Lumpy Labor Adjustment as a Propagation Mechanism of Business Cycles

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Abstract

This paper aims to study the quantitative significance of lumpy labor adjustment as a propagation mechanism for business cycles. In the baseline model, I introduce lumpy job turnover in the spirit of Taylor (1980) and Calvo (1983) in a DSGE framework and find that it performs as same as the quadratic-adjustment-cost model at the aggregate level, but different at firm’s level. In particular, it can capture lumpy labor adjustment at plant’s level through the ‘front-loading effect’. Then I implement the Weibull distribution in the same framework to incorporate the increasing hazards of the labor adjustment process, which is supported by the evidence from micro data. This extension represents a substantial improvement over benchmark models. It can replicate high volatility of employment, low volatile labor productivity and persistent dynamics in output. Based on these results, I conclude that intratemporal substitution between the two production factors and the aggregation mechanism play an important role in the propagation mechanism.

JEL Classification: E32; E24 ; E22

Keywords: Business cycles; Lumpy labor adjustment; Weibull distribution; Increasing hazard function
“...on the order of two-third of the business cycle is accounted for by movements in the labor input and one-third by the changes in technology. Thus, most business cycle theorists agree that an understanding of aggregate labor market fluctuations is a prerequisite for understanding how business cycles propagate over time.” — Kydland (1993)

Introduction

Over the last two decades, the standard RBC model has been extended in various directions to enhance its internal propagation mechanism. Recent developments emphasize the role of the labor market rigidity in propagating business cycle fluctuations. For example, the search and matching model (Merz (1995) and Andolfatto (1996)) generates persistence in the labor dynamics by assuming the matching friction in the labor market; The factor hoarding model (Burnside and Eichenbaum (1996)) assumes that extensive margins are predetermined, while the intensive margins can be only adjusted in a costly way; The habit formation model (Wen (1998)) emphasizes the role of household’s willingness to smooth the path of leisure; And the learning-by-doing model (Chang, Gomes, and Schorfheide (2002)) is motivated by the assumption that current labor input affects future output through worker’s skill accumulation.

These models have one feature in common, namely, they all introduced the lagged labor into the propagation mechanism in order to replicate persistence in output and employment. However, these partial-labor-adjustment models have to face a trade-off between persistence and volatility of dynamics. As a result, they usually need to be strengthened by other mechanisms to account for the magnitude of observed fluctuations. Wen (1998) combined the habit formation in leisure with increasing return to scale, and volatility of unemployment in matching models is enhanced by introducing wage rigidity (e.g. Shimer (2003) and Hall (2003)).

In this paper, I pursue an old source of labor market rigidity - the uncertainty regarding labor adjustment with a novel method, and show it has the ability to reconcile both persistence and magnitude of business cycle fluctuations.

Over the last decade empirical evidence has been accumulated showing that the labor de-

\[2\] For instance, models focused on the household’s intra- and intertemporal substitutions, e.g. nonseparable leisure (Kydland and Prescott (1982)), indivisible labor (Hansen (1985), Rogerson (1988)), home production (Greenwood and Hercowitz (1991)); and those models that incorporate lags and costs in capital adjustment, e.g. time-to-build model (Kydland and Prescott, 1982) and q-theoretic models of Baxter and Crucini(1993).
mand at the firm’s level is lumpy and asynchronous. Earlier evidence has been presented by Hamermesh (1989) and Caballero, Engel, and Haltiwanger (1997). Recently Letterie, Pfann, and Polder (2004) investigate the dynamic interrelation between factor demand with plant-level data for the Dutch manufacturing sector. They find that both adjustments of capital and labor are lumpy, and they are coordinated with each other in time. In addition, Varejão and Portugal (2006) find that large employment adjustments (larger than 10% of the plant’s labor force) account for about 66% of the total job turnover, and on average around 75% of all observed Portuguese employer do not change employment over an entire quarter.

In theoretical work, (S,s) models are popular to address the question as to the effect of lumpy factor adjustments. The earlier partial equilibrium (S,s) models of labor adjustment found that employment growth depends on the cross-sectional distribution of the employment deviation from optimal target. In particular, Caballero, Engel, and Haltiwanger (1997) found that the movements in distribution of firms are important in explaining unusually large deviations in total adjustment, and the adjustment hazard rises with large shocks and thus amplifies the shock’s effect in aggregate adjustment. This finding was taken as evidence that lumpy adjustment pattern at firm’s level matters for aggregate economy. However, recent development of the (S,s) approach in the general equilibrium framework shows that this considerable effect of non-convex costs at the plant level disappears with the equilibrium price changes. King and Thomas (2006) studied a (S,s) model in a general equilibrium framework and found that lumpy labor adjustment does not generate observationally different aggregate dynamics from a standard partial adjustment model.

In this paper, instead of using the general equilibrium (S,s) model, I first adopt the modeling strategy originally proposed by Kiyotaki and Moore (1997) to model lumpy labor adjustment. I assume that firm’s labor adjustment obeys a staggered rule in a spirit of Fischer (1977), Taylor (1980) and Calvo (1983). The advantage of this approach is twofold: first, it simplifies the theoretical structure of the model, and makes it more tractable. As a result, I can show analytically more insights implied by the model. Second, instead of solving the stationary distribution of firms on the basis of arbitrary free parameters, I directly calibrate the lumpy labor adjustment process using statistical duration theory and evidence from firm level data.

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3 Caplin and Spulber (1987) was the early work applying the (S,s) approach to macro models. See also Veracierto (2002) and Thomas (2002) among others.

4 See: e.g. Caballero and Engel (1993), Caballero, Engel, and Haltiwanger (1997).

5 Sveen and Weinke (2005) used the same idea to study the lumpiness in investment in a New-Keynesian model.
The key message conveyed by this model is that uncertainty in the labor adjustment process induces firms to make precautionary labor adjustment, which amplifies the volatility of factor demand. At the individual level, firm’s labor demand exhibits ‘front-loading effect’, i.e. when firms are facing positive persistent technology shocks, they hire more workers than they currently need to hedge the risk that they may not be able to re-optimize their labor input in the near future. However, at the aggregate level, this large labor adjustment is to some extent neutralized by the aggregation mechanism and the general equilibrium forces. To this end, simulation results show that the performance of the model in matching second moments of the U.S. business cycles is virtually as same as that of the quadratic adjustment cost model. In fact, I demonstrate that the aggregate labor demand equations derived from both models correspond to the same reduced form. Moreover, deep parameters of the two models have a one-to-one mapping to each other. This result confirms findings in the general equilibrium (S,s) literature.

Even though the baseline model does not outperform the quadratic cost model with respect to aggregate variables, its propagation mechanism exhibits a remarkable character compared to the latter, i.e. it has the ability to disentangle effects of lumpy labor adjustment at the micro-level dynamics from those on the aggregate level. Unlike the quadratic cost model, the propagation works in this model through two margins. At the micro level, it works through the front-loading effect that amplifies the optimal labor adjustment of firms; while, at the macro level, the aggregation mechanism implied by the Poisson process washes out the large effect of the individual labor adjustment and smooths aggregate dynamics. It implies that partial adjustment models like this one do not necessarily induce dampened aggregate dynamics. Instead, over-dampened aggregate dynamics may result from the restrictive aggregation mechanism.

Following this line of logic, in the second part of the paper I extend the baseline model to a more general case that incorporates an increasing hazard function for the stochastic labor adjustment process. In fact, I extend the Poisson labor adjustment process that underlies the Calvo assumption to the Weibull distribution, which is less restrictive and can encompass a broad range of hazard functions. The results show that the Weibull-adjustment model exhibits a stronger propagation mechanism in comparison to the benchmark model. In particular, it can generate high volatile aggregate employment as observed in the data. In addition, introducing rigidity in the labor market improves persistence of the model’s dynamics and gives rise to hump-shaped impulse responses. The reason why the Weibull model can make lumpy labor adjustment effective for the aggregate economy is as follows: on the one hand, at the micro level, increasing hazard function implies that agents are more sensitive to adjustment risk and consequently amplifies the front-loading effect fur-
ther, and hence firm’s labor adjustment reacts even more strongly than that in the Calvo case; on the other hand, at the aggregate level increasing hazard rate also affects the aggregation mechanism, which is crucial to linking micro-level features with implications for macro behavior. Based on these results, I conclude that the Weibull-adjustment RBC model possesses a quantitatively significant propagation mechanism and the aggregation scheme matters for the aggregate implication of the model.

