SFB 649 Discussion Paper 2009-015

Stochastic Mortality, Macroeconomic Risks, and Life Insurer Solvency

Katja Hanewald*
Thomas Post*
Helmut Gründl*

*Humboldt-Universität zu Berlin, Germany

This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

http://sfb649.wiwi.hu-berlin.de
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin
Spandauer Straße 1, D-10178 Berlin

SFB 649 ECONOMIC RISK BERLIN

STOCHASTIC MORTALITY, MACROECONOMIC RISKS, AND LIFE INSURER SOLVENCY

by Katja Hanewald, Thomas Post, and Helmut Gründl[1]

Humboldt-Universität zu Berlin

First Draft: February 13, 2009

This Draft: March 6, 2009

Abstract

Motivated by a recent demographic study establishing a link between macroeconomic fluctuations and the mortality index k_t in the Lee-Carter model, we assess the impact of macroeconomic fluctuations on the solvency of a life insurance company. Liabilities in our stochastic simulation framework are driven by a GDP-linked variant of the Lee-Carter mortality model. Furthermore, interest rates and stock prices are allowed to react to changes in GDP, which itself is modeled as a stochastic process. Our results show that insolvency probabilities are significantly higher when the reaction of mortality rates to changes in GDP is incorporated.

Keywords: Life insurance, asset-liability management, stochastic mortality, Lee-Carter model, business cycle

JEL classification: G22, G23, G28, G32, E32, J11.

[1] Katja Hanewald (contact author), Thomas Post, and Helmut Gründl are with the Humboldt-Universität zu Berlin, School of Business and Economics, Spandauer Str. 1, 10178 Berlin, Germany, Tel. +49-30-2093-5828, Fax: +49-30-2093-5665, E-mail: katja.hanewald@wiwi.hu-berlin.de. This work was supported by the German Research Foundation through the Collaborative Research Center 649 "Economic Risk."

1. Introduction

Assumptions about future mortality rates are an integral part of the pricing, reserving, and risk management of insurance companies or pension funds offering annuity and life insurance contracts. Systematic deviations of actual mortality rates from these assumptions can pose a serious threat to the financial stability of those businesses and, consequently, to the economic well-being of policyholders. Thus, there has been considerable recent research into developing models that allow for stochastic mortality, i.e., models that allow for systematic deviations from mortality trends.

In another stream of demographic research, several epidemiological studies find that mortality rates react to changes in macroeconomic conditions. By combining results from both fields of study, Hanewald (2009) shows that the mortality index k_t in the well-known Lee-Carter model is significantly correlated with macroeconomic changes. This insight is the inspiration for the present study, which assesses the overall impact of macroeconomic fluctuations on the financial stability of a life insurance company. We develop a dynamic asset-liability model in which both sides of the balance sheet are allowed to react to the state of the economy. Simulation results show that insolvency probabilities are considerably higher when the reaction of mortality rates to changes in the economic indicators is incorporated compared to scenarios where this relationship is ignored. This finding is robust to variations in the age of the insureds, the insurance portfolio size, the equity base, and the share of assets invested in stocks.

The paper is organized as follows. Section 2 contains a review of relevant literature. This is followed, in Section 3, by setting up the simulation model for the insurance company. Results of different simulation scenarios are presented in Section 4. A summary and conclusions are provided in Section 5.

2. Literature review

2.1 Stochastic mortality modeling

The development and status quo of stochastic mortality modeling is summarized in Cairns, Blake, and Dowd (2008), Booth (2006), and Booth and Tickle (2008). "The earliest model and still the most popular" (Cairns et al., 2008) was proposed by Lee and Carter (1992). This model is widely employed both in the academic literature and by practitioners working for pension funds, life insurance companies, and public pension systems. The original approach has seen several extensions (see, e.g., Lee and Miller, 2001; Brouhns, Denuit, and Vermunt, 2002; Renshaw and Haberman, 2006), and has been applied to mortality data of many countries, including the G7 countries (Tuljapurkar, Li, and Boe, 2000), Spain (Debón, Montes, and Puig, 2008), Australia (Booth, Maindonald, and Smith, 2002), and China and South Korea (Li, Lee, and Tuljapurkar, 2004). Variations of the Lee-Carter model have been employed to forecast other demographic variables, such as fertility rates or migration flows (Girosi and King, 2008; Härdle and Myšičková, 2009). For mortality modeling, however, the Lee-Miller variant is generally viewed as the standard (Booth and Tickle, 2008). It performs well in a 10-population comparison study of five variants or extensions of the Lee-Carter method (Booth et al., 2006).

2.2 The impact of macroeconomic changes on mortality

The key driver of mortality dynamics in the Lee-Carter model is the "index of the level of mortality" k_t (Lee and Carter, 1992). This variable is typically characterized as the "dominant temporal pattern in the decline of mortality" (Tuljapurkar et al., 2000), "a random period effect" (Cairns et al., 2008), or simply as a latent variable (Hári et al., 2008a). However, a recent cross-country study by Hanewald (2009) reveals that the mortality index in the Lee-Carter model is not merely an unobserved, latent variable that fluctuates erratically, but is driven to a considerable extent by external factors. The study shows that changes in k_t are significantly correlated with real GDP (gross domestic product) growth rates in Australia, Canada, and the United States, and with unemployment rate changes in Japan, over the period 1950–2005. These findings are in line with previous studies that relate age- and cause-specific mortality rates directly to macroeconomic conditions. Ruhm (2000) was the first to discover that mortality rates fluctuate pro-cyclically in the United States over the period 1972–1991. A similar pattern was observed for mortality rates in the United States, Spain, and Japan (Tapia Granados, 2005a, 2005b, 2008), for Sweden (Tapia Granados and Ionides, 2008), and for 23 OECD countries over the 1960–1997 period (Gerdtham and Ruhm, 2006). Neumayer (2004) and Hanewald (2008) also corroborate Ruhm's results using German data for 1980–2000 and 1956–2004, respectively. Further evidence for France, Japan, and the United States is provided by Reichmuth and Sarferaz (2008).

