TESTING STOCHASTIC CYCLES IN MACROECONOMIC TIME SERIES

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Abstract. A particular version of the tests of Robinson (1994) for testing stochastic cycles in macroeconomic time series is proposed in this article. The tests have a standard limit distribution and are easy to implement in raw time series. A Monte Carlo experiment is conducted, studying the size and the power of the tests against different alternatives, and the results are compared with those based on other tests. An empirical application using historical U.S. annual data is also carried out at the end of the article.

Key words: Stochastic cycles; Fractional roots; Unit root cycles

JEL Classification: C22

1. INTRODUCTION

Deterministic cycles seem to be inappropriate for modelling most macroeconomic time series. Stochastic cycles were proposed by Harvey (1985) amongst others, and they were generalized to allow for long memory by Gray et al. (1989, 1994). In particular, they considered processes like

$$(1 - 2\mu L + L^2)^d x_t = u_t, \qquad t = 1, 2, ... \tag{1}$$

where d can be any real number and u_t is an I(0) process, defined in this context as a covariance stationary process with spectral density function which is bounded and bounded away from zero at any frequency. Clearly, when d = 0, $x_t = u_t$, and we say then that x_t is "weakly autocorrelated", as opposed to d > 0 when the process is said to be "strongly autocorrelated" or also called "strongly dependent" because of the strong association between observations widely separated in time. Gray el al. (1989) showed that x_t in (1) is stationary if $|\mu| < 1$ and d < 0.50 or if $|\mu| = 1$ and d < 0.25. They also showed that the polynomial in (1) can be expressed in terms of the Gegenbauer polynomial $C_{j,d}(\mu)$ such that for all $d \neq 0$

$$(1 - 2\mu L + L^2)^{-d} = \sum_{j=0}^{[j/2]} C_{j,d}(\mu) L^j, \tag{2}$$

where

$$C_{j,d}(\mu) = \sum_{k=0}^{\infty} \frac{(-1)^k (d)_{j-k} (2\mu)^{j-2k}}{k!(j-2k)!}; \qquad (d)_j = \frac{\Gamma(d+j)}{\Gamma(d)},$$

and a truncation will be required below (2) to make (1) operational. Thus, the process in (1) becomes

$$x_t = \sum_{j=0}^{t-1} C_{j,d}(\mu) u_{t-j}, \quad t = 1, 2, ... \tag{3}$$

and when d = 1, we have

$$x_t \;=\; 2\mu\, x_{t-1} \;-\; x_{t-2} \;+\; u_t, \qquad t = 1, 2, ... \tag{4}$$

which is a cyclic I(1) process with the periodicity determined by μ. Tests of (4) based on autoregressive (AR) alternatives were proposed amongst others by Ahtola and Tiao (1987).

In this article we propose the use of the fractional structure (1) for testing cyclical unit root models like (4), using a particular version of the tests of Robinson (1994). These tests are explained in Section 2. Section 3 contains a Monte Carlo experiment, studying the size and the power of the tests in finite sample, and the results are compared with those based on Ahtola and Tiao's (1987) procedure. Section 4 applies the tests of Robinson (1994) to several macroeconomic time series while Section 5 contains some concluding remarks.

2. TESTING CYCLES WITH THE ROBINSON'S (1994) TESTS

Robinson (1994) proposes tests for unit roots and other forms of nonstationary hypotheses. He considers the regression model

$$y_t \;=\; \beta' z_t \;+\; x_t, \quad t = 1, 2, ... \tag{5}$$

where y_t is the time series we observe, β is a (kx1) vector of unknown parameters and z_t is a (kx1) vector of exogenous regressors. The regression errors x_t are such that

$$\rho(L;\theta)\; x_t \;=\; u_t, \quad t = 1, 2, ... \tag{6}$$

with $x_j = 0$ for $j \le 0$; u_t is a (possible weakly autocorrelated) I(0) process and ρ is a prescribed function of L and of the (px1) parameter vector θ, especifically,

$$\rho(L;\theta) \;=\; (1-L)^{d_1+\theta_1}\,(1+L)^{d_2+\theta_2}\,\prod_{j=3}^{p}(1 \,-\, 2\cos\, w_r\, L \,+\, L^2)^{d_j+\theta_j},$$

for given values of d_1, d_2, …, d_p, and w_r. He proposes a Lagrange multiplier (LM) test for testing the null

$$H_o: \ \theta = 0, \tag{7}$$

in (5) and (6). Thus, under (7), we can consider a wide range of possibilities to be tested in (6), for example:

a) I(1) processes: if $\rho(L; \theta) = (1 - L)^{1+\theta}$,

b) I(d) processes: if $\rho(L; \theta) = (1 - L)^{d+\theta}$,

c) Quarterly I(1): if $\rho(L; \theta) = (1 - L^4)^{1+\theta}$,

d) Cyclic I(1): if $\rho(L; \theta) = (1 - 2\,\mu L + L^2)^{1+\theta}$, and so on.

Specifically, the test statistic proposed by Robinson (1994) is given by

$$\hat{R} = \frac{T}{\hat{\sigma}^4}\, \hat{a}'\, \hat{A}^{-1}\, \hat{a} = \hat{r}'\hat{r}; \qquad \hat{r} = \frac{T^{1/2}}{\hat{\sigma}^2}\, \hat{A}^{-1/2}\, \hat{a} \tag{8}$$

where

$$\hat{a} = \frac{-2\pi}{T} \sum_j^* \psi(\lambda_j)\, g(\lambda_j; \hat{\tau})^{-1} I_{\hat{u}}(\lambda_j); \qquad \psi(\lambda_j) = \mathrm{Re}\left[\frac{\partial}{\partial \theta}\left(\log \rho(e^{i\lambda_j}; \theta)\right)\right]_{\theta=0};$$

$$\hat{A} = \frac{2}{T}\left(\sum_j^* \psi(\lambda_j)\psi(\lambda_j)' - \sum_j^* \psi(\lambda_j)\hat{\varepsilon}(\lambda_j)' \, x \left(\sum_j^* \hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)'\right)^{-1} x \sum_j^* \hat{\varepsilon}(\lambda_j)\psi(\lambda_j)'\right);$$

$$\hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log\, g(\lambda_j; \hat{\tau}); \qquad \hat{\sigma}^2 = \frac{2\pi}{T} \sum_j^* g(\lambda_j; \hat{\tau})^{-1} I_{\hat{u}}(\lambda_j);$$

