Limited Enforceable International Loans, International Risk Sharing and Trade

Almuth Scholl*

* Humboldt-Universität zu Berlin, Germany

This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

http://sfb649.wiwi.hu-berlin.de
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin
Spandauer Straße 1, D-10178 Berlin
Limited Enforceable International Loans, 
International Risk Sharing and Trade 

Almuth Scholl* 
Humboldt-University of Berlin 
ascholl@wiwi.hu-berlin.de

First version: June, 2002 
This version: August, 2005 

Abstract 

This paper analyzes the impact of limited enforceable international loans on international risk sharing and trade fluctuations in a two-country two-good endowment economy. Our specification of the punishment threat allows the exclusion from trade to last only finitely many periods and distinguishes between financial autarky and full autarky. Quantitative results show that limited enforceability substantially alters cross-country consumption correlations and the dynamics of net exports. In contrast to existing studies, risk sharing is low for large elasticities of substitution between the domestic and foreign goods. However, it remains challenging to explain the high volatility of the terms of trade empirically observed.

JEL: E32, D52, F34, F41

*Address: Humboldt-University of Berlin, School of Business and Economics, Institute of Economic Policy, Spandauer Str. 1, 10178 Berlin. This paper was previously circulated under the title “Do Endogenous Incomplete Markets Explain Cross-Country Consumption Correlations and the Dynamics of the Terms of Trade?”. This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 “Economic Risk”. I am especially grateful to Harald Uhlig for many helpful suggestions and comments. Furthermore, I thank Thomas Sargent, David Backus, Albert Marcet, Timothy Kehoe, Victor Rios-Rull, Marco Maffezoli, Mathias Trabandt, Stefan Ried, Emanuel Mönch, Ralf Brüggemann and participants at the North American Summer Meetings 2002, Los Angeles, at the EEA meetings 2002, Venice, the Workshop on Dynamics Macroeconomics 2003, Vigo, at the Winter Meetings of the Econometric Society 2004, Stockholm, and seminar participants at Humboldt-University, Mannheim University and New York University for helpful comments. All errors are mine.
1 Introduction

There are two well-known puzzles in international business cycle theory with complete markets: first, standard models generate cross-country correlations of consumption that are much higher than we find in the data and second, the terms of trade are less volatile than it is empirically observed. Backus, Kehoe and Kydland (1992, 1994) call the first puzzle the ‘quantity anomaly’ and the second the ‘price anomaly’.

In a recent study Kehoe and Perri (2002) show that financial frictions due to dynamic incentive constraints help to resolve the quantity anomaly. They argue that international loans are imperfectly enforceable in the sense that countries may not be willing to keep their promises to repay their debt. In a standard model with two countries producing perfect substitutes, international loans are restricted to be feasible only if, at any point in time, they are enforceable by the threat of an exclusion from future international borrowing and lending forever.

This paper analyzes the impact of limited enforceable international loans on international risk sharing and trade fluctuations. In a pure endowment economy the traded goods are assumed to be imperfect substitutes which yields endogenous and varying terms of trade. We focus on the specification of limited enforceability and make two substantive changes compared to Kehoe and Perri (2002) that have a considerable effect on consumption correlations and trade. First, since an infinite punishment would be hard to enforce, the exclusion from future trade is allowed to last only finitely many periods. Second, we analyze the impact of different punishment threats and distinguish between the exclusion from international borrowing and lending and the exclusion from all intertemporal and interstate trade.

A key issue to resolve is how to start a new and imperfectly enforceable contract after an exclusion period. Formally, all contracts solve social planning problems subject to enforcement constraints. We assume that the new contract is characterized by potential new initial welfare weights assigned to each country. The associated allocation can be understood as the outcome of renegotiations that countries conduct in order to arrive at a new agreement. We perform a sensitivity analysis with respect to the assignment of
the new weights and to the periods of exclusion and analyze in detail the impact of these new enforcement constraints on cross-country consumption correlations, trade fluctuations and welfare.

To solve the model we extend the approach by Marcet and Marimon (1998) who propose to introduce additional co-state variables that measure the binding pattern of the enforcement constraints. Our framework requires taking into account that after a finite period of exclusion a new and imperfectly enforceable contract is started.

Intuitively our model works as follows. First, suppose markets are complete and a shock occurs that increases domestic endowment. The price of the domestic good decreases and the terms of trade increase. Since the foreign good is relatively more expensive, imports decrease and exports increase, such that net exports are pro-cyclical. Complete markets perfectly insure country-specific risks and consumption is strongly smoothed across countries. However, international loans are imperfectly enforceable and countries might have incentives to default in order to fully exploit the gains of positive endowment shocks. To prevent the country from choosing this option, domestic consumption has to increase more and foreign consumption less compared to the complete markets allocation. This implies that international risk sharing is reduced. The increase in domestic consumption is realized by adjustments of imports. In the simple endowment setup it depends crucially on the severity of the punishment threat whether net exports behave pro-cyclically or counter-cyclically.

Quantitative results show that limited enforceable international loans substantially reduce international risk sharing. The specification of the value of default turns out to be critical for cross-country consumption correlations, trade fluctuations and welfare. Particularly, when considering full autarky as a punishment threat, the finite exclusion length is essential as the traded goods are imperfect substitutes and countries are dependent on trade. In this case a very long exclusion period poses a severe threat, such that the incentives to repudiate are low. The associated consumption co-movements and trade fluctuations are similar compared to the complete markets scenario. It turns out that the exclusion from international borrowing and lending is a weak punishment threat, such that the incentives to default are very
high independent of the exclusion length. In order to fulfill the enforcement constraint, consumption and imports are adjusted in such a way that virtually no risk sharing occurs. Moreover, in this case net exports appear to be counter-cyclical but with very low volatility. The severity of the punishment threat is highlighted by the associated welfare loss. The stronger the incentives to default the larger is the welfare loss generated by limited contract enforceability.

As also noted by e.g. Backus et al. (1994) and Heathcote and Perri (2002), the implications of this class of models are sensitive with respect to the elasticity of substitution between domestic and foreign goods. However, we find that limited enforceability of international loans generate lower cross-country consumption correlations for higher elasticities in contrast to complete markets that act reversely. With respect to the price anomaly we find that the volatility of the terms of trade is even lower compared to the complete markets outcome. This is due to the fact that import and export flows are adjusted in order to decrease the incentives to default.

This paper is related to the literature on sovereign debt, like e.g. Bulow and Rogoff (1989) and Eaton and Gersovitz (1981). Eaton and Fernandez (1995) provide a survey on the international debt literature. Furthermore, the approach of limited contract enforceability is connected to the literature on debt-constrained asset markets like e.g. Kehoe and Levine (1993, 2001), Kocherlakota (1996) and Alvarez and Jermann (1999, 2000). Marcet and Marimon (1992) have shown that enforcement constraints reduce investment patterns and economic growth. Other papers in different areas of economic research that consider limited contract enforceability are e.g. Aiyagari, Marcet, Sargent and Seppälä (2002), Krüger and Perri (2005), Cooley, Marimon and Quadrini (2003), Jeske (2001), Krüger and Uhlig (2005) and Kehoe and Perri (2004). In the international business cycle literature some papers introduce exogenous incomplete markets to solve the quantity anomaly. For example Kollmann (1996) and Baxter and Crucini (1995) exogenously restrict the number of traded assets. Among others, Kim, Kim and Levin (2003) investigate in detail the welfare implications of exogenous incomplete markets in a two-country endowment economy. Heathcote and Perri (2002) extend an approach by Cole and Obstfeld (1991) and compare complete mar-
kets, exogenous incomplete markets and financial autarky. They show that the extent of international borrowing and lending opportunities is important for the international business cycle. Other papers that maintain the assumption of complete markets are e.g. Stockman and Tesar (1995) and Betts and Kehoe (2001) who incorporate non-traded goods and taste shocks to explain international co-movements. Mazzenga and Ravn (1998) analyze the effects of including transportation costs whereas Burstein, Neves and Rebelo (2003) introduce distribution costs in international business cycle models. To explain the dynamics of the terms of trade Backus and Crucini (2000) study the impact of oil prices while Cunat and Maffezzoli (2004) introduce Heckscher-Ohlin trade features in a dynamic general equilibrium model.

The structure of the paper is as follows. Section 2 briefly discusses some stylized facts concerning international risk sharing and trade fluctuations. In Section 3 a two-country two-good pure endowment economy with limited enforceable international loans is developed and analyzed. Section 4 handles the computational method and parameterizations. Quantitative results are discussed in Section 5 and, finally, Section 6 concludes.