The remainder of the paper is organized as follows: Section 1 introduces the baseline model, in which I assume the firm’s adjustment process obeys the Poisson process; in section 2, I compare the labor dynamics of the Calvo-adjustment model with the quadratic adjustment cost model; Section 3 introduce the numerical solution method and calibration strategy, and then compare the Calvo-adjustment model’s quantitative implications with those from other benchmark models. In the section 4, I introduce the lumpy-labor-RBC model with the Weibull lifetime model and present the simulation results; Section 5 contains some concluding remarks.

1. The Baseline Model

In this section, I develop the baseline model in a standard RBC framework. The main feature of the model is that I allow for a staggered employment adjustment at the firm’s level similar in spirit to Calvo (1983). In the standard RBC model, firms can adjust their employment period-by-period without any costs. In contrast, I consider here an economy with a rigid labor market caused by unspecified economic frictions, which causes some random fixed fraction of firms not to be able to adjust their labor input. The labor market forms a common expectation of this ratio. In effect, more rigid the labor market is, lower the adjustment ratio is expected by all agents in the market.

Further I assume that firm can access a instantaneous rental market for capital which is supplied by households in any given period. This assumption is desired for a technical reason. Because the firm’s first order condition requires the value of a function of capital

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6 A rigid labor market is a common phenomenon in the world economy, according to OECD (1999), EPL provisions and practices are currently prevailing in 27 OECD countries. Indicators show that labor market rigidity is much higher in Southern European countries along with France and Germany than the English-speaking countries. Nickell and Numizita (2000) also found that, across OECD countries, labor demand adjusts less rapidly when the employment protection is more strict and union density is higher. Besides, this assumption is also motivated by Boldrin, Christiano, and Fisher (1999), which emphasizes the difficulty of shifting labor between economic sectors. Even though the RBC model has only one good and one sector, but one can also treat it as an economy with two sectors, i.e., one investment goods sector and one consumer goods sector.
and labor to be identical in the whole economy, the instantaneous capital market makes possible for those firms that can not change their employment to fulfill this requirement. However, the capital stock is still predetermined by the household (supply) side.

The third distinctive feature of my model is that agents use a decreasing-return-to-scale technology to produce output. This assumption enable me to show the lumpy effect at the plant’s level, however it brings also problems. First, using decreasing-return-to-scale production technology, firms can earn profits. Second, smaller firm is, the more efficient it becomes in the sense of profits per unit of production. Hence firm has incentive to be small. In order to set a minimum size of firm, I introduce an fixed cost of operation (i), which is equal to the profits earned at the stead state without this cost. In the stochastic environment, all firms earn positive profits in some periods and negative profits in the other periods. Since firms expect zero profits in the long run, no entry and exit occur in this economy and hence the number of firms is constant.

Staggered Labor Adjustment

In this subsection, I introduce formally the assumption of the staggered-employment-adjustment process in the context of statistical duration models. It is well known that the Calvo-type-adjustment assumption [Calvo (1983)] implies the Poisson process, however deriving it formally with the language of probability theory serves as a solid technical base for extending the baseline model to a theory that embodies the more general adjustment processes.

Here I consider a process in which firm’s employment adjustment occurs randomly in time. It turns out that under some basic assumptions that deal with independence and uniformity in time, this random process is governed by the Poisson process. This assumption simplifies the real-world continuous factor adjustment decisions in terms of a sequence of generic trials that satisfy the following assumptions:

- Each trial has two possible outcomes, called adjustment and non-adjustment.

---

7 Equation (12) shows that ratio of power functions of labor and capital only depends on the rental rate and aggregate shocks, hence it should be identical for all firms in the economy. However, the capital labor ratios in this model are different across firms.

8 The diseconomy of scale can be theoretically motivated in several ways. e.g. [Howitt and McAfee (1988)] emphasized the role of externalities, i.e. the marginal adjustment cost faced by a firm is positively related to the activity level already attained by its rivals.

9 In this paper, as I write the model in the discrete-time, the discretized adjustment process follows the Bernoulli trials process, which is the discrete version of the Poisson process.
The trials are memoryless, i.e. the outcome of one trial has no influence over the outcome of another trial.

For every firm, the probability of adjusting is $1 - \alpha$ and the probability of non-adjusting is $\alpha$.

Formally I define the labor adjustment process as a Bernoulli process as follows:

**Definition:** Given a probability space $(\Omega, Pr)$ together with a random variable $X$ over the set $\{0, 1\}$, so that for every $\omega \in \Omega$, $X_i(\omega) = 1$ with probability $\alpha$ and $X_i(\omega) = 0$ with probability $1 - \alpha$, where $\Omega = \{\text{adjusting, non-adjusting}\}$, a Bernoulli process is a sequence of integers $Z^\omega = \{n \in Z : X_n(\omega) = 1\}$.

Given the factor adjustment process follows the Bernoulli process, the probability of receiving zero adjusting signal in an interval of $j$ periods is:

$$Pr(0) = \binom{j}{0} (1 - \alpha)^0 \alpha^j = \alpha^j \quad \text{for } j = 0, 1, 2, ... \quad (1)$$

And, the probability that a duration spell terminates at the period $j$ is

$$Pr(j) = (1 - \alpha) \alpha^{j-1} \quad \text{for } j = 0, 1, 2, ... \quad (2)$$

From the statistical duration analysis (Lawless (1982)), we know that the hazard function is particularly useful lifetime distributions, because it describes the pattern of which the probability that a duration spell terminates changes over time. In applications, we can use qualitative information about the hazard function to help in selecting a life distribution model. For this reason, it is interesting to find out what kind of hazard function is implied by the Calvo-adjustment assumption.

The hazard function corresponding to the Bernoulli process is:

$$H(j) = \frac{\theta(j)}{1 - F(j)} = \frac{1 - \alpha}{\alpha} \quad (3)$$

The hazard function embedded in the Bernoulli distribution is constant. It implies that the probability of adjusting is independent of the time spell elapsed. However, as the micro empirical evidence is in favor of an increasing hazard function for the labor adjustment process, it is naturally desirable to extend this restrictive distribution-assumption to a more general one that encompasses both constant and increasing hazard cases. I will extend the baseline model following this line of thought in the section 4.
Distribution of Firms and Aggregation Mechanism

The economy is populated by a continuum of firms, which is normalized to one. At the beginning of each period, firms are differentiated by their stocks of employment, because each firm adjusts its labor input at the different time in the past. I index firms by \( j \) that corresponds to the spell of time that has been elapsed since the last adjustment occurred. I call it hereafter “vintage groups”. In fact, given the complete financial market, adjusting firms choose a common target labor adjustment at each period. As a result, firms in any vintage group share same amount of employment, and the aggregate stock of labor can be summed up with respect to the distribution of firms over the labor stocks, i.e. the aggregate labor is the weighted sum of all past optimal employment, and the weights are equal to the fractions of firms in each vintage group \( j \).

Define \( \Theta = \{\theta(j)\}_{j=0}^{\infty} \) as the distribution of firm over vintage groups. It can be easily shown that \( \theta(j) = (1 - \alpha)\alpha^j \) for \( j = 0, 1, 2, \ldots \).

Given \( \Theta \) and let \( l_{j,t} \) define as the optimal employment level that is determined \( j \) periods ago from time \( t \), aggregate employment is obtained by:

\[
L_t = \sum_{j=0}^{\infty} \theta(j)l_{j,t} = \sum_{j=0}^{\infty} (1 - \alpha)\alpha^j l_{j,t}
\]

Finally, since the fraction of firms that adjust their employment is randomly drawn across the population, we can easily iterate it and get the recursive law for aggregate employment.

\[
L_t = (1 - \alpha)l_{0,t} + \alpha L_{t-1}
\]

or equivalently,

\[
\Delta L_t = L_t - L_{t-1} = (1 - \alpha)(l_{0,t} - L_{t-1})
\]

This equation reveals the partial adjustment nature of this model, that the actual job

---

10 Because, by assumption there is \( 1 - \alpha \) fraction of firm in the group zero, and \( \alpha \) percent of them goes to group one, this gives the density of group one to be \( (1 - \alpha)\alpha \). Similarly, \( \alpha \) percent of units in group one goes to group two, so the density of group two is \( (1 - \alpha)\alpha^2 \), and so on.

