Auctions and Corruption[1]

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Abstract

In many auctions, the auctioneer is an agent of the seller. This delegation invites corruption. In this paper we propose a model of corruption, examine how corruption affects the auction game, how the anticipation of corruption affects bidding, and how it altogether changes the revenue ranking of typical auctions. In addition we characterize incentive schemes that may prevent corruption, and compare them to the fee schedules employed by major auction houses.

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1. INTRODUCTION

Whenever people face scarcity, some may be tempted to cheat. This is true for ordinary thieves as well as for bidders in an auction for a government construction job, or for those who compete for the right to host the Olympic games. In this paper we study corruption, as a special form of cheating in auctions.

Corruption is generally defined as the "misuse of a position of trust for dishonest gain." In an auction context, corruption refers to the lack of integrity of the auctioneer. It occurs whenever the auctioneer twists the auction rules in favor of some bidder(s), in exchange for bribes. Corruption may be a simple bilateral affair between one bidder and the auctioneer, but it may also involve collusion among several bidders who jointly strike a deal with the auctioneer.

Corruption is a frequently observed and well documented event in many government procurement auctions. For example, the bidding for the construction of a new metropolitan airport in the Berlin area was recently reopened after investigators found out that *Hochtief AG*, the winner of the auction, had illegally acquired the application documents of the rival bidder *IVG*.[1] As another example, in 1996 the authorities of Singapur ruled to exclude *Siemens AG* from all public procurements for a period of five years after they determined that Siemens had bribed Choy Hon Tim, the chief executive of Singapur's public utility corporation *PUB*, in exchange for supplying Siemens with information about rival bids for a major power station construction project.[2]

Of course, corruption in an auction cannot occur if the seller is also the auctioneer. Corruption is only an issue if the auctioneer is the agent of the seller. Such delegation occurs if the seller lacks the expertise to run the auction himself, or if the seller is a complex organization like the collective of citizens

[1] See *Wall Street Journal*, Aug. 19, 1999.
[2] See *Berliner Zeitung*, Feb. 2, 1996.

of a community, a state, or an entire nation. It does not matter whether the auctioneer-agent is a specialized auction house or a government employee. What matters alone is the fact that the auctioneer acts independently on behalf of the seller.

Corruption can also not work in an open-bid auction simply because it lacks secrecy. However, open auctions may not be feasible if the bids are complicated documents, as is the case of auctions for major construction jobs or for the right to host the Olympics. In such auctions sealed-bids seems to be the only feasible auction form. The fact that bids are sealed supplies the secrecy needed for corrupt games being played between the auctioneer and one or several bidders.

Given the quantitative importance of auctions and the temptation of corruption, three questions emerge. Are the usual sealed-bid auctions that are used for procurement vulnerable to corruption? Is corruption harmful in an efficiency sense? Can we make auctions corruption-proof? We will show that the answers to these questions are Yes, Maybe, and Yes. Standard sealed-bid auctions are indeed vulnerable to corruption, corruption may be socially inefficient, but fortunately there are ways to make auctions corruption-proof.

We also show that corruption has important distributional effects. Specifically, if the number of bidders is sufficiently large, bidders' expected equilibrium payoffs are unaffected by the possibility of corruption — a consequence of the revenue equivalence principle. However, there is a transfer of wealth from the seller to the agent who acts as auctioneer. Moreover, the seller also bears the cost of the whole expected punishment.

Furthermore, we find that revenue equivalence breaks down if widespread corruption schemes that involve all bidders are feasible. In that case, corruption also involves a transfer of expected payoffs from the seller to bidders. This case is interesting because it is congruent to other examples of breakdown of revenue equivalence studied in the literature, such as the analysis of collusive auction rings by Graham and Marshall (1987), of the breaking-up of partnerships (see Cramton, Gibbons, and Klemperer (1987)), and of the so-called Amish auctions (see Engelbrecht-Wiggans (1994)).

