TESTING OF FRACTIONAL COINTEGRATION IN MACROECONOMIC TIME SERIES

Luis A. Gil-Alana[*]

Humboldt Universität zu Berlin, Institut für Statistik und Ökonometrie, Berlin, Germany
University of Navarre, Department of Economics, Pamplona, Spain

ABSTRACT

We propose in this article a two-step testing procedure of fractional cointegration in macroeconomic time series. It is based on Robinson's (1994) univariate tests and is similar in spirit to the one proposed by Engle and Granger (1987), testing initially the order of integration of the individual series and then, testing the degree of integration of the residuals from the cointegrating relationship. Finite-sample critical values of the new tests are computed and Monte Carlo experiments are conducted to examine the size and the power properties of the tests in finite samples. An empirical application, using the same datasets as in Engle and Granger (1987) and Campbell and Shiller (1987) is also carried out at the end of the article.

JEL Classification: C15; C22.

Keywords : Fractional cointegration; Long memory.

1.   Introduction

Modelling macroeconomic time series is a matter that still remains controversial. From an univariate viewpoint, deterministic models based on linear (or quadratic) functions of time were initially proposed. Later on, following the work and ideas of Box and Jenkins (1970), stochastic models based on first (or second) differences of the series were considered, and they became popular, especially after the seminal paper of Nelson and Plosser (1982). In that paper, they showed, using tests of Fuller (1976) and Dickey and Fuller (1979), that many US macroeconomic time series contained a unit root. The tests of Dickey and Fuller (1979), however, were shown to have very low power against certain types of alternatives and several other unit-root tests were proposed in the following years (eg, Said and Dickey, 1984, 1985; Phillips, 1987; Phillips and Perron, 1988; Kwiatkowski et al., 1992, etc.). All these unit-root tests are nested in autoregressive (AR) alternatives. However, the AR model is merely one of the many models that nest a unit root. Robinson (1994) proposes tests for unit roots and other hypotheses which are embedded in a fractional model of form

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, ... \qquad (1)$$

where u_t is I(0), (defined for the purpose of the present paper as a covariance stationary process with spectral density function which is positive and finite at the zero frequency), and where the unit root hypothesis corresponds to the null: H_o: $d = 1$. Unlike most unit-root tests embedded in AR alternatives, Robinson's (1994) tests have standard null and local limit distributions and are easy to implement in raw time series.

In a multivariate framework, Engle and Granger (1987) noticed that many time series may have a common trend and suggested a technique called cointegration which implies that several series which are I(d) may be related in a way such that there exists at least one linear combination of them which is I(d-b) with b > 0. For the case of d = b = 1, they proposed a two-step testing strategy based on Dickey and Fuller's (1979) tests. More robust tests in this context of integers d and b were later proposed by Johansen (1988, 1995).

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In this article we propose a testing procedure for testing the null hypothesis of no cointegration against fractionally cointegrated alternatives. That is, we extend Engle and Granger's (1987) procedure to the case where d and b can be both possible real numbers. The outline of the paper is as follows: Section 2 describes the version of the tests of Robinson (1994) used in this article. Section 3 presents the procedure for testing fractional cointegration as well as finite-sample critical values of the tests obtained by simulation. Section 4 contains some Monte Carlo experiments in order to examine the size and the power properties of the new tests. Several illustrative examples are carried out in Section 5 while Section 6 contains some concluding comments.

2.   The tests of Robinson (1994)

Let's suppose that $\{x_t, t = 1,2,\ldots, T\}$ is the time series we observe and consider the model given by (1) with u_t with spectral density function given by

$$f(\lambda;\sigma^2;\tau) \;=\; \frac{\sigma^2}{2\pi} g(\lambda;\tau), \qquad -\pi \leq \lambda < \pi,$$

where $\sigma^2 = V(\varepsilon_t)$ and τ are unknown but g is of known form. For example, if u_t is white noise, $(u_t = \varepsilon_t)$, $g \equiv 1$, and if u_t is a stationary AR process of form: $\phi(L)u_t = \varepsilon_t$, $g = |\phi(e^{i\lambda})|^{-2}$, so that the AR parameters are function of τ.

Robinson (1994) proposes a Lagrange Multiplier (LM) test of:

$$H_o: \quad d \;=\; d_o, \qquad\qquad (2)$$

in (1) for any real value d_o. Specifically, the test statistic is given by

$$\hat{r} \;=\; \left(\frac{T}{\hat{A}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \qquad\qquad (3)$$

where

$$\hat{a} \;=\; \frac{-2\pi}{T}\sum_{j=1}^{T-1}\psi(\lambda_j)\,g(\lambda_j;\hat{\tau})^{-1}\,I(\lambda_j); \quad \hat{\sigma}^2 = \frac{2\pi}{T}\sum_{j=1}^{T-1}g(\lambda_j;\hat{\tau})^{-1}\,I(\lambda_j);$$

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$$\hat{A} = \frac{2}{T} \left(\sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j)\varepsilon(\lambda_j)' \times \left(\sum_{j=1}^{T-1} \varepsilon(\lambda_j)\varepsilon(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{T-1} \varepsilon(\lambda_j)\psi(\lambda_j)' \right);$$

$$\psi(\lambda_j) = \log \left| 2\sin\frac{\lambda_j}{2} \right|; \qquad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial\tau}\log g(\lambda_j;\hat{\tau}),$$

where $I(\lambda_j)$ is the periodogram of $\hat{u}_t = (1 - L)^{d_o} x_t$, evaluated at $\lambda_j = 2\pi j/T$, and $\hat{\tau}$ is obtained

by minimising $\sigma^2(\tau)$. Robinson (1994) showed that under certain regularity conditions:

$$\hat{r} \rightarrow_d N(0,1) \qquad as \ T \rightarrow \infty. \tag{4}$$

Thus, a one-sided $100\alpha\%$-level test of (2) against the alternative H_1: $d > d_o$ is given by

the rule: 'Reject H_o if $\hat{r} > z_\alpha$', where the probability that a standard normal variate exceeds

z_α is α, and, conversely, an approximate one-sided $100\alpha\%$-level test of (2) against the

alternative H_1: $d < d_o$ is given by the rule: 'Reject H_o if $\hat{r} < -z_\alpha$'. Furthermore, he shows that

the above tests are efficient in the Pitman sense, i.e. that against local alternatives of form: H_a:

