Private information, risk aversion, and the evolution of market research

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Abstract

On a homogeneous oligopoly market informed sellers are fully aware of market demand whereas uninformed sellers only know the distribution. We first derive the market results when sellers are risk averse, similarly to Ponssard (1979) who assumed risk neutrality throughout. With the help of these results evolutionary processes are formulated according to which sellers can switch to market research or refrain from it depending on the difference in profits of informed and uninformed sellers. We derive the evolutionarily stable number of informed sellers and discuss how it is influenced by market parameters.

JEL classification numbers: C72, D43, D82

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1. Introduction

The role of information has been often studied in two–stage models assuming risk neutral agents (see, e.g., Ponssard (1979), Li et al. (1987), Vives (1988), Ockenfels (1989), Chang and Lee (1992), Daughety and Reinganum (1994), Hwang (1993, 1995)). In these models the decisions to acquire information (taken at the first stage) are common knowledge when agents take other actions (e.g., quantities, prices, etc.) at the second stage. More recently, Hauk and Hurkens (in press) study information acquisition

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in a one–stage game (in the context of an oligopolistic market with uncertain demand) and show that relative to the two–stage game firms acquire less information.[1]

If market demand is stochastic, the decision whether or not to engage in active market research may crucially depend on the degree of risk aversion. Therefore, in our study we allow sellers to be risk averse. However, a simplifying assumption of our model is that market research signals market demand perfectly, i.e. in case of market research no ambiguity of market demand remains. Thus the sellers on the market either know the market demand function, we name them informed sellers, or only the a priori-probabilities of demand conditions. We refer to the latter sellers as uninformed sellers.

In the tradition of Ponssard (1979), who assumed risk neutral sellers, we first describe a homogeneous oligopoly market with linear demand and linear production costs. Different demand conditions correspond to parallel shifts of the demand curve. For a given parameter α of risk aversion we derive the market results for arbitrary numbers m of informed sellers among the n sellers on the market, $0 \leq m \leq n$.

With these result we then derive the profits of informed and uninformed sellers as well as their difference. All these variables are, of course, stochastic. They allow us to formulate an evolutionary process determining the number of informed sellers which is governed by the difference in profits of informed and uninformed sellers (as well as by a small probability ε of mutation).

Our evolutionary analysis first concentrates on the deterministic border case where sellers interact infinitely often with each time newly drawn market demand before being able to adapt their market research decision. Due to the infinite number of interactions with newly and randomly generated demand conditions, the difference in profits of informed and uninformed sellers is that one for the mean demand curve with 0-variance. Thus in case of no unintended mutation the evolutionary process is deterministic.

The other extreme assumption, which we consider, is just one interaction before adapting market research individually. To illustrate how to analyse the stochastic evolutionary process we focus on the simple case of just two sellers and derive the

[1] Due to the terminology introduced in Levine and Ponssard (1977) one distinguishes between private information acquisition (two–stage game) and secret information acquisition (one–stage game).

stationary distribution over the various numbers $m = 0, 1$, or 2 of informed sellers on the 2–seller market. The specific results allow us to discuss how the distribution depends on market parameters, especially on the parameter α of risk aversion and on the cost C of market research. The final remarks summarize our findings and discuss possible generalizations.

2. The market model

The market model is based on Ponssard (1979) who studied the interaction of sellers on a homogeneous market with stochastic demand where some sellers $i = 1, ..., m$ know and other sellers $j = m+1, ..., n$ do not know the exact demand condition. One possible interpretation of such information discrepancy is that the informed sellers are the market incumbents whereas the market entrants are the uninformed traders. Here, however, we assume that all $n(\geq 2)$ sellers are market incumbents who evolve as informed or uninformed sellers depending on past success.

Compared to Ponssard (1979) we introduce a new reason for acquiring better information about market demand, namely risk aversion. It is assumed that all n sellers $i = 1, ..., n$ have cardinal utilities which can be expressed in the form

$$u_i = E\{\Pi_i\} - \frac{\alpha}{2} V\{\Pi_i\} \text{ for } i = 1, ..., n \tag{2.1}$$

where Π_i denotes seller i's profit, $E\{\cdot\}$ the expectation and $V\{\cdot\}$ the variance operator. The non-negative parameter α measures the degree of risk aversion. If α is positive, seller i is risk averse. In Ponssard (1979) the α-parameter of sellers $i = 1, ..., n$ was assumed to be 0.