11 Note that equation 12 implies that firms in the vintage \( j \) group must also use same amount of capital. Thus the distribution of plants over labor is the same as over capital stocks. As a result, we can aggregate capital in the same way.

\[
K_t = \sum_{j=0}^{\infty} (1 - \alpha)\alpha^j k_{j,t}
\]
turnover is only a fraction of the optimal adjustment. The speed of adjustment is de-
pending on the extent of market rigidity $(1 - \alpha)$. If no friction exists in the labor market,
$\alpha = 0$, all firms can adjust their employment, and then this model reduces to the standard
RBC case.

**Firm’s optimization Problem**

At each period, the state of economy can be summarized by the vintage index $j$ with the
corresponding factor stocks $(l_{j,t})$.

Firms in a vintage group $j$ use capital $k_{j,t}$ and labor $l_{j,t}$ to produce output $y_{j,t}$, according
to the Cobb-Douglas production function:

$$y_{j,t} = Z_t l_{j,t}^a k_{j,t}^b$$

and $a + b < 1$ (8)

$Z_t$ summarizes the aggregate productivity shock, which consists of a trend component $\bar{Z}_t$
and the realization of a stochastic process $z_t$. The trend component $\bar{Z}_t$ evolves at constant
growth rate $g$, while $z_t$ follows an AR(1) process in logs:

$$Z_t = \bar{Z}_t z_t,$$

where $z_t = z_{t-1} e^{v_t}$, and $v_t \sim i.i.d.N(0; \sigma^2)$

Since I assume that only a fraction of firms adjust their employment, the labor force of a
firm evolves over time according to following scheme:

$$l_{j,t} = \begin{cases} 
  l_{0,t} & j = 0 \quad \text{(new re-optimized employment)} \\
  l_{j-1,t-1} & j > 0 \quad \text{(old employment from j periods ago)}
\end{cases}$$

(10)

In spite of heterogeneous nature of the problem, firms’ maximization problems can be
written in a representative firm fashion: a typical firm maximizes the expected discounted
real value of all future profits by choosing nonnegative value for current optimal labor
$l_{0,t}$ and a sequence of optimal capital stocks $\{k_{j,t}\}_{j=0}^\infty$, subject to the information set
$\Omega_t = \{\{l_{j,t}\}_{j=1}^\infty, Z_t\}$ and taking the real wage $W_t$ and real rental rate $R_t$ as given.

Note that this formulation is a simplified version of original problem. In fact, in the
original problem, we have heterogeneous firms indexed by the vintage label $j$. Firms in
the vintage group $j = 0$ should choose both optimal labor and capital $(l_{0,t}, k_{0,t})$, while the other firms in the vintage groups of $j > 0$ re-optimize their capital stock only, i.e. choosing \( \{k_{j,t}\}_{j=1}^{\infty} \). I simplify these serial optimal problems of different firms by a representative firm’s problem, in which it not only re-optimizes its current factor stocks, but also chooses all future optimal capital stocks taken into account that it will not adjust its employment in the near future with the probabilities $\alpha^j$.

\[
\max_{l_{0,t},(k_{j,t})_{j=0}^{\infty}} V_t = \sum_{j=0}^{\infty} E_t \{ \tilde{\beta}_{t,t+j} \alpha^j [F(l_{0,t}, k_{j,t}) - w_{t+j} l_{0,t} - r_{t+j} k_{j,t+j}] \Omega_t \} \quad (11)
\]

where $\tilde{\beta}_{t,t+j}$ is the stochastic discount factor, which equals $\beta E_t(U_c(C_{t+1}))/U_c(C_t)$.

Since at steady state real variables except for labor will grow at rate $g$ along the balanced growth path, from now on I work with detrended variables without changing the notions. First order conditions for this problem are:

\[
r_t = f_k(j, t) = b Z_t l_{0,t}^{a_j} k_{j,t}^{1-b_j}
\]

\[
\sum_{j=0}^{\infty} \alpha^j E_t [\tilde{\beta}_{t,t+j} (a Z_t^{a_j} r_{0,t}^{b_j-1} k_{j,t+j}^{b_j} - w_{t+j})] = 0
\]

Equation (12) shows that at any period some function of labor-capital-ratio has to be the same for all firms. This is the capital demand function give the rental rate and employment. Equation (13) characterizes optimal labor demand of a typical firm in case of adjusting at period $t$. Note that If I assume the production function is constant return to scale, this equation can only pin down the ratio of labor and capital, but not the levels.

To reveal the model’s implication of optimal labor demand at the firm’s level, I derive firm’s optimal employment demand by combining the first order conditions. The plant’s optimal labor demand $l_{0,t}$ at period $t$ is given by

\[
l_{0,t}^{1-a_j-b_j} = \frac{ab^{b_j-1-b_j} \sum_{j=0}^{\infty} \alpha^j E_t [\tilde{\beta}_{t,t+j} Z_{t+j}^{1-b_j} r_{t+j}^{b_j-1}] \Omega_t]^{1-b_j}}{\sum_{j=0}^{\infty} \alpha^j E_t [\tilde{\beta}_{t,t+j} w_{t+j}]}.
\]

Equation (14) characterizes the optimal demand of an adjusting firm. It shows, at the firm’s level, the optimal labor demand reacts to all future shocks and the equilibrium
prices. In particular, it is increasing in all expected future shocks $Z_{t+j}$ and decreasing in all expected future process of prices $W_{t+j}$ and $R_{t+j}$. This implies a ‘front-loading’ effect of labor demand due to the uncertainty in the labor adjustment process. In the partial equilibrium, where prices stay constant, it is easy to see that a positive persistent shock will make the individual labor adjustment higher than it would be in a frictionless economy. Plants hire more labor than they currently need to hedge the risk that they may not be able to re-optimize it in the near future, vice versa for the negative shocks. Additionally equation (14) also shows that larger the value of $\alpha$ is, the higher weight is attached on the future shocks. Thus the labor demand is more sensitive to the future shocks, when the friction in the labor market are more severe. In one word, the uncertainty about flexibility of the labor market acts to increase the volatility of employment at firm’s level and cause the lumpy fashion of adjustments.

**Dynamic Labor Demand Equations**

To gain further intuition of the firm’s behavior, I linearize the dynamic labor demand equations about the steady state for this Calvo-adjustment model. In contrast to the other partial adjustment model (e.g. the quadratic adjustment cost model and the habit formation model), Calvo-adjustment model implies different labor demand behaviors for the plant’s level and the aggregate level. As a result, it provides a suitable framework for estimation using both micro data and macro data.

After log-linearizing the FOCs (12) and (13) around the non-stochastic steady state, I get the following dynamic labor demand equation for individual firms$^{12}$

$$
\frac{(1 - a - b)\alpha\beta}{1 - \alpha\beta} E_t[\hat{l}_{t+1} - 1 - a - b] - \frac{1 - a - b}{1 - \alpha\beta} \hat{l}_{0,t} - \frac{bR}{r} \hat{R}_t - (1 - b)\hat{w}_t + z_t = 0
$$

(15)

Together with equation (6), the aggregate labor demand equation is as follows:

$$
\alpha\beta\kappa E_t[\hat{l}_{t+1}] - (1 + \alpha^2\beta)\kappa\hat{l}_t + \alpha\kappa\hat{l}_{t-1} - \frac{bR}{r} \hat{R}_t - (1 - b)\hat{w}_t + z_t = 0
$$

(16)

where \( \kappa = \frac{(1-a-b)}{(1-\alpha)(1-\alpha\beta)} \)

Equation (15) reveals that at firm’s level persistence of the labor demand depends on the parameter $\alpha$, and labor demand negatively depends on real prices and positively depends$^{12}$ Variables with hat are denoted as log deviation from the non-stochastic steady state, such as $\hat{x}_t = \log X_t - \log \bar{X}$; and the derivation is shown in a technical appendix, which is available upon request.
on aggregate technology shocks. By contrast, the aggregate labor demand (equation 16) exhibits more complex dynamics, which involve a AR(2) process. The labor market rigidity parameter $\alpha$ affects the dynamic property of labor demand, while the capital share $b$ has influence on the elasticities of labor demand with respect to prices. In particular, when the capital share is large, the elasticity to interest rate becomes bigger, while the elasticity to wage decreases.