2.3 Accounting for stochastic mortality in life insurance companies

The financial impact of stochastic mortality (i.e., systematic mortality risk) on a life insurer or pension fund is analyzed in several models. Gründl, Post, and Schulze (2006) and Cox and Lin (2007) examine natural hedging opportunities in the annuity and life insurance business; Dowd, Cairns, and Blake (2006), Hári et al. (2008b), and Bauer and Weber (2008) assess the impact of stochastic mortality on an insurer's risk exposure. None of the studies, however, accounts for the systematic dependency of mortality rates on the economic environment as proxied by real GDP, which is the focus of this contribution.

3. The simulation framework

3.1 The formal model for the insurance business

Our aim is to assess the overall impact of macroeconomic fluctuations on a life insurer's solvency. We set up a dynamic asset-liability model of a life insurance company as described below.

Consider a newly founded life insurance company. At the beginning of its first year, in $t = 0$, it writes I_0 homogeneous term-life contracts[2] with annually constant premium P per contract. All insureds are assumed to be of age x. The contract duration is for T years and the death benefit is B. For each contract, the premium P is collected immediately. Shareholders contribute a fixed proportion γ of the premium income $I_0 \cdot P$ as equity capital E_0. The sum of premiums and equity comprise the insurer's assets A_0.

We assess the insurer's financial stability by its insolvency probability. Insolvency occurs when the firm's equity—measured at market value—is negative at the end of the year. Insolvent insurance firms are not allowed to continue operating. Therefore, the target variable of our analysis is the multi-period insolvency probability Ψ_t of the insurance firm, which is defined as follows:

$$\Psi_t = \Pr[E_t < 0 \vee \Psi_{t-1} = 1]. \tag{1}$$

Equity capital at time t is the difference between the market value of assets A_t and the liabilities L_t at the end of the year:

$$E_t = A_t - L_t. \tag{2}$$

Asset values are given by:

$$A_t = (A_{t-1} + P \cdot I_{t-1}) \cdot R_t - B \cdot (-\Delta I_t) - D_t, \tag{3}$$

where R_t is the stochastic investment return (i.e., exp(rate of return)), $B \cdot (-\Delta I_t)$ are the claims payments, Δ is the lag operator, and D_t is the annual dividend paid to shareholders.

[2] Hanewald (2009) shows that the reaction of mortality rates to real GDP is stronger for the working-age population than for retirees. Therefore, we look at term-life contracts and not, e.g., at annuities.

With $\text{PV}_t[\cdot]$ denoting a present value operator, which is specified in Subsection 3.2, the market value of year-end liabilities L_t is given by:

$$L_t = \text{PV}_t[\textit{Future claims payment}] - \text{PV}_t[\textit{Future premium income}]. \quad (4)$$

Dividends D_t at the end of the year are assumed to be a constant fraction d of the insurer's net income for that year N_t when N_t is positive; zero otherwise. Formally, D_t is given by:

$$D_t = \max\{d \cdot N_t, 0\}, \quad (5)$$

where net income N_t is defined as:

$$N_t = A_{t\text{-}1} \cdot (R_t - 1) + P \cdot I_{t\text{-}1} \cdot R_t - B \cdot (-\Delta I_t) - \Delta L_t. \quad (6)$$

3.2 Random variables and stochastic processes

We now define the stochastic processes driving our model. Real GDP is introduced first; it is the fundamental link between the other random variables, i.e., the number of surviving insureds I_t (driven by the mortality index k_t) and capital market returns R_t.

Following Kruse, Meitner, and Schröder (2005), a lognormal distribution is assumed for real GDP. Thus annual changes in log real GDP are given by:

$$\Delta\ln(\textit{real GDP}_t) = \mu_{GDP} + \sigma_{GDP} \cdot \varepsilon_{GDP,\, t}\,, \quad (7)$$

where μ_{GDP} and σ_{GDP} denote the mean and standard deviation of real GDP growth rates and $\varepsilon_{GDP,\, t}$ is a standardized normal random variable.

The number of deaths at the end of each year $-\Delta I_t$ is assumed to follow a binomial distribution $\text{B}(I_{t\text{-}1},\ q_{x+t\text{-}1,\, t})$. We hereby account for unsystematic mortality risk, i.e., the fact that the actual number of deaths might deviate from the expected number. The probability for each insured aged $x + t - 1$ at the beginning of a year to die at the end of the year t is denoted as $q_{x+t-1,\ t}$. Considering stochastic mortality, i.e., accounting for systematic mortality risk, the probability $q_{x+t-1,\ t}$ itself is also a random variable realizing in t. Age-specific mortality probabilities $q_{x,\ t}$ are derived from the central death rates $m_{x,\ t}$ of a Lee-Carter-type model, using the approximation:[3]

$$q_{x,\ t} = m_{x,\ t} / (1 + 0.5 \cdot m_{x,\ t}). \tag{8}$$

According to the Lee-Carter approach, and abstracting from age-specific shocks,[4] central death rates $m_{x,t}$ are given by:

$$m_{x,\ t} = \exp(a_x + b_x \cdot k_t), \tag{9}$$

where a_x is an age-specific constant and b_x describes the sensitivity of age-specific mortality rates to changes in the mortality index k_t, which is a random variable.

As in the original Lee and Carter (1992) model, the stochastic process for the mortality index k_t is modeled as a random walk with drift:

$$\Delta k_t = \theta + \sigma_k \cdot \varepsilon_{k,\ t}, \tag{10}$$

with $\varepsilon_{k,\ t}$ being a standardized normal random variable.

In summary, there are two sources of randomness in our model for the number of deaths. One is based in the uncertainty regarding the path of the underlying mortality index k_t. The other source of randomness results from sampling the insurance portfolio.

---

[3] See Cairns, Blake, and Dowd (2008).

[4] It is common in the literature to ignore the age-specific error term $\varepsilon_{x,\ t}$ at this stage of the model (see, e.g., Cairns, Blake, and Dowd, 2008). A justification is provided by Lee and Carter (1992) themselves, who show that up to 90 percent of the standard errors of age-specific death rate forecasts are accounted for by uncertainty in k_t (Lee and Carter, 1992, Table.B2, forecast horizon of 10 years).

