The optimal industry structure in a vertically related market

Raffaele Fiocco*
The optimal industry structure in a vertically related market

Raffaele Fiocco*

April 19, 2010

Abstract

We consider a vertically related market characterized by downstream imperfect competition and by the monopolistic provision of an essential facility-based input, whose price is set by a social-welfare maximizing regulator. Our model shows that the regulatory knowledge about the cost for providing the monopolistic input crucially affects the design of the optimal industry structure. In particular, we compare ownership separation, which prevents a single company from having the control of both upstream and downstream operations, and legal separation, under which these activities are legally unbundled but common ownership is allowed. As long as the regulator has full information, the two industry patterns yield the same social welfare level. However, under asymmetric information about the input costs legal separation can make the whole society better off.

Keywords: access charge, legal separation, ownership separation, regulation.

JEL Classification: D82, L11, L51.

Humboldt-Universität zu Berlin, Institute for Microeconomic Theory, Spandauer Str. 1, D-10178 Berlin, Germany. E-mail: raffaele.fiocco@staff.hu-berlin.de. I am extremely grateful to the Center for Research on Energy and Environmental Economics and Policy (IEFE), Bocconi University (Milan). In particular, I am indebted to the Research Fellows at IEFE Federico Pontoni and Antonio Sileo who read the first draft of the paper and gave insightful suggestions. I would like to thank Clara Poletti and Susanna Dorigoni, Director and Research Fellow at IEFE respectively. I also owe special thanks to Roland Strausz for valuable advice. This work was supported by Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

* Humboldt-Universität zu Berlin, Institute for Microeconomic Theory, Spandauer Str. 1, D-10178 Berlin, Germany. E-mail: raffaele.fiocco@staff.hu-berlin.de. I am extremely grateful to the Center for Research on Energy and Environmental Economics and Policy (IEFE), Bocconi University (Milan). In particular, I am indebted to the Research Fellows at IEFE Federico Pontoni and Antonio Sileo who read the first draft of the paper and gave insightful suggestions. I would like to thank Clara Poletti and Susanna Dorigoni, Director and Research Fellow at IEFE respectively. I also owe special thanks to Roland Strausz for valuable advice. This work was supported by Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".
1. Introduction

The large-scale liberalization process occurred over last decades has affected many sectors where naturally monopolistic and potentially competitive activities are vertically related. This is often the case, for instance, in network industries, like the electricity, natural gas, railways and water utilities. The supply of the service to final consumers, which admits competition at least to some extent, requires the use of an essential facility-based input - the network - provided by a monopolistic firm.

One of the most interesting issues in policy debates is how to design the industry structure following the liberalization process. In practice, this question has received different answers. The Electricity Act of 1989 divided the Central Electricity Generating Board (CEGB) of England and Wales, which operated as a vertically integrated statutory monopoly, in four public limited companies and transmission grid activities were separated from generation. The same approach was followed in USA, where, after some important legislative measures, the Order 888 issued by the Federal Energy Regulatory Commission (FERC) in 1996 mandated that owners of regional transmission networks act as common carriers of electric power. Rather than having one vertically integrated provider of electricity, retail customers can now access the wholesale power market directly and purchase unbundled distribution and transmission services from their local utility to deliver power.

On the contrary, in 1984 British Telecommunications (BT) was privatized as a vertically integrated monopoly and only in 1995 there was the accounting separation of its operations into network and retail businesses. Also the privatization of British Gas (BG) in 1986 occurred without restructuring. Even though the government did not follow the 1993 Monopolies and Mergers Commission’s recommendation for breaking up the company, now BG provides its pipeline services through a separate unit.\(^1\)

More recently, the European Union has focused on the design of market structure in network industries. The European directives 2009/72/EC and 2009/73/EC,\(^2\) which concern common rules for the internal market in electricity and natural gas respectively, provide that a transmission system owner, which is part of a vertically integrated undertaking, must be inde-

---

1. Newberry [10] provides a precise account of the most important regulatory reforms of network utilities in the USA and the UK. See also the overview of Viscusi, Harrington and Vernon [13], which focuses on the case of the United States.

2. These directives, issued on 13 July 2009, repeal the directives 2003/54/EC and 2003/55/EC.
dependent *at least* in terms of its legal form, organization and decision-making from other activities not relating to transmission. These rules do *not* create an obligation to separate the ownership of assets of the transmission system from the other activities, even though the European Commission had strongly recommended the actual separation of production from network services.

This discussion emphasizes that we can identify two main approaches to the problem of designing the industry structure in markets where regulated and competitive activities are vertically related. The first one, which prohibits the upstream regulated monopolist from participating (directly or indirectly) in the downstream competitive segment, is known as *ownership separation*. The alternative solution, according to which upstream and downstream operations must be legally unbundled but common ownership is allowed, is defined as *legal separation*. As Vickers points out, *despite its importance for policy, the question of whether a regulated monopolist should be allowed also to operate in a vertically related industry has received relatively little theoretical attention* [12, p. 16].

This paper aims at giving a contribution to the debate about the choice of the type of unbundling to implement in vertically related markets and advancing some policy suggestions.

The topic is so broad and complex that it cannot be treated in an exhaustive way. We want to address only some relevant aspects of the problem at hand. Vogelgang [14] stresses that asymmetric information between regulators and regulated firms, despite its importance for the access pricing in network industries, has so far played a minor role in the policy debate. Armstrong and Sappington also have recently recognized that *further research is warranted on the design of regulatory policy in vertically-integrated industries when regulators are less omniscient* [4, p. 1684].

We intend to investigate how the presence of limited regulatory knowledge about the cost for providing an upstream facility-based input can affect the optimal industry structure, when the regulator is charged with determining the input access price paid by downstream competitive firms.

---

3 We recognize that a way to organize such an industry is to have one vertically integrated monopolist and to regulate the final product price. However, we assume that policy makers want to promote liberalization in potentially competitive segments, consistently with the practical examples quoted above.

4 Armstrong and Sappington [3] emphasize that there are other important issues that should be take into account when the potential benefits of legal and ownership separation are assessed, like their different impact on the economies of scope or the quality of the upstream input. The evaluation of these aspects is outside the scope of this paper.
In other terms, we want to answer the following question. When the input access price is regulated in a situation of asymmetric cost information, is it better to have legal separation or to require ownership unbundling between upstream and downstream operations?

Economic literature has recognized that one of the most important benefits from a policy of ownership separation is the prevention of anticompetitive practices in the unregulated market. When it operates (directly or indirectly) in the retail market, the input monopolist will generally anticipate greater profits from its downstream activities as the costs of its rivals increase. If the regulator is uncertain about the cost for supplying the input, the monopolist will seek to rise the costs of its downstream competitors by exaggerating its input cost. Vickers [12] shows analytically that vertical integration can complicate the regulator’s critical control problem, since it increases the monopolist’s incentives to overstate the access costs.

In this paper we consider a vertically related industry in which two firms - an incumbent and an entrant - compete in the downstream market. This is of course a shortcut since in reality imperfect competition takes forms which are much more complex. However, we believe that such an assumption is able to capture in a simple way two main aspects that characterize downstream sectors in many network industries. The first feature is the presence of a limited number of firms which can make positive profits. The second element is the existence of a dominant firm (typically monopolist before liberalization) and one or more weaker competitors which have recently entered the market.

The upstream essential input is provided by a monopoly. It is important to stress that under either industry pattern this is the only firm which is subject to price regulation. Under legal separation, the downstream incumbent and the upstream monopolist belong to the same company, even though they are independent in terms of their legal form. On the contrary, ownership separation implies a stronger pattern of unbundling since the two

---

5 Vickers [12] considers a setting where the number of downstream firms is determined endogenously by free entry, which implies zero profits. He shows that deregulation of the downstream sector may lead to excessive entry and duplication of the fixed costs.

6 In Vickers’s [12] model, also the monopolist’s profits arising from the downstream competitive activity are constrained by regulatory arrangements. However, as Vickers himself recognizes, regulatory bodies in the UK and elsewhere generally control only the monopolistic activities and allow the firm independently to operate in the deregulated sector, without affecting the outcome of competition there.

7 Empirical evidence shows that in most cases the company which runs the infrastructure segment is actually dominant also in the downstream sector when it is allowed to operate there.
undertakings cannot be subject to the same control.

In such a setting, our model shows that the two industry regimes yield the same social welfare level as long as the regulator is fully informed about the input costs. However, we find that the presence of asymmetric information can make it more desirable to implement legal separation. The idea is that the upstream monopolist’s greater profit from exaggerating input costs can be (at least in part) offset by the losses of the downstream branch which pays a higher access price. Consequently, a trade-off occurs between the incentive to overstate the input costs and the incentive to understate. This relaxes the regulator’s critical control problem and increases (expected) social welfare.

The policy implication of our model is that ownership separation should not be thought of as the best solution to mitigate the upstream monopolist’s incentive to overstate its costs. Indeed, we find that legal separation creates countervailing incentives within the vertical group that the regulator can exploit to make the society better off.

The plan of the paper is as follows. In Section 2 we describe the basic structures of the model. Section 3 compares the outcomes under legal and ownership unbundling in the benchmark case of complete information. In Section 4 we study how the presence of asymmetric information can affect the choice between the two regimes. This enables us to draw policy recommendations. Section 5 is devoted to some concluding remarks.

2. Basic structures

We examine a vertically related industry which supplies a single homogeneous final product, whose inverse demand function is given by

\[ p(Q) = \alpha - \beta Q, \] (2.1)

where \( Q \) denotes total quantity in the downstream market and \( \alpha, \beta > 0 \) are parameters.

