Disaster Risk in a New Keynesian Model

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Abstract

This paper develops a simple New Keynesian model incorporating a small time-varying probability that the economy is struck by a disaster in the future. The model’s main prediction is that a small increase in the disaster probability causes a recession in the economy, specifically due to limited saving opportunities inasmuch as the model abstracts from capital accumulation. By contrasting its findings to the ones of a comparable real business cycle model, this paper evaluates how the disaster hypothesis has been used and modelled in the existing literature.

Keywords: time-varying risk, disasters, rare events, nominal rigidities

JEL code: E21, E31, E32
1 Introduction

Several economic crises or so-called disasters have characterised and significantly influenced the economic world in the twentieth and twenty-first century. As pointed out by Barro (2006), a disaster may, beside other possible interpretations, represent an economic downturn due to significant destruction of physical capital, technology or output per se as typically occurring during wars and natural catastrophes. Some countries might have experienced fewer national disasters or have been less affected during international critical episodes in past decades, but due to global interdependencies of economies and their markets it is reasonable to argue that individual probabilities of entering a state of crisis or disaster varied by a significant amount in the past decades. In this context it is interesting to ask how agents of an economy change their behaviour by experiencing or perceiving a higher chance for the economy to collapse, particularly to question what are the channels through which variations in the likelihood of a future disaster translate.

Including the risk of an economy-wide downturn finds its origin in the work of Rietz (1988), who demonstrates that the inclusion of a low probability event representing a state of disaster in macroeconomic models can account for the high puzzling risk premia observed by Mehra and Prescott (1985). To justify the implementation and calibration of disaster parameters in the macro-finance literature, Barro (2006) empirically exposes that national and international disasters or crises occur frequently and are of sufficiently large scale finding a baseline value of a 1.7% chance per year to arrive in a state of crisis. Barro (2005) firstly criticises this assumption of a constant disaster probability as well as permanent disasters and proposes the incorporation of stochastic disaster probabilities for future research on macro-finance puzzles. Gabaix (2008), Gourio (2008b) and Wachter (2008) consider endowment economies with a small chance of a consumption disaster following the setup of Barro-Rietz, i.e. the disaster is modelled to hit the economy as a permanent partial destruction or loss of consumption. While Wachter (2008) captures consumption disasters by a Poisson process, Gabaix (2008) additionally considers dividend disasters and allows the intensity of both disasters to vary over time. In Gourio (2008b) the disaster probability jumps between two discrete values indicating a high or low probability of a disaster in the future period, that is the probability follows a two-state Markov chain. Gourio (2009) firstly introduces the disaster hypothesis in a standard Real Business Cycle model as a natural extension of the standard models.

1 “A natural next step is to extend the model to incorporate stochastic, persisting variations in the disaster probabilities [...].” (Barro 2005, p.39).
(RBC) model with capital accumulation in which he defines the state of crisis as a partial destruction of either the stock of capital, technology, or both where the disaster risk probability is assumed to follow an autoregressive process. His work can be seen as being the closest to this paper’s undertaking in implementing the Barro-Rietz assumption in a DSGE framework and delivers a suitable benchmark for comparisons of the model design and, partially, qualitative results.

Even though previously mentioned works reveal a considerable amount of variation in modelling assumptions concerning the disaster and its probability, none of these authors incorporate nominal rigidities or market imperfections in their model setups. Even though Keen and Pakko (2011) and Niemann and Pichler (2011) embody rare disasters in a New Keynesian framework, they do not focus on dynamics in the model by allowing the probability for the disaster state to vary, but on implications for the policy maker when the disaster actually realises.

This paper’s contribution is twofold. Firstly, it seeks to incorporate a disaster risk in a New Keynesian model, and hence fill a gap in the existing literature, in order to analyse effects induced by variations in its probability. Secondly, by comparing the dynamics to a flexible price benchmark model and results obtained by Gourio (2009) in the RBC setting, this paper reveals and discusses shortcomings of the Barro-Rietz hypothesis in the DSGE framework in terms of modelling assumptions and its implications. Most importantly, this paper’s attempt is to motivate the reader to critically review the success of the disaster hypothesis in economic models.

The paper proceeds as follows: Section 2 presents the setup of the model by considering the agents in the two possible states of the economy, normal times and the disaster. Section 3 reveals the qualitative results in terms of impulse response functions followed by a sensitivity analysis and a critical comment on the disaster hypothesis in DSGE models. Section 4 concludes.

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2 The inclusion of these features is firstly suggested by Gourio (2009) who concludes that “it would be interesting to consider the effect of a time-varying risk of disaster in a richer business cycle model, e.g. [...] a standard New Keynesian model” (p. 28).
2 The Model

This section deals with the setup of the model that allows for the analysis of a time-varying disaster probability in a DSGE framework with nominal rigidities. In order to clarify the environment in which the agents of the economy operate, general assumptions about the disaster state need defining in advance. Moreover, this section’s focus lies on the appropriate outline of general and state-contingent optimality conditions for agents acting in the economy.

2.1 The Disaster State

The economy stands in an environment of two possible states of nature — a disaster state and normal times. In normal times, as the name suggests, the economy operates under regular conditions where there is no current loss or destruction of capital, consumption, or output. However, there exists a small probability for the economy to enter such a state of misfortune, i.e. to experience a large downturn of economic activity as a consequence of a war or a natural catastrophe.

