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RESEARCH ARTICLE

Spying and imperfect commitment in first-price auctions: a case of tacit collusion

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Abstract

We analyze Stackelberg leadership in a first-price auction. Leadership is induced by an information system, represented by a spy, that leaks one bidder's bid before others choose their bids. However, the leader may secretly revise his bid with some probability; therefore, the leaked bid is only an imperfect signal. Whereas leadership with perfect commitment exclusively benefits the follower, imperfect commitment yields a collusive outcome, even if the likelihood that the leader may revise his bid is arbitrarily small. This collusive impact shows up in all equilibria and is strongest in the unique pooling equilibrium which is also payoff dominant.

Keywords Auctions · Tacit collusion · Espionage · Second-mover advantage · Signaling · Incomplete information

JEL Classification L12 · L13 · L41 · D43 · D44 · D82

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1 Introduction

The present paper studies Stackelberg leadership in a first-price auction subject to incomplete information and imperfect commitment. There, sequential bidding is induced by an information system, represented by a spy, who observes one bidder's bid, albeit imperfectly, and reports it to a rival bidder before the latter submits his own bid.

The presence of a spy is common knowledge. Therefore, bidders know that the spied-at bidder is Stackelberg leader and the bidder served by the spy is follower. However, the spied-at bidder is able to secretly revise his bid with some commonly known probability.

Unlike in a usual Stackelberg game, leadership and the leader's commitment to a particular choice of action is not by that bidder's intent. Instead, it is an inescapable consequence of the presence of the spy, and the thus induced commitment is imperfect because the leader may be able to secretly revise his bid.

Our analysis begins with two special cases which serve as benchmarks: the case when the spy's report is perfectly informative and the case when the spy's signal is uninformative. Not surprisingly, if the spy's report is a perfect signal, spying benefits only the follower; whereas, if the signal is uninformative, the presence of the spy is inconsequential and one obtains the equilibrium of the first-price auction without spying.

As the leader is able to secretly revise his bid with some probability, the follower cannot be sure that the leaked bid is the leader's true bid. This ambiguity gives rise to a peculiar signaling game where both sender and receiver of messages have private information, beliefs are multi-dimensional, and the sender has a chance to secretly revise his action with some probability.

That game admits a unique pooling equilibrium and multiple partially separating equilibria. There, spying benefits both the leader and the follower, even if the likelihood that the leader is able to secretly revise his bid is arbitrarily small. This collusive impact shows up in all equilibria and is strongest in the pooling equilibrium which is also the payoff dominant equilibrium. Therefore, our analysis makes a strong case for tacit collusion.

Spying out rivals' bids or prices is abundant in competitive environments. The incentive for spying is particularly strong in a first-price auction where the winner-takes-all and the winner has to pay his bid, which drives bidders to strategically shade their bids. Generally such bid shading leads to *ex post* regret, either because the winner could have lowered his bid and yet won or some loser could have raised his bid and won while making a positive profit, which could have been avoided if he had known rivals' bids.

Although spying is intrinsically a secret operation, evidence of spying has surfaced on numerous occasions. For example, bidding for the construction of a new metropolitan airport in Berlin was reopened after investigators found out that *Hochtief AG*, the winner of the auction, had illegally acquired the application documents of the rival bidder *IVG*. Similarly, in 1996 *Siemens AG* was excluded from all public procurements in Singapore for a period of five years after the authorities determined that Siemens had acquired information about rival bids for a major power station construction project.

Circumstantial evidence of bid leakage abounds. Andreyanov et al. (2017) observed that in first-price procurement auctions a bidder is likely to have observed leaked bids if that bidder bid last, close to the deadline, and if, conditional on winning, his bid was close to the runner-up. Based on this pattern they suspect widespread bid leakage in at least 10% of a large sample of 4.3 million procurement auctions that took place in Russia between 2011 and 2016. Using weaker indicators and more sophisticated techniques, Ivanov and Nesterov (2020) confirmed these findings based on another data set of 600,000 Russian procurement auctions that took place between 2014 and 2018 and estimated that the outcome was influenced by leaked bids in around 9% of these auctions.

Typically, the spy who leaks a rival's bid is a "mole" or trusted insider who is driven by financial motives, or takes revenge for unfair treatment as an employee, or because he has been blackmailed into handing over sensitive information. Sometimes a gullible staff member is lured into passing over inconsequential information and, after having committed a minor offense, is blackmailed into leaking sensitive information. However, the spy may also be a corrupt agent auctioneer who has access to early bids or, for that matter, a "malware tool" that exploits vulnerabilities in computer software to transmit prospective or actual bids prepared on a PC or submitted online.

The techniques used by spies range from low-key activities such as searching through wastebaskets, gaining access to unattended PCs and laptops, planting sophisticated malware that is able to secretly switch on cameras or recording devices of computers and mobile phones,[1] to participation in the bid preparation by a mole.

Spying may also trigger counter-spying. When the identity of a spy has been exposed, the spy may find himself between "Scylla and Charybdis", faced with the agonizing choice between either punishment or being "doubled", and, after being doubled, serves the spied-at party by leaking distorted information.[2]

Our analysis relates to various strands of the literature.

Xu and Ligett (2018) analyze the impact of commitment in first-price auctions, assuming a Stackelberg game where, in the first stage, one player publicly commits to a mixed bidding strategy, and, in the second stage, after having observed that strategy, all others submit their bids simultaneously. Focusing on bidding games under complete information, they show that in the subgame perfect equilibrium both first- and second-movers benefit in expectation.[3] Interestingly, all bidders benefit equally so that they do not care who is selected as first-mover.

Their approach differs from ours in several regards: whereas they assume that bidders observe the mixed bidding strategy of the first-mover, but not the bid drawn from that strategy, we assume that bidders observe the first-mover's bid, and whereas they assume perfect commitment, we assume that the first-mover may secretly revise his bid with some probability. Moreover, whereas they assume that one player proactively commits to a mixed strategy, our analysis assumes that commitment is simply an inescapable consequence of the activity of a spy who reveals the bid, albeit imperfectly.

[1] For a detailed account of the many ways in which smartphone may be misused for spying see Farrow (2022).

[2] For a collection of case studies of the different kinds and techniques of economic espionage and counter-espionage see Nasheri (2005) and Andrew (2019).

[3] The authors also address games of incomplete information which however yield weaker results.

In another related contribution, Fischer et al. (2021) compare the effects of spying in first- and second-price auctions, using the framework of Arozamena and Weinschelbaum (2009), where a corrupt auctioneer leaks a rival's bid to a favored bidder. However, while Arozamena and Weinschelbaum (2009) focus on first-price auctions, and show that the impact of leaked information on the behavior of the spied-at bidder is driven by the curvature of the probability distribution of valuations,[4] the authors assume a uniform distribution which implies that the spied-at bidder's bidding is not affected by spying, and otherwise focus on the role of behavioral assumptions in first- and second-price auctions and their testing in controlled lab experiments.

One limitation of their approach is the implicit assumption that the spy faithfully reports the bid of the spied-at bidder. This is where the present paper steps in. The distinct feature of our analysis is that we take into account that the spied-at bidder may be able to secretly revise his bid which in turn induces him to convey strategically distorted information.

