Money and Banks: Some Theory and Empirical Evidence for Germany

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February 2002

Abstract

In a world with imperfect competition, market externalities or asymmetric information, the impact of money and monetary policy on the real sector depends on the way money is created. Two conflicting views of money supply can be distinguished in the literature: the endogeneity view and the exogeneity view. In this paper, both views are discussed and compared from a theoretical and from an empirical point of view.

An industrial organization model of the money-creating sector with endogenous money is presented and compared to the money multiplier approach. The concept of a heterogeneous oligopoly is used to model the credit market and the deposits market. Using the aggregated balance sheet of the banking sector, endogenous money is explained by its counterparts, especially loans. The monetary base is determined endogenously, too, and a money multiplier equivalent expression can be derived.

A cointegrated vector autoregressive model for the development of the nominal money stock in Germany in the period of monetary targeting from 1975 to 1998 is estimated. The implications of the theoretical model are compared to the properties of the estimated VAR. It turns out that both the money multiplier approach and the presented model of the money-creating sector with endogenous money fail to explain all the empirical evidence from the VAR model.

Keywords: Endogenous money, industrial organization approach to banking theory, monetary policy, money multiplier, vector error correction model.

JEL classification: C32, E51, E52.

* This paper is a modified version of a chapter of my dissertation on “Vector Autoregressive Analysis and Monetary Policy” (Aachen: Shaker, forthcoming). I am grateful to Helmut Lütkepohl, Rainer Schulz, Jürgen Wolters and to participants of seminars at Freie Universität Berlin and Humboldt-Universität zu Berlin for helpful comments and suggestions. Financial support from the Deutsche Forschungsgemeinschaft is gratefully acknowledged.
1 Introduction

In a Walrasian equilibrium model with perfect markets, money is a veil. The money stock and the money growth rate have no impact on real variables, and monetary policy does not affect real activity. But in reality, markets are not perfect; we observe imperfect competition, market externalities, asymmetric information, and nominal rigidities. Therefore, it has been broadly discussed whether monetary policy systematically affects real activity. Friedman (1995, p. 2) states that today “economists do know that monetary policy systematically affects real activity.” But we do not know how money affects real activity. A prerequisite of the analysis of real effects of money is the analysis of the money supply process. An exogenous money stock that is controlled by a monetary authority does not play the same role in the transmission mechanism like an endogenous money stock that is demand determined. The purpose of this paper is to analyze the theoretical implications of the exogeneity view and the endogeneity view of money and to compare these implications to empirical evidence.

The paper is structured as follows. In section 2, the money multiplier approach is discussed, and the development of the money stock in Germany from 1975 to 1998 is analyzed under consideration of the monetary policy strategy of the Deutsche Bundesbank. Afterwards, the modeling of endogenous money in the literature is reviewed. In section 3, an industrial organization model of the money-creating sector with endogenous money is developed. In section 4, a vector error correction model (VECM) for the nominal money stock, the monetary base and related variables is estimated and analyzed. The empirical results are compared to the implications of both the money multiplier approach and the model presented in section 3. Finally, section 5 concludes.

2 Exogeneity and Endogeneity View of Money Supply

2.1 The Money Multiplier Approach

The standard textbook approach explaining the money stock outstanding and its growth rate, is the money multiplier model. Many versions of this model are in use. They have in common that the money stock \( M \) is determined by the monetary base (or high-powered money, \( H \)) and the money multiplier \( mm \):\(^1\)

\[
M = mm \cdot H. \tag{2.1}
\]

The monetary base is controlled by the central bank, and the money multiplier depends on the behavior of the public (constant currency-deposit ratio, \( d = CU/D \)), the commercial banks (reserve ratio as a function of interest rates and uncertainty), and the central bank (minimum reserve requirements). These behavioral determinants enter the money multiplier in a nonlinear way. The simplest version of the money multiplier is the following one: The money stock consists of currency in use (\( CU \)) and deposits (\( D \)): \( M = CU + D = mm \cdot H \); and the monetary base consists of currency in use and reserves of banks (\( R \)): \( H = CU + R \). The (required) reserve rate is \( r = R/D \) such that

\[
mm = \frac{CU/D + 1}{CU/D + R/D} = \frac{d + 1}{d + r}. \tag{2.2}
\]

If the central bank is able to forecast the money multiplier correctly and is also able to control the monetary base it can control the money stock. Under these circumstances, the money stock

\(^1\) The money multiplier approach is explained in many macroeconomic textbook. The following description draws from Dornbusch and Fischer (1994).
Notes: Money Multiplier \( mm = \frac{M}{H} \), where \( M \) is the money stock M3 and \( H \) is the monetary base, see data appendix.

is an exogenous variable assuming that the supply of deposits by the public is not restricted such that \( D = R/r \). Exogeneity of the money stock in this context means the ability of the central bank to control the money stock.

The money multiplier approach has some important drawbacks: First, the operating target of central banks in the USA and in Europe is not the monetary base but a money market interest rate (federal funds rate, euro overnight index average EONIA). A theory of money supply has to consider this and other institutional details. Second, according to studies of the relationship between the money stock and the monetary base in Germany by Willms (1993) and by Nautz (1998), a stable relationship between the money stock and the monetary base seems not to exist, see also figure 1. The increase of the money multiplier from about 4.5 to 7.1 shows that the share of the monetary base in M3 has decreased, and that book money created by financial intermediaries has become more and more important. Therefore, it may be appropriate to model the behavior of banks explicitly instead of reducing it to variations of the money multiplier. This is proposed inter alia by Tobin (1967). In his terminology, the money multiplier approach is the “old view” of money supply, while the “new view” interprets financial intermediaries as firms which optimize their portfolios given the optimizing behavior of non-banks. That is, financial intermediaries do not possess the ability to expand deposits without limit like it is assumed in the money multiplier approach above. Thus, the amount of deposits and the money stock are endogenous variables determined by the portfolio selection process of commercial banks and the public. Corresponding to the use of the notion of exogeneity in this paper, endogeneity of money in this context means that the central bank is not able to control the money stock.\(^2\)

While optimizing their portfolio, banks and non-banks have to consider the conditions set by the central bank. Advocates of the money multiplier approach refuse the new view, and find “no reason to look beyond the balance sheets of commercial banks.” (Meltzer, 1969, p. 39). Albeit weaker and not as explicit as in this quotation, this view can also be found in a more recent work (Meltzer, 1995). The money multiplier approach is also supported by Rasche (1993) who admits that (p. 32) “The principle that the algebraic components of the money multiplier, however formulated, vary in response to the economic decisions of both depository institutions and the public is now widely accepted among monetary analysts.” but claims that the variations of the money multiplier are unsystematic, and of short-run nature, and that (p. 47) “Over the longer run, such random movements tend to average out, so that changes in base money are the most important source of changes in transactions money.”

\(^2\) See also Müller (1993); for an analysis of endogeneity, causation, and their relation see Müller (1998).
Notes: Thick line: Money growth rate in %, shaded area: announced target (point target from 1975 to 1978, and in 1989, other years: upper and lower bound). From 1975 to 1987, the money stock under consideration has been central bank money, from 1988 to 1998, M3. Up to the complete year 1990, the targets and the realized growth rates are for West Germany. From 1991 on, the targets and the realized growth rates are for united Germany. The data are taken from Leschke and Polleit (1997), and from the monthly bulletin of the Deutsche Bundesbank.

2.2 Monetary Targeting in Germany from 1975 to 1998

The exogeneity or controllability assumption of the money multiplier approach forms the basis of the monetary policy strategy of monetary targeting. A monetary targeting strategy has been adopted for example by the Deutsche Bundesbank from 1975 to 1998. In Deutsche Bundesbank (1995, p. 91 ff.) it is described how the Bundesbank has controlled the money stock. The Bundesbank refers implicitly to the money multiplier approach and states that its monopoly for bank notes and the minimum reserves requirement imply long-run controllability of the money stock by means of controlling the monetary base.

