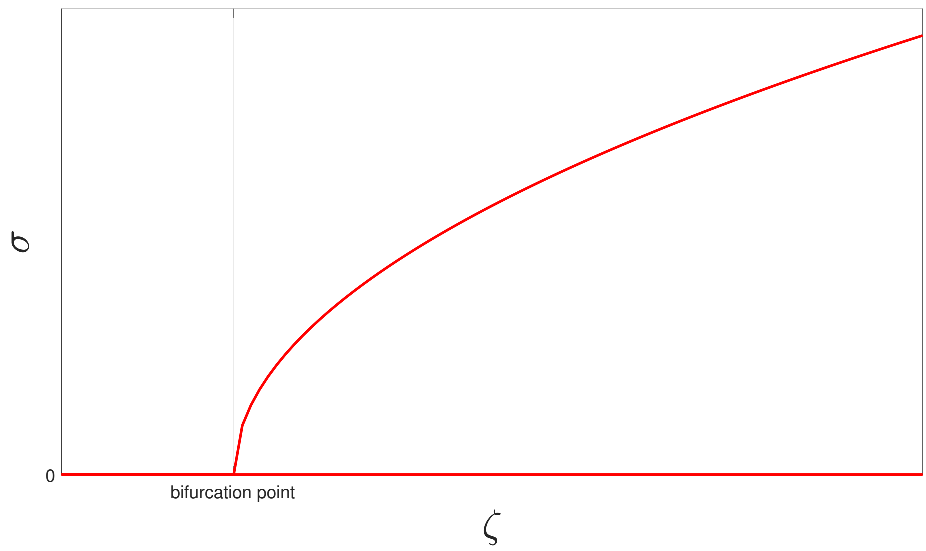


Figure 4.1: Intuition behind bifurcation methods. *The red curve corresponds to the solution set of the hypothetical portfolio choice problem.*

out of the infinitely many potential asset allocations, that is consistent with the solution set in the stochastic environment. Given this bifurcation portfolio, one can apply perturbation techniques to approximate the policy function for asset holdings around this point.

Nonlinear Moving Average Approximation

Instead of using state space approximation methods, I use the nonlinear moving average perturbation developed by Lan and Meyer-Gohde (2013). This technique has three advantages in a setup where risk plays an important role. First, it provides a cumulative uncertainty correction, contrary to the state space methods, providing one-step-ahead correction. Second, it starts the approximation at the stochastic steady state. Finally, it delivers stable nonlinear impulse responses and simulations and thus no pruning algorithm (Kim et al., 2008; Andreasen et al., 2017) is necessary.

To explain the method, I will cast the underlying model into a general form

$$E_t \left[f\left(y_{t+1}, y_t, y_{t-1}, \epsilon_t \right) \right] = 0, \tag{4.49}$$

where $f : \mathbb{R}^{ny} \times \mathbb{R}^{ny} \times \mathbb{R}^{ny} \times \mathbb{R}^{ne} \to \mathbb{R}^{ny}$ is assumed to be analytic, $y_t \in \mathbb{R}^{ny}$ stands for the vector containing both endogenous and exogenous variables, and $\epsilon_t \in \mathbb{R}^{ne}$ is a vector of zero-mean iid shocks. The nonlinear moving average represents a solution to (4.49) as a direct mapping of the history of shocks to model variables, i.e.,

$$y_t = y(\sigma, \epsilon_t, \epsilon_{t-1}, ..), \tag{4.50}$$

where σ refers again to the perturbation parameter. The third-order Taylor approximation of this policy function, given a symmetric distribution of shocks and $\sigma = 1$, is given by:

$$y_t^{(3)} = \bar{y} + \frac{1}{2}y_{\sigma^2} + \sum_{i=0}^{\infty}\left(y_i + \frac{1}{2}y_{\sigma^2 i}\right)\epsilon_{t-i} + \frac{1}{2}\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty} y_{i_1,i_2}\left(\epsilon_{t-i_1}\otimes\epsilon_{t-i_2}\right)$$
$$+\frac{1}{6}\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty} y_{i_1,i_2,i_3}(\epsilon_{t-i_1}\otimes\epsilon_{t-i_2}\otimes\epsilon_{t-i_3}) \tag{4.51}$$

where $\bar{y}$ denotes the deterministic steady state of the model and y_i, y_{i_1,i_2}, y_{i_1,i_2,i_3}, y_{σ^2}, $y_{\sigma^2,i}$ refer to partial derivatives of the policy function evaluated at the deterministic steady state. The expression $y_{SS} + \frac{1}{2}y_{\sigma^2}$ corresponds to the third-order accurate stochastic steady state.[16] Moreover, $y_{\sigma^2,i}$ adjusts the approximate responses of endogenous variables to shock realizations for the risk of future disturbances.

4.3.2 Calibration

My aim is to use the model to provide a general qualitative assessment of the empirical evidence presented above. Therefore, for all parameters I choose values previously used in the literature, with Gertler and Karadi (2011) being the major source. Table 4.1 reports the calibration.

The inverse of Frish elasticity of labor supply, φ, is set to 0.276, whereas γ is equal to 2. The choice of the value for χ ensures that labor supply in deterministic steady state equals 0.33. The capital share, α, in production is 0.33 and the depreciation rate, δ, is set to 0.025.

θ is set to 0.955, implying an average horizon of bankers of almost 6 years. Following Gertler and Karadi (2011), λ and ω are chosen to hit the following two targets: interest rate spread of one hundred basis points per year and banks' leverage ratio of four in the deterministic steady state.

The autocorrelation parameters of the level shocks are set in accordance with Devereux and Yetman (2010) and Afrin (2017). Moreover, the unconditional mean of their respective standard deviations is normalized to 0.01. Parametrization of stochastic volatility processes follows estimation results of chapter 2. Finally, υ_A and υ_A^* are chosen to replicate the empirical observation that emerging market countries are characterized by higher macroeconomic uncertainty (Aguiar and Gopinath, 2007).

[16] As shown by Andreasen (2012), the third-order constant term, y_{σ^3}, corrects the approximation for the skewness of the shocks. Since I assume symmetric distributions, it is equal to zero and thus omitted from (4.51).

Parameter		Value	Justification
Household			
Stst value of the discount factor	β	0.99	Gertler & Karadi (2011)
Inverse intertemporal elasticity of substitution	γ	2	Devereux & Yetman (2010)
Inverse Frish elasticity of labor supply	φ	0.276	Gertler & Karadi (2011)
Relative utility weight of labor	χ	4.7041	$\bar{L} = \frac{1}{3}$
Parameter of the endogenous discount factor	η_β	0.022	Devereux & Yetman (2010)
Parameter of the endogenous discount factor	ω_β	1.0023	$\beta = 0.99$
Nonfinancial Firms			
Effective capital share	α	0.3	Gertler & Karadi (2011)
Depreciation rate	δ	0.025	Gertler & Karadi (2011)
Inverse elasticity of investment w.r.t. the price of capital	η	1.5	Gertler & Kiyotaki
Financial Sector			
Divertable fraction in stst	λ	0.3196	$\bar{\phi} = 4$ & $\bar{R}_k - \bar{R} = 0.0025$
Starting-up transfer	ω	0.0065	$\bar{\phi} = 4$ & $\bar{R}_k - \bar{R} = 0.0025$
Survival rate of bankers	θ	0.955	Assumed
Shock Processes			
Persistence - TFP	ρ_A	0.9	Devereux & Yetman (2010)
Persistence - Survival probability	ρ_θ	0.9	Afrin (2017)
Persistence - Stochastic volatility	ρ_{σ^θ}	0.9	See chapter 2
Unconditional mean of S.D.	$\bar{\sigma}^i$	0.01	Normalization
S.D. - Stochastic volatility	τ_{σ^θ}	0.00045	See chapter 2
Relative size of the volatility - TFP	$\upsilon_A(\upsilon_A^*)$	1 (2)	Aguiar & Gopinath (2007)
Relative size of the volatility - Survival probability	$\upsilon_\theta(\upsilon_\theta^*)$	1 (1)	Assumed

Table 4.1: Calibration

4.4 Results

In this section, I discuss predictions of the underlying model. I start with steady-state portfolios and distinguish between the bifurcation portfolio and asset holdings in the stochastic steady state. Then, the dynamics of cross-border portfolios are outlined. Finally, I present the resulting impulse responses of macroeconomic variables.

4.4.1 Bifurcation Portfolio

	Benchmark	Only TFP Shocks	RBC
Share of Domestic Assets in Home Portfolio	53.69 %	41.78 %	-106.12 %

Table 4.2: Bifurcation portfolio

Optimal asset allocation is determined by risk properties of available assets and banks' hedging needs. This can be seen by inspecting the portfolio selection equation (4.48), which can be rewritten as

$$0 = Cov_t\left(\tilde{\Omega}_{t+1} - \tilde{\Omega}^*_{t+1}, R_{xt+1}\right) + E_t\left[\tilde{\Omega}_{t+1} - \tilde{\Omega}^*_{t+1}\right] E_t\left[R_{xt+1}\right], \tag{4.52}$$

with $Cov_t(\cdot)$ referring to the covariance operator conditional on the information set available in period t. As discussed before, one requires a second-order approximation of (4.52) to compute the bifurcation portfolio. Recall that $E_t\left[R_{xt+1}\right] \approx 0$ up to first-order of accuracy. Then, the following condition emerges

$$Cov_t\left(\tilde{\Omega}_{t+1} - \tilde{\Omega}^*_{t+1}, R_{xt+1}\right) \approx 0. \tag{4.53}$$

Since financial intermediaries are owned by households, they automatically inherit the consumption smoothing objective. Hence, they dislike assets associated with negative conditional covariance between the marginal value of net worth, expressed in terms of utility, and the excess return. The reason for this is that such assets enhance fluctuations in both consumption and the marginal value of net worth. Optimal asset allocation is achieved when the aforementioned covariance is equal to zero. If this condition is violated and the covariance is, for instance, positive, home assets tend to pay higher return when home banks profit relatively more from an additional unit of resources. Thus, financial intermediaries in home country have a relatively stronger incentive to provide funds to domestic firms.

