Preemption in Capacity and Price Determination – A Study of Endogenous Timing of Decisions for Homogeneous Markets –*

Sandra Güth† and Werner Güth‡

March, 1998
revised version: October, 1998

Abstract

The theory of industrial organization has experienced an impressive boom by using the methods of (non-cooperative) game theory. The conclusions depend, however, crucially on subtle details of the market decision processes about which there exist no or little empirical information. Studies of endogenous timing could help since they derive the time structure of decision making instead of assuming it as exogenously given. In our study we consider a homogeneous market where, like in the model of Kreps and Scheinkman (1983), sellers determine "sales capacities" before prices. To avoid rationing sellers must serve customers, but at higher costs when demand exceeds "capacity". Our model allows for preemption in "capacity" as well as in price determination. Since preemption means to decide before the random choice of cost parameters reflecting the stochastic nature of (excess) "capacity" costs, preemptive commitments are no obviously better timing dispositions.

*The authors completed this study at the faculty of social economics at the University of Bergen, Norway, whose inspiring research atmosphere is gratefully acknowledged. We thank the Ruhrgas Foundation for supporting this research visit.

†University of Bielefeld, Department of Economics, PF 10 01 31, D - 33501 Bielefeld, Germany

‡Humboldt-University of Berlin, Department of Economics, Institute of Economic Theory III, Spandauer Straße 1, D - 10178 Berlin, Germany

1. Introduction

Ever since Cournot (1838) quantity competition on homogeneous markets has played an important role in micro-economics. On markets with especially designed trade institutions like commodity or stock exchanges it is often possible to sell a certain amount and leave it to the market at which price this amount is sold. Here institutions have been created allowing sellers to abstain from own pricing policies. If such institutions exist, one should, however, incorporate them when modelling homogeneous markets with quantity competition.

When costly institutions like (commodity or stock) exchanges do not exist, quantity competition is hardly an acceptable idea: What is a seller supposed to answer to the first customer asking for the price? To overcome the obvious absurdity of quantity competition one sometimes refers to an (Cournot-)auctioneer who substitutes special institutions like exchanges. This, however, only avoids the need to justify quantity competition.

Kreps and Scheinkman (1983) have, however, offered a natural reinterpretation of quantity competition on homogeneous markets and of the results by Cournot (1838). The idea is to imagine quantity competition as the first stage of a **two stage-market decision process**: First sellers state their sales capacities which then become commonly known. Then, knowing all the available capacities, sellers choose their individual sales prices.

According to the play of the subgame perfect equilibrium (Selten, 1965 and 1975) sellers choose capacities matching the quantities of traditional quantity competition models (Cournot, 1838) and rely, on the second stage, on the price for which demand equals the sum of these capacities. Kreps and Scheinkman (1983) analyse only duopoly markets and assume a **special rationing scheme**: In subgames with larger capacities than the (Cournot-) solution quantities the seller with the lower price may not be able to serve all his customers. The residual demand for the seller with the higher price then depends crucially on whom the cheaper seller serves, respectively whom he does not serve. Kreps and Scheinkman (1983) rely on the rationing scheme **maximizing consumer surplus**, i.e. the cheaper seller serves customers with the higher willingness to pay (see the partly critical discussion by Davidson and Deneckere, 1986).

In our view, the basic idea of Kreps and Scheinkman (1983) is very intuitive and should not be questioned by debatable assumptions of rationing which imply equilibria in mixed pricing strategies (in case of “too large” capacities prices have to be chosen randomly). One way to do so is to rely on less rigorous cost functions. According to Kreps and Scheinkman a capacity is an upper bound for sales. This can be rephrased by saying that at the capacity level the costs of production are prohibitively large. What we will consider here is a similar jump in the cost of production at the capacity level, but only a finite one.

Given that capacities pose no longer absolute upper bounds for sales one can avoid the need of rationing and of analyzing equilibria in mixed pricing strategies by assuming that sellers must serve their customers. It can be shown for generic regions of parameter constellations that sellers, on the second stage, use prices at which their capacities are demanded and that, on the first stage, “capacities” resemble the ones of traditional quantity competition (Güth, 1995).

Studies of endogenous timing and indirect evolution try to derive – instead of imposing exogenously – the timing of market decisions (see van Damme and Hurkens, 1996). In our model sellers can determine their “capacities” before or after the random choice of the constant unit (capacity) costs, i.e. the first stage of the Kreps and Scheinkman-model now can consist of three successive substages (preemption, chance move, adjustment). By determining his sales capacity earlier a seller can try to preempt his competitor, similar to the sequential duopoly solution (von Stackelberg, 1934). Although this will be less important, we also allow for three substages on the second stage of the Kreps and Scheinkman-model (preemption in price setting, random choice of constant unit (excess capacity) cost, adjustment).

In section 2 we introduce our market model relying on this more complex market decision process which we solve in section 3 for all possible constellations of timing dispositions for determining capacities **and** prices. These results then enable us in section 4 to define an evolutionary game or truncation by which, in section 5, we can derive the evolutionarily stable or optimal timing dispositions. Before concluding section 6 illustrates that there can exist multiple price equilibria and how to select one of these.

2. The model

On the homogeneous duopoly market the two sellers $i = 1, 2$ first have to determine their planned sales amounts k_i and then their actual sales prices. To allow for an easy terminology we refer to the planned sales amounts k_i as **capacities** although actual sales amounts x_i can be higher (as well as lower). Due to our distinction between capacities k_i and actual sales amounts x_i a cost function $C_i(\cdot)$ must assign a cost level $C_i(k_i, x_i)$ to every constellation (k_i, x_i) of capacity k_i and sales amount x_i. For the sake of simplicity we rely on (piecewise) **linear cost functions** of the form

$$\text{(II.1)} \qquad C_i(k_i, x_i) = Ck_i + (C + D)\max\{0, x_i - k_i\} \text{ for } i = 1, 2$$

where $k_i \in \left[0, \frac{1}{2}\right]$ and $x_i \in \left[0, \frac{1}{2}\right]$ for $i = 1, 2$ and where the positive parameters C and D are assumed to be stochastic variables which are the same for both sellers, realizations of C and D are denoted by c and d.

