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Lumpy Labor Adjustment as a Propagation Mechanism of Business Cycles

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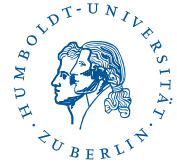


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Lumpy Labor Adjustment as a Propagation Mechanism of Business Cycles

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Abstract

I explore the aggregate effects of micro lumpy labor adjustment in a prototypical RBC model, which embeds a stochastic labor duration mechanism in the spirit of Calvo(1983), and it extends this approach by introducing a Weibull-distributed labor adjustment process to capture the increasing hazard function corroborated by the micro data. My principal findings are: The aggregate labor demand equation derived from the baseline Calvo-style model corresponds to the same reduced form as the quadratic-adjustment-cost model and deep parameters have a one-to-one mapping. However, this result does not hold in general. When introducing the Weibull labor adjustment, the aggregate dynamics vary with the extent of increasing hazard function, e.g., the volatility of aggregate labor is increasing, but the persistence is decreasing in degree of the increasing hazard of the labor adjustment.

JEL Classification: E32; E24; C68

Keywords: business cycles; heterogeneous labor rigidity; increasing hazard function; Weibull distribution

Introduction

Micro evidence shows that, at the plant level, non-convex adjustment costs cause firms to discretely adjust their production factor at infrequent intervals of stochastic length, i.e., labor adjustment at the firm level exhibits lumpy, asynchronous pattern. Earlier evidence has been presented by Hamermesh (1989). Recently, Letterie, Pfann, and Polder (2004) investigate the dynamic interrelation between factor demand with plant-level data for the Dutch manufacturing sector. They find that both adjustments of capital and labor are lumpy, and they are coordinated with each other in time. In addition, Varejão and Portugal (2006) find that large employment adjustments (larger than 10% of the plant's labor force) account for about 66% of the total job turnover, and on average around 75% of all observed Portuguese employer do not change employment over an entire quarter.

In macroeconomics, the (S,s) model is frequently used to investigate the aggregate effects of the lumpy factor adjustment². The major theme discussed in the literature is whether the micro-level lumpy factor adjustment has a significant impact on the aggregate dynamics. The earlier partial equilibrium (S,s) models of labor adjustment (See: e.g. Caballero and Engel, 1993, Caballero, Engel, and Haltiwanger, 1997) found that employment growth depends on the cross-sectional distribution of the employment deviation from optimal target. In particular, Caballero, Engel, and Haltiwanger (1997) found that the adjustment hazard rises with large shocks and thus amplifies the shock's effect in aggregate adjustment. These findings were taken as evidence that lumpy adjustment pattern at firm's level matters for aggregate economy. However, the recent development of the general equilibrium (S,s) models show that this considerable effect of lumpiness at the plant level disappears with changes in the equilibrium prices. King and Thomas (2006) construct a general equilibrium (S,s) model of discrete employment adjustment and find that simulation results are 'observationally' equivalent to the quadratic-adjustment-cost model³.

In this paper, I pursue the business cycle implications of the lumpy labor adjustment from a new perspective. It is motivated by the evidence of the empirical hazard function of labor adjustment⁴. Varejão and Portugal (2006) estimated parameters of a Weibull hazard function with the Portuguese employer survey data on labor adjustments, they found that the shape parameter lies in the range between 1.174 and 1.309, indicating an

² Caplin and Spulber (1987) was the early work applying the (S,s) approach to macro models.

³ Similar results have been also found in the capital adjustment context. See, e.g., Veracierto (2002) and Thomas (2002).

⁴ To quantify the concept of lumpy labor adjustment, Caballero, Engel, and Haltiwanger (1997) used a hazard function in terms of economic deviations from optimal targets.

increasing hazard function in the elapsed time.

Motivated by this evidence, I tackle the issue in a novel way, in which I embeds a general stochastic labor duration mechanism in a prototypical RBC model. The essence of the model is that, when making the firm's adjustment probability depend on the amount of time that has elapsed since the last adjustment, the labor dynamics in different vintage groups are heterogeneous with respect to both persistence and volatility. In these circumstances, aggregation mechanism (the distribution of labor vintages) matters for the aggregate behavior, so that the propagation mechanism of the model is significantly enriched.

To formalize this idea, in the benchmark model I introduce the firm's stochastic labor adjustment in the spirit of Calvo (1983), which implies that the underlying labor adjustment process is characterized by a constant hazard function. As a result, even though the 'front-loading' effect helps amplify the volatility of labor at the micro level, the large labor adjustment is neutralized by the restrictive aggregation mechanism implied by the Calvo-style labor adjustment. To this end, I show analytically that the aggregate labor demand equations derived from the Calvo-adjustment model and the quadratic-adjustment-cost model correspond to the same reduced form, and deep parameters have a one-to-one mapping of each other. With these results I confirm the finding by King and Thomas (2006) discussed above.

In the second part of the paper, I extend the baseline model to a more general case, in which I implement a Weibull-distributed labor adjustment process to capture features of increasing hazard rates corroborated by micro evidence. This extension has a significant impact on both the persistence and the magnitude of business cycles. When calibrating the model with the empirically plausible hazard function, adjustment probabilities vary across labor vintages. The longer a firm remains inactive, the more likely it adjusts its labor in the current period. As a result, heterogeneous labor dynamics emerge naturally from the underlying labor adjustment process, and as shown in the numerical results, the model matches several important aspects of the U.S. business cycles. In particular, the model can jointly account for persistent aggregate labor, smoothing real wages and features observed in both micro and macro labor adjustment data: i.e. at the micro level, labor adjustment exhibits a lumpy pattern in response to the technology shock, while the aggregate employment reacts smoothly and sluggishly. In addition, sensitivity analysis shows that aggregate dynamics vary with the extent of increasing hazard function, e.g., the volatility of aggregate labor is increasing, but the persistence is decreasing in degree of the increasing hazard of the labor adjustment.

My model is intrinsically related to the (S,s) approach with respect to many modeling concepts, it contributes to the literature, however, in the sense that it uses a more tractable framework to generate the findings of the general equilibrium (S,s) models, and then it extends the approach in an empirically plausible direction, showing that the micro lumpy labor adjustment could play an important role in propagating business cycles.

The remainder of the paper is organized as follows: Section 1 introduces the baseline model with a staggered employment adjustment at the firm's level ; In section 2, I show some analytical results to reveal the key mechanism underlying the model; Section 3 extends the basic model to the Weibull-adjustment model; and in section 4 I introduce the calibration of model parameters and present simulation results; Section 5 contains some concluding remarks.

1 The Baseline Model

In this section, I set up the baseline model in a RBC framework. The main feature of this basic model is to introduce the lumpy labor adjustment in the spirit of Calvo(1983). Even though this modeling idea has been existing for a long time and it is familiar to most researchers in macroeconomics, I formulate it here formally in the context of the statistical duration model, which also serves as the solid theoretical base for the extension in the next section.

