Rodeo or Ascot:
Which Hat to wear at the Crypto Race?

Master's Thesis submitted

to

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1 Abstract

The contribution of this paper is twofold. Firstly, it evaluates the performance of several cryptocurrency (CC) indices by two criteria: accuracy in representing the CC sector and statistical properties. Multiple measures are applied to account for the non-normally distributed returns of the CCs. Among the analyzed indices, the CRIX developed by Trimborn and Härdle [2018] reveals the most stable characteristics, whereas Bitwise 10 is the closest in tracking the overall market dynamics. The second contribution of this paper lies in the econometric analysis of the CC market. By modeling its dynamics via an SVCJ model with a rolling window, it is possible to catch the extreme ups and downs of the CC market and to understand the functioning of its dynamics. Parameter estimates are not robust over time and vary with the window size. However, several recurring patterns are observable, which are robust to changes of the window size and supported by clustering of parameter estimates: during bullish periods, volatility stabilizes at low levels and the size and volatility of jumps in mean decreases. In bearish periods though, volatility increases and takes longer to return to its long-run trend. Furthermore, jumps in mean and jumps in volatility are independent. With the rise of the CC market in 2017, a level shift of the volatility of volatility occurred. All codes are available on Quantlet.de

Contents

2 Introduction

A widely used tool for investments are index-funds. For the rising cryptocurrency (CC) market, there is no central authority that issues an index. In recent years, several research groups and companies developed indices for the CC-market, each of them with different methodological approaches and ideas for tracking the highly volatile and dynamic CC market. The first part of this paper aims to compare and evaluate the characteristics of the currently existing CC-Indices: Bloomberg Galaxy Crypto Index (BGCI), Bitwise 10, the CRIX, CCi30 and F5 crypto index.
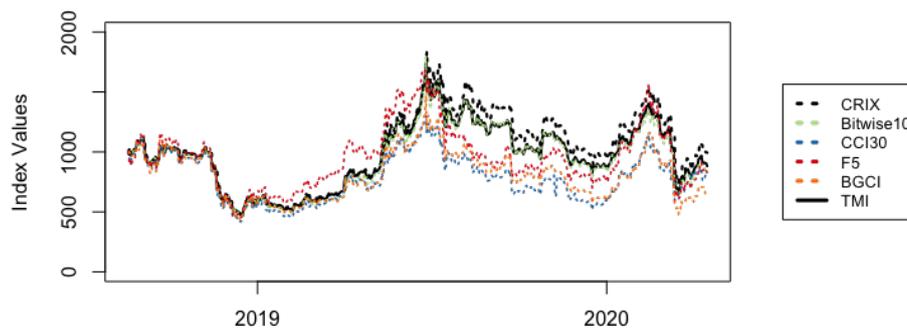

Figure 1: Scaled CC indices (dotted lines) and the total market index (TMI, black solid line). The indeces deviate from the TMI due to methodological differences in the construction and composition. Q SVCJrw_Indices_SharpeR

The methodological differences in the construction of the indices are analyzed in section 3.2. Major differences lie in the weighting of each CC to the respective index and the number of constituents. There is no 'right' or 'wrong' in constructing an index, each approach has its raison d'être, but they yield by construction different index values with dissimilar properties. A complete index should contain all available CCs weighted by their market capitalization, though this is practically not possible because of trading costs. A trade-off between low deviations from the total CC market and a sparse number of constituents is a logical consequence.

To compare the indices, I suggest two comparison criteria: 1. Accuracy in representing the CC sector and 2. investor suitable characteristics. The first is tackled by constructing a total market index (TMI) and an examination of the correlation and deviation of each index to the TMI. Figure 1 shows the indices under review (dotted lines) and the TMI (black solid line), scaled to 1000 points at mid-2018. The dynamics of the indices obviously differ, although they all start at 1000 points in 2018 and end there in mid-2020. An examination of the indices (presented in Section 4) shows that the Bitwise 10 CC index and the CRIX developed by Trimborn and Härdle [2018] are the most accurate indices in tracking the CC market dynamics. The key to success of the CRIX lies in its composition: by an iterative procedure, the returns of several portfolios that contain a varying number of CCs are compared to the returns of the TMI; the optimal number of constituents is then chosen by some information criteria (AIC or BIC).

The latter criteria is more difficult to address, as it is subject to the investors which properties of an index are 'suitable'. Generally, high returns and low risk/volatility are desirable, but this alone would not meet the challenges of non-normally distributed higher moments of CCs (cf. Zhang et al. [2018]). To account for the non-normal nature of CCs (cf. Table 2), the Probabilistic Sharpe Ratio (PSR) introduced by Bailey and Lopez de Prado [2012] is applied to compare the performance of indices. The PSR uses confidence bands that are adjusted by skewness and kurtosis that deviate from the normal distribution, thereby allowing to assess estimates of Sharpe Ratios probabilistically. The estimates are presented in Table 2; the low returns are due to the period of analysis of mid-2018 to mid-2020 (the recent growth of Bitcoin is not included). Not surprisingly, the Sharpe Ratio estimates are quite similar for all indices, though the PSR of the CRIX and F5 CC index outperform their competitors.