The idea of such cost functions is that a seller plans for a specific sales volume k_i. The costs for "capacity" k_i are sunk, i.e. must be paid even in case of lower sales than k_i. In case of a positive excess demand $x_i - k_i$ delivery is not excluded, but implies (by $d > 0$) higher unit cost. The assumption that demand must be served avoids complicated and debatable assumptions concerning demand rationing in case of excess demand and, even more importantly, subgames with equilibria in mixed pricing strategies (see Kreps and Scheinkman, 1983, and the discussion of their model by Davidson and Deneckere, 1986).

Rigid capacity constraints in the sense of $D = +\infty$ are rather unlikely. Nevertheless we readily admit that the higher cost of excess demand $x_i - k_i$ will often question that excess demand is served. Here we concentrate on the possibly less likely situation where one always serves excess demand.

Since the market is homogeneous, a demand function $X(p)$ must assign a total (non-negative) demand level X to any non-negative price p not exceeding the prohibitive price. To allow for a simple and parameter free description we assume a linear demand function whose prohibitive price and satiation level are standardized to 1 (by an appropriate choice of the monetary unit as well as of the unit amount). Thus the **linear demand function** can be written as

(II.2) $$X(p) = 1 - p \text{ for all } 0 \leq p \leq 1.$$

Market clearing implies, of course,

(II.3) $$X(p) = x_1 + x_2.$$

Furthermore, due to the **homogeneity of the market** one has

(II.4) $$p = \min\{p_1, p_2\}$$

where for $i = 1, 2$ the sales price is denoted by p_i. The profit π_i of seller $i = 1, 2$ is determined by k_i, p_i and p as follows:

(II.5) $$\Pi_i(k_i, p_i, p) = \begin{cases} -C_i(k_i, 0) \text{ for } p_i > p = p_j \\ px_i - C_i(k_i, x_i) \text{ for } p_1 = p_2 = p \\ p(1-p) - C_i(k_i, 1-p) \text{ for } p_i = p < p_j \ (j \neq i) \end{cases}$$

where

(II.6) $$x_i = max\left\{0; k_i + \tfrac{1-p-k_1-k_2}{2}\right\}$$

Except for the special case $p_1 = p = p_2$ our assumptions are standard ones. For $p_1 = p = p_2$ demand is distributed such that each seller encounters a demand level as close to his capacity as possible. If, for instance, $X(p) = k_1 + k_2$ both sellers $i = 1, 2$ will sell $x_i = k_i$ even when $k_i \neq k_j$ for $i \neq j$. Thus in case of equal prices total demand is distributed such that no redistribution is profitable. By assumption (II.6) one avoids to model explicitly redistribution with such an outcome.

The **decision process** describes the timing of decisions and what is learnt about them, i.e. when sellers make which choices under which information conditions. The process is graphically illustrated and explained by Figure 1.

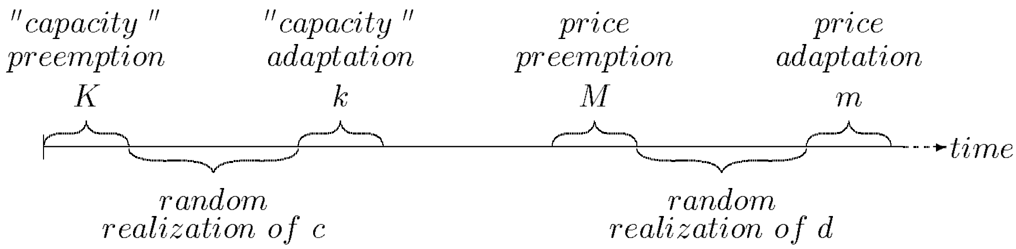

Figure 1: The market decision process (seller $i = 1, 2$ can determine his capacity k_i either in period K or, after the random realization of c, in period k; thereafter the sales prices p_i are chosen either in period M or, after the random choice of d, in period m; when deciding all former decisions are known)

In view of indirect evolution the "decisions" when to determine k_i or p_i are, of course, inherited timing dispositions. Such dispositions represent actual forward looking choices before period K when interpreting our model as an exercise in endogenous timing.

The two stochastic variables C and D are assumed to have distributions concentrated on $\left[\frac{1}{2}; 1\right]$

$$\text{(II.7)} \qquad C \in \left[\tfrac{1}{2}, 1\right] \text{ and } D \in \left[\tfrac{1}{2}, 1\right].$$

We refer to their means as $\bar{c}$ and $\bar{d}$. Of course, one generally needs conditions guaranteeing $0 \leq p \leq 1$ and non-negative individual sales levels as well as capacities. We will confine ourselves to check these conditions for the solution outcomes only. Sellers are assumed to be risk neutral.

3. Optimal behavior for given timing dispositions

What has to be derived here are the optimal capacities k_i^* as well as the optimal prices p_i^* for all situations resulting for the three constellations

$$(m_1, m_2) = \begin{cases} (m, m) \\ (M, M) \\ (M, m) \text{ or } (m, M) \end{cases}$$

of timing dispositions in pricing as well as for the three analogous constellations

$$(n_1, n_2) = \begin{cases} (k, k) \\ (K, K) \\ (K, k) \text{ or } (k, K) \end{cases}$$

of timing dispositions for choosing capacities. Since capacities are known when choosing prices, backward induction in the sense of **subgame perfect equilibria** (Selten, 1965, 1975) requires to solve first the (m_1, m_2)-constellations before investigating capacity choices.

a) The case $(m_1, m_2) = (m, m)$

What one encounters here is a deterministic (both, c and d are commonly known in period m) duopoly market with piecewise linear cost functions. If $p_1 \neq p_2$, one seller would encounter 0-demand. Thus there can be no equilibrium in pure pricing strategies with $p_1 \neq p_2$.

