Testing for short and long-run causality:  
The case of the yield spread and economic growth

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Abstract
To assess the predictive content of the interest rate term spread for future economic growth, we distinguish short-run from long-run predictability by using two different approaches. First, following Dufour and Renault (1998) a test procedure is proposed to test for causality at different forecast horizons. Second, the framework of Geweke (1982) and Hosaya (1991) is used to construct a simple test for causality in the frequency domain. This methodology is applied to investigate the predictive content of the yield spread for future output growth. For U.S. data we observe good leading indicator properties at frequencies around one year and typical business cycle frequencies. Using German data we found a (rather weak) predictability at low frequencies only.

Keywords:  Causality, Time series, Frequency domain, Prediction

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1 Introduction

There is already a rich literature that demonstrates the remarkable predictive power of interest rate spreads for real economic growth. Examples include Bernanke (1990), Estrella and Hardouvelis (1991), Estrella and Mishkin (1995), Haubrich and Dombrosky (1996), Davis and Fagan (1997), Boulier and Stekler (2000), to name but a few. Stock and Watson (1989, 1993) and Funke (1997) found that interest rate spreads are among the most promising leading variables from the perspective of business cycle forecasting for the U.S. and Germany.

The yield spread is determined by the financial market’s expectation of future interest rates and a term premium (e.g. Litterman, Scheinkman and Weiss, 1991; Hamilton and Kim, 2000). If, for instance, the central bank adopts a contractionary monetary policy, then market participants expect a temporary rise of short-term interest rates and, therefore, the yield spread tends to decrease. On the other hand, the monetary contraction affects real activity through various channels causing economic growth to slow. Converse conclusions can be drawn from an expansionary scenario.

To achieve attractive indicator properties it is important that the rise in interest rates immediately affects the term structure, whereas the contradictory effects of the monetary policy affects real activity with some delay. The empirical literature shows that there indeed exists a time lag of 2 – 4 quarters so that the yield spread is a reliable leading indicator of economic activity up to one year ahead.

Economic theory and empirical evidence suggest that the relationship between the yield spread and economic growth is dominated by short-term effects. Since the spread mainly reflects the stance of monetary policy, it cannot be expected to be a reliable indicator for the output trend. Therefore, it is more appealing to focus on the short-run rather than long-run predictability. To this end we adopt recent approaches that allow us to decompose measures of predictability for short and long run forecast horizons. Dufour and Renault (1998) suggest a causality concept for different forecast horizons, which permits the distinction between short-run and long-run causality. Geweke (1982) and Hosaya (1991) propose a frequency domain decomposition of the causality measure. In this paper we follow their approach and suggest a simple test procedure to test the
predictive power at particular frequencies.

Applying these different techniques to disentangle short-run and long-run predictability we found that in the U.S. the yield spread is a powerful predictor for short-run fluctuations of economic growth. For German data the frequency domain analysis does not reveal short-run predictive content of the spread. On the other hand we find predictive power of the term spread at typical business cycle frequencies in both countries. No predictive power is observed for cyclical fluctuations between one and two years.

The plan of the paper is as follows. Section 2 briefly reviews the previous evidence on the predictive content of the yield spread. The methodology of Dufour and Renault (1998) is presented in section 3 and the frequency domain approach of causality is introduced in section 4. Section 5 presents the results of our empirical study using U.S. and German data. Section 6 offers some conclusions.

2 Previous evidence

The yield spread is defined as the difference between yields on long-term and short-term bonds. It is determined by expectations of the market participants and the term premium. Assume that the monetary authorities decide to stimulate activity. An expansionary monetary policy implies a decrease in interest rates that serve as instruments of monetary policy (e.g. the Federal Funds rate in the U.S.). Consequently, market participants expect a temporary fall in short-term interest rates and, therefore, the long-run interest rate tends to decrease. However, according to the expectation hypothesis of the term structure of interest rates, the long-term rate will decrease less than the short-term rate. At the same time, the monetary expansion increases spending in interest sensitive sectors leading to a stimulation of economic growth. An opposite scenario applies to a contractionary monetary policy.

