A Money Demand System for M3 in the Unified Germany*

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Abstract

A small macroeconomic model is constructed starting from a German money demand relation for M3 based on quarterly, seasonally unadjusted data for the period from 1976 to 1996. In contrast to previous studies we build a vector error correction model for M3, GNP, an inflation rate and an interest rate spread variable to represent opportunity costs of holding money. Furthermore, import price inflation is added as an exogenous variable. The model is used to analyze the relation between money growth and inflation by means of an impulse response analysis.

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1 Introduction

For more than 20 years the Deutsche Bundesbank has used a money growth target to control the price level in Germany. A stable money demand relation is an important prerequisite for such a policy strategy. Since the late eighties this policy is based on the money stock variable M3. Therefore a number of studies have analyzed the stability of money demand relations for M3 for the unified Germany (e.g., Tullio, de Souza & Giucca (1996), Hansen & Kim (1995), Issing & Tödter (1995), Deutsche Bundesbank (1995), Scharnagl (1996) and Kole & and Meade (1995)). All these studies focus on a single equation analysis. Stability of a money demand relation can be viewed as a necessary condition for using a money growth target for controlling the price level because without such a relation the current monetary policy has little theoretical basis. On the other hand, a stable money demand relation is not a sufficient requirement for this policy strategy. In this study we will therefore construct a small macro model for the money stock, income, inflation and an opportunity cost variable for holding money based on an interest rate spread. This system enables us to get a more complete picture of the effects of monetary policy.

Systems analyses for German M3 with a similar objective have also been conducted by Hansen & Kim (1996), Juselius (1996), Hubrich (1996) and Deutsche Bundesbank (1997). All these studies differ in important respects from our present analysis. Hansen & Kim (1996) use seasonally adjusted data and analyze the preunification period only. Obviously, an event like the German unification is expected to result in major shifts and adjustment processes in the economic system. Since the Bundesbank has continued its policy of monetary targeting after the German monetary unification (GMU) on July 1, 1990, it is of importance to check whether the conditions in the underlying economic system are still suitable. Moreover, Hansen & Kim (1996) include the beginning of the seventies in their sample period when the Bundesbank did not pursue a policy of monetary targeting. Including two different policy regimes in the sampling period may distort the results and estimators. It may in fact require appropriate modifications of the model.

Juselius (1996) uses quarterly data from 1975 to 1994 and finds a break in her model in 1983. She focusses on analyzing the possible differences in the monetary mechanisms in the two regimes. The analysis starts from a full vector autoregressive model and centers on the long-run relations. Of course, a shift in a model may be due to many factors including
misspecification. Hubrich (1996) uses quarterly data from 1979 to 1994 and also analyzes the long-run relations in a small money demand system for German M3. She finds some evidence for stable long-run relations for the period before and after the GMU. This obviously contrasts with the results of Juselius (1996). Based on a full vector error correction system the Deutsche Bundesbank (1997) computes impulse responses which show significant effects of shocks in M3 on the price level but not vice versa. This result supports the view that M3 is a useful indicator for controlling inflation. Details of other interactions between the variables of the model are not provided.

For a more complete picture of the channels of monetary policy it is necessary, however, to consider the dynamics of the system in more detail. Therefore we will attempt to specify a small model with a more complete specification of the dynamic structure. We use seasonally unadjusted data for the period 1976(1) to 1996(4). The money demand relation is the central function of interest here. Therefore the ingredients of such a function are the core variables of our system, namely money, income, prices and interest rates. Since M3 is the intermediate target of the Bundesbank we use this variable here as our measure of the money stock. Clearly a model which fully explains all the variables of interest in the money demand relation would require modelling the whole economy. Since this is an infeasible task we focus on the variable of main interest, namely M3, and specify the other variables only partially.

In the next section we will briefly review the theoretical background of our model and the general modelling framework. In Section 3 the empirical model is specified and estimated. An analysis of the dynamic interrelationships between the variables of the model is presented in Section 4 and conclusions are given in Section 5.