2 Properties of International Business Cycles

To set the stage, we briefly summarize some stylized facts of international business cycles. For more detailed studies of international data the reader is referred to e.g. Backus, Kehoe and Kydland (1995) and Zimmermann (1997).

Table 1 and 2 list moments of international co-movements, the terms of trade and net exports considering the economies of Australia, Canada, France, Germany, Italy, Japan, Norway, the United Kingdom and the United States. The quarterly data are taken from the International Monetary Fund, Financial Statistics. The sample period starts in the first quarter of 1970 and ends in the fourth quarter of 1998. After taking the natural logarithm (except for net exports) the time series are Hodrick-Prescott filtered. Table 1 reports correlations of consumption and, for comparability, of output across countries. First of all, it is evident that cross-country correlations of output are higher than cross-country correlations of consumption. The fairly low consumption correlations clearly indicate that international risk
sharing is greatly inhibited. Across most countries consumption correlations are positive but for some country pairs - e.g. Germany and Canada, US and Australia, or Japan and Australia - they are negative. Table 2 reports business cycle statistics with respect to the terms of trade and net exports. We follow Backus et al. (1994) and define the terms of trade as the relative price of imports to exports. This corresponds to the convention concerning the real exchange rates in international macroeconomics. The high volatility of the terms of trade is evident for all countries. Moreover, it is striking that the correlations between the terms of trade and domestic output are negative for all countries except for Italy. This is also noted by Cunat and Maffezzoli (2003): in standard international business cycle models the correlations between output and the terms of trade are positive and high whereas in the data the correlations are low or negative. With respect to net exports Table 2 shows that they behave counter-cyclically.

3 The Model

3.1 The Environment

The environment of our model is similar to the one in Backus et al. (1994). There are two countries, \( i = 1, 2 \), each inhabited by a large number of infinitely-lived identical agents who maximize utility. The two countries trade two different, imperfectly substitutable intermediate goods which are transformed into country-specific final goods. Country 1 and 2 are assumed to be the domestic and foreign country, respectively. Fluctuations are driven by country-specific stochastic shocks to endowment.

In each country preferences of the representative agent are given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}), \quad i = 1, 2,
\]

with \( 0 < \beta < 1 \). \( c_{i,t} \) denotes period \( t \) consumption in country \( i \). The utility function is assumed to be \( u(c_{i,t}) = c_{i,t}^{1-\sigma}/(1 - \sigma) \) where \( \sigma \) is the parameter of relative risk aversion. Country 1 and country 2 are endowed with intermediate goods \( a_t \) and \( b_t \), respectively. \( a_{i,t} \) and \( b_{i,t} \) denote the use of the
two intermediate goods in country \(i\). Country 1 consumes \(a_{1,t}\) and \(b_{1,t}\) and exports \(a_{2,t}\) while country 2 consumes \(b_{2,t}\) and \(a_{2,t}\) and exports \(b_{1,t}\).

Consumption \(c_{i,t}\) is a composite of foreign and domestic goods:

\[
c_{i,t} = G^i(a_{i,t}, b_{i,t}), \quad i = 1, 2, \tag{1}
\]

where \(G^i(a_{i,t}, b_{i,t})\) is an aggregator proposed by Armington (1969):

\[
G^1(a_{1,t}, b_{1,t}) = (\omega_1 a_{1,t}^{-\rho} + \omega_2 b_{1,t}^{-\rho})^{-1/\rho}
\]
\[
G^2(a_{2,t}, b_{2,t}) = (\omega_2 a_{2,t}^{-\rho} + \omega_1 b_{2,t}^{-\rho})^{-1/\rho}. \tag{2}
\]

\(\omega_1\) and \(\omega_2\) denote the weights specifying the domestic and foreign content of domestic consumption. The elasticity of substitution between foreign and domestic goods is given by \(\eta = 1/(1 + \rho)\). The Armington aggregator transforms the intermediate goods \(a\) and \(b\) into a country-specific final good. Note that \(G^1\) and \(G^2\) are defined symmetrically for the two countries.

The resource constraints are given by

\[
\begin{align*}
z_{1,t}y_1 &= a_{1,t} + a_{2,t} \\
z_{2,t}y_2 &= b_{1,t} + b_{2,t}, \tag{3}
\end{align*}
\]

where \(z_{i,t}\) are exogenous stochastic shocks to endowment \(y_i, i = 1, 2\).

### 3.2 Limited Enforceable International Loans

Standard international business cycle models as e.g. Backus et al. (1994) proceed under the assumption of perfect markets. In particular, countries can go into debt repaying it at a future point. In an international context, though, a large literature has argued that international loans are imperfectly enforceable: countries only repay their debt if they have an incentive to do so (see e.g. Bulow and Rogoff (1989)). In a recent study Kehoe and Perri (2002) assume that international loans are feasible only if, at any point in time, they are enforceable by the threat of an exclusion from future intertemporal and interstate trade forever. However, such an infinite exclusion would probably be hard to enforce, particularly in a framework with imperfectly substitutable traded goods. Therefore, we allow for the possibility that countries renegotiate and conclude a new trade agreement even though one
country has chosen to default. We assume that repudiation is followed by a stay in autarky that lasts only finitely many periods. After such an exclusion period a new and imperfectly enforceable contract is started. Moreover, we distinguish between the exclusion from international borrowing and lending and the exclusion from all intertemporal and interstate trade.

We introduce enforcement constraints that restrict international loans to be feasible only if, at any point in time, they are enforceable by the threat of staying in autarky until a new trade contract is concluded. The exclusion from trade lasts \( K \) periods. Formally, we assume that the following social planning problem is solved at date \( s \):

\[
\{ c_t(\alpha, z_s), a_t(\alpha, z_s), b_t(\alpha, z_s) \}_{t=s}^{\infty} = \arg \max \{ c_t, a_t, b_t \}_{t=s}^{\infty} \sum_{t=s}^{\infty} \alpha_i u(c_{i,t}) \tag{4}
\]

s.t.

\[
E_t \sum_{t=0}^{\infty} \beta^t u(c_{i,t+i}) \geq E_t \left( \sum_{m=0}^{K-1} \beta^m u(\tilde{c}_{i,t+m}) + \sum_{n=0}^{\infty} \beta^n u(c_{i,t+n}(\tilde{\alpha}_i, z_{i,t+K})) \right) \tag{5}
\]

(1), (2) and (3).

\( \alpha = (\alpha_1, \alpha_2) \) are the weights that the social planner puts on the welfare of the countries. Throughout the paper we assume \( \alpha_1 = \alpha_2 \). \( z_s = (z_{1,s}, z_{2,s}) \) are the realizations of the endowment shocks known at date \( s \). \( \{ \tilde{c}_{i,t+m} \}_{m=0}^{K-1} \) denotes consumption during autarky. \( \{ c_{t+n}(\hat{\alpha}, z_{t+K}), a_{t+n}(\hat{\alpha}, z_{t+K}), b_{t+n}(\hat{\alpha}, z_{t+K}) \}_{n=0}^{\infty} \) is the allocation associated with the new and imperfectly enforceable contract that is concluded after the \( K \)-period stay in autarky. It depends on the realizations of the endowment shocks known at date \( t+K \) and on new welfare weights \( \hat{\alpha} = (\hat{\alpha}_1, \hat{\alpha}_2) \). \( \hat{\alpha}_i \) is the weight put on the welfare of the repudiating country \( i \) and \( \hat{\alpha}_j = 1 - \hat{\alpha}_i, j \neq i \), is the weight assigned to the other country \( j \). In the following we discuss how to specify the autarky allocation, the length of time of exclusion from trade \( K \) and the new welfare weights \( \hat{\alpha} \).