In the literature, spying on rivals' bids has also been analyzed in the context of corruption, where a dishonest agent auctioneer either allows a predetermined favored bidder to adjust his bid after reporting rivals' bids to him (as, for example, in Burguet and Perry 2007; Arozamena and Weinschelbaum 2009) or flexibly seeks a deal with the bidder who gains the most by either lowering or increasing his bid (as in Lengwiler and Wolfstetter 2010).[5]

Spying out a rival's choice of action has also been analyzed in the context of Bertrand market games with differentiated products and incomplete information about firms' unit cost (Fan et al. 2022, 2023a), and in entry deterrence games (Solan and Yariv 2004; Barrachina et al. 2014, 2021). Spying out a rival's type is analyzed in Wang (2020). Altogether, spying on a rival's *type* tends to increase competition and benefits consumers, whereas spying on a rival's *actions* tends to support higher prices.

The plan of the paper is as follows: In Sect. 2 we state the binary base model. Section 3 summarizes two benchmark games: the game with perfectly informative and with uninformative signals. In Sect. 4 we solve the game with imperfect commitment that yields imperfectly informative signals and fully characterize the unique pooling equilibrium and the family of partially separating equilibria. Section 5 summarizes the collusive impact of spying under imperfect commitment on players' payoffs and offers an intuitive interpretation. In Sect. 6 we explain why both kinds of equilibria are not driven by unreasonable off-equilibrium beliefs and show that the unique pooling equilibrium is payoff dominant. In Sect. 7.1 we characterize alternative partially separating equilibria and in Sect. 7.2 generalize our parsimonious binary model to more than two states and, in both extensions, show that essential results are preserved. The paper closes in Sect. 8. Several long proofs are relegated to the downloadable Online-Appendix Fan et al. (2023b) (which can also been obtained upon request from the authors).

[4] Specifically, they find that the spied-at bidder's behavior is not affected if the inverse of the reverse hazard rate function is linear (which occurs for the uniform distribution), whereas that bidder bids more/less aggressively if the inverse of the reverse hazard rate function is strictly convex/concave.

[5] If the spread between the two highest bids is sufficiently large, it is most profitable to let the highest bidder match the second highest bid, whereas if that spread is sufficiently small, due to bid shading it is most profitable to let the second highest bidder match the highest bid.

2 Model

Consider a first-price auction with two risk neutral bidders, 1 and 2, who compete for buying one unit of a valuable good. Each bidder knows his own valuation for that good but not that of the other.

Bidder 2 has access to an information system, represented by a spy, who observes the bid of bidder 1, albeit imperfectly, before bidder 2 submits his bid, and this fact is common knowledge. The presence of a spy induces a sequential game where the spied-at bidder 1 is Stackelberg leader and bidder 2 is follower.

However, bidder 1 is able to secretly revise his bid with some commonly known probability, and in that event makes two bids: the first-round bid observed and reported by the spy and the secretly revised (true) bid. Therefore, when bidder 2 receives the spy's message he does not know whether he has learned the true bid or just a decoy.

To keep the initial analysis as simple as possible, bidders' values, $V_i \in \{0, v\}, v > 0$, are binary random variables that are independently drawn with positive probabilities. This assumption is generalized in Sect. 7.2.

We refer to the bidder 1 who is committed to stick to his bid and cannot revise it as type c (mnemonic for "committed") and the bidder 1 who can secretly revise his bid as type n (mnemonic for "not committed"), and to bidders with value v as type h and with value zero as type ℓ (mnemonic for "high" and "low"). Therefore, the type set of bidder 1 is $T_1 = \{n\ell, nh, c\ell, ch\}$ and that of bidder 2 is $T_2 = \{\ell, h\}$.

The time-line is as follows:

1. Nature independently draws bidders' types, $(t_1, t_2) \in T_1 \times T_2$, and each bidder privately observes his own type.
2. Bidder 1 chooses his first-round bid, b_1, which the spy reports to bidder 2. That bid is the true submitted bid if bidder 1 is type c and a decoy if he is type n.
3. Bidder 2 submits his bid, b_2, and bidder 1 type n revises and submits his (true) second-round bid, b_1^r (mnemonic for "revised bid").
4. The auctioneer selects the winner based on true bids: (b_1, b_2) if bidder 1 is type c, and (b_1^r, b_2) if bidder 1 is type n, and collects payments according to the rules of the first-price auction.

Note that it does not matter when bidder 1 type n literally revises his bid; what matters is that bidder 2 and bidder 1 type n choose their (true) bids "simultaneously".

Bidders' prior probability of drawing the high value, v, is equal to $\theta \in (0, 1)$ and that of bidder 1 being type n is $\mu \in (0, 1)$. Values V_1 and types of bidder 1 are stochastically independent and the prior probabilities of T_1 are $(\Pr\{nh\}, \Pr\{n\ell\}) = \mu\,(\theta, 1-\theta)$, $(\Pr\{ch\}, \Pr\{c\ell\}) = (1-\mu)\,(\theta, 1-\theta)$.

We denote bids (actions) by the letters b, b^r, and pure bidding strategies that map bidders' values into bids by functions β, and mixed bidding strategies that prescribe a *c.d.f.* of bids by functions F and G.

As in other discontinuous games we need to invoke particular tie-breaking or sharing rules to assure existence of equilibrium. Simon and Zame (1990, p. 861) argue convincingly that "…payoffs should be viewed as only partially determined, and that whenever the economic nature of the problem leads to indeterminacies, the sharing rule should be determined *endogenously*, i.e., as part of the *solution* to the model rather than as part of the *description* of the model."

We follow this advice and carefully design tie-breaking rules that assure existence of equilibrium and consider them as part of the equilibrium. Unless stated otherwise, we apply the following type-dependent tie-breaking rule[6]:

Tie-breaking rule (T): *In the event of a tie the item is awarded to the bidder with the higher value and, if this fails to break the tie, the item is awarded to bidder 2.*

We stress that in order to assure existence of equilibrium we need a tie-breaking rule that thus favors bidder 2 (unless one invokes cumbersome discrete bids with a smallest monetary unit). Without it, there would be no best-reply of bidder 2. Note, however, that thus favoring bidder 2 is least favorable for explaining a collusive outcome where even the spied-at bidder benefits from spying. It is not surprising that the spying bidder 2 benefits from spying; the more challenging part is to explain that the spied-at bidder 1 benefits as well.

3 Benchmark games

We first consider two benchmark games: the standard first-price auction without spy, followed by the game with spy and a perfectly committed bidder 1. These models can be viewed as special cases of our model when the spy's message is either *uninformative* because bidder 1 can revise his bid with probability one ($\mu = 1$), or *perfectly informative* because bidder 1 cannot revise his bid with probability one ($\mu = 0$).

3.1 Benchmark without spy (A)

The benchmark bidding game without spy is a symmetric simultaneous moves game with the reduced type sets $T_1 = T_2 = \{\ell, h\}$; it is equivalent to a game with spy whose message is uninformative. It has a unique symmetric equilibrium: bidders type ℓ bid zero and bidders type h play the mixed bidding strategy $F(b)$ (see Maskin and Riley 1985, Sect. I):

$$\beta(0) = 0, \quad F(b) = \frac{(1-\theta)b}{\theta(v-b)}, \quad b \in [0, b^*], \; b^* := \theta v, \; F(b^*) = 1. \tag{1}$$

Randomization is essential because if a bidder type h could be predicted to submit a particular bid less than θv with certainty, the other bidder type h would match this bid if he is bidder 2 or outbid him if he is bidder 1 and win for sure. Of course, bidding more than θv is dominated by bidding 0. If bidder 1 type h bids θv, then the best response of bidder 2 type h is either 0 or θv to neither of which the bid of bidder 1 type h is a best response.

