

Real Estate Valuation According to Standardized Methods: An Empirical Analysis

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Abstract

Appraisals are needed for decision-making and for performance evaluation. Knowledge on the accuracy of valuation methods is of general interest for banks and investors. We assess the accuracy of the German Regulation on Valuation with monthly data on appraisals and prices for commercial apartment houses in Berlin, Germany from 1980 to 2000. The appraisals are compared with outcomes of the simpler capitalization method and are ranked better according to several prediction error measures. Nonparametric density estimates give the error distributions and investors can decide which method is preferable. Eventually, we explain short-run deviations between prices and appraisals by incompletely appraised object-specific factors and by market indicators.

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1 Introduction

“A rule of thumb, artfully employed, sometimes beats a complex discounted cash flow calculation hands down.” (Brealey and Myers, 2000, p. 82)

Real estate valuation is the task of appraising the prospective price of a site or building in the case of a sale. Such appraisals are important for investment decisions, for real estate funds and project developments. Additionally, real estate performance indexes like the German property index DIX are calculated with appraised values. As McParland, Adair, and McGreal (2002) found out, many European countries have their own national valuation standards. The internationalization of real estate suggests an investigation of such standards, because foreign investors need to understand the concepts that national appraisers use. In addition to the need of clarifying existing valuation concepts, there is also a need for a better *empirical assessment of the accuracy of property valuations*, see the statements of the *Carsberg Report* from the RICS (2002). Only such an assessment allows to evaluate the risk inherent in competing national standards. Eventually, due to the fact that German valuation standards for commercial real estate are discounted values, this study provides evidence if such values are reliable estimates of market values. Under this aspect, this study is of general interest for appraisers, banks and investors, irrespective if they need valuations of real estate, stocks or companies.

Generally, it is difficult to assess the accuracy of appraisals because every appraisal might be influenced by the practical knowledge of the respective appraiser. It has been said that appraising is more an art than a technique that produces inter-subjective outcomes. Indeed, it is not negligible that appraisals will be influenced by idiosyncratic effects, especially by the talent of the appraiser. Hence, it seems impossible to assess the quality of appraisals in general. However, by drawing this conclusion, one fails to recognize an important aspect: real estate markets do not behave totally irregular and transaction prices are not totally random. An appraisal *method* that mimics the regularities of the real estate market will be on average superior to the outcomes of any irregular and talented appraiser. Given such regulari-

ties, we should concentrate on *standardized methods* that can be applied—in principle—by everyone. The first step in assessing valuation standards is their plausibility and conformity with economic theory.

What are the regularities that govern the behavior of prices in the real estate market? If one assumes that investors are rational, then their decision to sell or buy should be based on the *present value* of the expected net proceeds from the object over the holding period. Many standardized methods to calculate appraisals are based more or less on this concept. However, whereas some methods use complicated formulas that try to incorporate as much information as possible, other methods are simple rules of thumb that capitalize the current proceeds by multiplying it with a constant factor. It is an important question if such simplified—and inexpensive—methods deliver accurate results. Thus, the second step in assessing valuation standards consists in an empirical evaluation of their accuracy.

Crosby (2000) gives an detailed overview on studies for Anglo-American countries that assess the accuracy of appraisals. In addition to studies that compare transaction prices and appraisals, there also exist studies that compare different appraisals for identical objects, see Graff and Young (1999). One can criticize such studies because they implicitly assess appraisers and not valuation methods. That critique is not applicable to our study. Unlike other European countries, appraisers in Germany are obliged in many cases to use prescribed methods. These methods are so prominent, that appraiser use them even for cases where their usage is not explicitly prescribed (McParland, Adair, and McGreal, 2002). The methods are codified in the *Regulation on Valuation* (Wertermittlungsverordnung) and the *Guidelines on Valuation* (Wertermittlungs-Richtlinien). We use data on transactions of commercial apartment buildings in Berlin, Germany, to evaluate the accuracy of the so-called *income valuation method* (Ertragswertverfahren). Income valuation is prescribed for commercial real estate. We compare outcomes from income valuation with outcomes of the simpler—and less expensive—*capitalization method* (Maklermethode).

Due to the fact that income valuation and capitalization method are oriented on the present value concept, our study is comparable with the

work of Kaplan and Ruback (1995). They use market values of companies and compare transaction prices with meticulously calculated discounted cash flows. As a result, Kaplan and Ruback derive that their discounted cash flows perform at least as well as comparable valuation methods.

The article proceeds as follows. In Section 1 we clarify the present value and show that the concept of income valuation is akin to the present value. The capitalization method is just a further simplification of this concept. In Section 2, we combine economic theory and valuation methods to derive some implications for the relationship between transaction prices and appraisals. Systematic short run deviations should be influenced by market indicators, that are new information about building permits, financial conditions and rents. In Section 3 we present the data, explore the accuracy and compare the different valuation methods. Section 4 presents estimates that explain the short-term deviation between prices and appraisals by market indicators. The final Section 5 summarizes and presents our conclusions.

One final remark is necessary: throughout the paper, we assume rational investors who use the present value to decide about favorable investments. This assumption is disputable because it excludes irrational behavior on the outset. Irrationality means, that investors act according to sentiments and do not base their decisions on present values (Shiller, 2000; Shleifer, 2000). Under irrationality it seems possible that prices and rents have different stochastic trends and no linear combinations of them will be stationary. If that is the case and prices are driven by irrational behavior and not by the stochastic behavior of rents, then any appraisal method that uses rents to predict prices is at least superfluous. In that case we should stop our work, because we would look for a relationship that does not exist. To derive meaningful results, we have to require that deviations between prices and appraisals are stationary. All statistical conclusions we conduct in this paper are derived under this requirement.

2 Concepts of valuation

2.1 Present value

The owner of an apartment house accrues every month the rents of the tenants. A part of the rents provide cover for maintenance, management and running cost. We denote the proceeds that remain after subtraction of the cost as *net proceeds*. The ownership of an apartment house entitles a stream of uncertain net proceeds over the holding period. Thus, given rational investors, the price of a house must be closely related to this stream of income. The relationship is given by the present value, a standard tool for evaluating investment opportunities (Hirshleifer, 1958; Brealey and Myers, 2000). In its generalized formulation (Cochrane, 2001), the present value for apartment house n in period t is given as

$$V_{n,t} \stackrel{\text{def}}{=} \mathcal{E}_t \left[\sum_{j=1}^{\infty} \frac{D_{n,t+j}}{\prod_{i=1}^j (1 + R_{t+i})} \right], \quad (1)$$

where $\mathcal{E}_t[\cdot]$ denotes the expectation operator given all information up to period t . $D_{n,t+j}$ are the net proceeds for the apartment house in period $t + j$ and R_{t+j} are discount rates for income in that period. The discount rates are returns that investors require for investments in commercial real estate. These rates are equal to returns of other investments that share the same risk as an investment in commercial real estate. If the house n can be utilized only $T_{n,t}$ periods, it has in period $t + T_{n,t}$ additionally a wreckage value that can be incorporated into $D_{n,t+T_{n,t}}$, and $D_{n,t+T_{n,t}+j} = 0$ for $j > 0$. With no demolition costs, the lower bound of the wreckage value in $t + T_{n,t}$ is just the price of the site.

So far, we have discussed the present value as a decision tool for individual investors. A potential investor should buy apartment house n , when the price for that house is at most as large as the present value. A house owner should sell his house, if he can retrieve at least the present value. In both cases, investors are free to use discount rates that account for the composition and the risk of their own portfolio. However, as we will show

later on, standardized German valuation methods prescribe discount factors that have to be used for valuation. These discount rates are calculated from observed transaction prices. The crucial—and disputable point—for any valuation method that uses prescribed discount factors is, that observed transaction prices and present values must coincide. Given well informed investors, their expectations about the rents will be the same (although the owner of a house might have better information). If one assumes furthermore that the marginal price—the transaction price—is just equal to the present values (1) of buyer and seller, then their discount rates must be the same. Given a complete capital market and no-arbitrage, discount rates will be the same Kruschwitz (1995). However, what happens if we have to incorporate transaction cost? These cost can be fees for real estate agents and notaries and purchase taxes on real estate. In most cases such cost are proportional to the transaction price. Let δ^B denote the relative cost for a buyer and accordingly δ^S the relative cost for a seller, then a buyer is indifferent if

$$(1 + \delta^B)P_{n,t} = V_{n,t}^B$$

and a seller is indifferent if

$$(1 - \delta^S)P_{n,t} = V_{n,t}^S$$

where $P_{n,t}$ is the transaction price and $V_{n,t}^i$ the present value for the buyer ($i = B$) and—respectively—the seller ($i = S$). A transaction will only occur if buyer and seller employ different discount rates. It is also easy to see that a correspondence between transaction price and present value still holds if we divide the present value of the buyer with $1 + \delta^B$ and the present value of the seller with $1 - \delta^S$. After the implicit multiplication with the relative cost, the expected net proceeds are discounted with adjusted discount factors that are the same for buyers and sellers.

2.2 Valuation in practice

In the practice of valuation, simplified versions of the present value (1) are used (Booth, Chadburn, Cooper, Haberman, and James, 1999). A first simplification replaces the time-varying discount rates with its long run average

R . One obtains with this restriction

$$F_{n,t} = \sum_{j=1}^{\infty} \frac{\mathcal{E}_t[D_{n,t+j}]}{(1+R)^j}. \quad (2)$$

Here, $F_{n,t}$ denotes the *fundamental value* of the discounted net proceeds. In a strict sense, the general present value given in equation (1) is also a fundamental value, because it excludes any rational bubble, see for example Campbell, Lo, and MacKinlay (1997). However, to provide an easy notation and for emphasizing the usage of the long-run discount rate, we label only (2) as fundamental value. Deviations between the generalized present value (1) and the fundamental value—given identical expectations on the net proceeds—are due only to short run deviations of the discount rates from its long run average.

One obtains an important simplification of the fundamental value with the assumption of a constant expected growth rate for the net proceeds. Let G denote the growth rate of the net proceeds with $G < R$, one obtains

$$F_{n,t}^{cT} = \frac{1}{\theta} \left\{ 1 - \left(\frac{1}{1+\theta} \right)^{T_{n,t}} \right\} D_{n,t} + \left(\frac{1}{1+\theta} \right)^{T_{n,t}} \sum_{j=1}^{\infty} \frac{\mathcal{E}_t[D_{n,T_{n,t}+j}]}{(1+G)^{T_{n,t}}(1+R)^j} \quad (3)$$

with $\theta \equiv (R-G)/(1+G)$. Here, $T_{n,t}$ is the expected number of periods during that the current building can be used. The first term on the RHS gives the fundamental value of the current building given that it will be used in the next $T_{n,t}$ periods. The second term is the discounted fundamental value of a new building in $t + T_{n,t} + 1$ and thus the discounted price that the seller can expect for selling the site in $t + T_{n,t} + 1$. The superscript cT indicates that the fundamental value is derived under the additional assumptions of constant growth rates and with a finite time of usage of the current building.

If the net proceeds expected for periods after $t + T_{n,t}$ are given by

$$\mathcal{E}_t[D_{n,T_{n,t}+j}] = (1+G)^{T_{n,t}+j} D_{n,t} \quad (4)$$

with $j > 0$, then we obtain a further simplification of the fundamental value

$$F_{n,t}^c = \frac{D_{n,t}}{\theta}. \quad (5)$$

F^c is known in the literature as the *capitalization method*. The meaning of the last label is easily understood if one rewrites (5) as

$$F_{n,t}^c = cD_{n,t} ,$$

where $c \stackrel{\text{def}}{=} 1/\theta$ is the so-called capitalization factor. We drop the superscript T to indicate that (5) is derived under the—implicit—assumption of an infinite usage of the current building.

However, even if the expected net proceeds for a new building on the site have no relationship to the net proceeds of the current building and (4) does not hold, $F_{n,t}^c$ will be approximately equal to $F_{n,t}^{cT}$ if the remaining time of usage is large. Thus, the capitalization method is just a simplified version of $F_{n,t}^{cT}$ that needs no information on $T_{n,t}$ and on the expected price of the site in $t + T_{n,t} + 1$.