The consumers’ surplus from purchasing \( Q \) units of output is then

\[ CS(Q) = \frac{1}{2} \beta Q^2. \] (2.2)

The downstream market is characterized by an incumbent firm and an entrant, whose profits are respectively equal to

\[ \pi_I(q_I, Q, a) = [p(Q) - c - a] q_I \] (2.3)

and
\[ \pi_E (q_E, Q, a) = [p(Q) - c - a] q_E, \] (2.4)

where \( Q \equiv q_I + q_E \). Expressions (2.3) and (2.4) show that the per-unit profit of each firm is given by the difference between the net revenue from the marketplace \((p - c)\) and the cost \( a \) incurred to purchase the access service. The level of downstream marginal costs \( c \) is constant and common to both producers.\(^8\)

Notice that both firms incur a payment \( a \) per unit of input to the upstream monopolist. What we are implicitly assuming is that they cannot bypass the monopolist’s access service, so that exactly one unit of upstream input is needed for each unit of the final product.\(^9\)

The upstream regulated monopolist, which provides the access to a crucial input (the network), has a profit equal to

\[ \pi_N (Q, a, S) = (a - c^u) Q + S, \] (2.5)

which is the sum of the net gain received from the two downstream firms plus a subsidy \( S \geq 0 \). The supply of the upstream service implies a constant marginal cost \( c^u \).\(^10\)

Under legal unbundling, the downstream incumbent and the upstream monopolist are separate only in legal terms. In fact, they constitute a single vertical group, whose aggregate profit from (2.3) and (2.5) is given by

\[ \pi_V (q_I, q_E, a, S) \equiv \pi_I + \pi_N = [p(Q) - c - c^u] q_I + (a - c^u) q_E + S. \] (2.6)

We assume that the access price \( a \) and the subsidy \( S \) are set by a benevolent regulator, which is charged with maximizing social welfare \( W \), defined as the sum of the consumers’ surplus, the downstream firms’ profits and the upstream monopolist’s profits minus the subsidy \( S \). Formally, we have

\[ W \equiv CS + \pi_I + \pi_E + \gamma \pi_N - S, \] (2.7)

---

\(^8\)This is clearly a simplification as firms are likely to have different costs. However, such an assumption should not undermine our results in terms of social welfare comparison.

\(^9\)When downstream firms have some ability to substitute away from the monopolist’s input, their constant marginal cost \( \varphi(a) \) for producing a unit of their own retail service is no longer equal to \( a + c \) [which implies \( \varphi''(a) = 0 \)] but it is a concave function of \( a [\varphi''(a) < 0].\)

\(^10\)We can imagine that there are fixed costs upstream which make the activity naturally monopolistic. However, these costs are not excessively large in relation to consumers’ valuation of the product, so that they do not play no role in the analysis and can be ignored.
where $\gamma \in [0, 1]$ is a weight on monopoly earnings, which reflects the regulator’s distributional concern in favour of consumers and firms which operate in the competitive market. It is important to stress that under legal separation the agency still regulates only the upstream firm, since it represents the legal entity charged with monopoly operations, while downstream activities occur in a liberalized market. Notice also that the downstream firms count as consumers for social welfare. The idea is that the regulator finds it so important to have competition as to give the profits of the downstream firms the same weight as consumers’ surplus.

Our regulatory model can be represented as a sequential game. At the first stage, the regulator makes a take-it-or-leave-it offer of a regulatory mechanism $\{a, S\}$, which the upstream monopolist can either accept or reject. If the firm refuses the proposed policy the regulatory interaction ends. In case of acceptance, at the second stage the downstream incumbent determines its production and at the last stage another firm decides to enter the market.

3. Complete information

To study in a suitable way the impact of the regulatory knowledge about the input costs on the choice between legal and ownership separation, we first derive the regulatory outcomes under both regimes in the benchmark case of complete information.

3.1. Legal separation

Applying the backward induction procedure, we start by deriving the entrant’s strategy at the last stage. Substituting (2.1) into (2.4), we can write down the entrant’s maximization problem as follows

$$\max_{q_E} (\alpha - \beta q_E - \beta q_I - c - a) \cdot q_E. \quad (3.1.1)$$

The first-order condition for $q_E$ immediately yields the entrant’s best reply function

$$q_E(q_I, a) = \frac{1}{2\beta} (\alpha - \beta q_I - c - a). \quad (3.1.2)$$

Using (2.6) and (3.1.2), the maximization problem of the vertical group at the second stage is
From (3.1.3) the first-order condition for $q_I$ can be written as

$$-\beta q_I + \frac{1}{2\beta}(a - c^u) = 0. \quad (3.1.4)$$

Solving (3.1.4) for $q_I$ yields the downstream output produced by the vertical group

$$q^{LS}_I = \frac{1}{2\beta}(\alpha - c - c^u), \quad (3.1.5)$$

where $\alpha - c - c^u \geq 0$, which is the difference between the consumers’ maximum willingness to pay $\alpha$ and the total (marginal) costs $(c + c^u)$, can be interpreted as the whole market size. Not surprisingly, the quantity supplied by the vertical group does not depend on the access charge $a$, which represents a mere internal transfer for the entire group.

If we replace (3.1.5) into (3.1.2) we obtain

$$q_E(a) = \frac{1}{4\beta}(\alpha - c + c^u - 2a). \quad (3.1.6)$$

It is immediate to see from (3.1.6) that entrant’s quantity decreases with $a$, since the input price represents a cost for the firm.

At the first stage, the regulator has to determine the access price $a$ and the transfer $S$ in order to maximize social welfare, as defined by (2.7). Using (2.2), (2.3), (2.4), (2.5) and substituting (3.1.5) and (3.1.6) into (2.7) yields after some manipulations

$$\max_{a,S} \left( \frac{3\alpha - 3c - c^u - 2a}{4\beta} \right)^2 + \left( \alpha - \beta \frac{3\alpha - 3c - c^u - 2a}{4\beta} - c - a \right) \cdot \left( \frac{3\alpha - 3c - c^u - 2a}{4\beta} \right) + \gamma \left[ (a - c^u) \cdot \frac{3\alpha - 3c - c^u - 2a}{4\beta} + S \right] - S \quad (3.1.7)$$

s.t. $(PC_C), (PC_E), (PC_I), (PC_N)$,
where \((PC_C), (PC_E), (PC_I)\) and \((PC_N)\) are nonnegative utility constraints which guarantee the participation in the market of the consumers, the entrant, and the downstream and upstream branches of the vertical group, respectively. Notice that both parts of the aggregate firm are assumed to receive a nonnegative profit, since they are independent in terms of their legal form.

We can replace the policy instrument \(S\) with \(\pi_N\), since from (2.5) there is a bijective correspondence between the two variables for a given \(a\). Ignoring all the participation constraints but \((PC_N)\),\(^{11}\) the regulator’s maximization program in (3.1.7) becomes

\[
\max_{a,\pi_N} \frac{\beta}{2} \left( \frac{3\alpha - 3c - c^u - 2a}{4\beta} \right)^2 + \left( \alpha - \beta \frac{3\alpha - 3c - c^u - 2a}{4\beta} - c - a \right).
\]

\[
\frac{3\alpha - 3c - c^u - 2a}{4\beta} + (a - c^u) \cdot \frac{3\alpha - 3c - c^u - 2a}{4\beta} - (1 - \gamma) \pi_N \tag{3.1.8}
\]

\[s.t. \ (PC_N).\]

Notice that the objective function in (3.1.8) is decreasing in \(\pi_N\), so the regulator finds it optimal to give zero profits to the branch of the vertical group which provides the essential input \((\pi_{NS} = 0)\).

Standard calculations show that the first-order condition for \(a\) is given by

\[
- \frac{1}{8\beta} (3\alpha - 3c - c^u - 2a) - \frac{1}{8\beta} (3\alpha - 3c - c^u - 2a) + \\
- \frac{1}{8\beta} (\alpha - c + c^u - 2a) + \frac{1}{4\beta} (3\alpha - 3c - c^u - 2a) - \frac{1}{2\beta} (a - c^u) = 0. \tag{3.1.9}
\]

From (3.1.9) we can derive the complete-information optimal access charge under legal separation, which may be written after some manipulations as

\[
a^{LS} = c^u - \frac{1}{2} (\alpha - c - c^u). \tag{3.1.10}
\]

\(^{11}\)It can be easily seen that they are all satisfied in equilibrium.
It is immediate to see from (3.1.10) that in equilibrium the input price $a^{LS}$ is below its marginal cost $c^u$. In other terms, the regulator finds it optimal to subsidize the input access. As we will see, the access pricing policy below costs is designed to offset the potential distortion of the (unregulated) downstream price arising from the presence of imperfect competition.\footnote{This access pricing policy can be implemented as long as transfers to the monopolist are feasible. Armstrong and Sappington [3] warn against the use of this sort of subsidies in the long-run, because they introduce at least two important problems. First, subsidized access to infrastructure can distort the technological choices of the competitor if the latter decides to use the existing network even though it would employ fewer social resources by building and running its own infrastructure. This issue refers to the provision to the entrant of the right make-or-buy incentives. Second, subsidies may permit an inefficient firm to operate profitably in the market, thereby increasing industry costs and reducing social welfare. Indeed, following the \textit{efficient component pricing rule} (ECPR) - of which Armstrong, Doyle and Vickers [1] give a brilliant synthesis - the access price which prevents inefficient entry should be equal in our setting to $a^{LS}_{ECPR} = c^v$, that is the difference between the direct cost of proving access ($c^v$) minus the opportunity cost of proving access ($p - c - c^u = 0$, since $p = c + c^u$, as we will see below). However, neither the possibly inefficient by-pass nor the threat of inefficient entry are considered in our model.}

Substituting (3.1.10) into (3.1.6) yields the quantity supplied by the entrant

$$d^{LS}_E = \frac{1}{2\beta} (\alpha - c - c^u). \tag{3.1.11}$$

Notice from (3.1.11) and (3.1.5) that in equilibrium the entrant and the vertical group will produce the same quantity in the downstream market. The subsidization of the access charge definitely benefits the entrant, which can increase its production and offset its strategic disadvantage with respect to the rival.

Substituting (3.1.5) and (3.1.11) into (2.1) we find that final consumers pay a price equal to

$$p^{LS} = c + c^u, \tag{3.1.12}$$

so that the marginal cost pricing is implemented. Even if it cannot intervene directly in the liberalized downstream sector, the regulator charges an input price below costs which eliminates any allocative inefficiency arising from imperfect competition.

If we replace (3.1.11) and (3.1.5) into (2.4) and (2.6) respectively and recall that $\pi^{LS}_N = 0$, we get

$$\pi^{LS}_E = \pi^{LS}_I = \pi^{LS}_V = \frac{1}{4\beta} (\alpha - c - c^u)^2. \tag{3.1.13}$$
The two downstream firms earn the same profit in equilibrium. This is a straightforward consequence of the even division of the market between them and of the unprofitability of monopoly regulated operations for the vertical group.

Using (2.2), the consumers’ surplus amounts to

\[ CS^{LS} = \frac{1}{2\beta} (\alpha - c - c^a)^2, \]  

which is increasing in the market size.

From (2.5) the subsidy received by the monopolist is given by

\[ S^{LS} = \frac{1}{2\beta} (\alpha - c - c^a)^2. \]  

Notice that (3.1.15) is equal to the sum of the profits in (3.1.13) of the two downstream firms. Hence, the transfer to the upstream monopolist indirectly finances the profits of its branch in the competitive market and those of its rival.

