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Abstract

In this study we forecast the term structure of FIBOR/EURIBOR swap rates by means of recursive vector autoregressive (VAR) models. In advance, a principal components analysis (PCA) is adopted to reduce the dimensionality of the term structure. To evaluate ex–ante forecasting performance for particular short, medium and long term rates and for the level, slope and curvature of the swap term structure, we rely on measures of both statistical and economic performance. Whereas the statistical performance is investigated by means of the Henrikkson–Merton statistic, the economic performance is assessed in terms of cash flows implied by alternative trading strategies. Arguing in favor of local homogeneity of term structure dynamics, we propose a data driven, adaptive model selection strategy to 'predict the best forecasting model' out of a set of 100 alternative implementations of the PCA/VAR model. This approach is shown to outperform forecasting schemes relying on global homogeneity of the term structure.

Keywords: Principal components, Factor Analysis, Ex–ante forecasting, EURIBOR swap rates, Term structure, Trading strategies.
JEL classification: C32, C53, E43, G29.

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1 Introduction

Being much larger in value than equity markets fixed income markets have been attracting huge interest in both, financial theory and applied financial econometrics. As a result, term structure modeling has experienced substantial progress over the last two decades. Numerous studies focus on some factor approach. Nelson & Siegel (1987) introduce a parsimonious model for the dynamics of yield curves comprised by three factors representing short, mid and long term components, respectively. They show that their model is able to produce empirically observed shapes of yield curves, such as monotonic, humped or S–shaped patterns. The number of factors necessary to capture the main dynamic features of the term structure dynamics faces an ongoing discussion. By means of factor analysis respectively PCA Litterman & Scheinkman (1991) and Steeley (1990) support empirically that three factors, interpreted as level, slope and curvature factors are sufficient to describe the dynamics of the term structure. Knez, Litterman & Scheinkman (1994) extract even a fourth factor that is interpreted as a private issue credit factor. Duffie & Singleton (1997) develop a multi factor econometric model of the term structure of interest rate swap yields and favor a two factor version of this specification. Liu, Longstaff & Mandell (2004) use this framework to estimate a five factor affine term structure model and investigate interest rate swap spreads. Niffikeer, Hewins & Flavell (2000) decompose the term structure of swap rates into two synthetic factors to estimate the Value–at–Risk associated with a portfolio of interest rate swaps. Among others, Elton, Gruber & Blake (1995) and Ang & Piazzesi (2003) present yield curve models with traditional latent yield variables and observable macroeconomic covariates. By means of a VAR approach they find that macroeconomic factors explain up to 85% of the forecast error variance for long horizon forecasts of rates with short and medium maturities.

Another issue in term structure econometrics is model stability. In particular, if dynamic models are used for ex–ante forecasting invariance of the model parameters is inevitable to justify common forecasting schemes. To investigate if factor loadings are stable over time, Bliss (1997) divided a sample of government interest rates beginning in January 1970 and ending in December 1995 into three subperiods of different lengths and finds that while the pattern of factor loadings is almost stable, factor volatilities differ over the subperiods. Implementing a three–factor model with conditionally heteroskedastic factors, Audrino, Barone–Adesi & Mira (2004), however, conclude that factor loadings of US zero coupon bond yields are unstable over the period January 1986 to May 1995.

Diebold & Li (2003) point out that the term structure literature is rarely concerned with forecasting issues, in particular out–of–sample forecasting. Relying on the Nelson & Siegel (1987) methodology, Diebold & Li (2003) use autoregressive models for the factors to obtain ex–ante forecasts of changes of term structure shapes. Whereas Diebold & Li (2003) provide evidence that VARs outperform naive forecasts based on the random walk model, Duffee (2002) finds that affine term structure models fail to outperform naive ex–ante forecasts. Incorporating volatility driven risk premia in 'essentially' affine models, however, proves useful in terms of forecasting accuracy. Using Heath–Jarrow–Morton (Heath, Jarrow & Morton 1992) models with different swap rate volatility specifications Driessen, Klaassen & Melenberg (2003) conclude that a three factor model results in the best out–of–sample forecasting in terms of cap and swaption prices. Moreover, time–varying volatility specifications yield better results in comparison to a constant volatility approach.

In this paper, we decompose the term structure of FIBOR/EURIBOR swap rates by means of PCA and employ VAR models for ex–ante forecasting. We allow both the number of factors and the VAR order to vary. In order to account for possibly unstable factor loadings, we estimate factor decompositions based on alternative time windows. We consider a battery of empirical model specifications which we compare in terms of statistical and economic performance. The statistical measure is the Henrikkson–Merton test on the ability to forecast the sign of changes of speculative prices or portfolio values (Henrikkson & Merton 1981). Measuring economic performance we evaluate cash flows realized via alternative trading strategies. We will motivate that owing to dynamic heterogeneity of the term structure, one may hardly expect one particular PCA/VAR implementation to uniformly outperform the remaining specifications of factor based VAR models. Instead, similar to Härdle, Herwartz & Spokoiny (2003) we argue in favor of local homogeneity of the term structure. We propose a data driven procedure to 'predict the best forecasting model' which is shown to outperform forecasting schemes building on global homogeneity of the term structure.

The remainder of the paper is organized as follows. The econometric methodology and the applied performance measures are discussed in the next Section. In Section 3 we introduce the data mostly relying on descriptive statistics. Moreover, we will motivate that the dynamic relationships characterizing the swap term structure are hardly homogeneous over the entire sample period. Sections 4 and 5 will provide the empirical results and a comparative discussion of forecasting performance. In addition, we motivate and employ a data driven adaptive model selection strategy designed to find the most suitable prediction model. Section 6 concludes.

2 Econometric model

2.1 A factor model for the swap rate structure

Over a sample period of approximately 6 years, we will consider daily movements of swap rates for the following $M = 10$ maturities: 3m (3 months), 6m, 1yr (1 year(s)), 2yr, 3yr, 5yr, 7yr, 10yr, 12yr and 15yr. Owing to the large dimension M factor models are in widespread use when modeling term structures. One may a–priori question the adequacy of structurally invariant dynamic models to hold over a sample period of that length. Therefore we will adopt a view of local structural invariance implementing the factor model in rolling windows of size τ over time. To be specific we formalize the following model which is mainly used to provide recursive forecasts for the swap rate structure or the underlying factors:

$$\tilde{y}_t = \Gamma_K F_t + \xi_t, \quad t = T^* - \tau + 1, \ldots, T^*, \tag{2.1}$$

$$\Delta F_t = \nu + \Phi_1 \Delta F_{t-1} + \ldots + \Phi_p \Delta F_{t-p} + \eta_t. \tag{2.2}$$

In (2.1) $\tilde{y}_t = (\tilde{y}_{1t}, \tilde{y}_{2t}, \ldots, \tilde{y}_{Mt})'$ is a 10–dimensional vector of swap rates over 10 maturities measured in terms of deviations from their unconditional mean, $\tilde{y}_t = y_t - \bar{y}_{T^*}$, $\bar{y}_{T^*} = 1/\tau \sum_{t=T^*-\tau+1}^{T*} y_t$. F_t is a K–dimensional vector of factors governing the term structure which itself exhibits VAR dynamics. The error terms ξ_t and η_t are treated as serially uncorrelated and independent by assumption. To formalize the matrix Γ_K in (2.1) we adopt a PCA decomposing the unconditional covariance matrix of $\tilde{y}_t$ observed over the period $t = T^* - \tau + 1, \ldots, T^*$, i.e.

$$\hat{\Sigma}_{T^*} = \frac{1}{\tau} \sum_{t=T^*-\tau+1}^{T^*} \tilde{y}_t \tilde{y}_t', \quad \hat{\Sigma}_{T^*} = \Gamma \Lambda \Gamma'. \tag{2.3}$$

In (2.3) Λ is a diagonal matrix of eigenvalues of $\hat{\Sigma}_{T^*}$ in decreasing order and the columns of Γ contain the corresponding eigenvectors, respectively. Then, the matrix Γ_K given in (2.1) contains the first K columns of Γ thereby accounting for the variation in $\tilde{y}_t$ driven by K principal components. Being aware of the differences in the concepts underlying principal component and factor analysis (see e.g. Johnson and Wichern 2002) we will in the following consider principal components as factors, and thus use both terms interchangeably.

Although the approach of extracting principal components from the mean adjusted levels of interest rates may be seen as a common practice a word of caution seems appropriate. Note that in the econometric literature interest rates are seen to share some features of so–called integrated processes whereas the covariance estimator in (2.3) is suitable only in case of stationary swap rates. In our case the latter argument is particularly relevant since we describe the dynamics of highly persistent rates over (short) local time windows. Given the potential of nonstationarity, some of the extracted eigenvectors will allow a similar interpretation as (unidentified) cointegration parameters (Johansen 1995). In addition, in case of a nonstationary swap term structure PCA will result in extracting at least some nonstationary factors. We will briefly come back to this point when discussing stylized facts of the swap term structure and, in particular, of extracted factors and loading statistics.

When implementing the VAR it turned out that for the purpose of forecasting F_{T^*+h} (and thus y_{T^*+h}) conditional on information contained in a set $\Omega_{T^*,\tau} = \{y_t \mid t = T^* - \tau + 1, \ldots, T^*\}$ a model specified in first differences of the factors yield more stable results in comparison with a VAR in levels of F_t. This experience, again, supports the view that interest rates are (locally) nonstationary or, at least, highly persistent. Note that for the model in (2.2) ν is essentially a drift parameter, in turn implying a linear trend to be present in the interest rate levels. Clearly such a property is at odds with empirical features of interest rates in the long run. In our local model, however, the drift parameter in (2.2) is thought to capture local trends which may be fruitful to exploit when it comes to ex–ante forecasting of interest rates.

To formalize ex–ante forecasting of the swap rates we readjust for the unconditional in sample mean after forecasting the conditional expectation $\hat{F}_{t+h} = E[F_{t+h}|\Omega_{T^*,\tau}]$, as

$$\begin{aligned} \hat{y}_{t+h} &= E[y_{t+h}|\Omega_{T^*,\tau}] \\ &= \Gamma_K \hat{F}_{t+h} + \bar{y}_{T^*}. \end{aligned} \tag{2.4}$$

Implementing the local model given in (2.1) and (2.2) the analyst has to choose the parameters τ, K and p. In this paper we will employ a variety of window selections $\tau \in \{42, 63, 126, 189, 252\}$ corresponding to trading periods of 2, 3, 6, 9 and 12 months. The number of relevant factors is varied over $K = 1, 2, 3, 4, 5$, and potential lag orders are $p =$

$0, 1, 2, 3$. Alternative selections of the latter parameters provide a battery of 100 competing forecasting models. Forecasting exercises are performed for horizons from $h = 1$ (one day ahead forecasting) to $h = 5, 10$ (one and two weeks ahead forecasting, respectively). Since numerous alternative parameter selections are employed for the forecasting exercises we will evaluate the overall statistical and economic performance by means of an analysis of variance (ANOVA, Johnson & Wichern 2002). The latter is implemented by regressing some key measures of statistical or economic performance on a set of dummy variables describing the features of particular models namely the choice of τ, K and p, respectively. Providing a measure of statistical performance the Henrikkson–Merton statistic (Henrikkson and Merton 1981) will be used as the dependent variable of the ANOVA. With respect to economic significance we will rely on the cash flows achieved from particular investment strategies conditioned upon trading signals extracted from recursive forecasts. We now provide these measures of statistical and economic significance in turn.

