Sticky Information vs. Sticky Prices: A Horse Race in a DSGE Framework*

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Abstract

Mankiw and Reis (2002) have proposed sticky information as an alternative to Calvo sticky prices in order to model the conventional view that i) inflation reacts with delay and gradually to a monetary policy shock, ii) announced and credible disinflations are contractionary and iii) inflation accelerates with vigorous economic activity. We develop a fully-fledged DSGE model with sticky information and compare it to Calvo sticky prices, allowing also for dynamic inflation indexation as in Christiano, Eichenbaum and Evans (2001). We find that both models do equally well in delivering the conventional view.

Key words: sticky information, sticky prices, inflation indexation, DSGE

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1 Introduction

A large strand of literature in monetary economics regards nominal rigidities as a desirable modelling feature to explain the effects of monetary policy. A leading framework has been provided by Calvo (1983) and used e.g. by Woodford (1996), Yun (1996), Goodfriend and King (1997), Clarida, Gali and Gertler (1999), Gali (2002) and Woodford (2003). Recently, Mankiw and Reis (2002) have proposed random information arrival and slow information diffusion as an alternative paradigm. They argue that models based on sticky information can more easily reproduce the following conventional views:

1. Inflation inertia: inflation reacts with delay and gradually to a shock in monetary policy (see e.g. Christiano, Eichenbaum and Evans (2001)).

2. Announced and credible disinflations are contractionary (see Ball (1994)).

3. Acceleration phenomenon: the change in inflation is positively correlated with output (see e.g. Able and Bernanke (1998)).

The present paper closely reexamines their claim and compares the ability of similarly sophisticated models to replicate the three "conventional wisdom" effects. To that end, we develop a fully-fledged dynamic stochastic general equilibrium (DSGE) model with sticky information and compare the results to those, when Calvo sticky prices are assumed instead. This modifies the comparison envisioned by Mankiw and Reis (2002) in two important dimensions. First, by employing a DSGE model, aggregate demand now arises from an intertemporal household maximization problem rather than from an
exogenously assumed static demand curve as in Mankiw and Reis (2002). Second, we allow also for dynamic inflation indexation in the Calvo sticky price model as it has been proposed in the recent literature, see Christiano, Eichenbaum and Evans (2001) and Smets and Wouters (2002).

Regarding the sticky information model our results confirm the finding by Mankiw and Reis (2002): all three effects listed above can be replicated in the DSGE model as well. However, we show that a Calvo sticky price model without indexation can already match the effects 2 and 3 as well. Finally, allowing for dynamic inflation indexation in the Calvo sticky price model works just as well as Mankiw and Reis (2002) in delivering all three effects.

We conclude that while one may want to view Mankiw and Reis (2002) as providing a micro foundation for the particular choice of dynamic inflation indexation in Calvo sticky price models, these models are also perfectly capable of replicating the conventional wisdom.

The paper is organized as follows. In section two we lay out the DSGE model. Results are discussed in section three and finally section four concludes.

2 The DSGE Model

In the following section we develop a fully-fledged DSGE model with intertemporally optimizing households, a government and either sticky information or Calvo sticky price firms.
2.1 Households

Similar to Woodford (2003), the representative household is infinitely lived and has preferences about consumption, real money holdings and hours worked. The household receives wage income from supplying specialized labor input to the firms, obtains the nominal payoff from a state contingent portfolio, receives nominal cash transfers from the government and gets profits from the firms. Further, the agent holds nominal money carried over from last period and pays lump-sum taxes to the government. Finally, the household decides about an investment in a state contingent portfolio. See appendix 1 for the specific formal representation of the households optimization problem.

2.2 Government

The government issues nominal money $M_t$ and nominal bonds $B_t$, pays cash transfers $S_t$ to the households and collects lump sum taxes $T_t$ to finance its expenditures $G_t$,

$$P_t G_t = T_t + B_t - R_{t-1}B_{t-1} - S_t$$

where $S_t = M_{t-1}(\xi_t - 1)$. $P_t$ is the aggregate price level, $R_{t-1}$ denotes the nominal interest rate from period $t - 1$ to period $t$ and $\xi_t = \frac{M_t}{M_{t-1}}$ is nominal money growth. We assume $\xi_t$ and $G_t$ to follow exogenous AR(1) processes.
2.3 Firms

We assume a continuum of firms \( i \in [0, 1] \) in monopolistic competition each producing a differentiated good according to a Cobb-Douglas production technology. Labor of type \( i \) supplied by the household is used to produce differentiated good \( i \). Technology is the same for all firms and follows an exogenous AR(1) process. As in Woodford (2003), we assume that the firms are wage-takers. Now, we consider four different variants for the price setting behavior by firms.