Let $x_i\,(\geq 0)$ denote seller i's sales amount and

$$X = x_1 + ... + x_n \tag{2.2}$$

the total market supply. The price p resulting for X is a stochastic variable according to

$$p = D - aX \tag{2.3}$$

where a is a positive parameter and D a stochastic variable with realizations d in the range

$$\overline{d} \geq d \geq \underline{d} \text{ with } \overline{d} > \underline{d}. \tag{2.4}$$

Production costs are assumed to be linear and identical for all sellers $i = 1, ..., n$. We denote by c with

$$\underline{d} \geq c \geq 0 \tag{2.5}$$

the constant marginal costs of all sellers.[2]

A seller, who did not invest in market research and does not know the realization d of D, has no other cost. The profit of an uninformed seller $j = m + 1, ..., n$ is thus

$$\Pi_j = (d - c - aX)\, x_j. \tag{2.6}$$

Since such a seller j does not know d, he can only maximize his expected utility as described by equation (2.1).

An informed seller i who knows the realization d of D when choosing x_i, has additional cost $C\,(> 0)$ of information acquisition, i.e. seller $i = 1, ..., m$ will maximize

$$\Pi_i = (d - c - aX)\, x_i - C. \tag{2.7}$$

The continuous distribution $\varphi(d)$ over the interval $\left[\underline{d}, \overline{d}\right]$ satisfying (2.4) and (2.5) is commonly known. We will rely on the shorthand $\mu = E\{D\}$ and $\sigma^2(m) = V\left(\frac{D}{m+1}\right)$. Clearly μ must satisfy $\overline{d} \geq \mu \geq \underline{d}$. Since an uninformed seller j only knows the a priori-expectation $\varphi(\cdot)$ of D his strategy is simply his sales amount x_j. An informed seller i can react to actual demand, described by d, i.e. a strategy of such seller i assigns a sales amount $x_i(d)$ to all possible realizations d of D.

In the next section we solve this market model for any number m of informed sellers i with $0 \leq m \leq n$. These results then allow us to investigate how risk aversion will influence information acquisition and the actual market results.

[2] If $c > \underline{d}$ would hold, sellers might prefer to not sell at all what could lead to monopolistic behavior which we want to rule out.

3. The solution for given information conditions

For any informed seller i the optimal decision must satisfy

$$ax_i^*(d) = d - c - aX \tag{3.1}$$

in view of equation (2.7). Thus all m informed sellers i supply the same amount

$$x^*(d) = x_i^*(d) \text{ for all realizations } d \text{ of } D. \tag{3.2}$$

An uninformed seller j maximizes

$$E\{(D - c - aX)\,x_j\} - \frac{\alpha}{2}V\{(D - c - aX)\,x_j\} \tag{3.3}$$

or

$$\int_{\underline{d}}^{\overline{d}} (d - c - aX)\,x_j d\varphi(d) - \frac{\alpha}{2}\int_{\underline{d}}^{\overline{d}} [(d - aX)\,x_j]^2\,d\varphi(d) + \frac{\alpha}{2}\left[\int_{\underline{d}}^{\overline{d}} (d - aX)\,x_j d\varphi(d)\right]^2. \tag{3.4}$$