2.2 Statistical measure of forecast performance

We consider the problem of evaluating the accuracy of competing forecasts of linear combinations of swap rates $g(a, y)_{T^*+h} = a'y_{T^*+h} - a'y_{T^*}$, where $a \in R^M$ is an M–dimensional column vector of known constants. For instance a could be used to formalize h–step ahead forecasts of changes of particular swap rates or, similarly, of signals used for particular trading strategies. To measure forecasting accuracy, $g(a, y)_{T^*+h}$ and the corresponding predictor, $\hat{g}(a, y)_{T^*+h} = a'\hat{y}_{T^*+h} - a'y_{T^*}$, may be regarded as dichotomous random variables. From this point of view a forecasting model is accurate if the distributional properties of the forecasts $\hat{g}(a, y)_{T^*+h}$ come close to the respective features of the actual quantities $g(a, y)_{T^*+h}$. Intuitively appealing to summarize key features of the joint distribution contingency tables are often used in applied statistics. As a formal criterion measuring the information content of a contingency table we consider the so–called Henrikkson–Merton statistic (hm). The latter is just the sum of the conditional probabilities of correctly forecasting a positive or negative value $\hat{g}(a, y)_{T^*+h}$ whenever the actual realization in time $T^* + h$ is positive or negative, i.e.

$$\begin{aligned} \text{hm} &= \text{Prob}(\hat{g}(a, y)_{T^*+h} \geq 0 \wedge g(a, y)_{T^*+h} \geq 0 | g(a, y)_{T^*+h} \geq 0) \\ &\quad + \text{Prob}(\hat{g}(a, y)_{T^*+h} < 0 \wedge g(a, y)_{T^*+h} < 0 | g(a, y)_{T^*+h} < 0). \end{aligned} \tag{2.5}$$

A successful forecasting scheme should deliver hm–statistics which exceed unity. Critical values for the hm–statistic depend on the number of available predictions. For this study we use simulated critical values taking the number of predictions that will be available (1240) into account. To determine critical values we generate 10000 sequences of bivariate and independent Gaussian random sequences of length 1240 and used one of these as a forecast for the other. Thereby we obtain the following quantiles of hm and particular order statistics which will be of interest in Section 5:

[Insert Table 1 about here]

2.3 Economic measure of forecast performance

We complement the statistical analysis of forecasting accuracy with an investigation of the performance of some trading strategies over time horizons of $h = 1, 5, 10$ days, respectively. We consider six trades, each based alternatively on a rate based and a factor based signal, thereby providing 12 strategies in total.

First, we consider trades based on the single 2yr swap rate. For example, if we proceed in time T^* from the expectation that the 2yr rate will increase we set up a *2yrSingleTrade* by entering a 2yr payer swap. This corresponds to a rate based trading signal $\hat{g}(a_{2yr}, y)_{T^*+h} > 0$, with $a_{2yr} = (0, 0, 0, 1, 0, 0, 0, 0, 0, 0)$. Alternatively, factor based signals exploit a forecast for the factor(s) and an assumption on the correlation between factors and rates. For instance, if we expect the first factor to increase and the correlation of the first factor with the 2yr rate is positive (negative), we enter a 2yr payer (receiver) swap. To formalize the latter correlation we employ the following estimator and relate it to time T^*:

$$\begin{aligned} \hat{\rho}_{km}^{T^*} &= \mathrm{Cor}(f_{kT^*}, y_{mT^*}) \\ &= \frac{\sqrt{\hat{\lambda}_k}\gamma_{km}}{\hat{\sigma}_m}. \end{aligned} \tag{2.6}$$

In (2.6) $\hat{\lambda}_k$ is the k–th eigenvalue of the covariance matrix $\hat{\Sigma}_{T^*}$, γ_{km} is the loading of factor k on swap rate m extracted from Γ and $\hat{\sigma}_m$ is the estimated standard deviation of swap rate m. For ease of notation we neglect in the upper definition the dependence of all quantities on τ, the size of the time window used to implement the PCA. Thus, positive correlation between the first principal component and a particular rate is indicated if $\gamma_{1m} > 0$. In complete analogy to the *2yrSingleTrade* we also implement trading strategies for mid– and long–term maturities, namely a *5yrSingleTrade* (a_{5yr}) and a *10yrSingleTrade* (a_{10yr}).

Note that in a payer swap, we pay the fixed and receive the floating rate in the agreement. Conversely, in a receiver swap we receive the fixed and pay the floating rate. Now consider, for instance, two 5yr payer swaps A and B sharing the same features except that swaps A and B have fixed rates of, say, 4.5% and 5.0%, respectively. Then, swap A is worth more than swap B as in both cases the floating rate is the same, but in agreement A the investor pays a lower fixed rate than in B. Moreover, the fair value swap rate is the fixed rate in a payer/receiver swap agreement delivering a zero present value of the agreement. The swap term structure is supposed to represent fair value swap rates over different maturities.

In addition to the trades introduced above, we also consider trades based on slightly more complex linear combinations of single rates, which we will call *LevelTrade*, *SlopeTrade* and *CurvatureTrade*. We set up a *LevelTrade* if we expect the level of the term structure to move up (down), by entering payer (receiver) swaps with 2yr, 5yr and 10yr maturities. We rely on two signals to predict level increases. Firstly, using the forecasts of the 2yr, 5yr and 10yr swap rates, we compute a rate based signal $\hat{g}(a_{level}, y)_{T^*+h}$ with $a_{level} = (0, 0, 0, \frac{1}{3}, 0, \frac{1}{3}, 0, \frac{1}{3}, 0, 0)$. Consequently, $\hat{g}(a_{level}, y)_{T^*+h} > 0$ indicates a level increase. Secondly, according to the interpretation of the first factor as measuring the level of the term structure (Steeley 1990, Litterman & Scheinkman 1991) VAR predictions of this factor are used as factor based signals. A few remarks on the stability of factor estimates and their interpretation will be made in Section 3.

Next, to initiate a *SlopeTrade* we enter a 2yr receiver (payer) swap and a 10yr payer (receiver) swap if we assume the slope of the term structure to increase (decrease). The corresponding rate based signal is $\hat{g}(a_{slope}, y)_{T^*+h} > 0$, $a_{slope} = (0, 0, 0, -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0)$, whereas forecasts of the second principal component serve as a signal for factor based trades. Similarly, a *CurvatureTrade* is characterized by $\hat{g}(a_{curve}, y)_{T^*+h} > 0$, $a_{curve} = (0, 0, 0, \frac{1}{4}, 0, -\frac{1}{2}, 0, \frac{1}{4}, 0, 0)$ (rate based) or forecasts of the third principal component (factor based). In case we expect the curvature of the term structure to increase (decrease), we enter 2yr and 10yr payer (receiver) swaps, and a 5yr receiver (payer) swap where we weight the former swaps with 0.25 (−0.25) and the latter with −0.5 (0.5).

To facilitate the presentation and discussion of the forecasting results each trade will be numbered. Distinguishing rate and factor based signals, trades will be denoted with indices R1 to R6 and F1 to F6, respectively. Table 2 summarizes the strategies.

[Insert Table 2 about here]

3 The data and stylized facts

So far empirical term structure modeling has a distinct focus on treasury and bond yield curves. Empirical models of swap curves are rare. Dai & Singleton (2000) point out that U.S. swap and treasury markets share similar stylized features, although the institutional structure of both markets is different. They find, for instance, that PCA yield similar 'level', 'slope' and 'curvature' factors for both markets. In this paper we analyze the FIBOR/EURIBOR (Frankfurt/European interbank offered rate) swap curve since the European swap market has become highly liquid. Moreover, the EURIBOR is widely used as the underlying interest rate for Euro–denominated derivative contracts.

In order to work out some stylized facts for the Euro area and highlight our view of structural variation of model parameters we present in this section the data set. We provide a brief description of the actual swap rate and factor dynamics. Further, we briefly illustrate empirical features of term structure forecasts and forecast errors obtained when employing the methodology proposed in Section 2.

3.1 Descriptive analysis

The investigated data set comprises 1552 daily observations, $y_t = (y_{1t}, y_{2t}, \ldots, y_{Mt})'$, of FIBOR swap rates from January 14, 1997 to December 31, 1998 and EURIBOR swap rates from January 4, 1999 to January 7, 2003 for $M = 10$ maturities as given in Section 2. The rates are observed daily at 3pm.

The first 252 days are exclusively used for initial training samples. The data driven model selection procedure given in Section 5 will need another 42 initial training days plus 10 days for the maximum forecasting horizon considered. To ensure that all PCA/VAR forecasting models are employed over the same subsample of available observations the recursive analysis will start in time $T^* = 303$ and deliver 1240 forecasts at horizons $h =, 1, 5, 10$.

The evolution of the daily term structure is shown in Figure 1. As can be seen, the shape of the swap curve varies over time with respect to its level, slope and curvature. Table 3 documents that the average swap term structure is increasing and rates at short maturities are more volatile than those at the long end. Approximating the curvature as 0.25*2yr - 0.5*5yr + 0.25*10yr the evidence is less clear. The empirical mean signifies a slightly positive (i.e. convex) curvature whereas the corresponding median would suggest a slightly negative curvature. Thus, on average one may conclude that the swap term structure is a straight line, although it locally reveals distinct curvature as shown in Figure 1.

[Insert Figure 1 about here]

[Insert Table 3 about here]

3.2 Factors and correlations

In a first step, we compute sequentially for each trading day over the subsample used for recursive forecasting ($T^* = 303, \ldots, 1542$) the principal components $F_t = (f_{1t}, \ldots, f_{Kt})' = \Gamma'_K \tilde{y}_t$, $t = T^* - \tau + 1, \ldots, T^*$ from historic windows of time with alternative sizes $\tau \in \{42, 63, 126, 189, 252\}$. To have a first illustration of the empirical properties of extracted factors Figure 2 shows the three principal components ($K = 3$, f_{1t}, f_{2t} and f_{3t}) estimated at April 26, 1999 using $\tau = 42$ and $\tau = 252$ observations, respectively. The estimated eigenvectors obtaining the principal components at April 26, 1999 are

$$\Gamma'_{3,\tau=42} = \begin{bmatrix} 0.4649 & 0.4243 & 0.3833 & 0.3501 & 0.3312 & 0.2878 & 0.2456 & 0.1730 & 0.1659 & 0.1553 \\ -0.3053 & -0.2655 & -0.1725 & -0.1012 & 0.0018 & 0.1609 & 0.3049 & 0.4578 & 0.4723 & 0.4948 \\ 0.4099 & 0.3120 & 0.1141 & -0.4781 & -0.5151 & -0.3519 & -0.0703 & 0.1902 & 0.1826 & 0.1716 \end{bmatrix}$$

and

$$\Gamma'_{3,\tau=252} = \begin{bmatrix} 0.1986 & 0.2498 & 0.3140 & 0.3877 & 0.4068 & 0.3879 & 0.3457 & 0.2930 & 0.2733 & 0.2307 \\ -0.6193 & -0.5185 & -0.3120 & -0.0300 & 0.0732 & 0.1769 & 0.2253 & 0.2534 & 0.2266 & 0.2154 \\ 0.2978 & 0.1622 & -0.0983 & -0.4261 & -0.4078 & -0.1877 & 0.0787 & 0.3007 & 0.3831 & 0.4989 \end{bmatrix}.$$

From the composition of weights it is natural to interpret the first factor (f_{1t}) as representing the level of the term structure since each maturity enters with some positive weight. Similarly f_{2t} is obtained giving positive weights for higher and negative weights for lower maturities thereby measuring the slope of the term structure. Using mostly negative weights for midterm maturities the third principal component (f_{3t}) approximates the curvature of the swap term structure. Although both matrices $\Gamma_{3,\tau=42}$ and $\Gamma_{3,\tau=252}$ are estimated from overlapping windows of observations the reported weighting coefficients show considerable differences. For instance, using the small time window the level of the term structure is determined with weights that are decreasing in the maturity. From the larger time window a U–shaped weighting scheme over alternative maturities is obtained.

To underscore the issue of local homogeneity and, implicitly, global inhomogeneity factor paths obtained for both window sizes $\tau = 42$ and $\tau = 252$ are given jointly in the three panels of Figure 2. Using a time window of $\tau = 252$ observations the level of the term structure at April 26, 1999 is clearly underestimated in comparison with the respective result derived from the smaller time window ($\tau = 42$). Similarly, using one year of observations signals a positive slope of the term structure at April 26, 1999 whereas according to the smaller time window the slope is almost zero.

By construction the unconditional mean of all factors is zero but apparently the factor variation around the zero mean is highly persistent. Although nonstationarity is most striking for the first two factors we conjecture all three principal components shown in Figure 2 to be driven by stochastic trends. We refrain from providing formal ADF tests (for all subsamples $T^* = 303, \ldots, 1542$), window sizes or factors. Instead, as formalized in (2.2) we adopt a VAR model specified in first differences of F_t to implement recursive forecasts of the principal components.

