FRACTIONAL INTEGRATION AND THE DYNAMICS OF UK UNEMPLOYMENT

Luis A. Gil-Alana
Humboldt Universität zu Berlin, Institute for Statistik und Ökonometrie, Berlin, Germany

and

Brian Henry
Centre for Economic Forecasting, London Business School, London, England.

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This article is concerned with the dynamic behaviour of UK unemployment. However, instead of using traditional approaches based on I(0) stationary or I(1) (integrated and/or cointegrated) models, we use the fractional integration framework. In doing so, we allow for a more careful study of the low frequency dynamics underlying the series. The conclusions suggest that the UK unemployment may be explained in terms of lagged values of the real oil prices and the real interest rate, with the order of integration of unemployment ranging between 0.50 and 1. Thus, unemployment shows the characteristics of long memory but is mean reverting.

Keywords: Unemployment; Long memory; Fractional integration

JEL Classification: C22, C32, C52, E24.

Contents

1. Introduction

Although the study of unemployment behaviour has been a major preoccupation for macroeconomists and labour market economists it is fair to say that the general view is that it is still not well understood. Recent contributions echoing this pessimistic conclusion are found in Carruth, Hooker and Oswald (1998) in a study of US unemployment, and Bean (1994) and Nickell (1997) in their general surveys of unemployment models. Two problems have increasingly become evident. In one, models using a large set of labour supply and institutional factors to account for movements in equilibrium unemployment appear structurally (i.e. parameter) unstable. Second, the hypothesised link between deviations of unemployment from estimated NAIRUs and inflation rates has not coincided with the observed behaviour of inflation and unemployment in many countries. These problems, which have bedevilled the empirical literature are, in our view, traceable to difficulties in distinguishing the long and short run behaviour of unemployment. Such a view may appear odd, given that the distinction between long run and short run behaviour of unemployment has been a major focus in the economic literature for decades.

We contend, however, that many of the clues to understanding unemployment lies in a more refined empirical distinction between its long and short run movement than has been the case up to now, and we illustrate this with an application to UK unemployment. In large part, the issues are empirical ones. Thus there is a broad consensus on the general nature of dynamic model involved; most models argue that various "rigidities" may lead to protracted responses in the labour market following major exogenous shocks. This is true whether a shirking or union-firm bargaining approach is used to provide the theoretical underpinnings of the estimated model. From either approach, a reduced form for unemployment can be obtained which is a dynamic autoregression depending on weakly exogenous shift variables (examples are described more fully below)[1]. It is in implementing this general idea that the major differences arise. At one end of the spectrum of empirical models, a pure hysteresis model attributes the major role to temporary exogenous shocks in bringing about changes to the level of the unemployment rate. Here, unemployment has a unit root, so there is no equilibrium rate (Blanchard and Summers (1986)). The most popular alternative approach to hysteresis is enshrined in familiar NAIRU (Non Accelerating Inflation Rates of Unemployment) models, which allow for both exogenous shocks and persistence mechanisms, with an equilibrium (NAIRU) rate given by steady state values of exogenous variables, when all dynamic adjustments have worked through. In this model, there is an important distinction between its transitory and its equilibrium components. In one version – the most common – much of the movement in unemployment is "accounted" for by movements in its equilibrium rate. Transitory movements are I(0), and are treated as relatively unimportant in the overall story. Examples are numerous, but include Layard, Nickell and Jackman (1991), and Minford (1994). These empirical models treat variables such as real unemployment benefits (or the replacement ratio), the duration of benefits, measures of union power, indices of corporateness, and measures of employment protection as exogenous determinants of unemployment. As an aside, these variables determine the dynamic responses of the labour market to other - genuinely exogenous-shocks, although adopting this approach is rarely done. (See particularly Bean and Layard (1988) together with the earlier citation for examples where the variables change the equilibrium, and Nickell (1997) for an attempt to do the alternative – where they affect the dynamics). Returning to orthodox NAIRU models, there is

[1] Distinctions arising from structural versus reduced – form estimation of the labour market are described in Layard et al (1991) and in Henry and Nixon (1999). They are not important for our purpose, which concentrates on reduced forms only.

an important set of propositions involved here. Thus, the typical NAIRU model attributes an important role to movements in the equilibrium rate when accounting for movements in actual unemployment (u), and this is most clearly seen in empirical applications using cointegration or related concepts. In this model the unemployment rate is non-stationary (I(1)), and its movements related to other I(1) variables, so the "long run" equilibrium rate (u^*) is "explained" by variables from the list just noted (most of which are I(1)). The disequilibrium term $u_t - u^*$ is then I(0) by construction. When accounting for the large changes in actual unemployment, proponents argue that the NAIRU has changed very substantially over the last two and a half decades. But the estimated disequilibrium term, ($u - u^*$) is often found not to correlate well with inflation changes. More importantly, some of the variables which are thought important in accounting for changes in u^* have not moved in the predicted way. For example, when unemployment in the UK rose strongly in the first part of the 1980s, unionisation rates fell, while the replacement ratio showed no clear trend. More generally, the treatment of unemployment as an I(1) variable is questionable. Many of the univariate tests used, have little power, so the alternative that the root in the unemployment series is close to, but actually less than one, is often rejected. These considerations have led to an alternative approach which though it is based on a more careful treatment of the time-series characteristics of the series involved in unemployment modelling (including the unemployment rate itself), have vitally important economic implications. It starts from the proposition that the root in the unemployment rate may be close to, but is not equal to, unity. In section 3 we describe more precisely how this may be established using fractional integration. This leads to the model where the reduced form for unemployment may have very considerable persistence but where its equilibrium (which we refer to as u^{**} to differentiate it from a NAIRU) is shifted by only a small number of exogenous shocks such as oil prices or real interest rates. Models of this sort are reported by Henry and Nixon (op.cit), Funke (1999), and Henry, Karanassou and Snower (1999).