Using (3.1.13), (3.1.14) and (3.1.15), we can compute the complete-information social welfare under legal separation

\[ W^{LS} = \frac{1}{2\beta} (\alpha - c - c^a)^2. \]  

It appears from (3.1.16) and (3.1.14) that social welfare is equal to the consumers’ surplus.

3.2. Ownership separation

To solve the regulatory game under ownership separation we adopt again the backward induction procedure. While (3.1.2) at the last stage still holds, the second-stage maximization problem of the incumbent in the downstream market must be reformulated, since the leader is now a separate firm which is independent from the upstream monopolist even in terms of ownership. Using (2.3) and (3.1.2), the incumbent’s maximization program becomes

\[ \max_{q_I} \left( \alpha - \beta q_I - c - a \right) \cdot q_I. \]  

From (3.2.1) the first-order condition for \( q_I \) is given by

\[ -\beta q_I + \frac{1}{2} (\alpha - c - a) = 0. \]
Using (3.2.2) we get

\[ q_I (a) = \frac{1}{2} \left( \alpha - c - a \right). \]  

(3.2.3)

Notice from (3.2.3) that now the incumbent also incurs the essential input cost to produce its quantity. Substituting (3.2.3) into (3.1.2) yields

\[ q_E (a) = \frac{1}{4} \left( \alpha - c - a \right). \]  

(3.2.4)

At the first stage, using (3.2.3) and (3.2.4) the regulator’s maximization problem of social welfare in (2.7) may be rewritten after some computations as

\[
\begin{align*}
\max_{a,S} & \quad \frac{\beta}{2} \left[ \frac{3}{4} (\alpha - c - a) \right]^2 + \left[ \alpha - \beta \frac{3}{4} (\alpha - c - a) \right] - c - a \\
& \quad + \frac{3}{4} (\alpha - c - a) + \gamma \left[ (a - c^u) \cdot \frac{3}{4} (\alpha - c - a) + S \right] - S
\end{align*}
\]  

s.t. \ (PC_C), (PC_E), (PC_I), (PC_N). \quad (3.2.5)

Ignoring all the participation constraints but (PC_N)\(^{13}\) and replacing from (2.5) the choice variable \(S\) with \(\pi_N\), the maximization problem in (3.2.5) becomes

\[
\begin{align*}
\max_{a,\pi_N} & \quad \frac{\beta}{2} \left[ \frac{3}{4} (\alpha - c - a) \right]^2 + \left[ \alpha - \beta \frac{3}{4} (\alpha - c - a) \right] - c - a \\
& \quad + (a - c^u) \cdot \frac{3}{4} (\alpha - c - a) - (1 - \gamma) \pi_N.
\end{align*}
\]  

s.t. \ (PC_N). \quad (3.2.6)

Since the objective function in (3.2.6) is decreasing in \(\pi_N\), the regulator finds it optimal to give zero profits to the input monopolist \((\pi_N^{OS} = 0)\).

The first-order condition for \(a\) is given by

\[- \frac{9}{16} \beta (\alpha - c - a) - \frac{3}{16} \beta (\alpha - c - a) - \frac{3}{16} \beta (\alpha - c - a) + \]

\[ \text{It can be easily seen that they are all satisfied in equilibrium.} \]
After some manipulations, the complete-information access charge under ownership separation can be written as

\[ a^\text{OS} = c^u - \frac{1}{3} (\alpha - c - c^u). \]  

(3.2.8)

We can immediately see from (3.2.8) that even under ownership unbundling the input price is below its costs in equilibrium. A comparison between (3.2.8) and (3.1.10) reveals that \( a^{LS} < a^\text{OS} \) and so the price distortion below marginal costs is higher under legal separation. To reach its objective of minimization of allocative inefficiency in the downstream market, the regulator finds it optimal to subsidize access more when the downstream imperfect competition is further undermined by the (indirect) participation of the monopolist in the retail market. This is because in such a case only the quantity in (3.1.6) produced by the entrant depends on the regulated input price, while under ownership separation the regulator can (indirectly) affect the outputs in (3.2.3) and (3.2.4) of both firms and then the need for subsidizing the access service is lower.

Substituting (3.2.8) into (3.2.3) and (3.2.4) we find the quantities supplied by the incumbent and the entrant, which are respectively given by

\[ q^\text{OS}_I = \frac{2}{3\beta} (\alpha - c - c^u) \]  

(3.2.9)

and

\[ q^\text{OS}_E = \frac{1}{3\beta} (\alpha - c - c^u). \]  

(3.2.10)

Notice that under ownership unbundling the dominant firm produces more than the entrant in the downstream sector, since now both firms benefit from access subsidization and the incumbent can fully exploit its dominant position in the market. Consequently, as emerges from the comparison of (3.2.9) and (3.2.10) with (3.1.5) and (3.1.11) respectively, in equilibrium the independent incumbent supplies a higher quantity than the vertical group (\( q^\text{OS}_I > q^\text{LS}_I \)), while the entrant proportionally reduces its sales (\( q^\text{OS}_E < q^\text{LS}_E \)).

Substituting (3.2.9) and (3.2.10) into (2.1) we find the downstream market price, which is equal to

\[ p^\text{OS} = p^{LS} = c + c^u. \]  

(3.2.11)
Expression (3.2.11) shows that the marginal cost pricing applies under both regimes. This means that the total production is unchanged and the industry pattern only affects the allocation of the output between the two firms in equilibrium.

Using (2.3) and (2.4), the profits of the incumbent and the entrant are respectively equal to

\[
\pi_L^{OS} = \frac{2}{9\beta} (\alpha - c - c^u)^2 \quad (3.2.12)
\]

and

\[
\pi_E^{OS} = \frac{1}{9\beta} (\alpha - c - c^u)^2. \quad (3.2.13)
\]

Consistently with the results in (3.2.9) and (3.2.10), ownership separation allows the incumbent to earn more than the entrant, even if both firms bear a reduction in their profits relative to the case of legal unbundling [compare (3.2.12) and (3.2.13) with (3.1.13)]. The rationale is that now the higher input cost erodes the firms’ profit margin, while the final price is unchanged.

The consumers’ surplus is equal to

\[
C^{OS} = \frac{1}{2\beta} (\alpha - c - c^u)^2. \quad (3.2.14)
\]

After a quick look at (3.2.14) and (3.1.14) we get the following result.

**Lemma 1** When the regulator has full information about a vertically related market, the legal and ownership unbundling between the upstream monopolist and the downstream incumbent yield the same consumers’ surplus.

The result in Lemma 1 is a straightforward consequence of (3.2.11).

If we use (2.5) and recall that \(\pi_X^{OS} = 0\), the transfer to the monopolist amounts to

\[
S^{OS} = \frac{1}{3\beta} (\alpha - c - c^u)^2. \quad (3.2.15)
\]

Notice that (3.2.15) is equal to the sum of (3.2.12) and (3.2.13). Also the subsidy under ownership separation - which is lower than that under legal separation in (3.1.15) - just matches the profits of the two downstream firms.
Using (3.2.12), (3.2.13), (3.2.14) and (3.2.15), we can now compute the complete-information social welfare under ownership unbundling

\[ W^{OS} = \frac{1}{2\beta} (\alpha - c - c^u)^2. \]  

(3.2.16)

If we compare (3.2.16) and (3.1.16), we find the following result.

**Lemma 2** When the regulator has full information about a vertically related market, the legal and ownership unbundling between the upstream monopolist and the downstream incumbent yield the same social welfare level.

The result in Lemma 2 indicates that a fully-informed regulator is able to replicate the same social-welfare outcome under both regimes by implementing a different allocation of the total output between the downstream firms through the regulation of access charge. As we will see, this conclusion no longer holds when the monopolist can use strategically its private information about the input costs.

4. Asymmetric cost information

The observations emphasized in Lemmas 1 and 2 have been derived under the condition that the regulator is fully informed. As Baron and Myerson argue, “this assumption is unlikely to be met in reality, since the firm would be expected to have better information than would the regulator” [6, p. 911].

We suppose now that there is asymmetric information about the upstream marginal costs \( c^u \).\(^{14}\) The revelation principle ensures that, without any loss of generality, the regulator may be restricted to direct incentive compatible policies, which require the firm to report its cost parameter and which give the firm no incentive to lie.\(^{15}\) The regulatory problem can be reduced to the design of a mechanism \( M = \{a(\bar{c}^u), S(\bar{c}^u), \bar{c}^u \in [c^-_u, c^+_u]\} \), which determines the access charge \( a(.) \) and the subsidy \( S(.) \) to the firm as functions of its report \( \bar{c}^u \in [c^-_u, c^+_u] \), by inducing the firm to reveal honestly its private information, so that in equilibrium \( \bar{c}^u = c^u \). The regulator is supposed to have only imperfect prior knowledge about \( c^u \), represented

\(^{14}\)See Lewis and Sappington [9] for an investigation of the optimal access tariffs when the regulator is uncertain about the production costs of the firm which recently entered the market.

\(^{15}\)For an application of the revelation principle to regulation, see the seminal paper of Baron and Myerson [6].
by a density function $f(c^u)$, which, to avoid technical problems, is continuous and positive on its domain $[c_u^-, c_u^+]$. The corresponding cumulative distribution function is given by $F(c^u) = \int_{c_u^-}^{c_u^+} f(\tilde{c}^u) d\tilde{c}^u \in [0, 1]$.

### 4.1. Legal separation

We know that the regime of legal unbundling implies that the upstream monopolist and the downstream incumbent form a vertical group which acts as a single entity, even though the two firms are separate in legal terms and only the upstream operations are regulated.

Economic theory has emphasized that a regulated firm has a natural incentive to overstate its costs if the regulator ignores asymmetric information and implements the regulatory policy discussed in the previous section. This conclusion can be definitely applied to the upstream monopoly. To see that, let us compute the extraprofit $\Delta \pi_N^{LS}(\tilde{c}^u, c^u)$ that the vertical group obtains from its monopoly activities when declaring $\tilde{c}^u$ rather than its true costs $c^u$.

$$\Delta \pi_N^{LS}(\tilde{c}^u, c^u) \equiv \pi_N^{LS}(\tilde{c}^u, c^u) - \pi_N^{LS}(c^u) = [a(\tilde{c}^u) - c^u] \cdot Q^{LS}(\tilde{c}^u) + S^{LS}(\tilde{c}^u) +$$

$$-\pi_N^{LS}(c^u) = \pi_N^{LS}(\tilde{c}^u) + (\tilde{c}^u - c^u) \cdot Q^{LS}(\tilde{c}^u) - \pi_N^{LS}(c^u) = (\tilde{c}^u - c^u) \cdot \frac{\alpha - c - \tilde{c}^u}{\beta},$$

(4.1.1)

where $\pi_N^{LS}(\tilde{c}^u) \equiv \pi_N^{LS}(\tilde{c}^u, \tilde{c}^u) = \pi_N^{LS}(c^u) = 0$, since any type of firm which reports the truth gets zero profits when the complete-information regulatory policy is applied, and $Q^{LS}(\tilde{c}^u) = \frac{\alpha - c - \tilde{c}^u}{\beta} \geq 0$ is the total output derived from (3.1.5) and (3.1.11) for $c^u = \tilde{c}^u$. It is evident that the monopolist has an incentive to exaggerate its costs ($\tilde{c}^u > c^u$), because doing so guarantees positive profits $[\Delta \pi_N^{LS}(\tilde{c}^u, c^u) > 0$ if $Q^{LS}(\tilde{c}^u) > 0$].