Another possible channel for the link between the yield spread and economic activity is considered by Harvey (1988) and Hu (1993), among others. This strand of literature employs a CAPM model to attribute the correlation between the term spread and future economic growth to inter-temporal consumption smoothing. If market par-
participants anticipate a recession and future lower rates of return to investment, then
they will increase current savings in order to boost future income. It follows that, ac-
cording to the expectation hypothesis, the yields on long-term bonds will fall relative
to short-term yields, indicating a slowdown in economic activity.

Finally, Hamilton and Kim (2000) emphasize that also the term premium may be
an indicator of economic activity. For example, if interest rates become more volatile
at the end of an expansion, this may affect the term premium of interest rates. As a
consequence long rates might fall relative to short rates because the cyclical volatility
warrants a change in the risk premium.

A large empirical literature has documented the excellent leading indicator prop-
ties of the yield spread for future economic activity. The dominant approach to assess
the predictive power is based on a simple regression of the form

$$(400/k) \log\left(\frac{y_{t+k}}{y_t}\right) = \alpha_k + \beta_k s_t + \gamma_k x_t + \nu_t,$$

where $y_t$ is real GDP, $s_t$ is the yield spread and $x_t$ represents additional variable(s).
Estrella and Hardouvelis (1991), Plosser and Rouwenhorst (1994), Bonser-Neal and
a predictive content of the spread for future GDP growth at lags from 2 up to 16
quarters, with a best predictive power around 3 quarters.

An alternative methodology is to apply Granger causality tests in a vector autore-
gression (VAR) with the term spread and economic growth. Bernanke (1990) and
Friedman and Kuttner (1993), inter alia, show that lagged values of the yield spread
are highly significant up to a lag length of 3 quarters.

Another approach employs a probit model for the prediction of whether or not
the economy will be in a recession $k$ quarters ahead referring to the NBER business
cycle datation (e.g. Estrella and Mishkin 1995, Duerer 1997, Dotsey 1998, Boulier and
Stekler 2000). The findings suggest that among a set of alternative predictors, the
yield spread remains the single best recession predictor up to a forecast horizon of one
year.
3 Testing for causality at different lag horizons

As has been argued by Lütkepohl (1993), Dufour and Renault (1998), and Burda (2001) it is in general not sufficient to consider a one-step-ahead forecast model when testing for causality. Specifically, if a variable has an indirect effect via a third variable with a time lag of more than one period, the usual causality tests may fail to indicate the causal relationship. To overcome this problem, Lütkepohl (1993) has introduced a causality concept that is based on the sequence of impulse response functions and Dufour and Renault (1998) extend the concept of Granger causality to higher forecast horizons. In this section we outline an empirical test procedure to test for causality at different lag horizons and discuss the relationship to the causality framework suggested by Lütkepohl (1993).

Let $z_t = [x_t, y_t]'$ be a two-dimensional vector of time series observed at $t = 1, \ldots, T$. It is assumed that $z_t$ has a finite order vector autoregressive (VAR) representation of the form:

$$\Theta(L)z_t = \varepsilon_t , \quad (1)$$

where $\Theta(L) = I - \Theta_1 L - \cdots - \Theta_p L^p$ is a $2 \times 2$ lag polynomial with $L^k y_t = y_{t-k}$. We assume that the error vector $\varepsilon_t$ is white noise with $E(\varepsilon_t) = 0$ and $E(\varepsilon_t \varepsilon_t') = \Sigma$, where $\Sigma$ is positive definite. For the ease of exposition we neglect any deterministic terms in (1) although in empirical practice the model typically includes a constant, trend or dummy variables.

The usual definition of causality due to Granger (1969) is based on the forecast variance. Let $\sigma^2(x_{t+1}|\mathcal{X}_t)$ denote the forecast variance of $x_{t+1}$ conditional on $\mathcal{X}_t = \{x_t, x_{t-1}, \ldots\}$. Similarly, we define $\sigma^2(x_{t+1}|\mathcal{X}_t \cup \mathcal{Y}_t)$, where $\mathcal{Y}_t = \{y_t, y_{t-1}, \ldots\}$. The variable $y_t$ is (Granger) causal for $x_t$ if

$$\sigma^2(x_{t+1}|\mathcal{X}_t) > \sigma^2(x_{t+1}|\mathcal{X}_t \cup \mathcal{Y}_t) .$$