**Flexible Price - Full Information Firms**

In the absence of any nominal and informational frictions firms choose prices \( P_i^*(t) \) each period to maximize profits.

**Sticky Information Firms**

Following Mankiw and Reis (2002), firms obtain new information with probability \( 1 - \lambda_1 \). These firms are able to find the profit maximizing price \( P_i^*(t) \). With probability \( \lambda_1 \) firms do not obtain new information. In this case, they use the information set they updated \( k \) period’s ago and set the price \( P_t(i) = E_{t-k}[P_i^*(t)] \).

**Calvo Sticky Price Firms**

According to Calvo (1983), sticky price firms can set their profit maximizing price \( \tilde{P}_t(i) \) with probability \( 1 - \lambda_2 \). With probability \( \lambda_2 \) firms cannot set
their optimal price. These firms have to keep last period’s price and set
\[ P_t(i) = P_{t-1}(i). \]

**Calvo Sticky Price Firms With Dynamic Indexation**

Two recent contributions by Christiano, Eichenbaum and Evans (2001) and Smets and Wouters (2002) propose dynamic inflation indexation as a modification of the standard Calvo sticky price approach. With probability \( 1 - \lambda_3 \) firms can set their optimal price \( \tilde{P}_t^*(i) \). With probability \( \lambda_3 \) firms cannot set their optimal price. Following Christiano, Eichenbaum and Evans (2001), these firms set the price \( P_t(i) = \Pi_{t-1} P_{t-1}(i) \). The non-optimizers apply a rule of thumb by updating last period’s price \( P_{t-1}(i) \) with yesterday’s gross inflation rate \( \Pi_{t-1} \).

Appendix 2 summarizes the formal description of each variant of price setting behavior by firms.

### 2.4 Equilibrium

In equilibrium all markets clear. We log-linearize our equilibrium conditions. Hat-variables denote percentage deviations from steady state. The DSGE framework can be characterized by the following set of equations: an intertemporal IS equation, a real money demand equation, a real money supply equation, an equation for the flexible price - full information real interest rate and the equations for the exogenous AR(1) processes for technology, nominal money growth and government expenditures. See appendix 3 for a formal description.
Into this DSGE framework, we throw in either one of the following three Phillips curves:

1. Under sticky information one can derive the so-called Sticky Information Phillips curve

$$\hat{\pi}_t = \frac{1 - \lambda_1}{\lambda_1} \zeta \hat{x}_t + (1 - \lambda_1) \sum_{k=0}^{\infty} \lambda_1^k E_{t-k-1} [\hat{\pi}_t + \zeta \triangle \hat{x}_t]$$  \hspace{1cm} (2)

where $\zeta = \frac{\omega + \sigma_s^{-1}}{1 + \theta \omega}$. $\hat{\pi}_t$ is the gross inflation rate and $\hat{x}_t$ denotes the output gap, defined as the difference between distorted and flexible price - full information output.

2. Under standard Calvo sticky prices we obtain the so-called New Keynesian Phillips curve

$$\hat{\pi}_t = \beta E_t [\hat{\pi}_{t+1}] + \kappa \hat{x}_t$$  \hspace{1cm} (3)

with $\kappa = \frac{(1 - \lambda_2)(1 - \lambda_2 \beta)}{\lambda_2} \zeta$.

3. Finally, under Calvo sticky prices with indexation we arrive at the so-called New Keynesian Phillips curve with dynamic indexation or hybrid New Keynesian Phillips curve

$$\hat{\pi}_t = \frac{1}{1 + \beta} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta} E_t [\hat{\pi}_{t+1}] + \frac{\kappa'}{1 + \beta} \hat{x}_t$$  \hspace{1cm} (4)

with $\kappa' = \frac{(1 - \lambda_3)(1 - \lambda_3 \beta)}{\lambda_3} \zeta$.

According to the Sticky Information Phillips curve, inflation is determined by current economic activity and by past expectations about current inflation.
and current economic activity. If new information arrives only some firms will be informed and change prices accordingly whereas most firms still set prices based on outdated information. If time passes by the fraction of firms that set prices based on new information increases and therefore, it is likely that inflation behaves inertial in response to new information.

By contrast, in the New Keynesian Phillips curve inflation is determined by current expectations about future inflation and by current economic activity. Thus, the New Keynesian Phillips curve is entirely forward looking and therefore inflation will immediately jump on impact rather than reacting with delay in response to new information. This lack of inflation inertia has been heavily discussed in the literature.

Empirical studies, see e.g. Gali and Gertler (1999) and Gali, Gertler and Lopez-Salido (2003), suggest that lagged inflation is an important determinant for the New Keynesian Phillips curve. Therefore, Christiano, Eichenbaum and Evans (2001) propose dynamic inflation indexation in a Calvo sticky price model. Non-optimizing firms apply a rule of thumb by updating last period’s price by last period’s inflation. The resulting New Keynesian Phillips curve with dynamic indexation shows that inflation is determined by past inflation, by current expectations about future inflation and by current economic activity. These forward and backward looking components make it likely that inflation behaves inertial in response to new information.