**Household**

There is a continuum of identical households, who are endowed with $k_0$ units of capital at $t = 0$ and one unit of time in each period, which can be spent on either working or leisure. The infinitely-lived representative household receives labor and rental income, which can be spent on consumption and investment. Consumption, labor supply and investment are chosen to maximize following life cycle utility:

$$U = \max_{\{C_t, L_t, I_t\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (U(C_t) - V(L_t)) \right\}. \quad (17)$$

subject to the sequence of budget constraints at each period:

$$C_t + I_t \leq W_t L_t + R_t K_t + T_t \quad (18)$$

Where $T_t$ is the lump-sum transfer of profits resulting from ownership of firms. The instantaneous utility $U(.)$ and $V(.)$ are bounded, continuously differentiable, strictly increasing and strictly concave in consumption and leisure $(1 - L_t)$. We take the following function forms for instantaneous utility:

$$U(C_t) - V(L_t) = \log C_t - \chi L_t \quad (19)$$

Where the linear disutility function is motivated by the indivisible labor assumption in spirit to Hansen (1985) and Rogerson (1988).

The capital stock evolves according to the following law of motion:

$$K_{t+1} = (1 - \delta) K_t + I_t \quad (20)$$

Finally, I restrict that no capital is left unused at the end of the life, and the transversality
condition is added:

\[
\lim_{T \to \infty} E_0 \left[ \prod_{t=0}^{T} R_{t,t+1}^{-1} \right] K_{T+1} = 0, \tag{21}
\]

The optimal conditions of household are summarized in the following FONCs:

\[
\chi L^\phi_t C^\eta_t = W_t, \tag{22}
\]

\[
1 = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\eta} (r_{t+1} + 1 - \delta) \right], \tag{23}
\]

**Equilibrium**

Given an exogenous stochastic process for aggregate technology shocks and the common knowledge of the firms’ distribution across vintage groups \( \Theta \), I define the competitive equilibrium as a set of stochastic processes of the endogenous variables \( \{Y_t, C_t, L_t, I_{j,t}, k_{j,t}, I_t, K_t, w_t, r_t\}_{t=0}^{\infty} \) such that:

1. Given \( K_t \) and the market prices \( \{w_t, r_t\}_{t=0}^{\infty} \), sequences \( \{C_t^s, L_t^s, I_t^s\}_{t=0}^{\infty} \) solve the representative household’s maximization problem (17) subject to (16)-(18).

2. Given \( \{w_t, r_t\}_{t=0}^{\infty}, \{l_{j,t}, k_{j,t}\}_{t=0}^{\infty} \) solve the Firms’ profits maximization problem (11) subject to production technology (8) and exogenous technology shock process (9).

3. Aggregate demands for employment \( L_t^d \) and capital \( K_t^d \) are determined by (5) and (4) respectively.

4. Markets clear: \( L_t^s = L_t^d = L_t \) in labor market, \( K_t^s = K_t^d = K_t \) in capital market and \( C_t + I_t = Y_t \) in the goods market.

5. Finally, markets’ equilibrium determine the equilibrium real wage and rental rate \( \{w_t, r_t\}_{t=0}^{\infty} \)

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13 Here, superscript \( s \) denotes “supply”; Similar notation \( d \) for “demand”
2. Equivalence of the Partial Adjustment Models

The quadratic adjustment cost model has lost influence in the macroeconomic literature because economists have grown disenchanted with its smoothing and synchronous implication on the firm-level factor adjustment. As discussed in the introduction, mounting micro evidence show that firms adjust their labor in a discrete and asynchronous fashion. Despite of this fact, the quadratic adjustment cost model has been used widely in theoretical and empirical work, because they are easily solved and produce aggregate equations in a form suitable for estimation. By contrast, as I have shown in the equation (14), the Calvo-adjustment model can capture lumpy and asynchronous features in firm’s labor adjustment, while aggregate labor demand of this model is characterized by a smoothing AR(2) dynamic process(see: equation (16)). The key question addressed in this section is whether the quadratic adjustment cost model is equivalent to the Calvo-adjustment model as to aggregate dynamics? If so, the quadratic cost model can be treated as a reduced form model, and is still valid in the empirical work using aggregate data.

The Quadratic Adjustment Cost Model

I first derive the aggregate labor demand equation from a textbook quadratic-adjustment-cost model(See e.g. Hamermesh (1993)).

In this economy, each firm is assumed to maximize the expected discounted real value of all future profits by choosing nonnegative values for optimal sequence of labors $l_{t+i}$ and optimal sequence of capital stocks $k_{t+i}$, subject to the quadratic labor adjustment costs.

The objective function of firm is:

$$
\max_{l_{t+i}, k_{t+i}} V_t = \sum_{i=0}^{\infty} E_t \{ \beta_t \{ F(l_{t+i}, k_{t+i}) - w_{t+i}l_{t+i} - r_{t+i}k_{t+i} - d (l_{t+i} - l_{t+i-1})^2 \} \} 
$$

(24)

where $d$ is denoted as the adjustment cost parameter.

subject to

$$
y_t = Z_t^{a_t} l_t^{b_t}
$$

(25)

and the total productivity shock $Z_t$ and the household’s problem are as same as in the Calvo adjustment model.
The first order conditions are:

\[ r_{t+i} = F_K(t + i) = bZ_{t+i} \frac{\tilde{t}_{i+1}}{k_{t+i}} \]  \hspace{1cm} (26)

\[ \tilde{\beta}_{t+i}[F_L(t + i) - w_{t+i} - d(l_{t+i} - l_{t+i-1})] + E_t[\tilde{\beta}_{t+i+1}d(l_{t+i} - l_{t+i-1})] = 0 \]  \hspace{1cm} (27)

It follows:

\[ a Z_{t+i} \tilde{t}_{i+1}^{a-1}k_{t+i}^{b} - w_{t+i} + \beta d l_{t+i+1} - d(1 + \beta)l_{t+i} + d l_{t+i-1} = 0 \]  \hspace{1cm} (28)

If I log-linearize these FOCs around the steady state, I get the following dynamic labor demand equation:

\[ \gamma \beta E_t[\hat{l}_{t+1}] - [(1 - a - b) + \gamma(1 + \beta)] \hat{l}_t + \gamma \hat{l}_{t-1} - \frac{bR}{r} \hat{R}_t - (1 - b)\hat{w}_t + z_t = 0 \]  \hspace{1cm} (29)

Where I denote \( \gamma = \frac{\alpha\kappa}{\theta} (1 - b) \).

**Equivalence**

As [Rotemberg (1987)] has shown the equivalence between the Calvo model and the quadratic cost model in the price adjustment context, I demonstrate here analytically that aggregate labor demand equations derived from both models conform to the same reduced form. In addition, the deep parameters of two models have a one-to-one mapping to each other.

Comparing the equation (29) to the dynamic labor demand equation which I derived from Calvo-adjustment model, I find that these two equations can be put into the following reduced form equation, so that the aggregate data alone can not differentiate them.

\[ \varphi_1 E_t[\hat{l}_{t+1}] + \varphi_2 \hat{l}_t + \varphi_3 \hat{l}_{t-1} + \varphi_4 \hat{R}_t + \varphi_5 \hat{w}_t + z_t = 0 \]

In addition, if I set \( \alpha\kappa = \gamma \), i.e. the correspondence among parameters in both models follows the expression (30), then the Calvo-adjustment model is equivalent to the quadratic adjustment cost model, and hence they generate exactly same aggregate dynamics, given
other aspects of both models are same.

\[
\frac{d\bar{n}}{\bar{w}} = \frac{\alpha(1 - a - b)}{(1 - a)(1 - \alpha)(1 - \alpha\beta)(1 - b)}
\]  

(30)

Note that both parameters $d$ and $\alpha$ govern the rigidity of the labor adjustment process in both models, this equation gives the exact mapping between these two rigidity parameters.

3. Calibration and Simulation Results

To make the quantitative statements of this baseline model, I apply the log-linear approximation method of King, Plosser, and Rebelo (1988), which produces linear decision rules depending on the state variables. Firstly I calculate the steady state, and then I apply log-linear approximation around the non-stochastic long-run trend to get the linear system of equations. Next, the linearized system is then stacked into a first order vector autoregressive stochastic difference equation, which can be further diagonalized by the method of the eigenvalue decomposition. After checking the existence of the unique equilibrium, I derive the recursive law of motion for the state variables. In the following, I will firstly discuss the calibration method for this model, and then present the quantitative results and impulse response functions.