[Insert Figure 2 about here]

For the three principal components f_{kt}, $k = 1, 2, 3$, Figure 3 shows average correlations with the rates,

$$\hat{\rho}_{km} = \frac{1}{1240} \sum_{T^*=303}^{T^*=1542} \hat{\rho}_{km}^{T^*},$$

over maturities 3m to 15yr. Time specific correlations $\hat{\rho}_{km}^{T^*}$ are defined in (2.6). Note, that the trading signals for F1, F2 and F3 presented in the previous Section rely on the time dependent correlation $\hat{\rho}_{1m}^{T^*}$.

Similar to stylized features of treasury and bond term structures the upper panel of Figure 3 confirms for the FIBOR/EURIBOR that the first principal component measures the level of the swap term structure since, on average, it is positively correlated with interest rates at all maturities. Moreover, correlations over maturities for the second and third principal component allow an interpretation as factors representing the swap terms structure's slope and curvature, respectively. The former shows average correlations increasing in maturity from negative to positive values, whereas the latter shows some asymmetric U–shaped pattern. It is worthwhile to mention that the latter findings do not hold uniformly for all trading days in the sample. For some particular time windows, $T^* - \tau + 1, \ldots, T^*$ the first factor, for instance, reveals a correlation pattern similar to the average correlation pattern of the slope factor illustrated in the medium panel of Figure 3. Opposite to the three principal components the corresponding correlation patterns obtained for the fourth and fifth factor do not allow any obvious interpretation holding uniformly over the sample period.

[Insert Figure 3 about here]

[Insert Figure 4 about here]

To finally underscore the case for structural variation Figure 4 displays the fraction of explained variances obtained from three principal components via recursive PCA modeling with time windows of length $\tau = 42$. The fraction of data variability explained by the first factor is obviously time varying between a lower and upper bound of about 60% and 98%, respectively. Over periods with a relatively small degree of explanation achieved with the first factor the second factor is contributing up to 40% of explained data variation. Throughout, the contribution of the third factor is rather small. With respect to ex–ante forecasting of the swap term structure, one may a–priori conjecture from Figure 4 that over certain periods of time one factor models may provide sufficiently accurate forecasts whereas, for instance, in the second third of the forecasting period model implementations with more than one factor might give superior results. Since model parsimony is positively related with forecasting efficiency one may, moreover, expect that higher VAR orders for the dynamic factor model may be suitable in case the factor dimension is small ($K = 1$). In multiple factor models forecasting precision will be higher when the autoregressive order of the factor VAR in (2.2) is small.

3.3 Term structure forecasts and forecast error statistics

In the previous section we investigated some properties of the factors extracted from particular time windows. Before we continue with systematic recursive forecasting exercises it is worthwhile to illustrate if forecasts from a particular implementation of PCA/VAR models come close to the actual variations of the swap term structure. Therefore, Figures 5 and 6 show the 1240 forecasted term structures and the associated errors for $T^* = 303, \ldots, 1542$ exemplary for a model specification with $\tau = 42, K = 1, p = 0$ and forecasting horizon $h = 5$. In Section 4 the latter implementation will serve as the benchmark model for an ANOVA investigation of recursive forecasting performance over all competing specifications.

From visual inspection of Figure 5 it appears that the ex–ante forecasted term structure shares key features with the corresponding actual pattern shown in Figure 1. Table 4 quantifies these similarities providing a few descriptive statistics for both the forecasted and the observed term structure. Displaying forecasting errors Figure 6 uncovers a few outlying estimates. Although the latter results informally indicate that the forecasting strategy adopted in this paper may give reliable estimates of future interest rates the graphical and unconditional evaluation may hide the potential of systematic forecast errors when concentrating at particular maturities, periods or states of the term structure. To clarify the case and nature of systematic forecasting errors we perform a regression of forecast errors $e_{m,T^*+h} = \hat{y}_{m,T^*+h} - y_{m,T^*+h}$ and squared forecast errors e^2_{m,T^*+h} observed at horizons $h = 1, 5$ and $h = 10$ on a set of dummy variables. The dummy variables indicate particular maturities and periods of time (1998, 1999, 2000, 2001). Characterizing key features of the state of the term structure we use, in addition, three dummy variables taking a value of one in time T^* if the level, slope, or curvature, respectively, exceed their median. Estimation results for the latter regression are shown in Table 5.

[Insert Figures 5 and 6 about here]

[Insert Table 4 about here]

Whereas forecast errors for $h = 5, 10$ step ahead forecasts do not seem to contain any systematic pattern over maturities short run forecasts ($h = 1$) are significantly smaller at the short end of the term structure relative to the 3yr benchmark maturity. One step ahead forecast errors are of similar size in 1998, 1999, 2000 and 2001. At the 5% significance level we obtain for $h = 5, 10$ that forecast errors in 1999, 2000, 2001 are significantly smaller in comparison with 2002. Note that our sample is comprised by the FIBOR until 1998. Thus, significance of estimated coefficients of dummy variables indicating the EURIBOR part of the sample (1999 to 2001) may signal the institutional change of our swap rate measure.

Interestingly, for the short forecasting horizon the sample appears quite homogeneous in this respect. Interpreting the regression results given for $h = 5, 10$ as an indication of structural variation provides an additional argument for our view at the term structure to be locally but not globally homogeneous. Moreover, it can be seen from Table 5 that forecast errors significantly tend to increase with the level and the slope of the term structure when forecasting at higher horizons ($h = 5, 10$). Owing to persistence of the term structure's level and slope as illustrated in Figure 2, the latter are convenient means to characterize the (local) state of the system. As a consequence, significance of the corresponding parameter estimates may also favor a local approach to model and forecast the term structure.

With regard to systematic patterns of squared forecasting errors the right hand side panel of Table 5 reveals systematic heteroskedasticity in numerous directions. On average the highest forecasting uncertainty is obtained over the years 1999 and 2001. Whereas the level of the term structure does not seem to affect forecasting uncertainty in a systematic fashion squared forecast errors are significantly higher in states of a relatively steep term structure. Opposite to the latter impact forecasting uncertainty is significantly reduced over states of a relatively large curvature.

[Insert Table 5 about here]

4 Forecasting performance

After the provision of methodological issues in Section 2 and a more descriptive view at the swap term structure in Section 3 we now turn to a systematic investigation of the forecasting performance obtained over all considered implementations of the PCA/VAR model in (2.1) and (2.2). We will present ANOVA results for the measures of statistical and economic performance, hm–statistics and the outcomes of alternative trading strategies.

4.1 Statistical forecasting performance

As mentioned hm–statistics exceeding unity indicate that predictions obtained from a particular forecasting scheme share key distributional properties with the forecasted variable. Since we implement 100 alternative model specifications to forecast six linear combinations of swap rates over alternative time horizons ($h \in \{1, 5, 10\}$) we will mostly refrain from providing model specific test results but relate the overall performance to particular model features via an ANOVA. Estimation results from such an analysis are given in Table 6 (rate based trades) and Table 7 (factor based trades), respectively. The first row of both Tables gives the hm–statistic for $h = 5$ days ahead forecasts obtained for recursive implementations of local models using mostly a benchmark specification with $\tau = 42, K = 1, p = 0$. For factor based trades F5 and F6 the benchmark specifications are characterized by $K = 2$ and $K = 3$, respectively, since the trading signals exploited for these strategies are directly taken from the second and third principal component. According to the critical values given in Table 1 6 (8) out of 12 hm–statistics (R1, R4, R6, F1, F2, F4) are larger than unity with 5% (10%) significance. The benchmark implementation for trade F6 (*CurvatureTrade*) shows a hm–statistic which is insignificantly smaller than one. Measured in terms of averaged hm–statistics the forecasting accuracy is similar for rate and factor based signals. Varying the model parameters τ, K or p we detect hardly any uniform impact on the forecasting performance holding over all considered trades.

The choice of locally homogeneous time windows is crucial from the viewpoint of ex–ante forecasting performance. For instance, relative to the benchmark choice ($\tau = 42$) time windows covering one year of daily quotes ($\tau = 252$) reduce average hm–statistics significantly in case of four rate based trading signals (R1, R2, R4, R5). For three strategies (R1, R2, R4) our results strongly support the choice of small time windows ($\tau = 42$) to obtain rate based trading signals. For the factor based trades the evidence supporting the choice of narrow local time windows is similarly strong. For only two factor based trading signals (F4 and F6) significant positive improvements of the average hm–statistic are diagnosed for wider time windows of size $\tau = 126$ and $\tau = 252$, respectively.

The effect of including more than one factor ($K > 1$) on the average forecasting performance is mostly positive and significant for rate based signals. Although the optimal number of factors to include varies from $K = 2$ (R3, R5, R6), $K = 3$ (R2) to $K = 5$ (R1, R4) the marginal gain in forecasting accuracy when using more than $K = 3$ factors is small. Different results are obtained for factor based signals. Here it turns out that choices of $K > 1$ yield only small and insignificant improvements or even deteriorations of the average performance of trading signals for F1 to F4. Note that the benchmark specification used for the ANOVA only contains the particular factor which is used to derive the trading signal. Including additional factors appears to increase the uncertainty of factor based trading signals.

Considering the VAR order used to model factor dynamics, ANOVA estimates indicate that higher autoregressive orders mostly involve negative impacts on the hm–statistic for the trades R1 to R4 and F1 to F4. Note that the rates underlying these signals show locally features of a nonstationary random walk with drift. For these trades one may conjecture that the sign of the forecasts is likely to be determined by the local drift parameter. Regarding the remaining trades, *SlopeTrade* (F5 and R5) and *CurvatureTrade* (F6 and R6), the sign of the underlying feature of the swap term structure will not depend on a (local) drift parameter which is canceled out in the respective linear combination. Interestingly, we obtain for the corresponding trading signals that, on average, higher order VAR models improve the hm–statistics significantly.

ANOVA results in Tables 6 and 7 are shown for $h = 5$ as a benchmark forecast horizon. In comparison with $h = 5$ most considered trading signals at the higher forecasting horizon $h = 10$ show stronger correspondence with the respective realized rates in comparison with $h = 5$. Comparing medium with short term predictability the ANOVA documents higher accuracy of the former.

In sum, the obtained hm–statistics underscore forecasting ability of the class of PCA/VAR models. Regarding minimum hm–statistics, however, shows that an unsuitable specification of the forecasting model has downside risk with respect to the implied hm–statistic. Under the null hypothesis of no predictability the minimum and maximum hm–statistics obtained over the set of 100 PCA/VAR implementations will have a quite different distribution in comparison to the statistic obtained unconditionally from some single model. To provide approximate critical values for the former we simulate 10000 replications of 101 sequences of bivariate Gaussian noise comprising 1240 observations. From the latter we take 100 sequences as forecasts for the remaining 101st draw and obtain minimum and maximum values over the set of 100 hm–statistics. Critical values for these order statistics are also shown in Table 1. At the 5% level downside potential indicated by the minimum hm–statistic is significant for only one (R5) out of 12 considered trading signals. Conversely the maximum hm–statistics are significant at the 5% level for 9 trading signals (all but F3, F5, F6). As a further indication of overall predicability one may consult the mean hm–statistics exceeding one significantly in 11 out of 12 cases (all but F6). Particular model features having an adverse (positive) impact on the average forecasting performance, as e.g. higher (smaller) VAR–orders or large (small) time windows for trades R1 to R4 and F1 to F4, are recovered when regarding the particular specification of the worst (best) performing model.

[Insert Tables 6 and 7 about here]

4.2 Economic forecasting performance

In order to evaluate the performance of the portfolio strategies introduced in Section 2.3 we implement for each model specification recursive trading schemes. To provide a measure of economic forecasting performance for a particular PCA/VAR implementation we add up the cash flows realized over all trading days giving the total cash flow. As an illustration for the implementation of the dynamic trading strategy consider, for example, a $h = 1$ day ($h = 5, 10$ days) ahead forecast. If we expect in some instant of time (T^*) the term structure level to increase, we enter 2yr, 5yr and 10yr payer swaps. Then, in time $T^* + h$ we close the positions entering receiver swaps with the same reduced (by one, five, ten days) time to maturity. Since we do not have data for maturities other than 3m, 6m, 1yr, 2yr, 3yr, 5yr, 7yr, 10yr, 12yr and 15yr, we approximate the fixed rate for a, say, 2yr minus one day (five, ten days) swap by the 2yr rate observed in $T^* + 1$ ($T^* + 5, T^* + 10$). From the difference in both rates we compute the present value of the cash flows over the remaining time to maturity. We account for accrued interest but not for transactions costs. Starting with an interest–free bank account of 100, we enter each day swap agreements adding up to a notional of 100. For instance, in case of the level trade the 2yr, 5yr and 10yr swaps each have a notional of 33.33 (see Table 2 for the details of trade specific weighting schemes).