Thus, it is the rule of thumb behavior of non-optimizing Calvo sticky price firms that potentially produces the desired inertial reaction of inflation. But which rule of thumb should by applied? Christiano, Eichenbaum and
Evans (2001) assume that last period’s inflation is used to update prices of non-optimizing firms. Thus, these firms use information that is outdated by one period. Clearly, one could assume instead that non-optimizers use inflation observed two period’s ago to update their prices. It is also conceivable that they could use even older information to update their prices. Thus, the particular choice how old the information is that firms use to update their prices is ad-hoc in the Calvo sticky price model with dynamic indexation. By contrast, the sticky information model implies that the choice of inflation indexation depends on the particular information sets that are available to heterogenous firms. Some firms may be forced to use past period’s information set also including past period’s inflation rate. Other firms may be forced to use even older information sets also including even older inflation rates. All these firms use their particularly outdated information sets with the corresponding outdated inflation rates to update yesterday’s prices. Therefore, one might want to view Mankiw and Reis (2002) as providing a micro foundation for the particular choice of indexation in Calvo sticky price models.

However, the focus of this paper is to compare the sticky information model with the Calvo sticky price model with dynamic indexation in a DSGE framework taking the conventional wisdom as a measuring instrument.

2.5 Calibration

Table 1 summarizes the calibration of our model. We restrict ourself to conservative values widely used in the existing literature. Time is taken to be
quarters. The subjective discount factor is set to 0.99. Steady state inflation is set to zero. The coefficient of relative risk aversion of consumption is set to 2. The elasticity of (dis-) utility from supplying labor is calibrated to 1.5. We set the elasticity of utility with respect to real money holdings equal to 2. By equation (14) in appendix 3, this implies a unit income elasticity of real money demand as it is often found in empirical studies. The labor share in the Cobb-Douglas production function is calibrated to \( \frac{2}{3} \). As in Mankiw and Reis (2002), the degree of information rigidity (\( \lambda_1 \)) respectively the degree of price stickiness (\( \lambda_2, \lambda_3 \)) is set to 0.75. Thus, in case of the Calvo sticky price model, firms set optimal prices on average once a year. In case of the sticky information model, firms obtain on average new information once a year. We assume a markup over marginal costs of 20 percent. The steady state consumption to output ratio is set to 0.7, a value that corresponds to the US average for the period from 1960:1 to 2001:4. The process for technology is calibrated to standard values with an autocorrelation of 0.95 and a standard deviation of 0.71 percent. The AR(1) process for nominal money growth is specified with a persistence parameter of 0.5 and a standard deviation of 0.8 percent, similar to Mankiw and Reis’ (2002) calibration. Finally, as in Backhus, Kehoe and Kydland (1995) the autocorrelation and standard deviation of the government expenditures is set to 0.95 and 0.6 percent.

2.6 Solution Method

Before turning to the results of our horse race we want to sketch our solution method. We use Uhlig (1999) to solve our models. However, the Sticky
Information Phillips curve generates a potentially infinite state space, since we face an infinite sum of past expectations (see equation (2)). We will pursue the following strategy in order to solve the sticky information model.

1. Start with the Sticky Information Phillips curve with only the first lagged expectation \( E_{t-1} \) and compute the recursive equilibrium law of motion (RELOM).

2. Add the second lagged expectation \( E_{t-2} \) to the Sticky Information Phillips curve from above and compute the new RELOM.

3. Proceed adding lagged expectations as long as the coefficients of the RELOM change by more than a specified tolerance.

