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1-Semi-quasihomogeneous Singularities of Hypersurfaces in Characteristic 2

Abstract:
In arbitrary characteristic different from 2, the singularities with semi-quasihomogeneous equations characterized by the condition to have Saito-invariant 1 are the "classical" quasihomogeneous ones, known over the field of complex numbers as simple elliptic singularities (Saito, [10]). Here we find them in characteristic 2 as well: In odd dimensions and for weights $\tilde{E}_6$ and $\tilde{E}_8$ non-quasihomogeneous equations appear.

0. The problem
$k$ denotes an algebraically closed field. Let $X$ be a finite set of indeterminates $x$ equipped with positive weights $w(x) \in \mathbb{Q}$ and $f \in k[[X]]$ be a formal power series consisting of monomials of weight $\geq 1$ such that $f_1 (:= \text{sum of terms of total degree 1})$ defines an isolated singularity (i.e. the partial derivatives generate an ideal which is primary for the maximal ideal in $k[[X]]/(f)$). Then we associate to $f$ the "Saito-invariant" $s := \left|X\right| - 2\sum_{x \in X} w(x)$. We say "$f$ is $s$-semi-quasihomogeneous" (or short: "s-sqh") with respect to the given weights. For $f = f_1$, $f$ is said to be "1-quasihomogeneous". The case of $s < 1$ gives the rational double points (the simple singularities or, equivalently, the absolutely isolated Cohen-Macaulay double points, cf. [3], [6], [4]). Here the "boundary case" of $s = 1$ is considered, which corresponds in the complex-analytic case to the simple elliptic singularities ([10]). Note however, that for $\text{char } k = 2$ not all of those singularities arise from dimension 2, so here they better will be referred to only as 1-semi-quasihomogeneous. As for the simple singularities, the case of characteristic 2 is most complicated in the sense of stable equivalence for different dimensions. From the point of representations (considering the Auslander-Reiten quiver of maximal Cohen-Macaulay modules over the local ring of the singularity), the usual Knörrer-periodicity has to be replaced by Solberg's periodicity (taking dimensions $\text{mod } 2$), and the results of Kahn (cf. [5]) may apply at least to some of the singularities found here.

1. The quasihomogeneous case
Write $X = \{X_0, \ldots, X_n\}$ and $w(X_i) = w_i$. We always assume $w_i \leq \frac{1}{2}$, this is no loss of generality (cf. e.g. [6]). Let $k[X]$ denote the polynomials which are sum of monomials of weight 1 (set of "quasihomogeneous polynomials" with respect to the given weights). Then we have the following

Cancellation property: Let $f, g \in k[X]$ define isolated singularities, and let $q_1, q_2 \in k[Y]$ be nondegenerate quadratic forms in a finite set $Y$ of new variables of weight $\frac{1}{2}$. Suppose $f + q_1$ can be transformed into $g + q_2$ by an automorphism
Choose $\Phi$ of $k[X, Y]$ preserving the grading. Then there exists an automorphism $\Psi$ of $k[X]$ which preserves the grading and such that $f = g \circ \Psi$.

This is a consequence of the following (cf. [6])

**Proposition (Saito, Knop):** Choose $f \in k[[X]]$ defining an isolated singularity.

(i) If $Y \subseteq X$, then one of the following is satisfied:

(a) There exists $X^a \in supp(f)$ such that $X^a \in k[Y]$, or
(b) There exists an injective map $\varphi : Y \mapsto X - Y$ and a map $\psi : Y \mapsto IV$ such that $Y^{\psi(y)} \cdot y \cdot \varphi(y) \in supp(f)$ for every $y \in Y$.

(ii) Assume $f$ is quasihomogeneous of weight 1. Then up to an automorphism of $k[X]$ which preserves the grading, $f = f_1 + \sum_{x \in A} x \phi(x)$, where $A = \{x \in X, w(x) > \frac{1}{2}\}$ and $\phi : A \mapsto X - A$ is an injection, $f_1 \in k[X -(A \cup \phi(A))]$. Now, choose all $w_i \leq \frac{1}{2}$ and denote $Q := \{x \in X | w(x) = \frac{1}{2}\}$, $R := X - Q = \{x \in X | w(x) < \frac{1}{2}\}$. Up to a graded automorphism, $f$ is of the following form:

(a) $f = f_1 + q$, $f_1 \in k[R]$, and $q \in k[Q]$ a nondegenerate quadratic form.
(b) $char k = 2$, and there exists $x_0 \in Q$ such that $f = f_1 + f_2 \cdot x_0 + x_0^n + q$, where $q \in k[Q - \{x_0\}]$, $f_i \in k[R]$ for $i = 1, 2$.