[6] This rule can be implemented without knowing bidders' valuations. One way is to endow bidders with a voucher of small value with the proviso that: (1) the voucher can either be used as a bid (in which case it is paid to the seller regardless of winning or losing) or traded-in to the seller for money, and (2) a voucher bid wins against a bid equal to zero, and a positive bid wins against a voucher bid.

The equilibrium outcome is efficient and bidders' *interim* equilibrium expected payoffs,[7] $\pi^A(t), t \in \{\ell, h\}$, their *ex ante* equilibrium expected payoff, Π^A, and the seller's expected revenue, Π_0^A, are:

$$\pi^A(\ell) = 0, \quad \pi^A(h) = (1-\theta)\, v, \quad \Pi^A = \theta(1-\theta)v \tag{2}$$

$$\Pi_0^A = v\left(1-(1-\theta)^2\right) - 2\Pi^A = \theta^2 v. \tag{3}$$

3.2 Benchmark game with perfect commitment (St)

The benchmark game with spy but with perfect commitment is a Stackelberg game with reduced type sets $T_1 = \{c\ell, ch\}$, $T_2 = \{\ell, h\}$ where the spy's message is perfectly informative. The perfect equilibrium of that bidding game is: bidder 1 bids zero, regardless of his type and bidder 2 matches the bid of bidder 1 unless it exceeds his own value:

$$\beta_1(V_1) = 0, \quad \beta_2(V_2, b_1) = \min\{V_2, b_1\}, \quad V_1, V_2 \in \{0, v\}. \tag{4}$$

In equilibrium bidder 1 wins if and only if $V_1 = v$ and $V_2 = 0$; otherwise, bidder 2 wins. In either case, the highest bid is equal to zero and the equilibrium outcome is efficient. Therefore,

Proposition 1 *The equilibrium outcome of game St exhibits a strong second-mover advantage; the spying bidder 2 fully extracts the seller's payoff while the payoff of the spied-at bidder 1 is not affected by spying:*

$$\pi_1^{St}(c\ell) = \pi_2^{St}(\ell) = \pi^A(\ell), \quad \pi_1^{St}(ch) = \pi^A(h), \quad \pi_2^{St}(h) = v > \pi^A(h) \tag{5}$$

$$\Pi_2^{St} = \left(\theta(1-\theta) + \theta^2\right) v = \Pi^A + \Pi_0^A, \quad \Pi_0^{St} = 0, \quad \Pi_1^{St} = \theta(1-\theta)v = \Pi^A. \tag{6}$$

Only the spying bidder 2 gains. Spying does not yield a collusive outcome for which all *bidders would have to gain, at the expense of the seller.*

The intuition is straightforward: It is optimal for bidder 2 to match the bid of bidder 1 (unless it exceeds his value); therefore, bidder 1 type h has only a chance to win if he faces bidder 2 type ℓ who must bid zero, and in that event, due to tie-rule T, bidding zero makes bidder 1 type h win and earn the greatest possible payoff equal to v. Therefore, bidding zero is a best response to the strategy of bidder 2 and *vice versa*.

4 The game with imperfect commitment

We now assume that bidder 1 is type n (able to secretly revise his bid) with probability $\mu \in (0, 1)$.

[7] Throughout this paper interim equilibrium expected payoffs are defined as function of bidders' type t.

After observing the first-round bid, b_1, reported by the spy, bidder 2 updates his beliefs about the type of bidder 1. His posterior beliefs are denoted by $\Pr\{t_1 \mid b_1\}$, $t_1 \in T_1 = \{n\ell, nh, c\ell, ch\}$.

The fact that bidder 1 may revise his bid with some probability gives rise to a non-standard signaling game where both sender and receiver of messages have private information and the sender has a chance to make an unobserved move. It admits pooling and partially separating perfect equilibria.

4.1 Pooling equilibrium

In a *pooling* equilibrium all types of bidder 1 submit the same first-round bid; consequently, the first-round bid is uninformative. Because bidder 1 type ℓ must bid zero, it follows naturally that the equilibrium first-round bid of bidder 1 is equal to zero, regardless of his type.

After observing the equilibrium first-round bid, in the second round the players are bidder 1 type n and bidder 2. For bidders type h randomization is essential for the same reason that explains randomization in benchmark game A. The main difference is that the mixed strategy of bidder 2 now has a mass point at zero because bidder 2 can thus take advantage of the fact that bidder 1 is bound by his zero bid if he is type c, together with the fact that in this event bidder 2 type h wins with a zero bid because of the way in which the tie-rule favors bidder 2.

An immediate implication is that the mixed strategy of bidder 1 type nh cannot also have a mass-point at zero, because if it did, bidder 1 type nh could increase his expected payoff by moving that probability mass upwards to a bid slightly greater than zero.

The equilibrium is supported by a non-restrictive variety of off-equilibrium beliefs and, given these beliefs, the pooling equilibrium turns out to be unique.

Applying the concept of a perfect equilibrium we obtain the following unique pooling equilibrium:

Proposition 2 (Pooling equilibrium) First-round bids: *Bidder 1 bids zero regardless of his type.*
Second-round bids: *Bidder 1 type $n\ell$ bids zero; type nh plays the mixed strategy:*

$$F_1(b) = \frac{(1-\theta\mu)b}{\theta\mu(v-b)}, \quad b \in [0, \bar{b}], \quad \bar{b} = \mu\theta v, \;\; F_1(\bar{b}) = 1. \tag{7}$$

Bidder 2 type ℓ bids zero; type h plays the mixed strategy, F_2, if $b_1 = 0$ has been observed:

$$F_2(b) = F_2(0) + \frac{(1-\theta\mu)b}{\theta(v-b)}, \; b \in [0, \bar{b}], \quad F_2(0) = 1-\mu, \quad F_2(\bar{b}) = 1, \tag{8}$$

and the equilibrium strategy of benchmark game A, $F(b)$, if $b_1 > 0$ has been observed.

Posterior beliefs: *If the spy reported the first-round bid $b_1 = 0$ prior beliefs are confirmed:* $\Pr\{t_1 \mid b_1\} = \Pr\{t_1\}$ *for all $t_1 \in T_1$; if he reported an off-equilibrium bid,*

$b_1 > 0$, *bidder 1 is believed to be type* nh *with* $\Pr\{nh \mid b_1\} = \theta$, *and type* $n\ell$ *with* $\Pr\{n\ell \mid b_1\} = 1 - \theta$.[8]

Proof It is obvious that bidders type ℓ must bid zero and that the belief system is consistent with equilibrium strategies and Bayes's rule. As a working hypothesis (which will confirm) suppose bidder 1 type nh plays the second-round mixed strategy F_1 and bidder 2 type h plays the mixed strategy F_2. By a standard argument F_1 and F_2 must have the same support, $[0, \bar{b}]$; therefore, one must have $F_1(\bar{b}) = F_2(\bar{b}) = 1$.