Calibration

In this model, general equilibrium is generated in markets where the household’s consumption-leisure choice meets firm’s factor demand decisions under restriction on staggered labor adjustment. I want to use this model as a laboratory to discover the impact of employment rigidity on business cycles. In order to address this question properly, I follow the tradition of RBC literature and calibrate our optimal growth model so that it is consistent with the long-run growth facts in the U.S. data, and then we study its short-run dynamics by investigating the statistical properties of simulated time series and impulse responses functions.

For most parameters in the model, we take the standard values in the literature, because it is more convenient to compare the results of different models when they are calibrated in the same manner. For quarterly discount rate $\beta$ I take 0.9902 to reflect that the real rate of interest in U.S. economy is around 4% per annum. The depreciation rate $\delta$ is 0.025.
indicating annual rate of 10%. Given these two values, I select the capital share \( b \) to be 0.329 to match the average capital-output ratio of 2.353 (Thomas and Khan (2004)), and the labor share of output \( a \) is set to be 0.58, which is consistent with direct estimates for the U.S. economy. (King, Plosser, and Rebelo (1988)).

Following Hansen (1985), Rogerson (1988), I assume indivisible labor, and the log utility function of consumption. The resulting momentary utility of the representative household is \( U(C, L) = \log C - \chi L \). This assumption makes it convenient to interpret total hours worked as the number of employment, since individual workers can either work for a fixed number of hours or not at all. Empirical studies also show that the variation in the total hours worked in the U.S. is primarily caused by changes in the number of employed workers. The parameter \( \chi \) is set to be 3.6142, indicating that, on average, 20 percent of the total time is contributed to market work (Thomas (2002)) by household.

The labor adjustment parameter is calibrated according to empirical work on estimating hazard function using aggregate net flow data. Caballero and Engel (1993) used U.S. manufacturing employment and job flow data (1972:1-1986:4) to estimate constant hazard function. Their results suggest that, on average, 22.9% of firms in the U.S. adjust their employment per quarter. As a result, I choose 0.77 as the value for \( \alpha \) in my model, and hence the mean duration of employment amounts to 4.35 quarters.

Finally, I select the values of \( \varsigma \) and \( \sigma_\epsilon \) for aggregate technology shocks. I choose \( \varsigma = 0.95 \) and standard deviation of 0.007, which are estimated parameters of Solow residuals that are commonly used in the RBC literature (King and Rebelo (2000)).

The summary of calibration is listed in Table 1:

<table>
<thead>
<tr>
<th>Model</th>
<th>Policy Preference</th>
<th>technology</th>
<th>shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calvo-Ad. RBC</td>
<td>( \alpha ) = 0.77</td>
<td>( \phi ) = 0</td>
<td>( \chi ) = 3.6142</td>
</tr>
</tbody>
</table>

Table 1: Calibration Values

---

14 Veracierto (2002) found the standard deviation of shocks should be smaller due to the decreasing return to scale assumption. He chose 0.0063 given his parameter values of labor and capital shares. However, since we are interested in the relative volatilities between variables to output, the scale of the standard deviation is not very important.
To evaluate the quantitative performance of the Calvo-adjustment RBC model, I compare
second moments generated by this model with two other benchmark models - the standard
RBC model and a habit formation in leisure RBC model. The habit formation model
is interesting, because it is another widely-used approach to introduce employment lags
into model’s dynamics. However, it is different from the Calvo-adjustment model, as it
introduces lags from the labor supply side through the habit formation assumption. So the
key questions addressed in this section is what difference of the propagation mechanism
underlying the Calvo-adjustment model in compared to the other theoretical models.

In Table(3)-(6), I report the second moments of U.S.data and those generated by the
three theoretical models. In all cases, the moments are for HP-filtered time series. For
each of these models, three sets of statistics are reported: first, absolute and relative
standard deviation; second, contemporaneous correlation coefficients relative to output;
and third, the cross correlation of output with 3 lags and leads.

Beginning with persistence of dynamics, Cogley and Nason (1995) has shown that the
standard RBC models fail to account for the observed positive serial correlation in the
growth rate of output. By construction, both Calvo-adjustment model and habit for-
mation model enhance persistence of business cycles by introducing lagged labor in the
dynamics. As seen in the tables, the standard RBC model only generate two third of
persistence observed in the data. By contrast, given same shocks, both Calvo-adjustment
model and habit formation models can remarkably replicate autocorrelations for employ-
ment in the data and a little weaker persistence for output.

As to cyclical volatility, I find that all three theoretical models can capture the general
pattern of volatilities in the data. In particular, the investment is about 3 times more
volatile than output, and capital and consumption are less volatile than output. However,
three theoretical models perform differently regarding volatility of total hours (employ-
ment). Not enhanced by the indivisible labor assumption, the standard RBC model only
generates a half of volatility in the data. The habit formation model performs even worse
as to this aspect. It accounts for only one third of volatility of total hours. In contrast, the
Calvo-adjustment model can generates strong fluctuations (125% of the level relative to
output) at the firm’s level and 77 percent of volatility observed in aggregate employment.

15 In the Appendix, I set up a habit formation model, which is similar in spirit to the model by [Wen
1998].

16 Each statistics is based on a 10,000-period simulation, so that the moments statistics for the simulated
time series can roughly converge to their population values.
Furthermore, the Calvo-adjustment model can account for low relative volatility of the labor productivity.

These results illustrate the strength of the Calvo-adjustment model in propagating business cycles. Different to other partial adjustment models, the Calvo-type assumption does not necessarily lead to dampened volatility of the employment dynamics. At the firm’s level, it generates strong lumpy labor adjustment through the ’front-loading’ effect. The intuition is as follows. Recalling the firm’s labor demand equation \( (15) \), the firm’s optimal demand depends on expectations of all future prices and shocks. Suppose that in some period \( t \) firms experience a temporary positive productivity shock, some firms are labor constrained, so they have to increase their demand of capital in the rental market, while, on the supply side, household capital stock is predetermined in the previous period. This leads to an increase in interest rates for the whole economy and rises in household savings. We can observe in the tables that volatility of interest rate in higher in the Calvo-adjustment model than the other models. On the other hand, those labor-unconstrained firms will adjust labor more than they currently need in order to hedge the adjustment-risk in the future. This in turn drives real wage up. Put them together, all those rises in productivity and prices can be expected by rational agents, so that the adjusting firms will, in addition to their front-loading motive, demand even more workers. Moreover, if labor supply is elastic, rise in the interest rate triggers additionally the substitution effect in the labor supply side, because wage tomorrow is discounted at a higher rate, and household is willing to enjoy less leisure today thus supply more labor. In conclusion, both labor and investment increase sharply at the micro level. However, at aggregate level, this strong effect is to some extent neutralized by the equilibrium price changes and the aggregation mechanism underlying in the Poisson process.

Impulse Response Functions

A different way to assess the importance of lumpy adjustment as a propagation mechanism is to consider the impulse response functions of different variables to the aggregate technology shock.

Figure[1] depicts the impulse response functions of employment and output to the aggregate technology shock. We can observe that, except of the standard RBC model, both Calvo-adjustment model and habit formation model can generate lumped-shaped impulse responses for aggregate employment and output. In addition, the Calvo-adjustment model can capture different responses for the firm’s and the aggregate level of employment, i.e.
the impulse response of aggregate employment is humped-shaped, while employment at
the firm’s level reacts immediately to the shock and by a large scale (dashed line in the
left panel). This result suggests that the Calvo-adjustment model is able to replicate
labor-adjustment features observed in the data: i.e. At the micro level, labor adjustment
exhibits a lumpy pattern in response to shocks; while, at the aggregate level, employment
reacts smoothly and with some delay back to the steady state.

4. Extension: Weibull Adjustment Probabilities

In this section, I extend the Calvo-adjustment RBC model to a more general case, in which
the labor adjustment process is characterized by an increasing hazard function. As I have
shown in the previous section, the Calvo-adjustment model can distinguish the effect of
imperfections on the micro-level dynamics from aggregate level dynamics, so that we can
discuss the role of partial adjustment at the firm’s level and the aggregate effect separately.
It seems that firm’s partial labor adjustment introduced by the Calvo assumption increases
the volatility of labor demand through the ‘front-loading effect’. However, this effect in
the baseline model is not strong enough to overcome the smoothing effect of aggregate
forces. As a result, aggregate employment implied by the model is as same as the one
predicted by the quadratic adjustment cost model. Moreover, the Calvo-assumption has
been criticized for its implication of a constant hazard rate in the adjustment process.
Motivated by these facts, I extend the baseline model by replacing the Poisson distribution
by a more general statistical duration model.
Weibull-adjustment Model

The statistical duration model I use to model lumpy labor adjustment process is the Weibull distribution\textsuperscript{17}, which is frequently used in statistical analysis of duration phenomena. Due to its flexibility, the Weibull distribution can therefore model a great variety of data and duration characteristics\textsuperscript{18}.