We deduce a

**Proof of the cancellation property:**

Let $f + q_1 = (g + q_2) \circ \Phi$. In case of part (ii) (a) of the preceding proposition, we may assume $X = R$, i.e. $f, g \in (X_0, \ldots, X_n)^3$, $w(X_i) < \frac{1}{2}$, thus $\Phi(X_i) \in k[X]$, and after a linear change of coordinates in $Y$, $\Phi(Y_i) = Y_i$.

Now let $char k = 2$ and suppose $f$ has the form (ii) of (b), $f + q_1 = f_1 + f_2 X_0 + X_0^n + q$, where $f_i \in k[X_1, \ldots, X_n]$ and $q \in k[Y]$ is a nondegenerate quadratic form. We may assume $g + q_2 = g_1 + g_2 X_0 + X_0^n + q$, and also $g_i, f_i \in k[Y]$, $|Y| = m$ even and $q = Y_1 Y_2 + \ldots + Y_{m-1} Y_m$ (classification of quadratic forms in characteristic 2). Then, if $f = g \circ \Phi$, $\Phi$ graded. We obtain $\Phi(R) \subseteq k[R]$, $R = \{X_0, \ldots, X_n\}$. $\Phi$ induces a linear transformation in the variables $\{X_0, \ldots, X_n\}$, fixing $X_0^n + q(Y) \ mod(X_1, \ldots, X_n)^2$. Thus we may assume $\Phi(X_0) = X_0 + \phi_0$, $\Phi(Y_i) = Y_i + \phi_i$, $\phi_i \in k[X]$ of weight $\frac{1}{2}$. But $q$ is nondegenerate, thus $\phi_1 = \ldots = \phi_m = 0$.

**Definition:** Choose $f \in k[[X]]$ and $g \in k[[X]]$.

(i) $f, g$ are said to be right equivalent if $X = X'$ and there exists an automorphism $\Phi$ of $k[[X]]$ such that $f = g \circ \Phi$. In this case, we write $f \sim g$ (without loss of generality, $\Phi$ can be chosen homogeneous of degree 0 if $f \in k[X]$, $g \in k[X]$, for a fixed weight $w$).
(ii) Assume there exist nondegenerate quadratic forms $q \in k[Z], \ q' \in k[Z']$ respectively in finite sets $Z$, resp. $Z'$ of new variables such that $f + q \sim g + q'$. Then $f, g$ are said to be stable-equivalent. We write $f \sim g$. The polynomials $f + q, \ g + q'$ respectively will be referred to as "quadratic suspensions" of $f, g$ respectively.

Thus, the above cancellation property says: If $f, g$ (as above) have the same number of variables and $f \sim g$, then $f \sim g$.

If $f \sim g$, then $s(f) = s(g)$, and always $0 \leq s(f) < [X]$. The classes of $f$ having $s(f) < 1$ are precisely the quasihomogeneous forms of the simple singularites ADE (cf. [6], [4]); their behavior under the canonical local resolution is studied in [7].

For the 1-qh polynomials we have the following

**Theorem:** Let $f \in k[X]$ be a polynomial defining an isolated singularity such that $f$ is quasihomogeneous for some weight $w$ with $s = 1$.