The codified German valuation method for commercial real estate is closely related to the F^{cT} given in (3). In the next Subsection, we present the method in detail. Furthermore, we present the so-called real estate agent method, which is just the capitalization method and which is used as rule of thumb by real estate agents.

2.3 Valuation according to WertV

In Germany, real estate valuation is codified through the Regulation on Valuation (Wertermittlungsverordnung, WertV) and the Guidelines on Valuation (Wertermittlungs-Richtlinien, WertR 91). Gottschalk (1999) gives an comprehensive description.

The central figure in the WertV is the *market value* (Verkehrswert). The market-value is—according to the *Building Law* (Baugesetzbuch, BauGB)—the transaction price that one should expect on average for an object given its characteristics and given the general market conditions (§ 194 BauGB). The market value is thus assumed to be equivalent and representable by the above given present value (1).

An appraisal of the market-value could be necessary for sales, for borrowing, and for property taxation (Berens and Hoffjan, 1995). The determination

of the market value is explicitly prescribed for calculating prices of shares on real estate funds (§ 34 KAAG), for compensations after expropriation (§ 95 BauGB), and as a relative ceiling for forced sales (§ 74a ZVG) (Thomas, 1995).

The determination of the market value of commercial real estate according to WertV follows a two-step procedure. In the first step, the *income value* (Ertragswert) of the respective house has to be determined (§ 7 WertV, Nr. 3.1.2 WertR 91). In the second step, the calculated income value has to be adjusted for general market conditions. Thus, whereas the first step—the appraisal of the income value—tries to figure out the value of the object with all its characteristics, the second step adjusts for influences on the price that are the same for all objects that are transacted in a given period.

In effect, the two-step procedure divides the market value into two components. The first component gives the fundamental value of the object, calculated with expected net proceeds and by using a constant discount factor. The second step adjusts for the fact that the returns that investors require might be currently higher or lower than they will be in the long run. In Subsection 3.3 we present a model that relates the concept of WertV to the concept of the generalized version of the present value (1). There we will show with an approximation that a proportional market correction factor is reasonable.

To calculate the income value of apartment house n according to WertV, the appraiser needs four figures: the expected lasting net proceeds $D_{n,t}$, the remaining time of usage $T_{n,t}$, the value of the site $B_{n,t}$ and the discount rate θ_t . Given all relevant information on the object and his knowledge of the market, the appraiser can assess the expected lasting net proceeds and the remaining time of usage. The guidelines of the WertR 91 give him further rules at hand how to assess both figures. The value of the site and the discount rate will be described in detail in the next paragraph. Given all four figures, the income value (Ertragswert) according to § 15, 16 WertV is

$$E_{n,t} = \frac{1}{\theta_t} \left\{ 1 - \left(\frac{1}{1 + \theta_t} \right)^{T_{n,t}} \right\} (D_{n,t} - \theta_t B_{n,t}) + B_{n,t} .$$

The value of the site $B_{n,t}$ (Bodenwert) has to be determined by using transaction prices of comparable sites (§ 13 Abs. 1 WertV). If that is not possible, the appraiser must use approximate values. Such values are delivered by the regional Surveyor Commissions for Real Estate, see the detailed description on the Commission in Subsection 4.1. These Commissions also deliver figures for the discount rate θ_t . They derive θ_t by calculating the *internal rates of return* with formula (6) after replacing E with observed historical transaction prices, see Gottschalk (1999, B III). Averaging over all internal rates for a class of houses yields the discount rate. The discount rates are effective for several years. For example, the current discount rates for apartment houses in Berlin are calculated with historical data from 1996 up to 1999 and it replaces the discount rates that were calculated in 1996 (Senatsverwaltung für Stadtentwicklung, 2000).

We obtain for the income value according to WertV after a simple reformulation

$$E_{n,t} = \frac{1}{\theta_t} \left\{ 1 - \left(\frac{1}{1 + \theta_t} \right)^{T_{n,t}} \right\} D_{n,t} + \left(\frac{1}{1 + \theta_t} \right)^{T_{n,t}} B_{n,t} . \quad (6)$$

The similarity of the income value according to WertV (6) and the fundamental value (3) is remarkable. However, there are two differences. First, the discounted value of the site after the house is used up is the discounted current value of the site. Second, the discount rate is not a long-run average of returns for apartment buildings but depends on short run fluctuations of the real estate market. These differences are inconsistent with the derivation of the fundamental value. It is not clear why the expected price of the site after the current building is used up should be equal to the current value of the site. According to economic theory, the expected price of the site should be equal to the present value of its efficient usage (perhaps plus an option value of waiting). It is doubtful whether the value of the site is a good proxy for the expected price in $t + T_{n,t} + 1$. However, it is an open question if there exists a plausible alternative. The definite advantage of the current regulation is, that site values are relatively easy to observe. Using the value of the site may not be the optimal rule, but at least it is a rule.

The second difference is still more confusing. The fundamental value was derived by plugging in the long-run average discount rate. If the discount rates are time-varying around the long-run average R , θ_t will be equal to its long-run rate only by sheer coincidence. Thus, the method of calculating the discount rate θ_t with historical data from a short time interval is at odds with the theoretical model. However, if the discount rate is persistent, then income values calculated with θ_t instead of θ might improve the accuracy of appraisals. In Appendix A we shed some light on this point.

In the second step, the calculated income value has to be adjusted for general market conditions. The conditions are influenced by the current situation of the economy, by financial conditions and special conditions of the respective region (§ 3 Abs. 3 WertV). Whereas the determination of the income value is codified in detail, the market adjustment is in principle open for the judgement of the appraiser. In many cases, valuers use proportional correction factors that are calculated by professional bodies or surveyor commissions, see Gottschalk (1999, C V).

2.4 Valuation with the capitalization method

In addition to valuation according to WertV, German appraisers use other valuation methods (Rüchardt, 2001, I.4). A very simple method is the so-called *real estate agent method* (Maklermethode), which is in effect the capitalization method given with equation (5). The capitalization factor is provided by real estate professional bodies, calculated with information given by its members. Or appraisers use historical prices and gross or net rents for calculating the capitalization factor. Given a capitalization factor, the appraiser just has to multiply the respective rent figure with the corresponding capitalization factor to derive the fundamental value. As Gottschalk (1999, C VII) emphasizes, this method

- ignores distributable costs and running cost,
- ignores discounts due to access rights or rights to way,

- ignores concrete conditions of the object that might influence the expected proceeds.

Thus, the real estate agent method is a *rule of thumb*, that needs only two figures for providing an appraisal of the fundamental value. No additional information has to be gathered than the rent figure and the corresponding capitalization factor. The price for this easy usability is that not all relevant information is incorporated in the appraisal. But does this really matter?

3 How to evaluate different valuation methods?

3.1 What is accuracy?

We have seen that the codified valuation method of the WertV is in accordance with the present value concept. However, the present value concept per se is too general to be used in concrete valuations. It is only practicable if simplifying assumptions are made. These assumptions are a constant discount rate for the investment horizon and a constant growth rate of the rents. However, the figure used for calculations is not simply the observed current rent, but the adjusted net proceed. The calculation of the net proceeds can be a difficult task and is explained in detail in the WertR 91. A simple question comes up: Are the detailed rules in the WertV necessary for accurate valuations? Or might a simpler method like the real estate agent method generate at least comparable results?

The biggest problem for comparing different valuation methods is the correct definition of accuracy. It is clear that even the best possible forecast can deviate by chance from the final transaction price (Mallinson and French, 2000). There might be unusual circumstances during the dealing, new information that unfolds after the appraisal or just an irrational market participant. However, difficulties do not arise due to uncertainty per se. Uncertainty accompanies every appraisal and every valuation method generates a distribution of outcomes. It is easy to calculate statistical measures for

such distributions, but—as Diebold and Mariano (1995) remark in a different context—it is important to recognize that the “*economic* loss associated with a forecast may be poorly assessed by the usual *statistical* metrics”. This point is easily seen for valuation methods that are used for appraising the collateral value of real estate for lending purposes. A method that undervalues on average might be preferable to a method that is unbiased, but exhibits outliers that overestimate the true collateral value at a high degree. Whereas the first method is too conservative, because it rejects some good applicants, the second method generates high losses in the case of default. Stated an asymmetric loss given default function, the first method might be economically preferable (Shiller and Weiss, 1999).

3.2 Intentions of WertV

The difficulties of evaluating different valuation methods arise because economic loss functions are context-sensitive and do not always coincide with statistical forecast evaluation measures. Due to the fact that income valuation is in wide use, there might exist many different context-sensitive loss functions. Which one should we choose? We try to circumvent this problem by concentrating on the inherent intentions of the WertV and use them as measure for evaluation. The intentions are: income values should be on average unbiased forecasts of transaction prices and short run deviations should happen only in a systematic way.

Let us show how these intentions are incorporated in the WertV. By doing this, we derive the statistical model that will be used in the ongoing. Let $V_{n,t}$ denote the market value of object n in period t . It represents the price one would expect in the case of a sale given usual business dealings (§ 194 BauGB). Any deviation between $V_{n,t}$ and the transaction price $P_{n,t}$ should be unsystematical (§ 6 WertV). Such deviations might happen if one of the contracting parties has a special interest in obtaining the object or if there are personal relationships between seller and buyer. In the practice of valuation, it is quite common to assume in such cases proportional discounts or surcharges from the market value (§ 14, 25 WertV). Let $U_{n,t}$ denote the

unsystematical component, where $\mathcal{E}_t[U] = 1$. Thus, the price for object n in period t is given as

$$P_{n,t} = V_{n,t}U_{n,t} . \quad (7)$$

On average, the transaction price is equal to the market value

$$\mathcal{E}_t[P_{n,t}] = V_{n,t} .$$

According to WertV, the market value is the income value adjusted for general market conditions (§ 7 WertV). Let M_t denote this adjustment factor, it follows

$$V_{n,t} = E_{n,t}M_t . \quad (8)$$

The definition of the general market condition (Nr. 1.5.3 WertR 91) implies that

$$\mathcal{E}[M_t] = 1 . \quad (9)$$

M_t does not depend on n and it must be independent of the unsystematical component $U_{n,t}$. We define the ratio between price and income value as

$$Q_{n,t} \stackrel{\text{def}}{=} \frac{P_{n,t}}{E_{n,t}} , \quad (10)$$

where we assume that the income value is always positive. We obtain with (7) and (8)

$$Q_{n,t} = M_tU_{n,t} \quad (11)$$

and thus

$$\mathcal{E}[Q_{n,t}] = 1 . \quad (12)$$

That is reasonable: the unconditional expected deviation between price and income value must be zero. Prices and income values—that are: fundamental values—must coincide in the long run. However, for the short run we have

$$\mathcal{E}_t[Q_{n,t}] = M_t . \quad (13)$$

The expected deviation between price and income value in the short run is given by the general market condition M_t .

Extracting these implicit assumptions out from the WertV leads to two testable hypotheses: the average ratio of price to income value should be

equal to one for a sample that covers several years. The income values calculated according to WertV must be unbiased on average, see (12). Berlin's Surveyor Commission prefers the ratio Q . This ratio is automatically generated in its data base. However, one might object that the reciprocal of Q is also of interest. In his study on the market comparison method, Dotzour (1988) uses this figure for evaluating residential appraisal errors. Recall, that it follows from Jensen's inequality that $\mathcal{E}[Q] > 1$ if $\mathcal{E}[1/Q] = 1$. Thus, if we reject (12) for our data, we should also check the hypothesis

$$\mathcal{E}[1/Q_{n,t}] = 1 . \tag{14}$$

Such ambiguities arise because we have no economic loss function at hand. The second hypothesis is robust against such objections. It states that any deviation between prices and income values for shorter periods must be systematic and not explainable by characteristics of the respective houses. The value of such characteristics must be included in the income value, see (13). Additionally, we can extend the above stated hypotheses by a third one: the deviations of the income values around the prices should be smaller than the deviations that are generated by any other valuation method. In principle, we are not only interested in the average outcome, but also in the spread around the average. Even a method that produces outcomes that are better on average might be worse if it produces also some severe outliers. Thus, we should also compare the mean square prediction error. However, even this measure uses only the first and the second moment for assessing the accuracy of a valuation method. Clapp and Giaccotto (2002) argue that estimated distributions are good graphical devices for comparing different forecast methods. Entire distributions are also advantageous for comparing different valuation methods. Investors can apply their own weights to appraisal errors for choosing their preferred valuation method.