The second contribution of this paper lies in the statistical analysis of the dynamics of the CC market. Investors need to understand the mechanisms and

dynamics of this asset class. Chen et al. [2016] have shown that it is not possible to capture these dynamics using standard time series methods. The approach in this paper builds on the work of Duffie et al. [2000] and assumes a stochastic movement of the mean index value as well as a stochastic movement of its volatility. Also, we consider correlations between jumps in the mean and jumps in volatility. The parameter estimates of this so-called SVCJ model (stochastic volatility with correlated jumps) are performed using the Metropolis-Hastings algorithm. In Bayesian estimation settings, parameter estimates are very sensitive to changes in the input data. To ensure robustness of the estimates, I use a rolling window approach and estimate multiple models (even with varying window sizes), thereby obtaining time series for each parameter. In general, parameter estimates are time-varying and sensitive to window size. However, several recurring patterns are observable that are robust to changes in window size and are supported by k-means clustering of the parameter estimates: First, volatility remains at a low level during bullish CC market movements and rises in times of bearish markets. In addition, when volatility is already on a high level, it needs longer to return to its long-run trend. Second, in times of bullish markets, the size of jumps in mean return decreases, and its volatility stabilizes as well at low levels. Third, a level shift of the volatility of volatility parameter occurred simultaneously to the rise of the CC market at the turn of the year 2017/18.

The first part of this paper is structured as follows: Section 3 explains the CC indices data and their composition, Section 4 conducts the comparison of the indices by two criteria: accuracy in representing the CC market and statistical properties. In the second part of this paper, the dynamics of the CC market are analyzed by statistical tools: Section 5.1 introduces the methodology and estimation approach and Section 5.2 presents the estimation results and their robustness. Section 5.3 reveals the dependencies among parameter estimates by k-means clustering, and thereby identifies several stylized facts on the CC market dynamics. All codes are available on **Q** Quantlet.de

3 Data

3.1 CC data and Index data

Two data sets are used in the analysis: daily data of all publicly traded CCs is obtained from coingecko.com. The author thankfully acknowledges their freely accessible API that allows retrieving information on prices, volumes and market capitalization for each CC. In addition, data on CC-indices is thankfully provided by bitwiseinvest, Bloomberg, f5 crypto capital, CCi30 and thecrix.de. As the CC-market is not centrally organized, no central authority issues an industry benchmark index. The before mentioned index issuers and data providers are partially research units and partially private companies. Their methodological approaches differ significantly, which is the topic of the following chapter.

The period of the analysis is restricted to 2018-2020. The reasoning is rather practically than theoretically founded, because many of the indices have been just recently issued and their historical values are thus not available. However, one advantage of the selected time period is that the rise and fall of the CC market at the turn of the year 2017/ 2018 is not included, which makes the performance analysis in Section 4 more robust.

3.2 Compositon of Indices

Bitwise 10 is a CC index and an investible fund by San Francisco-based company Bitwise Asset Management. Their CC index Bitwise 10 covers the ten biggest CCs by market capitalization, not including stablecoins (=CCs that are pegged to a fiat currency). Unfortunately, their methodological approach is not publicly disclosed, so the composition and weighting scheme of Bitwise 10 is not known to the author.

Bloomberg Galaxy Crypto Index is issued by Bloomberg in cooperation with Galaxy Crypto. The index consists of at most 12 constituents, which are

the biggest CCs ranked by market capitalization. The contribution of each constituent to the final index is capped at 40% and floored by 1%. Unfortunately, Bloomberg does not publish its guidelines on which base they determine the number of constituents of its index. The index values are calculated by

$$BGCI_t = \frac{\sum_{i=1}^{x} P_{i,t} \times CS_{i,m} \times CF_{i,m}}{D}$$

where x is the number of constituents of the index, $CS_{i,m}$ the circulating supply of each constituent i in month m and $CF_{i,m}$ the cap/ floor correction factor; D is a divisor for scaling. If a constituent exceeds the cap, its remaining weight is redistributed among the other constituents relative to their market capitalization.

CRIX The CRyptocurrency IndeX has been developed at the Blockchain Research Center at Humboldt University Berlin by Trimborn and Härdle [2018] and is constructed as a Laspeyres index that weights the market capitalization of its constituents relative to the base year 2015.

$$CRIX_t = \frac{\sum_i P_{it} Q_{i0}}{\sum_i P_{i0} Q_{i0}}$$

The number of included coins is adjusted dynamically, to ensure that the CRIX represents the total CC market accurately. As selection criterion, the AIC is applied to balance the CRIX between a sparse number of constituents and an accurate representation of the whole CC market. Furthermore, only liquid CCs are eligible to be included on the index.

CCi30 Cryptocurrency Index 30 selects the top 30 CCs by adjusted market capitalization, excluding stablecoins. The market capitalization is adjusted such that it represents a moving average over the past days, thereby smoothing out

fluctuations.

$$M^*(t) = \frac{\sum_{i=0}^{\infty} M(T-i)e^{-\alpha i}}{\sum_{i=0}^{\infty} e^{-\alpha i}}$$

The contribution of each CC to the index is computed by the square root of its adjusted market capitalization relative to the market capitalization of the other constituents. The issuers (Rivin and Scevola [2018]) of the index argue that thereby the dominance of Bitcoin and Ethereum is reduced. However, this weighting scheme yields a distortive representation of the market. The index values are calculated by

$$I_t = \sum_{j=1}^{30} \frac{\sqrt{M_j^*(t)}}{\sum_{i=1}^{30} \sqrt{M_i^*(t)}} \frac{P_j(t)}{P_j(0)}.$$

F5 crypto index The F5 crypto index issued by Berlin-based start-up F5 Crypto Capital (cf. Elendner [2018]) is a CC index as well as an investible fund. It consists of the 12 biggest CCs by market capitalization. Excluded are stablecoins, anonymous CCs and CCs that are traded less than 100 days. The weights of each constituent to the F5 index is computed by its *momentum*:

$$momentum_t = price_t - price_{t-n}$$

The weight allocation by the momentum strategy already reveals the purpose of the index as an investment tool: the best performing CCs get more weight. This approach is questionable, as the momentum is a measure of past and not future performance. The side effect is a less accurate representation of the whole CC market.

A tabular comparison of the index methodologies is summarized in Table 1. The main differences between the approaches concern the number of constituents and their weighting scheme to the respective index. Thereby the motivation of the index issuers seems to have an impact on the construction of the indices: whereas

some of the indices only aim to track the dynamics of the CC sector, others work already as investment vehicles. This impacts the level of transparency and the degree of scientific basedness of the approaches.