$d = d_o + \delta T^{-1/2}$, for $\delta \neq 0$, the limit distribution is normal with variance 1 and mean which

cannot (when u_t is Gaussian) be exceeded in absolute value by that of any rival regular

statistic. This version of the tests of Robinson (1994) was used in several empirical

applications by Gil-Alana and Robinson (1997) and Gil-Alana (2000a,b). Other versions of

his tests, based on seasonal (quarterly and monthly) and cyclical models can be found

respectively in Gil-Alana and Robinson (2000) and Gil-Alana (1999, 2000c). In the following

section, we present a procedure for testing fractional cointegration which is based on

Robinson's (1994) univariate tests described above.

3.   Testing of fractional cointegration

The components of a (nx1) vector X_t are said to be fractionally cointegrated of order d,b, ($X_t \sim$

CI(d,b)) if a): all components of X_t are integrated of order d ($X_{it} \sim I(d)$), and b): there exists a

vector r ($r \neq 0$) such that $N_t = r'X_t$ is integrated of order d-b ($N_t \sim I(d-b)$) with b > 0. The vector

r is called the cointegrating vector and r'X_t will represent an equilibrium constraint operating on the long-run component of X_t. If X_t has more than two components, then there may be more than one cointegrating vector r, though in what follows we will assume that X_t does have only two components, so that $X_t = (X_{1t}, X_{2t})'$, where X_{1t} and X_{2t} correspond to the variables to be analysed later.

We present here a testing procedure that follows a similar methodology to the one proposed by Engle and Granger (1987). First, we test that both individual series are integrated of the same order, (let's say, eg., $\bar{d}$). This can be done using Robinson's (1994) univariate tests described in Section 2. Alternatively, there exists a multivariate version of the tests of Robinson (1994), (see, Gil-Alana, 1998), which may also be used for this purpose. Once we have checked this, we can estimate the cointegrating parameters from the cointegrating regression. Since all the linear combinations of X_{1t} and X_{2t} except the one defined by the cointegrating relation will be integrated of order $\bar{d}$, the least squares (LS) estimate from the regression of X_{1t} on X_{2t}, under cointegration, will produce a good estimate of it. In standard cointegration analysis (in which $d = b = 1$), Stock (1987) showed that the LS estimate of the cointegrating parameter was consistent and converged in probability at the rate $T^{1-\delta}$ for any $\delta > 0$, rather than the usual rate $T^{1/2}$. Cheung and Lai (1993), Robinson and Marinucci (1998) and others extended the analysis to the case of fractional cointegration, and showed that the LS estimate was also consistent though with possible different convergence rates, according to the cointegrating order. Given the consistency of the LS estimate of the cointegrating regression, we can use Robinson's (1994) tests for testing the order of integration in the equilibrium errors e_t, where $e_t = X_{1t} - \hat{\alpha} X_{2t}$, with $\hat{\alpha}$ as the OLS estimate of the cointegrating parameter, and the test statistic will still remain with the same standard limit distribution. Thus, we could consider the model

$$(1 - L)^d e_t = v_t, \qquad t = 1, 2, \dots \qquad (5)$$

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with I(0) v_t, and test the null hypothesis

$$H_o : \quad d \; = \; \overline{d} \tag{6}$$

against the alternative:

$$H_o : \quad d \; < \; \overline{d}, \tag{7}$$

and the test statistic will have an asymptotic null N(0,1) distribution. Rejections of (6) against (7) will imply that X_{1t} and X_{2t} are fractionally cointegrated, given that the equilibrium errors display a smaller degree of integration than that of the individuals series. However, given that the equilibrium errors are not actually observed but obtained from minimizing the residual variance of the cointegrating regression, in finite samples, the residual series might be biased toward stationarity, and thus, we would expect the null hypothesis to be rejected more often than suggested by the nominal size of Robinson's (1994) tests. A similar problem arises in Engle and Granger (1987) and Cheung and Lai (1993) when testing cointegration. In order to cope with this problem, the empirical size of Robinson's (1994) tests for cointegration in finite samples is obtained using a simulation approach.

(Table 1 about here)

In Table 1 we report finite-sample critical values of Robinson's (1994) tests for cointegration, with T = 50, 100, 200 and 300. We use the Monte Carlo method in 50,000 replications, assuming that the true system is of two I(d) processes with Gaussian independent white noise disturbances that are not cointegrated, and take values of d ranging from 0.6 through 1.5 with 0.1 increments. For simplicity, we assume that v_t in (5) is white noise, though we could also have extended the analysis to cover the case of weakly autocorrelated disturbances. We see in this table that the critical values are similar across d. They have a negative mean and the critical values corresponding to the left-hand side distribution, (which is the one required to test (6) against (7)), are smaller than those given by the Normal distribution, which is consistent with the earlier discussion that, when testing H_o (6) against (7), the use of the standard critical values will result in the cointegration tests rejecting the

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null hypothesis of no cointegration too often. We also see in this table that the empirical distributions are positively skewed with kurtosis greater than 3, though increasing the sample size, the three statistics (mean, skewness and kurtosis) approximate to the values corresponding to the Normal distribution.