Figure 1 illustrates the solution algorithm. It shows the impulse responses of inflation to a one percent shock in nominal money growth for a stepwise inclusion of lagged expectations in the Sticky Information Phillips curve. The first plot in the top row shows the response of inflation if the model uses \( \hat{\pi}_t = \frac{1-\lambda_1}{\lambda_1} \zeta \hat{x}_t + (1 - \lambda_1) E_{t-1}[\hat{\pi}_t + \zeta \Delta \hat{x}_t] \). The second plot in the top row shows the response of inflation if the model takes an additional lagged expectation into account i.e. \( \hat{\pi}_t = \frac{1-\lambda_1}{\lambda_1} \zeta \hat{x}_t + (1 - \lambda_1) E_{t-1}[\hat{\pi}_t + \zeta \Delta \hat{x}_t] + (1 - \lambda_1) \lambda_1 E_{t-2}[\hat{\pi}_t + \zeta \Delta \hat{x}_t] \). Thus, the last plot in the bottom row shows the response of inflation if the sticky information model uses \( \hat{\pi}_t = \frac{1-\lambda_1}{\lambda_1} \zeta \hat{x}_t + (1 - \lambda_1) \sum_{k=0}^{11} \lambda_1^k E_{t-k-1}[\hat{\pi}_t + \zeta \Delta \hat{x}_t] \). Obviously, figure 1 illustrates that the shape of the response of inflation converges to a smooth hump-shaped pattern as \( k \) becomes larger and larger. As an approximation we look for that \( k \) where the recursive law of motion for all model variables does not change by more than a specified tolerance/critical value.
Technically, we apply the QZ-decomposition to obtain the recursive law of motion. Following Uhlig (1999), the model coefficient matrices $\Delta$ and $\Xi$ can be decomposed into unitary matrices $Y$ and $Z$ and uppertriangular matrices $\Sigma$ and $\Phi$ such that $Y'\Sigma Z = \Delta$ and $Y'\Phi Z = \Xi$. The recursive law of motion coefficient matrix $P$ which is needed to solve for the other recursive law of motion coefficient matrices, can be obtained by $P = -Z_{21}^{-1}Z_{22}$ where $Z_{21}$ and $Z_{22}$ are partitions of matrix $Z$, defined as in Uhlig (1999). $P$, $Z_{21}$ and $Z_{22}$ increase in their dimensions as $k$ - the number of included lagged expectations - increases. Additionally $P$ and $Z_{22}$ are singular. Therefore, to check for convergence of the recursive law of motion we look for that $k$ when the determinant of $Z_{21}^{-1}$ does not change more than a critical value. As an alternative, one could also check for convergence by numerically comparing impulse response functions for different $k$’s for all model variables.

For our problem we choose the tolerance/critical value to be 1.0e-25 units. This algorithm seems to be robust. We achieve convergence of the recursive equilibrium law of motion after including the 20th lag. This result is also intuitively reasonable. The Sticky Information Phillips curve can be interpreted as the geometric sum of past expectations with weights $(1 - \lambda)\lambda^k$. For our parametrization these weights cumulate to around 99.5 percent after including the 20th lagged expectation.

As a remark, it should be mentioned that one could derive a Sticky Information Phillips curve with a finite number of lagged expectations by e.g. allowing only for a finite number of adjustment prices in the aggregate price level (see equation (9) in appendix 2). However, this would imply a finite
horizon profit maximization problem for the sticky information firms which in turn implies that first order necessary conditions would change. We decide not to follow this strategy since it departs too much from Mankiw and Reis’ (2002) original specification of the Sticky Information Phillips curve. Instead, we found a (fairly accurate) algorithm to approximate the originally infinite geometric sum of lagged expectations of the Sticky Information Phillips curve with a finite number of lagged expectations.

3 Results

In this section we discuss the results by examining the models ability to deliver the three conventional views stated in the introduction.

3.1 Inflation Inertia

Figure 2 plots the responses of inflation, the output gap, the nominal interest rate and hours worked to a one percent nominal money growth shock for all three models. The sticky information model delivers a hump-shaped pattern of inflation with a maximum impact around the 7th quarter. However, the initial jump is much larger as in Mankiw and Reis (2002). This is due to the fact that households optimize intertemporally. They expect future inflation to be higher and thus adjust their consumption plans today which in turn generates a little more inflation on impact. Nevertheless, the Sticky Information Phillips curve seems to have a very strong internal propagation mechanism in response to a quickly dying out nominal money growth shock.
Interestingly, this result contrasts Keen (2003). He develops a model where households have imperfect information about the stance of monetary policy in a DSGE framework with sticky information firms also including a variety of other frictions such as a cash in advance constraint, portfolio adjustment costs and capital adjustment costs. As a special case Keen (2003) shows that if households have perfect information, the response of inflation is not hump-shaped when firms face sticky information. However, it is not clear which friction is responsible for his finding. Instead, our model delivers clear cut insights about the effects of sticky information in a standard DSGE framework, similar to the frameworks developed in Gali (2002) and Woodford (2003).

Inflation in the standard Calvo sticky price model immediately jumps on impact to its maximum effect and then decreases monotonically. By contrast, the response of the sticky price model with dynamic indexation also reacts with delay and gradually to a nominal money growth shock since it is both - forward and backward looking. The maximum impact occurs around the 5th quarter and is more pronounced than in the sticky information model.