Distribution of the asset return R_t depends on the insurer's asset allocation decisions. Following Kling, Richter, and Ruß (2007), we allow for two lognormally distributed investment opportunities: stocks and bonds. Let $r_{s,t}$ denote the stock log-return in period t and $r_{b,t}$ the bond log-return, and let $\alpha \in [0, 1]$ be the fraction of assets invested in stocks. Then, the return of the annually rebalanced asset portfolio R_t is given by:

$$R_t = \alpha \cdot \exp(r_{s,t}) + (1 - \alpha) \exp(r_{b,t}), \tag{11}$$

where:

$$r_{s,t} = \mu_s + \sigma_s \cdot \varepsilon_{s,t}, \text{ and} \tag{12}$$
$$r_{b,t} = \mu_b + \sigma_b \cdot \varepsilon_{b,t},$$

with μ_s, μ_b, σ_s, and σ_b denoting the mean and standard deviation of log-returns, and $\varepsilon_{s,t}$ and $\varepsilon_{b,t}$ being standardized normal random variables.

In a last step, we specify the value of the insurer's liabilities at the end of each year L_t, which were introduced in Equation (4). At the end of every year, the insurer observes the realized bond returns and the current level of the mortality index k_t. The insurer uses the latter as a starting point for projecting future mortality rates; observed bond returns are used to discount expected liabilities. Thus, the market value of liabilities—in the "second" between that year's claim payments and next year's premium income—is given by:

$$L_t = I_t \cdot \mathrm{E}_t \left[B \cdot \sum_{\tau=t+1}^{T} \frac{{}_{\tau-t}q_{x+t,t}}{\exp(r_{b,t})^{\tau-t}} - P \cdot \sum_{\tau=t}^{T-1} \frac{{}_{\tau-t}p_{x+t,t}}{\exp(r_{b,t})^{\tau-t}} \right], \tag{13}$$

where ${}_iq_{x,t}$ is the probability that an insured aged x will die after age $x + i - 1$, while ${}_ip_{x,t}$ is the probability that an insured aged x will survive at least another i years. The insurer calculates both probabilities conditional on the information available at time t. In

Equation (13), the present value calculus is specified by taking the expectation of future cash flows with respect to the real-world probability measure without further risk adjustments. Thus, we assume that the insurer is unaware of any correlations between mortality and GDP or the capital market development, i.e., the insurer does not consider the systematic nature of mortality risk.

In summary, economic and demographic randomness in our model are induced by the following random variables: the mortality index k_t, real GDP growth rates, and bond and stock returns. The main contribution of this paper is to account for the interaction of these factors, especially the dependency between mortality rates and economic conditions, which we accomplish by allowing for a correlation matrix with nonzero off-diagonal elements for the random variables $\varepsilon_{k,\,t}$, $\varepsilon_{s,\,t}$, $\varepsilon_{b,\,t}$, and $\varepsilon_{GDP,\,t}$.[5]

3.3 Numerical calibration of the model

Calibration of the model involves estimating parameters of the stochastic processes from empirical data, as well as setting insurance contract and management parameters. We begin with a base scenario, but will vary several of the parameters later on in the analysis.

3.2.1 Management assumptions

The fixed proportion γ of the first premium income $I_0 \cdot P$ raised as initial equity capital E_0 is set to 0.1. The dividend ratio d, i.e., the constant fraction of the insurer's net income paid out to shareholders, is set to 0.1. The asset fraction α that is invested in stocks is set to 0.3. This parameter set results in reasonably small one-period insolvency probabilities.

[5] Demographic studies show that the impact of changes in the macroeconomic conditions on mortality rates is primarily contemporary (see, e.g., Tapia Granados, 2005). Our data confirm this finding. Correlation between the mortality index and GDP growth rates of the previous year turned out to be close to zero. Therefore, we only account for correlations at time t in our model.

3.2.2 Contract characteristics

We consider a term-life insurance contract with a duration of T = 10 years and a death benefit of B = $100,000. This contract is sold to I_0 = 10,000 insureds in t = 0. In the base scenario, all insureds are male and of age x = 40. Mortality data are available up until 2005; therefore, the simulation starts with t = 0 at the beginning of 2006.

The fair premium for an individual contract is calculated by solving Equation (14) for P_{fair}:

$$E_0\left[P_{fair}\cdot\sum_{\tau=0}^{T-1}\frac{{}_{\tau}p_{x,0}}{\exp(r_{b,0})^{\tau}}\right]=E_0\left[B\cdot\sum_{\tau=1}^{T}\frac{{}_{\tau}q_{x,0}}{\exp(r_{b,0})^{\tau}}\right]. \tag{14}$$

Thus, the same assumptions used to calculate future liabilities in Equation (13) apply for premium calculation. The contract is sold at a premium P that includes a proportional loading λ on the fair premium, which, in the base scenario, is set to 0.1:

$$P = (1 + \lambda) \cdot P_{fair}. \tag{15}$$

3.2.3 Stochastic processes

Death rates and population size for the United States were obtained from the Human Mortality Database (University of California and Max Planck Institute, 2008), where they are available up until 2005. A series for “GDP in billions of chained 2000 dollars” was obtained from the website of the U.S. Bureau of Economic Analysis (Bureau of Economic Analysis, 2008). For calibration of the return processes we use annual total returns of large company stocks and U.S. treasury bills, which are published in Morningstar (2008). In the following, these series are referred to as real GDP, stock returns, and bond returns.

The Lee-Carter model was estimated with the R package “demography” by Hyndman et al. (2008). The Lee-Miller (2001) variant was chosen; it has been widely adopted as the standard Lee-Carter method (Booth and Tickle, 2008) and involves estimating the model for the latter half of the twentieth century[6]. Male and female forecasts are treated as two separate applications of the basic Lee-Carter approach (Lee, 2000). The model is estimated with the same upper age limit as in the original (85 years) article by Lee and Carter (1992) and a minimum age of 30. Fig. 1 plots the estimated parameters a_x and b_x that are needed to derive age-specific death rates $m_{x,t}$ from the mortality index k_t. Fig. 2 plots the mortality index k_t that was extracted for U.S. males for 1950–2005, together with the 2006–2015 forecast.

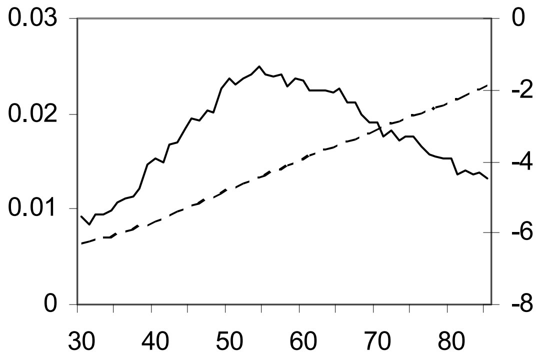

Fig. 1 Fitted values for a_x (dashed, right axis) and b_x (solid, left axis) for ages 30–85.

