SFB 649 Discussion Paper 2006-052

Forecasting the Term Structure of Variance Swaps

Kai Detlefsen*
Wolfgang Härdle*

* Center for Applied Statistics and Economics (C.A.S.E.),
School of Business and Economics,
Humboldt-Universität zu Berlin, Germany

This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

http://sfb649.wiwi.hu-berlin.de
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin
Spandauer Straße 1, D-10178 Berlin

Forecasting the Term Structure of Variance Swaps

K. Detlefsen[1], W. K. Härdle[2],

[1]Center for Applied Statistics and Economics, Humboldt Universität zu Berlin, Spandauer Straße 1, 10178 Berlin, Germany; e-mail: detlefsen@wiwi.hu-berlin.de; phone: +49(0)30 2093-5654

[2]Center for Applied Statistics and Economics, Humboldt Universität zu Berlin, Spandauer Straße 1, 10178 Berlin, Germany; e-mail: haerdle@wiwi.hu-berlin.de; phone: +49(0)30 2093-5630

Abstract

Recently, Diebold and Li (2003) obtained good forecasting results for yield curves in a reparametrized Nelson-Siegel framework. We analyze similar modeling approaches for price curves of variance swaps that serve nowadays as hedging instruments for options on realized variance.

We consider the popular Heston model, reparametrize its variance swap price formula and model the entire variance swap curves by two exponential factors whose loadings evolve dynamically on a weekly basis. Generalizing this approach we consider a reparametrization of the three-dimensional Nelson-Siegel factor model. We show that these factors can be interpreted as level, slope and curvature and how they can be estimated directly from characteristic points of the curves. Moreover, we analyze a semiparametric factor model.

Estimating autoregressive models for the factor loadings we get term-structure forecasts that we compare in addition to the random walk and the static Heston model that is often used in industry. In contrast to the results of Diebold and Li (2003) on yield curves, no model produces better forecasts of variance swap curves than the random walk but forecasting the Heston model improves the popular static Heston model. Moreover, the Heston model is better than the flexible semiparametric approach that outperforms the Nelson-Siegel model.

JEL classification: G1, D4, C5

Keywords: Term structure; Variance swap curve; Heston model; Nelson-Siegel curve; Semiparametric factor model

1 Introduction

In the last 30 years we have witnessed major advances in the modeling of option pricing models as well as their estimation. Although there are models like the local volatility model of Dupire (1994) that permit in practice a good and stable fit of observed plain vanilla surfaces these models imply dynamics not observed on the markets. Hence, the dynamic evolution of the markets has become a benchmark for models. The dynamics of implied volatility surfaces have been analyzed and modeled often by factor approaches, see Fengler (2005) or Cont and da Fonseca (2001). As modern approaches to option pricing, see Bergomi (2005), are also based on the variance swap market we analyze in this paper the dynamics of variance swap curves with a focus on factor modeling.

Variance swap markets have become quite liquid and these products serve hence - just like the plain vanilla options - as hedging instruments for some modern options like calls on realized variance. Thus, the analysis of variance curves is as important as the study of the evolution of implied volatility surfaces. In addition, the variance swap market is connected to the implied volatility surfaces by the results of Neuberger (1992), it is basically term structure of the surfaces. Thus, understanding the evolution of variance curves helps also modeling the plain vanilla market.

In this paper we analyze modeling approaches for variance curves focusing on the forecasting perspective because the dynamics are nowadays the essential model criterion in option pricing. First we consider the Heston model that may be regarded as a standard benchmark in option pricing. When we fix the mean reversion it leads to a two factor structure for the variance swap curves. Moreover, we consider a generalization that gives a reparametrization of the model of Nelson and Siegel (1987). In addition, we fit a semiparametric model in order to check the structure of the parametric models. The in-sample fit of the Heston model is of course worse than the fit of the generalized model. The flexible semiparametric model outperforms the Nelson-Siegel approach for long maturities. But the Heston model has the best out-of-sample performance. The static Heston model that is often used in industry leads to bigger forecasting errors than the Heston model with parameter forecasts. But the random walk has the best out-of-sample results. Hence, we can conclude that variance swap curve modeling seems to be more difficult than yield curve forecasting. Moreover, we show that the static Heston model leads to a wrong dynamic of variance swap curves. Forecasting the parameters improves the variance swap curve dynamics in the Heston model.

Although variance swaps are analyzed for some time there are only a few empirical works. Demeterfi et al. (1999) give a comprehensive survey over the practical aspects of these products and Carr and Madan (1998) consider the hedging and trading of volatility. Recently, Bergomi (2005) proposed an option pricing model based on the variance swap market and Bühler (2006) analyzed theoretical modeling questions for the joint plain vanilla and variance swap market. Duffie and Kan (1996) and Diebold and Li (2003) have analyzed similar forecasting problems for yield curves.

We proceed as follows. In section 2 we give a detailed description and derivation of the modeling framework, which comprises the popular Heston model, a generalization of this model and a semiparametric approach, and we analyze how these approaches can replicate stylized facts of variance curves. In section 3 we conduct an empirical analysis, describing the data, estimating the models and forecasting the variance curves. Moreover, we consider some other natural approaches for forecasting. In section 4 we draw our conclusions and compare our results with yield curve modeling.

2 Modeling the Term Structure

In this section we introduce variance swaps, explain the construction of variance swap curves and describe the approach that we use for fitting and forecasting the variance swap curves. We start with the popular stochastic volatility model of Heston (1993) and derive the corresponding model for the variance curves. Besides this two parameter model we consider a generalization with three parameters. A discussion of the factors is provided for both models. Moreover, we describe a semiparametric factor model. Finally, we see how good stylized facts are replicable in these models.

2.1 Constructing variance swap curves

Variance swaps are forward contracts on future realized volatility. They exchange at expiration the realized annualized variance of the log returns of an underlying against a predefined strike. These contracts vary in several respects: they may or may not assume zero mean of the log returns, they differ with respect to the annualization factor and it must be specified when the underlying is sampled. We assume here a zero mean of the returns, use $c = 252$ trading days for annualization with daily sampling and focus w.l.g. on zero strikes.

Given an underlying S, the price of such a variance swap for the period

$[0,T]$ with business days $0 = t_0 < \ldots < t_n = T$ is given by

$$\sigma_R^2(T) \stackrel{\mathrm{def}}{=} \frac{c}{n} \sum_{i=1}^{n} (\log \frac{S_{t_i}}{S_{t_{i-1}}})^2.$$

At time $t \in (0,T)$, the first part of the variance is already realized while the second is still uncertain. Hence, the prices are composed of the value of the realized variance and the price of the uncertain variance. In our analysis, we will focus on the uncertain part and denote the price for the not annualized variance that still has to be realized by $V_t(T)$. This point of view is supported by the usual quotation of variance swaps in volatility strikes, i.e. $\sqrt{V_t(T)/(T-t)}$.

Such price quotation indicates how closely variance swaps are related to (zero strike) volatility swaps that have payoff profiles $\sqrt{\sigma_R^2}$. Actually, there is a variety of options that depend on realized variance, e.g. calls on realized variance with payoffs $\max\{\sigma_R^2 - K, 0\}$. Variance swaps are often used not only for directly speculating on variance but also as hedging instruments for such products.

At a point in time we observe the prices of variance swaps $V(x_i)/x_i$ with expiries $x_1, \ldots, x_n$. The variance swap curve at this time is then given by the mapping $T \rightarrow V(T)/T$. We call V the variance curve and V' the forward variance curve. The variance swap curve quoted in volatility strikes is given by $T \rightarrow \sqrt{V(T)/T}$. Many approaches for modeling variance swap curves are based on forward variance curves, see e.g. Bergomi (2005). But in practice, variance curves or forward variance curves are not observed. Instead, they must be estimated from a discrete set of observed variance swap prices. It is practice in industry to use (piecewise) polynomial functions for interpolation between the observations. Although this gives a reasonable fit for the variance curves the forward variance curves show a high variation. This instability of the derivatives makes this approach seem unsuitable for theoretical analysis of variance swaps.