In earlier work, one of the authors argued for treating the unemployment rate (u) as I(0), but highly persistent, basing this argument on evidence of the logistic transformation of u, where this is subject to a small set of mean shifts due to oil price and real interest rate changes. (See Henry and Nixon (op cit)). It was found there that a model of this sort out performed a more standard NAIRU model with up to seven "explanatory" variables, in terms of econometric tests including parameter stability. In this persistence model the root of (the logistic transformation of) the unemployment rate was around 0.95, and only a few exogenous shocks, associated with the oil price hikes in 1973 and 1979, and the monetary tightening in 1989, seemed to be needed to explain the behaviour of UK unemployment.

The contribution which the present paper makes is to extend models of this latter sort, by investigating the case for low frequency dynamics in the reduced form for unemployment using fractional integration methods. As noted, when properly interpreted existing labour market models strongly suggest that there will be prolonged persistence in unemployment following shocks. Henry and Nixon (op.cit) note some of the econometric problems in identifying low frequency responses using traditional integration and cointegration methods. The aim of current paper is to overcome these shortcomings, and apply fractional integration methods to investigate the persistence of the unemployment rate.

The rest of the paper sets out some of the claims made so far in a more formal way, and some of the technical issues involved. Section 3, sets out the details on the estimation procedure, and Section 4 presents the application to UK unemployment. Section 5 concludes.

2. Modelling unemployment

The observation that high levels of unemployment can co-exist with broadly stable levels of inflation implies that either the equilibrium unemployment rate has moved in line with actual unemployment or actual unemployment is able to diverge from a relatively constant equilibrium rate, for considerable periods of time. This observation suggests that modelling unemployment can be grouped into two distinct categories: one that stresses equilibrium unemployment (and, by implication, factors which change this) and another, which stresses the persistence of unemployment between equilibria.

As already mentioned we focus on reduced-form dynamic models of the unemployment rate in what follows. The motivation for these models is quite general. Thus Bean and Layard (1988) obtain a second-order autoregressive equation for u_t based upon a model of long-term unemployment and insider effects on wages and prices. Dynamics arise in their model because short-term unemployment depends on lagged employment levels, and because insider effects on wages are also postulated to depend on lagged employment. The unemployment equation then takes the form

$$(1-(1-s-ch)\,L-(a+s\,(1-c)\,h)L^2)\,u_t = bZ_{t-1} \qquad \textbf{(1)}$$

where L is the backward shift operator and the parameters are defined as follows. This period inflow into unemployment from employment is given by s, thus, $U^s = s\ N_{t-1}$ where U^s is short term unemployment. The parameter c indexes the effectiveness of workers when exiting unemployment, where the longer they remain unemployed, the lower is their effectiveness. (Hence the short run unemployed have unit effectiveness so for them c = 1). Lastly h is the slope of the hiring function. The right hand side of (1) is a vector of shift variables. Bean and Layard (1988) take these to include supply (such as benefits) and demand shocks. As is clear from (1), the degree of persistence in unemployment depends upon how quickly workers flow into and out of unemployment, and the strength of insider effects. Henry and Nixon (1999) extend this to allow for hiring and firing dynamics and capital constraints on employment. Before leaving this point, a few further comments on the composition of Z are in order. Many empirical examples use predominantly supply side variables in Z. It is often a large list. Nickell (1998) for example has up to seven. One issue which arises is whether using such an extensive list tends to reduce the estimated degree of persistence in equations like (1) above. We will argue that a limited set of shocks – oil price and real interest rates – are all that is needed, once the possibility of considerable persistence in unemployment is allowed. To allow for the latter involves fractional integration.

Formally, the fractionally integrated structure imposes a slow rate of decay on the autocorrelations (in fact, these are given by a hyperbolic weighting function), unlike standard autoregressions were the decay is exponential and usually fairly rapid. Furthermore, this way of specifying the model, allows us to consider both the I(1) and the I(0) specifications as particular cases of a much more general class of model, called I(d), where d can be any real number.

In order to bring both ordinary and fractionally integrated formulations together, consider a reduced form equation for the unemployment rate (u_t),

$$\Phi(L)\ u_t \ =\ \beta_1{'}\ \Theta(L)\ x_t \ +\ \varepsilon_t\,, \qquad \textbf{(2)}$$

where x_t is a set of variables affecting unemployment and ε_t is a white noise process. If it is assumed that there is no change in the unemployment equilibrium level, then u_t would be an I(0) stationary process, and thus, $\Phi(L)$ an AR polynomial with all its roots lying outside the unit circle. Even with this specification, x_t might include some 'mean-shifting' exogenous variables, and then there would be temporary changes in unemployment. On the other hand, if we assume that equilibrium unemployment changes, u_t must have a unit root. Thus, $\Phi(L) = \Phi^*(L)$ (1-L), where $\Phi^*(L)$ is now an AR stationary polynomial. If x_t is also I(1), there may be a cointegrating relationship between u and x, and the model can be expressed in terms of an error correction form.