3.3 What are the general market conditions?

In most cases of real estate valuation the figure of interest is the market value. Up to now, we have concentrated on the first step in calculating

this value. However, as we have already mentioned, whereas the method for appraising the income value according to WertV is prescribed in detail, the adjustment for general market conditions is not. Good or bad adjustment seems to depend on the talent of the appraiser. Is it a good idea to leave this important adjustment at the will of an appraiser? Can we answer this question when we have no data of market values at hand? In principle, we can: pretending that we could extract the general market conditions M_t , we can try to find reasonable economic indicators that explain the behavior of this component. By doing this, we can show that the idea behind the adjustment procedure outlined in the WertV is reasonable. Thus, we can not test if an individual appraiser uses the right indicators to assess the general market conditions. But we can test if the idea is a good one and if the indicators that are quoted in the WertV (§ 3) and in the Reports of the GAA (Geschäftsstelle des Gutachterausschusses für Grundstückswerte in Berlin, 2001) are reasonable.

We can show that the idea of general market conditions is in accordance with the present value concept. We now derive an instructive interpretation of the general market conditions.

For the derivation, we use well-known log-linearized versions of the fundamental and the present value (Campbell, Lo, and MacKinlay, 1997; Cochrane, 2001).

$$\log F_{n,t} \approx \sum_{j=0}^{\infty} \rho^j \{k + (1 - \rho)\mathcal{E}_t[d_{n,t+1+j}] - r\} \quad (15)$$

is a first order approximation of the fundamental value (2). Here, $d_{n,t}$ denotes the logarithm of the net proceeds and $r \stackrel{\text{def}}{=} \ln(1 + R)$. k and ρ are approximation constants and $0 < \rho < 1$. We assume that this fundamental value corresponds to the income value (6) of the EWV. The corresponding approximation of the market value (8) is

$$\log V_{n,t} \approx \sum_{j=0}^{\infty} \rho^j \{k + (1 - \rho)\mathcal{E}_t[d_{n,t+1+j}] - \mathcal{E}_t[r_{t+1+j}]\} .$$

Let $q_{n,t}$ denote the log ratio of price to income value (10). We obtain with

(7) and the above stated approximations

$$q_{n,t} = \kappa - \sum_{j=0}^{\infty} \rho^j \mathcal{E}_t[r_{t+1+j} - r] + \varepsilon_{n,t} \quad (16)$$

where the new constant κ guarantees that $\varepsilon_{n,t} \sim (0, \sigma_\varepsilon^2)$ (recall with Jensen's inequality, that $\mathcal{E}[\ln U] < \ln \mathcal{E}[U] = 0$). We obtain for the log general market condition with (11)

$$\mathcal{E}_t[q_{n,t}] = m_t \quad (17)$$

where κ is included. Thus, we have

$$m_t = \kappa - \sum_{j=0}^{\infty} \rho^j \mathcal{E}_t[r_{t+1+j} - r] .$$

Short run deviations between prices and income values can be understood as short run deviations of the expected returns from its long run average. These deviations might be influenced by proxies of risks that influence returns. Ling and Naranjo (1997) study risk factors for real estate returns and Ferson and Harvey (1995) for stock and bond returns. We denote the risk proxies in the ongoing as market indicators. In that case, the expected return deviations can be modelled as

$$\mathcal{E}_t[r_{t+1} - r] = x_t ,$$

where x_t is a stationry process with

$$\alpha(L)x_t = s_t^\top \gamma + \xi_t . \quad (18)$$

Here, $\alpha(L)$ is a polynomial in the lag operator L with $L^j x_t = x_{t-j}$. The vector s_t comprises news in market indicators. ξ_t comprises unsystematical influences on the expected return deviation. We obtain for the sum of discounted expectations with Theorem 12.6 from Gouriéroux and Monfort (1997)

$$- \sum_{j=0}^{\infty} \rho^j \mathcal{E}_t[r_{t+1+j} - r] = \psi(L)\alpha(L)x_t \quad (19)$$

where

$$\psi(L) \stackrel{\text{def}}{=} \frac{L\alpha(L)^{-1} - \rho\alpha(\rho)^{-1}}{\rho - L}$$

is a stationary lag polynomial. Plugging this expression into (17) yields

$$m_t = \kappa + \psi(L)\alpha(L)x_t .$$

Eventually, we obtain with (18)

$$\phi(L)m_t = \tilde{\kappa} + s_t^\top \gamma + \xi_t \tag{20a}$$

with $\phi(L) \stackrel{\text{def}}{=} \psi(L)^{-1}$ and $\tilde{\kappa} \stackrel{\text{def}}{=} \kappa\phi(1)$. It is easy to see that $\mathcal{E}[m_t] = \kappa$. Using (16) and (17), we have furthermore

$$q_{n,t} = m_t + \varepsilon_{n,t} \tag{20b}$$

The system of equations (20) resembles the idea of the WertV: the deviations between log prices and income values are the general market conditions. They follow an autoregressive process and are influenced by economic market indicators.

4 Empirical Investigation

4.1 Data

For the empirical analysis, we use different data sets. The main data set contains 4150 transaction observations of apartment houses from January 1980 to May 2000 for Berlin, Germany. It is provided by the *Gutachterausschuss für Grundstückswerte in Berlin* (GAA), the Surveyor Commission for Real Estate in Berlin. In the ongoing, we use these 4150 for assessing the accuracy of valuation according to WertV. For comparing prices and rents, we use the yearly rent mirror of the *Ring Deutscher Makler* (RDM), the largest real estate professional body in Germany. Furthermore, we use different indices of the Statistical Office Berlin (Statistisches Landesamt Berlin, StaLa) and of the Deutsche Bundesbank.

According to the Building Law, surveyor commissions have to collect all relevant information on real estate transactions in its respective state (§§192-199 BauGB). In addition to collecting and storing the data, the surveyor

commissions have to calculate and to publish approximate values (Bodenrichtwerte) for sites and discount factors θ_t for different types of buildings, see Senatsverwaltung für Stadtentwicklung (2000, 2001). To obtain these figures, the commissions also calculate income values for commercial real estate according to WertV. Before calculating the income value, the reported figures are checked for consistency. If information on a transaction is insufficient, a questionnaire is sent to the owners of the building to gather the relevant figures. If the questionnaire is not returned, the appraisers of the commission impute figures from experience for the missing numbers. Most of these figures are explicitly specified in the WertR 91. However, in some cases it is not possible to calculate an income value.

It is a common problem in valuation accuracy studies that appraisals will use knowledge of prices from preceding transaction if one was made after the date of transaction (Crosby, 2000). Asked for that problem, the GAA assured that its appraisals are done 'mechanically' according to WertV and that transaction prices are not used. Two further arguments strengthen this statement: by definition of the WertV, the income value is the initial step for appraising the market value—systemic deviations between both figures are likely. In periods where markets use discount rates above or below the average rate, transaction prices are no benchmark at all for income values and using price information is at least useless. Nevertheless, one could argue that participants on the real estate market believe that income values should coincide with transaction prices. That might put some pressure on the objectivity of property consultants who earn fees with their work. However, the valuers of the GAA are independent and have no incentives to fulfill beliefs of any instructing clients. Thus, we conclude that income values in our data are calculated without the knowledge of the price. Another problem exists because all appraisals in our data set are made after the date of transaction. The GAA assured that only information is explicitly considered that was known at that date. However, the effect of 'implicit' information is difficult to assess. In the ongoing analysis, whenever it is necessary, we try to control for information leads.

Table 1 gives the number of observations with appraised income values per

year. Before 1994, we have only transactions from the West part of Berlin. Due to capacity restrictions of the commission and diminishing return of questionnaires, the number of calculated income values decreases in the last decade.

Table 1: *Number of observations with appraised income value per year.*

Year										
1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
251	197	193	382	335	253	265	287	393	315	236
Year										
1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	Total
158	115	110	107	70	62	78	212	105	26	4150

Note: Year 2000 comprises only observations for January-May.

However, the yearly number of transactions on the real estate market varies also during the years, see Geschäftsstelle des Gutachterausschusses für Grundstückswerte in Berlin (2001, Fig. 5). According to personal communication with the surveyors of the GAA, the number of transactions is mostly influenced by tax incentives and subsidies. Figure 1 shows the monthly number of observations from our data set. The information revealed by this plot

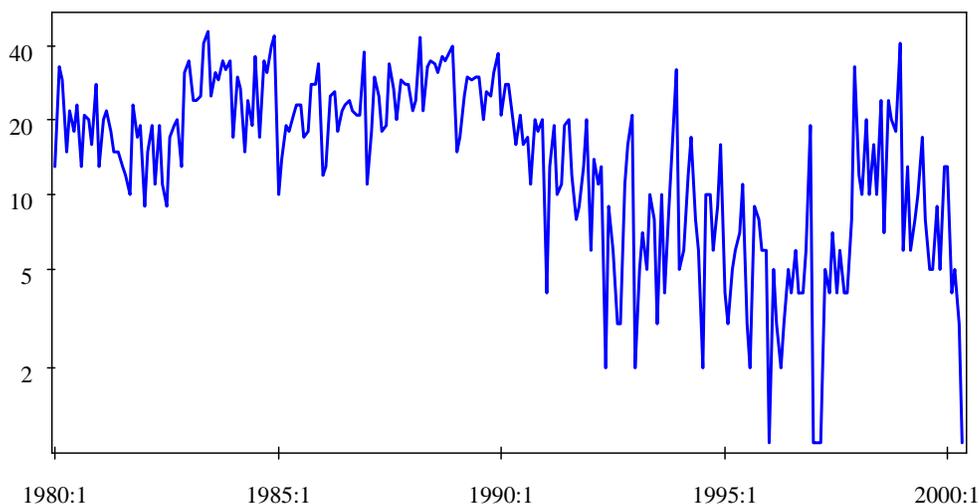


Figure 1: *Number of observations per month, logarithmic scale.*

should be treated with care because we observe only transactions where it was possible to appraise an income value.

In addition to the transaction price and the income value we observe gross or—respectively—net rents, age of the building, size of the floor space and size of the lot. Table 2 gives some summary statistics of the relevant variables. The age of the building is the age at the time of transaction. Before 1995, it was common to report the yearly gross rent. The gross rent includes cost that are distributable to the tenants, for example: cost for the housekeeper, elevator and other technical devices. Since 1995 it is more common to report the yearly net rent, that is the gross rent without distributable costs.

Table 2: *Summary statistics for transacted apartment houses in Berlin, Germany between 1980 to 2000.*

	Mean	Median	Std. Dev.	Min	Max	Units
Lot size	982.2	767	7920.8	186	56332	<i>Square metres</i>
Floor space	2168.9	1867.5	2637.2	128	89614	<i>Square metres</i>
Age	73.9	81	29.2	0	186	<i>Years</i>
Price	1.411	0.97	2.192	0.105	80.0	<i>Mio. DM</i>
Income value	1.295	0.891	2.839	0.094	142.0	<i>Mio. DM</i>
Gross rent	107.3	84.7	176.0	12.0	8332.8	<i>Thsd. DM</i>
Net rent	121.1	65.8	256.1	6.9	3000.1	<i>Thsd. DM</i>

Note: Currency units are *Deutsche Mark* (DM). 3835 observations have information on the gross rent and 315 on the net rent. Income values are calculated by the surveyors of the GAA according to WertV.