As in the previous section we perform an ANOVA regression on dummy variables representing specification parameters and forecasting horizons to uncover the effects of model features on the outcome of particular trading strategies. Tables 8 and 9 show the results for rate and factor based trades, respectively. The upper parts of both tables contain parameter estimates with t–statistics in parentheses underneath. In the lower part, minimum, mean and maximum cash flows for each trade obtained over all PCA/VAR forecasting models are reported. The particular model implementations delivering the latter cash flows are indicated in parentheses underneath. Furthermore, cash flows for the optimal specification are compared to a model sharing the same specification except for the autoregressive order which is set to $p = 0$, such that the factor model only includes a drift term. The relative performance of the latter model may give a valuable benchmark for the gains that can be addressed to VAR dynamics as formalized in (2.2).

ANOVA estimates indicate that choosing small time windows such as $\tau \in \{42, 63\}$ appears to be optimal for the single rate trades (R1, R2, R3, F1, F2, F3). For the more complex trades (R4, R5, R6, F4, F5, F6) the overall result is different. In five (R5, R6, F4, F5, F6) out of six cases a significant improvement is diagnosed that can be addressed to using time windows of 126 trading days or more.

The influence of the number of factors exploited for recursive trading is not uniform when comparing the outcome of rate and factor based trading signals. For rate based trades including more than one factor ($K > 1$) improves the strategies R1 to R4, but deteriorates the performance of R5 and R6. For factor based trades inverse results are obtained, i.e. the impact of using more than one factor is negative for F1 to F4, and positive for F5 and F6. With regard to F5 and F6 including more than two respectively three factors does not contribute significantly to a performance improvement. Note that similar results have already been discussed with regard to the impact of K on the statistical forecasting performance (see Tables 6 and 7).

Concerning the VAR order p the ANOVA results favor a model without any autoregressive dynamics ($p = 0$) for R1 to R4 and F1 to F4 which are all showing a negative average impact of higher VAR orders on the final bank account. For the remaining *SlopeTrades* and *CurvatureTrades* autoregressive dynamics generally obtain positive excess cash flows, although the improvements over the model with $p = 0$ is only significant for F6. Interpreting the outcome of PCA/VAR implementations with $p = 0$ it is worthwhile to recall that the corresponding forecasts are not naive predictions based on a pure random walk but rely on some local drift instead.

Furthermore, 11 trades (all except F6) perform significantly better at a forecasting horizon of $h = 10$ days ahead. Assuming that the magnitude of rate changes increases with the forecast horizon, this result could be addressed to the fact that we forecast only the direction and not the size of a movement.

Generally, all trades yield positive cash flows when averaging over all specifications. Comparing minimum and maximum cash flows, however, all PCA/VAR models have upside and downside potential. Regarding trade R3, for example, the specifications $\tau = 189, K = 1, p = 1$ and $\tau = 126, K = 2, p = 0$ generate the minimum and maximum cash flows at the $h = 10$ days forecast horizon of -71.06 respectively 215.06. It appears that negative cash flows are smaller in absolute value than positive cash flows. The latter asymmetry is even more apparent for factor based trades in comparison with rate based trades.

Overall, the specification implied by the ANOVA is close to the PCA/VAR implementation providing the maximum total cash flow. In three cases (R1, R6, F1) the ANOVA filters out the model specification providing the maximum cash flow. For seven out of 12 trading strategies the ANOVA implied best forecasting models almost double the stake, implying an annualized total return of approximately 15%.

For 'ex–post' selection of an outperforming PCA/VAR implementation one may be tempted just to choose the best performing model from the set of all models. However, the outcome of such an approach will not take systematic influences of model features on forecasting performance into account. Yet, we regard an ANOVA as a suitable means to uncover model features that are essential for accurate predictions. The proximity of the ANOVA implied and the unconditionally best performing model documented in Tables 8 and 9 provides a further support employing ANOVA for model specification. Building on the latter argument we will exploit ANOVA regressions to adaptively 'predict accurate forecasting models' in the following section.

[Insert Tables 8 and 9 about here]

5 An adaptive modeling strategy

In the previous sections we investigated the performance of recursive ex–ante forecasts for numerous model specifications. More precisely, we examined the performance of each model specification over the entire sample period implicitly assuming that the relationships governing the swap term structure are uniformly well approximated by a particular choice of model parameters. From the various illustrations of potential structural variation of the swap term structure dynamics given in Section 3 it seems unlikely that one particular model implementation will perform homogeneously over the entire sample period. There might be time periods in which dynamics are better captured by specific model features, such as, for example, parsimonious VAR orders or a low dimensional space of principal components. Moreover, structural variation may motivate the use of smaller time windows when extracting the principal components. Conversely, using large time windows, of one year say, will be justified over long periods of structural homogeneity or of only slight structural variation. As a consequence, we will propose a data driven adaptive model selection strategy designed to select the best forecasting model or at least a model which is likely to give positive cash flows in the near future.

To illustrate the procedure consider, say, trade R3 (*10yrSingleTrade*) and a forecasting horizon $h = 10$. At a specific trading day T^* we know the performance of each of the 100 PCA/VAR implementations ($\tau \in \{42, 63, 126, 189, 252\}$, $K = 1, 2, 3, 4, 5$, $p = 0, 1, 2, 3$) which we have used 10 days ago ($T^* - h$) to forecast today's (T^*) 10yr swap rate (swap curve). Similarly, we observed the performance of all models used in ($T^* - 1 - h$) to forecast the 10yr swap rate in ($T^* - 1$) etc. Our adaptive strategy will use a second window of $\tilde{\tau} = 42$ observations comprised by the recent history to evaluate forecasting performance. Then, it is feasible to assess current forecasting accuracy of each model specification over the period $T^* - \tilde{\tau} + 1$ to T^*. There are different approaches conceivable to measure forecast quality. One could think of a rate based criterion such as the mean squared or mean absolute forecast error. Likewise it is possible to use a cash flow based measure such as total cash flow over the trading days $t = T^* - \tilde{\tau} + 1, \ldots, T^*$. Also, a combination of both measures may be used. In the following we report results for the adaptive model selection strategy using a cash flow based criterion. As a means to implement time dependent model rankings, we compute for each model specification the total cash flow over a most recent period of $\tilde{\tau} = 42$ trading days. Then, the locally best model specification is chosen according to an ANOVA regression on dummy variables representing the specification features τ, K, p. To keep our results comparable with the unconditional model specific forecasting performance we employ the adaptive model selection over the same subsample period of 1240 trading days as in Section 4.

In Tables 10, and 11 we show measures of statistical and economic performance obtained from the model selection strategy previously described. Performance measures are shown for each trade and forecasting horizon. The first part contains the hm–statistic and some statistics describing the cash flow. Secondly, we compare the outcome of the adaptive strategy with the best, worst and average performance obtained when evaluating (non–adaptive) 'singular' PCA/VAR implementations as in Section 4. In the following we will refer to the latter as 'unconditional' modeling.

Regarding the total cash flow (CF) the adaptive strategy outperforms the best 'unconditional' model specification in 8 cases (F1 for $h = 5$ and R1, R2, R4, F1 to F4 for $h = 10$) out of 36. In 8 further cases (F1 for $h = 1$, R1, R2, R6, F2, F3, F4 for $h = 5$ and R1, R3, R6 for $h = 10$) its performance comes close (at least 80 %) to the best 'unconditional' specification. Most interesting are the trades R2, R3, R4, F2, F3, F4 implemented at a $h = 10$ forecasting horizon. All these strategies realize an annualized return in excess of 20%. Moreover, in only three cases (R3 for $h = 1$ and F6 for $h = 5, 10$) there is a negative total cash flow achieved via the adaptive procedure. In addition, looking at the minimum value of the bank account over the 1240 trading days no trade seems to have a great downside potential when implemented adaptively. For all trades and all forecasting horizons the bank account is never below 89.20.

Owing to the 'ex–post' character of the non–adaptive approach from Section 4 the latter comparison of the adaptive strategy with particular 'unconditional' model specifications is to some extent inadequate from a practical point of view. As the analyst does not know a priori which model will perform best, choosing the best specification appears to resemble a random experiment. Assuming that a–priori each model implementation has the same probability of performing best the probability of a successful choice is 0.01. Even if, according to the results given in Section 4 particular models may be preferred, as e.g. three factor specifications with small autoregressive order, the probability of choosing an outperforming specification is hardly larger than 8/36, the realized frequency of outperforming adaptive implementations. In this sense the 'unconditional' approach taken in Section 4 is not feasible in practice, thereby giving the adaptive model selection procedure another edge. Thus, from a probabilistic point of view, it is sensible to compare the performance of the adaptive strategy with the average performance over all 'unconditional' model specifications. Such a comparison reveals that the adaptive model selection procedure outperforms in 30 cases. The exceptions are F6, R4, R5 for $h = 1$, F4, F6 for $h = 5$ and F6 for $h = 10$.

Measured in terms of hm–statistics the statistical performance of the adaptive forecasting procedure is slightly inferior in comparison with the reported economic performance. In 18 cases the hm–statistic obtained from adaptive model choices is significantly larger than unity at the 5% level. Two of these hm–statistics (F3, F4 for $h = 10$) exceed the corresponding statistic of the unconditionally best performing specification. In six further cases (R6, F1 for $h = 5$ and R2, R4, F3, F4 for $h = 10$) the adaptive strategy generates hm–statistics close to the best 'unconditional' implementation. However, the adaptive strategy also obtains in 10 cases (R1, R4, F2, F6 for $h = 1$, R5, F4, F5 for $h = 5$ and R5, F5, F6 for $h = 10$) hm–statistics insignificantly smaller than unity. In comparison with the average performance over all 'unconditional' models the adaptive strategy obtains a larger hm–statistic in 22 out of 36 cases.

Given the superior performance of adaptive model selection it might be of interest to have a closer look at the empirical frequencies of particular model features over all 1240 recursive forecasting steps. Such empirical distributions of best performing model features would support our view of local homogeneity if the relative frequencies were flat over the range of admissible parameter selections. In turn, global homogeneity is likely indicated by unimodal histograms of best performing model features. We refrain from providing detailed results for the empirical distributions of particular model features to economize on space, but assure that adaptively selected model specifications are far from homogeneous over the considered forecasting period. As one may already imagine in the light of the empirical results discussed in Section 4, there is some overall tendency to favor small VAR–orders, in particular $p = 0$. Similarly, for implementing trades F1 to F4 at forecast horizons $h = 1, 5$ and $h = 10$ the adaptive strategy obtains a choice $K = 1$ in 17% to 51% of all 1240 forecast periods. The general impression obtained from the empirical frequencies of best performing model features is, however, more indicative of time dependence and, thus, of local homogeneity.

[Insert Tables 10 and 11 about here]

6 Conclusion

In this paper we analyze the FIBOR/EURIBOR swap term structure adopting PCA/VAR models. From a more descriptive initial analysis we conjecture that swap term structure dynamics are likely time varying. Evaluating the forecasting performance over a battery of 100 PCA/VAR implementations we fail to find any uniformly dominating single specification in terms of both, statistical or economic performance. The latter finding may also be seen as an implication against global dynamic homogeneity of the swap term structure. Building upon an assumption of locally homogeneous dynamics we motivate a data driven adaptive strategy to ex–ante determine a particular implementation of a factor model which is likely to provide accurate trading signals. Evaluating the latter strategy in terms of economic performance it mostly outperforms static designs of trading the swap term structure. Moreover, opposite to the universe of implemented static designs the adaptive procedure shows only small downside risk but promises considerable compounded returns of up to 25% per annum.

Our study shows that PCA techniques can be fruitfully exploited for term structure forecasting. As indicated in the term structure literature there might be some additional potential to improve ex–ante forecasts via incorporation of fundamental macroeconomic variables. Recent methodological and empirical contributions on dynamic factor models (Stock and Watson 2002) are also supportive for this vein of research which we regard as a key issue for future work.