To describe the extension of the fractionally integrated model, suppose we have

$$u_t \;=\; \beta_1' \,\Theta(L)\; x_t \;+\; v_t\,, \qquad \textbf{(3)}$$

$$\Phi^*(L)\;(1-L)^d\; v_t \;=\; w_t\,, \qquad \textbf{(4)}$$

with I(0) w_t and where d is a given real number and $(1\text{-}L)^d$ is expressed in terms of its Binomial expansion

$$(1-L)^d \;=\; \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j\, L^j\,,$$

such that

$$(1-L)^d\; v_t \;=\; v_t \;-\; d\; v_{t-1} \;+\; \frac{d\,(d-1)}{2}\, v_{t-2} \;-\; \frac{d\,(d-1)\,(d-2)}{6}\, v_{t-3} \;+\; \ldots .$$

Substituting (4) into (3), we obtain

$$\Phi^*(L)\;(1-L)^d\; u_t \;=\; \beta_1'\, w(L)\; x_t \;+\; w_t\,,$$

where $w(L) = \Phi^*(L)\;\Theta(L)\;(1\text{-}L)^d$. Then, if d = 0, u_t is an I(0) stationary process and if d = 1, we have an integrated model for unemployment; the two models already described. But as d can be any real number, this permits a richer characterisation of the dynamics affecting unemployment compared with the restrictiveness imposed by the I(1) and I(0) specifications. Furthermore, if d in (4) belongs to the interval (0, 0.5), the series is still covariance stationary but the autocorrelations take far longer to decay to zero than those based on d = 0. In addition, if $d \in [0.5, 1)$, the process is not longer stationary but still will be mean-reverting, with shocks affecting the series but this returns to its original level sometime in the future.

Thus, d plays a crucial role in explaining the degree of persistence of the series. Processes like (4) with positive non-integer d (and $\Phi^*(L) = 1$) are called fractionally integrated and when w_t is ARMA(p, q), v_t is a fractionally ARIMA (ARFIMA(p, d, q)) process. This type of model was introduced by Granger and Joyeaux (1980), Granger (1980, 1981) and Hosking (1981) and were justified theoretically in terms of aggregation by Robinson (1978) and Granger (1980). In the following section, we use a testing procedure suggested by Robinson (1994) for testing the applicability of models of this type to the UK unemployment rate.

Before moving to the application, we first note the testing procedure for the order of fractional integration which we use, based on the approach put forward by Robinson (1994).

3. The testing procedure for fractional integration

To define the testing procedure for d, consider the model given by a simplification of (3) and (4) above, namely

$$u_t = \beta' x_t + v_t , \tag{5}$$

$$(1 - L)^d \; v_t = w_t , \tag{6}$$

where u_t is the observed dependent variable and x_t is a (kx1) vector of weakly exogenous variables. The error term w_t is an I(0) process with parametric spectral density f, which is a given function of frequency λ and of unknown parameters,

$$f(\lambda; \sigma^2; \tau) = \frac{\sigma^2}{2\pi} g(\lambda; \tau) \qquad -\pi < \lambda \leq \pi ,$$

where the scalar σ^2 and the (qx1) vector τ are unknown but the function g is assumed to be known. For example, in the AR case, if $\sigma^2 = V(\varepsilon_t)$, we have

$$g(\lambda; \tau) = \left| \Phi(e^{i\lambda}) \right|^{-2} ,$$

where Φ is the AR polynomial, so that the AR coefficients are functions of τ.

Robinson (1994) defines tests of the null hypothesis:

$$H_o : d = d_o \tag{7}$$

for any given real number d_o in (5) – (7). To derive the test statistic, where x_t is non-empty, we form

$$z_t = (1 - L)^{d_o} \; x_t ; \qquad x_t = 0, \qquad t \leq 0.$$

Based on the null differenced model, the least-squares estimate of β and residuals are

$$\hat{\beta} = \left(\sum_{t=1}^{T} z_t \, z_t' \right)^{-1} \sum_{t=1}^{T} z_t \, (1 - L)^{d_o} \, u_{t,}$$

$$\hat{w}_t = (1 - L)^{d_o} \, u_t - \hat{\beta}' z_t , \quad t = 1, 2, \ldots .$$

In the univariate model u_t, there are no regressors, so the $\hat{\beta}' z_t$ term does not appear.

The periodogram of $\hat{w}_t$ is

$$P(\lambda) \;=\; \left| (2\,\pi\,T)^{-1/2} \sum_{t=1}^{T} \hat{w}_t\, e^{i\,t\,\lambda} \right|^2 .$$