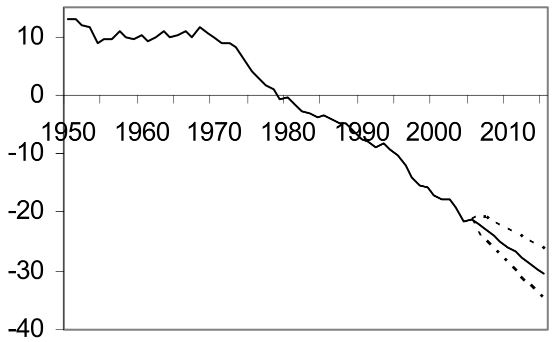

Fig. 2 Mortality index k_t, fitted values 1950–2005, forecast 2006–2015 with 95% confidence band.

The extracted time series for the mortality index k_t, together with the time series for GDP and returns, are used to estimate the parameters and correlation structure of the four exogenous stochastic processes. Based on results from Hanewald (2009), the period 1989–2005 was chosen for estimation. Hanewald’s study documents a structural change in the correlation structure between real GDP growth rates and the mortality index in six OECD countries over the period 1950–2005. For the United States, a significant break point was identified for 1989 using dummy variable regressions.

[6] Furthermore, the approach involves adjustment of the mortality index k_t to life expectancy e_0 instead of total deaths, and forecasting forward from observed (rather than fitted) rates.

Table 1 summarizes the estimated parameters and correlation structure.

Table 1 Estimation Results, 1989–2005.

	Real GDP Growth Rates	Stock Returns	Bond Returns	Changes in the Mortality Index
	$\Delta\ln(real\ GDP_t)$	$r_{s,\,t}$	$r_{b,\,t}$	Δk_t
Mean	0.029	0.110	0.043	-0.955
Standard Deviation	0.013	0.167	0.020	0.828
Correlation Matrix				
Real GDP	1.000	0.282	0.050	-0.395
Stock Returns		1.000	0.266	-0.286
Bond Returns			1.000	-0.195
Mortality index				1.000

4. Simulation results

The insurance company model was simulated with 100,000 iterations using the Latin Hypercube technique (McKay, Conover, and Beckman, 1979).

As a benchmark for comparison, we first simulate a version of the model that ignores the impact of macroeconomic changes on mortality rates. This scenario assumes that the mortality index k_t in the Lee-Carter model is uncorrelated with economic conditions as reflected by the processes for GDP, stocks, and bonds, i.e., entries in the last column of the correlation matrix in Table 1 are set to 0 (except the last value, which is 1). Next, the scenario employing the full correlation structure is simulated. The difference in insolvency probabilities between the two scenarios is a measure of model misspecification risk. Results are given in Fig. 3.

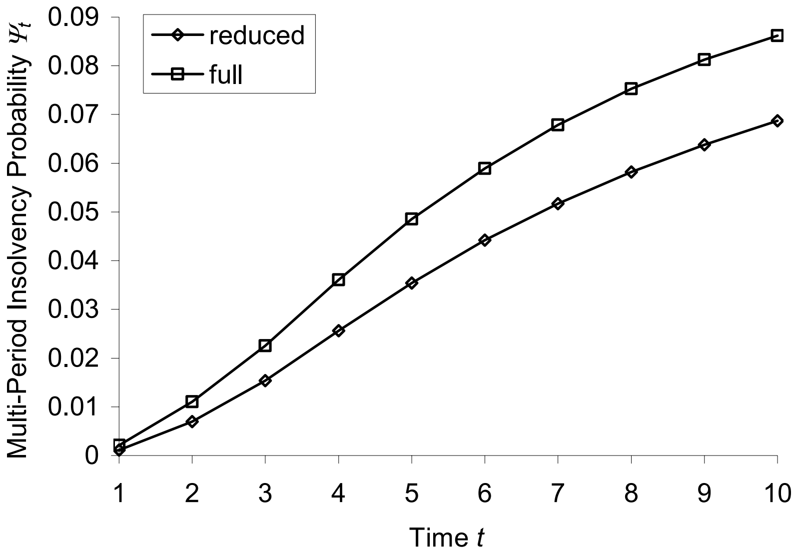

Fig. 3 Multi-period insolvency probability Ψ_t, base parameter calibration, full correlation structure vs. reduced correlation structure.

Multi-period insolvency probabilities increase over time in both scenarios. There are two reasons for this: first, confidence intervals for the realizations of the random variables, e.g., for the mortality index k_t (c.p., Fig. 2), broaden with an increasing time horizon; and second, insolvency probabilities cumulate because firms that become insolvent remain insolvent.

Looking at Fig. 3 reveals that employing the full correlation structure increases the insolvency probability for every time horizon considered. Thus, ignoring the correlations between the mortality index k_t and the economic variables will result in a systematic underestimation of the true insolvency probability. This will occur because the true correlation structure links assets and liabilities in a very unfavorable way: a drop in GDP, in tendency, coincides with lower stock and bond returns, i.e., with a shrinking asset base, along with a higher mortality index k_t, resulting in higher liabilities. Both effects take a toll on equity capital. In absolute numbers, the difference in insolvency probabilities between the two scenarios increases from 0.1 percentage points in $t = 1$ to 1.8 percentage points in $t = 10$.

Fig. 4 plots multi-period insolvency probabilities for four different initial ages x of insureds.