For the sake of accessibility this paper follows the assumption of the Barro-Rietz model in which the disaster is an absorbing state, i.e. once entered the economy stays in the disaster state forever.\footnote{This assumption is clearly unrealistic as it is commonly observed that large economic downturns are often followed by at least partial, recovery of economic activity. However, for the sake of simplicity the possibility of recoveries as dealt with in \cite{Gourio2008a} and \cite{Nakamura2010} is neglected.}

The assumption of a permanent disaster is also needed in order to compose a deterministic disaster state, since the general model setup abstracts from other stochastic elements such as technology or preference shocks. The deterministic design allows for a simple solution of allocations in the disaster state which, in a second step, is needed to solve the normal times economy. This argument will become clear once the optimality conditions are outlined. Except for the variations in the disaster risk probability, the abstraction from other stochastic innovations also applies to the normal times state. The subsequent subsections introduce the setup of the model by firstly sketching general assumptions, which are valid in both states, followed by a differentiation of optimality conditions for the two distinct states of nature.
2.2 Households

The economy consists of a continuum of identical and infinitely lived households who maximise their lifetime utility

$$E_t \left( \sum_{j=0}^{\infty} \beta^j U(C_{t+j}, N_{t+j}) \right)$$

with $\beta \in (0, 1)$ being the subjective discount factor and $E_t$ the mathematical expectation operator conditional on information available at time $t$. The utility function is assumed to be separable in consumption $C_t$ and labour $N_t$ and increasing and strictly concave in both its arguments. The following assumes that the utility function takes the form of

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\kappa}}{1+\kappa}$$

Following a standard notation, $\sigma$ denotes the coefficient of relative risk aversion and $\kappa$ the inverse of the Frisch elasticity of labour supply. Due to the presence of monopolistic competition, as will be further discussed later, general consumption $C_t$ is defined as a basket containing slightly differentiated consumption goods $C_t(i)$ which are substitutes to each other. Formally, it is defined as in Dixit and Stiglitz (2000)

$$C_t = \left( \int_0^1 C_t(i) \epsilon^{-1} di \right)^{\epsilon^{-1}}$$

where $\epsilon$ measures the price elasticity of demand. In order to maximise their lifetime utility, households seek to optimally allocate their consumption expenditures. Hence, there exists a first stage optimisation problem faced by the household: the cost minimisation problem for consumption good expenditures, namely

$$\min_{C_t(i)} \int_0^1 C_t(i) P_t(i) di = C_t P_t \quad \text{subject to} \quad C_t = \left( \int_0^1 C_t(i) \epsilon^{-1} di \right)^{\epsilon^{-1}}$$

By defining the aggregate price level as $P_t = \left( \int_0^1 P_t(i) \epsilon^{-1} di \right)^{1-\epsilon}$ the former optimisation problem yields the following demand for consumption good $i$

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$

It is accessible that demand for consumption good $i$ is a fraction of total consumption, where the weight is determined by the relation of good $i$’s price $P_t(i)$ to the aggregate price level $P_t$.

By supplying labour to the firms, the household earns a nominal wage $W_t$ which is assumed
to be determined in a perfectly competitive labour market. Additionally, the household can lend or borrow $B_t$ such that the budget constraint for the second stage optimisation problem, the utility maximisation, in real terms can be written as

$$C_t + \frac{B_t}{(1 + i_t)P_t} = \frac{W_t N_t}{P_t} + \tau_t + \frac{B_{t-1}}{P_t}$$

where $\tau_t$ denotes lump-sum taxes and $i_t$ the nominal interest rate paid on $B_t$.

### 2.2.1 Disaster State

Due to its absorbing character, the disaster state can be entirely separated from the normal times state and hence features the very standard structure of a New Keynesian model. The first order conditions are the intratemporal Euler equation characterising the within-period labour-leisure choice and the intertemporal Euler equation capturing the trade-off between consumption in period $t$ and consumption in period $t + 1$ reading

$$\frac{N_{D,t}}{C_{D,t}} = \frac{W_{D,t}}{P_{D,t}}$$

$$\left( \frac{C_{D,t}}{C_{D,t+1}} \right)^{-\sigma} = \beta E_t \left[ (1 + i_{D,t}) \frac{P_{D,t}}{P_{D,t+1}} \right]$$

These optimality conditions equate marginal rates of intra- and intertemporal substitution to their relative prices being the real wage and the discounted real return on bonds. The $D$-subscript indicating the presence of the disaster state is necessary to include in order to differentiate between disaster and normal times variables in the optimality conditions in the subsequent paragraphs.