Unless g is a completely known function (e.g., $g \equiv 1$, so that w_t is white noise), we have to estimate the nuisance parameter vector τ. The estimate must be a Gaussian one, that is it must have the same limit distribution as the efficient Maximum Likelihood estimate based on the assumption that $w_1, w_2, \ldots, w_T$ is Gaussian. One such estimate, which fits naturally into our frequency domain setting, is

$$\hat{\tau} \;=\; \arg\min_{\tau}\; \sigma^2(\tau)$$

where the minimisation is carried out over a suitable subset of R^q, and $\sigma^2(\tau) = 2\pi\ 1'h(\tau)/T$, where $h(\tau)$ is the (T-1)-dimensional column vector with j^{th}-element $P(\lambda_j)/g(\lambda_j;\tau)$, 1 is the (T-1)x1 vector of 1's and $\lambda_j = 2\pi j/T$. Next, $\hat{a}$ and $\hat{b}$ are given by

$$\hat{a} \;=\; -\frac{2\pi}{T} m' h(\hat{\tau}), \qquad \hat{b} \;=\; \frac{2}{T}\,\{m'm \;-\; m'M\,(M'M)^{-1}\,M'm\},$$

in which m is the (T-1)x1 vector with j^{th}-element given by $\log\,|2\sin(\lambda_j/2)|$, and M is the (T-1)xq matrix with j^{th}-row $(\partial/\partial\tau)\log\ g(\lambda_j;\hat{\tau})$. Next, we write

$$\hat{s} \;=\; \left(\frac{T}{\hat{b}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \qquad \textbf{(8)}$$

where $\hat{\sigma}^2 = \sigma^2(\hat{\tau})$. Under the null hypothesis (7), Robinson (1994) established under regularity conditions that

$$\hat{s} \;\to_d\; N(0,1) \qquad as\ T \;\to\; \infty. \qquad \textbf{(9)}$$

The conditions on w_t in (8) are far more general than Gaussianity, with a moment condition only of order 2 required. From these it follows that an approximate one-sided 100α%-level test of (7) against the alternatives

$$H_1: \quad d \;>\; d_o \qquad \textbf{(10)}$$

is given by the rule

"Reject H_o if $\hat{s} > z_\alpha$," **(11)**

where the probability that a standard normal variate exceeds z_α is α. Conversely, an approximate one sided 100α%-level test of (7) against alternatives

$$H_1: \quad d \;<\; d_o \qquad \textbf{(12)}$$

is given by the rule

"Reject H_o if $\hat{s} < -z_{\alpha}$" . **(13)**

As these rules indicate, we are in a classical large sample testing situation for reasons described by Robinson (1994), who also showed that the above tests are efficient in the Pitman sense that against local alternatives: H_1: $d = d_o + \delta T^{-1/2}$ for $\delta \neq 0$, $\hat{s}$ has an asymptotic normal distribution with variance 1 and mean which cannot (when w_t is Gaussian) be exceeded in absolute value by that of any rival regular statistic.

A notable feature of Robinson's (1994) tests is that the null N(0, 1) distribution of $\hat{s}$ holds across a broad class of exogenous regressors x_t, unlike most of unit root tests embedded in AR alternatives, where the null limit distribution can vary with features of the regressors. In the following section we use this framework to test (7) in a model given by (3) and (4).

4. The empirical application: UK Unemployment

In this section the testing procedure described earlier is used to identify the dynamics of the UK unemployment rate. Firstly, we investigate its univariate behaviour, estimating and testing its order of integration. We find there is evidence that it is fractionally integrated, so exhibits extensive persistence when shocked. Next, we investigate what the major source of these shocks are. To do this, a set of weakly exogenous regressors are included in the dynamic model of unemployment. In other words, we estimate a model based on (3) and (4) for different values of d.

The unemployment series used in this paper is the logistic transformation of the unemployment rate in the UK[2]. The data are quarterly and the sample size is 1966q1 - 1997q4.

(i) A univariate model

First, to investigate the univariate properties of unemployment, as already anticipated we model this as an ARFIMA(p,d,q) model, with p and q each taking values up to and equal to 3. That is, u_t is modelled as

$$\phi_p(L)\,(1 - L)^d\, u_t \;=\; \theta_q(L)\, \varepsilon_t \,,$$

where $\phi_p(L)$ and $\theta_q(L)$ $(p, q \leq 3)$ are respectively the AR and MA polynomials. Two approaches to estimating and testing this are implemented in what follows: the first a ML procedure and the second the testing procedure suggested by Robinson (1994) described in the previous section. Table 1 summarises the estimated values of d (and of the remaining parameters) when the ML procedure is used for alternative values of p and q. This estimation uses Sowell's (1992) procedure of estimating by maximum likelihood in the time domain. The results clearly indicate that practically all the estimated values of d are higher than 1. However, in eleven out of the sixteen models, the unit root hypothesis (d = 1) cannot be rejected. On the other hand, the null d = 0 is rejected in all cases. The Akaike and Schwarz

[2] See Wallis (1987) for a justification based on the logistic transformation being defined between $\pm\infty$ so that standard distributions apply.

information criteria both indicate that the best model specification might be an ARFIMA(0, 1.18, 3), and the unit root null hypothesis cannot be rejected in this case.[3]

