On the Job Search and the Wage Distribution*

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Abstract: The estimates of the structural parameters of a job separations model derived from the theory of on-the-job search are reported in this papers. Given that each employer pays the same wage to all observably equivalent workers and that wage dispersion across employers exists in the sense that different employers offer different wages to the same worker, the theory implies that a firms separations outflow is the sum of an exogenous flow to unemployment and a job-to-job flow that decreases with the employer's wage. We find that the model provides a good description of job separation flows in our cross-firm sample drawn from the Danish Pay and Performance database for the year 1994-1995. The estimates also explain most and in some cases all of the employment effect, defined as the difference between median wage earned by employed workers and the median wage offered by employers. Finally, the empirical results also provide estimates of the curvature of the cost of search function as well as the parameters of the separations equations.

1 Introduction

Ample evidence suggests that employers pay observably similar workers very different wages.[1] There are two principal alternative explanations: Ei-

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[1] Papers that provide empirical documentation include Kruger and Summers (1988), Katz and Summers (1989), Davis and Haltiwanger (1991), Doms, Dunne and Troske (1997), Abowd, Kramarz and Marolis (1999) and Oi and Idson (1999).

ther employers pursue different wage policies and/or high wage firms attract more able workers. Recent empirical studies by Abowd and Kramerz (2000a, 2000b), based on the analysis of matched employer-worker data for both the U.S. and France, conclude that these two explanations are equally important as explanations for well known industry and size differentials.

To the extent that employer wage policies differ, the typical worker has an incentive to seek out the relatively higher paying firms. Indeed, on-the-job search motivated by wage dispersion provides an explanation for the commonly observed negative association between cross-firm wages paid and separation flows.[2] The prediction that the expected wage earned by an employed worker increases with the length of time employed since the last unemployment spell is a consequence of job to a job movements in such an environment. This implied employment effect on the wage earned provides an alternative interpretation of positive tenure and experience coefficients in empirical wage equations.

The purpose of this paper is to estimate a structural model of worker separations based on the theory of on-the-job search using cross-firm observations and to test the associated implications of the theory for the differences between the wages earned by employed workers and the wages offered job applicants. Generally, the theory provides a parsimonious interpretation of the observations in two senses. First, the model fits the separations data at the establishment level and the implied estimates of the structural parameters make sense. Second, the estimated separations behavior explains much of the average employment effect, specifically defined as the difference between the median wage earned and the median wage offered by employers.

Burdett's [1978] provides the original treatment of search on the job given wage dispersion across employers. In his model, employer's pursue a stationary wage policy by assumption, an unemployed worker accepts the first offer received above some reservation wage, and an employed worker moves to a higher paying job when the opportunity arises. Mortensen [1990] demonstrates that the process by which workers move from one job to another will generate a distribution of wages earned over employed workers which stochastically dominates the distribution of wages offered outsiders. The location difference between the two distributions, called the employment effect on wages implied by the model, is a consequence of the fact that employed workers move up the "job ladder" by flowing from lower to higher paying jobs without intervening spells of unemployment. The formal model used in the estimation is a extension of Burdett's theory that allows for an endogenously chosen search intensity. The data strongly supports the need for incorporating this feature into the model.

The data used in the estimation are drawn from the Danish Integrated Database for Labour Market Research (IDA). This matched employer-employee data source, a product of Statistics Denmark, includes all employment, wage and characteristics for employees in all workplaces in Denmark since 1980. The data of interest for this paper includes cross section information on the total number of workers employed in each establishment in November of 1994, the

[2] For a review of examples, see Farber (1999).

number of these who are still employed one year later, and the wage paid each employee during a survey year, November 1994 to November 1995. Information on the occupation membership of each employee is also available in the data set and is used in this paper to create the sub-samples studied. An employer's wage is defined as the average paid to its employees of the type under study. The distribution of wages earned is the size weighted distribution of employer wages while the distribution of wage offers is weighted by the relative number of workers hired from non-employment. Employer separation functions and wage offer distributions are estimated under the maintained assumption that all workers in the specified sub-sample under study are equally productive in every employing firm. In other words, wage differences across firms represent pure dispersion is the maintained hypothesis. The results are reported for sub-samples defined by worker occupation as well as for the total sample.

The estimates of the separation model parameters imply a negative relationship between search intensity and wage for all industries and occupations, as implied by the theory. In other words, search intensity is high in low wage jobs but drops off, typically quite dramatically, as the wage earned by an employed worker increases within the support of the wage offer distribution. As empirical separation rates could be negatively related to the wage even if search intensities were to increase as wages increase, these results offer support for the theory and a structural interpretation of the reduced form negative relationship between firm separation rates and wages paid.

As a consequence of worker movements from employment at a specific wage to non-employment and to employment at a higher wage, one can show that the distribution of wages earned by employed workers follows a law of motion that depends only on the wage offer distribution and the wage-contingent separation function. As a consequence, the estimated separation function can be used to solve for a theoretical steady state distribution of wages earned by employed workers and that predicted distribution can be compared with the actual distribution constructed from the data. As a consequence of movement from lower to higher paying jobs, the distribution of wages earned stochastically dominates the distribution of wages offered.

