FRACTIONAL COINTEGRATION AND TESTS OF PRESENT VALUE MODELS*

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This paper tests the validity of Present Value (PV) models of stock prices by employing a two-step strategy for testing the null hypothesis of no cointegration against alternatives which are fractionally cointegrated. Monte Carlo simulations are conducted to evaluate the power and size properties of this test, which is shown to outperform existing ones, and to compute appropriate critical values for finite samples. It is found that stock prices and dividends are both I(1) nonstationary series, but they are fractionally cointegrated. This implies that, although there exists a long-run relationship which is consistent with PV models, the equilibrium errors exhibit slow mean reversion. As the error correction term possesses long memory, deviations from equilibrium are highly persistent.

Keywords: *Efficient Markets Hypothesis (EMH), Present Value (PV) models, Fractional integration, Fractional cointegration*

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1. Introduction

One of the central propositions of modern finance theory is the efficient markets hypothesis (EMH), which in its simplest formulation states that the price of an asset at time t should fully reflect all the available information at time t.[1] This has often been tested by using the present value (PV) model of stock prices, since, if stock market returns are not forecastable, as implied by the EMH, stock prices should equal the present value of expected future dividends. As pointed out by Campbell and Shiller (1987a) in their seminal paper, this implies that stock prices and dividends should be cointegrated, and recent studies of PV models have mainly used cointegration techniques. However, the discrete options I(1) and I(0) offered by classical cointegration analysis are rather restrictive, which might explain why the available empirical evidence is inconclusive. Adjustment to equilibrium might in fact take a longer time than suggested by standard cointegration tests. In other words, stock prices and dividends might be tied together through a fractionally integrated I(d)-type process such that the equilibrium errors exhibit slow mean reversion.

The contribution of the present paper is two-fold. First, we propose a two-step testing strategy for testing the null of no cointegration against alternatives which are fractionally cointegrated. We conduct Monte Carlo simulations in order to evaluate the size and power properties of this test, which is shown to outperform existing ones, and to compute appropriate critical values for finite samples. Second, we apply the new methodology to an updated version of the Campbell and Shiller's (1987a) dataset to test the adequacy of PV models of stock prices. We find that stock prices and dividends are both I(1) nonstationary series, but they are fractionally cointegrated. This implies that, although there exists a long-run relationship which is consistent with PV models, the error

[1] See Fama (1970) for a definition of weak, semi-strong and strong efficiency, and Fama (1991) for alternative definitions in terms of return predictability.

correction term possesses long memory, and hence deviations from equilibrium are highly persistent.

The layout of the paper is the following. Section 2 briefly reviews the existing literature on PV models. Section 3 initially describes the concepts of fractional integration and cointegration. A procedure for testing the null hypothesis of no cointegration against fractionally cointegrated alternatives is then proposed in this section, and its properties are investigated by conducting Monte Carlo experiments. This methodology is applied in Section 4 to test PV models, and Section 5 offers some concluding remarks.

2. Review of the literature

The literature on PV models has rapidly grown in the last two decades. Early studies, such as Shiller (1981) and LeRoy and Porter (1981), assumed that dividends were trend-stationary. They carried out variance bounds tests, finding that prices were too volatile to be consistent with the present value of rationally expected future dividends discounted by a constant real interest rate. Subsequent studies pointed out that the assumption of trend-stationarity for stock prices and dividends might be invalid. In particular, Marsh and Merton (1986) showed that the results were reversed if the variables of interest were in fact integrated. Kleidon (1986) then developed variance bounds tests which were valid under integration, and reported that the evidence was not inconsistent with the EMH. West (1988a) also tested for excess volatility by developing a method valid under either integration or trend-stationarity, though its parameter estimates were not consistent if dividends were trend-stationary as opposed to difference-stationary as he assumed.[2]

The above tests crucially depend on being able to establish the order of integration of the variables. But, as it was shown in the econometric literature, unit root tests have

[2] See also Flavin (1983), who argued that volatility tests are vitiated by small sample bias.

very low power in finite samples, and it is practically impossible to distinguish between a unit and a near-unit root (see West, 1988b, Campbell and Perron, 1991 and McCallum, 1993). It was generally felt that alternative tests for excess volatility in stock price might be more useful for investigating market efficiency (see Cochrane, 1991).

In a seminal paper, Campbell and Shiller (1987a) tested the PV model of stock prices adopting Engle and Granger's (1987) cointegration procedure, an approach which is valid provided stock prices and dividends are stationary in first differences rather than in levels.[3] They used the Standard and Poor's (S&P's) dividends and value-weighted and equally-weighted New York Stock Exchange (NYSE) 1926-1986 datasets. In the case of the S&P series they were unable to reject the null for stock prices, but rejected it for dividends, whilst they could not reject it in both cases when using the NYSE data. As for cointegration, their results were also mixed, some test statistics rejecting the null hypothesis of no-cointegration, other failing to reject it. In a companion paper (Campbell and Shiller, 1987b), they also showed that excess volatility directly implied forecastability of infinite-period returns, and that a long moving average of real earnings helps to forecast future real dividends. It should be noted, though, that failure to find cointegration could indicate that there are speculative bubbles, or a non-stationary time-varying discount rate, or some other form of misspecification. Diba and Grossman (1988) tested for bubbles by constructing the component of stock prices which is determined by the dividend and unobservable variables, and argued that if the latter two are stationary in first differences, stock prices should also be so in the absence of bubbles. In other words, bubbles can be ruled out if both dividends and prices are integrated series and they are

[3] The links between cointegration and market efficiency are discussed in Caporale and Pittis (1998), where it is pointed out that if one adopts alternative definitions of efficiency, for instance in terms of no risk-free returns above opportunity costs (as in Dwyer and Wallace, 1992), cointegration can be found in efficient markets. They then suggest that examining the implications of cointegration for predictability of asset prices might be more useful.

cointegrated. However, Evans (1991) showed that in fact cointegration is not inconsistent with the existence of an important class of bubbles.