We now consider constellations $p_1 = p_2 = p$. For any capacity vector (k_1, k_2) let $p(k_1, k_2)$ denote the price for which

$$\text{(III.a.1)} \qquad X(p(k_1, k_2)) = k_1 + k_2$$

holds. It is interesting that our model allows for more than just one pricing equilibrium (p_1, p_2) with $p_1 = p_2 = p$. A seller i, who underbids the common price p, has to serve the whole market demand at the lower price p. Thus the positive cost of serving positive excess demand in the sense of

$$X(p_i) = 1 - p_i > k_i$$

can prevent any attempt to underbid a common price $p = p_1 = p_2$. Here we will neglect the multiplicity of pricing equilibria and simply impose the solution $p_1^* = p_2^* = p(k_1, k_2)$ for which we now prove the equilibrium property. The chances of (p_1^*, p_2^*) to result from equilibrium selection will be discussed in Section 6 below. Clearly, $p_i > p_i^*$ can never be optimal. For

$$\text{(III.a.2)} \qquad p_1^* = p_2^* = p(k_1, k_2)$$

to be in equilibrium one therefore only has to guarantee that a marginal price decrease from (III.a.2) is worse than $p_i^* = p(k_1, k_2)$. Comparing

$$\text{(III.a.3)} \qquad p_i^* k_i - C_i(k_i, k_i)$$

and

$$\text{(III.a.4)} \qquad p_i(1 - p_i) - C_i(k_i, 1 - p_i),$$

where p_i is only marginally smaller than $1 - k_i - k_j$, shows that no marginal price cut pays if

$$\text{(III.a.5)} \qquad (c + d) k_j \geq (1 - k_i - k_j) k_j$$

or, for $k_j > 0$,

$$\text{(III.a.5')} \qquad (c + d) \geq p(k_1, k_2) = 1 - k_1 - k_2.$$

Due to $c, d \geq \frac{1}{2}$ and $k_i \geq 0$ for $i = 1, 2$ this condition is always fulfilled.

b) The case $(m_1, m_2) = (M, M)$

All what is changed here compared to the case $(m_1, m_2) = (m, m)$ is that sellers $i = 1, 2$ do not know the actual realization of d. Proceeding in the same way thus yields the condition

(III.b.1) $c + \overline{d} \geq 1 - k_1 - k_2$

which is less stringent than (III.a.5') and thus always fulfilled.

c) The case $(m_1, m_2) = (M, m)$

Assume that seller 1 does not know d, but seller 2 does. Clearly, in case of (III.a.5') and thus also of (III.b.1) neither seller has an incentive to slightly undercut the price $p(k_1, k_2)$. When deriving the results for the various timing constellations of "capacity" choices the results of the later decision stage are anticipated.

d) The case $(n_1, n_2) = (k, k)$

In case of $(n_1, n_2) = (k, k)$ capacities are chosen knowing the realization c. Assuming that always the prices $p(k_1, k_2)$ result (see the section a), b) and c) above) seller i's profit depends on k_i as follows:

(III.d.1) $\Pi_i(k_i, k_j) = (1 - k_i - k_j) k_i - ck_i$

for $i = 1, 2$ and $j \neq i$. The equilibrium choices are

(III.d.2) $k_i^* = \frac{1-c}{3}$ for $i = 1, 2$,

i.e. the well-known duopoly solution (Cournot, 1838). Since $c \leq 1$ the optimal "capacities" are non-negative. The profit expectations resulting from (III.d.2) are

(III.d.3) $E\left\{\left(\frac{1-c}{3}\right)^2\right\}$ for $i = 1, 2$

where $E\{\cdot\}$ denotes the expectation operator.

e) The case $(n_1, n_2) = (K, K)$

Here c in (III.d.2) must simply be substituted by its average $\overline{c}$ so that

(III.e.1) $\qquad k_i^* = \frac{1-\overline{c}}{3}$ for $i = 1, 2$

guaranteeing a well-defined solution for all realizations of $C \in \left[\frac{1}{2}, 1\right]$ and all distributions of C on $\left[\frac{1}{2}, 1\right]$. The profit expectations resulting from (III.e.1) are

(III.e.2) $\qquad \left(\frac{1-\overline{c}}{3}\right)^2$ for $i = 1, 2$

which are obviously non-negative since $\overline{c} \in \left[\frac{1}{2}, 1\right]$.

f) The case $(n_1, n_2) = (K, k)$

Assume that seller 1 does not yet know c whereas seller 2 does. From

(III.f.1) $\qquad \Pi_2(k_1, k_2) = (1 - k_1 - k_2)\, k_2 - ck_2$

one obtains seller 2's reaction function

(III.f.2) $$k_2^*(k_1) = \begin{cases} \frac{1-c-k_1}{2} & \text{for } 0 \leq k_1 \leq 1-c \\ 0 & \text{for } k_1 > 1-c. \end{cases}$$

In order to determine seller 1's optimal capacity level k_1^* we first have to calculate his expected profit as a function of k_1. This function $E\{\pi_1(k_1)\}$ depends crucially on the distribution of C. In the following we will carry out the analysis for various different distributions. First we examine the case in which the distribution of C has a continuous Lebesgue-density and solve the special case of a uniform distribution explicitly. Then, as an example for measures with finite support, we study one-point measures and give one numerical example for a two-point measure.

f1) Continuous distributions

In this case seller 1's expected profit is

$$\text{(III.f.3)} \qquad E\left\{\Pi_1(k_1)\right\} = \begin{cases} \int\limits_{\frac{1}{2}}^{1-k_1} \left(1 - k_1 - \frac{1-c-k_1}{2}\right) k_1 \varphi(c)\, dc + \\ \\ \int\limits_{1-k_1}^{1} (1-k_1) k_1 \varphi(c)\, dc - k_1 \int\limits_{\frac{1}{2}}^{1} c\varphi(c)\, dc. \end{cases}$$

where $\varphi(\cdot)$ denotes the density of C on $\left[\frac{1}{2}, 1\right]$ and $\Psi(\cdot)$ its distribution with $\Psi'(c) = \varphi(c)$ and $\Psi\left(\frac{1}{2}\right) = 0$, $\Psi(1) = 1$. Equation (III.f.3) can be rewritten as

$$\text{(III.f.3')} \qquad E\left\{\Pi_1(k_1)\right\} = \begin{cases} \int\limits_{\frac{1}{2}}^{1-k_1} \frac{1-k_1}{2} k_1 \varphi(c)\, dc + \frac{k_1}{2} \int\limits_{\frac{1}{2}}^{1-k_1} c\varphi(c)\, dc \\ \\ + (1-k_1) k_1 \int\limits_{1-k_1}^{1} \varphi(c)\, dc - \bar{c} k_1 \end{cases}$$

or

$$\text{(III.f.3'')} \qquad E\left\{\Pi_1(k_1)\right\} = \begin{cases} -\frac{1-k_1}{2} k_1 \Psi(1-k_1) + \frac{k_1}{2} \int\limits_{\frac{1}{2}}^{1-k_1} c\varphi(c)\, dc \\ \\ + (1-k_1) k_1 - \bar{c} k_1 \end{cases} .$$