### 2.2.2 Normal Times

In normal times the household’s problem mirrors the previously presented one in terms of assumptions and functional forms. In order to understand what changes in the first order conditions it needs to be emphasised that a possible disaster is solely anticipated and hence translates through expectations made by the household. While in a classic one-state economy in period $t$ the agent exclusively forms expectations about variables in period $t + 1$, in the two-state economy the agent additionally needs to consider that she may not find herself in the normal times state but in the disaster state in the next period. Anticipated consumption levels as well as the real return on bonds in period $t + 1$ will be

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4 For a comprehensive overview of a standard New Keynesian model see Gali (2002).
significantly different across the two states of nature.
As a result, expected values need to be split up forming a weighted average of the normal
times and disaster state future variables whenever they appear in optimality conditions.
The weight is determined by the probability of the respective state of nature, where in
the following $\psi_t$ denotes the probability to enter a disaster in period $t + 1$ and hence
$1 - \psi_t$ the probability to remain in normal times. After this reflection it is clear that what
changes significantly in the normal times state is the intertemporal Euler equation, while
the intratemporal Euler equation stays unaffected in its form, since it does not feature
expectations about future variables.\footnote{Hence, one can simply drop the D-index from the intratemporal Euler equation in the disaster state to obtain the normal times optimality conditions.} Following the argument above, the normal times
intertemporal Euler equation can be written as
\[
C_t^\sigma = \psi_t \beta E_t \left[ C_{D,t+1}^\sigma (1 + i_t) \frac{P_t}{P_{D,t+1}} \right] + (1 - \psi_t) \beta E_t \left[ C_{t+1}^\sigma (1 + i_t) \frac{P_t}{P_{t+1}} \right]
\]
Marginal utility of consumption at time $t$ equals the weighted average of expected marginal
utilities of consumption at time $t + 1$ in the two distinct states of nature.
Clearly, all previously presented arguments and optimality conditions for the household are
also valid under a flexible price setting assumption as in a basic RBC model. This equality
is practical in the sense that it allows the comparison of dynamic effects of alterations in
$\psi_t$ in both price setting environments as the household’s side stays unaffected by the price
setting regime and the only difference materialises on the supply side of the economy as
presented in the following.

2.3 Firms
A continuum of firms indexed on the unit interval $i \in [0, 1]$ produces differentiated goods
in monopolistic competition, i.e. firms are assumed to possess market power allowing them
to choose their prices. Aggregate output, mirroring the consumption basket $C_t$, is defined
as in Dixit and Stiglitz (2000)
\[
Y_t = \left( \int_0^1 Y_t(i) \frac{\epsilon}{\epsilon - 1} \, di \right)^{\frac{\epsilon}{\epsilon - 1}}
\]
where $Y_t(i)$ denotes output of firm $i$ and $\epsilon$ the elasticity of substitution across differentiated
goods. As found before from the cost-minimising behaviour of the household, firm $i$ faces
a certain demand for the good it produces. Under market clearing, namely $Y_t(i) = C_t(i)$
for all $i$ and hence $Y_t = C_t$, the demand faced by firm $i$ reads

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t$$

where $P_t(i)$ is the price charged by firm $i$ and $P_t$ the general price level of the economy defined as $P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ as stated earlier in the context of the households.

It is assumed that firm $i$ produces its output through a simplistic production function which is linear in individual labour input $N_t(i)$ as in

$$Y_t(i) = N_t(i)$$

The nominal wage $W_t$ paid on one unit of labour input in the production is taken as given by the firms. The price setting of firms is assumed to be rigid in the form of quadratic adjustment costs $\frac{\theta}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2$ à la Rotemberg (1982) and Schmitt-Grohe and Uribe (2004) with $\theta$ measuring the strength of stickiness in the economy.

### 2.3.1 Short Comment on the Modelling Choice of the Nominal Rigidity

Besides the presented cost-motive, there exist several other possibilities to model the nominal rigidity. Most prominently used in New Keynesian models are the assumption of price setting à la Calvo (1983), in which with a constant probability a firm has the opportunity to reset its price, or Taylor (1980) staggered price contracts, in which a price is set in advance for a specific number of periods. Both examples appear to be inconsistent with the two-state economy discussed in this paper, since both imply an average lifetime of a price or contract. Considering the actual realisation of an economic collapse it seems hard to defend the idea that a previously set contract on a specific price will stay in place. More realistically one would assume that contracts agreed upon in normal times would be re-optimised in case of a disaster rather than firms holding on to them. Theoretically, this could be modelled by special clauses for the event of the economic disaster such that, for instance, the Calvo parameter, which is the probability of being able to reset one’s price, changes or the average contract length is exogenously reduced once the disaster realises. Considering these clauses would create an immense room for possible calibrations and hence manipulations of the results which is not the purpose of this paper. Instead, the argument that price adjustments are costly appears to be valid in both states of nature. Consequently, using the assumption of Rotemberg-pricing to model the nominal rigidity is believed to deliver a more unified picture of the dynamics.\(^6\)

\(^6\) One could still argue that $\theta$ decreases once the disaster hits, but for the sake of uniformity this aspect will not be considered in this paper.
2.3.2 Disaster State

In the disaster state it is assumed that a certain fraction $\gamma$ of productivity is destroyed over the duration of the disaster state. In the context of physical disasters such as wars or natural catastrophes the loss of productivity might be motivated by arguing that factories and their utilized technology are damaged. Moreover, the loss of productivity could be an endogenous phenomenon resulting from fruitless investment policies or a reorganisation of the economy’s industries leaving certain technologies worthless. The framework of the realisation of the actual disaster resembles Gourio (2009) in the sense that he modelled the disaster as a partial destruction of either the capital stock, total factor productivity or both, whereas this work simply abstracts from capital. Interestingly, the results in the RBC setting obtained by Gourio (2009) for a disaster which is purely characterised by a large decline in total factor productivity will significantly contrast the findings obtained under the sticky price assumption.