We choose a non parametric method for constructing the curves, see e.g. Härdle et al. (2004). We apply a local quadratic regression to the variance prices $V(x_i)$ leading to the following minimization problem:

$$\min_{\beta} \sum_{i=1}^{n} \{V(x_i) - \beta_0 - \beta_1(x_i - x) - \beta_2(x_i - x)^2\} K_h(x_i - x)$$

where the vector $\beta = (\beta_0, \beta_1, \beta_2)$ depends on x. The result $\hat{\beta}(x)$ is a weighted least squares estimator where the variance curve is given by $\hat{\beta}_0$ and its first derivative by $\hat{\beta}_1$. We use the quartic kernel K and choose the bandwidth h by a rule of thumb described in Fan and Gijbels (1996). Moreover, confidence intervals can be constructed, see e.g. Härdle et al. (2004). Alternatively, higher order kernels can be used but they have an inferior finite sample bias.

2.2 Modeling variance swap curves

On each day, we construct a variance curve to which we fit two parametric models and a semiparametric model. The resulting time series of factor loadings are the basis for the forecasting. We use the functional form derived from the Heston model and in addition we consider an extension that leads to the form of Nelson and Siegel (1987). These model are convenient and parsimonious exponential factor approximations. Moreover, we analyze a semiparametric approach.

In the Heston model

$$\begin{aligned} \frac{dS_t}{S_t} &= r dt + \sqrt{\zeta_t} dW_t^{(1)} \\ d\zeta_t &= \kappa(\theta - \zeta_t)dt + \nu\sqrt{\zeta_t} dW_t^{(2)} \end{aligned}$$

with correlated Wiener processes $W^{(1)}$ and $W^{(2)}$ the prices of (annualized) variance swaps $V(T)/T$ are given by

$$\theta + (\zeta_0 - \theta)\frac{1 - \exp(-\kappa T)}{\kappa T}.$$

Hence, only the short variance ζ_0, the long variance θ and the mean reversion speed κ determine the variance swap prices. In the Heston model the smile of the implied volatility surfaces is controlled by two parameters: The correlation between the Brownian motions and the volatility of variance ν. These two parameters do not enter the formula of variance swap price.

The corresponding model for the forward variance curve is given by

$$v(T) \stackrel{\text{def}}{=} V'(T) = \theta + (\zeta_0 - \theta)\exp(-\kappa T).$$

This forward variance curve model implies exactly the above variance swap prices because of the constraint $V(0) = 0$. Reparametrizing this model and writing it in factor notation we get for the prices of variance swaps $V(T)/T$:

$$z_1 + z_2 \frac{1 - \exp(-\kappa T)}{\kappa T}. \tag{1}$$

where $z = (z_1, z_2)$ are the two factor loadings. They correspond to the model parameters $(\theta, \zeta_0 - \theta)$ for the volatility. The reparametrization can be described formally in terms of a reparametrization matrix by

$$\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \theta \\ \zeta \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}.$$

This model for forward variance curves is also called linearly mean-reverting (forward) variance curve model, see Bühler (2006).

We want to compare our results on variance curve modeling with the results of Diebold and Li (2003) on yields curves. As Diebold and Li (2003) apply the Nelson-Siegel parametrization we generalize the above model in such a way that the resulting variance swap prices have a Nelson-Siegel parametrization:

$$v(T) = z_1 + z_2 \exp(-\kappa T) + z_3 \kappa T \exp(-\kappa T)$$

This model is called the double mean-reverting (forward) variance curve model. The variance swap prices $V(T)/T$ are given in this model by

$$z_1 + z_2 \frac{1 - \exp(-\kappa T)}{\kappa T} + z_3 \{ \frac{1 - \exp(-\kappa T)}{\kappa T} - \exp(-\kappa T) \}. \qquad (2)$$

because of

$$\frac{V(T)}{T} = \frac{1}{T} \int_0^T v(t) dt.$$

Thus, the generalized Heston model leads exactly to a Nelson-Siegel parametrization for the prices of variance swaps.

While the linearly mean-reverting model is basically the Heston model, the second approach was considered by Bühler (2006) who analyzed conditions for an arbitrage free joint market of variance swaps and stock. His considerations imply that the mean reversion speed κ should be constant. In practice, a constant mean reversion speed is important for stability of the parameters. Because of these theoretical and practical reasons we fix this parameter and use $\kappa = 2$ as in Bergomi (2004).

We interpret z_{1t}, z_{2t} and z_{3t} as latent dynamic factor loadings for the prices of variances swaps $V(T)/T$. As the two factors in the Heston parametrization (1) equal the first two factors in the generalized Heston model (2) it is sufficient to discuss the generalized model.

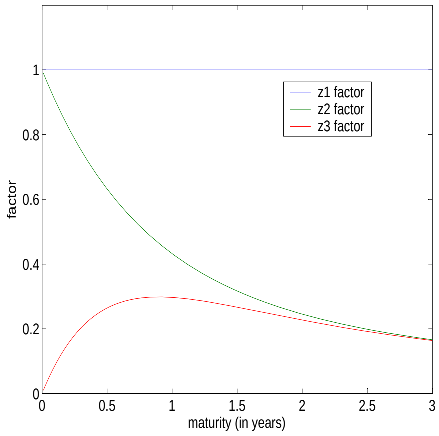

Figure 1: Factors for variance curves in the Heston model and its generalization.

The factor on z_{1t} is the constant 1. As this factor does not decay to zero in the long run it can be interpreted as a long-term factor. The factor on z_{2t} is $\{1 - \exp(-\kappa T)\}/(\kappa T)$. This function is monotonically decreasing from 1 to 0. As it influences only the short end of the curve it can be interpreted as a short-term factor. Besides these two factor the generalized model controls also the medium-term. The factor on z_{3t} is $\{1-\exp(-\kappa T)\}/(\kappa T)-\exp(-\kappa T)$. This mapping increases monotonically from 0 to a peak and then decreases to zero in the long-term in a similar way as the second factor. This form explains the interpretation as a medium-term factor. These three factors are presented in figure 1.

The interpretation of these factors corresponds to their meaning in the Heston model: z_1 is the long variance and z_1+z_2 stands for the short variance. Moreover, these quantities can be recovered from the variance swap curve: from the limits $\lim_{T\to 0} V(T)/T = z_1 + z_2$ and $\lim_{T\to\infty} V(T)/T = z_1$ we see that z_1 is the long variance (i.e. θ) and $z_1 + z_2$ is the short variance (i.e. ζ_0). Hence, we have used the parametrization $\zeta_0 = z_1 + z_2$ and $\theta = z_1$. Thus, the "original" parameters of the Heston model (θ, ζ) can be recovered by multiplying the invers of the reparametrization matrix with the factor loadings (z_1, z_2).

Moreover, the parameters have interpretations as level, slope and curvature of the curves. As an increase in z_1 increases the whole curve by the same amount the factor on z_1 represents the level of the curve. An increase of the short-term factor increases the curve more at the short end than at the long end. Hence it controls the slope of the curve. Finally, the third factor moves the middle of the curve while keeping the ends (almost) fixed. In this way it changes the curvature of the curve. Hence, the difference between the Heston model and its three factor generalization is the capability to control the curvature.

Besides these parametric approaches we analyze a semiparametric model described in Fengler (2005). It offers a low-dimensional representation of variance swap curves that are approximated by basis functions. These basis functions are unknown and have to be estimated from the data. The dynamics of the curves are described by the time series of the corresponding factor loadings.

Let $Y_{i,j}$ be an observed price of a variance swap on day i with maturity $T_j \in \{0.12, 0.25, 0.5, 0.75, 1.0, 1.5, 2.0\}$. Let $X_{i,j}$ be a one-dimensional variable representing the time-to-maturity. Then the model regresses $Y_{i,j}$ on $X_{i,j}$ by

$$Y_{i,j} = m_0(X_{i,j}) + \sum_{l=1}^{L} \beta_{i,l} m_l(X_{i,j}),$$

where m_0 is an invariant basis function, m_l ($l = 1, ..., L$) are the "dynamic" basis functions and $\beta_{i,l}$ are the factor weights depending on time i. We describe the estimation procedure and the obtained basis functions in section 3.2 where we use the data.

2.3 Stylized facts of variance swap curves

A model of variance swap curves should at least have a reasonable in-sample fit, i.e. it should reproduce the variety of observed shapes of variance swap curves. A good model should moreover correctly reflect the dynamics of the curves, i.e. have a reasonable out-of sample performance. The goal is to meet these two points in a parsimonious model.