4.   The power of the tests in finite samples

We next examine the power properties of the new tests described in Section 3 relative to the ADF and Geweke and Porter-Hudak (GPH, 1983) tests for cointegration. We consider a bivariate system, claimed to be non-cointegrated under the null hypothesis. The ADF unit-root test recommended by Engle and Granger (1987) is given by the usual t-statistic for b_0 in

$$(1 - L)e_t = b_0 e_{t-1} + b_1 (1 - L)e_{t-1} + ... + b_p (1 - L)e_{t-p} + \varepsilon_t,$$

where e_t are the equilibrium errors and the lag parameter p can be selected using some model-selection procedures such as the Akaike and Schwarz information criteria. The GPH test for cointegration proposed by Cheung and Lai (1993) is based on the estimation of the fractional differencing parameter d, in the linear regression

$$\ln\left(I(\lambda_j)\right) = \beta_0 + \beta_1 \ln\left(4\sin^2\left(\frac{\lambda_j}{2}\right)\right) + u_t,$$

where $\lambda_j = 2\pi j/T$ and $I(\lambda_j)$ is the periodogram of e_t evaluated at the ordinate j. Given that the LS estimate of β_1 provides a consistent estimate of 1-d (see Robinson, 1995), hypothesis testing concerning the value of d is based on the t-statistic of the regression coefficient.

Table 2 reports results of the power function of the three tests (ADF, GPH and Robinson) for cointegration against fractional and AR alternatives. Results for ADF and GPH tests have been taken from Cheung and Lai (1993). The power of a test is measured as the percentage of the time the test can reject a false null hypothesis of no cointegration, and the Monte Carlo experiment conducted here is described in Appendix I. We perform Robinson's

(1994) test statistic, assuming that the differenced series are white noise and AR processes of orders 1, 2 and 3, for 5% and 10% significance levels.

(Table 2 about here)

When testing against fractional alternatives, Robinson's (1994) tests perform better than the ADF and the GPH tests, and this is observed for white noise disturbances but also if they follow AR processes. The highest rejection frequencies are obtained with white noise disturbances if the integration order ranges between 0.05 and 0.75. but when this parameter approximates to 1, better results are obtained for weakly autocorrelated disturbances.

When testing against AR alternatives, again better statistical power properties are observed in Robinson (1994) relative to ADF and GPH tests, with higher rejection frequencies obtained at all values of the AR parameter. If this parameter ranges between 0.05 and 0.55, results are better when the disturbances are white noise, but if it ranges between 0.55 and 0.95, the tests behave better for weakly parametrically autocorrelated disturbances. The relative pronounced difference in power between Robinson's (1994) and the ADF and GPH tests for cointegration should not be surprising given that the ADF test assumes a strict I(0) and I(1) distinction and the GPH test requires estimation of the differencing parameter, whereas Robinson (1994) tests do allow fractional differencing and do not require estimation of the fractional differencing parameter. The performance of Robinson's (1994) tests in this context of cointegration is examined in the following section.

5.   Illustrative examples

We consider several pairs of variables that have been widely analysed in the literature in order to detect the presence of cointegrating relationships. In particular, we analyse the common behaviour between consumption and income, wages and prices, and nominal GNP and money, using the same dataset as in Engle and Granger (1987), and the relationship between stock prices and dividends, using the data in Campbell and Shiller (1987). All these pairs of

variables have been analysed by many authors. Thus, the relationship between consumption and income has been studied in Davidson et al. (1978) and also in Hall (1978), Campbell and Mankiw (1990), Qin (1991) and Ermisch and Westaway (1994) among others. The relation between wages and prices appears in Hall (1988), Mehra (1991) and Darrat (1994). An article relating stock prices and dividends is Campbell and Shiller (1987) and those relating nominal GNP and money include, amongst others, Hafer (1984) and Sims (1994). All these groups of variables were also analysed from a Bayesian viewpoint in DeJong (1992). The description of the time series data used in this application is given in Appendix II.

Table 3 reports the results of Robinson's (1994) tests for cointegration. The first two lines of each pair of variables correspond to the analysis of the individual series while the other two lines correspond to the results based on the residuals from the OLS regressions in both directions. We look at $\hat{r}$ given by (3), testing H_o (2) for values $d_o = 0.6$, (0.1), 1.5, with white noise disturbances.

(Table 3 about here)

Starting with consumption and income, we observe that for the individual series, H_o (2) cannot be rejected when $d_o = 0.9$, 1 and 1.1. However, looking at the residuals from the OLS regressions, these hypotheses are strongly rejected in favour of alternatives with smaller orders of integration. In fact, the null hypothesis cannot be rejected now when $d_o = 0.6$, 0.7 and 0.8, the lowest statistic appearing in both cases when $d_o = 0.7$. Thus, we find certain evidence of fractional cointegration between consumption and income, with the deviations from an equilibrium following a nonstationary fractional process with the order of integration smaller than one. Engle and Granger (1987) examined the same data, testing the null hypothesis of nonstationarity I(1) in the estimated residuals from the OLS regressions. Using the Cointegration Regression Durbin-Watson (CRDW) test, the null was rejected at 5% significance level but hardly at 1%, and using the ADF tests, it was rejected for the regression of consumption on income but hardly for the reverse even at 5% significance level. As

mentioned before, a problem with these testing procedures is that they only concentrate on the cases of I(1) and I(0) residuals and do not consider other fractionally integrated possibilities.

The results for the relationship between prices and wages clearly indicate a lack of cointegration. In fact, H_o (2) cannot be rejected when $d_o = 1$ and 1.1 for the individual series, and the same result is obtained when testing on the estimated residuals. This result is completely in line with the findings in Engle and Granger (1987). They found that the CRDW test from the cointegrating regression in either direction was 0.0054, suggesting that it was insignificantly different from zero. The ADF test of the regression of prices on wages was – 0.6 and for the reverse regression 0.2. None of these values approximated the critical value of 3.2 and therefore, their evidence accepted the null hypothesis of no cointegration.