A precondition for an exogenous money stock is a flexible exchange rate. This precondition has not been given due to the more or less fixed exchange rates within the European Monetary System (EMS, founded 1979), but the Bundesbank has been able to sterilize interventions on the foreign exchange market like it has been the case in the EMS crisis of September 1992. Nevertheless, as can be seen from figure 2, the Deutsche Bundesbank was not always able to achieve its announced monetary target. In the 24 years of monetary targeting from 1975 to 1998, the observed monetary growth rate deviated from the announced growth rate eleven times. Among others, there are two possible reasons for a deviation of the money growth rate from the announced target: first, the Bundesbank has also had other objectives. The money growth rate has not been an ultimate goal but only an intermediate target. The ultimate goal has been price stability measured in terms of the inflation rate. In some periods there may have been trade-offs between the announced monetary target, the price stability target, and other objectives, like the exchange rate. Second, money could be endogenous. That is, the central bank is not able to set the money growth rate as the money multiplier approach or exogeneity view of money suggests. While the exogeneity view assumes that the money stock determines one or all of income, prices, and interest rates, the endogeneity view postulates that the money stock is determined by one or all of income, prices, and interest rates, see for example Desai.

The money multiplier approach does not necessarily imply exogeneity of the money stock. If the money multiplier exhibits unpredictable and endogenous variations, the money stock is endogenous. For reasons of simplicity, it is supposed here that the money stock is exogenous in the money multiplier approach. A money multiplier model with endogenous money can be found in Jarchow (1998), for example.
In a modern open economy with a sophisticated profit-maximizing banking system, a non-banking financial sector, and rapid international capital flows, it is at least questionable whether money is exogenous.

The endogeneity view is also supported by the explanations of the Bundesbank for the differences between announced target and observed money growth rate since 1992/93. Before 1992/93, the explanations of the Bundesbank for deviations from the money growth rate target were reasons for a more expansive or more restrictive monetary policy than announced. That is, the actions of the Bundesbank have been responsible for the deviations of the exogenous money growth rate from the announced target. Since 1992/93, the explanations refer to unforeseen changes in the demand for money implying that the endogenous money stock has been determined by the demand for money. The following explanations have been given for deviations:

1. From 1975 to 1978, the money growth rate was higher than the announced target. The reason was a policy of low interest rates to increase the low level of real economic activity.
2. The excess money growth from 1986 to 1988 have been the stabilization of exchange rates (DM/US-Dollar\(^5\), EMS) and provision of liquidity to avoid a recession after the stock market crash in October 1987.
3. In 1993, a flight into currency and into short-term deposits has been the result of the introduction of a withholding tax on interest yields. Therefore, the demand for money increased.
4. In 1995, the only year with a lower money growth rate than the announced target, the demand for money decreased as a consequence of the permission of money market fund shares which are not part of M3. And in 1996, the money growth rate was too high because of interest rate driven portfolio variations from money capital to time deposits.

According to these explanations, at the end of the period of monetary targeting in Germany, demand side forces have been the reasons for deviations of the money growth rate from the announced target. This supports the endogeneity view.

### 2.3 Modeling of Endogenous Money in the Literature

It has already been stated that an important contribution to the discussion about endogenous money has been Tobin (1967) who emphasized the importance of optimizing behavior of commercial banks and the public for the money supply process. Essentially the same argument is used in the Post Keynesian literature on money supply endogeneity.\(^6\) Moore (1988, p. 381) points out that "Banks are price setters and quantity takers in both their retail loan and their deposit markets. As a result both loans and deposits are demand determined." and (p. 383) "Banks may be assumed to adjust their borrowing and lending rates on profit maximizing grounds." Moore assumes that the demand for money, given the interest rates set by central bank and commercial banks, is always satisfied. This implies a horizontal money supply curve in a di-

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\(^4\) The explanations of the monetary development are taken from von Hagen (1998) and Baltensperger (1998).

\(^5\) While the US-Dollar has been very strong in 1985 (average exchange rate of 2.94 DM/USD), the Dollar dropped sharply after the New York Plaza announcement of the G-5 on September 22, 1985. The depreciation of the US-Dollar ended only when Japan and Germany intervened on the foreign exchange market. The US-Dollar stabilized in early 1987, also due to the Louvre accord of February 22, 1987. For details see Krugman and Obstfeld (1994), p. 582 ff.

\(^6\) Endogeneity of money is one of the core propositions that are common to all Post Keynesians. These core propositions are according to Palley (1996, p. 9): "(a) the significance of social conflict over income distribution, (b) the centrality of aggregate demand in the determination of the level of economic activity, (c) the inability of nominal wage adjustment to ensure full employment, (d) the endogenous nature of money, (e) the importance of debt finance in the macroeconomic process, (f) the fundamentally mutable nature of expectations about the uncertain future." The Post Keynesian discussion about money supply endogeneity can be found in Journal of Post Keynesian Economics, Journal of Economic Issues, Economie et Sociétés, Economie Appliquée. For an overview see Musella and Panico (1995).
agram with money on the horizontal axis and interest rate on the vertical axis (horizontalism, accommodating endogeneity). Another view of money supply endogeneity within the Post Keynesian literature is held inter alia by Pollin (1991). This view is called structural endogeneity. The two approaches have in common that money and credit are demand determined but the structural endogeneity approach assumes that the money supply curve has a positive slope because the central bank imposes quantity constraints on the high-powered money it supplies. These constraints lead to a more sophisticated liability management and financial innovations such that given reserves are compatible with a higher quantity of deposits. The interest rate rises because the liability management causes additional costs.

Money is also endogenous in the workhorse model of modern monetary macroeconomics which is a dynamic stochastic general equilibrium model with or without nominal rigidities (sticky wages and/or prices). In this kind of models, the central bank usually follows an interest rate rule, and the money stock is determined by money demand, see for example Taylor (1999, section 2.1) and the literature cited there.

The endogeneity of nominal money can also be modeled in other frameworks, for example the overlapping generations (OLG) model. Richter (1990) uses an extended version of the OLG model to illustrate the implications of the “new view”. In this model, the endogeneity of money is a consequence of the relation between central bank money and commercial bank money (deposits). They are not perfect substitutes and therefore the money market can be divided into a market for central bank money and a market for deposits. While the central bank offers exogenously central bank money, the amount of deposits is determined endogenously by commercial banks and the public.

Here, the so-called (Freixas and Rochet, 1997) industrial organization approach to banking theory is applied to model the endogeneity of money. Central elements are submodels of the credit market and the deposits market. Reduced form equations for the quantity of loans and the quantity of deposits are developed and inserted into the aggregated balance sheet of commercial banks. As a consequence, the monetary base and the money stock are endogenous. Before the details of this model are explained, section 3.1 gives a brief overview of the industrial organization approach to banking theory.

### 3 An Industrial Organization Model of the Money-Creating Sector

#### 3.1 The Industrial Organization Approach to Banking Theory

The industrial organization approach is one possible framework to address the questions of contemporaneous banking theory. The definition of a bank in banking theory is mainly the legal definition of a commercial bank in the United States of America (U.S. Banking Act of 1971): banks are financial intermediaries that receive (demand) deposits and originate loans. The following functions are fulfilled by banks: offering access to a payment system, transforming assets, managing risk, and processing information and monitoring borrowers. The main questions in banking theory are summarized by Bhattacharya and Thakor (1993). Their review is organized around six major issues: existence of financial intermediaries, credit allocation, liquidity transformation, maturity transformation, bank regulation, and borrower’s choice of financing structure and market microstructure. All these topics are analyzed separately, and the relation between the different activities of banks is not inspected in detail. Santomero (1984) gives the