We consider here some stylized facts of variance swap curves and see how the described approaches can model these characteristic shapes: The average variance swap curve is increasing and concave. The slope factor can replicate the increasing structure easily. The concavity can be modeled in the generalized model by the third factor. But in the Heston model control of concavity is not directly possible. Variance swap curves show many different shapes in different markets over the time: They can be upward- or downward sloping and some have a hump. These shapes can be replicated by varying the three factor accordingly. The short end of the curves is more volatile than the long end. This is reflected in the models because two factors (z_1

and z_2) control the short end while only one factor (z_1) models the long end. The Heston model can replicate many pattern but not the humps and the curvature while the generalized model has in principle the capability to replicate all stylized facts. The semiparametric model should better reflect these stylized facts because of its more flexible structure.

3 Forecasting the Term Structure

In this section, we describe the data, estimate the factor loadings, model them and compare the forecasted variance swap curves.

3.1 The data

The data set studied contains prices of variance swaps on the S & P 500 index between 1 October 2003 and 30 September 2005. These swaps use daily closing prices of the index, have 252 business days as annualization factor and assume a zero mean for the calculation of the variance of the returns. The prices are quoted in volatility strikes and represent the mid market prices of a large financial institution. Hence, we observe on every trading day prices $\sqrt{V(x_i)/x_i},\ i = 1, \ldots, n$ of variance swaps with times-to-maturity $x_1, \ldots, x_n$. We have around $n = 7$ observations in the mean per day. Although variance swap prices can be generated synthetically our data consists of real market prices.

Our analysis does not require the use of fixed maturities because we always model the entire variance swap curve. But we use fixed maturities in order to simplify the following variance swap curve forecasts. Hence we first create from the discrete data real curves by local quadratic smoothing as described in section 2.1. Then we extract the data for the fixed maturities 1.5, 3, 6, 9, 12, 18 and 24 months.

The variance swap prices (not quoted in volatility strikes) and level, slope and curvature of these curves are the basis for the following. In figure 2 we present the smoothed variance swap curves. The figure also shows the variance swap curves quoted in volatility strikes. We estimate and forecast the variance curves but in industry people quote these prices in volatility strikes. In figure 3 we show the corresponding variance and forward variance curves. The parametric models all start from the forward variance curve and derive the variance curve. The variation of the level is clearly visible for the variance swap curves, the changes in the slope and curvature are less apparent.

We provide some descriptive statistics of the variance swap curves in

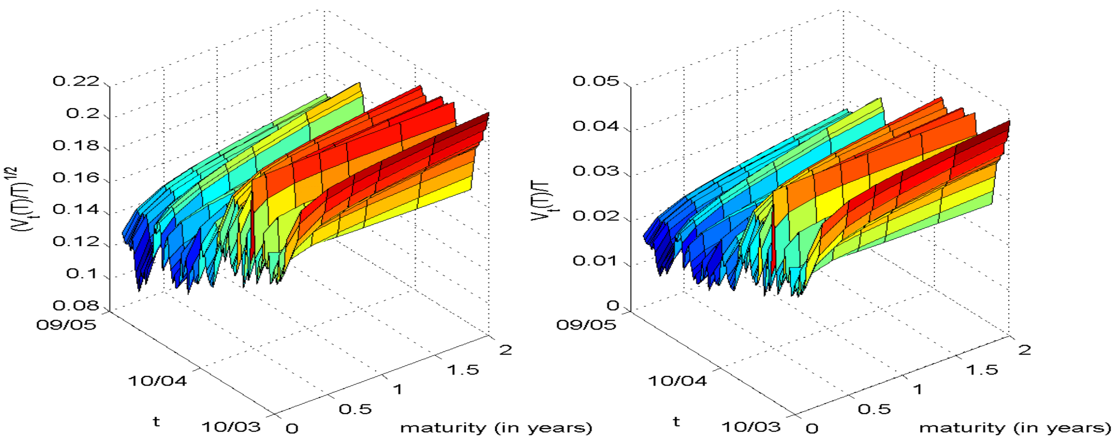

Figure 2: Variance swap curves quoted in volatility strikes (left) and variance swap curves (right), 01/10/03 - 30/09/05. The sample consists of weekly curves from October 2003 to September 2005 at maturities 1.5, 3, 6, 9, 12, 18 and 24 months.

table 1. Here, we present also the level (defined as the 24 months price), the slope (defined as the 24 months price minus the 1.5 month price) and the curvature (defined as twice the 6 months price minus the 1.5 month price minus the 24 months price). We see later that these empirical factor loadings are highly correlated with the loadings of the parametric models. In figure 4 we show the median variance swap curve (and its quotation in volatility strikes) together with pointwise interquartile ranges. The earlier-mentioned upward sloping and concave form is clearly visible.

3.2 Fitting the variance swap curves

We estimate the Heston model and its generalization by minimizing the difference between the observed variance swap curves and the model prices. In the Heston model, these prices are given by

$$z_1 + z_2 \frac{1-\exp(-\kappa T)}{\kappa T}.$$

and in the generalized Heston model by

$$z_1 + z_2 \frac{1-\exp(-\kappa T)}{\kappa T} + z_3 \{ \frac{1-\exp(-\kappa T)}{\kappa T} - \exp(-\kappa T) \}.$$

The factor loadings z and the parameter κ can be estimated by nonlinear least squares. In the approach of Nelson and Siegel (1987) for interest rates it

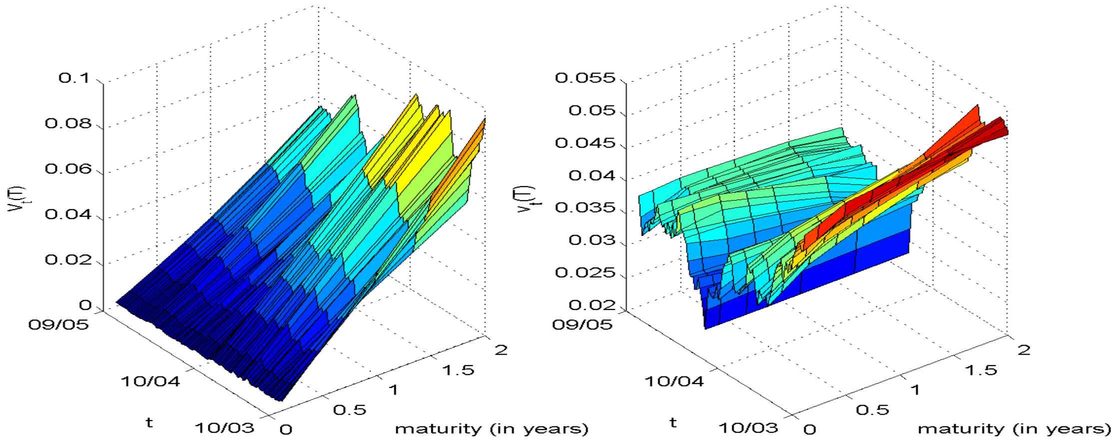

Figure 3: Variance curves (left) and forward variance curves (right), 01/10/03 - 30/09/05. The sample consists of weekly curves from October 2003 to September 2005 at maturities 1.5, 3, 6, 9, 12, 18 and 24 months.

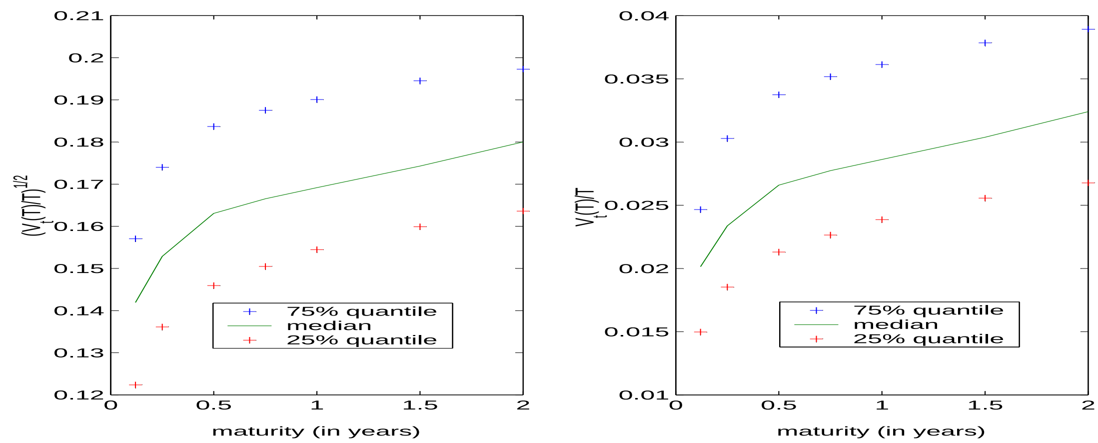

Figure 4: Median data-based forward variance curve quoted in volatility strikes with pointwise interquartile range. For each maturity, we plot the median along with the 25th and 75th quantiles.