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Appendix

Figures

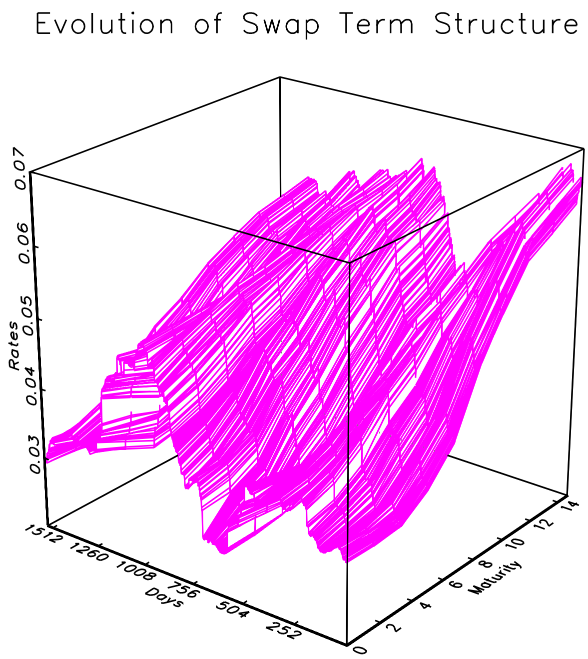

Figure 1. Evolution of the actual swap term structure for the period from January 14, 1997 to January 7, 2003. We removed one extreme outlier, the 12yr swap rate on January 2, 1998.

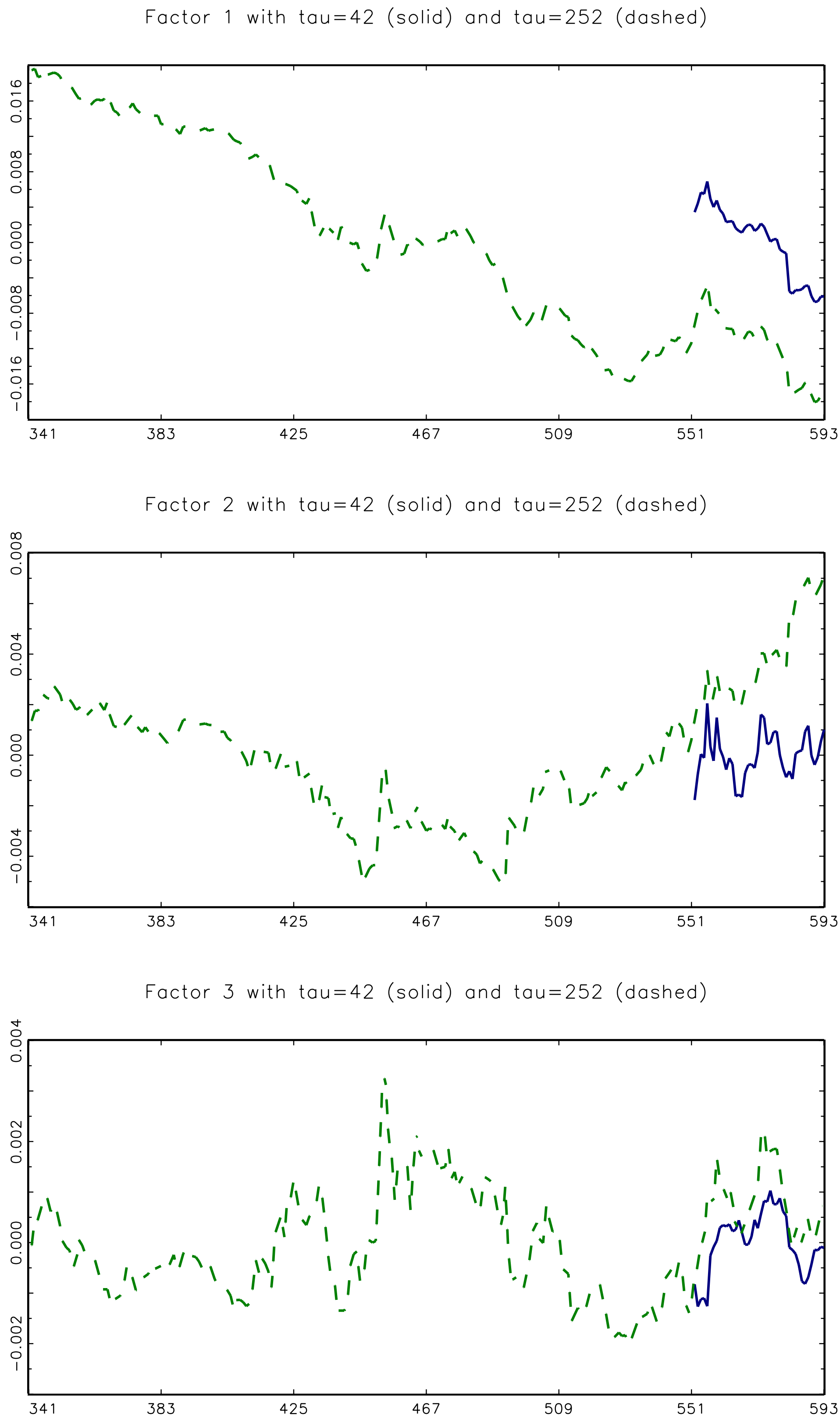

Figure 2. Time series of factors estimates f_{kt}, $k = 1, 2, 3$ estimated in $T^* = 593$ (April 26, 1999). Solid and dashed lines depict factor estimates for $\tau = 42$ and $\tau = 252$, respectively.

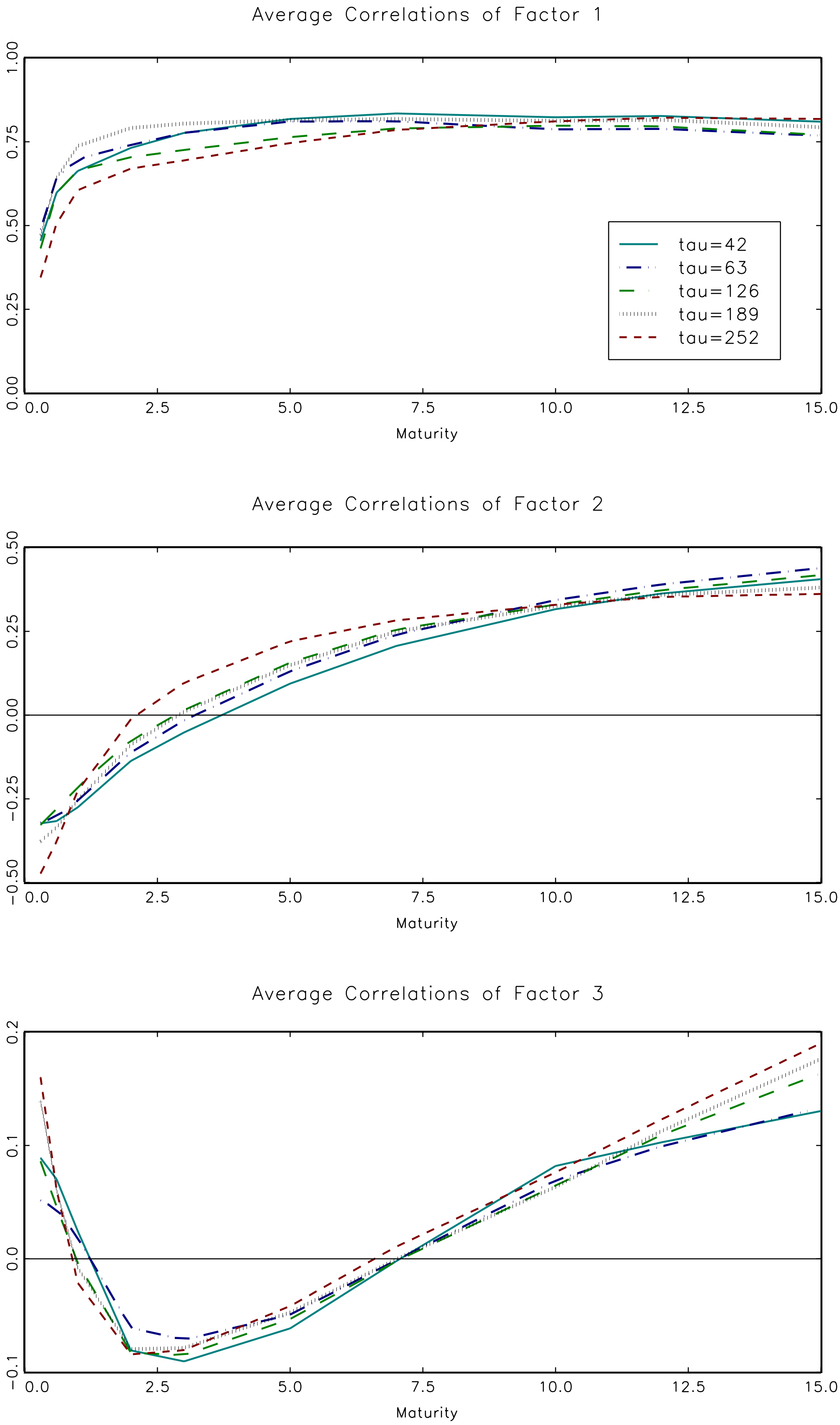

Figure 3. Average correlations $\hat{\rho}_{km} = 1/1240 \sum_{T^*=303}^{T^*1542} \hat{\rho}_{km}^{T^*}$ of three principal components with swap rates of maturities $m = 1, \ldots, M$ averaged over the period from March 12, 1998 ($T^* = 303$) to December 12, 2002 ($T^* = 1542$). Alternative choices for $\tau \in \{42, \ldots, 252\}$ are considered.

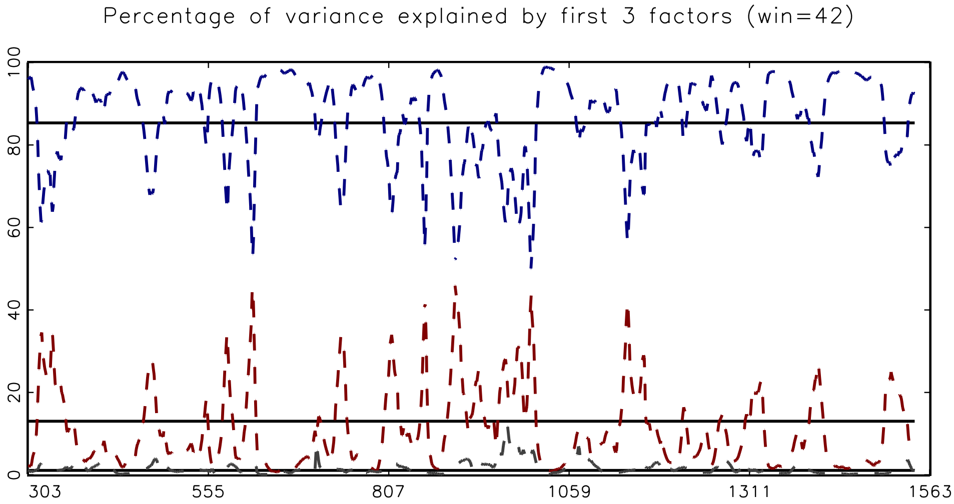

Figure 4. Fractions of explained variances for the period from March 12, 1998 ($T^* = 303$) to December 20, 2002 ($T^* = 1542$). Dashed lines depict explained variances for three principal components obtained from recursive factor decompositions with $\tau = 42$. Solid lines show the corresponding fractions implied by the unconditional covariance matrix.

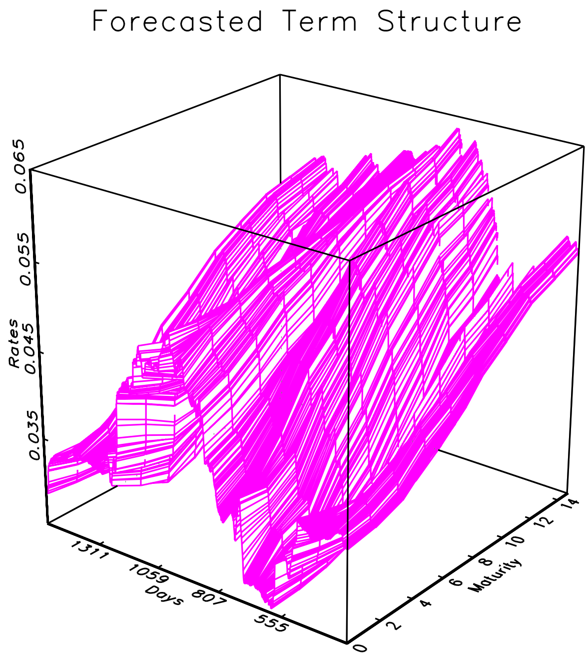

Figure 5. Recursively forecasted term structure ($\tau = 42, K = 1, p = 0, h = 5$) for the period from March 12, 1998 ($T^* = 303$) to December 20, 2002 ($T^* = 1542$).