Subsequent papers either used Bayesian methods or exploited further advances in cointegration analysis. Bayesians argue that classical integration tests give strong prior probability to explosive roots. DeJong and Whiteman (1991) therefore developed a Bayesian approach aimed at analysing the type of prior needed to support difference-stationary (DS) representations in preference to trend-stationary (TS) ones. Their procedure is based on the likelihood principle, and the inferences are *conditional* on the given data. They find that TS specifications are much more likely than DS ones. Specifically, integration is inferred when the deterministic trend is restricted to be zero, a restriction which makes unit root tests inconsistent against TS alternatives, and which is not justified even from a Bayesian viewpoint. By contrast, trend stationarity is not rejected either by classical or Bayesian tests when the trend is included. Therefore, according to DeJong and Whiteman (1991), stock prices and dividends are in fact stationary series, and the evidence against the EMH presented by Shiller (1981) and others should be considered valid.

DeJong (1992) also developed a Bayesian approach to cointegration analysis which is based on the same idea, namely examining the relative support the data give to integrated, cointegrated and TS alternatives. Having investigated the integration properties of the series as in DeJong and Whiteman (1991), he then tested for cointegration by looking at the dominant roots of a bivariate VAR representation and again exploiting the likelihood principle. He concluded that integration and cointegration are only inferred when TS alternatives are given zero prior probability; if this restriction is not imposed, the evidence points to trend stationarity, indicating that the series being considered (such as stock prices and dividends) share common deterministic trends. A Bayesian methodology was also used by Koop (1991), who once again concluded that stock prices and dividends do not contain unit roots, and hence questioned the usefulness of classical cointegration analysis for testing the EMH.

As for papers still relying on classical cointegration approaches, but using procedures other than Engle and Granger's (1987), a study by Han (1996) pointed out that if log dividends and log prices are DS and cointegrated, the cointegrating vector (1, -1) eliminates both stochastic and deterministic trends. One can test the latter (deterministic cointegration) restriction separately applying the Canonical Cointegrating Regression (CCR) (see Park, 1992), a procedure which involves testing the null of cointegration and has desirable small sample properties. Han (1996) used Johansen's (1991) maximum likelihood (ML) method, and found that the deterministic cointegration restriction can be rejected on the basis of the CCR tests, and that stochastic cointegration is also rejected. He interpreted these findings as evidence that the PV model does not hold, and that bubbles might exist in the deterministic components of stock prices.

Yuhn (1996) argued that in fact the PV model requires non-linear cointegration between stock prices and market fundamentals when it is linked to the flow of information. He claimed therefore that Campbell and Shiller's (1987a) tests for linear cointegration are not appropriate for investigating the EMH. He suggested that the evidence supports non-linear cointegration, which means that deviations of US stock prices from their long-run equilibrium are temporary, and the market is efficient in the sense that stock prices are not affected by public information about market fundamentals.[4]

In studies relying on standard cointegration analysis the equilibrium errors are restricted to be an I(0) process, which is not persistent. However, it might be the case that the equilibrium errors respond more slowly to shocks, which results in highly persistent deviations from equilibrium. Therefore, we introduce below a testing procedure which

[4] Other studies using cointegration techniques include Cerchi and Havenner (1988), Bossaerts (1988), Taylor and Tonks (1989), Rappoport and White (1991) and Kasa (1992).

allows for the possibility of a long-memory cointegrating relationship, and which enables us to gain a better understanding of the relationship between stock prices and dividends.

3. Testing for fractional cointegration

3.1 Fractional integration and cointegration

For the purpose of the present paper, we define an I(0) process u_t, t = 0, ±1, ….., as a covariance stationary process with spectral density which is positive and finite at zero frequency. In this context, an I(d) process, x_t, t = 0, ±1, …., is defined by

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots , \tag{1}$$

$$x_t = 0 \qquad t \leq 0 \tag{2}$$

where L is the lag operator. The macroeconomic literature focuses on the cases d = 0 and d = 1 (see, e.g., Nelson and Plosser, 1982), whereas we define $(1 - L)^d$ for all real d by

$$1 + \sum_{j=1}^{\infty} \frac{\Gamma(d+1)(-L)^j}{\Gamma(d-j+1)\,\Gamma(j+1)} = 1 - d\,L + \frac{d\,(d-1)}{2} L^2 - \frac{d\,(d-1)\,(d-2)}{3!} L^3 + \dots .$$

The process u_t in (1) could be a stationary and invertible ARMA sequence, with an exponentially decaying autocovariance function, or it could decay much slower than exponentially. The latter property can be said to characterise a "weakly autocorrelated" process. When d = 0, $x_t = u_t$, so a "weakly autocorrelated" x_t is allowed for. When d = 1, x_t has a unit root, while for a general integer d, x_t has d unit roots. For 0 < d < 0.5, x_t is still stationary, but its lag-j autocovariance γ_j decreases very slowly, like the power law j^{2d-1} as $j \rightarrow \infty$, and so the γ_j are non-summable. The distinction between I(d) processes with different values of d is also important from an economic point of view: if a variable is an I(d) process with $d \in [0.5, 1)$, it will be covariance nonstationary but mean-reverting since an innovation will have no permanent effect on its value. This is in contrast to an

I(1) process which will be both covariance nonstationary and not mean-reverting, in which case the effect of an innovation will persist forever.