Then $w$ (up to permutation) one of the weights

$$\tilde{E}_6 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{2}, \ldots, \frac{1}{2}), \ \tilde{E}_7 = (\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, \ldots, \frac{1}{2}), \ \tilde{E}_8 = (\frac{1}{3}, \frac{1}{6}, \frac{1}{2}, \ldots, \frac{1}{2})$$

and $f$ is stable-equivalent with one of the following polynomials $(t \in k$ denotes a parameter):

**Case A:** $\text{char}(k) \neq 2$

$$\tilde{E}_6: \quad f = X_1(X_1 - X_0)(X_1 - tX_0) - X_0X_2^2, \quad t \neq 0, 1$$
$$\tilde{E}_7: \quad f = X_0X_1(X_1 - X_0)(X_1 - tX_0), \quad t \neq 0, 1$$
$$\tilde{E}_8: \quad f = X_0(X_0 - X_1^2)(X_0 - tX_1^2), \quad t \neq 0, 1$$

**Case B:** $\text{char}(k) = 2$

1. $n$ odd

$$\tilde{E}_6(0): \quad X_0^3 + X_1^2X_2 + X_1X_2^2 + X_3^2$$
$$\tilde{E}_6(t): \quad X_0^3 + tX_2^3 + X_1^2X_2 + X_0X_1X_2 + X_3^2, \quad t \neq 0$$
$$\tilde{E}_7(t): \quad X_0X_1(X_1 + X_0)(X_1 + tX_0), \quad t \neq 0, 1$$
$$\tilde{E}_8(t): \quad X_0(X_0 + X_1^2)(X_0 + tX_1^2), \quad t \neq 0, 1$$

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Note, the condition implies that the total number of variables has to be the correct one.
2. \( n \) even

\[
\begin{align*}
\tilde{E}_0(0) & : \quad X_0^3 + X_1^2 X_2 + X_1 X_2^2 \\
\tilde{E}_0(t) & : \quad X_0^3 + tX_2^3 + X_1^2 X_2 + X_0X_1X_2, \quad t \neq 0 \\
\tilde{E}_7,1(t) & : \quad X_0^2 + X_0X_1^2 + X_1 X_2^2(tX_1 + X_2) \\
\tilde{E}_7,2(t) & : \quad X_0^2 + X_0X_1X_2 + X_1X_2(tX_1 + X_2)^2, \quad t \neq 0 \\
\tilde{E}_8(t) & : \quad X_0^2 + X_0X_1X_2 + X_1(X_1 + X_2^2)(X_1 + tX_2^2), \quad t \neq 0
\end{align*}
\]

**Proof:** To start with, we need the following

**Lemma:** With the previous notations, assume \( s = |X| - 2 \sum_{x \in X} w(x) = 1 \), i.e.

\[
\sum_{x \in R} w(x) = \frac{1}{2}(|R| - 1)
\]

and such that there exists a polynomial \( f \in k[X]_1 \) with an isolated singularity. Then

(i) \( |R| \neq 0, 1 \)

(ii) \( S := \{x \in X \mid \frac{1}{3} < w(x) < \frac{1}{2}\} = \emptyset \)

(iii) \( |R| \leq 3 \) with equality at most if \( w_0 = w_1 = w_2 = \frac{1}{3} \) (up to permutation of indices of the \( X_i \)).

(iv) If \( |R| = 2 \), then \( w_0 = w_1 = \frac{1}{4} \), or \( w_0 = \frac{1}{3}, w_1 = \frac{1}{6} \) (again, indices may permute).

**Proof of the Lemma:** (i) is an obvious consequence of \( s = 1 \).

To show (ii), (iii), apply (i) in the above proposition: Choose maps \( \varphi, \psi \) with the property (b) and obtain:

\[
\begin{align*}
\frac{1}{2}(|R| - 1) & = \sum_{x \in R} w(x) \\
& = \sum_{x \in S} w(x) + \sum_{x \in S} w(\varphi(x)) + \sum_{x \in R - (S \cup \varphi(S))} w(x) \\
& = \sum_{x \in S} w(x) + \sum_{x \in S} (1 - w(x) - w(S^{\psi(x)})) + \sum_{x \in R - (S \cup \varphi(S))} w(x) \\
& = \sum_{x \in S} (1 - w(S^{\psi(x)})) + \sum_{x \in R - (S \cup \varphi(S))} w(x) \leq \frac{2}{3} |S| + \frac{1}{3}|R - S \cup \varphi(S)|
\end{align*}
\]
(note that $S^{[x]} \cdot x \cdot \varphi(x) \in \text{supp}(f)$ for all $x$, i.e. $w(S^{[x]}) + w(x) + w(\varphi(x)) = 1$; also, $\varphi(S) \subseteq R$). Thus

$$\frac{1}{2}(|R| - 1) \leq \frac{1}{3}(2|S| + |R - (S \cup \varphi(S))|) = \frac{1}{3}|R|.$$ 