2. STRUCTURE OF THE GAME

There is one seller of a single good who faces n risk neutral potential buyers. The seller has hired an auctioneer to run either a sealed-bid first-price or a sealed-bid second-price auction. From the auctioneer's perspective bidders' valuations are *iid* random variables. Therefore, the auction game is a standard symmetric independent private values model, which is however supplemented by a corruption game.

In the sealed-bid first-price auction, the corruption game is as follows: After bids have been submitted, the auctioneer reveals the second highest bid to the highest bidder. The auctioneer allows the highest bidder to lower his bid to the level of the second highest bid, in exchange for a bribe.

In the sealed-bid second-price auction, the corruption game is slightly more involved: The auctioneer reveals the three highest bids to the highest and the second highest bidder. The three parties agree on removing the second highest bid in exchange for side payments.

Corruption is detected with probability δ, in which case the auctioneer pays a penalty p_0, the winning bidder pays a penalty p_1, and, if the second highest bidder is also involved, he pays a penalty p_2. The penalty takes the form of a jail sentence or the like; it is not a payment to the seller. Also, if corruption is detected, the original bids have to be paid, i.e. b_1 in the first-price and b_2 in the second-price auction.

After the auction, the price paid is published, in order to make sure that the auctioneer does not simply ignore the highest or the two highest bidders, respectively.

Valuations $v_1, \ldots, v_n$ are independent draws (no affiliation) from the distribution F with support $[0,1]$. We denote the distribution of the highest valuation of n draws with F_1, and the distribution of the highest valuation of $n-1$ draws with G_1. By the *iid* assumption, $F_1 = F^n$ and $G_1 = F^{n-1}$. We denote the joint density of the ith and jth highest valuations of n draws with f_{ij}. $b_1, \ldots, b_n$ denote the bids. W.l.o.g. we order bidders in such a way that $b_i \geqslant b_{i+1}$.

A *bidding strategy* is a map $\beta^i : v_i \mapsto b_i$. An *equilibrium* is a profile of strategies $(\beta^1, \ldots, \beta^n)$ such that β^i is a best reply for i given the strategies of all other bidders. An equilibrium is *symmetric* if all bidders use the same strategy, $\beta^1 = \cdots = \beta^n$.

We denote the equilibrium strategy of the symmetric equilibrium of the first- and second-price auction with corruption with β_1 and β_2, respectively. We call the corresponding auctions in which corruption is not part of the game the *standard first- and second-price auctions*, respectively, and denote the respective symmetric equilibrium bid functions with B_1 and B_2.[3]

3. SEALED-BID FIRST-PRICE AUCTIONS

The surplus from corruption that the auctioneer and the winning bidder can share is the difference of the winning and the highest losing bid times the probability of not being detected, minus the expected penalty imposed on the winning bidder and on the auctioneer if corruption is detected,

$$S(b_1, b_2) := (1-\delta)(b_1 - b_2) - \delta(p_0 + p_1). \tag{1}$$

We assume proportional sharing of this surplus and denote the share received by the auctioneer with $\alpha S(b_1, b_2)$.

[3]For a survey of the results of the standard first- and second-price auctions without corruption see McAfee and McMillan (1987) and Wolfstetter (1999, Chapter 8).

The winning bidder's payoff if he does not engage in corruption is

$$u_n(v_1, b_1) := v_1 - b_1,$$

and if he does engage in corruption his expected payoff is

$$u_c(v_1, b_1, b_2) := u_n(v_1, b_1) + (1 - \alpha)S(b_1, b_2).$$

Corruption pays if the surplus is positive, i.e. if the following *bribe condition* is satisfied,

$$b_1 - b_2 > \frac{\delta(p_0 + p_1)}{1 - \delta} =: \gamma. \tag{2}$$

Therefore, corruption occurs if and only if the winning bid exceeds the second bid by γ or more.