From the first order condition one obtains

$$\begin{aligned} &\int_{\underline{d}}^{\overline{d}} (d - c - aX - ax_j)\,d\varphi(d) \\ = \;& \alpha\int_{\underline{d}}^{\overline{d}} (d - aX)\,x_j\,(d - aX - ax_j)\,d\varphi(d) \\ & -\alpha\left[\int_{\underline{d}}^{\overline{d}} (d - aX)\,x_j d\varphi(d)\right]\int_{\underline{d}}^{\overline{d}} (d - aX - ax_j)\,d\varphi(d) \end{aligned} \tag{3.5}$$

for all $n - m$ uninformed sellers $j = m + 1, ..., n$. Rearranging (3.5) according to

$$\int_{\underline{d}}^{\overline{d}} (d - c - aX)\,d\varphi(d) =$$

$$
\begin{aligned}
&= x_j \left[a + \alpha \left(\int_{\underline{d}}^{\overline{d}} (d - aX)^2 \, d\varphi(d) - a x_j \int_{\underline{d}}^{\overline{d}} (d - aX) \, d\varphi(d) \right.\right. \qquad (3.6) \\
&\qquad \left.\left. - \left(\int_{\underline{d}}^{\overline{d}} (d - aX) \, d\varphi(d) \right)^2 + a x_j \int_{\underline{d}}^{\overline{d}} (d - aX) \, d\varphi(d) \right) \right] \\
&= x_j \left[a + \alpha \int_{\underline{d}}^{\overline{d}} (d - aX)^2 \, d\varphi(d) - \alpha \left(\int_{\underline{d}}^{\overline{d}} (d - aX) \, d\varphi(d) \right)^2 \right]
\end{aligned}
$$

implies that

$$x_j^* = x^* \text{ for all uninformed sellers } j = m+1, ..., n \tag{3.7}$$

must hold. Because of (3.1) and (3.2) we can thus express $x_i^*(d)$ by

$$a x_i^*(d) = \frac{d - c}{m+1} - a \frac{n-m}{m+1} x^*. \tag{3.8}$$

After inserting (3.7) and (3.8) equation (3.6) becomes an equation with one unknown x^* whose solution is

$$x^* = \frac{\mu - c}{(n+1)\, a + \alpha\, (m+1)\, \sigma^2(m)}. \tag{3.9}$$

Inserting (3.9) into (3.8) finally yields

$$x^*(d) = \frac{d - c}{(m+1)\, a} - \frac{n-m}{m+1} \frac{\mu - c}{(n+1)\, a + \alpha\, (m+1)\, \sigma^2(m)}. \tag{3.10}$$

Thus the a priori expected sales amount of informed sellers i is

$$\overline{x}^* := \int_{\underline{d}}^{\overline{d}} x^*(d) \, d\varphi(d) = (\mu - c) \left[\frac{1}{(m+1)\, a} - \frac{(n-m)\, /\, (m+1)}{(n+1)\, a + \alpha\, (m+1)\, \sigma^2(m)} \right]. \tag{3.11}$$

For $\alpha = 0$ one has

$$x^* = \frac{\mu - c}{(n+1)\, a} = \overline{x}^*. \tag{3.12}$$

Whereas $\overline{x}^*$ increases, the amount x^* decreases with a growing parameter α. Thus, the market will be more dominated by the m informed sellers i when the uninformed sellers are more risk averse, i.e. for high values of α. Whether, however, this will increase the number m of informed sellers i may very well depend on the cost C of information acquisition. Moreover, let $X^*(\alpha) = m\overline{x}^* + (n-m)x^*$ be the expected industry output. Since $\frac{\partial X^*(\alpha)}{\partial \alpha} = -\sigma^2(m)\,(\mu - c)\,(n-m)/\left(a(n+1) + \alpha\sigma^2(m)(m+1)\right)^2 < 0$ for $m < n$, the market is less efficient for greater α when not all sellers are informed.

How risk aversion as expressed by $\alpha > 0$ and the cost C of information acquisition together influence the success and thus the future number m of informed traders on the market with a constant number n of sellers will be explored next.