From the first order condition of a local maximum of (III.f.3) one obtains

$$\text{(III.f.4)} \qquad 1 - 2k_1 - \left(\tfrac{1}{2} - k_1\right) \Psi(1-k_1) + \tfrac{1}{2} \int\limits_{\frac{1}{2}}^{1-k_1} c\varphi(c)\, dc = \bar{c}$$

as an implicit formula for the interior maximum k_1^*. Moreover

$$\frac{d^2}{dk_1^2} E\left\{\pi_1(k_1)\right\} = -2 - \Psi(1-k_1) - \frac{k_1}{2} \varphi(1-k_1) - \bar{c} < 0 \text{ for all } k_1 \in \left[0, \frac{1}{2}\right].$$

Thus $E\left\{\pi_1(k_1)\right\}$ is strictly concave, i.e. the first order conditions are necessary and sufficient for a global maximum, and the left-hand side of equation (III.f.4) is strictly decreasing. By inserting $k_1 = 0$ and $k_1 = \frac{1}{2}$ into (III.f.4) one can now easily show that, for every continuous distribution of C, the equation has a unique solution in $\left[0; \frac{1}{2}\right]$. For the special case of the uniform density $\varphi(\cdot)$ on $\left[\frac{1}{2}, 1\right]$, i.e. $\Psi(c) = 2\left(c - \frac{1}{2}\right)$ for $\frac{1}{2} \leq c \leq 1$, this implies, for instance,

$$\text{(III.f.5)} \qquad k_1^2 + \tfrac{2}{3} k_1 = \tfrac{7}{12} - \tfrac{2}{3}\bar{c} = \tfrac{7}{12} - \tfrac{2}{3} \cdot \tfrac{3}{4} = \tfrac{1}{12}$$

or

(III.f.5') $\quad k_1^* = -\frac{1}{3} + \frac{\sqrt{25-24\bar{c}}}{6} = \frac{\sqrt{7}-2}{6} = .1076252.$

Let Π_1^* denote the profit expectation of seller 1 resulting from k_1^* and the subsequent choice $k_2^*(k_1^*)$ by seller 2. Let, furthermore, Π_2^* be the corresponding profit expectation of seller 2 who is second in determining the sales capacity. For the special case of the uniform density $\varphi(\cdot)$ on $\left[\frac{1}{2}, 1\right]$ we obtain

(III.f.6) $\quad E\{\Pi_1^*\} = k_1^* \left[(1-k_1^*)\left(1+\frac{k_1^*}{2}\right) - \frac{7}{8}\right] \approx 0.007$

and

(III.f.7) $\quad E\{\Pi_2^*\} = (1-k_1^*)\left[\frac{1}{8} - \frac{1}{6}(1-k_1^*)\left(\frac{1}{2}+k_1^*\right)\right] - \frac{1}{48} \approx 0.01,$

i.e. if preemption takes place, on average, the seller, who first determines his capacity, is worse off.

f2) One-point measures

We now look at the situation where the distribution of C is a one-point measure, i.e. where the cost parameter c is no longer random. Obviously this case is equivalent to the classical (von Stackelberg, 1934) leadership model.

Maximizing seller 1's expected profit function

$$E\{\pi_1(k_1)\} = \begin{cases} (1-k_1-k_2^*(k_1)) \cdot k_1 - \bar{c} \cdot k_1 & \text{if } 0 \leq k_1 \leq 1-\bar{c} \\ (1-k_1) \cdot k_1 - \bar{c} \cdot k_1 & \text{if } k_1 > 1-\bar{c} \end{cases}$$

leads to

(III.f.8) $\quad k_1^* = \frac{1-\bar{c}}{2}$

(III.f.9) $\quad k_2^* = \frac{1-\bar{c}}{4}$

(III.f.10) $$E\{\pi_1^*\} = \frac{(1-\overline{c})^2}{8}$$

and

(III.f.11) $$E\{\pi_2^*\} = \frac{(1-\overline{c})^2}{16}.$$

Clearly, in contrast to the case of a uniform distribution, the position of the leader is more profitable than the one of the follower.

f3) Two-point measures

As a final example we analyze the case where C can have exactly two values c_1 and c_2 with $c_1 < c_2$. Since solving the maximization problems for arbitrary two-point measures involves a complex case distinction, we restrict ourselves to one numerical example. Let the distribution of C be the measure assigning probability $1/2$ to the numbers $c_1 = 0.69$ and $c_2 = 0.81$, respectively.

The expected profit of seller 1 is

(III.f.12) $$E\{\pi_1(k_1)\} = \begin{cases} \frac{k_1}{2} \cdot \{1 - k_1 - \overline{c}\} & \text{if } c_2 \leq 1 - k_1 \\ k_1 \cdot \left\{\frac{3}{4}(1 - k_1) + \frac{1}{4}c_1 - \overline{c}\right\} & \text{if } c_2 > 1 - k_1 \text{ and } c_1 \leq 1 - k_1 \\ k_1 \cdot \{1 - k_1 - \overline{c}\} & \text{if } c_1 > 1 - k_1 \end{cases}$$

By maximizing this function we obtain

(III.f.13) $$k_1^* = \frac{1}{8}$$

(III.f.14) $$k_2^* = \begin{cases} 0.0925 & \text{if } c = c_1 \\ 0.0325 & \text{if } c = c_2 \end{cases}$$

(III.f.15) $$E\{\pi_1^*\} = \frac{1}{128} \approx 0.0078$$

(III.f.16) $$E\{\pi_2^*\} \approx 0.0048$$

Here again, as in the case of one-point measures, the seller who preempts is more successful than his opponent.