To specify the autarky allocation we consider two punishment scenarios. In the first scenario the defaulting country is punished by a \( K \)-period exclusion from all intertemporal and interstate trade, i.e. \( \{ \tilde{c}_{1,t+m} \}_{m=0}^{K-1} = G^1(\hat{a}_{1,t+m}, 0) \) and \( \{ \tilde{c}_{2,t+m} \}_{m=0}^{K-1} = G^2(0, \hat{b}_{2,t+m}) \). We label this scenario as ‘full autarky’. In the second scenario the defaulting country is punished by a \( K \)-period exclusion from intertemporal trade. In this scenario countries keep
trading but they do not have international borrowing and lending opportunities, i.e. trade has to be balanced every period. Following Cole and Obstfeld (1991) we call this scenario ‘financial autarky’. In financial autarky country 1 and country 2 solve the following maximization problems at time $t$ given the terms of trade $\tilde{p}_{t+m}$:

$$\max E_t \sum_{m=0}^{K-1} \beta^m u(\tilde{c}_{1,t+m}) \quad \text{s.t.} \quad \tilde{a}_{1,t+m} + \tilde{p}_{t+m} \tilde{b}_{1,t+m} = z_{1,t+m} y_1$$

$$\tilde{c}_{1,t+m} = G^1(\tilde{a}_{1,t+m}, \tilde{b}_{1,t+m})$$

$$\max E_t \sum_{m=0}^{K-1} \beta^m u(\tilde{c}_{2,t+m}) \quad \text{s.t.} \quad \tilde{a}_{2,t+m}/\tilde{p}_{t+m} + \tilde{b}_{2,t+m} = z_{2,t+m} y_2$$

$$\tilde{c}_{2,t+m} = G^2(\tilde{a}_{2,t+m}, \tilde{b}_{2,t+m})$$

Market clearing determines $\tilde{p}_{t+m}$, such that the resource constraints (3) of both countries are fulfilled. Note that if the traded goods are perfect substitutes, financial autarky is equivalent to full autarky. Assuming full autarky as a punishment can be motivated by trade sanctions that are imposed on repudiating countries. One might argue that full autarky seems to be an unrealistically strong punishment, especially in a world economy with imperfectly substitutable goods. Note, however, that assuming financial autarky instead is a less severe threat that creates higher incentives to default in this model.

We treat the length of time in autarky $K$ and the welfare weights of the new contract $\hat{\alpha}_i$ as exogenously given. The allocation associated with $\hat{\alpha}_i$ can be interpreted as the outcome of renegotiations that the two countries conduct after a $K$-period stay in autarky. We assume that the contract that was in place before repudiation does not influence the bargaining process after repudiation. This is a reasonable assumption if one supposes that both countries are interested in a recommencement and want to revive trade connections. The implication is that, at any point in time, renegotiations result in the same outcome. Therefore, the new weight on the welfare of the repudiating country $\hat{\alpha}_i$ is taken to be independent of the initial welfare weights $\alpha$ and constant over time. As a benchmark we consider new welfare weights that are equal for both countries, $\hat{\alpha}_i = \hat{\alpha}_j$, since symmetric countries are likely to have equal bargaining power independent of repudiation. However, it also seems reasonable to assign a lower weight to the country that has chosen to default, $\hat{\alpha}_i < \hat{\alpha}_j$, which can be interpreted as a general punishment for repudiation. Alternatively, this can be viewed as the amount of debt that the defaulting country has to pay back. On the other hand one
might argue that renegotiations result in a larger weight on the welfare of the repudiating country, $\hat{\alpha}_i > \hat{\alpha}_j$, since imperfect substitutability of traded goods implies that the other country is also hurt by the collapse of trade and is likely to be interested in a fast agreement. Even though it is costly, the other country might want to induce the repudiating country to enter a new trade agreement by accepting a contract with a lower weight. Later on we perform a detailed sensitivity analysis with respect to the choice of $\hat{\alpha}$ and $K$.

### 3.3 Analysis

A key condition in standard dynamic programming techniques is that only past realizations of the variables can influence the set of feasible current actions. Here the enforcement constraints (5) include future actions. To solve the social planning problem we follow the approach developed by Marcet and Marimon (1998) and introduce additional co-state variables:

$$w_{i,t} = w_{i,t-1} + \gamma_{i,t}, \quad w_{i,s-1} = \alpha_i, \quad i = 1, 2. \quad (6)$$

$\gamma_{i,t} \geq 0, \quad i = 1, 2,$ denote the Lagrange multipliers on the enforcement constraints. The co-state variables $w_{i,t}, \quad i = 1, 2,$ measure the potentially complicated binding patterns of the enforcement constraints of the domestic and foreign country. If country $i$’s enforcement constraint is not binding at time $t$, $\gamma_{i,t}$ is equal to zero and $w_{i,t}$ is determined by past binding patterns. If country $i$’s enforcement constraint is binding at time $t$, $\gamma_{i,t}$ is strictly greater than zero and $w_{i,t}$ increases. The planner’s problem can be transformed into the following saddle-point formulation by using the co-state variables (6), the law of iterated expectations and simple algebra:

$$\min_{\{\gamma_t \geq 0\}_{t=s}} \max_{\{c_t, a_t, b_t\}_{t=s}} E_s \sum_{t=s}^{\infty} \beta^{t-s} \left( \sum_{i=1,2} w_{i,t} u(c_{i,t}) - \gamma_{i,t} D_{i,t} \right) \quad (7)$$

s.t.

$$D_{i,t} = \left[ \sum_{m=0}^{K-1} \beta^m u(\tilde{c}_{i,t+m}) + \sum_{n=K}^{\infty} \beta^n u(c_{i,t+n}(\hat{\alpha}_i, z_{t+K})) \right] \quad (1), \ (2), \ (3) \text{ and } (6).$$

This formulation clearly shows that the co-state variables $w_{i,t}$ work as welfare weights. If country $i$ has an incentive to repudiate, the social planner has
to increase the welfare weight \( w_{i,t} \), such that the enforcement constraint of country \( i \) is fulfilled.

\[
\{ c_t(\alpha, z_s), a_t(\alpha, z_s), b_t(\alpha, z_s), \gamma_t(\alpha, z_s) \}_{t=\infty}^{\infty} \text{ is given by the first order conditions}
\]

\[
\frac{u_{c_{1,t}}}{u_{c_{2,t}}} \left( - \frac{G^1_{a_{1,t}}}{G^2_{a_{1,t}}} \right) = \frac{w_{2,t}}{w_{1,t}}
\]

\[
\frac{G^1_{a_{1,t}}}{G^1_{b_{1,t}}} = \frac{G^2_{a_{1,t}}}{G^2_{b_{1,t}}}
\]

together with the the constraints (1) to (3), the law of motion for the co-state variable (6), the complementary slackness conditions and \( G^2(a_{1,t}, b_{1,t}) = G^2((z_{1,t}y_1 - a_{1,t}), (z_{2,t}y_2 - b_{1,t})) \). \( u_{c_{i,t}} \) is the abbreviation for \( \frac{\partial u(c_{i,t})}{\partial c_{i,t}} \) (similarly for other terms). The outside option \( D_{i,t} \) depends on the allocation associated with the new, imperfectly enforceable contract \( \{ c_{t+n}(\hat{\alpha}, z_{t+K}), a_{t+n}(\hat{\alpha}, z_{t+K}), b_{t+n}(\hat{\alpha}, z_{t+K}), \gamma_{t+n}(\hat{\alpha}, z_{t+K}) \}_{n=0}^{\infty} \) which is given by the optimality conditions (8), (9), the constraints (1) to (3), the complementary slackness conditions at date \( t + n \) and \( w_{i,t+n} = w_{i,t+n-1} + \gamma_{i,t+n} \) with \( w_{i,t+K-1} = \hat{\alpha}_i, i = 1, 2 \).

The terms of trade \( p_t \) are given by the relative price of imports to exports and are equal to the marginal rate of transformation between the two goods in country 1 evaluated at equilibrium quantities:

\[
p_t = \frac{p_{b,t}}{p_{a,t}} = \frac{G^1_{b_{1,t}}}{G^1_{a_{1,t}}} = \frac{\omega_2}{\omega_1} \left( \frac{a_{1,t}}{b_{1,t}} \right)^{\rho+1}
\]

The optimal allocation requires the efficient choice of the time-dependent welfare weights \( w_{i,t} \) such that the enforcement constraints are fulfilled. Suppose both enforcement constraints are never binding: \( \gamma_{1,t} = \gamma_{2,t} = 0 \ \forall \ t \). Then the left-hand side of (8) is equal to \( \alpha_2/\alpha_1 \) which implies perfect insurance of country-specific risks. Now suppose a shock occurs that increases domestic endowment such that country 1 has an incentive to repudiate. To prevent the country from taking this option, the social planner has to rise domestic consumption by increasing the weight \( w_{1,t} \). Formally, the enforcement constraint of country 1 is binding, \( \gamma_{1,t} > 0 \), and \( w_{1,t} \) increases, such that the ratio of marginal utilities decreases (see equation (8)). Hence, consumption sharing is reduced.
The impact of limited enforceable international loans on risk sharing and trade depends on the value of default $D_{i,t}$. Clearly, the higher the value of the outside option the larger the incentive to repudiate. Since the optimality conditions and complementary slackness conditions form a dynamic nonlinear system with occasionally binding constraints, and the value of default depends on the periods of trade exclusion and on new and imperfectly enforceable contracts, we solve the model numerically.