A bidder's expected payoff, given that all rival bidders play the strategy β_1, is a weighted sum of expected payoffs in the corruption and no corruption regimes,

$$\begin{aligned} u(v, b) &:= \int_0^{\beta_1^{-1}(\max\{b-\gamma,0\})} u_c(v, b, \beta_1(v_2))dG_1(v_2) \\ &\quad + \int_{\beta_1^{-1}(\max\{b-\gamma,0\})}^{\beta_1^{-1}(b)} u_n(v, b)dG_1(v_2) \\ &= (v - b)G_1(\beta_1^{-1}(b)) + (1 - \alpha)\int_0^{\beta_1^{-1}(\max\{b-\gamma,0\})} S(b, \beta_1(v_2))dG_1(v_2). \end{aligned} \tag{3}$$

PROPOSITION 1 (WHO WINS AND WHO LOSES) *Consider a symmetric equilibrium. The possibility of corruption does not affect bidders' expected payoffs, it makes the auctioneer better off, and the seller worse off. Corruption also causes a social deadweight loss if the penalty for corruption is a jail sentence; this deadweight loss equals the expected disutility of penalties.*

PROOF: Bidders' equilibrium payoffs are $u^*(v) := u(v, \beta_1(v))$. By the Envelope Theorem together with (3) we have $u^{*\prime}(v) = \frac{\partial u}{\partial v}(v, \beta_1(v)) = G_1(v)$.

Integration yields

$$u^*(v) := \int_0^v G_1(x)dx + u^*(0).$$

G_1 is the probability of winning the auction. Evidently, $u^*(0) = 0$, because by bidding some $b > 0$, the zero-valuation bidder might win the auction and get something which is worthless to him; if he loses the auction, he never receives a bribe because corruption involves only the winning bidder and the auctioneer. Therefore, bidders' equilibrium payoffs are determined by the allocation rule G_1 only, as in the standard auction.

The auctioneer must be weakly better off by the possibility of corruption, because otherwise he would not participate. Therefore, the seller must be worse off. Since the allocation rule is unchanged, the expected gain from trade is unchanged as well. Thus, every expected gain of the auctioneer must be matched by a corresponding loss of the seller.

There is, however, another source of losses. If corruption occurs there is a positive probability that someone will be punished. If the penalty takes the form of "burning utility" (for instance by sending the winner and the auctioneer to jail), corruption entails a deadweight loss equal to the expected disutility of the penalty. This loss is borne by the seller alone, in addition to the transfer from the seller to the auctioneer. □

Altogether, corruption induces bidders to compete for the gain from corruption by bidding more aggressively to such an extent that they do not benefit from it. The auctioneer is the only one gaining in expectations, and his gain is paid for by the seller. The seller, however, pays more than what the auctioneer receives (in expectations), because he also loses the expected value of the penalties $\delta(p_0 + p_1)$ if the penalties take the form of a jail sentences (as opposed to payments to the seller). Thus, depending on the from of the penalties, corruption may reduce social welfare.

PROPOSITION 2 (CORRUPTION MAKES BIDDERS AGGRESSIVE) *Bidders with sufficiently low valuations bid the same as in the standard first-price auction, whereas bidders with higher valuations bid more aggressively, but still below their true valuation. Formally, let* $v^* := B_1^{-1}(\gamma)$*, then* $\forall v \leqslant v^*$ $\beta_1(v) = B_1(v)$*, and* $\forall v > v^*$ $B_1(v) < \beta_1(v) < B_2(v)$*.*

PROOF: (*i*) "$\beta_1(v) = B_1(v)$ for $v \leqslant v^*$." If $b \leqslant \gamma$, bidders' payoff function (3) reduces to

$$u(v, b) = (v - b)G_1(\beta_1^{-1}(b)).$$

This is simply the payoff function of the standard first-price auction. Hence, for sufficiently small valuations, the equilibrium bid function equals B_1. This is true if $B_1(v) \leqslant \gamma$, or equivalently if $v \leqslant B_1^{-1}(\gamma) =: v^*$.

(*ii*) "$\beta_1(v) > B_1(v)$ for $v \geqslant v^*$." Suppose $v > v^*$, then we know that corruption occurs with positive probability. Hence, if bidders would bid as in the standard first-price auction, $\beta_1(v) = B_1(v)$, their expected payment in the game with corruption would be less than their expected payment in the standard first-price auction, for otherwise corruption would not pay and would therefore never occur. But this contradicts revenue equivalence. Thus, $\beta_1(v) > B_1(v)$ for all $v > v^*$.