2 The Theoretical Model

For a period covering German unification, Wolters, Teräsvirta & Lütkepohl (1997) (henceforth WTL) and Wolters & Lütkepohl (1997) (henceforth WL) find a stable long-run money demand relation for M3 of the following form

\[ (m - p)_t = \beta_1 y_t + \beta_2 \Delta p_t + \beta_3 (R - r)_t + \nu + \varepsilon_t, \]

(2.1)

where \( m_t \) is the logarithm of M3, \( y_t \) is the logarithm of real GNP, \( p_t \) is the logarithm of the GNP deflator, hence, \( (m - p)_t \) is the logarithm of real M3 and \( \Delta p_t := p_t - p_{t-1} \) is the
quarterly inflation rate. Moreover, $R_t$ is a long-term interest rate (‘Umlaufsrendite’) and $r_t$ is the own rate of M3 so that $(R - r)_t$ represents the opportunity costs of holding M3 rather than longer term bonds.¹ The models in WTL and WL are based on quarterly seasonally unadjusted data for the periods 1976(1) to 1994(2) and 1976(1) to 1996(1), respectively. The series are plotted in Fig. 1 for 1976(1) to 1996(4) which is the observation period we will use in the following. Looking at the graphs of the time series $(m - p)_t$ and $y_t$ a remarkable level shift due to the GMU in 1990(3) becomes apparent. It turns out that this feature can be captured by a shift in the intercept term $\nu$ in the relation (2.1). On the other hand, $\Delta p_t$ and $(R_t - r_t)$ do not show apparent breaks due to the GMU.

Obviously, in (2.1) money demand depends on income $y_t$ which represents the transactions volume and on variables representing opportunity costs for holding money. This is obvious for the interest rate differential $(R - r)_t$. There is some ambiguity in the interpretation of the role of the inflation rate $\Delta p_t$ (see also Goldfeld & Sichel (1987)). Normally it is seen as a measure of opportunity costs for holding real assets. However, it may also represent the kind of adjustment process used by agents. More precisely, it may capture whether agents actively adjust their nominal portfolios only or also passively adjust their assets induced by changes in the price level. WL show within a cost minimization framework based on the approaches by Hendry & von Ungern-Sternberg (1981) and Hwang (1985) that therefore the inflation rate enters the long-run relation even if it does not appear in the desired long-run money demand relation.

WTL found $(m - p)_t$, $y_t$ and $\Delta p_t$ to be $I(1)$ variables, that is, variables which are stationary after differencing once, whereas $(R - r)_t$ is stationary. It was also found that there is just one cointegrating relation for $m_t$, $y_t$ and $\Delta p_t$ which may be interpreted as a long-run money demand function. Thus a relation $(m - p)_t - \beta_1 y_t - \beta_2 \Delta p_t$ is stationary if a level shift for the post-GMU period is properly accounted for. Because further stationary variables may influence a cointegration relation and because the interest rate differential turned out to be an important opportunity cost variable in the fully specified equation it was interpreted as part of the long-run relation. Given this relation the central variables in the transmission mechanism for monetary policy are $(m - p)_t$, $y_t$, $\Delta p_t$ and $(R - r)_t$. Hence, in the following we will construct a four equation model for these variables.

¹For the precise definitions of the variables and the data sources see the Appendix.
We will do so in the general context of a multivariate error correction model (ECM) of the form

$$
\Gamma_0 \Delta z_t = \alpha \beta' z_{t-1} + \Gamma_1 \Delta z_{t-1} + \cdots + \Gamma_p \Delta z_{t-p} + \Phi x_t + \Xi D_t + u_t,
$$

(2.2)

where \(z_t = [(m - p)_t, y_t, \Delta p_t, (R - r)_t]'\), \(x_t\) is a vector of exogenous variables, \(D_t\) includes all deterministic terms and \(u_t\) is the error vector which is assumed to be serially uncorrelated with zero mean and constant nonsingular covariance matrix. The \(\Gamma_i\) are structural coefficient matrices which is emphasized by the fact that \(\Gamma_0\) is not a priori assumed to be an identity matrix. The first term on the right-hand side of the system (2.2) is the error correction term with \(\beta' z_{t-1}\) representing the cointegration relations and \(\alpha\) being the loading matrix or vector containing the weights of the cointegration relations in the equations of the system. In our case we have basically just one cointegration relation, namely the long-run money demand function. In addition the interest rate spread \((R - r)_t\) is stationary and, hence, may be regarded as a cointegration relation in a set-up like (2.2). However, alternatively one may just reparameterize a model such as (2.2) by eliminating all differences of stationary variables. Such a reparameterization complicates the notation when the full model is of interest in ECM form. It is quite natural when the individual equations are considered, however. Therefore we will use such a form in the empirical analysis of the next section where we focus on the individual equations of the system. This way we just have to consider one cointegration relation in all the equations.