The third example illustrates the relation between nominal GNP and nominal money. This is upon the quantity theory equation: $M \cdot V = P \cdot Y$, and most empirical applications stem from the assumption that velocity is constant or at least stationary. Under this general condition, log M, log P and log Y should be cointegrated with known unit parameters, and similarly, nominal GNP and nominal money should also be cointegrated. Engle and Granger (1987) failed to find evidence of cointegration using M1 as the monetary aggregate. The results in Table 3 show that a certain degree of fractional cointegration may appear, with the orders of integration of the individual series ranging between 0.9 and 1.1, but ranging between 0.7 and 0.9 for the residuals from the OLS regressions.

Finally, we examine the relationship between stock prices and dividends, using the same dataset as in Campbell and Shiller (1987). They applied the ADF tests on both individuals series and their results suggested that both series were integrated of order 1. Using DF and ADF tests on the residuals from the cointegrating regressions, their results were mixed: the former test rejected the null hypothesis of no cointegration at the 5% level while the latter narrowly failed to reject it at the 10%. The results in Table 3 again indicate that this pair of variables may be fractionally cointegrated. Looking at the individual series, the orders

of integration range between 0.9 and 1.1 for the stock prices and between 0.9 and 1.3 for dividends, with the lowest statistics appearing in both series at the unit root case (i.e, $d_o = 1$). However, the results for the estimated residuals suggest that the orders of integration are between 0.6 and 0.9, with the lowest statistics appearing in both cases at $d_o = 0.7$, and thus, implying mean reversion in the long run equilibrium relationship.

6.   Concluding comments

We have presented in this article a procedure for testing the null hypothesis of no cointegration against alternatives which are fractionally cointegrated. It is based on Robinson's (1994) univariate tests and it follows in spirit the same methodology as the one proposed by Engle and Granger (1987). Thus, we initially test the order of integration of the individual series and, if all them have the same order, we again test the degree of integration but this time on the residuals from the cointegrating regression. There will be a cointegrating relationship if the order of integration of these residuals is smaller than that of the individual series. Finite-sample critical values of the new tests are computed and several experiments conducted via Monte Carlo show that they have better power properties against both fractional and autoregressive alternatives in relation to other existing tests for cointegration.

The tests of Robinson (1994) were employed to analyse the relationship between consumption and income, CPI and wages, nominal GNP and money, and stock prices and dividends, using the same datasets as in Engle and Granger (1987) and Campbell and Shiller (1987). The results indicate that all these variables may be individually I(1), and testing the order of integration of the residuals from the OLS regressions, the results show that all pairs of variables (except CPI and wages) may be fractionally cointegrated, with the order of integration of the residuals being greater than 0.5 but smaller than 1. Note that (though it is not shown in Table 3), the tests of Robinson (1994) rejected H_o (2) with $d_o = 0.5$ against $d > 0.5$ for all estimated residuals in all series. Thus, the equilibrium errors are nonstationary but

display slow mean reversion, unlike the individual series where shocks seems to persist forever. These results are interesting in that they seem to overcome the mixing conclusions obtained in Engle and Granger (1987) and Campbell and Shiller (1987), the reason being that they only concentrated on I(0) and I(1) specifications and did not consider other possible fractional possibilities.

This article can be extended in several directions. The finite-sample critical values obtained in Section 3 can be extended to permit more than two variables and also to allow weakly parametrically autocorrelated disturbances. Other semiparametric methods of estimating and testing the fractional differencing parameter may also be applied on the estimated residuals from the cointegrating regressions. However, these methods may be too sensitive to the choice of the bandwidth parameter and, in that respect, a fully parametric model like this may be more appropriate. Extensions of the multivariate version of the tests of Robinson (1994) which permit us to test fractional cointegration in an unified treatment is now under way.

References

Box, G.E.P. and G.M. Jenkins, 1970, Time Series Analysis: Forecasting and Control, San Francisco, Holden Day.

Campbell, J.Y. and N.G. Mankiw, 1990, Permanent income, current income and consumption, Journal of Business and Economic Statistics 8, 265-279.

Campbell, J.Y. and R.J. Shiller, 1987, Cointegration and tests of present value models, Journal of Political Economy 95, 1062-1088.

Cheung, Y.W., and K.S. Lai, 1993, A fractional cointegration analysis of purchasing power parity, Journal of Business and Economic Statistics 11, 103-112.

Darrat, A.F., 1994, Wage growth and the inflationary process: A reexamination, Southern Economic Journal 61, 181-190.

Davidson, J.E., D.F. Hendry, F. Srba and S. Yeo, 1978, Econometric modelling of the aggregate time series relationship between consumer's expenditure and income in the UK, Economic Journal 88, 661-692.

DeJong, D.N., 1992, Cointegration and trend stationary in macroeconomic time series, Journal of Econometrics 52, 347-370.

Dickey, D.A. and W.A. Fuller, 1979, Distribution of the estimators for autoregressive time series with a unit root, Journal of the American Statistical Association 74, 427-431.

Engle, R.F. and C.W.J. Granger, 1987, Cointegration and error correction: Representation, estimation and testing, Econometrica 55, 251-276.

Ermisch, J. and P. Westaway, 1994, The dynamics of aggregate consumption in an open economy life cycle model, Scottish Journal of Political Economy 41, 113-127.

Fuller, W.A., 1976, Introduction to statistical time series, Willey Series of Probability and Mathematical Statistics, Willey, New York.

Geweke, J. and S. Porter-Hudak, 1983, The estimation and application of long memory time series models, Journal of Time Series Analysis 4, 221-238.

Gil-Alana, L.A., 1998, Multivariate tests of fractionally integrated hypotheses, EUI Working Paper No. 98/19, Florence.