As predicted by the theory, the actual distribution of wages earned always lies to the right of the distribution of wages offered. Furthermore, the observed distribution of wages earned and that predicted by the estimated model are remarkably close for the full sample and for not all of the sub-samples studied. As a bi-product of the estimation, we are also able to make inferences about the unobserved shape of the cost of search effort function. Although there is substantial variation across the sub-samples, a quadratic cost of search effort function is a good approximation for the merged samples.

2 Job Search and Wages

2.1 A Model of Match Separation

Following Burdett (1978), we represent the set of workers in the labor market by the unit interval, all of whom are labor market participants. All are identical, live forever, and act to maximize expected wealth. Let w represent an employed worker's current wage and let $F(w)$ represent the probability that a randomly selected wage offer is less than w where the employer's weight implicitly reflects employer recruiting effort. In other words, $F(w)$ is the fraction of vacancies that offer wage w or less. To simplify the derivations below, the wage offer distribution is regarded as continuous.

Offers arrive at a Poisson frequency λs where s is a measure of endogenous search effort.[3] Each worker chooses search effort subject to a twice differential increasing convex cost function $c(s)$ with the properties $c(0) = c'(0)$. Finally, suppose that job matches end for exogenous reasons at the exponential job destruction rate δ and that the worker flows to non-employment as a consequence. Then, under the assumption that worker act to maximize expected wealth, the current wage contingent value of employment, $W(w)$, solves the continuous time Bellman equation

$$rW(w) = \max_{s\geq 0} \left\{ \begin{array}{c} w - c(s) + \lambda s \int [\max \langle W(x), W(w) \rangle - W(w)] \, dF(x) \\ +\delta[U - W(w)] \end{array} \right\} \quad (1)$$

where U is the value of non-employed search.

The difference between wage and search cost on the right side of equation (1) is the net current income. The next term on the right side represents the expected capital gain associated with the possible arrival of an outside offer and the last term is the expected capital loss attributable to job destruction. Hence, the equation is an arbitrage condition which defines the asset value of being employed to be that which equates the riskless return on the asset value of the search while employed option to current net income plus expected capital gains and losses associated with the option. This relationship is a continuous time equivalent of the well known Bellman equation of dynamic programming. Indeed, because equation (1) can rewritten as

$$W(w) = \max_{s\geq 0} \left\{ \frac{w - c(s) + \delta U + \lambda s \int \max \langle W(x), W(w) \rangle \, dF(x)}{r + \delta + \lambda s} \right\}$$

and because the right hand side satisfies Blackwell's sufficient conditions for a contraction on the space of differentiable and increasing real valued functions, the value function is the unique fixed point of the contraction map on that space. (See Lucas and Stockey (1989).)

[3] There is no loss of generality in the linearity of the relationship. However, the implicit assumption that worker who do not make an e•ort recieve no o•ers does have content.

Because the solution $W(w)$ to (1) is increasing in w, an employed worker accepts any offer greater than her current wage. Indeed,

$$W'(w) = \frac{1}{r + \delta + \lambda s(w)[1 - F(w)]} > 0$$

by the envelope theorem where $s(w)$ is the optimal search effort choice. Given the first order condition for an interior solution, integration by parts, and an appropriate substitution, it follows that

$$\begin{aligned} c'(s(w)) &= \lambda \int_{w}^{\overline{w}} [W(x) - W(w)]\, dF(x) = \lambda \int_{w}^{\overline{w}} W'(x)[1 - F(x)]dx \quad (2) \\ &= \lambda \int_{w}^{\overline{w}} \frac{[1 - F(x)]dx}{r + \delta + \lambda s(x)[1 - F(x)]} \end{aligned}$$

where $\overline{w}$ is the upper support of the wage offer distribution. In other words, the optimal search effort function is the unique particular solution to this integral equation. Because the uniqueness of the value function implies a unique search effort strategy, it follows that this functional equation has a unique solution. As an implication of the equation, search effort, $s(w)$, is strictly decreasing and continuous in the wage earned by convexity of the cost of search function.

Consider the same worker when non-employed. The value of unemployment solves the analogous asset pricing equation

$$rU = \max_{s \geq 0} \left\{ b + \lambda s \int [\max \langle W(x), U \rangle - U]\, dF(x) - c(s) \right\} \quad (3)$$

where b represents income forgone when employed, e.g., the unemployment benefit. The worker's reservation wage, R, is the solution to

$$W(R) = U.$$

Under the assumption that the costs of search effort is the same whether employed or not, a comparison of equations (1), (2) and (3) imply that optimal search effort when unemployed, denoted as s_0, equals search effort when employed at the worker's reservation wage and, consequently, the worker's reservation wage is simply the unemployment compensation, i.e.,

$$s_0 = s(R) \quad (4)$$

and

$$R = b. \quad (5)$$

Note that this result is sensitive to the assumption that search technology is not contingent on employment status.

In sum, the overall job duration hazard for any workers employed by an employer paying wage w is

$$d(w) = \delta + \lambda s(w)[1 - F(w)]. \quad (6)$$

Under the assumption that an employer pays all workers the same wage and the cost of search is the same for all workers, the sample function represents the employer's separation rate as well.