4.2 Overview of the market

As we have already mentioned, the assumption of rational investors is a prerequisite for valuation methods oriented on the present value concept. Table 2 shows that the average price for an apartment building in Berlin was about 1.4 Million Deutsche Mark (DM). Thus, the participants at the market are wealthy private investors, investment funds, insurance companies, cooperative and commercial house-building societies. It is quite natural to assume that they are well-informed about the Berlin real estate market and use well-founded valuation methods to decide about their investments. In addition,

even if we do not know how investors base their decisions, we know at least that property consultants use well-founded valuation methods. According to the poll of McParland, Adair, and McGreal (2002, Table VIII), 74.2% use the capitalization approach as their preferred valuation method. Here, the income valuation according to WertV is comprised under this heading, see McParland, Adair, and McGreal, p. 139. Valuers use also other standard valuation methods like the comparative method to derive a final appraisal. My personal communications with valuers and a project developer underline the guess that investors make every effort to derive reliable appraisals and behave rationally.

Figure 2 shows the average price-rent ratio per year. The price-rent ratio is the number of times that a house's yearly rent go into the current market price. The solid line gives the average over all houses sold and the dashed

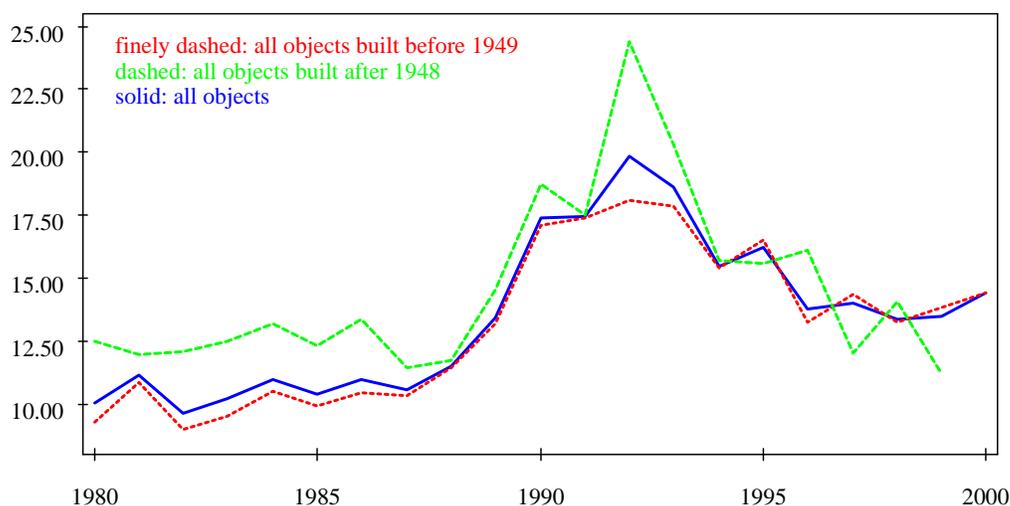


Figure 2: Average yearly price-rent ratios. Calculated with all 3835 observations with information about gross rents.

lines give the average of house built before 1949 and—respectively—after 1948. The average ratio for all houses is 12.5 over the whole period. Rents are the only flow of money that the owner obtains during the holding interval. According to the present value and given a constant discount rate, the higher the current price-rent ratio, the higher must be the market's expectation of future rent growth. This is easily seen if we divide the fundamental value (2)

by the current net proceed $D_{n,t}$. We obtain that

$$\frac{P_{n,t}}{D_{n,t}} = \sum_{j=1}^{\infty} \frac{\mathcal{E}_t[1 + G_{n,t+j}]}{(1 + R)^j},$$

where $G_{n,t+j}$ is the growth rate of the net proceeds. Under the assumption of proportional rents and net proceeds and a constant discount rate, Figure 2 reveals that market participants must have expected higher rent growth at the beginning of the 1990ties. A plausible explanation is the reunification of Germany in 1990 and the decision for Berlin as the Capital of the reunified Germany in 1991. After that decision it was clear that the Parliament, the Government, federations and professional bodies will move with their staff from the former Capital Bonn to Berlin. Some pundits expected that Berlin will grow in population and importance up to its position before the Second World War.

Was the boom in apartment house prices backed by rational expectations or by irrational sentiments? There are several studies that try to test efficiency in real estate markets, see Cho (1996). However, as Englund and Ioannides (1997, p. 126) state “it is in general not possible to distinguish between inefficiency and time-varying return requirements”. Thus, we can give at least only an indication if the rent growth that investors have expected was rational. An decrease in the discount rate at the beginning 1990ties is a competing explanation.

To assess if the increase in the average price-rent is backed by expectations of higher rent growth, we use the rent mirror of the RDM. The RDM does a poll with its members every year at the end of March and asks them about current net rents for apartments of different quality types. The RDM publishes the mirror after discussing the figures in a committee consisting of 15 members, where 5 are independent sworn valuers. The mirror reports only rents for new leasing contracts. It is plausible to presume that the rent mirror partially reflects expectations of future rent development. Table 3 presents the yearly growth rates for the rent mirror, where nominal rents are deflated with the price index of the StaLa for average income families with four persons in Berlin West.

Table 3: *Real growth rates of different rent indices in percent.*

Year	RDM					
	StaLa	Built before 1949			Built after 1948	
		Simple	Middle	Good	Middle	Good
1981	-1.11	-6.10	0.60	4.62	7.31	12.67
1982	-0.23	31.13	10.15	8.94	-5.58	-21.32
1983	3.64	-3.83	-3.83	-3.83	2.17	-3.83
1984	6.85	16.23	10.69	7.62	2.55	11.39
1985	4.14	-3.19	-3.19	-3.19	-3.19	-3.19
1986	3.31	-0.71	4.25	-0.71	4.80	3.60
1987	2.46	-0.55	6.55	-0.55	4.68	-0.55
1988	1.71	-1.09	-1.09	-1.09	-1.09	7.14
1989	0.81	68.46	44.70	45.78	-2.80	4.66
1990	1.06	2.95	1.69	10.32	55.74	39.05
1991	0.94	2.37	2.63	-1.77	3.15	6.79
1992	2.21	1.50	6.29	6.05	2.19	0.90
1993	3.42	1.32	-3.58	-3.66	-20.14	-16.67
1994	2.65	-1.88	-0.86	-1.61	-9.13	-12.37
1995	3.37	2.94	1.34	-5.70	-12.14	-12.53
1996	2.93	2.22	6.23	2.54	-8.63	-13.10
1997	-0.21	-0.98	-0.98	-0.98	-5.28	-8.05
1998	0.36	-0.19	-11.04	-5.44	-13.80	-11.71
1999	0.55	-7.23	-2.53	-0.09	-0.09	-0.09
2000	0.06	-0.87	-0.87	-0.87	-0.87	-0.87

Note: Data sources are Statistisches Landesamt Berlin (StaLa) and Ring Deutscher Makler (RDM). Simple, middle and good stand for the quality of the house. The prices are deflated with the price index of the StaLa for average income families comprising four persons in Berlin West.

The Stala rent index before 1995 is calculated for households comprising 4 persons with average income in Berlin West. Starting with 1995, it is calculated for all private households in Berlin. It reflects the average rent for existing contracts. Figure 3 shows the growth rates calculated with the figures from the RDM mirror. It is obvious that the RDM rent mirror must overstate the development of future rent growth in several years. A yearly

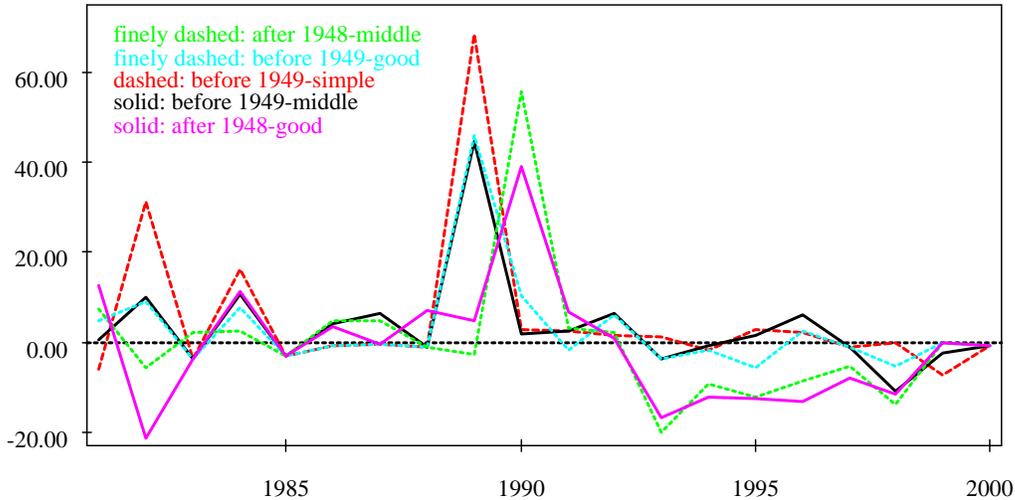


Figure 3: Real growth rates for the RDM Rent Mirror for different vintages and qualities.

growth rate of more than 20% is definitely too large.

Generating simple scatter plots that relate the current price-rent ratio to the current real rent growth or to the real rent growth in the following year do not show that high price-rent ratios are followed by high growth rates (Campbell and Shiller, 1998, 2001). In most cases with ordinary regressions, no or a slight negative relationship exists. The inclusive results are definitely influenced by the outliers of the rent mirror. Even robust regressions with least trimmed squares (Čížek and Víšek, 2000) deliver results that neither corroborates nor contradicts the claim that high price to rent ratios are followed by high rent growth.

Figure 4 shows a scatter plot of real RDM rents for such houses with middle quality on the lagged price-rent ratio for houses constructed before 1949. The shape is similar for the other rent measure of the RDM. Figure 5 shows a scatter plot of real RDM rents for houses with middle quality on the lagged price-rent ratio for houses constructed after 1948. Both figures reveal that high price rent ratios are followed by high real rents. However, the results have to be interpreted carefully. The question of an irrational bubble in the Berlin real estate market is important, because it can invalidate the scrutiny of the accuracy of existing valuation methods. However, as we have

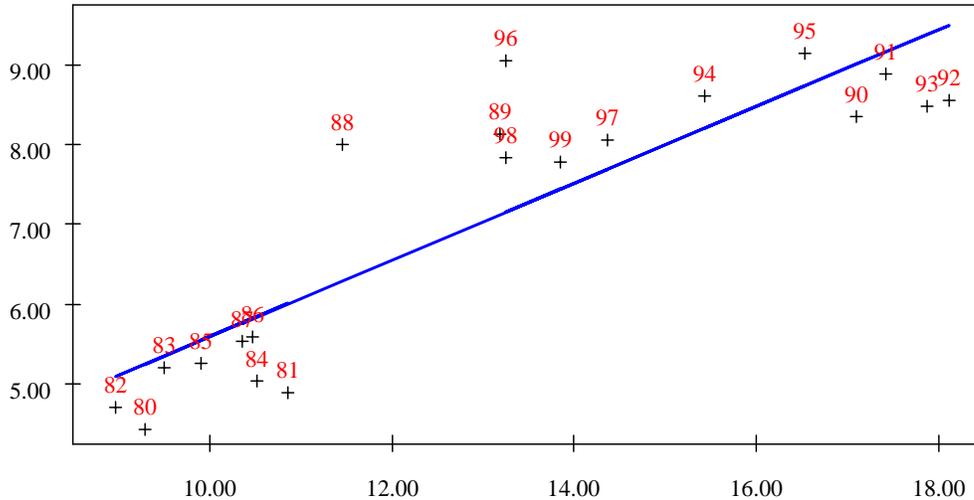


Figure 4: Scatter plot for lagged average price-rent ratios for houses constructed before 1949 and real RDM Rent Mirror for such houses with middle quality.

already mentioned, soaring price rent ratios can be explained equally with low discount rates. Perhaps investors were so confident after the reunification that they required historically low returns for an investment in Berlin's real estate.