4. The truncated or evolutionary game

In general, the market decision process in Figure 1 allows for four constellations of individual timing dispositions, namely (K, M), (K, m), (k, M), and (k, m). Here the task of deriving the stable timing dispositions is, however, reduced to the problem whether both will be of type K or k or whether the bimorphisms with one being of type K and the other of type k are stable. This is implied by our restriction to a parameter region where equilibrium prices on the second stage (of the Kreps-Scheinkman-model) induce full capacity utilization, i.e. $p_i^* = p(k_1, k_2)$ for $i = 1, 2$. If this holds for all constellations (k_1, k_2) of capacities, it obviously does not matter whether one is earlier in price setting and whether this is done before or after the parameter d is randomly determined.

For stable timing monomorphisms (K^*, K^*) or (k^*, k^*) or stable timing bimorphisms (K^*, k^*) or (k^*, K^*) we offer two (related) interpretations: According to indirect evolution stability means that timing dispositions regarding capacities are not consciously chosen, but rather evolve. The stable timing constellation is thus viewed as the final result of an evolutionary process.

Endogenous timing assumes instead that timing dispositions are consciously and independently determined before stage K in Figure 1 and then publicly announced. Thus the following symmetric 2-person matrix game is the truncation, i.e. the game describing seller 1's incentives for all four constellations (k, k), (k, K), (K, k), and (K, K) by anticipating the rational decision behavior as derived in section 3 above. In other words: The entries of Table 2 are just the expected payoffs of seller 1 resulting from the optimal capacity vectors (k_1^*, k_2^*) and the resulting equilibrium prices $p_1^* = p_2^* = p(k_1^*, k_2^*)$ for the four possible constellations (k, k), (k, K), (K, k), and (K, K) of timing dispositions.

seller 1 \ seller 2	k	K
k	$\frac{E\left\{(1-c)^2\right\}}{9}$	$E\left\{\Pi_2^*\right\}$
K	$E\left\{\Pi_1^*\right\}$	$\frac{(1-\bar{c})^2}{9}$

Table 2: The symmetric evolutionary game or truncation as defined by Π_1^*

In view of indirect evolution the symmetric matrix game in Figure 2 is an evolutionary game describing the (reproductive) success of mutants k or K when confronting another individual which can be of type k or K, too (see Hammerstein and Selten, 1994, for a survey of evolutionary game theory and Güth and Kliemt, 1998, for a conceptional discussion of indirect evolution).

For the case at hand the distinction between indirect evolution and endogenous timing matters more for the interpretation rather than for the nature of the results. To give an example assume that (k^*, K^*) and thus (due to symmetry) also (K^*, k^*) is stable. For indirect evolution this would mean that both bimorphisms are stable and that it depends on the initial state of the evolutionary process and possibly on random results which of the two will actually prevail, i.e. the final results would be path dependent.

In view of endogenous timing such a result would be more troublesome since one cannot recommend which timing disposition a seller should choose without applying a theory of equilibrium selection (e.g. Harsanyi and Selten, 1988). Notice, furthermore, that there would also exist a mixed strategy equilibrium which one might want to consider as a normative prescription, especially in view of the symmetry of the game.

For the special case of the uniform distribution with $\varphi(c) = 2$ for all $\frac{1}{2} \leq c \leq 1$ Table 2 becomes

seller 2 / seller 1	k	K
k	$\frac{1}{108} = .009$	$.01$
K	$.007$	$\frac{1}{144} = .007$

Table 2': The evolutionary game or truncation of Table 2 for the special case of the uniform density $\varphi(c) = 2$ for $\frac{1}{2} \leq c \leq 1$

If C is constant, i.e. the distribution of C is a one-point measure with $C \equiv \bar{c}$, Table 2 is of the form

seller 2 / seller 1	k	K
k	$\frac{(1-\bar{c})^2}{9}$	$\frac{(1-\bar{c})^2}{16}$
K	$\frac{(1-\bar{c})^2}{8}$	$\frac{(1-\bar{c})^2}{9}$

Table 2": The evolutionary game for the case of a one-point distribution $C \equiv \bar{c}$

For the numerical case of the two-point measure introduced in section 3.f3) Table 2 is of the form

seller 2 / seller 1	k	K
k	0.0073	0.0048
K	0.0078	0.0069

Table 2"': The evolutionary game for the two-point measure where $C = 0.69$ and $C = 0.81$, each with probability $\frac{1}{2}$

We now turn to the question whether k or K will finally evolve, respectively be chosen strategically.

5. The evolutionarily stable or optimal timing constellation

For the special case of the uniform density the unambiguous result is the constellation (k, k). This is true since the expected profit for k is always larger than the one for K (see Table 2'). Thus k strictly dominates K and is both, the unique evolutionarily stable strategy of Table 2' as well as the only optimal timing disposition.

If the distribution of C is concentrated on $\overline{c}$ we again obtain a unique evolutionarily stable strategy as long as $\overline{c} \neq 1$. Table 2" shows that K strictly dominates k. This result is very intuitive: The realization of c is of no interest at all, thus (K, K) and (k, k) lead to equal profits. But choosing K instead of k offers each seller the chance to substitute a (Cournot) duopoly by a (von Stackelberg) leadership model with him in the leading position and avoids the risk of ending up as a follower.

Table 2"' proves that in our example of a two-point measure, as in the case of one-point measures, K strictly dominates k. Whereas in the deterministic case $C \equiv \overline{c}$ the capacities in the unique strict equilibrium (K, K) equaled those for the timing disposition (k, k), we have now found an example where ex post equilibrium capacities deviate from the deterministic (Cournot, 1838) duopoly solution $k_i = \frac{1-c}{3}$. What is justified by (K, K) is the analogous solution of a stochastic duopoly market with cost uncertainty where sellers must choose their sales amounts before learning how costly this is. This result does not hold for all two-point measures. For other values of c_1 and c_2 and other probabilities for these states (k, k) can also be a unique strict equilibrium. Unfortunately we could neither find nor exclude bimorphic equilibria.

