Banks' Supply of Loans
When Future Monetary Policy is Uncertain

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Abstract

The most important policy instruments of the Bundesbank and of the coming European Central Bank involve lending to domestic credit institutions. In this monetary setup, banks use short–term central bank credits extensively in order to refinance long–term loans to the public, which makes them vulnerable to sudden monetary policy changes. We develop a loan supply model that captures distinguishing features of the European money supply process and show how money supply responds when future monetary policy is expected to become tighter or more uncertain. The results indicate that the controllability of borrowed reserves is of crucial importance for monetary policy practice.

Keywords: Loan and money supply, central bank lending, monetary policy instruments of the ECB, interest rate risk.

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1 Introduction

In September 1997, the European Monetary Institute published the 'General documentation' on monetary policy instruments and procedures of the projected European Central Bank (ECB) which basically reveals that the ECB takes the current monetary setup of the German Bundesbank as an institutional model.[1] In particular, the ECB's most important policy instruments will involve lending to 'domestic' credit institutions. This is in sharp contrast to monetary policy practice in the United States and the United Kingdom, where open market operations are predominant and central bank credits play only a minor role. In Germany, however, borrowed reserves clearly exceed total reserves, see Figure 1. Obviously, banks borrow reserves extensively in order to refinance loans, and not only to hold them as required or excess reserves. The present paper sheds light on how the predominance of borrowed reserves in the European monetary setup affects the money supply process and the conduct of monetary policy.

Figure 1: Major components of the German monetary base: 1980–1997

Notes: End of year figures in billions DM. Source: Monthly reports of the Deutsche Bundesbank.

Similar to the Bundesbank, the ECB will provide the bulk of central bank credits

[1] See European Monetary Institute (1997).

by weekly auctions of short–term securities repurchase aggreements (repos).[2] The extensive use of repos ensures the flexibility of the ECB's money market management.[3] However, a flexible monetary policy design that fine–tunes banks' refinancing conditions increases the importance of banks' expectations and guesstimates for the money supply process. For example, for European banks it will be of considerable interest whether the ECB will perform its upcoming repo auctions as variable rate tenders (where the repo rate is uncertain) or as fixed rate tenders where the repo rate is already known at the outset. As a consequence, the central bank may influence money supply just by being more or less vague or determined about the future course of monetary policy. We are therefore particularly interested in the question how banks cope with uncertainty about future monetary policy.

To that aim, we will develop a loan supply model which captures the distinguishing features of the European monetary setup. In particular, it will be assumed that the central bank sets the refinancing *rate* and banks choose the *amount* of central bank credits, so that the monetary base becomes endogenous.[4] Endogeneity of the monetary base is limited, however, since the central bank has the power to set a quantitative limit on its lending. The monetary base is *controllable* in the sense that "the central bank can pull the string but cannot push it". In this framework, banks

[2] In a repo, the central bank buys securities on condition that the seller (the bank) simultaneously repurchases the securities forward. Hence repos are borrowed reserves collaterized with securities. The typical maturity of a repo credit is two weeks. In addition, the ECB will provide repo credit with a maturity of three months. However, comparable to the former discount credit of the Bundesbank, these longer–term credits will not play a central role in the ECB's ongoing money market management. In Germany, the use of short–term repos increased dramatically since the mid–eighties, see Figure 1.

[3] Repos allow a flexible money market management for two reasons. First, in a repo auction the central bank determines the maturity of the repo, the date of refinancing, and even the volume of reserves banks can borrow. Second, repos allow a fine–tuning of banks' refinancing conditions since they mature and are renewed at relatively short intervals, see e.g. Deutsche Bundesbank (1994), Neumann and von Hagen (1993), or Nautz (1998a).

[4] A similar view of the money supply process is offered by Bofinger and Schächter (1995). They focus on alternative operating procedures for monetary policy (interest rate vs. monetary base targeting) but do not address the flexibility of money market management and the maturity transformation by banks.

care about the future course of monetary policy, since they use a sequence of short–term central bank credits to refinance long–term loans. Due to this sort of 'maturity transformation' banks are exposed to interest rate risk because loans granted today have to be refinanced at an uncertain interest rate in the future. Moreover, if access to central bank credit will turn out to be rationed, valuable future loan opportunities will be lost.

In this framework, we show that money supply decreases if banks expect their future refinancing conditions to become tighter or more uncertain. Moreover, the controllability of borrowed reserves is shown to be crucial for the dependence of money supply on both interest rate risk and the money multiplier.