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7 See Freixas and Rochet (1997, p. 2).
The commercial bank maximizes the expected value of an objective function:

\[
\max E \left[ V \left( \tilde{W}_{t+\tau} \right) \right]
\]  

subject to

\[
\tilde{W}_{t+\tau} = W_t (1 + \tilde{\pi}_{t+1}) (1 + \tilde{\pi}_{t+2}) \cdots (1 + \tilde{\pi}_{t+\tau})
\]  

\[
\tilde{\pi}_{t+k} = \frac{\sum_i \tilde{i}^{A_i} A_i - \sum_j \tilde{i}^{D_j} D_j - C \left( \sum_i A_i, \sum_j D_j \right)}{\tilde{W}_{t+k-1}} = \frac{\tilde{\Pi}_{t+k}}{\tilde{W}_{t+k-1}},
\]

where

\( V(\cdot) \) = objective function, \( \partial V/\partial W_{t+\tau} > 0 \) and \( \partial^2 V/\partial W_{t+\tau}^2 \leq 0 \)

\( \tilde{W}_{t+\tau} \) = value of terminal wealth at the horizon time \( \tau \)

\( \tilde{\pi}_{t+k} \) = stochastic profit per unit of capital during period \( t + k \), \( 0 \leq k \leq \tau \)

\( \tilde{\Pi}_t \) = stochastic profit in period \( t \)

\( \tilde{i}^{A_i} \) = stochastic return from asset \( i \)

\( \tilde{i}^{D_j} \) = stochastic cost for deposit \( j \)

\( A_i \) = asset category \( i \), where \( 1 \leq i \leq n \)

\( D_j \) = deposit category \( j \), where \( 1 \leq j \leq m \)

\( C(\cdot) \) = operations cost function

The models that analyze assets and liabilities as well as its possible dependencies can be divided into subsets. One subset is the industrial organization approach. In this set of models, a special focus is laid on the structure of the banking market and the competition between banks. The banks are modeled as optimizing agents on the market for loans and the market for deposits. The optimizing behavior is modeled as expected profit maximization, that is banks are risk neutral (\( \partial^2 V/\partial W_{t+\tau}^2 = 0 \)). This approach can be extended with assumptions about the cost function \( C(\cdot) \). Baltensperger (1980) reviews models that consider the costs of real resources, especially labor, and Bofinger et al. (1996) use a quadratic cost function to model credit default risk. Another subset of models uses the theory of portfolio selection, where banks are assumed to be risk averse (\( \partial^2 V/\partial W_{t+\tau}^2 < 0 \)), see for example Freixas and Rochet (1997, chapter 8).

Two articles, Klein (1971) and Monti (1972), build the basic setup of the industrial organization approach to banking. Klein (1971, p. 207) seeks to explain “(1) the equilibrium scale of the bank, (2) the composition of the bank’s asset portfolio, (3) the composition of the bank’s liabilities, (4) the rate of interest on bank loans, (5) the yield the bank offers on its time and demand deposit accounts.” Monti (1972) has the same purpose but analyzes additionally the impact of the objective function on bank behavior. The general framework is as follows. The analyzed economy consists of four sectors:

<table>
<thead>
<tr>
<th>Government / Central Bank</th>
<th>Households</th>
</tr>
</thead>
<tbody>
<tr>
<td>public dept</td>
<td>securities (S)</td>
</tr>
<tr>
<td>monetary base</td>
<td>deposits (D)</td>
</tr>
</tbody>
</table>

\(^8\) See Freixas and Rochet (1997, p. 53).
While the bond market is assumed to be perfectly competitive, the loan and the deposits markets are not perfectly competitive. The profit function of the banking firm is:

$$\Pi = \sum_i i^{A_i} A_i - \sum_j i^{D_j} D_j - C \left( \sum_i A_i, \sum_j D_j \right), \quad (3.4)$$

where $A_1 = S, A_2 = L$, and $D_1 = D$. This is also the objective function, banks are assumed to be risk neutral. Klein (1971) uses the expected value of the profit function, considers a third asset (bank’s cash holdings) and does also distinguish between two types of deposits. The bank decision variables are the interest rate on deposits, the quantity of loans and the quantity of bonds in Klein (1971) and the interest rate on deposits, the interest rate on loans, and the quantity of bonds in Monti (1972). The cost function is not modeled explicitly. If the demand for loans by firms and the supply of deposits by households are specified, the first order conditions of the profit maximization problem can be used to determine the stocks of bank assets and liabilities and the corresponding interest rates. If the demand for loans and the supply of deposits are independent of each other, the cross derivatives of the cost function are zero ($\partial^2 C/\partial L \partial D = \partial^2 C/\partial D \partial L = 0$), and if no further assumptions about the three markets (loans, deposits, bonds) are made, the decision problem of the bank can be divided into two independent problems: the optimal choice of interest rate/quantity on the loan market and the optimal choice of interest rate/quantity on the deposits market.

Freixas and Rochet (1997, p. 60) formulate this model with the quantities of loans and deposits as decision variables and interpret it as “a model with imperfect competition with two limiting cases: $N = 1$ (monopoly) and $N \to +\infty$ (perfect competition)”, where $N$ is the number of commercial banks.

The Monti-Klein model has been expanded in several ways. Dermine (1986) adds bankruptcy risk and deposit insurance and Prisman et al. (1986) analyze uncertainty and liquidity requirements. The result of both papers is that the separability result for loans and deposits breaks down.

In the following, a version of the Monti-Klein model with the interest rates on loans and deposits as decision parameters of banks is used.

### 3.2 The Model of the Money-Creating Sector: Market Participants

The market participants in this model of the money-creating sector are the central bank, commercial banks and households.

The behavior of the central bank is exogenous. It fixes the interest rate on the money market $i$. The balance sheet of the central bank consists of central bank credit ($CBC$) on the assets’ side and currency in use ($CU$) and reserves ($R$) on the liabilities’ side:

$$CBC = CU + R. \quad (3.5)$$

The commercial banks are profit-maximizing firms on an oligopolistic banking market. Banks buy loans ($L$) and sell deposits ($D$). Their profit function is

$$\Pi_n = i^L_n L_n - i^D_n D_n - i CBC_n + i IBP_n, \quad (3.6)$$
where $i_n^L$ denotes the interest rate of bank $n$ on loans, $i_n^D$ the interest rate on deposits, and $IBP_n$ the net position on the interbank money market. The balance sheet equation of a commercial bank is

$$R_n + L_n + IBP_n = D_n + CBC_n$$

such that the net position can be defined as

$$IBP_n = D_n \cdot (1 - r) + CBC_n - L_n$$

when it is assumed that banks only hold required reserves ($R_n = r \cdot D_n$). The profit function can also be written as

$$\Pi_n = i_n^L L_n + i (1 - r) D_n - i L_n - i_n^D D_n.$$  

(3.9)

Commercial banks are price setters and quantity takers on the credit and the deposits market. This type of simultaneous Bertrand competition between banks is discussed in Yanelle (1988, 1989). One problem of simultaneous Bertrand competition is the existence of a competitive equilibrium “because a monopolist in one market automatically becomes a monopolist in the other market” (Yanelle, 1988, p. XV). If a single commercial bank offers a higher interest rate on deposits than all other commercial banks, it gets all deposits and becomes also a monopolist on the credit market. But in a model with a central bank that offers high powered money at a fixed interest rate there is a second refinancing possibility for commercial banks besides deposits. On this market (the money market), the commercial bank is a price taker.

The third group of agents are the households. They have linear demand functions for loans of every bank $n$:

$$L_n = \alpha_0 + \alpha_1 \cdot i_n^L + \alpha_2 \cdot (i_{-n}^L - i_n^L) + \alpha_3 \cdot Y$$

(3.10)

with

$$\alpha_0, \alpha_2, \alpha_3 > 0, \quad \alpha_1 < 0$$

and for deposits of every bank $n$:

$$D_n = \beta_0 + \beta_1 \cdot i_n^D + \beta_2 \cdot (i_{-n}^D - i_n^D) + \beta_3 \cdot Y$$

(3.11)

with

$$\beta_0, \beta_1, \beta_3 > 0, \quad \beta_2 < 0.$$  

$i_n^L$ and $i_n^D$ are the interest rates on the credit market and the deposits market set by bank $n$, respectively. $Y$ is income and $i_{-n}^L$ and $i_{-n}^D$ are average interest rates of the other banks:

$$i_{-n}^L = \frac{1}{N - 1} \sum_{m=1}^{N} i_m^L \quad \text{and} \quad i_{-n}^D = \frac{1}{N - 1} \sum_{m=1}^{N} i_m^D,$$

Furthermore, it is assumed that $\frac{\alpha_0}{\alpha_1} > i$ and $\frac{\beta_0}{\beta_1} > i$.