(*iii*) "$\beta_1(v) < B_2(v)$." Assume $v > v^*$, $\delta(p_0 + p_1) = 0$ (hence $\gamma = 0$), and $\alpha = 0$. Then corruption takes place for sure. Moreover, corruption is free to the winning bidder because he will not be punished, $\delta p_1 = 0$, and the auctioneer participates for free, $\alpha = 0$. In this case, the game is a standard second-price auction and therefore bidding must be the same, $\beta_1(v) = B_2(v)$. Now consider an increase in the cost of corruption, i.e. $\delta(p_0+p_1) > 0$ or $\alpha > 0$. Since $v > v^*$ corruption occurs with positive probability, but corruption is costly. This is consistent with revenue equivalence only if bids are less aggressive than in the second-price auction, thus $\beta_1(v) < B_2(v)$. □

Clearly if corruption is detected with certainty, $\delta = 1$, corruption never pays and the game collapses to the ordinary first-price auction. In this case $\gamma = \infty$, or equivalently $v^* = \infty$, hence everyone has a valuation smaller than v^* and so corruption never occurs in equilibrium. This finding can be strengthened.

Corollary 3 (corruption-free equilibria) *All bidders bid as in the standard first-price auction and corruption does not occur, with certainty, if and only if* $\delta \geqslant \frac{B_1(1)}{B_1(1)+p_0+p_1} =: \delta^*$.

Proof: "If." If $\delta \geqslant \delta^*$, then $\gamma \geqslant B_1(1)$, hence $v^* = B_1^{-1}(\gamma) \geqslant 1$, and $v \leqslant v^*$ for all v.

"Only if." If $\delta < \delta^*$, then $\gamma < B_1(1)$, hence $v^* = B_1^{-1}(\gamma) < 1$, and, with positive probability, some bidder has a valuation that exceeds v^*. These bidders will bid more than in the standard first-price auction and engage in corruption, with positive probability. □

This result says that one only needs sufficient monitoring ($\delta^* \leqslant \delta \leqslant 1$), not perfect monitoring ($\delta = 1$), to rule out corruption.

We conclude that the sealed bid first-price auction is vulnerable to corruption, that it hurts only the seller, and may give rise to a deadweight loss. An immediate resolution of this problem is to run an open auction instead. However, in many circumstances, this may not be feasible, for instance if the bids are complicated documents, such as bids for major construction jobs or Olympic games. A sealed-bid second-price, or Vickrey auction, however, could help. It is not vulnerable to the kind of corruption we have been studying because the winning bidder is supposed to pay the second bid anyway. To make corruption work in the Vickrey auction requires the second bidder to be involved in the corruption scheme as well. This is the topic of the next section.

4. SEALED-BID SECOND-PRICE AUCTIONS

The sealed-bid second-price auction is not vulnerable to a corruption scheme that involves only the auctioneer and the winning bidder because they alone cannot change the price. They need the collaboration of the second bidder.

Instead of allowing the winning bidder to lower his bid, bidder 2 is bribed to withdraw or lower his bid. If this scheme succeeds the winning bidder pays only the third bid. Altogether, this requires the collaboration of three parties who must share the gain from corruption: the auctioneer, bidder 1, and bidder 2.

Assume $n \geqslant 3$. The surplus to be shared by the auctioneer and bidders 1 and 2 is equal to

$$S(b_2, b_3) := (1 - \delta)(b_2 - b_3) - \delta \bar{p}, \tag{4}$$

where $\bar{p} := p_0 + p_1 + p_2$ and p_0, p_1, p_2 are the penalties imposed upon the auctioneer and bidders 1 and 2, respectively. Again, we assume proportional sharing of the surplus and denote the respective shares with α_0, α_1, α_2, with $\alpha_0 + \alpha_1 + \alpha_2 = 1$.