The present paper may be seen as complementary to Nautz (1998a) who derives similar results in the context of an extended reserve management model.[5] That model assumes that banks borrow reserves exclusively in order to hold them as required or excess reserves, which is, however, in contrast to the refinancing practice in many European countries and the coming EMU.

The plan of the paper is as follows. In the next section we present and analyze the basic loan supply model where banks use a sequence of two short–term central bank credits to refinance a two–period loan. We show how banks' loan supply reacts when refinancing conditions are expected to become tighter or when there is increasing uncertainty about them. In Section 3, we allow for loan market repercussions, check the robustness of the comparative statics, and show that the equilibrium loan rate increases when uncertainty about monetary policy increases. Concluding remarks are provided in Section 4.

[5] For a survey of earlier contributions to the reserve management model see Baltensperger (1980).

2 Loan Supply and Central Bank Borrowing

2.1 A Loan Supply Model with Maturity Transformation

Consider a two–period model with many identical banks acting as price–takers. Banks use deposits (D_t) and central bank credits (B_t) in order to refinance their loans (L_t), $t = 1, 2$. Loans granted in period 1 are long–term, i.e. they last for both periods, whereas central bank credits are always short–term, i.e. they have to be repaid at the end of each period. Thus, the model captures the maturity transformation implied by long–term loans and short–term refinancing. For a single bank, say bank j, the balance sheet identities for the two periods are

$$D_1^j + B_1^j = L_1^j \tag{1}$$

$$D_2^j + B_2^j = L_1^j + L_2^j. \tag{2}$$

Bank profits in the two periods are given by

$$\Pi_1^j = r_1 L_1^j - C(L_1^j) - z_1 D_1^j - i_1 B_1^j \tag{3}$$

$$\Pi_2^j = r_1 L_1^j + r_2 L_2^j - C(L_2^j) - z_2 D_2^j - i_2 B_2^j \tag{4}$$

where C is a cost function for the production of loans with $C' \equiv c > 0$, $c' > 0$, and $c'' \leq 0$. The bank takes the loan rates r_t, deposit rates z_t, and the central bank's refinancing rates i_t as given. Following traditional reserve management models, we assume that a single bank cannot influence its own level of deposits, D_t^j. In each period, the bank decides upon the level of loans to its customers L_t^j, and the corresponding level of central bank refinancing B_t^j follows from the budget constraint.[6]

The level of deposits is determined by the whole banking sector, i.e. by the *representative* bank, via the usual multiplier process. Let m be the multiplier and B_t

[6] We mention that the model can easily be extended by reserve requirements. Excess reserves, on the other hand, are neglected since they are almost negligible in most countries (they rarely exceed one percent of required reserves). Thus, this loan supply model can be seen as complementary to traditional reserve management models.

the representative bank's amount of refinancing (variables which do not carry the superscript j refer to the representative bank), then

$$D_t^j = mB_t. \tag{5}$$

Note that it would be incorrect to equate D_t^j with mB_t^j, since multiple deposit creation does not work for a single atomistic bank. In deriving optimal loan supply, careful distinction between a single bank j on the one hand and the representative bank on the other hand is therefore required.[7]

Due to the central bank's flexible money market management, future refinancing conditions are uncertain. Consequently, the refinancing rate i_2 as well as the aggregate quantity of reserves available in the future are random and realize at the beginning of period 2. Assuming risk–neutrality, bank j maximizes its expected two–period profit

$$E\Pi^j(L_1^j, L_2^j) = \Pi_1^j + \delta E\Pi_2^j \tag{6}$$

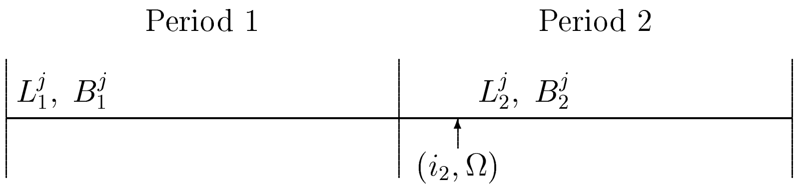
by choice of L_1^j and L_2^j (or, equivalently, B_1^j and B_2^j), where δ is a discount factor. Due to limited access to central bank credit, there might be a liquidity constraint in period 2.[8] Letting Ω denote the maximum amount of central bank credit available, expressed in per bank terms, bank j has to satisfy $B_2^j \leq \Omega$. Of course, a single bank cannot influence whether the whole banking sector is liquidity–constrained or not. Therefore, rationing will occur if and only if $B_2 = \Omega$.