4. The incentives for market research

The incentive for market research is what an informed seller earns more than an uninformed seller when disregarding the cost C of market research. Let us first investigate how the incentives for market research depend on market parameters, especially on the number m of informed sellers and on the parameter α of risk aversion. To do so we first compare the a priori expected payoff

$$u_i = E\left\{\Pi_i^*(m, \alpha, d, C = 0)\right\} - \frac{\alpha}{2} V\left\{\Pi_i^*(m, \alpha, d, C = 0)\right\}$$

of an informed seller i with the expected payoff

$$u_j = E\left\{\Pi_j^*(m, \alpha, d)\right\} - \frac{\alpha}{2} V\left\{\Pi_j^*(m, \alpha, d)\right\}$$

of an uninformed seller j where $\Pi_k^*(\cdot)$ indicates that we rely on optimal decisions as derived above. For the sake of simplicity assume $[\underline{d}, \overline{d}] = [0, 1]$ and a uniform density $\varphi(\cdot)$ over $[0, 1]$ so that $\mu = \frac{1}{2}$ and $\sigma^2(m) = 1/12(m+1)^2$.

Even for such simplifying assumptions the difference $u_i - u_j$ is a rather clumsy expression of market parameters which is, however, surely positive. An easy way for investigating how $u_i - u_j$ depends on m is to consider m as a continuous variable and to explore the derivative of $u_i - u_j$ with respect to m whose value $-2\sigma^2(m)/(m+1)$ for $\alpha = 0$ is negative, i.e. the incentive for market research decreases with an increasing number m of informed sellers for small values of α.

According to Ponssard (1979, Theorem 2), the fact that $u_i - u_j$ decreases with m does not depend on the number n of sellers on the market. Does this invariance still hold when sellers are risk averse, i.e. when α is positive? The question can be answered by evaluating the derivative of $u_i + C - u_j$ with respect to α at $\alpha = 0$ what yields

$$\frac{\partial}{\partial \alpha}(u_i + C - u_j)\Big|_{\alpha=0} = -\frac{1}{1440a^2}\frac{4n^2 + 8n + 15m^2 + 30m + 19}{(m+1)^4 (n+1)^2} < 0.$$

The larger n the less reactive is $u_i + C - u_j$ to changes of α at $\alpha = 0$. Thus risk aversion, i.e. a positive parameter α, implies a negative relationship between the incentive $u_i + C - u_j$ for market research and the number n of sellers on the market. In other words, the invariance result of Ponssard (1979) holds only for the special case $\alpha = 0$.

At first sight it appears counter-intuitive that $\frac{\partial}{\partial \alpha}(u_i + C - u_j)$ depends negatively on α in a neighbourhood of 0. One would have naturally expected that risk aversion inspires engaging in market research, i.e. that a larger α implies larger incentives $u_i + C - u_j$ for market research. The puzzling effect is due to the definition of $u_i + C - u_j$ as the difference of the a priori expected payoff $u_i + C$ of an informed seller $i = 1, 2, ..., m$ and the actually expected payoff u_j of an uninformed seller $j = m + 1, ..., n$. Since an informed seller i does not face any uncertainty, allowing for risk aversion, as in the definition of u_i in equation (2.1), is misleading.[3] To determine the incentive for market research one therefore should explore the derivative of

$$E\left\{\Pi_i^*(m, \alpha, d, C = 0)\right\} - u_j$$

with respect to α. Since

$$\frac{\partial}{\partial \alpha}\left(E\left\{\Pi_i^*(m, \alpha, d, C = 0)\right\} - u_j\right)\Big|_{\alpha=0} = \frac{1}{32a^2 (m+1)^2 (n+1)^2} > 0,$$

our intuition is confirmed: Risk aversion, i.e. a positive parameter α, increases the incentive for market research. Note, furthermore, that

$$\frac{\partial}{\partial m}\left(E\left\{\Pi_i^*(m, \alpha, d, C = 0)\right\} - u_j\right)\Big|_{\alpha=0} = -\frac{1}{6a (m+1)^3} < 0,$$

i.e., as above, the incentive for market research decreases with an increasing number m of informed sellers.

[3] Both approaches can, however, be justified by appropriate decision processes. The incentive above would rely on a stochastic game where all sellers face initially the same uncertainty. The other interpretation of private information assumes that this chance move is purely fictitious so that a priori-expectations of informed sellers do not make sense.