$g(\lambda_j; \tau)$ is the function appearing in the spectral density of u_t: $f(\lambda_j; \tau) = (\sigma^2/2\pi)\, g(\lambda_j; \tau)$, evaluated at $\hat{\tau}$ = arg min $\sigma^2(\tau)$, and $I_{\hat{u}}(\lambda_j)$ is the periodogram of $\hat{u}_t$ defined as:

$$I_{\hat{u}}(\lambda_j) = \frac{1}{2\pi T}\left|\sum_{j=1}^{T} \hat{u}_t\, e^{i\lambda_j t}\right|^2;$$

$$\hat{u}_t = \rho(L)\, y_t - \hat{\beta}' w_t; \quad w_t = \rho(L)\, z_t; \quad \hat{\beta} = \left(\sum_{t=1}^{T} w_t\, w_t'\right)^{-1} \sum_{t=1}^{T} w_t\, \rho(L)\, y_t,$$

where $\rho(L) = \rho(L; \theta = 0)$ and the summation on * in the above expressions are over $\lambda \in M$ where $M = \{\lambda: -\pi < \lambda < \pi, \lambda \notin (\rho_l - \lambda_1, \rho_l + \lambda_1), l=1,2,\ldots s\}$, such that ρ_l, $l = 1,2,\ldots s < \infty$ are the distinct poles of $\psi(\lambda)$ on $(-\pi, \pi]$.

Robinson (1994) showed that under very general conditions,

$$\hat{R} \;\rightarrow\; X_p^2, \qquad as \;\; T \;\rightarrow\; \infty, \tag{9}$$

and the same limit distribution holds whether or not deterministic regressors are included in (5). Furthermore, he shows that the above tests are efficient in the Pitman sense, that against local alternatives of form: H_a: $\theta = \delta\, T^{-1/2}$, for $\delta \neq 0$, the limit distribution is $\chi_p^2(\upsilon)$ with a non-centrality parameter, υ, which is optimal under Gaussianity of u_t. In this article we are concerned with the presence of unit root cycles in macroeconomic time series. Therefore, we can particularize the above tests to the case where $d_3 = 1$ and $d_j = 0$ for all $j \neq 3$. Then,

$$\rho(L;\theta) \;=\; (1 \,-\, 2\cos w_r\, L \,+\, L^2)^{1+\theta} \tag{10}$$

and substituting (10) in (6), we obtain, under the null hypothesis (7), the cyclic I(1) model (4) with $\mu = w_r$. In this context

$$\psi(\lambda_j) \;=\; \log\left|\, 2\left(\cos\lambda_j \;-\; \cos\, w_r\right)\right|$$

and $p = 1$. Thus, a one-sided test of (7) against the alternatives:

$$H_a:\; \theta \;>\; 0, \tag{11}$$

will be given by the rule:

"Reject H_o if $\hat{r} \;>\; z_\alpha$",

where the probability that a standard normal variate exceeds z_α is α. Conversely, a test of (7) against alternatives:

$$H_a:\; \theta \;<\; 0, \tag{12}$$

will be given by the rule:

“Reject H_o if $\hat{r} < -z_\alpha$”.

The tests of Robinson (1994) were applied to an extended version of the Nelson and Plosser’s (1982) dataset in Gil-Alana and Robinson (1997), testing the presence of unit roots and other long memory processes when the singularity at the spectrum occurred at the zero frequency. Other versions of Robinson’s (1994) tests, involving quarterly and monthly data, were respectively studied in Gil-Alana and Robinson (1998) and Gil-Alana (1999). However, testing cyclical unit root models with the tests of Robinson (1994) still remained without examination; one by-product of this work is its emergence as an alternative way of testing stochastic cycles in raw time series.

3. A MONTE CARLO SIMULATION STUDY

This section examines the finite-sample behaviour of versions of the above tests by means of Monte Carlo simulations, studying the size and the power of the tests when directed against fractionally and non-fractionally alternatives. In Robinson (1994), a finite-sample experiment was also performed. In that paper, he looked at the rejection frequencies when the model was a pure random walk (i.e., $(1 - L)\, x_t = \varepsilon_t$), and the alternatives were either fractional (i.e., $(1 - L)^{1+\theta}\, x_t = \varepsilon_t$), or autoregressive (i.e., $(1 - (1+\theta)L)\, x_t = \varepsilon_t$), for different values of θ.

In this section we investigate the power of Robinson’s (1994) tests when the true model is a cyclic I(1) process of form as in (4) and the alternatives are firstly

$$(1 - 2\mu L + L^2)^{1+\theta}\, x_t \;=\; u_t$$

for different values of θ. That is, the alternatives have the roots at the same frequencies as in the true model and thus, the number of periods per cycle remains the same under both the null and the alternative hypotheses. However, we will also examine cases where the number of periods per cycle changes under the alternative hypothesis. These results will be then compared with those obtained using the Ahtola and Tiao's (1987) procedure. Their tests are based on autoregressive alternatives of form:

$$x_t \;=\; \phi_1 \, x_{t-1} \;+\; \phi_2 \, x_{t-2} \;+\; u_t, \qquad (13)$$

which, under the null H_o: $|\phi_1| < 2$ and $\phi_2 = -1$, becomes the cyclic I(1) model (4).

In Tables 1 – 5 we look at the rejection frequencies of Robinson's (1994) tests when the null model consists of (5) – (7) and (10) with $\beta = 0$ a priori (i.e., $y_t = x_t$), and $w_r = 2\pi r/T$, with r = T/2, T/4, T/8, T/10 and T/20, i.e., we look at unit root cycles occurring each 2, 4, 8, 10 and 20 periods. The alternatives will be fractional with θ = -1; -0.8; -0.6; ….(0.2) …0,6; 0.8; 1 when T = 40 and non-fractional with θ = -1; 0; 1 when T > 40, and r = T/2; T/4; T/8; T/10 and T/20 in all cases. Thus, the rejection frequencies corresponding to $\theta = 0$ when the same r is taken under the null and the alternative hypothesis will indicate the sizes of the tests. In these cases we calculated both the one and the two sided test statistics. We generate Gaussian series generated by the routines GASDEV and RAN3 of Press, Flannery, Teukolsky and Vetterling (1986), with 10,000 replications of each case. The sample sizes were initially T = 40, 80, 120 and 160. However, given the similarities in the last three cases, we only report in the tables the results for T = 40 and 160. In all cases the nominal size is 5%.