Independence between the refinancing variables i_2 and Ω is not required in the model. Actually, assuming negative correlation between the interest rate and the upper bound on central bank credits is more realistic. Figure 2 summarizes the structure of the model.

[7] For ease of reference, we call m the multiplier, although the conventional money multiplier equals $(1+m)$, according to the balance sheet identity $L = D + B = mB + B$. It is reduced to $(1 + m - rrm)$ when banks' reserve ratio rr is taken into account. Notice further that the inclusion of exogenous non–borrowed reserves would have no impact on the results of the paper.

[8] The ECB will provide emergency credit at a 'penalty rate' (comparable to the Lombard rate in Germany), see EMI (1997). However, this penalty rate should be too high for refinancing purposes. Concerning period 1, we focus on the more interesting case that access to borrowed reserves is not restricted in that period.

Figure 2: The timing of the model



Notes: The choice variables of bank j are L_t^j, the loans granted in period t, and B_t^j, its refinancing credits. The exogenous random variables are the refinancing rate i_2 and the upper bound on central bank credits Ω in period 2.

2.2 Loan Supply and Deposit Creation in Period 2

At the beginning of period 2, bank j chooses its level of loans, L_2^j. At that point of time it knows the realizations of the refinancing conditions i_2 and Ω.[9] If access to central bank credit is not constrained, bank j's optimal loan supply in period 2 is $L_2^j = c^{-1}(r_2 - i_2)$, by differentiation of the period 2 profit (4). On the other hand, if $B_2^j = \Omega$, the balance sheet identity (2) implies $L_2^j = \Omega - L_1^j + D_2^j$.

Now consider the outcomes of all banks' actions. Since all banks are identical, one obtains for the representative bank:

$$L_2 \quad = \quad \min\left\{c^{-1}(r_2 - i_2),\ \Omega - L_1 + D_2\right\}. \tag{7}$$

By the multiplier process, (5), the representative bank's amount of deposits is $D_t = mB_t$. Accordingly, the representative bank's loan supply can be rewritten as:

$$L_2 \quad = \quad \min\left\{c^{-1}(r_2 - i_2),\ (1+m)\Omega - L_1\right\}, \tag{8}$$

and its demand for central bank credit follows again from the budget identity (2):

$$B_2 \quad = \quad \min\left\{\frac{c^{-1}(r_2 - i_2) + L_1}{1+m},\ \Omega\right\}. \tag{9}$$

Whether the banking sector is constrained ($B_2 = \Omega$) or not depends obviously on the realizations of i_2 and Ω. We therefore define $\mathcal{R}$ as the set of refinancing conditions

[9] Since central banks generally act as lenders of last resort, we further assume that $\Omega \geq L_1^j - D_2^j$, i.e. banks can always refinance their outstanding credits.

where the refinancing constraint is binding:

$$
\begin{aligned}
(i_2, \Omega) \in \mathcal{R} \quad :\Longleftrightarrow \quad & B_2 = \Omega && (10)\\
\Longleftrightarrow \quad & \Omega \leq \Omega^R := \frac{c^{-1}(r_2 - i_2) + L_1}{1+m} && (11)\\
\Longleftrightarrow \quad & i_2 \leq i^R := r_2 - c\Big((1+m)\Omega - L_1\Big) && (12)
\end{aligned}
$$

By (8), long–term credits L_1 have an impact on banks' loan supply in period 2 if and only if the liquidity constraint is binding. This already indicates that it might be profitable to reduce loan supply in period 1, since this will leave more room for fresh loans in period 2 in the case that access to central bank credit will be limited. Note that, when analyzing optimal loan supply in period 1, we have to take the perspective of a *single* bank again, not that of the representative bank. For example, if bank j considers increasing L_1^j by an amount, say ΔL^j, its period 2 demand for refinancing will not increase by $\Delta L^j/(1+m)$, as (9) might suggest, but by the larger amount ΔL^j, see budget constraint (2).

2.3 Loan Supply in Period 1

In period 1, bank j chooses L_1^j in order to maximize its expected two–period profit, which is given by (6). Its expected profit only depends on L_1^j since period 2 variables are given by their optimal values (see proof of Lemma 1 below).