Robinson (1994) proposes LM tests for testing unit roots and other forms of nonstationary hypotheses, embedded in fractional alternatives. He tests the null hypothesis:

$$H_o: \ \theta \ = \ 0 \qquad (3)$$

in the model

$$y_t = \beta' z_t + x_t \qquad t = 1, 2, \ldots \qquad (4)$$

$$(1 - L)^{d + \theta} x_t \ = \ u_t, \qquad t = 1, 2, \ldots , \qquad (5)$$

where y_t is a raw time series; z_t is a (kx1) vector of deterministic regressors that may include, for example, an intercept ($z_t \equiv 1$) and/or a linear time trend ($z_t = (1, t)'$); u_t is an I(0) process, and d is a given value that may be 1 but also any other real number. Specifically, the score test statistic proposed by Robinson (1994) takes the form:

$$\hat{r} \ = \ \left(\frac{T^{1/2}}{\hat{\sigma}^2} \right) \hat{A}^{-1/2} \, \hat{a}, \qquad (6)$$

where T is the sample size and

$$\hat{a} \ = \ \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) \, g(\lambda_j ; \hat{\tau})^{-1} \, I(\lambda_j)$$

$$\hat{A} \ = \ \frac{2}{T} \left(\sum_{j=1}^{T-1} \psi(\lambda_j)^2 \ - \sum_{j=1}^{T-1} \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \left(\sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \psi(\lambda_j) \right)$$

$$\hat{\sigma}^2 \ = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j ; \hat{\tau})^{-1} \, I(\lambda_j); \quad \psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|; \ \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j ; \hat{\tau}),$$

where $I(\lambda_j)$ is the periodogram of $\hat{u}_t \ = \ (1 - L)^d \ y_t - \hat{\beta}' w_t$, evaluated at $\lambda_j = 2\pi j/T$, with

$\hat{\beta} = \left(\sum_{t=1}^{T} w_t \, w_t' \right)^{-1} \sum_{t=1}^{T} w_t \, (1 - L)^d \ y_t ; \quad w_t \ = (1 \ - \ L)^d \ z_t$, and g is a known function coming from the spectral density function of $\hat{u}_t$: $f(\lambda_j;\tau) = \dfrac{\sigma^2}{2\pi} g(\lambda_j;\tau)$, with $\hat{\tau}$ obtained by minimising $\sigma^2(\tau)$. Robinson (1994) showed that under certain regularity conditions:

$$\hat{r} \;\rightarrow_d\; N(0,1) \qquad as\ T \rightarrow \infty. \tag{7}$$

Thus, a one-sided 100α%-level test of (3) against the alternative H_1: $\theta > 0$ is given by the rule: 'Reject H_o if $\hat{r} > z_\alpha$', where the probability that a standard normal variate exceeds z_α is α, and, conversely, an approximate one-sided 100α%-level test of (3) against the alternative H_1: $\theta < 0$ is given by the rule: 'Reject H_o if $\hat{r} < -z_\alpha$'. Furthermore, he shows that the above tests are efficient in the Pitman sense, i.e. that against local alternatives of the form: H_a: $\theta = \delta\, T^{-1/2}$, for $\delta \neq 0$, the limit distribution is normal with variance 1 and mean which cannot (when u_t is Gaussian) be exceeded in absolute value by that of any rival regular statistic.[5]

Having defined fractional integration and described a way of testing I(d) statistical models, we next introduce the concept of fractional cointegration. The components of a (nx1) vector X_t are said to be fractionally cointegrated of order d,b, ($X_t \sim CI(d,b)$) if a): all components of X_t are integrated of order d ($X_{it} \sim I(d)$), and b): there exists a vector r ($r \neq 0$) such that $N_t = r'X_t$ is integrated of order d-b ($N_t \sim I(d-b)$) with $b > 0$.[6] The vector r is called the cointegrating vector and $r'X_t$ will represent an equilibrium constraint operating on the long-run component of X_t. If X_t has more than two components then there may be more than one cointegrating vector r, though in what follows we will assume that X_t does have only two components, so that $X_t = (X_{1t}, X_{2t})'$, where X_{1t} and X_{2t} correspond to the variables to be analysed later.

[5] An empirical application of this testing procedure using historical U.S. annual data can be found in Gil-Alana and Robinson (1997).

[6] A more general definition of fractional cointegration, allowing different integration orders for each series, can be found in Marinucci and Robinson (1998). They define $X_t \sim CI(d_1, d_2, d_n, b)$ if $X_{it} \sim I(d_i)$ for all i, and there exists a vector $r \neq 0$ such that $N_t = r'X_t \sim I(d)$, where $d = \max_{1\leq i\leq n} (di - b)$. Note that this property is possible and meaningful if and only if $b > (\max_{1\leq i\leq n} d_i - \min_{1\leq i\leq n} d_i)$.