More generally, stability or optimality of (k, k) in the sense of (k, k) being a strict equilibrium of Table 2 requires

$$\text{(V.1)} \qquad E\left\{(1-c)^2\right\}/9 > E\left\{\Pi_1^*\right\}.$$

For (K, K) the condition is

(V.2) $\frac{(1-\overline{c})^2}{9} > E\{\Pi_2^*\}$

whereas for the bimorphism (k, K) or (K, k) the conditions are

(V.3) $E\{\Pi_1^*\} > E\left\{(1-c)^2\right\}/9$ and $E\{\Pi_2^*\} > \frac{(1-\overline{c})^2}{9}$.

6. Equilibrium selection for price competition or homogeneous markets

It has already been mentioned that due to the obligation to serve all customers price competition on homogeneous markets may have other equilibria than just $p_1^* = p_2^* = p(k_1, k_2)$ on which our previous analysis has been based. Here we want to investigate this possibility in more detail. By applying the theory of equilibrium selection (Harsanyi and Selten, 1988) we also explore whether one can justify our solution candidate $p_1^* = p_2^* = p(k_1, k_2)$. As we have seen before, in equilibrium both sellers set the same price p. We first compare the equilibrium $p_1 = p_2 = p(k_1, k_2)$ to equilibria $p_1 = p_2 = p$ with $p < p(k_1, k_2)$, then we analogously analyse the case $p > p(k_1, k_2)$.

Case VI.1: $p < p(k_1, k_2)$

Let (p_1, p_2) be an arbitrary strategy constellation $p_1 = p_2 = p$ with $p < p(k_1, k_2)$. Analogously to the proof of the equilibrium property of $p_1^* = p_2^* = p(k_1, k_2)$ in section 3 one can show that this constellation is an equilibrium if and only if, at price p, production beyond the capacity k_i causes losses. More formally, $p_1 = p_2 = p$ with $p < p(k_1, k_2)$ is a strict equilibrium if and only if

$$p(1-p) - (c + d_i)(1 - p - k_i) < pk_i + (p - c - d_i)\frac{1 - p - k_1 - k_2}{2} \text{ for } i = 1, 2$$

or

(VI.1.A) $p < c + d_i$ for $i = 1, 2$

where d_i is the value of D individual i expects when setting his price, i.e. $\overline{d}$ if seller i has timing disposition M and d if this timing disposition is m.

Since we only want to compare $p(k_1, k_2)$ to other equilibria, we focus on those prices p which fulfill the equilibrium condition (VI.1.A). In the cases where timing dispositions are asymmetric, i.e. $(m_1, m_2) = (m, M)$ or (M, m), it is obvious which of the two equilibrium prices p and $p(k_1, k_2)$ should be selected, namely the one which is payoff dominant. Anticipating that the second mover sets the same price the first mover clearly prefers the price which yields higher profits for both sellers.

It remains to analyse the symmetric cases (M, M) and (m, m), i.e. the situations where $d_1 = d_2 = \overline{d}$, respectively $d_1 = d_2 = d$ with d denoting the realization of the random variable D. The payoff implications of all price constellations p_i with $p_i = p(k_1, k_2)$ or $p_i = p$ for $i = 1, 2$ can be represented as a 2 x 2-bimatrix game (Table 3) where one can neglect the cost of "capacity" k_i since these cost are sunk on the price setting stage. Of course, one cannot neglect cost which result from selling more than one's capacity.

p_1 \ p_2	$p(k_1, k_2)$	p
$p(k_1, k_2)$	$(1 - k_1 - k_2) k_1$ $(1 - k_1 - k_2) k_2$	0 $(1 - p)(p - c - d_2) + (c + d_2) k_2$
p	$(1 - p)(p - c - d_1) + (c + d_1) k_1$ 0	$pk_1 + (p - c - d_1) \frac{1-p-k_1-k_2}{2}$ $pk_2 - (p - c - d_2) \frac{1-p-k_1-k_2}{2}$

Table 3: The 2 x 2-bimatrix game for $p < p(k_1, k_2)$

The equilibrium (p, p) would be **payoff dominated** by (p_1^*, p_2^*) when

(VI.1) $$(1 - k_1 - k_2) k_i > pk_i + (p - c - d_i) \tfrac{1-p-k_1-k_2}{2}$$

or

$$\text{(VI.1')} \qquad (1-p-k_1-k_2)(2k_i+c+d_i-p)>0$$

holds for $i=1,2$. It is interesting that here the condition for payoff dominance depends on the sum $c+d_i$ whereas the corresponding condition (VI.7) for the case $p>p(k_1,k_2)$ is completely independent of the cost parameters. Due to $1-p>k_1+k_2$ this is equivalent to

$$\text{(VI.2)} \qquad 2k_i+c+d_i>p.$$

Due to $k_i \geq 0$ and the equilibrium condition (VI.1.A) this is always fulfilled. This shows that strict equilibria (p_1,p_2) with $p_1=p_2<p(k_1,k_2)$ are always payoff dominated by (p_1^*,p_2^*).

Payoff dominance completely neglects the risks implied by coordinating on a specific strict equilibrium. Such risks are, however, carefully considered by **risk dominance** (Harsanyi and Selten, 1988) which, for the case at hand, is axiomatically characterized by three requirements, namely best reply and isomorphic invariance and monotonicity. Actually the axioms can be constructively used when deriving which of the two strict equilibria risk dominates the other.

p_1 \ p_2	$p(k_1,k_2)$	p
$p(k_1,k_2)$	$(1-k_1-k_2-c-d_1)k_1-(1-p)(p-c-d_1)$ $(1-k_1-k_2-c-d_2)k_2-(1-p)(p-c-d_2)$	0 0
p	0 0	$pk_1+(p-c-d_1)\frac{1-p-k_1-k_2}{2}$ $pk_2+(p-c-d_2)\frac{1-p-k_1-k_2}{2}$