Lemma 1 *The representative bank's optimal loan supply is characterized by*

$$
\begin{aligned}
0 \;=\; & r_1 - i_1 - c(L_1^j) + \delta(r_1 - E[i_2]) \\
& - \delta E\Big[r_2 - i_2 - c\Big((1+m)\Omega - L_1^j\Big)\Big|(i_2, \Omega) \in \mathcal{R}\Big] \qquad (13)
\end{aligned}
$$

evaluated at $L_1^j = L_1$.

Proof

Using the budget constraint (1) to substitute B_1^j, and the multiplier relation (5) to substitute D_1^j resp. D_2^j, bank j's expected two–period profit (6) can be restated as

$$\begin{aligned} E\Big[\Pi^j(L_1^j)\Big] &= r_1 L_1^j - C(L_1^j) - z_1 m B_1 - i_1(L_1^j - mB_1) \\ &\quad + \delta E\Big[r_1 L_1^j + r_2 L_2^j - C(L_2^j) - z_2 m B_2 - i_2 B_2^j\Big]. \end{aligned} \tag{14}$$

The derivative with respect to L_1^j is

$$r_1 - c(L_1^j) - i_1 + \delta r_1 + \delta E\left[\Big(r_2 - c(L_2^j)\Big)\frac{\partial L_2^j}{\partial L_1^j} - i_2 \frac{\partial B_2^j}{\partial L_1^j}\right]. \tag{15}$$

To complete the proof, one has to consider how L_2^j and B_2^j depend on L_1^j.[10] The balance sheet identity (2) implies

$$\frac{\partial B_2^j}{\partial L_1^j} = 1 + \frac{\partial L_2^j}{\partial L_1^j}. \tag{16}$$

If the banking sector is not constrained, the optimal loan policy in period 2, $L_2^j = c^{-1}(r_2 - i_2)$, is independent of L_1^j. Hence, by (10) and (16),

$$\text{for } (i_2, \Omega) \notin \mathcal{R}: \qquad \frac{\partial L_2^j}{\partial L_1^j} = 0 \quad \Longrightarrow \quad \frac{\partial B_2^j}{\partial L_1^j} = 1.$$

On the other hand, if the banking sector is constrained, bank j will, by symmetry, request the maximum amount of refinancing available, i.e. $B_2^j = \Omega$. Then $L_2^j = (1+m)\Omega - L_1^j$ follows immediately, and (16) implies

$$\text{for } (i_2, \Omega) \in \mathcal{R}: \qquad \frac{\partial B_2^j}{\partial L_1^j} = 0 \quad \Longrightarrow \quad \frac{\partial L_2^j}{\partial L_1^j} = -1.$$

Collecting terms in (15) gives (13). Finally, differentiating (13) shows that the second order condition is satisfied. □

The Lemma states that, of course, expected marginal cost equals expected marginal revenue at the optimum: If loans in period 1 are marginally increased, the two–period profit from *these* loans raises by $r_1 - i_1 - c(L_1^j) + \delta(r_1 - E[i_2])$, the first line

[10]Note that $\mathcal{R}$, and thus the probability of being rationed, is independent of a single bank's L_1^j.

of (13). Next period's discounted profit is unaffected by an increase of L_1^j if access to central bank credit will not be constrained, which occurs if $(i_2, \Omega) \notin \mathcal{R}$. However, if $(i_2, \Omega) \in \mathcal{R}$, the availability of reserves constrains loan supply in period 2 and profitable lending opportunities are lost, i.e. $r_2 > i_2 + c(L_2^j)$ (compare (12)). The expected opportunity cost of long–term loans due to the possibility of rationing are therefore $\delta \mathrm{E}[r_2 - i_2 - c(L_2^j) | (i_2, \Omega) \in \mathcal{R}]$ with $L_2^j = (1+m)\Omega - L_1^j$; this is the second line of (13).

2.4 Comparative Statics under Uncertainty

There is no closed–form solution for the optimal loan supply, not even in case of simple distribution functions. Yet, it is possible to predict how banks' loan supply reacts if expectations of future refinancing conditions become more pessimistic or more uncertain. In the following we derive comparative statics for the optimal loan supply when i_2 resp. Ω are subjected to first or second order stochastic dominance transformations.[11]

Proposition 1 *Banks' loan supply, and thus the demand for central bank credits, decreases when*

a) refinancing conditions are expected to become tighter or when

b) uncertainty about future refinancing conditions increases.