We study now the impact of such a strategic behaviour on the downstream branch of the vertical group. The difference in profit $\Delta \pi_I^{LS}(\tilde{c}^u, c^u)$ when $\tilde{c}^u$ is reported instead of $c^u$ is given by

$$\Delta \pi_I^{LS}(\tilde{c}^u, c^u) \equiv \pi_I^{LS}(\tilde{c}^u, c^u) - \pi_I^{LS}(c^u) =$$

$$= p(Q(\tilde{c}^u)) - c - a(\tilde{c}^u) \cdot q_I^{LS}(\tilde{c}^u) - \pi_I^{LS}(c^u) =$$

$$= \frac{1}{4\beta} \left[(\alpha - c - \tilde{c}^u)^2 - (\alpha - c - c^u)^2\right] =$$

16
where the last two equalities are derived by using (3.1.12). Notice from (4.1.2) that downstream activities benefit from an understatement of the upstream costs, as $\Delta \pi^L \geq 0$ if $\bar{c}^u < c^u$ (the bracketed expression is the sum of two nonnegative terms). This is not surprising, since a declared lower value for $c^u$ reduces the access charge and thus increases the profit margin of the downstream branch.

It is evident that the vertical group faces a trade-off when it lies. Exaggerating the input costs will be desirable when the extra profit in the upstream market more than offsets the losses on the downstream operations. Such is the case if and only if

$$\Delta \pi^L \geq 0 \quad \text{implies} \quad c^u < \bar{c}^u,$$

(4.1.3)

Substituting (4.1.1) and (4.1.2) into (4.1.3) yields after some manipulations

$$(\bar{c}^u - c^u) \cdot (\alpha - c - \bar{c}^u) > \frac{1}{3} (\bar{c}^u - c^u) \cdot (\alpha - c - c^u).$$

(4.1.4)

After dividing both sides of (4.1.4) by $\bar{c}^u - c^u > 0$, we can see that the vertical group will find it optimal to overstate its upstream costs if and only if

$$\frac{\alpha - c - \bar{c}^u}{\alpha - c - c^u} > \frac{1}{3}.$$  

(4.1.5)

We may rewrite (4.1.5) as

$$\bar{c}^u < c^u,$$

(4.1.6)

where $c^u (\bar{c}^u) \equiv \frac{2}{3} (\alpha - c) + \frac{1}{3} c^u$ if $\frac{2}{3} (\alpha - c) + \frac{1}{3} c^u \leq c^u$ and $c^u (\bar{c}^u) \equiv c^u$ otherwise, since the firm’s declaration $\bar{c}^u$ cannot be outside the interval $[c^u, c^u]$. Condition (4.1.6) shows that the vertical group will not report a value for $\bar{c}^u$ higher than the threshold $c^u (\bar{c}^u)$, otherwise it would incur losses for its statement.

\textit{It is important to stress that the firm does not have any incentive to understate its costs ($\bar{c}^u < c^u$). The condition for this to be the case $|\Delta \pi^L | < \Delta \pi^L$ implies $\frac{\alpha - c - c^u}{\alpha - c - \bar{c}^u} < \frac{1}{3}$ which is never met (since the left-hand side is greater than one for $\bar{c}^u < c^u$).}
We suppose that \( c_u (c^u) = c^u_+ \) if and only if \( c^u = c^u_+ \), which implies that \( \alpha - c - c^u_+ = 0 \). In other terms, the highest-cost firm is so inefficient that production cannot occur. In Figure 4.1.1 we depict graphically this situation. The area above the bisecting (broken) line represents the case of firm’s overstatement of its costs, that is \( \tilde{c}^u > c^u \). The part of the graph under the other (solid) line captures condition (4.1.6), that is \( \tilde{c}^u < c^u_+ (c^u) \).

![Graph](image)

Fig. 4.1.1. Incentives to lie under legal separation

Any type of the firm (with \( c^u < c^u_+ \)) is willing to report a cost parameter \( \tilde{c}^u \in (c^u, c^u_+ (c^u)) \) which is strictly lower than \( c^u_+ \). This observation has crucial implications for the following analysis.

We are ready to solve now the regulatory game. We substitute the outcomes at the last two stages in (3.1.5) and (3.1.6), which still hold, into the regulator’s maximization problem in (2.7) at the first stage and get after some manipulations

\[
\max_{a(c^u), S(c^u)} \int_{c^u}^{c^u_+} \left\{ \frac{\beta}{2} \left[ \frac{3\alpha - 3c - c^u - 2a(c^u)}{4\beta} \right]^2 + \left[ \alpha - \beta \frac{3\alpha - 3c - c^u - 2a(c^u)}{4\beta} - c - a(c^u) \right] \right. \cdot \left. \frac{3\alpha - 3c - c^u - 2a(c^u)}{4\beta} \right\} \, dc^u
\]

Such an assumption entails that there is not a continuum of the firm’s types that are willing to declare \( c^u_+ \). This shortcut guarantees the differentiability of \( c^u_+ (c^u) \) on the domain \([c^u_-, c^u_+]\).

The lines in the graph are depicted by assuming \( \alpha - c = 10, c^u = 2, c^u_+ = 10 \).
\[ + \gamma \left[ a(c^u) - c^u \right] \cdot \frac{3\alpha - 3\sigma - c^u - 2a(c^u)}{4\beta} + S(c^u) \right] - S(c^u) \right\} \right\} \right\} f(c^u) \, dc^u \]

\[ \text{s.t.} \]

\[(PC_C), (PC_E), (PC_I), (PC_N) \]

and

\[ \pi_{LS}^{LNS}(c^u) = \pi_{LS}^{LNS}(c^u(c^u)) + \int_{c^u}^{c^u(c^u)} \frac{3\alpha - 3\sigma - c^u - 2a(c^u)}{4\beta} \, dc^u, \quad (\text{ICC}_{LNS}^{L}) \]

where (ICC\textsubscript{LNS}^{L}) is the incentive compatibility constraint of the network provider under legal separation, whose formal derivation is provided in Appendix A.

We can ignore all the participation constraints but (PC\textsubscript{N}).\textsuperscript{19} Substituting (ICC\textsubscript{LNS}^{L}) into the objective function in (4.1.7) and replacing the choice variable \( S(c^u) \) with \( \pi_{LS}^{LNS}(c^u(c^u)) \) yields

\[ \max_{a(c^u), \pi_{LS}^{LNS}(c^u(c^u))} \left\{ \frac{\beta}{2} \left[ \frac{3\alpha - 3\sigma - c^u - 2a(c^u)}{4\beta} \right]^2 + \right. \]

\[ \left. + \left[ a(c^u) - \sigma \cdot \frac{3\alpha - 3\sigma - c^u - 2a(c^u)}{4\beta} - \sigma - a(c^u) \right] \right\} \!ight\} f(c^u) \, dc^u \]

\textsuperscript{19} It can be easily shown that they are all satisfied in equilibrium.
Since the objective function in (4.1.8) is decreasing in \( \pi_N(c_u(c^u)) \), the regulator finds it optimal to set \( \pi_{LS}^N(c_u(c^u)) = 0 \), still satisfying (PC\(_N\)) since the integral in (ICC\(_N\)) is nonnegative.\(^{20}\)

After integrating by parts as shown in Appendix B, the maximization problem becomes

\[
\max_{a(c^u)} \int_{c^u}^{c^u_+} \left\{ \frac{\beta}{2} \left[ \frac{3\alpha - 3c - c^u - 2a(c^u)}{4\beta} \right]^2 + \right. \\
\left. + \left[ \alpha - \beta \frac{3\alpha - 3c - c^u - 2a(c^u)}{4\beta} - c - a(c^u) \right] \frac{3\alpha - 3c - c^u - 2a(c^u)}{4\beta} \right. \\
\left. + \left[ a(c^u) - c^u \right] \cdot \frac{3\alpha - 3c - c^u - 2a(c^u)}{4\beta} + \right. \\
\left. - (1 - \gamma) \frac{20(\alpha - c) - 8c^u - 12a(c^u)}{36\beta} \left\{ \frac{H(c^u)}{f(c^u)} \right\} f(c^u) dc^u, \right\} 
\]

where \( H(c^u) \equiv \frac{F(c^u)}{f(c^u)} \) is the hazard rate.\(^{21}\)

The first-order condition for \( a(c^u) \) is given by

\[
- \frac{1}{8\beta} [3\alpha - 3c - c^u - 2a(c^u)] - \frac{1}{8\beta} [3\alpha - 3c - c^u - 2a(c^u)] + \\
- \frac{1}{8\beta} [\alpha - c + c^u - 2a(c^u)] + \frac{1}{4\beta} [3\alpha - 3c - c^u - 2a(c^u)] + \\
- \frac{1}{2\beta} [a(c^u) - c^u] + (1 - \gamma) \frac{1}{3\beta} H(c^u) = 0. \tag{4.1.10}
\]

\(^{20}\)In particular, the integral is strictly positive for \( c^u < c^u_+ \) since the integrand function is positive (as long as production occurs in equilibrium) and \( c^u_+ (c^u) > c^u \), while it vanishes for \( c^u = c^u_+ \) as \( c^u_+ (c^u) = c^u_+ \).

\(^{21}\)The hazard rate \( H(c^u) \) is supposed to be increasing in \( c^u \). This monotonicity property, which is met by the most usual distributions, may be interpreted as a decrease in the conditional probability that there are further cost reductions, given that there has already been a cost marginal reduction, as the firm becomes more efficient.
After some manipulations, we derive the asymmetric-information optimal access charge under legal separation

$$\pi^{LS}(c^u) = c^u - \frac{1}{2} (\alpha - c - c^u) + \frac{4}{3} (1 - \gamma) H(c^u). \quad (4.1.11)$$

Not surprisingly, expression (4.1.11) is higher than (3.1.10), so the input price is distorted above its complete-information level according to the level of regulatory uncertainty, measured by $H(c^u)$, and the regulator’s distributional concern, captured by the parameter $\gamma$. Hence, in principle we cannot predict whether the input will be subsidized in equilibrium or not.

