Empirical modeling of the DEM/USD and DEM/JPY foreign exchange rate: Structural shifts in GARCH-models and their implications

Helmut Herwartz[1] and Hans-Eggert Reimers

Institute of Statistics and Econometrics
Humboldt–University Berlin
Spandauer Str. 1
10178 Berlin
GERMANY
Tel.: (+30) 2093-5725

Hochschule Wismar
University of Technique, Business and Design
Postfach 12 10
23952 Wismar
GERMANY
Tel.: (+3841) 753601

Abstract

We analyze daily changes of two log foreign exchange (FX) rates involving the Deutsche Mark (DEM) for the period 1975 - 1998, namely FX-rates measured against the US dollar (USD) and the Japanese yen (JPY). To account for volatility clustering we fit a GARCH(1,1)–model with leptokurtic innovations. Its parameters are not stable over the sample period and two separate variance regimes are selected for both exchange rate series. The identified points of structural change are close to a change of the monetary policies in the US and Japan, the latter of which is followed by a long period of decreasing asset prices. Having identified subperiods of homogeneous volatility dynamics we concentrate on stylized facts to distinguish these volatility regimes. The bottom level of estimated volatility turns out be considerably higher during the second part of the sample period for both exchange rates. A similar result holds for the average level of volatility and for implied volatility of heavily traded at the money options.

JEL Classification: F31, C22, E44

Keywords: Foreign exchange market volatility; GARCH; Structural stability.

[1]The research of this paper was carried out within the Sonderforschungsbereich 373 at Humboldt University Berlin and was printed using funds made available by the Deutsche Forschungsgemeinschaft. Helpful comments from Christian Hafner are gratefully acknowledged.

1 Introduction

Price variations observed at speculative markets are typically found to exhibit positive autocorrelation. Periods of higher and lower volatility alternate. This phenomenon is well known and generated a vast body of econometric literature after the seminal contributions by Engle (1982), Bollerslev (1986), and Taylor (1986) introducing the (generalized) autoregressive conditionally heteroskedastic ((G)ARCH) process and the stochastic volatility model, respectively. Often these specifications are valid only locally within a particular window of time. Hamilton and Susmel (1994) point out that invariant parametric specifications, GARCH say, are often inconvenient to model long return series and, thus, advocate models with switching parameters. Moreover, Mikosch and Starica (2000) show formally that GARCH-models with parameter shifts may generate autocorrelation patterns of squared returns carrying the risk to overestimate persistence in volatility. Completely in line with the latter finding is the contribution of Lamoureux and Lastrapes (1990) considering volatility persistence in the US stock market. In their work time homogeneous estimates of persistence exceed substantially corresponding quantities obtained from a volatility model which allows deterministic shifts in the unconditional variance.

Apart from big events as the stock market crash in October 1987 at least two driving forces behind structural variation of volatility dynamics are interesting to consider. Firstly, Chang, Cheng and Pinegar (1999) and Harris (1989) (among others) address the impact of introducing new financial instruments like options or futures on volatility dynamics. Whereas Harris (1989) finds marginal increases of stock volatility after the introduction of S&P 500 index futures Chang et al. (1999) provide ambiguous results for the Japanese stock market. Secondly, and more important for this paper, changes in monetary policies might affect FX-volatility (Diebold 1986, Lastrapes 1989).

A further issue often raised in the empirical analysis of financial market volatility is that the unconditional distribution of returns remains leptokurtic even after adjustment for conditional heteroskedasticity (Bollerslev 1987). Since the convenience of Quasi Maximum Likelihood (QML) has been well established for financial analysis (Bollerslev and Wooldridge 1992, Lumsdaine 1996, Hansen and Lee 1994) a violation of the common normality assumption is hardly crucial for the purpose of parameter estimation and inference. The matching of the theoretical model and the empirical distribution becomes more important, however, when the return distribution is of direct interest to the analyst. As a particular example one may regard the evaluation of portfolio risk (Jorion 2000).

The purpose of this paper is paper is twofold. In the first place the convenience of the GARCH-approach to describe the (conditional) distribution of FX-returns is illustrated. For this purpose both generalizations mentioned before, structural variation and conditional leptokurtosis, will be of essential importance. Secondly, we use the identified subperiods of homogeneous dynamics to compare a few stylized facts of volatility over time. Thereby we contribute to the view that FX-uncertainty has been smaller at the beginning of the sample period compared to its end.

We analyze the dynamics of two daily FX–rate series (DEM/USD and DEM/JPY) during the period from 1975 through 1998. When accounting for the potential of structural variation of volatility dynamics we do not a priori assume exogenous (Lastrapes 1990) or deterministic (Lamoureux and Lastrapes 1990) time points of change but test formally the null hypothesis of structural invariance by means of a Lagrange Multiplier (LM) test (Chu 1995, Lin and Yang 2000). Having rejected the latter hypothesis we determine the time point of structural variation by means of a likelihood ratio (LR) criterion. We show that a two regime GARCH(1,1)-model driven by leptokurtic innovations describes accurately the empirical distribution of FX-returns. Our empirical results are almost uniform for both investigated FX-series. Interestingly, we find for both series that the identified time points of structural change correspond to major changes of monetary policies adopted by the Federal Reserve System (Fed) and the Bank of Japan (BoJ).

The implications of the estimated structural changes are illustrated by means of implied volatilities, which financial market practitioners regard as a measure of uncertainty. We obtain that a time invariant GARCH(1,1)-specification provides implied volatilities which differ considerably from their time depending counterparts. Considering European call options we obtain that implied volatility of heavily traded at the money options is higher within the second subperiod. When considering short times to maturity it turns out that implied volatility of out of the money options is higher at the beginning of the sample period. The latter result mirrors that volatility responds stronger to lagged returns during the identified first subsamples.

The paper is organized as follows: Section 2 provides a brief descriptive analysis of the investigated FX-rates. Estimating time homogeneous volatility dynamics and distinguishing the scope of the GARCH-approach under conditional normality and conditional leptokurtosis are central issues addressed in Section 3. In Section 4 we sketch the methodological framework when testing parameter stability and detecting the time points of structural change. Moreover, the identified volatility dynamics are related to real economic events. In Section 5 the GARCH option pricing model is briefly motivated and we discuss empirically its implications in presence of structural change. Section 6 concludes the paper.

2 Descriptive statistics of the data

We analyze daily figures of two FX-rates, the DEM/USD- and the DEM/JPY-rate for the period January 2, 1975 to December 30, 1998. Log rates are depicted in Figure 1. Both processes are nonstationary which is confirmed by formal ADF tests (Dickey and Fuller 1979). The ADF statistics (test regression of lag order 1 with intercept term) for the DEM/USD and DEM/JPY log-rates are -1.33 and -1.58, respectively. The corresponding results for log-rate changes are -34.04 and -53.08. Thus, both exchange rates are integrated of order 1.

As it is typical for price variations observed at speculative markets large log-return changes (of either sign) are followed by further large changes (of either sign), periods of higher and lower volatility tend to cluster. The latter property can be inferred from descriptive statistics. Table 1 reports the third and fourth order moments and a few statistics testing particular features of the FX-return processes. In particular, the Jarque-Bera (JB) statistic (Jarque and Bera 1987) on unconditional normality, the Ljung-Box (LB) statistics (Ljung and Box 1978) against joint serial correlation up to order 8, 16 and 24, and ARCH-LM test (Engle 1982) results on homoskedasticity are given. According to the JB test the null hypothesis of normality is rejected at any reasonable level. The unconditional distribution of DEM/USD-returns shows a slightly negative skewness and, as a consequence of volatility clustering, a high degree of excess kurtosis. Moreover, DEM/USD log-returns do not exhibit significant autocorrelation. Testing a homoskedastic variance against conditional heteroskedasticity by means of an ARCH-LM(1) test, however, indicates higher order dependence of the FX log-returns.

Empirical results for DEM/JPY-returns are similar to interpret. However, the latter process exhibits significant autocorrelation. To estimate an uncorrelated residual series generating volatility dynamics we consider a linear autoregressive subset model. Starting from a maximum lag order of 12 we impose zero restrictions on coefficient estimates which are not significant at the 5% significance level. Following these lines a convenient representation for log-returns of the DEM/JPY-rate (e_t) is found to be:

$$\Delta e_t = \underset{(-1.141)}{8.92\text{E-}05} + \underset{(1.918)}{0.041}\,\Delta e_{t-1} + \underset{(1.763)}{0.032}\Delta e_{t-8} + \underset{(3.318)}{0.034}\Delta e_{t-9} + \hat{\epsilon}_t, \tag{1}$$

where Δ is short for the first difference operator, i.e. $\Delta e_t = e_t - e_{t-1}$. In (1) heteroskedasticity consistent $t-$ratios (White 1980) for the parameter estimates are given in parentheses. As shown in Table 1 the estimated residuals $\hat{\epsilon}_t$ are uncorrelated but still show dependence in higher order moments. The following analyses concentrate on DEM/USD log-returns and residuals from (1) when modeling the DEM/JPY-rate. There are 6015 and 6006 observations available for the DEM/USD- and the DEM/JPY-rate, respectively, since we use 9 observations as presample values for the latter rate.