Proof

Let us first assume that i_2 and Ω are stochastically independent. In that case, the criteria for comparative statics under uncertainty derived by Ormiston (1992) can be applied.[12] He considers an objective function $V(a, X)$ where the outcome

[11] A *first order stochastic dominance* (FSD) transformation moves probability from smaller to larger realizations so that the transformed random variable gets *stochastically larger* (in particular, its mean always increases). On the other hand, a *second order stochastic dominance* (SSD) transformation typically moves probability from the tails of the distribution to its centre so that the transformed random variable gets less volatile and thus *less risky.*

[12] Ormiston (1992) considers the comparative statics of a *single* random variable. However, in line with the generalization of stochastic dominance for higher dimensional random variables, see

depends not only on the action variable a, but also on the realization of the random variable X. The optimal choice a^* maximizes $E_X[V]$. The following criteria state how a^* changes when X is altered by FSD or SSD transformations, i.e. when X gets stochastically larger or less risky. Subscripts of V denote derivatives.[13]

FSD criterion: *The optimal action increases for FSD transformations of X if $V_{ax} > 0$, and it decreases if $V_{ax} < 0$.*

SSD criterion: *Consider a mean preserving SSD transformation of X. Then the optimal action increases if $V_{axx} < 0$ and it decreases if $V_{axx} > 0$.*

In our application we have $a = L_1^j$. In case of independence banks' expected profit can be rewritten as

$$E_{i_2\Omega}[\Pi^j] = E_{i_2}\left[E_\Omega[\Pi^j]\right] = E_\Omega\left[E_{i_2}[\Pi^j]\right], \tag{17}$$

where the expectations operator E carries as indices the random variables which are integrated over. Therefore, referring to $V = E_\Omega[\Pi^j]$ for $X = i_2$, and to $V = E_{i_2}[\Pi^j]$ for $X = \Omega$, gives in both cases $E_X[V] = E_{i_2\Omega}[\Pi^j]$, and the above criteria can be applied.

Letting $f \equiv F'$ the distribution of Ω, and $g \equiv G'$ that of i_2, Lemma 1 leads to

$$V_{L_1^j} = \frac{\partial E_\Omega \Pi^j}{\partial L_1^j} = r_1 - i_1 - c(L_1^j) + \delta(r_1 - i_2) - \delta \int_{\Omega \le \Omega^R} \left[r_2 - i_2 - c\Big((1+m)\Omega - L_1^j\Big)\right] f(\Omega)\, d\Omega \tag{18}$$

resp.

$$V_{L_1^j} = \frac{\partial E_{i_2} \Pi^j}{\partial L_1^j} = r_1 - i_1 - c(L_1^j) + \delta(r_1 - E[i_2]) - \delta \int_{i_2 \le i^R} \left[r_2 - i_2 - c\Big((1+m)\Omega - L_1^j\Big)\right] g(i_2)\, di_2. \tag{19}$$

Refinancing conditions are expected to become 'tighter' if the refinancing rate i_2 gets stochastically larger (FSD transformation) or Ω gets stochastically smaller (inverted

Fishburn and Vickson (1978, p.93), the successive consideration of i_2 and Ω is feasible if the random variables are stochastically independent.

[13] For a detailed presentation of these criteria, see also Nautz (1998a, Appendix).

FSD transformation). From (18) and (19) one obtains

$$V_{L_1^j i_2} = \frac{\partial^2 E_\Omega \Pi^j}{\partial L_1^j \partial i_2} = -\delta\Big(1 - F(\Omega^R)\Big) < 0 \tag{20}$$

$$V_{L_1^j \Omega} = \frac{\partial^2 E_{i_2} \Pi^j}{\partial L_1^j \partial \Omega} = \delta(1+m) \int_{i_2 \leq i^R} c'\Big((1+m)\Omega - L_1\Big) g(i_2)\, di_2 > 0, \tag{21}$$

where all derivations are evaluated at $L_1^j = L_1$. According to the FSD–criterion, the signs of (20) and (21) imply that L_1 decreases as a result of an FSD transformation of i_2 and increases in case of Ω (part a). Note that (20) is proportional to the probability of not being rationed for a given i_2, see (11). Similarly, for a quadratic cost function, (21) is proportional to $G(i^R)$, the probability of being rationed for a given Ω, see (12).