5. On the evolution of m

For any m with $0 < m < n$ and any realization d of the stochastic variable D let

$$\Pi_j^* (m, \alpha, d) = [d - c - amx^* (d) - a (n - m) x^*] x^* \tag{5.1}$$

and

$$\Pi_i^* (m, \alpha, d, C) = [d - c - amx^* (d) - a (n - m) x^*] x^* (d) - C \tag{5.2}$$

denote the realized profits of the $n - m$ uninformed sellers j, respectively of the m informed sellers i. The evolution of m is supposed to depend on the difference

$$\triangle (m, \alpha, d, c) = \Pi_j^* (m, \alpha, d) - \Pi_i^* (m, \alpha, d, C) \tag{5.3}$$

of these two payoffs.[4] If $\triangle (m, \alpha, d, C)$ is positive, one naturally will expect the number m of informed sellers i to decrease whereas m should increase when $\triangle (m, \alpha, d, C)$ is negative. As $\Pi_j^* (m, \alpha, d)$ and $\Pi_i^* (m, \alpha, d, C)$ also $\triangle (m, \alpha, d, C)$ is a stochastic variable.

Before investigating the stochastic evolutionary process of m let us first consider the border case where the n sellers interact infinitely often on the market where after each interaction d is (independently and identically) randomly chosen according to $\varphi (\cdot)$. Only after experiencing an infinite number of interactions with each time independently chosen d, but with a constant composition of the market by m informed and $n - m$ uninformed sellers, adjustment of market research, i.e. a change of m can take place. Since $\Pi_j^* (\cdot)$ depends linearly on d (see equation (5.1 above), this means that we just have to consider the value of $\Pi_j^* (\cdot)$ for $d = \mu$. On the other hand $\Pi_i^* (\cdot)$ depends quadratically on d (see equation (5.2)) so that its expected value will also depend on the variance of D. The results can be illustrated in the m, α-diagram of Figure 5.1 where we rely on the parameter constellation

$$a = 1, \ c = 0, \ \underline{d} = 0, \ \overline{d} = 1, \ n = 10, \ \mu = \frac{1}{2} \text{ and } \sigma^2(m) = \frac{1}{12(m+1)^2}. \tag{5.4}$$

[4] This means that we distinguish between payoff (the cardinal utility of a seller), governing his market behavior, and (reproductive) success (profit) on which the diffusion of his information type (informed versus uninformed) depends.

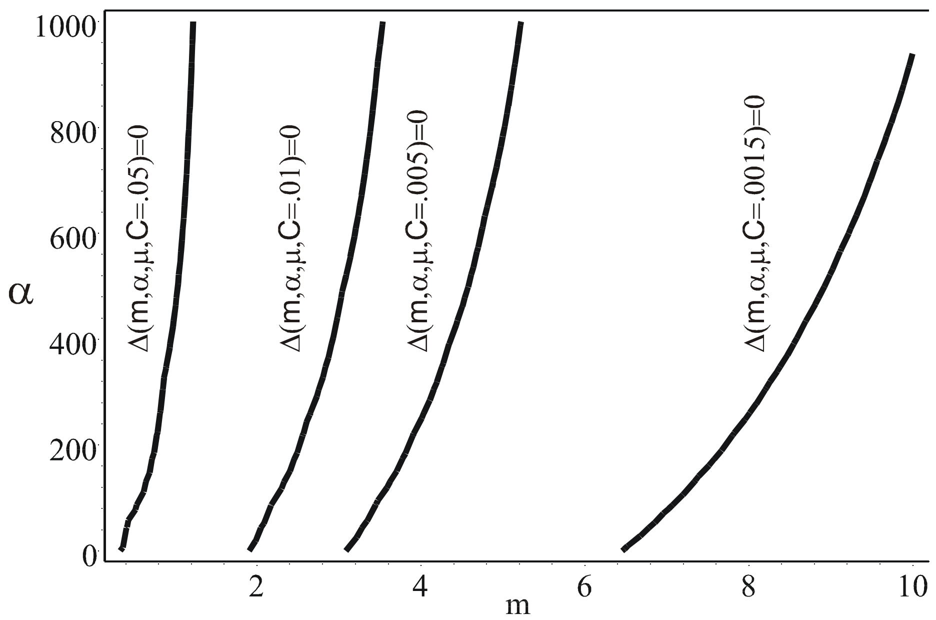

Figure 5.1: The dynamics of m

Whereas above the $\triangle(m, \alpha, \mu, C) = 0$-curves the number m of informed sellers i increases, below these curves m tends to decrease. This shows that constellations m^* such that