Gil-Alana, L.A., 1999, Testing fractional integration with monthly data, Economic Modelling 16, 613-629.

Gil-Alana, L.A., 2000a, Mean reversion in the real exchange rate, forthcoming in Economics Letters.

Gil-Alana, L.A., 2000b, A fractionally integrated exponentially spectral model for the UK unemployment, forthcoming in Journal of Forecasting.

Gil-Alana, L.A., 2000c, Testing stochastic cycles in macroeconomic time series, forthcoming in Journal of Time Series Analysis.

Gil-Alana, L.A. and P.M. Robinson, 1997, Testing of unit roots and other nonstationary hypotheses in macroeconomic time series, Journal of Econometrics 80, 241-268.

Gil-Alana, L.A. and P.M. Robinson, 2000, Seasonal fractional integration in the UK and Japanese consumption and income, forthcoming in Journal of Applied Econometrics.

Hafer, R.W., 1984, The money-GNP link: Assessing alternative transaction measures, Federal Reserve Bank of St. Louis Review 66, 19-27.

Hall, R.E., 1978, Stochastic implications of the life cycle-permanent income hypothesis: Theory and evidence, Journal of Political Economy 86, 971-987.

Hall, R.E., 1988, The relation between prices and marginal cost in US industry, Journal of Political Economy 96, 921-947.

Johansen, S., 1988, Statistical analysis of cointegrating vectors, Journal of Economics Dynamics and Control 12, 231-254.

Johansen, S., 1995, Likelihood-based inference in cointegrated vector autoregressive models, Oxford University Press, New York.

Kwiatkowski, D., P.C.B. Phillips, P. Schmidt and Y. Shin, 1992, Testing the null hypothesis of stationary against the alternative of a unit root, Journal of Econometrics 54, 159-178.

Mehra, Y.P., 1991, Wage growth and the inflation process: An empirical note, American Economic Review 81, 931-937.

Nelson, C.R. and C.I. Plosser, 1982, Trends and random walks in macroeconomic time series, Journal of Monetary Economics 10, 139-162.

Phillips, P.C.B., 1987, Time series regression with a unit root, Econometrica 55, 277-301.

Phillips, P.C.B. and P. Perron, 1988, Testing for a unit root in a time series regression, Biometrika 75, 335-346.

Qin, D., 1991, Aggregate consumption and income in China: An econometric study, Journal of Comparative Economics 15,132-141.

Robinson, P.M., 1994, Efficient tests of nonstationary hypotheses, Journal of the American Statistical Association 89, 1420-1437.

Robinson, P.M., and D. Marinucci, 1998, Semiparametric frequency domain analysis of fractional cointegration, Sticerd, Discussion Paper No. 348.

Robinson, P.M., 1995, Log-periodogram regression of time series with long range dependence, Annals of Statistics 23, 1048-1072.

Said, E. and D.A. Dickey, 1984, Testing for unit roots in autoregressive-moving average models of unknown order, Biometrika 71, 599-607.

Said, E. and D.A. Dickey, 1985, Hypothesis testing in ARIMA(p,1,q) models, Journal of the American Statistical Association 80, 369-374.

Sims, C.A., 1994, A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy, Economic Theory 4, 381-399.

Stock, J.H., 1987, Asymptotic properties of least squares estimators of cointegrating vectors, Econometrica 55, 1035-1056.

Appendix I

To illustrate the potential difference in power between the tests of Robinson (1994) and the GPH and the ADF tests of cointegration, a Monte Carlo experiment, similar to that in Engle and Granger (1987) and Cheung and Lai (1993) is conducted. We consider a bivariate system where X_{1t} and X_{2t} are given by

$$X_{1t} + X_{2t} = U_{1t}, \quad t = 1, 2, ... \tag{A1}$$

$$X_{1t} + 2 X_{2t} = U_{2t}, \quad t = 1, 2. ..., \tag{A2}$$

where $(1 - L) U_{1t} = \varepsilon_{1t}$, and U_{2t} is generated, alternatively, as an autoregressive process

$$(1 - \rho L) U_{2t} = \varepsilon_{2t}, \quad t = 1, 2, ... \tag{A3}$$

or as a fractional white noise process

$$(1 - L)^d U_{2t} = \varepsilon_{2t}, \quad t = 1, 2, ... , \tag{A4}$$

where the innovations ε_{1t} and ε_{2t} are generated as independent standard normal variates. Thus, if $\rho = 1$ in (A3) or $d = 1$ in (A4), the two series are I(1) and non-cointegrated; if U_{2t} is generated by (A3) and $|\rho| < 1$, X_{1t} and X_{2t} are cointegrated, and (A2) is their cointegrating relationship; alternatively, if U_{2t} is generated by (A4) and $d < 1$, X_{1t} and X_{2t} are fractionally cointegrated. As in Engle and Granger (1987) and Cheung and Lai (1993), we used samples os size T = 76, and sample series of X_{1t} and X_{2t} were generated setting the initial values of U_{1t} and U_{2t} equal to zero, creating 126 observations, of which the first 50 were discarded to reduce the effect of the initial conditions.

Appendix II

C_t: US quarterly real per capita consumption on non-durables from 1947.I to 1981.II

Y_t: US quarterly real per capita disposable income from 1947.I to 1981.II.

CPI_t: Log of the US monthly Consumer Price Index from 1950.1 to 1979.12.

W_t: Log of the US monthly production worker wage in manufacturing from 1950.1 to 1979.12.

GNP_t: Log of the US quarterly nominal Gross National Product from 1959.I to 1981.II

$M1_t$:   Log of the US quarterly nominal M1 from 1959.I to 1981.II

SP_t: US real annual stock prices from 1871 to 1986.

D_t: US real annual dividends from 1871 to 1986.

The first six series has been taken from Engle and Granger (1987) and the remaining two from Campbell and Shiller (1987).