Many DSGE models imply a foreign bias in "steady-state" asset holdings that is not supported by the empirical evidence.[17] In contrast, bifurcation portfolios exhibit a home bias in the underlying framework. The share of domestic assets in home banks' portfolios amounts to 53.69 %. This outcome is not driven by

[17] See, e.g., Baxter and Jermann (1997), Devereux and Sutherland (2009), Devereux and Yetman (2010), as well as Yao (2012)

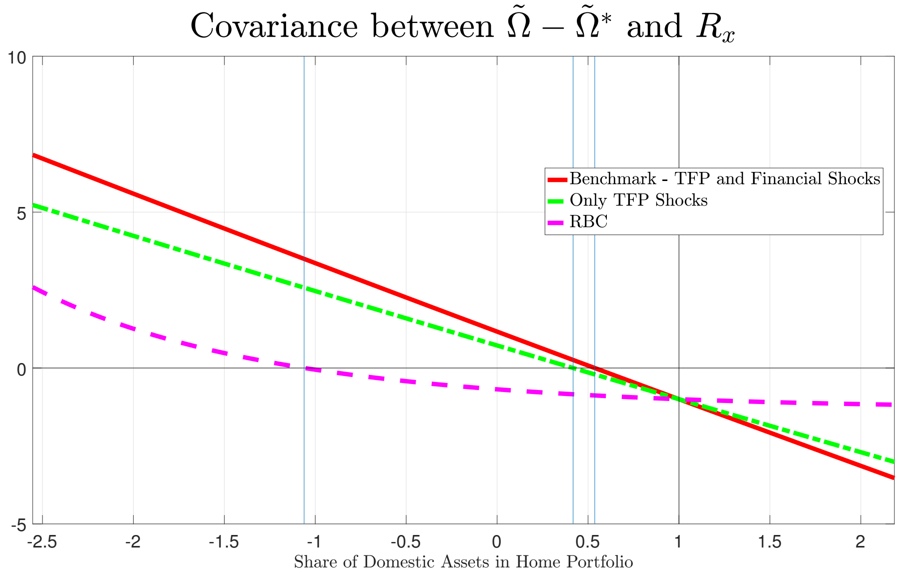


Figure 4.2: Cross-model comparison of covariance. *The covariance is normalized by its autarky-value to simplify the exposition. Optimal portfolio are characterized by the respective zero-covariance condition.*

uncertainty shocks, as the bifurcation portfolio represents an asymptotic point being reached when the size of uncertainty goes towards zero. Rather, the home bias emerges due to the presence of financial frictions and financial level shocks. To explain this, I compare the benchmark model to two other setups: a basic RBC model and the underlying framework with only TFP shocks. Table 4.2 summarizes results of this exercise. In the RBC model, the portfolio decision is made by households and thereby only consumption risk is taken into account. Because of strong positive correlation between capital and labor income under Cobb-Douglas production function, home households short-sell home equity to purchase foreign assets. The implied share of domestic assets in home agents' portfolio amounts to -106.12 %. On the other hand, a model with financial frictions and only TFP shocks implies much smaller foreign bias. The share of domestic assets in this case equals to 41.78 %. Hence, the presence of financial frictions à la Gertler and Karadi (2011) has a significant impact on the bifurcation portfolio because it alters the pattern of the covariance between the relative adjusted marginal utility, $\tilde{\Omega}_t - \tilde{\Omega}_t^*$, and the excess return, R_{xt}. Figure 4.2 depicts this covariance for different portfolio holdings. The RBC model implies a convex covariance function with a relatively small slope under complete home bias. In contrast, the framework with financial frictions generates a concave pattern of the covariance with a larger slope, when home banks hold only domestic assets. Thus, smaller purchases of foreign assets are necessary to obtain the optimal hedge in the presence of financial frictions. This outcome arises because of the financial accelerator mechanism which implies nonlinear and more volatile dynamics of the relative adjusted marginal utility. In particular, starting from a complete home bias, purchasing an unit of foreign assets yields a larger saving of resources in case of a negative TFP shock because saved capital can be leveraged.

Finally, introducing financial shocks to the model increases the slope of the covariance function under full home bias. Since financial shocks directly affect net worth of the entire banking system, their presence reinforces the effect of the financial accelerator mechanism on the portfolio choice problem. As a consequence, even smaller changes in asset holdings are required to obtain an allocation with optimal hedging properties.

4.4.2 Stochastic Steady State

	Deterministic Steady State	Stochastic Steady State
$Share$	53.69 %	89.81 %
S_h	4.5328	6.6220
S_f	3.9092	0.7517
NFA	0	-1.2129
R_k	1.0126	1.0126
R_k^*	1.0126	1.0129
N	2.11	1.90341
N^*	2.11	2.4269
K	8.442	8.5866
K^*	8.442	8.1921

Table 4.3: Deterministic vs stochastic steady state

One advantage of the chosen solution method is the possibility to compute both net foreign asset position and portfolio composition in the stochastic steady state. In other words, we can capture the direct effect of uncertainty on external asset holdings. This is important, especially in a setup with asymmetric countries.

Since the assumed asymmetry exists only in a stochastic environment, I compute the approximation to the true solution around net foreign asset position being equal to zero. However, due to the use of higher-order approximation techniques, this balanced position is adjusted for the presence of different hedging motives across countries.

Compared to the bifurcation point, both foreign and home financial intermediaries reduce their international exposure due the presence of uncertainty (See table 4.3). In particular, higher uncertainty in foreign country discourages home banks from purchasing cross-border assets and consequently they hold more domestic claims. As a result, the home bias in home country is larger and holdings of domestic assets amount almost to 90 % of their entire portfolio. On the other hand, foreign banks have to accommodate the local demand for credit and consequently increase their lending to domestic firms. Since foreign assets yield a higher rate of return, they are compensated for higher risk on their balance sheets, which allows them to accumulate more net worth. Finally, because of a stronger precautionary motive and the presence of additional resources, foreign intermediaries do not reduce their international exposure as much as home banks. This is reflected by negative net foreign assets of home country amounting to 124 % of domestic GDP.

4.4.3 Dynamics of Portfolio Holdings

In this section, I discuss the responses of external asset holdings to global uncertainty shocks. First- and second-order accurate responses of international portfolio are determined by changes in covariance between the relative adjusted marginal utility and the excess rate of return.[18]

Figure 4.3 depicts the response of external banking assets in the model economy to one standard deviation increase in the volatility of a) TFP and b) financial level shocks. The responses represent second-order accurate percentage deviations of asset holdings from their respective stochastic steady states. The analysis suggests that the origin of uncertainty plays a key role. In particular, a rise in macroeconomic uncertainty induces an increase in external assets, whereas a higher financial uncertainty reduces cross-country portfolios. These contrasting results reflect the fact that the two types of second-moment shocks affect the covariance between the relative adjusted marginal utility and the excess return in different ways (see figure 4.4). From the perspective of the financial sector, macroeconomic uncertainty can be viewed as uncertainty about available assets. Since their balance sheets are ex-ante riskier, foreign financial intermediaries suffer more strongly from an increase in TFP volatility, i.e., $\tilde{\Omega}_t - \tilde{\Omega}_t^*$ falls. At the same time, they have a stronger hedging desire compared to their counterparts in home country. For that reason, they have an incentive to shift their lending towards home firms, which generates an increase in the excess return. Finally, because of a relative increase in the return on their portfolio, home banks can absorb higher risk of loans to foreign firms and therefore increase their cross-country asset holdings as well.

On the other hand, financial uncertainty is more strongly related to banks' ability to intermediate funds. As discussed in chapter 2 in a closed-economy framework, it is propagated to the real economy mainly via a tightening of the funding constraint. The excess return falls in this case because foreign banks are prevented from expanding their activities abroad due to tighter financial conditions and foreign

[18] Note that the higher-order level effects are also important because of the presence of expectations in (4.52). Yet, time-varying covariance plays a major role.

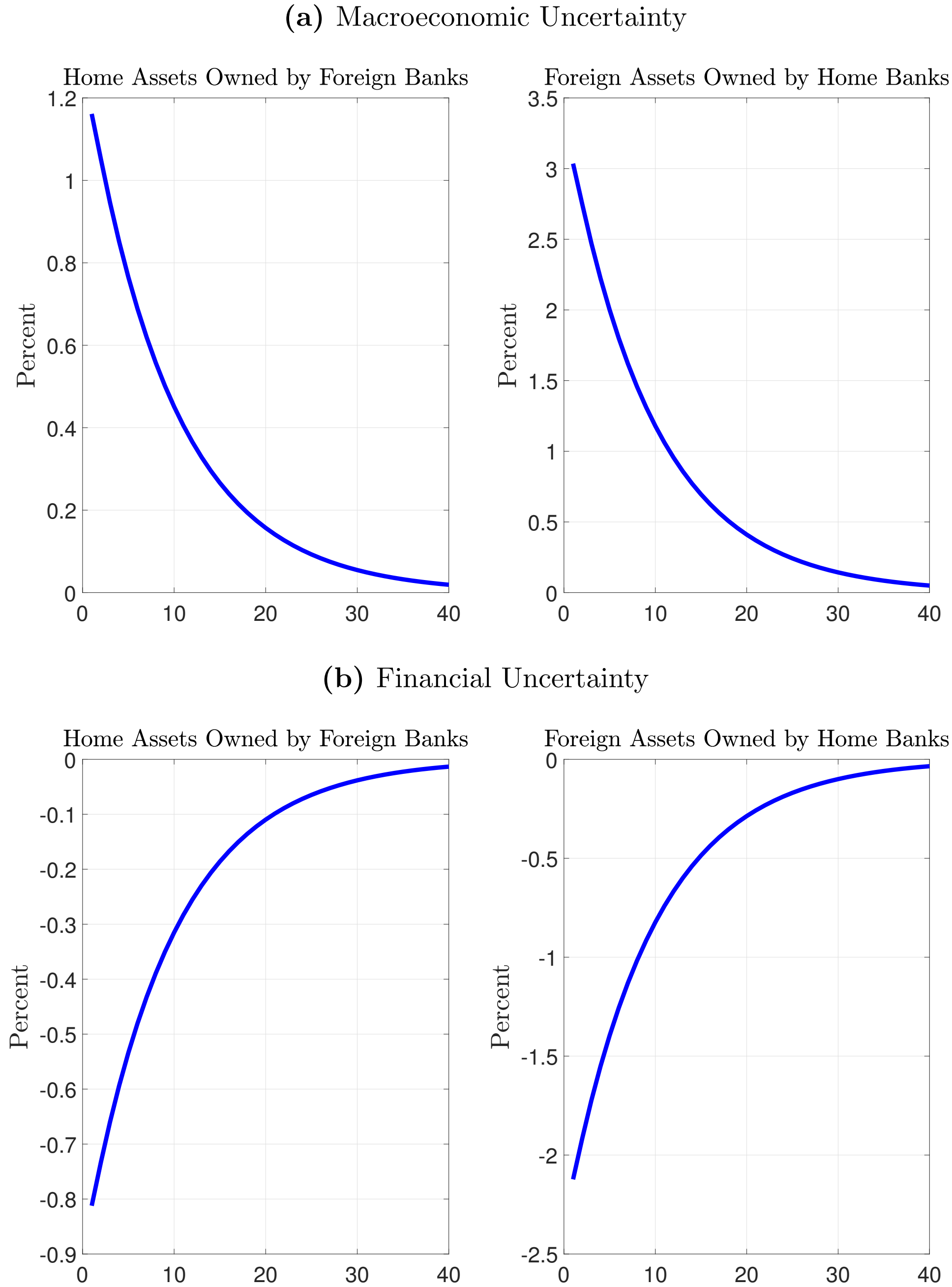


Figure 4.3: Response of cross-country asset holdings to global uncertainty shocks *Level shocks are held constant. Horizontal axes indicate quarters. All responses are in percent.*

