Macroeconomic Policy in a Heterogeneous Monetary Union

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Abstract
We use a two-country model with a central bank maximizing union-wide welfare and two fiscal authorities minimizing comparable, but slightly different country-wide losses. We analyze the rivalry between the three authorities in seven static games. Comparing a homogeneous with a heterogeneous monetary union, we find welfare losses to be significantly larger in the heterogeneous union. The best-performing scenarios are cooperation between all authorities and monetary leadership. Cooperation between the fiscal authorities is harmful to both the whole union’s and the country-specific welfare.

JEL-classification: E52, E61, F42
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1 Introduction

A country participating in a currency union has to abstain from sovereign monetary policy. A union-wide central bank conducts monetary policy for the whole currency area and cannot pay individual attention to every specific country in its decision-making. In contrast, national fiscal policies typically care about their single country and not the union as a whole. This gives rise to a variety of possible strategic behaviors: National fiscal policies can help monetary policy to maximize union-wide welfare (Gali and Monacelli 2002, 2005, Benigno 2004), they can try to adjust the outcomes of monetary policy to maximize nationwide welfare (Dixit 2001, Uhlig 2002), or they can be used to maximize the probability of the current government staying in office after the next elections (Beetsma and Uhlig 1999).

In this paper we merge these three strands of the literature. We propose a model that allows us to incorporate all three possibilities. We analyze monetary and fiscal policy interactions in a monetary union under various scenarios and elaborate which scenarios are preferable from a welfare perspective. We find that from the viewpoint of welfare maximization, joint cooperation of all policy makers produces the smallest losses. The second best scenario is one in which the monetary authority has a first-mover advantage. Cooperation between the fiscal authorities is harmful not only to union-wide welfare, but also to the welfare of each individual region. We demonstrate that the more asymmetric the regions, the larger the overall losses and the higher the relative gains from a first mover advantage of monetary policy.

The literature on monetary and fiscal policy in a monetary union is vast, so we only refer to articles of special importance for our paper. Dixit and Lambertini (2003b) consider monetary-fiscal policy interactions in a monetary union. They assume that the participating regions and their policy goals are symmetric and in line with the common central bank’s target. Accordingly, optimal output and inflation levels can be achieved – even without coordination of the fiscal authorities and the common central bank and without the need for monetary commitment. Dixit (2001), Dixit and Lambertini (2003a) and Lambertini (2004, 2006a) check the implications of this model for the case where monetary policy is conservative in the sense of Rogoff (1985). One of their major findings is that fiscal discretion destroys the positive effect of monetary commitment, while fiscal cooperation typically leads to less efficient outcomes than discretionary fiscal policies.

Lombardo and Sutherland (2004) construct a symmetric, two-country model that features government spending in the utility function. They find that the last
result can be overturned if the share of steady-state government spending in output is positive and supply shocks are not perfectly negatively correlated. Nonetheless, for plausible parameter values the welfare gains of fiscal cooperation are small.

Dixit and Lambertini (2001) allow for some heterogeneities by assuming that fiscal and monetary authorities may have conflicting output and inflation goals. They show that without commitment or leadership by either authority the ideal points of output and inflation cannot be attained.

Chari and Kehoe (2004) take a closer look at the desirability of fiscal debt constraints. They find that such constraints are undesirable if monetary commitment is possible, whereas the opposite holds if the central bank cannot commit to its policy. The latter is the result of a time-inconsistency problem of monetary policy, which leads to free-riding behavior by the fiscal authorities.

In the very recent literature, the topic of monetary and fiscal interactions has also been dealt with in dynamic, stochastic general-equilibrium models. However, the emphasis in most of these papers is not so much on strategic behavior and game-theoretical scenarios. Gali and Monacelli (2005) e.g. analyze optimal fiscal and monetary policies in a monetary union where all policy agents care about union-wide variables, and Ferrero (2005) considers a two region model and compares the optimal policies to simple policy rules, where all policy agents care about union-wide variables. Canzoneri et al. (2005) study the interactions between monetary and fiscal policy in a monetary union and compare the results of their New Keynesian model with the data. They also assess the effects of regional asymmetries on welfare, but they assume that fiscal policy is described by exogenously given processes for government spending and distortionary taxes. Lambertini (2006b) attempts to combine the game theoretical approach of the static models with features of dynamic models. To do so, she assumes that fiscal authorities can commit to their policies. Also, she assumes that government spending is exogenously given.

In a series of papers, van Aarle et al. (2001) and (2002), Engwerda et al. (2002) and Garretsen et al. (2005) focus on macroeconomic policy interactions of national fiscal policies and the monetary policy of a common central bank by using a New Keynesian framework. Of these papers, van Aarle et al. (2002) is the one most closely related to our model. They compare the outcomes of different scenarios by distinguishing between non-cooperation, partial cooperation, and full cooperation between monetary and fiscal policies. They find that the stability of coalitions depends strongly on the policy makers’ preferences. When the countries

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2As alternative specifications they consider fiscal policy rules making movements in the budget deficit lead to reactions either in government spending or in tax rates. In our model, by contrast, the government budget is always balanced.
are very heterogeneous, non-cooperative behavior is the most likely outcome.

In this paper we consider a two-country model with a single currency and one monetary policy conducted by a common central bank. Each country or region has its own fiscal policy authority that maximizes its objective function with the arguments of output and inflation. The equations of the basic model and the loss functions are derived from microfoundation by enhancing and modifying the Dixit and Lambertini (2003a) and (2003b) approach. Our contribution here is to accurately model the possibility of various differences between two countries in a heterogeneous monetary union.

As an application of the theory, the countries participating in the European Monetary Union (EMU) are far from being homogeneous. Both, the differentials of output growth and inflation dispersion, have been significant and rather persistent. The spread of the key macroeconomic indicators in the participating countries will presumably become even larger when the ten new EU member states adopt the Euro. Hence, it seems appropriate to incorporate those heterogeneities when analyzing the interactions of monetary and fiscal policies in a currency area like the Euro area.

We do this in two steps: First, we derive the output equation from a microfoundation and state that the terms of trade (i.e. inflation differentials) and a country-specific productivity shock both affect the region-specific output levels. Second, we take the view that national fiscal policies are concerned with national output and inflation targets, whereas they are not directly concerned with output growth and price changes in other parts of the union unless they decide to cooperate. As a simple illustration relating to the European Monetary Union (EMU), the Greek finance minister considers the current wage and house-price increases in Ireland not to be of major importance for his economy. Additionally, we assume that fiscal authorities have target rates for output and inflation that are higher than the welfare-optimal rates. Fiscal policy makers aim at reducing monopolistic distortions by granting production subsidies, i.e. we consider a supply side oriented fiscal policy. Monetary policy is assumed to aim at union-wide optimal rates in terms of welfare.

We analyze the fiscal policy makers’ and central bank’s losses in various scenarios: Policies can be conducted under discretion, simultaneously in the Nash scenario, or sequentially in Stackelberg leadership scenarios for each policy. Alternatively, policies can be coordinated between some or all authorities. We investigate the implications for output, inflation, and various policy loss functions in a numerical analysis, and show that the ranking of the scenarios is relatively robust across different degrees of heterogeneity.

We find that from the viewpoint of welfare maximization, joint cooperation between all policy makers and monetary leadership produce the smallest losses. Increasing the heterogeneities between the regions implies larger overall losses. Finally,
we show that the larger the heterogeneities, the higher the relative gains from a first mover advantage of monetary policy.

The remainder of the paper is structured as follows. Section 2 presents the model, Section 3 the various policy scenarios and Section 4 parameterization, evaluation method, results, and the sensitivity analysis. The final section concludes.

2 Model

We consider a general-equilibrium monetary model with monopolistic distortions and staggered prices. The model is closely related to Dixit and Lambertini (2003b) and Benigno (2004). In the economy, households derive utility from consumption and from holding real money balances. Each household, henceforth referred to as “producer-consumer”, produces a specific good and consumes a bundle of goods. There exists a continuum of consumption goods over the unit interval which are imperfect substitutes. There are two regions, home \( H \) and foreign \( F \), with the population on the segment \([0, n)\) belonging to the home region \( H \) and the remaining population belonging to the foreign region \( F \), with \( 0 \leq n \leq 1 \).

2.1 The Problem of a Producer-Consumer

A producer-consumer \( j \) in region \( i \in \{H, F\} \) derives utility

\[
U^j_i = \left( \frac{C^j_i}{\gamma} \right)^\gamma \left( \frac{M^j_i}{P_i} \right)^{1-\gamma} \left( d_i \right)^\frac{\beta}{\beta} (Y^j_i)^\beta, \quad \gamma \in (0, 1), \; d_i > 0, \; \beta \geq 1. \tag{1}
\]

The utility function depends on consumption, real money balances and labor. The producer-consumer derives positive utility from consumption of goods and from the stock of real money, while the parameter \( \gamma \) captures the elasticity of substitution between the two. Labor, which, for simplicity, is assumed to be a linear function of output and is, therefore, replaced by output itself, contributes negatively to the utility of agent \( j \). Here, \( 1 + \beta \) is the elasticity of the marginal disutility of labor. The stochastic variable \( d_i \) captures both the scaling of disutility of labor and the fluctuations in total factor productivity. Changes in this variable may be interpreted as changes in technology.\(^3\) The total consumption of agent \( j \) – who for reasons of

\[^3\]The two-country setting is taken from Benigno (2004). Other related models are Lombardo and Sutherland (2004), Ferrero (2005), and Gali and Monacelli (2005b). In general, our model can be traced back to the seminal work of Blanchard and Kiyotaki (1987).

\[^4\]To see this, assume a production function of \( Y^j_i = A_i N^j_i \) with total factor productivity \( A_i \) and hours \( N^j_i \). Then, rewrite the second summand in the utility function as \( \frac{d_i}{\beta} (N^j_i)^\beta \) with the help of
exposition is assumed to live in region $H$ – is given by

$$C^j = \frac{(C^j_H)^\nu (C^j_F)^{1-\nu}}{(\nu^H)^\nu (1-\nu^H)^{1-\nu^H}},$$

(2)

where $\nu^H$ is a preference shifter with $n \leq \nu^H \leq 1$ that allows for a home bias in consumption. We assume that both regions exhibit the same home bias, i.e. we henceforth use $\nu \equiv \nu^H = \nu^F$.

Consumption of goods from each region is given by

$$C^j_H = \left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n c^j(h)^{\theta-1} dh \right]^\frac{\theta}{\theta-1}, \quad C^j_F = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_0^1 c^j(f)^{\theta-1} df \right]^\frac{\theta}{\theta-1},$$

(3)

where $h$ is a generic good produced in region $H$, $f$ a generic good produced in region $F$, and $\theta > 1$ the elasticity of substitution between different goods in the same region. The elasticity of substitution of the home and foreign bundles of goods equals one. The corresponding consumer price indices – with subscripts denoting the place of production and superscripts denoting variables specific to agent $j$ or region $i$ – are

$$P^H \equiv (P^H_H)^\nu (P^H_H)^{1-\nu} \quad \text{and} \quad P^F \equiv (P^F_F)^\nu (P^F_H)^{1-\nu},$$

(4)

where

$$P^i_H \equiv \left[\frac{1}{n} \int_0^n p^i(h)^{1-\theta} dh \right]^\frac{1}{1-\theta} \quad \text{and} \quad P^i_F \equiv \left[\frac{1}{1-n} \int_0^1 p^i(f)^{1-\theta} df \right]^\frac{1}{1-\theta}$$

(5)

denote the market-price indices of goods consumed in region $i$ and produced in region $H$ and $F$, respectively. Note that the price index $P^H$ is defined as the minimum expenditure necessary for purchasing goods leading to a consumption index $C^j$ of size one, and the price indexes $P^i_H$ and $P^i_F$ are defined as the minimum expenditure

the definition $d_i \equiv \delta_i A^{-\beta}$, where $\delta_i$ captures the disutility of labor. In the welfare derivation we will define $d_i \equiv \delta_i \xi_i$, where for simplicity $\delta_i = 1$ and $\xi_i$ is a stochastic variable capturing technological progress.

5 For an agent $j$ living in region $F$, total consumption is given by $C^j \equiv \frac{(C^j_H)^\nu (C^j_F)^{1-\nu}}{(\nu^F)^\nu (1-\nu^F)^{1-\nu^F}}$ for all $j \in [n, 1]$.

6 To our knowledge, this model is the first two-region model of a monetary union that features the possibility of more than proportional demand for goods produced in the agent’s home economy.

7 The weights $(1/n)^{1/\theta}$ and $(1/(1-n))^{1/\theta}$ are a “normalization with the implication that an increase in the number of products does not affect marginal utility after optimization”. See Blanchard and Kiyotaki (1987), p. 649.

8 The same argument also holds for region $F$.  

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required to purchase goods resulting in consumption indexes $C_H^j$ and $C_F^j$, which equal one.