Therefore, we conclude that the qualitative result of Mankiw and Reis (2002) is also robust in our DSGE framework: inflation reacts with delay and gradually to a monetary policy shock in the sticky information model whereas it does not in the standard Calvo sticky price model. Furthermore, we show that the Calvo sticky price model with dynamic indexation performs equally well as the sticky information model.
For completeness, figure 3 depicts the effects of a technology shock and figure 4 shows the response of the models to a government expenditures shock. As for the monetary policy shock the reaction of inflation is highly inertial in the sticky information model and the Calvo sticky price model with dynamic indexation. But is inflation in the data as inertial as in our models? Figure 5 compares the hp-filtered crosscorrelations of the model variables to output in the presence of technology, monetary and fiscal shocks to their counterparts in the data. We use quarterly hp-filtered US time series from 1960:1 to 2001:4. Inflation is the quarterly change in the log CPI (all items) and output is log real GDP. We find that inflation lags up to 4-5 quarters behind output in the data. The standard Calvo sticky price model is not able to deliver this feature. By contrast, the sticky information model and the Calvo sticky price model with dynamic indexation perform equally well and are able to match the empirical evidence for inflation quite convincingly.

3.2 Announced Disinflations

Let us turn to the disinflationary boom issue. Similar to Mankiw and Reis (2002), in period $t = 0$ the central bank announces credibly that it will reduce nominal money growth temporarily from period $t = 2$ (respectively the 8th quarter) onwards. The credibly announced fall in nominal money growth is temporary in the sense that we assume the same stationary process for nominal money growth as before. Figure 6 shows the impulse responses to the announced temporary fall in nominal money growth. Again, our DSGE model confirms Mankiw and Reis’ (2002) result that in the sticky
information model a credibly announced disinflation is contractionary. We also show that in the Calvo sticky price model with dynamic indexation announced credible disinflations are contractionary too. However, Mankiw and Reis (2002) as well as Ball (1994) find that for standard Calvo sticky price models announced and credible disinflations cause booms rather than recessions. This result is not robust in a fully-fledged DSGE framework. The reason for this is the forward looking behavior of the households. In contrast to Mankiw and Reis (2002) and Ball (1994) who assume a static quantity equation representing aggregate demand, our DSGE framework generates a forward looking IS curve that represents aggregate demand. Households have complete information and thus know that the central bank will lower nominal money growth from period $t = 2$ onwards. They know that economic activity in the future will decrease and with that their future consumption. In order to smooth consumption they already start lowering consumption from the announcement period onwards. Thus, the output gap falls in response to the announced disinflation in all three models.

Regarding inflation, the standard New Keynesian Phillips curve generates an immediate jump down when the announcement is made. In contrast to that, the Sticky Information Phillips curve as well as the New Keynesian Phillips curve with dynamic indexation lead to a gradual downward adjustment of inflation. It should be stressed that this result is different from Mankiw and Reis’ (2002) finding. They show that there is absolutely no reaction of inflation in response to the announcement. The variables react only when policy comes into place. By contrast, we show that inflation
starts reacting when the announcement is made due to perfectly informed and forward looking households.

The reaction of the nominal interest rate is worth to be mentioned here. It decreases during the announcement period before it increases when policy is implemented. One might argue that lower interest rates would fuel inflation whereas the actual aim was to lower inflation. However, one should recognize that policy follows an exogenous nominal money growth rule. Lower nominal interest rates will not fuel inflation since nominal money supply is exogenously fixed.

To sum up, we have shown that credibly announced disinflations are contractionary in all three models.

3.3 Acceleration Phenomenon

Table 2 provides values for the correlation between output and the annual change of quarterly inflation for the data and the model variables. The data suggest a positive correlation of about 0.25. All models deliver a significantly higher but positive correlation. Thus, sticky information and Calvo sticky prices with dynamic indexation in a DSGE framework are qualitatively able to explain the third conventional view that vigorous economic activity speeds up inflation. Moreover, the standard Calvo sticky price model also generates a positive correlation in a DSGE framework which contrasts the finding by Mankiw and Reis (2002) who report a negative correlation. Again, the forward looking behavior of households produces this result. It can be easily verified that the intertemporal IS curve representing aggregate
demand in a fully-fledged DSGE framework relates output positively to the change in inflation.

3.4 Still Improvable...

This section discusses results of the models that do not match the data. Figure 5 shows that the crosscorrelation of nominal interest rates and real marginal costs with output cannot be explained within our framework. To account for this one may want to introduce limited participation and nominal labor market frictions to the model. Further, the introduction of real frictions like habit formation might also help to improve the match with the data for these variables. However, these extensions are beyond the scope of this paper.

4 Conclusion

Mankiw and Reis (2002) have proposed sticky information as an alternative to Calvo sticky prices in order to model the conventional view that i) inflation reacts with delay and gradually to a monetary policy shock, ii) announced and credible disinflations are contractionary and iii) inflation accelerates with vigorous economic activity. We develop a fully-fledged DSGE model with sticky information and compare it to Calvo sticky prices, allowing also for dynamic inflation indexation.