4 Numerical Solution

4.1 Parameterizing the Outside Option

Key parameters of the model are those that specify the value of the outside option. In the following we discuss the parameterizations of $K$ and $\hat{\alpha}_i$.

$K$ is the parameter that denotes the number of periods countries have to stay in autarky after default. Suppose a shock occurs that increases domestic endowment. Then for low values of $K$ one expects the domestic country to have high incentives to repudiate since it will not take long until a new trade contract is concluded. The consequences of default are mild. Hence, the domestic country is likely to be better off by abandoning trade connections and reducing risk sharing in order to fully exploit the gains of positive shocks. On the other hand, because countries trade imperfectly substitutable goods, long periods of exclusion from trade are likely to hurt the domestic country, such that the incentive to repudiate is low. As a strategy for parameterizing $K$ we start with a one-period exclusion from trade and increase $K$ as long as there is still an incentive to default.

$\hat{\alpha}_i$ is the new weight that is initially put on the welfare of the repudiating country after an exclusion period. As argued before, the new welfare weight is assumed to be independent of the contract that was in place before repudiation and constant over time. We simply define $\hat{\alpha}_i = 0.5 + \zeta$, $\hat{\alpha}_j = 1 - \hat{\alpha}_i$. As a benchmark we take $\zeta = 0$ which implies equal new weights on the welfare of both countries, $\hat{\alpha}_i = \hat{\alpha}_j$. This corresponds to the assumption that both countries have equal bargaining power independent of repudiation. Assuming $\zeta < 0$, i.e. $\hat{\alpha}_i < \hat{\alpha}_j$, implies a punishment since renegotiations result in a
new contract that is less favorable for the repudiating country. The value of the outside option and, hence, the incentive to default is lower compared to $\zeta = 0$. On the other hand, $\zeta > 0$, i.e. $\hat{\alpha}_i > \hat{\alpha}_j$, implies that even though it is costly, the other country wants to induce the repudiating country to enter a new trade agreement by accepting a contract with a lower weight. The value of the outside option is larger compared to $\zeta = 0$, and countries experiencing positive shocks are more likely to default. Already small values of $\zeta$ influence the results strongly. We vary $\zeta$ between $-0.01$ and $+0.01$ and analyze the associated properties of the theoretical economy.

Alternatively one could assume that the new initial weight on the welfare of the repudiating country $\hat{\alpha}_i$ depends on the level of the co-state variable before repudiation, $w_{i,t-1}$, that measures the past binding patterns of the enforcement constraints. This corresponds to the assumption that countries do not forget and do not forgive past defaults. Quantitative results look similar for both parameterizations. However, we find it more appropriate to rule out dependencies on former contracts and continue with welfare weights that are constant over time as argued before.

### 4.2 Calibration

In addition to the parameters of the outside option, the elasticity of substitution between domestic and foreign goods $\eta$ is a crucial parameter in this model. Since $\eta$ measures the importance of trade, it influences the incentives to repudiate. In the empirical literature we find a wide range of estimates for $\eta$. While Whalley (1985) and Shiells and Reinhart (1993) find values between 0.8 and 1.9 and 0.1 and 1.1, respectively, Hummels (1999a, 1999b) and Trefler and Lai (1999) propose elasticities between 5 and 6. Harrigan (1993) reports estimates between 5 and 12 (see also the discussion by Obstfeld and Rogoff (2000)). While Backus et al. (1994) choose values around 1.5, we try different values between 1 and 7.

In a symmetric steady state $y_1 = y_2$ and $b_1 = a_2$, such that the ratio $a_1/b_1$ is determined by $(1 - b_1/y_1)/(b_1/y_1)$ where $b_1/y_1$ is the import share of GDP. Using the steady state version of the optimality condition (10) that determines the terms of trade, the value of the Armington weights $\omega_i$ can
be found. Import shares of 0.09, 0.12 and 0.15 are used where the latter is employed as a benchmark. The parameter of relative risk aversion equals 2 and the time preference $\beta$ is set to 0.99.

The endowment shocks of the two countries $(z_{1,t}, z_{2,t})$ are assumed to follow a first order vector autoregressive process

$$
\begin{pmatrix}
    z_{1,t} \\
    z_{2,t}
\end{pmatrix}
= \begin{pmatrix}
    \phi_{11} & \phi_{12} \\
    \phi_{21} & \phi_{22}
\end{pmatrix}
\begin{pmatrix}
    z_{1,t-1} \\
    z_{2,t-1}
\end{pmatrix}
+ \begin{pmatrix}
    \epsilon_{1,t} \\
    \epsilon_{2,t}
\end{pmatrix},
$$

where the error terms $\epsilon_{i,t}$ are Gaussian white noise. As a benchmark calibration we use $\phi_{11} = \phi_{22} = 0.97$, $\phi_{12} = \phi_{21} = 0$, $\text{var}(\epsilon_1) = \text{var}(\epsilon_2) = (0.007)^2$ and $\text{corr}(\epsilon_1, \epsilon_2) = 0.25$ which is in line with studies by Baxter and Crucini (1995), Kollmann (1996) and Cuñat and Maffezzoli (2003) who find little evidence for spill-over effects across countries. In the sensitivity analysis, we consider $\phi_{11} = \phi_{22} = 0.99$ as well as $\phi_{11} = \phi_{22} = 0.95$ to analyze the impact of higher and lower persistence on the economy. Moreover, we allow for spill-over effects, $\phi_{12} = \phi_{21} = 0.025$.

### 4.3 Computational Method

For convenience we reduce the number of state variables by considering the relative weight of the two countries $q_t = \frac{w_{2,t}}{w_{1,t}}$ and defining the normalized multiplier $d_{i,t} = \frac{\gamma_{i,t}}{w_{i,t}}$ for $i = 1, 2$. The optimality conditions can be rewritten as:

$$
\begin{align*}
\frac{u_{c_{1,t}}}{u_{c_{2,t}}} \left( -\frac{G^1_{a_{1,t}}}{G^2_{a_{1,t}}} \right) &= q_t \\
\frac{G^1_{a_{1,t}}}{G^2_{b_{1,t}}} &= \frac{G^2_{a_{1,t}}}{G^2_{b_{1,t}}} \\
q_t &= \frac{1 - d_{1,t}}{1 - d_{2,t}} q_{t-1} \\
q_{t-1} &= \frac{\alpha_2}{\alpha_1}
\end{align*}
$$

together with the constraints (1) to (3) and the complementary slackness conditions.

Let efficient allocations at time $t$ be described by time invariant policy functions dependent on the state variable $q_{t-1}$ and the shock $z_t$. To solve
for the policy functions we apply a numerical algorithm that is similar to the one used by Kehoe and Perri (2002). We discretize the state space and approximate the continuous-valued shock process by a two-state Markov chain using the method suggested by Tauchen (1986). Let \( s = (q, z) \) be the state. We define the value function

\[
V_i(\alpha, s) = u(c_i(\alpha, s)) + \beta E[V_{i'}(\alpha, s')], \quad i = 1, 2,
\]

where the prime denotes the next period. Let \( \tilde{s} \) denote the state after \( K \) periods in autarky. The value function of the new, imperfectly enforceable contract is given by

\[
V_i(\tilde{\alpha}, \tilde{s}) = u(c_i(\tilde{\alpha}, \tilde{s})) + \beta E[V_{i'}(\tilde{\alpha}, \tilde{s}')], \quad i = 1, 2.
\]

We start with an initial guess \( V_i^{(0)}, i = 1, 2 \). The new value functions \( V_i^{(1)}, i = 1, 2 \), are found as follows. First, we assume that neither enforcement constraint binds and compute the corresponding allocations using the normalized first order conditions (11) and (12) and the resource constraints (1) to (3) under the assumption that both multipliers are equal to zero. We use \( V_i(\tilde{\alpha}, \tilde{s}), i = 1, 2 \), to check whether the constructed allocations satisfy the enforcement constraints (5). If e.g. the enforcement constraint of the foreign country is satisfied but not the one for the domestic country, the multiplier of the foreign country is set to zero. The domestic multiplier and the allocation are recalculated using the normalized first order conditions and the enforcement constraint of the domestic country fulfilled with equality. If the enforcement constraint of the domestic country is satisfied but not the one for the foreign country, the multiplier of the domestic country is set to zero, and the foreign multiplier as well as the allocation are recalculated analogously. The algorithm is iterated until the value functions converge on every grid point.