1.1 Household

There is a continuum of identical households, who are endowed with K_0 units of capital at $t = 0$ and then with one additional unit for each subsequent period of time, which can be spent on either working or leisure. The infinitely-lived representative household chooses consumption, labor supply and investment to maximize the expected discounted utility:

$$U = \max_{\{C_t, L_t, I_t\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (U(C_t) - V(L_t)) \right\}. \quad (1)$$

The instantaneous utility $U(\cdot)$ and $V(\cdot)$ are bounded, continuously differentiable, strictly increasing and strictly concave in consumption and leisure. I take the following function

form for instantaneous utility:

$$U(C_t) - V(L_t) = \frac{C_t^{1-\eta}}{1-\eta} - \chi \frac{L_t^{1+\phi}}{1+\phi} \quad (2)$$

In each period, households receive wage income, rental payment for their capital stock and a lump-sum transfer of net profits resulting from firm ownership, which can be spent on consumption and investment in capital stocks. Due to the assumption of complete financial markets, all households can perfectly share their idiosyncratic income risk, so that they consume and invest the same amount. Consequently, the sequence of aggregate budget constraints is given by:

$$C_t + I_t \leq W_t L_t + R_t K_t + T_t \quad (3)$$

The capital stock evolves according to the following law of motion:

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (4)$$

Finally, I impose the transversality condition for the capital stock:

$$\lim_{T \rightarrow \infty} E_0 \left[\prod_{t=0}^T R_{t,t+1}^{-1} \right] K_{T+1} = 0, \quad (5)$$

Based on this setup, the following first order conditions must hold in an equilibrium:

$$\chi L_t^\phi C_t^\eta = W_t, \quad (6)$$

$$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\eta} (r_{t+1} + 1 - \delta) \right], \quad (7)$$

1.2 Firms

The economy is populated by a continuum of firms, which is normalized to one. Firms operate in a rigid labor market, where some unspecified frictions cause a fixed ratio of firms not to adjust their labor input in each period. In effect, the more rigid the labor market is, the lower the adjustment ratio is, as expected by agents in the market. Due to this rigid labor adjustment process, firms are differentiated with respect to the amount

of time that has elapsed since the last adjustment and hence by their stocks of labor force. I index firms by j , corresponding to the "time-since-last-adjustment". I call them hereafter "labor vintages". Furthermore, given the complete financial market, adjusting firms choose a common target labor adjustment at each period. Firms in any labor vintage share an equal amount of employment, and hence the state of the economy can be summarized by the vintage index j with the corresponding labor stock $(l_{j,t})$.

1.2.1 Stochastic Labor Adjustment Process and Distribution of Firms

Now I formally introduce the staggered labor adjustment process in the context of the statistical duration model.

Here I consider a process in which the firm's employment adjustment occurs randomly over time. It turns out that under some basic assumptions with respect to independence and uniformity in time, this random process is governed by the Poisson process⁵. This assumption simplifies the real-world continuous factor adjustment decisions in terms of a sequence of generic trials that satisfy the following assumptions:

- Each trial has two possible outcomes, called adjustment and non-adjustment.
- The trials are memoryless, i.e. the outcome of one trial has no influence over the outcome of another trial.
- For every firm, the probability of adjusting is $1 - \alpha$ and the probability of non-adjusting is α .

Formally I define the labor adjustment process as a Bernoulli process as follows:

Definition: Given a probability space (Ω, Pr) together with a random variable X over the set $\{0, 1\}$, so that for every $\omega \in \Omega$, $X_i(\omega) = 1$ with probability α and $X_i(\omega) = 0$ with probability $1 - \alpha$, where $\Omega = \{\text{adjusting, non-adjusting}\}$, a Bernoulli process is a sequence of integers $Z^\omega = \{n \in Z : X_n(\omega) = 1\}$.

Given the factor adjustment process follows the Bernoulli process, the probability of receiving zero adjusting signal in an interval of j periods is:

$$Pr(0) = \binom{j}{0} (1 - \alpha)^0 \alpha^j = \alpha^j \quad \text{for } j = 0, 1, 2, \dots \quad (8)$$

⁵ In this paper, as I write the model in the discrete-time, the discretized adjustment process follows the Bernoulli trials process, which is the discrete version of the Poisson process.

And, the probability that a duration spell terminates at the period j is

$$Pr(j) = (1 - \alpha)\alpha^{j-1} \quad \text{for } j = 0, 1, 2, \dots \quad (9)$$

Define $\Theta = \{\theta(j)\}_{j=0}^{\infty}$ as the distribution of firm over labor vintages. It can be easily shown that $\theta(j) = (1 - \alpha)\alpha^j$ for $j = 0, 1, 2, \dots$ ⁶.

The hazard function corresponding to the Bernoulli process is:

$$H(j) = \frac{\theta(j)}{1 - F(j)} = 1 - \alpha \quad (10)$$

The hazard function embeded in the Bernoulli distribution is constant. It implies that the probability of adjusting is independent of the period time elapsed. The aggregate stock of labor can be summed up with respect to the distribution of firms over labor vintages, i.e. the aggregate labor is the weighted sum of all past optimal labor demands, and weights are equal to the probability density function over vintages j .

Finally aggregate labor is obtained by⁷:

$$L_t = \sum_{j=0}^{\infty} \theta(j)l_{j,t} = \sum_{j=0}^{\infty} (1 - \alpha)\alpha^j l_{j,t} \quad (12)$$

Since the fraction of firms that adjust their employment is randomly drawn across the population, it follows that the recursive law for aggregate employment is obtained by:

$$L_t = (1 - \alpha)l_{0,t} + \alpha L_{t-1} \quad (13)$$

or equivalently,

$$\Delta L_t = L_t - L_{t-1} = (1 - \alpha)(l_{0,t} - L_{t-1}) \quad (14)$$

⁶ Because, by assumption there is $1 - \alpha$ fraction of firm in the group zero, and α percent of them goes to group one, this gives the density of group one to be $(1 - \alpha)\alpha$. Similarly, α percent of untis in group one goes to group two, so the density of group two is $(1 - \alpha)\alpha^2$, and so on.

⁷ Note that equation 18 implies that firms in the vintage j group must also use same amount of capital. Thus the distribution of plants over labor is the same as over capital stocks. As a result, we can aggregate capital in the same way.

$$K_t = \sum_{j=0}^{\infty} (1 - \alpha)\alpha^j k_{j,t} \quad (11)$$

This equation reveals the partial adjustment nature of this model, that the actual job turnover is only a fraction of the optimal adjustment. The speed of adjustment depends on the extent of market rigidity $(1 - \alpha)$. If no friction exists in the labor market $(\alpha = 0)$, all firms re-optimize their labor by $l_{0,t}$, where this model is then reduced to the standard RBC case.

1.2.2 Capital Market and Technology

Furthermore I assume that firms can access an instantaneous rental market for capital, which is supplied by households in any given period. This assumption is desirable because the firm's first order condition requires the capital and labor ratio to be identical in the entire economy⁸, the instantaneous capital market makes possible for those firms that can not change their employment to fulfill this requirement. The aggregate capital stock, however, is still predetermined by the household.

Firms use a decreasing-return-to-scale technology to produce output⁹.

$$y_t = Z_t l_t^a k_t^b - \iota \quad \text{and} \quad a + b < 1 \quad (15)$$

Where (ι) denotes the fixed cost of operation, which is equal to the profits earned in the steady state. Consequently, firms expect zero profit and thus the number of firms is constant in the long run.

Z_t summarizes the aggregate productivity shock, which consists of a trend component \bar{Z}_t and a realization of a stochastic process z_t . The trend component \bar{Z}_t evolves at a constant growth rate g , while z_t follows an AR(1) process in logs:

$$Z_t = \bar{Z}_t z_t, \quad (16)$$

$$\text{where} \quad z_t = z_{t-1}^\xi e^{v_t}, \quad \text{and} \quad v_t \sim i.i.d.N(0; \sigma^2)$$

⁸ This is the case when the production function is constant-return-to-scale, however, when assuming decreasing-return-to-scale, as shown in equation(18), a power function of labor and capital depends only on the rental rate and aggregate shocks, hence it should be identical for all firms in the economy.

⁹ In the equation (21), it shows that some extent of DRTS is needed to show the lumpy effect at the firm level. However, my main numerical results do not crucially depend on this assumption.