Hence, with the loss of technology being present in the disaster state the production function reads

$$Y_{D,t}(i) = (1 - \gamma)N_t(i) = (1 - \gamma)Y_t(i)$$

Apart from the factor $1 - \gamma$ in the production function, the firm’s problem in the state of a crisis, as for the households, follows the basic setup of a New Keynesian model. Firms maximise their profits by choosing the output level, labour input and the price they want to charge in the current period. The first order conditions of the profit maximisation problem of firm $i$ with respect to individual output $Y_{D,t}(i)$, labour input $N_{D,t}(i)$ and charged price $P_{D,t}(i)$ are given by

$$\lambda_{D,t}(i) = P_{D,t}(i) - \mu_{D,t}$$

$$W_{D,t} = \mu_{D,t}(1 - \gamma)$$

$$0 = Y_{D,t}(i) - \theta\left(\frac{P_{D,t}(i)}{P_{D,t-1}(i)} - 1\right) \frac{Y_{D,t}P_{D,t}}{P_{D,t-1}(i)} - \epsilon\lambda_{D,t}(i) \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-(1+\epsilon)} \frac{Y_{D,t}}{P_{D,t}}$$

$$+ E_t \left[ Q_{D,t+1}(i) \left(\frac{P_{D,t+1}(i)}{P_{D,t}(i)} - 1\right) \frac{P_{D,t+1}(i)}{P_{D,t}(i)}^{2} Y_{D,t+1}P_{D,t+1}\right]$$

where $\mu_{D,t}$ denotes the Lagrange multiplier on the production function measuring nominal marginal costs $MC_{D,t}^n$, $\lambda_t(i)$ the Lagrange multiplier on the demand measuring the nominal profit of firm $i$ and $Q_{D,t+1}$ being the stochastic discount factor.

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This aspect will be highlighted in section 3.4.
2.3.3 Normal Times

Obviously, in normal times there is no current loss or destruction of productivity, but firms operate in their usual setting with the linear production function presented in the general setting of the firms. This in turn causes an immediate but minor alteration in one of the firm’s first order conditions, namely the one obtained from the differentiation of the profit function with respect to labour input $N_t(i)$, which in normal times simply reads

$$W_t = \mu_t$$

with $\mu_t$ again denoting the Lagrange multiplier on the production function measuring nominal marginal costs $MC^n_t$ in normal times.

The dominant change on the firm’s side of the economy lies in the first order condition with respect to one’s chosen price $P_t(i)$ as it features an expected value. By forming expectations about variables in $t+1$ the firm needs to assign a probability or weight of $\psi_t$ to the disaster state and a weight of $1 - \psi_t$ to the scenario in which the economy remains in normal times. Hence, as brought up in the intertemporal Euler equation, the expected value needs to be split up yielding the following first order condition for $P_t(i)$ for the normal times state

$$0 = Y_t(i) - \theta \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right) \frac{Y_t P_t}{P_{t-1}(i)} - \epsilon \lambda_t(i) \left( \frac{P_t(i)}{P_t} \right)^{-(1+\epsilon)} Y_t \frac{P_t}{P_t}$$

$$+ \psi_t \mathbb{E}_t \left[ Q_{t,t+1} \theta \left( \frac{P_{D,t+1}(i)}{P_t(i)} - 1 \right) \frac{P_{D,t+1}(i)}{P_t^2(i)} Y_{D,t+1} P_{D,t+1} \right]$$

$$+ (1 - \psi_t) \mathbb{E}_t \left[ Q_{t,t+1} \theta \left( \frac{P_{t+1}(i)}{P_t(i)} - 1 \right) \frac{P_{t+1}(i)}{P_t^2(i)} Y_{t+1} P_{t+1} \right]$$

This rather unpleasant looking optimality condition appears to be challenging to analyse. So far, it does not deliver a clear image of how a shock on $\psi_t$ transmits through the supply side of the economy and whether firms will increase or decrease their prices in response to an increase in $\psi_t$. However, the discussion on optimal monetary policy will deliver a strong simplification resulting in a convenient log-linearised version of the optimality condition stated above.

2.4 Monetary Authority

To close the setup of the theoretical model, this section deals with the monetary authority operating in the previously described economy. It is convenient to firstly consider optimal
monetary policy in the transition period when the economy jumps from the normal times state into the disaster state and its implications for the inflation rate in the transition $\frac{P_{D,t+1}}{P_t}$. From the perspective of normal times it is known that agents anticipate a sharp drop in output, and hence consumption, if a disaster truly realises, where this negative outlook might cause agents to extremely react to changes in $\psi_t$ by the intertemporal Euler equation as a matter of risk aversion or simply anxiety about the future. These overreactions should not be in the general interest of the monetary authority. Rather one could think about a monetary authority trying to let the disaster scenario appear less disastrous in order to ease agent’s expectations about the future. Such a relaxation of facts is expected to happen when the monetary authority commits to a policy in which they allow for a relatively high transition inflation rate $\frac{P_{D,t+1}}{P_t}$. As consumption in the disaster state will be only a fraction of the normal times consumption level, marginal utility of consumption in the disaster state will be very high such that, in order to smooth the relation of consumption at time $t + 1$ in the two distinct states of nature, the real return on borrowing or lending should be low when the crisis hits.