$$\triangle(m^*, \alpha, \mu, C) = 0 \tag{5.5}$$

are the candidates for an evolutionarily stable number m of informed sellers i. Of course, m^* will usually not be an integer so that m may cycle around such non-integer values m^*. Such minor changes seem, however, of little relevance and will therefore be neglected. Note that in our numerical example presented in Figure 5.1, for high costs for market research, e.g. $C = .05$, no seller will gather information if the parameter of risk aversion α is below 500. On the other hand when the costs for market research are low, e.g. $C = .0015$, the number of informed sellers varies between 6 and 10 for α varying between 0 and 1000.

Let us now explore the actual stochastic process when assuming that a positive value of $\triangle(\cdot)$ leads more or less surely to becoming an informed type whereas the tendency is reversed for $\triangle(\cdot) < 0$. Let the probability by which an uninformed seller j switches to market research at cost C, respectively by which an informed seller i gives up market research be determined as

$$(1-\varepsilon)\, P_{+}\,(m,\alpha,C) = (1-\varepsilon)\, Prob\,\{\triangle\,(m,\alpha,d,C) > 0\}\,, \tag{5.6}$$

respectively

$$(1-\varepsilon)\, P_{-}\,(m,\alpha,C) = (1-\varepsilon)\, Prob\,\{\triangle\,(m,\alpha,d,C) < 0\}\,. \tag{5.7}$$

Here ε with $0 < \varepsilon < 1$ is the (unintended and therefore typically small) mutation probability.

For given ε these two probabilities define a stochastic Markov-process whose stationary distributions $\sigma = (\sigma_0, ..., \sigma_n)$ with

$$\sigma T = \sigma \tag{5.8}$$

specifying the probabilities σ_m for the possible values of m in the long run. Here $T = (t_{k,l})$ denotes the transition matrix specifying for each number $m_t = k$ of informed sellers in period t with $0 \le m_t \le n$ how likely the result $m_{t+1} = l$ is for the following period $t+1$. To define T also for the border cases $m_t = 0$ and $m_t = n$ let for $m_t = 0$ a seller j switch to market research with the small (unintended) mutation probability $\frac{\varepsilon}{2}\,(> 0)$. Correspondingly, for $m_t = n$ the switch of an informed seller i to no market research occurs with the same mutation probability $\frac{\varepsilon}{2}$. In the next section we will illustrate the evolutionary stable distributions σ, defined in (5.8), for the special situation $n = 2$ with three possible realisations of m, namely $m = 0, 1$, and 2.

5.1. An example

As in oligopoly theory where the duopoly market is used as the paradigm of interaction of finitely many strategically interacting sellers the case $n = 2$ here serves as the paradigmatic case of truly stochastic evolution on finite markets. In the special case $n = 2$ (neglecting the null-event $\triangle\,(\cdot) = 0$) the transition matrix T is given by

$$T = \begin{pmatrix} (1-\frac{\varepsilon}{2})^2 & 2(1-\frac{\varepsilon}{2})\frac{\varepsilon}{2} & \left(\frac{\varepsilon}{2}\right)^2 \\ \left(\frac{\varepsilon}{2} + (1-\varepsilon)P\right)^2 & 2\left(\frac{\varepsilon}{2} + (1-\varepsilon)P\right)\left(\frac{\varepsilon}{2} + (1-\varepsilon)(1-P)\right) & \left(\frac{\varepsilon}{2} + (1-\varepsilon)(1-P)\right)^2 \\ \left(\frac{\varepsilon}{2}\right)^2 & 2(1-\frac{\varepsilon}{2})\frac{\varepsilon}{2} & (1-\frac{\varepsilon}{2})^2 \end{pmatrix}$$

where $P = P_+(1,\alpha,C)$. Here $t_{k,l}$ $(k,l = 0,1,2)$ is the probability that the system switches from $m_t = k$ at time t to $m_{t+1} = l$ at time $t+1$. For example, if $m_t = 0$ then with probability $1 - \frac{\varepsilon}{2}$ a buyer simply keeps his type and with probability $\frac{\varepsilon}{2}$ he unintendedly mutates to an informed seller. Since the random moves are stochastically independent across sellers we have for example $t_{0,0} = (1 - \frac{\varepsilon}{2})^2$.