Because Germany is an open economy we use the logarithm of an import price index \(p_{m_t}\) as an exogenous variable which may be seen as a measure of the real exchange rate and therefore is an important variable for influencing the monetary transmission mechanism in Germany (see also Issing & Tödter (1995), Hansen & Kim (1996) and Deutsche Bundesbank (1997)). Since \(p_{m_t}\) is also \(I(1)^2\) and we have taken care of the nonstationarities of the endogenous variables with a cointegration relation already, we include \(\Delta p_{m_t}\), the inflation rate of import prices, which is a stationary variable. A period of unusually slow and even negative growth of the money stock due to an enormous substitution into short-term interest bearing assets not included in M3 has occurred in 1994 and 1995. WL found that this phenomenon could be well explained by allowing for a higher interest rate elasticity in this

\(^2\)An augmented Dickey-Fuller test for \(\Delta p_{m_t}\) with a constant and four lags shows a \(t\)-value of \(-3.53\), thus rejecting nonstationarity of \(\Delta p_{m_t}\) at the 1% level.
period. This effect can be captured by a variable \(d_t(R - r)_t\), where \(d_t = 1\) for 1994(3) - 1995(4) and zero elsewhere. The variable \(d_t(R - r)_t\) is therefore active for just a period of six quarters. It may also be thought of as a component of the vector of exogenous variables for notational convenience although it is, of course, part of the endogenous variable \((R - r)_t\).

The vector of deterministic terms \(D_t\) includes an intercept term, seasonal dummies and other dummies representing special events. For instance, the shift in the long-run relation due to the GMU has to be matched by a shift dummy and its first differences which results in an impulse dummy for 1990(3) in the short-term part of the model. Since the deterministic terms appearing in the individual equations differ from each other, we will discuss them in more detail when we present the individual equations in the empirical analysis of the next section.

3 The Empirical Model

3.1 The Money Demand Equation

In modelling and estimating the system of interest we use quarterly, seasonally unadjusted data for the period 1976(1) to 1996(4) (84 observations). In addition we use the data for 1975 as presample values whenever they are needed to include lags as regressors. Details on the data sources are given in the appendix. The computations reported in this section were done by EVIEWS. We first estimated the money demand cointegration relation. We do not simply use the relation from WL here because our data set has been extended. Moreover, in that study the maximum lag length of the stationary variables was two. Given the strong seasonal pattern of the inflation rate (see Fig. 1), a lag length of two is clearly insufficient for the full system. Therefore we allow for up to four lags in the multivariate model. Hence, for consistency, we also use four lags of the stationary variables initially in setting up the money demand relation. More precisely, we use the simple Stock (1987) estimation approach, regressing in our case \(\Delta(m-p)_t\) on \((m-p)_{t-1}, y_{t-1}, \Delta p_{t-1}\), all differences of these variables up to lag order four, the interest rate differential up to lag order four, \(d_t(R - r)_t\), an intercept term, seasonal dummy variables (denoted by \(d_{jt}\)), a step dummy
$S90q3_t$ and impulse dummies $I90q3_t$, $I92q4_t$ and $I93q3_t$.\textsuperscript{3} The following equation results by going through a model reduction procedure as in WTL and WL where successively the least significant variables are eliminated:\textsuperscript{4}

\[
\Delta(m - p)_t = -0.104 \ (m - p - y)_{t-1} - 1.399 \Delta p_{t-1} \\
- 0.205 \Delta y_{t-1} \\
- 1.093 \Delta^2 p_t - 0.230 \Delta^2 p_{t-4} \\
- 0.582 (R - r)_{t-1} + 0.390(R - r)_{t-3} - 0.363 (R - r)_{t-4} \\
- 0.407 \delta_{t}(R - r)_t \\
+0.176 - 0.101 \delta_{1t} - 0.072 \delta_{2t} - 0.055 \delta_{3t} \\
+0.014 S90q3_t + 0.131 I90q3_t - 0.022 I92q4_t - 0.018 I93q3_t + \tilde{u}_{mt} \\
\]

\[
\begin{align*}
T &= 84 \ [1976(1) - 1996(4)] \\
R^2 &= 0.93 \quad SE = 0.0062 \quad JB = 1.00 (0.606) \\
LM(1) &= 0.73 (0.40) \quad LM(4) = 0.64 (0.64) \quad LM(8) = 0.56 (0.80) \\
ARCH(1) &= 0.12 (0.73) \quad ARCH(2) = 0.24 (0.79) \quad ARCH(4) = 0.51 (0.73)
\end{align*}
\]