We propose here a two-step testing procedure based on a methodology similar to that put forward by Engle and Granger (1987) (for more details, see Gil-Alana, 1997). We initially test the order of integration of the original series using Robinson's (1994) univariate tests, and, if all of them have the same order of integration (say d), we then test the degree of integration of the estimated residuals of the cointegrating structure. Since all linear combinations of X_{1t} and X_{2t} except the one defined by the cointegrating regression will be integrated of order d, the least squares estimate from the regression of X_{1t} on X_{2t} (or viceversa), under cointegration, will produce a good estimator of that relationship. In standard cointegration analysis, (where cointegration of order 1,1 is considered), Stock (1987) showed that the least squares estimate of the cointegrating parameter was consistent and converged in probability at the rate $T^{1-\delta}$ for any $\delta > 0$, rather than the usual $T^{1/2}$. Cheung and Lai (1993) and others extended the analysis to the case of fractional cointegration, and showed that the least squares estimate was also consistent, though with possible different convergence rates, depending on the cointegration order. In particular, they showed that under the general hypothesis of cointegration of order d,b with $d > 0.5$ and $b > 0$, the least squares estimate was consistent and converged at the rate T^{b-d}, Stock's (1987) result being a special case with $b = 1$.

Given the consistency of the least squares estimates, it is legitimate to use Robinson's (1994) univariate tests for testing the integration order of the equilibrium errors

$$e_t \;=\; X_{1t} \;-\; \hat{\alpha}\, X_{2t}, \qquad t \;=\; 1, 2, \ldots \qquad (8)$$

where $\hat{\alpha}$ is the least squares estimate of the cointegrating parameter, and the test statistic will have the same limit normal distribution. Thus, one can consider the model:

$$(1 - L)^{d+\theta}\, e_t \;=\; v_t \qquad t \;=\; 1, 2, \ldots, \qquad (9)$$

with I(0) v_t and test the null hypothesis:

$$H_o: \theta = 0 \quad (10)$$

against the alternative

$$H_o: \theta < 0 \quad (11)$$

and the test statistic will have an asymptotic null N(0,1) distribution. Rejections of (10) against (11) will imply that X_{1t} and X_{2t} are fractionally cointegrated, given the fact that the equilibrium errors e_t exhibit a smaller degree of integration than the original series. However, as the equilibrium errors are not actually observed but obtained from minimising the residual variance of the cointegrating regression, in finite samples the residual series might be biased towards stationarity, and thus we would expect the null hypothesis to be rejected more often than suggested by the nominal size of the Robinson's (1994) test. A similar problem arises in Engle and Granger (1987) and Cheung and Lai (1993) when testing for cointegration. In order to deal with this problem, we obtain the empirical size of Robinson's (1994) tests for cointegration in finite samples by using a simulation approach.

3.2 Monte Carlo analysis

In Table 1 we report the critical values of Robinson's (1994) tests for cointegration corresponding to different sample sizes (T = 50, 100, 200 and 300). We use a Monte Carlo approach based on 50,000 replications, assuming that the true model consists of two I(1) processes with Gaussian independent white noise disturbances that are not cointegrated.[7] For simplicity, we also assume that v_t in (9) is white noise, though one can extend the analysis to cover the case of weak parametric autocorrelation in v_t. We observe that the distribution has a negative mean and the critical values are smaller than those given by the normal distribution, which is consistent with the earlier discussion pointing

[7] The experiment was also extended to allow the true model to consist of two I(d) non-cointegrated processes with d = 0.6, (0.1), 1.5, the critical values being displayed in Gil-Alana (1997).

out that, when testing (10) against (11) in (8) and (9), the use of the standard critical values will result in the cointegration tests rejecting the null hypothesis of no cointegration too often. We also see that the empirical distribution is positively skewed with kurtosis greater than 3, though as the sample size increases the three statistics (mean, skewness and kurtosis) approximate to the values corresponding to the normal distribution.

(Table 1 about here)

Table 2 examines the power properties of Robinson's (1994) tests for cointegration relative to the ADF and Geweke and Porter-Hudak (GPH, 1983) tests. The ADF unit root test recommended by Engle and Granger (1987) is given by the usual t-statistic for b_0 in

$$(1 - L)\,e_t \;=\; b_0\, e_{t-1} \;+\; b_1\,(1 - L)\,e_{t-1} \;+\; \ldots \;+\; b_p\,(1 - L)\,e_{t-p} \;+\; \varepsilon_t$$

where e_t are the equilibrium errors and the lag parameter p can be selected using some model-selection procedures such as the Akaike and Schwarz information criteria. The GPH test for cointegration proposed by Cheung and Lai (1993) is based on the estimation of the fractional differencing parameter d in the linear regression

$$\ln I(\lambda_j) \;=\; \beta_0 \;+\; \beta_1\, \ln\left(4 \sin^2 \frac{\lambda_j}{2}\right) \;+\; \varepsilon_j$$

where $\lambda_j = 2\pi j/T$ and $I(\lambda_j)$ is the periodogram of e_t at the ordinate j. Given that the least squares estimate of β_1 provides a consistent estimate of 1 - d (see, e.g., Robinson, 1995a), hypothesis testing on the value of d can be carried out using the t-statistic of the regression coefficient.