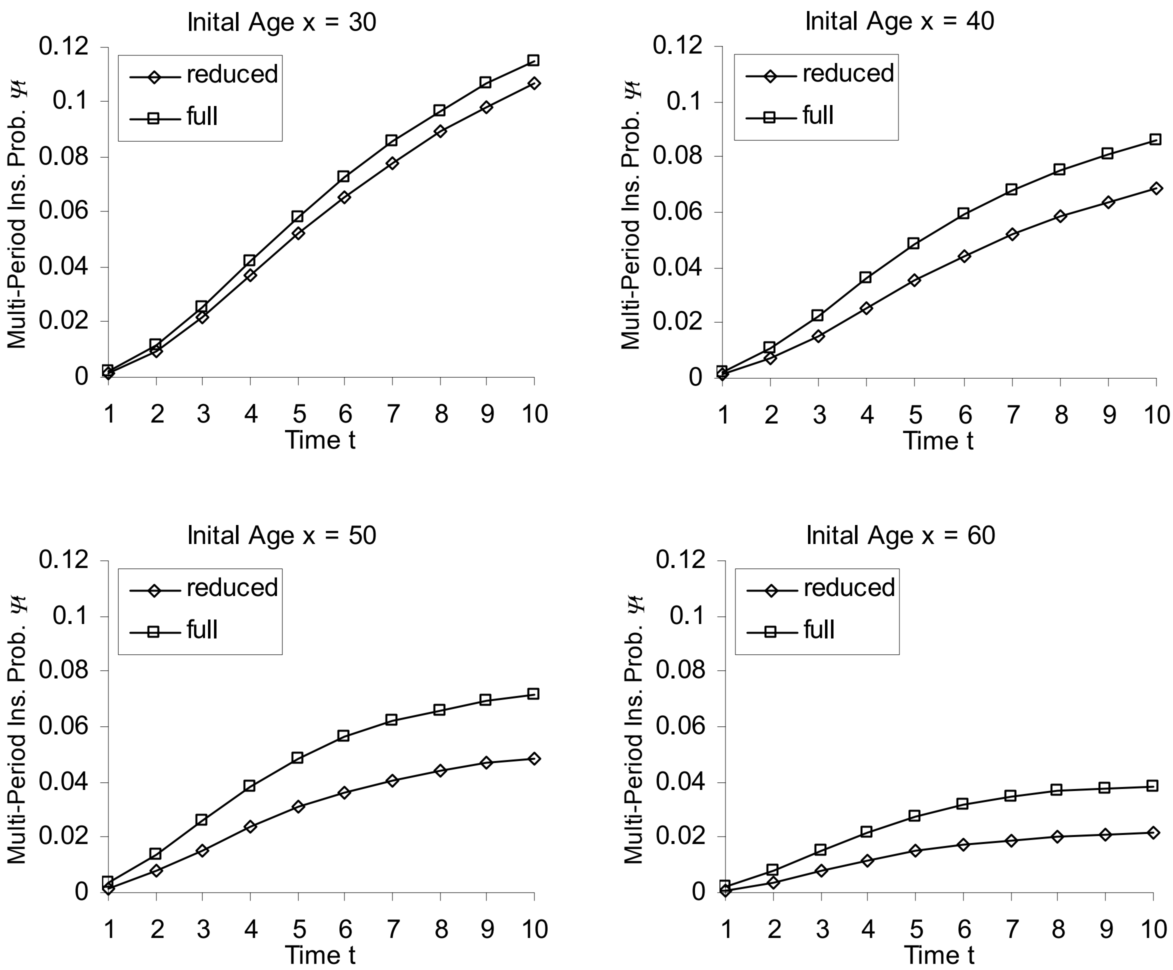

Fig. 4 Multi-period insolvency probabilities Ψ_t, reduced and full correlations structure, different initial ages x.

For all four ages, we again observe higher insolvency probabilities under the full correlation structure, meaning that our results are robust to changes in age. However, there are two noteworthy effects that result from varying the age parameter. First, insolvency probabilities decrease in initial age. This is due to the fact, that for higher ages generally the variation of the number of deaths around the (now higher) mean in relative terms, i.e., the variation coefficient, is smaller. Second, the increase in insolvency probabilities from switching to the full correlation scenario is greater at higher ages x, except for age x = 60. This effect is explained by the different sensitivity of the age-specific death rates to shocks in the mortality index k_t, which is controlled by b_x (c.p., Equation (9)). As can be seen in Fig. 1, the parameter b_x exhibits a hump-shaped profile, peaking around age 50. For that reason, we observe a smaller absolute increase in insolvency probability for age $x = 60$ in comparison to age $x = 50$.

The effect of different initial insurance portfolio sizes I_0 is illustrated in Fig. 5.

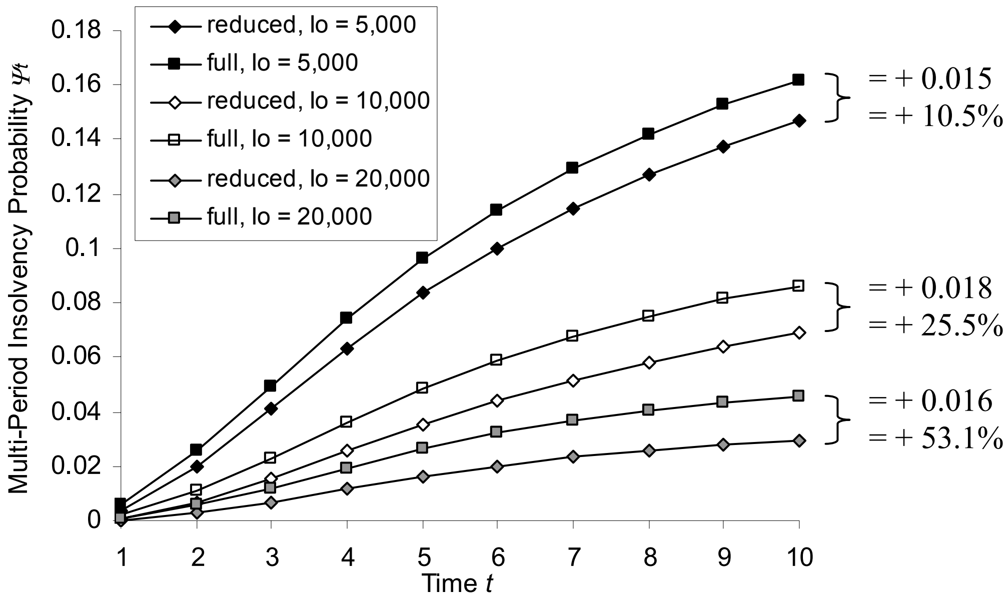

Fig. 5 Multi-period insolvency probabilities Ψ_t, reduced and full correlations structure, different initial numbers of insureds I_0.