(1) Suppose the first-round bid $b_1 = 0$ has been observed. If bidder 1 type nh and bidder 2 type h submit the second-round bids $b_1^r, b_2 \in [0, \bar{b}]$ their expected payoffs are:

$$\pi_1^{nh}(b_1^r) = \left(1 - \theta + \theta F_2(b_1^r)\right)(v - b_1^r)$$
$$\pi_2^h(b_2) = \left(1 - \mu + \mu\,(1 - \theta + \theta F_1(b_2))\right)(v - b_2)\,.$$

By the tie-rule F_1 cannot have a mass point at zero, i.e., $F_1(0) = 0$, and by the indifference property of a mixed strategy equilibrium all bids $b_1^r \in [0, \bar{b}]$ and all bids $b_2 \in [0, \bar{b}]$ must yield the same expected payoffs. In particular, one must have:

$$\pi_1^{nh}(0^+) = \pi_1^{nh}(\bar{b}), \quad \text{i.e.,} \quad (1 - \theta + \theta F_2(0))\, v = v - \bar{b}$$
$$\pi_2^h(0) = \pi_2^h(\bar{b}), \quad \text{i.e.,} \quad (1 - \mu\theta)\, v = v - \bar{b},$$

where $\pi_1(0^+)$ is obtained by taking the limit of $\pi_1^{nh}(b)$ for $b \to 0$ from above. Therefore,

$$\bar{b} = \mu\theta v, \quad F_2(0) = 1 - \mu. \tag{9}$$

Finally, combining (9) with the indifference requirements: $\pi_1^{nh}(b_1^r) = \pi_1^{nh}(\bar{b})$, $\pi_2^h(b_2) = \pi_2^h(\bar{b})$, for all $b_1^r, b_2 \in (0, \bar{b}]$ yields the asserted equilibrium strategies F_1, F_2.

(2) Suppose the first-round bid $b_1 > 0$ has been observed. Then bidder 1 is believed to be type n with probability one; hence, bidder 2 ignores the bid reported by the spy and both bidders play the benchmark game A.

(3) Suppose bidder 1 type ch deviates and bids $b_1 > 0$. Then, his payoff is at most equal to the equilibrium payoff in game A, which is equal to $(1 - \theta)v$. Whereas if he does not deviate and bids zero, his expected payoff is $(1 - \theta)v$, because he wins against bidder 2 with value zero, by the tie-breaking rule T. Therefore, it does not pay to deviate.

(4) If bidder 1 type nh deviates and makes a positive first-round bid, his payoff is equal to his equilibrium payoff in game A, i.e., $(1 - \theta)v$, which is less than what he earns if he does not deviate, $(1 - \mu\theta)v$.

(5) The equilibrium is unique because the game admits no equilibrium in pure strategies and there exists no other pair of mixed strategies, F_1, F_2, that satisfies all indifference requirements of an equilibrium in mixed strategies. □

[8] The assumed off-equilibrium beliefs are not restrictive because the equilibrium is supported by all off-equilibrium beliefs with the property $b_1 > 0 \Rightarrow \Pr\{n\ell \mid b_1\} + \Pr\{nh \mid b_1\} = 1$ and $\Pr\{nh \mid b_1\} \in (\theta, 1)$.

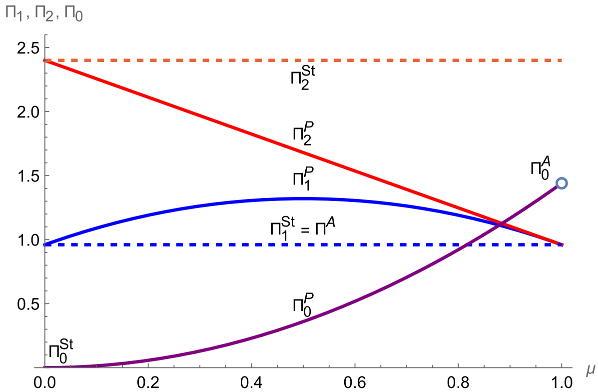

Fig. 1 Ex-ante equilibrium expected payoffs

The *interim* and *ex ante* equilibrium expected payoffs and their relationship to those of the benchmark games are (there P is mnemonic for "pooling equilibrium")[9]:

$$\pi_1^{St}(ch)=\pi_1^P(ch)=\pi^A(h), \quad \pi_2^{St}(h)>\pi_2^P(h)=\pi_1^P(nh)=(1-\mu\theta)v>\pi^A(h) \tag{10}$$

$$\Pi_1^P = \theta\left(\mu\pi_1^P(nh)+(1-\mu)\pi_1^P(ch)\right) = (1-\theta+\theta\mu(1-\mu))\,\theta v \tag{11}$$

$$\Pi_2^P=\theta\pi_2^P(h)=\theta\,(1-\theta\mu)\,v, \quad \Pi_0^P=\left(1-(1-\theta)^2\right)v-\Pi_1^P-\Pi_2^P=\theta^2\mu^2 v \tag{12}$$

$$\Pi_1^P > \Pi_1^{St} = \Pi^A, \quad \Pi_2^{St} > \Pi_2^P > \Pi^A, \quad \Pi_0^{St} < \Pi_0^P < \Pi_0^A. \tag{13}$$

Of course, the payoffs of bidders type ℓ are equal to zero.

Both bidders benefit from spying at the expense of the seller: $\Pi_i^P > \Pi^A$, $i \in \{1, 2\}$ and $\Pi_0 < \Pi_0^A$ for all $\mu \in (0, 1)$. The spied-at bidder 1 benefits because Π_1^P is strictly concave in μ and equal to Π^A at $\mu \in \{0, 1\}$, and the spying bidder 2 benefits because Π_2^P is decreasing in μ and equal to Π^A at $\mu = 1$. Figure 1 illustrates this ranking of ex-ante equilibrium expected payoffs as a function of μ.[10]

We conclude: *The unique pooling equilibrium yields a collusive outcome where both the spying and the spied-at bidder mutually benefit from spying at the expense of the seller.*

The collusive outcome and the fact that Π_1^P is not monotone in μ have a simple intuitive interpretation which is explained in Sect. 5.

4.2 Partially separating equilibria

The game admits no fully separating equilibrium. Because, if it did, bidder 2 would believe that bidder 1 is type ch if he observed the first-round bid $b_1 = \beta_1(ch)$, respond

9 Invoking the indifference property of equilibrium mixed strategies one finds, for example, $\pi_1^P(nh) = \pi_1^{nh}(\bar{b})$.

10 The plots assume $(\theta, v) = (0.6, 4)$.

with matching that bid, and win the auction. In turn, bidder 1 type nh would have an incentive to deviate, mimic bidder 1 type ch and thus bid $b_1 = \beta_1(ch)$ in lieu of $b_1 = \beta_1(nh)$, and, in the second round, revise his bid to slightly exceed $\beta_1(ch)$ and win.

However, the game admits partially separating equilibria where the first-round bid conveys some information about the type of bidder 1 that allows updating of prior beliefs. We now confirm that such equilibria exist.

Unlike in the pooling equilibrium, in a partially separating equilibrium only bidder 1 type $c\ell$ submits a first-round bid equal to zero with probability one. The other types of bidder 1 bid either zero or $b^* > 0$. Specifically, we consider one family of equilibria where bidder 1 type $n\ell$ bids b^* with probability one, whereas types nh and ch randomize between bidding zero and b^*.[11] Therefore, when bidder 2 observes b^* he infers that bidder 1 cannot be type $c\ell$. Moreover, observing either zero or b^* allows updating of prior beliefs, using the equilibrium probabilities with which bidder 1 randomizes his bid between zero and b^*.

After the first-round bid $b_1 \in \{0, b^*\}$ has been observed and beliefs have been updated, bidder 1 type $n\ell$ and bidder 2 type ℓ bid zero, whereas bidder 1 type nh and bidder 2 type h play mixed strategies. As it turns out, these mixed strategies do not depend upon the observed first-round bid of bidder 1.