(Table 2 about here)

We analyse the case of a bivariate I(1) system, assumed to be non-cointegrated under the null hypothesis, and compare the power function of the three tests for cointegration (Robinson's, ADF and GPH) against different fractional alternatives. Results for the ADF and GPH tests have been taken from Cheung and Lai (1993). We consider $X_t = (X_{1t}, X_{2t})'$, where

$$X_{1t} + X_{2t} = u_{1t} \tag{12}$$

and

$$X_{1t} + 2 X_{2t} = u_{2t} \tag{13}$$

where

$$(1 - L)u_{1t} = \varepsilon_{1t} \tag{14}$$

and u_{2t} is generated as a fractional noise process, i.e.

$$(1 - L)^d u_{2t} = \varepsilon_{2t} \tag{15}$$

where the innovations ε_{1t} and ε_{2t} are generated as independent standard normal variates. Thus, if d = 1 in (15), the two series are I(1) and non-cointegrated, whereas if d < 1 in (15), X_{1t} and X_{2t} are fractionally cointegrated, and (13) is their cointegrating relationship. As in Engle and Granger (1987) and Cheung and Lai (1993), we use samples of size T = 76, and sample series for X_{1t} and X_{2t} were generated setting the initial values of u_1 and u_2 equal to zero, creating 126 observations, the first 50 of which were discarded to reduce the effect of the initial conditions. We report the rejection frequencies of Robinson's (1994) test statistics with d in (15) equal to 0.05, (0.10), 0.95, for four different possibilities, assuming that the differenced series are white noise and AR processes of order 1, 2, and 3, for the 5% and 10% significance levels, based on 10,000 replications.

We can see in Table 2 that Robinson's (1994) tests perform better than the ADF and GPH tests, regardless of whether the disturbances are white noise or follow AR processes. The highest rejection frequencies are obtained with white noise disturbances if the integration order ranges between 0.05 and 0.75, but when this parameter approximates 1 better results are obtained for weakly parametrically autocorrelated disturbances. The relatively pronounced difference in power between the tests of Robinson (1994) and the ADF and GPH tests for cointegration is not surprising, since the ADF test is based on an I(0) versus I(1) dichotomy and the GPH test requires the estimation of the fractionally differencing parameter, whereas Robinson's (1994) tests do allow fractional differencing and do not require the estimation of the fractional differencing parameter.[8]

4. Testing present value models of stock prices

The validity of PV models of stock prices is tested in this section by means of fractional cointegration techniques. In particular, we use the methodology described in Section 3, testing initially for the order of integration of the individual series, and then testing for the possibility of their being fractionally cointegrated.

The dataset we use is the standard one in the equity premium literature, and has been updated compared to Campbell and Shiller (1987a), covering the sample period 1871-1995. Both series are annual. The price series is the Standard & Poor's Monthly Composite Stock Price Index (S&P 500) for January divided by the January Produced Price Index. The series has been extended back to 1871 using data from Cowles (1939). The dividend series is the total dividends for the calendar year for a portfolio including the stocks in the index divided by the January Produced Price Index. Data from Cowles (1939) have been used prior to 1926, from which year the S&P series is available.[9]

We start by performing Robinson's (1994) univariate tests on the individual series. Using the notation y_t for the log of each series, we employ throughout the model given by (4) and (5) with $z_t = (1, t)'$, $t \geq 1$, $z_t = (0, 0)$ otherwise, so that under H_o (3),

$$y_t = \beta_1 + \beta_2 t + x_t \qquad t = 1, 2, \ldots, \qquad (16)$$

[8] A similar experiment but based on AR alternatives, (i.e., with $(1 - \rho L)u_{2t} = \varepsilon_{2t}$ in (15)) for ρ = 0.05, (0.10), 0.95 was also conducted in Gil-Alana (1997), Robinson's (1994) tests again outperforming ADF and GPH tests for cointegration.

[9] More details on the data can be found in Campbell (2000).

$$(1 - L)^d x_t \quad = \quad u_t \qquad\qquad t = 1, 2, \ldots, \qquad\qquad (17)$$

treating separately the cases $\beta_1 = \beta_2 = 0$ a priori, β_1 unknown and $\beta_2 = 0$ a priori, and (β_1, β_2) unknown. We model the I(0) process u_t to be both white noise and to have parametric autocorrelation. Thus, if u_t in (17) is white noise, when d = 1, for example, the differences $(1 - L)y_t$ behave, for t > 1, like a random walk when $\beta_2 = 0$, and a random walk with drift when $\beta_2 \neq 0$. However, we report test statistics not merely for the null with d = 1 in (17) but with d = 0.50 (0.25) 2.00, thus including also a test for stationarity (d = 0.50) and for I(2) (d = 2), as well as other possibilities.

(Table 3 about here)

The test statistic reported in Table 3 is the one-sided one given by (6), so that significantly positive values are consistent with greater values of d and, conversely, significantly negative ones are consistent with smaller d's. Starting with dividends (in the upper part of the table), it can be noted that the results are quite similar across the different specifications in (16). Thus, whether or not we include an intercept and/or a linear time trend, the null (3) cannot be rejected when d = 0.75 and 1 if u_t is white noise or AR(1), being strongly rejected for the remaining values of d. If u_t is AR(2), these two values are also not rejected along with d = 1.25. Looking at stock prices (in the lower part of the table), we can see that if u_t is white noise, the unit root is the only non-rejection value across the different d's, and this is observed whether or not deterministic regressors are included in (16). Allowing u_t to follow AR processes, we observe a few more non-rejection values, occurring then when d = 0.75, 1 and 1.25, and in all cases, the lowest statistics occur in the unit root case (i.e., d = 1). In view of these results, we can conclude that both series may contain a unit root[10].

[10] Unit root tests based on AR alternatives (like Dickey and Fuller, 1979, and Phillips and Perron, 1988) were also performed on these series, obtaining further evidence in favour of the presence of a unit root.