To integrate the Weibull distribution into our theoretical framework, I assume that firms in the labor market know that the labor adjustment signal will be drawn from the Weibull distribution. As a result, the hazard function of the time-since-last-adjustment is given by the following expression:

$$H(j) = \frac{\tau}{\lambda} \left( \frac{j}{\lambda} \right)^{\tau-1}$$  \hspace{1cm} (31)

where \( \tau \) and \( \lambda \) are the two parameters in the Weibull distribution and \( j \) is denoted as the spell of time counted from the last adjusting period. Note that this hazard function is constant when the shape parameter \( \tau \) equals one, and increasing when \( \tau \) is greater than one. So the Weibull model is more general than the Poisson model.

Furthermore, firms calculate the expected future probabilities of labor adjustment by using the survival function of the Weibull distribution in their optimization problem.

$$\max_{l_0,t,(k_j,t)_{j=0}^{\infty}} V_t = \sum_{j=0}^{\infty} S(j) E_t \{ \tilde{\beta}_{t,t+j}[f(l_{0,t}, k_{j,t}) - W_{t+j}l_{0,t} - R_{t+j}k_{j,t+j}]|\Omega_t \}$$

where \( S(j) \) denotes the probability that firm’s newly adjusted labor force will survival for \( j \) periods in the future, which is obtained by the expression \( S(j) = 1 - F(j) = \exp \left( - \left( \frac{j}{\lambda} \right)^{\tau} \right) \).

In fact, all equations I derived from the Calvo-adjustment model are intact except that future adjustment probabilities and the resulting distribution of firms over vintage groups \( \Theta \) should be adjusted according to this new assumption.

\textsuperscript{17} For detailed discussion on Weibull distribution, see technical appendix \textsuperscript{[B]}

\textsuperscript{18} However, the Weibull distribution is a kind of the reduced-form duration model, though it is interesting to derive the micro-foundation of the Weibull parameters, this subject is clearly out of the scope of this paper. I leave it for the future research agenda.
Aggregate Accounting

If all firms adjust their employment according to hazard function (49), then the distribution of firm over vintage groups $\Theta$ follows the probability density function of the Weibull distribution. So we can use it to aggregate the economy as we did in the baseline model.

$$L_t = \theta(0) l_t^* + \theta(1) l_{t-1}^* + ... + \theta(J) l_{t-J}^* = \sum_{j=0}^{J} \theta(j) l_{t-j}^*$$  \hfill (32)

Where $\theta(j)$ denotes fractions of adjusting firms on the time period $j$. The parameter $J$ is determined by the time, at which the density of firms becomes close to zero. Aggregate employment is calculated by multiplying the past optimal labor inputs with their corresponding fractions, and $\theta(j)$ is obtained by:

$$\theta(j) = F(j + 1) - F(j) = \exp \left( - \left( \frac{j}{\lambda} \right)^\tau \right) - \exp \left( - \left( \frac{j + 1}{\lambda} \right)^\tau \right),$$  \hfill (33)

Labor Demand Equation in the Weibull-adjustment Model

The dynamic labor demand equation implied by the Weibull-adjustment Model is as follows:\textsuperscript{19}

$$\Psi(1 - a - b) \tilde{l}_{0,t} = \Psi \beta \bar{\varpi}(1 - a - b) E_t [\tilde{l}_{0,t+1}] - \frac{b \bar{R}}{\bar{F}} \tilde{R}_t - (1 - b) \tilde{w}_t + z_t$$  \hfill (34)

Where $\Psi = \sum_{j=0}^{\infty} S(j) \beta^j$ and $\bar{\varpi}$ is an constant whose value is dependent on the values of the Weibull parameters. Analog to the Calvo-adjustment model, parameter $\Psi$ and $\bar{\varpi}$, whose values are determined by two parameters in the Weibull distribution, govern the dynamic properties of the labor adjustment. This equation also implies that the optimal individual labor adjustment is increasing in all expected future shocks $Z_{t+j}$ and decreasing in all expected future process of prices $W_{t+j}$ and $R_{t+j}$, and thus the 'front-loading' effect is also at work here. The predictable future productivity shocks and price changes affect current labor demand immediately due to uncertainty in the future labor adjustment process.

Note that, as I’ve discussed in the baseline model, using constant-return-to-scale technology, the firm’s problem only solves the optimal ratio of capital and labor. Here I also

\textsuperscript{19} The derivation of this equation is available from the author upon request.
need some degrees of decreasing-return-to-scale to avoid this problem.

Calibration

To evaluate the quantitative performance of the Weibull-adjustment RBC model (hereafter 'WBLRBC'), I calibrate the model as follows. For the most parameters, I use same values as in the Calvo-case. Here I emphasize on how to calibrate the Weibull parameters using evidence from the empirical labor literature.

I set the shape parameter $\tau$ to be 1.2 that implies the increasing hazard function. Varejão and Portugal (2006) estimated the Weibull duration model using survey data of Portuguese employers, and found the shape parameter is in the range between 1.174 to 1.309. To calibrate the mean duration of employment, I use empirical evidence on average adjustment ratio from the aggregate net flow data. Caballero and Engel (1993) estimated a constant hazard function using U.S. manufacturing employment and job flow data(1972:1-1986:4), and got the result that, on average, 22.9% of firms in the U.S. adjust their employment per quarter. As a result, I choose 0.23 as the value of mean adjustment ratio, and it implies that the mean duration of employment amounts to 4.35 quarters.

Finally, the scale parameter $\lambda$ is obtained by using equation (48). The characteristic life of the Weibull distribution is equal to 1.38 quarters, given $\tau = 1.2$ and the average duration of 4.35 quarters.

A summary of calibration values can be found in the Table(3).

Numerical Results

In table(7), simulation results of the Weibull-adjustment model are presented. First of all, the most striking result is that aggregate employment in the WBLRBC model fluctuates almost as volatile as output. The relative volatility of aggregate employment is 93%, which is close to the level of total hours and aggregate employment in the data.

Second, as I assume capital can be adjusted along with labor at any periods, lumpy adjustment in the labor market is also infected to the capital market. Spikes of labor adjustments coordinate large adjustments in capital, so that volatility of investment is also high in this model. Letterie, Pfann, and Polder (2004) found that both labor and capital adjustments are lumpy in the data of dutch manufacturing sector, and they are interrelated with respect to the timing of adjustment.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9902</td>
<td>annual real rate 4%</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>annual depreciation rate 10%</td>
</tr>
<tr>
<td>$b$</td>
<td>0.329</td>
<td>to match capital to output ratio of 2.35 (Thomas and Khan (2004))</td>
</tr>
<tr>
<td>$a$</td>
<td>0.58</td>
<td>labor’s share of output (King, Plosser, and Rebelo (1988))</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1</td>
<td>$\log C$ common in the literature</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0</td>
<td>Indivisible labor assumption (Hansen, 1985)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.77</td>
<td>average adjustment rate of 0.23 (Caballero and Engel (1993))</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.38</td>
<td>characteristic life of the Weibull distribution</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1.2</td>
<td>increasing hazard function (Varejão and Portugal (2006))</td>
</tr>
<tr>
<td>$\varsigma$</td>
<td>0.95</td>
<td>Solow residual estimate,</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.007</td>
<td>Solow residual estimate,</td>
</tr>
</tbody>
</table>

Table 2: Calibration Values

Based on these facts, we can conclude that the WBLRBC model possesses a strong propagation mechanism both in terms of persistence and amplification.

![Impulse responses to a shock in technology](image)

Figure 2: Impulse Responses of the Weibull-adjustment model

In figure 2, impulse response functions confirm what we found in the second-moment table. On the one hand, peaks of the impulse responses show that aggregate employment reacts almost as strong as the output, on the other hand, response of individual firm’s labor adjustment is lumpy in the sense of strong and rapid reaction to shocks. In addition,
responses of output and employment to shocks are humped-shaped indicating persistence in dynamics generated by the model.