As noted, for example, by Lütkepohl and Burda (1997), this definition is restrictive as it rules out causes that operate with a time lag larger than one time period. Dufour and Renault (1998) suggest to define \textit{causality at horizons} $h$ as

$$\sigma^2(x_{t+h}|\mathcal{X}_t) > \sigma^2(x_{t+h}|\mathcal{X}_t \cup \mathcal{Y}_t) \quad h = 1, 2, \ldots . \quad (2)$$
This approach is closely related to the impulse response analysis advocated by Sims (1980). If all roots of the characteristic equation \(|\Theta(L)| = 0\) are outside the complex unit circle, then there exists a moving average (MA) representation of the form

\[
    z_t = \Phi(L)\varepsilon_t
    = \begin{bmatrix}
        \Phi_{11}(L) & \Phi_{12}(L) \\
        \Phi_{21}(L) & \Phi_{22}(L)
    \end{bmatrix}
    \begin{bmatrix}
        \varepsilon_{t1} \\
        \varepsilon_{t2}
    \end{bmatrix},
\]

where \(\Phi(L) = I + \Phi_1 L + \Phi_2 L^2 + \cdots = \Theta(L)^{-1}\). The \(h\)-step forecast error results as

\[
    \sigma^2(x_{t+h}|\mathcal{X}_t \cup \mathcal{Y}_t) = e_1 \text{var}(\varepsilon_{t+h} + \Phi_1 \varepsilon_{t+h-1} + \cdots + \Phi_{h-1} \varepsilon_{t+1})e_1
    = e_1 (\Sigma + \Phi_1 \Sigma \Phi_1' + \cdots + \Phi_{h-1} \Sigma \Phi_{h-1}')e_1,
\]

where \(e_1 = [1, 0]^T\). To orthogonalize the errors we apply a Choleski decomposition such that \(\varepsilon_t = G \eta_t\), where \(G\) is a lower triangular matrix and \(E(\eta_t \eta_t') = I\). This gives

\[
    \sigma^2(x_{t+h}|\mathcal{X}_t) = \sigma^2(x_{t+h}|\mathcal{X}_t) + g_{11}^2 \left( \sum_{j=1}^h \phi_{12,j}^2 \right),
\]

where \(\sigma^2(x_{t+h}|\mathcal{X}_t) = g_{11}^2 \left( \sum_{j=1}^h \phi_{11,j}^2 \right)\), \(g_{11}\) is the upper-left element of \(G\) and \(\phi_{11,j}\) (\(\phi_{12,j}\)) denotes the upper left (right) element of \(\Phi_j\). The coefficient \(\phi_{12,j} = \partial x_{t+h}/\partial \eta_{2t}\) measures the impulse response of \(x_{t+h}\) with respect to the innovation \(\eta_{2t}\). From (4) it follows that \(y_t\) is not causal for \(x_t\) if and only if all impulse responses of \(x_t\) with respect to an innovation in \(y_t\) up to a lag horizon of \(h\) are equal to zero. Therefore, this causality concept is similar to the approach suggested by Lütkepohl (1993).

A test of the hypothesis that \(y_t\) is not causal for \(x_t\) at horizon \(h\) can be based on the regression

\[
    x_{t+h} = a_1 x_t + \cdots + a_p x_{t-p+1} + b_1 y_t + \cdots + b_p y_{t-p+1} + \varepsilon_{t+h}.
\]

The null hypothesis of no causality at horizon \(h\) implies that \(b_1 = \cdots = b_p = 0\), which can be tested by using a Wald test procedure. It is important to note, however, that for \(h > 1\) the error \(\varepsilon_t\) has a MA(\(h-1\)) representation so that the estimated covariance matrix of the coefficients must be estimated by using an autocorrelation robust approach as suggested, e.g., by Newey and West (1987). The resulting Wald
statistic is asymptotically $\chi^2$ distributed with $p$ degrees of freedom. Furthermore, Lütkepohl (1993) has shown that if $y_t$ is not causal for $x_t$ at lag $h^* = 2p$, then this is also the case for all $h > h^*$.