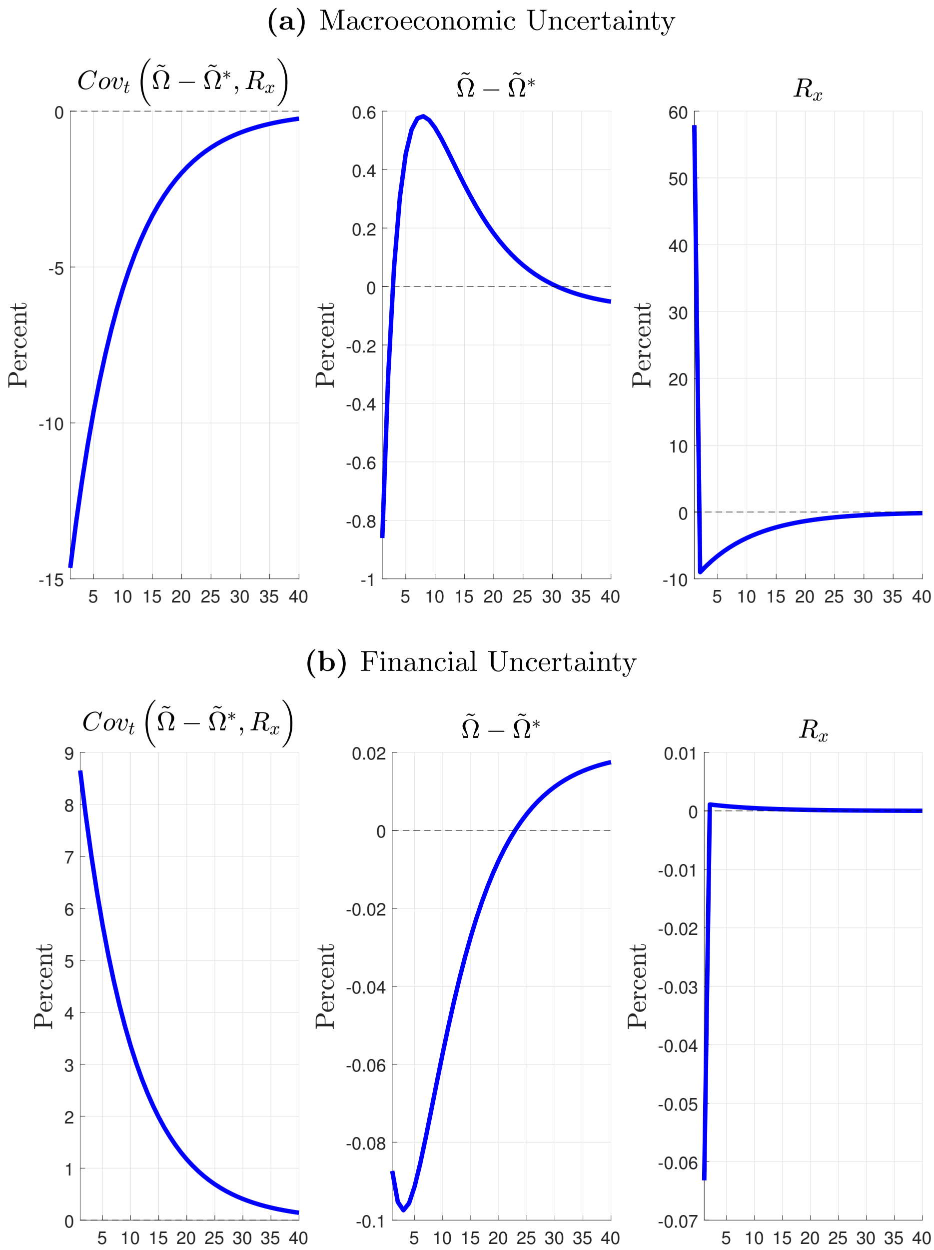


Figure 4.4: Response of covariance to global uncertainty shocks. *Level shocks are held constant. Horizontal axes indicate quarters. All responses are in percent.*

rate of return falls by less because of higher risk premium. Simultaneously, a relatively higher rate of return gives foreign banks a stronger incentive to conduct their business. Consequently, they would profit more from an additional unit of resources, i.e., $\tilde{\Omega}_t - \tilde{\Omega}_t^*$ falls again. In other words, the relative marginal value of net worth and the excess return co-move following an increase in financial uncertainty.

4.4.4 Dynamics of Non-portfolio Variables

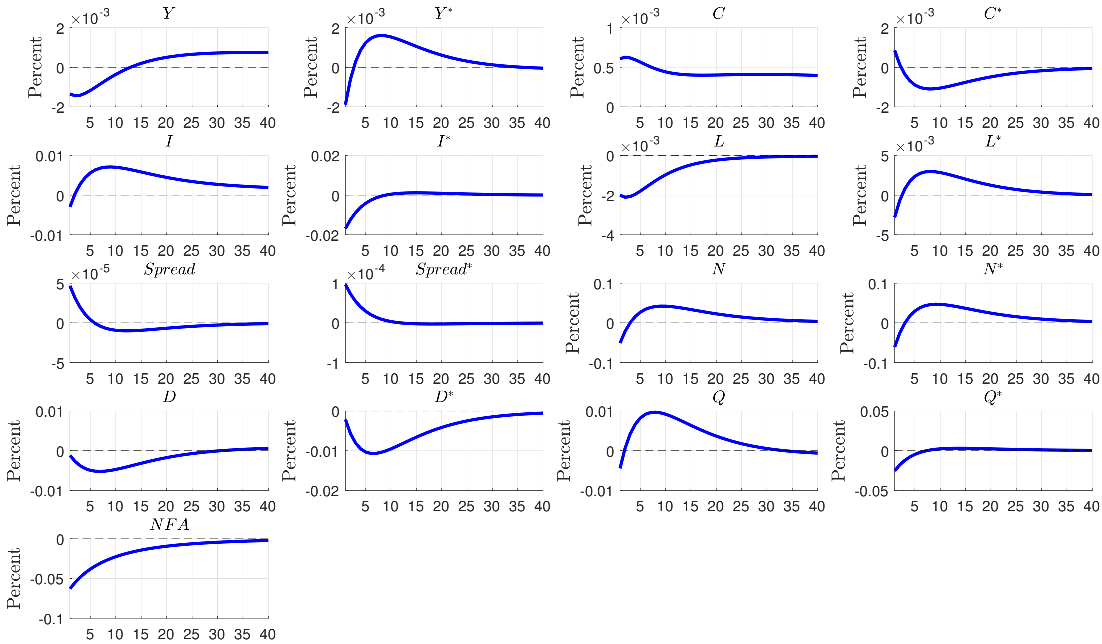


Figure 4.5: Dynamic consequences of TFP uncertainty shocks. *Level shocks are held constant. Horizontal axes indicate quarters. All responses are in percent, except for spreads, defined as the expected future rate of return on assets in a country relative to the domestic rate on deposits.*

During the global financial crisis, the great retrenchment was accompanied by a drop in GDP observed over the entire world. This section investigates whether the model can also replicate the worldwide decline in economic activity.

Figure 4.5 displays impulse responses of macroeconomic variables to a global increase in TFP volatility. External asset positions of both countries increase, however due to the heterogeneous precautionary motives, home NFA declines and thus the country's indebtedness increases. Additional resources stimulate home investment which recovers already in the second period. This drives up the price of capital and consequently the rate of return on home assets. Despite higher investment, home country experiences a persistent drop in GDP, as hours worked decrease. On the other hand, production and investment in foreign country fall on impact. However, foreign production recovers in the subsequent period and foreign country enters an expansionary stage. To summarize, an adverse shock to the volatility of TFP generates non-synchronized responses of the macroeconomic variables across countries.[19]

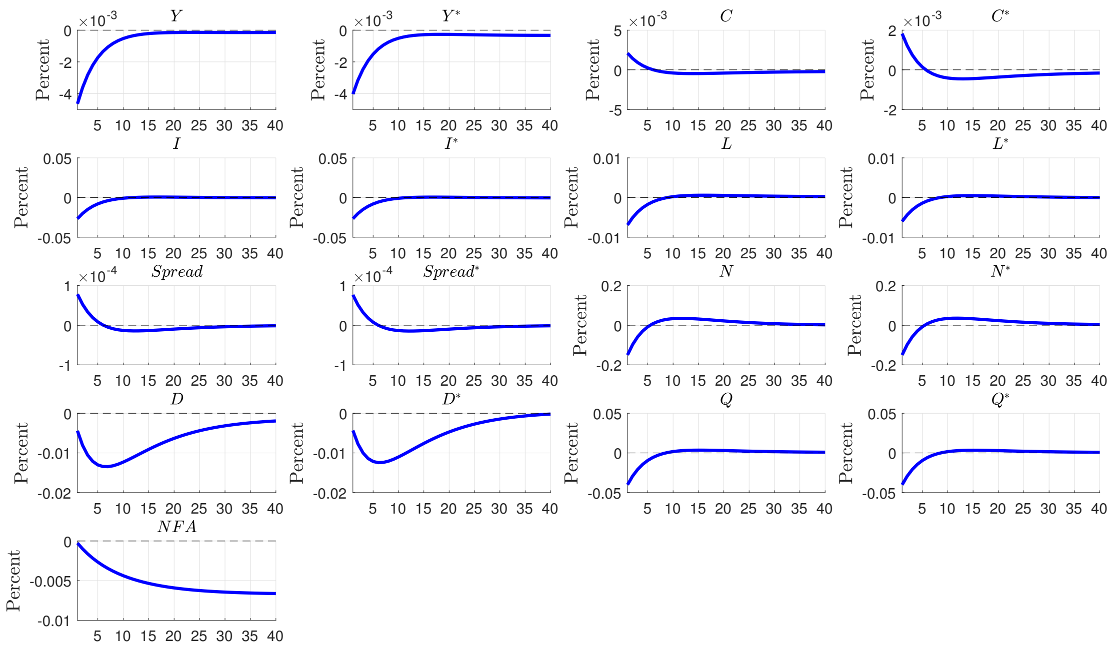


Figure 4.6: Dynamic consequences of financial uncertainty shocks. *Level shocks are held constant. Horizontal axes indicate quarters. All responses are in percent, except for spreads, defined as the expected future rate of return on assets in a country relative to the domestic rate on deposits.*

Figure 4.6 repeats the previous exercise for an increase in financial uncertainty. In contrast to TFP volatility, a higher standard deviation of disturbances to banks' survival probability implies a lasting reduction in production level in both countries. An increase in financial uncertainty leads to tighter funding conditions. As a result, home and foreign households reduce their bank deposits, which diminishes investment in both countries. Because of ex-ante higher macroeconomic uncertainty in foreign country, local banks reduce their cross-border assets not as strong as their home counterparts. Thus, net foreign assets fall, but, in contrast to a negative TFP volatility shock, home country's indebtedness increases gradually.

One could suspect that financial uncertainty shocks result in synchronized dynamics across countries because of the assumption that only foreign TFP shocks are twice as volatile as the disturbances in home economy. To address these concerns, I consider an adverse financial uncertainty shock in a framework where 1) only financial shocks are more volatile and 2) both financial and TFP disturbances

[19]Note that private consumption increases on impact in both countries. This outcome is common for real DSGE models with perfectly competitive labor markets.

are associated with a higher standard deviation in the foreign country. As shown in Appendix by figures C.2 and C.3, the impulse responses of the majority of macroeconomic variables obtained under these alternative assumptions do not substantially differ from the ones in the benchmark exercise. The only exception is home country's net foreign asset position, which increases on impact in this case. This positive response is caused by two factors. First, home country experiences much stronger fall in price of domestic assets (valuation effects). Second, foreign intermediaries become now much more constrained after an increase in financial uncertainty. Hence, they are forced to cut their lending more significantly than their home country's counterparts.

4.5 Conclusion

This paper employs an open-economy DSGE framework with two ex-ante asymmetric countries, financial frictions, and endogenous portfolio choice to investigate implications of uncertainty shocks for the global economy. The analysis supports the empirical evidence by providing a theoretical rationale for a reduction in international banking assets in response to a rise in uncertainty. It also suggests that financial markets were the source of uncertainty that caused *the great retrenchment* and contributed to the worldwide recession. An increase in financial uncertainty leads to a fall in the excess rate of return and simultaneously reduces the stochastic discount factor of home financial intermediaries relative to foreign banks. Consequently, home institutions reduce their holdings of foreign assets. Furthermore, an adverse financial uncertainty shock tightens funding conditions and leads to a recession in both countries, replicating thereby the experience of the global financial crisis.

Highly volatile capital inflows may create a tension between financial stability and macroeconomic stabilization and thus constitute a policy challenge, especially in a country characterized by institutional weaknesses (Nier et al., 2014). For this reason, it is of great importance, for both academics and policy makers, to understand the drivers and economic consequences of external portfolio decisions. The analysis conducted in this paper can be extended to include a variety of policy and regulatory tools aiming at protecting domestic economy from negative effects of volatile capital inflows. This extension would allow for a welfare-based evaluation of these tools.