Unlike the pooling equilibrium which is unique, we find a continuum of partially separating equilibria. We now characterize the family of partially separating equilibria, again using the concept of perfect equilibrium, and show that they are payoff dominated by the unique pooling equilibrium.

Proposition 3 (Partially separating equilibria) First-round bids: *Bidder 1 type $c\ell$ bids zero and type ch bids $b^* > \bar{b}$ with probability q and zero otherwise; type $n\ell$ bids b^* and type nh bids b^* with probability ρ and zero otherwise (where γ is introduced to simplify the exposition)*[12]*:*

$$q \in (0, \bar{q}],\ \ \bar{q} = \frac{1-\sqrt{1-4\theta\mu(1-\mu)}}{2\theta(1-\mu)},\ \ \rho = \frac{(1-\theta)\mu}{\gamma},\ \ \gamma := 1-\theta\big(q(1-\mu)+\mu\big). \tag{14}$$

Second-round bids: *Bidder 1 type $n\ell$ bids zero and type nh plays the mixed strategy, F_1. Bidder 2 type ℓ bids zero and type h plays the mixed strategy F_2 if he observed a first-round bid $b_1 \in \{0, b^*\}$, and the equilibrium strategy of benchmark game A if he observed $b_1 \notin \{0, b^*\}$*[13]*:*

$$F_1(b) = \frac{\gamma b}{\theta\mu(v-b)}, \quad F_1(\bar{b}) = 1, \quad \bar{b} = \frac{\theta\mu v}{\gamma + \theta\mu} < b^* := \theta v \tag{15}$$

$$F_2(b) = F_2(0) + \frac{\gamma b}{\theta\,(\gamma+\mu\theta)\,(v-b)},\ \ F_2(0) = \frac{(1-q\theta)(1-\mu)}{1-q\theta(1-\mu)} > 0,\ \ F_2(\bar{b}) = 1. \tag{16}$$

[11] Other equilibria are considered in footnote 12 and in Sect. 7.1.

[12] We also find other partially separating equilibria where bidder 1 type $n\ell$ bids b^* with probability less than 1. These equilibria yield the same ex ante expected payoffs as the one described here.

[13] Just like in the above stated pooling equilibrium we denote the upper bound of the supports of F_1 and F_2 by $\bar{b}$, although these bounds differ.

Table 1 Posterior beliefs

	$b_1 = 0$	$b_1 = b^*$	$b_1 \notin \{0, b^*\}$
$\Pr\{c\ell \mid b_1\}$	$\frac{(1-\mu)(1-\theta)}{\Pr\{0\}}$	0	0
$\Pr\{ch \mid b_1\}$	$\frac{(1-\mu)\theta(1-q)}{\Pr\{0\}}$	$\frac{(1-\mu)\theta q}{\Pr\{b^*\}}$	0
$\Pr\{n\ell \mid b_1\}$	0	$\frac{\mu(1-\theta)}{\Pr\{b^*\}}$	$1-\theta$
$\Pr\{nh \mid b_1\}$	$\frac{\mu\theta(1-\rho)}{\Pr\{0\}} = \frac{\theta\mu(1-\mu)(1-q\theta)}{\gamma \Pr\{0\}}$	$\frac{\mu\theta\rho}{\Pr\{b^*\}} = \frac{(1-\theta)\theta\mu^2}{\gamma \Pr\{b^*\}}$	θ
$\Pr\{b_1\}$	$1 - \Pr\{b^*\}$	$(1-\mu)\theta q + \mu(1-\theta(1-\rho))$ $= \left(\mu(1-\theta)+\theta q(1-q\theta)(1-\mu)^2\right)/\gamma$	0

Posterior Beliefs: *Table 1 summarizes the updated beliefs of bidder 2 after he has observed a first-round bid* b_1.[14]

Proof (1) It is obvious that bidders type ℓ must submit "true" bids equal to zero and that $F_1(b)$ and $F_2(b)$ must have the same support.

(2) Consider bidder 2 type h. We confirm that his asserted equilibrium strategy is indeed optimal for all observed bids, b_1, on and off the equilibrium path.

If bidder 2 type h observed $b_1 \in \{0, b^*\}$ he is indifferent between all bids $b \in [0, \bar{b}]$, because his payoffs, denoted by $\pi_2^h(b \mid b_1)$, are equal to[15]:

$$\pi_2^h(b \mid 0) = \left(\Pr\{c \mid 0\} + \Pr\{n\ell \mid 0\} + \Pr\{nh \mid 0\}F_1(b)\right)(v-b) = \frac{(1-q\theta)(1-\mu)v}{\Pr\{0\}} \tag{17}$$

$$\pi_2^h(b \mid b^*) = \left(\Pr\{n\ell \mid b^*\} + \Pr\{nh \mid b^*\}F_1(b)\right)(v-b) = \frac{\mu(1-\theta)v}{\Pr\{b^*\}}. \tag{18}$$

Therefore, he is also indifferent between all probability distributions of bids with support $[0, \bar{b}]$.

Bidder 2 type h may deviate and bid $b > \bar{b}$. If he observed $b_1 = 0$ deviating to bid higher than $\bar{b}$ is obviously dominated by bidding $b = \bar{b}$. If he observed $b_1 = b^*$, the same is true for all $b \in (\bar{b}, b^*)$ and $b > b^*$; however, if he deviates and bids $b = b^*$ he can win for sure (due to the assumed tie-rule) and earn a payoff equal to $v - b^* = v(1-\theta)$. Yet, because bidder 1 type ch bids $b = b^*$ only with probability $q \le \bar{q} < 1$, and[16]:

$$q \le \bar{q} \Rightarrow \pi_2^h(b \mid b^*) \ge v(1-\theta) \quad \text{for all } b \in [0, \bar{b}], \tag{19}$$

this deviation is not profitable either.

[14] Note, we characterize the family of partially separable equilibria in terms of $q \in (0, \bar{q}]$.

[15] Note: if b^* is observed in equilibrium bidder 2 loses if he meets bidder 1 type ch because $b^* > \bar{b}$.

[16] $\Pr\{b^*\}$ is increasing in $q \in (0, 1)$ and equal to μ at $q = \bar{q}$; therefore, $\pi_2^h(b \mid b^*)$ is decreasing in $q \in (0, 1)$ and equal to $v(1-\theta)$ at $q = \bar{q}$ for all $b \in [0, \bar{b}]$.

We conclude that if bidder 2 type h observed $b_1 \in \{0, b^*\}$ all mixed bidding strategies with support $[0, \bar{b}]$, and in particular $F_2(b)$, are best replies to the strategy profile of bidder 1.

If bidder 2 type h observed an off-equilibrium first-round bid $b_1 \notin \{0, b^*\}$, he believes that bidder 1 is type n. In that case he ignores the observed reported bid and plays the equilibrium strategy of benchmark game A and earns a payoff equal to $((1-\theta) + \theta F(b^*))\,(v - b^*) = v - b^* = v(1-\theta)$.

(3) Next consider bidder 1 type ch. If he bids $b = b^*$, he wins for sure and earns the payoff

$$\pi_1^{ch}(b^*) = v - b^* = (1-\theta)v,$$

because bidder 2 never bids more than $\bar{b}$ and $\bar{b} < b^*$. If he bids $b = 0$ he earns the same payoff because in that case he wins if and only if $V_2 = 0$. Therefore, he is indifferent between $b = b^*$ and $b = 0$, and thus also between all probability distributions of bids with support $\{0, b^*\}$.