To assess the strength of the causal link, a measure of causality is required. For a stationary variable it follows from (4) that $\sigma^2(x_{t+h}|X_t \cup Y_t) \leq \sigma^2(x_{t+h+j}|X_t \cup Y_t)$ for positive $h$ and $j$ so that the expected value of $R^2$ is monotonically increasing in $h$. Therefore, the $R^2$ of the forecast equation is not a valid measure of the strength of causality. Following Geweke (1982) we therefore define the measure of causality at horizon $h$ as

$$M^h_{y \rightarrow x} = \log \left[ \frac{\sigma^2(x_{t+h}|X_t)}{\sigma^2(x_{t+h}|X_t \cup Y_t)} \right].$$

(6)

Under normality the estimator of $M^h_{y \rightarrow x}$ is proportional to the pseudo likelihood ratio statistic of the hypothesis $b_1 = \cdots = b_p = 0$:

$$LR = T \log \left[ \frac{\sum_{t=p+1}^T \hat{e}_t^2}{\sum_{t=p+1}^T e_t^2} \right],$$

where $\hat{e}_t$ denotes the least-squares residuals from the regression (2) and $e_t$ is the least-squares residual from the (restricted) regression of $x_{t+h}$ on $x_t, \ldots, x_{t-p+1}$. For $h = 1$ and $M^h_{y \rightarrow x} = 0$ the LR statistic is asymptotically $\chi^2$ distributed with $p$ degrees of freedom. For $h > 1$ the error $e_t$ is serially correlated and the asymptotic distribution depends on the parameters of the model.

By construction $M^h_{y \rightarrow x} \in [0, \infty)$. In order to obtain a measure with values between 0 (no predictability) and 1 (perfect predictability) we may alternatively define the measure as

$$\overline{M}^h_{y \rightarrow x} = 1 - \left[ \frac{\sigma^2(x_{t+h}|X_t \cup Y_t)}{\sigma^2(x_{t+h}|X_t)} \right].$$

(7)

This measure is identical to the $R^2$ of a regression of the forecast errors of the restricted model $\hat{e}_{t+h}$ on $x_t, \ldots, x_{t-p+1}, y_t, \ldots, y_{t-p+1}$. Furthermore, for small values of $M^h_{y \rightarrow x}$ we have $M^h_{y \rightarrow x} \approx \overline{M}^h_{y \rightarrow x}$.

6
4 Testing for causality in the frequency domain

In the framework of Dufour and Renault (1998) short-run and long-run causality is distinguished with respect to different forecast horizons. Alternatively, a frequency domain approach may be used to define short and long-run causality. In what follows we make use of the causality measures of Geweke (1982) and Hosaya (1991) and propose a simple test procedure that allows us to test for causality at some pre-specified frequency.

As in the previous section, we apply a Choleski decomposition such that \( \varepsilon_t = G \eta_t \), where \( G \) is a lower triangular matrix and \( E(\eta_t \eta'_t) = I \). Hence, \( \eta_t = [\eta_{1t}, \eta_{2t}]' = [(\varepsilon_{1t}/\sigma_1), \eta_{2t}]' \), where \( \sigma_j^2 = E(\varepsilon_{jt})^2 \). The orthogonalized MA representation results as

\[
\begin{align*}
  z_t &= \Phi(L)G \eta_t \\
  &= \Psi(L) \eta_t \\
  &= \begin{bmatrix}
    \Psi_{11}(L) & \Psi_{12}(L) \\
    \Psi_{21}(L) & \Psi_{22}(L)
  \end{bmatrix}
  \begin{bmatrix}
    \eta_{1t} \\
    \eta_{2t}
  \end{bmatrix}.
\end{align*}
\]

(8)

Let

\[
  f_x(\omega) = \frac{1}{2\pi} \left\{ |\Psi_{11}(e^{-i\omega})|^2 + |\Psi_{12}(e^{-i\omega})|^2 \right\}
\]

denote the spectral density of \( x_t \). The measure of causality suggested by Geweke (1982) and Hosaya’s (1991) is defined as

\[
  M_{y \rightarrow x}(\omega) = \log \left[ \frac{2\pi f_x(\omega)}{|\Psi_{11}(e^{-i\omega})|^2} \right].
\]

The measure is zero if \( |\Psi_{12}(e^{-i\omega})| = 0 \). In this case we say that \( y_t \) does not cause \( x_t \) at frequency \( \omega \).