3 GARCH-Model

For the DEM/USD- and the DEM/JPY-rate the identified residual processes are shown in the upper panels of Figure 2 and Figure 3, respectively. Both processes show typical properties of (high frequency) price variations, in particular leptokurtosis and volatility clustering. GARCH-processes are convenient parametric models to capture these features. A Gaussian GARCH(p,q)-process ϵ_t obeys the following dynamic structure conditional on Ψ_{t-1}, the history generated by the process:

$$\epsilon_t = \xi_t\sigma_t,\ \xi_t \sim N(0,1),\ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q}\alpha_i\epsilon_{t-i}^2 + \sum_{i=1}^{p}\beta_i\sigma_{t-i}^2,\ t = 1,\ldots,T. \tag{2}$$

To guarantee positivity of the conditional variance of ϵ_t sufficient parameter restrictions are $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, $i = 1,\ldots,q$, $j = 1,\ldots,p$. The GARCH-model in (2) implies $\epsilon_t|\Psi_{t-1} \sim N(0,\sigma_t^2)$. In the empirical literature it turned out that for most applications the GARCH(1,1)-model suffices to model financial market returns (Bollerslev, Engle and Nelson 1994). Nelson (1990) discusses stationarity and ergodicity conditions of the GARCH(1,1)-process in detail. In particular, σ_t^2 and ϵ_t are strictly stationary and ergodic if $E[\ln(\beta_1 + \alpha_1 \xi_t^2)] < 0$. Note that this moment conditions depends on the distribution of ξ_t. Imposing the stronger restriction $E[\beta_1 + \alpha_1 \xi_t^2] < 1$ ($\Leftrightarrow \alpha_1 + \beta_1 < 1$) it can be shown that $E[\epsilon_t^2] = \sigma^2 < \infty$ and, thus, ϵ_t is weakly (covariance) stationary. Then an estimate of the unconditional variance is:

$$\hat{\sigma}^2 = \hat{\alpha}_0 (1 - \alpha_1 - \beta_1)^{-1}. \tag{3}$$

The specific case where α_1 and β_1 sum to unity has become popular as the so-called integrated GARCH(1,1)-model (IGARCH(1,1), Engle and Bollerslev 1986). Note that the result in Nelson (1990) implies that while the GARCH(1,1)-process with $\alpha_1 + \beta_1 < 1$ is covariance stationary, strictly stationary and ergodic, the IGARCH(1,1)-process is not covariance stationary but still strictly stationary and ergodic, thus distinguishing it from the random walk with drift case.

ML-estimation of GARCH-models requires numerical optimization routines, since the specification of the likelihood function is only feasible conditional on Ψ_{t-1}. The Gaussian log-likelihood function ($l(.) = \ln L(.)$) may be given as follows:

$$\begin{aligned} l(.) &= \sum_t l_t(.), \\ l_t(.) &= -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\sigma_t^2) - \frac{1}{2}\frac{\epsilon_t^2}{\sigma_t^2}. \end{aligned} \tag{4}$$

When initializing the iterative optimization procedure it is convenient to try alternative parameter choices, $\alpha_1 = 0.05, 0.09,\ \beta_1 = 0.8, 0.9$ say, to guard against local optimality of the numerical estimate. Given some pair of initial values α_1, β_1 and the unconditional means of ϵ_t^2 the corresponding parameter α_0 is obtained from (3). Then, to initialize the volatility path $\sigma_1^2 = \hat{\sigma}^2$ is suitable. Estimating GARCH(1,1)-specifications along these lines for both FX-series we obtain the following results (QML t-ratios in parentheses):

$$\text{DEM/USD: } \hat{\sigma}_t^2 = \underset{(2.29)}{5.56\text{E-07}} + \underset{(8.17)}{0.103}\epsilon_{t-1}^2 + \underset{(58.6)}{0.890}\hat{\sigma}_{t-1}^2, \text{ log-lik: } 21925.23, \tag{5}$$

$$\text{DEM/JPY: } \hat{\sigma}_t^2 = \underset{(1.83)}{5.30\text{E-07}} + \underset{(6.26)}{0.112}\epsilon_{t-1}^2 + \underset{(35.0)}{0.877}\hat{\sigma}_{t-1}^2, \text{ log-lik: } 22893.45. \tag{6}$$

A few diagnostic test results for the implied standardized residuals, $\hat{\xi}_t = \epsilon_t / \hat{\sigma}_t$, from (5) and (6) are also given in Table 1 (columns 3 and 6). Estimated innovations show some skewness and are still highly leptokurtic. The hypothesis of normally distributed innovations is rejected at any reasonable significance level.

The ARCH-LM(1) test on homoskedasticity of $\hat{\xi}_t$ indicates for the DEM/JPY-rate that the GARCH(1,1)-model may not capture conditional heteroskedasticity entirely. Increasing the model order towards a GARCH(1,2)- or GARCH(2,1)-process, however, neither provided significant coefficient estimates of the corresponding higher order parameters nor improved the maximum value of the Gaussian log-likelihood function considerably. Moreover, the ARCH-LM(5) statistic is not significant at the 5% level. Thus we regard the GARCH(1,1)-model to be an appropriate model describing both DEM/USD log-returns and linear residuals obtained from (1) for the log DEM/JPY-rate.

With respect to parameter estimation the violation of the normality assumption is not crucial. In this case the adopted estimation procedure has become popular as QML-estimation. Due to the large sample investigated here we expect the efficiency loss of QML compared to exact ML methods to be negligible. Asymptotic normality of the QML-estimator in the GARCH(1,1)- and IGARCH(1,1)-model is derived in Lumsdaine (1996) building upon former work by Bollerslev and Wooldridge (1992). Lee and Hansen (1994) prove asymptotic normality of QML-estimates even for the case where $\alpha_1 + \beta_1$ slightly exceeds unity. Given some standard regularity conditions the latter result is obtained mainly under the assumptions that the GARCH-process is strictly stationary ($E[\ln(\beta_1 + \alpha_1 \xi_t^2)] < 0$) and that the conditional fourth order moment of ξ_t is bounded ($E[\xi_t^4|\Psi_{t-1}] < \infty$). A sufficient condition for the latter requirement is that ξ_t is iid with finite fourth order moment.

The violation of the distributional assumptions underlying the GARCH-model in (2) is more crucial if one is interested in inference on ϵ_t. Given some (estimated) GARCH-specification and a particular history Ψ_{t-1} the practitioner might be interested in a confidence band for ϵ_t with significance level α. For the model in (2) such an interval is:

$$\mathrm{CI}(\epsilon_t|\Psi_{t-1})_N^\alpha = \{\tilde{\epsilon} : |\tilde{\epsilon}| \leq z_{1-\alpha/2}\, \sigma_t\}, \tag{7}$$

where $z_{1-\alpha/2}$ is the $(1-\alpha/2)$-quantile of the Gaussian distribution. In (7) the subscript N is used to indicate that the confidence interval of interest is determined under conditional normality. A violation of the normality assumption may involve empirical significance levels of $\mathrm{CI}(\epsilon_t|\Psi_{t-1})_N^\alpha$ which differ from their nominal counterparts. With $I(.)$ denoting an indicator function the empirical significance level is estimated as:

$$\hat{\alpha}_N = \frac{1}{T}\sum_{t=1}^{T} I(\epsilon_t \notin \text{CI}(\epsilon_t|\Psi_{t-1})_N^{\alpha}).$$

Due to the large sample sizes which are typical in financial practice powerful tests of the hypothesis $H_0 : \hat{\alpha}_N = \alpha$ can be performed. To illustrate this point Table 2 (second column) shows $\hat{\alpha}_N$ for alternative nominal significance levels.

The empirical significance levels $\hat{\alpha}_N$ allow a similar interpretation for both FX-rates. Due to excess kurtosis of the innovations ξ_t empirical significance levels violate their nominal counterparts significantly in almost all cases. For the DEM/USD-rate we obtain, for instance, that confidence bands with nominal coverage of 50% actually include 54.4% of all FX-returns, indicating that these confidence bands are too wide on average. The nominal level $\alpha = 0.50$ and its empirical estimate $\hat{\alpha}_N$ differ at the 1% significance level.

To allow valid inference for ϵ_t one may assume that the innovations ξ_t follow a leptokurtic distribution as, for instance, a standardized t-distribution with ν degrees of freedom ($\xi_t \sim t(0,1,\nu)$). Then ξ_t may be given formally as $\xi_t = Z^*\sqrt{(\nu-2)/\nu}$, where Z^* is Student-t distributed with ν degrees of freedom. It is easily verified that for such random variables excess kurtosis is inversely related to ν.