5 Quantitative Properties of the Model

5.1 When Do the Enforcement Constraints Bind?

To study the quantitative properties of the theoretical economy, we start by analyzing the question under what conditions countries do have incentives
to default on their debt. Whether the enforcement constraints bind depends crucially on the specification of the outside option. We start by considering the case of full autarky, i.e. repudiating countries are punished by a $K$-period exclusion from all intertemporal and interstate trade. We focus our attention on the number of exclusion periods $K$ and the initial welfare weights of the new trade agreement $\hat{\alpha}_i = 0.5 + \zeta$. Suppose asymmetric shocks occur that increase endowment in the domestic country. Considering $0 < K < 25$ and $-0.01 \leq \zeta \leq 0.01$, Figure 1 plots the multiplier on the domestic enforcement constraint at time $t$, $d_{1,t}$, under the assumption that in $t-1$ the welfare of both countries are weighted equally, $q_{t-1} = 1$. The figure shows that for values of $\zeta$ implying $\hat{\alpha}_1 < \hat{\alpha}_2$ the normalized domestic multiplier is decreasing in $K$. The interpretation is straightforward: new small initial welfare weights can be understood as a punishment, since the renegotiated contracts are of low value to the domestic country. Therefore, the incentives to default are low. The smaller $\zeta$, i.e. the lower the welfare weight $\hat{\alpha}_1$, the more severe is the penalty and the lower is the multiplier. Since high values of $K$ represent protracted renegotiations, i.e. a long stay in full autarky, the domestic country has low incentives to default. Interestingly, for large positive values of $\zeta$, i.e. $\hat{\alpha}_1 > \hat{\alpha}_2$, the multiplier shows a hump shape in $K$. To understand this, note that first, the shock process is very persistent and, second, positive values of $\zeta$ imply that renegotiations result in a more favorable new contract for the repudiating country. Because the shock persistence is high, the domestic country expects to experience another positive shock in the following period and at the same time anticipates the advantageous new agreement. Therefore, the multiplier is increasing in $K$. On the other hand, for values of $K$ large enough, the multiplier is falling in $K$. The higher $K$ the more the country expects to be hit by a bad shock that decreases the value of staying in autarky and compensates the effect of a large new welfare weight. The two opposing effects imply that the incentive to default is hump shaped in $K$. Note that for $K$ large enough the enforcement constraint is not binding anymore, $d_{1,t} = 0$, and the model collapses to the complete markets scenario. If repudiating countries are punished by a long period of exclusion from all trade, there are no incentives to default and the international loans arising under complete markets are self-enforcing.
In addition to the parameters of the outside option $K$ and $\hat{\alpha}$, the elasticity of substitution between domestic and foreign goods $\eta$ determines crucially the incentive to default since it indicates the importance of trade connections. In the following we focus on the impact of $\eta$ and compare full autarky and financial autarky as punishment threats. Figure 2 assumes asymmetric shocks that increase domestic endowment and holds the new initial welfare weights $\hat{\alpha}_1 = \hat{\alpha}_2$ and the punishment length $K = 4$ fixed. The value of the normalized domestic multiplier at time $t$, $d_{1,t}$, is plotted for different elasticities of substitution $\eta$. The figure shows that the domestic enforcement constraint does not bind for low elasticities of substitution if the repudiating country is punished by a stay in full autarky. The intuition behind this result is that the domestic country is very dependent on imports of the foreign good in order to produce the country-specific final consumption good. There is no incentive to default since any (short) exclusion from future trade is miserable. For $\eta \geq 4.5$ the multiplier $d_1$ is strictly greater than zero and is increasing in $\eta$. The higher the substitutability of domestic and foreign goods the higher is the country’s incentive to default on its debt because autarky is becoming a less harmful threat. If the defaulting country is punished by financial autarky, the multiplier is much larger. The incentives to default are higher because the exclusion from international lending and borrowing is a less severe punishment than full exclusion from trade. In contrast to full autarky, the enforcement constraint is binding for low elasticities of substitution since countries are still allowed to trade the commodities.

### 5.2 Limited Enforceable Loans and Risk Sharing

The question of interest is how limited enforceable international loans influence the degree of risk sharing across countries. Qualitatively, the model works as follows. Suppose a shock occurs that increases domestic endowment and default is an attractive choice. To prevent the domestic country from choosing this option, compared to the complete markets outcome, the social planner has to increase domestic consumption more and foreign consumption less by increasing the welfare weight of the domestic country (see equation (8)). This implies that risk sharing is reduced.
To study the quantitative impact of enforcement constraints, we consider different parameterizations of the outside option, simulate the model and calculate the theoretical moments. We compare the results relative to the complete markets outcome, i.e. we determine by how many percent enforcement constraints reduce the correlations if the correlations generated by complete markets serve as a benchmark.

First, we focus on full autarky as a punishment for default. Table 3 reports the relative percentage reductions in cross-country consumption correlations considering various values of the exclusion period $K$, of the new initial welfare weights $\tilde{\alpha}_i = 0.5 + \zeta$, and of the elasticity of substitution between domestic and foreign goods $\eta$. Clearly, limited enforceable loans reduce the amount of international risk sharing considerably. Assume $\zeta = 0$, i.e. $\tilde{\alpha}_i = \tilde{\alpha}_j$, and $K = 4$: consumption correlations are reduced by about 8, 21 and 33 percent for $\eta = 5, 6, 7$, respectively. It is evident that the shape of the multiplier directly transmits into the correlation patterns. The larger the multiplier, i.e. the larger the incentive to default, the greater is the percentage decrease in correlations relative to the complete markets outcome. For $\zeta \leq 0$ the reductions decrease in $K$ since a long stay in autarky that ends in a disadvantageous new contract decreases the value of repudiating the contract. For large positive values of $\zeta$ the correlations show a hump shape in $K$ as the multiplier does. The reductions of correlations first increase in $K$ and then approach zero again. E.g. for $\zeta = 0.01, \eta = 5$ and $K = 4$ the enforcement constraints reduce consumption correlations by 17 percent while for $K = 8$ the reduction amounts to 23 percent. Increasing $K$ beyond 16 yields the complete markets outcome.

Table 4 focuses on financial autarky as a punishment threat and reports the relative percentage change in cross-country consumption correlations for various values of $K$ and $\eta$. Even for low elasticities of substitution international risk sharing is greatly inhibited in correspondence to the very large values of the multiplier on the enforcement constraint as seen in Figure 2. E.g. assuming $\eta = 1.5$ results in a relative reduction of consumption correlations of 77 percent. The larger $\eta$ the larger the impact of enforcement constraints on consumption sharing since countries are less dependent on trade. In this scenario increasing the length of punishment $K$ has minimal
effects. This corresponds to the finding that financial autarky does not pose a severe punishment and is in line with the results in Cole and Obstfeld (1991) who find that the loss generated by an exclusion from international portfolio diversification is marginal.

The first columns of Table 5 shed light on the impact of limited enforceability on international risk sharing from another point of view. With particular regard to the importance of import shares and elasticities of substitution, cross-country consumption correlations are reported considering the two punishment scenarios and the complete markets case. Like noted by e.g. Backus et al. (1994) complete markets generate rising cross-country correlations of consumption as the elasticity of substitution increases. Table 5 shows that the introduction of enforcement constraints has the reverse effect: limited contract enforceability reduces cross-country correlations to fairly low levels, the higher the elasticity of substitution is. As an example consider $b_1/y_1 = 0.09$ and full autarky as punishment scenario. For $\eta = 5$ and $\eta = 7$ cross-country consumption correlations are about 0.56 and 0.38 whereas complete markets generate correlations around 0.92 and 0.95, respectively. Assuming financial autarky as a punishment scenario yields qualitatively the same effect, however, consumption correlations are close to zero, i.e. virtually no risk sharing can be observed.

Reducing the import share results in stronger impacts of enforcement constraints on international risk sharing. Intuitively, lower import shares indicate that economies are less open to trade. The lower the import share the less serious are the consequences of an exclusion from trade and the incentives to repudiate are high. Consider $\eta = 6$ and full exclusion from trade. Import shares of 0.15, 0.12 and 0.09 imply consumption correlations of 0.77, 0.62 and 0.24, respectively.