To prove (iv), we may assume $w_0 + w_1 = \frac{1}{2}$, $w_i > \frac{1}{2}$ for $i > 1$, i.e. for $w_0 = w_1$ we are done. Assume $w_0 > w_1$, then $\frac{1}{4} < w_0 < \frac{1}{2}$. If $X_0^3 \notin \text{supp}(f)$, then no power of $X_0$ is in $\text{supp}(f)$, and (i) (a) in the Proposition implies (using $Y = X_0$) that one of the monomials $X_0^{\alpha + 1}X_i$ ($\alpha \in \mathbb{N}$, $i \in \{1, \ldots, n\}$) is in $\text{supp}(f)$. This implies $w_0 = \frac{1}{2\alpha}$ or $w_0 = \frac{1}{2(\alpha + 1)}$ (contradiction, since $\alpha \in \mathbb{N}$). Thus $X_0^3 \in \text{supp}(f)$, i.e. $w_0 = \frac{1}{3}$, $w_1 = \frac{1}{6}$.

Now, a detailed case by case analysis gives the

**Proof of the Theorem:**

Choose e.g. the case of $E_6$ in even dimension, i.e. here without loss of generality in dimension 2. Then in coordinates $(x_0 : x_1 : x_2)$, the corresponding equation $f = 0$ defines a smooth curve $C$ of degree 3 in the projective plane. We obtain the above normal form after a linear change of coordinates. In $\text{char} \; k = 2$ we have two cases: $E_6(0)$ if the elliptic curve is supersingular, $E_6(t)$, with $t \neq 0$ otherwise.

For the weights $E_7$, $E_8$, a geometric analysis of the relevant forms is necessary, giving different equations in even and odd dimensions for $\text{char} \; k = 2$.

We apply the proposition to obtain the list of equations; choose e.g. $f$ of weight $E_7$, $\text{char} \; k = 2$:

We may assume $X = \{X_0, \ldots, X_n\}$ with

(a) $n = 1$, $w_0 = w_1 = \frac{1}{4}$, $f = f(X_0, X_1)$ homogeneous of degree 4 and defining an isolated singularity, i.e. $f$ with 4 different zeroes on $\mathbb{P}^1$.

(b) $n = 2$, $w_0 = \frac{1}{2}$, $w_1 = w_2 = \frac{1}{4}$, $f = x_0^2 + gx_0 + h$, $g \in k[X_1, X_2]$ homogeneous of degree 2, $h \in k[X_1, X_2]$ homogeneous of degree 4.

If (b1) $g = 0$, then coordinates can be choosen such that $X_1X_2^3 \notin \text{supp} \; h$, thus $V(X_1, X_0^2 + g(X_1, X_2)) \subseteq \text{sing}(f)$, i.e. the singular locus has positive dimension. Now assume (b2) $g = X_2^2$, then $f = X_0^2 + X_0X_1^2 + h(X_1, X_2)$. Write $h(X_1, X_2) = \sum_{\nu=0}^{1} h_{\nu}X_1^\nu X_2^{1-\nu}$. Then $f$ defines an isolated singularity iff $h_1 \neq 0$; we may assume $h_1 = 1$. A coordinate transformation $X_0 := X_0 + aX_1^2 + bX_1X_2 + cX_2^2$ brings $h$ into the form $h = X_1X_2^2(tX_1 + X_2)$.

The case (b3) $g = X_1X_2$ is done in a similar way.
Remark: Note that also in char $k = 2$, the equations for $\tilde{E}_6$ can be written in a form such that $\tilde{E}_6(0)$ and $\tilde{E}_6(t)$, $t \neq 0$ are in the same 1-parameter family: Take $n = 2$ and let $C(s)$ be the curve defined in the projective plane by

$$X_0^3 + X_1^3 + X_2^3 + sX_0X_1X_2 = 0$$

where $s \in k$. For $s^3 \neq 1$ this is an elliptic curve with absolute invariant $j = \frac{(s^3 + 1)^3}{s^3}$, and $\tilde{E}_6(0)$ is the cone over an elliptic curve with invariant 0, thus isomorphic to the cone over $C(0)$. For fixed $t \neq 0$, the equation $ts^{12} + s^9 + s^6 + s^3 + 1 = 0$ has 12 different solutions $s$. We obtain several $C(s)$ with invariant $j = \frac{1}{t}$.