Although producers would have an incentive to set different prices across regions because of the home bias in consumption, we exclude this possibility by assuming that goods-market arbitrage leads to identical prices across borders such that $P_H^H = P_H^F$ and $P_F^H = P_F^F$.\footnote{In our theoretical model, inflation differentials occur due to the home-bias effect, as the composition of the consumption bundles differ in both regions. This assumption is somewhat critical when referring to the Euro-zone, where significant price differences for the same product exist in different countries (also for tradeable goods).} With output produced by agent $j$ in region $i$ denoted by $Y^j_i$, the budget constraint for this agent is

$$\int_0^n p^j(h)c^j(h)dh + \int_n^1 p^j(f)c^j(f)df + M^j_i = p^j(j)Y^j_i(1 - \tau_i) - P_iT_i + \bar{M}^j_i \equiv I^j_i.$$  \hfill (6)

The budget constraint guarantees that the sum of consumption expenditures plus money demand equals nominal net income $I^j_i$, which is the sum of sale revenues from the good produced and beginning-of-period money holdings minus net tax payments.

In each region, a government pursues its fiscal policy by making use of four instruments: a tax rate $\tau_i$ proportional to sales, real lump-sum taxes $T_i$, government consumption $G^i$, and wasteful government expenditures $X^i$. Government consumption of goods $G^i$ is defined symmetrically to private consumption, as given in equation (3). Sale taxes could also be negative with the interpretation of subsidies. Also, lump-sum transfers $T_i < 0$ are possible. For the two regional government budget constraints we have

$$\int_0^n p^H(j)y(j)\tau_H dj + nP_HT_H = \chi^H[\nu P^H G^H + (1 - \nu)P^F G^F] + (1 - \chi^H)X^H$$
$$\equiv I^g_H.$$  \hfill (7)

$$\int_n^1 p^F(j)y(j)\tau_F dj + (1 - n)P_FT_F = \chi^F[\nu P^F G^F + (1 - \nu)P^H G^H] + (1 - \chi^F)X^F$$
$$\equiv I^g_F.$$  \hfill (8)

Following DIXIT and LAMBERTINI (2003b) we assume that the government can spend its budget on government consumption $G^i$ or it can be wasted, $X^i$, ruled by the weight $\chi^i \in [0, 1]$.

\subsection*{2.2 Terms of Trade, Inflation and Output}

As set out before, the law of one price holds in the economy considered, i.e. $p^H(h) = p^F(h)$ and $p^H(f) = p^F(f)$. Nonetheless, agents appreciate consumption of domestically produced goods more. Hence, the (consumer) price index in the home region
\( P^H \) includes a larger share of domestic goods than the (consumer) price index in the foreign region \( P^F \). This implies non-trivial terms of trade, which we define as follows.

**Definition 1.** The terms of trade for region \( i \), \( S_i \), are given by the price of imports relative to the price of exports. Using \( "-i" \) to denote "not \( i \)"

\[
S_i \equiv \frac{P^i_{-i}}{P^i_i}.
\] (9)

Note that this notation is the reciprocal of the usual definition.\(^{10}\) Here, \( P^i_{-i} \) is the price level of goods produced in region \( -i \) and consumed in region \( i \), i.e. imports, whereas \( P^i_{i} \) is the price level of goods produced in region \( i \) and consumed in region \( -i \), i.e. exports. The following lemmata apply.

**Lemma 1.** The terms of trade are equal to the ratio of producer price indices.

\[
S_i = \frac{P^i_{-i}}{P^i_i}.
\] (10)

**Proof.** The equality holds as the rate of substitution between domestic goods is constant in both economies, so that the basket of domestically produced goods has the same composition in both economies, though not the same relative size. Therefore, a change in the price index of domestically produced goods has the same impact on e.g. \( P^H_H \), the price index of domestically produced goods consumed in the foreign region, and on \( P^H_H \), and we can drop the superscript.

Using the definitions of the consumer price indices given in equation (4), we can relate the terms of trade to the consumer price indices \( P^H \) and \( P^F \) and to the price indices of goods produced in each region, \( P_H \) and \( P_F \) as follows:

\[
\frac{P^H}{P_H} = (S_H)^{1-\nu}, \quad \frac{P^H}{P_F} = \frac{1}{(S_H)^{\nu}}, \quad \frac{P^F}{P_H} = (S_H)^{\nu} \quad \text{and} \quad \frac{P^F}{P_F} = \frac{1}{(S_H)^{1-\nu}}.
\] (11)

In the case of an identical home bias in both regions, which we are assuming here, the ratios of the two measures of inflation are inversely related to each other\(^{11}\) \( S_i = 1/S_{-i} \). Movements in the terms of trade imply movements in relative prices and, therefore, shift demand across the border.

A loglinear approximation to the model equilibrium is given by the following two propositions.

\(^{10}\)See e. g. Obstfeld and Rogoff (1996), p. 242. The notation is in line with the standard literature from the viewpoint of the foreign economy.

\(^{11}\)See Gali and Monacelli (2002) for a similar treatment in a small open economy setting.
Proposition 1. Inflation of region $i$ is a function of the deviations of the domestic and the foreign tax rate from its respective steady state. It is also dependent on actual and expected technology and real money balances as well as inflation expectations, all subsumed in the variable $\mu_i$:

$$\pi_i = \mu_i + c_i^d\hat{\tau}_i + c_i^{-}\hat{\tau}_{-i}, \quad i \in \{H, F\}. \quad (12)$$

**Proof.** See Appendix A, notably Section A.5.

The parameters $c^d_i$ and $c^{-}_i$ denote the impact of domestic and foreign fiscal policy on inflation, respectively. This equation states that regional PPI inflation can be explained as the outcome of influences from monetary policy and stochastic events, from fiscal policy of the same region and from fiscal policy of the other region.

Proposition 2. The deviation of region $i$’s output from its steady state is dependent on changes in the domestic as well as in the foreign tax rate, domestic surprise inflation, the terms of trade and changes in the productivity differential between the domestic and the foreign region, and is given by

$$y_i = \bar{y}_i + a^i_\tau \hat{\tau}_i + a^{i,-}_\tau \hat{\tau}_{-i} + b^i (\pi_i - \bar{\pi}_i) + \kappa^i s_i + \phi_i, \quad (13)$$

where $\bar{y}_i = 0$, $a^i \equiv \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} - \frac{1}{2(\beta-1)}\right)\bar{\tau}_i$ captures the effect of the home country’s fiscal policy instrument and $a^{i,-}_\tau \equiv -\left(\frac{1-\theta}{2[1+\theta(\beta-1)]} + \frac{1}{2(\beta-1)}\right)\bar{\tau}_{-i}$ the effect of foreign fiscal policy on domestic output.

**Proof.** See Appendix A, Section A.6.

Note that the steady-state level of taxes $\bar{\tau}_i$ is negative as will be shown in Section 4. Therefore, an expansionary fiscal policy is given if $\bar{\tau}_i < \bar{\pi}_i$, i.e., if $\hat{\tau}_i = \frac{\bar{\tau}_i - \bar{\pi}_i}{\bar{\tau}_i} > 0$. It is important to keep this in mind to follow the fiscal policy description in the Section 3. The effect of domestic surprise inflation on output is captured by $b^i \equiv \frac{2\beta\rho}{(\beta-1)(1-\rho)}$, with $\bar{\pi}_i = \bar{\pi}_i = E[\pi_i]$, whereas the effect of a surprise change in the terms of trade, $s_i$, is measured by $\kappa^i \equiv \frac{\beta\rho}{(\beta-1)(1-\rho)}$. The variable $\phi_i$ replaces the effects of both productivity shocks, as given by

$$\phi_i = \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} - \frac{1}{2(\beta-1)}\right)\hat{d}_i - \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} + \frac{1}{2(\beta-1)}\right)\hat{d}_{-i}.$$ 

Henceforth, $\phi_i$ is denoted as the “region-specific” output shock. In the following section we will focus attention on the equations given in Proposition 1 and 2, which summarize the microeconomic model.
3 Policy Analysis

3.1 Framework

We consider a region to be defined by a set of countries characterized by a high degree of homogeneity and exposed to similar shocks. Thus, fiscal policies within a specific region can be considered as being coordinated, as each region has to optimize a similar problem. Instead of home region $H$ and foreign region $F$, we will from now on denote the two regions as region $A$ and region $B$ to take a neutral point of view.

In the whole currency area, the population is given by a continuum of agents on the interval $[0,1]$, with $[0,n]$ living in region $A$ and $[n,1]$ in region $B$. The fiscal authority in region $i$ chooses a policy variable $\tau_i$, with $i = A, B$, where $\tau_i$ is a shortcut to $\hat{\tau}_i$, the notation used in the previous section. Fiscal policy affects national output, $y_i$, and national inflation, $\pi_i$, as well as union-wide output, $y$, and inflation, $\pi$. Union-wide variables are given by the weighted sum of the region-specific levels, where the weights of the regions are given by $n$ and $(1-n)$, respectively. In the following, we show the essential building blocks of our model:

Output Equation for Region $i$

Output in region $i$ was derived in the micro-model in Section 2 and is explicitly given by equation (A.53) in the Appendix. For convenience, we restate it here:

$$y_i = \bar{y}_i + a^i \tau_i + a^{ij} \tau_j + b^i (\pi_i - \pi_e^i) + \kappa^i s + \phi_i ,$$

(14)

where $j$ denotes “not region $i$”. According to KYDLAND and PRESCOTT (1979) and BARRO and GORDON (1983), surprise inflation may generate an increase in the national output level. Workers demand nominal wages that are sufficiently high to cover expected average future price increases. When the inflation rate reaches an unexpectedly high level, i.e. $\pi > \pi_e$, it leads ex post to lower real wages and increases employment and, thereby, output. Therefore, $b_i$ has a positive sign.

A higher $\tau_i$ corresponds to a more expansionary fiscal policy. It can be interpreted as subsidies granted by the fiscal policies to reduce the frictions stemming from monopolistic power. Additionally, fiscal policies have positive spill-over effects

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12 Alternatively, one region could capture one specific country of interest, while the other region refers to the remainder of the monetary union.

13 More precisely, $y_i$ denotes the percentage deviation of output from its steady state. Henceforth, we use “output” for reasons of brevity.

14 This interpretation of $\tau_i$ is in line with our microfoundation in Section 2 and is also typically used in New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models.
onto the other region. Therefore, for the supply-side fiscal policy considered both $a^i$ and $a^{ij}$ have a positive sign.

The term $\kappa^i s_i$ denotes the change in the current account, where $\kappa^i > 0$ and the terms of trade, $s_i$, from the perspective of region $i$, which is given by the log-linear approximation of equation (10):

$$s_i = (\pi_j - \pi_i).$$

(15)

We know from empirical studies that the terms of trade effect also depends on the region’s size. This means that a smaller region typically has a higher $\kappa^i$, implying that inflation differentials have a greater effect on output, something that is missing here.15 A higher inflation rate in region $j$ than in region $i$ corresponds to a real depreciation of region $i$ and thus increases its net exports. This shift of consumption from foreign goods (region $j$) to domestic goods (region $i$) increases domestic income.

Finally, a random shock $\phi_i$ enters the output equation, which is an i.i.d. shock with zero mean and a variance $\sigma^2_{\phi_i}$. In the microfounded model we show that this shock is the weighted sum of the deviations of the two regional (stationary) productivity processes from their respective steady states.

**Inflation Equation of Country $i$**

Inflation differences within the monetary union are caused by asymmetric shocks and country-specific fiscal policy actions. Thus, inflation in region $i$ evolves according to

$$\pi^i = \mu + c^i \tau_i + c^{ij} \tau_j,$$

(16)

as derived in Section 2 and before equation (A.48) in the appendix. The central bank influences a policy variable $\mu$, where we assume that monetary policy has the same impact on inflation in both regions.16 Analogously to DIXIT and LAMBERTINI (2003a), “$\mu$ stands for some actual policy variable such as the base money supply or a nominal interest rate, and determines a component of the price level,” (p. 1525). Therefore, a higher $\mu$ implies a more expansionary monetary policy.

The parameter $c^i$ refers to the influence of national fiscal policies on inflation, and $c^{ij}$ measures the spill-over effects on region $i$’s inflation stemming from foreign fiscal policy.

15Note that we implicitly assume that the intensity of trade inside the currency area is high enough for effects from outside the union to be neglected. Another possibility for eliminating outside effects is to assume that all regions within the monetary union have similar trade relations with the rest of the world, such that these are negligible for our results.