Uncertainty about future refinancing conditions *decreases* if the refinancing rate or the constraint Ω get less risky (SSD transformations). Using the SSD criterion for $X = i_2$, (20) leads to

$$V_{L_1^j i_2 i_2} = \frac{\partial^3 E_\Omega \Pi^j}{\partial L_1^j \partial^2 i_2} = \delta f(\Omega^R) \frac{d\Omega^R}{di_2} < 0 \tag{22}$$

because Ω^R is decreasing in i_2, see (11). Analogously, (21) implies for $X = \Omega$

$$V_{L_1^j \Omega\Omega} = \frac{\partial^3 E_{i_2} \Pi^j}{\partial L_1^j \partial^2 \Omega} = \delta(1+m) g(i^R) c'\Big((1+m)\Omega - L_1\Big) \frac{di^R}{d\Omega} + \delta(1+m)^2 \int_{i_2 \leq i^R} c''\Big((1+m)\Omega - L_1\Big) g(i_2)\, di_2 < 0 \tag{23}$$

since $c'' \leq 0$ and i^R decreases in Ω by (12). According to the SSD criterion, banks' loan supply increases if uncertainty about Ω decreases (part b).

We have thus shown that loan supply decreases if i_2 gets stochastically larger *independent* of Ω and if Ω gets stochastically smaller *independent* of i_2. As a consequence, the comparative statics remain valid in the more realistic case of negative correlation of i_2 and Ω. If Ω tends to decrease if i_2 increases (due to negative correlation), the comparative statics are even reinforced.

□

According to part a) of the proposition, banks increase their loan supply if expectations of future refinancing conditions become more optimistic. However, as stated in part b), an increase in loan supply can also be the result of reduced *uncertainty* about banks' future refinancing conditions. And vice versa: When the central bank does not attempt to reduce uncertainty about its future course of monetary policy, loan supply L_1 and banks' demand for refinancing B_1 will decrease. Moreover, money supply, defined as $(1+m)B_1$, will decrease too. This parallels the results of the extended reserve management model of Nautz (1998a), although the economic mechanisms behind the results are different.

To get the intuition behind Proposition 1, recall that in case of a binding refinancing constraint the return of a loan granted in period 2 strictly exceeds its cost, $r_2 > i_2 - c(L_2)$. Thus, the danger of being rationed in the future raises the opportunity cost of incurring large fixed obligations (L_1) because future rents might be lost. For example, if Ω is expected to be smaller, rationing gets more likely so that opportunity cost increase and current loan supply decreases. A higher expected refinancing rate i_2 obviously raises the cost of refinancing L_1 in period 2. This explains part a) of the proposition. Part b) states that banks, even though they are risk–neutral, respond to an increase in uncertainty by reducing loan supply. This is explained by the SSD–criteria (22) and (23): Loan supply is reduced because the probability of being rationed increases if bank's future refinancing conditions get more risky.

2.5 The Controllability of Central Bank Credit

In accordance with Nautz (1998a), interest rate uncertainty affects banks' behavior only if banks attach a positive probability to being rationed in their access to central bank credit. To see this, suppose that the maximum amount of central bank credit in period 2 cannot be controlled by the central bank. In that case, the first order condition on L_1^j, given in Lemma 1, simplifies to

$$c(L_1^j) + i_1 + \delta E[i_2] \quad = \quad (1+\delta)r_1, \tag{24}$$

and

$$L_1^j = c^{-1}\Big((1+\delta)r_1 - i_1 - \delta E[i_2]\Big) \tag{25}$$

follows immediately.[14] In accordance with part a) of Proposition 1, loan supply (25) decreases if the expected value of the future refinancing rate increases. However, without the refinancing quota, loan supply does not depend on the *degree of uncertainty* about i_2. This demonstrates that the central bank's ability to set a quantitative limit on its lending is a key feature of the European repo–based monetary setup.

Since $(1+m)B_1 = L_1 = L_1^j$, equation (25) also implies that money and loan supply are independent of the money multiplier if the access to central bank credit is not controllable. Thus, under a policy of pure interest rate targeting, a decrease of the multiplier m is simply compensated by an increase of borrowed reserves B_1 because the *marginal* loan is always refinanced by central bank credits at the given rate i_1 (as we assumed throughout). If rationing is possible, however, a decrease of m increases the probability of being rationed in period 2 because the reduced deposit creation lowers the availability of total refinancing, $(1+m)\Omega$. According to Lemma 1, a decrease of the multiplier raises the opportunity cost of current loans, and loan and money supply go down.[15] In fact, Lemma 1 can also be used to derive comparative statics under uncertainty for the multiplier. Assuming that its value in period 2 is uncertain (and that rationing is possible), it can be shown that loan supply in period 1 decreases if the multiplier is expected to decrease or if it gets more risky.[16]

[14]Note that Proposition 1 implies that (25) gives an upper bound for optimal loan supply in the general case of stochastic refinancing conditions.