From the perspective of normal times, such inflationary promises in the state of transition seem to be desirable but it remains questionable whether such a policy is time-consistent or subject to re-optimisation at another point in time. Considering the actual realisation of the disaster, and hence the transition period, the inflationary promises do not appear to be a policy the monetary authority will undoubtedly stick to. Due to its deterministic design the monetary authority has the ability to implement the flexible price allocation in the disaster state and allowing for a high transition inflation would solely create a high degree of distortion in the economy. As a consequence, it seems to be more reasonable to think that monetary policy will be re-optimised in the transition period in order to eliminate any distortion arising from its inflation rate, namely to set $\frac{P_{D,t+1}}{P_t}$ equal to 1.

This assumption of a discretionary monetary policy in the transition when the disaster actually hits causes a strong simplification of the first order conditions of the households and the firms. By log-linearisation around a zero-inflation and symmetric steady state one obtains the two New Keynesian Phillips Curves (NKPC) which read

$$\pi_{D,t-1,t} = \beta E_t \hat{\pi}_{D,t,t+1} + \frac{\epsilon - 1}{\theta} (\sigma + \kappa) \hat{y}_{D,t} \quad \text{and} \quad \hat{\pi}_{t-1,t} = (1 - \psi) \beta E_t \hat{\pi}_{t,t+1} + \frac{\epsilon - 1}{\theta} (\sigma + \kappa) \hat{y}_t$$

where $\psi$ denotes the steady state level of the disaster probability. The similarity and standard character of the two NKPCs for the two states of nature are a practical feature for choosing an appropriate form of monetary policy. By following a Taylor (1993) rule,
usually expressed as

\[ i_t = \phi_x \hat{\pi}_{t-1,t} + \phi_y \hat{y}_t \quad \text{and} \quad i_{D,t} = \phi_x \hat{\pi}_{D,t-1,t} + \phi_y \hat{y}_{D,t} \]

the monetary authority can ensure local determinacy by restricting the values of \( \phi_y \) and \( \phi_x \), which denote the preferences of the monetary authority in terms of inflation and output stabilisation\(^8\). In conclusion, it is assumed that the monetary authority follows a Taylor (1993) rule in both states of nature, while in the transition from normal times to the state of disaster she is assumed to deviate from this commitment in order to limit the distortion arising from the transition inflation as discussed in the foregoing paragraphs\(^9\).

### 2.5 Equilibrium

Concerning both states in which the economy exists, as used in the determination of optimal monetary policy, this paper focuses on symmetric equilibria, that is all firms charge the same price such that \( P_t(i) = P_t \) and hire the same amount of labour. Since all households are assumed to be identical the market clearing condition for the credit market is

\[ B_t = 0 \quad \forall t \]

that is borrowing or lending has to be balanced on aggregate. In order to clear the goods market it has to hold in both states of nature that

\[ C_t(i) = Y_t(i) \]

which in turn implies

\[ C_t = Y_t \]

In other words, all produced output is consumed by the households. Moreover, the third market clearing condition considers the labour market where optimality requires that

\[ N_t = \int_0^1 N_t(i) di \]

\(^8\) See, for example, Gali and Gertler (1999) for a discussion of advantages and shortcomings arising from preferences of the monetary authority.

\(^9\) The concrete form of this discretionary policy is not derived in this paper as only its result that \( \frac{P_{D,t+1}}{P_t} = 1 \) is needed for the solution of the model.
Lastly, in equilibrium the nominal interest rate needs to be non-negative,

\[ i_t \geq 0 \]

for otherwise households would benefit from arbitrage opportunities. Combining these statements with the optimality conditions of the households and firms derived in the previous subsections and the lastly found simplification of \( \frac{P_{D,t+1}}{P_t} = 1 \) delivers a parsimonious set of equilibrium conditions which after log-linearisation around the zero-inflation steady state, in which it holds that \( C^D = (1 - \gamma)C \), read

\[ \hat{\pi}_{D,t-1,t} = \frac{\epsilon - 1}{\theta} (\sigma + \kappa) \hat{y}_{D,t} + \beta E_t \hat{\pi}_{D,t,t+1} \]
\[ \hat{y}_{D,t} = E_t \hat{y}_{D,t+1} - \frac{1}{\sigma} (\hat{i}_{D,t} - E_t \hat{\pi}_{D,t,t+1}) \]
\[ i_{D,t} = \phi_x \hat{\pi}_{D,t-1,t} + \phi_y \hat{y}_{D,t} \]
\[ \hat{\pi}_{t-1,t} = \frac{\epsilon - 1}{\theta} (\sigma + \kappa) \hat{\pi}_t + (1 - \psi) \beta E_t \hat{\pi}_{t,t+1} \]
\[ -\sigma \hat{y}_t = -\sigma \psi \beta (1 + i)(1 - \gamma) E_t \hat{y}_{D,t+1} \]
\[ -\sigma (1 - \psi) \beta (1 + i) E_t \hat{y}_t + \hat{i}_t - (1 - \psi) \beta (1 + i) E_t \hat{\pi}_{t,t+1} \]
\[ + [\beta (1 - \gamma)^{-\sigma} (1 + i) - \beta (1 + i)] \psi \hat{\pi}_t \]
\[ i_t = \phi_x \hat{\pi}_{t-1,t} + \phi_y \hat{y}_t \]

The first three equations characterise the disaster state by a very basic setup of a standard New Keynesian model, while the last three equations capture the economy in normal times. While equation 4, the NKPC in normal times, does not largely differ from its disaster counterpart in equation 1, it is evident that the log-linearised Euler equation, or New Keynesian IS curve, in normal times displayed by equation 5 heavily differs from its standard representation as in equation 2 as it also features the disaster variable \( \hat{y}_{D,t+1} \) as well as it is directly affected by percentage deviations of the disaster probability from its steady state \( \hat{\psi}_t \).