Having found that both individual series exhibit unit root behaviour, we next examine the possibility of their being fractionally cointegrated. Table 4 reports results for $\hat{r}$ defined as in (6) when using the model given by (16) and (17), with d = 1 + θ, and y_t in (16) replaced by e_t, where e_t are the residuals from the cointegrating regressions

$$\underset{}{d_t} = \underset{(0.005)}{0.063} + \underset{(0.0012)}{0.026}\; p_t .$$

and

$$\underset{}{p_t} = \underset{(0.2452)}{-1.643} + \underset{(0.0012)}{33.831}\; d_t$$

Clearly, if we cannot reject the null given by (3), the estimated residuals will be I(1), indicating that there is no cointegration. On the other hand, rejections of (3) against the alternative H_1: $\theta < 0$, will imply that the estimated residuals are fractionally integrated, with stock prices and dividends being (possibly fractionally) cointegrated.

(Table 4 about here)

Again we present the results for the cases of no regressors ($\beta_1 = \beta_2 = 0$); an intercept ($\beta_1 \equiv 1$) and a linear time trend (β_1 and β_2 unknown), with white noise and autoregressions of orders 1, 2 and 3, using the finite sample critical values obtained in Table 1.[11] It can be seen from Table 4 that the null hypothesis of no cointegration is practically always rejected. In fact, the only non-rejection case occurs when regressing stock prices on dividends with AR(1) disturbances. For all the remaining specifications, the null (3) is always decisively rejected in favour of alternatives which are less integrated, suggesting that the estimated residuals from the cointegrating regression exhibit an order of integration smaller than one.

[11] The critical values allowing weakly autocorrelated disturbances and deterministic regressors like an intercept and a linear time trend were also calculated using a Montecarlo simulation approach, obtaining values very similar to those given in Table 1 (only the second decimal digit being affected).

As mentioned above, the fact that the least square estimate of the cointegrating parameter is a consistent estimate of the true value under cointegration, allows us to use the asymptotic critical values given by the normal distribution when testing d on the estimated residuals above. Table 5 extends the results of Table 4 for values of d = 0, (0.10). 0.90, again with different regressors and different types of disturbances. As we should expect, the results are similar whether we regress prices on dividends or vice versa. If v_t is white noise, the non-rejection values range between 0.60 and 0.90 with the lowest statistics occurring in all cases when d = 0.70. However, when allowing for autocorrelated disturbances, one finds a somewhat smaller degree of integration, with d fluctuating between 0.30 and 0.80 with AR(1) disturbances, and between 0.30 and 0.70 with AR(2) v_t. Higher order autoregressions were also allowed obtaining results similar to those in the AR(2) case and are not reported here. We see that the lowest statistics across d are obtained in practically all cases when d = 0.50 (and d = 0.60 in some cases), suggesting that the estimated residuals still may be nonstationary. On the other hand, if d = 0, H_o (3) is always decisively rejected in favour of alternatives of form H_1: $\theta > 0$, indicating that classical cointegration between stock prices and dividend does not apply.

(Table 5 about here)

We can conclude therefore that there is evidence in favour of fractional cointegration between stock prices and dividends, with the estimated residuals from the cointegrating regression showing long-memory behaviour. As a result, deviations from equilibrium will be long-lived, with mean reversion occurring very slowly.

5. Conclusions

We have shown in this paper that the cointegrating relationship between stock prices and dividends possesses long memory, i.e., it can be characterised as a fractionally cointegrated I(d)-type process. This is an important finding, as it means that, although these two variables are linked in the long run, adjustment to equilibrium takes a long time. Consequently, the validity of PV models of stock prices is confirmed, but only over a long horizon. Failure to take into account this slow adjustment process explains why the evidence from standard cointegration tests is often contradictory, as for instance in the seminal study due to Campbell and Shiller (1987a). An implication of our results is that investment strategies should allow for the slow response to shocks and the persistence of deviations from equilibrium.

We have also made a methodological contribution by proposing a two-step testing strategy for fractional cointegration. This procedure is based on Robinson's (1994) univariate tests, and it involves initially testing the order of integration of the individual series, and then testing the degree of integration of the estimated residuals from the cointegrating regression. As the Monte Carlo analysis shows, the suggested test has higher power than alternative tests for the null of cointegration against fractional alternatives.

It should also be mentioned that we did not attempt in this paper to select a specific model for the residuals from the cointegrating regression. In fact, our approach generates simply computed diagnostics for departures from any real d. Thus, given the continuity of d on the real line, it is not surprising that, when fractional hypotheses are considered, the evidence should appear to be supportive. Other methods for estimating d, based on semiparametric approaches, have been recently proposed (see, e.g., Robinson, 1995a, 1995b), and they could also be usefully employed for analysing financial series (as in the present study) or other macroeconomic time series.