Maturity (Months)	Mean	Std.dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(4)$	$\hat{\rho}(12)$
1.5	2.03	0.65	0.91	4.36	80.1	64.5	51.1
3	2.48	0.70	1.39	4.39	90.1	79.3	65.4
6	2.75	0.73	1.51	4.41	93.3	82.6	64.7
9	2.89	0.73	1.60	4.45	94.1	82.5	61.3
12	2.98	0.72	1.69	4.54	94.5	81.8	57.1
18	3.14	0.70	1.85	4.68	94.9	79.6	47.3
24	3.27	0.69	2.02	4.79	9.50	77.1	36.8
Slope	1.24	0.44	-0.24	2.21	74.1	38.8	-16.6
Curvature	0.20	0.31	-0.50	0.98	83.6	62.0	49.5

Table 1: Descriptive statistics of variance swap curves $[E^{-2}]$.

is common to fix the parameter κ. As the generalized Heston model leads to variance swap prices in the form of the Nelson-Siegel it makes sense that we also fix this parameter κ. Moreover, it is practice to fix this parameter in the Heston model for the modeling, pricing and hedging of options. Hence, we use $\kappa = 2$ as in Bergomi (2004). Keeping this parameter constant simplifies considerably the numerics and increases the reliability of the results because the factor loadings z are given by the OLS formula.

On every day we apply an ordinary least squares to the variance swap curves. In this way we get we a time series of estimated factor loadings $(\hat{z}_1, \hat{z}_2, \hat{z}_3)^\top$. As we do not use explicitly weights the short end is more important for the estimation because we sample more observation with short maturities. We estimate the variance swap curves (and not the variance swap curves quoted in volatility strikes) because the Heston model gives directly a factor model for the variance swap curves.

We estimate the factors in the semiparametric model from the first year of our time series. The factors or basis functions $\widehat{m}_l$ and the factor loadings $\widehat{\beta}_{i,l}$ are estimated by minimizing the following least squares criterion ($\beta_{i,0} = 1$):

$$\sum_{i=1}^{I} \sum_{j=1}^{J_i} \int \left\{ Y_{i,j} - \sum_{l=0}^{L} \widehat{\beta}_{i,l} \widehat{m}_l(u) \right\}^2 K_h(u - X_{i,j})\, du,$$

where K_h denotes a kernel function. The minimization procedure searches through all functions $\widehat{m}_l : \mathbb{R} \longrightarrow \mathbb{R}$ $(l = 0, ..., L)$ and time series $\widehat{\beta}_{i,l} \in \mathbb{R}$ $(i = 1, ..., I; l = 1, ..., L)$ by an iterative procedure. Afterwards the estimates are orthogonalized and normalized, see Fengler (2005) for details. The estimated

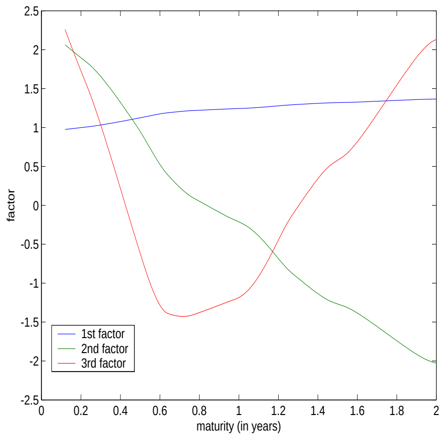

Figure 5: Factors in the semiparametric model estimated from the variance swap curves 01/10/03 - 30/09/04.

factors are plotted in figure 5. They can be interpreted as in the parametric models as level, slope and curvature (see section 2.2). A comparison with the parametric factors in figure 7 reveals that the estimated factors have a different scaling and they become negative. The positivity of the parametric factors allows to ensure the positivity of the resulting curve by imposing simple constraints on the loadings. After the factors have been estimated the factor loadings can be estimated by ordinary least squares as in the parametric models.

Information about the in-sample fit of the models is presented in figure 6. It shows that the Heston and the semiparametric model have problems fitting the short end of the curves. This systematic deviation holds also for the generalized model but is less pronounced. The prices for long maturities show no systematic error for all models. The semiparametric model leads to the smallest errors in this region while the Heston model still has big errors. In table 2 we present descriptive statistics of the variance swap curve residuals that support the above interpretation. Although the semiparametric model is quite flexible it has problems fitting the short maturities. This can be explained by the fact that the semiparametric model puts less weight on the short maturities than the other models. Another reason can be seen in the numerically more involved estimation procedure.

In figure 7 we show the time series of the estimated factor loadings in the Heston model, in the generalized Heston model and in the semiparametric factor model. In the figure we have plotted the negative slope loadings of the models and we have scaled curvature loadings of the models by 0.3. As

Maturity (months)	Mean	Std.dev.	Minimum	Maximum	MAE
Heston					
1.5	-0.17	0.12	-0.46	0.06	0.17
3	0.10	0.05	-0.03	0.22	0.10
6	0.11	0.08	-0.04	0.30	0.11
9	0.05	0.06	-0.08	0.19	0.06
12	0.00	0.03	-0.09	0.07	0.03
18	-0.05	0.03	-0.11	0.01	0.05
24	-0.04	0.07	-0.22	0.13	0.07
generalized Heston					
1.5	-0.09	0.04	-0.18	0.02	0.09
3	0.11	0.05	-0.03	0.24	0.11
6	0.04	0.02	-0.01	0.08	0.04
9	-0.03	0.02	-0.07	0.01	0.03
12	-0.06	0.03	-0.11	0.01	0.06
18	-0.03	0.01	-0.05	0.01	0.03
24	0.05	0.02	-0.01	0.09	0.05
semiparam. model					
1.5	-0.26	0.12	-0.53	0.07	0.26
3	0.09	0.05	-0.03	0.22	0.09
6	0.10	0.04	-0.03	0.20	0.10
9	0.01	0.01	-0.00	0.02	0.01
12	0.04	0.01	-0.01	0.06	0.04
18	0.00	0.00	-0.01	0.01	0.00
24	0.01	0.01	-0.02	0.02	0.01

Table 2: Descriptive statistics of variance swap curves residuals $[E^{-2}]$.

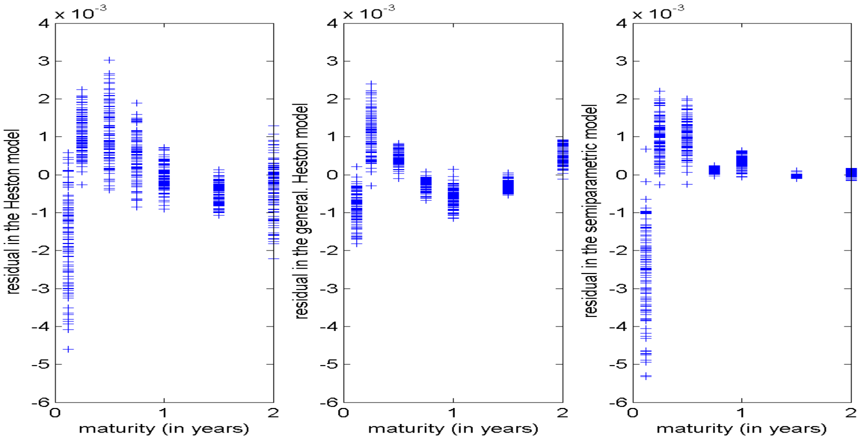

Figure 6: Variance swap curve residuals, 01/10/03 - 30/09/05. left: Heston, middle: generalized Heston, right: semiparametric model.

the factors have interpretations as level, slope and curvature we compare them to the empirical level, slope and curvature as defined in section 3.1. The graphs proof that our interpretations are correct because the empirical and the estimated factor loadings are highly correlated in all cases. The empirical level factor is very similar to the level loadings in the generalized Heston model. The corresponding loadings of the Heston model lie above, the loadings of the semiparametric model below the empirical levels. The empirical slope is a good estimator for slope loadings in the Heston and in the generalized Heston model. The empirical curvature differs from the loadings of the models. But in all three cases the model and the empirical values are highly correlated.