ML-estimation of GARCH-processes under $t(0,1,\nu)-$distributed innovations is advocated in Bollerslev (1987). In comparison to (2) the $t(0,1,\nu)-$model requires to estimate an additional parameter, namely the degrees of freedom coefficient ν. Instead of (4) the contribution of a single observation to the sample log-likelihood is given as (Johnson and Kotz 1972):

$$l_t(.) = -\ln c - \frac{1}{2}\ln(\omega_t) - \frac{\nu+1}{2}\ln\left(\nu + \frac{\hat{e}_t^2}{\omega_t}\right), \tag{8}$$

where

$$c = \frac{\pi^{0.5}\Gamma(\frac{\nu}{2})}{\nu^{\frac{\nu}{2}}\Gamma(\frac{\nu+1}{2})},\ \omega_t = \frac{\nu-2}{\nu}\sigma_t^2,$$

and $\Gamma(\cdot)$ is the gamma function. A particular feature of this model is that not the conditional variance (σ_t^2) but a rescaled version of it (ω_t) enters the log-likelihood function. Obviously ω_t is only defined if $\nu > 2$, the necessary condition for a $t-$distributed random variable to have a finite variance. Estimating a GARCH(1,1)-model under the assumption of leptokurtic innovations we obtain estimates given in Table 3 (columns 2 and 5).

The numerical differences between these estimates and those obtained under conditional normality (see (5) to (6)) are rather small. The estimated degrees of freedom, $\hat{\nu}$, are 6.3 and 7.1 for the DEM/USD- and DEM/JPY-rate, respectively. Taking the corresponding standard error estimates (not reported) into account both results indicate that the underlying innovations are only poorly approximated by a standard normal distribution. Note that both estimates $\hat{\nu}$ imply the existence of $E[\xi_t^4]$ which has been one of the main assumptions to establish asymptotic normality of QML-estimates. For both investigated series the value of the log-likelihood function is considerably higher under leptokurtic innovations indicating that the $t(0,1,\nu)$-model provides a closer fit to the observed data.

Similar to the case of conditional normality confidence intervals for ϵ_t can be provided under the leptokurtic distribution. Analogously to (7) we have formally:

$$\mathrm{CI}(\epsilon_t|\Psi_{t-1})^\alpha_{t_\nu} = \{\tilde{\epsilon} : |\tilde{\epsilon}| \leq t_{1-\alpha/2}(\nu)\sqrt{\frac{\nu-2}{\nu}}\,\sigma_t\}, \tag{9}$$

where $t_{1-\alpha/2}(\nu)$ is the $(1-\alpha/2)$-quantile of the Student's t-distribution with ν degrees of freedom.

The corresponding empirical significance level is:

$$\hat{\alpha}_{t_\nu} = \frac{1}{T}\sum_{t=1}^{T} I(\epsilon_t \notin \mathrm{CI}(\epsilon_t|\Psi_{t-1})^\alpha_{t_\nu}).$$

Applying (9) we again compute confidence bands with alternative nominal levels to test equivalence of $\hat{\alpha}_{t_\nu}$ and α. As can be seen from Table 2 (third column) all estimates $\hat{\alpha}_{t_\nu}$ cannot be distinguished from α at the 5% significance level. Complementary to the improved accuracy of fit offered by the GARCH(1,1)-specification with leptokurtic innovations the latter result provides a strong support for the convenience of this model in empirical practice.

4 Stability of the GARCH-parameters

In the foregoing section the GARCH(1,1)-model was found to provide a powerful tool to capture volatility clustering of FX-rates. Since we analyze a rather long sample period one may question the adequacy of a time homogeneous model (Hamilton and Susmel 1994). Therefore we now turn to the issue of testing on time homogeneity and detecting a potential change point of the dynamic model. Having identified a point of structural change we illustrate the gain in modelling empirical returns by means of a time inhomogeneous model. Moreover, we provide a few stylized facts characterizing FX-volatility in both identified subsamples. Finally we relate our characterization of volatility dynamics to economic events and shifts in monetary policies.

4.1 Evidence of structural variation

LM-tests are convenient candidates to test the null hypothesis of structural invariance of GARCH-processes. In this framework the test is typically performed once an estimate of the invariant model, (2) say, is available. Such an approach is especially sensible if the specification of the model under the alternative is difficult to justify by a-priori reasoning. Parametric as well as nonparametric tests of structural stability of GARCH-processes are given in Chu (1995) and Lin and Yang (2000).

The supremum LM-test (Chu 1995) we apply here essentially compares the unconditional variance of ϵ_t before and after a set of break points located on a prespecified grid. Detailed test results are given in Table 4. We reject structural stability of volatility dynamics for both FX-rates at the 1% significance level. Applying a supremum test to the distance between the empirical distribution functions of standardized residuals $\hat{\xi}_t$ before and after some prespecified break point (Lin and Yang 2000) we find strong evidence in favor of a structural break of DEM/USD-volatility. With respect to the DEM/JPY-rate this framework delivers evidence in favor of a structural shift at the 5% significance level but not at the 2.5% level.

Although supremum LM-tests deliver evidence against the stable GARCH-model it is not immediately informative with respect to the time point of structural variation. In principle, performing the test on a dense grid could guide the analyst to detect the location of a (single) breakpoint since the supremum test should reach its maximum at the true break point. To identify the change point we follow a supremum likelihood ratio (LR) approach which is asymptotically equivalent to the LM procedure. The advantage of the LR approach is that the obtained log-likelihood estimates directly indicate the improvement of the empirical model going back to the implementation of a structural change.

We perform QML-estimation of the GARCH-model under normality and assume that one structural shift occurs during the sample period. The break point τ^* is found by maximizing the Gaussian log-likelihood over two subsamples, i.e.

$$\tau^* = \max_{\tau} \Big(\sum_{t=1}^{\tau} l_t(\hat{\theta}_1) + \sum_{t=\tau+1}^{T} l_t(\hat{\theta}_2) \Big),$$

where $\hat{\theta}_1$ and $\hat{\theta}_2$ denote the vectors of estimated GARCH-parameters operating until and after $t = \tau$. For reasons of computational convenience we initially assume the point of structural change to be found on a grid: $\tau^* = 100, 200, \ldots$. Then, as intermediate change point estimates we take the three grid points providing the highest log-likelihood estimates. Around these estimates we use a grid of only 10 observations to further improve the intermediate results. Finally we employ a daily scheme to detect a particular point of structural change. Along these lines we find $\tau^*_{\text{USD}} = 1432$ (September 8, 1980) and $\tau^*_{\text{JPY}} = 3717$ (November 7, 1989) to maximize the Gaussian log-likelihood under the assumption of two alternative volatility regimes governing the dynamics of the DEM/USD- and DEM/JPY-rate, respectively. The log-likelihood values are 21971.85 and 22913.97 which can be directly compared with the corresponding results in (5) and (6).

Using standard LR-tests to infer on time homogeneity we obtain $\lambda_{\text{USD}} = 2(21971.85 - 21925.23) = 93.24$ and $\lambda_{\text{JPY}} = 2(22913.97 - 22893.45) = 41.04$. If the detected break points were known a-priori it would be sensible to compare λ_{USD} and λ_{JPY} with critical values from a $\chi^2(3)$-distribution. In this case it is evident that the assumption of structural invariance has to be rejected. In the present case, however, we actually perform a family of dependent tests such that the $\chi^2(3)$-distribution cannot be applied. The values of the statistics are so large, however, that it is hard to think of a reasonable distribution for which both statistics were not significant.

Estimating the time point of structural change along the lines in Lin and Yang (2000) we find similar but not identical break points ($\tau^*_{\text{USD}} = 1287$ and $\tau^*_{\text{JPY}} = 3538$). With respect to the second subsample identified for the DEM/JPY-rate the GARCH(1,1)-model estimated under standardized t-distributed innovations fails to provide confidence bands with nominal and empirical coverage probability of 95%. For this reason we concentrate on LR-estimates of τ^* in the following.

Although the change point GARCH(1,1)-model under normality improves the accuracy of the time invariant counterpart considerably the model still delivers leptokurtic residuals $\hat{\xi}_t$ and, as can be inferred from the log-likelihood estimates, is inferior to the time homogeneous model with $t(0,1,\nu)$-innovations. For these reasons we consider now a change point model with underlying leptokurtic innovations.

