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Topological susceptibility from $N_f = 2+1+1$ lattice QCD at nonzero temperature

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Abstract. We present results for the topological susceptibility at nonzero temperature obtained from lattice QCD with four dynamical quark flavours. We apply different smoothing methods, including gradient Wilson flow and over–improved cooling, before calculating the susceptibility. It is shown that the considered smoothing techniques basically agree among each other, and that there are simple scaling relations between flow time and the number of cooling/smearing steps. The topological susceptibility exhibits a surprisingly slow decrease at high temperature.

The non–trivial topological structure of gauge fields and the computation of the topological susceptibility χ_{top} is discussed in lattice QCD since long time. Recent considerations (see [1] and references therein) ranging from the restoration of the $U_A(1)$ symmetry at high temperature (or density) to the abundance of cosmic axions [2] are calling for a better knowledge of χ_{top} as a function of the temperature and quark masses.

In this study we calculate χ_{top} in the temperature range $150 < T < 500$ MeV using lattice configurations generated with $N_f = 2+1+1$ dynamical Wilson twisted mass fermions at finite temperature [3]. The heavy doublet of s and c quarks has its mass parameters matching the physical K and D meson masses, while the HMC simulations still require light quarks being unnaturally heavy. The configurations are generated at three coupling values $\beta = 1.90$, 1.95, and 2.10 (for later continuum extrapolations) with the Wilson twisted mass fermionic action tuned to maximal twist, taking benefit from automatic $\mathcal{O}(a)$ improvement. Several charged pion masses are considered, but here we restrict ourselves to one value $m_{\pi^\pm} \simeq 370$ MeV.

We focus on the comparison between different smoothing methods for lattice gauge fields necessary to get χ_{top} by the gluonic method. The smoothing techniques under consideration are the Wilson flow [4], Wilson and over–improved cooling [5], and stout–link smearing [6]. We use the Wilson flow to set two stopping scales determined by

$$t^2\langle E\rangle\big|_{t=t_0} = 0.3 \quad \text{or} \quad t^2\langle E\rangle\big|_{t=t_1} = 0.66, \qquad E = \frac{1}{2N_\tau N_\sigma^3}\sum_x \mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}(x)], \tag{1}$$

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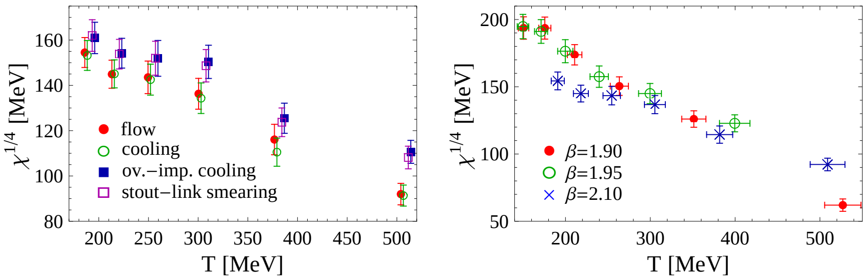

Figure 1. T dependence of $\chi_{top}^{1/4}$ from Wilson flow. Left: comparison with all other smoothing methods for $\beta = 2.10$ and t_0. Right: at stopping time t_0 for three different β values.

where $F_{\mu\nu}(x)$ is the field strength tensor on the lattice. We match ensemble averages $\langle E\rangle$ obtained in other methods to the values measured with the Wilson flow at t_0 and t_1 in order to relate these to corresponding numbers of cooling steps. Empirically we find in a good agreement for both the stopping criteria t_0 and t_1 that $N_{\rm cool}^{\rm Wilson} \simeq 3\tau$ [7], $N_{\rm cool}^{\text{ov.-imp.}} \simeq 5\tau$ and $N_{\rm smear}^{\rm stout} \simeq 12\tau$, the latter for our choice $\rho_{\rm smear} = 0.06$ (τ defined by $t = a^2\tau$, a is the lattice spacing).

If the diffusion radius $R = \sqrt{8t}$ at t_0 or t_1 satisfies the condition $R \ll 1/T$ the topological susceptibility $\chi_{top}(T)$ can be safely determined as

$$\chi_{top} = \frac{\langle Q_{top}^2\rangle}{V}, \qquad Q_{top} = \frac{a^4}{32\pi^2}e^{\mu\nu\rho\sigma}\sum_x \mathrm{Tr}[F_{\mu\nu}F_{\rho\sigma}(x)], \qquad V = a^4 N_\tau N_\sigma^3. \tag{2}$$

The spatial and temporal lattice sizes are $N_\sigma = 24$, 32 and $N_\tau = 4\ldots16$, respectively, and the statistics is varying in the range $200\ldots1000$ configurations, depending on the respective temperature. One can see from the left panel of Figure 1, where the methods are compared for the finest lattice spacing (at $\beta = 2.10$), that the pairs of Wilson flow and Wilson cooling, as well as over–improved cooling and stout–link smearing, give almost indistinguishable $\chi_{top}(T)$ values throughout the considered temperature range. In the same manner, the agreement between the two stopping criteria t_0 and t_1 can be checked independently for each algorithm. The results for χ_{top} from Wilson and over–improved cooling agree (within errors) for low temperatures. Starting from approximately $T \gtrsim 300$ MeV, the over–improved result for χ_{top} turns out somewhat larger than the results for Wilson flow and Wilson cooling as one would expect.

In the right panel of Figure 1, $\chi_{top}^{1/4}(T)$ is shown as calculated at three different β values (different lattice spacings a). The curves beyond $\sim$200 MeV can be reasonably approximated with linear fits, with slopes visibly flattening with the growth of β (towards $a \to 0$). Note that the value of crossover temperature obtained from the chiral susceptibility is $T_c = 184(4)$ MeV. Thus, we come to the preliminary conclusion that for $N_f = 2+1+1$ and $m_{\pi^\pm} \simeq 370$ MeV the topological susceptibility decreases very slowly beyond T_c in the continuum limit, in contrast to the rapid fall–off observed in the quenched approximation ($N_f = 0$), and to the gradual descent taking place in the $N_f = 2$ case [8].

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