To illustrate this condition it is useful to consider the univariate forecast error \( u_{t+1} = x_{t+1} - E(x_{t+1} | x_t) \). From

\[
  x_t = E(x_t | x_{t-1}) + u_t \\
  = E(x_t | \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots) + u_t \\
  = \Psi_{11}(L) \eta_{1t} + \Psi_{12}(L) \eta_{2t} \\
  = \left( \varepsilon_{1t} + \sum_{j=1}^{\infty} \psi_{11,j} \eta_{1t} \right) + \Psi_{12}(L)
\]

\( 7 \)
\[
\sigma_1^{-1} \sum_{j=1}^{\infty} \psi_{11,j}\varepsilon_{1,t-j} + u_t
\]

it follows that

\[ u_t = \xi_t + \varepsilon_{1t} , \]

where \( \xi_t = \Psi_{12}(L)\eta_{2t} \). This equation relates the univariate forecast error \( u_t \) to the multivariate forecast error \( \varepsilon_{1t} \). It shows how much of the univariate forecast error can be reduced by using additional information of the past of \( y_t \). Accordingly, we can define a measure of predictability as the \( R^2 \) of (9). It is not difficult to verify that this gives the measure \( \hat{M}_{y\to x}^1 \) as defined in (7).

It is possible to construct a respective measure in the frequency domain. Since \( \varepsilon_{1t} \) and \( \eta_{2t} \) are orthogonal it follows that

\[ f_u(\omega) = (2\pi)^{-1}\sigma^2_1 + (2\pi)^{-1}|\Psi_{12}(e^{-i\omega})|^2 \]

and, therefore, a measure of predictability at frequency \( \omega \) can be constructed as:

\[ M_{y\to x}(\omega) = \frac{|\Psi_{12}(e^{-i\omega})|^2}{2\pi f_u(\omega)} . \]

This measure expresses how much of the spectral density of the univariate forecast error can be reduced by extending the information set using the past of \( y_t \).

To construct an empirical test procedure for the hypothesis \( M_{y\to x}(\omega) = 0 \) it is useful to consider the implication of non-causality at frequency \( \omega \) for the AR representation of the process. From \( \Psi(L) = \Theta(L)^{-1}G \) and

\[ \Psi_{12}(L) = \frac{g_{22}\Theta_{12}(L)}{[\Theta(L)]} \]

it follows that \( y_t \) does not cause \( x_t \) at frequency \( \omega \) if\(^1\)

\[ |\Phi_{12}(e^{-i\omega})| = 0 . \]

\(^1\)Note that \( g_{22} \) is positive by the assumption that \( \Sigma \) is positive definite.
This implies that the coefficients of the lag polynomial \( \Phi_{12}(L) = \phi_{12,1} L + \cdots + \phi_{12,p} L^p \) obeys the following two restrictions (cf. Breitung and Candelon, 2000):

\[
\sum_{s=1}^{p} \phi_{12,s} \cos(\omega^* s) = 0 \quad (12)
\]
\[
\sum_{s=1}^{p} \phi_{12,s} \sin(\omega^* s) = 0 . \quad (13)
\]

Therefore, the hypothesis that \( y_t \) does not cause \( x_t \) at frequency \( \omega \) implies two linear restrictions for the coefficients in the regression

\[
x_t = \alpha_1 x_{t-1} + \cdots + \alpha_p x_{t-p} + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p} + u_t . \quad (14)
\]

The hypothesis \( M_{y \rightarrow x} (\omega) = 0 \) is equivalent to

\[
H_0 : \quad R(\omega) \beta = 0 ,
\]

where \( \beta = [\beta_1, \ldots, \beta_p]' \) and

\[
R(\omega) = \begin{bmatrix}
\cos(\omega) & \cos(2\omega) & \cdots & \cos(p\omega) \\
\sin(\omega) & \sin(2\omega) & \cdots & \sin(p\omega)
\end{bmatrix}.
\]

The ordinary \( F \) test for (15) is approximately distributed as \( F(2, T - 2p) \).