3 Results

This section deals with the results of shocks on the disaster probability in the New Keynesian model graphically illustrated by impulse response functions. The subsequent paragraphs present a baseline calibration of the model with which the first results are obtained.
3 RESULTS

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</tr>
<tr>
<td>Weight on inflation stabilisation in Taylor rule</td>
<td>$\phi_\pi$</td>
<td>1.5</td>
</tr>
<tr>
<td>Weight on output stabilisation in Taylor rule</td>
<td>$\phi_y$</td>
<td>0.125</td>
</tr>
</tbody>
</table>

Table 1: Calibration of parameter values for the baseline model

and displayed. Furthermore, a sensitivity analysis with respect to the nominal rigidity is executed in order to motivate a following critical reception on the disaster hypothesis.

3.1 Calibration

Parameters and their calibration are listed in Table 1 where one period is set to be one quarter. To account for disagreements on parameter values for $\sigma$ and $\kappa$ in the existing literature the assumption of log-utility and a unit Frisch elasticity of labour supply is considered as an acceptable compromise for a baseline calibration. In the appendix, different parameter values are considered serving as a robustness check of the results obtained from the baseline calibration.

Crucial elements such as the steady state value for the probability of entering a state of disaster $\psi$ follows the empirical finding of 1.7% per year by Barro (2006), and hence 0.4% per quarter, as well as the baseline size for the destruction rate of 43% in a disaster which has also been used in Gourio (2009). As shown in the setup of the model, the inclusion of the disaster state significantly alters the intertemporal Euler equation and consequently the determination of certain parameters. In the steady state the intertemporal Euler equation reads

$$(1 + i) = \frac{1}{\beta} \psi (1 - \gamma)^{-\sigma} + (1 - \psi)$$

revealing a slight restriction of the model regarding its calibration. By fixing specific parameter values one variable has to be chosen which will be determined endogenously. As this paper seeks to assign the stated values for $\gamma$ and $\psi$ found by Barro (2006) either $i$
or $\beta$ needs to be determined endogenously. In order to avoid possible negative values of $i$ which would arise for certain parameter combinations by fixing the value of $\beta$, this paper takes a commonly assumed value for the steady state annual real return of 4% by allowing $\beta$ to vary with parameter variations.\(^{10}\)

The evolution of percentage deviations of the disaster probability from its steady state value is modelled as an AR(1) process, i.e.

$$\hat{\psi}_t = \rho \hat{\psi}_{t-1} + \xi_t$$

with $\rho$ measuring the persistence, being calibrated to a value of 0.9, and $\xi_t$ denoting the stochastic innovation.

As arguably troublesome to calibrate stands the Rotemberg parameter $\theta$ as it does not feature an empirical counterpart, as an average contract length or lifetime of a price, allowing the measurement of appropriate values. This deficiency has been addressed by Keen and Wang (2007) who show a convenient and intuitive way to express $\theta$ by $\beta$, $\epsilon$ and the Calvo parameter $\alpha$ measuring the average lifetime of a price by $1/(1 - \alpha)$. As a baseline value it is assumed that on average firms change or adjust their prices once a year or every four quarters as found by Rotemberg and Woodford (1995), which designates a value of 116.51 for $\theta$. Due to general disagreements in the literature on the strength of price stickiness in general or on an industry-specific level, the sensitivity analysis discusses variations of $\theta$ as well as the flexible price scenario in which $\theta$ is equal to zero.

Lastly, Table 1 lists the parameters of the Taylor rule $\phi_\pi$ and $\phi_y$. The chosen calibration lies in a standard range being in line with Christiano et al. (2005) and Keen and Pakko (2011), emphasising the focus on inflation stabilisation via $\phi_\pi$ rather than responses of the interest rate to output deviations by $\phi_y$. Since this paper’s focus does not lie on the analysis of monetary policy, variations in the last two mentioned parameters will not be explicitly discussed in the following. However, and for the sake of completeness, a sensitivity analysis of impulse responses for different combinations of $\phi_\pi$ and $\phi_y$ is displayed in Figure 6 in appendix A.

\(^{10}\)As a side note, as $\gamma \to 0$ or $\psi \to 0$ one obtains the usual relation of $1 + i = 1/\beta$ which would imply the usually taken value of 0.99 for $\beta$.

\(^{11}\)The formula stated by Keen and Wang (2007) to determine $\theta$ by observables reads $\theta = \frac{[\epsilon - (1 - \alpha)\gamma]}{[(1 - \alpha)(1 - \beta\alpha)]}$.\[\]
3.2 Response to an Increase in the Disaster Probability

After declaring the baseline calibration, the key exercise of this paper, to exogenously increase the probability of a disaster, is performed, where it is assumed that the probability of entering a disaster doubles at time $t = 0$ from 0.425% to 0.85% per quarter. Figure 1 displays the impulse responses of output, the nominal interest rate, and inflation rate. Output decreases and initially falls to approximately 0.3% below its steady state level accompanied with a drop in the price level displayed by negative inflation in the center panel. According to the Taylor principle the nominal interest rate drops as both output and inflation are below their steady state levels. Geometrically declining with the persistence parameter of the probability process all series move back to their steady state. Emphasis has to be put on the fact that these impulse responses reveal that an arguably small increase in the chance to enter a disaster to 0.85% per quarter leads to a recession in the economy.