1.2.3 Firm's optimization Problem

In spite of heterogeneous nature of the problem, the firms' maximization problem can be written in a representative fashion: a typical firm maximizes the expected discounted real value of all future profits by choosing nonnegative values for current optimal labor $l_{0,t}$ and a sequence of optimal capital stocks $\{k_{j,t}\}_{j=0}^{\infty}$, taking the real wage w_t and real rental rate r_t as given.

$$\max_{l_{0,t}, \{k_{j,t}\}_{j=0}^{\infty}} V_t = \sum_{j=0}^{\infty} E_t \{ \tilde{\beta}_{t,t+j} \alpha^j [F(l_{0,t}, k_{j,t}) - w_{t+j} l_{0,t} - r_{t+j} k_{j,t+j}] | \Omega_t \} \quad (17)$$

where $\tilde{\beta}_{t,t+j}$ is the stochastic discount factor, which is defined according to equation (7).

Since, at the steady state, all real variables except for labor grow at rate g along the balanced growth path, I will work with detrended variables without changing the notions from now on.

First order conditions for the firm's optimization problem are:

$$r_t = f_k(j, t) = b z_t \frac{l_{j,t}^a}{k_{j,t}^{1-b}} \quad (18)$$

$$\sum_{j=0}^{\infty} \alpha^j E_t [\tilde{\beta}_{t,t+j} (a z_{t+j} l_{0,t}^{a-1} k_{j,t+j}^b)] = \sum_{j=0}^{\infty} \alpha^j E_t [\tilde{\beta}_{t,t+j} (w_{t+j})] \quad (19)$$

Equation (19) shows that the optimal labor demand is determined by balancing all future discounted marginal benefits of adding one more worker (marginal product of labor) and the marginal costs of having a worker (real wage). This condition contrasts to the standard RBC case, where real wage is equal to labor productivity period by period. Due to the labor adjustment friction the optimal labor demand in this model becomes a forward-looking condition.

To reveal the model's implication for the optimal labor demand at the firm level, I derive the firm's optimal employment demand by combining first order conditions and solving for the plant's optimal labor demand $l_{0,t}$ at period t :

$$l_{0,t}^{\frac{1-a-b}{1-b}} = \frac{ab^{b/1-b} \sum_{j=0}^{\infty} \alpha^j E_t [\tilde{\beta}_{t,t+i} z_{t+j}^{1/1-b} / r_{t+j}^{b/1-b}]}{\sum_{j=0}^{\infty} \alpha^j E_t [\tilde{\beta}_{t,t+i} w_{t+j}]}, \quad (20)$$

Equation(20) shows that, at the firm level, the optimal labor demand reacts to all future shocks and the equilibrium prices. In the case of the first-order-approximation, it is increasing in all expected future shocks z_{t+j} and decreasing in all expected future prices w_{t+j} and r_{t+j} . In the partial equilibrium, where prices are constant, it is easy to show that a positive persistent shock will make the individual labor adjustment higher than that in the frictionless economy. Firms hire more labor than they currently need to hedge the risk they might not be able to re-optimize it in the near future and vice versa for the negative shocks. It is the 'front-loading' effect of the labor demand under the uncertainty in the labor adjustment process. Note that the magnitude of the front-loading effect is dependent on the labor rigidity parameter α . The larger the value of α is, the higher weight is attached on the expectations of future variables. In another words, when frictions in the labor market are more severe, the labor demand is more sensitive to the future economic state.

1.3 Equilibrium

Given an exogenous stochastic process for aggregate technology shocks and the common knowledge of the firms' distribution across vintage groups Θ , I define the competitive equilibrium as a set of stochastic processes of endogenous variables $\{Y_t, C_t, L_t, l_{j,t}, k_{j,t}, I_t, K_t, w_t, r_t\}_{t=0}^{\infty}$ such that:

1. Given K_t and the market prices $\{w_t, r_t\}_{t=0}^{\infty}$, the sequences $\{C_t^s, L_t^s, I_t^s\}_{t=0}^{\infty}$ ¹⁰ solve the representative household's maximization problem (1) subject to (2)-(5).
2. Given $\{w_t, r_t\}_{t=0}^{\infty}$, $\{l_{j,t}, k_{j,t}\}_{t=0}^{\infty}$ solve the Firms' profits maximization problem (17) subject to production technology (15) and exogenous technology shock process (16).
3. Aggregate demands for employment L_t^d and capital K_t^d are determined by (12) and (11) respectively.
4. Markets clear: $L_t^s = L_t^d = L_t$ in labor market, $K_t^s = K_t^d = K_t$ in capital market and $C_t + I_t = Y_t$ in the goods market.
5. Finally, market's equilibrium determines the equilibrium real wage and rental rate $\{w_t, r_t\}_{t=0}^{\infty}$

¹⁰ Here, superscript s denotes "supply"; Similar notation d for "demand"

2 Analysis

2.1 Dynamic Labor Demand Equations

To gain further intuition of the firm's behavior, I log-linearizing the FOCs (18) and (19) around the non-stochastic steady state¹¹. In contrast to the other partial adjustment model, the Calvo-adjustment model implies different labor demand behaviors at different aggregation levels.

$$\hat{l}_{0,t} = \alpha\beta E_t[\hat{l}_{0,t+1}] - \frac{b(1-\alpha\beta)}{1-a-b} \hat{r}_t - \frac{(1-b)(1-\alpha\beta)}{1-a-b} \hat{w}_t + \frac{1-\alpha\beta}{1-a-b} z_t \quad (21)$$

Equation (21) reveals that at the firm level optimal adjustment is forward-looking and a trade-off exists between the weights assigned to the current shock and future shocks. When α is large, firms put more weight on future shocks than on current shocks.

Together with equation (13), the aggregate labor demand equation is obtained by:

$$\alpha\beta\kappa E_t[\hat{l}_{t+1}] - (1 + \alpha^2\beta)\kappa\hat{l}_t + \alpha\kappa\hat{l}_{t-1} - b\hat{r}_t - (1-b)\hat{w}_t + z_t = 0 \quad (22)$$

where $\kappa = \frac{(1-a-b)}{(1-\alpha)(1-\alpha\beta)}$. The aggregate labor demand (22) exhibits more complex dynamics, which are not only dependent on the forward-looking component, but also on the lagged labor. Moreover, it demonstrates that equilibrium prices work here as a counter factor to the technology shock. In this equation, one can explicitly see that, when the aggregate technology shock, real wage and interest rate all rise by 1%, then the total effect of those changes on the aggregate labor are exactly cancelled.

Note that both equations require some degree of decreasing-returns-to-scale ($1-a-b > 0$) to ensure that the size of labor demand is determined.

2.2 Equivalence to the Quadratic-adjustment-cost Models

The quadratic-adjustment-cost model has lost footing in macroeconomic literature because economists have grown disenchanted with its smoothing and synchronous implication relating to the firm-level factor adjustment. As discussed in the introduction, mounting micro evidence shows that firms adjust their labor in a discrete and asynchronous fashion. Despite this fact, the quadratic adjustment cost model has been used

¹¹ Variables with hat are denoted as log deviation from the non-stochastic steady state, such as $\hat{x}_t = \log X_t - \log \bar{X}$; and the derivation is shown in a technical appendix, which is available upon request.

widely in theoretical and empirical work, because they are easily solved and produce aggregate equations in a form suitable for estimation. By contrast, as I have shown in the equation (20), the Calvo-adjustment model can capture lumpy and asynchronous features in firm's labor adjustment, while aggregate labor demand in this model is characterized by a smoothing AR(2) dynamic process (see: Equation 22). The key question addressed in this subsection is whether the quadratic-adjustment-cost model is equivalent to the Calvo-adjustment model concerning the aggregate dynamics. If this is true, it can be treated as a reduced form model and is still valid in the empirical work using aggregate data.