Obviously, if \( y_t \) is not Granger causal for \( x_t \), then \( y_t \) does not cause \( x_t \) at all frequencies \( \omega \in [0, \pi] \) and the hypothesis is equivalent to the usual Granger causality test with \( \beta = 0 \).

### 5 Empirical Results

For the empirical application we use quarterly data of real Gross Domestic Product (\( Y_t \)), government 10-year bond yield (\( R_t \)) and the 3-month bond yield (\( r_t \)) for the U.S. economy extracted from the Saint-Louis Federal Reserve Bank database. The German data are provided by the Bureau of Census (\textit{Statistische Bundesamt}) in Germany. The sample period is 1959q1 – 1998q4 for the U.S. and 1968q1 – 1998q1 for Germany. Since we found a unit root in real GDP, we use first differences of the logged series (i.e. the
growth rates). The spread \((s_t)\) is constructed as the difference of the long run \((R_g)\) and short-run \((r_t)\) interest rates.\(^2\)

In a first step, we compute the cross-correlations between the spread and economic growth. From Figure 1 it is seen that for a small number of lags the correlation is substantial for the U.S. data. The maximal correlation is found at three quarters. This confirms the short-run predictive power of the interest spread well documented in the literature (e.g. Estrella and Hardouvelis 1989). The results for Germany suggest a relatively weaker link between the term spread and economic activity. The short run cross-correlations are generally smaller and have a maximum at four lags.

In order to analyse the temporal relationship between both series in the frequency domain, we present the estimated coherence and phase (see Figure 2). The coherence is a measure of the degree to which the series are jointly influenced by cycles at a particular frequency. The phase represents the time delay (in quarters) between the cycles at a common frequency. A relevant leading indicator should have a high coherence associated with a substantial phase shift.

The results suggest that for the U.S. there is a strong coherence at frequencies with a cycle length of 1 up to 4 years, that is, at typical business cycle frequencies. Nevertheless, the estimated phase indicates that for such frequencies the phase shift is less than 1 quarter. This suggests that at business cycle frequencies the series exhibit a mere contemporaneous comovement which can hardly be used for forecasting. On the other hand appealing leading indicator properties can be observed for frequencies \(\omega > 2.2\) (corresponding to roughly 3 quarters), where the phase shift is larger than two quarters. This suggests that the yield spread is a powerful short-term predictor for economic activity.

The results are quite different for German data. The estimated phase shift is smaller than one quarter for frequencies \(\omega > 0.6\) (corresponding roughly to 11 quarters), indicating that the spread has nearly no short-run predictive content for future growth. At lower frequencies \(\omega < 0.6\), the phase shift approaches 3 quarters suggesting a possible predictability of the yield spread for economic fluctuations, longer than 3 years.

\(^2\)Applying unit root tests to the spread series we found that the spread is stationary. The results of the unit root tests are available from the authors upon request.
However, it turns out that for low frequencies the coherence is fairly low (around 0.2). In sum, this preliminary analysis points to a powerful short-run predictability for the U.S., whereas for Germany the predictability is fairly weak at short and to a lesser extent at long lag horizons.

Next we consider empirical tests for the causality concept proposed by Dufour and Renault (1998). The Akaike information criterion (AIC) suggests a VAR model of order 6 for the U.S. and 4 for German data. The LM test statistic for serial correlation in the residuals up to a lag order of 10 is 11.83 (U.S.) respective 13.93 (Germany) for the output equation and 16.80 (U.S.A) respective 5.60 (Germany) for the yield spread equation. All these test statistics are insignificant with respect to a 0.05 significance level. A breakpoint Chow test does not reveal any structural break in the output equation for both countries. On the other hand, for the U.S. yield spread equation the test indicates a structural break around 1980. However, it is important to note that all following test procedures are based on the output equation so that a possible structural break in the yield spread equation does not affect the outcomes.