In Table 1 the true model is given by

$$(1 - 2 \cos w_r \, L + L^2)^{1+\theta} \, x_t \;=\; \varepsilon_t, \qquad (14)$$

with r = T/2; $\theta = 0$ and white noise ε_t, i.e., we have a unit root cycle occurring each two periods. If the alternatives are also modelled with r = T/2, we look at the rejection frequencies for both the one and the two sided test statistics, (i.e., $\hat{r}$ and $\hat{R}$ in (8)). Looking at the one-sided tests, the sizes of $\hat{r}$ are too large for $\theta < 0$ but too small for $\theta > 0$, however, they improve considerably as we increase the number of observations. The size of $\hat{R}$ is also too large when T = 40, but it approximates to the nominal value with T. Looking at the rejection frequencies when r = T/2, a bias in favour of negative values of θ appears when the fractional alternatives are entertained, and taking r = T/4, T/8, T/10 and T/20, all the rejection frequencies become 1, even if the number of observations is relatively small (e.g., T = 40).

(Tables 1 and 2 about here)

Table 2 reports rejection frequencies when the true model is given by (14) with r = T/4 and θ = 0, i.e., we have cycles occurring every four periods. As in Table 1, the sizes of the one-sided tests are too large for positive θ but too small for negative values of θ, and the size of $\hat{R}$ is too large in all cases, though again improving considerably as we increase the number of observations. A bias in favour of negative values of θ also appears in this table and it is observed even when the alternatives are θ = ±1. If r ≠ T/4, the rejection frequencies are relatively high in all cases except for r = T/8 and θ > 0, where the values never exceed 0.500 with T = 40. Increasing the number of observations, the rejection frequencies also increase, and some of the pathological cases observed above, (r = T/8; θ = 1) improve considerably (0.999 when T = 160).

In Table 3 the true model consists of (14) with r = T/8 and θ = 0. Once more, the sizes for the one-sided tests are asymmetric, implying a bias toward negative values of θ. The size of $\hat{R}$ is 11.9% when T = 40, and 7.4% when T = 160, and higher rejection frequencies are observed when θ = -1 than when θ = 1. When T = 40 and r ≠ T/8, the results are quite good for alternatives with r = T/2 and T/4, however, if they include a higher number of periods per cycle, (i.e., with r = T/10 or T/20), the results are relatively poor when θ is close to zero. Increasing the sample size, the rejection frequencies appear fairly reasonable in all cases.

(Tables 3 – 5 about here)

Table 4 reports rejection frequencies when the true model is a cyclic I(1) process with cycles occurring each 10 periods, (i.e., (14) with r = T/10 and θ = 0). As in all the previous tables, a bias appears in $\hat{r}$ and $\hat{R}$, with higher values when θ < 0. The worst results are again obtained when the alternatives contain more periods per cycle than those observed from the true model, (i.e., when r = T/20), especially when the number of observations is small. We see in this table that if the alternatives are such that r = T/20 and θ = 0, the rejection frequency is 0.437 though it becomes 0.984 when T = 160. Finally Table 5 reports the results when the true model contains 20 periods per cycle. The sizes of $\hat{r}$ are again biased and the size of $\hat{R}$ is too large in all cases, though improving with T. The rejection frequencies are relatively high in all cases, especially when the alternatives are such that r = T/2 and T/4.

Table 6 reports results of the same experiment as in Tables 1 – 5, but using the Ahtola and Tiao's (1987) procedure. Their test is based on the least squares estimator of the second AR parameter in (13), the test statistic being $T(\hat{\phi}_2 + 1)$, whose distribution is tabulated. It is shown in the paper that the asymptotic distribution of $T(\hat{\phi}_2 + 1)$ does not depend on ϕ_1 and the frequencies and thus, asymptotic inference about the existence of complex roots in the unit circle can be based on the single distribution of $T(\hat{\phi}_2 + 1)$. This makes a different with respect to Robinson's (1994) tests, where the periodicity μ must be given for testing of a unit root cycle. Thus, instead of presenting the results below each of the previous tables, we have created a new one (Table 6), whose structure is exactly the same as Tables 1 – 5, i.e, showing the rejection frequencies of the test statistic when the null model is given by (14) with r = T/2, T/4, T/8, T/10 and T/20, and T = 40 and 160 with a nominal size of 5%.

(Table 6 about here)

Starting with T = 40, we see that the size is 10.3% if r = T/2, and it ranges around 5.5% for the remaining values of r. Thus, it is closer to the nominal size than the tests of Robinson (1994). These small sizes are also associated with some inferior rejection frequencies in many cases, especially when the alternatives are close to the null. This is not surprising if we take into account the efficiency property of Robinson's (1994) tests against local alternatives. We see, for example, that if θ = -0.2, the rejection frequency in Ahtola and Tiao's (1987) tests is 0.679 when r = T/2, and it never exceeds 0.300 for the remaining values of r. Similarly, if θ = 0.2, the rejection probability with r = T/2 is 0.516, and it is smaller than 0.250 in all the other cases. This is in sharp contrast with the results in Tables 1 – 5, where the rejection frequencies of Robinson's (1994) tests for the same two alternatives are in practically all cases higher than 0.300. On the other hand, if we concentrate on departures far away from the null, (i.e., $\theta = \pm 1$), the rejection frequencies in Ahtola and Tiao (1987) are higher in some cases than in Robinson (1994), though increasing the sample size, (T = 160), the latter outperforms the results in Ahtola and Tiao (1987) when θ = 1.

As a conclusion, the tests of Robinson (1994) seem to perform quite well when testing the null of cyclic I(1) models. When the sample size is small (eg., T = 40), the sizes of $\hat{r}$ are too large for $\theta < 0$ but too small for $\theta > 0$, and the size of $\hat{R}$ is too large in all cases. However, as we increase the number of observations, the sizes improve considerably, approximating to the nominal value. The rejection frequencies are relatively high, especially when the alternatives are such that the number of periods per cycle is smaller than that observed from the true model. This suggests that when testing unit root cycles with the tests of Robinson (1994), a plausible strategy might be to start by testing models containing a large number of periods per cycle and then, testing in a decreasing way. Comparing the tests of Robinson (1994) with Ahtola and Tiao (1987), the latter seems to be better in term of size, though the rejection frequencies are higher in Robinson (1994) against these fractional alternatives. The following section contains an empirical application based on the tests of Robinson (1994) for testing unit root cycles in macroeconomic time series.