As the disaster state becomes more probable, or agents become more pessimistic, they have a higher incentive to save in pursuance of insuring themselves against the higher perceived risk in the economy. The only tool of saving in the economy, borrowing or lending, is

\[ \hat{y}_t = \hat{c}_t = \hat{n}_t \]

and hence it is appropriate to solely report output responses.

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**Figure 1:** The effect on an increase in the disaster probability. Displayed are the impulse responses for the baseline calibration where at $t = 0$ the disaster probability doubles from 0.425% to 0.85% per quarter. The red line denotes the steady state.
3 RESULTS

constrained to be zero on aggregate, which leaves agents with no other choice than to reduce current consumption, and by market clearing output as well as labour input, in order to smooth out a possible huge drop in consumption if the disaster would actually realise. The drop in demand faced by the firms gives rise to price lowering motives causing the drop in the price level and hence deflation of around 0.4%. As the nominal interest rate responds to both outlined deviations it comes very close to the zero lower bound as to be seen in the initial shock period, where the level of the nominal interest rate drops to 0.33%. As the shock on $\psi_t$ has only been temporary the monetary authority does not face an actual liquidity trap, but it is recognisable that parameter alterations might easily cause the impotence of monetary policy when the zero lower bound becomes binding.

To summarise, all presented responses are the result of a disputable small increase in the disaster probability to 0.85% per quarter. Especially the reaction of output appears to be quite significant for this slight increase in agent’s perceived risk or pessimism towards the future. On the contrary, the deflation rate following the increase in the disaster probability appears to be reasonably small keeping in mind that the model is based on a zero-inflation steady state.

In the following, parameter variations in $\theta$ will be considered, determining the sensitivity of the previously presented results with respect to the nominal rigidity.  

3.3 Robustness with Respect to $\theta$

The basic calibration of $\theta$ implied an average lifetime of a price of a year or 4 quarters, which denotes quite a solid degree of stickiness in the economy. The considerations in this paragraph are twofold as $\theta$ will be calibrated to values implying average price lifetimes of 2 and 5 quarters as well as the flexible price benchmark economy attained by setting $\theta$ equal to zero. Figure 2 displays the impulse responses to a 10% increase in the disaster probability for the different calibrations of $\theta$ where the red line, denoting the steady state, coincides with the response of output in the flexible price benchmark economy (dashed line).

\[ \text{For sensitivity checks of } \sigma, \kappa, \phi_*, \phi_y \text{ and } \gamma \text{ please see appendix A.} \]

\[ \text{In these alterations the steady state markup remains fixed to } 10\%. \]

\[ \text{An unpleasant by-product with the initially used size of the shock, i.e. a } 100\% \text{ increase of the disaster probability, and higher values of } \theta \text{ the monetary authority hits the zero lower bound, that is has to set the nominal interest rate to zero. To allow for a comparable graphical illustration the size of the shock has been reduced to a } 10\% \text{ increase in the disaster probability.} \]
Generally, it can be said that with a rising strength of the nominal rigidity, the decline in output below its steady state level amplifies while the responses of inflation and the nominal interest rate are dampened. With a rigidity implying an average price lifetime of 5 quarters ($\theta = 192.31$) placing a 10% increase in the probability of a disaster causes a decline in output of more than 0.04% below its steady state level. Concerning the flexible price benchmark economy, where $\theta$ equals zero, it is apparent that there are no real effects in the economy as the probability of a future disaster increases. The shock entirely transmits through inflation and the nominal interest rate.

In conclusion it can be said that variations in $\theta$ drive the relative transmission of shocks on the disaster probability. As the degree of price stickiness increases the effects of the shock are carried out by the real economy while in a flexible price economy the real economy stays unaffected and solely the nominal interest rate and inflation rate respond.

### 3.4 The Barro-Rietz Hypothesis in a DSGE framework

The foregoing sensitivity check revealed that the sign of the responses to an increase in the disaster probability is robust to variations in the parameter calibration, while the magnitude as well as the relative responses of nominal and real variables are subject to parameter values. After achieving these results, it is interesting to compare the sign of the
obtained responses in the sticky price model of this paper to the ones obtained by Gourio (2009) in the RBC setting. As mentioned earlier, he considers three possible disaster scenarios being characterised by either a large destruction of the capital stock, total factor productivity, or both. By considering the disaster scenario in which, as well as in this work, total factor productivity is destroyed he finds the reverse response of output, that is an increase in the probability of the disaster leads to a boom in the economy. On the other hand, by inspecting the disaster scenarios in which a certain fraction of the capital stock is erased he finds the negative response of output identical to the response displayed in the antecedent paragraphs of this section. In order to correct the sign of the response of output and match key business cycle statistics he needed to add large adjustment costs to the disaster scenario of pure productivity destruction.