16In this context, ADÁO et al (2004) show that monetary policy cannot be used to offset idiosyncratic shocks within different countries belonging to a monetary union, as common monetary policy affects the monetary union as a whole.
Note that the parameters $c^i$ and $c^{ij}$ have a negative sign. Dixit and Lambertini (2003a) indicate that the sign of the parameters may become negative when tax cuts and subsidies raise the supply of goods and are at the same time financed by income taxes, which lead to a crowding out of private demand. This is in line with the microfounded model of Section 2. By contrast, a positive sign is likely to appear when fiscal policies are characterized by demand-side policies. This effect may be stronger if government expenditures are financed by distortionary production taxes reducing supply. We focus on supply-side fiscal policy. Accordingly, $c^i$ and $c^{ij}$, both, have a negative sign, but the absolute value of $c^i$ is higher than that of $c^{ij}$, i.e., direct effects from fiscal policies are stronger than the resulting spill-over effects to the other region.

Rational Expectations

The private sector has rational expectations about inflation, i.e. the following condition holds:

$$\pi_i^e = E(\pi_i).$$

(17)

Target Functions of Fiscal Authorities

Fiscal authorities minimize a quadratic loss function that aims at national inflation and national output. The functional form of the loss function is identical to that of regional welfare, derived in an appendix available from the authors upon request.

$$L_{Fi} = \frac{1}{2} \left[ (\pi_i - \pi_{Fi})^2 + \theta_{Fi}^i (y_i - y_{Fi})^2 \right].$$

(18)

Note that $\pi_{Fi}^i$ is the fiscal policy’s inflation target in region $i$, and $y_{Fi}^i$ is the desired output level of the fiscal authority in region $i$. According to the utility-based welfare criterion, these reference values should be equal to zero for inflation and to the flexible price output plus the steady state deviation from the efficient steady state in the case of output. If both fiscal authorities and the monetary authority agree on the targets, the first-best situation with the highest possible welfare can be obtained. This is demonstrated in Dixit and Lambertini (2003b) and corresponds to the joint cooperation case in our model, which will be introduced later.

However, EMU national governments and the ECB have often disagreed about the appropriate strategy for their policies. Therefore, we deviate from the microeconomic model by presuming that the fiscal targets deviate from the socially optimal level. More specifically, for inflation and output we assume target levels that are

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17With some simplifying assumptions, the optimal target for output is also zero.
both above the socially optimal levels. This may be justified by the fiscal policy makers’ desire to attain greater government size (cf. Fatas and Rose, 2001) or their incentive to maximize reelection probability (cf. Beetsma and Uhlig, 1999). To illustrate this, one can imagine that fiscal authorities are able to deceive their voters about the socially optimal targets, particularly during election campaigns. This would be especially true of a monetary union, where fiscal policy communicates with the domestic society, while monetary policy is centralized and concerned with the whole society of the monetary union. Accordingly, it communicates with the private sector of the individual regions from a greater distance.

Furthermore, the inflation and output targets of fiscal policies in both regions may differ. Economically intuitive reasons for considering different inflation targets on the part of the agents may be given (i) by home-bias effects in the consumption of goods, (ii) by different elasticities of substitution in the representative agents’ utility function across regions, or (iii) by different proportions of tradeable and non-tradeable goods in both regions. In our microeconomic model we have incorporated a home-bias effect in consumption and considered region-specific productivity shocks, which represent possible reasons for different fiscal targets in the two regions.

**Target Function of the Common Central Bank**

The common central bank is assumed to optimize the union-wide social welfare function\(^{18}\). Using a notation with indices \( M \) to denote monetary policy, we have

\[
L_M = \frac{1}{2} \left[ n \left( (\pi_A - \pi^A_M)^2 + \theta^A_M (y_A - y^A_M)^2 \right) \\
+ (1 - n) \left( (\pi_B - \pi^B_M)^2 + \theta^B_M (y_B - y^B_M)^2 \right) \right].
\]

(19)

In the case of excessive fiscal targets, as motivated above, we can state that the central bank is relatively conservative in comparison to fiscal policies, given by \( \pi^i_M < \pi^i_F \) and \( y^i_M < y^i_F \) for all \( i \). Our model differs in that respect from the approach of Dixit and Lambertini (2003b): They assume that fiscal policies act in a socially optimal manner and the central bank is too conservative, whereas we claim that the central bank maximizes union-wide welfare and fiscal policies act in too expansionary a way.

The different weights on output stabilization and the different output and inflation targets of monetary and fiscal policies give rise to trade-offs among policy makers. Whereas the fiscal authorities attach greater importance to output stabilization (and to pushing output and inflation above their natural levels), the common central bank sets a relatively higher weight on stabilization of inflation. These conflicting

\(^{18}\)The derivation is available from the authors upon request.
targets induce strategic behavior among the policy makers, which is examined in the following.

3.2 Scenarios of Simultaneous Decision-Making

In this subsection, we consider the scenario in which both fiscal authorities and the common central bank choose their optimal policies simultaneously. As the analytical results are dreadfully tedious, we restrict our policy analysis to a numerical examination undertaken in Section 4.

3.2.1 Nash Behavior

First, we consider the scenario of uncoordinated fiscal and monetary policies. The policy makers decide upon their optimal policies after having observed the realizations of the region-specific shocks. Thus, they take the households’ expectations on inflation as given. For better understanding, the sequence is depicted in Figure 1.

Figure 1: Time Structure for Simultaneous Decision-Making

Country A’s fiscal policy maker optimizes the social loss function (18) with respect to $\tau_A$, while taking the decision of the other region’s fiscal policy, $\tau_B$, and the policy choice of the common central bank, $\mu$, as given. Accordingly, country $B$ optimizes (18) with respect to $\tau_B$, while taking the policy choices of fiscal policy in country $A$ ($\tau_A$) and that of the common central bank ($\mu$) as given.

Simultaneously, monetary policy optimizes the union-wide social loss function (19), taking the fiscal policy actions and the expectations of the private sector as given.

3.2.2 Cooperation of Monetary and Fiscal Policies

According to many economists and politicians, coordination plays a crucial role. This is emphasized by the fact that regions and international organizations create institutions like the Stability and Growth Pact and aim at further common targets like tax harmonizations, which are only a few examples of coordination instruments. In this subsection, we analyze the scenario of coordination under discretion characterized by an agreement of the political authorities on common policy goals, i.e.
\[ \pi_F^A = \pi_F^B = \pi_M = \pi_{JC}, \ y_F^A = y_F^B = y_M = y_{JC} \text{ and } \theta_F^A = \theta_F^B = \theta_M = \theta_{JC}, \] where the subscript \( JC \) denotes the “joint cooperation” scenario. The timing of political decision-making corresponds to the Nash scenario and is illustrated in Figure [1]. We assume here, that the policy makers share a combined loss function of the following kind:

\[ L_{JC} = n \frac{1}{2} [ (\pi_A - \pi_{JC})^2 + \theta_{JC} (y_A - y_{JC})^2 ] \]

\[ + (1 - n) \frac{1}{2} [ (\pi_B - \pi_{JC})^2 + \theta_{JC} (y_B - y_{JC})^2 ] . \]

The minimizing problem follows the same pattern as in the Nash scenario, the only difference being that all authorities face the same loss function. We, implicitly, treat the joint cooperation case as if the policy makers were committed to the socially optimal targets, i.e. we assume that all policy makers aim at attaining the social optimum in this scenario and that the private sector is aware of that when forming its expectations about inflation. We do not incorporate possible deviations from this strategy, though this could be an interesting enhancement of this model. Thus, the first-best optimum for the private agents is attainable under joint cooperation. Dixit and Lambertini (2003b) use the same assumption in their model. We return to this point in Section [4].

3.2.3 Independent Monetary Policy and Cooperation between Fiscal Policies

If fiscal policy makers agree on cooperation while monetary policy acts independently, the fiscal authorities optimize a similar loss function as in the joint cooperation scenario. The loss function differs in the target values of inflation and output above the socially optimal levels. The fiscal objective function of both regions is given by

\[ L_{FC} = n \frac{1}{2} [ (\pi_A - \pi_{FC})^2 + \theta_{FC} (y_A - y_{FC})^2 ] \]

\[ + (1 - n) \frac{1}{2} [ (\pi_B - \pi_{FC})^2 + \theta_{FC} (y_B - y_{FC})^2 ] , \]

where the subscript \( FC \) denotes “fiscal cooperation”. The monetary authority optimizes the loss function (19). The solution is obtained analogously to the previous cases.

3.3 Scenarios of Sequential Decision-Making

The policy choices made by monetary and fiscal authorities may possibly take place at different times due to certain pre-scheduled rules, bureaucracy, or special intrinsic features of the political institutions. Therefore, we focus here on interactions
between fiscal and monetary policies when both authorities act sequentially. The evaluation of the different scenarios follows in Section 4.

3.3.1 Stackelberg Leadership of Fiscal Policy

We begin with the scenario of fiscal leadership, i.e. fiscal policy makers have to decide on their policy actions before monetary policy has been implemented and after having observed the realization of the regional shocks $\phi_i$. Thereby, they take the household’s inflation expectation as given. Beetsma and Bovenberg (1998) argue that fiscal leadership seems to be more likely when monetary policy can be implemented and adjusted more quickly than fiscal policy. This may be applicable when choices for taxes and subsidies are accompanied by bureaucratic and legislative processes that provide the fiscal authority with leadership over monetary policy. The sequence in that scenario is depicted in Figure 2.

The solution of the game is obtained by backward induction. Solving the monetary policy’s optimization problem at the second stage of the game leads to the optimal choice of $\mu$ while taking the fiscal policy variables $\tau_A$ and $\tau_B$ as given. In the first stage, the fiscal policy maker of region $i$ optimizes $\tau_i$ to react to the action taken by the policy maker of region $j$, $\tau_j$, and subject to the monetary reaction function, which is derived from the second stage of the game.

3.3.2 Stackelberg Leadership of Monetary Policy

In contrast to the previous case, monetary policy attains Stackelberg leadership over fiscal policies if it only affects the economy with a lag of time exceeding the legislative and bureaucratic time needed for fiscal policy decision-making. The timing is shown in Figure 3. The solution is similar to the former scenario of fiscal leadership. In the second stage, fiscal policy makers minimize the loss function (18) analogously to the Nash scenario shown above, given the other region’s fiscal policy and the monetary policy variable $\mu$. The common central bank chooses $\mu$ in the first stage, given the best responses of the fiscal policies $\tau_A$ and $\tau_B$. 

Figure 2: Time Structure for Sequential Decision-Making (Fisc. Leadership)
3.3.3 Fiscal Cooperation and Sequential Policy Actions

Analogously to the fiscal corporation scenario where the policy makers choose their optimal policies simultaneously, one can also assume coordination between national fiscal policies when the decision-making on monetary and fiscal policies takes place at different stages. The motivation for a coordinated fiscal policy in a sequential policy game corresponds to that of fiscal coordination in a simultaneous game. Accordingly, we also analyze scenarios (i) fiscal cooperation when fiscal policy moves first and (ii) fiscal cooperation when monetary policy moves first.

The time structure of scenario (i) corresponds to the one in Figure 2, while the time structure of scenario (ii) corresponds to that in Figure 3. The optimization problem under both scenarios follows the same pattern as in the corresponding sequential scenarios without coordination and are, therefore, omitted in this section.

4 Results

In the following we derive numerical results for the seven scenarios of strategic behavior between monetary and fiscal authorities introduced in the previous section.

We, first, describe the calibration of the model. Second, we show the evaluation methods used for the ranking of the different scenarios. Third, we run simulations for the case of a homogeneous and a heterogeneous monetary union by using the structural parameters from the microfounded model of Section 2. In this case, fiscal policy aims at granting production subsidies and levying per-capita taxes to reduce the distortions caused by monopoly power. We use the results from the homogeneous monetary union as a reference case and compare the rankings of different scenarios in the heterogeneous case. Fourth, we strengthen our results by using a sensitivity analysis of both the structural parameters and the policy targets.

4.1 Calibration

We calibrate the structural parameters of the model in accordance with the standard literature, as referred to in Dixit and Lambertini (2003a, Appendix F). The
elasticity of marginal disutility of labor is set at 0.45, a value proposed by Blanchard and Fischer (1989). This implies that the disutility parameter $\beta$, which is one plus the inverse of the elasticity of marginal disutility of labor, has the value $\beta = 3.22$. The Calvo-stickiness parameters $\Phi^H$ and $\Phi^F$ are set at a moderate value of 0.5, implying an average price to be fixed for three periods. The elasticity of substitution between goods of the same region is set at $\theta = 11$, as in Dixit and Lambertini (2003a). Obstfeld and Rogoff (2001) discuss the literature that has found values between 1 and 20. Note that the elasticity of substitution between goods of different regions is set to unity, as in Benigno (2004). In setting the steady state of the technology parameter as $d_i = 1$ and the subjective discount factor as $\eta = 0.98$ we strictly follow Dixit and Lambertini (2003a). The steady-state value for the fiscal policy instrument is assumed to be set optimally, i.e. to offset monopolistic distortion. Via $\tilde{\tau}_i = 1/(1 - \theta)$ we obtain a subsidy rate of ten percent for both regions in the steady state.