**Discussion on the role of the shape parameter**

The key question addressed in this section is why the Weibull adjustment model can amplify aggregate employment compared to the Calvo-adjustment model. Note that the only change in the former is to assume labor adjustment process follows the Weibull distribution with the shape parameter \( \tau = 1.2 \), which implies the increasing hazard function, while the Calvo adjustment model has the implied Poisson adjustment process, which corresponds to the value of the shape parameter being equal to one. I will analyze the effects of the changing shape parameter on both micro- and macroscopic aspects respectively.

The Calvo adjustment model is characterized by the memoryless Poisson process, which gives rise to a monotone decreasing density function and a flat hazard function (dark blue line in both figures). By contrast, the Weibull model with the shape parameter greater than one features the increasing hazard function in time-since-last-adjustment. It amounts to assuming that the newly-adjusted firms remember that they just changed their labor input and expect that the next adjustment will not come soon. This can be seen when you compare the shapes of Weibull distribution densities with various values for the shape parameter. Formally, the mode of each distribution density is increasing in values of \( \tau \).

In contrast to the Calvo case, in the Weibull adjustment model, both distributional parameters (\( \tau \) and \( \lambda \)) affect dynamics of the model. I calibrate \( \alpha \) and \( \lambda \) such that they imply the same average duration of employment, so the underlying degree of employment rigidity is same in both models. Keeping this in mind, we can focus on the economic
interpretation of the shape parameter $\tau$. As shown above, rising $\tau$ shifts the peak of
the density function to the right. As a result, it can be interpreted as a measure of risk
attitude of agents in the labor market. If agents are risk-neutral, they will behave like in
the Calvo case, where they do not care about the timing of adjustment. By contrast, if
agents are risk-averse, they will not forget when they adjust their labor force and consider
the risk accordingly. This can be captured by a higher value of $\tau$. In sum, the Weibull
model has a more realistic setup that captures not only the average level of risk in the
market but also the attitude of agents toward the risk.

5. Conclusion Remarks

In this paper, I investigate quantitative significance of lumpy labor adjustment as a propa-
gation mechanism for business cycles. The key message conveyed by this paper is that
uncertainty in the labor adjustment process induce firms to make precautionary labor
adjustment, which amplifies the volatility of the individual factor demand. In addition,
the aggregation mechanism matters for the aggregate dynamics, and hence including the
information of distribution of agents enriches the propagation mechanism of the RBC
model.

Despite these successes, the model is subject to the following limitations: first, it is based
on the premise that the lumpy labor market is characterized by an stationary distribution
of firms over 'time-since-last-adjustment' groups. I assume that both hazard function and
the distribution of aged-factor stocks change only with some deep parameters that are
influenced by the underlying economic mechanisms instead of transitory shocks to the
economy. Whether this premise can be justified empirically or it serves only as a theo-
retical parsimony is still an open question; second, capital market is modeled too simple
in this model. Empirical evidence shows that both investment and labor adjustment are
lumpy, therefore it is interesting to see what effect of introducing adjustment costs on both
capital and labor market on business cycle fluctuations. third, the Weibull distribution
is a reduced-form model, so linking the value of its parameters with the micro-founded
structure of adjustment cost is interesting research topic for the future work.
A. Leisure Habit formation Model

In this appendix, I set up a habit formation model that is similar in spirit to the model by Wen (1998), where habit is assumed on leisure instead of on consumption. I also simplify the production technology to the constant-return-to-scale one to facilitate the comparison with the Calvo-adjustment model derived in this paper.

I consider a representative household’s problem, where infinite-lived agent maximizes the expected discounted utility by choosing nonnegative values for optimal sequences of consumption, leisure and capital stock of the next period, subject to the technology, resource constraint and the evolution equation of capital stock.

\[ U = \max_{\{C_t, L_t, K_{t+1}\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (U(C_t) + V(L_t)) \right\}. \]  

subject to the sequence of budget constraints at each period:

\[ C_t + I_t \leq Z_t N_t^a K_t^{1-a} \]  

The capital stock evolves according to the following law of motion:

\[ K_{t+1} = (1 - \delta) K_t + I_t \]  

and the time endowment constraint

\[ 1 = N_t + L_t \]  

The instantaneous utility \( U(.) \) and \( V(.) \) are bounded, continuously differentiable, strictly increasing and strictly concave in consumption and leisure \( (L_t) \). I take the following function forms for instantaneous utility:

\[ U(C_t) + V(L_t) = \log C_t + \chi \log (h(B)L_t) \]  

Where \( h(B) \) is a polynomial distributed lag satisfying:

\[ h(B) = 1 - \sum_{i=1}^{I} h_i B^i, \quad \text{where} \quad |h_i| < 1, \quad \sum_{i=1}^{I} |h_i| < 1 \quad I \leq \infty. \]

For simplicity, I set \( I = 1 \), which indicates habits formation follows a AR(1) process, so
that \( h(B)L_t = L_t - h_1 L_{t-1} \). In addition, I set the value of habit parameter \( (h_1) \) to be 0.75, according to the direct estimate by Wen (1998).

Finally, I restrict that no capital is left unused at the end of the life, and the transversality condition is added:

\[
\lim_{T \to \infty} E_0 \left[ \prod_{t=0}^{T} R_{t,t+1}^{-1} \right] R_{T+1} = 0;
\]

(41)

The equilibrium conditions are summarized in the following FONCs:

\[
\frac{1}{C_t} = \lambda_t;
\]

(42)

\[
1 = E_t \left[ \beta \left( \frac{\lambda_{t+1}}{\lambda_t} \right) R_{t+1} \right],
\]

(43)

\[
\frac{\chi}{L_t - h_1 L_{t-1}} = \lambda_t W_t + \beta \frac{\chi h_1}{L_{t+1} - h_1 L_t},
\]

(44)

Where \( \lambda_t \) is the lagrange multiplier associating to the resource constraint.

The final log-linearized equations for this model are:

\[
\beta h_1 \hat{l}_{t+1} = (1 + \beta h_1^2) \hat{l}_t - h_1 \hat{l}_{t-1} + (1 - \beta h_1)(1 - h_1)(\hat{w}_t - \hat{c}_t)
\]

(45a)

\[
\hat{w}_t = \hat{y}_t - \hat{n}_t
\]

(45b)

\[
0 = E_t [\hat{c}_t - \hat{c}_{t+1} + \hat{R}_{t+1}]
\]

(45c)

\[
\hat{R}\hat{R}_t = \hat{r}\hat{y}_t - \hat{r}\hat{k}_t
\]

(45d)

\[
\hat{y}_t = z_t + a\hat{n}_t + (1 - a)\hat{k}_t
\]

(45e)

\[
\hat{k}_{t+1} = \delta\hat{i}_t + (1 - \delta)\hat{k}_t
\]

(45f)

\[
\delta\hat{i}_t = \bar{Y}/\bar{K}\hat{y}_t - \bar{C}/\bar{K}\hat{c}_t
\]

(45g)

\[
\hat{n}_t = -\frac{\hat{L}}{\bar{N}}\hat{l}_t
\]

(45h)

\[
\hat{z}_{t+1} = \rho z_t + v_t
\]

(45i)
B. Weibull Distribution and Aggregation

B.1. Weibull Distribution

The PDF of Weibull distribution is given by the following expression:

\[ P_r(j) = \frac{\tau}{\lambda} \left( \frac{j}{\lambda} \right)^{\tau-1} \exp \left( - \left( \frac{j}{\lambda} \right)^{\tau} \right) \]  \hspace{1cm} (46)

and the cumulative probability function is:

\[ F(j) = 1 - \exp \left( - \left( \frac{j}{\lambda} \right)^{\tau} \right) \]  \hspace{1cm} (47)

The parameters that characterize the Weibull distribution are the scale parameter \( \lambda \) and the shape parameter \( \tau \). The shape parameter determined the shape of the Weibull's pdf function, e.g. when \( \tau = 1 \), it reduces to an exponential case; while \( \tau = 3.4 \), the Weibull amounts to the normal distribution. The scale parameter tells the characteristic life of the random process that amounts to the time, at which 63.2% of the firm will adjust their labor. This can be seen when you evaluate the cdf function of Weibull distribution at \( t \) equaling the scale parameter \( \lambda \), we have, \( F(\lambda) = 1 - e^{(-1)} = 0.632 \).