Table 4: A best reply invariant transformation of Table 3

The bimatrix game of Table 4 results from Table 3 by subtracting the non-equilibrium payoff from the equilibrium payoff for a given strategy of the other seller, i.e. after such a transformation the non-equilibrium payoff for a given strategy of the other player is 0. The mixed strategy equilibrium in Table 4 is the same as in Table 3. Thus both games have the same best reply structure, i.e. the same stability sets (the sets of mixed strategy vectors to which every component of a pure strategy vector is a best reply). By best reply invariance we can thus solve the game in Table 4 instead of the one in Table 3.

p_1 \ p_2	$p(k_1, k_2)$	p
$p(k_1, k_2)$	X, 1	0, 0
p	0, 0	1, Y

Table 5: An isomorphic transformation of Table 4 where

(VI.3) $$X = \frac{(1-k_1-k_2-c-d_1)k_1-(1-p)(p-c-d_1)}{pk_1+(p-c-d_1)\frac{1-p-k_1-k_2}{2}}$$

and

(VI.4) $$Y = \frac{pk_2+(p-c-d_2)\frac{1-p-k_1-k_2}{2}}{(1-k_1-k_2-c-d_2)k_2-(1-p)(p-c-d_2)}.$$

Finally, the game of Table 5 results from Table 4 by positively affine transformations of payoff functions, i.e. by an isomorphic transformation. If $X = Y$ would hold, isomorphic invariance in the form of symmetry invariance would prescribe the mixed strategy equilibrium as the solution. If, however, $X \neq Y$ monotonicity prescribes (p_1^*, p_2^*) for $X > Y$ and (p_1, p_2) for $Y > X$ as the solution. A change from a game with $X^0 = Y^0$, where no strict equilibrium is the solution, to

$X > X^0 = Y^0$ can be seen as strengthening the strict equilibrium (p_1^*, p_2^*) since player 1's incentive to coordinate on (p_1^*, p_2^*) is increased. Monotonicity requires that such strengthening makes (p_1^*, p_2^*) the solution. Now the condition $X > Y$ is equivalent to

$$\text{(VI.5)} \qquad \prod_{i=1}^{2} \left[(1-k_1-k_2-c-d_i)\,k_i-(1-p)\,(p-c-d_i)\right] > \prod_{i=1}^{2} \left[pk_i+(p-c-d_i)\,\tfrac{1-p-k_1-k_2}{2}\right].$$

or equivalently

$$\text{(VI.6)} \qquad \prod_{i=1}^{2} \left\{(c+d-p)\,(1-p)-k_i\left[c+d-p\,(k_1,k_2)\right]\right\} > \prod_{i=1}^{2} \left\{(c+d-p)\,\tfrac{p(k_1,k_2)-p}{2}-pk_i\right\}$$

revealing a complicated dependence on the various parameters k_1, k_2, $d = d_1 = d_2$, and p with $p < p\,(k_1, k_2)$.

Case VI.2: $p > p\,(k_1, k_2)$

As for the other case we first state the condition under which $p_1 = p_2 = p$ is an equilibrium and then explore the conditions for payoff and risk dominance of (p_1^*, p_2^*) over (p_1, p_2) for the cases of symmetric timing dispositions (M, M) and (m, m). In the asymmetric cases again the payoff dominant equilibrium will be selected.

The indicator function 1_A assumes the value 1 on A and 0 otherwise. The constellation $p_1 = p_2 = p$ is an equilibrium if and only if

$$p\,(1-p)-(c+d_i)\,(1-p-k_i)\cdot 1_{\{1-p-k_1>0\}} < p\frac{1-p+k_i-k_j}{2}$$

for $i = 1, 2$ and $j \neq i$ or

(VI.2.A) $1 - p > k_i$ for $i = 1, 2$

and

(VI.2.B) $(c+d_i) > \frac{p}{2}\left(1+\frac{k_j}{1-p-k_i}\right)$ for $i,j=1,2$ with $i \neq j$.

As mentioned earlier, the multiplicity of equilibria is due to the assumption that a seller i with the lower price must serve the whole market. If total demand exceeds his capacity, i.e. condition (VI.2.A) is fulfilled, then the cost of the excess demand $1-p-k_i$ might be higher than the additional surplus resulting from underbidding the common price. For this (VI.2.B) is a necessary and sufficient condition. The payoffs of all price constellations p_i with $p_i = p(k_1,k_2)$ or $p_i = p$ are again represented as a 2 x 2-bimatrix game in Table 6. As in Table 3 the cost of k_i is neglected. Clearly (p_1^*, p_2^*) and (p,p) are both strict equilibria of this game.

p_1 \ p_2	$p(k_1,k_2)$	p
$p(k_1,k_2)$	$(1-k_1-k_2)\,k_1$ $(1-k_1-k_2)\,k_2$	$(1-k_1-k_2)(k_1+k_2)-(c+d_1)\,k_2$ 0
p	0 $(1-k_1-k_2)(k_1+k_2)-(c+d_2)\,k_1$	$p\frac{1-p+k_1-k_2}{2}$ $p\frac{1-p-k_1+k_2}{2}$

Table 6: The 2 x 2-bimatrix game for $p > p(k_1,k_2)$

The strict equilibrium (p_1^*, p_2^*) with $p_1^* = p_2^* = p(k_1,k_2)$ payoff dominates (p_1,p_2) with $p_1 = p_2 = p > p(k_1,k_2)$ when

(VI.7) $\qquad (1-k_1-k_2)\,k_i > p\frac{1-p+k_i-k_j}{2}$ for $i,j=1,2$ and $i \neq j$.

For risk dominance of (p_1^*, p_2^*) one needs again that $X > Y$ where now

(VI.8) $\qquad X = \frac{(1-k_1-k_2)k_1}{p\frac{1-p+k_1-k_2}{2}-(1-k_1-k_2)(k_1+k_2)+(c+d_1)k_2}$

and

$$\text{(VI.9)} \qquad Y = \frac{p\frac{1-p+k_2-k_1}{2} - (1-k_1-k_2)(k_1+k_2) + (c+d_2)k_1}{(1-k_1-k_2)k_2}.$$

To illustrate the chances that (p_1^*, p_2^*) payoff dominates (p_1, p_2) assume $k_1 = k_2 = k$ so that (VI.7) simplifies to

$$\text{(VI.10)} \qquad (1-2k)\,2k > p\,(1-p).$$

Thus payoff dominance typically depends on whether (p_1^*, p_2^*) or (p_1, p_2) generates the larger total revenue. In other words: Only capacity vectors (k_1, k_2), whose sums do not exceed the monopoly supply of $1/2$, cannot be payoff dominated by price vectors (p_1, p_2) with $p_1 = p_2 = p > p\,(k_1, k_2)$.