### 3.3 Equilibrium on the Credit Market

As already mentioned, the commercial banks are price setters and quantity takers on the credit market and the deposits market. The quantities of loans and deposits in equilibrium are the
results of a price setting game. The following solution of this game is similar to the standard model of a heterogeneous oligopoly in Güth (1994).

The first derivative of the profit function of a commercial bank \( n \) \((3.9)\) with respect to the interest rate on the credit market is

\[
\frac{\partial \Pi_n}{\partial i_n^L} = L_n + i_n^L \cdot \frac{\partial L_n}{\partial i_n^L} - i \cdot \frac{\partial L_n}{\partial i_n^L} = (\alpha_0 + \alpha_1 i_n^L - \alpha_2 i_n^L - \alpha_2 i_n^L - \alpha_3 Y) + i_n^L (\alpha_1 - \alpha_2) - i (\alpha_1 - \alpha_2)
\]

(3.12)

and the first order condition is:

\[
(\alpha_0 + \alpha_2 i_n^L) - (\alpha_1 - \alpha_2) i + 2(\alpha_1 - \alpha_2) i_n^L + \alpha_3 Y = 0.
\]

(3.13)

The second order condition is satisfied:

\[
\frac{\partial^2 \Pi_n}{\partial i_n^{L2}} = 2(\alpha_1 - \alpha_2) < 0.
\]

(3.14)

By inserting the definition of \( i_n^L \) into the first order condition and rearranging we get:

\[
i_n^L = - \left( \frac{\alpha_0 + \frac{\alpha_3}{N-1} \sum_{m=1}^{N} i_m^L}{2(\alpha_1 - \alpha_2) - \frac{\alpha_2}{N-1}} \right) - (\alpha_1 - \alpha_2) i + \alpha_3 Y.
\]

(3.15)

The right-hand side is independent of \( n \), therefore the interest rate on the credit market is the same for all commercial banks:

\[
\forall n: \quad i_n^L = i^L.
\]

(3.16)

It follows that the interest rate on the credit market in equilibrium is

\[
i^L = - \frac{\alpha_0 - (\alpha_1 - \alpha_2) i + \alpha_3 Y}{2\alpha_1 - \alpha_2}.
\]

(3.17)

Inserting this interest rate into the demand for loans yields

\[
L^*_n = \alpha_0 + \alpha_1 i^*_n + \alpha_3 Y
\]

\[
= \frac{\alpha_0 (\alpha_1 - \alpha_2)}{2\alpha_1 - \alpha_2} + \frac{\alpha_1^2 - \alpha_1 \alpha_2}{2\alpha_1 - \alpha_2} i + \frac{(\alpha_1 - \alpha_2) \alpha_3 Y}{2\alpha_1 - \alpha_2}.
\]

(3.18)

The aggregated quantity of loans is

\[
L = c_0 + c_1 \cdot i + c_2 \cdot Y,
\]

(3.19)

where

\[
c_0 > 0, \quad c_1 < 0, \quad c_2 > 0
\]

are the coefficients of the expression for the quantity of loans of a single bank \((3.18)\) multiplied by \( N \), the number of commercial banks. The quantity of loans depends on the money market interest rate and on the income of the households.
3.4 Equilibrium on the Deposits Market

The solution concept on the deposits market is the same as on the credit market. Commercial banks are price setters and quantity takers. The first derivative of the profit function (3.9) of bank $n$ with respect to the interest rate on the deposits market is:

$$
\frac{\partial \Pi_n}{\partial i_n^D} = i \cdot (1 - r) \cdot \frac{\partial D_n}{\partial i_n^D} - D_n - i_n^D \cdot \frac{\partial D_n}{\partial i_n^D} \\
= i \cdot (1 - r) \cdot (\beta_1 - \beta_2) - (\beta_0 + \beta_1 i_n^D + \beta_2 (i_n^D - i_n^D) + \beta_3 Y) \\
- i_n^D \cdot (\beta_1 - \beta_2).
$$

(3.20)

We get the following first order condition:

$$
-(\beta_0 + \beta_2 i_n^D) + (\beta_1 - \beta_2) i(1 - r) - 2(\beta_1 - \beta_2) i_n^D - \beta_3 Y = 0
$$

(3.21)

and the second order condition is satisfied:

$$
\frac{\partial^2 \Pi_n}{\partial i_n^D^2} = -2(\beta_1 - \beta_2) < 0.
$$

(3.22)

In analogy to the credit market, every bank sets the same interest rate on deposits:

$$
i_n^D = -\frac{\beta_0 - (\beta_1 - \beta_2) i(1 - r) + \beta_3 Y}{2\beta_1 - \beta_2}
$$

(3.23)

and the demanded quantity of deposits for every bank is:

$$
D_n^* = \beta_0 + \beta_1 i_n^D + \beta_3 Y \\
= \frac{\beta_0 (\beta_1 - \beta_2)}{2\beta_1 - \beta_2} + \frac{\beta_2^2 - \beta_1 \beta_2}{2\beta_1 - \beta_2} i(1 - r) + \frac{(\beta_1 - \beta_2) \beta_3 Y}{2\beta_1 - \beta_2}.
$$

The aggregated quantity of deposits

$$
D = d_0 + d_1 \cdot i(1 - r) + d_2 \cdot Y
$$

(3.24)

with

$$
d_0 > 0, \quad d_1 > 0, \quad d_2 > 0
$$

depends on the money market interest rate $i$ and on income $Y$. The coefficients $d_i$ are again the coefficients of the expression for the individual quantities multiplied by $N$.

3.5 Monetary Base and Money Stock

The aggregated balance sheet of the commercial banks is:

$$
\sum_{n=1}^{N} (R_n + L_n + IBP_n) = \sum_{n=1}^{N} (D_n + CBC_n).
$$

With $R_n = r \cdot D_n$ and $\sum_{n=1}^{N} IBP_n = 0$ follows

$$
\sum_{n=1}^{N} L_n = \sum_{n=1}^{N} D_n - r \cdot \sum_{n=1}^{N} D_n + \sum_{n=1}^{N} CBC_n
$$
The monetary base $H$ equals central bank credit and it depends on the quantity of loans $L = \sum_{n=1}^{N} L_n$, the quantity of deposits $D = \sum_{n=1}^{N} D_n$ and the required reserve rate $r$. The monetary base is endogenous. The quantity of central bank lending is a result of the profit maximization of the commercial banks.

Aggregating the balance sheet of the banking sector yields the money stock $M$:

$$L = CU + D \equiv M.$$  

The money stock equals the aggregated quantity of loans, that is, it is determined by its counterparts. In a more detailed model of money supply, the other counterparts of the money stock which are neglected here, could be modeled, too. Neglected major counterparts are net foreign assets of the banking system ($FA$) and non-monetary liabilities of the banking system ($NML$):

$$
\begin{align*}
M &= L + FA - NML - O \\
&= (2262.1) + (5247.7) - (262.7) - (3005.2) - (243.0)
\end{align*}
$$

Numbers in parentheses are 1998 averages of German M3 and its counterparts in billions of DM, and $O$ denotes other counterparts.\textsuperscript{10}

3.6 \textit{Comparison of Money Multiplier Approach and Money Supply Endogeneity}

In the money multiplier approach, money is determined by the monetary base and the money multiplier. The monetary base is set exogenously by the central bank, and under the assumptions made here, money is exogenous. In the theory of endogenous money on the other hand, money equals credit demand and the monetary base is a result of the optimal behavior of commercial banks and households.