The numbers in parentheses behind the values of the test statistics are the corresponding $p$-values. JB is the Jarque-Bera test for normality, LM(1), LM(4) and LM(8) are Lagrange-multiplier (LM) tests for autocorrelation based on 1, 4 and 8 lags, respectively, and ARCH($k$) is an LM test for autoregressive conditional heteroscedasticity of order $k$, $k = 1, 2, 4$ (see, e.g., Hendry (1995) for more details on these tests). Obviously, in the present case all $p$-

\textsuperscript{3} The precise definition of the dummy variables is as follows: $S90q3_t$ is 1 from the third quarter of 1990 onwards and zero before that quarter. $I90q3_t$, $I92q4_t$ and $I93q3_t$ are 1 in 1990(3), 1992(4) and 1993(3), respectively, and zero elsewhere. Possible reasons for $I92q4_t$ to be important are the September 1992 crisis in the European exchange rate mechanism (ERM) and the reintroduction of a withholding tax on interest income in Germany, effective January 1, 1993, whereas $I93q3_t$ captures a further ERM crisis in July 1993 (see also Kole & Meade (1995) and WL).

\textsuperscript{4}$t$-ratios in parentheses underneath the estimated coefficients.
values exceed usual significance levels substantially and, hence, none of the test statistics is significant at conventional levels. In other words, we conclude that the residuals do not show signs of autocorrelation, conditional heteroscedasticity or nonnormality. Moreover, in Fig. 2 recursive residuals, CUSUM and CUSUM-of-squares tests are presented which overall support a stable relation for the period of interest here.

Note that an income elasticity of one was not rejected by the data and thus we have imposed this restriction by including \((m - p - y)_{t-1}\) as a single regressor. Equation (3.1) is similar to the specifications of WL although it includes further lags of \(\Delta^2 p_t\) and \((R - r)_t\) and thus has richer dynamics. The reason for the differences is again the change in the observation period and the fact that the starting point of our model reduction procedure was a model with four lags of the stationary variables whereas WL include two lags only. We have also used our extended data set to construct a model with at most two lags and ended up with the same specification as WL. Note that lags of \(\Delta pm_t\) were not significant at the 5% level in (3.1) and are therefore omitted. The deterministic terms are the same as in WL.

Normalizing on the coefficient of \((m - p - y)_{t-1}\) in (3.1) gives the following long-run relation:

\[
(m - p)_t = y_t - 13.50 \Delta p_t + 0.14590 q 3_t + \epsilon c_t. \tag{3.2}
\]

Adding the long-run impact of the interest rate differential and using the annual inflation rate \(\pi_t = 4 \Delta p_t\) instead of \(\Delta p_t\) gives the long-run money demand equation

\[
(m - p) = y - 5.35 (R - r) - 3.38 \pi + 0.14590 q 3.
\]

Comparing this relation with the results of WL,

\[
(m - p) = y - 4.84 (R - r) - 3.72 \pi + 0.15590 q 3,
\]

and WTL,

\[
(m - p) = y - 4.32 (R - r) - 3.57 \pi + 0.13590 q 3,
\]

shows that the parameter estimates are very similar. In particular, an income elasticity of one is supported by the data in all three cases.
3.2 The System

Since the estimators of the cointegration parameters obtained from (3.1) are superconsistent we use \( ec_t = (m - p - y)_t + 13.500\Delta p_t - 0.139890q3_t \) as an additional stationary variable in the sequel in specifying the systems equations for \( \Delta(m - p)_t, \Delta^2 p_t, \Delta y_t \) and \( (R - r)_t \) in our model. The starting point of the money demand equation is the same as in the single equation specification whereas ‘reduced form equations’ with four lags of the variables \( \Delta(m - p)_t, \Delta y_t, \Delta^2 p_t, (R - r)_t \) are specified for the remaining endogenous variables. Moreover, \( ec_t-1, d_t(R - r)_t \) as well as the deterministic terms are included in the initial equations. As in the single equation analysis we then eliminate insignificant variables successively according to the lowest \( t \)-values but always keeping the error correction term in each equation until the end. Then it was eliminated if it turned out to be insignificant at the 5% level. This strategy eventually resulted in the equations presented in the following. Estimation is done in the full system using iterated three-stage least squares (3SLS).