4.3 Assessment for valuations according to WertV

For the subsequent analysis, the figures of interest are the ratios of transaction prices and income values. Given our observations for more than 20 years, the average ratio should be equal to one. However, recall that systematical deviations of prices and income values might occur in periods where the market expects lower or higher returns. Panel A of Table 4 reports summary statistics for ratios of price to income value. On average, the appraisal error is 13.3% and prices were understated. The median appraisal error is much smaller than the average error and amounts 5.8%. The positive skewness underlines that the density of ratios is not symmetric. The excess kurtosis of 16.178 reveals a leptokurtic density with more mass in the middle compared with a normal distribution (Spanos, 1999). Panel B of Table reports summary statistics for ratios of income value to price. The mean of the ratios is less than one. On average, the relative appraisal error is -3.22% and prices were

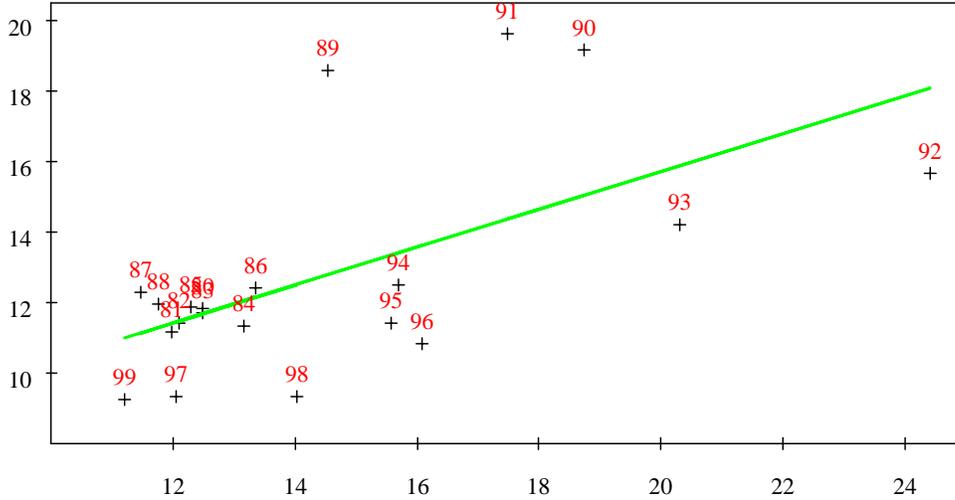


Figure 5: Scatter plot for lagged average price-rent ratios for houses constructed after 1948 and real RDM Rent Mirror for such houses with middle quality.

understated. The median error of -5.5% is even larger. The positive skewness underlines that the density of ratios is not symmetric and the excess kurtosis of 9.147 reveals a leptokurtic density with more mass in the middle compared with a normal distribution.

Table 4: Summary statistics for ratios of price to income value and for ratios of income value to price.

Panel A: Ratios of Price to Income Value				
Mean	Standard deviation	Minimum	Median	Maximum
1.133	0.392	0.247	1.058	4.941
10% Quantile	90% Quantile	Skewness	Kurtosis	Number of obs.
0.773	1.577	2.726	19.178	4150
Panel A: Ratios of Income Value to Price				
Mean	Standard deviation	Minimum	Median	Maximum
0.968	0.294	0.202	.945	4.044
10% Quantile	90% Quantile	Skewness	Kurtosis	Number of obs.
0.634	1.294	1.562	12.147	4150

Now we can check the first hypothesis on the unbiasedness of the income valuation according to WertV. Recall that we should have $\mathcal{E}[Q] = 1$ according to hypothesis (12). We reject this hypothesis with a t -statistic of more than 21.8 at the common levels of significance. For the competing hypothesis (14)—i.e. $\mathcal{E}[1/Q] = 1$ —we obtain a t -statistic of -7.06 with an average ratio of income value to price of 0.9678. Thus, we reject the hypothesis of an expected ratio equal to one at the common significance levels.

It is obvious that the income values according to WertV are smaller on average than prices. The result holds irrespective which ratio is used for assessing unbiasedness. The average deviations of about -3.2% for ratios of income values to prices appear to be large compared with the results of Dotzour (1988), who found an average appraisal error of 0.06% calculated with ratios of appraisal to price. As opposed to our study, Dotzour uses appraisals for residential real estate. Chinloy, Cho, and Megbolugbe (1997) found for US data, that appraisals are on average about 2% higher than prices. Their explanation for this result is, that appraisers have an incentive to overappraise, because they are paid only after a successful deal. Can we find comparable explanations for the average underappraisal of prices that we have found for our data? As we have already stated, there are no incentives for surveyors of the GAA to over- or understate income values. Income values should account for all object-specific characteristics that influence the present value of the respective apartment house. Thus, we should check if this is well-done. Inspection of the income value (6) reveals that there are at least four possible explanations—individual and in combination—for an understatement

- the discount rate θ_t is too large on average and prospected net proceeds are discounted too much
- prospected net proceeds $D_{n,t}$ are valued too low on average
- lot values $B_{n,t}$ are too low on average
- remaining times of usage $T_{n,t}$ are too low on average.

These partial effects are derived in Appendix A.2. Choosing a discount rate that is too large biases the accuracy of income value even if the other figures are assessed correctly. As we have already mentioned, the practise of calculating discount rates with a short interval of historical data is not in accordance with the fundamental value concept. But we have also seen in Appendix A that this must not necessarily lead to bad predictions. However, incorrect rating of discount factors is a simple explanation for the average understatement of prices. We will discuss this point further in Subsection 4.4.

Whereas the discount rate influences all appraised income values in the same way, the other three factors are specific to the house under valuation. Corresponding to hypothesis (13), the object-specific factors net proceeds, lot value, and remaining time of usage should have no explanatory power for individual deviations of price and corresponding income value. The log-linearized version of the hypothesis is given by the second equation in (20), that is

$$q_{n,t} = m_t + \varepsilon_{n,t} .$$

We test the hypothesis by running a simple regression of the log ratios on three proxies for the factors. Recall that $\ln(1/Q) = -q$ and so the qualitative results of the regression will not depend on whether $\ln Q$ or $\ln(1/Q)$ is the dependent variable.

To check for correct specification of the net proceeds, we use the real gross rents. Net proceeds are derived by the surveyors from the gross rents by adjusting for several sorts of costs. Regressing the unadjusted figure reveals if they over- or underadjust. When they adjust correctly, the real gross rents should be without influence. We deflate the gross rents with the yearly StaLa price index for household comprising four persons with average income in Berlin West. The base year is 1995. Recall that q is dimensionless and deflating controls for trending behavior of the rents. To explore any miss-adjustment of the lot value, we use as proxy the size of the lot in square metres. If the surveyors judge the remaining time of usage correctly, the age should have no influence.

Table 5 shows the result from a regression on the log ratios. The qualitative regression results remain unchanged if we include dummy variables for every month. These dummies control for possible correlations between factors and general market conditions, see equation (13).

Table 5: *Linear regression for log Q and object-specific factors.*

	Coefficient	t-Statistic	P-Value
Log real gross rent	0.055	4.65	0.000
Log lot size	-0.053	-3.77	0.000
Age	-0.001	-4.62	0.000
Constant	-0.138	-1.46	0.144
Regression Diagnostics			
R^2	0.009	\bar{R}^2	0.008
F-Stat.	11.99	P-Value(F-Stat.)	0.0000

Note: Gross rent is deflated with StaLa price index, base year is 1995. All 3835 objects with observed gross rents are used.

For example, such correlations might occur when in periods with high market conditions on average more older objects are sold. The coefficient of determination for the extended regression is $R^2 = 0.0973$ and $\bar{R}^2 = 0.0724$.

We see from Table 5 that all three factors influence the log deviation between price and income value. However, the total explanatory power of the regression is small, as the coefficient of determination of about 1% shows. Thus we should interpret the results carefully. The ratio of price to income value increases, *ceteris paribus*, by about 0.05% if the real gross rent increases by 1%. This result is explainable with the (prescribed) practice to use fixed relative figures for cost adjustments. Surveyors must use such figures when they retrieve no—or no reliable—data on maintenance and management cost for an object (Gottschalk, 1999, p. 278, Rn. 27). It is important to stress that fixed figures ignore any economics of scale. It is quite plausible that the proportion of such costs to gross rent decreases with the level of gross rents. When fixed average factors are used for adjustment, running costs of objects with little gross rents are understated and costs of objects with large gross rents are overstated. It follows, that net proceeds for the first are to

high and net proceeds for the latter are too low and income values are likely to overstate—respectively understate—the price.

The log ratio decreases by about 0.05% if the size of the lot increases by 1%. The size of the lot influences the income value indirectly as a result of its assessed value. Lot values are appraised with the comparative method or with approximate values. These appraisals will be more reliable for average-sized sites, because many sales will be observed. The reliability might decrease for small or large sites. Thus, *ceteris paribus*, assessed lot values might tend to be too low for houses with small lots and too large for houses with large lots.

When the age of a house increases by 1%, the log ratio decreases by 0.0008%. We should assume that the remaining time of usage is a decreasing function in the age of the building. Therefore, the income value will decrease, *ceteris paribus*, with the age of the building. If we assume that the remaining time of usage is assessed correctly for new objects, than the remaining time for older objects is too high.

Although our tentative interpretation of the regression results seems to be reasonable, it is an open question on how to filter out the correct assessment of the different factors. Stated in absolute terms, we have to reject the hypothesis that every factor of an individual building is assessed correctly in its income value. In relative terms, that might be the price for a regulation that tries to be usable for every case of valuation.

4.4 Comparison with the capitalization method

We have shown that income values are on average biased appraisals of prices. Nevertheless, concluding that valuations according to WertV are inaccurate is a little bit too hasty. As we have already discussed, it is difficult to assess the absolute accuracy of a valuation method. However, it is always possible to compare the outcomes of one method with the outcomes of another.

A simple valuation method that is used as rule of thumb by real estate agents is capitalization method. We have already discussed the method in Subsection 2.4. The capitalized rent of an object is just its fundamental value $F_{n,t}^c$. Recall that depending on the rent figures—gross or net—the

capitalization factor is 1 divided by a discount rate that is calculated with observed prices and rents. We use all observations of our data set with gross rents to calculate discount factors that guarantee that the average ratio of prices and appraised fundamental values is unbiased and thus equal to one.

Let θ_{PF} denote the discount factor that guarantees for $\overline{P/F^c} = 1$ on average and let θ_{FP} denote the discount factor that guarantees for $\overline{F^c/P} = 1$.

$$\theta_{PF} = \frac{\sum_{t=1}^T N_t}{\sum_{t=1}^T \sum_{n=1}^{N_t} \left(\frac{D_{n,t}}{P_{n,t}}\right)^{-1}}$$

is given by the harmonic mean of the ratios of gross rent to price. Here, t is the index for the periods and N_t is the number of observation per period. θ_{PF} is just the reciprocal of the average of price to gross rent ratios, see Subsection 4.2. We obtain analogously

$$\theta_{FP} = \frac{1}{\sum_{t=1}^T N_t} \sum_{t=1}^T \sum_{n=1}^{N_t} \frac{D_{n,t}}{P_{n,t}}$$

as the arithmetic mean of the ratios. Calculating these figures with our data yields $\theta_{PF} = 8\%$ and $\theta_{FP} = 9.17\%$. The corresponding capitalization factors for gross rents are 12.5 and 10.9. Adjusting the gross discount rates for running costs—comprising distributable running cost, maintenance and management cost—ranging from 30% to 60% delivers discount rates θ_{PF}^n between 3.2% and 5.6% and θ_{FP}^n between 3.7% and 6.4%. Here, the superscript indicates that the adjusted discount rates are for capitalizing net proceeds. The WertR 91 (3.5.5) proposes a discount rate of 5% for valuations where no other discount rate is at hand. With respect to the range deduced above, this figure seems to be reasonable. Discounting net proceeds with 5% appears to be a good alternative to time-varying discount rates θ_t . In addition, a fixed rate is in accordance with the derivation of the fundamental value.