4. AN EMPIRICAL APPLICATION

The extended version of the annual data set of fourteen U.S. macroeconomic variables analysed by Nelson and Plosser (1982) ends in 1988; as with their data, the starting date is 1860 for consumer price index and industrial production; 1869 for velocity; 1871 for stock prices; 1889 for GNP deflator and money stock; 1890 for employment and unemployment rate; 1900 for bond yield, real wages and wages; and 1909 for nominal and real GNP and GNP per capita. As in Nelson and Plosser (1982), all the series except the bond yield are transformed to natural logarithms.

Denoting any of the series y_t, we employ throughout the model (5); (6) and (10) with $z_t = (1, t)'$, $t \geq 1$, $z_t = (0, 0)'$ otherwise, so

$$y_t = \beta_1 + \beta_2 t + x_t, \qquad t = 1, 2, ... \tag{15}$$

$$(1 - 2\cos w_r L + L^2)^{1+\theta} x_t = u_t, \qquad w_r = 2\pi r / T, \qquad t = 1, 2, ... , \tag{16}$$

testing the null (7) for values of r = T, T/2, T/3, … , T/10, T/20, T/30 and T/40, i.e., allowing unit root cycles occurring at 1, 2, 3, …, 10, 20, 30 and 40 periods respectively. We treat separately the cases $\beta_1 = \beta_2 = 0$ a priori, β_1 unknown and $\beta_2 = 0$ a priori, and (β_1, β_2) unknown and model the I(0) disturbances to be both white noise and to have parametric autocorrelation.

We begin with the assumption that u_t in (16) is white noise. The test statistic reported in Table 7 (and also in Table 8) is the two-sided one given by $\hat{R}$ in (8). A notable feature of Table 7a, in which $\beta_1 = \beta_2 = 0$ a priori, is the fact that we cannot reject the unit root null in any of the series when the cycles occur every 6 periods. Similarly, if r = T/7 (i.e., 7 periods per cycle), the null is only rejected for velocity. Also, in nine series we observe non-rejection values if r = T/5, while unemployment is the only one in which the null is not rejected when r = T/3 and T/4. Tables 7b and 7c give results, respectively, with $\beta_2 = 0$ a priori, (i.e., no time trend in the undifferenced regression), and both β_1 and β_2 unrestricted, still with white noise u_t. In both tables the results are very similar and while there are sometimes large differences in the values of $\hat{R}$ across the number of periods per cycle, the conclusions suggested by both seem very similar, with the non-rejection values occurring practically always at the same series/periods per cycle combination. Imposing r = T/6, the unit root null cannot be rejected in any series except unemployment in Table 7b, and unemployment and velocity in Table 7c. Similarly, imposing r = T/7, the null is rejected for unemployment, velocity and stock prices, in Table 7b, and for these three series along with industrial production and bond yield in Table 7c. The shortest periods per cycle where the unit root null hypothesis cannot be rejected appear for unemployment, with the non-rejection values occurring when r = T/3, T/4 and T/5. We also observe across this table that unit root cycles occurring at periods smaller than three or greater than seven are always decisively rejected, suggesting that the efficiency property of Robinson's (1994) tests may hold not only against local alternatives but also when the alternatives include different numbers of periods per cycle.

(Tables 7 and 8 about here)

Table 8 reports the same statistic as in Table 7 but imposing an autoregressive (AR) structure on the disturbances u_t. Table 8a corresponds to AR(1) u_t, while Tables 8b and 8c refer respectively to AR(2) and AR(3) u_t. Higher order autoregressions were also performed obtaining similar results. Starting with Tables 8a and 8b, we see that the results are very similar. In fact, the non-rejection values occur at exactly the same series/periods per cycle combination in all except one single case, which corresponds to velocity. For this series, we see in Table 8a that if r = T/5 and T/6, the unit root null is not rejected while in Table 8b only the latter case results in rejection. Apart from this series, and also unemployment, (where the null hypothesis is always rejected), for the remaining series the unit root null cannot be rejected when r = T/6. If r = T/5 or T/7, the null is also non-rejected for industrial production, money stock, bond yield and stock prices, the latter two series allowing r = T/8 as well. Table 8c gives similar results though the proportion of non-rejection values is slightly smaller. On the whole, we observe twelve series where the unit root null hypothesis cannot be rejected when r = T/6, the only exceptional series being again unemployment, where the null is always rejected, and velocity, where r = T/6 is rejected but r = T/5 is not. We also observe that imposing an AR(3) process for u_t all the non-rejection values form a proper subset of those in Tables 8a and 8b. Moreover, in all except two series, only a single value of $\hat{R}$ is not rejected across the different r's. Thus, the unit root null hypothesis is not rejected when r = T/6 for real, nominal and real per capita GNP, industrial production, employment, GNP deflator, CPI, wages, real wages and money stock, and when r = T/5 for velocity. Finally, we observe several non-rejections for the bond yield, occurring at r = T/4, T/5, T/6 and T/7, and for stock prices at r = T/5 and T/6.

We can summarize the results obtained in this section by saying that unit root cycles are practically never rejected for the extended version of the Nelson and Plosser's (1982) series, with cycles occurring approximately every 6 periods. These results are obtained whether or not we include an intercept or an intercept and a linear trend in the regression model, and independently of the way of modelling the I(0) disturbances u_t, as white noise or autoregressions. Attempting to summarize the conclusions for individual series, we are left with the impression that the cycles occur every 3, 4 or 5 periods for unemployment; every 5 or 6 periods for the bond yield; 6 or 7 for industrial production and money stock; while for the remaining series, they occur almost exactly every 6 periods.