### 5.3 Limited Enforceable Loans and Trade Fluctuations

This section focuses on the impact of enforcement constraints on trade fluctuations and international prices. Table 6 contrasts the world economy with complete markets with the world economy where loans are imperfectly enforceable. The correlations of imports, exports and net exports with domestic
output are reported assuming \( K = 4, \hat{\alpha}_i = \hat{\alpha}_j \) and \( \eta = 5 \). First consider the world economy with complete markets and suppose a shock occurs that increases domestic endowment. Since the price of the domestic good decreases, the foreign good is relatively more expensive. Imports fall and exports rise, such that net exports are pro-cyclical. Because markets are complete, consumption in both countries increases. Now consider imperfectly enforceable loans and assume that the defaulting country is punished by a stay in full autarky. Again suppose a positive domestic endowment shock occurs making default an attractive option. To prevent the domestic country from taking this option, the social planner has to raise domestic consumption more and foreign consumption less. Imports decrease less and net exports are less pro-cyclical than in the world economy with complete markets. Next assume that the defaulting country is punished by an exclusion from international borrowing and lending. Since financial autarky does not pose a severe punishment, the incentive to repudiate is very high and we barely observe consumption sharing. To realize the high value of domestic consumption, imports have to increase, such that net exports behave counter-cyclically. However, in this case the volatility of net exports is very small.

The reactions of imports and exports in response to endowment shocks are reflected in the dynamics of the terms of trade. Considering the world economy with complete markets and the world economy with limited enforceable loans, Table 5 reports the correlations with output and standard deviations for different elasticities of substitution \( \eta \) and different import shares \( b_1/y_1 \).

Since a positive domestic endowment shock decreases the price of imports, the terms of trade defined as the relative price of imports to exports increase. This occurs independently of parameter values and punishment scenarios and is in contrast to the observed negative correlations of the terms of trade and output in the data. However, the reactions of the terms of trade are less strong, the more the enforcement constraints bind which is in line with the pattern of net exports. Consider e.g. the case \( \eta = 5 \) and \( b_1/y_1 = 0.15 \). The standard deviation of the terms of trade is around 1.2 percent in the complete markets scenario. Assuming limited enforceable international loans, the volatility decreases to 1.1 and 0.4 percent depending on the punishment scenario. Inspecting the volatility of the terms of trade found in the data reveals
the price anomaly: the standard deviations of the terms of trade generated by the model economies is far too low (compared with Table 2). Assuming limited enforceability of international loans leads to a more pronounced price anomaly.

5.4 Sensitivity Analysis

This section conducts some sensitivity experiments especially with respect to the specification of the shock process. To conserve space, we focus on the main insights and omit details. We find that considering lower persistence yields less strong effects of enforcement constraints. The lower the persistence of the shock process the less binding is the enforcement constraint since a future bad shock is likely to decrease the value of autarky. Consider the scenario in which the repudiating country is punished by a stay in full autarky. If one assumes \( \eta = 5, K = 4, \alpha = \alpha \) and \( \phi_{11} = \phi_{22} = 0.95 \), the model collapses to the complete markets scenario. On the other hand, if countries experience highly persistent shocks, \( \phi_{11} = \phi_{22} = 0.99 \), the incentives to default increase. The consumption sharing decreases to about 0.62. If spill-over effects of the endowment shocks are taken into account, the effect of enforcement constraints is lower. Assume \( \phi_{12} = \phi_{21} = 0.025 \) and consider \( \eta = 5, K = 4, \hat{\alpha}_i = \hat{\alpha}_j \) and financial autarky as the punishment threat. Cross-country consumption correlations decrease to 0.22 compared to 0.05 with no spill-over effects.

As an additional experiment perfect substitutes are considered in combination with an infinite exclusion from future trade. This corresponds to the enforcement constraints studied by Kehoe and Perri (2002) with the difference that we assume a pure endowment economy. Note that in the case of perfect substitutes financial autarky, i.e. the exclusion from international borrowing and lending, is equivalent to full autarky. The results show that consumption correlations are strongly reduced though default is punished by a stay in autarky forever. While Kehoe and Perri (2002) get consumption correlations equal to 0.28 in a fully-fledged business cycle model, our simple endowment economy results in consumption correlations around 0.08.
5.5 Welfare Analysis

In this section we analyze the impact of limited enforceable international loans on the welfare of the economy. As a welfare measure we consider the expected lifetime utility of the representative household at the beginning of period $t = s$ before the realization of the endowment shock is observed. In order to interpret the results we follow Lucas (1987) and use compensating variations to formulate differences in expected lifetime utility. In the following we express the welfare loss in terms of percentage deviation in certainty-equivalence consumption relative to the steady state.

Tables 7 and 8 consider full autarky and financial autarky as the punishment scenarios and varies the elasticity of substitution $\eta$ and the length of time in autarky $K$. The results show that the welfare loss generated by enforcement constraints is very sensitive to the value of default. As an example consider full autarky as the punishment scenario, $\eta = 7$ and different exclusion periods $K$. For $K = 4$ the welfare loss is about 0.0134 percent of steady state consumption while for $K = 16$ it reduces to 0.0037. Given $\hat{\alpha}_i = \hat{\alpha}_j$, the shorter the exclusion period $K$ the more consumption has to increase relative to the steady state in order to compensate for the welfare loss generated by enforcement constraints. The huge welfare losses that occur in a world economy where both countries are in full autarky emphasize the severeness of full autarky as the punishment threat.

A comparison with financial autarky as the punishment scenario reveals that the welfare loss is much higher since the incentives to repudiate are high. Considering e.g. $\eta = 5$ the welfare loss is 0.0677 percent of steady state consumption. As already noted before, increasing the number of exclusion periods does not have dramatic effects. Note that the welfare losses are of similar magnitude in a world economy without any access to international borrowing and lending. This highlights once again that a stay in financial autarky is not a serious punishment threat.

It is evident that the welfare loss of enforcement constraints is increasing in the elasticity of substitution $\eta$. This corresponds to the result that the incentives to repudiate are higher the more substitutable the traded goods are. If we consider perfect substitutes, i.e. $\eta \rightarrow \infty$ and $\omega_1 = \omega_2$, and infinite
exclusion from future trade, the welfare loss is about 0.08 percent of steady state consumption. This corresponds approximately to the welfare loss of autarky and is in line with the results in Kim et al. (2003). In a two-country one-good endowment economy they also have shown that the welfare loss of autarky is increasing in the persistence of the endowment shocks. Analyzing the sensitivity of the welfare loss of enforcement constraints to different specifications of the shock process results in similar conclusions.

6 Conclusions

This paper has studied the importance of limited enforceability of international loans for international risk sharing, trade fluctuations and welfare in a two-country two-good endowment economy. We have focused on the specification of punishment threats and made two substantive changes compared to previous studies: first, the exclusion from future trade has been allowed to last only finitely many periods and, second, we have distinguished between the exclusion from international borrowing and lending and the exclusion from all intertemporal and interstate trade.

Quantitative results have shown that the impact of limited enforceable international loans on international risk sharing and trade fluctuations is substantial. The severeness of the punishment threat has turned out to be critical for the dynamics of net exports and the size of cross-country consumption correlations. This is also highlighted by the associated welfare loss. However, it remains challenging to explain the high volatility of the terms of trade observed in the data.

This research can be extended in different directions. It seems to be particularly promising to analyze a fully-fledged international business cycle model since Backus et al. (1994) point out that the trade balance is crucially determined by international capital flows. They show that investment dynamics generate counter-cyclical net exports as they are observed empirically. In future research we analyze the impact of limited enforceable international loans on international capital flows and the implications for the terms of trade and the trade balance. Moreover, since this paper has shown that the specification of the punishment threat is crucial, it seems to be interesting
to endogenize the characteristics of the new contract that is restarted after the exclusion period. Furthermore, considering a multi-country setup could give new insights.
References