We look here at two different cases. In the first case, both regions have the same size ($n = 1 - n = 0.5$) and are completely symmetric, with identical structural parameters, identical fiscal policies, and no home bias ($\nu^H = \nu^F = 0.5$). In the second case, region $B$ accounts for only 30 percent of the union and displays more price rigidities. The latter assumption is based on the findings of Benigno and Lopez-Salido (2004). They estimate the price rigidity in five core EMU countries and identify substantial heterogeneities.

In the second case we presume that there is also a considerable home bias in consumption in both regions, thus following Anderson and van Wincoop (2003).

Given the values stated above, we can calculate the various parameters $a^i, b^i, c^i, \kappa^i$ in the model equations. Also, we can infer the values in the policy loss functions maximizing social welfare: In the symmetric case, these are target values for inflation and output, both equal zero, and a weight on output of $\theta^A_M = \theta^B_M = 0.00763$. In the asymmetric case, the output weight for region $B$ rises to $\theta^B_M = 0.01046$, while all other socially optimal target values remain the same.

As stated earlier, we assume that the common central bank sticks to these values, while the fiscal policy authorities may deviate from them. There may be various reasons for such deviation, for example systematic mismeasurement by the fiscal

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19 The authors discuss this parameter on page 341. Dixit and Lambertini (2003a) assume unit wage elasticity and thus less curvature.

20 The average price duration varies between around four quarters in the Netherlands and Germany and up to 17 quarters in Spain, implying price rigidity parameters between 0.75 and 0.94. We will choose numbers between 0.5 and 0.58, following the more conservative estimates of Bils and Klenow (2004). For a closer look at European data, the reader is referred to Dhyne et al. (2005).
Table 1: Calibration of the Baseline Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value*</th>
<th>Alternative*</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structural parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>0.50</td>
<td>0.70</td>
<td>Size of region A</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.50</td>
<td>0.80</td>
<td>Parameter capturing preference for home goods</td>
</tr>
<tr>
<td>$\beta$</td>
<td>3.22</td>
<td>3.22</td>
<td>One plus one over the elasticity of marginal disutility of labor</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>0.50</td>
<td>0.58</td>
<td>Fraction of firms that cannot adjust prices</td>
</tr>
<tr>
<td>$\theta$</td>
<td>11.00</td>
<td>11.00</td>
<td>Elasticity of substitution between goods</td>
</tr>
<tr>
<td>$d_i$</td>
<td>1.00</td>
<td>1.00</td>
<td>Technology parameter</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.98</td>
<td>0.98</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$\bar{\tau}_i$</td>
<td>-0.10</td>
<td>-0.10</td>
<td>Steady state value of taxes</td>
</tr>
<tr>
<td>Loss functions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta^i_M$</td>
<td>0.00736</td>
<td>0.01046</td>
<td>Central bank’s weighting factor for output gap</td>
</tr>
<tr>
<td>$\pi^i_M$</td>
<td>0.00</td>
<td>0.00</td>
<td>Inflation target of the central bank</td>
</tr>
<tr>
<td>$y^i_M$</td>
<td>0.00</td>
<td>0.00</td>
<td>Output gap target of the central bank</td>
</tr>
<tr>
<td>$\theta^i_F$</td>
<td>1.00</td>
<td>1.25</td>
<td>Fiscal policy’s weighting factor for output gap</td>
</tr>
<tr>
<td>$\pi^i_F$</td>
<td>0.02</td>
<td>0.03</td>
<td>Inflation target of fiscal policy</td>
</tr>
<tr>
<td>$y^i_F$</td>
<td>0.015</td>
<td>0.025</td>
<td>Output gap target of fiscal policy</td>
</tr>
</tbody>
</table>

Remarks: The term “Value” denotes the value chosen for both regions in the symmetric case and for region $A$ in the asymmetric case. “Alternative” denotes the value chosen for region $B$ in the asymmetric case. $i = A, B$.

Authorities or the fiscal authorities maximizing a different objective function they are able to conceal from the households. This was substantiated in Section 3. More particularly, we assume that the fiscal policy authorities put equal weight on output and inflation of unity. Furthermore, fiscal policies have higher target values for output $y^A_F = y^B_F = 0.015$ and inflation $\pi^A_F = \pi^B_F = 0.02$. In the asymmetric case, fiscal policy in region $B$ even puts a weight of $\theta^B_F = 1.25$ on output, sets its output target at $y^B_F = 0.025$ and its inflation target at $\pi^B_F = 0.03$, which could be seen as the result of its self-perception as a high-growth, catch-up region. Table 1 summarizes this calibration. As in Dixit and Lambertini (2003a), the stochastic term is calibrated to match the variance of output around its steady state as plus/minus six percent, as is the case for the U.S.

As set out in Section 3, we assume that the private sector has rational expectations about inflation. In our analytical calculations, we treat $\pi^A_A$ and $\pi^B_B$ as given. The inflation expectations of the private agents in both countries are determined...
in our model by iteration. In other words, we use an arbitrary starting value for
the inflation expectations in both countries and repeat the optimization calculations
until the inflation expectations differ from realized inflation by a value of less than
$10^{-10}$ for both countries, while keeping the shock at its expected value of zero. This
approach guarantees that $\pi_i^e = E(\pi_i)$ holds for $i = A, B$. After inflation expec-
tations are determined, we simulate our model by averaging over 100,000 random
draws of the stochastic processes.

4.2 Evaluation Method

The main purpose of our numerical approach is to rank the different scenarios of
strategic behavior of monetary and fiscal policies for the losses they induce. We
distinguish three approaches:

(i) Evaluation of the loss functions referring to the policy exercised by the fiscal
and monetary authorities. In each cooperation scenario, the corresponding
loss function is a compromise between the cooperating authorities.

(ii) Evaluation of the region-specific loss functions. In each cooperation scenario,
these are the region-specific loss functions the policy authorities would mini-
mize if they were not cooperating. This approach allows us to infer whether
cooperation scenarios are preferable for each participating policy authority.

(iii) Evaluation of social welfare. For each region, we calculate the welfare loss
that arises due to deviations in output and inflation from the socially optimal
values.

We show the losses involved in all three approaches in Table 2. In our discussion
we incorporate only the second and third approach. The reasoning behind this is
as follows: In approach (i), the losses of the three policy authorities are based on
the loss functions used in the optimization calculations. If the policy makers decide
to cooperate, they usually compromise on targets that differ from their own true
preferences. However, the “true losses” which the policy makers face are still based
on their specific preferences. Therefore, in approach (ii) we calculate the values
of the policy makers’ loss functions given by equations (18) and (19), irrespective
of the loss function used for optimization in the relevant scenarios. One should
also take these losses into account, when exploring whether joint cooperation among

21Note that by this definition the losses in case (ii) only differ from the losses in case (i) for the
joint cooperation scenario and the scenarios of fiscal cooperation.
all policy makers or cooperation between fiscal policy makers can take place on a voluntary basis.

The region-specific social welfare losses of approach (iii) are given by

\[
L_A = \frac{1}{2} \left( (\pi_A - \pi_M^A)^2 + \theta_M^A(y_A - y_M^A)^2 \right)
\]

\[
L_B = \frac{1}{2} \left( (\pi_B - \pi_M^B)^2 + \theta_M^B(y_B - y_M^B)^2 \right).
\]

Additionally, we express the region-specific social losses in terms of an equivalent reduction in region-specific consumption units, following the example of Lucas (2003). A scenario “performs best” when it shows the lowest reduction of consumption units compared to the consumption level in the social optimum. The calculation of the consumption-equivalent losses follows the approach of Adam and Billi (2005).

From our welfare derivation we know that for region \( A \)

\[
U^A = -\bar{Y}_A u_C L_A
\]

holds. To derive a relation between a permanent reduction of consumption (given by \( \delta_C^A \) percent) and the welfare loss, a second-order Taylor approximation of the utility loss is generated by

\[
U^A \approx \left( -\frac{u_C \bar{Y}_A \delta_C^A}{100} + u_{CC} \left( \frac{\bar{Y}_A \delta_C^A}{100} \right)^2 \right) = -u_C \bar{Y}_A \left( \frac{\delta_C^A}{100} - \frac{u_{CC} \bar{Y}_A}{u_C} \left( \frac{\delta_C^A}{100} \right)^2 \right)
\]

\[
= -u_C \bar{Y}_A \left( \frac{\delta_C^A}{100} + \frac{(1 - \gamma) \bar{Y}_A}{\bar{Y}_A} \left( \frac{\delta_C^A}{100} \right)^2 \right). \tag{23}
\]

Replacing \( \frac{U^A}{u_C \bar{Y}_A} \) by \( L_A \) yields

\[
L_A = \frac{(\delta_C^A)^2}{100^2} + \frac{\delta_C^A}{100}. \tag{24}
\]

To calculate the reduction of consumption equivalent to the social loss for region \( A \), we solve for \( \delta_C^A \) to obtain

\[
\delta_C^A = 100 \frac{-1 + \sqrt{1 + 4(1 - \gamma)L_A}}{2(1 - \gamma)}. \tag{25}
\]

The reduction of consumption equivalent to a certain welfare loss for region \( B \) can be obtained analogously. We use this transformation in the following subsections to make the welfare losses more tangible.

---

22Recall from Section 3 that the central bank is assumed to optimize the union-wide social loss, which is a region-sized weighted sum of the social losses of region \( A \) and \( B \).
4.3 Evaluation of Monetary and Fiscal Policies in the Different Policy Games

In the following we examine the results of the simulations. The model calibration was explained in Section 4.1 and is summarized in Table 1. A summary of the results is given in Table 2.

Homogeneous Monetary Union

We begin with a comparison of the losses for the monetary and fiscal policy authorities in the symmetric case. The first columns of Table 2 show that the fiscal authorities of both regions face the highest region-specific policy losses under cooperation and in the scenario where monetary policy moves first. The lowest fiscal losses occur when fiscal policies have the greatest influence, i.e., under the scenarios of fiscal cooperation when fiscal policies move first and under fiscal cooperation in the simultaneous scenario. The explanation is simple: Fiscal policies aim at higher inflation and higher output than the central bank, which targets socially optimal levels. Due to the low relative weight on output stabilization the central bank reacts strongly to offset inflation deviating from the socially optimum level. Fiscal policies themselves engage in a trade-off between inflation and output when fixing their own policy decisions. An expansionary fiscal policy pushes output above the socially optimal level by granting subsidies in order to lower production costs. Thus it decreases inflation at the same time. Accordingly, output is higher than natural output and lower than the desired fiscal targets. Inflation is below the fiscal target levels and slightly below the social optimum. Note, however, that the central bank reacts strongly to the downward pressure of inflation with an expansionary monetary policy on account of the high weight on inflation in the target function.

The loss in the Nash scenario is similar to that of the two scenarios where fiscal policies move first.

In the scenarios where monetary policy takes lead (with or without coordination of fiscal policies), fiscal policies internalize the fact that the central bank cannot offset a fiscal policy that is too expansionary. Therefore, fiscal policies are less expansionary, and output and inflation deviate from the fiscal targets to a higher degree than in the previously analyzed scenarios. This implies higher losses for the fiscal policy authorities. The highest losses occur when policy makers cooperate and agree on the socially optimal targets: On average, the realized value for inflation is close to zero (but still dependent on stochastics) and output is at its lowest compared to the desired levels. It is, therefore, questionable whether overall cooperation aiming at socially optimal targets can be implemented in this setting.
Table 2: Baseline Model – Analysis of Welfare and Policy Losses

<table>
<thead>
<tr>
<th>Policy</th>
<th>Symmetric case</th>
<th>Equivalent Consumption Reduction, %</th>
<th>Asymmetric case</th>
<th>Equivalent Consumption Reduction, %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated Policy Losses</td>
<td></td>
<td>Calculated Policy Losses</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$L_{FA}$</td>
<td>$L_{FB}$</td>
<td>$L_M$</td>
<td>$CR_A$</td>
</tr>
<tr>
<td>Nash</td>
<td>21.90936</td>
<td>21.90935</td>
<td>0.11895</td>
<td>0.012</td>
</tr>
<tr>
<td>Stackelberg, fiscal leadership</td>
<td>21.91073</td>
<td>21.91072</td>
<td>0.11582</td>
<td>0.012</td>
</tr>
<tr>
<td>Stackelberg, mon. leadership</td>
<td>23.63918</td>
<td>23.63917</td>
<td>0.01599</td>
<td>0.002</td>
</tr>
<tr>
<td>Cooperation</td>
<td>0.00001</td>
<td>0.00001</td>
<td>0.00001</td>
<td>0.000</td>
</tr>
<tr>
<td>... region-specific policy losses</td>
<td>31.25024</td>
<td>31.25020</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Fiscal coop., simultaneous</td>
<td>21.90926</td>
<td>21.90926</td>
<td>0.11848</td>
<td>0.012</td>
</tr>
<tr>
<td>... region-specific policy losses</td>
<td>21.90927</td>
<td>21.90926</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Fiscal coop., fiscal leadership</td>
<td>21.64560</td>
<td>21.64560</td>
<td>0.11056</td>
<td>0.011</td>
</tr>
<tr>
<td>... region-specific policy losses</td>
<td>21.64561</td>
<td>21.64559</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Fiscal coop., mon. leadership</td>
<td>31.24131</td>
<td>31.24131</td>
<td>0.00011</td>
<td>0.000</td>
</tr>
<tr>
<td>... region-specific policy losses</td>
<td>31.24143</td>
<td>31.24120</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Remarks: $L_{Fi}$ is fiscal loss in region $i$, $L_M$ loss of the common central bank, all multiplied by $10^5$. $CR_i$ denotes welfare loss measured in terms of an equivalent permanent percent reduction in consumption in region $i$. The numbers in parentheses denote standard deviations.
Our assumption of a welfare-maximizing monetary policy means that the rankings of the central bank losses correspond to the rankings of the union-wide social losses. The social losses, in turn, can be transformed into welfare equivalent consumption reductions relative to the social optimum. Accordingly, we consider only the consumption losses of the private agents in the following. We find that the ranking of the scenarios is quite different in comparison with the (fiscal) policy makers’ losses (see again Table 2). The first best can be attained in the cooperation scenario. The consumption loss is also very low in both monetary leadership scenarios, i.e. when fiscal policies do not cooperate and when fiscal policies are coordinated. The highest social losses occur when fiscal policies are dominant in the sense of being Stackelberg leaders, and in the Nash scenario. In line with the explanation for the fiscal policy makers’ losses, inflation and output levels are closest to the social optimum when monetary policy takes the lead (together, of course, with the joint cooperation case).