5. CONCLUDING REMARKS

A particular version of the tests of Robinson (1994) for testing stochastic unit root cycles in raw time series has been proposed in this article. The tests are nested in fractional alternatives of the form advocated by Gray et. al. (1989, 1994) and have standard null and local limit distribution. A finite sample experiment, based on Monte Carlo simulations was also computed and the results indicate that the tests perform relatively well for testing cyclic I(1) processes when the number of periods per cycle under the alternative is smaller than or equal to the number of periods per cycle under the null. However, if they are greater, the tests have relatively low power, especially if the number of observations is small. Thus, a plausible strategy when using these tests might be to test initially for a wide number of periods per cycle, and then testing the number of them in a decreasing way. Comparing these tests with those based on AR alternatives (Ahtola and Tiao, 1987), the results indicate that the latter has better size, though the rejection frequencies are higher in Robinson (1994), especially if the alternatives are close to the null.

The tests were also applied to an extended version of the Nelson and Plosser's (1982) dataset, and the results suggest that a cyclic I(1) model with approximately six periods per cycle seems to be a plausible way of modelling all these series, the only exceptions being unemployment and velocity, where the cycles seem to occur at a fewer number of periods.

The frequency domain version of the test statistic used in this article seems to be unpopular amongst econometricians. There also exist time domain versions (cf. Robinson, 1991). However, our preference here for the frequency domain set-up of Robinson (1994) is motivated by the somewhat greater elegance of formulae it affords, especially when the disturbances are autocorrelated. In addition, the fact that the article stresses the presence of cycles in macroeconomic time series makes the use of the frequency domain even more relevant.

Several other lines of research are under way which should prove relevant to the analysis of these and other macroeconomic data. Thus, for example, testing the order of integration of the series for any real value of d must be of interest if we want to determine the degree of dependence between the cycles. There also exist multivariate versions of the tests of Robinson (1994), (cf. Gil-Alana, 1997), and work is also proceeding on programming these multivariate tests in the context of fractional cycles.

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TABLE 1*

Rejection frequencies of the tests of Robinson (1994)

T = 40		True model: $(1 - 2\cos w_r L + L^2)^{1+\theta} x_t = \varepsilon_t$, with r = T/2 and $\theta = 0$											
		Values of θ											
r	Stat.	-1	-0.8	-0.6	-0.4	-0.2	0	0	0.2	0.4	0.6	0.8	1
T/2	$\hat{r}$	1.000	1.000	1.000	.997	.858	**0.133**	**0.021**	.566	.787	.809	.892	.961
T/2	$\hat{R}$	1.000	1.000	.999	.998	.718	**0.074**		.496	.730	.742	.831	.926
T/4	$\hat{R}$	1.000	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
T/8	$\hat{R}$	1.000	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
T/10	$\hat{R}$	1.000	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
T/20	$\hat{R}$	1.000	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000

T = 160		True model: $(1 - 2\cos w_r L + L^2)^{1+\theta} x_t = \varepsilon_t$, with r = T/2 and $\theta = 0$			
r	Statistic	$\theta = -1$	$\theta = 0$	$\theta = 0$	$\theta = 1$
T/2	$\hat{r}$	1.000	**0.084**	**0.033**	1.000
T/2	$\hat{R}$	1.000	**0.053**		1.000
T/4	$\hat{R}$	1.000	1.000		1.000
T/8	$\hat{R}$	1.000	1.000		1.000
T/10	$\hat{R}$	1.000	1.000		1.000
T/20	$\hat{R}$	1.000	1.000		1.000

*: 10,000 replications were used for each case. Sizes are in bold and the nominal size was 5%. $\hat{r}$ and $\hat{R}$ are Robinson's (1994) test statistics in (8).

TABLE 2*													
Rejection frequencies of the tests of Robinson (1994)													
T = 40		True model: $(1 - 2\cos w_r L + L^2)^{1+\theta} x_t = \varepsilon_t$, with r = T/4 and $\theta = 0$											
		Values of θ											
r	Stat.	-1	-0.8	-0.6	-0.4	-0.2	0	0	0.2	0.4	0.6	0.8	1
T/4	$\hat{r}$	1.000	.473	.443	.388	.128	**0.322**	**0.004**	.123	.376	.501	.452	.560
T/4	$\hat{R}$	.999	.369	.337	.243	.107	**0.200**		.085	.290	.403	.458	.465
T/2	$\hat{R}$	.999	.999	.998	.997	.996	.993		.989	.985	.973	.940	.813
T/8	$\hat{R}$	.951	.908	.841	.757	.667	.557		.442	.322	.203	.093	.026
T/10	$\hat{R}$	.996	.992	.985	.973	.957	.935		.904	.864	.791	.667	.452
T/20	$\hat{R}$	1.000	1.000	.999	.999	.998	.997		.996	.992	.983	.957	.862

T = 160		True model: $(1 - 2\cos w_r L + L^2)^{1+\theta} x_t = \varepsilon_t$, with r = T/4 and $\theta = 0$			
r	Statistic	$\theta = -1$	$\theta = 0$	$\theta = 0$	$\theta = 1$
T/4	$\hat{r}$	1.000	**0.156**	**0.015**	0.998
T/4	$\hat{R}$	1.000	**0.095**		0.995
T/2	$\hat{R}$	1.000	1.000		1.000
T/8	$\hat{R}$	1.000	1.000		0.999
T/10	$\hat{R}$	1.000	1.000		1.000
T/20	$\hat{R}$	1.000	1.000		1.000

*: 10,000 replications were used for each case. Sizes are in bold and the nominal size was 5%. $\hat{r}$ and $\hat{R}$ are Robinson's (1994) test statistics in (8).