For $n = 2$ the solution of equation (5.8) is given by

$$\begin{aligned}
\sigma_0(\varepsilon) &= \frac{1}{4}\frac{2P\varepsilon(1-\varepsilon) + 4P^2(1-\varepsilon)^2 + \varepsilon}{2P(P-1)(1-\varepsilon)^2 + 1}, \\
\sigma_1(\varepsilon) &= \frac{1}{2}\frac{(2-\varepsilon)\,\varepsilon}{2P\left(\varepsilon-1\right)^2\left(P-1\right)+1}, \\
\sigma_2(\varepsilon) &= \frac{1}{4}\frac{4P^2(1-\varepsilon)^2 + 2P(7\varepsilon - 3\varepsilon^2 - 4) + \varepsilon(2\varepsilon - 5) + 4}{2P\left(\varepsilon-1\right)^2\left(P-1\right)+1}.
\end{aligned}$$

Moreover, some computations show that

$$P_+(1,\alpha,C) = \begin{cases} 1 & \text{if } \frac{1}{4}\frac{15552+\alpha(576+\alpha)}{(288+\alpha)^2} < C < \frac{1}{4} \\ 2\delta - 1 & \text{if } C < \frac{1}{4}\frac{15552+\alpha(576+\alpha)}{(288+\alpha)^2} \end{cases}$$

where $\delta = \frac{96}{288+\alpha}\left(\frac{3}{2} + \sqrt{\frac{9}{16} + 4C\left(3 + \frac{1}{96}\alpha\right)^2}\right)$.

Thus for a positive, although small mutation probability ε all three states $m = 0, 1,$ and 2 will be reached with positive probability according to the stationary distribution $\sigma = (\sigma_0, \sigma_1, \sigma_2)$. Furthermore, one has

$$\lim_{\varepsilon\to 0} \sigma_0(\varepsilon) = \frac{P^2}{1 - 2P + 2P^2}, \quad \lim_{\varepsilon\to 0} \sigma_1(\varepsilon) = 0, \quad \text{and} \quad \lim_{\varepsilon\to 0} \sigma_2(\varepsilon) = \frac{1 - P^2}{1 - 2P + 2P^2},$$

so that the bimorphic population consisting of one informed and one uninformed seller is not represented in the limit distribution $\sigma = \lim_{\varepsilon\to 0} \sigma(\varepsilon)$. Note that $\sigma_0 > \sigma_2$ iff $P > \frac{1}{2}$.

Figure 5.2 displays for various values of the cost parameter C for market research how the components of the stationary distribution σ vary with a growing parameter α of risk aversion. (In these simulations the mutation rate is $\varepsilon = .001$.)[5] In all these

[5] Other simulations have shown that the composition of the stationary distribution hardly reacts to the parameter ε.