**Heterogeneous Monetary Union**

In our analysis of a heterogeneous monetary union we assume that the fiscal policy of region $A$ follows the same strategy as in the homogeneous case, whereas the fiscal policy of region $B$ targets higher levels of both inflation and output. Furthermore, we assume that region $B$ is smaller than region $A$ and is characterized by a slightly higher degree of price-stickiness. The exact parameter values for region $A$ are again depicted in the second column of Table 1, while the “alternative” parameter values for region $B$ are summarized in the third column of this table. Results for the heterogeneous case are shown in columns seven to eleven of Table 2.

Beginning with the losses for region $A$, we find that the values of the fiscal policy maker’s losses are much higher for all scenarios in the heterogeneous case except one: The cooperation scenario corresponds to the homogeneous case by definition, as all policy makers agree on the socially optimal targets. The ranking of the scenarios with respect to the region-specific fiscal policy makers’ losses is similar to that in the homogeneous case: The highest losses occur when monetary policy has the greatest influence (monetary leadership scenarios), the smallest losses occur in the scenarios in which fiscal policies have the greatest influence (fiscal cooperation when fiscal policy takes leadership, fiscal cooperation and simultaneous decision-making, and fiscal leadership when monetary policy is uncoordinated), and in the Nash scenario. The fiscal policy maker again faces the highest loss in the joint cooperation scenario. We observe almost the same ranking for region $B$, but the losses are higher compared to region $A$.

---

23 The (monetary) policy loss is slightly larger than zero because of the shock in our simulation.
We find that the losses of the common central bank and hence also the consumption losses of the private agents show also a similar ranking as in the homogeneous monetary union: The lowest losses are attained when monetary policy moves first or when all policy makers agree on the socially optimal targets (=first best). The highest losses occur when fiscal policies moves first (uncoordinated and coordinated) and when fiscal policies are coordinated and monetary and fiscal policy decisions take place simultaneously. This result seems, at first glance, to be contrary to the findings of Lombardo and Sutherland (2004), who state that fiscal cooperation is welfare-improving. But a closer look reveals that our calibration of a unit elasticity of substitution between domestic and foreign goods also implies in Lombardo and Sutherland (2004), according to their Proposition 1, that fiscal cooperation is no longer welfare-improving.

The welfare-equivalent consumption reductions under Nash, fiscal leadership, and the two fiscal cooperation scenarios with simultaneous actions or with fiscal leadership are about three times larger in the (smaller) region $B$. Also, the equivalent consumption reductions are relatively higher in the heterogeneous case compared to the homogeneous case, by about 50 percent for region $A$ and a factor of above four for region $B$. This implies that a model of a homogeneous monetary union that does not properly take into account heterogeneities possibly underestimates the welfare effects of certain policies. This finding also suggests that homogeneity is a desirable feature of the currency area for all policy makers (fiscal and monetary authorities) and the private agents.

### 4.4 Sensitivity Analysis

Are the results of the previous section robust to changes in the structural parameters of the model? To examine this, we vary the structural parameters within plausible ranges. In Figure 4 we plot the parameter variations that show the highest sensitivity of results. The corresponding parameters are the elasticity of marginal disutility of labor ($\text{emdl}$), price rigidity $\phi_i$, and the elasticity of substitution $\theta$. We plot their effects on fiscal policy makers’ losses and social welfare, which is equivalent to the central bank loss for both the symmetric and the asymmetric case.

\[ \text{24Note also that Lombardo and Sutherland (2004) features government consumption in the utility function.} \]

\[ \text{25In the figures we use the following abbreviations to save space: For the policy scenarios, Nash = Nash, Coop = cooperation, FCoo} \]

\[ \text{p} = \text{fiscal cooperation, FLead = fiscal leadership, MLead = monetary leadership, FCFL = fiscal cooperation with fiscal leadership, FCML = fiscal cooperation with monetary leadership. The labels on the x-axis denote emdl = elasticity of marginal disutility of labor, } \]

\[ \Phi = \text{Calvo parameter, i.e. the percentage of firms that cannot adjust their prices, and } \theta \]
Figure 4: Identical Parameter Variations in Region A and B

<table>
<thead>
<tr>
<th>Symmetric case</th>
<th>Asymmetric case</th>
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<tbody>
<tr>
<td>sym. $L_{FA}$</td>
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</table>

Diagram showing parameter variations for symmetrical and asymmetrical cases with different parameter labels.
Variation of the Elasticity of Marginal Disutility of Labor

We vary the elasticity of marginal disutility of labor (emd1) between zero and one, where the lower bound is given in Blanchard and Fischer (1989), while the upper bound is often used in New Keynesian models, see e.g. Gali and Monacelli (2005a). The effects of these variations on the policy losses in the three simultaneous scenarios are depicted in the first row of Figure 4, while the second row shows the effects in the four sequential scenarios.

Increasing elasticity of marginal disutility of labor leads to higher central bank losses. This result is obvious as, given the other parameters, the same outcome is produced at higher cost, meaning that the same effort in the production of goods leads now to a higher reduction of utility than before.

Referring to the homogeneous case, we see that the rankings for both the fiscal authorities’ losses and the central bank losses are stable: fiscal policies suffer from the smallest losses in the Nash scenario and if they obtain fiscal leadership, as in comparison with the other scenarios they are better able to pursue their inflation and output targets (above the socially optimal levels). The central bank’s welfare function shows the smallest losses in the joint cooperation case (which determines the first best) and in the scenario where monetary policy takes leadership. In the latter scenario, the fiscal policies are restrained, as too expansionary a fiscal policy would lead to low inflation, which will not be corrected by the central bank afterwards. Therefore, monetary leadership has a disciplining effect on supply-side-oriented fiscal policies. The fact that joint cooperation leads to the first best from a welfare perspective comes as no surprise as all policy makers agree upon the socially optimal targets, as mentioned in the previous section.

In the heterogeneous case, the losses are higher for the fiscal policies of both regions, the one with the more conservative and the one with the more aggressive targets, and also for the central bank. However, the rankings seem to be robust with two exceptions: (i) When monetary policy moves first fiscal losses are strongly increasing for higher values of the elasticity of disutility of labor. (ii) The losses in the fiscal cooperation fiscal leadership cases “explode” to a value of 0.4, which may be an indication that there is no equilibrium to which rational inflation expectations could converge. It would be interesting to take up this point in further research.

Variation of Price Rigidity

The third and fourth rows of Figure 4 examine the effect of varying price rigidities on fiscal and monetary losses. The figure shows that the ranking of the scenarios is

\[\text{elasticity of substitution between different goods produced in the same region.}\]
stable in the homogeneous and heterogeneous case for almost the whole parameter set, and it is in line with the results of Table 2. Fiscal policies incur the smallest loss under fiscal leadership, whereas monetary policy suffers from the smallest losses when it takes leadership and, of course, under the joint cooperation scenario. Again, the fiscal cooperation fiscal leadership scenario leads to dramatically increasing losses for more rigid prices, a factor that calls for an analysis in future research.

Variation of the Elasticity of Substitution of Consumption Goods

In the fifth and sixth rows of Figure 4 we consider the effect of changes from the elasticity of substitution of consumption goods, $\theta$, on the losses over the range discussed by Obstfeld and Rogoff (2001). The figure confirms one intuitive result, i.e. that an increasing $\theta$ leads to smaller fiscal policy and welfare losses: higher substitutability between goods implies fewer distortions from monopoly power. There is again one interesting exception. For a relatively small value of $\theta$ below 10 the losses explode, which again may conceivably induce indeterminacy of equilibria.

Summary of the Findings

For all parameter variations over the ranges used in the standard literature (see our calibration), we find that the rankings of the different scenarios illustrated by Table 2 are relatively robust. The sensitivity analysis has also confirmed that the losses in a heterogeneous monetary union tend to be higher. From the perspective of welfare maximization, joint cooperation and monetary leadership are the best-performing scenarios.

5 Conclusion

In this paper we have examined the interactions of fiscal and monetary policies in a monetary union. One main focus was to derive a theoretical model that allows for capturing heterogeneities among the different countries participating in a monetary union, and for analyzing strategic interactions of fiscal and monetary authorities. Why do heterogeneities matter? The answer is relatively simple. By adopting the Euro, the participating countries abstain from a monetary policy of their own and fiscal policy remains the only instrument for pursuing region-specific goals and

---

26 The variations of the intertemporal discount factor $\eta$, which determines the importance of “pseudo-future” periods relative to the present period in the producer-consumers price-setting behavior, show almost the same results as those indicated for variations of the price rigidity parameter. We, therefore, abstain from depicting and discussing the figures for $\eta$. 

27
stabilizing region-specific shocks. The common central bank has to implement a monetary policy that is most appropriate for the whole monetary union, and cannot respond to idiosyncratic shocks and country-specific political targets. This makes the role of fiscal policies more important and leaves room for strategic behavior in achieving national goals.

To examine these heterogeneities we have enhanced the model of Dixit and Lambertini (2003b). From the microfoundation we have established that a region-specific productivity shock and terms of trade have an impact on regional output. In Section 3 we introduced different possible scenarios for strategic interactions between fiscal and monetary policies. In this context we assumed that fiscal policies deviate from optimizing regional welfare, aiming instead at higher inflation and output compared to the union-wide central bank. By contrast, monetary policy is assumed to maximize union-wide welfare.

We have used simulations to evaluate the different scenarios of strategic behavior for supply-side fiscal policies in line with the micro-model. These aim at granting subsidies to increase output financed by per-capita taxes. We have thus considered a heterogeneous monetary union comprising two different regions: a “conservative region” and a “catch-up” region. We have assumed that the desired inflation and output targets of the “conservative region” are relatively closer to the social optimum.

To evaluate the different policy games, we have used a calibration of our micro-model drawing upon the parameters from the standard economic literature. We have shown that the losses of fiscal policies are relatively small in the Nash scenario, in the fiscal leadership scenario (for both cooperation of fiscal policies and independently acting fiscal policies), and when fiscal policies cooperate and all policy makers move simultaneously. In these scenarios, fiscal policies achieve an output level closest to their preferred levels, whereas inflation is stabilized close to the socially optimal level by the common central bank.

The losses of monetary policy, which correspond to the welfare losses of the private agents, are lowest when monetary policy moves first. The first-best situation is attained when all policy makers agree upon the socially optimal levels. But as the central bank and fiscal policy makers consider different scenarios optimal such, an agreement appears to be unrealistic on a voluntary basis.

In the EMU, fiscal policies appear primarily to track national interests. However, the analysis has shown that fiscal policies in a heterogeneous monetary union can contribute to high welfare losses. From a welfare perspective, monetary leadership or cooperation would then be a desirable scenario for both types of fiscal policy.

To summarize the main findings, we state that if the authorities’ preferences do not coincide, or are at least relatively far apart, worse outcomes are likely to occur.
In such a case, designing the institutions so that monetary policy plays a lead role generates the smallest losses for the agents living in both regions, even with existing heterogeneities.