Table 1: Comparison of the Composition of CC Indices

	BGCI	Bitwise 10	CCi30	CRIX	F5 crypto index
Number of Constituents	≤ 12	10	30	depending on AIC	12
Index & Weights	$BGCI_t = \frac{\sum_{i=1}^{x} P_{i,t} \times CS_{i,m} \times CF_{i,m}}{D}$	NA	$I_t = \sum_{j=1}^{30} \frac{\sqrt{M_j^*(t)}}{\sum_{i=1}^{30} \sqrt{M_i^*(t)}} \frac{P_j(t)}{P_j(0)}$	$CRIX_t = \frac{\sum_i P_{it} Q_{i0}}{\sum_i P_{i0} Q_{i0}}$	$momentum_t = price_t - price_{t-n}$
Self-image		"industry standard" "like the S&P 500 of crypto"	"industry benchmark"	"the first"	"scientific approach"

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Abbreviations:

$M^* =$ moving-average adjusted Market Capitalization

$P_{it} =$ price of coin i at time t

$Q_{it} =$ number of coins at time t

$CS_{i,m} =$ circulating supply of each constituent i at month m

$CF_{i,m} =$ Cap/Floor correction factor: cap maximum weight of each coin at 40%, min floor 1%

4 Evaluation of CC Indices

The objective of issuing an index can either be to track a market segment as accurately as possible or to construct an investment instrument on it that allows to diversify coin-specific risks and to benefit from overall market gains. Irrespective of the motivation, two comparison criteria are proposed to measure the performance of the CC Indices under review with regard to both motives.

Firstly, tackling the issue of accuracy, the correlation of each index to a scaled total market index (TMI) is taken as comparison criteria. This benchmark TMI is composed of all available CCs, weighted by their market capitalization and normed to 1000 points as starting value. Such a TMI is an ideal theoretical construct, however, it is not feasible to implement it in practice since there are minimum trading amounts and trading fees.
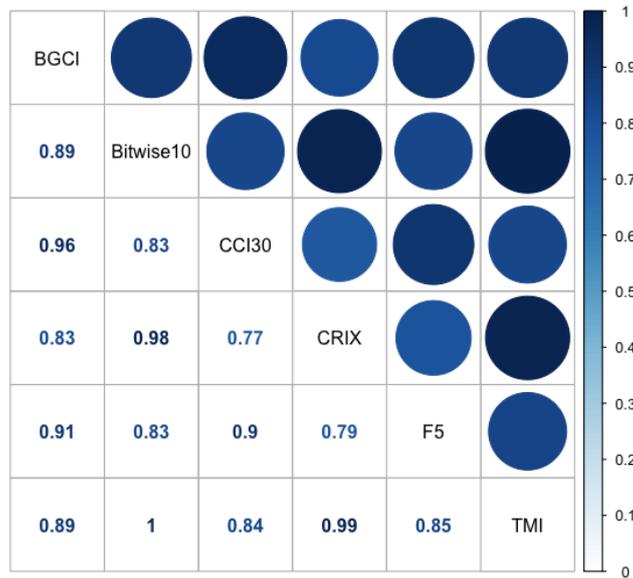

Figure 2: Correlation matrix of CC Indices and Total Market Index (TMI).
Q SVCJrw_Indices_SharpeR

As can be seen in the correlation matrix of Figure 2, all CC indices are highly correlated to the TMI. The highest correlation reveal Bitwise 10 and the

CRIX. It is not surprising that all the indices are correlated, however, it is surprising that Bitwise 10 reveals a higher correlation to the TMI than CCi30. As mentioned previously in Section 3.2, Bitwise 10 is composed of the ten biggest CC (by market capitalization) and CCi30 by the biggest 30. This finding indicates that it is enough to track the Top 10 coins and to discard the rest. Hu et al. [2019] found similar results, their study showed that the returns of CC are highly correlated, especially to the returns of Bitcoin.

Secondly, as the statistical properties of each index differ due to methodological differences of their composition, an analysis of their statistical properties is conducted, with a special focus on their moments. As comparison criteria, the Probabilistic Sharpe Ratio (PSR) introduced by Bailey and Lopez de Prado [2012] is used as a performance measure. A normal Sharpe Ratio as a measure of return to risk is a point estimate constructed on empirical estimates based on historical values. In settings where returns are non-normal, the classical Sharpe Ratio is not reliable, because higher moments have an impact on the confidence intervals of the estimated Sharpe Ratios, and thus on their statistical significance. The PSR instead corrects the confidence intervals for the non-normal higher moments.

As outlined by Lo [2002], the estimated variance of a Sharpe Ratio under the assumption that returns are normally distributed is given by

$$\hat{\sigma}(\widehat{SR}) = \sqrt{\frac{1}{n}\left(1 + \frac{1}{2}\widehat{SR}^2\right)} \qquad (1)$$

However, in the overall CC market, returns are highly non-normal. As shown in Table 2, the properties of all indices differ from the normal distribution. Mertens [2002] suggests to adjust the confidence bands of Sharpe Ratios that are estimated based on non-normally distributed returns by higher moments. Loosening the assumption of normal returns (cf. [Bailey and Lopez de Prado,