The money demand equation is

\[
\Delta(m - p)_t = -0.111 \, ec_{t-1} \\
- 0.069 \, \Delta(m - p)_{t-4} \\
- 1.262 \, \Delta^2 p_t - 0.251 \, \Delta^2 p_{t-4} \\
- 0.220 \, \Delta y_{t-1} \\
- 0.568 \, (R - r)_{t-1} + 0.427 \, (R - r)_{t-3} - 0.406 \, (R - r)_{t-4} \\
- 0.430 \, d_t(R - r)_t \\
+ 0.187 \, - 0.115 \, d_{1t} - 0.077 \, d_{2t} - 0.059 \, d_{3t} \\
(10.7) \quad (-10.0) \quad (-10.0) \quad (-14.8) \\
+ 0.145 \, I90q3_t - 0.021 \, I92q4_t - 0.018 \, I93q3_t + \hat{u}_{mt} \\
(24.1) \quad (-3.6) \quad (-3.1)
\]

\( T = 84 \) \([1976(1) - 1996(4)]\)

\( R^2 = 0.93 \) \( SE = 0.0061 \) \( JB = 0.807 \) (0.668)

\( LM(1) = 0.34 \) (0.56) \( LM(4) = 0.31 \) (0.87) \( LM(8) = 0.56 \) (0.81)

\( ARCH(1) = 0.01 \) (0.91) \( ARCH(2) = 0.03 \) (0.97) \( ARCH(4) = 0.21 \) (0.93)
The test statistics given here are computed from the residuals of the estimated system. The results are very similar to those of equation (3.1), that is, none of the diagnostic test statistics is significant at conventional levels and, hence, the residuals appear to be normally distributed as well as free of autocorrelation and autoregressive conditional heteroscedasticity. As expected, equation (3.3) is very similar to (3.1). The only material difference is an additional lag of $\Delta(m - p)_{t}$. It is also easy to check that the long-run effect of the interest rate spread is clearly negative, as theoretically expected. The negative coefficient of the EC term implies that excess money lowers money growth, as one would expect in a stable model.

The income equation turns out to be

$$\Delta y_{t} = 0.044 e_{t-1}$$

(2.3)

$$+ 0.269 \Delta(m - p)_{t-1} + 0.172 \Delta(m - p)_{t-2}$$

(3.2)

$$- 0.323 \Delta y_{t-1} + 0.243 \Delta y_{t-4}$$

(3.7)

$$- 0.007 - 0.083 d_{1t} - 0.012 d_{2t} - 0.008 d_{3t} + 0.108 I_{90} Q_{3t} + \hat{u}_{gt}$$

(3.4)

$T = 84 [1976(1) - 1996(4)]$

$R^2 = 0.92$ \hspace{1cm} $SE = 0.012$ \hspace{1cm} $JB = 2.451 (0.294)$

$LM(1) = 0.00 (1.00)$ \hspace{1cm} $LM(4) = 1.52 (0.20)$ \hspace{1cm} $LM(8) = 1.18 (0.32)$

$ARCH(1) = 1.47 (0.23)$ \hspace{1cm} $ARCH(2) = 1.11 (0.34)$ \hspace{1cm} $ARCH(4) = 1.05 (0.39)$

Again the diagnostics do not indicate any problems with autocorrelation, conditional heteroscedasticity or nonnormality. The income equation includes the EC term with the expected positive sign so that excess money stimulates growth in real income. There is quite a bit of interaction from the changes in the other variables of the system. Obviously, there is dynamic feedback between income and money with lags of both variables appearing in both equations. Also lagged inflation influences output growth whereas no direct interest rate and import price effects are significant. The deterministic terms are the same as in the money equation except for the impulse dummies $I_{92} Q_{4t}$ and $I_{93} Q_{3t}$, which are not needed in the income equation.
The inflation equation was found to be