He may deviate and bid $\tilde{b} \in (0, b^*)$. In that case bidder 2 believes that bidder 1 is type n and plays the equilibrium strategy of benchmark game A. This yields:

$$\tilde{\pi}_1 = \big(1 - \theta + \theta F(\tilde{b})\big)(v - \tilde{b}) = (1-\theta)v = \pi_1^{ch},$$

which is not an improvement. He may also deviate and bid $b > b^*$, which is however dominated by bidding $b = b^*$.

We conclude that the asserted equilibrium strategy, q, is a best reply to the other players' strategy profile. (We already explained why q is less than or equal to $\bar{q}$.)

(4) Finally, consider bidder 1 type nh. If he plays the asserted equilibrium strategy, for all first-round bids $b_1 \in \{0, b^*\}$ and all revised bids $b_1^r \in [0, \bar{b}]$ his payoff is equal to

$$\pi_1^{nh}(b_1^r) = \big(1 - \theta + \theta F_2(b_1^r)\big)(v - b_1^r) = \frac{\gamma v}{\gamma + \theta\mu}.$$

Therefore, he is indifferent between all probability distributions of first-round bids, b_1, with support $\{0, b^*\}$ and all probability distributions of revised bids, b_1^r, with support $[0, \bar{b}]$.

He may deviate and set an off-equilibrium first-round bid, $b_1 \notin \{0, b^*\}$. In that case bidder 2 believes that he is type n and plays the equilibrium strategy of the benchmark game A. Then, the payoff of bidder 1 type nh is $\tilde{\pi}_1 = (1 - \theta + \theta F(b^*))\,(v - b^*) = v(1-\theta)$, which is less than π_1^{nh}, because

$$\tilde{\pi}_1 - \pi_1^{nh}(\bar{b}) = -\frac{v\theta(1 - q\theta)(1-\mu)}{\gamma + \theta\mu} < 0.$$

He may also deviate and submit a revised bid greater than $\bar{b}$. In that case, the only relevant deviation is to bid $b = b^*$ (which is greater than $\bar{b}$). Then, he wins for sure, yet reduces his payoff to $v - b^* = v(1-\theta)$.

We conclude that the asserted equilibrium strategy, $(\rho, F_1(b))$, is a best reply to the profile of the other players' equilibrium strategies. □

Remark We mention that in constructing the family of partially separating equilibria we considered that the second-round equilibrium strategies, F_1, F_2, may depend upon the first-round equilibrium bid, $b_1 \in \{0, b^*\}$. However, after substituting the solution of ρ we find that F_1, F_2 are indeed independent of b_1.[17] This is a peculiarity of the binary case. If there are more than two states, the second-round equilibrium bidding strategies depend upon which equilibrium first-round bid has been observed (see the Online-Appendix B.2.2).

The resulting *interim* equilibrium expected payoffs and their relationship to the pooling equilibrium are (there S is mnemonic for "partially separating equilibrium")[18]:

$$\pi_1^P(ch) - \pi_1^S(ch) = 0, \quad \pi_1^P(nh) - \pi_1^S(nh) = \frac{q\theta^2\mu(1-\mu)v}{\gamma + \mu\theta} > 0 \tag{20}$$

$$\pi_2^S(h) = \Pr\{0\}\,\pi_2^h(\bar{b} \mid 0) + \Pr\left\{b^*\right\}\pi_2^h(\bar{b} \mid b^*) = \gamma v$$

$$\pi_2^P(h) - \pi_2^S(h) = \theta v q(1-\mu) > 0. \tag{21}$$

It follows that all partially separating equilibria are payoff dominated by the unique pooling equilibrium in terms of interim equilibrium expected payoffs and therefore also in terms of ex ante equilibrium expected payoffs.

Altogether we find the following ranking of ex ante equilibrium expected payoffs, where $\Pi_1^S = \theta\left(\mu\pi_1^S(nh) + (1-\mu)\pi_1^S(ch)\right)$, $\Pi_2^S = \theta\pi_2^S(h)$, and $\Pi_0^S = \left(1-(1-\theta)^2\right)v - \Pi_1^S - \Pi_2^S$:

$$\Pi_1^P > \Pi_1^S > \Pi_1^{St} = \Pi^A, \quad \Pi_2^{St} > \Pi_2^P > \Pi_2^S > \Pi^A \tag{22}$$

$$\Pi_0^A > \Pi_0^S > \Pi_0^P > \Pi_0^{St} = 0. \tag{23}$$

Again both bidders benefit from spying: $\Pi_i^S > \Pi^A$, $i \in \{1, 2\}$ for all $\mu \in (0, 1)$. The proof is essentially the same as that for the pooling equilibrium and the intuitive interpretation is discussed in Sect. 5.

5 Collusive impact of spying: intuitive interpretation

Spying has a "collusive impact" and can be viewed as a case of tacit collusion if it benefits both the spying and the spied-at bidder and both bidders have a vested interest to maintain spying. While it is not surprising that the spying bidder benefits from spying, and whereas perfectly informative spying exclusively benefits the spying

[17] A comprehensive analysis of the binary model where all computations are programmed in *Mathematica* is available upon request from the authors.

[18] Of course, the payoffs of bidders type ℓ are equal to zero.

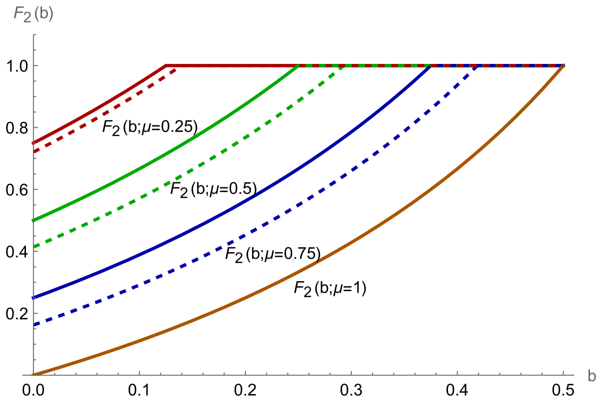

Fig. 2 *FSD* relationships: pooling (solid), partially separating (dashed)

bidder, imperfectly informative spying does yield a collusive outcome even though the assumed tie-rule favors the spying bidder 2.

That collusive impact shows up in all equilibria of the game with imperfect commitment. However, the pooling equilibrium has the strongest collusive impact. Among all partially separating equilibria (including the alternative equilibria characterized in Sect. 7.1), the collusive impact is diminishing in q and, as q is diminished and approaches zero, the equilibrium expected payoffs approach those of the unique pooling equilibrium:

$$\partial_q \Pi_i^S < 0,\ i \in \{1,2\},\ \ \partial_q \Pi_0^S > 0,\ \text{ and } \lim_{q \to 0} \Pi_i^S = \Pi_i^P,\ i \in \{0,1,2\}. \tag{24}$$

The collusive impact occurs even if the probability that the spy is able to secretly revise his bid is arbitrarily small, as long as that probability is positive.

The collusive impact of spying has a simple intuitive interpretation.

In the pooling equilibrium bidder 2 observes a zero bid with probability one. He knows that bidder 1 cannot revise his bid with positive probability; he also knows that, if he is type h, he will win against a zero bid by bidder 1 because the tie-rule favors him if he also bids zero. He takes advantage of this by bidding zero with sufficiently high probability. This benefits the spying bidder 2. Moreover, as μ is decreased, starting from the benchmark case of no spying, $\mu = 1$, the *entire* probability distribution of bids by bidder 2 type h shifts upwards, because $\partial_\mu F_2(b;\mu) = {-v}/{v-b} < 0$. Therefore, there is a first-order stochastic dominance (*FSD*) relationship (with strict inequality if $F_2(b;\mu') < 1$):

$$\mu' > \mu \Rightarrow F_2(b;\mu') \leq F_2(b;\mu). \tag{25}$$

In other words, as μ is decreased, bidder 2 type h bids less and less aggressively which benefits bidder 1 type nh and does not affect the other types of bidder 1.