Regarding the sticky information model our results confirm the finding by Mankiw and Reis (2002): all three effects listed above can be replicated
in the DSGE model as well. However, we show that a Calvo sticky price model without indexation can already match the effects ii) and iii) as well. Finally, allowing for dynamic inflation indexation in Calvo sticky price models works just as well as Mankiw and Reis’ (2002) sticky information model in delivering all three effects. We conclude that while one may want to view Mankiw and Reis (2002) as providing a micro foundation for the particular choice of inflation indexation in Calvo sticky price models, these models are also perfectly capable of replicating the conventional wisdom.

Appendix 1: Households

The representative agent maximizes the discounted sum of live-time utility,

\[
\max_{C_t,M_t,N_t(i),D_t+1} \mathbb{E}_t \sum_{t=0}^\infty \beta^t \left[ \frac{C_t^{1-\sigma} - 1}{1-\sigma} + \frac{\chi}{1-\nu} \left( \frac{M_t}{P_t} \right)^{1-\nu} - 1 \right] - \int_0^1 N_t(i)^{1+\phi} \, di
\]

subject to

\[
P_t C_t + M_t + E_t [Q_t,t+1D_t+1] \leq \int_0^1 W_t(i) N_t(i) \, di - T_t + M_{t-1} + D_t + S_t + \int_0^1 \Pi_t(i) \, di
\]  

(5)

where \( C_t \) denotes a composite consumption index which is defined as

\[
C_t = \left[ \int_0^1 C_t(i)^{\frac{\theta-1}{\sigma}} \, di \right]^{\frac{\sigma}{\theta-1}}. 
\]

This in turn implies the following for the aggregate price level: \( P_t = \left[ \int_0^1 P_t(i)^{1-\theta} \, di \right]^{1-\sigma} \). \( M_t \) denotes nominal money. We assume that each categorized good \( i \) is produced by specialized labor \( N_t(i) \) which is supplied by the representative household. \( W_t(i) \) is the wage that is
payed from firm $i$ to the household. As in Woodford (2003), the assumption of specific labor markets will generate strategic complementarities in firm’s pricing decisions. $D_{t+1}$ is a nominally denominated state contingent private bond that pays $D_{t+1}$ in period $t+1$. $Q_{t,t+k}$ is the stochastic discount factor from period $t$ to $t+k$ for nominal claims. $T_t$ denotes a lump-sum tax of the government and $S_t$ is a nominal money cash transfer. Finally, the household receives profits of the firms. The household is endowed with one unit of time (normalized) to be allocated between hours of work and leisure. Information is complete for the agent.

**Appendix 2: Firms**

A continuum of firms $i \in [0,1]$ in monopolistic competition produce according to $Y_t(i) = Z_t N_t^a(i)$ with $Y_t(i)$ and $N_t(i)$ being categorized output and specific labor input of firm $i$. $Z_t$ denotes technology which is assumed to follow an exogenous AR(1) process. With price $P_t(i)$ for firm $i$ and $P_t$ as the aggregate price level, firm demand is given by $Y^d(P_t(i); P_t, C_t, G_t) = Y^d_t(i) = (\frac{P_t(i)}{P_t})^{-\theta} (C_t + G_t)$. Required labor input for firm $i$ is given by $N(P_t(i); Y^d_t, Z_t) = N_t(i) = (\frac{Y^d_t(i)}{Z_t})^{1-\theta}$ which implicitly assumes that firms are wage-takers as in Woodford (2003).

**Flexible Price - Full Information Firms**

In the absence of any nominal and informational frictions firms choose prices
each period to maximize profits,

$$\pi_t(i) = \max_{P_t(i)} P_t(i) Y^d_t(i) - W_t(i) \left( \frac{Y^d_t(i)}{Z_t} \right)^{\frac{1}{\alpha}}. \quad (6)$$

The solution to this problem gives us the standard markup over marginal costs pricing rule.

**Sticky Information Firms**

The profit maximizing optimal price $P^*_t(i)$ in absence of any nominal and informational frictions is the solution to the flexible price - full information firms problem (equation (6)). The solution can be written as

$$P^*_t(i) = \frac{\theta}{\theta - 1} \frac{1}{\alpha} W_t(i) Z_t^{\frac{1}{\alpha}} Y^d_t(i)^{\frac{1}{\alpha} - 1}. \quad (7)$$

A firm that updated its information $k$ period’s ago sets the adjustment price

$$P^\text{adj}_{k,t}(i) = E_{t-k}[P^*_t(i)]. \quad (8)$$

Finally, the aggregate price level is the average of all adjustment prices

$$P_t = \left[ (1 - \lambda_1) \sum_{k=0}^{\infty} \lambda_1^k P^\text{adj}_{k,t}(i)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (9)$$

Combining the last three equations gives us the explicit form of the ag-
aggregate price level in presence of information rigidities

\[ P_t = \left[ (1 - \lambda_1) \frac{\theta}{\theta - 1} \frac{1}{\alpha} \sum_{k=0}^{\infty} \lambda_2^k E_{t-k} \left[ W_t Z_t^{-\frac{1}{\alpha}} Y_{t}^{d}(i)^{\frac{1}{\alpha} - 1} \right] \right]^{1/\theta}. \]  

(10)

The last equation can be modified to obtain the so-called Sticky Information Phillips curve.