Would it be advantageous to use the capitalization method for valuation? Capitalization of the gross rent is a much simpler method than the one outlined in WertV. If we could conclude that the outcomes are moreover better, then one should use the easier and better method for appraisals. The rates

θ_{FP} and θ_{PF} are constructed in such a way that they deliver unbiased fundamental values for our data set. However, as we have already discussed in Subsection 3.2, accuracy is more than unbiasedness. An accurate valuation method should be unbiased, but it should also have a small variance. A convenient measure for evaluating this relation is the mean squared prediction error

$$\mathcal{E} [(X - a)^2] = \mathcal{V}[X] + (\mathcal{E}[X] - a)^2 .$$

This error measures the expected squared distance between the realizations of the predictor X and the predictive target a . It is composed of two terms: the variance of the predictor $\mathcal{V}[X]$ and the squared bias of the predictor. Applied to our implementation, the price to appraisal ratios—respectively the appraisal to price ratios—are the predictors and $a = 1$ is the predicting target. Panel A in Table 6 gives the mean squared prediction errors for the different valuation methods.

Table 6: Comparison of mean squared prediction errors and mean absolute errors of different valuation methods.

Panel A: Mean squared prediction errors (MSPE)			
	Variance	Bias	MSPE
WertV price to income value	0.129	0.135	0.147
Capitalization with θ_{PF}	0.167	0	0.167
WertV income value to price	0.079	-0.042	0.080
Capitalization with θ_{FP}	0.126	0	0.126
Panel B: Mean absolute prediction errors (MAPE) in percent			
	MAPE	Percentage within 15%	
WertV price to income value	25.72%	44.64%	
Capitalization with θ_{PF}	29.56%	31.94%	
WertV income value to price	21.17%	45.34%	
Capitalization with θ_{FP}	27.79%	32.46%	

Note: Calculated for all 3835 objects with information on the gross rent.

Although the appraisals calculated with the capitalization method are unbiased, they are ranked inferiorly to the income values according to the MSPE. The ratios of price to income value have an MSPE of 14.7% compared with

16.7% for ratios of price to capitalized value. The ratios of income value to price have an MSPE of 8% compared with 12.6% for ratios of capitalized value to price.

Panel B of Table 6 gives the mean absolute prediction errors for the different valuation methods, where the absolute prediction error is $|X - 1|$. In both cases, valuations according to WertV deliver smaller mean absolute prediction errors and larger fractions of prediction errors that are at most 15%. The errors of income values are slightly larger than the errors generated by Kaplan and Ruback (1995, Table II) for their meticulously prepared valuations of companies. Under the assumption that we can compare valuations of companies and real estate, this reveals that complex discounted cash flow calculation might have generated better results than the simpler income valuation according to WertV and the even more simpler capitalization method.

Up to now, we have only focussed on the first and second moments of the appraisal errors. As we have discussed in Subsection 4.4, investors may be sensitive to the entire distribution of appraisal errors. To shed more light on the distribution of the ratios generated by the different valuation methods, we estimate the densities with kernel smoothing techniques (Härdle, 1991; Simonoff, 1996). A kernel density estimate at point Q is a weighted average probability mass of all observations Q_n in the neighborhood of Q . The weighting function—the kernel—is itself a density function, where different functions can be used. The size of the neighborhood is controlled with the bandwidth h . The choice of the kernel is not really important, whereas the choice of h is. Several criteria for selecting h have been proposed, for a comprehensive discussion, see Wand and Jones (1995). Due to the fact that the market component M_t is dependent over time, our observations are not independent. Hart and Vieu (1990) have shown that the Least-Squares Cross Validation Criterion gives the asymptotically optimal bandwidth for dependent observations. We use that criterion for choosing the optimal bandwidth. Figure 6 shows kernel density estimates with uniform confidence bands (Müller, 2000, p.182) for ratios of price to income value and for ratios of price to fundamental value, where the fundamental value is

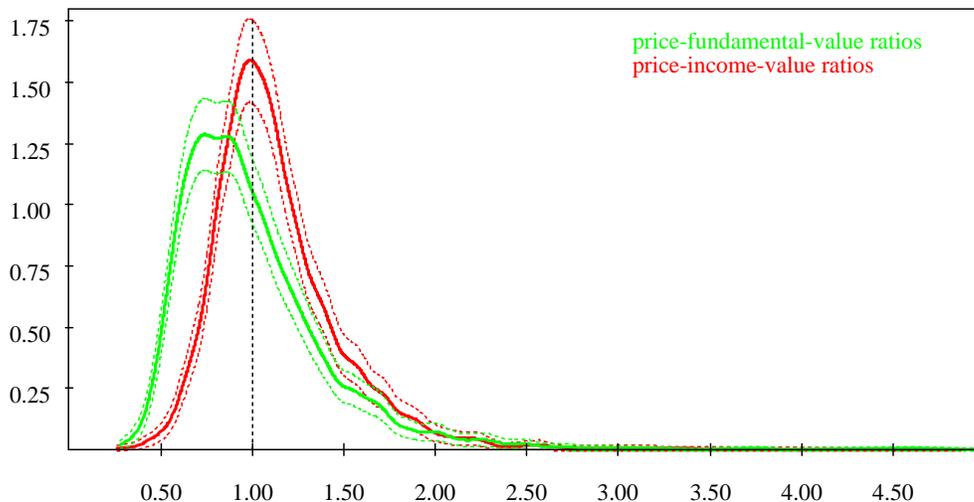


Figure 6: *Nonparametric density estimates for ratios of price to income value and price to fundamental value, where the fundamental values are calculated with θ_{PF} . Uniform confidence bands are at the 95% level.*

the capitalized gross rent. The capitalization factor is the reciprocal of θ_{PF} . Whereas the density of price to income value ratios peaks close to 1—the modus is at about 0.974—the most likely realizations of price to fundamental value ratios are definitely smaller than 1. Both densities are skewed to the right. Such skewness arises obviously when symmetric relative deviations between prices and income values are required. For example, if prices and corresponding income values can diverge by maximal 200%, then the ratios will fall in the interval $[0.5, 2]$. An average ratio of 1 implies a density skewed to the right. Price to fundamental value ratios show more probability mass for ratios smaller than one. About 60% of price to fundamental value ratios are below 1, compared with 40% of the price to income value ratios. In that sense, these ratios are more ‘bullish’ than ratios of prices to income values, because more ratios overstate than understate the price. However, it is difficult to assess which distribution is more favorable. We have to conclude that it depends on the objectives of the investor which distribution of appraisal errors is more ‘tolerable’.

A ranking of the distribution for ratios of income value to prices and fundamental value to prices is more obvious. Figure 7 shows kernel density

estimates for ratios of income value to price and for ratios of fundamental

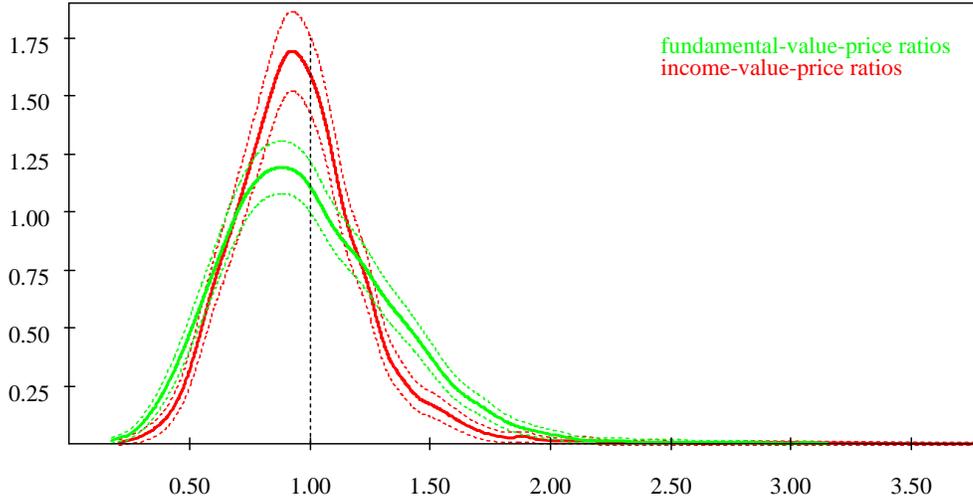


Figure 7: *Nonparametric density estimates for ratios of income value to price and fundamental value to price, where the fundamental values are calculated with θ_{FP} . Uniform confidence bands are at the 95% level.*

value to price, where the fundamental value is the capitalized gross rent. The capitalization factor is the reciprocal of θ_{FP} . Once again, both densities are skewed to the right and the appraisals calculated with the capitalization method are more ‘bullish’ than the income values. Whereas for the latter about 60% of the ratios lie below 1 (income values are smaller than prices), only 55% of the former lie below 1. It is obvious that large deviations from the true price are more likely for appraisals calculated with the capitalization method. The tails of the corresponding density dominate the tails of the density of the ratios of income values to price. If investors dislike the occurrence of large appraisal errors, they prefer valuation according to WertV.

Let us conclude: we have ranked the appraisals according to WertV above the appraisals calculated with the capitalization method. For this ranking we have used MSPE and MAPE. Additional to that, we have compared the distributions of ratios generated by the different methods. We have seen that the valuation according to WertV gives better results.

How should we assess this result? Can we conclude that valuation according to WertV is definitely better? First of all, the capitalization method is

an easy rule of thumb that needs little information. It ignores the remaining time of usage and object-specific cost factors. Valuation according to WertV is much more information-intensive and needs such information. However, more information should lead to better appraisals. So, our result is not surprising. But, we are back to the problem that we already have discussed. To rank different valuation methods according to the economic loss that is implied, we must know the circumstances for which the method will be used. However, we have seen that kernel density estimates are a good advise for a graphical representation of the whole distribution. The investor can use such estimates for deciding about the preferable method.

5 The general market conditions

To derive the market value for an object according to WertV, the appraiser calculates the income value as the first step. The second step consists in adjusting the income value with the general market condition. In Subsection 3.3 we have discussed that such an adjustment has to be checked for its empirical relevance. We use the system of equations (20). For convenience, we reproduce here both equations

$$\phi(L)m_t = \tilde{\kappa} + s_t^\top \gamma + \xi_t$$

and

$$q_{n,t} = m_t + \varepsilon_{n,t} .$$

The first equation gives the unobserved general market conditions m_t as a function of its own lagged values, observed market indicators comprised in the vector s_t and an innovation term. The second equation gives the log ratio of price to income value of house n that is sold in period t as the general market condition for that period plus an idiosyncratic noise term. Taking conditional expectation on $q_{n,t}$ gives

$$\mathcal{E}_t[q_{n,t}] = m_t .$$

Thus, a simple way for estimating $\{m_t\}_{t=1}^T$ is a regression of the ratios $q_{n,t}$ on constants for the respective periods. Figure 8 shows the results of a regression

on monthly time-dummies. It appears as if the volatility of the general

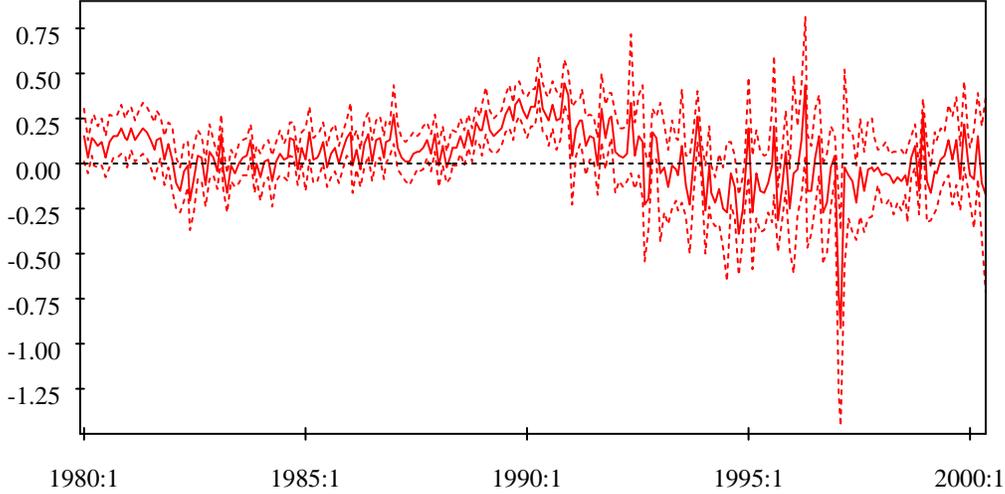


Figure 8: General market conditions \hat{m}_t . Estimated with a regression on time-dummies, confidence bands are at the 95% level.

market condition increases through time. However, this effect is partially caused by the decreasing number of observations. This is obviously revealed by larger confidence bands. A correct specified model has to incorporate the fact that the number of observations is different for different month.