Similarly, the family of partially separating equilibria exhibits the same first-order stochastic dominance relationship, regardless of whether bidder 2 observes either $b_1 =$

0 or $b_1 = b^*$, because: $\partial_\mu F_2(b;\mu,q) = {}^{-v(1-\theta q)}/_{(v-b)(1-\theta q(1-\mu))^2} < 0$. This explains why bidder 1 benefits from spying also in all partially separating equilibria (including the alternative equilibria characterized in Sect. 7.1). Moreover, F_2 is decreasing in q: $\partial_q F_2(b;\mu,q) = {}^{-v\theta(1-\mu)\mu}/_{(v-b)(1-\theta q(1-\mu))^2} < 0$, and as q goes to zero, the F_2 function converges to that of the pooling equilibrium. Therefore, for all $q \in (0,\bar{q}]$ bidder 2 type h bids more aggressively than in the pooling equilibrium. This also explains why the collusive impact of spying is the strongest in the unique pooling equilibrium.

These results are illustrated in Fig. 2. There the solid curves plot the F_2 functions that solve the unique pooling equilibrium and the dotted curves plot the F_2 functions that solve the corresponding partially separating equilibrium.[19] As μ is decreased, starting from the benchmark case without spy, $\mu = 1$, bidder 2 bids less and less aggressively, but this effect is significantly stronger in the pooling equilibrium. We also stress that if one picks a partially separating equilibrium with a lower value of q, the dashed curves move continuously closer towards the corresponding solid curves and coincide as q approaches zero.

Even though bidder 2 bids less and less aggressively as μ is decreased, the equilibrium expected payoff of bidder 1 is however not monotone decreasing in μ. As μ is decreased bidder 1 benefits from the less aggressive bidding of bidder 2; however, he is then also less likely able to revise his bid. Π_1 is an average of $\pi_1(ch)$ and $\pi_1(nh)$ and when μ is decreased more weight is given to the smaller term $\pi_1(ch)$. Therefore, there is a trade-off and bidder 1's net gain from spying reaches a maximum at some $\mu \in (0,1)$ (which is incidentally equal to $1/2$ in the pooling equilibrium).

6 Equilibrium selection

Like other signaling games, the spying game with imperfect commitment has multiple equilibria – a unique pooling equilibrium and a family of partially separating equilibria.

In classical signaling games the multiplicity of equilibria is driven by "unreasonable" out-of-equilibrium beliefs and one can eliminate pooling equilibria by invoking standard equilibrium refinements such as the intuitive criterion by Cho and Kreps (1987). This does not apply to the present game in which, unlike in standard signaling games, both sender and receiver of messages have private information, the type space of bidder 1 is two-dimensional, and the sender, bidder 1, has a chance to make an unobserved move, his revised bid, independent of his first-round bid.

The idea of the intuitive criterion is that a belief system is unreasonable if one can identify a type $t \in \{c\ell, ch, n\ell, nh\}$ of bidder 1 who can convince bidder 2, by choosing a particular off-equilibrium action, that he should recognize him as the type who he is, because triggering this belief change is beneficial only to him. We now sketch briefly why, in the present model, this criterion has no bite.

Consider the pooling equilibrium. Obviously, type $c\ell$ cannot benefit from an off-equilibrium first-round bid $b_1 > 0$ that triggers bidder 2 to recognize his type. If type ch makes an off-equilibrium bid $b_1 > 0$ and triggers bidder 2 to recognize him, then

[19] The plots are computed assuming $(\theta, v, q) = (1/2, 1, \bar{q})$.

bidder 2 type h will match his bid, and he cannot be better off. Similarly, type $n\ell$ cannot benefit either.

If type nh makes an off-equilibrium first-round bid, $b_1 > 0$, and is thus recognized by bidder 2, bidder 2 will ignore the spy's report, and bidder 2 type h plays the equilibrium mixed strategy of game A, while bidder 1 plays a mixed strategy with mass point at zero (see Maskin and Riley 1985, p. 153) which yields the payoff $(1-\theta)v$, which is smaller than his equilibrium payoff, $(1-\mu\theta)v$. Therefore, the equilibrium satisfies the intuitive criterion also in this case.

While there is a continuum of partially separating equilibria, their equilibrium payoffs converge to those of the unique pooling equilibrium, as q approaches zero. As one can see from (20)–(21) and (22), that pooling equilibrium is payoff dominant which suggests that it may be a plausible selection of equilibrium.

7 Extensions

7.1 Other partially separating equilibria

A disturbing feature of the above family of partially separating equilibria is that bidder 1 type ch may bid zero with a considerably high probability even if the probability that bidder 2 is type ℓ is close to zero. Bidder 1 type ch cannot revise his bid and if he bids zero he can only win in the event when bidder 2 is type ℓ because of the way in which the tie-rule favors bidder 2. Nevertheless bidder 1 type ch bids zero with probability $1-q$, $q \leq \bar{q}$ which can be as high as 1 (as q approaches 0), even if the probability that bidder 2 is type ℓ is arbitrarily close to zero. For example, if $\mu = 1/2$ bidder 1 type ch may bid zero with probability 1 even if he then stands practically no chance to win because $1-\theta$ is arbitrarily close to zero and $1-q$ can be arbitrarily close to 1.

One may thus wonder whether one can find other partially separating equilibria in which bidder 1 type ch never makes a first-round bid equal to zero and fully separates himself from bidder 1 type $c\ell$.

The answer to this question is in the negative:

Proposition 4 *There is no equilibrium where bidder 1 type ch fully separates himself from type $c\ell$ by bidding $b_1 = b^*$ with probability 1.*

However, the above family of partially separating equilibria is not unique. All equilibria in this family have the property that $b^* > \bar{b}$. Allowing for $b^* \leq \bar{b}$, we find alternative equilibria. These other equilibria share the same essential properties:

Proposition 5 *The game has also partially separating equilibria where $b^* \leq \bar{b}$. Like the equilibria summarized in Proposition 3 (that exhibit $b^* > \bar{b}$) they are all payoff dominated by the unique pooling equilibrium, both the spying and the spied-at bidder benefit from spying, and bidder 1 type ch does not fully separate himself from type $c\ell$ (i.e., $q < 1$).*

The elaborate proofs of Propositions 4 and 5 are relegated to the Online-Appendix A.

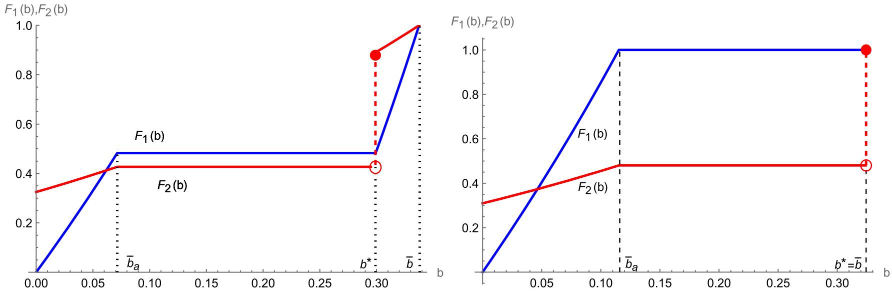

Fig. 3 Equilibrium mixed strategies. Left: with $b^* < \bar{b}$; Right: with $b^* = \bar{b}$

The equilibrium mixed strategies, F_1, F_2, of these alternative equilibria are illustrated in Fig. 3; the strategies of bidder 2, F_2, exhibit mass points at $b = 0$ *and* at $b = b^*$.