In Appendix (A), I derive the aggregate labor demand equation from a textbook quadratic-adjustment-cost model (See e.g. Hamermesh, 1993). As Rotemberg (1987) has shown that the equivalence between the Calvo model and the quadratic cost model in the price adjustment context, it can also be shown analytically that aggregate labor demand equations derived from both models conform to the same reduced form. In addition, the deep parameters of the two models have a one-to-one mapping of each other.

Equation (34) is the dynamic labor demand equation derived from the quadratic-adjustment-cost model:

$$\gamma\beta E_t[\hat{l}_{t+1}] - [(1 - a - b) + \gamma(1 + \beta)]\hat{l}_t + \gamma\hat{l}_{t-1} - b\hat{r}_t - (1 - b)\hat{w}_t + z_t = 0$$

where I denote $\gamma = \frac{d\bar{n}}{\bar{w}}(1 - b)$.

And it is the dynamic labor demand equation derived from the Calvo-adjustment model:

$$\alpha\beta\kappa E_t[\hat{l}_{t+1}] - (1 + \alpha^2\beta)\kappa\hat{l}_t + \alpha\kappa\hat{l}_{t-1} - b\hat{r}_t - (1 - b)\hat{w}_t + z_t = 0$$

where $\kappa = \frac{(1-a-b)}{(1-\alpha)(1-\alpha\beta)}$

Comparing these two equations, I find that these two equations can be put into the following reduced form equation, so that the aggregate data alone can not differentiate between them.

$$\varphi_1 E_t[\hat{l}_{t+1}] + \varphi_2 \hat{l}_t + \varphi_3 \hat{l}_{t-1} - b\hat{r}_t - (1 - b)\hat{w}_t + z_t = 0$$

When I set $\alpha\kappa = \gamma$, For example, the correspondence among parameters in both models is expressed by equation (23). Then the Calvo-adjustment model is equivalent to the quadratic-adjustment-cost model with respect to the aggregation relations and they con-

sequently generate the exact same aggregate dynamics, given that all other aspects of both models are equal.

$$\frac{d\bar{n}}{\bar{w}} = \frac{\alpha(1-a-b)}{(1-\alpha)(1-\alpha\beta)(1-b)} \quad (23)$$

Note that both parameters d and α govern the rigidity of the labor adjustment process in both models and this equation gives the exact mapping between these two rigidity parameters.

3 Extension

In this section, I extend the baseline model to a more general case in which the labor adjustment process is characterized by an increasing hazard function. In particular, I apply the Weibull distribution¹² to model the firm's labor adjustment process. Because of its flexibility, the Weibull distribution is frequently used in statistical analysis of duration phenomena. In fact, it enables the incorporation of a wide range of hazard functions by using various values of the shape parameter.

3.1 The Weibull-adjustment Model

To integrate the Weibull-labor-adjustment into the RBC framework, I only have to modify the firm's problem, while keeping the household's optimal conditions (6) and (7) as they are in the baseline model.

I consider an economy with a continuum of perfectly competitive firms, which are differentiated with respect to the time elapsed since their last labor adjustments, indexed by $j \in \{0, J\}$ ¹³. I assume that the stochastic labor duration follows a Weibull distribution. According to the statistical duration theory, the distribution of firms with respect to time-since-last-adjustment (vintage groups) is summarized by the density function of the Weibull distribution, and the hazard rate in the vintage group j is obtained by:

$$h(j) = \frac{\tau}{\lambda} \left(\frac{j}{\lambda} \right)^{\tau-1} \quad \forall j \leq J \quad (24)$$

¹² For detailed discussion on Weibull distribution, see technical appendix (B)

¹³ $J = \lambda \left(\frac{\lambda}{\tau} \right)^{1/\tau-1}$ is the maximum number of vintage groups, which is obtained through equaling the hazard rate of the group J to one ($H(J) = 1$).

where τ and λ are the parameters of the Weibull distribution and j is the amount of time that has elapsed since the last adjustment. Note that this hazard function is increasing when τ is greater than one, thereby the adjustment probability in each vintage is dependent on the vintage index j . The longer a firm remains inactive, the more likely it adjusts its labor in the current period.

When resetting its labor $l_{0,t}^*$ at time t , a firm uses the survival function of the Weibull distribution to access the probabilities that its reseted labor input will remain unchanged in the future, and the firm chooses an optimal labor adjustment to maximize:

$$\max_{l_{0,t}^*, \{k_{j,t+i}\}_{j=0}^J} V_t = \sum_{i=0}^J S(i) E_t \{ \tilde{\beta}_{t,t+i} [f(l_{0,t}^*, k_{j,t+i}) - W_{t+i} l_{0,t} - R_{t+i} k_{j,t+i}] | \Omega_t \}$$

where $S(i)$ denotes the probability that firm's newly adjusted labor force will survive for i periods in the future, which is obtained by the formula $S(i) = 1 - F(i) = \exp(-(\frac{i}{\lambda})^\tau)$.

The first order necessary condition gives us the optimal labor demand:

$$l_{0,t}^* \frac{1-a-b}{1-b} = ab^{b/1-b} \frac{\sum_{i=0}^J S(i) E_t [\tilde{\beta}_{t,t+i} z_{t+i}^{1/1-b} / r_{t+i}^{b/1-b}]}{\sum_{i=0}^J S(i) E_t [\tilde{\beta}_{t,t+i} w_{t+i}]}, \quad (25)$$

Equation(25) has the same form as in the baseline model, except that the survival function is now a more complex function of the elapsed inactive time. This change enriches the labor dynamics of the model, but as the same time it also puts a challenge to the computation of the solution.

I log-linearize equation(25) for the labor demand as follows¹⁴:

$$\hat{l}_{0,t}^* = \tilde{\alpha} \beta E_t [\hat{l}_{0,t+1}^*] - \frac{b}{\Psi(1-a-b)} \hat{r}_t - \frac{1-b}{\Psi(1-a-b)} \hat{w}_t + \frac{1}{\Psi(1-a-b)} z_t \quad (26)$$

Analog to the Calvo-adjustment model, $\tilde{\alpha}$ governs dynamic properties of the labor demand. Given my calibration values of the model's parameters, $\tilde{\alpha}$ is equal to 0.75, which is slightly less than its counterpart (0.77) in the Calvo-adjustment model. As in the baseline model, the optimal labor adjustment is increasing in all expected future shocks z_{t+j} and decreasing in all expected future prices w_{t+j} and r_{t+j} , and thus the 'front-loading' effect is also at work here. It is important to note that the parameters in this equation nest

¹⁴ The derivation of this equation is shown in the Appendix (C).

those in the corresponding equation (21), where the hazard function is constant.

To aggregate the labor demand, I use a two-stage aggregation scheme. First I define a dummy sectoral labor demand as $\hat{l}_{j,t}$, which is the sum of labor demand in a labor vintage before reshuffling firms into the new vintage groups, and let $\alpha_j = 1 - h(j)$ denote the probability of non-adjusting.

$$\hat{l}_{j,t} = (1 - \alpha(j)) \hat{l}_{0,t}^* + \alpha(j) \hat{l}_{j,t-1} \quad (27)$$

In equation (27), we can see that the heterogeneous sectoral labor demands arise as a result of the non-constant hazard function. Because the hazard rates $\alpha(j)$ are disparate across vintages due to the increasing-hazard function, each vintage labor group is composed of the optimal labor adjustments ($\hat{l}_{0,t}^*$) and the lagged sectoral labor demand with different compositions. As a result, heterogeneity in labor emerges naturally from the underlying labor adjustment process in this economy. Given the increasing hazard rate in the time-since-last-adjustment, the labor demand in the younger labor vintage is more persistent, but less volatile than those in the older labor vintage.