Only the most efficient input provider (for which $c^u = c^u$) will receive the same price as under complete information [$H(c^u) = 0$] because this is the type of the firm for which no one else wants to pass itself off. Moreover, notice that (4.1.11) is a decreasing function of $\gamma$, so the more weight the regulator gives to the monopolist’s profits the lower will be the price. As emerges from (2.7), the rationale for this result arises from the loss $(1 - \gamma)$ that the society incurs for a transfer of a unit of money to the monopolist. Only if there is no distributional issue ($\gamma = 1$), the optimal access charge does not need to be distorted in equilibrium.

Substituting (4.1.11) into (3.1.6) yields the quantity produced by the entrant

$$q^{LS}_E = \frac{1}{2\beta} \left[ \alpha - c - c^u - \frac{4}{3} (1 - \gamma) H(c^u) \right]. \quad (4.1.12)$$

It is immediate to notice that expression (4.1.12) is lower than (3.1.11). The higher asymmetric-information access charge results in a reduction in the quantity produced by the entrant.

From (3.1.5) and (4.1.12) the downstream market price is equal to

$$p^{LS} = c + c^u + \frac{2}{3} (1 - \gamma) H(c^u). \quad (4.1.13)$$

A quick look at (4.1.13) and (3.1.12) shows that the price is distorted above the complete-information level. This is a direct consequence of the increase in the access charge in (4.1.11). However, notice that the distortion of the input price translates into a lower raise in the final price.

---

22 Notice that a mean-preserving spread of the distribution implies an increase in $H$ for given costs.

23 This property is known in the literature as *non distortion at the top* and indicates that the only efficient price is that designed for the agent with the "best" characteristic.
Now we compute the profit of the downstream branch of the vertical group. Using (3.1.5), (4.1.11) and (4.1.12) we get

\[ \pi_I^{LS} = \frac{1}{4\beta} (\alpha - c - c^u) \left[ \alpha - c - c^u - \frac{4}{3} (1 - \gamma) H(c^u) \right]. \quad (4.1.14) \]

Notice that expression (4.1.14) is lower than (3.1.13), which means that the incumbent is worse off because of asymmetric information. Indeed, if we take the difference between (4.1.14) and (3.1.13) we obtain

\[ \Delta \pi_I^{LS} \equiv \pi_I^{LS} - \pi_I^{T} = -\frac{1}{3\beta} (\alpha - c - c^u) (1 - \gamma) H(c^u) \leq 0. \quad (4.1.15) \]

Replacing (4.1.12) into (2.4) we find the profit of the entrant

\[ \pi_E^{LS} = \frac{1}{4\beta} \left[ \alpha - c - c^u - \frac{4}{3} (1 - \gamma) H(c^u) \right]^2. \quad (4.1.16) \]

We can easily see that the entrant is also penalized by the situation of asymmetric information, since its profit in (4.1.16) is smaller than the one in (3.1.13). Subtracting (3.1.13) from (4.1.16) yields after some manipulations

\[ \Delta \pi_E^{LS} \equiv \pi_E^{LS} - \pi_E^{T} = -\frac{2}{3\beta} (1 - \gamma) H(c^u) \left[ \alpha - c - c^u - \frac{2}{3} (1 - \gamma) H(c^u) \right] \leq 0, \quad (4.1.17) \]

since the nonnegativity condition on \( q_E^{LS} \) in (4.1.12) implies that the term in square brackets in (4.1.17) must be positive. This is the result of two combined effects. The first one is the reduction in the quantity produced by the entrant seen before. The second factor is the decrease in the profit margin. Indeed, the higher downstream price from which the firm benefits is more than offset by the greater access price that it has to pay.

Comparing (4.1.17) and (4.1.15) immediately yields \( |\Delta \pi_E^{LS}| \geq |\Delta \pi_I^{LS}| \) (as long as \( q_E^{LS} \geq 0 \)). In other words, the incumbent is relatively less penalized by the situation of asymmetric information than the entrant. The motivation is that, even if it incurs the same reduction in the profit margin as its competitor, the quantity in (3.1.5) is unchanged \( (\bar{q}_I^{LS} = q_I^{LS}) \) since it does not depend on the access charge and thus cannot be distorted by the regulator in equilibrium.

Substituting (4.1.12) and (3.1.5) into (ICC_N^{LS}) we find the profit of the input monopolist

22
\[ \pi_N = \int_{c^u}^{\overline{c}^u} \frac{\alpha - c - \overline{c}^u - \frac{2}{3} (1 - \gamma) H(\overline{c}^u)}{\beta} d\overline{c}^u. \]  

(4.1.18)

The downward distortion of the entrant’s production in (4.1.12) entails a reduction in the total output, captured by the integrand in (4.1.18), which allows the regulator to curb the socially costly rent (if \( \gamma < 1 \)) that the monopolist extracts for its informational advantage.

As shown in Appendix C, expression (4.1.18) satisfies the standard property of decreasing monotonicity in \( c^u \). This corresponds to the intuitive notion that the profit should increase in the efficiency of the firm.

Using (2.2), we derive the consumers’ surplus

\[ CS = \frac{1}{2\beta} \left[ \alpha - c - c^u - \frac{2}{3} (1 - \gamma) H(c^u) \right]^2. \]  

(4.1.19)

The situation of asymmetric information makes the consumers worse off, since their utility in (4.1.19) is lower than that in (3.1.14). The gains from reducing the informational rents in (4.1.18) come at the expense of a decrease in the consumers’ surplus, since they imply allocative inefficiency.

From (2.5) the subsidy received by the monopolist is equal to

\[ S = \int_{c^u}^{\overline{c}^u} \frac{\alpha - c - \overline{c}^u - \frac{2}{3} (1 - \gamma) H(\overline{c}^u)}{\beta} d\overline{c}^u + \frac{\alpha - c - c^u - \frac{2}{3} (1 - \gamma) H(c^u)}{2\beta} \cdot \left[ \alpha - c - c^u - \frac{8}{3} (1 - \gamma) H(c^u) \right]. \]  

(4.1.20)

Notice that (4.1.20) may be less than zero for values of \( c^u \) high enough, since the integral is quite low and the last term is likely to be negative. This means that, unlike in the complete-information case, the regulator may find it optimal to tax rather than subsidize the monopolist in exchange for a higher price.

Using (4.1.14), (4.1.16), (4.1.18), (4.1.19) and (4.1.20), we derive after some computations the asymmetric-information social welfare under legal separation

\[ W = \frac{1}{2\beta} \left[ (\alpha - c - c^u)^2 - \frac{4}{9} (1 - \gamma)^2 H^2(c^u) \right] + \]

23
\[
-(1 - \gamma) \cdot \frac{c_u^{2}(c_u)}{\int c_u^{2}(c_u) \cdot \frac{\alpha - c - \omega^{u} - \frac{2}{3}(1 - \gamma) H(\omega^{u}) d\omega^{u}}{\beta}.
\] (4.1.21)

It is immediate to see from (4.1.21) and (3.1.16) that the situation of asymmetric information is social-welfare detrimental, as long as the regulator has some distributional concern \((\gamma < 1)\). There are two elements of distortion with respect to the complete-information case. The first one, captured by the term in square brackets, concerns the reduction in the consumers’ surplus and in the downstream firms’ profits. The second factor, represented by the integral, refers to the part of the informational rent of the monopolist which represents a mere loss from the social-welfare point of view.

4.2. Ownership separation

The existence of asymmetric information does not change the outcomes at the last two stages of the regulatory game. Hence, still applying the backward induction procedure, we substitute (3.2.3) and (3.2.4) into (2.7) and write down the regulator’s maximization problem when the privately-informed upstream monopolist is independent of the downstream incumbent even in terms of ownership

\[
\max_{a(c^u), S(c^u)} \int_{c_u^{2}} \beta \left[ \frac{\alpha - c - a(c^u)}{4\beta} \right]^{2} + \left[ \alpha - 3\beta a(c^u) - c - a(c^u) \right] - c - a(c^u) \cdot \frac{\beta}{4\beta} \frac{\alpha - c - a(c^u)}{4\beta} - S(c^u) f(c^u) dc^u.
\] (4.2.1)

s.t.

\((PC_C), (PC_E), (PC_I), (PC_N)\)

and

24
\[ \pi_{OS}^N (c^u) = \pi_{OS}^N (c^u_+) + \int_{c^u_-}^{c^u_+} \frac{3}{4\beta} \left( \alpha - c - a (\tilde{c}^u) \right) d\tilde{c}^u. \]  

(ICC\textsubscript{OS}^N)

In Appendix E we give some details about (ICC\textsubscript{OS}^N), which represents the incentive compatibility constraint of the network provider under ownership separation.

We can ignore all the participation constraints but (PC\textsubscript{N}).\textsuperscript{24} After substituting (ICC\textsubscript{OS}^N) into the objective function in (4.2.1), we replace from (2.5) the choice variable \( S (c^u) \) with \( \pi_{OS}^N (c^u_+) \) and integrate by parts so as to get

\[
\max_{a(c^u), \pi_{OS}^N (c^u_+)} \int_{c^u_-}^{c^u_+} \left\{ \frac{\beta}{2} \left[ \frac{3}{4\beta} (\alpha - c - a (c^u)) \right]^2 + \left[ \alpha - 3 \beta \frac{\alpha - c - a (c^u)}{4\beta} - c - a (c^u) \right] \cdot \frac{3}{4\beta} \alpha - c - a (c^u) \cdot \frac{3}{4\beta} \right\} f (c^u) \, dc^u. \tag{4.2.2}
\]

\[ s.t. \quad (PC_N). \]

Since the objective function in (4.2.2) is decreasing in \( \pi_{OS}^N (c^u_+) \), the regulator finds it optimal to give zero profits to the most inefficient firm \( [\pi_{OS}^N (c^u_+)] = 0 \), still satisfying (PC\textsubscript{N}).