Table 1

Maximum likelihood estimation of ARFIMA(p,d,q) models for unemployment											
		t-tests’		AR parameters			MA parameters			Criterions	
ARMA	d	$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)	1.92	21.3	10.2	--	--	--	--	--	--	221.6	220.1
(1, 0)	1.19	7.43	1.18’	0.70	--	--	--	--	--	225.7	222.8
(0, 1)	1.74	13.3	5.69	--	--	--	0.19	--	--	221.7	218.9
(1, 1)	1.15	5.00	0.65’	0.71	--	--	0.03	--	--	224.7	220.4
(2, 0)	0.60	6.00	0.66’	1.60	-0.63	--	--	--	--	225.3	221.0
(0, 2)	1.50	12.5	4.16	--	--	--	0.33	0.44	--	226.1	221.8
(2, 1)	1.25	6.25	1.25’	0.03	0.44	--	0.59	--	--	223.9	218.2
(1, 2)	1.14	7.12	0.87’	0.71	--	--	-3013	-8764	--	223.7	218.0
(2, 2)	1.14	2.85	0.35’	0.72	-0.23	--	0.01	0.50	--	228.0	220.9
(3, 0)	0.98	3.76	-0.07’	0.89	0.06	-0.21	--	--	--	226.8	221.1
(0, 3) (*)	1.18	10.7	1.63’	--	--	--	0.77	1.65	0.92	230.3	224.6
(3, 1)	1.02	3.09	0.06’	0.92	0.02	-0.21	-0.06	--	--	225.8	218.7
(3, 2)	0.97	3.12	-0.09’	0.85	-0.51	0.28	0.05	1.39	--	228.3	219.7
(1, 3)	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
(2, 3)	1.86	2.38	1.10’	0.92	-0.23	--	-0.85	2.08	-2.12	227.1	218.6
(3, 3)	1.88	4.70	2.20	0.41	-0.06	0.18	0.67	1.56	0.63	227.8	217.8

-----: Convergence was not achieved after 240 iterations. ‘: Non-rejection values of the null hypothesis: d = 1 at the 95% significance level. *: Best model specification according to the AIC and SIC criteria.

Turning now to the Robinson procedure, we take the model given by (5) and (6), where β = 0 (i.e. the model is univariate). A range of different forms for w_t are tried, including where it is pure white noise, autoregressive (AR(1), AR(2)) and seasonal autoregressive (AR(1) and AR(2))[4]. Higher order autoregressions were also performed obtaining similar results.

Table 2, then gives the estimated orders of integration of unemployment according to Robinson’s (1994) tests. Using the test of d given by (7) for d_o = 0.00, … (0.25) ….2.00, we observe that the null hypothesis d = 1 is never rejected, though we also observe several non-rejection values when d = 0.75 and 1.25.

The conclusion of both of these univariate procedures applied to unemployment is that the unit root null hypothesis cannot be rejected when modelling unemployment alone. However, this feature may not be robust to extensions in the model, particularly when the likely determinants of unemployment are used in a multivariate model. We turn to consider extensions to the univariate model next.

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[3] Note that this estimation procedure is based on Maximum Likelihood and thus, conventional tests based on the statistic $(d - \hat{d}) / SE(\hat{d})$ can be performed.

[4] By seasonal autoregressions we mean processes of form $w_t = \sum_{j=1}^{p} \phi_j \, w_{t-js} + \varepsilon_t$, with s = 4, (the data are quarterly), and p = 1 and 2.

Table 2

Testing the order of integration of unemployment with the tests of Robinson (1994)									
u_t	Values of d_o								
	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White noise	26.13	20.12	12.70	5.24	0.28’	-2.33	-3.72	-4.51	-5.00
AR(1)	8.98	8.97	6.18	3.27	0.59’	-1.86’	-1.98	-2.02	-2.59
AR(2)	6.39	2.39	2.31	1.68’	-0.91’	-1.61’	-2.24	-3.09	-3.99
Seasonal AR(1)	11.87	6.58	3.83	1.81’	-0.06’	-2.37	-3.75	-4.52	-5.03
Seasonal AR(2)	8.58	4.22	2.51	1.15’	-0.14’	-2.46	-3.77	-4.52	-5.03

‘: Non-rejection values of the null hypothesis at the 95% significant level.

(ii) A multivariate model

Turning now to the economic modelling of unemployment as a reduced form in terms of weakly exogenous variables, we focus on a set of contending explanatory variables, which include labour supply variables (like the union density), real interest rates, the terms of trade and real oil prices. There are two reasons for selecting this specification. First, in a series of econometric tests Henry and Nixon (1999), find that a model based on this restricted set of driving variables is preferred to a commonly used model based on a much wider set. Second, the emphasis on real oil prices and the real interest rate enables us to compare our findings with those proposed by Phelps (1994), Carruth et al (op.cit) and Blanchard (1999), each of which has recently placed emphasis on at least one of these variables as a primary determinant of unemployment. The sample for our model runs from 1966q1 to 1997q4. We initially employ the model given by equations (5) and (6), testing (7) for d_o = 0; 0.10; 0.20; …(0.10) … 0.90 and 1.00, with x_t being the set of weakly exogenous variables just defined. Initially up to five lags is allowed in each variable. So,

$$u_t \;=\; \alpha \;+\; \sum_{k=0}^{5} \beta_k \, r_{t-k} \;+\; \sum_{k=0}^{5} \gamma_k \, p_{t-k} \;+\; \sum_{k=0}^{5} \delta_k \, t_{t-k} \;+\; \sum_{k=0}^{5} \lambda_k \, s_{t-k} \;+\; v_t \qquad \textbf{(14)}$$

$$(1 - L)^d \; v_t \;=\; w_t \,, \qquad t \;=\; 1,\, 2, ... \qquad \textbf{(15)}$$

where r_t is the real interest rate; p_t is real oil prices; t_t the terms of trade and s_t corresponds to union density.