At last, the aggregate labor demand can be derived by using the sectoral labor demand and the Weibull density function:

$$\hat{l}_t = \int_0^J \theta(j) \hat{l}_{j,t} dj. \quad (28)$$

Equation (28) reveals that, given the heterogeneous nature of the economy, the aggregation mechanism plays an important role in forming aggregate dynamics. In this model dynamics properties in the different labor vintages are divergent, and their contributions to the aggregate behavior depend on their weights that are given by the distribution of labor vintages $\theta(j)$.

4 Calibration and Simulation Results

In this paper, I investigate quantitative significance of lumpy labor adjustment as a propagation mechanism for business cycles. In order to address this question properly, I follow the tradition of RBC literature and calibrate my optimal growth model such that it is consistent with long-run growth facts in U.S. data, and then study its short-run dynamics by investigating the statistical properties of simulated time series and impulse responses

functions. In the following sections, I address the calibration method for this model and then present the quantitative results and impulse response functions.

4.1 Calibration

For most parameters in the model, I take the standard values in the RBC literature. As for special parameters of the Weibull distribution, I refer to evidence of empirical studies using micro employment data.

For the quarterly discount rate β I use 0.9902 to reflect that the real rate of interest in the U.S. economy is around 4% per annum. The depreciation rate δ is 0.025, indicating an annual rate of 10%. Given these two values, I select the capital share b to be 0.329 to match the average capital-output ratio of 2.353 (Thomas and Khan, 2004), and the labor share of output a is set to be 0.58, which is consistent with direct estimates for the U.S. economy. (King, Plosser, and Rebelo, 1988).

As to the preference parameters, I choose $\phi = 0.25$ implying that the average household allocates one quarter of the time to productive activities (Benhabib and Farmer, 1992), and $\sigma = 1$, which gives rise to a log utility function for consumption.

The labor adjustment parameter is calibrated according to empirical work estimating the hazard function using aggregate net flow data. Caballero and Engel (1993) used U.S. manufacturing employment and job flow data (1972:1-1986:4) to estimate the constant hazard function. Their results suggest that on average, 22.9% of firms in the U.S. adjust their employment per quarter. As a result, I choose 0.77 as the value for α in the baseline model, which implies that the mean duration of employment is 4.35 quarters.

The Weibull parameters are set as follows: In the standard case, I set the shape parameter τ to be 1.2, implying an increasing hazard function. This value is based on Varejão and Portugal (2006), in which they found that the shape parameter is in the range between 1.174 to 1.309¹⁵. Since there is yet no standard value for this parameter in the literature, I will test the sensitivity of my results to the value of τ in the later part of this section. To calibrate the scale parameter λ , I apply the equation (37), implying that the characteristic life of the Weibull distribution is equal to 4.62 quarters, given $\tau = 1.2$ and the average duration of 4.35 quarters.

Finally, I select the values of ς and σ_ϵ for aggregate technology shocks. I choose $\varsigma = 0.95$

¹⁵ Since Portuguese labor market emerges as the most regulated in Europe in all existing rankings of indexes of employment protection (OECD,1999), this evidence may be thought of as lower-bounds for the slope of the hazard function.

and a standard deviation of 0.007, which are estimated parameters of Solow residuals that are commonly used in the RBC literature (King and Rebelo, 2000).

Parameters	Values	Interpretation
β	0.9902	Annual real rate 4%
δ	0.025	Annual depreciation rate 10%
b	0.329	To match capital to output ratio of 2.35(Thomas and Khan (2004))
a	0.58	Labor's share of output (King, Plosser, and Rebelo (1988))
η	1	$\log C_t$, common in the literature
ϕ	0.25	On average one quarter of the time are allocated to productive activities(Benhabib & Farmer,1992)
λ	4.62	Average duration of employment of 4.35 quarters ($\alpha = 0.77$)
τ	1.2	Increasing hazard function Varejão and Portugal (2006)
ς	0.95	Solow residual estimate,
σ^2	0.007	Solow residual estimate,

Table 1: Calibration Values

4.2 Simulation Results

To evaluate the quantitative performance of the Weibull-adjustment model, I apply the log-linear approximation method of King, Plosser, and Rebelo (1988), which produces linear decision rules depending on the state variables, and then solve the rational expectation equilibrium by using the standard algorithm¹⁶.

In table (4)-(6), I report the second moments of U.S. data and those generated by the theoretical models. In all cases, the moments are for HP-filtered time series. For each of these models, three sets of statistics are reported: first, absolute and relative standard deviation; second, contemporaneous correlation coefficients relative to output; and third, the cross correlations with respect to output.

In Table (2), I summarize some results regarding variables for the labor market.

It is well documented in the RBC literature that the standard RBC model fails to match some important aspects of the U.S. business cycle facts. For example, Cogley and Nason (1995) has shown that the standard RBC models fail to account for the observed positive

¹⁶ See, for example, Blanchard and Kahn (1980) and Uhlig (2001)

Labor	Relative S.D.	Cross Correlation with output				
		-2	-1	0	1	2
U.S. data(Hours)	0.98	0.54	0.78	0.92	0.90	0.78
U.S. data(Employment)	0.82	0.47	0.72	0.89	0.92	0.86
RBC model	0.47	0.47	0.70	0.98	0.61	0.32
Weibull model	0.45	0.55	0.76	0.96	0.88	0.67
Real wage		-2	-1	0	1	2
U.S. data	0.44	0.58	0.66	0.68	0.59	0.46
RBC model	0.54	0.37	0.64	0.99	0.72	0.49
Weibull model	0.44	0.43	0.69	0.96	0.83	0.68

Table 2: Statistics for labor and output

serial correlation in the output growth rate and aggregate labor, and its persistent dynamics rely on the high autocorrelation of the productivity shocks. Introducing stickiness in the labor adjustment improves the model's performance with regard to the persistence of aggregate labor and output. As shown in the table, when propagating the same aggregate technology shocks, the standard RBC model generates low volatile and nonpersistent aggregate labor and output. By contrast, with empirically plausible labor rigidity, the Weibull-adjustment model replicate procyclical and persistent labor dynamics.

Moreover, the Weibull-adjustment model can also replicate the stylized facts regarding real wage. As seen in the lower panel of the table (2), it implies smoothing real wage even in a Walrasian labor market setting. As discussed in the search and matching literature (e.g., Shimer, 2005 and Hall, 2005), the real wage rigidity plays an important role in propagating business cycles in the labor market. Because this mechanism is missing in my model, it is not able to replicate highly volatile labor and acyclical real wage. The new insight revealed by this model, however, is that there is a smoothing effect of labor rigidity on real wage. The reason is that in this model the direct link between productivity and real wage is weakened by the forward-looking labor adjustment behavior. As seen in Equation (19), all future real wages appear to be the cost for the current labor adjustment, therefore firms have incentive to smooth real wage by their labor demand decisions. These results reveal that stickiness in the labor adjustment can be a source of real wage rigidity.

4.3 Impulse Responses

Figure (1) compares the responses (percent deviations from steady state) of the Weibull-adjustment model to a one percent increase in the aggregate technology shock.