The first-order condition for \( a (c^u) \) is equal to

\[
- \frac{9}{16\beta} [\alpha - c - a (c^u)] - \frac{3}{16\beta} [\alpha - c - a (c^u)] - \frac{3}{16\beta} [\alpha - c - a (c^u)] +
\]

\[ + \frac{3}{4\beta} [\alpha - c - a (c^u)] - \frac{3}{4\beta} [a (c^u) - c^u] + \frac{3}{4\beta} (1 - \gamma) H (c^u) = 0. \tag{4.2.3} \]

After some computations, we find the asymmetric-information access charge under ownership separation

\textsuperscript{24}It can be easily seen that they are all satisfied in equilibrium.
\[
\sigma^{OS}(c^u) = c^u - \frac{1}{3}(\alpha - c - c^u) + \frac{4}{3}(1 - \gamma) H(c^u). \tag{4.2.4}
\]

It appears from (4.2.4) and (3.2.7) that even under ownership unbundling the optimal input price is set above the complete-information level. Notice from (4.2.4) and (4.1.11) that the difference between the access charges under the two regimes is the same as that with complete information, since the regulator applies the same distortion [equal to \(\frac{4}{3}(1 - \gamma) H(c^u)\)] in response to the asymmetric-information problem. This result is a bit surprising. We have argued in Section 3.2 that the complete-information input price is less distorted below the marginal cost under ownership separation because the regulator can (indirectly) affect the outputs of both downstream firms rather than only that of the entrant in order to minimize allocative inefficiency. Following this reasoning, we would have expected a lower distortion arising from asymmetric information under ownership separation. Then, why does not this occur in equilibrium? One possible answer is that \textit{ceteris paribus} - that is, before considering the different impact of the regulatory policy in the two regimes on the quantities produced - the regulator’s need for distorting the price upwards is actually higher under ownership separation than under legal separation. We recognize that this explanation is incomplete and a bit cryptic, but its rationale will become clearer when we compare the informational rents received by the monopolist under the two industry regimes.

Replacing (4.2.4) into (3.2.3), we get the output supplied by the downstream incumbent

\[
q^{OS}_I = \frac{2}{3\beta} \left[\alpha - c - c^u - (1 - \gamma) H(c^u)\right]. \tag{4.2.5}
\]

It is immediate to see from (4.2.5) and (3.2.8) that the higher input price leads to a reduction in the production of the incumbent under asymmetric information.

Taking the difference between (4.2.5) and (3.1.5) we find that \(q^{OS}_I > q^{LS}_I = q^{LS}_I\) if and only if

\[
\alpha - c - c^u - 4(1 - \gamma) H(c^u) > 0. \tag{4.2.6}
\]

Notice that condition (4.2.6) holds when the access charge in (4.2.4) is distorted below the marginal cost (\(\sigma^{OS} < c^u\)). In other terms, the incumbent will produce more under ownership separation as long as the input is subsidized. This may not be the case under asymmetric information, so
we cannot predict under which regime the incumbent’s production will be higher.

If we substitute (4.2.4) into (3.2.4) we derive the quantity produced by the entrant

\[ q_{OS}^E = \frac{1}{3\beta} [\alpha - c - c^u - (1 - \gamma) H (c^u)]. \] (4.2.7)

A quick look at (4.2.7) and (3.2.9) shows that the entrant will produce less because of the asymmetric-information problem.

If we subtract (4.1.12) from (4.2.7) we find that \( q_{OS}^E > q_{LS}^E \) if and only if

\[ \alpha - c - c^u - 2 (1 - \gamma) H (c^u) < 0. \] (4.2.8)

Condition (4.2.8) is satisfied for \( H \) high enough. Therefore, we can state that a sufficiently high level of regulatory uncertainty implies that the entrant will produce more under ownership separation because of the greater downward distortion of its quantity that occurs under legal unbundling [see from (4.2.7) and (4.1.12) that \( \frac{4}{3} (1 - \gamma) H (c^u) \geq (1 - \gamma) H (c^u) \)]. Put another way, the advantage of a higher input subsidization that the entrant enjoys under legal separation in the case of complete information is more than offset by a decrease in quantity under asymmetric information when the level of regulatory uncertainty is high enough.

It is also important to stress that conditions (4.2.6) and (4.2.8) cannot be satisfied simultaneously. Hence, under neither regime both downstream firms can increase their quantities.

If we substitute (4.2.5) and (4.2.7) into (2.1) we derive the price in the downstream market

\[ p^{OS} = c + c^u + (1 - \gamma) H (c^u). \] (4.2.9)

As under legal separation, the increase in the access charge in (4.2.4) arising from asymmetric information yields a lower distortion in the final price. More importantly, we see from (4.2.9) and (4.1.13) that consumers are worse off under ownership separation, since they pay a higher price. The rationale for this result will be analyzed when we derive the monopolist’s informational rents.

Replacing (4.2.4), (4.2.5) and (4.2.9) into (2.3), we can compute the profit of the incumbent firm

\[ \pi^{OS}_I = \frac{2}{9\beta} [\alpha - c - c^u - (1 - \gamma) H (c^u)]^2. \] (4.2.10)
It is immediate to see from (4.2.10) and (3.2.11) that the asymmetric information problem penalizes the incumbent. If we take the difference between (4.2.10) and (3.2.11) we find

\[ \Delta \pi_I^{OS} \equiv \pi_I^{OS} - \pi_I^{OS} = \frac{2}{9\beta} (1 - \gamma) \ H (c^u) [(1 - \gamma) H (c^u) - 2 (\alpha - c - e^u)] \leq 0, \]

(4.2.10)

since downstream quantities in (4.2.5) and (4.2.7) must be nonnegative. Using (4.2.4), (4.2.7) and (4.2.9) we find the profit of the entrant

\[ \pi_E^{OS} = \frac{1}{9\beta} [\alpha - c - e^u - (1 - \gamma) H (c^u)]^2. \]

(4.2.11)

Not surprisingly, the entrant also is worse off under asymmetric information. Subtracting (3.2.12) from (4.2.11) yields

\[ \Delta \pi_E^{OS} \equiv \pi_E^{OS} - \pi_E^{OS} = \frac{1}{9\beta} (1 - \gamma) \ H (c^u) [(1 - \gamma) H (c^u) - 2 (\alpha - c - e^u)] \leq 0. \]

(4.2.12)

It appears from (4.2.12) and (4.2.10) that the incumbent is more penalized than the entrant by asymmetric information. This is the opposite of what we found under legal separation. The rationale is that now both firms incur a reduction in their profit margin and output, so the incumbent will suffer relatively more from the problem of asymmetric information.

Substituting (4.2.5) and (4.2.7) into \((ICC_N^{OS})\) we find the monopolist’s profit

\[ \pi_N^{OS} = \int_{e^u}^{c^u} \frac{\alpha - c - e^u - (1 - \gamma) H (e^u)}{\beta} de^u. \]

(4.2.13)

Notice that the range between boundaries of the integral in (4.2.13) is higher than that in (4.1.18), as \(c^u_+ > c^u_- (c^u)\) for \(c^u \in [c^u_-, c^u_+]\). The rationale is that under ownership separation the monopolist with costs \(c^u\) has an incentive to report \(\bar{e}^u \in (c^u, c^u_+), \) i.e. to mimic every more inefficient type of the firm, and it has to be accordingly remunerated in order to reveal the truth. Under legal separation, this incentive is weaker, since the network provider does not find it profitable to declare \(\bar{e}^u > c^u_- (c^u)\). This implies a higher distortion of total output under ownership separation in order to curb the monopolist’s informational rents, as is evident from the comparison
between the integrand functions in (4.2.13) and (4.1.18). Consequently, consumers will pay higher prices, as we found in (4.2.9).

Subtracting (4.1.18) from (4.2.13) we get after some manipulations

\[
\Delta \pi_N \equiv \pi_N^{OS} - \pi_N^{LS} = \int_{c^u}^{c^e} \frac{\alpha - c - c^u - (1 - \gamma) H (c^u)}{\beta} d\bar{c}^u +
\]

\[
- \frac{1}{3\beta} (1 - \gamma) \cdot \int_{c^u}^{c^e} H (\bar{c}^u) d\bar{c}^u.
\]

(4.2.14)

The sign of (4.2.14) is ambiguous. This is the outcome of the trade-off between the higher incentive to lie under ownership separation, which leads \textit{ceteris paribus} to an increase in the informational rent, and the greater downward distortion in the total output, which is aimed at achieving the opposite result. Differentiating (4.2.14) yields

\[
\frac{d\Delta \pi_N}{dc^u (c^u)} = - \frac{1}{\beta} \left[ \alpha - c - c^u (c^u) - \frac{2}{3} (1 - \gamma) H (c^u (c^u)) \right] \leq 0,
\]

(4.2.15)

since the expression (4.2.15) is the opposite of total output under legal separation when we replace \( c^u \) with \( c^u (c^u) \). This result shows that the difference in the informational rents under the two regimes reduces as the highest possible cost overstatement under legal separation \( c^u (c^u) \) increases and thus approaches \( c^u \), which is the highest cost that the monopolist is willing to report under ownership unbundling.

Using (2.2), we derive the consumers’ surplus

\[
\text{CS}^{OS} = \frac{1}{2\beta} [\alpha - c - c^u - (1 - \gamma) H (c^u)]^2.
\]

(4.2.16)

A comparison (4.2.16) and (4.1.19) gives the following result.

\textbf{Proposition 3} When the regulator has asymmetric information about the production costs of the upstream monopolist in a vertically related market, final consumers benefit from the legal separation between the input monopolist and the downstream incumbent.
This is a straightforward consequence of the greater output distortion which occurs under ownership separation.

From (2.5) we derive the subsidy received by the monopolist, which is equal to

\[
S^{OS} = \int_{c^u}^{\bar{c}u} \frac{\alpha - c - \bar{c}u - (1 - \gamma) H(\bar{c}u)}{\beta} dc^u + \frac{\alpha - c - c^u - (1 - \gamma) H(c^u)}{3\beta}.
\]

Notice that even under ownership separation the regulator may find it optimal to tax rather than subsidize the monopolist. This can occur when the firm is highly inefficient.

Substituting (4.2.10), (4.2.11), (4.2.13), (4.2.16) and (4.2.17) into (2.7), we derive after some computations the asymmetric-information social welfare under ownership separation

\[
W^{OS} = \frac{1}{2\beta} \left[ (\alpha - c - c^u)^2 - (1 - \gamma)^2 H^2(c^u) \right] + (1 - \gamma) \cdot \int_{c^u}^{\bar{c}u} \frac{\alpha - c - \bar{c}u - (1 - \gamma) H(\bar{c}u)}{\beta} dc^u.
\]

From (4.2.18) and (4.1.21) we can see that the presence of asymmetric information still produces two effects. The first one, which appears in the expression in square brackets, concerns the distortion in total output. Not surprisingly, the bracketed term in (4.2.18) is lower than that in (4.1.21). The second factor, which is captured by the integral, refers to the monopolist’s information rent. Consistently with what we found before, we cannot know a priori whether this effect is stronger under ownership or legal separation.