TABLE 3*

Rejection frequencies of the tests of Robinson (1994)

T = 40		True model: $(1 - 2 \cos w_r L + L^2)^{1+\theta} x_t = \varepsilon_t$, with r = T/8 and $\theta = 0$											
		Values of θ											
r	Stat.	-1	-0.8	-0.6	-0.4	-0.2	0	0	0.2	0.4	0.6	0.8	1
T/8	$\hat{r}$	1.000	1.000	.999	.983	.772	**0.199**	**0.008**	.175	.540	.664	.632	.577
T/8	$\hat{R}$	1.000	1.000	.998	.966	.657	**0.119**		.110	.418	.560	.533	.472
T/2	$\hat{R}$	1.000	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
T/4	$\hat{R}$	.998	.985	.989	.993	.994	.996		.996	.996	.991	.982	.953
T/10	$\hat{R}$	1.000	.999	.960	.686	.291	.337		.666	.893	.974	.989	.987
T/20	$\hat{R}$	.941	.739	.425	.169	.071	.136		.346	.597	.809	.938	.979

T = 160		True model: $(1 - 2\cos w_r L + L^2)^{1+\theta} x_t = \varepsilon_t$, with r = T/8 and $\theta = 0$			
r	Statistic	$\theta = -1$	$\theta = 0$	$\theta = 0$	$\theta = 1$
T/8	$\hat{r}$	1.000	**0.121**	**0.019**	0.997
T/8	$\hat{R}$	1.000	**0.074**		0.992
T/2	$\hat{R}$	1.000	1.000		1.000
T/4	$\hat{R}$	1.000	1.000		1.000
T/10	$\hat{R}$	1.000	0.955		1.000
T/20	$\hat{R}$	1.000	0.956		1.000

*: 10,000 replications were used for each case. Sizes are in bold and the nominal size was 5%. $\hat{r}$ and $\hat{R}$ are Robinson's (1994) test statistics in (8).

TABLE 4*

Rejection frequencies of the tests of Robinson (1994)

T = 40		True model: $(1 - 2 \cos w_r L + L^2)^{1+\theta} x_t = \varepsilon_t$, with r = T/10 and $\theta = 0$											
		Values of θ											
r	Stat.	-1	-0.8	-0.6	-0.4	-0.2	0	0	0.2	0.4	0.6	0.8	1
T/10	$\hat{r}$	1.000	1.000	.999	.992	.811	**0.181**	**0.011**	.253	.665	.739	.682	.614
T/10	$\hat{R}$	1.000	1.000	.999	.982	.694	**0.105**		.170	.554	.644	.585	.513
T/2	$\hat{R}$	1.000	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
T/4	$\hat{R}$	1.000	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
T/8	$\hat{R}$	1.000	.975	.749	.316	.341	.706		.926	.986	.997	.998	1.000
T/20	$\hat{R}$	.998	.963	.737	.332	.191	.437		.750	.932	.987	.998	.998

T = 160		True model: $(1 - 2\cos w_r L + L^2)^{1+\theta} x_t = \varepsilon_t$, with r = T/10 and $\theta = 0$			
r	Statistic	$\theta = -1$	$\theta = 0$	$\theta = 0$	$\theta = 1$
T/10	$\hat{r}$	1.000	**0.117**	**0.022**	0.997
T/10	$\hat{R}$	1.000	**0.070**		0.991
T/2	$\hat{R}$	1.000	1.000		1.000
T/4	$\hat{R}$	1.000	1.000		1.000
T/8	$\hat{R}$	1.000	1.000		1.000
T/20	$\hat{R}$	1.000	0.984		1.000

*: 10,000 replications were used for each case. Sizes are in bold and the nominal size was 5%. $\hat{r}$ and $\hat{R}$ are Robinson's (1994) test statistics in (8).

TABLE 5*

Rejection frequencies of the tests of Robinson (1994)

T = 40		True model: $(1 - 2 \cos w_r L + L^2)^{1+\theta} x_t = \varepsilon_t$, with r = T/20 and $\theta = 0$										
		Values of θ										
r	Stat.	-1	-0.8	-0.6	-0.4	-0.2	0	0	0.2	0.4	0.6	0.8	1
T/20	$\hat{r}$	1.000	1.000	.999	.998	.881	**0.180**	**0.014**	.431	.798	.811	.764	.734
T/20	$\hat{R}$	1.000	1.000	.999	.993	.765	**0.101**		.343	.736	.750	.692	.648
T/2	$\hat{R}$	1.000	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
T/4	$\hat{R}$	1.000	1.000	1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000
T/8	$\hat{R}$	.251	.389	.643	.856	.956	.993		.999	.999	1.000	1.000	1.000
T/10	$\hat{R}$	.813	.466	.354	.615	.882	.981		.998	.999	1.000	1.000	1.000

T = 160		True model: $(1 - 2\cos w_r L + L^2)^{1+\theta} x_t = \varepsilon_t$, with r = T/20 and $\theta = 0$			
r	Statistic	$\theta = -1$	$\theta = 0$	$\theta = 0$	$\theta = 1$
T/20	$\hat{r}$	1.000	**0.110**	**0.022**	1.000
T/20	$\hat{R}$	1.000	**0.063**		1.000
T/2	$\hat{R}$	1.000	1.000		1.000
T/4	$\hat{R}$	1.000	1.000		1.000
T/8	$\hat{R}$	0.955	1.000		1.000
T/10	$\hat{R}$	0.963	1.000		1.000

*: 10,000 replications were used for each case. Sizes are in bold and the nominal size was 5%. $\hat{r}$ and $\hat{R}$ are Robinson's (1994) test statistics in (8).

TABLE 6*												
Rejection frequencies of the Ahtola and Tiao (1987)'s tests												
T = 40		Null model: $(1 - 2\cos w_r L + L^2)^{1+\theta} x_t = \varepsilon_t$, with $\theta = 0$.										
		Values of θ										
r	Stat.	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
T/2	A - T	1.000	1.000	1.000	.999	.679	**0.103**	.516	.820	.936	.976	.989
T/4	A – T	1.000	.999	.934	.501	.128	**0.051**	.196	.434	.636	.776	.860
T/8	A – T	1.000	1.000	.995	.707	.172	**0.060**	.216	.415	.653	.787	.862
T/10	A – T	1.000	1.000	.998	.797	.198	**0.056**	.219	.456	.651	.786	.867
T/20	A – T	1.000	1.000	1.000	.949	.299	**0.058**	.163	.499	.673	..787	.859

T = 160		Null model: $(1 - 2\cos w_r L + L^2)^{1+\theta} x_t = \varepsilon_t$, with $\theta = 0$.		
r	Statistic	θ = -1	θ = 0	θ = 1
T/2	A – T	1.000	**0.091**	0.997
T/4	A – T	1.000	**0.048**	0.881
T/8	A – T	1.000	**0.050**	0.995
T/10	A – T	1.000	**0.052**	0.983
T/20	A - T	1.000	**0.051**	0.983

*: 10,000 replications were used for each case. Sizes are in bold and the nominal size was 5% . A – T means Ahtola and Tiao's (1987) test statistic.