The European Central Bank aggressively pursues the price-stability goal, meaning that the inflation rate should not exceed 2%. Accordingly, it appears to act as a first mover, which is beneficial for welfare. At the same time, fiscal policies are restricted in their actions by the Stability and Growth Pact, which leaves less room for pursuing excessive fiscal targets and implies a reduction of the trade-offs caused by strategic behavior. Recent experience, however, has shown that in bad times meeting the stability criteria may not be a very credible option for fiscal policies, especially, when the culprits judge their own sanctions, as it has happened in the European Union. Therefore, reducing heterogeneities and bringing fiscal policies’ targets closer to the socially optimal levels is an essential task in achieving a longer-term stability guarantee for the EMU.

References


Appendix: Details of the Microfounded Model

A.1 First Order Conditions and Aggregate Demand

Consumption maximization is done in two steps: first, suppose that $C^j_H$ is a single good instead of an aggregate. Then, utility maximization of agent $j$ in region $H$ subject to the corresponding aggregated budget constraint implies the two first-order conditions

$$\lambda_{BC} = \left( \frac{C^j}{\gamma} \right)^{\gamma-1} \left( \frac{M^j_i/P^H}{1-\gamma} \right)^{1-\gamma} \nu \frac{C^j}{P^H C^j_H}, \quad (A.1)$$

$$\lambda_{BC} = \left( \frac{C^j}{\gamma} \right)^{\gamma} \left( \frac{M^j_i/P^H}{1-\gamma} \right)^{-\gamma} \frac{1}{P^H}. \quad (A.2)$$

Equalizing the two equations by replacing the Lagrange multiplier $\lambda_{BC}$ and noting that $\frac{P_{ij} C^j}{\gamma} = \frac{M^j_i}{1-\gamma} = I^j_i$ leads to

$$C^j_H = \nu \left( \frac{P^H}{P^H} \right) C^j. \quad (A.3)$$

Second, maximizing $C^j_H$ with respect to two generic elements $c^j(h)$ and $c^j(h')$, subject to $\int_0^n P^j_i(h)c^j(h)dh = Z$, leads to

$$c^j(h) = \left( \frac{p^j_i(h)}{p^j_i(h')} \right)^{-\theta} c^j(h'). \quad (A.4)$$

Then, replacing $c^j(h)$ in equation (3) by the right-hand side of the previous equation gives

$$C^j_H = \left[ \left( \frac{1}{n} \right)^{\theta} \int_0^n \left( \frac{p^j_i(h)}{p^j_i(h')} \right)^{-\theta} c^j(h') \frac{n^\theta}{P^j_i} \right]^{\frac{\theta}{\theta-1}} \frac{n^\theta}{P^j_i},$$

which implies

$$c^j(h) = \frac{1}{n} \left( \frac{p^j_i(h)}{P^j_i} \right)^{-\theta} C^j_H. \quad (A.5)$$

27This is a result of the Cobb-Douglas structure of the utility function.
Adding steps one and two plus the symmetric results for the foreign good – for ease of exposition agent $j$ is still assumed to live in region $H$ – results in:

\[
c^j(h) = \frac{\nu}{n} \left( \frac{p^H(h)}{p^H_H} \right)^{-\theta} \frac{P^H_H}{P^H_H} C^j \quad \text{and} \quad c^j(f) = \frac{1 - \nu}{1 - n} \left( \frac{p^H(f)}{P^H_F} \right)^{-\theta} \frac{P^H_H}{P^H_F} C^j. \tag{A.6}
\]

We assume that government spending is subject to the same home bias as private consumption expenditures. This assumption lies between the extreme positions of no home bias in government expenditures, as proposed by Lombardo and Sutherland (2004), and complete home bias, as proposed by Beetsma and Jensen (2004), Benigno (2004) and Gali and Monacelli (2005a). The symmetric results for optimal expenditures of the home government are:

\[
g^H(h) = \frac{\nu}{n} \left( \frac{p^H(h)}{p^H_H} \right)^{-\theta} \frac{P^H_H}{P^H_H} G^H \quad \text{and} \quad g^H(f) = \frac{1 - \nu}{1 - n} \left( \frac{p^H(f)}{P^H_F} \right)^{-\theta} \frac{P^H_H}{P^H_F} G^H. \tag{A.7}
\]

Using the terms of trade and the fact that $C^j = \gamma I^j_i P_i$, we can rewrite the first-order condition of the producer-consumers with respect to their consumption of a single good and – in a similar manner – to their money holdings $M^j_i$ as:

\[
c^j(h) = \frac{\nu}{n} \left( \frac{p^H(h)}{P^H_H} \right)^{-\theta} \gamma I^j_i \frac{P^H_H}{P_H}, \tag{A.8}
\]

\[
c^j(f) = \frac{1 - \nu}{1 - n} \left( \frac{p^H(f)}{P^H_F} \right)^{-\theta} \gamma I^j_i \frac{P^H_F}{P_F}, \tag{A.9}
\]

\[
c^j(h) = \frac{1 - \nu}{n} \left( \frac{p^H(h)}{P^H_H} \right)^{-\theta} \gamma I^j_F \frac{P^H_H}{P_H}, \tag{A.10}
\]

\[
c^j(f) = \frac{\nu}{1 - n} \left( \frac{p^H(f)}{P^H_F} \right)^{-\theta} \gamma I^j_F \frac{P^H_F}{P_F}, \tag{A.11}
\]

\[
M^j_i = (1 - \gamma) I^j_i. \tag{A.12}
\]

The first two equations determine a home resident’s optimal choice of home and foreign goods, the next two equations determine the analog for a foreign resident, while the last equation shows the optimality condition with respect to money holdings.

\[\text{28} \text{An agent } j \text{ of region } F \text{ would demand } c^j(h) = \frac{1 - \nu}{n} \left( \frac{p^H(h)}{P^H_H} \right)^{-\theta} \frac{p^F_F}{P^H_F} C^j \text{ and } c^j(f) = \frac{\nu}{1 - n} \left( \frac{p^F(f)}{P^F_F} \right)^{-\theta} \frac{p^F_F}{P^H_F} C^j.\]

\[\text{29} \text{Our solution is in line with the comment by Leith (2004) alluded to by Lombardo and Sutherland (2004) in footnote 8. Gali and Monacelli (2005a) cite “evidence on a strong home bias in government procurement” in their footnote 8.}\]
Total nominal expenditure by consumers in region $H$ is $I_H = \int_0^1 I_H^t dj$, while in region $F$ it is $I_F = \int_0^1 I_F^t dj$. The demand function for a good $h$ is given by

$$Y^d(h) = \int_0^1 c^j(h) dj + \chi^H g^H(h) + \chi^F g^F(h)$$

$$= \left( \frac{p^H(h)}{P_H} \right)^{-\theta} \frac{1}{n} \times \left[ \gamma \nu I_H + (1-\nu) I_F + \nu^H \frac{p^H}{P_H} G^H + (1-\nu)^\chi^F \frac{P^F}{P_F} G^F \right]. \quad (A.13)$$

Similarly, the demand for a certain foreign good $f$ is given by

$$Y^d(f) = \int_0^1 c^j(f) dj + \chi^H g^H(f) + \chi^F g^F(f)$$

$$= \left( \frac{p^F(f)}{P_F} \right)^{-\theta} \frac{1}{1-n} \times \left[ \gamma (1-\nu) I_H + \nu I_F + \nu^F \frac{p^F}{P_F} G^F + (1-\nu)^\chi^F \frac{P^H}{P_F} G^F \right]. \quad (A.14)$$

Again, denoting “not $i$” by $-i$, we define a variable proportional to “wealth”:

$$W \equiv \gamma \nu I^i_H + (1-\nu) I^i_F - \nu \chi^i \frac{P^i}{P} G^i + (1-\nu)^\chi^i - \nu \chi^i \frac{P^i}{P} G^i. \quad (A.15)$$

At this point it is useful to note that this definition includes the terms of trade between domestic and foreign goods, as $I_i = \frac{P^i}{\gamma}$ measures the nominal consumption expenditures using the level of the consumer price index (CPI), while the denominator involves the level of the producer price index (PPI) as a reference. Using the identities from (11), one can easily transform this notation into one that includes real expenditures and the terms of trade $S$:

$$W = \begin{cases} \nu S^{-\nu} (\gamma \frac{I^H}{P^H} + \chi^H G^H) + (1-\nu) S^{\nu-1} (\gamma \frac{I^F}{P^F} + \chi^F G^F) & \text{if } i = H, \\ \nu S^{\nu-1} (\gamma \frac{I^F}{P^F} + \chi^F G^F) + (1-\nu) S^{-\nu} (\gamma \frac{I^H}{P^H} + \chi^H G^H) & \text{if } i = F. \end{cases}$$

To obtain a single equation for demand, we define the following weights:

$$w_i = \begin{cases} n & \text{if } i = H, \\ 1-n & \text{if } i = F. \end{cases}$$

Then, demand for a specific good $j$ from region $i$ amounts to

$$Y^d(j) = \left( \frac{p^i(j)}{P_i} \right)^{-\theta} W \frac{w_i}{w_i}. \quad (A.16)$$
Analogously to Benigno (2004), the smaller a region is (i.e. the higher the degree of openness), the larger the terms of trade effect will be on regional output (included in the $W$ term).

### A.2 Price Setting

When selling the product each producer is a monopolist. The producer, therefore, decides upon the price of the product by maximizing the indirect utility function. The indirect utility function is obtained by plugging $C_j = \gamma I_j$ and $M_j = (1 - \gamma)I_i$ into the utility function (1), replacing $I_j$ by the right-hand side of the budget constraint, replacing the price ratio with the help of equation (A.16), and simplifying:

$$U_j = (1 - \tau_i) \left( \frac{W}{w_i} \right) \left( Y_j \right) \frac{\theta - 1}{\theta} + \frac{\beta I_j}{P_i} \left( Y_j \right)^\beta. \quad \text{(A.17)}$$

The indirect utility function of agent $j$ is maximized with respect to the price $p_i(j)$, noting that the output produced by agent $j$ is equal to its demand, i.e. $Y_j = Y^d(j)$.

We obtain the optimal ratio of prices as

$$\left( \frac{p_i(j)}{P_i} \right) = \left( \frac{-d_i \theta \left( \frac{W}{w_i} \right)^{\beta - 1}}{\theta - 1 - \tau_i} \right)^{\frac{1}{\theta + \theta - 1}} \left( \frac{\theta d_i}{(\theta - 1)(1 - \tau_i)} \right)^{\frac{1}{\theta + \theta - 1}}. \quad \text{(A.18)}$$

Furthermore, we assume that some prices are fixed in advance, comparable to a static version of the staggered price-setting introduced by Calvo (1983). A fraction $\Phi_i$ of producers cannot change their prices and thus have to charge the same prices as in the past, whereas a fraction $(1 - \Phi_i)$ of producers are able to set their prices freely after the realization of the shocks in region $i$. The price level of goods from region $H$ is a weighted sum of the average of pre-set prices $E[\bar{p}_H^H(h)]$ and the newly set prices $\tilde{p}_H^H(h)$, which due to symmetry are equal for all producers. Based on equation (5), we obtain

$$P_{H}^{1-\theta} = \Phi_H(E\bar{p}_H^H(h))^{1-\theta} + (1 - \Phi_H)(\tilde{p}_H^H(h))^{1-\theta}. \quad \text{(A.19)}$$

---

30 Note that our demand functions are more complicated than the ones in Benigno because of the preference parameter $\nu$. This destroys the identity $P_H = P^F$ that holds in Benigno (2004) as long as $\nu_H \neq n$. If $\nu_H = \nu^F = n$ and $1 - \nu_H = 1 - \nu^F = 1 - n$, the consumer price indices of both regions are identical, and the demand functions become as simple as in Benigno.