Not surprisingly, we find that insolvency probabilities are generally higher for smaller portfolios due to a less pronounced risk pooling. However, in relative terms, ignoring the true correlation structure leads to a more severe underestimation of the true insolvency probability for larger portfolios. For example, the relative change in the level of the 10-year insolvency probability amounts to +10.5% for $I_0 = 5{,}000$ insureds versus +53.1% for $I_0 = 20{,}000$ insureds. This effect can be explained by noting that in small portfolios less unsystematic risk is eliminated compared to large portfolios. By accounting for the full correlation structure, a similar amount of systematic risk (in absolute terms) is added to the risk exposure of both small and large portfolios, leading to a higher relative increase in the overall risk, measured by the insolvency probability, for large portfolios. In other words, for both small and large portfolios the diversification potential decreases, but with more severe consequences for a portfolio originally believed to be well-diversified.

Fig. 6 plots multi-period insolvency probabilities for three different fractions γ used when calculating initial equity E_0.

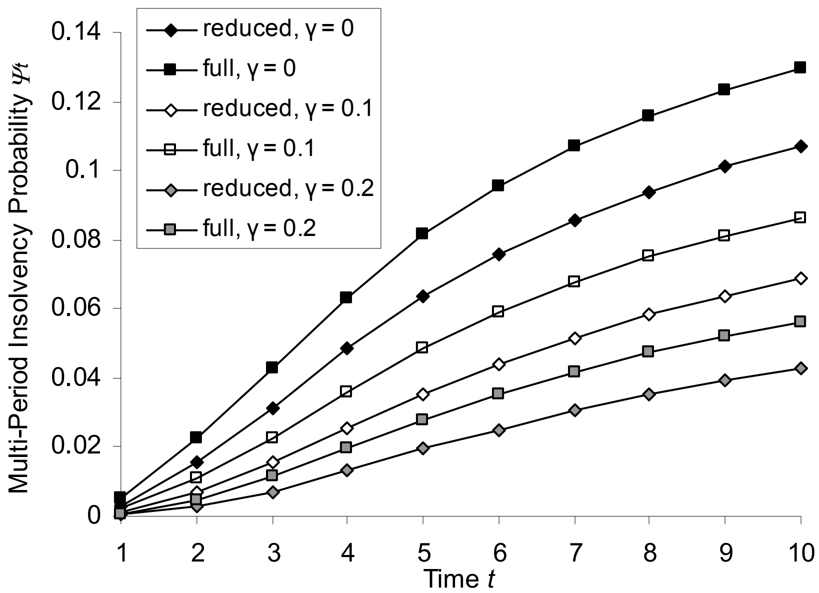

Fig. 6 Multi-period insolvency probability Ψ_t, reduced and full correlations structure, different equity buffer factors γ.

Insolvency probabilities in Fig. 6 are similar, and for similar reasons, to those shown in Fig. 5. A higher equity buffer, i.e., a higher constant fraction γ of initial premium income raised as equity capital, improves the insurer's solvency situation. Adding, through implementing the full correlation structure, a similar absolute amount of systematic risk on top of the three considered risk exposures leads to a larger relative increase in risk for higher initial amounts of equity capital. In this sense, safer firms, meaning those with greater equity capital, suffer more from the described model risk.

Multi-period insolvency probabilities for different fractions α of assets invested in stocks are plotted in Fig. 7.

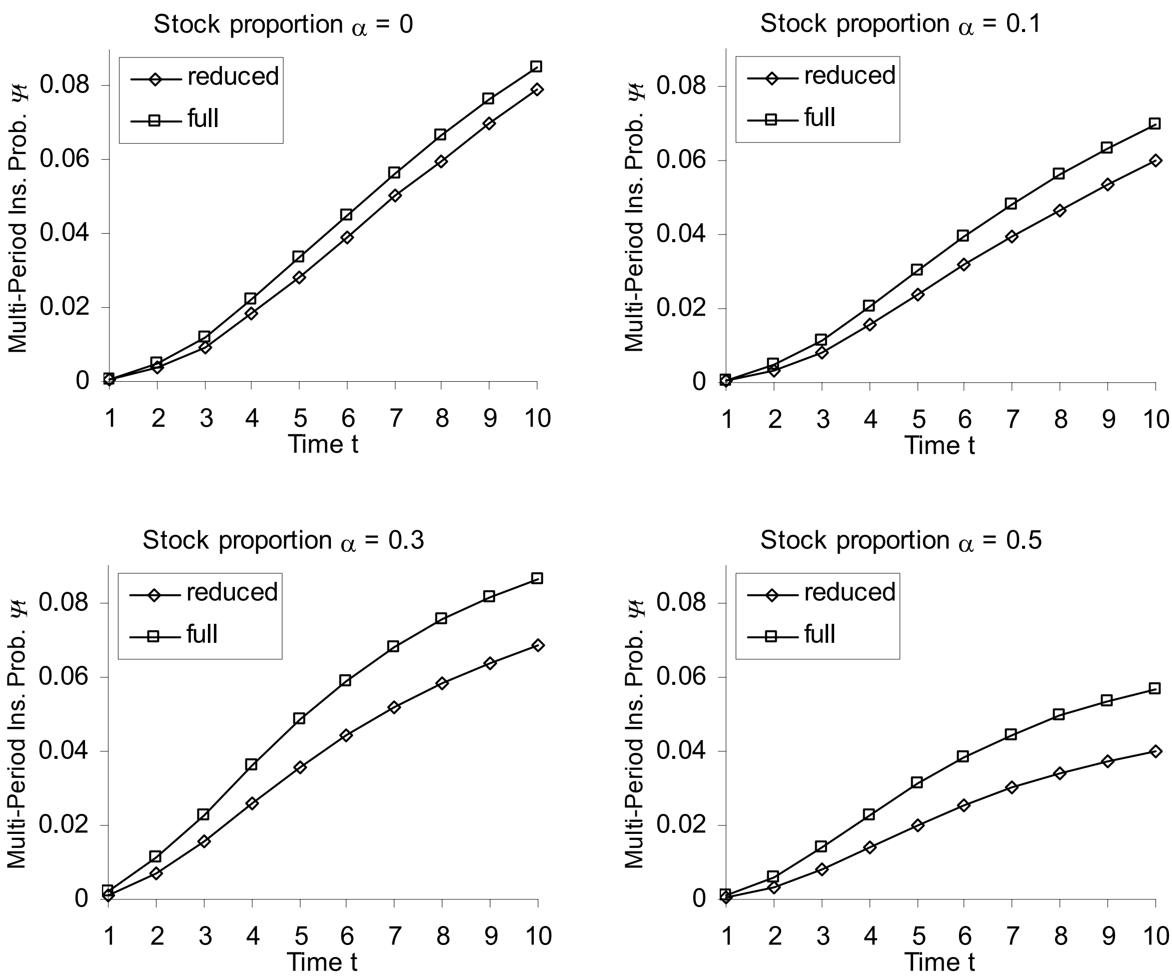

Fig. 7 Multi-period insolvency probabilities Ψ_t, reduced and full correlations structure, different proportions α of assets invested in stocks.