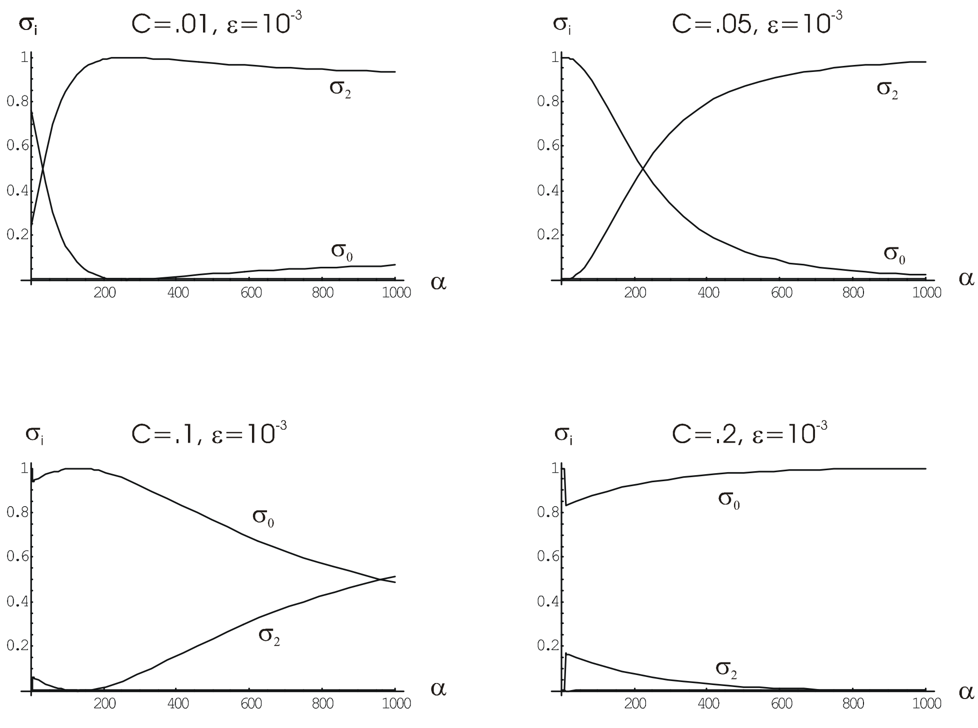

Figure 5.2: Stationary distributions for varying α.

simulations the component σ_1 indicating the bimorphic population is virtually zero. Therefore, the other two lines indicating the monomorphic populations are mirror images to each other.

A similar analysis for finite numbers $n > 2$ of sellers on the market as for $n = 2$ would just require more complex calculations. Especially the result $\lim_{\varepsilon \to 0} \sigma_1(\varepsilon)$ for $n = 2$ should generalize to $\lim_{\varepsilon \to 0} \sigma_m(\varepsilon)$ for all $0 < m < n$ for $n \geq 3$ for the same reason, namely the instability of bimorphism, as for $n = 2$.

6. Discussion

Starting from the oligopoly market with informed and uninformed sellers, as studied by Ponssard (1979), we have shown

- that some of his results, like the invariance of $u_i + C - u_j$ with respect to the number n of firms in the market, does not hold for risk averse sellers,
- that more risk aversion decreases $u_i + C - u_j$, but not $E\{\Pi_i^*(m, \alpha, d, C = 0)\} - u_j$, and
- how such results allow us to investigate the evolutionary dynamics of m, i.e. the composition of the market by informed and uninformed traders.

In our evolutionary analysis we have mainly concentrated on two border cases, namely

- the one of infinitely many interactions before an m-adjustment implying deterministic evolutionary dynamics (except for the effect of rare random mutation) and
- the special case of just two sellers as the paradigmatic situation of finitely many sellers on the same market where the evolutionary dynamics are stochastic and where the stationary solution for positive mutation rates depends in a rather complex way on the various parameters.

Whereas most applications of indirect evolution focus on endogenous preference formation, we thus have illustrated how other aspects like the information conditions of the various sellers can be endogenously derived. In our study a seller either is completely aware of market demand or just knows the distribution by which market demand is randomly determined (see Güth (1998) for a study of arbitrary belief evolution in case of a deterministic market).

Altogether such studies of indirect evolution can help to overcome the fundamental dilemma of industrial economics that nearly all market results can be justified by a rational choice approach. If some rules, like who is informed about market demand and who not, cannot be endogenously derived, this can increase tremendously the cutting power of our theoretical predictions.[6] As an example we can point to the fact that low mutation rates render bimorphic markets very unlikely, i.e. in the long run all sellers will either be uniformly aware or unaware of market demand.

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[6] Of course, such endogenous determination of institutional aspects requires certain assumptions in order to derive well specified transition matrices. In our view, our assumption that realized profits determine the adaptation of information types seems rather natural. This, however, cannot be claimed for the mutation process as captured by a positive ε. At least the specification of institutional aspects can be traced to deeper reasons.

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