31 As the decision of a single individual has only marginal impact on terms of trade and the price indices, this effect is neglected in the optimization.
For goods produced in region \( F \) the equivalent equation is

\[
P^{\theta}_F = \Phi^F (E \rho^F (f))^{1-\theta} + (1 - \Phi^F) (p^F (f))^{1-\theta}.
\]  

(A.20)

For convenience, the price ratio in region \( i \) may be defined to be

\[
\lambda_i \equiv \Phi^i \left( \frac{E \rho^i (j)}{P_i} \right)^{1-\theta} + (1 - \Phi^i) \left( \frac{\rho^i (j)}{P_i} \right)^{1-\theta} = 1.
\]  

(A.21)

In line with equation (4) the aggregate consumer price index in region \( i \) is given by

\[
P^H = \left[ \Phi^H (E \rho^H (h))^{1-\theta} + (1 - \Phi^H) (\rho^H (h))^{1-\theta} \right]^{1-\theta}
\]

\[
\times \left[ \Phi^F (E \rho^F (f))^{1-\theta} + (1 - \Phi^F) (\rho^F (f))^{1-\theta} \right]^{1-\theta}.
\]  

(A.22)

\[
P^F = \left[ \Phi^F (E \rho^F (f))^{1-\theta} + (1 - \Phi^F) (\rho^F (f))^{1-\theta} \right]^{1-\theta}
\]

\[
\times \left[ \Phi^H (E \rho^H (h))^{1-\theta} + (1 - \Phi^H) (\rho^H (h))^{1-\theta} \right]^{1-\theta}.
\]  

(A.23)

This can be written in terms of the overall price level\(^{32}\)

\[
P \equiv (P^H)^{n} (P^F)^{1-n}
\]

\[
= \left[ \Phi^H (E \rho^H (h))^{1-\theta} + (1 - \Phi^H) (\rho^H (h))^{1-\theta} \right]^{n\nu + (1-n)(1-\nu)}
\]

\[
\times \left[ \Phi^F (E \rho^F (f))^{1-\theta} + (1 - \Phi^F) (\rho^F (f))^{1-\theta} \right]^{n\theta + (1-n)\theta}.
\]  

(A.24)

### A.3 Aggregate Output and Fiscal Policy

Aggregate output in each region is defined by the following equations:

\[
Y_H \equiv \int_0^n H \rho^H (h) Y(h) \, dh \quad \text{and} \quad Y_F \equiv \int_n^1 F \rho^F (f) Y(f) \, df.
\]  

(A.25)

Using the demand functions (A.13) and (A.14) as well as the price index definitions (5), and denoting the lower and upper integral limits of each region \( i \) by \( lli \) and \( uli \), respectively\(^{33}\), aggregate output produced in region \( i \) can be rewritten as

\[
Y_i = \int_{lli}^{uli} \frac{\rho^i (j)}{P_i} \left( \frac{\rho^i (j)}{P_i} \right)^{-\theta} W \, dj = \left[ \int_{lli}^{uli} \left( \frac{p^i (j)}{P_i} \right)^{1-\theta} \right] \frac{W}{w_i}.
\]  

(A.26)

\(^{32}\)Note that the numerators of the exponents add up exactly to one.

\(^{33}\)I.e., \( lli = \begin{cases} 0 & \text{if } i = H; \\ n & \text{if } i = F; \end{cases} \text{ and } uli = \begin{cases} n & \text{if } i = H; \\ 1 & \text{if } i = F. \end{cases} \)
Essentially, this implies that the goods’ supply in region \( i \) is equal to its demand, which according to equation (A.15) originates from both regions. Total output is given as the geometric average of output in both regions:

\[
Y \equiv Y_H^n Y_F^{1-n}.
\]  

(A.27)

We specify fiscal policy as follows: Each fiscal authority uses per-capita taxes \( T_i \) to subsidize production, i.e., \( T_i > 0, \tau_i < 0 \). We assume for the moment, that there is no other government spending, i.e. \( \chi_i = X^i = G^i = 0 \). In this case, wealth \( W \) simplifies to

\[
W = \gamma \nu I_i + (1 - \nu) L_i \quad \Leftrightarrow \quad W = \gamma \frac{M}{P} \frac{1}{1 - \gamma \nu \Pi_i} - \gamma \frac{1 - \nu}{\Pi_i} S_i,
\]

where we assume identical beginning-of-period real money holdings for all agents \( \bar{M}/P = \bar{M}_i/P_i \) and for all \( i,j \). This fiscal policy uses distortionary taxation to offset market distortion due to monopolistic competition. Therefore, this type of fiscal policy is closest to the theoretical optimum. Nonetheless, our framework allows for various other fiscal policies.

A.4 Log-Linear Equilibrium Fluctuations: Price Setting

We log-linearize the model as follows: First, note that a linear approximation of equation (4) around \( P_i = P^i = P \) for all \( i \) results in

\[
\pi^H = \nu \pi_H + (1 - \nu) \pi_F \quad \text{and} \quad \pi^F = \nu \pi_F + (1 - \nu) \pi_H,
\]

(A.28)

where the inflation rates are defined as percentage deviations of the respective price level from its steady-state level\(^{36}\), i.e.

\[
\pi^i \equiv \log(P^i) - \log(\bar{P}^i), \quad \text{given} \ \bar{P}^i \neq 0.
\]

(A.29)

\(^{34}\)Without the assumption of internationally identical money holdings \( \bar{M}/P \) has to be replaced by \( [nM_i + (1 - n)M_{-i}]/P_i \).

\(^{35}\)Two alternative fiscal policies – with distortionary taxation that is either wasted or used for government spending – are analyzed in \textsc{Dixit} and \textsc{Lambertini} (2003b). In the first, \( \tau_i > 0, \chi_i = G^i = T_i = 0 \) and \( X^i > 0 \). In the second, \( \tau_i > 0 \) (as long as \( G^i > 0 \)), \( \chi_i = 1, T_i = 0 \). Analyzing the effects of these policies might be a useful topic for future research.

\(^{36}\)Under the assumption that \( \bar{P}^i \equiv 1 \), one can equivalently define \( \pi^i \equiv \log(P^i) \).
Then, equations (A.19) and (A.20) linearize\footnote{To appreciate this, compare the following procedure undertaken with a simplified, yet similar equation: \( P^b = \phi Q^b + (1 - \phi) R^b \Rightarrow \bar{P}^b e^{b\pi} = \phi \bar{Q}^b e^{b\pi} + (1 - \phi) \bar{R}^b e^{b\pi} \), which is approximately equal to \( \bar{P}^b (1 + b\pi) = \phi \bar{Q}^b (1 + b\pi) + (1 - \phi) \bar{R}^b (1 + b\pi) \Rightarrow b\pi = \frac{\phi}{\bar{P}^b} b\bar{\pi} + (1 - \phi) \frac{\bar{R}^b}{\bar{P}^b} b\bar{\pi} \). As the fractions are equal to unity, this simplifies to \( \pi = \phi \bar{\pi} + (1 - \phi) \bar{\pi} \).} to
\[
\pi_H = \Phi^H \bar{\pi}_H + (1 - \Phi^H) \bar{\pi}_H \quad \text{and} \quad \pi_F = \Phi^F \bar{\pi}_F + (1 - \Phi^F) \bar{\pi}_F. \tag{A.30}
\]
Combining the results gives
\[
\pi_H = \nu(\Phi^H \bar{\pi}_H + (1 - \Phi^H) \bar{\pi}_H) + (1 - \nu)(\Phi^F \bar{\pi}_F + (1 - \Phi^F) \bar{\pi}_F) \tag{A.31}
\]
\[
\pi_F = \nu(\Phi^F \bar{\pi}_F + (1 - \Phi^F) \bar{\pi}_F) + (1 - \nu)(\Phi^H \bar{\pi}_H + (1 - \Phi^H) \bar{\pi}_H). \tag{A.32}
\]
Now, we turn to the optimal price a producer would set if he could choose the price freely. According to Dixit and Lambertini (2003a), we refer to the idea of Calvo-staggered pricing, which reflects a dynamic setting (for details see Calvo, 1983). Analogously to the procedure proposed by Dixit and Lambertini, we introduce a discount factor \( \eta \) with \( \eta < 1 \) (which means that pseudo-future period utilities have a lower weight than present utility). We, first, assume that \( \eta \) equals unity to explain the “intuitive proceeding”. In the case where prices are allowed to change, the optimal log price equals
\[
\bar{\pi}_H = (1 - \Phi^H) \pi^j_H + \Phi^H \bar{\pi}_H, \tag{A.33}
\]
\[
\bar{\pi}_F = (1 - \Phi^F) \pi^j_F + \Phi^F \bar{\pi}_F, \tag{A.34}
\]
where \( \pi^j_i \) is the log steady-state deviation of the price that would be optimal if prices could be adjusted freely. The log price set by producer \( j \) is a sum of the weighted optimal price of producer \( j \), if prices were fully flexible, and the weighted price that maximizes the expected indirect utility, if prices are to be fixed in future periods. The weights equal the probability of being able, \( (1 - \Phi^i) \), or not being able, \( \Phi^i \), to change the price in the following period(s).

Now we come back to the discount factor \( \eta < 1 \). As already mentioned, the individuals place lower weight on future utilities. Therefore, the fact that the producer cannot change the price in future periods with a certain probability is expressed by a lower weight than the pure probability of future price setting (given by \( \eta \Phi^i \)) and a higher weight for the present period \( (1 - \eta \Phi^i) \). Hence, we obtain
\[
\bar{\pi}_H = (1 - \Phi^H \eta) \pi^j_H + \Phi^H \eta \bar{\pi}_H, \tag{A.33}
\]
\[
\bar{\pi}_F = (1 - \Phi^F \eta) \pi^j_F + \Phi^F \eta \bar{\pi}_F. \tag{A.34}
\]
In the case of $\eta = 0$, this setting would be purely static: Here, the (deviation from the steady state of the) optimal price once an individual is allowed to change price $\tilde{\pi}_i$ is identical to the price that is optimal for the current period only, as there are no future periods to form expectations about.

Using equations (A.33) and (A.34) to replace the optimal prices in the consumer price indices (A.31) and (A.32) gives

$$\pi^H = \nu \Phi^H [1 + (1 - \Phi^H) \eta] \tilde{\pi}_H + \nu (1 - \Phi^H) (1 - \Phi^H \eta) \pi^j_H$$

$$\pi^F = \nu \Phi^F [1 + (1 - \Phi^F) \eta] \tilde{\pi}_F + \nu (1 - \Phi^F) (1 - \Phi^F \eta) \pi^j_F$$  \hspace{1cm} (A.35)

The overall inflation rate can be calculated by using the previous equations together with equation (A.24):

$$\pi = n \pi^H + (1 - n) \pi^F$$  \hspace{1cm} (A.37)

$$= [n \nu + (1 - n)(1 - \nu)] \tilde{\pi}_H + [n(1 - \nu) + (1 - n)\nu] \pi_F.$$  \hspace{1cm} (A.38)

Equation (A.37) states that union-wide inflation is the sum of the regional CPI inflation weighted by the size of each region. The second equation (A.38) links union-wide inflation to the PPI inflation rates in each region, where the influence of regional PPI inflation depends on both the size of the region and the preference of agents for goods from that region.

### A.5 Proof of Proposition 1: Inflation Determination

In general, a producer sets its price by maximizing the indirect utility function which results in equation (A.18) above. A log-linear approximation of this equation around the steady state, solved for the relative deviation of wealth from its steady state level, $\hat{W}$, is

$$\hat{W} = \frac{1 + \theta (\beta - 1)}{\beta - 1} (\bar{p}^j(j) - \pi_i) - \frac{1}{\beta - 1} \hat{d}_i - \frac{\bar{\tau}_i}{\beta - 1} \hat{\tau}_i,$$  \hspace{1cm} (A.39)

where $\pi_i \equiv \hat{P}_i$, and a “hat” above a variable denotes percentage deviations of the variable from its steady state.\footnote{For the approximation of the fiscal policy term, note that $(1 - \tau_i) = \frac{\bar{\tau}_i}{1 - \tau_i} \hat{\tau}_i.$} To replace $\hat{W}$ in the last expression, we log-linearize the policy dependent wealth equation.
For the fiscal policy considered here, we use equation (A.28), and obtain the result
\[ \hat{W} = \frac{\gamma \bar{m}}{\omega} \hat{m} + \frac{\gamma(1 - \nu)}{\omega w_{-i}} s_i, \]  
(40)
where \( \omega \) is given by \( \omega \equiv 1 - \gamma \left[ \frac{w_i}{w_{-i}} + \frac{1 - \nu}{w_{-i}} \right] \) and \( s_i \equiv \hat{S}_i = \pi_i - \pi_i \). \( \hat{m} = \hat{M}/\hat{P} \) is the change in the beginning-of-period real money holdings.