Estimates of GARCH(1,1)-models assuming two different volatility regimes under conditional leptokurtosis are given in Table 3. We obtain considerable improvements of the log-likelihood function for the time inhomogeneous model in comparison to its time homogeneous counterpart. Complementary to parameter estimates $\hat{\alpha}_1$ and $\hat{\beta}_1$ we also provide the sum $\hat{\alpha}_1 + \hat{\beta}_1$ and the corresponding standard error. Both time invariant specifications are close to the IGARCH(1,1)-model. $\hat{\alpha}_1 + \hat{\beta}_1$ sum to 1.001 and 0.990 for the DEM/USD- and DEM/JPY-rate, respectively. Due to the rather small corresponding standard errors both estimates differ at the 1% significance level from unity. In analogy to former results in Lamoureux and Lastrapes (1990) and Lastrapes (1990) splitting the sample period weakens the evidence in favor of high persistence in volatility to some degree. For the DEM/USD-rate ϵ_t^2 is covariance stationary in the second subsample and with respect to the DEM/JPY-rate $\hat{\alpha}_1 + \hat{\beta}_1$ is significantly smaller in both subperiods compared to the time homogeneous estimator. For both investigated time series the estimated α_1 coefficient, governing the impact of lagged innovations on current volatility, is considerably larger in the first subsample relative to the second period. Note that the value of α_1 is of particular importance for excess kurtosis of ϵ_t (Bollerslev 1986). In the Gaussian GARCH(1,1)-model conditional heteroskedasticity and thus leptokurtosis of ϵ_t vanishes in the limit as $\alpha_1 \to 0$.

Together with the corresponding standard errors average values of $\hat{\sigma}_t$ as estimated from competing model specifications are also shown in Table 3. Since $\hat{\alpha}_1 + \hat{\beta}_1 \geq 1$ for some (sub)samples these estimates should be treated with care since then an unconditional variance does not exist. The descriptive results, however, allow an analogous interpretation for both FX-rates. On average, volatility is larger in the second subperiod compared to the first. Taking the empirical standard errors of the latter averages into account we conjecture that the unconditional levels of volatility differ significantly from each other. A similar conclusion can be drawn when considering the minimum levels of $\hat{\sigma}_t$ which are attained in longer periods of small price variations. As reported in Table 3 these lower volatility bounds are higher in the second subsample compared to the first period for both FX-rates. Moreover, a graphical inspection (lower panels of Figure 2 and Figure 3) supports the view that the bottom level of volatility is higher in the second subsamples compared to the beginning of the sample period.

As for the time invariant models we use the estimated change point processes to sequentially estimate confidence bands for ϵ_t. The obtained empirical significance levels are again shown in the right hand side of Table 2. Similar to the time homogeneous specification empirical and nominal significance levels cannot be distinguished by statistical tests. Estimating $\hat{\alpha}_{t_\nu}$ for the time homogeneous and the change point model within the identified subperiods, it turns out that, for instance, the empirical coverage of $\mathrm{CI}(\epsilon_t|\Psi_{t-1})^{.50}_{t_\nu}$ obtained for the time invariant model is the weighted average of empirical size estimates which are too small in the first subperiod and too large in the second. Subperiod specific estimates $\hat{\alpha}_{t_\nu}$ differ significantly from the nominal 50% level. Allowing for two volatility regimes driven by leptokurtic innovations we obtain for both FX-rates empirical size estimates which are insignificant within subperiods and over the entire sample period.

4.2 Volatility dynamics and economic events

The sample covers periods of different macroeconomic policy regimes and inflation rate phases. Following the first oil price shock in 1973 a world wide acceleration of inflation rates is observed (Krugman and Obstfeld 1996, Chapter 19). Major central banks adopted a monetary targeting policy to stabilize inflation rates. The less tight monetary policy in the US appears to weaken the USD against currencies of other major industrial countries from 1976 to 1979 (see Figure 1). In 1979 the second oil price shock induced a world wide recession. Monetary growth was restricted in most industrial countries to limit inflation rates. The USD appreciated against most currencies from 1982 to 1985. According to the Louvre accord in February 1987 the governments set up target zones for the DEM/USD- and JPY/USD-rate. A period of relatively stable exchange rates ended with the stock market crash in October 1987. Afterwards the DEM/USD-rate varied between 1.34 and 1.95.

The medium and lower panels of Figure 2 (Figure 3) display estimates of the conditional standard deviations of the DEM/USD- (DEM/JPY-) rate. Three important peaks of estimated DEM/USD-volatility occurring in January 1978, November 1978 and April 1980 are concentrated in the first subsample. Within the second subsample periods of high DEM/USD-volatility are September 1985 and May 1995. In September 1985 the Group of Five announced to intervene jointly on the foreign exchange market in order to depreciate the USD (Plaza announcement).

High DEM/USD-volatility in May 1995 coincides with the all time low of the DEM/USD-rate. The Bank for International Settlements (BIS) (1996) states that different business cycle states in the US and Germany and forward interest rate differentials may explain the exchange rate movement. The peak in estimated DEM/USD-volatility coincides with extremely high implied volatility determined from currency options with 1 month maturity (BIS 1996).

The DEM/JPY-volatility reaches its highest values in December 1979, April 1984, September 1984, September 1985 and October 1998. Between April 1979 and March 1980 the BoJ raised its discount rate from 3.5 to 9.0 percent. During this period FX-volatility culminated in December 1979. The coincidence of high volatility states in both series in September 1985 illustrates the interdependence of financial markets. The volatility peak in October 1998 corresponds to the crisis of the "Long-Term Capital Management" hedge fund. In the following the JPY appreciated abruptly against the USD due to better growth expectations in Japan compared to the US (BIS 1999).

Both dates of structural change (September 8, 1980 for the DEM/USD and November 7, 1989 for the DEM/JPY) do not correspond to outstanding events of financial market developments like the crash in October 1987. Nevertheless, they fall in periods of important changes of monetary policies. On October 6, 1979 the Board of the Fed announced to abandon the federal funds rate targeting. In its place a new operating procedure, so-called nonborrowed reserves targeting, was adopted to improve monetary control. Afterwards the inflation rate fell from its single-digit range and remained fairly stable between 3 and 6 percent. Being one effect of nonborrowed reserves targeting large variations of interest rates occurred from 1979 to 1981. Since money demand decreased in 1981 the Fed terminated this practice and switched to the borrowed reserve targeting in October 1982. Similar to Lastrapes (1989) we conjecture that the change of monetary control instruments caused the observed shift of volatility dynamics. In contrast to the latter work, however, we have not fixed possible time points of structural variation a-priori but chose a data driven procedure to determine it. Moreover, in testing time homogeneity of the entire model specification, we do not restrict the attention to variation in just one parameter (α_0) as, for instance, Lastrapes (1989) and Lamoureux and Lastrapes (1990).

Interestingly also the change point of DEM/JPY-volatility almost coincides with a change to a more tighten monetary policy conducted by the BoJ (Jinushi et al. 2000). Inflation in Japan picked up in 1989 reflecting the expansive monetary policy of the BoJ during the period 1986-1989. In May 1989 the BoJ began to raise the discount rate from 2.5 to 6 percent in August 1990. Moreover the time point of structural change is close to the highest values of the Nikkei stock index in the end of August 1989. In the second half of the 1980s, Japanese stock and real estate prices doubled and tripled within a few years (Ito and Iwaisako 1995). Apart from fighting inflationary pressures the BoJ tried to puncture the asset price bubble. In the first half of the 1990s, most asset prices plummet. The Nikkei stock price index lost more than half its value between 1990 and 1992. The sharp fall in asset prices threw Japan's banking system into a crisis and deteriorated real economic growth rates.

5 Time variation of implied volatilities

The foregoing section illustrated how the GARCH-model may be employed to fully describe price variations of FX-rates. As an economic application of the particular GARCH-estimates one may regard pricing of derivatives with FX-rates being the underlying assets. Hafner and Herwartz (2001) extend the GARCH option pricing model (Duan 1995 and 1999), to account for typical properties of empirical return processes, as, for instance, positive autocorrelation of ϵ_t or leptokurtosis of ξ_t. Under stochastic volatility option pricing is no longer preference free as it is in the homoskedastic Black and Scholes (1973) model. The GARCH option pricing model (Duan 1995) generalizes the traditional risk neutral valuation methodology to the case of conditional heteroskedasticity. Applying some pricing measure Q for local risk neutralization the representative agent maximizes expected utility, for instance, if his utility function is either linear or nonlinear with constant relative risk aversion. Moreover, in the latter case, relative changes in aggregate consumption have to be normally distributed. Then, the current price of a European call option (C_t) with exercise price K and time to maturity $\tau = T - t$ is:

$$C_t = (1+r)^{-\tau} E^Q[\max(S_T - K, 0) \mid \Psi_t]. \tag{10}$$

In (10) S_T denotes the price of the underlying asset in time T and r is the risk free interest rate. For a slightly different specification of the GARCH(1,1)-model under normality as used here (see (2)) Heston and Nandi (2000) provide a closed form solution for the expectation in (10). Since the Gaussian GARCH-model is found in Section 3 to provide only a poor approximation to the investigated FX-rates we prefer to evaluate $E^Q[\max(S_T - K, 0) \mid \Psi_t]$ by simulation along the lines in Duan (1999) and Hafner and Herwartz (2001) which also incorporate leptokurtic innovations.