Table 3 displays the results of the one-sided statistic $\hat{s}$ given by (8) above in the model (14) and (15) when w_t is either assumed to be white noise or an autoregressive process of orders 1 or 2. Higher order autoregressions were also performed obtaining similar results to those in the AR(2) case, and are not reported here. When modelling w_t as white noise the null hypothesis given by (7) always results in a rejection across the different values of d_o. But allowing w_t to follow an autoregressive process, the results differ. Thus, if w_t is AR(1), the null is practically never rejected, and modelling w_t as an AR(2) process, the only non-rejection cases occur when d = 0.50 and 0.60. Interestingly, in this table, we see that the only significant regressors seem to be the lagged real oil price (p_{t-5}) and union density (s_t) when d = 0, 0.10, 0.20 and 0.30; and these together with the real interest rate (r_t) when d = 0.40 and 0.50; lagged real oil prices and the current real interest rate when d = 0.60 and 0.70; and finally lagged real oil prices, current real interest rate and lagged terms of trade (p_{t-5}, r_t and t_{t-3})

when d = 0.80 and 0.90. A striking feature in this table is the lack of monotonic decrease in the value of $\hat{s}$ with respect to d_o. Such monotonicity is a characteristic of any reasonable statistic, given correct specification and adequate sample size. For example, if d = 0.50 is rejected against d > 0.50, an even more significant result in this direction could be expected when d = 0.40 or d = 0.30 is tested. We interpret this lack of monotonicity in the case of an AR(2) for w_t, (and also in some cases for AR(1) w_t), as reflecting possible misspecification of the model due to the inclusion of non-significant variables. So, in the appendix we report the same statistic as in Table 3 but this time only including those regressors that were significant in that table across all the different values of d_o. Monotonicity is not expected in this case, since the elements of x_t differ between the equations. (See Table 1 Appendix).

Table 3

Testing (7) in the model given by (14) and (15)				
d_o	Significant regressors	White noise w_t	AR(1) w_t	AR(2) w_t
0.00	p_{t-5}; s_t	12.18	0.83'	-8.25
0.10	p_{t-5}; s_t	11.23	0.65'	-6.34
0.20	p_{t-5}; s_t	10.93	0.56'	-5.30
0.30	p_{t-5}; s_t	10.61	0.57'	-4.05
0.40	p_{t-5}; s_t; r_t	10.45	0.61'	-2.63
0.50	p_{t-5}; s_t; r_t	10.29	0.83'	-0.92'
0.60	p_{t-5}; r_t	10.20	1.12'	0.98'
0.70	p_{t-5}; r_t	10.12	1.29'	2.65
0.80	t_{t-3} ; p_{t-5}; r_t	9.92	1.08'	3.25
0.90	t_{t-3} ; p_{t-5}; r_t	9.51	0.46'	2.48
1.00	r_t;	7.50	-2.43	-6.88

': Non-rejection values of the null hypothesis at the 95% significance level.

The results in the appendix table are very similar to those given in Table 3. The null d = 0 is not rejected if x_t consists of real oil prices and unionisation (p_{t-5} and s_t), and w_t is assumed to be an AR(1), but this hypothesis is strongly rejected in case of white noise or higher order autoregressive disturbances.

Table 4 summarises the selected models according to results shown in the appendix. That is, we write the estimated models based on (14) and (15), in which the null hypothesis (7) was not rejected and all the coefficients were significantly different from zero. We see that Models 1, 2 and 3 are consistent with stationary unemployment. In such situations, the real oil prices lagged five periods, along with the union density appear as significant regressors, and the coefficients are rather similar in the three models. For all the other specifications, d is greater than 0.60, indicating that unemployment may be a nonstationary series. We also see that lagged oil prices appears as a significant regressor in practically all the models, (in fact, in all except when d = 1.00). Surprisingly, we also observe across these models that the higher the order of integration d is, the lower the coefficient on oil prices. Thus, for example, setting d = 0 (in Model 1), the coefficient for p_{t-5} is 0.96; setting d = 0.30 (in Model 3), it becomes 0.78; and setting d = 0.9 (in Model 6) the coefficient reduces to 0.07. This may indicate that there may exist some kind of competition between the lagged real oil prices and the order of integration in describing the UK unemployment behaviour.