2.2 Steady State Wage Distribution

Given the wage offer distribution, $F(w)$, and the model of worker flows developed in the theory section, the distribution of wage across employed workers, denote as $G(w)$, converges over time to a unique steady state in a stationary environment. The separation theory outlined above predicts that the wages of employed workers generally exceed the wages offered workers by employers in the sense that $G(w)$ stochastically dominates $F(w)$. The purpose of this section is to derive the formal relationship between the two distributions. The empirical interest in this relationship arises because both are observable in our data.

Workers flow from unemployment to employment at rate $\lambda s_0[1-F(R)]$, equal to the product of the offer arrival rate and the probability that a randomly generated offer exceeds the reservation wage R. Workers flow from employment to unemployment at exogenous rate δ in the modeled market. Hence, if the total number of participants is fixed, then the steady state fraction who are unemployed, u, balances these two flows, i.e., u solves

$$\frac{u}{1-u} = \frac{\delta}{\lambda s_0[1-F(R)]} = \frac{\delta}{\lambda s_0} \tag{7}$$

since $F(R) = 0$ when all workers have the same reservation wage or their reservation wages are less than the lower support of the wage offer distribution.

By analogous reasoning, the flow of unemployed workers who obtain a job paying w or less is $s_0\lambda[F(w) - F(R)]u$. The flow out of this subset, defined as employed workers who are paid w or less which has measure $(1-u)G(w)$, is the flow of those who lose their jobs, equal to $\delta G(w)(1-u)$, plus the flow who find jobs paying more than w. Since the rate at which workers search depends the current wage, the flow that finds a higher wage is

$$\lambda \int_{\underline{w}}^{w} s(z)[1-F(w)](1-u)dG(z)$$

where $z \in [\underline{w}, w]$ represents a wage in the interval of interest and $(1-u)dG(z)$ is the measure of workers who are earning that wage. Hence, the steady state solution for the distribution function $G(w)$ solves the integral equation

$$\delta G(w) + \lambda[1-F(w)] \int_{\underline{w}}^{w} s(z)dG(w) \tag{8}$$
$$= \frac{\lambda s_0[F(w) - F(R)]u}{(1-u)} = \delta F(w).$$

where the last equality is implied by $F(R) = 0$ and equation (7).

Equation (8) has qualitative implications of considerable interest for the predicted relationship between the distribution of wages offered to new employees and the distributions of wages paid to workers who are already employed. Namely,

$$\frac{F(w) - G(w)}{1-F(w)} = \frac{\lambda}{\delta} \int_{\underline{w}}^{w} s(z)dG(w) > 0 \text{ for all } w \in (\underline{w}, \overline{w}) \tag{9}$$

implies that the wages paid employed workers are higher than those offered to new hires in the sense that $G(w)$ stochastically dominates $F(w)$. This difference can be interpreted as an employment premium or employment effect. It arises because worker flow from lower to higher paying jobs in the model without intervening periods of unemployment. Note that the premium declines with the job destruction rate but increases with the offer arrival rate parameter because workers return to unemployment more frequently as δ increases but move to higher paying jobs more rapidly as λ increases.

3 Estimating the Separation Function

3.1 Maximum Likelihood Procedure

If workers are identical in the sense that they face the same wage offer distribution and the same search cost, then the labor force separation rate for any firm i takes the form

$$d_i = \delta + \lambda s(w_i)[1 - F(w_i)] \tag{10}$$

where w_i is the firm's wage, δ is the exogenous job destruction rate, λ is an offer arrival parameter, $s(w)$ is the optimal search effort of a worker employed at wage w, and $F(w)$ is the wage offer cdf. The purpose of this section is to report estimates the separation equation using cross firm wage and separations data. The search intensity function is the unique solution the functional equation

$$s(w) = \phi\left(\lambda \int_w^{\overline{w}} \frac{[1 - F(x)]dx}{r + \delta + \lambda s(x)[1 - F(x)]}\right)$$

by virtue of equation (2) where $\phi(\cdot)$ is the inverse of the marginal cost function $c'(\cdot)$. The estimates that follow assume that a cost function of the form

$$c(s) = c_0 \frac{(s)^{1+\frac{1}{\gamma}}}{1 + \frac{1}{\gamma}}$$

so that the search effort function solves the functional equation

$$s(w) = \frac{1}{c_0}\left(\lambda \int_w^{\overline{w}} \frac{[1 - F(x)]dx}{r + \delta + \lambda s(x)[1 - F(x)]}\right)^{\gamma}.$$

In other words, we assume that current search effort is an iso-elastic function of the expected future marginal return to search effort.