First, Fig. 7 again confirms that insolvency probabilities are always higher under the full correlation structure.

Second, we observe some general effects of increasing the proportion of stocks in the asset portfolio. On the one hand, the higher expected return of stocks can lead to reduced insolvency probabilities as assets accumulate more rapidly (compare $\alpha = 0$ with $\alpha = 0.1$, and $\alpha = 0.3$ with $\alpha = 0.5$). On the other hand, the higher volatility of stocks can worsen the insurer's solvency situation (compare $\alpha = 0.1$ with $\alpha = 0.3$).

Third, the proportion of stocks influences the difference in insolvency probabilities between the two scenarios both in absolute and in relative terms. A larger fraction of stocks induces a higher exposure of the insurer to the unfavorable dependency between GDP, assets, and mortality, thus liabilities, that was described under Fig. 1. Hence, ignoring the full correlation structure results in a more severe underestimation of insolvency probability by insurers heavily invested in stocks.

5. Conclusion and outlook

Based on demographic findings establishing a link between macroeconomic conditions and mortality, we assess the impact of macroeconomic fluctuations on the financial stability of a life insurance company. We develop a dynamic asset-liability model that allows both sides of the balance sheet to react to the state of the economy. Stochastic drivers in our model are real GDP, mortality, bond returns, and stock returns.

We find that multi-period insolvency probabilities are considerably higher when taking into account the dependencies between the mortality index k_t in the Lee-Carter model and economic conditions. Thus, ignoring the existing dependency structure will lead an insurer to systematically underestimate its true insolvency probability. This result is robust to variations in the age of insureds, portfolio size, equity base, and asset allocation. Through the systematic nature of mortality risk, the relative increase in insolvency probability is higher for insurers with a generally lower insolvency probability. In our model, these are the insurers that have a high equity buffer, relatively mature insureds, and have written a large number of contracts. Additionally, the underestimation risk is more severe for insurers heavily invested in stocks, both in absolute and relative terms.

Therefore, the interaction between mortality and macroeconomic conditions needs to be an integral part of life insurers' internal risk models, of capital allocation decision making, and of solvency assessment by rating agencies and regulatory authorities.

Taking this crucial relationship into consideration will make assessments of an insurer's risk situation more accurate and thus more effectively protect policyholders.

Other useful applications of our model could involve investigating a more general insurance portfolio that includes a mixed age structure or annuity contracts. For a mixed age structure, we expect the following: Because all age-specific mortality rates react in the same direction to changes in GDP, the resulting effect on insolvency probabilities would be a mixture of the age-specific increases shown in Fig. 4. Including annuities, i.e., contracts written on the opposite site of mortality risk, would give rise to natural hedging opportunities, as analyzed in Gründl, Post, and Schulze (2006) and Cox and Lin (2007). Additionally, dependencies between lapse and surrender rates and macroeconomic conditions (Browne, Carson and Hoyt, 2001; Kim, 2005) could be accounted for.

References

Bauer, D., and Weber, F. (2008), Assessing investment and longevity risks within immediate annuities, *Asia-Pacific Journal of Risk and Insurance*, 3: 89–111.

Booth, H. (2006), Demographic forecasting: 1980 to 2005 in review, *International Journal of Forecasting*, 22: 547–581.

Booth, H., Hyndman, R. J., Tickle, L., and de Jong, P. (2006), Lee-Carter mortality forecasting: A multi-country comparison of variants and extensions, *Demographic Research*, 15: 289–310.

Booth, H., Maindonald, J., and Smith, L. (2002), Applying Lee-Carter under conditions of variable mortality decline, *Population Studies*, 56: 325–336.

Booth, H., and Tickle, L. (2008), Mortality modelling and forecasting: A review of methods, *Annals of Actuarial Science* (forthcoming).

Brouhns, N., Denuit, M., and Vermunt, J. K. (2002), A Poisson log-bilinear regression approach to the construction of projected lifetables, *Insurance: Mathematics and Economics*, 31: 373–393.

Browne, M. J., Carson, J. M., and Hoyt, R. E. (2001), Dynamic financial models of life insurers, *North American Actuarial Journal*, 5: 11–26.

Bureau of Economic Analysis (2008), National Economic Accounts, Gross Domestic Product, available at: http://www.bea.gov/national/index.htm#gdp.

Cairns, A. J. G., Blake, D., and Dowd, K. (2008), Modelling and management of mortality risk: A review, *Scandinavian Actuarial Journal*, 79–113.

Cox, S. H., and Lin, Y. (2007), Natural hedging of life and annuity mortality risks, *North American Actuarial Journal*, 11: 1–15.

Debón, A., Montes, F., and Puig, F. (2008), Modelling and forecasting mortality in Spain, *European Journal of Operational Research*, 3: 624–637.

Dowd, K., Cairns, A., and Blake, D. (2006), Mortality-dependent financial risk measures, *Insurance: Mathematics and Economics,* 38: 427–440.

Gerdtham, U. G., and Ruhm, C. J. (2006), Deaths rise in good economic times: Evidence from the OECD, *Economics & Human Biology*, 4: 298–316.

Girosi, F., and King, G. (2008), *Demographic Forecasting*, Princeton University Press, Princeton.

Gründl, H., Post, T., and Schulze, R. N. (2006), To hedge or not to hedge: Managing demographic risk in life insurance companies, *Journal of Risk and Insurance*, 73, 19–41.

Hanewald, K. (2008), Beyond the business cycle—Factors driving aggregate mortality rates, *SFB 649 Discussion Paper 2008-031*, Humboldt-Universität zu Berlin.