The pricing measure Q can be derived from the empirical measure P which is the data generating process specified and estimated for the underlying asset. Under specific assumptions, concerning the conditional expectations of S_t and the distribution of ξ_t the measures Q and P are identical. Since the normal and the $t(0,1,\nu)$-distribution are unimodal and symmetric equivalence of Q and P follows if both the conditional mean of empirical returns ($E[\Delta e_t|\Psi_{t-1}]$) and the risk free rate are zero ($r = 0$). With respect to the first assumption note that DEM/USD-returns were found in Section 2 to be uncorrelated, thus the conditional expectation for this series is zero. Weak correlation has been detected for the DEM/JPY-rate as it is documented in (1). Since we are mainly interested in uncovering the impact of volatility misspecification we neglect this weak correlation pattern to facilitate the numerical evaluation of (10). Assuming a zero risk free rate is convenient when analyzing high frequency (daily) data.

We assume coincidence of Q and P, i.e. we determine option prices by simulating FX-rates according to the GARCH-parameter estimates given in Table 3. We use $R = 200000$ replications to evaluate

$$C_t = (1+r)^{-\tau} E^Q[\max(S_T - K, 0) \mid \Psi_t]. \tag{11}$$

Throughout we set $S_0 = 1$ and initialize the volatility paths with a medium volatility level. To be precise we use $\sigma_1 = 6.77\text{E-}03$ ($\sigma_1 = 5.73\text{E-}03$) for simulating the DEM/USD-rate (DEM/JPY-rate). Option prices are determined for given times to maturity $\tau = 20, 40, 60, 120$ and varying degrees of moneyness $0.85 \leq S_0/K \leq 1.15$, which is the typical range of traded options.

To illustrate the results from a viewpoint often taken by practitioners, we provide implied volatilities. Here the implied volatility is the particular volatility parameter that, when plugged into the Black and Scholes (1973) formula, yields the option price generated by the GARCH-model. Derived under the assumption of homoskedasticity the Black and Scholes (1973) option price is:

$$C_{BS}(S_t, t, \sigma^2) = S_t \Phi(d_1) - K e^{-r\tau} \Phi(d_2), \tag{12}$$

where

$$d_1(\sigma^2) = \frac{\ln(S_t/K) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, \; d_2 = d_1 - \sigma\sqrt{\tau},$$

and $\Phi(.)$ denotes the standard normal distribution function.

For $\tau = 20$ the left hand side panels of Figure 4 provide volatilities $(100 \cdot \sigma)$ implied by the option prices generated from the GARCH(1,1)-models with $t-$distributed innovations as listed in Table 3. For both FX-rates the implied volatilities show a U-shaped pattern which is the prominent volatility smile measured towards the moneyness of a particular call option. Options out of the money and far in the money have higher implied volatilities than options at the money. Since we are interested in the effects of falsely applying a time homogeneous model the right hand side panels of Figure 4 show implied volatilities obtained from the change point model relative to the simulation results generated under time homogeneity. In addition to the case $\tau = 20$, Figure 5 provides relative volatility measures for both FX-rates and maturities $\tau = 20, 40, 60, 120$. To facilitate the comparison of results across maturities and FX-rates all graphs are shown on identical scales.

For both rates and all maturities we obtain that the implied volatility of at the money options is higher during the second compared to the first subsamples. Note that these options are more heavily traded than options which are out of the money or far in the money. Considering short maturities, $\tau = 20$ say, we have the interesting result that options being out of the money show a higher implied volatility during the first subsample. This result can be related to the estimates for the GARCH-parameter α_1 which are larger for both rates in the first subperiod. Governing the impact of lagged squared returns on current volatility the larger α_1 the higher is the probability for an out of the money option to end in the money. As mentioned, the latter result is confined to the short maturity case. With respect to higher maturities the overall level of volatility becomes more and more important. As the latter is higher during the second subsamples we obtain for $\tau = 120$ and both FX-rates that implied volatility is higher for the entire range of moneyness during the second subsamples.

6 Conclusions

Analyzing a sample period from 1975 to 1998 we find that the empirical properties of the DEM/USD- and DEM/JPY-rate are quite similar. Firstly, the underlying volatility processes exhibit serial correlation. Dependence of higher order moments is accurately captured by a GARCH(1,1)-model with leptokurtic innovations. Secondly, GARCH-parameters, are not time invariant. Evidence of high persistence in volatility obtained from the time homogeneous model is mitigated to some extent when introducing structural variation. DEM/USD (DEM/JPY) volatility dynamics changed in September 1980 (in November 1989). Both time points fall into periods of changing of monetary policies adopted by the Fed or the BoJ, respectively. When comparing stylized facts of volatility across subperiods we find fourthly that the bottom and average level of (estimated) volatility is larger during the second subsamples compared to the beginning of the sample period. Finally, applying the GARCH option pricing model higher implied volatility during the second subsample is particularly diagnosed for at the money options. Determining option prices alternatively under structural invariance and assuming two homogeneous subperiods we find that falsely applying the time invariant model involves the largest deviations from time varying implied volatilities for at the money options with high maturity.

The higher volatility may give one explanation for the growing importance of derivatives trading on FX-markets since the mid of the eighties stated e.g. by the Bank for International Settlement (BIS 1999). A particular issue, relevant to future research, is to clarify if the increase of volatility affected the real economy. This topic is beyond the scope of this paper and deserves a multivariate framework.

References

BIS 65th Annual Report 1996, Bank for International Settlements, Basle.

BIS 69th Annual Report 1999, Bank for International Settlements, Basle.

Black F, Scholes M. The pricing of options and corporate liabilities, *Journal of Political Economy* 1973; **81**: 637–659.

Bollerslev T. Generalized autoregressive conditional heteroscedasticity, *Journal of Econometrics* 1986; **31**: 307-327.

Bollerslev T. A conditional heteroscedastic time series model for speculative prices and rates of return, *Review of Economics and Statistics* 1987; **69**: 542-547.

Bollerslev T, Engle RF, Nelson DB. ARCH Models. In: *Handbook of Econometrics, Vol. 4, Chapter 49*, Engle RF, McFadden DL (eds). Elsevier: Amsterdam, 1994, 2961-3038.

Bollerslev T, Wooldridge JM. Quasi-Maximum Likelihood estimation and inference in dynamic models with time–varying covariances. *Econometric Reviews* 1992; **11**: 143-172.

Chang EC, Cheng JW, Pinegar JM. Does futures trading increase stock market volatility? The case of the Nikkei stock index futures markets. *Journal of Banking and Finance* 1999; **23**: 727-753.

Chu CSJ. Detecting parameter shifts in GARCH models. *Econometric Reviews* 1995; **14**: 241-266.

Dickey DA, Fuller WA. Distributions of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association* 1979; **74**: 427-431.

Diebold FX. Modeling persistance in conditional variances: A Comment. *Econometric Reviews* 1986; **5**: 51–56.

Duan JC. The GARCH option pricing model. *Mathematical Finance* 1995; **5**: 13–32.

Duan JC. Conditionally fat-tailed distributions and the volatility smile in options. *Working paper, Department of Finance, Hong Kong University of Science and Technology* 1999.

Engle RF. Autoregressive conditional heteroscedasticity with estimates of the variance of UK inflation. *Econometrica* 1982; **50**: 987-1008.

Engle RF, Bollerslev T. Modelling the persistence of conditional variances. *Econometric Reviews* 1986; **5**: 81–87.

Hafner CM, Herwartz H. Option pricing under linear autoregressive dynamics, heteroskedasticity, and conditional leptokurtosis. *Journal of Empirical Finance*, 2001; **8**: 1-34.

Hamilton J, Susmel R. Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics*, 1994; **64**: 307-333.

Harris L. , S&P 500 cash stock price volatilities, *Journal of Finance* 1989; **44**: 1155-1175.

Heston SL, Nandi S. A closed form GARCH option valuation model, *The Review of Financial Studies* 2000; **13**: 585-625.

Ito T, Iwaisako T. Explaining asset bubbles in Japan. *NBER Working Paper 5358*, 1995.

Jarque CM, Bera AK, A Test for normality of observations and regression residuals. *International Statistical Review* 1987; **55**:163–172.

Jinushi T, Kuroki Y, Miyao R. Monetary Policy in Japan Since the Late 1980s: Delayed Policy Actions and Some Explanations. In: Japan's Financial Crisis and its Parallels to US Experience, Special report 13, Posen A, Mikitani R. (eds.). Institute of International Economics, Washington, 2000, 115-148.

Johnson NL, Kotz S. *Distributions in Statistics - Continuous univariate distributions.* Wiley, New York, 1972.

Jorion P. *Value at Risk: The New Benchmark for Controlling Derivatives Risk.* 2nd edition, McGraw-Hill, New York, 2000.

Krugman P, Obstfeld M. *International Economics, Theory and Policy* (4th edn) Addison-Wesley, Reading, 1996.

Lamoureux CG, Lastrapes WD. Persistence in variance, structural change, and the GARCH-model. *Journal of Business and Economics Statistics* 1990; **8**: 225-234.