According to the money multiplier approach, changes in income and changes in the money market interest rate should not cause changes in the money stock. However, this statement only holds in the very simple money multiplier model of section 2.1. In more sophisticated models, the money multiplier depends also on interest rates and on income. Therefore, the impact of interest rates and income on the money stock can not be used to distinguish between the two approaches. However, the effects of changes in the required reserve rate on the monetary base and on the money stock are different in both concepts. Whereas an exogenous monetary base does not depend on changes in the required reserve rate, there is a positive effect on the monetary base in the model of the money-creating sector presented in this section:

$$\frac{\partial H}{\partial r} = d_0 + 2d_1 i(1 - r) + d_2 Y > 0.$$  

A higher required reserve rate causes a decrease in the quantity of deposits but does not affect the quantity of loans. The banks ask for more central bank credit to finance the loans.\textsuperscript{11}

\textsuperscript{10} The data is taken from the monthly bulletin of the Deutsche Bundesbank, February 1999, table II.2.

\textsuperscript{11} Regardless of the considered theoretical model, the monetary base will always increase if the required reserve rate is increased, at least in the very short run. This is due to the definition of the monetary base which is the sum of currency in use and reserves. In the money multiplier approach, however, the required reserve rate and the monetary base are assumed to be more or less independent policy variables.
Another difference between the two concepts is the causality structure of money and income. In the exogeneity view, money is exogenous and causes nominal income via prices (monetarist view) or real income via the loan stock (credit view). In the endogenous money view, income determines the demand for money and monetary aggregates depend on income. It is not excluded that money causes income, therefore the causality structure could be bidirectional.

A money multiplier equation can also be written in the endogenous money framework:

\[
\frac{M}{H} = \frac{c_0 + c_1 i + c_2 Y}{c_0 + c_1 i + c_2 Y - (1 - r)(d_0 + d_1 i(1 - r) + d_2 Y)} = mm(Y, i, r).
\] (3.28)

The interpretation of (3.28), however, is different from the interpretation of (2.2). A discussion of the money multiplier approach, the counterparts approach and their relation can also be found in Artis and Lewis (1990).

\section{The Econometric Model}

\subsection{The Data}

Quarterly data for Germany from 1975-1998 is used in the econometric analysis. The variables are denoted as follows: \( m \) is the logarithmic money stock M3, \( h \) is the logarithmic monetary base, \( y \) is logarithmic gross domestic product in current prices, \( s \) is a short-term interest rate, \( \ell \) is a long-term interest rate and \( r \) is the average required reserve rate. The data is not adjusted for the German unification in 1990 and not seasonally adjusted. Further details can be found in the data appendix. The calculations in this paper are all performed using Mathematica 4.0 modules written by the author.

All variables that are analyzed here are found to be integrated of order one at a significance level of at least 10\% (with exception of the spread, for which the unit root hypothesis can be rejected at 10\% but cannot be rejected at a 5\% level), that is, they have to be differenced once to become stationary. The results of the augmented Dickey-Fuller unit root test are summarized in table 1.\footnote{Unit root test are for example discussed in Hamilton (1994, chapter 17).} For the time series with structural break (\( m, h, \) and \( y \)), the modification of the Dickey-Fuller unit root test proposed by Perron (1989), Perron (1990), and Perron and Vogelsang (1992) is applied. The interest rate series, and the required reserve series do not seem to exhibit a structural break, see figures in Appendix A. The results are quite robust to variations of the number of included lagged differences in the test regression and the inclusion of deterministic terms.

\subsection{Cointegration Analysis}

The econometric framework, that is used to confront the theoretical model of section 3 with empirical evidence, is a vector autoregressive model with \( p = 6 \) variables:

\[
x_t = \mu_t + \sum_{i=1}^{k} A_i x_{t-i} + u_t,
\] (4.1)
If cation. AIC seems to indicate a too high lag length in this case, and extended to allow for a structural break in the time series (German Unification, 1990), see Perron (1989), model information criteria is eight, and a constant and a linear trend are included in the VARM (1995). The process cointegration is performed using the reduced rank regression technique developed by Johansen (1990). Given the finding that the variables are integrated of order one, denoted by $x_t \sim I(1)$, a test on cointegration is performed using the reduced rank regression technique developed by Johansen (1995). The process $x_t$ is cointegrated if there exists a linear combination $\beta' x_t$ that is stationary. If $x_t$ is cointegrated, (4.1) can also be written as a vector error correction model (VECM):

$$\Delta x_t = \mu_t + \Pi x_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + u_t = \mu_t + \Pi x_{t-1} + \Gamma \Delta x_{t-1} + u_t,$$

where $k = 2$ is used, and $\Pi$ is of reduced rank $r$ ($0 < r < p$) and can be decomposed into two
matrices $\alpha$ and $\beta$ such that
\[
\Pi = \alpha \beta',
\]
\[
(p \times p) \quad (p \times r) \quad (r \times p)
\]
$r$ is called cointegration rank and is the number of stationary linear combinations $\beta'x_t$. The columns of $\beta$ are named cointegrating vectors. The $(p \times r)$ matrix $\alpha$ contains the adjustment parameters that describe to which extent the $p$ variables react if the observed levels differ from the $r$ implied long-run equilibria. In the case that is analyzed here the VECM (4.2) has to be slightly modified to consider the structural break in the data. It is assumed in the following that the structural break can be captured with a mean shift in the cointegrating relations. Therefore, the step dummy $d_t = DS903$, that has also been used in the unit root tests in section 4.1, is included into the model such that it occurs only in the long-run relations but not in the short-run dynamics of the model. In other words, the step dummy variable is restricted to the cointegration space. The change in mean in $x_t$ is observed as an outlier in $\Delta x_t$. Therefore, the dummy variable $\Delta d_t = DI903$ ($\Delta d_t$ is one in 1990:Q3 and zero in all other quarters) and its first lag is added in an unrestricted way. The first lag is considered because the lag length of the VECM representation is $k-1 = 1$. The VECM can now be written as:
\[
\Delta x_t = \nu_0 + [\Pi : \nu_1] \begin{bmatrix} x_{t-1} \\ d_{t-1} \end{bmatrix} + \Gamma \Delta x_{t-1} + \nu_2 \Delta d_t + \nu_3 \Delta d_{t-1} + u_t,
\]
\[(4.4)\]

The inclusion of an unrestricted intercept term $\nu_0$ allows for a linear trend in the data, because a constant in the equation for the first differences implies a linear trend in the levels. This specification seems reasonable because three of the six variables are clearly trending (money stock, monetary base, and GDP), see figures in Appendix A. The inclusion of a deterministic trend for the interest rates and the required reserve rate can be justified by the possibility that the constant in the respective equations can be zero. The cointegrating vectors can now be written as
\[
\begin{pmatrix} \beta \\ \mu_1 \end{pmatrix} \begin{pmatrix} x_t \\ d_t \end{pmatrix} \sim \Pi(0),
\]
\[(4.5)\]

where $\mu_1$ is a $(1 \times r)$ vector, that contains the coefficients of the step dummy in the long-run relations. The complete set of cointegration coefficients is stored in $\beta^* = (\beta', \mu_1')'$. Figure 3 shows the results of tests for the cointegration rank. The tests are calculated in a recursive way starting with the sample 1975:1-1981:4 and ending with the sample 1975:1-1998:4. In addition to the previously described deterministic specification some other relevant specifications are considered. Up to the German unification, the hypotheses that the cointegration rank is at most one is clearly rejected. Afterwards the test results are not that unambiguous. Specification (4.4) corresponds to case (c), and it is assumed that the cointegration rank is two though the trace statistic for the whole sample does not reject the hypothesis that the cointegration rank is at most one. This procedure can be justified by arguing that the adjustment process after the German unification is the reason for the fluctuations of the trace statistic around the critical value.