Thus any 2-dimensional quasihomogeneous singularity of type $\tilde{E}_6$ is obtained as cone over some $C(s)$.

Corollary: Let $f \in k[X]$ be quasihomogeneous of some weight $w = w(f)$ and assume $s = s(f) \leq 1$. Then $w$ is uniquely determined up to permutation in the class of quasihomogeneous functions which are stable equivalent $f$. Especially, the number $s$ is well defined on the equivalence class.

Remark: In the case considered here, $w$ (up to permutation) and therefore $s(f)$ depends only on the complete local ring of the singularity. It is not known to the author, if this is generally so for $s(f) > 1$ (but it is always true for $k = \mathbb{C}$ by [10]).

2. Normal forms of semiquasihomogeneous functions

Now let $f = f_1 + f_{>1}$ be a formal power series which contains no monomials of weight $< 1$ with respect to the given weight $w$. Put $f_1 := \text{sum of terms of weight 1 in } f$ and assume $f_1$ defines an isolated singularity.

$f$ is said to be contact equivalent with a power series $g$, if the $k$-algebras $k[[X]]/(f)$ and $k[[X]]/(g)$ of formal power series are isomorphic.

The following result reduces the part $f_{>1}$ into a normal form without changing $f_1$ and the contact equivalence class of $f$. $T(f_1)$ denotes the "Tjurina-algebra",

$$T(f_1) := k[[X]]/(f_1, \frac{\partial f_1}{\partial X_0}, \ldots, \frac{\partial f_1}{\partial X_n}).$$

We have $\dim_k(T(f_1)) < \infty$.

Theorem: Let $(\overline{e}_1, \ldots, \overline{e}_s)$ denote any maximal linear independent set of classes in $T(f_1)$ of monomials $e_i$ having weight $> 1$ ("superdiagonal monomials"). Then $f = f_1 + f_{>1}$ is contact equivalent with $f_1 + c_1e_1 + \ldots + c_se_s$, $e_i \in k$.

Proof: Let $w = (\frac{m_0}{d}, \ldots, \frac{m_n}{d})$ with positive integers $m_i$, $d$. Denote $a_m(h)$ the total order of the initial term of a power series $h \in k[[X_0, \ldots, X_n]]$ with respect to $(m_0, \ldots, m_n)$. 

6
If the classes of superdiagonal monomials \( \{e_1, \ldots, e_s\} \) form a basis of the subspace generated by all superdiagonal monomials in the Tjurina algebra \( T(f_1) \), then the similar assertion is true for any fixed order \( d' \), i.e. let \( \{e_i, \ldots, e_s\} \) be the subset of monomials such that \( o_m(e_i) = d' \), then this is a basis for the subspace in \( T(f_1) \) generated by the classes of all monomials having \( o_m = d' \) (\( f_1 \) is homogeneous).

Obviously, an inductive convergence argument gives the result, if we show the following

**Lemma:** Let (after some permutation) \( e_1, \ldots, e_r \) be the monomials of order \( o_m(e_i) = d' > d \) in \( \{e_1, \ldots, e_s\} \). Then \( f \) is contact equivalent with a series

\[
f_1 + f'_{>1} + \sum_{i=1}^{r} c_i e_i + h,
\]

where \( f'_{>1} \) is the sum of terms of order \( o_m < d \) in \( f'_{>1}, e_i \in k \) und \( h \in k[[X_0, \ldots, X_n]] \) has order \( o_m(h) > d' \).

(Note that the case is included, where \( \{e_1, \ldots, e_r\} \) is the empty set.)

**Proof of the Lemma:** Choose \( c_i \in k \) such that

\[
g - \sum_{i=1}^{r} c_i e_i = q \cdot f_1 + \sum_{i=0}^{n} v_i \frac{\partial f_1}{\partial X_i}
\]

for some \( q, v_i \in k[[X_0, \ldots, X_n]] \) and \( g \) the sum of monomials of order \( d' \) in \( f'_{>1} \).