Appendix A

Appendix to Chapter 2

A.1 Data Sources

I use the following data sources to estimate my VAR model. The data is available on the Federal Reserve Economic Database (FRED) unless specified otherwise:

1. **Financial Uncertainty Measure** - Monthly – Source: Ludvigson et al. (2015) – Data available from: https://www.sydneyludvigson.com/data-and-appendixes/

2. **Macro Uncertainty Measure** – Monthly – Source: Jurado et al. (2015) – Data available from: https://www.sydneyludvigson.com/data-and-appendixes/

3. **Nominal GDP** – Quarterly, Billions of Dollars, Seasonally Adjusted – FRED Code: GDP

4. **Personal Consumption Expenditures, Nondurable Goods** - Quarterly, Billions of Dollars, Seasonally Adjusted – FRED Code: PCND

5. **Personal Consumption Expenditures, Services** – Quarterly, Billions of Dollars, Seasonally Adjusted – FRED Code: PCESV

6. **Nominal Gross Private Investment** – Quarterly, Billions of Dollars, Seasonally Adjusted – FRED Code: GPDI

7. **Nonfarm Business Sector: Hours of all Persons** – Quarterly, Index 2009=100, Seasonally Adjusted – FRED Code: HOANBS

8. **GDP: Implicit Price Deflator** – Quarterly, Index 2009=100, Seasonally Adjusted – FRED Code: GDPDEF

9. **Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity** – Monthly, Percent –FRED Code: BAA10YM

10. **Effective Federal Funds Rate** – Monthly, Percent – FRED Code: FEDFUNDS
11. **Civilian Noninstitutional Population** – Monthly, Thousands of Persons – FRED Code: CNP16OV
12. **Credit Spread by Gilchrist and Zakrajšek (2012) (GZ)** – Monthly – Source: Gilchrist and Zakrajšek (2012) – http://people.bu.edu/sgilchri/Data/data.htm
13. **Shadow Federal Funds Rate** – Monthly – Source: Wu and Xia (2016) – https://sites.google.com/view/jingcynthiawu/shadow-rates

All variables reported at a monthly frequency are converted to a quarterly frequency by applying time averages. All nominal variables are converted to real terms by applying the GDP deflator. Finally, I express aggregate quantities in per-capita terms by dividing them by the civilian noninstitutional population.

A.2 Robustness of the VAR Results

A.2.1 Different ordering of the variables

To assess the robustness of the VAR evidence, I firstly use a different ordering of variables with the measure of financial uncertainty ordered last. This identification scheme allows for a contemporaneous effect of macroeconomic variables on financial uncertainty. Note that it is not consistent with my theoretical model. Moreover, it is not supported by findings of Ludvigson et al. (2015). However, by employing this alternative ordering, I can show that the empirical evidence provided in this paper does not rely solely on the baseline identification strategy (see figure A.1).

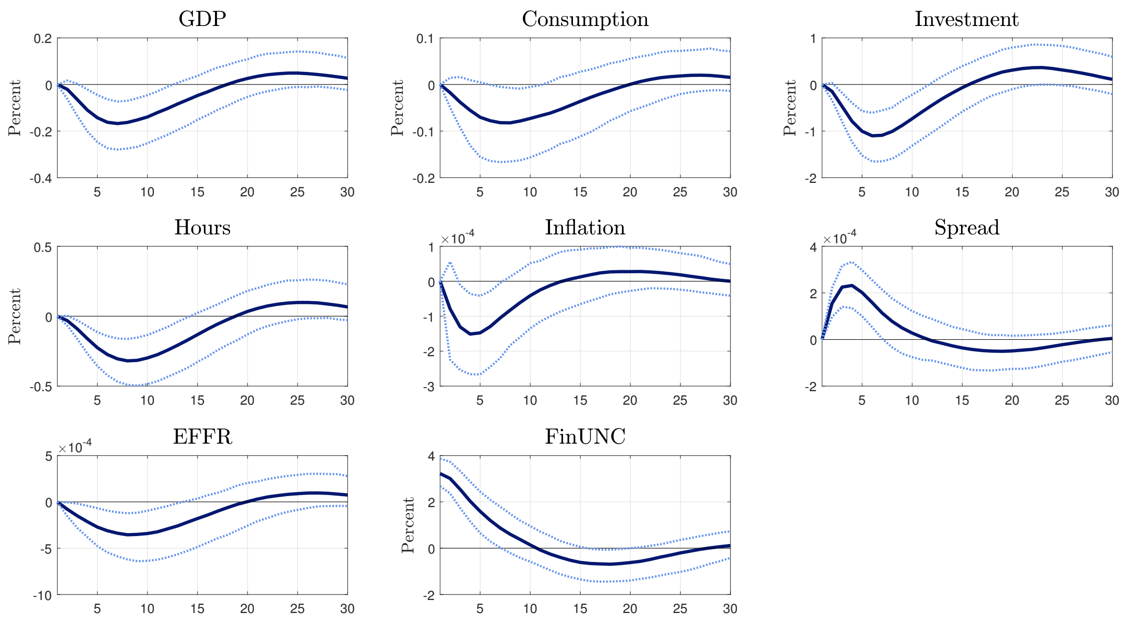


Figure A.1: SVAR robustness: financial uncertainty is ordered last. *Horizontal axes indicate quarters. The solid curve denotes the median response, whereas the dotted curves refer to the 95 % confidence interval. Responses of all variables except inflation, the risk premium, and the federal funds rate are in percent.*

A.2.2 Different measures for monetary policy stance and credit spread

Second, I replace the BAA spread by the measure constructed by Gilchrist and Zakrajšek (2012). Figure A.2 presents the results of this robustness check. Moreover, I use the shadow federal funds rate proposed by Wu and Xia (2016) as a measure of monetary policy stance. This exercise aims to account for the zero lower bound episode included in the sample. The results remain virtually the same, as depicted in figure A.3.

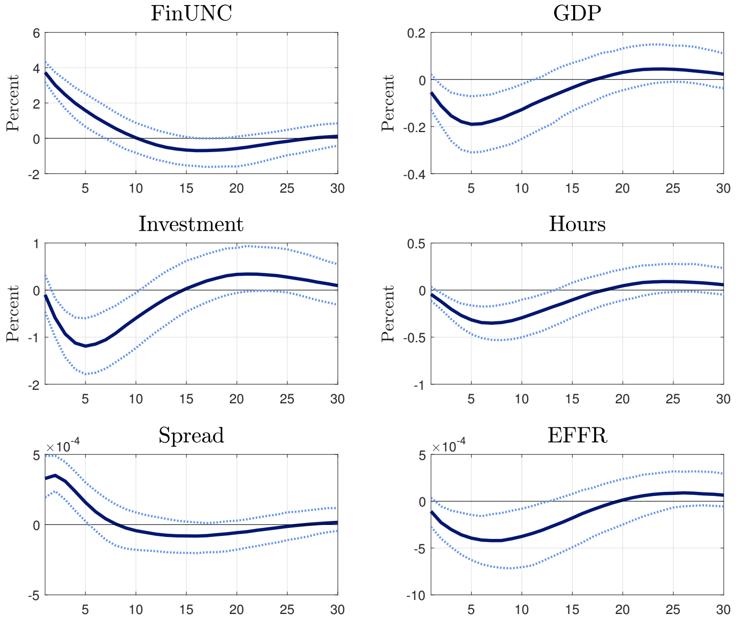


Figure A.2: SVAR robustness: BAA spread is replaced by the GZ indicator. *Horizontal axes indicate quarters. The solid curve denotes the median response, whereas the dotted curves refer to the 95 % confidence interval. Responses of all variables except inflation, the risk premium, and the federal funds rate are in percent.*

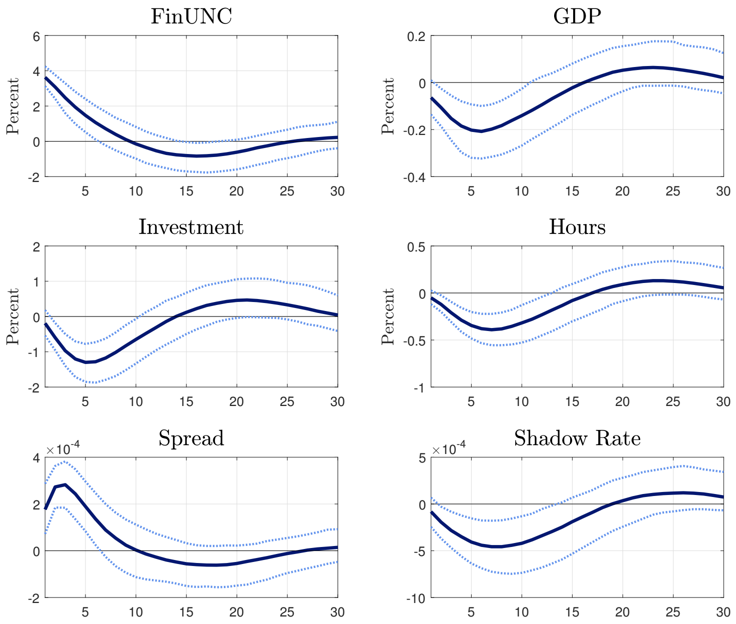


Figure A.3: SVAR robustness: shadow federal funds rate as a measure of monetary policy stance. *Horizontal axes indicate quarters. The solid curve denotes the median response, whereas the dotted curves refer to the 95 % confidence interval. Responses of all variables except inflation, the risk premium, and the federal funds rate are in percent.*

A.2.3 Including a measure of macroeconomic uncertainty

Finally, I extend the set of variables used in the estimation exercise by including a measure of macroeconomic uncertainty constructed by Jurado et al. (2015).

Given findings of Ludvigson et al. (2015), the measure of financial uncertainty is ordered first and is followed by the proxy for macro uncertainty and remaining macroeconomic variables. As shown by figure A.4, the results remain virtually unchanged.