2012]), the estimated variance of the Sharpe Ratio extends to

$$\hat{\sigma}(\widehat{SR}) = \sqrt{\frac{1}{n-1}\left(1 + \frac{1}{2}\widehat{SR}^2 - \gamma_3\widehat{SR} + \frac{\gamma_4 - 3}{4}\widehat{SR}^2\right)} \qquad (2)$$

where γ_3 is the skewness and γ_4 the kurtosis. It is basically an Edgeworth expansion that adjusts for the non-normal higher moments. The PSR by Bailey and Lopez de Prado [2012] applies this standard deviation to assess the significance of the estimated Sharpe Ratios. Given a predefined benchmark SR^*, the PSR of Bailey and Lopez de Prado [2012] is defined as

$$PSR\left(SR^*\right) = \text{Prob}[SR^* \leq \widehat{SR}] \qquad (3)$$

which can be estimated by

$$\widehat{PSR}\left(SR^*\right) = Z\left[\frac{\left(\widehat{SR} - SR^*\right)}{\hat{\sigma}(\widehat{SR})}\right] = Z\left[\frac{\left(\widehat{SR} - SR^*\right)\sqrt{n-1}}{\sqrt{1 + \frac{1}{2}\widehat{SR}^2 - \gamma_3\widehat{SR} + \frac{\gamma - 3}{4}\widehat{SR}^2}}\right] \qquad (4)$$

where Z refers to the cdf of the standard normal distribution.

	sharpe_ratio	returns	sd	skewness	kurtosis	PSR
BGCI	0.005	0.02 %	0.043	-0.224	8.989	0.549
Bitwise10	0.013	0.05 %	0.040	-0.166	8.154	0.625
CCi30	0.012	0.05 %	0.041	-0.155	8.965	0.614
CRIX	0.021	0.02 %	0.040	-1.063	16.191	0.691
F5	0.017	0.07 %	0.046	-0.990	13.928	0.659
TMI	0.009	0.02 %	0.031	-1.292	17.314	0.587

Table 2: Descriptive Statistics on daily level for the CC indices under review. Period of analysis 08/2018-04/2020. PSR given the benchmark $SR^* = 0$ (probability of positive returns). **Q** SVCJrw_Indices_SharpeR

The estimates in Table 2 are supporting the findings of Zhang et al. [2018] who report heavy tails of return distributions for many CCs. The estimated returns seem quite low, which is due to the period of analysis and one has to consider that these are daily returns, not annualized returns. Skewness and kurtosis differ for all indices by far from the normal distribution, which supports

the use of PSR. Interestingly, the F5 crypto index reveals the highest return, which indicates that the *momentum* strategy is effective. The highest PSR is achieved by the CRIX, followed by F5 crypto index. This finding, combined with the high correlation of the CRIX to the TMI, indicates that the CRIX is the most suitable CC index.

5 Dynamics of the Cryptocurrency Market

It is of general interest to understand the dynamics of CC markets. Chen et al. [2016] have shown that standard econometric time series methods like ARIMA and GARCH cannot catch the heavy tails of the return distributions of CCs. Chen et al. [2018] however caught the dynamics of the CC market very accurately by using a model with stochastic volatility and correlated jumps (in mean price and volatility), denoted as SVCJ. The following analysis combines the SVCJ model framework of Perez [2018] and a rolling window approach to examine the dynamics and robustness of the CC market. Thereby, time series estimates for each parameter are obtained and some dependencies among them are identified. To start with, a short description of the SVCJ model and its estimation procedure will be explained, Section 5.2 presents time series for each parameter and discusses their behavior. Section 5.3 illustrates the correlations among parameters by visualizing them with k-means cluster analysis.

5.1 SVCJ - Model and Estimation Approach

The SVCJ model, introduced by Duffie et al. [2000] adds a jump process to the stochastic volatility model of Heston [1993]. In this setting, the mean index value is modeled by a geometric Wiener process, extended by a jump process:

$$d \log \left(S_t \right) = \mu dt + \sqrt{V_t} dW_t^{(s)} + Z_t^{(y)} dN_t \tag{5}$$
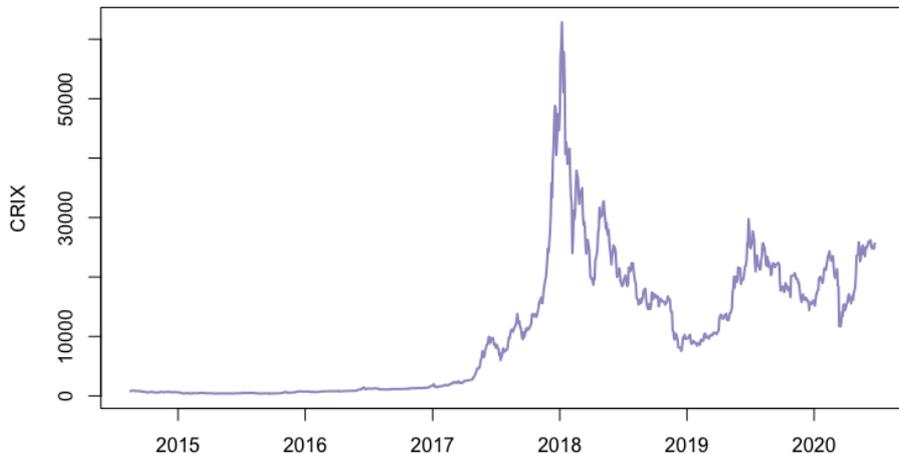
Figure 3: CRIX, 07-2014 to 07-2020. Data source: thecrix.de

where S_t denotes the index value, μ the trend or drift, V_t the volatility, $W_t^{(s)}$ a Wiener process and N_t is a pure jump process with a constant mean-jump arrival rate λ, such that $P\left(dN_t = 1\right) = \lambda dt$. The random jump size $Z_t^{(y)}$ follows a normal distribution (cf. Equation 7a).