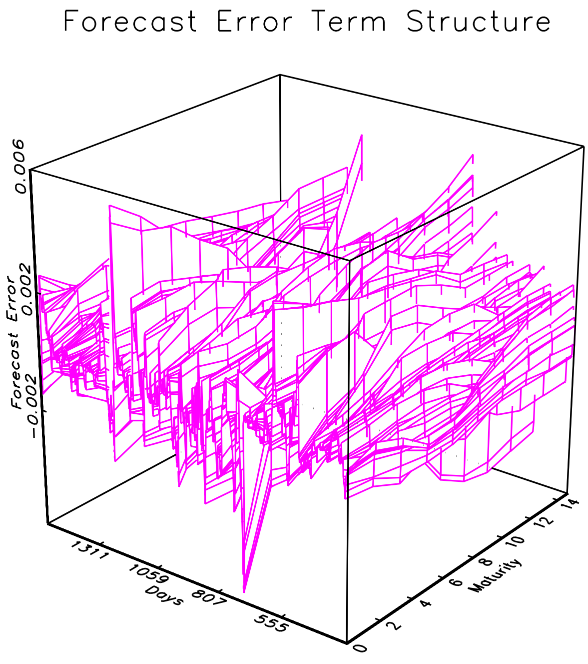

Figure 6. Term structure of forecast errors $e_{m,T^*+h} = \hat{y}_{m,T^*+h} - y_{m,T^*+h}$ ($\tau = 42, K = 1, p = 0, h = 5$) for the period from March 12, 1998 ($T^* = 303$) to December 20, 2002 ($T^* = 1542$).

Tables

	1%	2.5%	5%	95%	97.5%	99%
hm	0.933	0.945	0.954	1.046	1.055	1.067
min	0.895	0.902	0.907	0.947	0.949	0.952
max	1.048	1.051	1.053	1.093	1.099	1.105

Table 1. Simulated critical values of the hm–statistic and particular order statistics.

ID	TradeName	Signal	2yr	5yr	10yr
R1	*2yrSingleTrade*	2yr up	1Pay	0	0
R2	*5yrSingleTrade*	5yr up	0	1Pay	0
R3	*10yrSingleTrade*	10yr up	0	0	1Pay
R4	*LevelTrade*	($\frac{1}{3}$2yr + $\frac{1}{3}$5yr + $\frac{1}{3}$10yr) up	$\frac{1}{3}$Pay	$\frac{1}{3}$Pay	$\frac{1}{3}$Pay
R5	*SlopeTrade*	($\frac{1}{2}$10yr - $\frac{1}{2}$2yr) up	$\frac{1}{2}$Rec	0	$\frac{1}{2}$Pay
R6	*CurvatureTrade*	($\frac{1}{4}$2yr - $\frac{1}{2}$5yr + $\frac{1}{4}$10yr) up	$\frac{1}{4}$Pay	$\frac{1}{2}$Rec	$\frac{1}{4}$Pay
F1	*2yrSingleTrade*	first factor up & $\mathrm{Cor}(f_1, 2yr) > 0$	1Pay	0	0
F2	*5yrSingleTrade*	first factor up & $\mathrm{Cor}(f_1, 5yr) > 0$	0	1Pay	0
F3	*10yrSingleTrade*	first factor up & $\mathrm{Cor}(f_1, 10yr) > 0$	0	0	1Pay
F4	*LevelTrade*	first factor up	$\frac{1}{3}$Pay	$\frac{1}{3}$Pay	$\frac{1}{3}$Pay
F5	*SlopeTrade*	second factor up	$\frac{1}{2}$Rec	0	$\frac{1}{2}$Pay
F6	*CurvatureTrade*	third factor up	$\frac{1}{4}$Pay	$\frac{1}{2}$Rec	$\frac{1}{4}$Pay

Table 2. Summary of trading strategies.

	3m	6m	1yr	2yr	3yr	5yr	7yr	10yr	12yr	15yr	Level	Slope	Curvature
Mean	3.67	3.71	3.79	4.03	4.26	4.64	4.95	5.27	5.41	5.58	4.65	0.62	0.0017
Median	3.51	3.56	3.73	4.03	4.29	4.74	5.03	5.31	5.44	5.59	4.73	0.58	−0.0073
Min	2.60	2.60	2.67	2.78	2.95	3.36	3.66	4.02	4.20	4.46	3.44	0.18	−0.0725
Max	5.17	5.20	5.27	5.43	5.50	5.62	5.78	6.29	6.53	6.76	5.61	1.32	0.0950
StD*100	0.64	0.62	0.62	0.61	0.58	0.54	0.52	0.51	0.51	0.51	0.52	0.24	0.0354

Table 3. Descriptive statistics of location and dispersion for actual swap rates and shape parameters for the period from January 14, 1997 ($T^* = 1$) to January 7, 2003 ($T^* = 1552$). The standard deviation is multiplied by 100 (StD*100). Level, slope and curvature are measured by $\frac{2\text{yr} + 5\text{yr} + 10\text{yr}}{3}$, $\frac{10\text{yr}}{2} - \frac{2\text{yr}}{2}$ and $\frac{2\text{yr}}{4} - \frac{5\text{yr}}{2} + \frac{10\text{yr}}{4}$, respectively. We removed one extreme outlier, the 12yr swap rate on January 2, 1998 ($T^* = 254$).

	Observed swap rates						Forecasted swap rates					
	2yr	5yr	10yr	Level	Slope	Curvature	2yr	5yr	10yr	Level	Slope	Curvature
Mean	4.05	4.58	5.14	4.59	0.55	0.0064	4.05	4.58	5.14	4.59	0.55	0.0051
Median	4.10	4.67	5.20	4.65	0.54	0.0014	4.08	4.67	5.18	4.65	0.53	0.0003
Min	2.78	3.36	4.02	3.44	0.18	−0.0725	2.73	3.31	3.99	3.41	0.14	−0.0659
Max	5.43	5.62	5.97	5.61	0.90	0.0950	5.45	5.67	6.02	5.64	0.88	0.0901
StD*100	0.67	0.58	0.47	0.56	0.16	0.0375	0.67	0.60	0.49	0.57	0.17	0.0400

Table 4. Descriptive statistics of location and dispersion for forecasted swap rates and shape parameters for $h = 5$ days and corresponding actual rates and shape parameters for the period from March 12, 1998 ($T^* = 303$) to December 20, 2002 ($T^* = 1542$). The standard deviation is multiplied by 100 (StD*100). Level, slope and curvature are measured by $\frac{2\text{yr} + 5\text{yr} + 10\text{yr}}{3}$, $\frac{10\text{yr}}{2} - \frac{2\text{yr}}{2}$ and $\frac{2\text{yr}}{4} - \frac{5\text{yr}}{2} + \frac{10\text{yr}}{4}$, respectively.

h	$e_{m,T^*+h} = \hat{y}_{m,T^*+h} - y_{m,T^*+h}$ 1	5	10	$e^2_{m,T^*+h} = (\hat{y}_{m,T^*+h} - y_{m,T^*+h})^2$ 1	5	10
Const	0.402	−0.275	−0.095	0.051	0.746	1.524
	(1.267)	(−0.473)	(−0.117)	(1.271)	(7.116)	(7.649)
3m	−0.702	−0.832	−0.980	0.222	−0.174	−0.632
	(−2.012)	(−1.404)	(−1.179)	(3.917)	(−1.291)	(−2.666)
6m	−0.605	−0.720	−0.857	0.024	−0.463	−1.050
	(−1.989)	(−1.303)	(−1.087)	(0.741)	(−4.548)	(−5.302)
1yr	−0.597	−0.705	−0.829	−0.027	−0.377	−0.805
	(−2.071)	(−1.260)	(−1.030)	(−0.957)	(−3.898)	(−4.174)
2yr	−0.267	−0.309	−0.352	0.024	−0.024	−0.090
	(−0.915)	(−0.518)	(−0.410)	(0.896)	(−0.228)	(−0.421)
5yr	−0.058	−0.095	−0.158	−0.021	−0.061	−0.150
	(−0.209)	(−0.161)	(−0.187)	(−0.828)	(−0.588)	(−0.699)
7yr	−0.192	−0.286	−0.434	−0.009	−0.156	−0.391
	(−0.676)	(−0.497)	(−0.526)	(−0.316)	(−1.538)	(−1.920)
10yr	−0.241	−0.382	−0.602	0.080	−0.153	−0.538
	(−0.784)	(−0.660)	(−0.740)	(2.320)	(−1.501)	(−2.736)
12yr	−0.394	−0.575	−0.831	0.143	−0.065	−0.419
	(−1.221)	(−0.977)	(−1.010)	(3.605)	(−0.623)	(−2.106)
15yr	−0.662	−0.902	−1.214	0.331	0.215	−0.001
	(−1.826)	(−1.451)	(−1.419)	(5.735)	(1.733)	(−0.003)
Year98	−0.502	−0.189	0.101	0.163	0.279	0.502
	(−1.984)	(−0.452)	(0.179)	(5.159)	(3.953)	(4.114)
Year99	−0.374	−2.015	−3.593	0.345	1.013	2.225
	(−1.414)	(−4.372)	(−5.566)	(7.982)	(11.015)	(13.170)
Year00	−0.265	−1.360	−2.485	0.295	0.632	1.109
	(−0.960)	(−2.856)	(−3.795)	(9.444)	(7.375)	(6.954)
Year01	0.046	−2.209	−4.403	0.447	1.153	2.365
	(0.146)	(−4.009)	(−5.811)	(11.118)	(10.146)	(10.565)
Level	0.193	2.594	4.760	0.002	−0.136	−0.167
	(0.846)	(6.536)	(8.600)	(0.043)	(−1.630)	(−1.065)
Slope	0.297	1.152	1.257	0.247	0.433	0.725
	(1.487)	(3.549)	(2.743)	(9.312)	(7.391)	(6.768)
Curvature	0.009	0.388	0.126	−0.167	−0.451	−0.848
	(0.044)	(1.134)	(0.267)	(−6.445)	(−6.661)	(−6.602)

Table 5. Regression of forecast errors from a particular PCA/VAR implementation ($\tau = 42, K = 1, p = 0$) for the period from March 12, 1998 to December 20, 2002. Estimated coefficients for e_{mt} and e^2_{mt} are multiplied with 10^4 respectively 10^6. HAC consistent t–ratios are given in parentheses, Newey & West (1987).