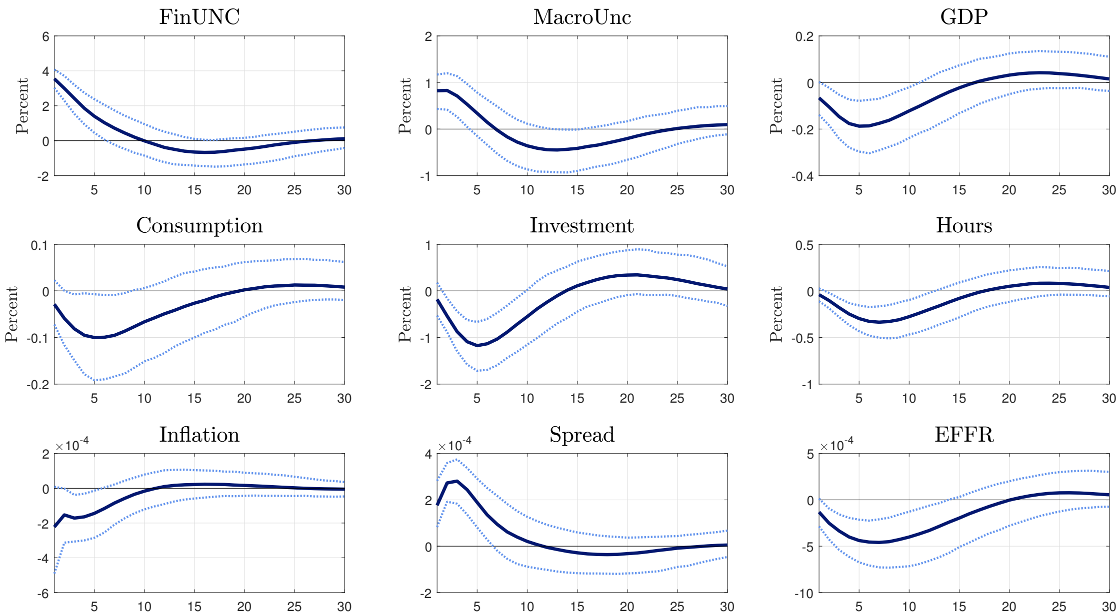


Figure A.4: SVAR robustness: macroeconomic uncertainty is included in the estimation. *Horizontal axes indicate quarters. The solid curve denotes the median response, whereas the dotted curves refer to the 95 % confidence interval. Responses of all variables except inflation, the risk premium, and the federal funds rate are in percent.*

Appendix B

Appendix to Chapter 3

B.1 Proof of the Bifurcation Theorem

Proof. The bifurcation theorem can be proven by dividing the H-function by its singularity (See Zeidler, 1986 and Judd and Guu, 2001). Define the following function $\tilde{H}$

$$\tilde{H}(\alpha,\sigma) = \begin{cases} \frac{H(\alpha,\sigma)}{\sigma^2} & if \quad \sigma \neq 0 \\ \frac{\partial^2 H(\alpha,\sigma)}{(\partial\sigma)^2} & if \quad \sigma = 0 \end{cases}.$$

Since H is analytic, and $H(\alpha,\sigma) = 0$ for all α, it follows that $H(\alpha,\sigma) = \tilde{H}(\alpha,\sigma)\sigma^2$ and $\tilde{H}$ is analytic in $(\alpha,\, \sigma)$. Implicit differentiation yields

$$H_{\sigma\sigma}|_{\sigma=0} = \tilde{H}|_{\sigma=0}, \tag{B.1}$$

and

$$H_{\sigma\sigma\alpha}|_{\sigma=0} = \tilde{H}_{\alpha}|_{\sigma=0}. \tag{B.2}$$

Therefore, to obtain a root of $\tilde{H}|_{\sigma=0}$, $H_{\sigma\sigma}|_{\sigma=0}$ must be set equal to the zero vector. Moreover, the implicit function theorem can be applied to $\tilde{H}$ if and only if $det\,(H_{\sigma\sigma\alpha}(\alpha_0,\sigma_0)) \neq 0$. □

B.2 Computing Derivatives of the Portfolio Equation - State Space Approach

B.2.1 General Relationship

Recall the portfolio equation

$$H\left(\sigma, \alpha_t, y_t^{state}, \epsilon_{t+1} | g_{guess}^{\alpha}\right) \equiv$$
$$E_t\left[m\left(g^{\mu}\left(\sigma, \alpha_t, y_t^{state}, \epsilon_{t+1}\right| g_{guess}^{\alpha})\right) \otimes b\left(g^{r}\left(\sigma, \alpha_t, y_t^{state}, \epsilon_{t+1} | g_{guess}^{\alpha}\right)\right)\right].$$

Furthermore, let $\tilde{H}\left(\sigma, \alpha_t, y_t^{state}, \epsilon_{t+1} | g_{guess}^{\alpha}\right)$ be the function determined by the bifurcation theorem. Then, the following holds

$$H\left(\sigma, \alpha_t, y_t^{state}, \epsilon_{t+1} | g_{guess}^{\alpha}\right) = \tilde{H}\left(\sigma, \alpha_t, y_t^{state}, \epsilon_{t+1} | g_{guess}^{\alpha}\right) \sigma^2. \tag{B.3}$$

The policy function for α is defined by

$$\tilde{H}\left(\sigma, g^{\alpha}\left(\sigma, y_t^{state}\right), y_t^{state}, \epsilon_{t+1} | g^{\alpha}\right) = 0. \tag{B.4}$$

B.2.2 First-Order Coefficients

Implicit differentiation yields

$$\tilde{g}_{y^{state}}^{\alpha} = -\bar{\tilde{H}}_{\alpha}^{-1} \bar{\tilde{H}}_{y^{state}}, \tag{B.5}$$

and

$$\tilde{g}_{\sigma}^{\alpha} = -\bar{\tilde{H}}_{\alpha}^{-1} \bar{\tilde{H}}_{\sigma}. \tag{B.6}$$

To find the corresponding derivatives of $\tilde{H}$, I implicitly differentiate (B.3). As a result, the following relationships are obtained: $\bar{\tilde{H}}_{\sigma} = \frac{1}{6}\bar{H}_{\sigma\sigma\sigma}$, $\bar{\tilde{H}}_{y^{state}} = \frac{1}{2}\bar{H}_{\sigma\sigma y^{state}}$, and $\bar{\tilde{H}}_{\alpha} = \frac{1}{2}\bar{H}_{\sigma\sigma\alpha}$. Inserting these expressions into (B.5) and (B.6) yields (3.35) and (3.36).

In the last step, exact expressions of the respective derivatives of the H-function need to be found. They will be pruned to avoid unnecessary higher-order terms that may lead to an explosive behavior and thus, deteriorate accuracy of the approximation. In particular, to obtain the first-order approximation of gross asset holdings, the derivatives will include only third-order components. This approach follows the ideas of Samuelson (1970) and also underlies the procedure of Devereux and Sutherland (2010, 2011).

Implicit differentiation and omitting components of order higher than three yield

$$\begin{aligned}\bar{H}_{\sigma\sigma\sigma} &= 3\left[\left(m_{\mu\mu}\left(g_\epsilon^\mu \otimes g_\epsilon^\mu\right)\right) \otimes \left(b_r g_\epsilon^r\right)\right]\Sigma_3 \\ &+ 3\left[\left(m_\mu g_\epsilon^\mu\right) \otimes \left(b_{rr}\left(g_\epsilon^r \otimes g_\epsilon^r\right)\right)\right]\Sigma_3 \\ &+ 3\left[\left(m_\mu g_{\epsilon\epsilon}^\mu\right) \otimes \left(b_r g_\epsilon^r\right)\right]\Sigma_3 \\ &+ 3\left[\left(m_\mu g_\epsilon^\mu\right) \otimes \left(b_r g_{\epsilon\epsilon}^r\right)\right]\Sigma_3, \end{aligned} \tag{B.7}$$

where $\Sigma_3 \equiv E_t\left[\epsilon_t^{\otimes[3]}\right]$ denotes a matrix of third moments of the underlying shock structure.

Let $p = [y^{state}, \alpha]^\top$ be a $np \times 1$ vector, then

$$\begin{aligned}\bar{H}_{\sigma\sigma p} &= 2\left[\left(m_{\mu\mu}\left(g_p^\mu \otimes g_\epsilon^\mu\right)\right) \otimes \left(b_r g_\epsilon^r\right)\right]\left[I_{np} \otimes vec(\Sigma)\right] \\ &+ 2\left[\left(m_\mu g_{\epsilon p}^\mu\right) \otimes \left(b_r g_\epsilon^r\right)\right]\left[I_{np} \otimes vec(\Sigma)\right] \\ &+ 2\left[\left(b_r g_{\epsilon p}^r\right) \otimes \left(m_\mu g_\epsilon^\mu\right)\right]\left[I_{np} \otimes vec(\Sigma)\right] \\ &+ \left[\left(m_\mu g_p^\mu\right) \otimes \left(b_{rr}\left(g_\epsilon^r \otimes g_\epsilon^r\right)\right)\right]\left[I_{np} \otimes vec(\Sigma)\right] \\ &+ \left[\left(m_\mu g_p^\mu\right) \otimes \left(b_r g_{\epsilon\epsilon}^r\right)\right]\left[I_{np} \otimes vec(\Sigma)\right] \\ &+ \left[\left(m_\mu g_p^\mu\right) \otimes \left(b_r g_{\sigma\sigma}^r\right)\right], \end{aligned} \tag{B.8}$$

where I_{np} stands for the identity matrix of dimension $np \times np$ and Σ denotes the variance-covariance matrix of the underlying shock structure. Note that combining (B.6) with the above derivatives yields the expression for the first-order uncertainty correction term presented in (3.37) with the skew tolerance given by

$$\begin{aligned}\tau = 3\bar{H}_{\sigma\sigma\alpha}^{-1} &\left[\left(m_{\mu\mu}\left(g_\epsilon^\mu \otimes g_\epsilon^\mu\right)\right) \otimes \left(b_r g_\epsilon^r\right)\right. \\ &+ 3\left(m_\mu g_\epsilon^\mu\right) \otimes \left(b_{rr}\left(g_\epsilon^r \otimes g_\epsilon^r\right)\right) \\ &+ 3\left(m_\mu g_{\epsilon\epsilon}^\mu\right) \otimes \left(b_r g_\epsilon^r\right) \\ &\left. + 3\left(m_\mu g_\epsilon^\mu\right) \otimes \left(b_r g_{\epsilon\epsilon}^r\right)\right]. \end{aligned}$$

B.2.3 Second-Order Coefficients

Given symmetrically distributed shocks, second-order coefficients can be computed as

$$\tilde{g}_{\sigma\sigma}^\alpha = -\bar{\tilde{H}}_\alpha^{-1}\bar{\tilde{H}}_{\sigma\sigma}. \tag{B.9}$$

$$\tilde{g}_{y^{state}y^{state}}^\alpha = -\bar{\tilde{H}}_{\sigma\sigma\alpha}^{-1}\Gamma. \tag{B.10}$$

with

$$
\begin{aligned}
\Gamma \equiv\ & \bar{\tilde{H}}_{\alpha\alpha} \left(\tilde{g}^{\alpha}_{y^{state}} \otimes \tilde{g}^{\alpha}_{y^{state}}\right) \\
& + \bar{\tilde{H}}_{\alpha y^{state}} \left(I_{nystate} \otimes \tilde{g}^{\alpha}_{y^{state}}\right) \left(I_{nystate^2} + K_{nystate,nystate}\right) + \bar{\tilde{H}}_{y^{state}y^{state}}
\end{aligned}
$$