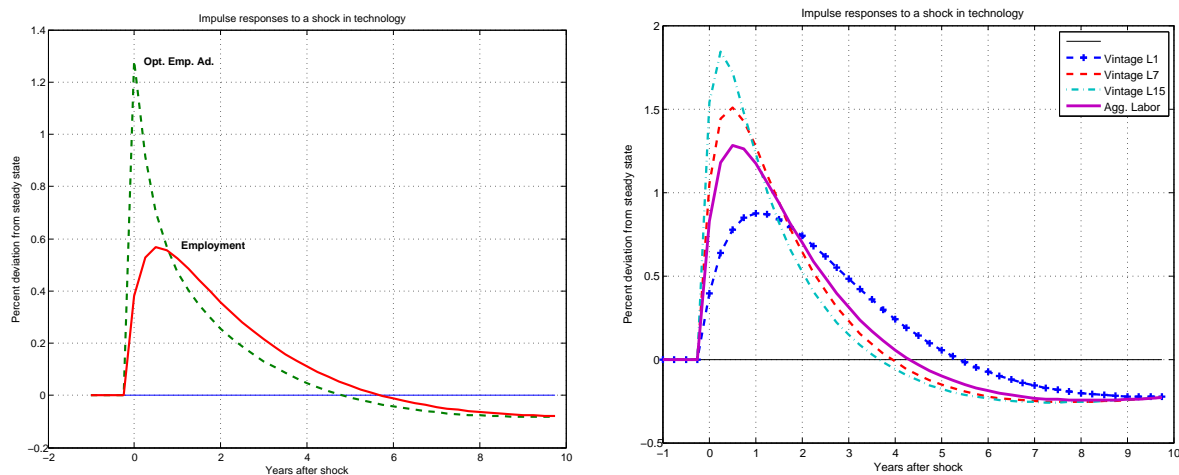


Figure 1: Impulse responses of the Weibull model

First, in the left panel we can observe that the individual firm’s labor adjustment and the aggregate labor respond to the aggregate technology shock differently. While the impulse response of aggregate labor is humped-shaped (The solid line), the labor input at the firm’s level reacts to the shock immediately and by a large amount (The dash line). These results illustrate that the lumpy-adjustment models are able to reconcile features observed in both micro and macro labor adjustment data: i.e. at the micro level, labor adjustment exhibits a lumpy pattern in response to the technology shock, while the aggregate employment reacts smoothly and sluggishly.

These results manifest the unique feature of the lumpy adjustment model in propagating business cycles. Different to other partial adjustment models, the Calvo-type assumption does not necessarily lead to the dampened volatility of labor dynamics. At the firm level, it generates strong lumpy labor adjustment through the ‘front-loading’ effect. The intuition is as follows. The firm’s optimal demand depends on expectations of all future prices and shocks. Suppose that in some period t firms experience a positive productivity shock, some firms are labor-adjustment constrained, so they have to increase their demand of capital in the rental market, while, on the supply side, the household’s capital stock is predetermined. This leads to an increase in interest rates for the whole economy and rises household savings. On the other hand, those labor-unconstrained firms will adjust labor more than they currently need in order to hedge the adjustment-risk in the future. This

in turn drives real wage up. Put them together, all those rises in productivity and prices can be expected by rational agents, so that the adjusting firms will, in addition to their risk-hedging motive, demand even more workers. Moreover, if labor supply is elastic, rise in the interest rate triggers the intertemporal substitution effect in the labor supply side, because real wage is higher today and wage tomorrow is discounted at a higher rate, the household is willing to enjoy less leisure today thus supply more labor. Consequently, both labor and investment rise sharply at the micro level. However, at the aggregate level, this strong effect is to a large extent neutralized by the underlying aggregation mechanism.

To further illustrate the important role played by the heterogeneous labor and the aggregation mechanism in this model, I show in the right panel of the figure (1) the impulse response functions of aggregate labor along with the responses of labor in different vintage groups. Recalling the aggregate labor demand equation (28), the aggregate labor is a weighted average of vintage labor demands, where the weights correspond to the probability density function of the Weibull distribution. This can be visualized in this figure. The aggregate labor (the solid thick line) is composed of the sectoral labor from different vintages (Dashed lines). As discussed in the previous section, given the increasing hazard rates, the labor demand in the younger labor vintage is more persistent, but less volatile than those in the older labor vintage. IRFs of the sectoral labor vary from the persistent but less volatile younger vintage labor (The vintage $L1$) to the volatile but less persistent older vintage labor (e.g. the vintage $L15$).

4.4 Sensitivity Analysis

Now I use numerical results to test how sensitive my results are in response to the key parameter τ , which measures the shape of the Weibull distribution. In Table (3), I report the relative volatility of aggregate labor to output and the first-order autocorrelations of aggregate labor that are generated by a wide range of values of the shape parameter¹⁷.

In general, I find that the value of the shape parameter exerts an important influence on the aggregate labor dynamics. As the shape parameter increases, the relative volatility of labor to output rises, while the persistence of labor decreases. These results confirm the intuition of the model, in which the higher is τ , the less likely firms sustain a fix amount of labor for a long period of time, and hence the labor market is less rigid. On the other hand,

¹⁷ Here I check the range in which the hazard function of the Weibull distribution is increasing and 2.2 is the maximum value that guarantees an unique stable solution of this dynamic system, given other parameters' value that I specify in the calibration section.

The shape parameter τ	1.1	1.3	1.5	1.7	1.9	2.2
Relative S.D. to y_t	0.446	0.455	0.476	0.484	0.496	0.50
Autocorr. $Corr(L_t, L_{t-1})$	0.89	0.88	0.87	0.86	0.83	0.81

Table 3: Sensitivity Analysis for τ

with the increasing value of τ , the economy becomes more heterogeneous with respect to the labor adjustment risk. In Figure(2), we can see that as the value of τ increases, the

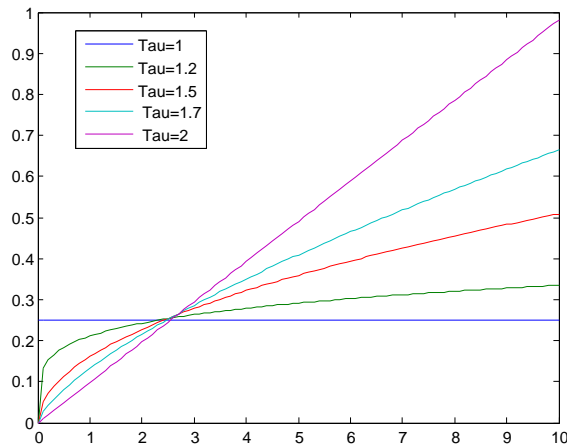


Figure 2: Hazard Function with different shape parameters

hazard function becomes steeper, which implies a trade-off between the probability of adjusting today and the probability of adjusting later. If I use the extent of changes in the hazard rates to measure the economic risk in the labor market, then the economic risk associated to the high value of τ is higher than that in the Calvo case, where the probabilities of adjusting are equal. In another words, economic risk is high in the sense that, for a given time horizon, the volatility of hazard rates is larger. Consequently, firms will adjust more to hedge the higher risk in the labor market, and hence the aggregate labor also becomes more volatile. This mechanism serves as an example, in which the aggregation mechanism plays an important role in forming aggregate dynamics when the economy is featured by heterogeneous labor demand.

5 Concluding Remarks

In this paper general equilibrium is generated in markets where the household's consumption-leisure choice meets the firm's factor demand decision under a stochastic labor adjustment process. The innovation of the model is to apply the statistical duration analysis to extend the well-established time-dependent adjustment scheme in the spirit of Calvo (1983) in a DSGE framework. Using the increasing-hazard Weibull distribution, the model generates heterogeneous labor vintages, which are different not only in the time of adjustment, but also in terms of the volatility and the persistence of dynamics.

The key message conveyed in this paper is that impediment in the labor adjustment process induces firms to make precautionary labor adjustments, and non-constant hazard adjustment process brings about heterogeneity in the economy. In addition, given the heterogeneous nature of the economy, the underlying aggregation mechanism play a crucial role in forming the aggregate dynamics. Serial studies by Hamermesh (e.g. Hamermesh, 1989, Hamermesh, 1993 and Hamermesh and Pfann, 1996) have shown that information about the distribution of sub-units is crucial to linking micro-level features with implications for macro behavior deduced by determining the correct mechanism for aggregation. Thus my model is an endeavor to illustrate how this mechanism works in propagating realistic business cycle fluctuations.