TABLE 1									
Finite-sample critical values of Robinson (1994) tests for cointegration[*]									
T = 50									
Perc ./ d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
0.1%	-2.94	-2.94	-2.95	-2.93	-2.93	-2.93	-2.93	-2.92	-2.93	-2.92
0.5%	-2.65	-2.66	-2.66	-2.67	-2.66	-2.66	-2.66	-2.66	-2.65	-2.65
1%	-2.51	-2.52	-2.53	-2.52	-2.52	-2.52	-2.52	-2.51	-2.50	-2.50
2.5%	-2.29	-2.30	-2.31	-2.30	-2.30	-2.30	-2.29	-2.29	-2.28	-2.27
5%	-2.09	-2.10	-2.11	-2.11	-2.10	-2.09	-2.08	-2.08	-2.07	-2.07
10%	-1.84	-1.85	-1.85	-1.84	-1.84	-1.84	-1.83	-1.82	-1.82	-1.81
Mean	-0.70	-0.72	-0.72	-0.72	-0.71	-0.70	-0.70	-0.69	-0.68	-0.68
Skewness	0.59	0.59	0.59	0.59	0.58	0.57	0.56	0.56	0.55	0.54
Kurtosis	3.67	3.68	3.69	3.70	3.68	3.64	3.60	3.59	3.53	3.50
T = 100									
Perc ./ d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
0.1%	-2.96	-2.95	-2.95	-2.97	-2.96	-2.94	-2.95	-2.96	-2.96	-2.96
0.5%	-2.64	-2.65	-2.64	-2.63	-2.63	-2.62	-2.62	-2.61	-2.60	-2.60
1%	-2.48	-2.49	-2.48	-2.48	-2.47	-2.47	-2.46	-2.45	-2.45	-2.44
2.5%	-2.23	-2.24	-2.24	-2.23	-2.23	-2.22	-2.21	-2.21	-2.20	-2.20
5%	-2.01	-2.00	-2.00	-2.01	-2.00	-2.00	-1.99	-1.99	-1.99	-1.98
10%	-1.74	-1.75	-1.75	-1.74	-1.74	-1.72	-1.71	-1.71	-1.71	-1.70
Mean	-0.56	-0.57	-0.58	-0.57	-0.56	-0.56	-0.55	-0.54	-0.54	-0.53
Skewness	0.46	0.46	0.46	0.45	0.45	0.45	0.45	0.44	0.44	0.45
Kurtosis	3.41	3.40	3.39	3.39	3.40	3.40	3.38	3.35	3.34	3.34
T = 200									
Perc ./ d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
0.1%	-3.04	-3.08	-3.07	-3.14	-3.19	-3.20	-3.12	-3.12	-3.10	-3.06
0.5%	-2..71	-2.73	-2.70	-2.70	-2.66	-2.64	-2.62	-2.61	-2.63	-2.64
1%	-2.50	-2.48	-2.47	-2.46	-2.45	-2.46	-2.45	-2.44	-2.44	-2.43
2.5%	-2.21	-2.20	-2.20	-2.21	-2.20	-2.20	-2.20	-2.19	-2.18	-2.18
5%	-1.95	-1.97	-1.97	-1.97	-1.97	-1.96	-1.94	-1.94	-1.93	-1.93
10%	-1.64	-1.65	-1.67	-1.66	-1.66	-1.65	-1.63	-1.62	-1.62	-1.61
Mean	-0.44	-0.46	-0.46	-0.46	-0.46	-0.45	-0.44	-0.43	-0.43	-0.42
Skewness	0.31	0.30	0.30	0.31	0.32	0.32	0.33	0.34	0.35	0.36
Kurtosis	3.18	3.17	3.18	3.23	3.26	3.27	3.28	3.28	3.30	3.31
T = 300									
Perc. / d	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
0.1%	-2.96	-2.96	-3.04	-3.12	-3.19	-3.22	-3.19	-3.17	-3.17	-3.15
0.5%	-2.52	-2.56	-2.63	-2.61	-2.60	-2.59	-2.60	-2.59	-2.61	-2.61
1%	-2.41	-2.42	-2.44	-2.45	-2.44	-2.44	-2.44	-2.44	-2.44	-2.44
2.5%	-2.17	-2.18	-2.19	-2.20	-2.18	-2.17	-2.16	-2.14	-2.13	-2.13
5%	-1.90	-1.91	-1.92	-1.92	-1.91	-1.90	-1.89	-1.88	-1.87	-1.87
10%	-1.59	-1.60	-1.60	-1.61	-1.60	-1.60	-1.60	-1.59	-1.58	-1.58
Mean	-0.37	-0.39	-0.39	-0.39	-0.39	-0.38	-0.38	-0.37	-0.37	-0.36
Skewness	0.31	0.29	0.28	0.28	0.29	0.29	0.30	0.30	0.30	0.30
Kurtosis	3.07	3.08	3.10	3.13	3.15	3.15	3.15	3.14	3.13	3.13

[*]: The empirical distribution has been obtained using 50,000 replications in simulation, assuming that the true system is of two non-cointegrated I(d) processes. The test statistic is $\hat{r}$ in (3).