**Sticky Price Firms**

Calvo sticky price firms solve

\[
\max_{P_t(i)} \sum_{k=0}^{\infty} \lambda_2^k E_t \left[ Q_{t,t+k} \left( P_t(i) Y_{t+k}^{d}(i) - W_{t+k}(i) \left( \frac{Y_{t+k}^{d}(i)}{Z_{t+k}} \right)^{\frac{1}{\alpha}} \right) \right].
\]

The aggregate price level in case of Calvo sticky prices can be written as

\[ P_t = \left[ (1 - \lambda_2) \tilde{P}_t^{1-\theta} + \lambda_2 P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \]  

(11)

with \( \tilde{P}_t \) as the solution to the above maximization problem. After some algebra, we obtain the so-called New Keynesian Phillips curve.

**Sticky Price Firms With Dynamic Indexation**

The profit maximization problem of the sticky price firms with dynamic indexation reads as follows

\[
\max_{P_t(i)} \sum_{k=0}^{\infty} \lambda_2^k E_t \left[ U_{t,k} P_t(i) Y_{t+k}^{d}(i) - W_{t+k}(i) \left( \frac{Y_{t+k}^{d}(i)}{Z_{t+k}} \right)^{\frac{1}{\alpha}} \right]
\]
with $U_{t,k} = \Pi_t \times \Pi_{t+1} \times \ldots \times \Pi_{t+k-1}$ and firm $i$’s demand schedule $Y^d_{t+k} = \left(\frac{U_{t,k} P(i)}{P_{t+k}}\right)^{-\theta} (C_{t+k} + G_{t+k})$. The aggregate price level in the presence of sticky prices and dynamic inflation indexation can be written as

$$P_t = \left[(1 - \lambda_3)(\hat{P}_t^*)^{1-\theta} + \lambda_3(\Pi_{t-1} P_{t-1})^{1-\theta}\right]^\frac{1}{1-\theta} \quad (12)$$

with $\hat{P}_t^*$ as the solution to the above dynamic programming problem.

After some algebra, we arrive at the so-called New Keynesian Phillips curve with dynamic indexation or hybrid New Keynesian Phillips curve.

### Appendix 3: The DSGE Framework

We obtain the following set of log-linearized equilibrium conditions. The consumer Euler equation can be manipulated to obtain an intertemporal IS relation,

$$\hat{x}_t = E_t[\hat{x}_{t+1}] - \frac{s_c}{\sigma} \left[\hat{R}_t - E_t[\hat{\pi}_{t+1}] - \hat{r}_f\right] \quad (13)$$

where $\hat{x}_t$ denotes the output gap, defined as the difference between distorted and flexible price - full information output. $E_t[\hat{\pi}_{t+1}]$ is the expected gross inflation rate, $\hat{R}_t$ denotes the nominal interest rate, $\hat{r}_f$ is the flexible price - full information real interest rate and $s_c$ is the steady state consumption to output ratio. Real money demand in this economy can be derived as a function of the output gap, exogenous disturbances and the nominal interest rate,

$$\hat{m}_t = \frac{\sigma}{\nu} \hat{x}_t + \frac{\sigma}{s_c \nu \varphi} \hat{z}_t - \gamma_0 \hat{g}_t - \frac{1}{\nu(R-1)} \hat{R}_t \quad (14)$$
where $$\varphi = \frac{\omega + \sigma s c^{-1}}{1 + \omega}$$, $$\omega = \frac{\phi}{\alpha} + 1 - 1$$, $$\gamma_g = \frac{\sigma(1 - s c)}{s c \nu} (1 - \frac{\sigma s c^{-1}}{\omega + \sigma s c^{-1}})$$.

Real money supply is given as

$$\hat{m}_t = \hat{m}_{t-1} - \hat{\pi}_t + \hat{\xi}_t.$$  \hspace{1cm} (15)

The flexible price - full information real interest rate can be expressed as

$$r^{f}_t = \mu_{rg} \hat{g}_t + \mu_{rz} \hat{z}_t.$$  \hspace{1cm} (16)

with $$\mu_{rg} = \frac{\sigma(\rho_g - 1)}{s c} \frac{\sigma(1 - s c)}{s c \omega + \sigma} + s c - 1$$ and $$\mu_{rz} = \frac{\sigma(1 + \omega)(\rho_z - 1)}{s c \omega + \sigma}$$.