A comparison between the expected values of social welfare under the two regimes yields a relevant result, which is summarized in the following Proposition.

**Proposition 4** When the regulator has asymmetric information about the production costs of the upstream monopolist in a vertically related market, the regime of legal separation between the upstream monopolist and the downstream incumbent yields a higher expected social welfare level.
The proof is shown in Appendix F. Proposition 4 emphasizes that when the regulator has limited knowledge of the industry it is desirable from an expected social welfare viewpoint to separate the input monopolist from the downstream incumbent just in legal terms so that they can still belong to the same company. Such a kind of separation, unlike the ownership one, generates a trade-off between the incentive to exaggerate private information by the monopolist in order to have higher profits and the incentive to understate this information by the downstream firm in order to pay a lower access charge. Hence, legal separation yields countervailing incentives within the vertical group which allow the regulator to reduce the output distortion to curb informational rents. The higher allocative efficiency, emphasized in Proposition 3, leads to a greater expected social welfare level.

The result in Proposition 4 has crucial implications. Often in the literature and policy debates the regime of ownership separation between the input monopolist and the downstream incumbent is thought of as the best solution to the regulator’s critical control problem, since it should remove the monopolist’s practice of exaggerating the input costs in order to worsen the competitiveness of the downstream rivals. However, the monopolist’s incentives to exploit its private information still play a relevant role. Our model shows that legal separation can be (expected) social welfare improving, since it creates a conflict of interests within the vertical group, which reduces the negative effects of asymmetric information.

5. Concluding remarks

In this paper we have dealt with the problem of how to design the industry structure in a vertically related market when the regulator is charged with setting the price for the access to an upstream monopolistic input and there is imperfect competition downstream. Although the literature on the access pricing is quite extensive, this is an issue that, despite its importance after the liberalization process, has been by and large ignored in the economic research.

Empirical evidence shows that there are two main industry patterns that have so far been implemented. Under legal separation upstream and downstream operations are legally unbundled but common ownership is permitted. In contrast, ownership separation prevents a single company from controlling both activities. We have studied the impact on social welfare generated by these two industry structures of a problem that has so far played a minor role in the policy debate about access pricing: the asymmetric in-
formation about the industry on the part of the regulator. We have found that, while under complete information the two regimes yield the same social welfare, regulatory limited knowledge about the monopolist’s input costs implies that legal separation is (expected) social welfare improving. The idea is that a trade-off occurs between the monopolist’s incentive to overstate its costs in order to get higher profits and incentive of the downstream branch to understate them to pay a lower access charge. The policy implication of this result is that ownership separation should not be considered as the best solution to deal with the problem of the monopolist’s incentive to raise the input costs. Under legal separation the regulator can exploit the conflict of interests that emerges between the two branches of the vertical group and reduce the detrimental effects of asymmetric information.

We believe that the analysis can be extended in a variety of directions. We would like to mention three suggestions which are left for future research.

First of all, we have considered only two downstream firms, one incumbent and one entrant. However, in the literature imperfect competition is usually captured by assuming a dominant firm and a competitive fringe which makes zero profit. Would our results change in this case?

Other development would be to examine a more realistic setting where the regulator is uncertain not only on the costs of the upstream monopolist but also on those of the downstream firms.

Finally, it would be interesting to investigate in our setting the issues of the possible by-pass of the infrastructure by entrants and the impact on production efficiency of increased competition.

**Appendix A**

We derive now the incentive compatibility constraint (ICC$_N$) of the network provider for the profit function in (2.5) and show that this represents a local necessary condition which is also globally sufficient in equilibrium if $Q(c^u)$ is nonincreasing.

The class of global incentive compatible mechanisms must satisfy the following set of conditions

$$\pi_N(e^u, c^u) \equiv \pi_N(\hat{c}^u, c^u) \geq \pi_N(\bar{c}^u, c^u), \forall \bar{c}^u, e^u \in [c^u_-, c^u_+].$$  \hspace{1cm} (A.1)

In order to induce a firm not to lie, the profit $\pi_N(e^u, c^u)$ obtained by telling the truth has to be at least as great as the profit $\pi_N(\hat{c}^u, c^u)$ that the firm could get for any report $\hat{c}^u$. 

32
Following the Baron [5] approach, we use (2.5) and rewrite $\pi_N(\bar{c}^u, c^u)$ as

$$
\pi_N(\bar{c}^u, c^u) = [a(\bar{c}^u) - c^u] \cdot Q(\bar{c}^u) + S(\bar{c}^u) = \pi_N(\bar{c}^u) + (\bar{c}^u - c^u) \cdot Q(\bar{c}^u),
$$

(A.2)

where $\pi_N(\bar{c}^u) \equiv \pi_N(\bar{c}^u, \bar{c}^u)$. Substituting $\pi_N(\bar{c}^u, c^u)$ from (A.2) into (A.1) and combining terms yields

$$
\pi_N(c^u) - \pi_N(\bar{c}^u) \geq (\bar{c}^u - c^u) \cdot Q(\bar{c}^u), \quad \forall \bar{c}^u, c^u \in [c^-_u, c^+_u].
$$

(A.3)

Reversing the roles of $c^u$ and $\bar{c}^u$ implies

$$
\pi_N(c^u) - \pi_N(\bar{c}^u) \leq (\bar{c}^u - c^u) \cdot Q(c^u), \quad \forall \bar{c}^u, c^u \in [c^-_u, c^+_u].
$$

(A.4)

Since (A.3) and (A.4) must hold simultaneously for any $\forall \bar{c}^u, c^u \in [c^-_u, c^+_u]$, we may write

$$(\bar{c}^u - c^u) \cdot Q(\bar{c}^u) \leq \pi_N(c^u) - \pi_N(\bar{c}^u) \leq (\bar{c}^u - c^u) \cdot Q(c^u).$$

If we divide by $\bar{c}^u - c^u > 0$ and take the limit as $\bar{c}^u \to c^u$ we get by applying de l’Hospital’s theorem

$$
\frac{d\pi_N(c^u)}{dc^u} = -Q(c^u).
$$

(A.5)

Since a derivative is a local property of a function, (A.5) is a local condition which indicates that for any incentive compatible mechanism the profit of the firm viewed across the possible types is a decreasing function of $c^u$. By integrating both sides in (A.5), we find the local necessary condition for the incentive compatibility (ICC) seen in the paper

$$
\pi_N(c^u) = \pi_N(c^u_*(c^u)) + \int_{c^u}^{c^u_*(c^u)} Q(\bar{c}^u) d\bar{c}^u,
$$

(A.6)

where $Q(\bar{c}^u) = \frac{3\alpha - 3c - \bar{c}^a - 2a(\bar{c}^u)}{4\beta}$.

If the firm’s profit function satisfies the sorting (or Spence-Mirrlees) condition $\frac{\partial^2 \pi_N(Q, c^u_*)}{\partial Q \partial c^u} < 0$ ($\frac{\partial^2 \pi_N(Q, c^u_*)}{\partial Q \partial c^u} > 0$, respectively), then the function $Q(c^u)$ is implementable, or glabally incentive compatible, if it is monotone nonincreasing (nondecreasing, respectively). In equilibrium we have $\pi_N(c^u_*(c^u)) = \pi_N(c^u)$. 

33
0, so condition (A.6) boils down to $\pi_{N}^{L}(c^{u}) = \int_{c_{u}^{L}}^{c_{u}} Q(c^{u}) \, dc^{u}$. Since
\[
\frac{\partial^{2} \pi_{N}^{L}(Q,c^{u})}{\partial Q \partial c^{u}} = \frac{\partial}{\partial Q} \left[ \frac{\partial}{\partial c^{u}} \int_{c_{u}^{L}}^{c_{u}} Q(c^{u}) \, dc^{u} \right] = \frac{\partial}{\partial c^{u}} [c_{u}^{L}(c^{u}) - c^{u}] = \frac{3}{4} - 1 = -\frac{2}{3} < 0,
\]
then condition (A.6) is globally incentive compatible if $Q(c^{u})$ is non-increasing.

**Appendix B**

Integrating by parts yields
\[
\int_{c_{u}^{L}}^{c_{u}} \frac{3\alpha - 3c - \beta c^{u} - 2a}{4\beta} \, dc^{u} \, d(c^{u}) = \int_{c_{u}^{L}}^{c_{u}} \left[ \frac{c_{u}^{L}(c^{u})}{4\beta} \frac{3\alpha - 3c - \beta c^{u} - 2a}{dc^{u}} F(c^{u}) \right] + \\
- \int_{c_{u}^{L}}^{c_{u}} \frac{c_{u}^{L}(c^{u})}{dc^{u}} \frac{d}{dc^{u}} \left[ \frac{3\alpha - 3c - \beta c^{u} - 2a}{4\beta} \, dc^{u} \right]. \tag{B.1}
\]

Notice that the first addend in (B.1) vanishes since $c_{u}^{L}(c_{u}^{L}) = c_{u}^{L}$ and $F(c_{u}^{L}) = 0$. If we apply some properties of the integrals and the Torricelli-Barrow theorem, we may rewrite
\[
\frac{d}{dc^{u}} \int_{c_{u}}^{c_{u}^{L}} \frac{3\alpha - 3c - \beta c^{u} - 2a}{4\beta} \, dc^{u} = \frac{d}{dc^{u}} \left[ \int_{c_{u}}^{c_{u}^{L}} \frac{k \alpha - 3c - \beta c^{u} - 2a}{4\beta} \, dc^{u} \right] + \\
+ \frac{d}{dc^{u}} \int_{k}^{c_{u}^{L}} \frac{3\alpha - 3c - \beta c^{u} - 2a}{4\beta} \, dc^{u} = \left( \frac{3\alpha - 3c - \beta c^{u} - 2a}{4\beta} \right) + \\
+ \frac{d}{dc^{u}} \int_{k}^{c_{u}^{L}} \frac{3\alpha - 3c - \beta c^{u} - 2a}{4\beta} \, dc^{u}, \tag{B.2}
\]
where $k$ is a constant which belongs to $(c_{u}, c_{u}^{L}(c_{u}))$.