Lastrapes WD. Exchange rate volatility and U.S. monetary policy: An ARCH application. *Journal of Money, Credit and Banking* 1989; **21**: 66-77.

Lee SW, Hansen BE. Asymptotic theory for the GARCH(1,1) Quasi-Maximum Likelihood estimator. *Econometric Theory* 1994; **10**, 29-52.

Lin SJ, Yang J. Testing shifts in financial models with conditional heteroskedasticity: An empirical distribution function approach, *Discussion paper presented at the 7th World Congress of the Econometric Society, Seattle* 2000.

Ljung GM, Box GEP. On a measure of lack of fit in time series models, *Biometrika* 1978; **65**: 297–303.

Lumsdaine RL. Consistency and asymptotic normality of the quasi maximum likelihood estimator in IGARCH(1,1) and covariance stationary GARCH(1,1) models. *Econometrica* 1996; **64**: 575–596.

Mikosch T, Starica C. Change of structure in financial time series, long range dependence and the GARCH model, *Working Paper 58, University of Aarhus*, Aarhus Scool of Business, 2000.

Nelson DB. Stationarity and persistence in the GARCH(1,1)-model. *Econometric Theory* 1990; **6**: 318–334.

Taylor SJ. *Modelling Financial Time Series*, John Wiley, Chichester, 1986.

White H. A heteroskedasticity consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica* 1980; **48**: 817–838.

Table 1: Tests on normality, homoskedasticity and serial correlation.

	DEM/USD		DEM/JPY		
	$\Delta e_t = \epsilon_t$	$\hat{\xi}_t$	Δe_t	$\hat{\epsilon}_t$	$\hat{\xi}_t$
	Unconditional distribution				
Skewness	-0.17	0.16	0.82	0.74	0.22
Kurtosis	7.07	4.98	14.6	13.8	4.57
JB	4179.7***	1008.3***	34124.9***	29954.2***	668.4***
ARCH-LM(1)	51.7***	1.83	866.5***	952.0***	5.63**
ARCH-LM(5)	203.8***	5.36	885.4***	967.6***	8.76
	Serial correlation				
LB(8)	14.96*	3.91	21.79***	4.10	12.87
LB(16)	22.56	16.52	38.09***	11.64	19.47
LB(24)	28.44	24.06	43.76***	16.40	24.03

The sample period is January 2, 1975 to December 30, 1998. FX-returns (Δe_t), linear residuals ($\Delta\hat{\epsilon}_t$) and GARCH(1,1)-innovations (ξ_t) are distinguished. JB and ARCH-LM(q) are the Jarque-Bera test on normality, and the LM-test of homoskedasticity against conditional heteroskedaticity, respectively. The Ljung-Box statistic (LB(q)) tests against joint serial correlation up to order q. *, **,*** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 2: Empirical significance levels of sequential confidence bands $CI(\epsilon_t|\Psi_{t-1})_N^\alpha$ and $CI(\epsilon_t|\Psi_{t-1})_{t_\nu}^\alpha$

		Time invariant model				Change point model		
		T	T	1st	2nd	T	1st	2nd
	α	$\hat{\alpha}_N$	$\hat{\alpha}_{t_\nu}$			$\hat{\alpha}_{t_\nu}$		
	obs.	6015	6015	1431	4584	6015	1431	4584
	0.05	.056*	0.049	0.045	0.050	0.048	0.052	0.047
DEM/	0.10	.096	0.102	0.087*	0.107	0.100	0.102	0.099
USD	0.50	.456***	0.507	0.457***	0.523***	0.507	0.506	0.507
	0.90	.886***	0.901	0.890	0.905	0.901	0.901	0.901
	0.95	.939***	0.946	0.941	0.947	0.947	0.950	0.946
	obs.	6005	6005	3715	2291	6005	3715	2291
	0.05	.053	0.047	0.044*	0.053	0.047	0.049	0.043
DEM/	0.10	.096	0.101	0.092*	0.115**	0.098	0.099	0.096
JPY	0.50	.454***	0.499	0.481**	0.530***	0.497	0.493	0.504
	0.90	.890**	0.903	0.897	0.912**	0.903	0.900	0.907
	0.95	.945*	0.951	0.949	0.953	0.951	0.951	0.951

obs. denotes the number of available observation for the entire sample (T) and two subsamples (1st and 2nd). *, **,*** indicate significance at the 10%, 5%, and 1% level, respectively.

Table 3: GARCH(1,1)-estimates under conditional leptokurtosis

	DEM/USD			DEM/JPY		
	time inv.	1st sub	2nd sub	time inv.	1st sub	2nd sub
log–lik.	22067.04	22110.46		22998.25	23014.98	
$\hat{\nu}$	6.26 (13.7)	5.23 (7.33)	7.43 (10.2)	7.18 (12.2)	7.33 (8.54)	8.49 (6.80)
$\hat{\alpha}_0$	4.0e-07 (2.85)	5.9E-07 (2.43)	1.4E-06 (3.51)	5.8E-07 (3.68)	8.8E-07 (3.23)	1.1E-06 (2.33)
$\hat{\alpha}_1$	0.107 (10.1)	0.215 (6.24)	0.077 (7.39)	0.114 (9.10)	0.134 (6.91)	0.086 (5.61)
$\hat{\beta}_1$	0.894 (87.9)	0.790 (27.7)	0.899 (61.1)	0.876 (62.3)	0.837 (33.7)	0.894 (42.4)
$\hat{\alpha}_1 + \hat{\beta}_1$	1.001 (1.6E-04)	1.006 (3.02E-05)	0.977 (7.91E-04)	0.990 (4.99E-04)	0.971 (1.48E-03)	0.980 (1.15E-03)
	Descriptive statistics for $\hat{\sigma}_t$					
mean	6.77E-03 (3.27E-05)	4.91E-03 (7.13E-05)	7.22E-03 (2.71E-05)	5.73E-03 (3.06E-05)	4.85E-03 (2.80E-05)	7.06E-03 (5.03E-05)
min	2.18E-03	1.84E-03	3.19E-03	2.53E-03	2.55E-03	3.69E-03

Estimated t-values (standard errors) in parentheses underneath parameter estimates $\hat{\nu}, \hat{\alpha}_0, \hat{\alpha}_1, \hat{\beta}_1$ $(\hat{\alpha}_1 + \hat{\beta}_1)$. For estimates of average volatility $(\hat{\sigma}_t)$ the empirical standard errors are given in parentheses.

Table 4: Tests on structural invariance of the GARCH(1,1)

	DEM/USD				DEM/JPY			
$k \setminus g$	1	5	11	21	1	5	11	21
0.1	48.0	36.0	26.7	20.2	10.2	7.29	5.93	4.83
0.2	44.2	33.2	24.6	18.6	15.3	10.9	8.88	7.24
0,3	15.7	11.8	8.74	6.59	5.72	4.09	3.33	2.71
0.4	21.5	16.1	12.0	9.05	16.2	11.5	9.41	7.67
0.5	0.58	0.44	0.32	0.24	18.2	13.0	10.6	8.64
0.6	1.19	0.89	0.66	0.50	44.9	32.1	26.1	21.3
0.7	0.23	0.17	0.13	0.10	44.4	31.7	25.8	21.0
0.8	7.40	5.55	4.13	3.11	23.7	16.9	13.8	11.2
0.9	10.6	7.96	5.91	4.46	61.9	44.3	36.0	29.3
max	48.0***	36.0***	26.7***	20.2***	61.9***	44.3***	36.0***	29.3***

The LM-test is performed under the null hypothesis that the unconditional variance is identical before and after k*100% of the sample. Centered and squared error terms (ϵ_t^2) enter the test statistic. Its variances are estimated by means of g Bartlett-weights. Critical values are 12.96, 14.82, and 18.84 for the 10%, 5%, and 1% significance level, respectively (Chu 1995). *** significant at the 1% level.