Now consider a bidder with valuation v who makes the bid b. If he happens to win the auction, his payoff is equal to u_{1n} if he does not engage in corruption,

$$u_{1n}(v_1, b_2) := v_1 - b_2,$$

and equal to u_{1c} if he does engage in corruption

$$u_{1c}(v_1, b_2, b_3) := u_{1n}(v_1, b_2) + \alpha_1 S(b_2, b_3).$$

If he loses the auction he may still gain something. Indeed, in the event that b is the second highest bid and corruption occurs, his payoff is equal to the bribe he receives,

$$u_{2c}(b, b_3) := \alpha_2 S(b, b_3).$$

Corruption pays if the surplus is positive, i.e. if the following *bribe condition* is satisfied,

$$b_2 - b_3 > \frac{\delta \bar{p}}{1-\delta} =: \gamma. \tag{5}$$

In words, corruption occurs only if the second bid exceeds the third bid by γ or more.

Taking rivals' strategies as given, bidders' expected payoffs are

$$\begin{aligned} u(v,b) := & \int_0^{\beta_2^{-1}(b)} \int_{\beta_2^{-1}(\max\{\beta_2(v_2)-\gamma,0\})}^{v_2} u_{1n}(v,\beta_2(v_2)) f_{23}(v_2,v_3)\, dv_3\, dv_2 \\ & + \int_0^{\beta_2^{-1}(b)} \int_0^{\beta_2^{-1}(\max\{\beta_2(v_2)-\gamma,0\})} u_{1c}(v,\beta_2(v_2),\beta_2(v_3)) f_{23}(v_2,v_3)\, dv_3\, dv_2 \\ & + \int_{\beta_2^{-1}(b)}^1 \int_0^{\beta_2^{-1}(\max\{b-\gamma,0\})} u_{2c}(b,\beta_2(v_3)) f_{13}(v_1,v_3)\, dv_3\, dv_1. \end{aligned}$$

Substituting the payoff functions yields

$$\begin{aligned} u(v,b) = & \int_0^{\beta_2^{-1}(b)} (v - \beta_2(v_2))\, dG_1(v_2) \\ & + \alpha_1(1-\delta) \int_0^{\beta_2^{-1}(b)} \int_0^{\beta_2^{-1}(\max\{\beta_2(v_2)-\gamma,0\})} (\beta_2(v_2) - \beta_2(v_3) - \gamma) f_{23}(v_2,v_3)\, dv_3\, dv_2 \\ & + \alpha_2(1-\delta) \int_{\beta_2^{-1}(b)}^1 \int_0^{\beta_2^{-1}(\max\{b-\gamma,0\})} (b - \beta_2(v_3) - \gamma) f_{13}(v_1,v_3)\, dv_3\, dv_1. \end{aligned} \tag{6}$$

PROPOSITION 4 (SECOND-PRICE AUCTION) *Suppose $n \geqslant 3$. Then Proposition 1 applies also to the second-price auction.*

PROOF: We only need to show that revenue equivalence applies. Similar to the proof of Proposition 1 we can show that

$$u^*(v) := \int_0^v G_1(x)\, dx + u^*(0).$$

Therefore, bidders' equilibrium payoffs are as in the standard auctions if and only if $u^*(0) = 0$. But this follows immediately from (6) because $\beta_2^{-1}(\beta_2(0)) = 0$, which entails that, for $v = 0$, (6) is an integral over a null set.

The rest of the proof is analogous to the proof of Proposition 1 and therefore omitted. □

Interestingly the above result does not apply if there are only two bidders. In this case revenue equivalence is destroyed, and it may happen that a bidder who stands no chance of winning the auction bids quite aggressively, because he speculates on earning a bribe for withdrawing his bid, which lowers the price paid all the way down to zero.