The loadings of the semiparametric factor model differ the most from the empirical factor loadings. This can partly be explained by the different scaling in the semiparametric factor model. In table 3 we give some summary statistics of the time series of factor loadings. These statistics confirm the different scaling of the parametric approaches and the semiparametric model. Moreover, we have found that these time series are only weakly correlated. The two factor loadings in the Heston have an empirical correlation of -0.39. This quantity is in the generalized Heston model -0.33 while the other two correlations of the model are below 0.2. The correlations in the semiparametric factor model are similar.

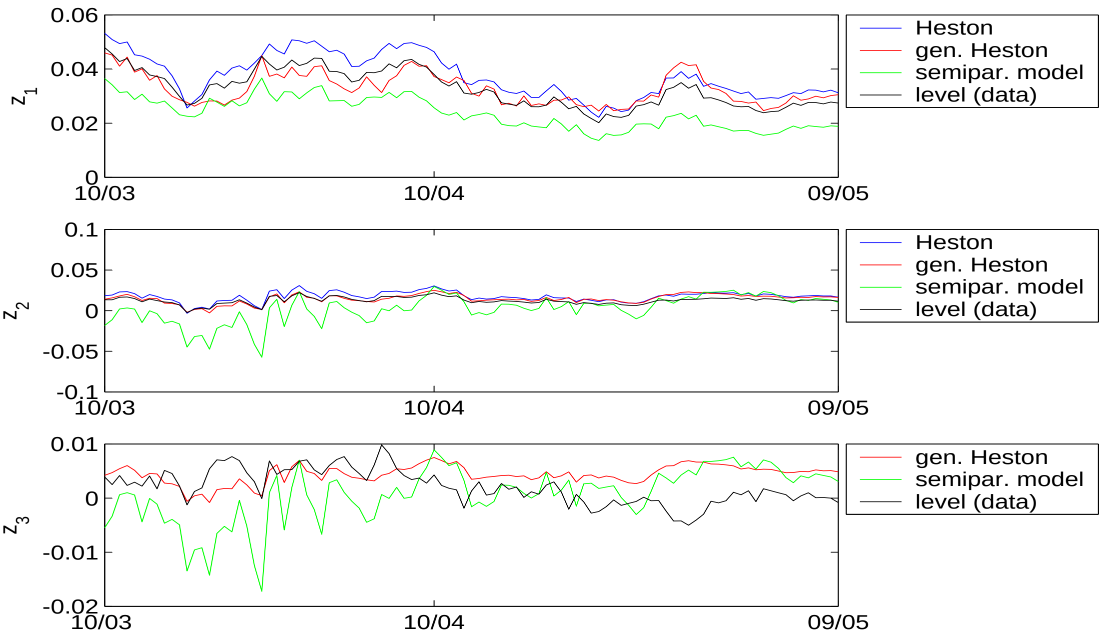

Figure 7: Factor loadings in the models and in the data.

Factor	Mean	Std.dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(4)$	$\hat{\rho}(12)$
Heston							
z_1	3.74	0.80	2.21	5.32	95.4	75.2	31.1
z_2	-1.74	0.61	-3.08	0.33	74.1	38.1	-17.9
generalized Heston							
z_1	3.27	0.58	2.45	4.60	87.9	51.9	-26.6
z_2	-1.46	0.56	-2.50	0.27	79.8	49.0	-11.2
z_3	1.34	1.30	-1.37	4.61	81.3	57.6	45.7
semiparam. model							
z_1	2.37	0.58	1.36	3.67	93.5	81.1	58.1
z_2	-0.01	0.10	-0.17	0.33	73.1	50.0	23.1
z_3	0.00	0.03	-0.06	0.06	70.0	36.9	24.8

Table 3: Descriptive statistics of the factor loadings $[E^{-2}]$.

3.3 Modeling the time series of factor loadings

Diebold and Li (2003) model the factor loadings of the Nelson-Siegel framework by univariate AR(1) processes. Also Cont and da Fonseca (2001) use these models for their factor loadings in a principal components analysis of implied volatility surfaces. Hence, we follow this accepted approach. These processes can be viewed as the standard method for parsimonious modeling. Moreover, more complex ARMA models did not improve the forecasting results. Thus, the choice of these simple models seems justified. We do not consider multivariate AR processes because there is only little correlation between the factor loadings (see section 3.2). In addition, the use of AR(1) processes allows us to compare our results to the findings of Diebold and Li (2003).

The resulting forecasts of the variance swap curves τ weeks ahead are then given by

$$\widehat{V_{t+\tau}(T)}/T = \hat{z}_{1,t/t+\tau} f_1(T) + \hat{z}_{2,t/t+\tau} f_2(T) + \hat{z}_{3,t/t+\tau} f_3(T)$$

where $\hat{z}_{i,t/t+\tau}$ are the forecasts of the i-th factor loading and f_1, f_2, f_3 are the factors. These factor loading forecasts can be computed by regressing the loadings at $t+h$ on the loadings at t. But our results improved for repeated 1-day forecasts. Hence we have used the second approach.

In figure 8 we show the autocorrelation functions of the residuals of the AR(1) models. The results for the Heston model indicate that the AR(1) models describe accurately the time series for the level and the slope factor loadings because only a few autocorrelations lie slightly outside the 95% confidence interval. The autocorrelation functions for the generalized Heston model show that the level and the slope are could be modeled differently. But we want to compare our results to Diebold and Li (2003) and they have also used AR(1) processes although they faced similar modelling problems. Finally, the loadings can be modelled quite well in the semiparametric factor model by AR(1) processes, only a few autocorrelations lie slightly outside the confidence bands.

3.4 Forecasting the variance swap curves

Besides a good in-sample fit models are judged by their dynamic properties. This is true notably at the moment for option pricing models because of the popularity of forward started options. In order to assess the quality of the models in this dynamic respect we perform out-of-sample forecasts.

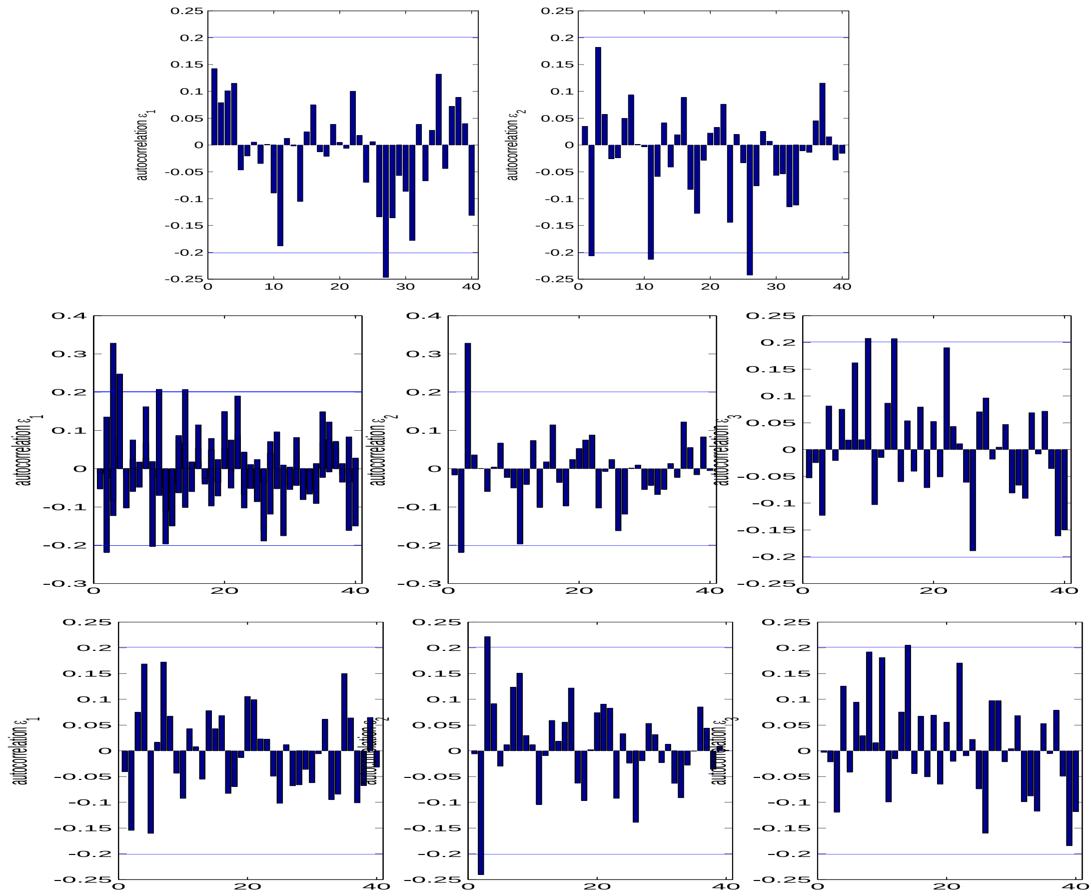

Figure 8: Factor loadings in the Heston model (up), in the generalized Heston model (middle) and in the semiparametric factor model (down).