TABLE 7												
$\hat{R}$ in (8) with white noise u_t												
	Periods per cycle											
a): $\beta_1 = \beta_1 = 0$	1	2	3	4	5	6	7	8	9	10	20	40
Real GNP	56.55	11.76	52.56	24.10	**3.80**	**.0004**	**1.93**	5.15	8.29	15.84	71.40	147.8
Nominal GNP	56.85	16.81	24.20	18.83	**3.45**	**.0001**	**1.82**	5.15	8.23	15.52	71.91	140.1
Real pcap. GNP	55.35	18.66	30.03	17.38	**3.06**	**.0006**	**1.58**	4.41	7.34	13.39	60.52	128.8
Ind. Production	58.82	12.53	50.55	35.76	5.38	**0.016**	**2.85**	6.40	9.00	23.32	92.35	175.3
Employment	55.20	16.72	19.71	14.66	**2.82**	**.0000**	**1.46**	4.49	7.45	12.88	56.55	124.7
Unemployment	51.84	24.80	**0.34**	**1.41**	**0.19**	**1.51**	**3.49**	5.24	6.65	14.40	15.28	219.0
GNP deflator	55.95	17.72	64.64	28.51	4.32	**.0004**	**1.84**	5.15	7.72	15.36	77.44	137.8
C.P.I.	55.65	16.72	65.12	30.03	4.08	**.0004**	**1.74**	5.19	7.89	14.28	78.49	137.1
Wages	57.00	18.06	38.56	22.46	**3.68**	**.0001**	**1.79**	5.42	8.64	15.13	75.56	143.2
Real wages	55.95	15.13	64.80	28.51	3.88	**.0001**	**1.76**	5.10	8.46	14.13	69.22	146.1
Money stock	60.99	15.52	60.21	38.44	5.52	**.001**	**2.82**	8.23	10.56	23.13	104.0	186.3
Velocity	38.81	16.32	12.25	9.00	**0.84**	**.193**	4.84	5.61	7.12	18.57	38.93	40.19
Bond yield	50.55	32.14	23.42	11.49	**1.34**	**.435**	**2.43**	7.07	14.59	12.60	86.30	137.3
Stock prices	58.06	10.75	17.13	31.58	**2.37**	**.028**	**3.45**	8.58	6.70	22.84	75.86	170.0
$\beta_2 = 0$	1	2	3	4	5	6	7	8	9	10	20	40
Real GNP	259.2	6.30	65.93	40.70	7.34	**.0009**	**3.49**	9.30	14.97	29.59	116.6	268.6
Nominal GNP	259.2	42.77	65.77	40.96	7.39	**.0004**	**3.61**	10.04	18.40	28.62	118.8	267.9
Real pcap. GNP	259.2	52.99	66.09	40.96	7.18	**.0001**	**3.27**	5.80	9.36	29.26	110.2	248.3
Ind. Production	252.8	219.0	62.41	36.96	7.07	**.040**	**3.45**	10.95	12.60	29.70	118.8	268.6
Employment	259.2	39.81	66.25	41.08	7.12	**.0004**	**3.27**	7.67	10.75	29.48	116.6	253.1
Unemployment	262.4	262.4	**2.34**	**2.10**	**1.87**	5.76	23.52	56.40	35.64	45.83	70.72	219.9
GNP deflator	256.0	216.0	65.77	40.70	7.29	**.0009**	**3.61**	10.43	17.55	25.40	116.6	266.0
C.P.I.	256.0	210.2	65.93	40.70	7.34	**.0009**	**3.57**	9.67	16.40	25.00	118.8	267.6
Wages	259.2	85.95	66.09	40.96	7.34	**.0004**	**3.57**	9.48	18.40	29.05	118.8	268.6
Real wages	259.2	219.0	66.09	40.57	7.12	**.0001**	**2.89**	8.12	16.72	27.45	118.8	262.4
Money stock	252.8	210.2	64.80	40.32	7.07	**0.002**	**3.68**	11.02	18.49	29.48	118.8	264.3
Velocity	252.8	259.2	43.83	28.51	**0.92**	**3.76**	5.80	25.90	11.35	33.75	39.94	271.2
Bond yield	243.3	204.5	24.01	11.56	**1.36**	**0.53**	**2.43**	8.58	22.27	26.83	108.1	210.5
Stock prices	249.6	240.2	49.42	32.26	4.04	**0.048**	3.88	9.73	9.73	24.20	96.04	223.8
β_1 and $\beta_1 \neq 0$	1	2	3	4	5	6	7	8	9	10	20	40
Real GNP	424.3	12.67	66.09	40.83	7.45	**.0009**	**3.68**	11.42	19.00	29.59	118.8	268.9
Nominal GNP	424.3	16.89	66.09	40.96	7.45	**.0004**	**3.61**	11.35	19.0	29.70	118.8	268.9
Real pcap. GNP	428.5	20.88	66.25	41.08	7.45	**.0001**	**3.61**	11.35	18.57	29.59	118.8	269.2
Ind. Production	424.3	292.4	63.20	38.56	7.23	**0.04**	4.20	12.18	19.00	29.70	118.8	268.6
Employment	428.5	24.70	66.25	41.08	7.50	**.0001**	**3.57**	11.22	18.57	29.59	121.0	269.2
Unemployment	424.3	412.1	**2.84**	**2.49**	**3.80**	5.76	26.41	57.60	39.31	48.16	139.2	259.8
GNP deflator	424.3	20.70	65.77	40.70	7.34	**.0009**	**3.64**	11.42	18..66	29.81	118.8	268.9
C.P.I.	424.3	32.26	65.93	40.70	7.34	**.0009**	**3.64**	11.35	18.66	29.81	118.8	268.9
Wages	428.5	19.00	66.09	40.96	7.39	**.0004**	**3.64**	11.35	18.57	29.70	118.8	268.9
Real wages	428.5	68.65	66.25	40.83	7.45	**.0001**	**3.64**	11.35	18.66	29.70	118.8	269.2
Money stock	424.3	75.47	65.44	40.44	7.12	**.002**	**3.68**	11.62	18.92	29.81	118.8	266.6
Velocity	420.2	219.0	17.13	28.51	**1.90**	5.15	22.37	31.02	34.80	34.92	123.2	271.5
Bond yield	408.0	353.4	35.64	21.16	**2.92**	**0.57**	5.42	12.81	22.37	36.12	118.8	266.3
Stock prices	420.2	353.4	50.12	33.75	4.66	**0.05**	4.88	14.36	21.62	31.69	118.8	271.2

In bold: The non-rejection values of the one-sided tests at the 95% significant level.