The Torricelli-Barrow theorem and the chain rule imply that the second addend in (B.2) is equal to
\[
\frac{d}{dc^{u}} \int_{k}^{c_{u}^{L}} \frac{3\alpha - 3c - \beta c^{u} - 2a}{4\beta} \, dc^{u} = \frac{d c_{u}^{L}(c^{u})}{dc^{u}} \cdot \frac{3\alpha - 3c - c_{u}^{L}(c^{u}) - 2a}{4\beta} = \\
= \frac{1}{3} \cdot \frac{3\alpha - 3c - \left[ \frac{2}{3} (\alpha - c) + \frac{1}{3} c_{u}^{L} \right] - 2a}{4\beta} = \frac{7 (\alpha - c) - c^{u} - 6a}{36\beta}. \tag{B.3}
\]
Substituting (B.3) into (B.2) yields

$$\frac{d}{dc^u} \int_{c^u}^{c_0(c^u)} \frac{3\alpha - 3c - c^u - 2a}{4\beta} dc^u = -\left(\frac{3\alpha - 3c - c^u - 2a}{4\beta}\right) + \frac{7(\alpha - c - c^u - 6a)}{36\beta} = -\frac{20(\alpha - c - 8c^u - 12a)}{36\beta}. \tag{B.4}$$

Finally, replacing (B.4) into (B.1) implies

$$\int_{c^u}^{c_0(c^u)} \frac{3\alpha - 3c - c^u - 2a}{4\beta} dc^u f(c^u) = \int_{c^u}^{c_0} \frac{20(\alpha - c - 8c^u - 12a)}{36\beta} dc^u. \tag{B.5}$$

Appendix C

Taking the derivative of $\Pi_{LS}^N$ with respect to $c^u$ yields

$$\frac{d\Pi_{LS}^N}{dc^u} = \frac{d}{dc^u} \int_{c^u}^{c_0(c^u)} \frac{\alpha - c - c^u - \frac{2}{3}(1 - \gamma) H(c^u)}{\beta} dc^u =$$

$$= \frac{d}{dc^u} \left[ \int_{c^u}^{c_0(c^u)} \frac{k(\alpha - c - c^u - \frac{2}{3}(1 - \gamma) H(c^u)}{\beta} dc^u + \int_{k}^{c_0(c^u)} \frac{(\alpha - c - c^u - \frac{2}{3}(1 - \gamma) H(c^u)}{\beta} dc^u \right], \tag{C.1}$$

where $k$ is a constant which belongs to interval $(c^u, c^0(c^u))$. If we apply the Torricelli-Barrow theorem and the chain rule, we may rewrite (C.1) as follows

$$\frac{d\Pi_{LS}^N}{dc^u} = -\frac{\alpha - c - c^u - \frac{2}{3}(1 - \gamma) H(c^u)}{\beta} +$$

$$+ \frac{dc^u(c^u)}{dc^u} \cdot \frac{(\alpha - c - c^u - \frac{2}{3}(1 - \gamma) H(c^u))}{\beta} =$$

$$= -\frac{\alpha - c - c^u - \frac{2}{3}(1 - \gamma) H(c^u)}{\beta} + \frac{\alpha - c - \frac{2}{3}(\alpha - c) + \frac{1}{3}c^u - \frac{2}{3}(1 - \gamma) H(c^u)}{3\beta}. \tag{C.2}$$
Combining terms implies

\[
\frac{d\pi^{LS}_N}{dc^u} = -\frac{2}{3\beta} \left[ \frac{4}{3} (\alpha - c - c^u) - (1 - \gamma) H (c^u) + \frac{1}{3} H (c^u (c^u)) \right]. \tag{C.3}
\]

As \( \overline{q}_E^{LS} \geq 0 \) the expression in square brackets in (C.3) is positive, so \( \pi^{LS}_N \) is decreasing in \( c^u \).

**Appendix E**

To derive (ICC\(_N^{OS}\)), which represents a local necessary condition of the incentive compatibility, we follow exactly the same procedure as that in Appendix A, but in the end we integrate from \( c^u \) to \( c^u + \) and get

\[
\pi^{OS}_N (c^u) = \pi^{OS}_N (c^u + c^u) \frac{c^u}{c^u} + \frac{1}{c^u} \int Q (\overline{c}^u) \, d\overline{c}^u, \tag{E.1}
\]

where \( Q (\overline{c}^u) = \frac{3 (\alpha - c - a (\overline{c}^u))}{4\beta} \).

In equilibrium we have \( \pi^{OS}_N (c^u + c^u) = 0 \), so condition (E.1) boils down to \( \pi^{LS}_N (c^u) = \int_{c^u}^{c^u + c^u} Q (\overline{c}^u) \, d\overline{c}^u \). Since \( \frac{\partial^2 \pi^{OS}_N (Q, c^u)}{\partial Q \partial c^u} = \frac{\partial}{\partial Q} \left[ \int_{c^u}^{c^u + c^u} Q (\overline{c}^u) \, d\overline{c}^u \right] = 0 \), then condition (E.1) is globally incentive compatible if \( Q (c^u) \) is nonincreasing.

**Appendix F**

After taking the expected value of (4.1.21) and (4.2.17), we can write

\[
\Delta E [\overline{W}] = E [\overline{W}^{LS}] - E [\overline{W}^{OS}] = \int_{c^u}^{c^u} \left\{ \frac{1}{2\beta} \left[ (\alpha - c - c^u)^2 - \frac{4}{9} (1 - \gamma)^2 H^2 (c^u) \right] + \right.
\]

\[
- (1 - \gamma) \int_{c^u}^{c^u} \frac{c^u (c^u)}{c^u} \frac{1}{3} (1 - \gamma) H (\overline{c}^u) \, d\overline{c}^u \right\} f (c^u) \, dc^u +
\]

\[36\]
Combining and manipulating terms in (F.1) implies

\[ \Delta E \left[ \mathcal{W} \right] = (1 - \gamma) \int_{c^-_u}^{c^+_u} \left\{ \int_{c^-_u}^{c^+_u} \frac{\alpha - c - \bar{c}^u - (1 - \gamma) H(\bar{c}^u)}{\beta} d\bar{c}^u \right\} f(c^u) dc^u. \]  

(F.1)

Notice that sufficient condition for (F.2) to be positive for \( \gamma \in [0, 1] \) is that the expression in square brackets is also positive. Integrating by parts yields

\[ \int_{c^-}^{c^+} \int_{c^-}^{c^+} H(\bar{c}^u) d\bar{c}^u f(c^u) dc^u = \left[ \int_{c^-}^{c^+} H(\bar{c}^u) d\bar{c}^u \cdot F(c^u) \right]_{c^-}^{c^+} + \]

\[ - \int_{c^-}^{c^+} F(c^u) \frac{d}{dc^u} \int_{c^-}^{c^+} H(\bar{c}^u) d\bar{c}^u dc^u = \]

\[ = \int_{c^-}^{c^+} F(c^u) \left[ H(c^u) - \frac{1}{3} H(c^u(c^u)) \right] dc^u, \]  

(F.3)

where the last inequality arises from the Torricelli-Barrow theorem and the chain rule.
Using (F.3), the sufficient condition for (F.2) to be positive becomes after summing and subtracting by $H(c^u)$

$$\int_{c_-^u}^{c_+^u} \frac{5}{6} H^2(c^u) f(c^u) \, dc^u > \int_{c_-^u}^{c_+^u} \left\{ \frac{2}{3} H(c^u) F(c^u) - \frac{1}{3} F(c^u) [H(c^u (c^u)) - H(c^u)] \right\} \, dc^u.$$  (F.4)

As $H(c^u) \equiv \frac{F(c^u)}{f(c^u)}$, it is immediate to see that the expression on the left-hand side is greater than the first addend on the right-hand side. The increasing monotonicity of the hazard rate implies that the second term in curly brackets is positive, so we can conclude that (F.4) is always satisfied.

References


SFB 649 Discussion Paper Series 2010

For a complete list of Discussion Papers published by the SFB 649, please visit http://sfb649.wiwi.hu-berlin.de.

001 "Volatility Investing with Variance Swaps" by Wolfgang Karl Härdle and Elena Silyakova, January 2010.
002 "Partial Linear Quantile Regression and Bootstrap Confidence Bands" by Wolfgang Karl Härdle, Ya’acov Ritov and Song Song, January 2010.
003 "Uniform confidence bands for pricing kernels" by Wolfgang Karl Härdle, Yarema Okhrin and Weining Wang, January 2010.
004 "Bayesian Inference in a Stochastic Volatility Nelson-Siegel Model" by Nikolaus Hautsch and Fuyu Yang, January 2010.
005 "The Impact of Macroeconomic News on Quote Adjustments, Noise, and Informational Volatility" by Nikolaus Hautsch, Dieter Hess and David Veredas, January 2010.
006 "Bayesian Estimation and Model Selection in the Generalised Stochastic Unit Root Model" by Fuyu Yang and Roberto Leon-Gonzalez, January 2010.
008 "Characterising Equilibrium Selection in Global Games with Strategic Complementarities" by Christian Basteck, Tijmen R. Daniels and Frank Heinemann, January 2010.
009 "Predicting extreme VaR: Nonparametric quantile regression with refinements from extreme value theory" by Julia Schaumburg, February 2010.
010 "On Securitization, Market Completion and Equilibrium Risk Transfer" by Ulrich Horst, Traian A. Pirvu and Gonçalo Dos Reis, February 2010.
011 "Illiquidity and Derivative Valuation" by Ulrich Horst and Felix Naujokat, February 2010.
012 "Dynamic Systems of Social Interactions" by Ulrich Horst, February 2010.
013 "The dynamics of hourly electricity prices" by Wolfgang Karl Härdle and Stefan Trück, February 2010.
015 "Estimation of the characteristics of a Lévy process observed at arbitrary frequency" by Johanna Kappusl and Markus Reiß, February 2010.
016 "Honey, I’ll Be Working Late Tonight. The Effect of Individual Work Routines on Leisure Time Synchronization of Couples" by Juliane Scheffel, February 2010.
017 "The Impact of ICT Investments on the Relative Demand for High-Medium-, and Low-Skilled Workers: Industry versus Country Analysis" by Dorothée Schneider, February 2010.
018 "Time varying Hierarchical Archimedean Copulae" by Wolfgang Karl Härdle, Ostap Okhrin and Yarema Okhrin, February 2010.
019 "Monetary Transmission Right from the Start: The (Dis)Connection Between the Money Market and the ECB’s Main Refinancing Rates" by Puriya Abbassi and Dieter Nautz, March 2010.
021 "Nonparametric Estimation of Risk-Neutral Densities" by Maria Grith, Wolfgang Karl Härdle and Melanie Schienle, March 2010.
SFB 649 Discussion Paper Series 2010

For a complete list of Discussion Papers published by the SFB 649, please visit http://sfb649.wiwi.hu-berlin.de.

022 "Fitting high-dimensional Copulae to Data" by Ostap Okhrin, April 2010.
024 "The optimal industry structure in a vertically related market" by Raffaele Fiocco, April 2010.