The tests of the restrictions considered in Dufour and Renault (1998) are similar to the usual Granger causality tests. Instead of considering a forecast horizon of $h = 1$ we estimate a forecast equation for different values of $h$. Since for $h > 1$ the errors are serially correlated, the Wald statistics are computed by using the Newey-West covariance matrix estimator with a truncation lag of 10.

The results reported in Table 1 suggest that U.S. economic growth can be predicted by the yield spread at lag horizons from 2 up to 6 quarters, which corresponds well to the outcomes of the cross-correlations analysis. For Germany, the test statistics suggest a predictability at almost all lags up to ten quarters.

The results of the causality tests in the frequency domain are presented in Figure 3. These figures report the test statistics along with their critical value for all frequencies in the interval $\omega \in (0, \pi)$. It turns out that for the U.S., the null hypothesis of no predictability is rejected in the range $\omega \in [1.8, 2.4]$ corresponding to a cycle length of 2.5 and 3.5 quarters. Such a result is in line with the former findings that the spread is a powerful predictor for economic activity at a lag horizon of 2 and 3 quarters.

We also find predictability at frequencies less than 0.8 which corresponds to (busi-
ness cycle) frequencies with a wave length of more than 2 years. This result suggests that the cyclical behavior at business cycle frequencies is well reproduced in the one-step-ahead forecasts of the economic growth. Accordingly, the spread variable is a useful predictor of the stance of the business cycle. For Germany, the null hypothesis is rejected for $\omega \leq 0.6$. Accordingly, the low frequency fluctuations of output growth can be predicted by the yield spread.

It is interesting to consider the test at frequency zero. In this case the set of hypotheses (12) and (13) reduces to a one-dimensional restriction of the form $\sum_{\alpha=1}^{p} \phi_{12,\alpha} = 0$. This restriction can be tested by running the regression

$$x_t = \alpha_1 x_{t-1} + \cdots + \alpha_p x_{t-p} + \beta_1 y_{t-1} + \beta_2 \Delta y_{t-1} + \cdots + \beta_p \Delta y_{t-p+1} + u_t.$$ 

The hypothesis that $x_t$ does not help to predict $y_t$ at frequency zero is equivalent to the restriction $\beta_1 = 0$ which can be tested by using an ordinary $t$-test (see also Granger and Lin, 1995). For the U.S. data, this test yields a $t$-statistic of 2.118 and for Germany the $t$-statistic is 2.962. It follows that for both countries the yield spread is also a predictor of economic growth at frequency zero.

6 Conclusion

In this paper we consider two different approaches that can be used to distinguish short-run and long-run causality (predictability). In the time domain, the framework of Dufour and Renault (1998) is adopted. We propose a simple empirical test procedure based on a $h$-step forecast equations and a $R^2$ type of measure for predictability at horizon $h$ is suggested. In the frequency domain, we follow Geweke (1982) and Hosaya (1991) and suggest a simple test procedure that allows us to test for predictability at some pre-specified frequency.

This methodology is applied to investigate the predictive content of the yield spread for future output growth. For U.S. data we observe good leading indicator properties at frequencies around one year and typical business cycle frequencies ($2 - 8$ years cycle), whereas for German data our analysis suggests a rather weak predictability at low frequencies only.
References


Figure 1: Cross-Correlations of $s_t$ and $\Delta Y_{t+k}$

a) U.-S.

b) Germany
Figure 2: Coherence and Phase

a) U.S.

b) Germany
Figure 3: Causality Test in the Frequency domain

a) U.-S.

b) Germany


**Table 1:** Causality test at various forecast horizons

<table>
<thead>
<tr>
<th>lags $h$</th>
<th>$R^2$</th>
<th>Wald</th>
<th>$M^h_{y \rightarrow x}$</th>
<th>$\widetilde{M}^h_{y \rightarrow x}$</th>
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<td>15.157*</td>
<td>0.1632</td>
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**Note:** $R^2$ refers to the goodness-of-fit measure of the forecast equation (2). The Wald statistic is the test of the hypothesis $b_1 = \ldots = b_p = 0$. For $h > 1$ the HAC covariance matrix estimator of Newey-West (1987) with 10 lags is used. The causality measures $M^h_{y \rightarrow x}$ and $\widetilde{M}^h_{y \rightarrow x}$ are defined in (6) and (7).