\[
\Delta^2 p_t = -0.058 \Delta (m - p)_{t-1} + 0.089 \Delta (m - p)_{t-3} \\
- 1.086 \Delta^2 p_{t-1} - 1.044 \Delta^2 p_{t-2} - 0.747 \Delta^2 p_{t-3} - 0.258 \Delta^2 p_{t-4} \\
+ 0.075 \Delta y_{t-3} + 0.096 \Delta y_{t-4} \\
+ 0.065 \Delta pm_{t-4} \\
+ 0.021 - 0.034 d_{1t} - 0.027 d_{2t} - 0.031 d_{3t} + \hat{u}_{pt}
\]

\[
(2.8) \\
(-11.3) \\
(-9.5) \\
(-6.9) \\
(-2.7) \\
(3.5) \\
(2.6) \\
(5.3) \\
(-4.6) \\
(-4.4) \\
(-4.8)
\]

\[T = 84 \ [1976(1) - 1996(4)]\]

\[R^2 = 0.98 \quad SE = 0.0046 \quad JB = 0.972 \ (0.615)\]

\[LM(1) = 0.05 \ (0.82) \quad LM(4) = 0.54 \ (0.71) \quad LM(8) = 0.67 \ (0.72)\]

\[ARCH(1) = 0.18 \ (0.67) \quad ARCH(2) = 0.75 \ (0.48) \quad ARCH(4) = 0.68 \ (0.61)\]

The diagnostic tests do not indicate any specification problems for this equation either. In addition to the own lags, the estimated equation also includes lags of income and money. Thus, there is obviously considerable interaction between inflation, money and income, as one would expect. However, the EC term turned out to be insignificant in this equation and hence the inflation rate is weakly exogenous. The precise nature of the interaction between the variables is difficult to see directly from the coefficient estimates. Therefore we will perform an impulse response analysis in the next section in order to get a better picture of the channels by which disturbances affect the system. Given that we have used unadjusted seasonal data the importance of the seasonal lags in the foregoing equations is not surprising. Moreover, as expected, the import prices have an impact on German inflation which has a delay of one year (four quarters), however. In this case the deterministic terms consist of an intercept and seasonal dummies only. The impulse dummy for the GMU period is not needed here because there is no break in the inflation rate due to the GMU (see Fig. 1).
Finally the equation for the interest rate spread is

\[
(R - r)_t = 0.836 (R - r)_{t-1} - 0.200 (R - r)_{t-4} \\
+ 0.055 \Delta pm_{t-4} \\
+ 0.014 + 0.004 d_{1t} + 0.004 d_{2t} + 0.003 d_{3t} + \hat{u}_{rt}
\]

\[T = 84 \quad [1976(1) - 1996(4)]\]

\[R^2 = 0.67 \quad SE = 0.0043 \quad JB = 6.441 (0.040)\]

\[LM(1) = 0.05 (0.83) \quad LM(4) = 0.43 (0.79) \quad LM(8) = 0.53 (0.83)\]

\[ARCH(1) = 1.80 (0.18) \quad ARCH(2) = 1.11 (0.34) \quad ARCH(4) = 0.86 (0.49)\]

Again the diagnostic tests do not indicate specification problems except that there is some evidence against normally distributed disturbances, a result which is quite common for financial data. Since the normal distribution is only of limited importance for our inference we do not regard this result as problematic. The interest rate spread is seen to depend on own lags and a lag of the import inflation rate only. Thus, the interest rate spread is exogenous in the equations for \((m - p)_t, y_t\) and \(\Delta p_t\). Although the Bundesbank uses interest rates such as the discount rate, the Lombard rate and especially the repo rate to control the money stock it is generally acknowledged that there is an impact on the market rates which in our model may be seen as measures of the interest rate policy which is reflected in our interest rate equation. Again we have an intercept term and seasonal dummies only as deterministic terms. Since there is no level shift due to the GMU in the interest rates it is not surprising that the corresponding dummy is not needed here. However, it may be a bit surprising that the seasonal dummies are significant in this equation. This may be due to the seasonal pattern of the import inflation variable (see Fig. 1).

Note that we are using the interest rate spread as the dependent variable in equation (3.6) rather than its first differences. The latter variable would be consistent with the general model set-up in (2.2). As mentioned in the discussion of that model, because \((R - r)_t\) is a stationary variable, it is more natural to include it directly and not in first differences.