As we do not observe and measure search effort directly, the scale parameter c_0 is not identified by our data. Equivalently, we can only estimate the function up to an arbitrary choice of the unit in which search effort is measured. Given the fact that it is convenient to normalize the search effort choice of the non-employed at unity, i.e., $s(\underline{w}) = 1$ by assumption, the structural search effort function estimated in the paper solves the parametric functional equation

$$s(w) = \left(\frac{\int_{w}^{\overline{w}} \frac{[1-F(x)]dx}{r+\delta+\lambda s(x)[1-F(x)]}}{\int_{\underline{w}}^{\overline{w}} \frac{[1-F(x)]dx}{r+\delta+\lambda s(x)[1-F(x)]}} \right)^{\gamma}. \tag{11}$$

The solution, $s(w)$, can be interpreted then as the search effort of a worker currently employed at wage w relative to the effort made by the same worker were he employed at the lowest wage paid in the market $\underline{w}$.

The Danish Pay and Performance data contains cross firm observations on the wages paid each employee as well the number of workers employed in November, 1994, the number of these who remain employed during the year ending in November, 1995, and the number of unemployed workers hired during the year. Let w_i represent the average wage paid of employer $i \in \{1, 2, ...N\}$, let n_i denote the number of employees and x_i represent the number of "stayers", defined as those who were employees both initially and in the following November. The implications of the theory for the probability distribution of stayers in each firm conditional on the firm's wage and size are used for form a likelihood function for these firm level data conditional on the model's unknown parameter vector $(\gamma, \delta, \lambda)$ and "market prices" represented by the interest rate r and the offer distribution $F(w)$ which are in principle observable.

As the duration of employment at firm i is exponential with expectation $1/d_i$ for any worker under the assumption that all are identical, the probability of observing a worker who does not leave during the year is $p_i = e^{-d_i}$. As x_i is the realized number of stayers out of the total possible, x_i is binomial with probability of "success" p_i and "sample size" n_i, i.e.,

$$\Pr\{x = x_i | n = n_i\} = \begin{pmatrix} n_i \\ n_i^s \end{pmatrix} (e^{-d_i x})(1 - e^{-d_i})^{n_i - x} \tag{12}$$

Subject to equations (10) and (11) and conditional on r and F , the maximum likelihood estimates of the parameters $(\gamma, \delta, \lambda)$ are obtained by maximizing

$$\ln L(\alpha, \delta, \lambda) = \sum_{i=1}^{N} \left[\ln \begin{pmatrix} n_i \\ x_i \end{pmatrix} - d_i x_i + (n_i - x_i) \ln(1 - e^{-d_i}) \right]. \tag{13}$$

There are three additional complications in the actual procedure used to obtain the estimates reported below. First, wages, new hires, and employment are observable only for the firms in our sample. We use these data to form a sample analogue of the market offer distribution function $F(w)$ by weighting each firm's wage by the relative number of workers hired by that firm from non-employment. Only the non-employed are included in forming the weights because the theory implies a sample selection problem for the employed. Namely, according to the theory, no employed worker who was offered a wage less than or equal to the one earned will be observed among the new hires. Hence, if all new hires were included, the resulting distribution would be biased upward in the sense that it would stochastically dominate the true sample distribution. Because the non-employed all accept any offer above the common reservation wage and because all wages offered in the market by participating employers must be no less than this minimum, there is no selection problem for these workers. Although, point estimates obtained are conditional on this first state sample estimate of F, the confidence intervals are obtained by a boot strap procedure which account for the joint sampling uncertainty in F and the parameters. A full description of that procedure is included in the Appendix.

The interest rate r could be regarded as a parameter to be estimated. Alternatively, one can set its value equal to a good market estimate based on a priori grounds. The approach is adopted here since initial experimentation with the alternative implies an unreasonably large and not well determined estimate. The predetermined value chosen is 4.9% per year. Variation in this number between zero and say 10% per year has no significant effect on the resulting estimates of the other parameters.

Finally, the functional equation (11) does not yield a closed form solution for the search effort function $s(w)$. Hence, an iterative procedure is needed to solve for the maximum likelihood estimates of the underlying structural parameters. Essentially, the algorithm uses a hill climbing search procedure in the parameter space such that at stage t in the procedure the solution to (11) for $s(w)$ is that implied by the state $t-1$ values of the parameters. This solution is obtained using a numerical differential equation routine because $s(w)$ is also the unique solution to the ordinary differential equation

$$c''(s(w))s'(w) = c_0 \gamma s(w)^{\frac{1-\gamma}{\gamma}} s'(w) = \frac{-\lambda[1-F(w)]}{r+\delta+\lambda s(w)[1-F(w)]}$$

associated with the boundary condition $s(\overline{w}) = 0$ and the normalization $s(\underline{w}) = 1$, a fact that follows by differentiation of (2).

3.2 Data Description

The establishment included in the study are all those in the Danish private sector. The overall sample is referred to as the private sector sample. Sub-samples are constructed by partitioning the pooled sample by worker occupation. There are four occupational categories: skilled and unskilled workers, managers and salaried workers. The union of the first two categories can be regarded as the set of "blue collar" workers while the union of the third and fourth are "white collar" workers. A wage offer distribution, F, and a wage earned distribution, G, are constructed separately for each sub-sample. Specifically, for each establishment represented in the sub-sample, first an average hourly wage is constructed by averaging the Statistics Denmark estimate of the hourly wage earned by each worker of the occupational type employed by the establishment during the November 1994 to November 1995 year. Given this number, denoted w_i in the case of firm i above, F is constructed by weighting these by the fraction of all non-employed worker hired by firm i during the year. The wage earned distribution, G, uses the same firm wages but weights them by firm's relative employment size in November 1994.