I_n denotes an $n \times n$ identity matrix and $K_{n,n}$ is a commutation matrix with dimension $n^2 \times n^2$ (Magnus and Neudecker, 1979). To find required derivatives of $\tilde{H}$, I implicitly differentiate (B.3). As a result, the following relationships are obtained $\bar{\tilde{H}}_{\sigma\sigma} = \frac{1}{12}\bar{H}_{\sigma\sigma\sigma\sigma}$ and $\bar{\tilde{H}}_{pp} = \frac{1}{2}\bar{H}_{\sigma\sigma pp}$, with $p \equiv [y^{state}, \alpha]$. Furthermore, applying the procedure described in the previous section yields

$$
\begin{aligned}
\bar{H}_{\sigma\sigma\sigma\sigma} =\ & 4\left[\left(m_{\mu\mu}\left(K_{nmu,nmu} + 2I_{nmu^2}\right)\left(g^{\mu}_{\epsilon} \otimes g^{\mu}_{\sigma\sigma}\right)\right) \otimes \left(b_r g^{r}_{\epsilon}\right)\right] vec\left(\Sigma\right) \\
& + 12\left[\left(m_{\mu} g^{\mu}_{\sigma\sigma\epsilon}\right) \otimes \left(b_r g^{r}_{\epsilon}\right)\right] vec\left(\Sigma\right) \\
& + 6\left[\left(m_{\mu\mu}\left(g^{\mu}_{\epsilon} \otimes g^{\mu}_{\epsilon}\right)\right) \otimes \left(b_r g^{r}_{\sigma\sigma}\right)\right] vec\left(\Sigma\right) \\
& + 6\left[\left(m_{\mu} g^{\mu}_{\sigma\sigma}\right) \otimes \left(b_r\left(g^{r}_{\epsilon} \otimes g^{r}_{\epsilon}\right)\right)\right] vec\left(\Sigma\right) \\
& + 6\left[\left(m_{\mu} g^{\mu}_{\epsilon\epsilon}\right) \otimes \left(b_r g^{\mu}_{\sigma\sigma}\right)\right] vec\left(\Sigma\right) \\
& + 6\left[\left(m_{\mu} g^{\mu}_{\sigma\sigma}\right) \otimes \left(b_r g^{\mu}_{\epsilon\epsilon}\right)\right] vec\left(\Sigma\right) \\
& + 6\left(m_{\mu} \otimes b_r\right)\left(g^{\mu}_{\sigma\sigma} \otimes g^{\mu}_{\sigma\sigma}\right) \\
& + 4\left[\left(m_{\mu} g^{\mu}_{\sigma\sigma}\right) \otimes \left(b_{rr}\left(K_{nr,nr} + 2I_{nr^2}\right)\left(g^{r}_{\epsilon} \otimes g^{r}_{\sigma\sigma}\right)\right)\right] vec\left(\Sigma\right) \\
& + 12\left[\left(m_{\mu} g^{\mu}_{\epsilon}\right) \otimes \left(b_r g^{r}_{\sigma\sigma\epsilon}\right)\right] vec\left(\Sigma\right), \qquad \text{(B.11)}
\end{aligned}
$$

and

$$
\begin{aligned}
\bar{H}_{\sigma\sigma pp} = & \, b_r g^r_{pp} \otimes \left[\left(m_{\mu\mu} \left(g^\mu_\epsilon \otimes g^\mu_\epsilon \right) + m_\mu g^\mu_{\epsilon\epsilon} \right) vec\left(\Sigma\right) \right] \\
& + b_r g^r_{pp} \otimes \left(m_\mu g^\mu_{\sigma\sigma} \right) \\
& + 2 \left(m_{\mu\mu\mu} \otimes b_r \right) \left(g^\mu_p \otimes g^\mu_p \otimes \left[\left(g^\mu_\epsilon \otimes g^r_\epsilon \right) vec\left(\Sigma\right) \right] \right) \\
& + 2 \left(m_{\mu\mu} \otimes b_r \right) \left(g^{\mu\mu}_{pp} \otimes \left[\left(g^\mu_\epsilon \otimes g^r_\epsilon \right) vec\left(\Sigma\right) \right] \right) \\
& + 2 \left(m_{\mu\mu} \otimes b_r \right) \left(g^\mu_p \otimes \left[\left(g^\mu_{\epsilon p} \otimes g^r_\epsilon \right) \left(I_{np} \otimes vec\left(\Sigma\right) \right) \right] \right) \\
& + 2 \left[\left(m_\mu g^\mu_{\epsilon pp} \otimes \left(b_r g^r_\epsilon \right) \right) \left(I_{np^2} \otimes vec\left(\Sigma\right) \right) \right] \\
& + 2 \left(m_{\mu\mu} \otimes b_r \right) \left(g^\mu_p \otimes \left[\left(g^\mu_\epsilon \otimes g^r_{\epsilon p} \right) \left(I_{np} \otimes vec\left(\Sigma\right) \right) \right] \right) \\
& + \left(m_\mu \otimes b_r \right) \left(g^\mu_{\epsilon p} \otimes g^r_{\epsilon p} \right) \left(I_{np^2} \otimes vec\left(\Sigma\right) \right) \\
& + 2 \left(b_{rr} \otimes m_\mu \right) \left(g^r_{pp} \otimes \left[\left(g^r_\epsilon \otimes g^\mu_\epsilon \right) vec\left(\Sigma\right) \right] \right) \\
& + 2 \left[\left(b_r g^r_{\epsilon pp} \right) \otimes \left(m_\mu g^\mu_\epsilon \right) \right] \left[I_{np^2} \otimes vec\left(\Sigma\right) \right] \\
& + \left[m_{\mu\mu} \left(g^\mu_p \otimes g^\mu_p \right) \right] \otimes \left[b_{rr} \left(g^r_\epsilon \otimes g^r_\epsilon \right) vec\left(\Sigma\right) \right] \\
& + \left[m_{\mu\mu} \left(g^\mu_p \otimes g^\mu_p \right) \right] \otimes \left[b_r \left(g^r_{\sigma\sigma} + g^r_{\epsilon\epsilon} vec\left(\Sigma\right) \right) \right] \\
& + \left[m_\mu g^\mu_{pp} \right] \otimes \left[b_{rr} \left(g^r_\epsilon \otimes g^r_\epsilon \right) vec\left(\Sigma\right) \right] \\
& + \left[m_\mu g^\mu_{pp} \right] \otimes \left[b_r \left(g^r_{\sigma\sigma} + g^r_{\epsilon\epsilon} vec\left(\Sigma\right) \right) \right] \\
& + \left[m_\mu g^\mu_p \right] \otimes \left[b_{rr} \left(g^r_{\epsilon p} \otimes g^r_\epsilon \right) \left(I_{np} \otimes vec\left(\Sigma\right) \right) \right] \\
& + 2 \left[m_\mu g^\mu_p \right] \otimes \left[b_r \left(g^r_{\sigma\sigma p} + g^r_{\epsilon\epsilon p} \left(I_{np} \otimes vec\left(\Sigma\right) \right) \right) \right]
\end{aligned} \tag{B.12}
$$

Finally, implicit differentiation implies the following expression for second-order coefficients on cross terms

$$
\tilde{g}^\alpha_{\sigma y^{state}} = -\bar{\tilde{H}}_\alpha \left[\bar{\tilde{H}}_{\alpha\alpha} \left(g^\alpha_\sigma \otimes g^\alpha_{y^{state}} \right) + \bar{\tilde{H}}_{\sigma\alpha} g^\alpha_{y^{state}} + \bar{\tilde{H}}_{\sigma y^{state}} \right], \tag{B.13}
$$

where $\bar{\tilde{H}}_{\sigma p} = \frac{1}{6} \bar{H}_{\sigma\sigma\sigma p}$, with $p \equiv [y^{state}, \alpha]$. Recall that $g^\alpha_\sigma = 0$. Moreover, $\bar{H}_{\sigma\sigma\sigma p}$ is a zero matrix as well which can be inferred from (B.7). Hence, $\tilde{g}^\alpha_{\sigma y^{state}} = 0$.

B.3 Computing Derivatives of the Portfolio Equation - Nonlinear Moving Average

B.3.1 General Relationship

Recall the following relationship

$$
H^{nlma} \left(\sigma, \alpha_t, \epsilon_{t+1}, \epsilon_t, \epsilon_{t-1}, ... \right) = H \left[\sigma, \alpha_t, y^{state}_t \left(\sigma, \epsilon_t, \epsilon_{t-1}, ... \right), \epsilon_{t+1} \right] \tag{B.14}
$$

Moreover, note that the function determined by the bifurcation theorem is defined by

$$H^{nlma}(\sigma, \alpha_t, \epsilon_{t+1}, \epsilon_t, \epsilon_{t-1}, ...|\alpha_{guess}) = \tilde{H}^{nlma}(\sigma, \alpha_t, \epsilon_{t+1}, \epsilon_t, \epsilon_{t-1}, ...|\alpha_{guess})\,\sigma^2. \tag{B.15}$$

B.3.2 First-Order Coefficients

The starting point for the computation of the first-order coefficients on state variables is the derivation of coefficients with respect to ϵ_{t-j}[1]

$$\alpha_j = -\Phi \bar{\tilde{H}}_j^{nlma}, \tag{B.16}$$

where Φ denotes the inverse of $\bar{\tilde{H}}_\alpha^{nlma}$. Implicit differentiation of (B.15) yields $\bar{\tilde{H}}_j^{nlma} = \frac{1}{2}\bar{H}_{\sigma\sigma j}^{nlma}$ and $\bar{\tilde{H}}_\alpha^{nlma} = \frac{1}{2}\bar{H}_{\sigma\sigma\alpha}^{nlma}$. Moreover, differentiating (B.14) leads to

$$\bar{H}_{\sigma\sigma j}^{nlma} = \bar{H}_{\sigma\sigma y^{state}} y_j^{state} + \bar{H}_{y^{state}y^{state}}\left(I_{nystate} \otimes y_{\sigma\sigma}^{state}\right) y_j^{state}. \tag{B.17}$$

Combining (B.16) with (B.17) and exploiting the fact that $\bar{H}_{\sigma\sigma\alpha} = \bar{H}_{\sigma\sigma\alpha}^{nlma}$ yield

$$\alpha_j = -\bar{\tilde{H}}_{\sigma\sigma\alpha}^{-1}\left[\bar{H}_{\sigma\sigma y^{state}} + \bar{H}_{y^{state}y^{state}}\left(I_{nystate} \otimes y_{\sigma\sigma}^{state}\right)\right] y_j^{state}. \tag{B.18}$$

Since we are interested in first-order coefficients, (B.18) has to be equal to $g_{y^{state}}^{\alpha,nlma} y_j^{state}$. Thus,

$$g_{y^{state}}^{\alpha,nlma} = -\bar{\tilde{H}}_{\sigma\sigma\alpha}^{-1}\left[\bar{H}_{\sigma\sigma y^{state}} + \bar{H}_{y^{state}y^{state}}\left(I_{nystate} \otimes y_{\sigma\sigma}^{state}\right)\right] = -\bar{\tilde{H}}_{\sigma\sigma\alpha}^{-1}\bar{\tilde{H}}_{\sigma\sigma y^{state}}^{nlma}, \tag{B.19}$$

with

$$\begin{aligned}
\bar{\tilde{H}}_{\sigma\sigma y^{state}}^{nlma} = {} & 2\left[\left(m_{\mu\mu}\left(g_{y^{state}}^{\mu} \otimes g_{\epsilon}^{\mu}\right)\right) \otimes \left(b_r g_\epsilon^r\right)\right]\left[I_{nystate} \otimes vec(\Sigma)\right] \\
& + 2\left[\left(m_\mu g_{\epsilon y^{state}}^{\mu}\right) \otimes \left(b_r g_\epsilon^r\right)\right]\left[I_{nystate} \otimes vec(\Sigma)\right] \\
& + 2\left[\left(b_r g_{\epsilon y^{state}}^{r}\right) \otimes \left(m_\mu g_\epsilon^\mu\right)\right]\left[I_{nystate} \otimes vec(\Sigma)\right] \\
& + \left[\left(m_\mu g_{y^{state}}^{\mu}\right) \otimes \left(b_{rr}\left(g_\epsilon^r \otimes g_\epsilon^r\right)\right)\right]\left[I_{nystate} \otimes vec(\Sigma)\right] \\
& + \left[\left(m_\mu g_{y^{state}}^{\mu}\right) \otimes \left(b_r g_{\epsilon\epsilon}^r\right)\right]\left[I_{nystate} \otimes vec(\Sigma)\right] \\
& + \left[\left(m_\mu g_{y^{state}}^{\mu}\right) \otimes \left(b_r r_{\sigma\sigma}\right)\right].
\end{aligned} \tag{B.20}$$

[1]Note that α_t does not depend on realizations of ϵ_{t+1}.