The surplus from corruption simplifies in this case to

$$S(b_2) := (1-\delta)b_2 - \delta\bar{p}.$$

For $b \leqslant \gamma$, bidders' objective function is as in the standard second-price auction,

$$u(v,b) := \int_0^{\beta_2^{-1}(b)} (v - \beta_2(v_2))dG_1(v_2).^4$$

For $b > \gamma$, bidders' objective function becomes

$$\begin{aligned} u(v,b) := &\int_0^{\beta_2^{-1}(b)} (v - \beta_2(v_2))dG_1(v_2) \\ &+ \alpha_1(1-\delta)\int_{\beta_2^{-1}(\gamma)}^{\beta_2^{-1}(b)} (\beta_2(v_2) - \gamma)dG_1(v_2) \\ &+ \alpha_2(1-\delta)(1 - F_1(\beta_2^{-1}(b)))(b-\gamma). \end{aligned} \tag{7}$$

This last term is the expected value of the share of the surplus from corruption that the losing bidder receives. We will show that, depending upon the parameters of the game, this last term can upset revenue equivalence.

Proposition 5 (failure of revenue equivalence) *Revenue equivalence does not generally hold if $n = 2$.*

Proof: As in the case $n \geqslant 3$, we have

$$u^*(v) := \int_0^v G_1(x)dx + u^*(0). \tag{8}$$

[4] If $n = 2$, then $G_1 = F$, but for clarity we stick to the more general notation.

We now show that in a symmetric equilibrium one may have $u^*(0) > 0$, which together with (8) destroys revenue equivalence. Consider the following counterexample: Let $\delta = \alpha_0 = \alpha_1 = 0$, $\alpha_2 = 1$, then $\gamma = 0$. For $v = 0$, (7) becomes

$$u(0,b) = -\int_0^{\beta_2^{-1}(b)} \beta_2(v_2)dG_1(v_2) + (1 - F_1(\beta_2^{-1}(b)))b.$$

For $b < \beta_2^{-1}(1)$ we have

$$\begin{aligned} u(0,b) &\geqslant -b\int_0^{\beta_2^{-1}(b)} dG_1(v_2) + b - bF_1(\beta_2^{-1}(b)) \\ &= b\left[1 - G_1(\beta_2^{-1}(b)) - F_1(\beta_2^{-1}(b))\right]. \end{aligned} \tag{9}$$

It follows immediately that $u(0,b) > 0$ for a sufficiently small but strictly positive bid b. Therefore, in a symmetric equilibrium one must have $u^*(0) > 0$. □

REMARK: There is an interesting analogy between the present failure of revenue equivalence and the auctions with price-proportional benefits to bidders. Examples of such auctions are the Amish auction to settle an indivisible inheritance among family members (Engelbrecht-Wiggans, 1994), auction rings that divide the gain from collusion (Graham and Marshall, 1987), and the breaking-up of partnerships through auctions (Cramton et al., 1987). In his analysis Engelbrecht-Wiggans establishes that such auctions “unbalance revenue equivalence.” Just like in the present context, the reason for this result is that even the player who stands no chance to win the auction collects some payment. Hence $u^*(0) > 0$, which implies that bidders’ equilibrium payoffs $u^*(v)$ are greater than the level reached in standard auctions.

5. MORE ELABORATE AUCTIONS AND CORRUPTION SCHEMES

So far we have restricted the analysis of coalitions in corruption schemes to the smallest possible size, and we restricted the analysis to the two most common auction rules. In this section we consider some less common auctions and allow for more elaborate corruption schemes that involve larger coalitions.

If coalitions involve more bidders, the benefits from corruption can be raised by further lowering the equilibrium price, yet this gain has to be shared among more members. Moreover, with each illegal contact between two parties there is a risk of detection. Therefore, the larger the coalition, the more likely corruption is detected.

A general corruption scheme in a more general auction framework is as follows: Consider a k-price auction, $k \in \{1, \dots, n\}$, which awards the item to the highest bidder who is asked to pay the k-th highest price. The winning bidder has the option to offer to bidder i a bribe in exchange for withdrawing his bid. Let c be the number of bidders participating in the corrupt coalition. $c = 1$ means that the corruption scheme involves only the winning bidder and the auctioneer; $c = 5$ means that the winner has bribed four other bidders to drop their bids. $c = 0$ means that the winning bidder does not engage in corruption. If corruption is not detected, the winning bidder pays b_{k+c}, otherwise he pays b_k and all involved bidders and the auctioneer are penalized. This more complex corruption scheme is clearly also available in the first-price auction: Several loosing bidders can be bribed to drop their bids, and the winning bidder may lower his bid below the original highest losing bid.