TABLE 2										
Power of the ADF, GDH and Robinson tests for cointegration against fractional alternatives[*]										
Size	Test	0.95	0.85	0.75	0.65	0.55	0.45	0.35	0.25	0.15	0.05
5%	ADF (ρ = 4)	0.06	0.07	0.10	0.14	0.19	0.26	0.36	0.50	0.61	0.73
	GPH (μ = .55)	0.06	0.09	0.15	0.21	0.30	0.37	0.47	0.56	0.61	0.64
	GPH (μ = .575)	0.06	0.10	0.16	0.24	0.33	0.42	0.53	0.62	0.67	0.71
	GPH (μ = .60)	0.06	0.11	0.18	0.28	0.40	0.52	0.63	0.73	0.78	0.81
	ROB (Wh. N)	**0.07**	**0.22**	**0.50**	**0.78**	**0.94**	**0.99**	**0.99**	**1.00**	**1.00**	**1.00**
	ROB (AR (1))	**0.15**	**0.22**	**0.35**	**0.52**	**0.71**	**0.85**	**0.94**	**0.97**	**0.99**	**0.99**
	ROB (AR (2))	**0.22**	**0.26**	**0.31**	**0.41**	**0.54**	**0.67**	**0.78**	**0.86**	**0.92**	**0.95**
	ROB (AR (3))	**0.30**	**0.32**	**0.35**	**0.41**	**0.50**	**0.59**	**0.68**	**0.76**	**0.82**	**0.85**
10%	ADF (ρ = 4)	0.11	0.13	0.18	0.24	0.32	0.41	0.53	0.67	0.78	0.87
	GPH (μ = .55)	0.12	0.17	0.26	0.35	0.46	0.56	0.65	0.72	0.76	0.78
	GPH (μ = .575)	0.12	0.18	0.27	0.38	0.50	0.60	0.71	0.77	0.81	0.83
	GPH (μ = .60)	0.12	0.19	0.30	0.43	0.57	0.68	0.79	0.85	0.88	0.90
	ROB (Wh. N)	**0.16**	**0.37**	**0.66**	**0.88**	**0.97**	**0.99**	**1.00**	**1.00**	**1.00**	**1.00**
	ROB (AR (1))	**0.26**	**0.36**	**0.51**	**0.69**	**0.84**	**0.94**	**0.98**	**0.99**	**0.99**	**0.99**
	ROB (AR (2))	**0.32**	**0.37**	**0.45**	**0.57**	**0.69**	**0.81**	**0.89**	**0.94**	**0.97**	**0.98**
	ROB (AR (3))	**0.40**	**0.43**	**0.47**	**0.55**	**0.64**	**0.73**	**0.81**	**0.87**	**0.91**	**0.94**
Power of the ADF, GDH and Robinson tests for cointegration against autoregressive alternatives										
Size	Test	0.95	0.85	0.75	0.65	0.55	0.45	0.35	0.25	0.15	0.05
5%	ADF (ρ = 4)	0.07	0.16	0.29	0.42	0.53	0.61	0.66	0.73	0.75	0.77
	GPH (μ = .55)	0.07	0.17	0.33	0.49	0.59	0.64	0.67	0.69	0.68	0.66
	GPH (μ = .575)	0.07	0.17	0.35	0.52	0.63	0.69	0.73	0.75	0.74	0.72
	GPH (μ = .60)	0.07	0.18	0.37	0.56	0.70	0.76	0.81	0.84	0.83	0.83
	ROB (Wh. N)	**0.07**	**0.21**	**0.46**	**0.72**	**0.90**	**0.98**	**0.99**	**1.00**	**1.00**	**1.00**
	ROB (AR (1))	**0.18**	**0.36**	**0.59**	**0.76**	**0.88**	**0.94**	**0.97**	**0.98**	**0.99**	**0.99**
	ROB (AR (2))	**0.27**	**0.42**	**0.58**	**0.70**	**0.80**	**0.86**	**0.90**	**0.93**	**0.95**	**0.96**
	ROB (AR (3))	**0.37**	**0.49**	**0.60**	**0.69**	**0.75**	**0.80**	**0.83**	**0.86**	**0.87**	**0.88**
10%	ADF (ρ = 4)	0.14	0.28	0.46	0.60	0.71	0.78	0.82	0.86	0.88	0.89
	GPH (μ = .55)	0.14	0.29	0.50	0.66	0.75	0.78	0.81	0.82	0.81	0.79
	GPH (μ = .575)	0.14	0.30	0.52	0.69	0.78	0.82	0.85	0.86	0.85	0.84
	GPH (μ = .60)	0.14	0.30	0.54	0.72	0.82	0.87	0.90	0.91	0.92	0.91
	ROB (Wh. N)	**0.16**	**0.38**	**0.65**	**0.87**	**0.97**	**0.99**	**0.99**	**1.00**	**1.00**	**1.00**
	ROB (AR (1))	**0.30**	**0.54**	**0.76**	**0.89**	**0.95**	**0.98**	**0.99**	**0.99**	**0.99**	**0.99**
	ROB (AR (2))	**0.39**	**0.58**	**0.74**	**0.84**	**0.90**	**0.94**	**0.96**	**0.97**	**0.98**	**0.98**
	ROB (AR (3))	**0.47**	**0.63**	**0.74**	**0.82**	**0.87**	**0.90**	**0.92**	**0.93**	**0.94**	**0.95**

[*]: ADF is augmented Dickey-Fuller test statistic and p is the lag parameter selected using AIC and SIC criteria. GPH is Geweke and Porter-Hudak test statistic and μ is the value used in the sample size function $n=T^{\mu}$. Results for ADF and GPH have been taken from Cheung and Lai (1993), (pages 108 and 109). The critical values of Robinson's (1994) tests with white noise disturbances were taken from Table 1, while those corresponding to AR disturbances were obtained by simulation. The power of each test is based on 10,000 replications.