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TABLE 1				
Critical values of Robinson's (1994) tests for the null hypotheses of two I(1) non-cointegrated processes against fractional cointegration				
Perc. / T	T = 50	T = 100	T = 200	T = 300
0.1%	-2.93	-2.96	-3.19	-3.19
0.5%	-2.66	-2.63	-2.66	-2.60
1%	-2.52	-2.48	-2.45	-2.44
2.5%	-2.30	-2.23	-2.20	-2.18
5%	-2.10	-2.00	-1.97	-1.91
10%	-1.84	-1.74	-1.66	-1.60
20%	-1.50	-1.38	-1.28	-1.21
30%	-1.25	-1.11	-1.00	-0.92
40%	-1.02	-0.87	-0.74	-0.67
50%	-0.80	-0.64	-0.51	-0.43
60%	-0.57	-0.39	-0.26	-0.19
70%	-0.30	-0.12	0.01	0.07
80%	0.02	0.20	0.33	0.40
90%	0.51	0.67	0.79	0.87
95%	0.94	1.11	1.21	1.29
97.5%	1.37	1.49	1.55	1.69
99.0%	1.87	1.97	2.01	2.12
99.5%	2.25	2.29	2.33	2.41
99.9%	3.04	3.09	2.98	2.91
Mean	-0.71	-0.56	-0.46	-0.39
Skewness	0.58	0.45	0.32	0.29
Kurtosis	3.68	3.40	3.26	3.15

The critical values were obtained on the basis of 50,000 replications in simulation, assuming that the true system is two non-cointegrated I(1) processes.

TABLE 2											
Power of the ADF, GPH and Robinson tests for cointegration against fractional alternatives											
		Values of d									
Size	Test statistic	0.95	0.85	0.75	0.65	0.55	0.45	0.35	0.25	0.15	0.05
5 %	ADF (p = 4)	.06	.07	.10	.14	.19	.26	.36	.50	.61	.73
	GPH (μ=.55)	.06	.09	.15	.21	.30	.37	.47	.56	.61	.64
	GPH (μ=.575)	.06	.10	.16	.24	.33	.42	.53	.62	.67	.71
	GPH (μ=.60)	.06	.11	.18	.28	.40	.52	.63	.73	.78	.81
	ROB. (W.N.)	**.07**	**.22**	**.50**	**.78**	**.94**	**.99**	**.99**	**1.00**	**1.00**	**1.00**
	ROB: (AR1)	**.15**	**.22**	**.35**	**.52**	**.71**	**.85**	**.94**	**.97**	**.99**	**.99**
	ROB: (AR2)	**.22**	**.26**	**.31**	**.41**	**.54**	**.67**	**.78**	**.86**	**.92**	**.95**
	ROB: (AR3)	**.30**	**.32**	**.35**	**.41**	**.50**	**.59**	**.68**	**.76**	**.82**	**.85**
10 %	ADF (p = 4)	.11	.13	.18	.24	.32	.41	.53	.67	.78	.87
	GPH (μ=.55)	.12	.17	.26	.35	.46	.56	.65	.72	.76	.78
	GPH (μ=.575)	.12	.18	.27	.38	.50	.60	.71	.77	.81	.83
	GPH (μ=.60)	.12	.19	.30	.43	.57	.68	.79	.85	.88	.90
	ROB. (W.N.)	**.16**	**.37**	**.66**	**.88**	**.97**	**.99**	**1.00**	**1.00**	**1.00**	**1.00**
	ROB: (AR1)	**.26**	**.36**	**.51**	**.69**	**.84**	**.94**	**.98**	**.99**	**.99**	**.99**
	ROB: (AR2)	**.32**	**.37**	**.45**	**.57**	**.69**	**.81**	**.89**	**.94**	**.97**	**.98**

ADF is the augmented Dickey-Fuller test statistic and p is the lag parameter selected using the AIC and the SIC. GPH is the Geweke and Porter-Hudak test statistic and μ is the value used in the sample-size function $n=T^{\mu}$. Results for the ADF and GPH have been taken from Cheung and Lai (1993), (pages 108 and 109). ROB stands for Robinson's (1994) tests. The power of each test is based on 10,000 replications and the Monte Carlo experiment with the Fortran code is available from the authors upon request.

TABLE 3							
Univariate tests of Robinson (1994) for testing (3) in (4) and (5)							
Dividends							
i): with white noise u_t							
z_t / d	0.50	0.75	1.00	1.25	1.50	1.75	2.00
With no regressors	6.63	**1.27’**	**-1.72’**	-3.33	-4.21	-4.73	-5.07
Wiith an intercept	6.42	**0.54’**	**-1.88’**	-3.60	-4.38	-4.84	-5.15
With a linear trend	4.84	**0.48’**	**-1.86’**	-3.60	-4.39	-4.86	-5.17
ii): with AR(1) u_t							
z_t / d	0.50	0.75	1.00	1.25	1.50	1.75	2.00
With no regressors	1.99	**1.06’**	**-0.74’**	-2.23	-3.23	-3.89	-4.35
Wiith an intercept	2.70	**-0.10’**	**-1.45’**	-2.73	-3.61	-4.18	-4.56
With a linear trend	2.61	**0.04’**	**-1.42’**	-2.73	-3.64	-4.24	-4.63
ii): with AR(2) u_t							
z_t / d	0.50	0.75	1.00	1.25	1.50	1.75	2.00
With no regressors	2.17	**1.26’**	**0.55’**	**-0.54’**	-1.98	-2.29	-2.86
Wiith an intercept	6.89	**-0.27’**	**-0.33’**	**-1.21’**	-2.08	-2.76	-3.24
With a linear trend	1.98	**0.09’**	**-0.29’**	**-1.22’**	-2.15	-2.88	-3.40
Stock prices							
i): with white noise u_t							
z_t / d	0.50	0.75	1.00	1.25	1.50	1.75	2.00
With no regressors	9.65	4.33	**0.07’**	-2.38	-3.60	-4.21	-4.55
Wiith an intercept	8.36	3.54	**-0.39’**	-2.70	-3.82	-4.37	-4.67
With a linear trend	9.14	3.81	**-0.33’**	-2.71	-3.85	-4.38	-4.68
ii): with AR(1) u_t							
z_t / d	0.50	0.75	1.00	1.25	1.50	1.75	2.00
With no regressors	4.00	2.29	**1.12’**	**-1.15’**	-2.73	-3.55	-3.95
Wiith an intercept	10.54	**0.44’**	**0.30’**	**-1.66’**	-3.08	-3.79	-4.11
With a linear trend	2.18	**1.56’**	**0.39’**	**-1.71’**	-3.17	-3.84	-4.13
ii): with AR(2) u_t							
z_t / d	0.50	0.75	1.00	1.25	1.50	1.75	2.00
With no regressors	7.00	**1.29’**	**1.10’**	**-1.76’**	-2.26	-2.68	-3.43
Wiith an intercept	11.39	**1.27’**	**0.89’**	**-0.92’**	-1.97	-2.12	-2.73
With a linear trend	6.07	**0.48’**	**0.03’**	**-0.80’**	-1.97	-2.22	-2.78