TABLE 8												
$\hat{R}$ in (8) with AR(1) u_t												
Periods p. cylce	1	2	3	4	5	6	7	8	9	10	20	40
Real GNP	8.06	174.2	432.6	234.1	41.12	**0.006**	25.60	64.32	187.7	169.2	725.2	9999
Nominal GNP	8.29	9999	681.2	376.3	68.22	**0.003**	38.31	103.0	255.6	276.8	1194	9999
Real pcap. GNP	9.67	9999	8930	4886	882.1	**0.02**	506.2	135.7	38.01	35.29	9999	9999
Ind. Production	5.90	9999	31.58	16.64	**2.99**	**0.01**	**3.16**	5.10	14.59	12.67	52.56	8790
Employment	10.24	9999	9999	9999	9999	**0.01**	71.74	184.4	9999	9999	9999	9999
Unemployment	13.76	9999	143.2	840.4	1147	275.5	52.86	988.6	120.1	9999	6925	1525
GNP deflator	8.12	9999	196.0	112.3	20.07	**.0003**	10.95	31.02	63.68	85.74	344.4	667.7
C.P.I.	8.41	9999	262.4	151.3	27.45	**.0004**	14.89	41.99	79.92	119.0	494.4	9999
Wages	8.00	6674	670.8	225.0	40.57	**.0004**	23.23	61.15	153.7	163.5	726.3	12.40
Real wages	8.35	9999	519.8	272.2	48.72	**.0002**	31.24	75.69	237.4	196.5	808.8	12.90
Money stock	5.29	9999	25.70	13.46	**2.34**	**.0009**	**1.53**	3.92	11.42	10.04	44.35	86.11
Velocity	35.40	9999	17.64	9.48	**0.62**	**3.68**	9.61	23.52	16.40	12.88	14.28	24.60
Bond yield	9.24	278.2	13.42	**1.27**	**0.14**	**0.01**	**0.26**	**0.54**	4.11	4.90	5.07	5.15
Stock prices	5.38	9999	9.42	4.97	**0.67**	**.008**	**0.81**	**2.13**	4.92	4.88	16.81	21.43
$\hat{R}$ in (8) with AR(2) u_t												
Real GNP	4.20	9999	552.2	292.4	57.60	**0.008**	36.12	89.49	260.1	225.9	828.8	1509
Nominal GNP	3.88	9999	1156	519.8	96.43	**0.006**	54.46	143.5	356.0	370.1	13.69	2538
Real pcap. GNP	4.00	9999	9999	7022	1296	**0.03**	729.0	1836	502.3	4792	9999	9999
Ind. Production	4.70	9999	40.19	16.89	3.64	**0.02**	**2.99**	7.02	19.89	16.81	73.78	9999
Employment	5.88	9999	9999	9999	**1.93**	**0.02**	9999	9999	9999	9999	9999	9999
Unemployment	5.66	9999	9999	1417	1927	415.7	7435	115.1	152.7	9999	7274	184.6
GNP deflator	4.53	9999	234.1	9999	26.31	**0.004**	15.60	43.29	87.98	114.9	416.4	775.0
C.P.I.	4.53	9999	320.4	171.6	36.60	**0.006**	21.34	58.36	110.4	160.5	604.6	9999
Wages	4.04	9999	580.8	285.6	55.20	**0.003**	33.06	84.82	213.4	219.0	865.8	9999
Real wages	4.57	9999	630.0	309.7	66.25	**0.003**	44.22	104.2	330.5	264.3	913.2	9999
Money stock	4.41	9999	31.58	13.46	**2.78**	**0.001**	**2.07**	5.15	15.52	9.79	49.98	9999
Velocity	19.09	9999	12.81	15.52	**0.94**	5.56	12.60	24.80	19.27	18.40	19.71	25.80
Bond yield	5.52	9999	12.68	**1.63**	**0.21**	**0.60**	**0.37**	**0.73**	4.10	4.17	4.22	6.05
Stock prices	5.15	9999	15.36	5.06	**0.92**	**0.01**	**1.06**	**2.75**	6.40	6.50	19.44	26.31
$\hat{R}$ in (8) with AR(3) u_t												
Real GNP	20.61	9999	1451	249.6	73.78	**0.01**	148.3	538.7	1741	1812	6412	7871
Nominal GNP	18.92	9999	2246	404.0	118.8	**0.01**	222.0	863.7	2390	2956	9999	9999
Real pcap. GNP	19.62	9999	292.4	5285	1560	**0.08**	2941	9999	9999	9999	9999	9999
Ind. Production	22.84	9999	108.1	17.55	5.29	**0.06**	13.03	42.64	132.7	136.6	464.4	533.1
Employment	18.74	9999	9999	9999	225.0	**0.06**	9999	9999	9999	9999	9999	9999
Unemployment	29.59	9999	1780	864.9	1958	907.8	9999	9999	9999	9999	9999	956.6
GNP deflator	22.27	9999	670.8	113.8	35.16	**0.01**	63.52	260.1	594.8	912.0	3041	4057
C.P.I.	22.37	9999	936.3	161.3	48.02	**0.01**	85.93	351.1	746.3	1262	4366	5984
Wages	19.71	9999	1361	237.1	70.89	**0.008**	134.5	512.5	1436	1745	6368	7544
Real wages	22.27	9999	1764	292.4	85.19	**0.008**	180.6	629.5	2221	2101	7157	7860
Money stock	21.25	9999	88.92	14.21	4.12	**0.003**	8.94	32.14	106.5	107.5	390.8	519.8
Velocity	102.0	9999	2959	10.30	**1.10**	11.76	50.97	167.9	144.0	138.3	124.7	149.3
Bond yield	29.59	9999	625.0	**1.27**	**0.24**	**0.07**	**1.41**	4.45	7.61	19.09	31.58	30.91
Stock prices	25.40	9999	2883	5.24	**1.18**	**0.02**	4.75	17.72	42.77	51.69	148.1	9999

In bold: The non-rejection values of the one-sided tests at the 95% significant level; 9999 means that the value of the test statistic exceeds that quantity.