Sample	Private sector	Skilled	Unskilled	Managers	Salaried
No. of Establishments	113,332.0	37,034.0	60,332.0	41,393.0	47,835.0
min wage	69.0	69.0	69.0	69.0	69.0
max wage	389.0	306.0	304.0	508.0	288.0
median wage offer	131.7	137.9	115.3	185.7	124.0
mean wage offer	138.4	141.8	121.2	189.1	128.2
std of wage offer	31.0	25.9	25.3	46.5	24.2
median wage earned	141.7	142.0	121.3	196.5	131.4
mean wage earned	146.6	145.3	126.9	198.3	133.3
std of wage earned	30.6	25.2	26.5	42.8	23.4
min employment	1	1	1	1	1
max employment	15,870	1,708	8,856	4,069	7,163
mean employment	13.3	6.6	8.5	7.0	7.0
std of employment	125.8	30.5	68.7	49.2	75.1
min no. of stayers	0	0	0	0	0
max no. of stayers	10,751	1,527	4,330	3,584	6,237
mean stayer no.	9.3	5.3	5.7	5.5	5.5
std of stayer no.	96.9	25.3	44.3	41.7	63.5

Wages are expressed in 1994 Danish kroner per hour.
Employment and no of stayers are person counts.

Table 1: Summary Statistics

Sample statistics are represented in Table 1. In constructing the wages rates and the person counts on which these statistics are based, only workers between the ages of 18 and 65 years of age are included. Because there are good reasons to believe that the hourly wages for some individuals were abnormally low and for other abnormally high due to measurement error, the firm average hourly wage was constructed after excluding the wages rates for certain individuals as follows: All workers for whom reported wage rates were less than 69 DKK per hour were excluded. This figure is regarded as an estimate of the effective legal minimum wage. The wage rates of all individuals in the top one percent of the observed distribution were also excluded. Although their wage rates were excluded for the purpose of computing the firm wage average, the estimate of the firm's wage policy, all were included in the employment and stayer number person counts. (Note: This paragraph needs to be expanded.)

4 Results

4.1 Private Sector Sample

Parameter estimates for the pooled or full sample of all private sector establishments are reported in Table 2. The exogenous separation rate δ and the offer arrival parameter λ are expressed as annual rates while the parameter γ is the elasticity of the search effort with respect to the marginal expected economic payoff to effort. Equivalently, its inverse $1/\gamma$ is the elasticity of the marginal cost function with respect to search effort. For reference, $\gamma = 1$ is the case of a quadratic recruiting cost function.

The point estimates for the full samples reported in the first column of Table 2 are those obtained using the maximum likelihood procedure described above after substituting the constructed sample wage offer distribution for the market distribution, F. Although these estimates are consistent, the standard errors of the estimates must take into account the sampling variation implicit in the estimate of F. For this purpose, the distribution of each parameter estimate was constructed via bootstrapping techniques described in the appendix. The mean of the distribution for each statistic as well as the 95% confidence bounds are reported in the second through fourth columns of the table. Note that differences between the maximum likelihood point estimates and the mean of the distribution typically appear only after the first three significant digits. Plots of the distribution of each statistic are included in the appendix.

Table 2: Parameter Estimates, Private Sector

	Point Estimate	Mean of Bootstraps	Lower Bound	Upper Bound
δ	0.2863	0.2863	0.2843	0.2882
γ	1.1347	1.1333	1.0251	1.2134
λ	0.5825	0.5824	0.5681	0.5959

The job destruction rate estimate, δ, is 28.6% per year. According to the model, this estimate suggests that jobs last a little about four years, abstracting from voluntary job to job movements. The estimate of λ is 58.3% per year. As the sum, $\delta + \lambda = 0.869$, is the separation rate of worker employed in the lowest paying establishment, the expected duration of such a match is only slightly longer than one year. However, as the wage earned increases the quit rate decreases, both because workers search less intensively and because higher paying jobs are more difficult to find. The parameter γ is the elasticity of search effort with respect to return, which declines with the wage earned, and its inverse is the elasticity of the marginal cost function with respect to search effort. The estimate $\gamma = 1.13$ suggest a cost of effort function which is approximately quadratic. Finally, all parameters are highly significant. Indeed, the fact that the 95% confidence intervals are all quite tight suggests well determined estimates with clear structural interpretations.

The relationship between search intensity and the wage, the function $s(w)$ implicitly defined by equation (2), and between the separation rate and the wage, the function $d(w)$ as specified in equation (6), implied by these parameter estimates are plotted in the top two panels of Figure 1. The point estimate for each value of the wage represented by the central curve while the upper and lower curves indicate the 95% confidence bounds. Obviously, the hypothesis that search effort declines with the wage earned is consistent with the estimated model. Indeed, search effort is high for low wages, declines dramatically as wages rise, and finally converges to zero in the upper tail of the distribution. For this reason and because the probability of finding a higher wage declines with w the separation function has a similar shape.