Due to this fact, we model the behavior of the general market condition in the so-called *State Space Form* (SSF). Unknown coefficient can be estimated with Kalman filter techniques that can handle varying numbers of observations. In general, a SSF is given as

$$\alpha_t = c_t + T_t \alpha_{t-1} + \varepsilon_t^s \quad (22a)$$

$$y_t = d_t + Z_t \alpha_t + \varepsilon_t^m \quad (22b)$$

$$\varepsilon_t^s \sim (0, R_t), \quad \varepsilon_t^m \sim (0, H_t) . \quad (22c)$$

The notation partially follows Harvey (1989, 1993). The first equation is the *state equation* and the second is the *measurement equation*. The characteristic structure of state space models relates a series of unobserved values α_t

to a set of observations y_t . The unobserved values α_t represent the behavior of the system over time (Durbin and Koopman, 2001). The vectors c_t , d_t and Z_t contain—possibly time varying—parameters. The so-called transition matrix T_t governs the behavior of the state vector α_t .

In our case, the state equation models the behavior of the general market conditions. The transition matrix and the covariance matrix R_t allow the formulation of the general market conditions as an ARMA(p,q) process, see Harvey (1993). c_t is just $s_t^\top \gamma$. The measurement equation relates the observed log ratios of all houses sold in period t to the behavior of the return deviations. Let N_t denote the number of houses sold in month t and let ι_t denote an unit vector with dimension $N_t \times 1$. We split the general market conditions into two parts: the constant $\tilde{\kappa}$ and the deviations of the required returns from its long-run average. In that case, $d_t = \tilde{\kappa} \iota_t$ and $Z_t = \iota_t$. Eventually, the noise vector contains the idiosyncratic influences $\varepsilon_{n,t}$ and H_t is a $N_t \times N_t$ diagonal matrix with σ_ε^2 on its diagonal.

Generally, the estimators delivered by Kalman filter techniques have minimum mean-squared error among all linear estimators (Shumway and Stoffer, 2000, Chapter 4.2). If the initial state vector, the noise ε^m and ε^s are multivariate Gaussian, then the Kalman filter delivers the optimal estimator among all estimators, linear and nonlinear (Hamilton, 1994, Chapter 13). There exist two different techniques for filtering with missing observations, see Shumway and Stoffer (1982, 2000) and Koopman, Shephard, and Doornik (1999). However, it is possible to show that both methods deliver the same results, see Schulz and Werwatz (2002).

When some parameters of the system matrices of the SSF are unknown, they can be estimated via Maximum Likelihood. Under the assumption of normality, the likelihood function can be evaluated with Kalman filter techniques, see Harvey (1989). For our model, these parameters are the coefficients of the lag polynomial $\phi(L)$, the weights for the financial indicators γ , the constant $\tilde{\kappa}$, and the variances σ_ξ^2 and σ_ε^2 . Under the assumption of stationary process for the return deviations, we initialize the filter recursions with the unconditional distribution (Koopman, Shephard, and Doornik, 1999).

Thus, we set $a_0 = 0$ and the covariance matrix Σ is given implicitly as

$$\text{vec}(\Sigma) = (I - T \otimes T)^{-1} \text{vec}(R) .$$

In every step of the maximization procedure, Σ is recalculated with the current estimates of the unknown parameters.

5.1 Estimation

Table 7 shows summary statistics for the log ratios. Using the logarithm produces a more symmetric distribution compared with the distributions of Q or $1/Q$.

Table 7: *Summary statistics for log ratios of price to income value.*

Mean	Standard Deviation	Minimum	Median	Maximum
0.077	0.301	-1.397	0.057	1.597
10% Quantile	90% Quantile	Skewness	Kurtosis	Number of obs.
-0.258	0.456	0.414	5.002	4150

This is obviously revealed by the Kernel density estimate in Figure 9.

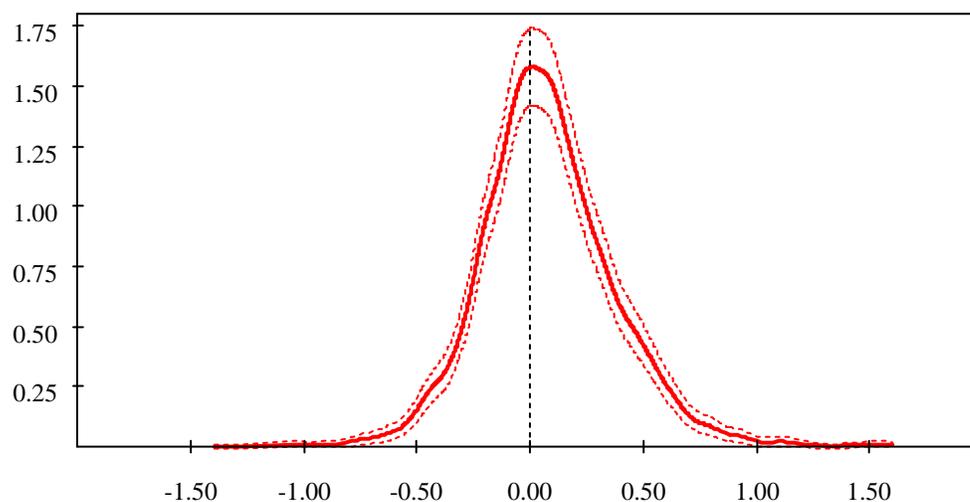


Figure 9: *Nonparametric density estimate for log ratios of price to income value. Uniform confidence bands are at the 95% level.*

According to the *Report on the Berlin real estate market* of the GAA (Geschäftsstelle des Gutachterausschusses für Grundstückswerte in Berlin, 2001), the general market conditions are influenced by five-year mortgage rates, interest rates on credits, development of building permissions, and the price index of the StaLa. Furthermore, the number of transactions is reported as a measure of the market conditions. To control for an information lead of the surveyors of the GAA, we include innovations in the rent index of the StaLa. All variables can be interpreted as proxies for economic risk that influences required returns of investments in commercial real estate. Alas, the Reports of the GAA do not describe explicitly the channels through which the market indicators influence the real estate market. We will give some tentative interpretations.

To model the financing conditions on the market, we use the spread of the five-year mortgage rate and the capital market rate with the same maturity. These series are only obtainable for the periods 1982:6-2000:5. As Nautz and Wolters (1996) have shown for periods up to 1994:8, both rates are cointegrated. Banks try to match the volume of mortgage credits by deposits with the same maturity. The spread can be interpreted as a risk premium for mortgage loans and thus for investments in real estate. The spread should have a positive influence on the required returns and thus a negative relationship with the general market conditions. Generally, the interest rate spreads show near unit root behavior. Test for a unit root are conducted with the Augmented Dickey Fuller (ADF). Including a constant and no lags gives a test statistic of -3.48. Using MacKinnon's critical values, we can reject the hypothesis of a unit root at the 1% level. Real estate investors often use checking accounts for interim financing (Brauer, 1999). We use the interest rate for such credits with a withdrawal between 0.2 up to 1 Million DM. Dividing by twelve and subtracting the expected monthly inflation rate gives roughly the real interest costs. Here, the expected monthly inflation rate is given by the fitted values of an AR specification. The real interest rate captures the state of short run investment opportunities.

To use a series of building permissions for the whole sample, we must use the number of building permits for Berlin West. However, as we have already

mentioned, the largest part of our data comprises houses from Berlin West. Thus, it should not matter that we use only the building permits for Berlin West. A higher number of new buildings will decrease the attractiveness of existent apartment houses and will increase the required return. Thus, increasing building permissions will depress the general market conditions.

To model the transaction volume of the market, we use the monthly log number of transacted apartment houses. The series is given in Figure 1. It is obvious that there are several breaks in that series. However, comparing the series with the estimated general market conditions in Figure 8, it seems that both series are co-breaking. The analysis of the residuals reveals that this guess is acceptable. Interpreting the transaction volume as an proxy for incentives to buy due to tax brackets and subsidies, a higher volume should accompanied by a higher general market conditions. Given the subsidies, the effective price of a house for a buyer is lower—given subsidies—and he is prepared to pay higher absolute prices. An alternative reason for the transaction volume is, that it captures “hot” and “cold” markets.

Eventually, to control for information leads of the surveyors, we fit an ARMA model for the inflation of the rent index and use the innovations—that are current values minus fitted values—as a measure of potential information leads of the surveyors.

Given our results about the incompletely appraised age and the influence of the log size of the lot on q , we include both variables in the measurement equation. We do not control for the real gross rent, because—as we have already mentioned—that figure is seldom observed after 1995. Fitting several specifications for the process of the general market conditions, comparing the value of the log likelihood function and the state residuals, we choose an ARMA(1,1) specification. Table 8 presents the results.

Table 8: *Estimated State Space Form for the General Market Conditions for 1982:6 -2000:5.*

	Coefficient	t-Statistic	P-Value
$\hat{\phi}$	0.925	32.309	0.000
$\hat{\theta}$	-0.689	-6.075	0.000

—continued on the next page—

Table 8: *Continued*

	Coefficient	t-Statistic	P-Value
$\widehat{\ln \sigma_{\xi}}$	-2.892	-20.560	0.000
$\widehat{\ln \sigma_{\varepsilon}}$	-1.265	-104.870	0.000
$\hat{\kappa}$	0.168	2.4384	0.015
log lot size	-0.015	-1.659	0.097
age	-0.0005	-2.668	0.008
spread5	-3.953	-2.902	0.004
real interest	5.759	1.687	0.092
building permissions	-0.027	-1.551	0.121
log number of transactions	0.015	2.529	0.011
rent index	-0.986	-0.937	0.349
Diagnostics			
Log likelihood	-614.636	Observations	3629

Note: 4150 observations are included. The market indicators are lagged by one month and demeaned. Building permissions is the deviation between the growth rate and its twelve-months moving average. Rent index gives the innovations of a fitted ARMA model for the inflation rate of the rent index. Spread5 is the difference between mortgage and interest rate with 5-year maturity. Real interest is the difference between the monthly interest rate for check accounts and the expected monthly inflation rate.

Using a significance level of 1%, the residuals behave like white noise and are normally distributed with a Jarque-Bera Statistic of 0.87 and a corresponding p-value of 0.65. The slight autocorrelation in the residuals decreases further by including the spread between the mortgage and interest rate with 10-years maturity. The coefficient for the spread is negative, but insignificant.

Figure 10 shows the smoothed general market conditions. The figure closely resembles the behavior of the simple averages given in Figure 8. It appears that there may be a unit root in the process of the general market conditions, which implies that the German Reunification might have suspend the equilibrating process between prices and rents. A bubble in real estate price during that time is a plausible explanation for the non-working of the equilibrating mechanism. Generally, Kalman filtering procedures can cope instationary processes. However, given such a process the asymptotic approximations may be poor (Engle and Watson, 1981). More important,

instationary general market conditions are at odds with the concept of stationary required returns. Taking an agnostic view, we interpret our results under the assumption of stationary required returns.