According to the result of the cointegration test, the VECM (4.4) is estimated, and $r = 2$ cointegrating relations are imposed.\(^{13}\) The unrestricted cointegrating vectors are given in table 2. The interpretation of the cointegrating vectors depends on the identifying assumptions that have to be made at this stage. As mentioned earlier it is assumed that log-linear approximations of the money stock equation and the monetary base equation are valid such that two stationary linear combinations exist. The inspection of the unrestricted estimators of $\beta$ suggests that one

\(^{13}\) Estimation of vector error correction models and related topics are summarized in Lütkepohl (2001).
Figure 3: Tests for the Cointegration Rank of $x_t = (m_t, h_t, y_t, \ell_t, s_t, r_t)^T$

Notes: The figures show the LR trace statistic for the hypothesis that the cointegration rank is at most zero (upper curve) and for the hypothesis that the cointegration rank is at most one (lower curve). Different specifications of the deterministic terms are used in cases (a) to (f); centered seasonal dummies are included in any case. Horizontal lines are 5% critical values, these are tabulated in the standard cases (a), (b), (d), (e) where no dummy variables are restricted to the cointegration space. For the cases (c) and (f), the critical values are generated with the program DisCo, see Johansen and Nielsen (1993), with time series of length $T = 400$ and a step dummy which is zero for the first 65% of the sample and 1 afterwards. Critical values are based on 10,000 replications of the simulation experiment.
Table 2: Long-Run Relations

<table>
<thead>
<tr>
<th>m</th>
<th>h</th>
<th>y</th>
<th>ℓ</th>
<th>s</th>
<th>r</th>
<th>DS903</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unrestricted cointegrating vectors</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-30.98</td>
<td>7.60</td>
<td>22.51</td>
<td>-81.06</td>
<td>56.36</td>
<td>-214.94</td>
<td>0.34</td>
</tr>
<tr>
<td>-25.20</td>
<td>33.79</td>
<td>-5.31</td>
<td>-72.66</td>
<td>50.62</td>
<td>-110.95</td>
<td>-2.81</td>
</tr>
<tr>
<td>just identified cointegrating vectors</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 0 -0.94 2.56 -1.78 7.50 0.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 -1 -0.08 2.80 -1.95 5.19 0.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>overidentified cointegrating vectors</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 0 -1.32 2.54 0 0 0</td>
<td>(0.04) (0.70)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 -1 0.21 3.76 -3.76 10.62 0.04</td>
<td>(0.07) (0.76) (0.76) (0.90) (0.03)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Reduced rank estimation of the VECM (4.4) as described in Johansen (1995) and Lütkepohl (2001), the cointegration rank is two, the number of lags is two. Centered seasonal dummies are included. The estimates reported are the entries of \( \beta^* \). Asymptotic standard errors in parentheses. The sample period is 1975:1-1998:4.

relation is a money demand function (\( M \) and \( Y \) have approximately coefficients proportional to 1 and \(-1\)) and that the other one is a money multiplier relation (\( M \) and \( H \) have approximately coefficients proportional to 1 and \(-1\)). Therefore, the following identifying restrictions are imposed:

\[
\beta^* = (H_1 \phi_1, H_2 \phi_2),
\]

where the matrices \( \phi_i \) contain the unrestricted estimates and \( H_1 \) and \( H_2 \) are defined as

\[
H_1' = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\quad \text{and} \quad
H_2' = \begin{pmatrix}
1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}.
\]

The restricted cointegrating relations can now be described by the two equations:

\[
m_t = - \beta^*_{31} y_t - \beta^*_{41} \ell_t - \beta^*_{51} s_t - \beta^*_{61} r_t - \beta^*_{71} d_t + e_{c1t},
\]

\[
m_t - h_t = - \beta^*_{32} y_t - \beta^*_{42} \ell_t - \beta^*_{52} s_t - \beta^*_{62} r_t - \beta^*_{72} d_t + e_{c2t},
\]

where \( e_{c1t} \) and \( e_{c2t} \) are the error correction terms which can be interpreted as deviations from the long-run equilibria. These restrictions satisfy the rank criterion given in Johansen and Juselius (1994, Theorem 1) such that they are identifying:

\[
\text{rk}(H_{11}' H_2) = \text{rk}(H_{21}' H_1) = 1 \geq 1.
\]

The corresponding estimates are given in the middle part of table 2. It can be tested if this is a valid identifying normalization scheme. The imposed restrictions imply that the variables \( y_t, \ell_t, s_t \) and \( r_t \) are not cointegrated. Luukkonen et al. (1999) suggest a test for a valid normalization.
using this implication. The test statistic that they propose is a likelihood ratio test statistic for certain zero restrictions in an auxiliary regression model. This test statistic has the same limiting distribution as the corresponding likelihood ratio trace statistic for cointegration. In the case considered here, the likelihood ratio test statistic has a value of $\lambda_{LR} = 49.01$. The 5% critical value (generated with DisCo, see notes to figure 3) is 55.63 such that the validity of the normalization is not rejected at a significance level of 5%.

The required reserve rate measures a policy variable that is assumed to be exogenous, that is, it is assumed that the central bank does not have a reaction function for setting the required reserve rate. Therefore, the model is now conditioned on the required reserve rate in order to get a slightly more parsimonious model. Further restrictions can be imposed. One implication of the theoretical model is that the required reserve rate does not occur in the money demand relation. Additionally, it is often sufficient to have one interest rate in the money demand relation, in general this is a long-term interest rate; and it turns out that the money demand relation remains stable after the structural break in 1990 such that the step dummy is not needed in this cointegration relation. Imposing these restrictions and calculating the cointegrating vectors suggests a further restriction on the second cointegrating vector. The coefficients of the long-term and the short-term interest rate are approximately proportional to 1 and $-1$ such that it can be assumed that the interest rate spread is part of the money multiplier relation. The overidentifying set of variables that is now imposed is characterized by

$$H'_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad H'_2 = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

or alternatively by

$$m_t = -\beta_{31}y_t - \beta_{31}^*\ell_t + ec_{1t}$$

$$m_t - h_t = -\beta_{32}y_t - \beta_{32}^*(\ell_t - s_t) - \beta_{62}^*r_t - \beta_{72}^*d_t + ec_{2t}$$

with

$$\text{rk}(H'_1, H'_2) = 4 \geq 1 \quad \text{and} \quad \text{rk}(H'_2, H'_1) = 2 \geq 1.$$  

The validity of these overidentifying restrictions can be tested by comparing the maximized likelihood function of the unrestricted and the restricted model. The corresponding likelihood ratio test statistic has a value of 6.19 and is $\chi^2$-distributed with 4 degrees of freedom such that the empirical significance level is 0.19. The restrictions are not rejected at usual significance levels, and the cointegrating vectors can be found in the lower part of table 2. Abstracting from deterministic terms they can be interpreted as a money demand function

$$m_t = 1.32y_t - 2.54\ell_t + ec_{1t}$$

and a money multiplier relation (recall that $m$, $h$ and $y$ are measured in logarithms)

$$m_t - h_t = 0.21y_t - 3.76(\ell_t - s_t) - 10.62r_t + ec_{2t}.$$  

These two long-run relations imply that

$$h_t = 1.11 y_t + 1.22 \ell_t - 3.76 s_t + 10.62 r_t + (ec_{1t} - ec_{2t}).$$

14 Partial VECMs are discussed in Harbo et al. (1998).

15 This test is described in Johansen and Juselius (1994) and in Johansen (1995).
This is a long-run relation between the monetary base and the variables that determine it in the theoretical model of section 3. It can be seen that all coefficients including the required reserve rate have the expected signs.

The money demand function (4.13) can not be compared directly to other money demand studies for Germany. While (4.13) is a demand function for nominal money, most other studies focus on real money balances. The general result that a stable money demand function can be specified for the unified Germany is also supported by Wolters et al. (1998), for example. These authors find a stable real money demand function (including the step dummy DS903) for the period 1976 to 1994. However, a stable relationship between the monetary base and the broad money stock M3 has not been found in other studies. The reason is presumably that the impact of the required reserve rate is modeled explicitly here while Willms (1993) and Nautz (1998), for example, do not include the required reserve rate in the model but use a monetary base that is adjusted for changes in the required reserve rate.