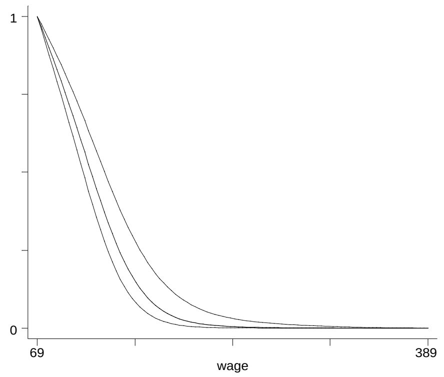


Figure 1a: Private Sector $s(w)$

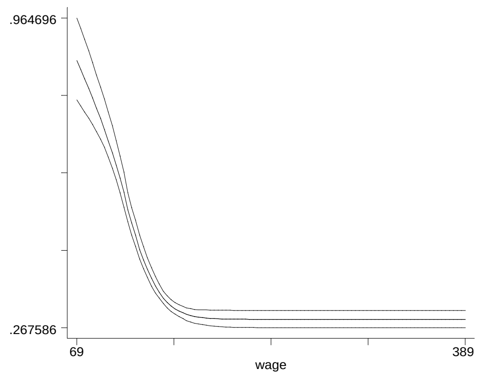


Figure 1b: Private Sector $d(w)$

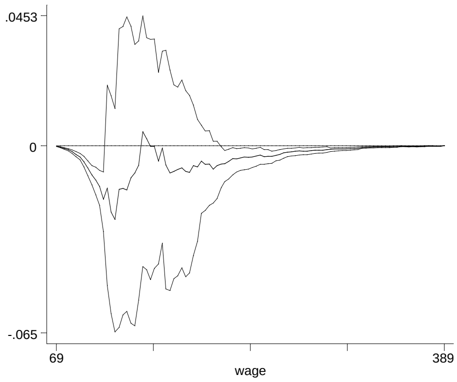


Figure 1c: $G(w)^{ss} - G(w)$

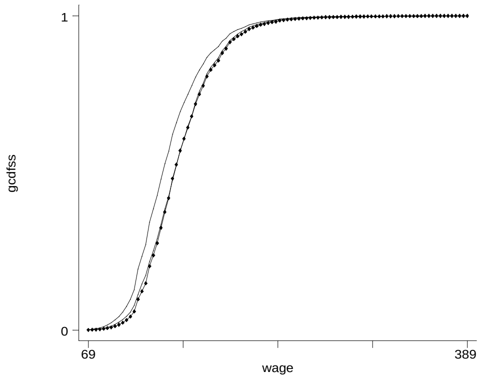


Figure 1d: $F(w), G(w), G(w)^{ss}$

The steady state condition, equation (8), together with our estimates of the separation function parameter vector $(\delta, \gamma, \lambda)$ and the observed offer distribution F can be used to compute an implied estimate of the steady state distribution of wages earned by employed workers, denoted G^{ss}. As the actual distribution G is observed in our data and not used in the estimation, a comparison of the two provides and out-of-sample test of the predictive power of the theory.

The actual wage offer distribution, $F(w)$, wage earned distribution, $G(w)$, and estimated steady state distribution, $G(w)^{ss}$, are plotted in the lower right panel of Figure 1. The wage offer distribution is represented by the curve that lies everywhere to the left of the other two, as the theory predicts it should. The estimated steady state wage distribution is represented by the curve containing the dots. It and the observed wage distribution, the remaining curve in the panel, are virtually coincident given the resolution of the chart. In short, for the structural parameters estimated, the model explains the entire employment effect, as represented by the magnitude of the vertical difference between the distributions of wage earned and offered. As measure by the differences in the medians of G and F reported in Table 1, the employment effect is 10 DKK per hour, about 7% of the 142 DKK median wage earned per hour.

The point estimate of the difference between the actual wage distribution function and the implied distribution, $G(w) - G^{ss}(w)$ is plotted for all values of the wage in the lower left panel of Figure 1. Specifically, the point estimate of the difference is the center curve while the other two represent 95% confidence interval bounds. Obviously, the hypothesis that there is no significant difference between the actual and the predicted distribution of earnings cannot be rejected for all wages with any significant mass. In sum, the theory seems to pass the test.

4.2 Occupational Sub-samples

In this section, the results for four mutually exclusive occupations are reported and compared. The occupation categories are skilled, unskilled, managerial and salaried workers. Interest in a partition of the full sample by these categories is motivated by the fact each corresponds broadly to separate non-competing segment of the Danish labor market. In other words, within each segment, the idea that all workers are more homogenous and are searching by sampling from a common offer distribution seems to make more sense. In contrast, this would not be the case for an analogous partition by industry since workers in any one of these occupations seek jobs in many industries.

As suggested by the comments above, each sub-sample is created by first constructing an establishment wage for the occupation by averaging the wages earned by all workers employed in the category. The occupation wage distribution is obtained by weighting each workplace wage by the numbers of workers employed in the occupation and the occupation offer distribution is constructed by weighting each workplace wage by the number of worker in each occupation hired from non-employment. Finally, the reported parameters are obtained using the methods outlined above for each occupation separately.