Naturally, this difference in the response of output stems from the involvement of capital accumulation in his model. When the disaster is assumed to not affect the capital stock, investments in this stock denote a device of saving and insurance against the disaster state. Consequently, agents increase investments due to a precautionary savings motive in response to an increase in the disaster probability causing a rise in the stock of capital and hence an increase in aggregate production explaining the reverse response of output in the RBC setting. Moreover, the abstraction from capital in this work puts emphasis on uninsured intertemporal smoothing as the increase in the disaster probability represents a negative wealth effect. Due to its simplistic design output, consumption and hours worked perfectly comove, while in Gourio (2009) the sign of the consumption response opposes the one of the other two when considering a disaster scenario with capital destruction. This denotes a small success of the sticky price model with the disaster hypothesis since it does not fail to generate a positive comovement of consumption and hours worked. Finally, this paper shows that by removing the nominal rigidity \( \theta = 0 \) alterations in the disaster probability transmit through nominal variables alone leaving the real economy unaffected, which highlights the importance of including capital in the RBC context in order to achieve success.

This comment on the different results of the disaster hypothesis by adapting particular modelling assumptions gives rise to a critical reception on the inclusion of the Barro-Rietz hypothesis in DSGE models. To begin with, in the review of the literature it became apparent that there exist numerous definitions and hence approaches to model the disaster hypothesis. In the foregoing comparison to the RBC setting this paper revealed that
depending on the general setup of the model one can obtain reverse responses of variables which are of interest. As there does not exist a unified picture on how to model or implement the disaster state in macroeconomic models one should question the validity of the hypothesis’ success in explaining phenomena in macroeconomics in general, specifically in the area of macro-finance. By combining different modelling assumptions the disaster hypothesis can be designed to be successful, which has been a common approach in the disaster literature illustrating a way of residualisation of the disaster aspect. Movements or variances of the disaster probability were set or backed out from model frameworks in order to optimally match phenomena or puzzles in macro-finance, where the critical examination of obtained disaster probability aspects as the size, time series behaviour, and the definition itself mostly fails to appear. Secondly, and more obviously, the difficulty to empirically assess the disaster probability and its movements continues to hinder the full acceptance of the Barro-Rietz hypothesis. The model considered in this paper comes with the deficiency that the tiniest movements in the disaster probability cause the zero lower bound to bind, which was the reason for the reduced size of the shock on the disaster probability in the sensitivity analysis. In order to appropriately judge and evaluate this theoretical drawback as well as to justify the quantitative assumptions which have been made on the Barro-Rietz hypothesis in the macro-finance literature, future focus has to be put on a convenient and convincing empirical assessment most favourably detached from theoretical containments.

4 Conclusion

This work shows that exogenous variations in the probability for a future disaster in a New Keynesian model causes significant responses of real and nominal variables. Most importantly the model suggests that an increase in the aggregate risk perceived by households and firms leads to a recession in the economy which stands in line with findings of Gourio (2009) in the context of an RBC model. The replication of this finding depends heavily on the definition of the disaster state and the limited saving opportunities of the households inasmuch as the presented model abstracts from capital accumulation. On that account, this work questions the validity and the success of the Barro-Rietz hypothesis in DSGE models believing that there exist several aspects which need to be adressed in future research.

To begin with, and in order to avoid the common approach of residualising the disaster
hypothesis, it would be interesting to find further empirical support, specifically quantifying movements in the disaster probability. Since these are captured in agents’ optimism or pessimism towards the future they might be extractable from, for example, business outlook surveys. By feeding these into a theoretical framework one could validate the success of the Barro-Rietz assumption.

Furthermore, while earthquakes or hurricanes certainly denote exogenous events it is more likely that for instance financial crises indicate an endogenous phenomenon in the economy. Accordingly, it would be interesting to endogenise variations in the probability for a disaster as well as its realisation allowing for economic crashes which have been induced by agents’ behaviour. In light of this aspect, future research could consider the Barro-Rietz assumption in models of heterogeneous agents concerning the degree of individual risk aversion in which the decisions of a small fraction of the population can significantly drive the disaster probability.
References


Figure 3: Sensitivity of impulse responses with respect to $\sigma$. Impulse responses to a 10\% increase in the probability of a disaster in $t = 0$ for $\sigma = 1$ (solid), $\sigma = 3$ (dotted) and $\sigma = 5$ (dotdashed). The red line denotes the steady state.
Figure 4: **Sensitivity of impulse responses with respect to $\kappa$.** Impulse responses to a 10% increase in the probability of a disaster in $t = 0$ for $\kappa = 1$ (solid), $\kappa = 3$ (dotted) and $\kappa = 5$ (dotdashed). The red line denotes the steady state.

Figure 5: **Sensitivity of impulse responses with respect to $\gamma$.** Impulse responses to a 10% increase in the probability of a disaster in $t = 0$ for $\gamma = 0$ (dashed), $\gamma = 0.1$ (dotted), $\gamma = 0.43$ (solid) and $\gamma = 0.6$ (dotdashed). The red line denotes the steady state.
Figure 6: Sensitivity of impulse responses with respect to $\phi_\pi$ and $\phi_y$. Impulse responses to a 10% increase in the probability of a disaster in $t = 0$ for pairs of $(\phi_\pi, \phi_y)$ of $(1.5, 0.125)$ (solid), $(1.5, 0)$ (dotted), $(5, 0)$ (dotdashed) and $(5, 1)$ (dashed). The red line denotes the steady state.
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