‘: Non-rejection values of the null hypothesis (3) at the 95% significance level.

TABLE 4		
Testing the null hypothesis of no cointegration against cointegration with the tests of Robinson (1994)		
v_t	Dividends / Stock prices	Stock prices. / Dividends
White noise v_t		
With no regressors:	-2.45’	-2.31’
With an intercept:	-2.79’	-2.55’
With a time trend:	-2.78’	-2.54’
AR(1) v_t		
With no regressors:	-2.06’	-1.92
With an intercept:	-2.71’	-2.39’
With a time trend:	-2.70’	-2.37’
AR(2) v_t		
With no regressors:	-2.06’	-2.04’
With an intercept:	-2.81’	-2.51’
With a time trend:	-2.84’	-2.63
AR(3) v_t		
With no regressors:	-2.17’	-2.15’
With an intercept:	-2.77’	-2.73’
With a time trend:	-2.73’	-2.80’

‘: Rejections of the null hypothesis of “no cointegration” at the 95% significance level.

TABLE 5										
Robinson’s (1994) tests on the estimated residuals from the cointegrating regression										
d_t = 0.063 + 0.026 p_t										
i): White noise v_t:										
z_t / d	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
With no regressors	13.33	10.38	7.77	5.56	3.73	2.20	**0.91’**	**-1.16’**	**-1.07’**	**-1.83’**
With an intercept	13.33	10.35	7.62	5.23	3.23	2.60’	**0.30’**	**-1.72’**	**-1.55’**	-2.23
With a time trend	7.81	6.46	5.12	3.80	2.52	2.31’	**0.22’**	**-1.73’**	**-1.55’**	-2.23
ii): AR(1) v_t:										
z_t / d	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
With no regressors	3.25	3.08	2.11	**1.28’**	**1.06’**	**0.68’**	**0.19’**	**-0.35’**	**-1.97’**	-2.01
With an intercept	4.50	3.06	2.97	**1.90’**	**1.41’**	**-0.20’**	**-0.80’**	**-1.34’**	**-1.93’**	-2.28
With a time trend	4.88	3.72	2.52	**1.84’**	**1.61’**	**-0.69’**	**-0.98’**	**-1.37’**	**-1.82’**	-2.27
i): AR(2) v_t:										
z_t / d	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
With no regressors	6.30	3.60	2.55	**1.24’**	**0.76’**	**0.44’**	**-0.99’**	**-1.15’**	-1.99	-2.14
With an intercept	6.30	3.55	3.33	**1.92’**	**1.19’**	**-0.90’**	**-1.05’**	**-1.44’**	-1.93	-1.99
With a time trend	6.77	3.87	3.44	**1.83’**	**1.34’**	**-0.37’**	**-0.77’**	**-1.54’**	-2.11	-2.16
p_t = -1.643 + 33.831 d_t										
i): White noise v_t:										
z_t / d	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
With no regressors	11.15	9.22	7.43	5.75	4.17	2.71	**1.39’**	**0.22’**	**-0.78’**	**-1.62’**
With an intercept	11.15	9.19	7.32	5.54	3.87	2.35	**1.01’**	**-0.13’**	**-1.10’**	**-1.90’**
With a time trend	9.89	8.43	6.93	5.40	3.87	2.41	**1.07’**	**-0.09’**	**-1.07’**	**-1.89’**
ii): AR(1) v_t:										
z_t / d	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
With no regressors	3.39	2.81	2.19	**1.02’**	**0.86’**	**-0.61’**	**-0.83’**	**-1.67’**	**-1.87’**	-1.99’
With an intercept	3.39	2.85	2.16	**1.37’**	**0.94’**	**-0.62’**	**-0.92’**	**-1.85’**	**-1.95’**	-2.09
With a time trend	4.43	3.79	2.18	**1.62’**	**0.95’**	**-0.68’**	**-0.98’**	**-1.74’**	-1.98	-2.06
i): AR(2) v_t:										
z_t / d	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
With no regressors	6.31	3.65	2.66	**1.35’**	**0.90’**	**0.04’**	**-0.31’**	**-1.37’**	-1.97	-2.04
With an intercept	6.31	3.70	2.83	**1.72’**	**1.09’**	**-0.22’**	**-0.95’**	**-1.46’**	-1.98	-2.01
With a time trend	7.76	4.66	2.43	**1.90’**	**1.10’**	**-0.37’**	**-0.56’**	**-1.67’**	-1.96	-2.12

‘: Non-rejection values of the null hypothesis at the 95% significance level.