The parameter estimates are reported in Tables 3a-3d. Although all estimates are interpretable and statistically different from zero, it is obvious that the point estimates vary substantially across the sub-samples. Some of these differences are perfectly sensible. For example, the job destruction rate, δ, is higher for unskilled than for skilled workers, and the point estimates for both of these blue collar occupations exceed those for the white collar occupations, managers and salaried workers. Hence, these differences seem to be consistent with priors concerning exposure to job destruction. Although the absolute differences are not all that large, the 95% confidence bounds are quite tight suggesting that they are meaningful none the less. For example, the lower bound for unskilled workers just exceeds the upper bound for skilled while the lower bound on the estimate for skilled is roughly equal to the upper bound for salaried workers and only slightly lower than the upper bound for managers.

The reasons for large cross occupation differences in the other parameter estimates are less obvious. Why is the offer arrival rate frequency λ almost five time larger for the unskilled than for the other occupational categories? Why is the elasticity of search effort with respect to the return to search as much as ten times larger for skilled workers than any of the other categories? Do estimated differences in λ truly reflect differences in market tightness and do differences γ suggest differences in rate at which cost increases with effort across occupation groups as the theory suggests? What ever the answer, it is clear that the "nice" estimates of these two parameters obtained for the complete sample are not representative of any one of the occupational sub-samples.

Table 3a: Skilled Worker Subsample Parameter Estimates

	Point Estimate	Mean of Bootstraps	Lower Bound	Upper Bound
δ	0.2309	0.2302	0.2215	0.2398
γ	3.4806	3.4552	2.0247	4.6890
λ	0.1129	0.1159	0.0673	0.1673

Table 3b: Unskilled Worker Subsample Parameter Estimates

	Point Estimate	Mean of Bootstraps	Lower Bound	Upper Bound
δ	0.2742	0.2751	0.2468	0.2956
γ	0.4446	0.4558	0.1844	0.7417
λ	0.4992	0.4958	0.4362	0.5505

Table 3c: Mangerial Worker Subsample Parameter Estimates

	Point Estimate	Mean of Bootstraps	Lower Bound	Upper Bound
δ	0.2192	0.2179	0.2054	0.2333
γ	0.3482	0.3448	0.0000	0.6865
λ	0.0748	0.0767	0.0410	0.1239

Table 3d: Salaried Worker Subsample Parameter Estimates

	Point Estimate	Mean of Bootstraps	Lower Bound	Upper Bound
δ	0.1992	0.1997	0.1819	0.2201
γ	0.1825	0.1888	0.0000	0.6843
λ	0.1126	0.1124	0.0778	0.1455

How well do the estimated models explain the employment effect, defined as the extent to which the actual distribution of wages earned is shifted to the right as a consequence of job-ladder climbing? Table 4 reports the horizontal difference between G and F, the employment effect, and between G^{ss} and F, the predicted employment effect, at the first, second, and third quartiles for both the full sample and for the total and for each occupation sub sample. The ratio of the predicted over the actual difference in each case is reported as the percent explained.

As noted above, the model does very well explaining the differences between the wages earned by the already employed and the wages offered new employees. Although the estimates of the employment effect vary over the three measures, the actual and predicted differences are essentially the same in all cases. This statement is far from true for any one of the sub-samples. Indeed, the model explains only a fraction, about 10%, of the effect in the case of skilled workers and predicts an employment effect almost twice the actual in the case of skilled workers. The model does better in the other two cases by explaining 50% of the difference in the managerial mean offer and wage and almost 70% of the difference between the median wage earned and offered by salaried workers.

Occupation	Private Sector	Skilled	Unskilled	Managerial	Salaried
Employment effect at 1st quartile	8.10	4.29	4.63	14.21	5.71
Employment effect at 2d quartile	9.95	4.10	5.98	10.85	7.47
Employment effect at 3d quartile	8.82	2.40	7.75	8.13	4.28
Predicted effect at 1st quartile	9.37	0.49	8.73	3.51	4.17
Predicted effect at 2d quartile	9.68	0.37	11.30	5.41	5.18
Predicted effect at 3d quartiel	9.69	0.26	11.32	4.98	5.28
Percent explained at 1st quartile	116%	11%	188%	25%	73%
Percent explained at 2d quartile	97%	9%	189%	50%	69%
Percent explained at 3d quartile	110%	11%	146%	61%	123%

Table 4: Employment effects by Occupation

The computed (vertical) differences between the predicted and actual wage distribution functions at each wage, $G(w)^{ss} - G(w)$, and the 95% confidence interval around these estimates of the difference in distributions are plotted in Figure 2. These graphs provide a statistical test for whether the model explains the actual distribution of wages earned. Specifically, if zero is inside the bounds for all wage rates, the hypothesis is not rejected. Consistent with the under prediction of the employment effect in the case of skilled workers, Figure 2a implies that the difference between the predicted and actual wage distribution functions is significantly different for zero for all wage rates around and below the median. Although the sign goes the other way, the same test result holds for unskilled worker, i.e., a zero difference is not likely. Finally, the hypothesis that there is no difference cannot be rejected for either the managerial and or the salaried worker sub sample in the sense that a zero difference is inside the 95% confidence interval for almost all wage rates.