In our model the only instantaneous endogenous variable entering the right-hand side is \(\Delta^2 p_t\) in the money demand equation. Thus most of the interaction between the variables
enters in a dynamic way through the lags of the endogenous variables. Therefore it is of interest to analyze the interactions in more detail. As mentioned earlier, they are difficult to see directly from the coefficients of the model. Therefore we will perform an impulse response analysis in the next section.

4 Impulse Response Analysis

In this section we will consider the effects of impulses hitting the system. In particular, we are interested in the effects of monetary policy on nominal money, the inflation rate and income. Therefore we remove all deterministic terms and the ‘exogenous variables’ \( pm_t \) and \( d_t(R - r)_t \) from the system and trace the marginal effects of impulses to the different equations. Removing the exogenous variables means that they are treated as fixed in the impulse response analysis. The constancy assumption is justified in a linear model if they are really exogenous and, hence, they are not affected by impulses hitting the system.

Generally impulse responses of the levels variables of the ECM (2.2) may be obtained by solving for the levels variables. We write the resulting model as

\[
A_0z_t = A_1z_{t-1} + \cdots + A_{p+1}z_{t-p-1} + \Phi x_t + \Xi D_t + u_t, \tag{4.1}
\]

The responses to an impulse \( u_0 \) at time zero are then obtained by removing the terms \( \Phi x_t \) and \( \Xi D_t \) and computing the forecasts for \( z_1, z_2, \text{etc.} \) conditionally on \( z_0 = A_0^{-1}u_0 \) and \( z_t = 0 \) for \( t < 0 \). Commonly the impulses \( u_0 \) are vectors with a unit in one position and zeros elsewhere. For instance, \( u_0 = (1, 0, \ldots, 0)' \) represents a unit impulse to the first equation.

This type of impulse response analysis has been criticized on the grounds that the residuals of a model may be correlated and hence isolated shocks to individual equations may not actually occur in the underlying system. Therefore the model cannot be expected to actually reflect the responses to such shocks adequately (see, e.g., Lütkepohl (1991) and Lütkepohl & Breitung (1997) for a more detailed discussion of impulse response analysis). Therefore it may be of interest to check the residual correlation matrix of our system which is

\[
\hat{R} = \begin{bmatrix}
1.00 & 0.10 & 0.21 & -0.06 & \Delta(m - p) \\
0.10 & 1.00 & 0.04 & 0.14 & \Delta y \\
0.21 & 0.04 & 1.00 & -0.18 & \Delta^2 p \\
-0.06 & 0.14 & -0.18 & 1.00 & R - r
\end{bmatrix}
\]
Clearly, all off-diagonal elements are relatively small and the actual correlations may well be zero. Hence, the impulse response analysis presented in the following is not prone to the critique related to correlated residuals.

In our system there is one deviation from the standard impulse response set-up which needs special consideration. The original variables in $z_t$ are $(m - p)_t$, $y_t$, $\Delta p_t$ and $(R - r)_t$. Because we are interested in the behaviour of nominal money, the log price level $p_t$ has to be regarded as a separate variable in the system and thus appears in levels and first differences. We therefore reparameterize the model such that we get a system similar to (4.1) but with endogenous variables $z^*_t = (m_t, y_t, \Delta p_t, (R - r)_t, p_t)'$ and a fifth equation $p_t = \Delta p_t + p_{t-1}$. Then we compute impulse responses for these variables and thereby get the responses of nominal money to impulses in the inflation rate equation etc. The resulting impulse responses of $m_t$, $\Delta p_t$ and $y_t$ are depicted in Fig. 3.

The dynamic interactions of the variables are largely in line with expectations. An impulse in the equation for nominal money has a lasting effect on the dependent variable, causes increased inflation in the long-run (after two quarters) and leads to income growth (see the first column in Fig. 3). An impulse in the inflation equation results in an initial decline in the money stock which may be a consequence of policy actions by the Bundesbank. As expected, in the long-run the nominal money stock increases due to an impulse in the inflation rate. The inflation variable itself reacts in a strongly seasonal way which may be a spurious effect, of course, because the impulse responses are computed from estimated coefficients so that there is some uncertainty regarding the sizes of the actual effects.\(^5\) Finally, after an initial income growth, an inflationary impulse leads to a long-term decline in the real income variable.