<table>
<tr><td colspan="11" align="center">TABLE 3</td></tr>
<tr><td colspan="11" align="center">Testing fractional cointegration with the tests of Robinson (1994)[*]</td></tr>
<tr><td colspan="11" align="center">Consumption (C_t) and Income (Y_t)</td></tr>
<tr><td>Series / d_o</td><td>0.6</td><td>0.7</td><td>0.8</td><td>0.9</td><td>1.0</td><td>1.1</td><td>1.2</td><td>1.3</td><td>1.4</td><td>1.5</td></tr>
<tr><td>C_t</td><td>7.13</td><td>4.81</td><td>2.74</td><td>1.00'</td><td>-0.40'</td><td>-1.49'</td><td>-2.33</td><td>-2.98</td><td>-3.50</td><td>-3.91</td></tr>
<tr><td>Y_t</td><td>6.74</td><td>4.40</td><td>2.36</td><td>0.65'</td><td>-0.73'</td><td>-1.80'</td><td>-2.63</td><td>-3.27</td><td>-3.77</td><td>-4.16</td></tr>
<tr><td>$C_t - 0.52 - 0.23\,Y_t$</td><td>0.98'</td><td>-0.24'</td><td>-1.27'</td><td>-2.12</td><td>-2.83</td><td>-3.40</td><td>-3.87</td><td>-4.26</td><td>-4.58</td><td>-4.85</td></tr>
<tr><td>$Y_t + 0.22 - 4.30\,C_t$</td><td>0.95'</td><td>-0.26'</td><td>-1.27'</td><td>-2.12</td><td>-2.81</td><td>-3.39</td><td>-3.86</td><td>-4.25</td><td>-4.57</td><td>-4.84</td></tr>
<tr><td colspan="11" align="center">Consumer Price Index (CPI_t) and Wages (W_t)</td></tr>
<tr><td>Series / d_o</td><td>0.6</td><td>0.7</td><td>0.8</td><td>0.9</td><td>1.0</td><td>1.1</td><td>1.2</td><td>1.3</td><td>1.4</td><td>1.5</td></tr>
<tr><td>CPI_t</td><td>17.44</td><td>11.40</td><td>6.41</td><td>2.60</td><td>-0.17'</td><td>-1.16'</td><td>-3.60</td><td>-4.66</td><td>-5.47</td><td>-6.10</td></tr>
<tr><td>W_t</td><td>27.84</td><td>16.41</td><td>8.91</td><td>3.93</td><td>0.57'</td><td>-1.73'</td><td>-3.33</td><td>-4.47</td><td>-5.32</td><td>-5.97</td></tr>
<tr><td>$CPI_t - 3.91 - 0.70\,W_t$</td><td>35.07</td><td>24.95</td><td>15.55</td><td>8.10</td><td>1.53'</td><td>-0.68'</td><td>-2.97</td><td>-4.49</td><td>-5.52</td><td>-6.26</td></tr>
<tr><td>$W_t + 5.31 - 13.6\,CPI_t$</td><td>32.40</td><td>22.63</td><td>13.86</td><td>7.04</td><td>1.22'</td><td>-1.02'</td><td>-3.16</td><td>-4.60</td><td>-5.60</td><td>-6.31</td></tr>
<tr><td colspan="11" align="center">Gross National Product (GNP_t) and Money ($M1_t$)</td></tr>
<tr><td>Series / d_o</td><td>0.6</td><td>0.7</td><td>0.8</td><td>0.9</td><td>1.0</td><td>1.1</td><td>1.2</td><td>1.3</td><td>1.4</td><td>1.5</td></tr>
<tr><td>GNP_t</td><td>5.48</td><td>3.72</td><td>2.10</td><td>0.70'</td><td>-0.44'</td><td>-1.35'</td><td>-2.05</td><td>-2.60</td><td>-3.03</td><td>-3.38</td></tr>
<tr><td>$M1_t$</td><td>5.65</td><td>3.80</td><td>2.13</td><td>0.70'</td><td>-0.46'</td><td>-1.37'</td><td>-2.07</td><td>-2.62</td><td>-3.05</td><td>-3.39</td></tr>
<tr><td>$GNP_t + 12.1 - 1.54\,M1_t$</td><td>3.15</td><td>1.26'</td><td>-0.22'</td><td>-1.32'</td><td>-2.13</td><td>-2.71</td><td>-3.13</td><td>-3.44</td><td>-3.67</td><td>-3.86</td></tr>
<tr><td>$GNP_t + 12.1 - 1.54\,M1_t$</td><td>3.24</td><td>1.31'</td><td>-0.19'</td><td>-1.31'</td><td>-2.12</td><td>-2.70</td><td>-3.12</td><td>-3.43</td><td>-3.67</td><td>-3.85</td></tr>
<tr><td colspan="11" align="center">Stock Prices (SP_t) and Dividends (D_t)</td></tr>
<tr><td>Series / d_o</td><td>0.6</td><td>0.7</td><td>0.8</td><td>0.9</td><td>1.0</td><td>1.1</td><td>1.2</td><td>1.3</td><td>1.4</td><td>1.5</td></tr>
<tr><td>SP_t</td><td>6.02</td><td>4.03</td><td>2.35</td><td>0.95'</td><td>-0.19'</td><td>-1.10'</td><td>-2.03</td><td>-2.40</td><td>-2.86</td><td>-3.23</td></tr>
<tr><td>D_t</td><td>5.41</td><td>4.03</td><td>2.84</td><td>1.79'</td><td>0.86'</td><td>-0.94'</td><td>-1.08'</td><td>-1.30'</td><td>-1.96</td><td>-2.30</td></tr>
<tr><td>$SP_t + 0.12 - 30.99\,D_t$</td><td>1.48'</td><td>0.21'</td><td>-0.79'</td><td>-1.58'</td><td>-2.20</td><td>-2.68</td><td>-3.06</td><td>-3.37</td><td>-3.62</td><td>-3.83</td></tr>
<tr><td>$D_t - 0.005 - 0.027\,SP_t$</td><td>1.10'</td><td>-0.02'</td><td>-0.90'</td><td>-1.60'</td><td>-2.15</td><td>-2.60</td><td>-2.97</td><td>-3.27</td><td>-3.52</td><td>-3.74</td></tr>
</table>

* ' and in bold: Non-rejection values at the 95% significance level. The critical values for the cases corresponding to the OLS regressions are those given in Table 1.