Additionally, the variance is modeled as a stochastic process, allowing for deviations from its long-run trend as described by Cox et al. [2005] and extended by a jump process

$$dV_t = \kappa\left(\theta - V_t\right) dt + \sigma_v \sqrt{V_t} dW_t^{(v)} + Z_t^{(v)} dN_t \tag{6}$$

where κ refers to the speed of convergence of the volatility towards its trend θ, σ_v denotes the volatility of the volatility parameter and $W_t^{(v)}$ is a Wiener process that is correlated to $W_t^{(s)}$ at rate ρ, $\mathrm{Cov}\left(dW_t^{(s)}, dW_t^{(v)}\right) = \rho dt$. The SVCJ model differs from the previously mentioned Cox-Ingersoll-Ross model by allowing for correlation between the jump size of the mean trend and the jump

size of the volatility:

$$Z_t^y \mid Z_t^v \sim N\left(\mu_y + \rho_j Z_t^v, \sigma_y^2\right) \tag{7a}$$

$$Z_t^v \sim \text{Exp}\left(\mu_v\right) \tag{7b}$$

where Exp denotes the exponential distribution, which ensures that jumps in volatility are positive.

Bayesian Estimation Procedure

The author thankfully acknowledges the work of Chen et al. [2018] and Perez [2018], who provide an implementation framework for the SVCJ model for CCs. The code of the basic model of Perez [2018] is available on **Q** Quantlet.de and has been extended for the present analysis.

The estimation procedure follows a Bayesian approach: we are interested in the distribution of the parameters Θ and covariates X given the CRIX index data Y.

$$p(\Theta, X \mid Y) = p(Y \mid \Theta, X)p(X \mid \Theta)p(\Theta)$$

where $\Theta = \{\mu, \kappa, \theta, \sigma_v\}$ and $X = \{V_t, Z_t^y, Z_t^v, N_t\}$. We use the Metropolis Hastings algorithm to obtain Markov chains that converge to the posterior distribution as the number of iterations increases. For a discussion of the burn-in rate and settings of the prior distributions, please refer to Perez [2018]. Several checks for autocorrelation of parameter estimates along the iterations of the Metropolis-Hastings algorithm are there discussed as well.

Implementation

The empirical calibration of equation 5 and 6 is realized by Euler discretization, rewriting it as

$$Y_t = \mu + \sqrt{V_{t-1}}\varepsilon_t^y + Z_t^y J_t \tag{8a}$$

$$V_t = \alpha + \beta V_{t-1} + \sigma_v \sqrt{V_{t-1}}\varepsilon_t^v + Z_t^v J_t \tag{8b}$$

where $Y_{t+1} = \log(S_{t+1}/S_t)$ denotes the log return. $\varepsilon_t^y, \varepsilon_t^v$ are discrete versions of the Wiener processes, distributed as $N(0,1)$ and correlated at rate ρ. The volatility is calibrated by $\alpha = \kappa\theta$ and $\beta = 1 - \kappa$. The implementation of the jump processes is implemented by jump sizes Z_t^y and Z_t^v, following the distributions of equation 7b and a Bernoulli random variable J_t, with $P(J_t = 1) = \lambda$.

5.2 Robustness of Parameter Estimates

As we are interested in the dynamics of the CC sector, we want to examine whether it is possible to precisely characterize this sector by the parameter estimates of the above-described model. To do so, two robustness measures are applied: firstly, a rolling window approach is conducted, which yields time series estimates for each estimated parameter. Optimally, parameter estimates would be time-invariant, which would allow a precise description of the CC sector. Secondly, several window sizes are tested, to see whether the estimates depend on the choice of the window size. Time series estimates for each parameter are presented in Figure 4 (window size: 150 days) and Figure 5 (window sizes 150, 300 & 600 days).

The time series estimates in Figure 4 are obtained by shifting a rolling win-

dow of 150 days through the data of the period 07/2014 to 07/2020[1]. Estimates are fluctuating a lot, which is a typical issue in Bayesian estimation as they are sensitive to changes in the input data. The fluctuating lines are parameter estimates, the solid lines in their center are moving averages of 20 days. In the next paragraph, a discussion about i) the trend, ii) volatility and iii) jumps is conducted.

Trend The estimates are mainly behaving as expected: μ, the trend of the return process (cf. Equation 8a), moves parallelly to the CRIX (cf. Figure 3). Especially the growth in 2017 and the drop in 2018 are well observable. However, the trend is always one step ahead of the index. This is due to the forward-looking nature of the estimation procedure: the rolling window reacts early to changes in future index values.

Volatility The coefficients of volatility (α and β, see Equation 8b) reveal interesting patterns: α oscillates at a low level until the end of 2017, then suddenly jumps to a high level ($\alpha =0.5$) at the turn of 2017/2018. A similar pattern occurs at the end of 2019: less strongly, but in the same direction, α rises again. It is interesting to note that the rise in α always correlates with downturns in the CRIX: when the CRIX falls, the volatility level rises, or in other words: when the CC market is bearish, volatility is high. An alternative interpretation is possible by the construction of $\alpha = \theta\kappa$: when the market is bearish, the volatility takes longer to return to its long-run trend (i.e. κ increases) or the long-run volatility trend θ climbs up to a higher level.

β, the coefficient of lagged volatility V_{t-1}, seems to be correlated as well to the trend of the CC market: before the rise in 2017, its values oscillate around -0.4, they stagnate at -0.2 throughout 2017, though towards the end of 2017 they drop to -0.8 . The dynamics of β allow for several interpretations: volatility detaches from its lagged values in times of rising markets (β close

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[1]The historical index values for the CRIX are available from 2014. The previous analysis was limited to the period 2018-2020, as the historical values for other indices were not available.

to zero througout 2017), and quickly returns to its long-run trend (since β is reversely related to the reversion rate towards the long-run trend: $\beta = 1 - \kappa$). In bearish periods (market downturn), the values of β get closer to -1 and α increases, which indicates that volatility takes longer to return to its long-run level.

Additionally, there seems to be a correlation between σ_v and β: when σ_v increases, β declines and vice versa. This, however, may rather be model inherent than a reportable insight, as both parameters are coefficients of lagged volatility. Furthermore, we can observe that σ_v behaves similarly to μ or the CRIX: when the market increases, the volatility of volatility also increases.