As the models describe the variance swap curves by the factor loadings z, we forecasts the loadings $(\hat{z}_{1t}, \hat{z}_{2t}, \hat{z}_{3t})$. Our data set consists of observations from 2003 and 2004. We use the first part of the data for the estimation of the factor loadings and forecast the variance swap curves of the second year. In the semiparametric model, the factors are estimated from the data of the first year. Then we keep these factors fixed for the forecasting. Actually they differ only a bit from the factors estimated from the whole data (see figure 7). If we want to forecast at time t the variance curve at time $t+\tau$ then we use whole history of the factor loadings up to time t, i.e. $(z_{11}, z_{21}, z_{31}), \ldots, (z_{1t}, z_{2t}, z_{3t})$.

In tables 4 - 7 we show the results for 1 week, 1 month, 3 months and 6 months ahead forecasts. As we have only one years for forecasting we cannot consider longer periods. In addition to the three models considered so far we analyze two simple competitors, the static Heston model and the random walk. As before we consider the variance swap curves at the maturities $1.5, 3, 6, 9, 12, 18$ and 24 months.

The two benchmark models are:

- The static Heston model
  In the industry, the Heston model is usually applied without forecasting the parameters. On every day the model is calibrated to observed prices and other prices are calculated on the basis of these fixed parameters. The forecasts τ weeks ahead in this model are

  $$\widehat{V_{t+\tau}(T)}/T = \frac{V_t(T+\tau) - V_t(\tau)}{T} \tag{3}$$

  where V_t denotes the variance curve at time t.

  Forward started options depend on the model parameters at the time when the options really starts. Because of the popularity of such options the dynamic properties of models are important. The variance swap forecast 3 is the price of a forward started variance swap in the Heston model when the parameter are fixed. In this way, we analyze if the prices of such forward started options should be calculated with fixed parameters (static Heston) or if parameter forecasts ((dynamic) Heston) give more accurate prices.

- The random walk
  This model proposes that the variance swap curves do not change:

  $$\widehat{V_{t+\tau}(T)}/T = V_t(T)/T$$

This is a natural benchmark. Duffie and Kan (1996) show that it is difficult for yield curve models to give better forecasts than the random walk. Diebold and Li (2003) conclude that the reparametrized Nelson-Siegel outperforms the random for yield curves. The Nelson-Siegel model corresponds to the generalized Heston model.

The forecast errors at time $t+\tau$ are defined as the difference between the observed variance swap curve and the forecasted curve:

$$\widehat{V_{t+\tau}(T)}/T - V_{t+\tau}(T)/T$$

for $T = 1.5, 3, 6, 9, 12, 18$ or 24 months. We examine a number of descriptive statistics of the errors, including the mean absolute error:

$$MAE \stackrel{\text{def}}{=} \frac{1}{n}\sum_t \|\widehat{V_{t+\tau}(T)}/T - V_{t+\tau}(T)/T\|$$

and the mean absolute relative error:

$$MARE \stackrel{\text{def}}{=} \frac{1}{n}\sum_t \|\frac{\widehat{V_{t+\tau}(T)}/T - V_{t+\tau}(T)/T}{V_{t+\tau}(T)/T}\|$$

where the index t runs over all forecasting days and n is the number of forecasting days.

The results of the 1-week forecasts are presented in table 4. We see that the random walk model has the smallest errors of all models. Moreover, these errors are bigger for small maturities than for long maturities. The static Heston model can be regarded as the second best model but the Heston model with parameter forecasts has only slightly bigger errors. The three factor models have the worst performance where the Nelson-Siegel framework leads to bigger errors for long maturities. These results show that the short end of the variance swap curves are harder to forecasts. This corresponds to the in-sample fits reported in table 2 .

The 1-month-ahead forecast errors in table 5 show qualitatively similar results. The random walk model outperforms the other models with the highest errors for small maturities. The errors are on average twice as big as the errors in the 1-week ahead forecasts. The static and the dynamic Heston model have a similar performance while the three factor models lead again to the worst results. The semiparametric factor model outperforms as before the generalized Heston model.

In table 6, we describe the errors of the 3-months-ahead forecasts. The best results yields the random walk with relative deviation from the observed

Maturity (months)	Mean	Std.dev.	MAE	MARE
Heston				
1.5	0.14	0.27	0.24	16.8
3	-0.04	0.21	0.16	08.4
6	-0.02	0.21	0.15	07.2
9	0.03	0.21	0.15	06.8
12	0.07	0.21	0.16	07.0
18	0.08	0.21	0.17	06.6
24	0.04	0.20	0.16	05.7
generalized Heston				
1.5	0.19	0.23	0.24	17.1
3	0.06	0.20	0.16	08.6
6	0.13	0.20	0.18	09.0
9	0.20	0.20	0.23	10.4
12	0.22	0.20	0.25	10.7
18	0.19	0.19	0.22	08.8
24	0.10	0.19	0.17	06.2
static Heston				
1.5	0.17	0.25	0.25	17.0
3	-0.01	0.21	0.15	08.1
6	-0.00	0.20	0.15	06.9
9	0.04	0.20	0.15	06.7
12	0.07	0.20	0.16	06.8
18	0.08	0.20	0.16	06.4
24	0.03	0.20	0.15	05.5
semiparam. model				
1.5	0.32	0.26	0.35	24.4
3	0.04	0.20	0.16	08.6
6	0.03	0.20	0.15	07.3
9	0.10	0.21	0.18	07.9
12	0.06	0.20	0.16	06.8
18	0.07	0.20	0.16	06.3
24	0.04	0.19	0.15	05.4
random walk				
1.5	0.00	0.24	0.18	12.0
3	0.01	0.21	0.15	08.2
6	0.02	0.20	0.15	07.2
9	0.02	0.20	0.15	06.7
12	0.02	0.20	0.15	06.3
18	0.02	0.19	0.15	05.7
24	0.02	0.19	0.14	05.2

Table 4: Out-of-sample 1-week-ahead forecasting results [E^{-2}].

Maturity (months)	Mean	Std.dev.	MAE	MARE
Heston				
1.5	0.21	0.43	0.39	27.7
3	0.05	0.38	0.31	17.2
6	0.08	0.37	0.32	15.3
9	0.14	0.37	0.33	15.2
12	0.18	0.38	0.35	15.2
18	0.19	0.39	0.37	14.6
24	0.15	0.40	0.36	13.5
generalized Heston				
1.5	0.43	0.33	0.45	33.4
3	0.35	0.29	0.37	21.4
6	0.45	0.30	0.46	23.0
9	0.53	0.31	0.53	24.8
12	0.55	0.32	0.55	24.5
18	0.49	0.33	0.51	21.1
24	0.37	0.34	0.44	16.7
static Heston				
1.5	0.35	0.38	0.42	30.2
3	0.16	0.34	0.30	16.8
6	0.15	0.33	0.29	14.5
9	0.18	0.34	0.31	14.4
12	0.20	0.35	0.33	14.4
18	0.18	0.37	0.35	13.7
24	0.12	0.39	0.34	12.7
semiparam. model				
1.5	0.52	0.36	0.54	38.9
3	0.26	0.32	0.34	19.3
6	0.27	0.33	0.35	17.3
9	0.33	0.35	0.39	18.1
12	0.28	0.35	0.38	16.4
18	0.26	0.37	0.38	15.3
24	0.19	0.39	0.37	13.6
random walk				
1.5	0.01	0.38	0.32	20.9
3	0.04	0.33	0.27	14.8
6	0.05	0.33	0.27	13.2
9	0.06	0.33	0.28	12.7
12	0.06	0.34	0.29	12.4
18	0.06	0.36	0.30	12.0
24	0.07	0.37	0.32	11.6

Table 5: Out-of-sample 1-months-ahead forecasting results [E^{-2}].