First-order uncertainty correction can be obtained by

$$\alpha_\sigma = -\Phi \bar{\tilde{H}}^{nlma}_{\sigma}. \tag{B.21}$$

Implicit differentiation of (B.15) yields $\bar{\tilde{H}}^{nlma}_{\sigma} = \frac{1}{6}\bar{H}^{nlma}_{\sigma\sigma\sigma}$ and $\bar{\tilde{H}}^{nlma}_{\alpha} = \frac{1}{2}\bar{H}^{nlma}_{\sigma\sigma\alpha}$. By differentiating (B.14) and exploiting the certainty equivalency of first-order approximation ($y_\sigma = 0$) we can obtain the following relation

$$\bar{H}^{nlma}_{\sigma\sigma\sigma} = \bar{H}_{\sigma\sigma\sigma}. \tag{B.22}$$

Therefore, both *BIF* and *BIFN* yield the same first-order risk adjustment term, i.e. $\alpha_\sigma = g^{\alpha}_{\sigma}$.

B.3.3 Second-Order Coefficients

Given symmetrically distributed shocks, applying the procedure from previous sections leads to the following expression for the second-order uncertainty correction

$$\alpha_{\sigma\sigma} = -\Phi \bar{\tilde{H}}^{nlma}_{\sigma\sigma}, \tag{B.23}$$

with

$$\bar{\tilde{H}}^{nlma}_{\sigma\sigma\sigma\sigma} = \frac{1}{12}\bar{H}^{nlma}_{\sigma\sigma\sigma\sigma}, \tag{B.24}$$

and

$$\bar{H}^{nlma}_{\sigma\sigma\sigma\sigma} = \bar{H}_{\sigma\sigma\sigma\sigma} + 6\bar{H}_{\sigma\sigma y^{state}} y^{state}_{\sigma\sigma} + 3\bar{H}_{y^{state}y^{state}}\left(y^{state}_{\sigma\sigma} \otimes y^{state}_{\sigma\sigma}\right). \tag{B.25}$$

Combining these three equations yields

$$\alpha_{\sigma\sigma} = g^{\alpha,nlma}_{\sigma\sigma} + g^{\alpha,nlma}_{y^{state}} y^{state}_{\sigma\sigma} = g^{\alpha}_{\sigma\sigma} + \Delta + g^{\alpha,nlma}_{y^{state}} y^{state}_{\sigma\sigma}, \tag{B.26}$$

with

$$\begin{aligned}\Delta \equiv -\Phi \Big[&\left(\left(h_{\mu\mu}\left(g^{\mu}_{\epsilon} \otimes g^{\mu}_{\epsilon}\right)\right) \otimes \left(b_r g^{r}_{y^{state}}\right)\right) vec(\Sigma) + \left(h_\mu g^{\mu}_{\sigma\sigma}\right) \otimes \left(b_r g^{r}_{y^{state}} y^{state}_{\sigma\sigma}\right) \\ &+ \left(\left(h_\mu g^{\mu}_{\epsilon\epsilon}\right) \otimes \left(b_r g^{r}_{y^{state}} y^{state}_{\sigma\sigma}\right)\right) vec(\Sigma) + 2\left(\left(h_\mu g^{\mu}_{\epsilon}\right) \otimes \left(b_{rr}\left(g^{r}_{y^{state}} y^{state}_{\sigma\sigma} \otimes g^{r}_{\epsilon}\right)\right)\right) vec(\Sigma)\Big].\end{aligned}$$

Δ accounts for the transition from the first to the second order of accuracy as excess returns do not depend on state variables up to first-order approximation.

Finally, the second-order coefficient on $\epsilon_{t-j} \otimes \epsilon_{t-j}$ can be expressed as[2]

$$\alpha_{jj} = -\bar{H}^{-1}_{\sigma\sigma\alpha}\tilde{\Gamma}^{nlma}, \tag{B.27}$$

[2]Note that I have already exploited the mappings between $\tilde{H}^{nlma}$, H^{nlma} and H to arrive at (B.27) and (B.28).

with

$$\begin{aligned}\tilde{\Gamma}^{nlma} \equiv\ & \bar{H}_{\sigma\sigma\alpha\alpha}\left(g^{\alpha,nlma}_{y^{state}} \otimes g^{\alpha,nlma}_{y^{state}}\right)\left(y^{state}_j \otimes y^{state}_j\right) + \bar{H}_{\sigma\sigma y^{state}y^{state}}\left(y^{state}_j \otimes y^{state}_j\right) \\ & + \bar{H}_{\sigma\sigma\alpha y^{state}}\left(y^{state}_j \otimes g^{\alpha,nlma}_{y^{state}} y^{state}_j\right)\left(I_{nepsilon} + K_{nepsilon,nepsilon}\right) \\ & + \bar{H}_{y^{state}y^{state}y^{state}}\left(I_{nystate^2} \otimes y^{state}_{\sigma\sigma}\right)\left(y^{state}_j \otimes y^{state}_j\right) \\ & + \left[\bar{H}_{\sigma\sigma y^{state}} + \bar{H}_{y^{state}y^{state}}\left(I_{nystate} \otimes y^{state}_{\sigma\sigma}\right)\right] y^{state}_{jj}. \end{aligned} \tag{B.28}$$

Then by using the relationship between the state-space approximation and the corresponding nonlinear moving average representation $\alpha_{jj} = g^{\alpha,nlma}_{y^{state}} y^{state}_{jj} + g^{\alpha,nlma}_{y^{state}y^{state}}\left(y^{state}_j \otimes y^{state}_j\right)$, we can obtain

$$\tilde{g}^{\alpha,nlma}_{y^{state}y^{state}} = -\bar{H}^{-1}_{\sigma\sigma\alpha}\Gamma^{nlma}, \tag{B.29}$$

with

$$\begin{aligned}\Gamma^{nlma} \equiv\ & \bar{H}_{\sigma\sigma\alpha\alpha}\left(\tilde{g}^{\alpha,nlma}_{y^{state}} \otimes \tilde{g}^{\alpha,nlma}_{y^{state}}\right) \\ & + \bar{H}_{\sigma\sigma\alpha y^{state}}\left(I_{nystate} \otimes \tilde{g}^{\alpha,nlma}_{y^{state}}\right)\left(I_{nystate^2} + K_{nystate,nystate}\right) \\ & + \bar{H}_{\sigma\sigma y^{state}y^{state}} + \bar{H}_{y^{state}y^{state}y^{state}}\left(I_{nystate} \otimes y^{state}_{\sigma\sigma}\right). \end{aligned}$$

Appendix C

Appendix to Chapter 4

C.1 Model Equations

The equilibrium is described by the following equations. For simplicity, only home country equations are presented. Equations (C.1) to (C.22), (C.26), and (C.27) have foreign country counterparts. This gives 53 equations in 52 variables: Y_t ,Y_t^*, C_t, C_t^*, I_t, I_t^*, W_t, W_t^*, L_t, L_t^*, T_t, T_t^*, U_{Ct}, U_{Ct}^*, $\Lambda_{t-1,t}$, $\Lambda_{t-1,t}^*$, R_t, R_t^*, A_t, A_t^*, K_t, K_t^*, Q_t, Q_t^*, R_{kt}, R_{kt}^*, S_{ht}, S_{ft}, S_{ht}^*, S_{ft}^*, D_t, D_t^*, N_t, N_t^*, υ_t, υ_t^*, μ_t, μ_t^*, μ_{ht}, μ_{ht}^*, μ_{ft}, μ_{ft}^*, Ω_t, Ω_t^*, ϑ_t, ϑ_t^*, N_{nt}, N_{nt}^*, N_{ot}, N_{ot}^*, σ_t^A, σ_t^θ, with one equation redundant by Walras' law.

Household

$$C_t + D_t = W_t L_t + R_{t-1} D_{t-1} + T_t, \tag{C.1}$$

$$W_t U_{Ct} = \chi L_t^\varphi \tag{C.2}$$

$$E_t\left[\Lambda_{t,t+1}\right] R_t = 1, \tag{C.3}$$

$$\Lambda_{t,t+1} \equiv \beta\left(C_t\right)\frac{U_{Ct+1}}{U_{Ct}} \tag{C.4}$$

$$U_{Ct} = C_t^{-\gamma} \tag{C.5}$$

$$\begin{aligned} T_t = {} & Q_t I_t - \left[1 + f_{inv}\left(I_t, K_{t-1}\right)\right] I_t \\ & + \left(1-\theta_t\right)\left[\left(R_{kt} - R_{t-1}\right) Q_{t-1} S_{ht-1} + \left(R_{kt}^* - R_{t-1}\right) Q_{t-1}^* S_{ft-1} + R_{t-1} N_{t-1}\right] \end{aligned} \tag{C.6}$$

Goods Producers

$$Y_t = A_t K_{t-1}^\alpha L_t^{1-\alpha}, \tag{C.7}$$

$$W_t = \left(1-\alpha\right)\left(K_{t-1}\right)^\alpha L_t^{-\alpha}. \tag{C.8}$$

$$R_{kt} = \frac{\alpha\frac{Y_t}{K_{t-1}} + Q_t\left(1-\delta\right)}{Q_{t-1}} \tag{C.9}$$

Capital Producers

$$Q_t = 1 + \kappa \left(\frac{I_t}{\delta K_{t-1}} - 1 \right) \tag{C.10}$$

$$K_t = (1 - \delta) K_{t-1} + I_t \tag{C.11}$$

Financial Intermediaries

$$Q_t S_{ht} + Q_t^* S_{ft} = D_t + N_t \tag{C.12}$$

$$Q_t S_{ht} + Q_t^* S_{ft} = \phi_t N_t \tag{C.13}$$

$$\phi_t \equiv \frac{\upsilon_t}{\lambda - \mu_t} \tag{C.14}$$

$$\mu_{ht} = E_t \left[\Lambda_{t,t+1} \Omega_{t+1} \left(R_{kt+1} - R_t \right) \right] \tag{C.15}$$

$$\mu_{ft} = E_t \left[\Lambda_{t,t+1} \Omega_{t+1} \left(R_{kt+1}^* - R_t \right) \right] \tag{C.16}$$

$$\mu_{ht} = \mu_{ft} \tag{C.17}$$

$$\upsilon_t = E_t \left[\Lambda_{t,t+1} \Omega_{t+1} \right] R_t \tag{C.18}$$

$$\Omega_{t+1} \equiv 1 - \theta_{t+1} + \theta_{t+1} \left(\upsilon_{t+1} + \phi_{t+1} \mu_{t+1} \right) \tag{C.19}$$

$$N_{nt} = \omega \left(Q_{t-1} S_{ht-1} + Q_{t-1}^* S_{ft-1} \right) \tag{C.20}$$

$$N_{ot} = \theta_t \left[\left(R_{kt} - R_{t-1} \right) Q_{t-1} S_{ht-1} + \left(R_{kt}^* - R_{t-1} \right) Q_{t-1}^* S_{ft-1} + R_{t-1} N_{t-1} \right] \tag{C.21}$$

$$N_t = N_{nt} + N_{ot} \tag{C.22}$$

Market Clearing Conditions

$$Y_t + Y_t^* = C_t + C_t^* + \left[1 + f_{inv} \left(I_t, K_{t-1} \right) \right] I_t + \left[1 + f_{inv} \left(I_t^*, K_{t-1}^* \right) \right] I_t^*, \tag{C.23}$$

$$K_t = S_{ht} + S_{ht}^* \tag{C.24}$$

$$K_t^* = S_{ft} + S_{ft}^*. \tag{C.25}$$

Shock Processes

$$A_t = (1 - \rho_A) + \rho_A A_{t-1} + \upsilon_A \sigma_{t-1}^A \epsilon_t^A, \tag{C.26}$$

$$\vartheta_t = (1 - \rho_\theta) + \rho_\theta \vartheta_{t-1} + \upsilon_\theta \sigma_{t-1}^\theta \epsilon_t^\theta. \tag{C.27}$$

$$\sigma_t^A = (1 - \rho_{\sigma^A}) \bar{\sigma}^A + \rho_{\sigma^A} \sigma_{t-1}^A + \tau_{\sigma^A} \epsilon_t^{\sigma^A}, \tag{C.28}$$

$$\sigma_t^\theta = (1 - \rho_{\sigma^\theta}) \bar{\sigma}^\theta + \rho_{\sigma^\theta} \sigma_{t-1}^\theta + \tau_{\sigma^\theta} \epsilon_t^{\sigma^\theta}, \tag{C.29}$$