7.2 More than two states

We have chosen a parsimonious model with binary valuations. This raises the question whether our main results hinge on the assumption of binary valuations or generalize to more than two valuations.

In order to answer this question we have extended the analysis to three valuations: $V_1, V_2 \in \{0, v, v'\}$, $v' > v > 0$, with associated prior probabilities $\{\theta_0, \theta, \theta'\}$, that are positive, less than one, and add up to one. This analysis also indicates how it can be adapted to cover arbitrarily many valuations.

As a natural extension of our notation, we denote bidder 1 type n with $V_1 = v'$ as type nH bidder 1 type c with $V_1 = v'$ as type cH and bidder 2 with $V_2 = v'$ as type H. Therefore, type sets are $T_1 = \{c\ell, ch, cH, n\ell, nh, nH\}$ and $T_2 = \{\ell, h, H\}$.

The extension is more involved than the above analysis of the binary model. However, the bulk of it employs essentially the same solution procedures. Therefore, in the following, we provide only a brief summary of results and elaborate only on issues that make the analysis different. The detailed analysis is relegated to the Online-Appendix B.

We focus on equilibria that are monotone (efficient) in the sense that they exhibit a monotone allocation rule, i.e., a high value bidder wins against a low value bidder except in the event of a tie.

The game has a unique monotone pooling equilibrium. Like in the binary case, the first-round bids of bidder 1 are equal to zero, regardless of his type, while, in the second round, bidder 1 type nh and nH now play distinct mixed strategies, F_1, G_1, and bidder 2 type h and H play distinct mixed strategies, F_2, G_2 with neighboring supports, as illustrated in Fig. 4.[20] Again, both bidders benefit from spying in the strong sense that spying increases their interim equilibrium expected payoffs.

The fact that the equilibrium strategy of bidder 2 type H, G_2, has a mass point at $\bar{b}$, $G_2(\bar{b}) = 1 - \mu$, implies that one can only assure existence of a monotone equilibrium

[20] The plots assume $(\theta_0, \theta, \theta', v, v', \mu) = (1/3, 1/3, 1/3, 1, 2, 1/2)$.

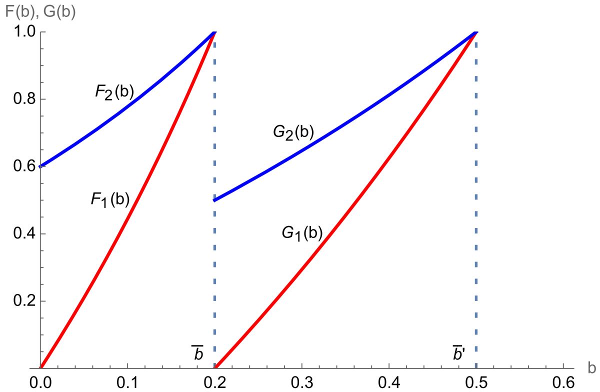

Fig. 4 Equilibrium mixed strategies F_1, F_2 and G_1, G_2

if one amends the auction rules as follows: Prior to bidding bidder 1 must pay an entry fee, $f = (\theta_0 + \theta)R$, and, after his final bid b has been submitted to the auctioneer, bidder 1 receives a refund, $R = \theta'(1 - \mu)(v - \bar{b})$, conditional on $b < \bar{b}$.

To explain the role of this amendment, note that, because G_2 has a mass point at $\bar{b}$, bidder 1 type nh can discontinuously increase his probability of winning by bidding slightly more than $\bar{b}$. However, by bidding more than $\bar{b}$ he no longer qualifies to collect the refund R which must be set in such a way that bidding more than $\bar{b}$ is not profitable. In turn, the refund may induce bidder 1 type nH to bid slightly lower than $\bar{b}$ and thus qualify to collect the refund. Therefore, the refund has to be set in such a way that it also deters bidder 1 type nH from bidding slightly less than $\bar{b}$. The prescribed R achieves both.

One might think that a monotone equilibrium could also be established without amending the auction rules by shifting the support of G_1, G_2 upwards to $[\hat{b}, \bar{b}']$ with $\hat{b} > \bar{b}$ to such an extent that bidder 1 type nh does not have an incentive to deviate to bid slightly more than $\hat{b}$. However, then bidder 2 type H would gain by shifting the probability mass $G_2(\hat{b})$ downwards to $\bar{b}$ and thus reduce his expected payment by $(\hat{b} - \bar{b})G_2(\hat{b})$ without reducing his probability of winning. Alternatively, one might think that a monotone equilibrium could be established by allowing for overlapping supports. However, then, for all bids in the intersection of the supports, the indifference requirements of mixed strategies would be violated, as we show in the Online-Appendix C.

The monotone pooling equilibrium is unique and yields a collusive outcome. The game has also a family of monotone partially separating equilibria which are, however, payoff dominated by the pooling equilibrium.

For some parameter values the game has also a non-monotone (inefficient) pooling equilibrium, where a low value bidder wins against a high value bidder with positive probability. This equilibrium does not require amending the auction rules because there G_2 has no mass point (see Fig. 5).

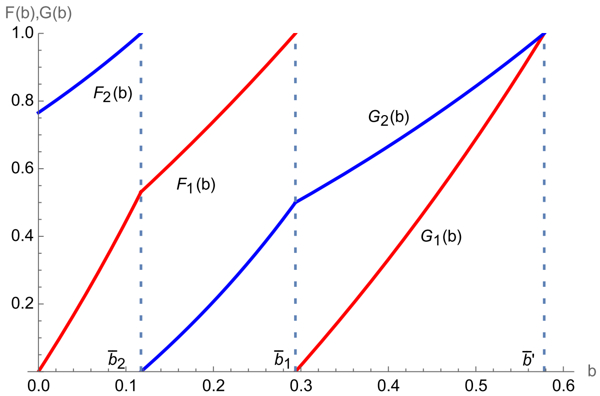

Fig. 5 Non-monotone (inefficient) equilibrium

8 Conclusion

In the present paper we examine the impact of spying in a first-price auction, assuming the spied-at bidder may secretly revise his bid with some known probability. Whereas perfectly informative spying exclusively benefits the spying bidder, the ambiguity about the spied-at bidder's type gives rise to a collusive outcome where both bidders benefit from spying.

The collusive impact suggests that once a bidder has procured the service of a spy, the spied-at bidder passively tolerates being spied-at by not taking measures to neutralize the spy and by not firing the spy if his identity has become known. This way tacit collusion is established and persists.

While bidders benefit from spying, the seller is hurt. He may thus contemplate to replace the first-price auction by a Vickrey auction in which spying has no effect.[21] However, Vickrey auctions pose their own problems and are susceptible to collusion (see, for example, Rothkopf et al. 1990; Ausubel and Milgrom 2006).

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[21] Unless bidders engage in spiteful bidding which has been observed in experiments by Fischer et al. (2021). However, one wonders whether spiteful bidding would also have occurred if they had assumed that the leaked information is subject to noise in which case spiteful bidders are at risk to suffer losses.

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