# Tables and Figures

## Table 1: International Co-movements

<table>
<thead>
<tr>
<th></th>
<th>AUS</th>
<th>CAN</th>
<th>FRA</th>
<th>GER</th>
<th>IT</th>
<th>JPN</th>
<th>NOR</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Correlations of Output Across Countries</strong>&lt;br&gt;$\text{corr}(y_i, y_j)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUS</td>
<td>1.0</td>
<td>.62</td>
<td>-0.01</td>
<td>-0.20</td>
<td>0.20</td>
<td>0.12</td>
<td>0.10</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td>CAN</td>
<td>1.0</td>
<td>.18</td>
<td>-0.28</td>
<td>0.39</td>
<td>0.12</td>
<td>0.09</td>
<td>0.42</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>FRA</td>
<td>1.0</td>
<td>.32</td>
<td>.49</td>
<td>0.54</td>
<td>0.01</td>
<td>0.49</td>
<td>0.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GER</td>
<td>1.0</td>
<td>.36</td>
<td>.37</td>
<td>0.22</td>
<td>0.05</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IT</td>
<td>1.0</td>
<td>.40</td>
<td>.22</td>
<td>0.19</td>
<td>0.18</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JPN</td>
<td>1.0</td>
<td>.25</td>
<td>.51</td>
<td>.46</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NOR</td>
<td>1.0</td>
<td>.21</td>
<td>.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>1.0</td>
<td>.55</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>AUS</th>
<th>CAN</th>
<th>FRA</th>
<th>GER</th>
<th>IT</th>
<th>JPN</th>
<th>NOR</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Correlations of Consumption Across Countries</strong>&lt;br&gt;$\text{corr}(c_i, c_j)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUS</td>
<td>1.0</td>
<td>.20</td>
<td>.02</td>
<td>-0.23</td>
<td>0.06</td>
<td>-0.24</td>
<td>-0.16</td>
<td>-0.10</td>
<td>-0.20</td>
</tr>
<tr>
<td>CAN</td>
<td>1.0</td>
<td>-0.02</td>
<td>-0.27</td>
<td>0.45</td>
<td>-0.02</td>
<td>0.11</td>
<td>0.33</td>
<td>0.41</td>
<td></td>
</tr>
<tr>
<td>FRA</td>
<td>1.0</td>
<td>-0.04</td>
<td>0.06</td>
<td>0.18</td>
<td>0.09</td>
<td>0.19</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GER</td>
<td>1.0</td>
<td>0.21</td>
<td>0.09</td>
<td>0.11</td>
<td>-0.11</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IT</td>
<td>1.0</td>
<td>0.07</td>
<td>0.12</td>
<td>0.27</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JPN</td>
<td>1.0</td>
<td>0.04</td>
<td>0.54</td>
<td>0.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NOR</td>
<td>1.0</td>
<td>0.02</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>1.0</td>
<td>0.44</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** The natural logarithm has been taken before data series are Hodrick-Prescott filtered (smoothing parameter = 1600). Variables are real gross domestic product $y$ and real consumption expenditures $c$. Data are quarterly from the International Monetary Fund IMF, Financial Statistics. The sample period is 1970:1 to 1998:4.
Table 2: Properties of Terms of Trade and Net Exports

<table>
<thead>
<tr>
<th></th>
<th>AUS</th>
<th>CAN</th>
<th>FRA</th>
<th>GER</th>
<th>IT</th>
<th>JPN</th>
<th>NOR</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Properties of Terms of Trade</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr($y,p$)</td>
<td>-.35</td>
<td>-.50</td>
<td>-.43</td>
<td>-.16</td>
<td>.17</td>
<td>-.25</td>
<td>-.34</td>
<td>-.22</td>
<td>-.46</td>
</tr>
<tr>
<td>% std ($p$)</td>
<td>6.59</td>
<td>3.07</td>
<td>3.08</td>
<td>3.49</td>
<td>4.03</td>
<td>7.67</td>
<td>6.29</td>
<td>3.17</td>
<td>3.08</td>
</tr>
<tr>
<td><strong>Properties of Net Exports</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr($nx,y$)</td>
<td>-.36</td>
<td>-.27</td>
<td>-.35</td>
<td>-.53</td>
<td>-.29</td>
<td>-.46</td>
<td>.11</td>
<td>-.34</td>
<td>-.25</td>
</tr>
<tr>
<td>% std ($nx$)</td>
<td>.47</td>
<td>.35</td>
<td>.93</td>
<td>.92</td>
<td>.75</td>
<td>.11</td>
<td>1.62</td>
<td>.94</td>
<td>.15</td>
</tr>
</tbody>
</table>

Notes: Except for net exports $nx$ the natural logarithm has been taken before data series are Hodrick-Prescott filtered (smoothing parameter = 1600). Variables are real gross domestic product $y$, real consumption expenditures $c$ and the terms of trade $p$ defined as the relative prices of imports to exports. $nx$ is the ratio of net exports to output. Data are quarterly from the International Monetary Fund, Financial Statistics. The sample period is 1970:1 to 1998:4.
Notes: This figure shows the value of the normalized multiplier of the domestic enforcement constraint \( d_1(q_{t-1}) \) if asymmetric shocks occur that increase domestic endowment. The new weight \( \hat{\alpha} \) is given by \( \hat{\alpha} = 0.5 + \zeta \). It is assumed that the defaulting country is punished by a \( K \)-period exclusion from intertemporal and interstate trade. The figure refers to \( q_{t-1} = 1, \eta = 5 \) and \( b_1/y_1 = 0.15 \).
Figure 2: Full Autarky Versus Financial Autarky: The Importance of $\eta$

Notes: This figure shows the value of the normalized multiplier of the domestic enforcement constraint $d_1(q_{t-1})$ if asymmetric shocks occur that increase domestic endowment. Full autarky refers to the scenario where the defaulting country is punished by exclusion from full trade. Financial autarky refers to the scenario where the defaulting country is punished by exclusion from international borrowing and lending. The figure refers to $q_{t-1} = 1$, $K = 4$, $\zeta = 0$, i.e. $\hat{\alpha}_i = \hat{\alpha}_j$, and $b_1/y_1 = 0.15$. 
### Table 3: Limited Enforceability and International Risk Sharing: Full Autarky and the Importance of $K$ and $\hat{\alpha}$

<table>
<thead>
<tr>
<th>$K/\zeta$</th>
<th>$\eta = 5$</th>
<th>$\eta = 6$</th>
<th>$\eta = 7$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.01</td>
<td>0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>-8.3</td>
<td>-17.4</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>-6.2</td>
<td>-23.1</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>0</td>
<td>-29.1</td>
</tr>
<tr>
<td>32</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Notes:** Relative change in correlations generated by imperfectly enforceable international loans is measured in percent of complete markets correlations. The new weight $\hat{\alpha}$ is given by $\hat{\alpha} = 0.5 + \zeta$. It is assumed that default is punished by a $K$-period stay in full autarky. The natural logarithm has been taken before simulated time series are Hodrick-Prescott filtered (smoothing parameter = 1600). Entries are averages of the correlations of 250 simulations of 500 quarters each where the first 50 observations are discarded. $c_1$ and $c_2$ denote domestic and foreign consumption, respectively. The results refer to $b_1/y_1 = 0.15$. 

32
Table 4: Limited Enforceability and International Risk Sharing: Financial Autarky and the Importance of $\eta$

<table>
<thead>
<tr>
<th>$K/\eta$</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-76.8</td>
<td>-86.7</td>
<td>-92.8</td>
<td>-94.9</td>
<td>-95.6</td>
</tr>
<tr>
<td>8</td>
<td>-76.8</td>
<td>-86.7</td>
<td>-92.8</td>
<td>-94.9</td>
<td>-95.6</td>
</tr>
<tr>
<td>16</td>
<td>-76.8</td>
<td>-86.7</td>
<td>-92.8</td>
<td>-94.8</td>
<td>-95.6</td>
</tr>
<tr>
<td>32</td>
<td>-76.5</td>
<td>-85.8</td>
<td>-91.4</td>
<td>-94.2</td>
<td>-95.4</td>
</tr>
</tbody>
</table>

Notes: Relative change in correlations generated by imperfectly enforceable international loans is measured in percent of complete markets correlations. It is assumed that default is punished by a $K$-period exclusion from international borrowing and lending. The natural logarithm has been taken before simulated time series are Hodrick-Prescott filtered (smoothing parameter = 1600). Entries are averages of the correlations over 250 simulations of 500 quarters each where the first 50 observations are discarded. The results refer to $b_1/y_1 = 0.15$ and $\zeta = 0$, i.e. $\hat{\alpha}_i = \hat{\alpha}_j$. 
Table 5: Limited Enforceability and the Quantity and Price Anomaly