What do we know about the validity of our results if such involved corruption schemes are available? Proposition 1 goes through if and only if revenue equivalence holds in these more complicated games as well. However, there are more constellations where revenue equivalence fails.

As an instructive digression let us study the reasons for the failure of revenue equivalence in Proposition 5 more closely. Considering a second-price auction, and assuming that the corruption scheme involves at most three players (the highest and the second-highest bidder and the auctioneer), we found that revenue equivalence holds if $n \geqslant 3$ (Proposition 4). The reason why this works is that in a symmetric equilibrium the bidder with a valuation equal to zero will never receive a bribe because the third bid exceeds his own with certainty; hence $u^*(0) = 0$. This is not true if $n = 2$. After all, with two bidders there is no third bid, so the bribe condition requires only that the losing bid be high enough. Under some parameter constellations, aggressive bidding by the zero value bidder is indeed part of a symmetric equilibrium. His probability of winning the auction is still zero, but he does always receive a bribe in equilibrium, thus $u^*(0) > 0$, destroying revenue equivalence.

If more involved corruption schemes with arbitrarily large coalitions are feasible, revenue equivalence can fail even if $n \geqslant 3$. In a k-price auction with $k > 1$, every coalition that includes $n - k + 2$ bidders drives the price down to zero, and therefore a bidder with valuation equal to zero can force a positive expected payoff by bidding aggressively enough, as demonstrated before for $n = 2$, $k = 2$. If corruption succeeds to lower the price all the way down to zero, the seller loses an extra chunk of revenue equal to $nu^*(0)$. Therefore, we conclude that the second-price auction is superior to third- and higher-price auctions on the ground that smaller coalitions suffice to bring the price down to zero in the higher-price auctions.

The argument does, however, not apply to the comparison of the first- and second-price auction. Both auction forms require all n bidders to participate in the corrupt coalition for revenue equivalence to fail. The second-price auction, however, has an advantage over the first-price auction because it rules out corrupt coalitions with only two members (the winner and the auctioneer), to which the first-price auction is susceptible. We conclude that, if corruption is an issue and an open-bid auction is not feasible, the seller should choose a second-price sealed-bid auction.

6. CORRUPTION-PROOF CONTRACTS

Corruption necessarily involves the auctioneer who acts as an agent on behalf of the seller. In principle, the seller could avoid corruption by running the auction himself or by setting up appropriate monitors. However, this solution is often not feasible because the seller is not qualified for the task, because ownership is diversified, or because the seller himself is an agent for some other party, as in the case of the public sector. Yet, even in these cases the seller can fight corruption by awarding the auctioneer an appropriate incentive contract. We now explain some properties of such incentive schemes, using the example of a sealed-bid first-price auction.

PROPOSITION 6 (CORRUPTION-PROOF INCENTIVE CONTRACT) *Consider a sealed-bid first-price auction. The seller can rule out corruption with a costless incentive contract* (s, s_0) *that offers the auctioneer a share* $s \geqslant 1 - \frac{\delta}{\delta^*}$ *of the profit in exchange for a flat fee* s_0*.*

PROOF: The auctioneer's compensation equals $sb_1 - s_0$. Set $s_0 := s \int_0^1 B_1(x)\, dF_1(x)$, so that the contract is costless if $\beta_1 = B_1$.

The total surplus of the corrupt coalition equals the expected reduction of the price paid to the seller, minus the expected penalty, minus the reduction of the auctioneer's compensation received from the seller,

$$\begin{aligned} S(b_1, b_2) &:= (1-\delta)(b_1 - b_2) - \delta(p_0 + p_1) - s(b_1 - b_2) \\ &= (1 - \delta - s)(b_1 - b_2) - \delta(p_0 + p_1). \end{aligned}$$

We want to find conditions that guarantee that this surplus is not positive.