Table 4

Selected models for unemployment according to Table 4		Diagnostic*
1.	$u_t = \underset{(0.28)}{2.49} + \underset{(0.04)}{0.96}\, p_{t-5} - \underset{(0.005)}{0.07}\, s_t + v_t$ $v_t = \underset{(0.04)}{0.79}\, v_{t-1} + \varepsilon_t .$	A; C
2.	$u_t = \underset{(0.32)}{2.07} + \underset{(0.04)}{0.87}\, p_{t-5} - \underset{(0.006)}{0.07}\, s_t + v_t$ $(1 - L)^{0.10} v_t = w_t ;\quad w_t = \underset{(0.04)}{0.76}\, w_{t-1} + \varepsilon_t$	A; B; C
3.	$u_t = \underset{(0.37)}{1.53} + \underset{(0.05)}{0.78}\, p_{t-5} - \underset{(0.007)}{0.06}\, s_t + v_t$ $(1 - L)^{0.20} v_t = w_t ;\quad w_t = \underset{(0.06)}{0.78}\, w_{t-1} + \varepsilon_t$	A; B; C
4.a	$u_t = \underset{(0.21)}{-2.62} + \underset{(0.04)}{0.16}\, p_{t-5} - \underset{(0.008)}{0.019}\, r_t + v_t$ $(1 - L)^{0.70} v_t = w_t ;\quad w_t = \underset{(0.05)}{0.75}\, w_{t-1} + \varepsilon_t$	A; B; C; D
4.b	$(1 - L)^{0.70} v_t = w_t ;\quad w_t = \underset{(0.08)}{0.78}\, w_{t-1} + \underset{(0.08)}{0.14}\, w_{t-2} \quad \varepsilon_t$	A; B; D
5.a	$u_t = \underset{(0.27)}{-2.52} + \underset{(0.03)}{0.11}\, p_{t-5} - \underset{(0.007)}{0.019}\, r_t + \underset{(0.36)}{0.83}\, tt_{t-3} + v_t$ $(1 - L)^{0.80} v_t = w_t ;\quad w_t = \underset{(0.06)}{0.73}\, w_{t-1} + \varepsilon_t$	A; B; D
5.b	$(1 - L)^{0.80} v_t = w_t ;\quad w_t = \underset{(0.05)}{0.76}\, w_{t-1} + \underset{(0.02)}{0.03}\, w_{t-2} + \varepsilon_t$	A; B; D
6.a	$u_t = \underset{(0.49)}{-1.66} + \underset{(0.03)}{0.07}\, p_{t-5} - \underset{(0.006)}{0.017}\, r_t + \underset{(0.32)}{0.73}\, tt_{t-3} + v_t$ $(1 - L)^{0.90} v_t = w_t ;\quad w_t = \underset{(0.06)}{0.72}\, w_{t-1} + \varepsilon_t$	A; B; D
6.b	$(1 - L)^{0.90} v_t = w_t ;\quad w_t = \underset{(0.05)}{0.74}\, w_{t-1} + \underset{(0.14)}{0.44}\, w_{t-2} + \varepsilon_t$	A; B; D
7.a	$u_t = \underset{(0.06)}{-1.33} + v_t$	
7.b	$(1 - L)\, v_t = w_t ;\quad w_t = \underset{(0.05)}{0.82}\, w_{t-1} + \varepsilon_t$	A; B; D

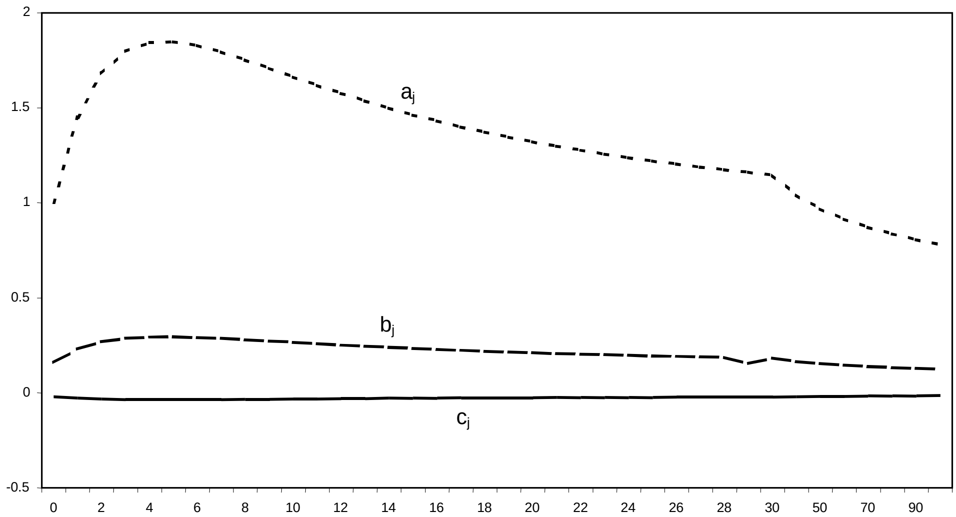
*: Non-rejections at the 99% significance level of A): No Serial Correlation; B): Functional form; C): Normality; and D): Homoscedasticity. Standard errors in parenthesis.

A more difficult task is to determine which is the correct model specification across the different models presented in that table. We display in the last column of Table 4 several diagnostic tests carried out on the residuals. We observe that if we assume that unemployment is I(0), the model fails in relation to tests of functional form and homocedasticity. However, allowing d to have a low degree of long memory, (with d = 0.1 or 0.2), the models fail then only in relation to the homocedasticity property. On the other hand, assuming nonstationarity for unemployment, (in Models 4 –7), the real interest rate becomes a significant regressor along with the terms of trade in some cases. We observe across these models that the only one which passes all the diagnostic tests on the residuals seems to be Model 4a, where unemployment is modelled as

$$u_t \;=\; \alpha \;+\; \gamma_5\, p_{t-5} \;+\; \beta_1\, r_t \;+\; v_t$$
$$(1 - L)^{0.70}\, v_t \;=\; w_t\,; \qquad w_t \;=\; \phi\, w_{t-1} \;+\; \varepsilon_t\,, \qquad \textbf{(16)}$$

giving the estimates: α = -2.62; γ_5 = 0.16; β_1 = -0.019; and ϕ = 0.75. Thus, the impact of prices and interest rates is quite slow, with the adjustment process modelled through both the fractional parameter and the autoregressive coefficient.