A Equivalence of the Partial Adjustment Models

I first derive the aggregate labor demand equation from a textbook quadratic-adjustment-cost model(See e.g. Hamermesh, 1993).

In this economy, each firm is assumed to maximize the expected discounted real value of all future profits by choosing nonnegative values for optimal sequence of labors l_{t+i} and optimal sequence of capital stocks k_{t+i} , subject to the quadratic labor adjustment costs.

The objective function of firm is:

$$\max_{l_{t+i}, k_{t+i}} V_t = \sum_{i=0}^{\infty} E_t \{ \tilde{\beta}_{t+i} [F(l_{t+i}, k_{t+i}) - w_{t+i} l_{t+i} - r_{t+i} k_{t+i} - \frac{d}{2} (l_{t+i} - l_{t+i-1})^2] \} \quad (29)$$

where d is denoted as the adjustment cost parameter.

subject to

$$y_t = Z_t l_t^a k_t^b \quad (30)$$

and the total productivity shock Z_t and the household's problem are the same as in the Calvo adjustment model.

The first order conditions are:

$$r_{t+i} = F_K(t+i) = b Z_{t+i} \frac{l_{t+i}^a}{k_{t+i}^{1-b}} \quad (31)$$

$$\tilde{\beta}_{t+i} [F_L(t+i) - w_{t+i} - d(l_{t+i} - l_{t+i-1})] + E_t [\tilde{\beta}_{t+i+1} d(l_{t+i} - l_{t+i-1})] = 0 \quad (32)$$

It follows:

$$a Z_{t+i} l_{t+i}^{a-1} k_{t+i}^b - w_{t+i} + \beta d l_{t+i+1} - d(1 + \beta) l_{t+i} + d l_{t+i-1} = 0 \quad (33)$$

If I log-linearize these FOCs around the steady state, I get the following dynamic labor demand equation:

$$\gamma \beta E_t [\hat{l}_{t+1}] - [(1 - a - b) + \gamma(1 + \beta)] \hat{l}_t + \gamma \hat{l}_{t-1} - \frac{b \bar{R}}{\bar{r}} \hat{R}_t - (1 - b) \hat{w}_t + z_t = 0 \quad (34)$$

Where I denote $\gamma = \frac{d \bar{n}}{\bar{w}} (1 - b)$.

B Weibull Distribution

The PDF of Weibull distribution is given by the following expression:

$$Pr(j) = \frac{\tau}{\lambda} \left(\frac{j}{\lambda}\right)^{\tau-1} \exp\left(-\left(\frac{j}{\lambda}\right)^\tau\right) \quad (35)$$

and the cumulative probability function is:

$$F(j) = 1 - \exp\left(-\left(\frac{j}{\lambda}\right)^\tau\right) \quad (36)$$

The parameters that characterize the Weibull distribution are the scale parameter λ and the shape parameter τ . The shape parameter determines the shape of the Weibull's pdf function, e.g. when $\tau = 1$, it reduces to an exponential case; while $\tau = 3.4$, the Weibull amounts to the normal distribution. The scale parameter defines the characteristic life of the random process that amounts to the time, at which 63.2% of the firm will adjust their labor. This can be seen with the evaluation of the cdf function of the Weibull distribution at j equaling the scale parameter λ . Then we have, $F(\lambda) = 1 - e^{(-1)} = 0.632$.

Note that it relates to the mean duration \bar{j} according to the following equation:

$$\bar{j} = \frac{1}{\bar{\alpha}} = \lambda \Gamma\left(\frac{1}{\tau} + 1\right), \quad (37)$$

where $\Gamma()$ is the Gamma function.

It follows that the hazard function of Weibull distribution is:

$$h(j) = \frac{\tau}{\lambda} \left(\frac{j}{\lambda}\right)^{\tau-1} \quad (38)$$

Note that this hazard is constant when the shape parameter τ equals one, and increasing when τ is greater than one.

C Derivation of the Dynamic Labor Demand Equation

First, log-linearize Equation 25:

$$\sum_{i=0}^J S(i) \beta^i E_t[(a-1)\hat{l}_{0,t}^* + b\hat{k}_{j,t+i} - \hat{w}_{t+i} + z_{t+i}] = 0 \quad (39)$$

$$\sum_{i=0}^J S(i) \beta^i E_t[b\hat{k}_{j,t+i} - \hat{w}_{t+i} + z_{t+i}] + (a-1)\hat{l}_{0,t}^* \sum_{i=0}^J S(i) \beta^i = 0$$

Let $\Psi = \sum_{i=0}^J S(i)\beta^i$ and rearrange this equation:

$$(1-a)\Psi \hat{l}_{0,t}^* = \sum_{i=0}^J S(i) \beta^i E_t[b\hat{k}_{j,t+i} - \hat{w}_{t+i} + z_{t+i}]$$

Then, substitute out $\hat{k}_{j,t+i}$ with the log-linearized Equation (18):

$$(1-a)\Psi \hat{l}_{0,t}^* = \sum_{i=0}^J S(i) \beta^i E_t\left[\frac{ab}{1-b}\hat{l}_{j,t+i} - \frac{b}{1-b}\hat{r}_{t+i} - \hat{w}_{t+i} + \frac{1}{1-b}z_{t+i}\right]$$

Note that $\hat{l}_{0,t}^* = \hat{l}_{j,t+i} \quad \forall j \in (0, J)$, we obtain:

$$\begin{aligned} \frac{1-a-b}{1-b}\Psi \hat{l}_{0,t}^* &= \sum_{i=0}^J S(i) \beta^i E_t\left[\underbrace{\frac{1}{1-b}z_{t+i} - \frac{b}{1-b}\hat{r}_{t+i} - \hat{w}_{t+i}}_{X_{t+i}}\right] \\ &= S(0)X_t + S(1)\beta X_{t+1} + S(2)\beta^2 X_{t+2} + S(3)\beta^3 X_{t+3} + \dots \end{aligned} \quad (40)$$

And, iterate Equation 40 one period forward, we obtain:

$$\begin{aligned} \frac{1-a-b}{1-b}\Psi \hat{l}_{0,t+1}^* &= \sum_{i=0}^J S(i) \beta^i E_t\left[\frac{1}{1-b}z_{t+i+1} - \frac{b}{1-b}\hat{r}_{t+i+1} - \hat{w}_{t+i+1}\right] \\ &= S(0)X_{t+1} + S(1)\beta X_{t+2} + S(2)\beta^2 X_{t+3} + \dots \\ &= \frac{S(0)}{S(1)}S(1)X_{t+1} + \frac{S(1)}{S(2)}S(2)\beta X_{t+2} + \frac{S(2)}{S(3)}S(3)\beta^2 X_{t+3} + \dots \end{aligned}$$

Multiply both sides of this equation by β :

$$\beta \frac{1-a-b}{1-b} \Psi \hat{l}_{0,t+1}^* = \frac{S(0)}{S(1)} S(1) \beta X_{t+1} + \frac{S(1)}{S(2)} S(2) \beta^2 X_{t+2} + \frac{S(2)}{S(3)} S(3) \beta^3 X_{t+3} + \dots$$

where $\frac{S(i)}{S(i+1)} = \exp\left(\frac{(i+1)^\tau - i^\tau}{\lambda^\tau}\right)$. Given my calibration values of the Weibull parameters, these values can be approximated to be a constant ($\tilde{\alpha}$).

$$\beta \frac{1-a-b}{1-b} \Psi \hat{l}_{0,t+1}^* = \frac{1}{\tilde{\alpha}} S(1) \beta X_{t+1} + \frac{1}{\tilde{\alpha}} S(2) \beta^2 X_{t+2} + \frac{1}{\tilde{\alpha}} S(3) \beta^3 X_{t+3} + \dots$$

$$\tilde{\alpha} \beta \frac{1-a-b}{1-b} \Psi \hat{l}_{0,t+1}^* = S(1) \beta X_{t+1} + S(2) \beta^2 X_{t+2} + S(3) \beta^3 X_{t+3} + \dots \quad (41)$$