4.3 Adjustment and Short-Run Dynamics

The cointegrating vectors do not show the complete information about the relationships between the variables in the system. It is still an open question which variables react on deviations from the long-run equilibria and how innovations or shocks affect the variables.

When the cointegrating vectors $\beta^*$ are known, the other parameters of the VECM can be estimated by OLS. The adjustment to the long-run equilibria can be characterized by the adjustment parameters that are stored in the matrix $\alpha$. These parameters are the coefficients of $e_{c_1,t-1}$ and $e_{c_2,t-1}$ in the equations for the first differences. The error correction terms are plotted in figure 4. The adjustment parameters are summarized in table 3, where the results of tests on weak exogeneity for each variable can be found, too. The test on weak exogeneity is a likelihood ratio test on restrictions on the loading matrix $\alpha$ as described in Johansen (1995). A variable is said to be weakly exogenous if its adjustment coefficients in front of all cointegration relations ($e_{c_1,t-1}$ and $e_{c_2,t-1}$) are zero: $\alpha_{i1} = \alpha_{i2} = 0$.\textsuperscript{16} If the money stock, $m$, is considered, the restricted loading matrix is:

$$\alpha = \begin{bmatrix} 0 & 0 \\ \alpha_{21} & \alpha_{22} \\ \vdots & \vdots \\ \alpha_{51} & \alpha_{52} \end{bmatrix}. \tag{4.16}$$

These tests suggest that both the money stock and the long-term interest rate are weakly exogenous. On the other hand, nominal income, the short-term interest rate and and the monetary...

\textsuperscript{16} For a discussion of (weak) exogeneity and cointegration see Ericsson et al. (1998).
Table 3: Adjustment Coefficients (Loading Matrix $\alpha$)

<table>
<thead>
<tr>
<th>$e_{c1,t-1}$</th>
<th>$\Delta m$</th>
<th>$\Delta h$</th>
<th>$\Delta y$</th>
<th>$\Delta \ell$</th>
<th>$\Delta s$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.08</td>
<td>0.01</td>
<td>0.17</td>
<td>-0.02</td>
<td>-0.00</td>
</tr>
<tr>
<td></td>
<td>(-1.89)</td>
<td>(0.15)</td>
<td>(3.47)</td>
<td>(-1.63)</td>
<td>(-0.08)</td>
</tr>
<tr>
<td>$e_{c2,t-1}$</td>
<td>-0.01</td>
<td>0.16</td>
<td>0.14</td>
<td>-0.02</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(-0.41)</td>
<td>(3.28)</td>
<td>(3.73)</td>
<td>(-1.83)</td>
<td>(2.00)</td>
</tr>
<tr>
<td>Weak Exogeneity</td>
<td>4.22</td>
<td>6.93</td>
<td>7.50</td>
<td>4.61</td>
<td>8.00</td>
</tr>
<tr>
<td>p-value</td>
<td>0.12</td>
<td>0.03</td>
<td>0.02</td>
<td>0.10</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Notes: Coefficients of the error-correction term in the equation for the variable in the first row. Ratio of coefficient and asymptotic standard errors in parentheses. The test on weak exogeneity is a likelihood ratio test of zero restrictions on $\alpha$. While $\alpha$ is calculated under consideration of the overidentifying restrictions on $\beta$, the weak exogeneity test statistic is calculated without imposing these restrictions. Weak exogeneity is rejected if the $p$-value is smaller than 5%.

base adjust in direction of the long-run equilibria if equilibrium-deviations occur. This constellation can be given the following economic interpretation. Consider an increase of the money stock such that $e_{c1t} > 0$. In the next period, nominal income and therefore demand for nominal money increase such that the equilibrium error is getting smaller. The initial increase of the money stock gives also rise to a positive error correction term in the money multiplier relation. In the next period, the monetary base is increasing such that the equilibrium error is getting smaller. This economic scenario contradicts the money multiplier approach as well as the endogenous money model of the money-creating sector. It seems to be the case that the nominal money stock has been under control of the central bank and that nominal income and the short-term interest rate as well as the monetary base have done the adjustment.

The reaction of the variables to innovations in other variables can be analyzed with impulse response functions and forecast error variance decompositions. They are calculated from the level representation:

$$x_t = \mu_t + A_1 x_{t-1} + A_2 x_{t-2} + u_t,$$

with $A_1 = \Gamma + \Pi + I_5$ and $A_2 = -\Gamma$. $\mu_t$ contains the deterministic terms. Because $A_1$ and $A_2$ are calculated from the VECM representation, the cointegration restrictions and the overidentifying restrictions are imposed on the level coefficients. Now, the impulse responses and their asymptotic standard errors as well as the forecast error variance decompositions can be calculated, see Appendix B. and Lütkepohl (1993, Chapter 11). The orthogonalized impulse responses and the forecast error variance decompositions are depicted in Appendix B. The ordering of the variables is $(m, \ell, s, y, h)$; the two variables that are weakly exogenous are the first ones in the ordering.\(^{17}\) Impulse responses are supposed to be significant if zero is not included within the confidence bands which are calculated by adding and subtracting two asymptotic standard errors.

Not all 25 impulse responses can be described here, but some important findings should be mentioned. First, innovations in the money stock have a positive impact on future nominal

\(^{17}\) The ordering of variables is important in the sense that a recursive structure of contemporaneous relations is assumed in the calculation of orthogonalized impulse responses. This is the case because the Cholesky decomposition of $\Sigma_u$ which gives a lower triangular matrix is used to orthogonalize the innovations. The ordering is not important in the sense that the interpretation of the impulse responses in this application depends crucially on the ordering of the variables.
income, that is, a monetary expansion does lead to higher prices and/or higher real income. The reverse impact can also be seen: nominal income seems to have a small but significant impact on the money stock. The forecast error variance decomposition, however, shows that this effect can be neglected. Second, innovations in the money stock have an impact on the monetary base but innovations in the monetary base do not lead to a significant response of the money stock. Third, the monetary base is negatively affected by innovations in the interest rates while the money stock is not. This implies that the money multiplier depends on the interest rates. All these findings are compatible with the economic scenario described in the previous section. That is, there is no evidence against the view that the money stock has been an exogenous, policy-determined variable during the period of monetary targeting. However, the money multiplier approach is strongly rejected. The monetary base seems to be caused by the money stock which is opposite to the causality expected from the money multiplier approach. Similar results are reported by Brand (2001) who estimates a state space model for interest rates, bank reserves, and the money stock. He concludes that (p. 114 f.) “it would be misleading to view the money supply process in terms of a money-multiplier model, since interest rates and money are exogenous to bank reserves and not vice versa.”

Further, the coefficient of $\Delta r$ is only significant in the monetary base equation of the VECM. Its value is 4.50 with a standard error of 0.69 such that the positive impact of the required reserve rate on the monetary base that is predicted from the theoretical model can be confirmed. The respective coefficient in the equation for the money stock has a value of $-0.12$, the standard error is 0.47.

The analysis of the short-run dynamics is completed by tests on Granger causality.\(^\text{18}\) The vector $x_t = (m_t, h_t, y_t, \ell_t, s_t)^T$ can be partitioned into three parts:

$$x_t = (x_{1t}^T, x_{2t}^T, x_{3t}^T)^T. \quad (4.18)$$

The number of elements are $n_1$, $n_2$, and $n_3$, respectively, with $n_1 + n_2 + n_3 = 5$. The null hypothesis that the first $n_1$ variables $x_{1t}$ are not Granger caused by the last $n_3$ elements $x_{3t}$ can be formulated as follows:

$$H_0 : \Pi_{13} = \Gamma_{13} = 0, \quad (4.19)$$

where $\Pi_{13}$ and $\Gamma_{13}$ are the upper-right $n_1 \times n_3$ submatrices of $\Pi$ and $\Gamma$. Toda and Phillips (1993) show that under the null, the Wald test statistic $F_{ML}$ is $\chi^2$-distributed with $n_1 n_3 k$ degrees of freedom, if certain rank conditions are fulfilled\(^\text{19}\) ($k = 2$ is the lag length of the VAR model in levels). The Wald test statistic is calculated for all subsets with $n_1 = n_3 = 1$, see table 4. From the first row follows that $m$ is not Granger caused by $y, \ell$ and $s$ at a significance level of at least 24%, and the second row shows that $h$ is Granger caused by all other variables. It can also be seen that money is Granger causing nominal income. With exception of the result that the monetary base Granger causes the money stock, the causality test results confirm the previous results.