	R1	R2	R3	R4	R5	R6
BenchHM	1.098**	1.074**	1.073**	1.055**	1.019	0.975
Const	1.056	1.047	1.049	1.055	1.003	1.083
$\tau = 63$	−0.0075	−0.0202	−0.0134	−0.0121	0.0070	0.0319
	(−1.00)	(−3.65)	(−2.77)	(−2.44)	(1.18)	(7.89)
$\tau = 126$	−0.0579	−0.0202	0.0024	−0.0141	**0.0111**	0.0348
	(−7.75)	(−3.65)	(0.50)	(−2.82)	(1.88)	(8.60)
$\tau = 189$	−0.0798	−0.0526	−0.0011	−0.0444	0.0010	**0.0358**
	(−10.68)	(−9.52)	(−0.23)	(−8.90)	(0.16)	(8.85)
$\tau = 252$	−0.1002	−0.0757	**0.0075**	−0.0617	−0.0288	0.0036
	(−13.41)	(−13.70)	(1.55)	(−12.37)	(−4.85)	(0.89)
$K = 2$	0.0583	0.0170	**0.0154**	0.0278	**0.0433**	**0.0068**
	(7.80)	(3.07)	(3.19)	(5.57)	(7.30)	(1.68)
$K = 3$	0.0664	**0.0377**	−0.0053	0.0329	0.0357	−0.0106
	(8.88)	(6.81)	(−1.09)	(6.61)	(6.02)	(−2.63)
$K = 4$	0.0663	0.0344	0.0036	0.0339	0.0393	−0.0007
	(8.87)	(6.23)	(0.75)	(6.80)	(6.62)	(−0.18)
$K = 5$	**0.0890**	0.0324	0.0019	**0.0424**	0.0408	−0.0226
	(11.91)	(5.86)	(0.39)	(8.50)	(6.87)	(−5.58)
$p = 1$	−0.0031	−0.0041	**0.0038**	−0.0086	**0.0065**	0.0065
	(−0.47)	(−0.82)	(0.88)	(−1.92)	(1.22)	(1.80)
$p = 2$	−0.0095	−0.0116	−0.0064	−0.0183	0.0054	0.0057
	(−1.42)	(−2.34)	(−1.49)	(−4.10)	(1.02)	(1.58)
$p = 3$	−0.0109	−0.0117	−0.0058	−0.0135	0.0053	**0.0085**
	(−1.63)	(−2.36)	(−1.33)	(−3.02)	(1.00)	(2.35)
$h = 1$	−0.0464	−0.0257	−0.0262	−0.0412	−0.0148	−0.0053
	(−8.01)	(−5.99)	(−7.00)	(−10.66)	(−3.22)	(−1.68)
$h = 10$	**0.0483**	**0.0193**	**0.0123**	**0.0294**	**0.0166**	−0.0473
	(8.34)	(4.50)	(3.28)	(7.60)	(3.60)	(−15.09)
min	0.9224	0.9278	0.9545	0.9442	0.8801	0.9462
($\tau/K/p/h$)	(126/1/3/10)	(189/2/2/5)	(189/1/1/10)	(252/1/2/1)	(252/1/3/10)	(42/5/0/10)
mean	1.0579	1.0289	1.0440	1.0423	1.0376	1.0866
(t–stat)	(13.434)	(10.751)	(23.572)	(15.149)	(16.065)	(41.808)
max	1.2354	1.1415	1.1467	1.1515	1.1485	1.1643
($\tau/K/p/h$)	(42/2/0/10)	(126/3/0/10)	(252/2/0/10)	(126/5/1/10)	(189/2/0/10)	(189/2/2/5)
anova	1.2280	1.1113	1.1393	1.1163	1.1234	1.1561
($\tau/K/p/h$)	(42/5/0/10)	(42/3/0/10)	(252/2/1/10)	(42/5/0/10)	(126/2/1/10)	(189/2/3/5)

Table 6. ANOVA results for hm–statistic for rate based signals for the period from March 12, 1998 to December 20, 2002. BenchHM are the hm-statistics for the specification $\tau = 42, K = 1, p = 0, h = 5$, where ** and * indicate significance of the hm–statistic at the 5% and 10% significance level (critical values given in Table 1). ANOVA estimates are given with t–statistics in parentheses underneath. Bold entries indicate model features providing the best forecasting results on average. The lower part shows minimum, mean and maximum hm–statistics for each trade obtained over all alternative forecasting models. hm–statistics for the ANOVA implied specification (anova) are given in the last line. For the latter PCA/VAR characteristics are shown in parentheses underneath, except for the mean hm–statistics where a t–statistic for the hypothesis $hm = 1$ is provided.

	F1	F2	F3	F4	F5	F6
BenchHM	1.066**	1.034	1.041	1.081**	1.046*	1.075**
Const	1.098	1.073	1.053	1.060	1.008	0.986
$\tau = 63$	**0.0380**	−0.0300	−0.0292	−0.0239	−0.0248	−0.0312
	(10.13)	(−8.51)	(−8.35)	(−5.05)	(−5.58)	(−5.17)
$\tau = 126$	−0.0071	−0.0131	−0.0042	−0.0065	−0.0188	**0.0213**
	(−1.90)	(−3.72)	(−1.21)	(−1.37)	(−4.23)	(3.53)
$\tau = 189$	−0.0265	−0.0575	−0.0287	−0.0310	−0.0007	−0.0055
	(−7.05)	(−16.34)	(−8.19)	(−6.55)	(−0.16)	(−0.92)
$\tau = 252$	−0.0043	−0.0465	−0.0131	**0.0247**	−0.0044	−0.0158
	(−1.15)	(−13.21)	(−3.73)	(5.23)	(−1.00)	(−2.63)
$K = 2$	−0.0008	**0.0003**	**0.0013**	**0.0005**	-	-
	(−0.22)	(0.08)	(0.39)	(0.10)		
$K = 3$	−0.0061	−0.0053	−0.0030	−0.0034	0.0048	-
	(−1.64)	(−1.51)	(−0.85)	(−0.72)	(1.21)	
$K = 4$	−0.0097	−0.0078	−0.0044	−0.0068	**0.0083**	−0.0080
	(−2.58)	(−2.22)	(−1.26)	(−1.43)	(2.08)	(−1.72)
$K = 5$	−0.0051	−0.0033	−0.0007	−0.0033	0.0052	−0.0033
	(−1.36)	(−0.93)	(−0.19)	(−0.69)	(1.29)	(−0.71)
$p = 1$	−0.0087	−0.0114	−0.0082	−0.0069	0.0219	0.0183
	(−2.59)	(−3.61)	(−2.60)	(−1.64)	(5.50)	(3.39)
$p = 2$	−0.0210	−0.0186	−0.0150	−0.0154	**0.0223**	**0.0238**
	(−6.26)	(−5.91)	(−4.78)	(−3.63)	(5.60)	(4.41)
$p = 3$	−0.0178	−0.0133	−0.0104	−0.0162	0.0223	0.0230
	(−5.31)	(−4.23)	(−3.32)	(−3.82)	(5.59)	(4.27)
$h = 1$	−0.0640	−0.0439	−0.0365	−0.0409	**0.0132**	−0.0097
	(−21.99)	(−16.10)	(−13.45)	(−11.16)	(3.84)	(−2.08)
$h = 10$	**0.0381**	**0.0269**	**0.0248**	**0.0198**	−0.0071	−0.0194
	(13.09)	(9.86)	(9.15)	(5.41)	(−2.06)	(−4.15)
min	0.9684	0.9221	0.9341	0.9388	0.9451	0.9077
($\tau/K/p/h$)	(252/5/2/1)	(252/3/3/1)	(252/3/3/1)	(252/3/3/1)	(63/2/3/10)	(63/4/3/5)
mean	1.0728	1.0235	1.0204	1.0334	1.0218	0.9827
(t–stat)	(24.218)	(9.860)	(10.036)	(13.948)	(12.542)	(−7.069)
max	1.2029	1.0990	1.0921	1.1405	1.0904	1.0793
($\tau/K/p/h$)	(63/1/1/10)	(42/4/1/10)	(252/5/3/10)	(252/5/3/10)	(42/5/1/1)	(126/3/3/5)
anova	1.1986	1.0687	1.0600	1.0999	1.0628	1.0458
($\tau/K/p/h$)	(63/1/0/10)	(42/2/0/10)	(42/2/0/10)	(252/2/0/10)	(42/4/2/1)	(126/3/2/5)

Table 7. ANOVA results for hm–statistic for factor based signals for the period from March 12, 1998 to December 20, 2002. For further notes see Table 6.

	R1	R2	R3	R4	R5	R6
Const	17.76	39.83	30.21	33.12	−14.60	9.69
	(5.35)	(7.80)	(4.40)	(7.63)	(−3.15)	(22.51)
$\tau = 63$	**3.30**	**1.23**	7.26	**2.30**	6.96	0.36
	(1.18)	(0.28)	(1.25)	(0.63)	(1.78)	(0.99)
$\tau = 126$	−9.38	−4.01	**22.12**	−2.31	39.60	0.19
	(−3.34)	(−0.93)	(3.81)	(−0.63)	(10.11)	(0.52)
$\tau = 189$	−20.74	−38.61	−7.16	−38.46	**42.75**	**1.49**
	(−7.39)	(−8.94)	(−1.23)	(−10.48)	(10.92)	(4.10)
$\tau = 252$	−31.58	−60.65	5.83	−47.95	33.87	−0.73
	(−11.25)	(−14.05)	(1.01)	(−13.07)	(8.65)	(−2.00)
$K = 2$	19.87	13.46	32.86	20.27	2.27	**1.11**
	(7.08)	(3.12)	(5.67)	(5.52)	(0.58)	(3.05)
$K = 3$	18.97	26.09	18.25	27.43	−0.86	−2.47
	(6.76)	(6.04)	(3.15)	(7.47)	(−0.22)	(−6.78)
$K = 4$	21.50	31.19	**42.73**	29.07	**3.27**	−0.16
	(7.66)	(7.23)	(7.37)	(7.92)	(0.83)	(−0.43)
$K = 5$	**26.16**	**34.53**	35.55	**33.98**	−1.64	−0.90
	(9.32)	(8.00)	(6.13)	(9.26)	(−0.42)	(−2.48)
$p = 1$	−1.48	−2.24	−1.38	−3.67	**1.64**	0.00
	(−0.59)	(−0.58)	(−0.27)	(−1.12)	(0.47)	(0.01)
$p = 2$	−4.23	−6.69	−5.55	−5.45	1.61	−0.32
	(−1.68)	(−1.73)	(−1.07)	(−1.66)	(0.46)	(−0.97)
$p = 3$	−5.95	−7.06	−5.83	−5.96	1.08	**0.13**
	(−2.37)	(−1.83)	(−1.12)	(−1.82)	(0.31)	(0.39)
$h = 1$	−18.95	−33.53	−48.88	−31.90	−8.13	−5.91
	(−8.72)	(−10.03)	(−10.88)	(−11.22)	(−2.68)	(−20.95)
$h = 10$	**38.33**	**47.67**	**51.58**	**50.61**	**14.19**	**2.23**
	(17.63)	(14.26)	(11.48)	(17.80)	(4.68)	(7.92)
min	−18.03	−34.19	−71.06	−20.93	−49.27	2.16
$(\tau/K/p/h)$	(252/1/2/5)	(252/3/0/10)	(189/1/1/10)	(252/1/1/5)	(63/4/3/10)	(63/3/2/1)
mean	26.93	41.19	59.40	40.46	13.74	8.20
max	110.63	177.79	215.06	156.25	90.03	19.06
$(\tau/K/p/h)$	(63/5/0/10)	(126/4/0/10)	(126/2/0/10)	(126/4/0/10)	(126/2/3/10)	(189/2/3/10)
anova	110.63	143.92	147.05	129.33	75.06	19.06
$(\tau/K/p/h)$	(63/5/0/10)	(63/5/0/10)	(126/4/0/10)	(63/5/0/10)	(189/4/1/10)	(189/2/3/10)

Table 8. ANOVA results for cash flows for rate based signals for the period from March 12, 1998 to December 20, 2002. The upper part shows regression results for the estimated coefficients and corresponding t–statistics in parentheses underneath. The lower part gives minimum, mean and maximum cash flows for each trade obtained over all alternative forecasting models. Cash flows for the ANOVA implied specification (anova) are given in the last line. Specifications are shown in parentheses below cash flows.