In the next step, equation (A.39) – with \( \hat{W} \) replaced by the fiscal-policy-dependent equation – is evaluated at both \( E[\hat{p}(j)] \equiv \bar{\pi}_i \), the (log deviation of the) price that maximizes the future indirect utility, and at \( \hat{p}_i^j \equiv \pi_i^j \), the (log deviation of the) price that maximizes the current period indirect utility. Starting with the first case \( \bar{\pi}_i \), we obtain
\[
\bar{\pi}_i = E[\pi_i] + \frac{1}{1 + \theta(\beta - 1)} E[\hat{d}_i] + \frac{\bar{\tau}_i}{1 + \theta(\beta - 1)} E[\hat{\tau}_i] \\
+ \frac{\beta - 1}{1 + \theta(\beta - 1)} E \left[ \frac{\gamma \bar{m}}{\omega} \hat{m} + \frac{\gamma(1 - \nu)}{\omega w_{-i}} s_i \right] \\
= \bar{\omega}_{0,i} + \omega_1 E[\hat{\tau}_i] + \omega_2 E[\hat{\tau}_{-i}] + \omega_3 E[\bar{\pi}_i] + \omega_4 E[\pi_{-i}],
\]  
(41)
where \( \bar{\omega}_{0,i} \equiv \frac{1}{1 + \theta(\beta - 1)} E[\hat{d}_i] + \frac{\beta - 1}{1 + \theta(\beta - 1)} \frac{\gamma \bar{m}}{\omega} E[\hat{m}], \) \( \omega_1 \equiv \frac{\bar{\tau}_i}{1 + \theta(\beta - 1)}, \) \( \omega_2 \equiv 0, \) \( \omega_3 \equiv \left( 1 - \frac{\beta - 1}{1 + \theta(\beta - 1)} \frac{\gamma(1 - \nu)}{\omega w_{-i}} \right) \) and \( \omega_4 \equiv \frac{\beta - 1}{1 + \theta(\beta - 1)} \frac{\gamma(1 - \nu)}{\omega w_{-i}} \) \(^{39} \) Note that \( s_i \) has been replaced by terms of \( \pi_i \) and \( \pi_{-i} \). Accordingly, for the price that maximizes the current period indirect utility only, we obtain
\[
\pi_i^j = \frac{1}{1 + \theta(\beta - 1)} \hat{d}_i + \frac{\beta - 1}{1 + \theta(\beta - 1)} \frac{\gamma \bar{m}}{\omega} \hat{m} + \frac{\bar{\tau}_i}{1 + \theta(\beta - 1)} \hat{\tau}_i \\
+ \left( 1 - \frac{\beta - 1}{1 + \theta(\beta - 1)} \frac{\gamma(1 - \nu)}{\omega w_{-i}} \right) \pi_i + \frac{\beta - 1}{1 + \theta(\beta - 1)} \frac{\gamma(1 - \nu)}{\omega w_{-i}} \pi_{-i},
\]  
(42)
(43)
Using equations (A.30), (A.33) and (A.34), we obtain an equation that expresses the regional producer inflation rate in terms of the log of the price that maximizes the future indirect utility and the prize that maximizes the current period indirect utility only:
\[ \pi_i = \rho^j \bar{\pi}_i + (1 - \rho^j) \pi_i^j, \quad \rho^j = \Phi^j [1 + (1 - \Phi^j) \eta]. \]  
(44)
Henceforth, we will neglect the superscript \( i \) for the parameter \( \rho \) for reasons of clarity, because the results derived in the following have exactly the same structure for both regions.

\(^{39}\)We add the term \( \omega_2 \) to show that under alternative fiscal policies this spillover effect can be non-zero.
We use (A.44) and combine the two log prices in equations (A.41) and (A.43):

\[ \pi_i = \rho [\omega_{0,i} + \omega_1 E[\tilde{\tau}_i] + \omega_2 E[\tilde{\tau}_i] + \omega_3 E[\pi_i] + \omega_4 E[\pi_i]] \\
+ (1 - \rho) [\omega_{0,i} + \omega_1 \tilde{\tau}_i + \omega_2 \tilde{\tau}_i + \omega_3 \pi_i + \omega_4 \pi_i], \quad \text{(A.45)} \]

For the other region, analog steps yield

\[ \pi_{-i} = \rho [\omega_{0,-i} + \omega_1 E[\tilde{\tau}_{-i}] + \omega_2 E[\tilde{\tau}_{-i}] + \omega_3 E[\pi_{-i}] + \omega_4 E[\pi_{-i}]] \\
+ (1 - \rho) [\omega_{0,-i} + \omega_1 \tilde{\tau}_{-i} + \omega_2 \tilde{\tau}_{-i} + \omega_3 \pi_{-i} + \omega_4 \pi_{-i}], \quad \text{(A.46)} \]

where \( \omega_{0,-i} \) differs only from \( \omega_{0,i} \) by the stochastic disutility of labor variable \( \hat{d}_{-i} \) instead of \( d_i \).

Combining (A.45) and (A.46) and solving this system of equations for the region-specific inflation rates, one gets

\[ \pi_i = \Omega \rho \left[ \omega_{0,i} + \frac{(1 - \rho) \omega_4}{1 - (1 - \rho) \omega_3} \omega_{0,-i} \right] \\
+ \Omega \rho \left[ \left( \frac{(1 - \rho) \omega_4}{1 - (1 - \rho) \omega_3} \right) E[\tilde{\tau}_i] + \left( \frac{(1 - \rho) \omega_4}{1 - (1 - \rho) \omega_3} \right) E[\tilde{\tau}_{-i}] \right] \\
+ \Omega \rho \left[ \left( \frac{(1 - \rho) \omega_4}{1 - (1 - \rho) \omega_3} \right) E[\pi_i] + \left( \frac{(1 - \rho) \omega_4}{1 - (1 - \rho) \omega_3} \right) E[\pi_{-i}] \right] \\
+ \Omega (1 - \rho) \left[ \omega_{0,i} + \frac{(1 - \rho) \omega_4}{1 - (1 - \rho) \omega_3} \omega_{0,-i} \right] \\
+ \Omega (1 - \rho) \left( \frac{(1 - \rho) \omega_4}{1 - (1 - \rho) \omega_3} \right) \tilde{\tau}_i \\
+ \Omega (1 - \rho) \left( \frac{(1 - \rho) \omega_4}{1 - (1 - \rho) \omega_3} \right) \tilde{\tau}_{-i}, \quad i \in \{H, F\}, \quad \text{(A.47)} \]

\[ \pi_i = \mu_i + c^i \tilde{\tau}_i + c^{-i} \tilde{\tau}_{-i}, \quad i \in \{H, F\}, \quad \text{(A.48)} \]

with \( \Omega \equiv \frac{1 - (1 - \rho) \omega_3}{(1 - (1 - \rho) \omega_3)^2 - (1 - \rho) \omega_4^2} \). Under the supply-side fiscal policy introduced above, \( \omega_2 = 0 \) we have\(^{40}\)

\[ \mu_i \equiv \Omega \rho \left[ \omega_{0,i} + \frac{(1 - \rho) \omega_4}{1 - (1 - \rho) \omega_3} \omega_{0,-i} \right] + \Omega \rho \left[ \omega_1 E[\tilde{\tau}_i] + \frac{(1 - \rho) \omega_4}{1 - (1 - \rho) \omega_3} E[\tilde{\tau}_{-i}] \right] \\
+ \Omega \rho \left[ \left( \frac{(1 - \rho) \omega_4}{1 - (1 - \rho) \omega_3} \right) E[\pi_i] + \left( \frac{(1 - \rho) \omega_4}{1 - (1 - \rho) \omega_3} \right) E[\pi_{-i}] \right] \\
+ \Omega (1 - \rho) \left[ \omega_{0,i} + \frac{(1 - \rho) \omega_4}{1 - (1 - \rho) \omega_3} \omega_{0,-i} \right]. \]

\(^{40}\)Note that the calculations made so far allow for a more general setting to facilitate enhancement of the micro-model with respect to other types of fiscal policies.
and
\[ c^i \equiv \Omega(1 - \rho)\omega_1 \quad \text{and} \quad c^{-i} \equiv \Omega(1 - \rho)\frac{(1 - \rho)\omega_4}{1 - (1 - \rho)\omega_3}\omega_1. \]

### A.6 Proof of Proposition 2: Output Determination

To obtain the equation for regional output \( y_i \), we start with equation (A.26) and plug in equation (A.18):

\[
Y_i = \int_{u_i}^{v_i} \left( \frac{p^j(j)}{P_i} \right)^{1 - \theta} W \frac{d_j}{w_i}
= \left( \frac{\theta \hat{d}_i}{(\theta - 1)(1 - \tau_i)} \left( W \beta^{-1} \right)^{\frac{1 - \theta}{\theta + \beta(\theta - 1)}} W \right) \frac{d_j}{w_i}.
\]

Log-linearizing this equation and using the notation \( y_i \equiv \hat{Y}_i \), we get

\[
y_i = \frac{1 - \theta}{1 + \theta(\beta - 1)} \hat{d}_i + \frac{1 - \theta}{1 + \theta(\beta - 1)} \hat{\tau}_i \hat{\tau}_i + \frac{(\beta - 1)(1 - \theta)}{1 + \theta(\beta - 1)} \hat{W} + \hat{\hat{W}}. \tag{A.49}
\]

Now we follow the procedure in Dixit and Lambertini (2003b) and apply equation (A.39) in two ways. First, we replace the first \( \hat{\hat{W}} \) in (A.39) with \( i \) indices and the second \( \hat{\hat{W}} \) with \(-i\) indices. We thus obtain

\[
y_i = \frac{1 - \theta}{1 + \theta(\beta - 1)} \hat{d}_i + \frac{1 - \theta}{1 + \theta(\beta - 1)} \hat{\tau}_i \hat{\tau}_i + \left[ \frac{1 + \theta(\beta - 1)}{\beta - 1} (\hat{p}^i(j) - \pi_i) - \frac{1}{\beta - 1} \hat{d}_i - \frac{1}{\beta - 1} \hat{\tau}_i \right] + \left[ \frac{1 + \theta(\beta - 1)}{\beta - 1} (\hat{p}^{-i}(j) - \pi_{-i}) - \frac{1}{\beta - 1} \hat{d}_{-i} - \frac{1}{\beta - 1} \hat{\tau}_{-i} \right]. \tag{A.50}
\]

Second, we do the same thing the other way round, leading to

\[
y_i = \frac{1 - \theta}{1 + \theta(\beta - 1)} \hat{d}_i + \frac{1 - \theta}{1 + \theta(\beta - 1)} \hat{\tau}_i \hat{\tau}_i + \left[ \frac{1 + \theta(\beta - 1)}{\beta - 1} (\hat{p}^i(j) - \pi_i) - \frac{1}{\beta - 1} \hat{d}_i - \frac{1}{\beta - 1} \hat{\tau}_i \right] + \left[ \frac{1 + \theta(\beta - 1)}{\beta - 1} (\hat{p}^{-i}(j) - \pi_{-i}) - \frac{1}{\beta - 1} \hat{d}_{-i} - \frac{1}{\beta - 1} \hat{\tau}_{-i} \right]. \tag{A.51}
\]

In the next step, we add up the two equations and divide by two. We evaluate \( \hat{p}^i(j) \) in both regions for the flexible price firms, i.e. we replace \( \hat{p}^i(j) \) by \( \pi_i^1 \), the price that
maximizes current period indirect utility only. Replacing \( \pi_i \) with equation (A.44) and simplifying leads to

\[
y_i = \left( \frac{1 - \theta}{2[1 + \theta(\beta - 1)]} - \frac{1}{2(\beta - 1)} \right) \hat{\tau}_i - \left( \frac{1 - \theta}{2[1 + \theta(\beta - 1)]} + \frac{1}{2(\beta - 1)} \right) \hat{\tau}_{-i} - i \]

\[
+ \frac{2\beta \rho}{(\beta - 1)(1 - \rho)} (\pi_i - \bar{\pi}_i) + \frac{\beta \rho}{(\beta - 1)(1 - \rho)} (s_i - \bar{s}_i)
\]

\[
+ \left( \frac{1 - \theta}{2[1 + \theta(\beta - 1)]} - \frac{1}{2(\beta - 1)} \right) \hat{d}_i - \left( \frac{1 - \theta}{2[1 + \theta(\beta - 1)]} + \frac{1}{2(\beta - 1)} \right) \hat{d}_{-i} \tag{A.52}
\]

The notation \( \bar{s}_i = E[s_i] \) is used to denote region \( i \)'s expected terms of trade. Given the steady state of \( \bar{P}_i = \bar{P}^i = \bar{P} \) for all \( i \), we have \( \bar{s}_i \equiv 0 \) so that we can drop this term. For ease of exposition, we rewrite the last equation as follows:

\[
y_i = \bar{y}_i + a^i \hat{\tau}_i + a^{i-1} \hat{\tau}_{-i} + b^i (\pi_i - \bar{\pi}_i) + \kappa^i s_i + \phi_i, \tag{A.53}
\]

where \( \bar{y}_i = 0 \), \( a^i \equiv \left( \frac{1 - \theta}{2[1 + \theta(\beta - 1)]} - \frac{1}{2(\beta - 1)} \right) \hat{\tau}_i \), \( a^{i-1} \equiv - \left( \frac{1 - \theta}{2[1 + \theta(\beta - 1)]} + \frac{1}{2(\beta - 1)} \right) \hat{\tau}_{-i} \),

\[
b^i \equiv \frac{2\beta \rho}{(\beta - 1)(1 - \rho)}, \text{ with } \pi_i = \bar{\pi}_i = E[\pi_i], \quad \kappa^i \equiv \frac{\beta \rho}{(\beta - 1)(1 - \rho)} \text{ and}
\]

\[
\phi_i = \left( \frac{1 - \theta}{2[1 + \theta(\beta - 1)]} - \frac{1}{2(\beta - 1)} \right) \hat{d}_i - \left( \frac{1 - \theta}{2[1 + \theta(\beta - 1)]} + \frac{1}{2(\beta - 1)} \right) \hat{d}_{-i}. \]
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