Chart 1: Impulse response function and impacts of real oil prices and real interest rates on unemployment



a_j corresponds to the impulse response function based on the polinomial: (1 – 0.75L) (1 – L)$^{0.70}$;

b_j and c_j represents respectively the impacts of oil prices and real interest rates on the unemployment

To evaluate the responses of unemployment to a shock, we need to derive the impulse response functions. We do this next. Let (1 – 0.75L) $(1 - L)^{0.70}$ = a(L), and calling k(L) = -2.62 a(L); b(L) = 0.16 a(L); and c(L) = -0.019 a(L), the model in (16) becomes

$$a(L)u_t \;=\; k(L)1 \;+\; b(L)\,p_{t-5} \;+\; c(L)r_t \;+\; \varepsilon_t\,,$$

and using a power expansion of a(L), b(L) and c(L) in terms of its lags, with $u_j = p_j = r_j = 0$ for $j \le 0$, we obtain

$$u_t \;=\; k(L)1 \;+\; \sum_{j=1}^{t-1} a_j\, u_{t-j} \;+\; \sum_{j=0}^{t-6} b_j\, p_{t-5-j} \;+\; \sum_{j=0}^{t-1} c_j\, r_{t-j} \;+\; \varepsilon_t\,, \qquad \textbf{(17)}$$

where a_j are the coefficients of the impulse response function, and b_j and c_j represents respectively the impacts of the real oil prices and the real interest rates on the unemployment. Chart 1 summarizes these values for j = 1,2,…,30, 40, 50, … 100. We observe through the a_j's that the effect of a shock on unemployment tends to die away in the long run though it takes a very long period to disappear completely. In fact, we see that even 30 periods after the initial shock, its complete effect still remains on the series and is only after around 50 periods that it becomes smaller than 1. The impact of real oil prices is around 16% five periods later; it increases up to around 30% in the following five periods, and then starts decreasing slowly. Similarly, the current impact of the interest rate is around –1.9% and then increases up to –3.5% before falling.

We can conclude by saying that the lagged values of the real oil prices in all models and the real interest rate in some of the models play an important role in explaining the movements in the UK unemployment. They have an immediate effect but this is coupled with an adjustment process which takes a very long time to disappear due to the persistence observed through the fractionally differencing parameter d, (which is 0.7), and the autoregressive parameter (which is also high, 0.75). That suggests that unemployment is a nonstationary series with shocks taking a very long time to decay, and there is evidence that the main shocks which have affected it in the last 30 years are fluctuations in real oil prices and real interest rates.

5. Conclusions

In this article we have examined the underlying dynamics affecting the UK unemployment. However, instead of using the classical approaches based on I(0) stationarity or I(1) cointegrating relationships, we have gone throughout a new different approach based on fractionally integrated models. This is an important development since it allows for the possibility that unemployment is highly persistent. Hence, it allows us to test whether unemployment behaviour is due to extreme persistence to a limited set of shocks, rather than changes in its equilibrium.

Looking at the univariate behaviour of unemployment, we find strong evidence in favour of a unit root. Estimating d within a fractionally integrated ARMA (ARFIMA) model, the null hypothesis of a unit root was almost never rejected and the ARFIMA(0, 1.18, 3) specification was chosen according to the likelihood criteria. Testing the order of integration of unemployment with the tests of Robinson (1994), the unit root hypothesis was also not rejected, though other alternatives, with d slightly smaller or higher than one were also plausible in some cases.

Including weakly exogenous regressors produced a different picture. The important regressors appear to be the real oil prices lagged five periods and the current real interest rate, and the order of integration of unemployment was found in these cases to be smaller than 1 but higher than 0.50. That means that unemployment may be modelled as a nonstationary series with a strong component of mean-reverting behaviour, and this strongly suggests that shocks affecting it take a very long time to disappear.

The next step in this work is modelling prices and real interest rates and unemployment in terms of the so-called fractional cointegration structure. This area, which is relatively new in econometrics, may lead to yet improved ways of explaining the adjustment process of unemployment due to variation in oil prices and real interest rates. Work in this direction is now in progress.

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APPENDIX

Table: Monotonicity Test

Testing (7) in model given by (14) and (15) including only significant regressors for each model				
d_o	Regressors (*)	White noise w_t	AR(1) w_t	AR(2) w_t
0.00	p_{t-5}; s_t	13.08	0.27’	-5.76
0.10	p_{t-5}; s_t	12.44	-0.54’	-4.31
0.20	p_{t-5}; s_t;	12.19	-1.09’	-3.74
0.30	p_{t-5}; s_t	12.30	-1.99	-3.59
0.40	p_{t-5}; (t_t); r_t	12.62	-2.01	-2.91
0.40	p_{t-5}; r_t	12.59	-1.98	-3.55
0.50	p_{t-5}; (t_t); r_t	13.00	-2.14	-2.90
0.50	p_{t-5}; r_t	13.09	-1.97	-2.98
0.60	p_{t-5}; r_t	13.57	-2.02	-2.13
0.70	p_{t-5}; r_t	12.82	-1.07’	-0.08’
0.80	t_{t-3}; p_{t-5}; r_t	11.63	-0.60’	0.29’
0.90	t_{t-3}; p_{t-5}; r_t	10.76	-0.70’	0.73’
1.00	(r_t)	9.58	-2.10	-4.78
	---	9.49	-1.90’	-4.05

* : In parenthesis, the non-significant regressors.
‘ : Non-rejection values of the null hypothesis at the 95% significance level.