Jumps The jump sizes (μ_v and μ_y) seem to interact with the overall CC market dynamics: in bullish periods, μ_y stabilizes at low levels and the volatility of jumps in mean σ_y stabilizes as well. Even its estimates do not fluctuate a lot. The jump arrival rate λ does not reveal any specific pattern and its values change only within a small interval (note that one would need scaled values to interpret the magnitude of the fluctuations).

In contrast to the study of Duffie et al. [2000], there is almost no correlation between the volatility of the mean trend and the volatility of the volatility, as the ρ estimates are close to zero. The estimates indicate that in certain intervals (at the end of 2015 and 2016) there are interactions between these parameters, though there is no overall effect observable.
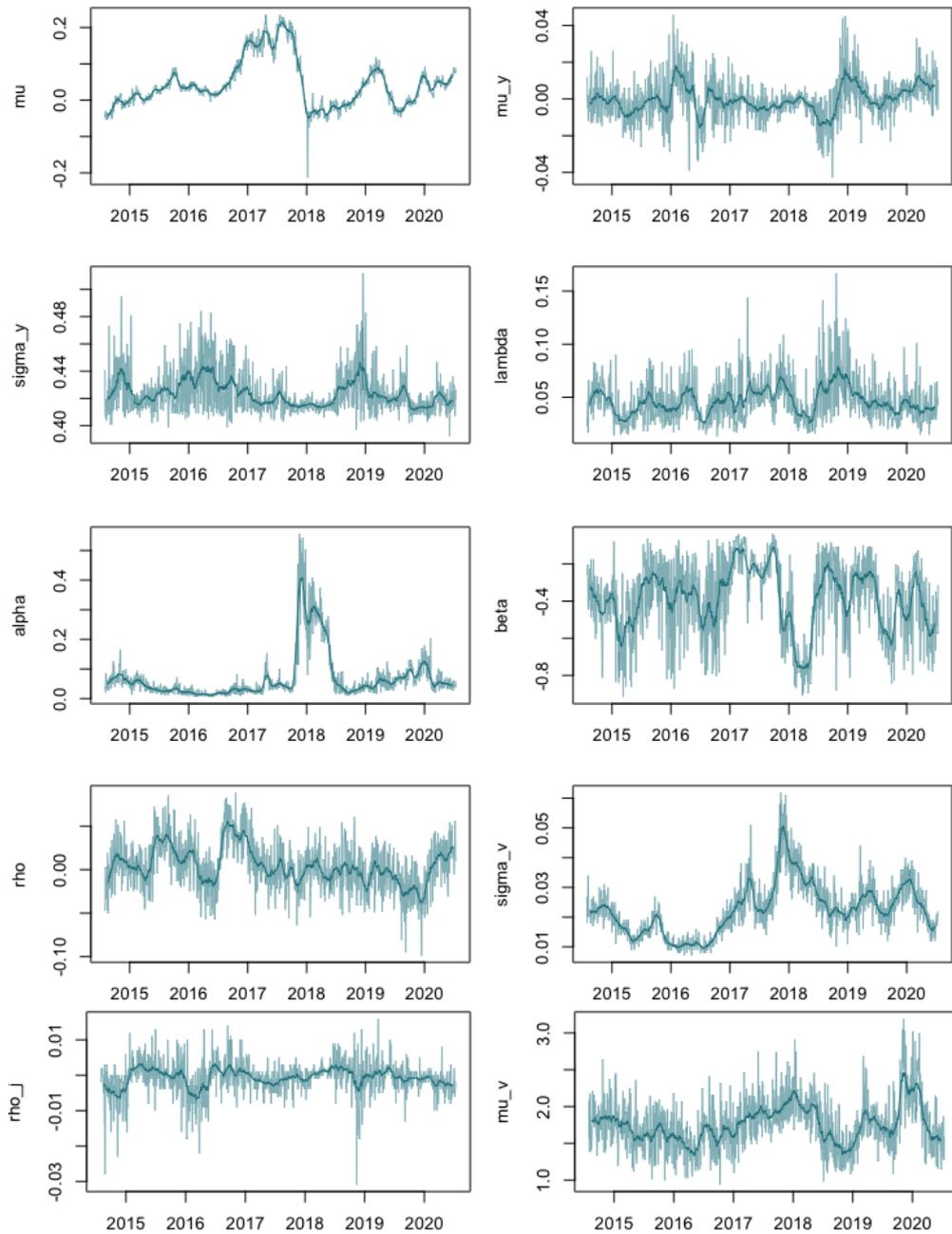
Figure 4: Parameter estimates of the SVCJ model with a rolling window of 150 days. The fluctuating lines represent actual parameter estimates, the solid lines in their center depict moving averages of 20 days. ⊙ SVCJrw_graph_parameters

To check whether the size of the rolling window has an impact on the parameter estimates, Figure 5 presents estimates for several rolling windows of size 150, 300, and 600 days. The three time series in each figure depict moving averages of 20 days for each window size.

The three time series are not identical and the estimates fluctuate a lot over time and are sensitive to the size of the rolling windows. The bigger the window, the more temporary fluctuations are smoothed, which is especially protruding for the parameters μ, α, ρ and σ_y. However one can observe common patterns among the three time series:

Volatility The time series of the parameters for volatility show matching dynamics. The dynamics of α are very robust, almost over the entire period of the analysis, the time series overlap. Only around the turn of 2017/18 does α skyrocket, to varying degrees for each window size. This suggests that volatility increases when the market is falling.

Similarly, the time series of β converge at a level close to Zero throughout 2017. This confirms that volatility gets detached from its lagged values when the market is bullish. By contrast, when there is no clear market direction or when the market is falling, β deviates from Zero, i.e. volatility needs some time to return to its long-run trend and persists in its former state.