Maturity (months)	Mean	Std.dev.	MAE	MARE
Heston				
1.5	0.42	0.55	0.60	43.2
3	0.27	0.51	0.48	27.6
6	0.30	0.53	0.51	25.8
9	0.36	0.55	0.57	26.8
12	0.40	0.57	0.61	27.3
18	0.40	0.62	0.65	26.7
24	0.36	0.65	0.65	24.9
generalized Heston				
1.5	0.73	0.33	0.73	54.3
3	0.68	0.31	0.68	39.6
6	0.80	0.33	0.80	41.0
9	0.87	0.36	0.87	41.8
12	0.88	0.38	0.88	40.3
18	0.80	0.41	0.80	33.9
24	0.65	0.43	0.69	27.5
static Heston				
1.5	0.82	0.53	0.87	64.5
3	0.60	0.49	0.67	39.1
6	0.53	0.52	0.63	32.0
9	0.51	0.55	0.64	30.5
12	0.49	0.59	0.66	29.5
18	0.42	0.65	0.66	27.0
24	0.33	0.69	0.64	24.3
semiparam. model				
1.5	0.82	0.40	0.83	60.9
3	0.58	0.38	0.61	35.8
6	0.61	0.43	0.64	32.9
9	0.68	0.46	0.71	34.1
12	0.63	0.49	0.68	31.1
18	0.59	0.53	0.67	28.5
24	0.51	0.57	0.65	25.5
random walk				
1.5	0.13	0.50	0.40	29.7
3	0.16	0.45	0.40	22.9
6	0.17	0.48	0.43	21.5
9	0.18	0.51	0.45	21.0
12	0.18	0.54	0.47	20.6
18	0.18	0.60	0.50	20.3
24	0.18	0.65	0.54	20.2

Table 6: Out-of-sample 3-months-ahead forecasting results [E^{-2}].

Maturity (months)	Mean	Std.dev.	MAE	MARE
Heston				
1.5	0.52	0.40	0.54	39.8
3	0.37	0.44	0.44	26.0
6	0.40	0.48	0.46	24.6
9	0.45	0.51	0.50	24.9
12	0.48	0.53	0.53	24.8
18	0.48	0.56	0.53	23.1
24	0.42	0.58	0.50	20.6
generalized Heston				
1.5	0.82	0.30	0.82	62.3
3	0.79	0.27	0.79	46.1
6	0.92	0.31	0.92	46.9
9	0.98	0.33	0.98	47.0
12	0.99	0.35	0.99	44.8
18	0.87	0.38	0.87	36.9
24	0.71	0.40	0.71	28.6
static Heston				
1.5	1.16	0.48	1.16	84.4
3	0.90	0.53	0.90	52.5
6	0.76	0.59	0.76	39.5
9	0.68	0.63	0.69	34.0
12	0.62	0.66	0.64	30.3
18	0.49	0.71	0.57	25.1
24	0.36	0.75	0.54	22.2
semiparam. model				
1.5	0.97	0.31	0.97	71.8
3	0.74	0.31	0.74	43.2
6	0.77	0.36	0.77	39.7
9	0.84	0.39	0.84	40.5
12	0.79	0.41	0.79	36.1
18	0.74	0.44	0.74	31.3
24	0.64	0.46	0.64	26.0
random walk				
1.5	0.18	0.35	0.33	24.0
3	0.21	0.38	0.32	18.9
6	0.21	0.47	0.36	18.9
9	0.20	0.52	0.38	18.9
12	0.18	0.55	0.40	18.8
18	0.14	0.62	0.44	19.1
24	0.11	0.67	0.48	19.2

Table 7: Out-of-sample 6-months-ahead forecasting results [E^{-2}].

curve vary between 20% and 30%. The Heston model with parameter forecasts gives better results than the static Heston model while the other two models perform badly.

Finally, we consider forecasts half a year ahead in table 7. For these long periods the dynamic Heston model performs quite good compared to the random walk that gives again the best forecasts. The static Heston model leads to rather big errors and the semiparametric model produces similar errors as the Nelson-Siegel approach.

4 Conclusion

We have analyzed the modeling and forecasting of variance swap curves. Reparametrizing the Heston model we consider a Nelson-Siegel framework with two factors, level and slope. Generalizing this approach we analyzed also the full Nelson-Siegel model with the three factors, level, slope and curvature. Moreover, we considered a three factor semiparametric model. We analyzed the in-sample and out-of-sample performance of these models and compared the results to two benchmark models: the random walk and the static Heston model that is often used in industry.

The in-sample fit (table 2) gives good results for long maturities but all models have problems in fitting the short end of the variance swap curves. The generalized Heston model naturally outperforms the Heston model and the flexible semiparametric factor model leads to the best fit for long maturities but also has significant problems with the short maturities. Actually, all models show some bias at the short end as is confirmed by the residuals (figure 6). Comparing these results to yield curve modeling we see that variance swap curves seem to be more difficult to model because the in-sample fits are worse. An explanation can be seen in the higher curvature of variance swap curves. The Nelson-Siegel model that leads to quite good fits for yield curves, see Diebold and Li (2003), has problems in modeling the short end of the variance swap curves. Moreover, variance swap curves lie in general below interest rate curves so that the higher absolute error gives an even bigger relative error.

We forecast the variance swap curves in these three models by forecasting the factor loadings. For the out-of-sample analysis we consider in addition two benchmark models: the random walk and the static Heston model. In the random walk the forecast is the curve observed today and in the static Heston model the forecast is computed without forecasting the parameters. None of the four models gives better results than the random walk. This corresponds the forecasting problems for yield curves described by Duffie and

Kan (1996). Thus we cannot confirm the results of Diebold and Li (2003) who conclude that the Nelson-Siegel framework outperforms the random walk in yield curve modeling for long forecasts. An explanation can be seen in the difficulties already encountered in the in-sample fit. The two factor models give better forecasts than the three factor approaches. The static Heston that is popular in industry produces worse forecasts than the dynamic Heston model for long periods. Hence, we can conclude that the parameters move significantly in the Heston model and this movements should be taken into account in order to model the dynamics of the model. Such dynamics are important for forward started options and should be priced into these products in the Heston model. The famous Nelson-Siegel approach for yield curves gives rather bad results for variance curves, the semiparametric factor model outperforms it even in the out-of-sample analysis.

We conclude that the modeling of variance swap curves is a challenging new topic because yield curve models show a bad performance for variance swap curves. We could not confirm the results of Diebold and Li (2003) in the out-of-sample analysis. These different results may be due to the different nature of the data, 15 years of monthly yield curves and 2 years of weekly variance swap curves. But we have seen that the Heston model gives better variance swap forecasts when the parameter are forecasted. This is essential for forward started options on realized variance. As the two factor models outperform the three factor models the results underline Zellner (1992)'s principle to keep it sophisticatedly simple. The bad performance of the Heston model for short maturities is well known so that future research could analyze other approaches (e.g. the model of Bates (1996)). Moreover, other forecasting techniques like moving windows could be considered in order to improve the forecasting results.

References

David S Bates. Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options. *Review of Financial Studies*, 9(1):69–107, 1996.

L. Bergomi. Smile dynamics. *RISK*, 17(9), 2004.

L. Bergomi. Smile dynamics ii. *RISK*, 18(10), 2005.

H. Bühler. Consistent variance curve models. *Finance and Stochastics*, 2006.

P. Carr and D. Madan. Towards a theory of volatility trading, 1998.

R. Cont and J. da Fonseca. Dynamics of implied volatility surfaces, 2001.

K. Demeterfi, E. Derman, M. Kamal, and J. Zou. More than you ever wanted to know about volatility swaps. Quantitative strategies research notes, Goldman Sachs, 1999.

Francis X. Diebold and Canlin Li. Forecasting the term structure of government bond yields. NBER Working Papers 10048, National Bureau of Economic Research, Inc, October 2003.

D. Duffie and R. Kan. A yield factor model of interest rates. *Mathematical Finance*, 6(4), 1996.

B. Dupire. Pricing with a smile. *Risk*, 7:327–43, 1994.

J. Fan and I. Gijbels. *Local Polynomial Modelling and Its Applications*. Chapman and Hall, London, 1996.

M. Fengler. *Semiparametric Modeling of Implied Volatility*. Springer, Berlin, 2005.

W. Härdle, M. Müller, S. Sperlich, and A. Werwatz. *Nonparametric and semiparametric models*. Springer, Heidelberg, 2004.