S reaches a maximum at $b_1 = \beta_1(1)$ and $b_2 = \beta_1(0)$. Therefore, $S(\beta(v_1), \beta(v_2)) \leqslant 0$ for all v_1, v_2 if and only if

$$s \geqslant s^* := 1 - \frac{\delta(\beta_1(1) - \beta_1(0) + p_0 + p_1)}{\beta_1(1) - \beta_1(0)}.$$

If $s \geqslant s^*$ the game collapses to the standard first-price auction without corruption. Thus, $\beta_1 = B_1$, and s^* simplifies to

$$s^* = 1 - \frac{\delta(B_1(1) + p_0 + p_1)}{B_1(1)} = 1 - \frac{\delta}{\delta^*},$$

as certified. □

If $\delta < \delta^*$ then, by Corollary 3, there is a positive probability that corruption will occur in equilibrium. But the seller may only have limited influence on the detection probability and penalties because they are controlled by the legal system. However, the seller can compensate the deficiency of the legal system by offering the auctioneer an incentive compatible compensation scheme. For instance, if the detection probability falls twenty percent short of the smallest level that rules out corruption, i.e. $\delta = 0.8\delta^*$, it suffices to offer the auctioneer a twenty percent profit share, $s^* = 0.2$. With this contract, S is non-positive and corruption is prevented at zero cost, and the expected revenues are the same as in the standard first-price auction. We mention that a similar condition for a corruption-proof compensation scheme can be developed for second- and higher-price auctions.

Incidentally, linear sharing rules are commonly applied by major auction houses. For example, Sotheby's takes a profit share of 12% to 37%.[5] Whether this is meant to be a simple markup or an anti-corruption measure is open to debate. In any case, such contracts reduce or even eliminate the incentives to engage in the kind of corruption that we analyzed in the present paper.

[5]For live auctions at Sotheby's salesrooms the buyer pays a commission of 10% to 20% of the hammer price. For internet-auctions at `sothebys.com` the fee is 10% (called "buyer's premium"). In addition, buyers and sellers pay a commission between 2% and 20% of the hammer prices of all purchases and sales within a calendar year (except sales by Sotheby's associates done over the internet). The seller also pays all agreed upon expenses (shipping, insurance, taxes). The total fees (for non-associates), net of expenses, are therefore 10% to 20% buyer's commission or premium, plus two times an amount between 2% and 20% of the hammer price. Thus, the fees collected by the auctioneer are between 14% and 60% of the hammer price, or between 12.3% and 37.5% of the totally paid price (hammer plus fees, net of expenses); see `http://auction.sothebys.com/conditions.html`, section "Certain Conditions Relating to Buyers," item 2 "Buyer's Premium and Payment," and `http://auction.sothebys.com/auctionslive/sell.html`, section "Standard Commission," and personal communication with Sotheby's customer assistance.

7. CONCLUSIONS

Our model establishes corruption as an equilibrium phenomenon of sealed-bid auctions, if the seller delegates the actual conduct of the auction to an auctioneer-agent. The model therefore explains the empirical fact that corruption does happen in public submissions and the like.

The model also shows that bidders do not benefit from corruption in terms of equilibrium expected payoffs. The prospect of participating in a profitable corruption scheme induces them to raise their bids to such an extent that bidders' entire surplus is competed away. Only the auctioneer-agent benefits from it. This is a consequence of the revenue equivalence principle. The entire cost of corruption, i.e. the excess profit of the auctioneer as well as the expected value of punishment, is borne by the seller. Thus, the seller has a strong interest to design and apply anti-corruption measures.

Such measures are available. All that is required is an appropriate incentive contract between the seller and the auctioneer. We show that such an incentive contract takes a simple form. It is just an ordinary linear profit sharing contract, which, incidentally, is the standard form of contract used by major auction houses, such as Sotheby's. For this reason we conclude that similar agreements should be used by government agencies for their public submissions.

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