C.2 Equations of the Rewritten Model

Similar to Devereux and Sutherland (2010, 2011), the chosen solution method requires that the model will be rewritten in terms of net foreign assets. Let $\zeta_{ht} \equiv Q_t (S_{ht} - K_t)$ and $\zeta_{ft} \equiv Q_t^* S_{ft}$. Then, the net foreign asset position of home country is defined as $NFA_t \equiv \zeta_{ht} + \zeta_{ft}$. (C.6), (C.12), (C.13), (C.20) and (C.21) are replaced by (C.30), (C.31), (C.32), (C.33) and (C.34), respectively. Market clearing conditions (C.24) and (C.26) boil down to equation (C.35). Finally, equation (C.17) and its foreign country counterpart are combined to pricing equation (C.36). Variables S_{ht}, S_{ft}, S_{ht}^*, S_{ft}^* are replaced by ζ_{ht}, ζ_{ft}, NFA_t and NFA_t^*. Thus, we have 52 variables and 53 equations (including definition of NFA and guessed policy function for ζ_{ht}) with one equation being again redundant by Walras' Law.

$$\begin{aligned} T_t = {} & Q_t I_t - [1 + f_{inv}(I_t, K_{t-1})] I_t \\ & + (1 - \theta_t) [R_{xt}\zeta_{ht-1} + R_{kt}^* NFA_{t-1} + R_{kt} Q_{t-1} K_{t-1} - R_{t-1} D_{t-1}] \\ & - \omega (NFA_{t-1} + Q_{t-1} K_{t-1}) \end{aligned} \tag{C.30}$$

$$NFA_t + Q_t K_t = D_t + N_t \tag{C.31}$$

$$NFA_t + Q_t K_t = \phi_t N_t \tag{C.32}$$

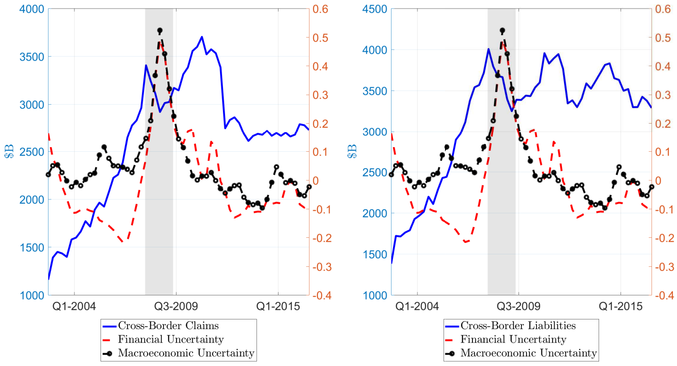
$$N_{nt} = \omega (NFA_{t-1} + Q_{t-1} K_{t-1}) \tag{C.33}$$

$$N_{ot} = \theta_t [R_{xt}\zeta_{ht-1} + R_{kt}^* NFA_{t-1} + R_{kt} Q_{t-1} K_{t-1} - R_{t-1} D_{t-1}] \tag{C.34}$$

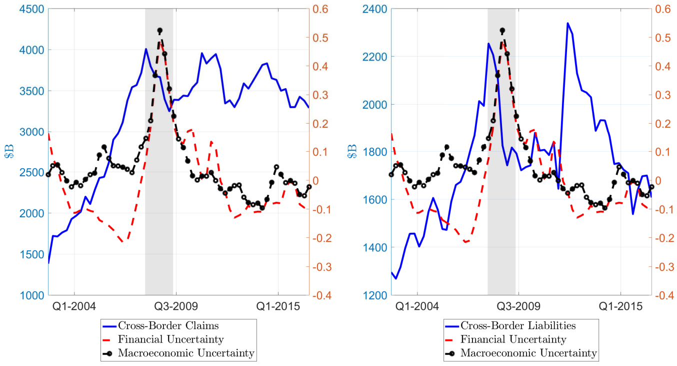
$$NFA_t = -NFA_t^* \tag{C.35}$$

$$\left(\Lambda_{t,t+1}\Omega_{t+1} + \Lambda_{t,t+1}^*\Omega_{t+1}^*\right) R_{xt+1} = 0 \tag{C.36}$$

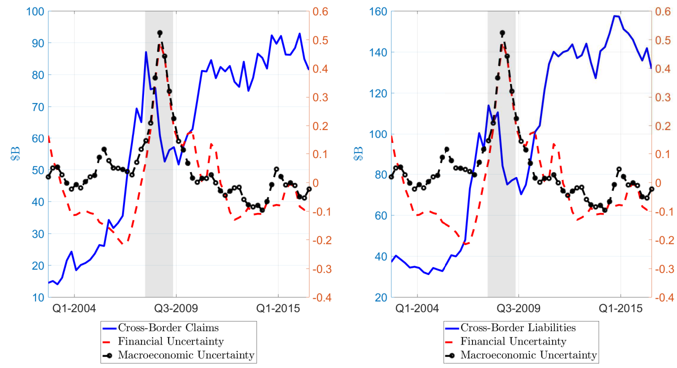
C.3 Additional Figures



(a) USA



(b) Germany



(c) Brazil

Figure C.1: Dynamics of international banking positions. *Sources: cross-border assets and liabilities: BIS Locational Statistics ; macroeconomic uncertainty: Jurado et al. (2015); financial uncertainty: Ludvigson et al. (2015). Cross-border banking positions are expressed in billions of US dollars, whereas uncertainty measures are presented as percentage deviations from the respective pre-crisis mean. The gray bars represent periods of recession defined by the NBER.*

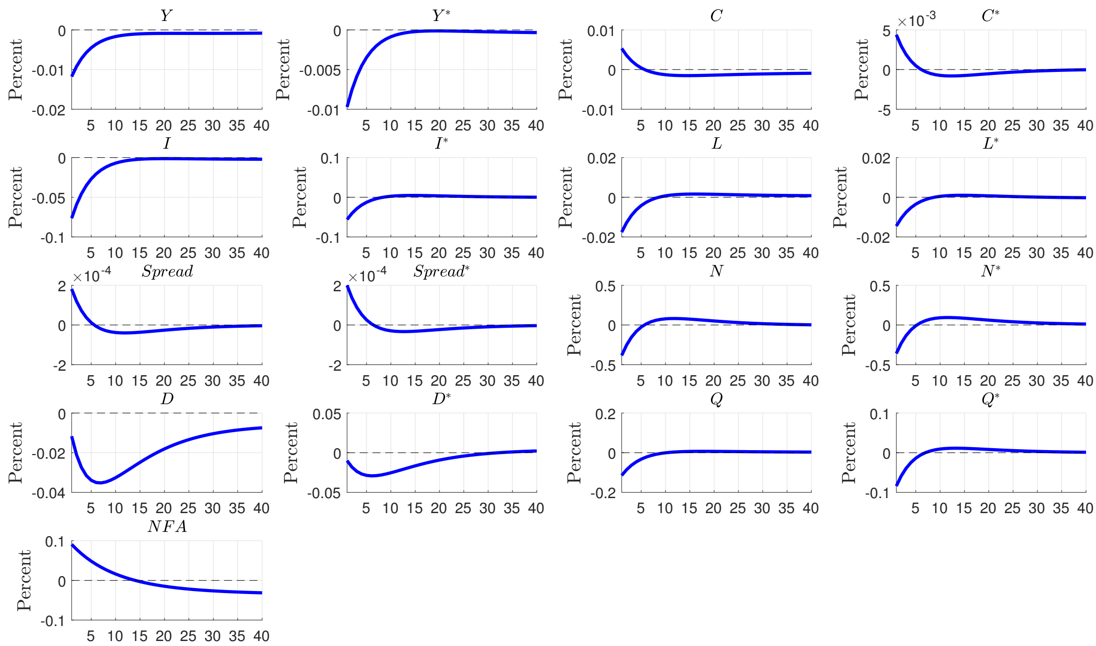


Figure C.2: Robustness check: foreign financial shocks are more volatile. *Only financial shocks are more volatile in foreign country. Level shocks are held constant. Horizontal axes indicate quarters. All responses are in percent, except for spreads, defined as the expected future rate of return on assets in a country relative to the domestic rate on deposits.*

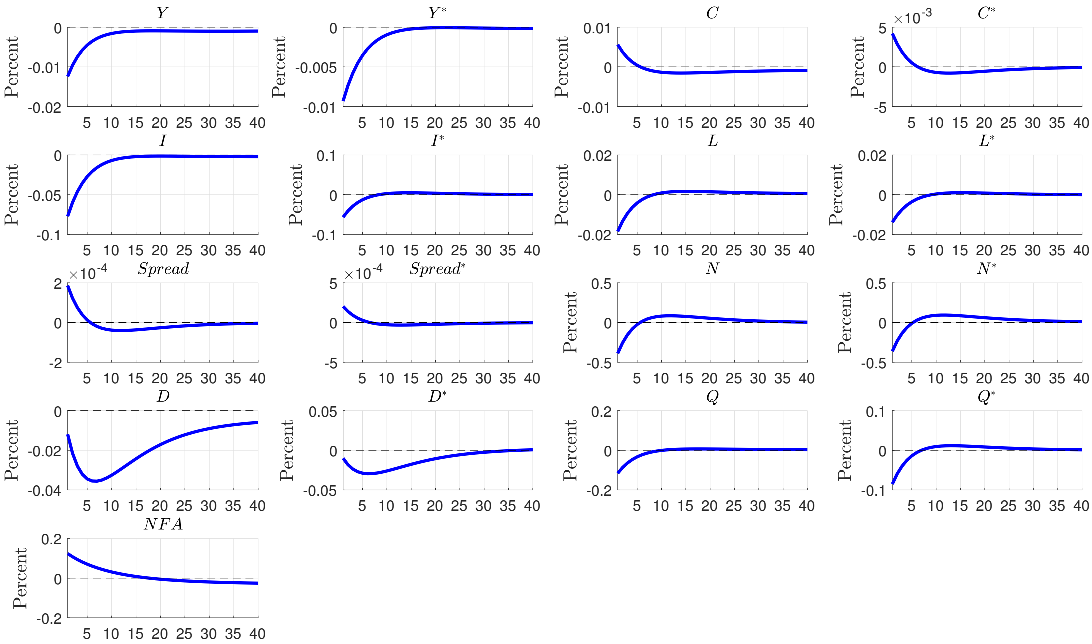


Figure C.3: Robustness check: both foreign TFP and financial shocks are more volatile. *Both financial and TFP shocks are more volatile in foreign country.Level shocks are held constant. Horizontal axes indicate quarters. All responses are in percent, except for spreads, defined as the expected future rate of return on assets in a country relative to the domestic rate on deposits.*

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Selbstständigkeitserklärung

Ich versichere, die von mir vorgelegte Dissertation selbständig und ohne unerlaubte Hilfe und Hilfsmittel angefertigt, sowie die benutzten Quellen und Daten anderen Ursprungs als solche kenntlich gemacht zu haben.

Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung meiner Dissertation benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

Berlin, 18. April 2019

Grzegorz Długoszek