Another finding relates to the volatility of volatility σ_v: there seems to have been a regime change around the 2017/2018 turn of the year. Until 2017, σ_v fluctuated at low levels and increased strongly simultaneously with the growth of the CC sector. After 2018, volatility remained at this elevated level. Based on the CRIX time series alone, one cannot explain this level shift, but it stands to reason that the opportunities in the CC market have attracted many investors since 2018, which may have increased applications of CCs as well as speculation, thereby increasing volatility.

Jumps In the previous section 5.2, we observed that the size of jumps in mean μ_y declined to zero whenever the market is rising. Interestingly, this finding can

be confirmed: all three time series converge simultaneously towards zero for the period of 2017-2018. And not only the size of the jumps, but also their volatility decreases rapidly when the market is rising: one can wonderfully observe how the estimates of the volatility of the jumps σ_y decrease in 2017. Due to the forward-looking nature of rolling windows, estimates respond early to market changes. Weakened, but in the same direction, a convergence of the time series can be observed for the second half of 2019.
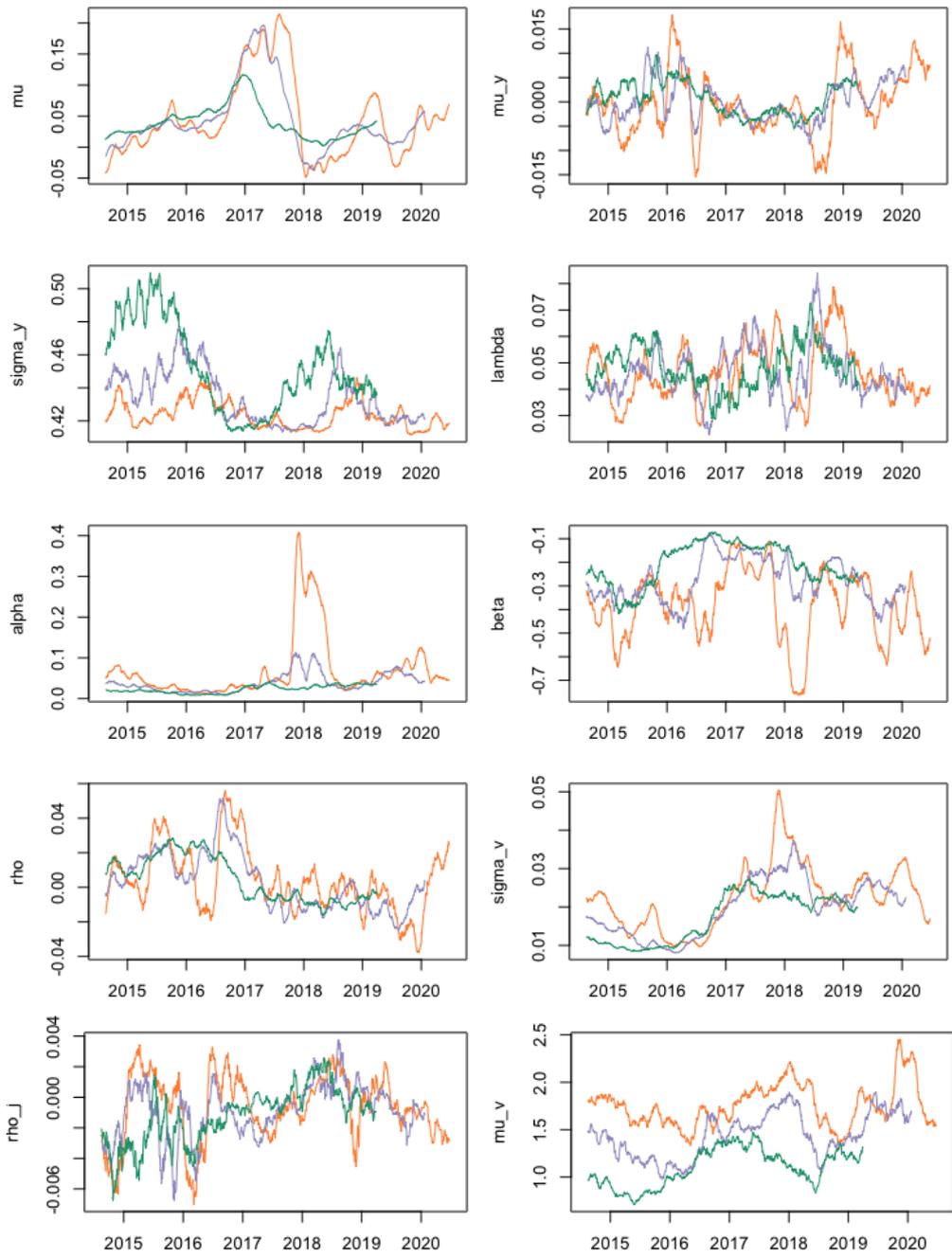
Figure 5: Paramter estimates for several window sizes (150, 300 & 600 days).
🔍 SVCJrw_graph_parameters

5.3 Cluster Analysis

Even though it is not possible to precisely characterize the CC sector by robust parameter estimates, some dependencies among the parameters are observable. The discussion of the previous section has already introduced some relationships between trend, volatility and jumps; in this section we will extend the analysis by an examination of clusters. Since the estimated time series are not independent, statistical inference is limited. However, some correlations between the time series of the parameters are recognizable, individual pairs of parameters move only in certain ranges, which reveal recurring patterns.

The focus of this section is on the interactions between trend and volatility. The time series estimates of μ, α, β and σ_v in Figure 4 revealed interesting patterns, the following cluster analysis will unravel their correlations. Figure 6 presents k-means clusters for the pair of parameters μ & β, respectively. The elbow method yielded the optimal number of $k = 3$ clusters. Below the clustered pair of parameters is the CRIX colored in the same colors as the clusters, which reveals the time dimension of the clustered time series.