### 5 Conclusions

A model of the money-creating sector with endogenous money has been developed. In this model, money equals credit, and the monetary base is determined by the profit-maximizing behavior of commercial banks.

---

18 Testing for Granger causality in cointegrated vector autoregressive models is covered in Toda and Phillips (1993), where the following paragraph draws from. An alternative approach to test for Granger causality in integrated VARs is the lag augmentation procedure which is described in Dolado and Lütkepohl (1996) and Toda and Yamamoto (1995).

19 The rank condition is satisfied in every case reported in table 4.
Table 4: Granger Causality Tests

<table>
<thead>
<tr>
<th></th>
<th>( m )</th>
<th>( h )</th>
<th>( y )</th>
<th>( \ell )</th>
<th>( s )</th>
</tr>
</thead>
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<td>0.25</td>
<td>0.42</td>
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<tr>
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<td>–</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>( y )</td>
<td>0.00</td>
<td>0.01</td>
<td>–</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>( \ell )</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>–</td>
<td>0.00</td>
</tr>
<tr>
<td>( s )</td>
<td>0.33</td>
<td>0.00</td>
<td>0.58</td>
<td>0.02</td>
<td>–</td>
</tr>
</tbody>
</table>

Notes: Null hypothesis of the test: The column variable is Granger noncausal for the row variable. The table shows the empirical significance levels of the Wald test statistic \( F_{ML} \) described in the text. Granger noncausality is rejected if the empirical significance level is smaller than the desired nominal significance level.

The implications of the theoretical model have been tested in the framework of a cointegrated vector autoregressive model. Some empirical evidence against the money multiplier approach has been found: A stable relationship between the monetary base and the money stock can be specified, but the nominal money stock does not adjust if deviations from this long-run relation occur; the adjustment is done by the monetary base instead. Furthermore, the required reserve rate has not the negative impact on the money stock that is predicted from the money multiplier approach. The model of the money-creating sector that has been presented has its drawbacks, too. Some of its implications are not found in the data. Especially the weak exogeneity of nominal money is a contradiction to the theoretical model. While it cannot be rejected that the monetary base is endogenously determined like it is implied in the theoretical model, the money stock can be supposed to be an exogenous (in the sense of policy-determined) variable. One further drawback is that the money stock is set equal to the quantity of loans, which are only one item of the counterparts of M3. There are also other important counterparts like net capital formation, for example.

The main conclusion of this analysis is therefore that the Bundesbank has been able to control the money stock M3 to a considerable extent but that the standard textbook money multiplier approach is not appropriate to describe how the Bundesbank has affected the development of M3. Because a stable money demand relation can be specified for the period of monetary targeting in Germany it is reasonable to suppose like for example Brand (2001) that the Bundesbank followed a policy of indirect monetary targeting by changing money market conditions.
Appendix A: Data

M3: End of month money stock M3 (currency in use plus sight deposits of domestic non-banks at domestic banks in Germany plus time deposits for less than four years of domestic non-banks at domestic banks plus savings deposits at three months’ notice of domestic non-banks at domestic banks in Germany) in billions of DM, seasonally unadjusted. Monthly data (TU0800) from the Compact Disc Deutsche Bundesbank (1998a), continued with data from the monthly bulletin of the Deutsche Bundesbank, table II.2. 1975:01-1990:5 West Germany, and 1990:06-1998:12 Germany, not adjusted for German unification. Quarterly data are end of quarter stocks.

Monetary base: sum of currency in use (TU0048), required and excess reserves (TU0062), liabilities of the Deutsche Bundesbank against domestic banks (TU0084) and cash of banks (OU0312), in billions of DM, seasonally unadjusted. Monthly data from the Compact Disc Deutsche Bundesbank (1998a), continued with data from the monthly bulletin of the Deutsche Bundesbank, tables II.2., III.2 and IV.1. Quarterly data are end of quarter stocks.


Short-term interest rate: Daily money market interest rate, Frankfurt/Main, monthly averages, fractions, monthly data (SU0101) from the Compact Disc Deutsche Bundesbank (1998a), continued with data from the monthly bulletin, table VI.4. Quarterly data are the respective values of the last month in a quarter.

Long-term interest rate: Yields on bonds outstanding issued by residents, monthly averages, fractions, monthly data (WU0017) from the Compact Disc Deutsche Bundesbank (1998a), continued with data from the monthly bulletin, table VII.5. Quarterly data are the respective values of the last month in a quarter.

Average required reserve rate: Ratio of required reserves (IU3006) and reserve base of banks subject to reserve requirements (IU3156), monthly data from the Compact Disc Deutsche Bundesbank (1998a), continued with data from the monthly bulletin of the Deutsche Bundesbank, table V.2. Quarterly data are the respective values of the last month in a quarter.
Figure 5: Data

- Logarithmic Money Stock M3
- Logarithmic GDP Deflator
- Logarithmic Nominal GDP
- Logarithmic Real GDP
- Logarithmic Monetary Base
- Average Required Reserve Rate
- Short-run Interest Rate
- Long-run Interest Rate
Appendix B: Short-Run Analysis

The impulse response functions and forecast error variance decompositions are calculated from the level representation of the VAR which is obtained by transforming the VECM estimated in section 4.2. This is a linear transformation, and the standard errors for the coefficients in the level representation can be calculated from the VECM standard errors. Details of this procedure can be found in Lütkepohl (1993, Chapter 11). When the level representation and the corresponding standard errors have been computed, the impulse responses, forecast error variance decompositions, and their standard errors can be calculated like in the case of a stationary VAR.

<table>
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<td>$m$</td>
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<tr>
<td></td>
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</tr>
<tr>
<td>2</td>
<td>0.99</td>
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<tr>
<td></td>
<td>(0.01)</td>
</tr>
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<td>3</td>
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<tr>
<td></td>
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<tr>
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</tr>
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</table>

Notes: Ordering $m, \ell, s, y, h$. Asymptotic standard errors in brackets.
Table 6: Forecast Error Variance Decomposition for $h$

<table>
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<tr>
<th>horizon</th>
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<th>$h$</th>
<th>$y$</th>
<th>$\ell$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>3</td>
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Notes: Ordering $m, \ell, s, y, h$. Asymptotic standard errors in brackets.

Table 7: Forecast Error Variance Decomposition for $y$

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<th>$y$</th>
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<th>$s$</th>
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Notes: Ordering $m, \ell, s, y, h$. Asymptotic standard errors in brackets.
Table 8: Forecast Error Variance Decomposition for $\ell$

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<th>$y$</th>
<th>$\ell$</th>
<th>$s$</th>
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</thead>
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</tr>
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<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
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<td>0.00</td>
<td>0.01</td>
<td>0.94</td>
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<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.03)</td>
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</tr>
<tr>
<td></td>
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<td>(0.00)</td>
<td>(0.02)</td>
<td>(0.05)</td>
<td>(0.04)</td>
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Notes: Ordering $m, \ell, s, y, h$. Asymptotic standard errors in brackets.

Table 9: Forecast Error Variance Decomposition for $s$

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<th>$y$</th>
<th>$\ell$</th>
<th>$s$</th>
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</tbody>
</table>

Notes: Ordering $m, \ell, s, y, h$. Asymptotic standard errors in brackets.
Figure 6: Orthogonalized Impulse Responses

Notes: Ordering \( m, \ell, s, y, h \). Dotted lines: \( \pm 2 \) asymptotic standard errors.