<table>
<thead>
<tr>
<th>( \eta )</th>
<th>( \text{corr}(c_1, c_2) )</th>
<th>( \text{corr}(p, y_1) )</th>
<th>% std(( p ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td><strong>Import Share</strong> ( b_1/y_1 = 0.09 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full Autarky</td>
<td>0.56</td>
<td>0.45</td>
<td>0.38</td>
</tr>
<tr>
<td>Financial Autarky</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>Complete Markets</td>
<td>0.92</td>
<td>0.94</td>
<td>0.95</td>
</tr>
<tr>
<td><strong>Import Share</strong> ( b_1/y_1 = 0.12 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full Autarky</td>
<td>0.75</td>
<td>0.62</td>
<td>0.52</td>
</tr>
<tr>
<td>Financial Autarky</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>Complete Markets</td>
<td>0.95</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td><strong>Import Share</strong> ( b_1/y_1 = 0.15 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full Autarky</td>
<td>0.89</td>
<td>0.77</td>
<td>0.66</td>
</tr>
<tr>
<td>Financial Autarky</td>
<td>0.05</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Complete Markets</td>
<td>0.97</td>
<td>0.98</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Notes: Full autarky refers to the scenario where the defaulting country is punished by an exclusion from intertemporal and interstate trade. Financial autarky refers to the scenario where the defaulting country is punished by an exclusion from international borrowing and lending. The natural logarithm has been taken before simulated time series are Hodrick-Prescott filtered (smoothing parameter = 1600). Statistics refer to averages over 250 simulations of 500 quarters each where the first 50 observations are discarded. \( c_1 \) and \( c_2 \) denote domestic and foreign consumption, respectively, \( p \) are the terms of trade and \( y_1 \) is domestic output. Results refer to \( \zeta = 0 \), i.e. \( \hat{\alpha}_i = \hat{\alpha}_j \), and \( K = 4 \).
<table>
<thead>
<tr>
<th></th>
<th>Complete Markets</th>
<th>Limited Enforceable Loans:</th>
<th>Full Autarky</th>
<th>Financial Autarky</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(c, y)</td>
<td>0.79</td>
<td>0.83</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>corr(im, y)</td>
<td>-0.43</td>
<td>-0.39</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>corr(ex, y)</td>
<td>0.89</td>
<td>0.90</td>
<td>0.79</td>
<td></td>
</tr>
<tr>
<td>corr(nx, y)</td>
<td>0.71</td>
<td>0.69</td>
<td>-0.32</td>
<td></td>
</tr>
<tr>
<td>% std(c_1)</td>
<td>0.71</td>
<td>0.74</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>% std(y_1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% std(nx)</td>
<td>1.30</td>
<td>1.15</td>
<td>0.002</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Full autarky refers to the scenario where the defaulting country is punished by an exclusion from intertemporal and interstate trade. Financial autarky refers to the scenario where the defaulting country is punished by an exclusion from international borrowing and lending. Except for net exports the natural logarithm has been taken before simulated time series are Hodrick-Prescott filtered (smoothing parameter = 1600). Statistics refer to averages over 250 simulations of 500 quarters each where the first 50 observations are discarded. c_1 and y_1 denote domestic consumption and output. im and ex are the imports b_1 and exports a_2. nx = (a_2 - pb_1)/y_1 denote net exports. Results refer to ζ = 0, i.e. α_i = α_j, K = 4, η = 5 and b_1/y_1 = 0.15.
Table 7: Welfare Loss: Full Autarky

<table>
<thead>
<tr>
<th>Limited Enforceable Loans</th>
<th>Punishment: Full Autarky</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K/\eta$</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>0.0008</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>32</td>
<td>0</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0</td>
</tr>
</tbody>
</table>

Full Autarky

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>2.2939</td>
<td>1.8254</td>
<td>1.5198</td>
</tr>
</tbody>
</table>

Notes: Welfare loss is measured relative to complete markets as percentage deviation in certainty-equivalence consumption relative to the steady state. Results associated with limited enforceability refer to $\zeta = 0$, i.e. $\hat{\alpha}_i = \hat{\alpha}_j$, and $b_1/y_1 = 0.15$.

Table 8: Welfare Loss: Financial Autarky

<table>
<thead>
<tr>
<th>Limited Enforceable Loans</th>
<th>Punishment: Financial Autarky</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K/\eta$</td>
<td>1.5</td>
</tr>
<tr>
<td>4</td>
<td>0.0322</td>
</tr>
<tr>
<td>8</td>
<td>0.0322</td>
</tr>
<tr>
<td>16</td>
<td>0.0322</td>
</tr>
<tr>
<td>32</td>
<td>0.0321</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.0319</td>
</tr>
</tbody>
</table>

Financial Autarky

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>0.0323</td>
<td>0.0451</td>
<td>0.0580</td>
<td>0.0643</td>
<td>0.0681</td>
</tr>
</tbody>
</table>

Notes: Welfare loss is measured relative to complete markets as percentage deviation in certainty-equivalence consumption relative to the steady state. Results associated with limited enforceability refer to $\zeta = 0$, i.e. $\hat{\alpha}_i = \hat{\alpha}_j$, and $b_1/y_1 = 0.15$. 

36
For a complete list of Discussion Papers published by the SFB 649, please visit http://sfb649.wiwi.hu-berlin.de.

001 "Nonparametric Risk Management with Generalized Hyperbolic Distributions" by Ying Chen, Wolfgang Härdle and Seok-Oh Jeong, January 2005.
003 "Competitive Risk Sharing Contracts with One-sided Commitment" by Dirk Krueger and Harald Uhlig, February 2005.
004 "Value-at-Risk Calculations with Time Varying Copulae" by Enzo Giacomini and Wolfgang Härdle, February 2005.
005 "An Optimal Stopping Problem in a Diffusion-type Model with Delay" by Pavel V. Gapeev and Markus Reiß, February 2005.
007 "Implied Trinomial Trees" by Pavel Čížek and Karel Komorád, February 2005.
008 "Stable Distributions" by Szymon Borak, Wolfgang Härdle and Rafal Weron, February 2005.
009 "Predicting Bankruptcy with Support Vector Machines" by Wolfgang Härdle, Rouslan A. Moro and Dorothea Schäfer, February 2005.
010 "Working with the XQC" by Wolfgang Härdle and Heiko Lehmann, February 2005.
011 "FFT Based Option Pricing" by Szymon Borak, Kai Detlefsen and Wolfgang Härdle, February 2005.
012 "Common Functional Implied Volatility Analysis" by Michal Benko and Wolfgang Härdle, February 2005.
013 "Nonparametric Productivity Analysis" by Wolfgang Härdle and Seok-Oh Jeong, March 2005.
015 "Robust Estimation of Dimension Reduction Space" by Pavel Čížek and Wolfgang Härdle, March 2005.
017 "A Two State Model for Noise-induced Resonance in Bistable Systems with Delay" by Markus Fischer and Peter Imkeller, March 2005.
021 "Dynamics of State Price Densities" by Wolfgang Härdle and Zdeněk Hlávka, March 2005.
022 "DSFM fitting of Implied Volatility Surfaces" by Szymon Borak, Matthias R. Fengler and Wolfgang Härdle, March 2005.
"Towards a Monthly Business Cycle Chronology for the Euro Area" by Emanuel Mönch and Harald Uhlig, April 2005.


"Duality Theory for Optimal Investments under Model Uncertainty" by Alexander Schied and Ching-Tang Wu, April 2005.

"Projection Pursuit For Exploratory Supervised Classification" by Eun-Kyung Lee, Dianne Cook, Sigbert Klinke and Thomas Lumley, May 2005.


"Notes on an Endogenous Growth Model with two Capital Stocks II: The Stochastic Case" by Dirk Bethmann, June 2005.

"Skill Mismatch in Equilibrium Unemployment" by Ronald Bachmann, June 2005.

"Uncovered Interest Rate Parity and the Expectations Hypothesis of the Term Structure: Empirical Results for the U.S. and Europe" by Ralf Brüggemann and Helmut Lütkepohl, April 2005.


"Discretisation of Stochastic Control Problems for Continuous Time Dynamics with Delay" by Markus Fischer and Markus Reiss, August 2005.


"Optimal Sticky Prices under Rational Inattention" by Bartosz Maćkowiak and Mirko Wiederholt, July 2005.

"Fixed-Prize Tournaments versus First-Price Auctions in Innovation Contests" by Anja Schöttner, August 2005.

"Bank finance versus bond finance: what explains the differences between US and Europe?" by Fiorella De Fiore and Harald Uhlig, August 2005.

"On Local Times of Ranked Continuous Semimartingales; Application to Portfolio Generating Functions" by Raouf Ghomrasni, June 2005.

"A Software Framework for Data Based Analysis" by Markus Krätzig, August 2005.


"Courtesy and Idleness: Gender Differences in Team Work and Team Competition" by Radosveta Ivanova-Stenzel and Dorothea Kübler, September 2005.

"Do Factor Shares Reflect Technology?" by Benjamin Bental and Domonique Demougin, September 2005.


"Relational Contracts and Job Design" by Anja Schöttner, September 2005.

"Explicit characterization of the super-replication strategy in financial markets with partial transaction costs" by Imen Bentahar and Bruno Bouchard, October 2005.

"Aid Effectiveness and Limited Enforceable Conditionality" by Almuth Scholl, August 2005.