	F1	F2	F3	F4	F5	F6
Const	30.56 (19.96)	58.73 (18.22)	68.89 (16.42)	48.68 (14.35)	2.58 (1.10)	−3.22 (−4.55)
$\tau = 63$	**14.55** (11.25)	−0.18 (−0.07)	−15.26 (−4.30)	−10.68 (−3.73)	7.08 (3.44)	−3.67 (−5.67)
$\tau = 126$	7.69 (5.94)	**5.86** (2.15)	**3.24** (0.91)	17.84 (6.23)	13.95 (6.77)	**2.26** (3.50)
$\tau = 189$	−3.70 (−2.86)	−36.95 (−13.56)	−55.52 (−15.65)	−19.54 (−6.82)	**19.14** (9.29)	−3.13 (−4.84)
$\tau = 252$	−1.19 (−0.92)	−23.43 (−8.60)	−17.13 (−4.83)	**28.18** (9.83)	15.54 (7.55)	0.20 (0.31)
$K = 2$	−0.35 (−0.27)	−0.16 (−0.06)	−0.15 (−0.04)	−0.20 (−0.07)	-	-
$K = 3$	−1.64 (−1.27)	−2.70 (−0.99)	−3.15 (−0.89)	−2.36 (−0.82)	0.06 (0.04)	-
$K = 4$	−2.17 (−1.68)	−3.63 (−1.33)	−3.59 (−1.01)	−3.25 (−1.13)	1.52 (0.83)	0.02 (0.03)
$K = 5$	−2.50 (−1.93)	−3.26 (−1.20)	−3.14 (−0.89)	−3.25 (−1.14)	**1.53** (0.83)	**0.09** (0.18)
$p = 1$	−1.45 (−1.25)	−2.78 (−1.14)	−3.57 (−1.13)	−2.07 (−0.81)	0.56 (0.31)	0.93 (1.61)
$p = 2$	−4.74 (−4.09)	−6.20 (−2.54)	−4.94 (−1.56)	−4.62 (−1.80)	**0.64** (0.35)	2.47 (4.27)
$p = 3$	−5.12 (−4.42)	−6.53 (−2.68)	−8.14 (−2.57)	−7.10 (−2.77)	−2.61 (−1.42)	**3.20** (5.53)
$h = 1$	−26.73 (−26.67)	−39.33 (−18.64)	−45.43 (−16.54)	−42.82 (−19.29)	−11.94 (−7.48)	**1.84** (3.67)
$h = 10$	**36.06** (35.97)	**51.04** (24.19)	**60.23** (21.92)	**55.89** (25.18)	**10.65** (6.67)	−4.64 (−9.27)
min ($\tau/K/p/h$)	−3.02 (252/3/3/1)	−13.23 (252/3/3/1)	−42.91 (189/5/3/5)	−11.42 (252/3/3/1)	−10.81 (42/3/2/5)	−16.76 (189/3/0/10)
mean	32.98	45.86	50.72	50.93	13.72	-3.34
max ($\tau/K/p/h$)	94.43 (63/1/0/10)	136.69 (126/1/1/10)	154.62 (42/3/1/10)	170.48 (252/5/2/10)	61.84 (189/4/2/10)	4.76 (126/5/3/5)
anova ($\tau/K/p/h$)	94.43 (63/1/0/10)	133.27 (126/1/0/10)	144.74 (126/1/0/10)	141.06 (252/1/0/10)	55.87 (189/5/2/10)	−0.18 (126/5/3/1)

Table 9. ANOVA results for cash flows for rate based signals for the period from March 12, 1998 to December 20, 2002. For further notes see Table 8.

	R1			R2			R3		
	$h=1$	$h=5$	$h=10$	$h=1$	$h=5$	$h=10$	$h=1$	$h=5$	$h=10$
HM	0.991	1.052*	1.161**	1.020	1.055**	1.134**	1.017	1.065**	1.098**
Bank	103.76	142.74	214.39	113.70	174.71	283.67	112.38	179.22	284.89
CF	3.76	42.74	114.39	13.70	74.71	183.67	12.38	79.22	184.89
Return	0.0074	0.0738	0.1648	0.0260	0.1181	0.2319	0.0236	0.1238	0.2329
MinBank	99.70	98.66	98.56	99.52	94.58	89.77	97.54	94.27	94.92
AvBank	102.59	118.37	147.07	106.99	135.68	173.51	107.20	138.52	217.08
MaxBank	104.29	142.74	214.39	113.70	174.71	283.67	114.59	179.22	285.41
MinHM	0.9599	0.9346	0.9224	0.9608	0.9279	0.9471	0.9627	0.9950	0.9545
$(\tau/K/p)$	(252/3/3)	(126/1/3)	(126/1/3)	(63/5/3)	(189/2/2)	(252/2/0)	(42/3/2)	(63/5/3)	(189/1/1)
AvHM	1.0109	1.0573	1.1056	1.0054	1.0310	1.0503	1.0224	1.0486	1.0609
MaxHM	1.0641	1.1834	1.2354	1.0499	1.1195	1.1415	1.0612	1.1188	1.1467
$(\tau/K/p)$	(42/4/0)	(42/2/2)	(42/2/0)	(189/3/2)	(42/2/1)	(126/3/0)	(63/1/3)	(126/2/0)	(252/2/0)
MinCF	−6.91	−18.03	−5.30	−11.93	−27.76	−34.19	−16.84	−33.38	−71.06
$(\tau/K/p)$	(252/3/3)	(252/1/2)	(252/1/2)	(189/2/1)	(189/2/2)	(252/3/0)	(42/3/2)	(189/1/0)	(189/1/1)
AvCF	1.51	20.46	58.80	2.95	36.48	84.15	9.62	58.51	110.08
MaxCF	7.32	54.28	110.63	19.00	91.38	177.79	26.61	121.70	215.06
$(\tau/K/p)$	(63/4/0)	(63/5/0)	(63/5/0)	(42/3/0)	(42/2/1)	(126/4/0)	(126/2/0)	(126/2/0)	(126/2/0)

	R4			R5			R6		
	$h=1$	$h=5$	$h=10$	$h=1$	$h=5$	$h=10$	$h=1$	$h=5$	$h=10$
HM	0.991	1.024	1.137**	1.007	0.996	0.957	1.091**	1.134**	1.050*
Bank	98.29	153.67	278.47	100.04	124.60	143.14	104.00	113.02	117.56
CF	-1.71	53.67	178.47	0.04	24.60	43.14	4.00	13.02	17.56
Return	-0.0034	0.0897	0.2273	0.0001	0.0450	0.0744	0.0079	0.0248	0.0329
MinBank	91.14	94.16	89.20	95.50	97.15	96.72	100.00	99.94	99.50
AvBank	96.90	125.87	180.03	101.00	115.83	129.21	101.98	106.58	109.99
MaxBank	104.81	153.67	278.48	104.92	127.66	152.40	104.01	113.15	118.27
MinHM	0.9442	0.9611	0.9744	0.9701	0.9413	0.8801	1.0497	0.9845	0.9462
$(\tau/K/p)$	(252/1/2)	(126/1/0)	(252/3/0)	(126/2/0)	(252/1/1)	(252/1/3)	(42/5/0)	(42/5/0)	(42/5/0)
AvHM	1.0050	1.0462	1.0756	1.0222	1.0370	1.0535	1.0989	1.1041	1.0569
MaxHM	1.0720	1.1225	1.1515	1.0935	1.1046	1.1485	1.1449	1.1643	1.1327
$(\tau/K/p)$	(63/5/0)	(126/4/0)	(126/5/1)	(42/5/2)	(126/5/3)	(189/2/0)	(63/5/2)	(189/2/2)	(63/1/3)
MinCF	−14.35	−20.93	−13.53	−8.28	−29.51	−49.27	2.16	5.78	5.22
$(\tau/K/p)$	(252/1/2)	(252/1/1)	(252/3/2)	(63/5/3)	(42/3/0)	(63/4/3)	(63/3/2)	(252/3/3)	(42/5/0)
AvCF	2.32	34.22	84.83	3.59	11.72	25.91	3.52	9.43	11.66
MaxCF	16.94	82.89	156.25	14.10	52.42	90.03	5.62	15.42	19.06
$(\tau/K/p)$	(63/5/0)	(126/4/0)	(126/4/0)	(126/4/1)	(189/5/3)	(126/2/3)	(63/5/2)	(189/2/3)	(189/2/3)

Table 10. Results for the adaptive model selection strategy implementing R1, R2 and R3 for the period from March 12, 1998 to December 20, 2002. The first part contains the hm–statistic and some statistics describing the cash flow. ** and * indicate significance of the hm–statistic at the 5% and 10% level (critical values given in Table 1).

	F1			F2			F3		
	$h = 1$	$h = 5$	$h = 10$	$h = 1$	$h = 5$	$h = 10$	$h = 1$	$h = 5$	$h = 10$
HM	1.004	1.118**	1.191**	0.990	1.059**	1.142**	1.009	1.054*	1.118**
Bank	107.05	155.06	214.03	109.66	174.78	282.60	101.72	176.76	313.30
CF	7.05	55.06	114.03	9.66	74.78	182.60	1.72	76.76	213.30
Return	0.0137	0.0917	0.1644	0.0186	0.1181	0.2309	0.0034	0.1207	0.2566
MinBank	99.23	99.15	98.83	98.81	96.56	89.81	94.27	96.47	94.70
AvBank	102.25	126.14	154.64	104.09	136.32	181.56	101.10	142.92	212.00
MaxBank	107.06	155.06	214.03	109.66	174.78	282.60	107.78	176.76	313.83
MinHM	0.9684	1.0172	1.0681	0.9221	0.9563	1.0030	0.9248	0.9554	0.9796
$(\tau/K/p)$	(252/5/2)	(189/5/3)	(189/5/1)	(252/3/3)	(189/5/3)	(189/5/1)	(252/3/3)	(189/5/3)	(189/5/1)
AvHM	1.0174	1.0814	1.1195	0.9853	1.0292	1.0561	1.0033	1.0163	1.0417
MaxHM	1.0619	1.1394	1.2029	1.0360	1.0978	1.0990	1.0494	1.0945	1.1179
$(\tau/K/p)$	(63/1/0)	(63/1/3)	(63/1/1)	(42/1/0)	(42/2/2)	(42/4/1)	(126/1/0)	(42/2/2)	(42/2/1)
MinCF	−3.02	8.75	43.04	−13.23	−12.92	35.49	−28.84	−42.91	17.87
$(\tau/K/p)$	(252/3/3)	(189/5/3)	(42/3/3)	(252/3/3)	(189/5/3)	(189/5/1)	(252/3/3)	(189/5/3)	(189/5/1)
AvCF	3.14	29.87	65.93	2.63	41.96	93.00	0.35	45.79	106.02
MaxCF	8.50	43.91	94.43	15.03	75.03	136.69	16.10	93.63	154.62
$(\tau/K/p)$	(126/1/0)	(63/2/2)	(63/1/0)	(126/1/0)	(126/1/0)	(126/1/1)	(126/1/0)	(42/2/2)	(42/3/1)

	F4			F5			F6		
	$h = 1$	$h = 5$	$h = 10$	$h = 1$	$h = 5$	$h = 10$	$h = 1$	$h = 5$	$h = 10$
HM	0.999	1.082**	1.158**	1.034	0.978	0.967	0.992	1.002	1.003
Bank	107.84	183.54	313.67	103.73	115.43	131.83	100.02	96.85	93.78
CF	7.84	83.54	213.67	3.73	15.43	31.83	0.02	-3.15	-6.22
Return	0.0152	0.1291	0.2569	0.0073	0.0291	0.0568	0.0000	-0.0064	-0.0128
MinBank	98.99	97.84	97.05	99.02	96.20	93.24	99.22	96.80	93.70
AvBank	102.56	143.78	210.37	101.20	106.22	118.75	100.18	99.49	97.60
MaxBank	107.84	183.54	313.67	104.46	116.82	143.99	101.08	101.34	101.06
MinHM	0.9388	0.9411	1.0059	0.9778	0.9579	0.9451	0.9454	0.9077	0.9122
$(\tau/K/p)$	(252/3/3)	(63/4/3)	(189/5/1)	(189/2/3)	(126/2/0)	(63/2/3)	(252/3/0)	(63/4/3)	(189/3/0)
AvHM	0.9995	1.0404	1.0603	1.0330	1.0198	1.0127	0.9826	0.9923	0.9730
MaxHM	1.0444	1.1055	1.1405	1.0904	1.0628	1.0717	1.0292	1.0793	1.0494
$(\tau/K/p)$	(126/1/0)	(252/1/3)	(252/5/3)	(42/5/1)	(189/4/2)	(189/3/3)	(126/3/2)	(126/3/3)	(126/3/3)
MinCF	−11.42	−7.63	51.34	−5.70	−10.81	−10.16	−2.43	−10.54	−16.76
$(\tau/K/p)$	(252/3/3)	(189/5/3)	(189/5/1)	(63/4/3)	(42/3/2)	(42/2/3)	(189/3/0)	(63/3/0)	(189/3/0)
AvCF	3.75	46.58	102.46	2.21	14.15	24.80	-0.57	-2.41	-7.05
MaxCF	15.52	83.95	170.48	9.33	39.82	61.84	2.02	4.76	3.95
$(\tau/K/p)$	(252/1/0)	(252/1/2)	(252/5/2)	(42/5/1)	(189/4/2)	(189/4/2)	(252/5/2)	(126/5/3)	(126/5/3)

Table 11. BestPractice results for strategies F1 to F6 for the period from March 12, 1998 to December 20, 2002. For further notes see Table 10.

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