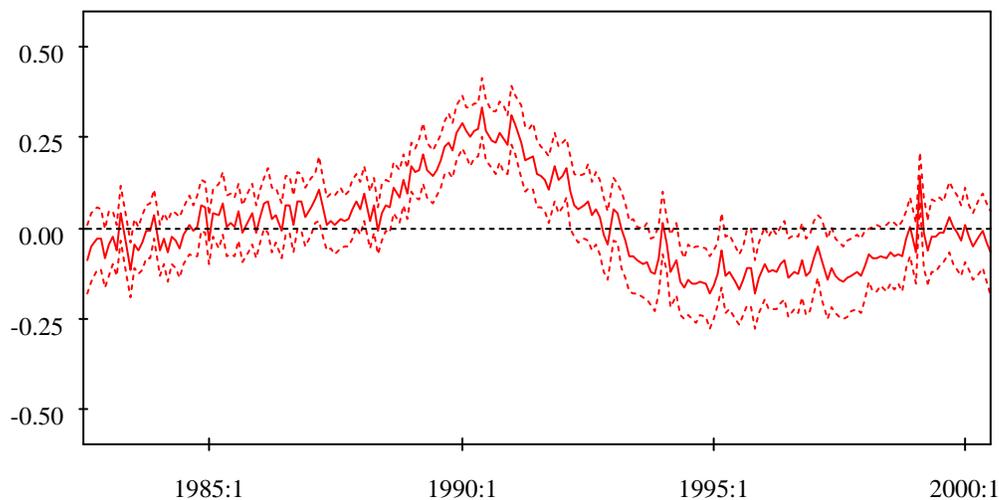


Figure 10: *Smoothed general market conditions 1982:6-2000:5. Confidence bands are at the 95% level.*

In Table 8 the first coefficient $\hat{\phi}$ gives the effect of the lagged general market conditions on its current value. It is relatively close to unity. $\hat{\theta}$ is the MA(1) coefficient. The standard deviations show that the larger part of variation of the log price to income value ratios amounts from the unsystematical component. The variance is about 0.08. Ignoring the explanatory variables in the state equation, the variance for an ARMA(1,1) process is (Harvey, 1993)

$$\sigma_m^2 = \frac{1 + \theta^2 + 2\phi\theta}{1 - \phi^2} \sigma_\xi^2 .$$

Calculated in this way, the variance of the general market conditions is 0.004. The variance of the log ratios is the sum of both variances—recall that $q = m + \varepsilon$ —and thus 0.084. The respective standard deviation is about 0.29. The largest part of variation in log ratios is due to unsystematical effects. The effect of the incompletely appraised age and the size of the lot resemble the result from the simple regression analysis. However, the size coefficient is insignificant at the 5% level.

The financial indicators affect the general market conditions directly. As conjectured, the spread between mortgage and interest rate has an depressing effect on the general market conditions. Interpreting the spread as risk premium, a higher risk premium increases required returns and thus has a negative effect on prices given the current expectations on net proceeds. The real interest rate is insignificant at the 5% level. The sign of the coefficient is positive. A higher real rate increases the general market conditions. On the one hand, this result is puzzling because a higher real rate makes interim financing more expensive. On the other hand, the real rate may reflect costs for short-run investments and higher costs may make long-run investments—like real estate—more attractive. The building permissions have an insignificant negative coefficient. The sign of the coefficient is plausible. As conjectured, a larger number of new buildings depress the general market conditions. The number of transactions has a positive influence on the general market conditions. Eventually, the innovations in the rent index have an insignificant negative coefficient. So we can reject the hypothesis that the surveyors of the GAA confound their backward-looking appraisals with current information.

After all, we found that some of the market indicators have an influence on the general market conditions. Although the Reports of the GAA only mention these indicators and give no explanation for their influence, we were able to derive some meaningful relations. Due to the fact that the WertV is silent on the correct usage of market indicators, it is nevertheless questionable if appraisers use the indicators in the correct way.

6 Conclusion

Our study of the German Regulation on Valuation has revealed that the intentions of the Regulation are in accordance with economic principles. The calculation of the income value (Ertragswert) is oriented on the present value concept and prescribes a simplified discounted cash flow method for valuation. The discounted cash flow method is widely accepted for estimating market values. The discount rate is provided by the average internal rate

of return of preceding observed transactions of commercial real estate. The accuracy of the backward-looking provision of discount rates needs definitely further investigation.

We have shown that the net proceeds are incompletely appraised. However, that might be the price of a standardized valuation method. Compared with the outcomes of the simpler capitalization method, we have shown that the outcomes of the income valuation is at least preferable. Income valuation—although biased—gives smaller mean squared prediction errors and smaller mean relative errors. Because the assessment of valuation accuracy depends on the investor specific loss function, we have also evaluated the complete distribution of outcomes. Density estimates can serve as a tool for investors to decide about their preferred valuation method. However, due to lack of reliable data on net proceeds, we should interpret our comparison carefully. The outcomes of our capitalization method might be biased because properly adjustment of the gross rents was impossible. In that sense our chosen constant discount factor might be confounded due to the auxiliary assumption of constant running costs.

Due to the fact that discounted cash flows react strongly on the chosen discount rate, further investigation on the optimal choice of that rate is necessary. It might be important to compare the outcomes of income valuation with valuations where the discount rate is determined in accordance with economic theory. Studies in the fashion of Kaplan and Ruback (1995) should shed more light on that problem. These authors use discount rates determined by the CAPM and evaluate the sensitivity of the outcomes with respect to different discount rates.

Income valuation is only the first step for appraising the market value of real estate according to WertV. The second step is the adjustment of the income value for general market conditions. Our interpretation of these conditions is that they reflect short-run deviations of required returns from its long-run average. Although we found evidence that the market conditions are influenced by market indicators, the general concept is nevertheless questionable. Further research has to explore if meticulously prepared discounted cash flows will lead to better results than the two-step procedure of

the WertV.

A Appendix

A.1 Accuracy of time-varying discount rates

In this Appendix, we check with a simple example if a time-varying discount rate for calculating the income value improves the accuracy. To do this, we use the log-linear framework and assume that appraisers use data up to $t - 1$ to calculate the discount factor θ_t that they use for discounting the rents in t . We will compare two different methods to calculate the discount factor: the long-run average of the price rent ratio and the current average price rent ratio. For the comparison, we use a reformulation of the log-linearized price (Campbell, Lo, and MacKinlay, 1997)

$$p_t = \frac{k}{1 - \rho} + d_t + \sum_{j=0}^{\infty} \rho^j (\mathcal{E}_t[\Delta d_{t+1+j}] - \mathcal{E}_t[r_{t+1+j}]) . \quad (23)$$

For illustrative purposes, we assume a zero growth rate of the rents. Thus, we eliminate all uncertainty on the rents. Let denote d the constant rent. Furthermore, we assume that the process of log real estate returns is given by the following AR(1) process

$$r_{t+j} = (1 - \phi)r + \phi r_{t+j-1} + u_{t+j} \quad (24)$$

with $|\phi| < 1$ and $u_{t+j} \sim (0, \sigma_u^2)$ is white noise. It follows that

$$\mathcal{E}_t[r_{t+1}] = r + \phi(r_t - r) . \quad (25)$$

Given these assumptions, one could calculate (23) with consecutive substitution. However, with $\alpha(L) = 1 - \phi L$ and $x_t = \phi(r_t - r)$, we use (19) and obtain

$$\sum_{j=0}^{\infty} \rho^j \mathcal{E}_t[r_{t+1+j}] = \frac{r}{1 - \rho} + \frac{\phi(r_t - r)}{1 - \rho\phi} \quad (26)$$

Plugging this expression into (23) yields

$$p_t = \frac{k - r}{1 - \rho} + d - \frac{\phi(r_t - r)}{1 - \rho\phi} . \quad (27)$$

The log income value for t calculated with information up to $t - 1$ is $d - \theta_t$ and the average difference between price and income value is just

$$\Delta_t \stackrel{\text{def}}{=} \frac{k - r}{1 - \rho} + \frac{\phi(r_t - r)}{1 - \rho\phi} - \theta_t .$$

Now let us compare the quality of different discount factors θ_t . The first way to calculate θ_t is just the long-run average of the price rent ratio. Using (27) yields

$$\theta_{t,1} = \frac{k-r}{1-\rho}$$

and thus

$$\Delta_{t,1} = \frac{\phi(r_t - r)}{1 - \rho\phi}.$$

Using the average price rent ratio of period $t-1$ yields

$$\theta_{t,2} = \frac{k-r}{1-\rho} + \frac{\phi(r_{t-1} - r)}{1 - \rho\phi}$$

and thus

$$\Delta_{t,2} = -\frac{\phi\{(1-\phi)(r_{t-1} - r) - u_t\}}{1 - \rho\phi}.$$

where we have used (24).

It is easy to check that both methods are unbiased. Furthermore, we derive with

$$\mathcal{V}[r_t] = \frac{\sigma_u^2}{1 - \phi^2}$$

that

$$\begin{aligned}\mathcal{V}[\Delta_{t,1}] &= \left(\frac{\phi}{1 - \rho\phi}\right)^2 \mathcal{V}[r_t] \\ \mathcal{C}[\Delta_{t,1}, \Delta_{t+k,1}] &= \left(\frac{\phi}{1 - \rho\phi}\right)^2 \phi^k \mathcal{V}[r_t]\end{aligned}$$

and

$$\begin{aligned}\mathcal{V}[\Delta_{t,2}] &= \left(\frac{\phi}{1 - \rho\phi}\right)^2 2(1 - \phi) \mathcal{V}[r_t] \\ \mathcal{C}[\Delta_{t,2}, \Delta_{t+k,2}] &= -\left(\frac{\phi(1 - \phi)}{1 - \rho\phi}\right)^2 \phi^{k-1} \mathcal{V}[r_t]\end{aligned}$$

where $\mathcal{C}[\cdot]$ denotes the covariance. It is easy to see that the constant discount factor used for calculating the income value delivers less volatile results if

$$\phi < 0.5.$$

This is intuitively plausible if one takes a look at the conditional expectation of the return process (25): the expected return for t is just a weighted average of r

and r_{t-1} . This average will be closer to r whenever $\phi < 0.5$. On the other hand, a value $\phi > 0.5$ makes discounting with a time-varying discount rate more favorable. In that case the process of the return rate is positively correlated and persistent.

Therefore, we can conclude for our example that a time-varying discount rate is better than a constant discount rate if the process of the expected returns is persistent. This will hold especially for houses with a short remaining time of usage. For a general statement, however, empirical studies are necessary that compare the outcomes of both discounting rates.

A.2 Partial effects on the income value

We want to inspect the partial effects of $D_{n,t}$, $B_{n,t}$, $T_{n,t}$, and θ_t on the income value (6). It is obvious that $E_{n,t}$ increases in the net proceeds $D_{n,t}$ and in the value of the lot $B_{n,t}$. Furthermore, we obtain

$$\frac{\partial E_{n,t}}{\partial T_{n,t}} = \left(\frac{1}{1 + \theta_t} \right)^{T_{n,t}} \ln \left(\frac{1}{1 + \theta_t} \right) \left(B_{n,t} - \frac{D_{n,t}}{\theta_t} \right).$$

This expression is positive—with $\theta_t > 0$ —if the last term is negative. § 20 WertV states that a surveyor has to set $E_{n,t} = B_{n,t}$ if $D_{n,t} \leq \theta_t B_{n,t}$ happens. In that case, changing the remaining time of usage does not influence the income value at all. On the other hand, when $D_{n,t} > \theta_t B_{n,t}$, the remaining time of usage increases the income value. For easier interpretation, we assume that $D_{n,t} > \theta_t B_{n,t}$ is fulfilled for all data and that a larger $T_{n,t}$ would have yielded a larger income value. Moreover, we obtain

$$\frac{\partial E_{n,t}}{\partial \theta_t} = - \left\{ 1 - \left(\frac{1}{1 + \theta_t} \right)^{T_{n,t}} \right\} \frac{D_{n,t}}{\theta_t} - T_{n,t} \left(\frac{D_{n,t}}{\theta_t} + B_{n,t} \right) \left(\frac{1}{1 + \theta_t} \right)^{T_{n,t}} < 0.$$

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