Note that it relates to the mean duration \( \bar{j} \) according to the following equation:

\[ \bar{j} = \frac{1}{\alpha} = \lambda \Gamma\left(\frac{1}{\tau} + 1\right), \]  \hspace{1cm} (48)

where \( \Gamma() \) is the Gamma function.

It follows that the hazard function of Weibull distribution is:

\[ H(j) = \frac{\tau}{\lambda} \left( \frac{j}{\lambda} \right)^{\tau-1} \]  \hspace{1cm} (49)

Note that this hazard is constant when the shape parameter \( \tau \) equals one, and increasing when \( \tau \) is greater than one.
Collection of Steady State Equations:

\[ \bar{L} = \frac{a[1 - \beta(1 - \delta)]}{\chi[1 - \beta(1 - \delta) - \delta \beta b]} \quad (50a) \]

\[ \frac{\bar{Y}}{\bar{K}} = \frac{1 - \beta(1 - \delta)}{b \beta} \quad (50b) \]

\[ \bar{K}^{1-b} = \frac{\bar{Z} \bar{K}}{\bar{Y} \bar{L}^s} \quad (50c) \]

\[ \frac{\bar{C}}{\bar{K}} = \frac{1 - \beta(1 - \delta) - \delta \beta b}{b \beta} \quad (50d) \]

\[ \bar{W} = \chi \bar{C} \quad (50e) \]

\[ \bar{R} = \frac{1}{\beta} \quad (50f) \]

\[ \bar{I} = \delta \bar{K} \quad (50g) \]

The Final Log-linearized Equations:

\[ \alpha \beta \kappa E_t[\hat{l}_{t+1}] = (1 + \alpha^2 \beta) \kappa \hat{l}_t - \alpha \kappa \hat{l}_{t-1} + \frac{b \bar{R}}{\bar{r}} \hat{R}_t + (1 - b) \hat{w}_t - z_t \quad (51a) \]

\[ \hat{l}_t = (1 - \alpha) \hat{l}_{0,t} + \alpha \hat{l}_{t-1} \quad (51b) \]

\[ \hat{w}_t = \phi \hat{l}_t + \hat{c}_t \quad (51c) \]

\[ 0 = E_t[\hat{c}_t - \hat{c}_{t+1} + \hat{R}_{t+1}] \quad (51d) \]

\[ \bar{R} \hat{R}_t = \bar{r} \hat{y}_t - \bar{r} \hat{k}_t \quad (51e) \]

\[ \hat{y}_t = z_t + a \hat{l}_t + b \hat{k}_t \quad (51f) \]

\[ \hat{k}_{t+1} = \delta \hat{i}_t + (1 - \delta) \hat{k}_t \quad (51g) \]

\[ \delta \hat{i}_t = \bar{Y} / \bar{K} \hat{y}_t - \bar{C} / \bar{K} \hat{c}_t \quad (51h) \]

\[ z_{t+1} = \varsigma z_t + v_t \quad (51i) \]
References


NICKELL, S., AND L. NUNZIATA (2000): “Employment Patterns in OECD Countries,” Cep discussion papers, Centre for Economic Performance, LSE.


### C. Tables

<table>
<thead>
<tr>
<th>Variables</th>
<th>Standard Deviation%</th>
<th>Relative S.D.</th>
<th>Cross Correlation with output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>-3</td>
</tr>
<tr>
<td>Hours*</td>
<td>1.69</td>
<td>0.98</td>
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<td>Employment*</td>
<td>1.41</td>
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<tr>
<td>Consumption</td>
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<td>0.74</td>
<td>0.57</td>
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<tr>
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<td>5.34</td>
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<td>0.43</td>
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<tr>
<td>Labor productivity</td>
<td>0.73</td>
<td>0.42</td>
<td>0.44</td>
</tr>
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Notes: all statistics are reported in Cooley (1995) Table(1.1)
*: Based on establishment survey.

Table 3: Business Cycle Statistics for the U.S. Economy

<table>
<thead>
<tr>
<th>Variables</th>
<th>Standard Deviation%</th>
<th>Relative S.D.</th>
<th>Cross Correlation with output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>-3</td>
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<tr>
<td>Hours</td>
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<tr>
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<td>0.26</td>
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<td>0.54</td>
<td>0.15</td>
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<tr>
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<td>0.38</td>
<td>0.31</td>
<td>0.02</td>
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<td>Output</td>
<td>1.24</td>
<td>1.00</td>
<td>0.22</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.04</td>
<td>0.04</td>
<td>0.32</td>
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<tr>
<td>Investment</td>
<td>3.84</td>
<td>3.10</td>
<td>0.27</td>
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<tr>
<td>Labor productivity</td>
<td>0.67</td>
<td>0.54</td>
<td>0.15</td>
</tr>
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Table 4: Business Cycle Statistics for the Standard RBC Model
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<thead>
<tr>
<th>Variables</th>
<th>Standard Deviation%</th>
<th>Relative S.D.</th>
<th>Cross Correlation with output</th>
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<tbody>
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<td></td>
<td></td>
<td></td>
<td>-3</td>
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<tr>
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<td>0.36</td>
<td>0.33</td>
<td>0.22</td>
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<td>0.30</td>
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<td>-0.33</td>
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<tr>
<td>Real wage</td>
<td>0.76</td>
<td>0.71</td>
<td>0.27</td>
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<tr>
<td>Consumption</td>
<td>0.36</td>
<td>0.33</td>
<td>0.08</td>
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<tr>
<td>Output</td>
<td>1.08</td>
<td>1.00</td>
<td>0.26</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.04</td>
<td>0.04</td>
<td>0.37</td>
</tr>
<tr>
<td>Investment</td>
<td>3.24</td>
<td>3.01</td>
<td>0.31</td>
</tr>
<tr>
<td>Labor productivity</td>
<td>0.76</td>
<td>0.71</td>
<td>0.27</td>
</tr>
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Table 5: Business Cycle Statistics for the Leisure Habit Formation Model

<table>
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<th>Standard Deviation%</th>
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</tr>
</thead>
<tbody>
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<td>Agg. employment</td>
<td>0.85</td>
<td>0.63</td>
<td>0.34</td>
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<tr>
<td>Firm’s employment</td>
<td>1.69</td>
<td>1.25</td>
<td>0.43</td>
</tr>
<tr>
<td>Capital</td>
<td>0.37</td>
<td>0.27</td>
<td>-0.34</td>
</tr>
<tr>
<td>Real wage</td>
<td>0.37</td>
<td>0.27</td>
<td>0.10</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.37</td>
<td>0.27</td>
<td>0.10</td>
</tr>
<tr>
<td>Output</td>
<td>1.35</td>
<td>1.00</td>
<td>0.30</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.06</td>
<td>0.04</td>
<td>0.42</td>
</tr>
<tr>
<td>Investment</td>
<td>3.74</td>
<td>2.77</td>
<td>0.35</td>
</tr>
<tr>
<td>Labor productivity</td>
<td>0.59</td>
<td>0.44</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 6: Business Cycle Statistics for the Calvo-adjustment RBC Model
<table>
<thead>
<tr>
<th>Variables</th>
<th>Standard Deviation%</th>
<th>Relative S.D.</th>
<th>Cross Correlation with output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>-3</td>
</tr>
<tr>
<td>Agg. employment</td>
<td>1.80</td>
<td>0.93</td>
<td>0.32</td>
</tr>
<tr>
<td>Firm’s employment</td>
<td>2.33</td>
<td>1.21</td>
<td>0.36</td>
</tr>
<tr>
<td>Capital</td>
<td>0.58</td>
<td>0.30</td>
<td>-0.35</td>
</tr>
<tr>
<td>Real wage</td>
<td>0.55</td>
<td>0.28</td>
<td>-0.01</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.55</td>
<td>0.28</td>
<td>-0.01</td>
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<td>Output</td>
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<tr>
<td>Interest rate</td>
<td>0.07</td>
<td>0.04</td>
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<td>Investment</td>
<td>6.67</td>
<td>3.46</td>
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<tr>
<td>Labor productivity</td>
<td>0.39</td>
<td>0.20</td>
<td>-0.29</td>
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Table 7: Business Cycle Statistics for the Weibull-Adjustment RBC Model
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