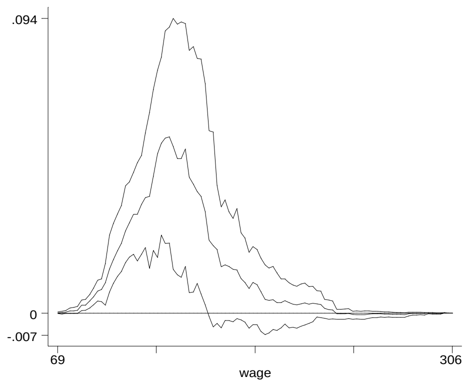


Fig 2a: Skilled $G(w)^{ss} - G(w)$

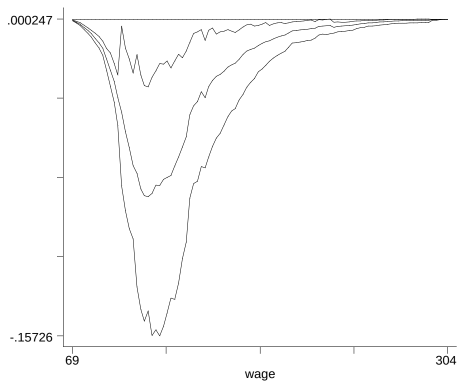
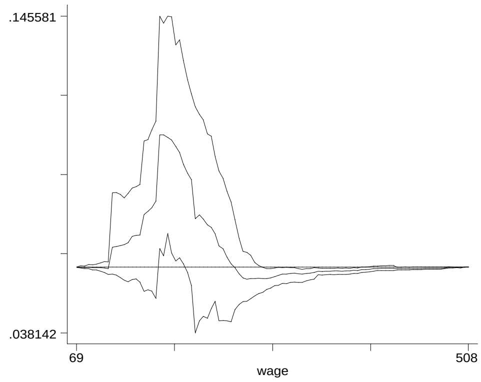


Fig 2b: Unskilled $G(w)^{ss} - G(w)$



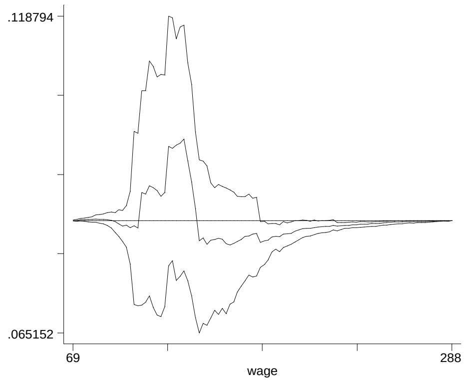


Fig 2c: Managerial $G(w)^{ss} - G(w)$ Fig 2d: Salaried $G(w)^{ss} - G(w)$

In sum, the results of the test are mixed. Futher more, the acceptance of the hypothesis that the models does explain the wage distribution in the case of the full sample may simply be a artifact of inappropriate aggregation across the occupations. Still, this test is a very stringent one.

5 Conclusions

Establishing a quantitative link between two well known empirical observations, that higher paying employers have lower turnover and that workers with more experience earn higher wages, is the principal empirical contribution of the paper. Given the existence of wage dispersion, a link is implied by the fact that workers have an incentive to seek higher paying jobs while the wage earned is positively correlated with the time since the last unemployment spell as a consequent outcome of worker search effort while employed.

The estimation exercise conducted in the paper is a one of estimating the parameters of a specific structural model of turnover using establishment level observation on separation flows, establishment wages, and the distribution of alternative wage offers. The model is an off the shelf on-the-job search model with endogenous search intensity. This exercise is highly successful in the sense that well determined coefficient estimates obtained that are consistent with the theory for both the full sample which regards all workers as identical as well as for the occupational sub-samples. However, there are important differences between the structural parameter estimates across the sub-samples.

Because workers flow from lower to higher paying jobs without intervening spells of non-employment, the on-the-job search model implies that the expected wage earned rises with experience as measured by the elapsed time since the last non-employment spell. The impact of this measure of experience on the wages of individual workers is reflected at the market level by the employment effect, defined as the horizontal difference in the distributions of wages earned by the employed and the distribution of wages offered. Conditional on the wage offer distribution and the structural parameter values, the model can be used to predict the employment effect. Since the wage distribution itself was not used in the estimation of the model's parameters, these predictions can be used as a test of the theory.

For the full sample of all workplaces with workers not distinguished by occupation, the theory passes the test with flying colors. Indeed, the theoretical prediction regarding the difference between the median wage earned and offered is within 3% of the actual difference. However, the same statement is not generally true for all the occupation sub-samples. Indeed, a test for no difference between the predicted and actual wage distributions fails at standard level of confidence for both the skilled and unskilled worker sub-sample. Still, the model does explain half or more of the observed employment effect and one cannot reject the hypothesis the predicted and observed wage distributions are the same given either the managerial or the salaried workers cases.

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