Finally, we collect the log-linearized exogenous stochastic processes for technology, $$\hat{z}_t = \rho_z \hat{z}_{t-1} + \epsilon_{z,t}$$, for nominal money growth, $$\hat{\xi}_t = \rho \hat{\xi}_{t-1} + \epsilon_{\xi,t}$$ and for government expenditures, $$\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g,t}$$.
References


**Calibration**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Subjective discount factor</td>
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<tr>
<td>$\sigma$</td>
<td>2</td>
<td>Coefficient of relative risk aversion</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.5</td>
<td>Elasticity of (dis-) utility from supplying labor</td>
</tr>
<tr>
<td>$\nu$</td>
<td>2</td>
<td>Elasticity of real money balances</td>
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<tr>
<td>$\alpha$</td>
<td>$\frac{3}{4}$</td>
<td>Labor share</td>
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<td>$\lambda_1 = \lambda_2 = \lambda_3$</td>
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<td>Degree of price stickiness resp. information rigidity</td>
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<tr>
<td>$\frac{\sigma}{\sigma - 1}$</td>
<td>1.2</td>
<td>Markup of 20 percent over marginal costs</td>
</tr>
<tr>
<td>$s_c$</td>
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<td>Steady state consumption to output ratio</td>
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<tr>
<td>$\rho_z$</td>
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<td>Autocorrelation of technology shock</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.71</td>
<td>Standard deviation of technology shock</td>
</tr>
<tr>
<td>$\rho_\xi$</td>
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<td>Autocorrelation of nominal money growth shock</td>
</tr>
<tr>
<td>$\sigma_\xi$</td>
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<td>Standard deviation of nominal money growth shock</td>
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<tr>
<td>$\rho_g$</td>
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<td>Autocorrelation of gov. expenditures shock</td>
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<tr>
<td>$\sigma_g$</td>
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<td>Standard deviation of gov. expenditures shock</td>
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**TABLE 1:** Benchmark calibration of the DSGE model.
### Acceleration Phenomenon

<table>
<thead>
<tr>
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<th>corr($\hat{y}<em>t, \hat{\pi}</em>{t+2} - \hat{\pi}_{t-2}$)</th>
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<tr>
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<tr>
<td>sticky information (DSGE)</td>
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<tr>
<td>sticky prices (index, DSGE)</td>
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<td>sticky prices (DSGE)</td>
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<tr>
<td>sticky information (Mankiw-Reis)</td>
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<tr>
<td>sticky prices (Mankiw-Reis)</td>
<td>-0.13</td>
</tr>
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</table>

**TABLE 2**: Correlation of output with the annual change of quarterly inflation.

*Notes*: We use logged and hp-filtered quarterly US CPI (all items) and real GDP data. We obtain hp-filtered crosscorrelation figures by simulating the models.
Solution Method

FIGURE 1: Solution Method.

Notes: Impulse responses of inflation to a nominal money growth shock for a stepwise inclusion of lagged expectations in the Sticky Information Phillips curve. The first plot in the top row shows the response of inflation if the model uses $\hat{\pi}_t = \frac{1-\lambda_1}{\lambda_1} \zeta \hat{x}_t + (1-\lambda_1)E_{t-1}[\hat{\pi}_t + \zeta \Delta \hat{x}_t]$. The next plot depicts the response of inflation if the model uses $\hat{\pi}_t = \frac{1-\lambda_1}{\lambda_1} \zeta \hat{x}_t + (1-\lambda_1)E_{t-1}[\hat{\pi}_t + \zeta \Delta \hat{x}_t] + (1-\lambda_1)\lambda_1 E_{t-2}[\hat{\pi}_t + \zeta \Delta \hat{x}_t]$ etc. The x-axis plots years, the y-axis plots percent deviations from steady state.
Nominal Money Growth Shock

FIGURE 2: Impulse responses to a one percent shock in nominal money growth.
Technology Shock

FIGURE 3: Impulse responses to a one percent shock in technology.
FIGURE 4: Impulse responses to a one percent shock in government expenditures.
Crosscorrelation

FIGURE 5: Crosscorrelation of variables (t+j) with output (t).

Notes: Frequency domain techniques are used to obtain crosscorrelations for the model variables. We use quarterly hp-filtered US time series from 1960:1 to 2001:4 (all in logs). Inflation is the quarterly change in the CPI (all items). The nominal interest rate is a three month government bond yield. We use a manufacturing employment index for hours worked. Output is real GDP and real marginal cost are CPI deflated unit labor cost.
FIGURE 6: Impulse responses of model variables to an announcement at $t = 0$ that nominal money growth will fall temporarily from period $t = 2$ onwards.