A larger sum $c + d_1 = c + d_2$ reduces X and increases Y. It thus does not only improve the chances of an alternative strict equilibrium (p_1, p_2) according to (VI.2.B), but also that this alternative solution risk dominates the solution (p_1^*, p_2^*).

By this we only wanted to illustrate how equilibrium selection can be used to derive a unique solution of the price competition subgames and on which parameters it will depend on whether the equilibrium (p_1^*, p_2^*) with $p_1^* = p_2^* = p\,(k_1, k_2)$ is selected or not. A complete solution for all possible subgames would overburden our study since due to the many parameters (k_1, k_2, c, d) defining such subgames and the many alternative common prices $p \neq p\,(k_1, k_2)$ it would have to rely on a (too) complicated case distinction.

7. Conclusions

Quantity competition requires special institutions like commodity or stock exchanges which, when they exist, should be appropriately captured by the market model. The alternative is to rely on the natural and intuitive idea of Kreps and

Scheinkman (1983) who justify quantity competition by a two stage-decision process of simultaneous decisions (first capacities, then sales prices).

Now simultaneous decision making on two stages is just one of several possibilities. In our model it is possible to preempt on both stages. Even when sellers decide simultaneously, they can do this early (before the random choice of cost) or later (after this chance move). What one would have hoped for is a clear advantage of either preemption or adaptation. Actually, uniform density (and because of the strict dominance of k in Figure 2 also nearby densities) of the decisive cost parameter c implies such a result: Both sellers prefer to adapt, i.e. one does not want to precommit to a certain capacity before the other and before learning the common cost level.

Unfortunately, other constellations of timing dispositions can result for other densities. Thus no general conclusion seems possible. Especially, up to now we have no general result justifying or rejecting the implicit assumption of Kreps and Scheinkman (1983) that sellers decide simultaneously. More specifically, we could not prove the impossibility of stable bimorphisms (k, K) nor find an example for which such bimorphisms are stable.

For all specific distributions, analysed here, the stable process of market decisions is essentially unique. The troublesome ambiguity, however, remains although now at a deeper level. Different cost densities might imply different market decision processes. Simultaneous or independent market decisions can, furthermore, avoid cost uncertainty (when both sellers adapt) or not (when both sellers preempt).

Our model differs from the one of Kreps and Scheinkman (1983) mainly by our different interpretation of "capacities" which appears more natural (usually production and sales can be more or less easily varied even beyond their planned levels) and greatly simplifies our analysis. Actually, an analysis like ours for the original Kreps and Scheinkman-model is very difficult or even practically impossible. Whereas our model can be easily extended to oligopoly markets (see Güth, 1995), the results of Kreps and Scheinkman (1983) still await such a generalization.

Such advantage has, of course, its price. The price which we have to pay are three crucial and debatable assumptions, namely: (i) In case of equal prices demand is distributed according to capacities (see equation (II.6)). (ii) Sellers must serve demand even beyond their planned sales level. (iii) On the price setting stage the solution $p_1^* = p_2^* = p(k_1, k_2)$ is just an ad hoc-selection (see section 6 above). One can justify (i) as a result of renegotiating the demand distribution by the two sellers. Assumption (ii) could be valid when not serving a customer would result in losing him forever. The ad hoc-selection (iii) relies mainly on the intuition that sellers coordinate on prices which fulfill their initial expectations as represented by their total planned sales level $k_1 + k_2$. Thus the three crucial assumptions are not outrageous, but their main justification is, of course, that they greatly simplify the analysis.

References

[1] Cournot, A. (1838): *Recherches sur les principes mathématiques de la théorie des riches*, Paris, English translation: (N. Bacon, trans.), Researches into the Mathematical Principles of the Theory of Wealth, New York: Macmillan & Company, 1897.

[2] Davidson, C. and R. Deneckere (1986): Long-run competition in capacity, short-run competition in price, and the Cournot model, *Rand Journal of Economics*, 17, 404 - 415.

[3] Güth, W. (1995): A simple justification of quantity competition and the Cournot-oligopoly solution, *ifo Studien*, 41/2, 245 - 257.

[4] Güth, W. and H. Kliemt (1998): The indirect evolutionary approach: Bridging the gap between rationality and adaptation, *Rationality and Society*, 10 (3), 377 - 399.

[5] Hammerstein, P. and R. Selten (1994): Game theory and evolutionary biology, in: *Handbook of Game Theory*, Vol. 2, R. Aumann and S. Hart (eds.) Amsterdam: Elsevier Science, 929 - 993.

[6] Harsanyi, J. C. and R. Selten (1988): *A general theory of equilibrium selection in games*, Cambridge Mass: M.I.T. Press.

[7] Kreps, D. and J. Scheinkman (1983): Quantity precommitment and Bertrand competition yield Cournot outcomes, *Bell Journal of Economics*, 14, 326 - 337.

[8] Selten, R. (1965): Spieltheoretische Behandlung eines Oligopolmodells mit Nachfrageträgheit, Teil I: Bestimmung des dynamischen Preisgleichgewichts, Teil II: Eigenschaften des dynamischen Preisgleichgewichts, *JITE (Zeitschrift für die gesamte Staatswissenschaft)*, 121, 301 - 324, 667 - 689.

[9] Selten, R. (1975): Reexamination of the perfectness concept for equilibrium points in extensive games, *International Journal of Game Theory*, 4, 25 - 55.

[10] van Damme, E. and S. Hurkens (1996): Endogeneous Stackelberg Leadership, *CentER Discussion Paper*, No. 96115, Tilburg University.

[11] von Stackelberg, H. (1934): *Marktform und Gleichgewicht*, Wien: Verlag von Julius Springer.