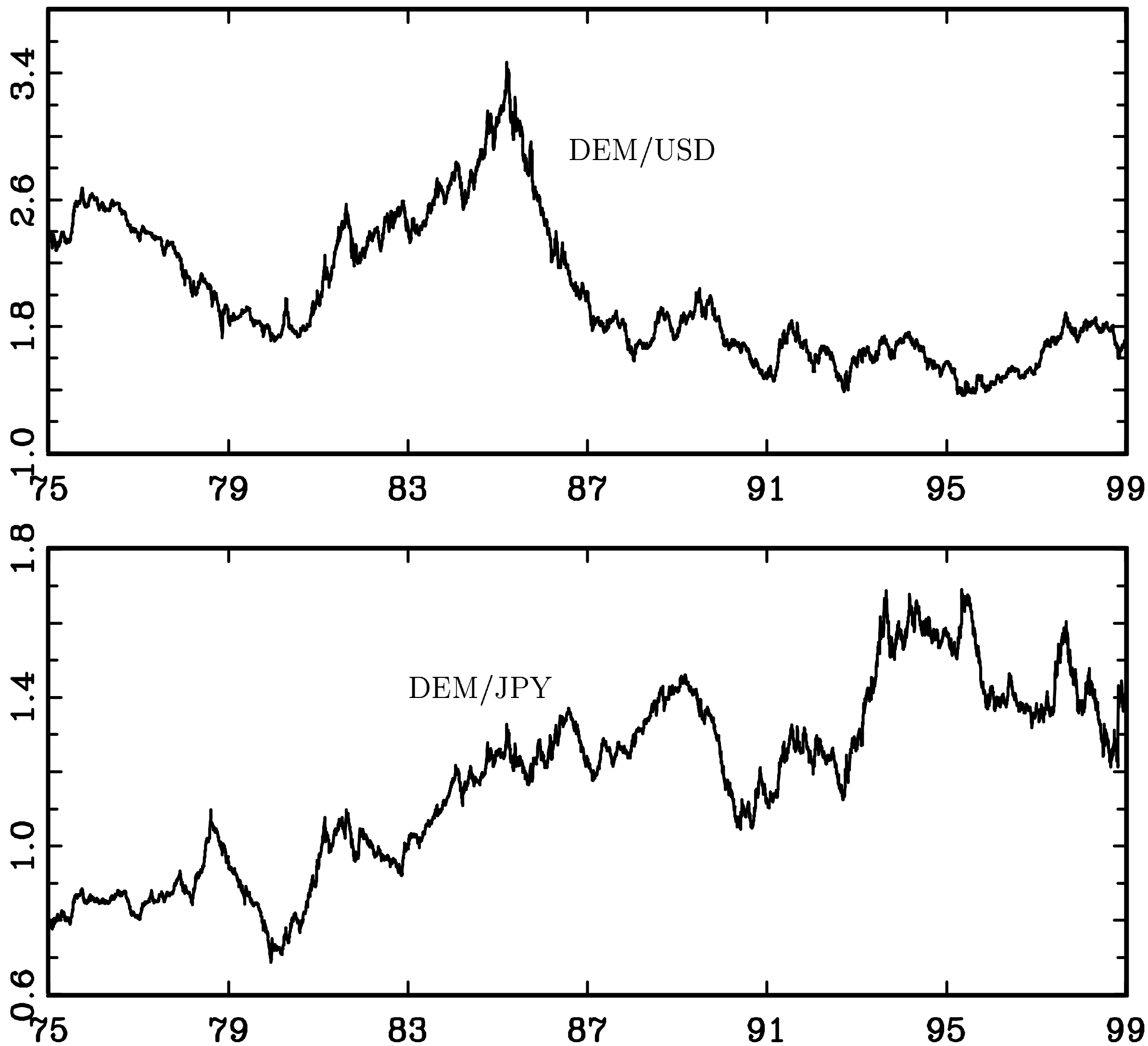

Figure 1: Log FX-rates

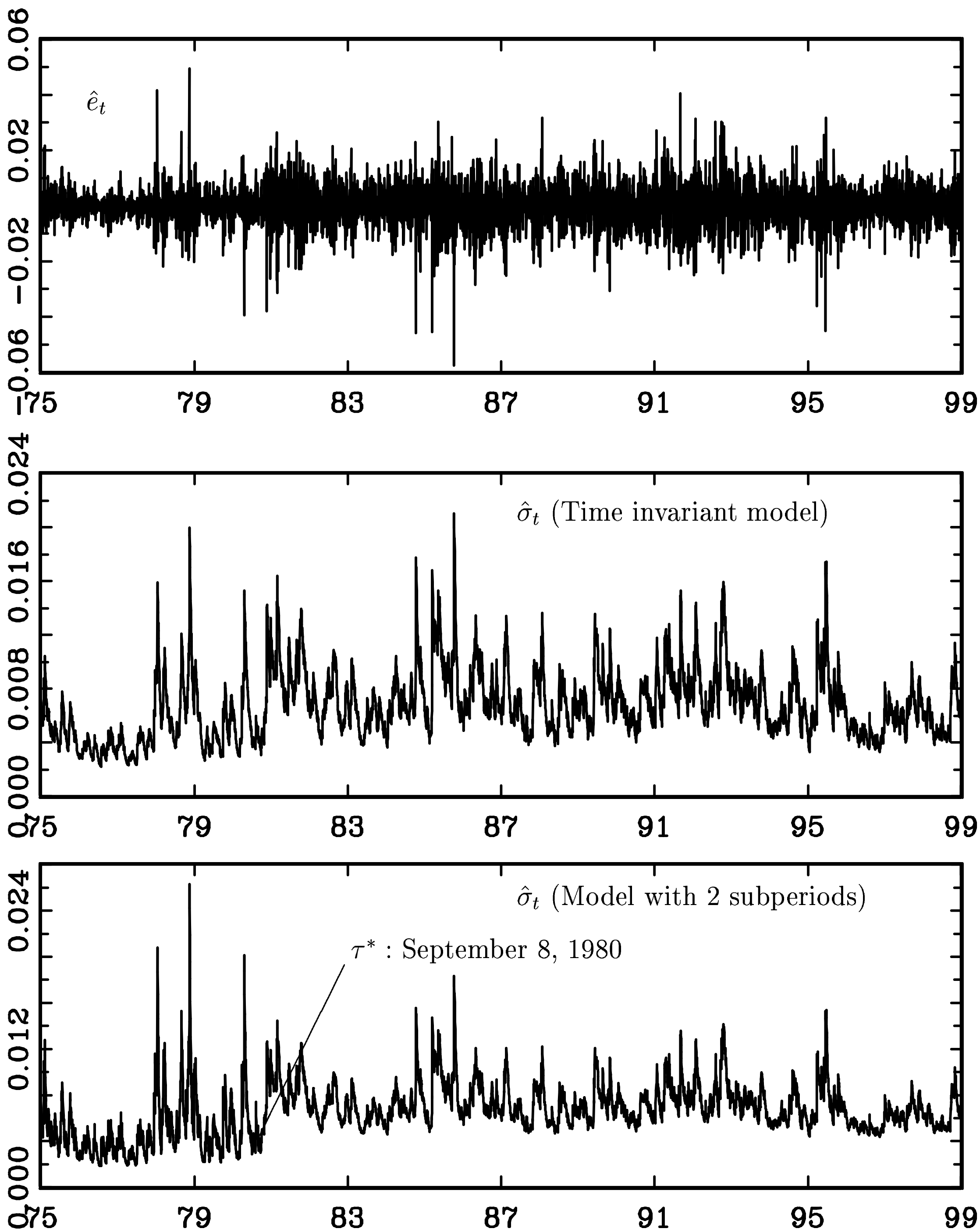

Figure 2: Changes of log DEM/USD-rates (upper panel) and estimated standard errors from GARCH(1,1)-models under conditional leptokurtosis.