Without loss of generality, \( q \) and \( v_i \) are quasihomogeneous for \((m_0, \ldots, m_n)\) of order

\[
o_m(q) = d' - d =: \delta > 0
\]

\[
o_m(v_i) = d' - (d - m_i) = \delta + m_i > m_i,
\]

respectively. We obtain

\[
(*) \quad f_1 + f'_{>1} + \sum_{i=0}^{r} c_i e_i = (1 - q)(f_1 + f'_{>1}) + qf'_{>1} - \sum_{i=0}^{n} v_i \frac{\partial f_1}{\partial X_i} + p,
\]

where in the right hand term \( o_m(qf'_{>1}) > d' \), \( v_i \frac{\partial f_1}{\partial X_i} \) is quasihomogeneous with \( o_m(v_i \frac{\partial f_1}{\partial X_i}) = d' \), and \( o_m(p) > d' \).

Assume without loss of generality \( m_0 \geq m_1 \geq \ldots \geq m_n \). Let \( X_i := X_i' - v_i(X') \), then \( o_m(v_i) > m_i = o_m(X_i) \) implies: The linear part of this coordinate transformation has a lower triangular matrix \((a_{ij})\) with \( a_{ii} = 1 \), and \( a_{ij} \neq 0 \) for \( i > j \) is possible only if \( m_i > m_j \). The above substitution sends

\[
(**) \quad f_1(X) \mapsto f_1(X') - \sum_{i=0}^{r} v_i(X') \frac{\partial f_1(X')}{\partial X_i} + \text{terms in } X' \text{ of order } o_m > d
\]

(if we take the same weights for the \( X' \)).
By (⋆), we have

\[
(⋆⋆⋆) \quad (1 - q(X)) f(X) \equiv f_1 + \sum_{i=0}^{n} e_i(X) \frac{\partial f_1}{\partial X_i} + (f'_{\geq 1}(X) + \sum_{i=0}^{r} c_i e_i(X))
\]

mod terms of order \( o_m > d' \). If we apply (⋆⋆) and remember \( o_m(v_i) > d' - d \), the substitution above transforms the right hand side of (⋆⋆⋆) into

\[
f_1(X') + f'_{\geq 1}(X') + \sum_{i=0}^{r} c_i e_i(X') + h(X'),
\]

where \( o_m(h) > d' \). This completes the proof.

Note that we do not need any assumption on \( char k \). If \( char k = 0 \), by Euler’s formula we have \( f_1 \in (\frac{\partial f_1}{\partial X_0}, \ldots, \frac{\partial f_1}{\partial X_n}) \), i.e. the Tjurina-algebra \( T(f_1) \) coincides with the Milnor-algebra \( M(f_1) = k[[X]]/(\frac{\partial f_1}{\partial X_0}, \ldots, \frac{\partial f_1}{\partial X_n}) \), and in this case the result coincides with ([1], 12.6).

3. Results in the 1-semiquasihomogeneous case

Using a computer\(^3\), from the theorem in section 2, we obtain easily:

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<tr>
<th>semiquasihomogeneous singularities with ( s = 1 ) in characteristic 2</th>
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<tr>
<td>type</td>
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<tr>
<td>case 1:</td>
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<td>( E_6(0) )</td>
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<td>( E_6(t) )</td>
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<td>( E_7 )</td>
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<td>case 2:</td>
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<td>( E_6(t) )</td>
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<td>( E_7,1(t) )</td>
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<td>( E_7,2(t) )</td>
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<td>( E_8(t) )</td>
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Thus e.g. for \( n \) odd, the 1-sqh singularities with first term \( \tilde{E}_8 \) (as in the theorem of section 1) are given by adding a constant multiple the monomial \( X_0X_1^5 \). If the

\(^3\)Calculations done with REDUCE 3.5
coefficient is not zero, an easy coordinate transformation leads to the only non quasihomogeneous 1-sqh singularity of that weight; it is given by the equation \( X_0(X_0 + X_1^2)(X_0 + tX_1^2) + X_0X_1^5 = 0 (t \neq \{0, 1\}) \) with Tjurina number 11.

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