A one-time impulse in the income equation has a persistent effect on income, hence, it also drives up nominal money and leads to some inflation in the long-run. Again there is a strongly seasonal pattern in the response function of the inflation rate. Finally, an impulse in the interest rate differential and, hence, an increase in the opportunity costs of holding M3 money results in a decline of the money stock, a long-term decrease in the inflation rate and a decline in real income. Hence, overall the reactions of the variables are plausible in the

\(^5\)We do not give confidence bands for our impulse responses because there are a number of problems related to the standard confidence bands which are often reported in the literature (see Benkwitz, Lütkepohl & Neumann (1997)).
light of economic theory and they also conform with the assumed interactions which form the basis of the monetary policy conducted by the Bundesbank. In contrast to the impulse response analysis presented by the Deutsche Bundesbank (1997) it seems, however, that in our system the reaction of the inflation rate to money shocks is not as pronounced as the reaction of money to inflation shocks. This result is in line with findings of Juselius (1996) for the post-1983 period where the empirical support for price inflation to be a monetary phenomenon is rather weak.

5 Conclusions

In this study we have constructed a small macroeconomic model for studying the dynamic relation between money growth and inflation in Germany based on seasonally unadjusted, quarterly data for 1976 to 1996. Thus, our sample period covers the post-GMU period. A money demand equation for M3 is the central relation in the system. This relation includes GNP, the GNP deflator as a measure for the price level and an interest rate spread variable which represents the opportunity costs of holding money. Moreover, the system contains an import price index as an exogenous variable reflecting international influences. The model pays special attention to the cointegration properties of the variables and is set up as an ECM with a long-run money demand relation as its centerpiece. To find this relation we have first constructed a structural single equation ECM for money demand and then we have used the error correction term obtained in this way in building up a full dynamic system. Then an impulse response analysis is performed to analyze the dynamic interactions between the variables and in particular the relation between inflation and money growth.

It turns out that shocks in the money growth variable do not have a very pronounced impact on inflation whereas there appears to be a quite strong effect of inflation shocks on money growth. Therefore, from this analysis it is not quite clear whether a strong and predictable relation between money and inflation really exists in Germany which can be exploited to control inflation. On the other hand, our empirical results clearly show that, for given influences from abroad, interest rates are exogenous and, hence, can be used as monetary policy instruments to control inflation and growth. However, using only these instruments, the typical trade-off exists. Higher interest rates reduce inflation but also cause a decline in real growth. In addition, it is also clear that interest rates are not under the full
control of the Bundesbank but depend to some extent on international influences.

Appendix. Variables and Data Sources

Seasonally unadjusted quarterly data for the period from the first quarter of 1976 to the fourth quarter of 1996 (84 observations) were used for the following variables taken from the given sources. All data refer to West Germany until 1990(2) and to the unified Germany afterwards.

M3: nominal monthly values from Monatsberichte der Deutschen Bundesbank; the quarterly values are the values of the last month of each quarter. The variable $m$ is log M3.

GNP: real ‘Bruttosozialprodukt’ quarterly values from Deutsches Institut für Wirtschaftsforschung, Volkswirtschaftliche Gesamtrechnung. The variable $y$ is log GNP.

Price index: GNP deflator (1991 = 100) from Deutsches Institut für Wirtschaftsforschung, Volkswirtschaftliche Gesamtrechnung. The variable $p$ is the logarithm of the price index.

Umlaufsrendite ($R$): monthly values from Monatsberichte der Deutschen Bundesbank; the quarterly value is the value of the last month of each quarter.

Own rate of M3 ($r$): the series was constructed from the interest rates of savings deposits ($rs$) and the interest rates of 3-months time deposits ($rt$) from Monatsberichte der Deutschen Bundesbank as a weighted average as follows:

$$r = \begin{cases} 
0.24rt + 0.42rs & \text{for } 1976(1) - 1990(2) \\
0.30rt + 0.33rs & \text{for } 1990(3) - 1996(4) 
\end{cases}$$

The weights are chosen according to the relative shares of the corresponding components of M3. The quarterly value is the value of the last month of each quarter.

Import price index: PM (1991 = 100) from Deutsches Institut für Wirtschaftsforschung, Volkswirtschaftliche Gesamtrechnung. The variable $pm$ is the logarithm of PM.
References