S. Heston. A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, 6 (2):327–43, 1993.

C. Nelson and A. Siegel. Parsimonious modeling of the yield curve. *Journal of Business*, 60:473–89, 1987.

A. Neuberger. Volatility trading, 1992.

A. Zellner. Statistics science and public policy. *Journal of the American Statistical Association*, 87, 1992.

SFB 649 Discussion Paper Series 2006

For a complete list of Discussion Papers published by the SFB 649, please visit http://sfb649.wiwi.hu-berlin.de.

001 "Calibration Risk for Exotic Options" by Kai Detlefsen and Wolfgang K. Härdle, January 2006.
002 "Calibration Design of Implied Volatility Surfaces" by Kai Detlefsen and Wolfgang K. Härdle, January 2006.
003 "On the Appropriateness of Inappropriate VaR Models" by Wolfgang Härdle, Zdeněk Hlávka and Gerhard Stahl, January 2006.
004 "Regional Labor Markets, Network Externalities and Migration: The Case of German Reunification" by Harald Uhlig, January/February 2006.
005 "British Interest Rate Convergence between the US and Europe: A Recursive Cointegration Analysis" by Enzo Weber, January 2006.
006 "A Combined Approach for Segment-Specific Analysis of Market Basket Data" by Yasemin Boztuğ and Thomas Reutterer, January 2006.
007 "Robust utility maximization in a stochastic factor model" by Daniel Hernández–Hernández and Alexander Schied, January 2006.
008 "Economic Growth of Agglomerations and Geographic Concentration of Industries - Evidence for Germany" by Kurt Geppert, Martin Gornig and Axel Werwatz, January 2006.
009 "Institutions, Bargaining Power and Labor Shares" by Benjamin Bental and Dominique Demougin, January 2006.
010 "Common Functional Principal Components" by Michal Benko, Wolfgang Härdle and Alois Kneip, Jauary 2006.
011 "VAR Modeling for Dynamic Semiparametric Factors of Volatility Strings" by Ralf Brüggemann, Wolfgang Härdle, Julius Mungo and Carsten Trenkler, February 2006.
012 "Bootstrapping Systems Cointegration Tests with a Prior Adjustment for Deterministic Terms" by Carsten Trenkler, February 2006.
013 "Penalties and Optimality in Financial Contracts: Taking Stock" by Michel A. Robe, Eva-Maria Steiger and Pierre-Armand Michel, February 2006.
014 "Core Labour Standards and FDI: Friends or Foes? The Case of Child Labour" by Sebastian Braun, February 2006.
015 "Graphical Data Representation in Bankruptcy Analysis" by Wolfgang Härdle, Rouslan Moro and Dorothea Schäfer, February 2006.
016 "Fiscal Policy Effects in the European Union" by Andreas Thams, February 2006.
017 "Estimation with the Nested Logit Model: Specifications and Software Particularities" by Nadja Silberhorn, Yasemin Boztuğ and Lutz Hildebrandt, March 2006.
018 "The Bologna Process: How student mobility affects multi-cultural skills and educational quality" by Lydia Mechtenberg and Roland Strausz, March 2006.
019 "Cheap Talk in the Classroom" by Lydia Mechtenberg, March 2006.
020 "Time Dependent Relative Risk Aversion" by Enzo Giacomini, Michael Handel and Wolfgang Härdle, March 2006.
021 "Finite Sample Properties of Impulse Response Intervals in SVECMs with Long-Run Identifying Restrictions" by Ralf Brüggemann, March 2006.
022 "Barrier Option Hedging under Constraints: A Viscosity Approach" by Imen Bentahar and Bruno Bouchard, March 2006.

SFB 649, Spandauer Straße 1, D-10178 Berlin
http://sfb649.wiwi.hu-berlin.de

This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

HUMBOLDT-UNIVERSITÄT · ZU BERLIN ·

023 "How Far Are We From The Slippery Slope? The Laffer Curve Revisited" by Mathias Trabandt and Harald Uhlig, April 2006.
024 "e-Learning Statistics – A Selective Review" by Wolfgang Härdle, Sigbert Klinke and Uwe Ziegenhagen, April 2006.
025 "Macroeconomic Regime Switches and Speculative Attacks" by Bartosz Maćkowiak, April 2006.
026 "External Shocks, U.S. Monetary Policy and Macroeconomic Fluctuations in Emerging Markets" by Bartosz Maćkowiak, April 2006.
027 "Institutional Competition, Political Process and Holdup" by Bruno Deffains and Dominique Demougin, April 2006.
028 "Technological Choice under Organizational Diseconomies of Scale" by Dominique Demougin and Anja Schöttner, April 2006.
029 "Tail Conditional Expectation for vector-valued Risks" by Imen Bentahar, April 2006.
030 "Approximate Solutions to Dynamic Models – Linear Methods" by Harald Uhlig, April 2006.
031 "Exploratory Graphics of a Financial Dataset" by Antony Unwin, Martin Theus and Wolfgang Härdle, April 2006.
032 "When did the 2001 recession *really* start?" by Jörg Polzehl, Vladimir Spokoiny and Cătălin Stărică, April 2006.
033 "Varying coefficient GARCH versus local constant volatility modeling. Comparison of the predictive power" by Jörg Polzehl and Vladimir Spokoiny, April 2006.
034 "Spectral calibration of exponential Lévy Models [1]" by Denis Belomestny and Markus Reiß, April 2006.
035 "Spectral calibration of exponential Lévy Models [2]" by Denis Belomestny and Markus Reiß, April 2006.
036 "Spatial aggregation of local likelihood estimates with applications to classification" by Denis Belomestny and Vladimir Spokoiny, April 2006.
037 "A jump-diffusion Libor model and its robust calibration" by Denis Belomestny and John Schoenmakers, April 2006.
038 "Adaptive Simulation Algorithms for Pricing American and Bermudan Options by Local Analysis of Financial Market" by Denis Belomestny and Grigori N. Milstein, April 2006.
039 "Macroeconomic Integration in Asia Pacific: Common Stochastic Trends and Business Cycle Coherence" by Enzo Weber, May 2006.
040 "In Search of Non-Gaussian Components of a High-Dimensional Distribution" by Gilles Blanchard, Motoaki Kawanabe, Masashi Sugiyama, Vladimir Spokoiny and Klaus-Robert Müller, May 2006.
041 "Forward and reverse representations for Markov chains" by Grigori N. Milstein, John G. M. Schoenmakers and Vladimir Spokoiny, May 2006.
042 "Discussion of 'The Source of Historical Economic Fluctuations: An Analysis using Long-Run Restrictions' by Neville Francis and Valerie A. Ramey" by Harald Uhlig, May 2006.
043 "An Iteration Procedure for Solving Integral Equations Related to Optimal Stopping Problems" by Denis Belomestny and Pavel V. Gapeev, May 2006.
044 "East Germany's Wage Gap: A non-parametric decomposition based on establishment characteristics" by Bernd Görzig, Martin Gornig and Axel Werwatz, May 2006.
045 "Firm Specific Wage Spread in Germany - Decomposition of regional differences in inter firm wage dispersion" by Bernd Görzig, Martin Gornig and Axel Werwatz, May 2006.

This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

046 "Produktdiversifizierung: Haben die ostdeutschen Unternehmen den Anschluss an den Westen geschafft? – Eine vergleichende Analyse mit Mikrodaten der amtlichen Statistik" by Bernd Görzig, Martin Gornig and Axel Werwatz, May 2006.
047 "The Division of Ownership in New Ventures" by Dominique Demougin and Oliver Fabel, June 2006.
048 "The Anglo-German Industrial Productivity Paradox, 1895-1938: A Restatement and a Possible Resolution" by Albrecht Ritschl, May 2006.
049 "The Influence of Information Costs on the Integration of Financial Markets: Northern Europe, 1350-1560" by Oliver Volckart, May 2006.
050 "Robust Econometrics" by Pavel Čížek and Wolfgang Härdle, June 2006.
051 "Regression methods in pricing American and Bermudan options using consumption processes" by Denis Belomestny, Grigori N. Milstein and Vladimir Spokoiny, July 2006.
052 "Forecasting the Term Structure of Variance Swaps" by Kai Detlefsen and Wolfgang Härdle, July 2006.

SFB 649, Spandauer Straße 1, D-10178 Berlin
http://sfb649.wiwi.hu-berlin.de

This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".