[15]In 1994 and 1995 the Bundesbank reduced required reserve ratios and thereby raised the money multiplier. The implied expansionary effect on the money supply was sterilized, however, by a reduction of borrowed reserves.

[16]This points to another channel of influence of expectations about future monetary policy. Although changes in reserve ratios do not play an important role in recent monetary policy practice, the empirical evidence provided by Garfinkel and Thornton (1991) indicates that the multiplier is not independent of the central bank's actions, even if the monetary base is adjusted for changes in reserve ratios.

3 Loan Market Repercussions

So far we have analyzed aggregate loan supply without considering its effects on the equilibrium loan rate. This amounts to assuming that the public's loan demand is infinitely elastic at an exogenously given rate. In the following, we check the robustness of the comparative statics results derived in Proposition 1 by considering a loan demand that decreases in the loan rate.[17]

Proposition 2 *Suppose that loan demand is less than perfectly elastic. Then the comparative statics derived in Proposition 1 are strengthened with regard to the refinancing constraint* Ω *and weakened with regard to the refinancing rate* i_2*.*

Proof
The proof which contains the analysis of loan market equilibrium in period 2 is given in the Appendix. □

These results are quite intuitive. For example, suppose that the banking sector is liquidity constrained by Ω in period 2. Since banks' loan supply is then rationed from the refinancing side and loan demand is less than perfectly elastic, the equilibrium loan rate r_2 increases. As a consequence, the loss due to rationing is also increased. The effects of expectations about Ω are therefore reinforced when r_2 reacts. Now consider the effects of expectations about the future refinancing rate i_2. To that aim, recall that the demand for central bank credits B_2, and thereby the probability of being rationed, increases in $(r_2 - i_2)$, see (9). Since a responsive loan rate r_2 will partly follow i_2 (see Appendix), the effect of expectations about i_2 on B_2 is mitigated when r_2 reacts.

If banks' loan supply decreases, the demand for borrowed reserves decreases accordingly. As a consequence, for given loan demand, the loan rate will *increase* and the money market rate will *decrease* if future refinancing conditions are expected

[17] Expectations about the future *deposit rate* z_2 have no effect on banks' behavior since the *marginal* refinancing rate of a loan in period 2 is i_2 rather than z_2, by assumption. The deposit rate z_2 has only an effect on banks' profits.

to become tighter or if uncertainty about them increases. The empirical relevance of this prediction has been demonstrated by Nautz (1998a) who uses an ARCH–M model of the day–to–day interest rate in order to capture the Bundesbank's impact on the degree of uncertainty.

4 Concluding Remarks

In the monetary setup of the Bundesbank and the prospected ECB, central bank lending is the predominant instrument of monetary policy. Banks borrow on a massive scale from the central bank and use the funds mainly to refinance loans to the public (and not only to hold them as required or excess reserves). Flexibility of money market management is ensured by the use of weekly repo auctions where the central bank can fine–tune the refinancing rate and the maximum amount of refinancing available to banks. However, since banks use revolving short–term central bank credits for the refinancing of long–term loans, the central bank's flexibility exposes them to interest rate risk. The management of interest rate risk in view of possible monetary policy changes is therefore a major concern for European banks.[18]

In order to capture the characteristic features of the European monetary setup, we developed a two–period loan supply model where banks refinance long–term loans by a sequence of two short–term central bank credits. Due to this maturity transformation, the expected profitability of a loan depends on banks' expectations about future refinancing conditions. Since the central bank sets the refinancing *rate* and banks determine the *amount* they borrow, the monetary base is endogenous. However, as in a repo auction, the central bank is able to set an upper limit on its lending.[19]

[18] The importance of interest rate risk due to maturity transformation has recently been emphasized by Hellwig (1994) and Wong (1997) who do not, however, address monetary policy issues.

[19] Note that the traditional multiplier approach to the money supply process, comprehensively surveyed by Brunner and Meltzer (1990), is based on the assumption that the monetary base is *perfectly* controlled by the central bank while commercial banks and the public can only influence the money multiplier. However, a recent empirical study by Nautz (1998b) showed that the German

In this framework, we showed that loans, central bank lending, and money supply decrease when refinancing conditions are expected to become tighter or when uncertainty about them increases. In accordance with Nautz (1998a), the impact of interest rate risk is proportional to the probability of being rationed in the access to central bank credit. Similarly, money supply depends on the money multiplier only if rationing by the central bank is possible. This indicates that the controllability of banks' access to central bank credit is a key feature of the repo–based monetary setup of the Bundesbank and the future ECB.