Substitute (41) into (40), we obtain:

$$\frac{1-a-b}{1-b} \Psi \hat{l}_{0,t}^* = X_t + \tilde{\alpha} \beta \frac{1-a-b}{1-b} \Psi \hat{l}_{0,t+1}^* \quad (42)$$

And, it follows the equation 26, which is introduced in the text.

$$\hat{l}_{0,t}^* = \tilde{\alpha} \beta E_t[\hat{l}_{0,t+1}^*] - \frac{b}{\Psi(1-a-b)} \hat{r}_t - \frac{1-b}{\Psi(1-a-b)} \hat{w}_t + \frac{1}{\Psi(1-a-b)} z_t$$

where $\tilde{\alpha}^{-1} = \frac{1}{J+1} \sum_{i=0}^J \exp\left(\frac{(i+1)^\tau - i^\tau}{\lambda^\tau}\right)$

References

- BENHABIB, J., AND R. E. FARMER (1992): “Indeterminacy and Increasing Returns,” Ucla economics working papers, UCLA Department of Economics.
- BLANCHARD, O. J., AND C. M. KAHN (1980): “The Solution of Linear Difference Models under Rational Expectations,” *Econometrica*, 48(5), 1305–11.
- CABALLERO, R. J., AND E. M. ENGEL (1993): “Microeconomic Adjustment Hazards and Aggregate Dynamics,” *The Quarterly Journal of Economics*, 108(2), 359–83.
- CABALLERO, R. J., E. M. ENGEL, AND J. HALTIWANGER (1997): “Aggregate Employment Dynamics: Building from Microeconomic Evidence,” *American Economic Review*, 87(1), 115–37.
- CALVO, G. (1983): “Staggered Prices in a Utility Maximizing Framework,” *Journal of Monetary Economics*, 12, 383–398.
- CAPLIN, A. S., AND D. F. SPULBER (1987): “Menu Costs and the Neutrality of Money,” *The Quarterly Journal of Economics*, 102(4), 703–25.
- COGLEY, T., AND J. M. NASON (1995): “Output Dynamics in Real-Business-Cycle Models,” *American Economic Review*, 85(3), 492–511.
- COOLEY, T. F. (1995): *Frontiers of Business Cycle Research*. Princeton University Press.
- HALL, R. E. (2005): “Employment Fluctuations with Equilibrium Wage Stickiness,” *American Economic Review*, 95(1), 50–65.
- HAMERMESH, D. S. (1989): “Labor Demand and the Structure of Adjustment Costs,” *American Economic Review*, 79(4), 674–89.
- HAMERMESH, D. S. (1993): *Labor Demand*. Princeton University Press.
- HAMERMESH, D. S., AND G. A. PFANN (1996): “Adjustment Costs in Factor Demand,” Cepr discussion papers, C.E.P.R. Discussion Papers.
- KING, R. G., C. I. PLOSSER, AND S. REBELO (1988): “Production, Growth and Business Cycles: The Basic Neoclassical Model,” *Journal Monetary Economics*, 21(2/3), 195–232.
- KING, R. G., AND S. T. REBELO (2000): “Resuscitating Real Business Cycles,” Nber working papers, National Bureau of Economic Research, Inc.

- KING, R. G., AND J. K. THOMAS (2006): “Partial Adjustment Without Apology,” *International Economic Review*, 47(3), 779–809.
- LETTERIE, W. A., G. A. PFANN, AND J. M. POLDER (2004): “Factor adjustment spikes and interrelation: an empirical investigation,” *Economics Letters*, 85(2), 145–150.
- ROTEMBERG, J. (1987): “The New Keynesian Microfoundations,” *NBER Macroeconomics Annual*, December(2), 69–104.
- SHIMER, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95(1), 25–49.
- THOMAS, J. K. (2002): “Is lumpy investment relevant for the business cycle?,” Discussion paper.
- THOMAS, J. K., AND A. KHAN (2004): “Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics,” Discussion paper.
- UHLIG, H. (2001): “A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily,” Discussion paper.
- VAREJÃO, J., AND P. PORTUGAL (2006): “Employment Dynamics and the Structure of Labor Adjustment Costs,” Iza discussion papers, Institute for the Study of Labor (IZA).
- VERACIERTO, M. L. (2002): “Plant-Level Irreversible Investment and Equilibrium Business Cycles,” *American Economic Review*, 92(1), 181–197.

D Tables

Variables	Standard Deviation%	Relative S.D.	Cross Correlation with output						
			-3	-2	-1	0	1	2	3
Hours*	1.69	0.98	0.38	0.54	0.78	0.92	0.90	0.78	0.63
Employment*	1.41	0.82	0.22	0.47	0.72	0.89	0.92	0.86	0.73
Real wage	0.76	0.44	0.47	0.58	0.66	0.68	0.59	0.46	0.29
Consumption	1.27	0.74	0.57	0.72	0.82	0.83	0.67	0.46	0.22
Output	1.72	1.00	0.38	0.63	0.85	1.00	0.85	0.63	0.38
Investment	5.34	3.10	0.43	0.63	0.82	0.90	0.81	0.60	0.35
Labor productivity	0.73	0.42	0.44	0.45	0.34	0.34	0.10	-0.09	-0.30

Notes: all statistics are reported in Cooley (1995) Table(1.1)

*: Based on establishment survey.

Table 4: Business Cycle Statistics for the U.S. Economy

Variables	Standard Deviation%	Relative S.D.	Cross Correlation with output						
			-3	-2	-1	0	1	2	3
Hours	0.59	0.47	0.29	0.47	0.70	0.98	0.61	0.32	0.10
Capital	0.32	0.26	-0.31	-0.16	0.06	0.36	0.54	0.63	0.65
Real wage	0.67	0.54	0.15	0.37	0.64	0.99	0.72	0.49	0.31
Consumption	0.38	0.31	0.02	0.24	0.53	0.90	0.75	0.60	0.46
Output	1.24	1.00	0.22	0.42	0.68	1.00	0.68	0.42	0.22
Interest rate	0.04	0.04	0.32	0.49	0.70	0.96	0.57	0.27	0.05
Investment	3.84	3.10	0.27	0.46	0.70	0.99	0.63	0.35	0.14
Labor productivity	0.67	0.54	0.15	0.37	0.64	0.99	0.72	0.49	0.31

Table 5: Business Cycle Statistics for the Standard RBC Model

Variables	Standard Deviation%	Relative S.D.	Cross Correlation with output						
			-3	-2	-1	0	1	2	3
Labor	0.55	0.45	0.36	0.55	0.76	0.96	0.88	0.67	0.43
Capital	0.37	0.30	-0.29	-0.13	0.09	0.35	0.54	0.66	0.71
Real wage	0.54	0.44	0.20	0.43	0.69	0.96	0.83	0.68	0.51
Consumption	0.42	0.34	0.14	0.37	0.64	0.92	0.79	0.66	0.53
Output	1.22	1.00	0.34	0.55	0.78	1.00	0.78	0.55	0.34
Interest rate	0.04	0.03	0.45	0.62	0.80	0.96	0.66	0.38	0.14
Investment	3.95	3.24	0.40	0.59	0.79	0.99	0.74	0.49	0.26
Labor productivity	0.70	0.57	0.31	0.52	0.75	0.98	0.65	0.42	0.26

Table 6: Business Cycle Statistics for the Weibull-Adjustment RBC Model

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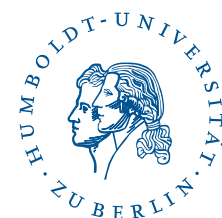
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