Hanewald, K. (2009), Lee-Carter and the macroeconomy, *SFB 649 Discussion Paper 2009-008*, Humboldt-Universität zu Berlin.

Härdle, W. K., and Myšičková, A. (2009), Stochastic population forecast for Germany and its consequence for the German pension system, *Working Paper*, Humboldt-Universität zu Berlin.

Hári, N., De Waegenaere, A., Melenberg, T., and Nijman, T. E. (2008a), Estimating the term structure of mortality, *Insurance: Mathematics and Economics,* 42: 492–504.

Hári, N., De Waegenaere, A., Melenberg, T., and Nijman, T. E. (2008b), Longevity risk in portfolios of pension annuities, *Insurance: Mathematics and Economics,* 42: 505–519.

Hyndman, R. J., Booth, H., Tickle, L., and Maindonald, J. (2008), Demography: Forecasting mortality and fertility data, R package, available at: http://robjhyndman.com.

Kim, C. (2005), Modeling surrender and lapse rates with economic variables, *North American Actuarial Journal*, 9: 56–70.

Kling, A., Richter, A., and Ruß, J. (2007), The interaction of guarantees, surplus distribution, and asset allocation in with-profit life insurance policies, *Insurance: Mathematics and Economics*, 40: 164–179.

Kruse, S., Meitner, M., and Schröder, M. (2005), On the pricing of GDP linked financial products, *Applied Financial Economics*, 15: 1125–1133.

Lee, R. D. (2000), The Lee-Carter method for forecasting mortality, with various extensions and applications, *North American Actuarial Journal,* 4: 80–93.

Lee, R. D., and Carter, L. (1992), Modeling and forecasting U.S. mortality, *Journal of the American Statistical Association,* 87: 659–671.

Lee, R. D., and Miller, T. (2001), Evaluating the performance of the Lee-Carter method for forecasting mortality, *Demography,* 38: 537–549.

Li, N., Lee, R., and Tuljapurkar, S. (2004) Using the Lee-Carter method to forecast mortality for populations with limited data, *International Statistical Review,* 72: 19–36.

McKay, M., Conover, W., and Beckman, R. (1979), A comparison of three methods for selecting values of input variables in the analysis of computer code, *Technometrics*, 211: 239–245.

Morningstar (2008), *Ibbotson SBBI 2008 Valuation Yearbook: Market Results for Stocks, Bonds, Bills, and Inflation,* Chicago, IL.

Neumayer, E. (2004), Recessions lower (some) mortality rates: Evidence from Germany, *Social Science & Medicine*, 58: 1037–1047.

Reichmuth, W. H., and Sarferaz, S. (2008), The influence of the business cycle on mortality, *SFB 649 Discussion Paper 2008-059*, Humboldt-Universität zu Berlin.

Renshaw, A. E., and Haberman, S. (2006), A cohort-based extension to the Lee-Carter model for mortality reduction factors*, Insurance: Mathematics and Economics*, 58: 556–570.

Ruhm, C. J. (2000), Are recessions good for your health? *Quarterly Journal of Economics*, 115: 617–650.

Tapia Granados, J. A. (2005a), Increasing mortality during the expansion of the U.S. economy 1900–1996, *International Journal of Epidemiology*, 34: 1194–1202.

Tapia Granados, J. A. (2005b), Recessions and mortality in Spain, 1980–1997, *European Journal of Population*, 21: 393–422.

Tapia Granados, J. A. (2008), Macroeconomic fluctuations and mortality in postwar Japan, *Demography*, 45: 323–343.

Tapia Granados, J. A., and Ionides, E. L. (2008), The reversal of the relation between economic growth and health progress: Sweden in the 19th and 20th centuries, *Journal of Health Economics*, 27: 544-563.

Tuljapurkar, S., Li, N., and Boe, C. (2000), A universal pattern of mortality decline in the G7 countries, *Nature*, 405: 789–792.

University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany) (2008), Human Mortality Database, available at: http://www.mortality.org.

SFB 649 Discussion Paper Series 2009

For a complete list of Discussion Papers published by the SFB 649, please visit http://sfb649.wiwi.hu-berlin.de.

001 "Implied Market Price of Weather Risk" by Wolfgang Härdle and Brenda López Cabrera, January 2009.
002 "On the Systemic Nature of Weather Risk" by Guenther Filler, Martin Odening, Ostap Okhrin and Wei Xu, January 2009.
003 "Localized Realized Volatility Modelling" by Ying Chen, Wolfgang Karl Härdle and Uta Pigorsch, January 2009.
004 "New recipes for estimating default intensities" by Alexander Baranovski, Carsten von Lieres and André Wilch, January 2009.
005 "Panel Cointegration Testing in the Presence of a Time Trend" by Bernd Droge and Deniz Dilan Karaman Örsal, January 2009.
006 "Regulatory Risk under Optimal Incentive Regulation" by Roland Strausz, January 2009.
007 "Combination of multivariate volatility forecasts" by Alessandra Amendola and Giuseppe Storti, January 2009.
008 "Mortality modeling: Lee-Carter and the macroeconomy" by Katja Hanewald, January 2009.
009 "Stochastic Population Forecast for Germany and its Consequence for the German Pension System" by Wolfgang Härdle and Alena Mysickova, February 2009.
010 "A Microeconomic Explanation of the EPK Paradox" by Wolfgang Härdle, Volker Krätschmer and Rouslan Moro, February 2009.
011 "Defending Against Speculative Attacks" by Tijmen Daniëls, Henk Jager and Franc Klaassen, February 2009.
012 "On the Existence of the Moments of the Asymptotic Trace Statistic" by Deniz Dilan Karaman Örsal and Bernd Droge, February 2009.
013 "CDO Pricing with Copulae" by Barbara Choros, Wolfgang Härdle and Ostap Okhrin, March 2009.
014 "Properties of Hierarchical Archimedean Copulas" by Ostap Okhrin, Yarema Okhrin and Wolfgang Schmid, March 2009.
015 "Stochastic Mortality, Macroeconomic Risks, and Life Insurer Solvency" by Katja Hanewald, Thomas Post and Helmut Gründl, March 2009.

SFB 649, Spandauer Straße 1, D-10178 Berlin
http://sfb649.wiwi.hu-berlin.de

This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".