Appendix: Proof of Proposition 2

In a first step, we have to analyze the loan market in period 2. Loan demand is denoted by $K_t^d(r_t)$ for $t = 1, 2$ with $K_t^{d\,\prime} \equiv k < 0$ and $k > -\infty$. If the banking sector is not constrained, $B_2 < \Omega$, the equilibrium condition, $c^{-1}(r_2 - i_2) = K_2^d(r_2)$, implies that r_2 is independent of Ω, L_1, and m. Evidently, these variables only determine the *availability* of funds for fresh loans. With respect to i_2, it follows that

$$\text{for } B_2 < \Omega: \qquad \frac{dr_2}{di_2} = \frac{1}{1 - kc'(c^{-1}(r_2 - i_2))} \in (0, 1). \tag{26}$$

The loan rate is tied to the refinancing rate, as expected, but the margin is decreasing.

On the other hand, if the banking sector's access to central bank credit is constrained, $B_2 = \Omega$, the equilibrium condition, $(1 + m)\Omega - L_1 = K_2^d(r_2)$, implies that r_2 is independent of i_2. In this case, only bank profits vary with i_2 whereas the amount of loans is rationed by the limited access to refinancing. However, r_2 is increasing in L_1 and v and decreasing in Ω. Thus, if more funds are available for fresh loans, the loan rate will decrease. Note that a more generous access to central bank credit involves the multiplier effect:

$$\text{for } B_2 = \Omega: \qquad \frac{dr_2}{d\Omega} = \frac{1 + m}{k} < 0. \tag{27}$$

monetary base is Granger–caused by money supply and not vice versa. This evidently contradicts the exogeneity assumption of the simple multiplier approach and thus indicates that the latter does not provide an appropriate view of the European money supply process.

Note that Lemma 1 extends to the case that r_2 is endogenous. From (13), one obtains the following derivatives which generalize (20)–(23) in the proof of Proposition 1 for the case that $k > -\infty$.

For the refinancing rate i_2:

$$\begin{aligned} \frac{\partial^2 E_\Omega \Pi^j}{\partial L_1^j \partial i_2} &= -\delta\Big(1 - F(\Omega^R)\Big) < 0, \\ \text{and} \quad \frac{\partial^3 E_\Omega \Pi^j}{\partial L_1^j \partial^2 i_2} &= \delta f(\Omega^R) \frac{d\Omega^R}{di_2} < 0, \end{aligned}$$

since $\frac{d\Omega^R}{di_2} = -\frac{1}{1+m} \frac{1}{c'(c^{-1}(r_2 - i_2))} \left(\frac{dr_2}{di_2} - 1\right) = \frac{1}{1+m} \frac{k}{1 - kc'(c^{-1}(r_2 - i_2))} < 0$, from (11) and (26), which is decreasing in absolute terms when $|k|$ goes down. Hence, the SSD–criterion 2 implies that more uncertainty about i_2 reduces loan supply, but that this effect is the smaller the lower $|k|$.

For the refinancing constraint Ω, using (27) (and recalling that $k < 0$):

$$\frac{\partial^2 E_{i_2} \Pi^j}{\partial L_1^j \partial \Omega} = \delta(1+m) \int_{i_2 \le i^R} \left[\frac{1}{-k} + c'\Big((1+m)\Omega - L_1\Big)\right] g(i_2)\, di_2 > 0,$$

which goes up if $|k|$ decreases. Thus, by the FSD–criterion, expectation of a lower Ω reduces loan supply, and the more so the lower $|k|$.

$$\begin{aligned} \frac{\partial^3 E_{i_2} \Pi^j}{\partial L_1^j \partial^2 \Omega} &= -\delta(1+m) g(i^R) \left[\frac{1}{-k} + c'\Big((1+m)\Omega - L_1\Big)\right] \frac{di^R}{d\Omega} \\ &\quad + \delta(1+m)^2 \int_{i_2 \le i^R} c''\Big((1+m)\Omega - L_1\Big) g(i_2)\, di_2 < 0, \end{aligned}$$

since $c'' \le 0$ and, using (12) and (27), $\frac{di^R}{d\Omega} = -(1+m)\left[\frac{1}{-k} + c'\Big((1+m)\Omega - L_1\Big)\right] < 0$, which is increasing in absolute terms when $|k|$ goes down. Thus, by SSD–criterion 1, more uncertainty about Ω reduces loan supply, and this effect is the larger the lower $|k|$.

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