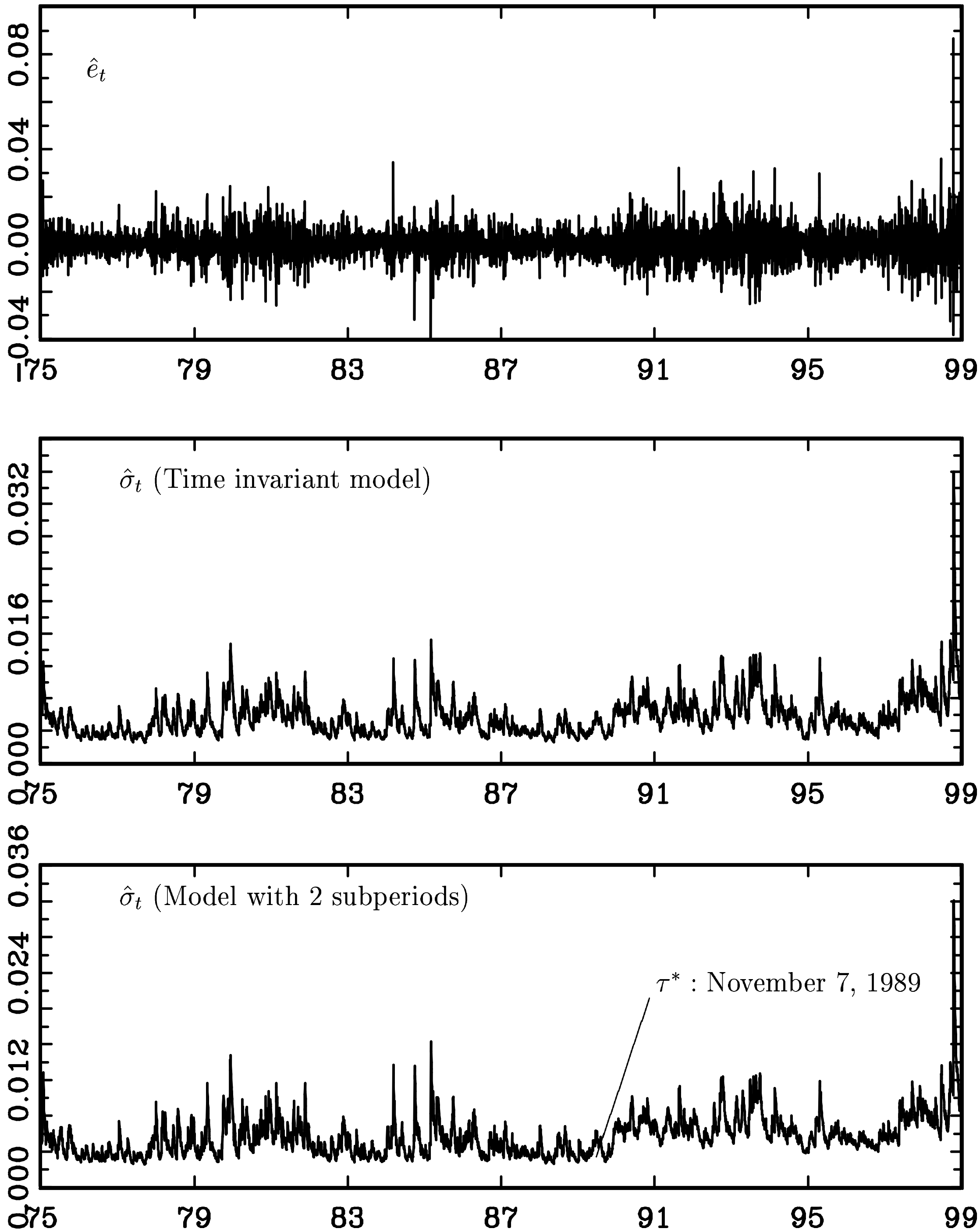

Figure 3: Residuals of a linear autoregressive subset model for changes of log DEM/JPY-rates (upper panel) and estimated standard errors from GARCH(1,1)-models under conditional leptokurtosis.

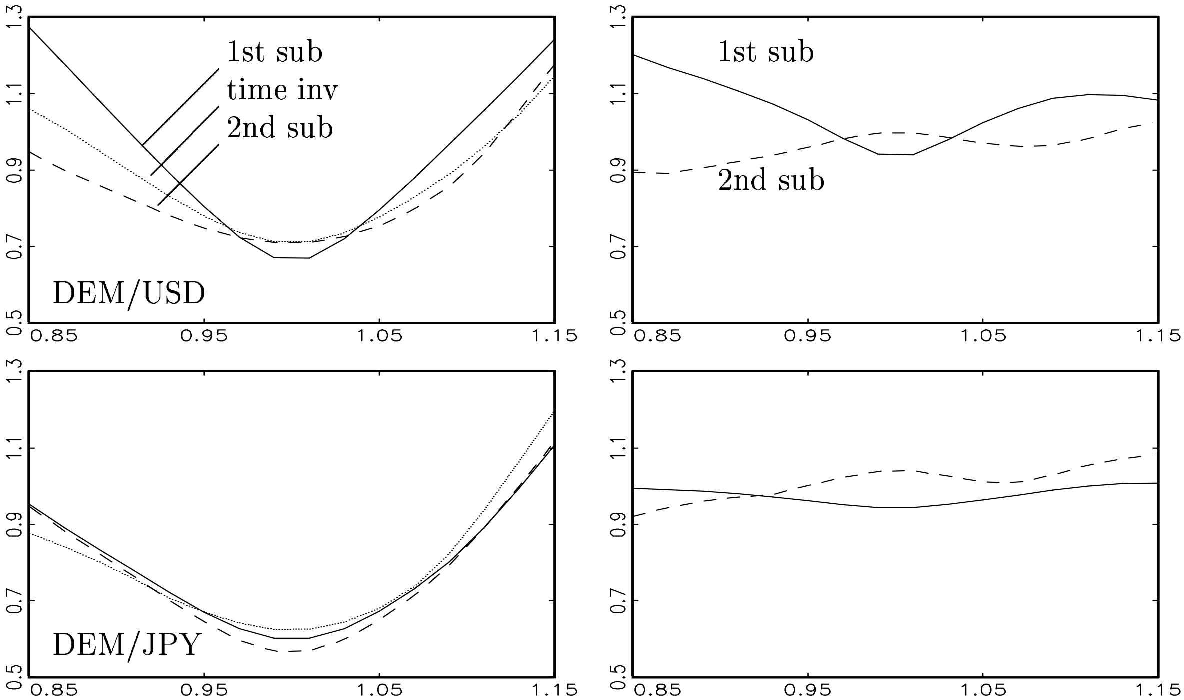

Figure 4: Implied volatilities (left hand side patterns) estimates for GARCH option prices with FX-rates as underlying assets. Time invariant and time varying GARCH-specifications are distinguished. Right hand side: Measures for two subsamples relative to the time invariant model. The x-axis is the moneyness S_0/K of the option, $S_0 = 1$.

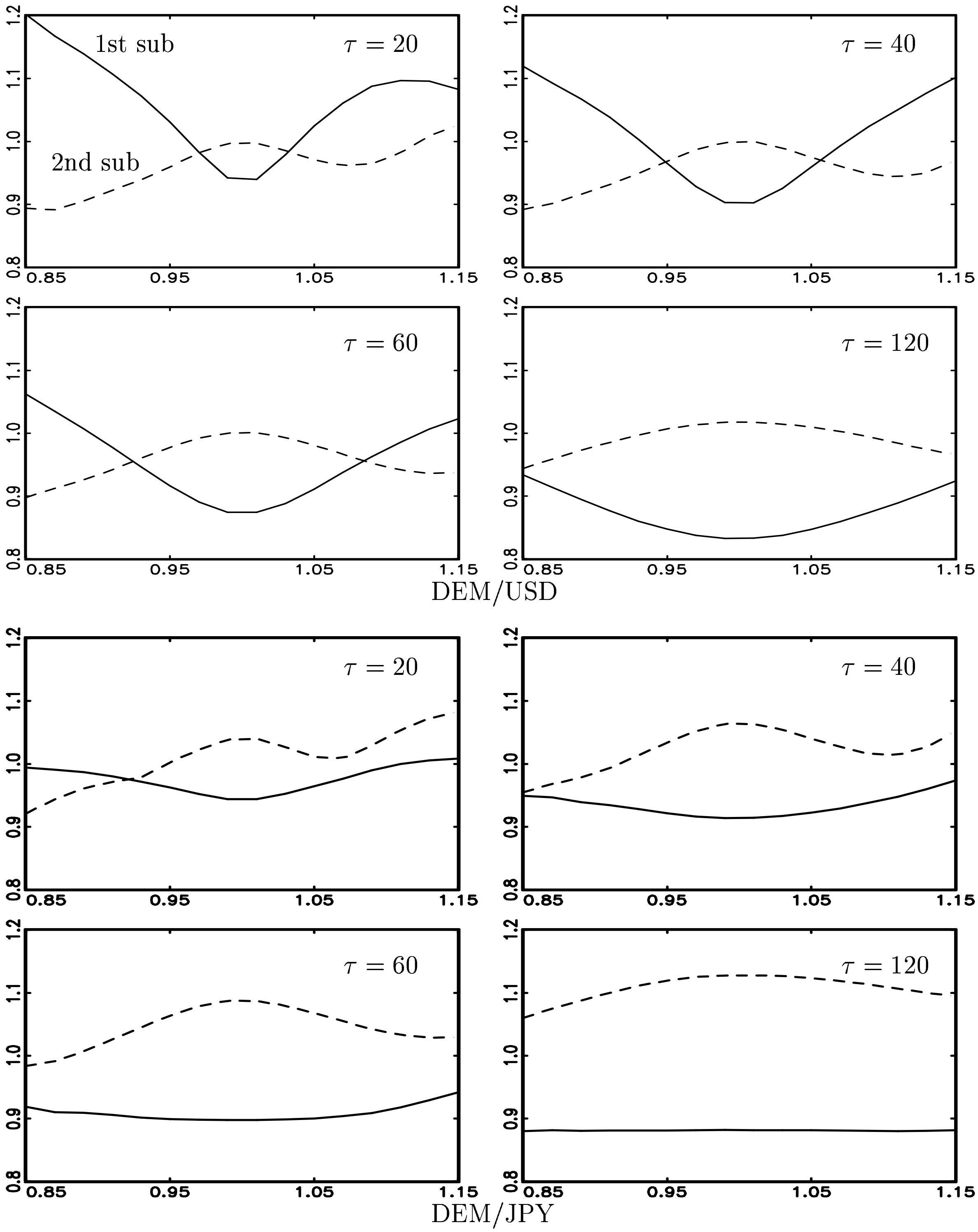

Figure 5: Implied volatilities for the time varying model relative to the time invariant model. The x-axis is the moneyness S_0/K of the option, $S_0 = 1$. Alternative maturities τ are distinguished.