As an example should serve the correlation between the trend μ and the volatility parameter β. The clustering of μ and β reveals interesting connections: there seems to be a linear relationship between the two parameters. This is impressive since the underlying CRIX data is highly non-stationary. Note that the forward-looking nature of the rolling window approach reacts early to future changes in the CRIX, which is the reason why the clusters do not clearly correspond to the rise and fall of the CRIX. Nevertheless, it is easy to see that volatility follows the trend. The increase in trend is accompanied by an increase in β, i.e. β converges to zero and the current volatility breaks away from its previous values (note that for clustering, variables were scaled). However, when the trend is negative, β also falls and volatility becomes more persistent.
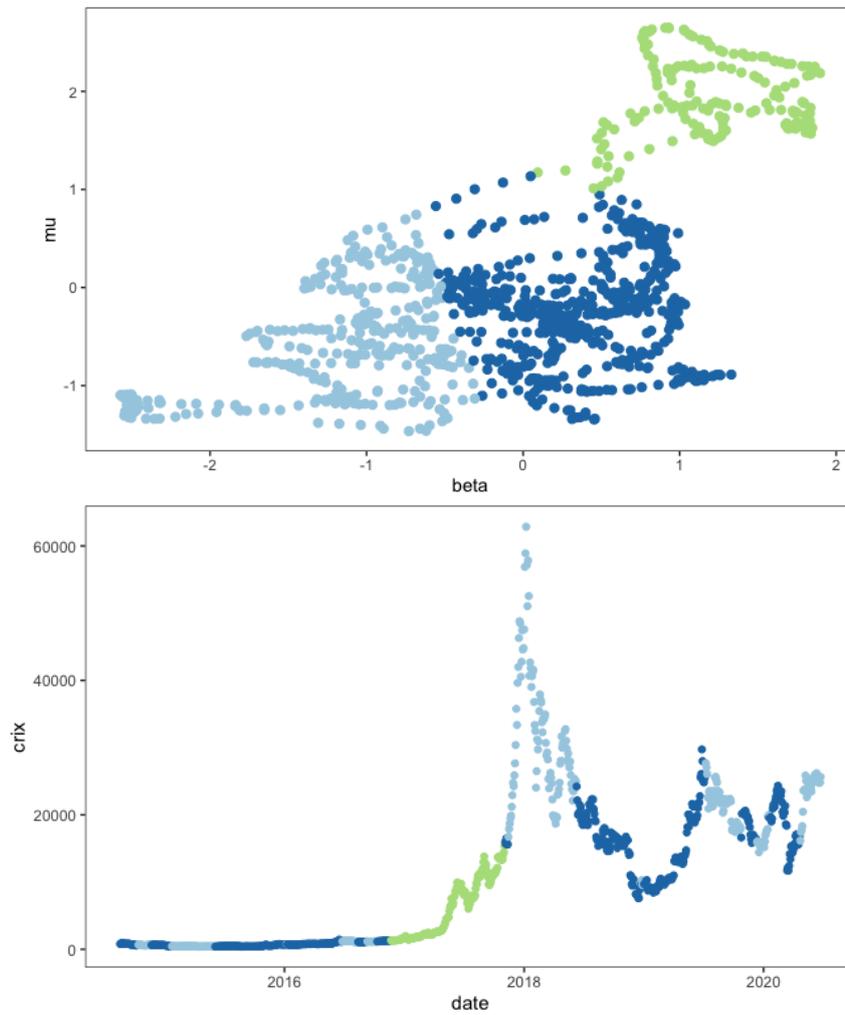
25

Figure 6: Top: k-means clusters of parameter estimates μ and β, k = 3. Bottom: the CRIX coloured by the respective clusters.

⌕ SVCJrw_clustered_parameters

6 Conclusion

The present thesis examined the CC sector in two ways: firstly, an analysis of the exisiting CC indices was conducted. A detailed assessment of their composition, methodological differences, statistical properties and accuracy in representing the CC sector yielded several insights: First, the major differences in the construction of the indices lie in the weighting scheme and the number of constituents in each index. Surprisingly, a larger number of CCs included in an index does not necessarily lead to higher accuracy in the representation of the CC market. The best example of this is Bitwise 10, the CRIX and CCi30: the correlation of the former two to the TMI is higher than for CCi30, which includes many more CCs in its index. Second, regarding the statistical properties of the indices, the use of PSR is justified due to the non-normally distributed returns of the CCs and the highest PSR at a benchmark of $SR^* = 0$ yields the CRIX. The key factor to success of the CRIX lies in its composition: at each date the CRIX is calculated, the return of the TMI is iteratively compared with a portfolio consisting of one, two, three, ... CCs and the optimal number of constituents is determined by AIC/BIC information criterion. This solves the challenging trade-off each index is facing: a low number of constituents and a high representation of the CC market.

The second part focused on the statistical modeling of the CC sector, and thereby on the characterization of its dynamics. Several SVCJ models are estimated in combination with a rolling window approach, yielding time series for each parameter of the model. The results reveal time-varying parameter estimates, which do not allow to precisely characterize the CC sector. However, some patterns among the parameter estimates are observable: First, volatility remains at a low level during bullish CC market movements and rises in times of bearish markets. In addition, when volatility is already on a high level, it needs longer to return to its long-run trend. Second, in times of bullish markets, the

size of jumps in mean return decreases, and its volatility stabilizes as well at low levels. Third, a level shift of the volatility of volatility parameter occurred simultaneously to the rise of the CC market at the turn of the year 2017/18. Finally, the jumps in mean and in volatility seem to be independent. The findings are robust to changes in the window size and confirmed by clustering of the parameters.

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